On the Design of Dynamical Cores for GCMs: Physical and Computational Challenges

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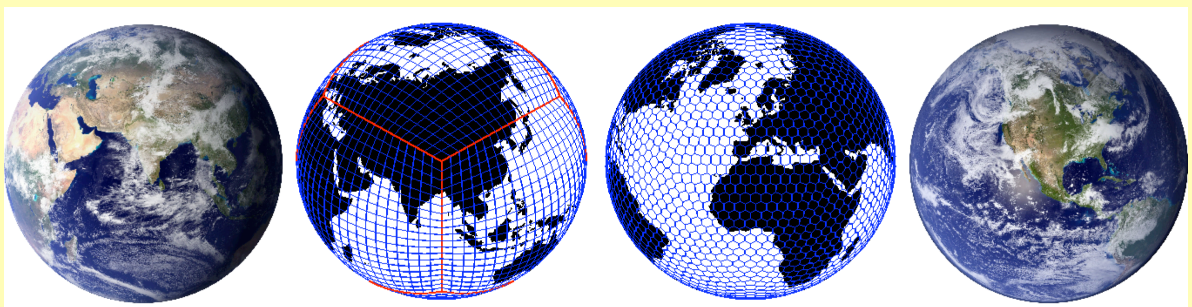
Overview of tutorial 2

- Evaluation techniques and selected dynamical core test cases
- Physical challenges: conservation, positive tracer advection
- Hidden features in the design process: subgrid-scale diffusive and filtering processes
- Computational aspects, computer architectures, scalability, efficiency, how it determines science decisions
- Trends in GCM modeling

Proposed Dynamical Core Test Suite used during the 2008 NCAR ASP Colloquium

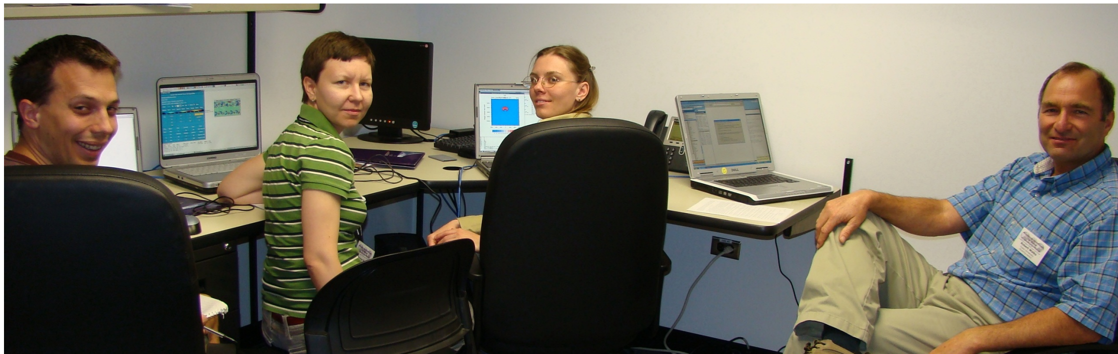
- All tests are formulated on the sphere
- Some have multiple test variants, e.g. rotation angle α

1. Steady-state test case (various rotations α)
2. Evolution of a baroclinic wave (various rotations α)
3. 3D advection experiments (various rotations α)
4. 3D Rossby-Haurwitz wave with wavenumber 4
5. Mountain-induced Rossby wave train
6. Pure gravity waves and inertial gravity waves

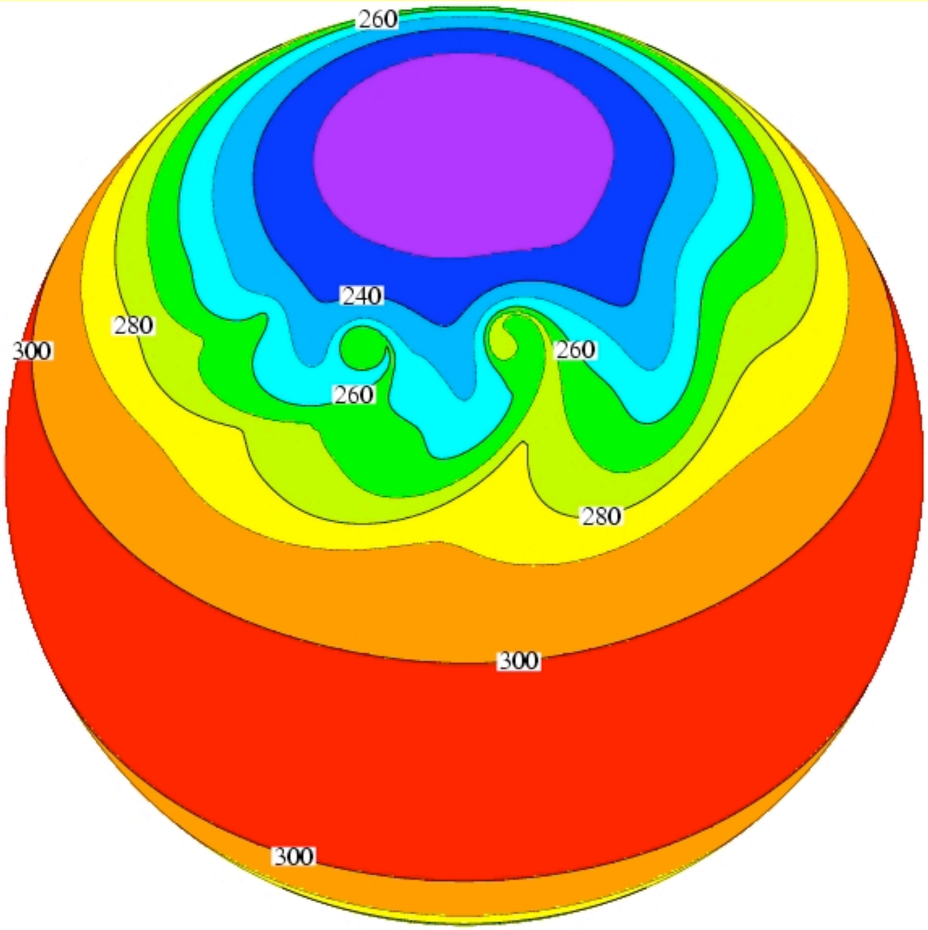


Examples of the red test cases are shown today

NCAR 2008
ASP Colloquium



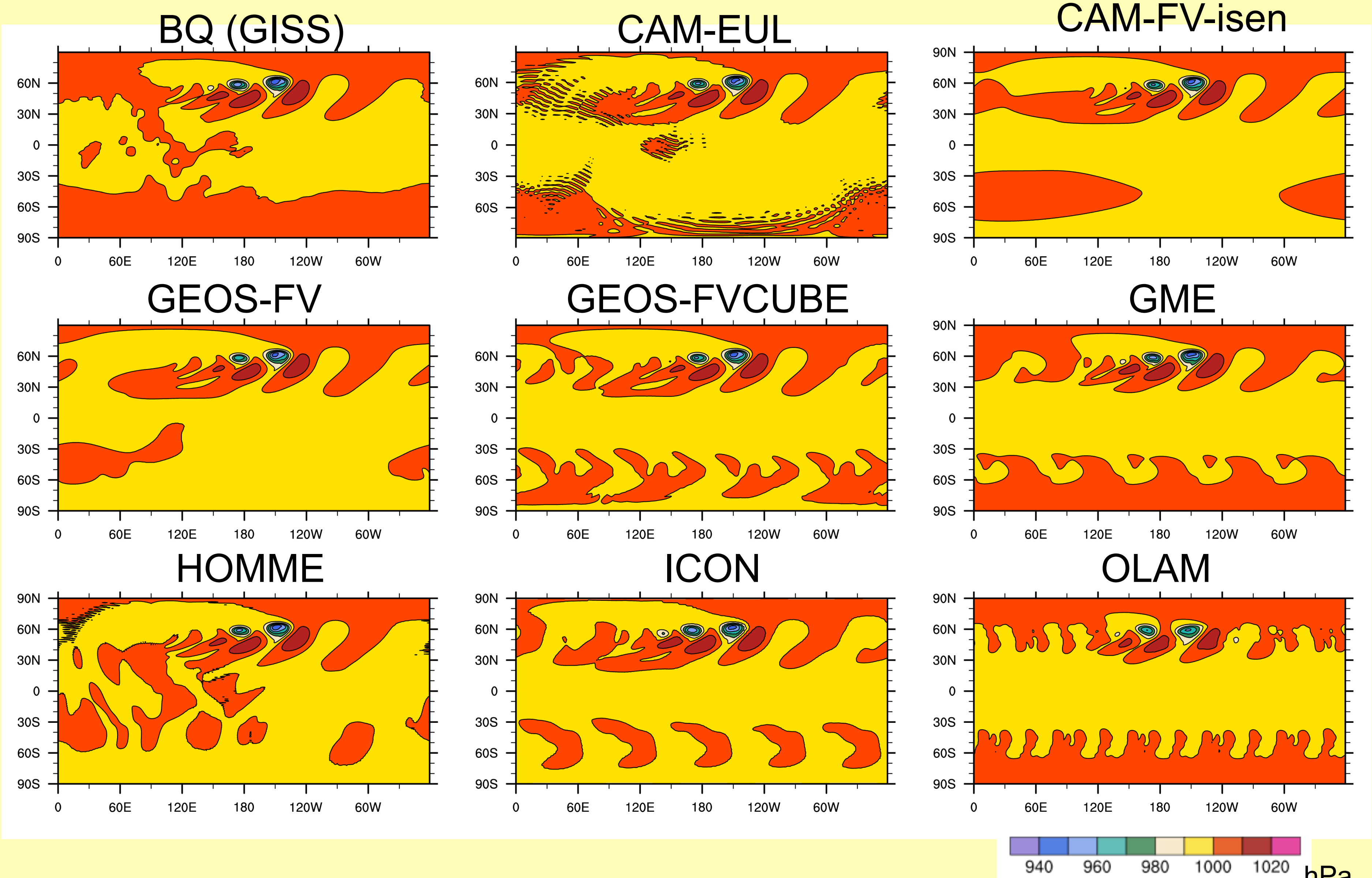
Test 2: Baroclinic Waves



- 850 hPa temperature field (in K) of an idealized baroclinic wave at model day 9
- Initially smooth temperature field develops strong gradients associated with warm and cold fronts
- Explosive cyclogenesis after day 7
- Baroclinic wave breaks after day 9
- Models start converging at 1°

Jablonowski and Williamson (QJ, 2006)

Test 2: Model Intercomparison, p_s at Day 9

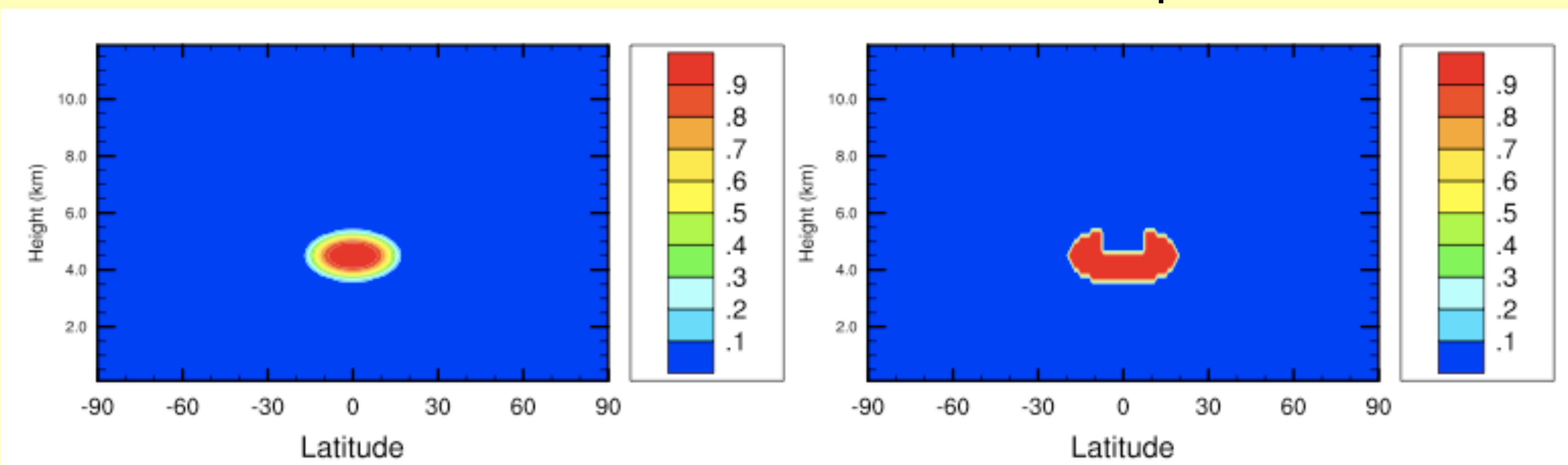


with α=0°, resolution ≈ 1°×1°L26

Test 3: 3D Advection Tests

- Prescribe the 3D wind field: Solid body rotation in 2D plus an overlaid vertical velocity
- Use different rotation angles α

- Prescribe two 3D tracer distributions: z-φ cross section

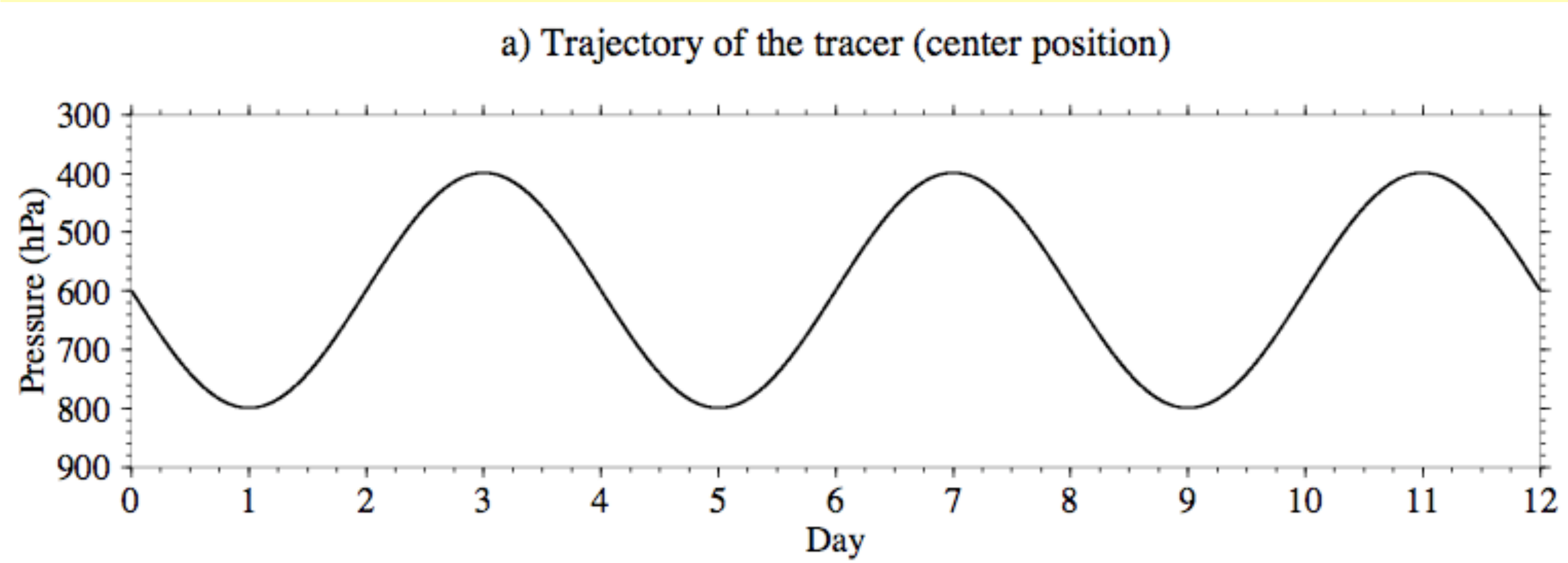


Smooth: Cosine bell **Non-smooth: Slotted ellipse**

Jablonowski et al. (2008)

Test 3: Vertical advection

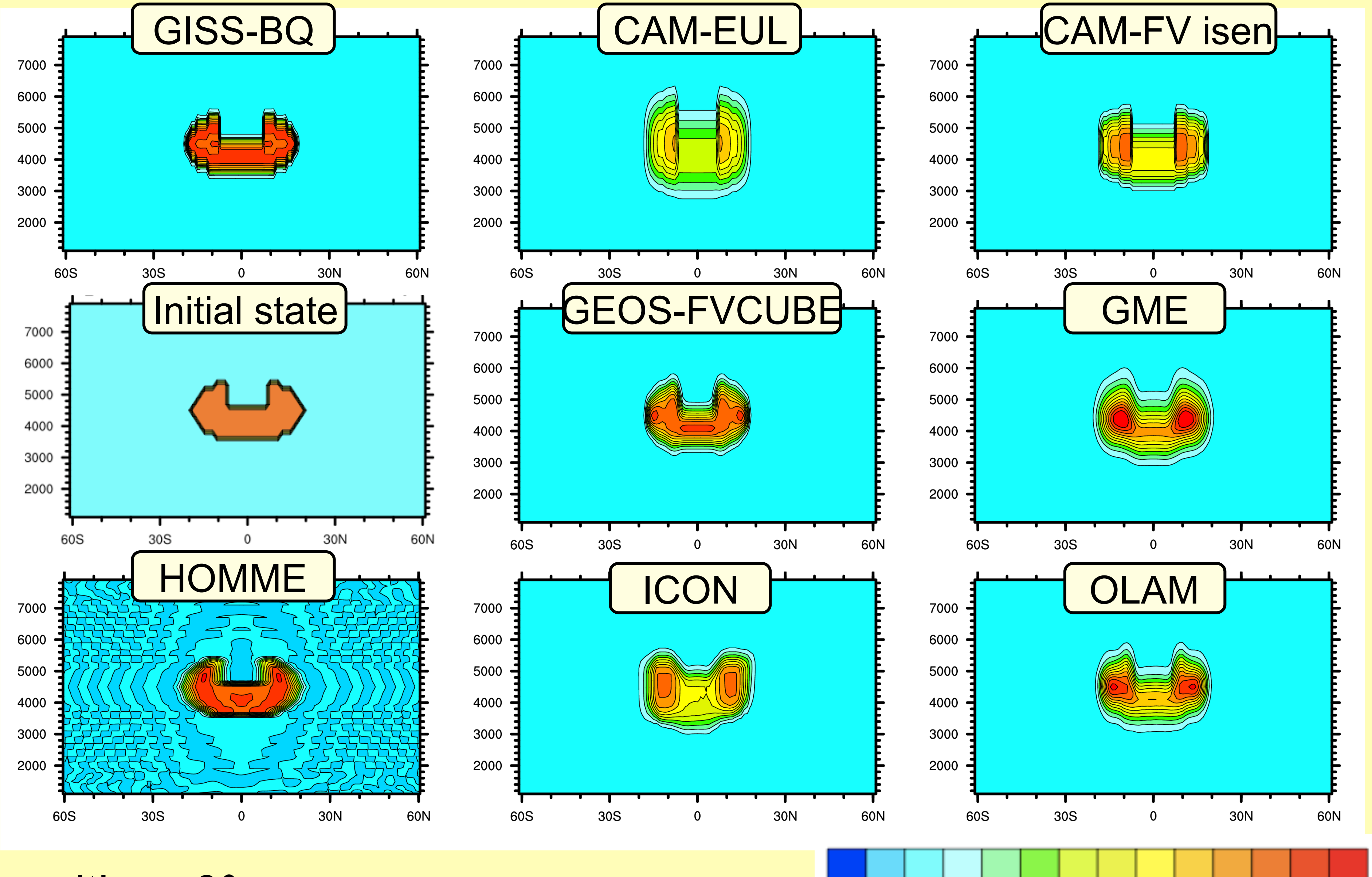
Tracers undergo 3 wave cycles in the vertical



Tracers return to initial position after 12 days:

Allows assessment of the diffusion

Test 3: Slotted Ellipse after 12 Days

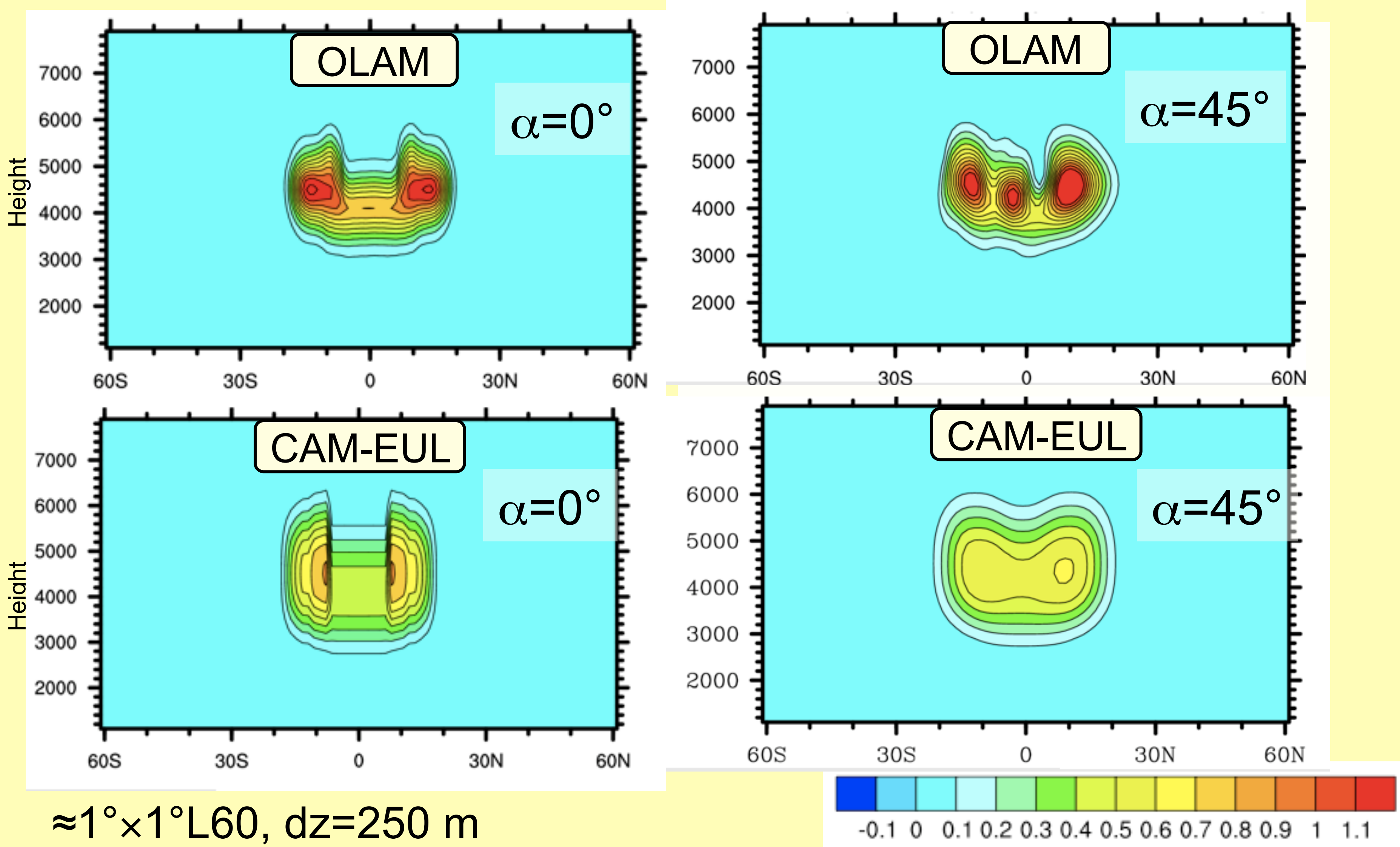


with $\alpha=0°$, (≈1°×1°L60, dz=250 m)

Test 3: Slotted Ellipse after 12 Days

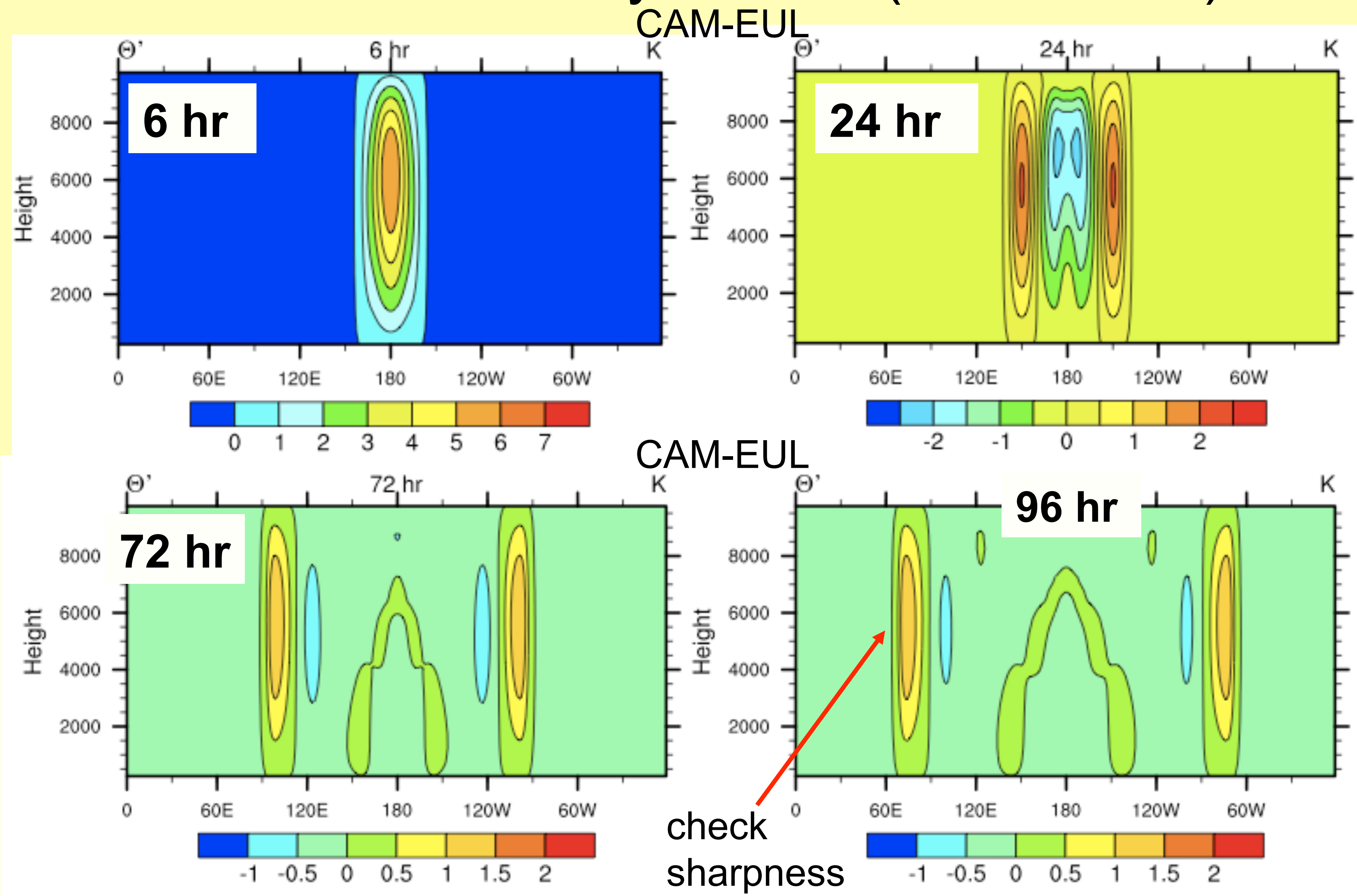
Rotation angles can matter

Most insensitive: models GISS-BQ, FVCUBE, HOMME



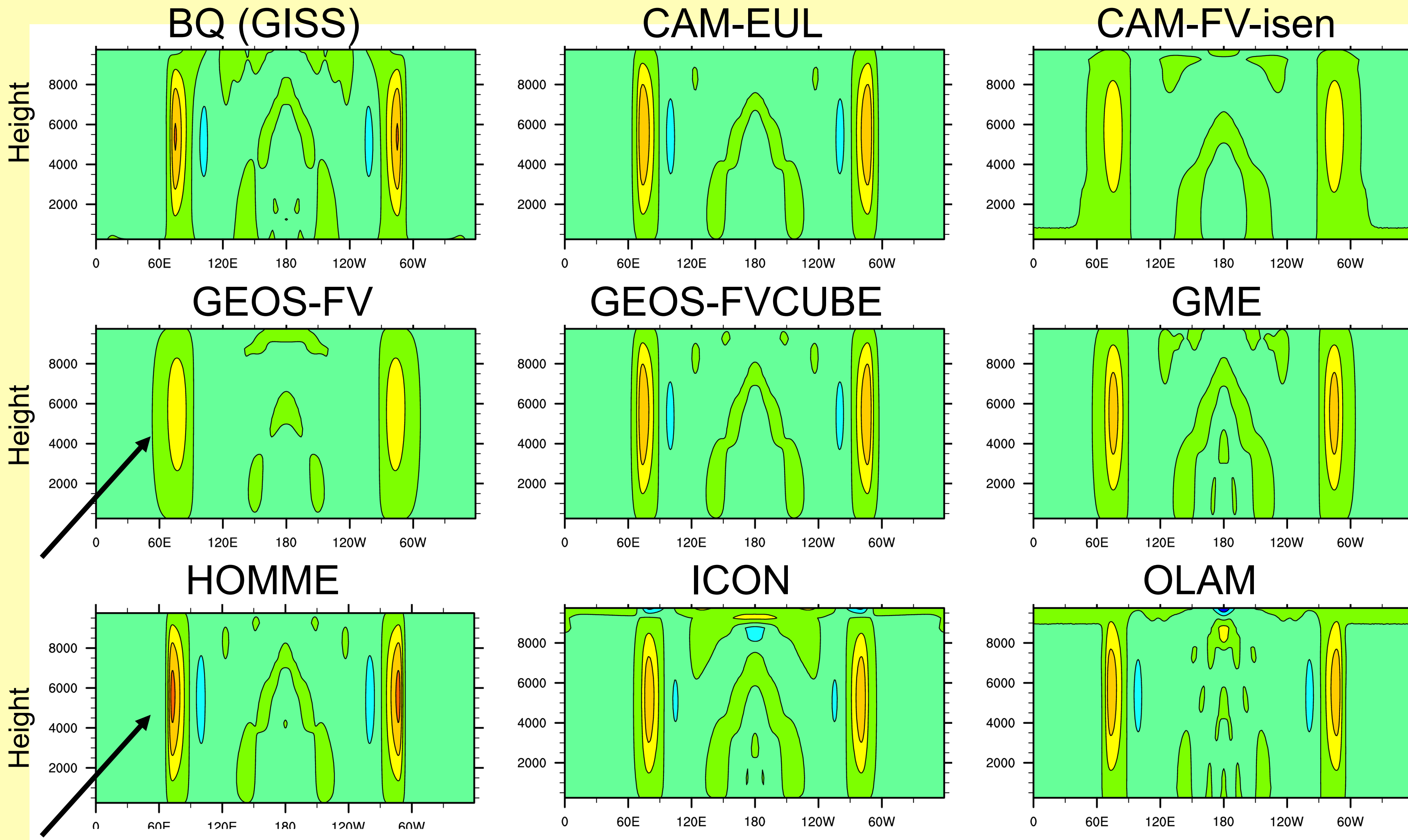
≈1°×1°L60, dz=250 m

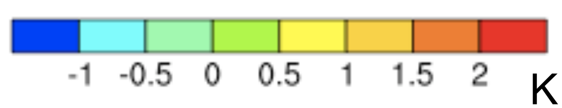
Test 6: Pure Gravity Waves (time series)



CAM-EUL T106 L20 with standard diffusion, Θ' cross section along equator

Test 6: Θ' cross section along the equator





Day 4, (≈1°×1°L20)

Check sharpness
amplitude, speed

What are the subgrid-scale (diffusive) processes that are included in dynamical cores?

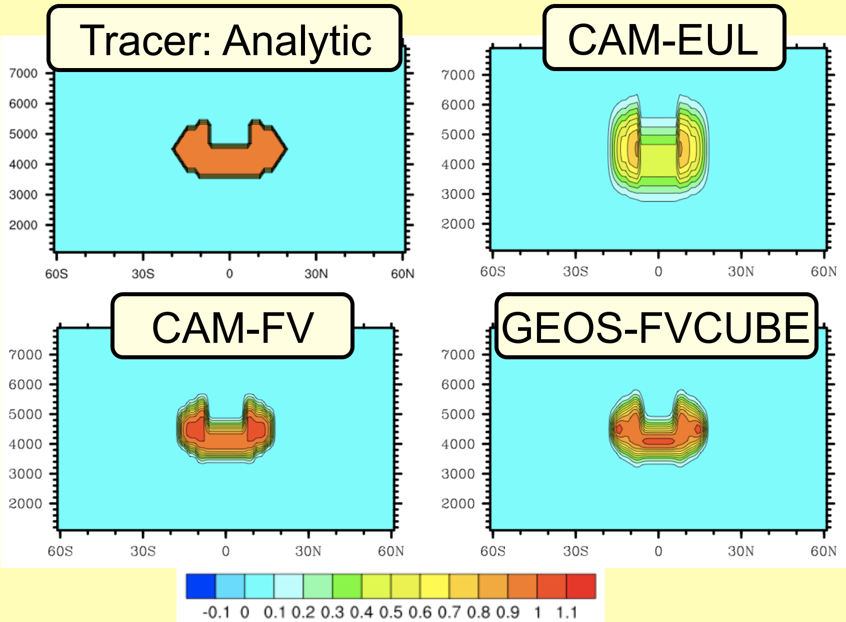
- Dynamical cores need diffusive processes for physical and numerical (stability) reasons
- These are the subgrid-scale processes in dynamical cores that are often hidden:

1) **Implicit Numerical Diffusion**
2) **Explicit diffusion** such as hyperdiffusion, 2nd-order diffusion, divergence damping or Rayleigh friction
3) **Numerical filters** in space and/or time
4) ad-hoc **Fixers** that ensure conservation principles

Engineering (unphysical) climate change

some **provocative** approaches from the dynamical core perspective (be careful!)

- Too much heavy rain: add **divergence damping**
- **Too warm** in the troposphere and **too cold** in the stratosphere: add a **mass fixer**
- **Cold bias** at the tropopause: add strong **explicit diffusion**
- Rely on **tracer advection? Understand what you get**



We take a systematic look at the subgrid-scale diffusion processes and their impact in dry dynamical cores.

Diffusion, Filters and Fixers

- Equations of motion: diabatic effects
- Diffusion
  - Explicit horizontal diffusion (neglecting vertical diffusion)
  - Implicit numerical diffusion
  - Divergence damping (2D or 3D)
  - External mode damping
- Spatial filters:
  - Polar filters / Fourier filters
  - Digital filters: e.g. Shapiro filters
- Time filters: Asselin-filter
- *a posteriori* Fixers:
  - Mass
  - Energy

The 3D Primitive Equations: diabatic effects

Horizontal momentum equation with $\vec{v}_h = (u, v)$

$$\frac{\partial \vec{v}_h}{\partial t} + \left(\vec{v}_h \vec{\nabla}_z\right)\vec{v}_h + w\frac{\partial \vec{v}_h}{\partial z} + f\vec{k}\times\vec{v}_h = -\frac{1}{\rho}\vec{\nabla}_z p + \boxed{\vec{F}_r}$$

temporal change | horizontal & vertical advection | Coriolis force | pressure gradient | friction

Hydrostatic equation:

$$\frac{\partial p}{\partial z} = -g\rho$$

Equation of state (with moisture effects):

$$p = \rho R T_v \quad (\text{with } \mathrm{T_v} = \mathrm{T}(1+0.608\mathrm{q}) \text{ virtual temperature})$$

The 3D Primitive Equations: diabatic effects

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}_z \bullet \left(\rho \vec{v} \right) = 0$$

Thermodynamic equation:

$$\frac{D\Theta}{Dt} = \frac{\partial \Theta}{\partial t} + (\vec{v}\vec{\nabla})\Theta = \boxed{\frac{1}{c_p}\left(\frac{p_0}{p}\right)^{R_d / c_p} Q}$$

Q: diabatic heating

Conservation of water vapor (specific humidity) q:

$$\frac{\partial q}{\partial t} + \vec{v}_h \bullet \nabla_z q + w\frac{\partial q}{\partial z} = \boxed{S_q}$$

S_q: sources/sinks

+ Conservation laws for liquid water + ice

Explicit Horizontal Diffusion

- Diffusion applied to the prognostic variables
  - Regular diffusion ∇^2 - operator
  - Hyper-diffusion ∇^4, ∇^6, ∇^8 - operators: more scale-selective
  - Example: Temperature diffusion, i = 1, 2, 3, …

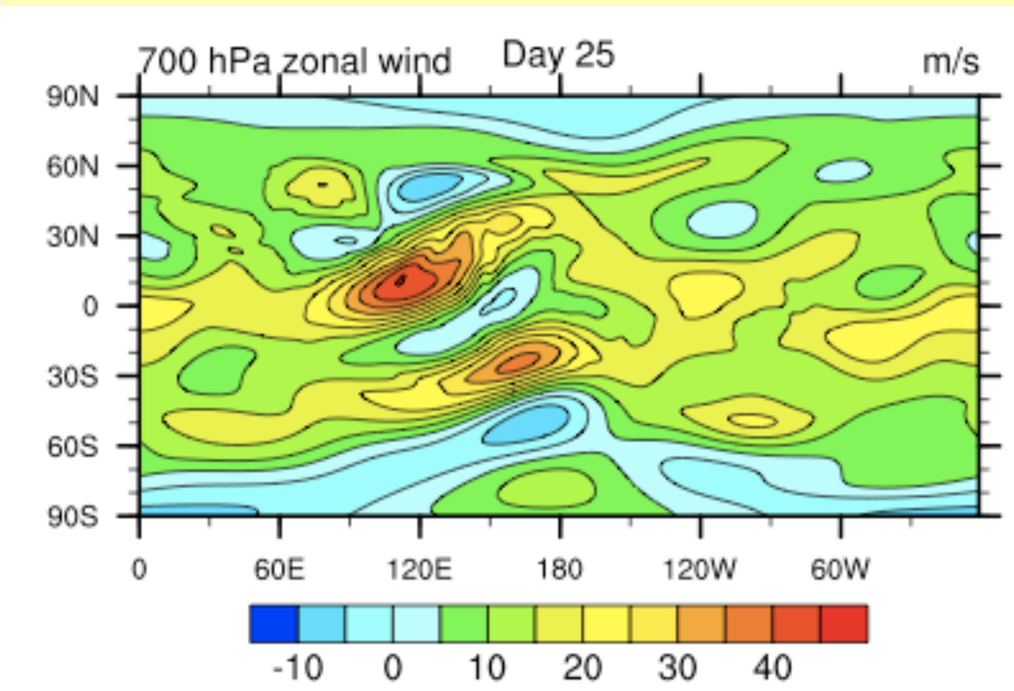
$$\frac{\partial T}{\partial t} = \cdots - \left(-1\right)^i \; K^{(2i)} \left(\nabla^{(2i)} \; T\right)$$

  - K: diffusion coefficients, e-folding time dependent on the resolution
  - Choice of the prognostic variables and levels
- Divergence damping

Effects of Horizontal Diffusion

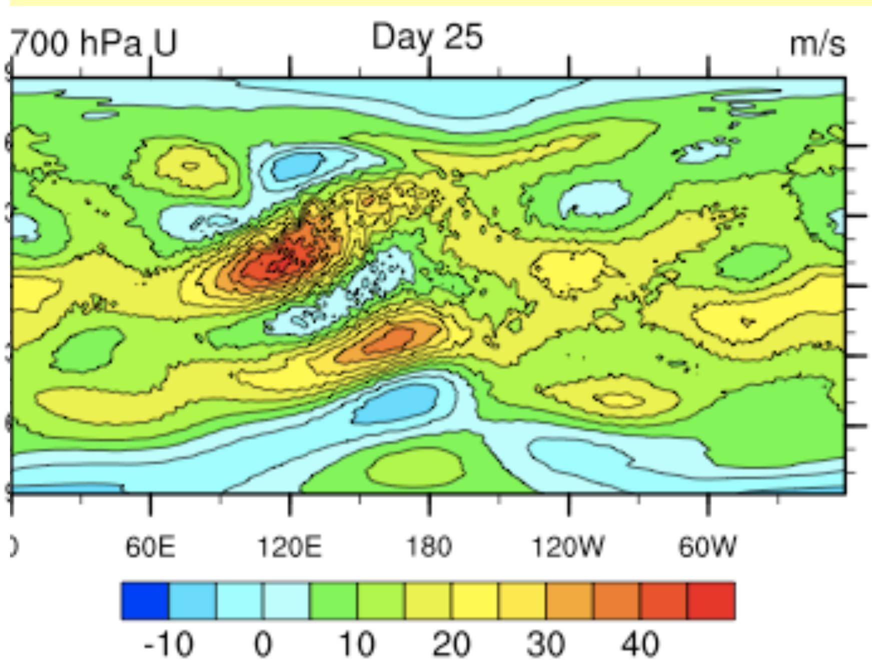
- Comparison of the 700 hPa zonal wind at day 25 in CAM FV and CAM EUL with mountain-wave test (5)

CAM FV 1ºx1ºL26



with monotonicity constraint, divergence damping

CAM EUL T106L26



with standard horizontal diffusion

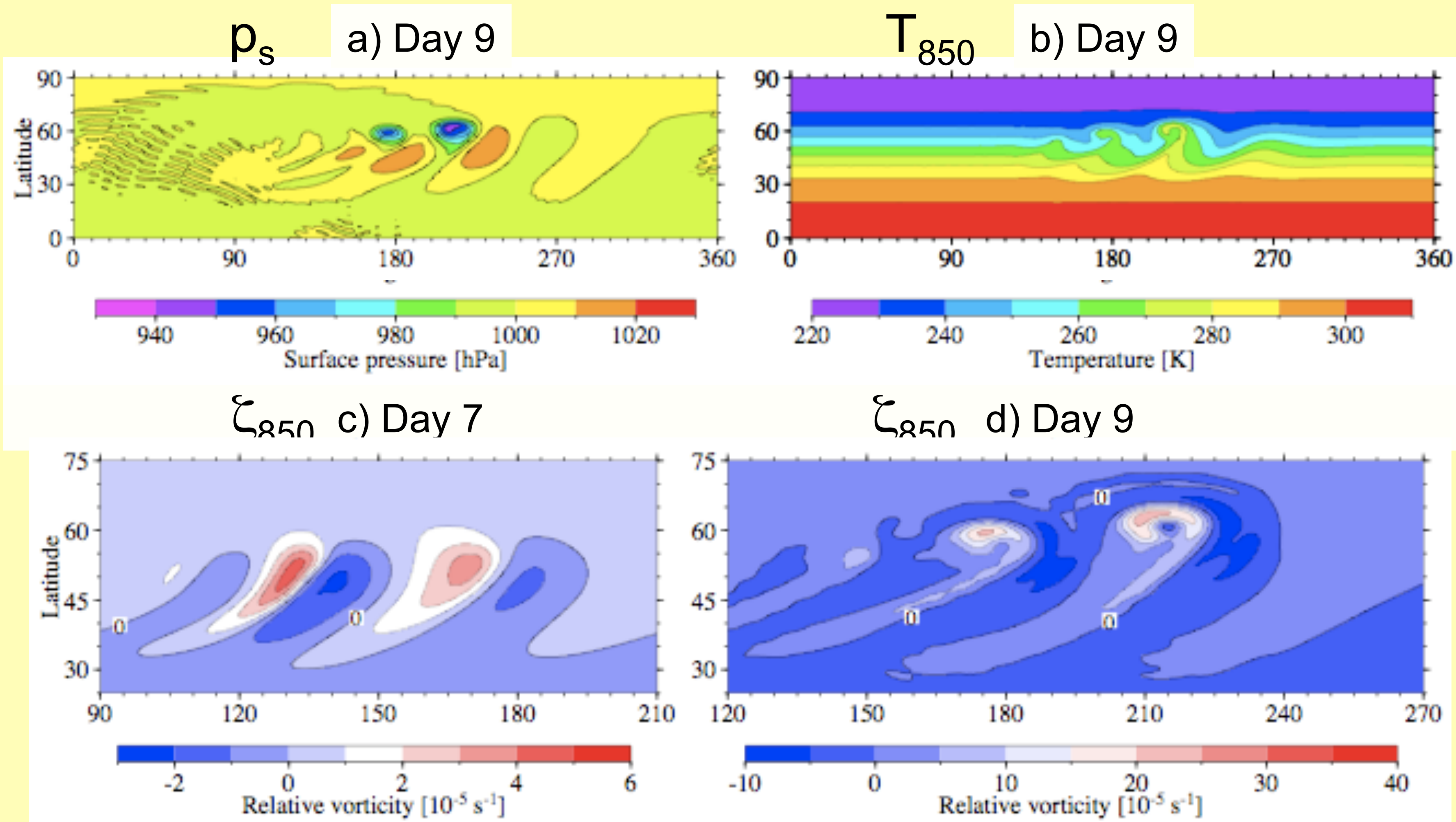
Horizontal Diffusion Coefficients

- Diffusion coefficients are scale-dependent
- Are guided by the so-called e-folding time: How quickly are the fastest waves damped so that their amplitude decrease by a factor of 'e'?
- Typical 4th-order diffusion coefficients K_4 for CAM EUL

Eulerian spectral transform dynamical core(EUL)				
Spectral Resolution	# Grid points lat $\times$ lon	Grid distance at the equator	Time step Δt	Diffusion coefficient K_4 (m^4 s^{-1})
T21	32×64	625 km	2400 s	2.0×10^{16}
T42	64×128	313 km	1200 s	1.0×10^{16}
T85	128×256	156 km	600 s	1.0×10^{15}
T106	160×320	125 km	450 s	0.5×10^{15}
T170	256×512	78 km	300 s	1.5×10^{14}
T340	512×1024	39 km	150 s	1.5×10^{13}

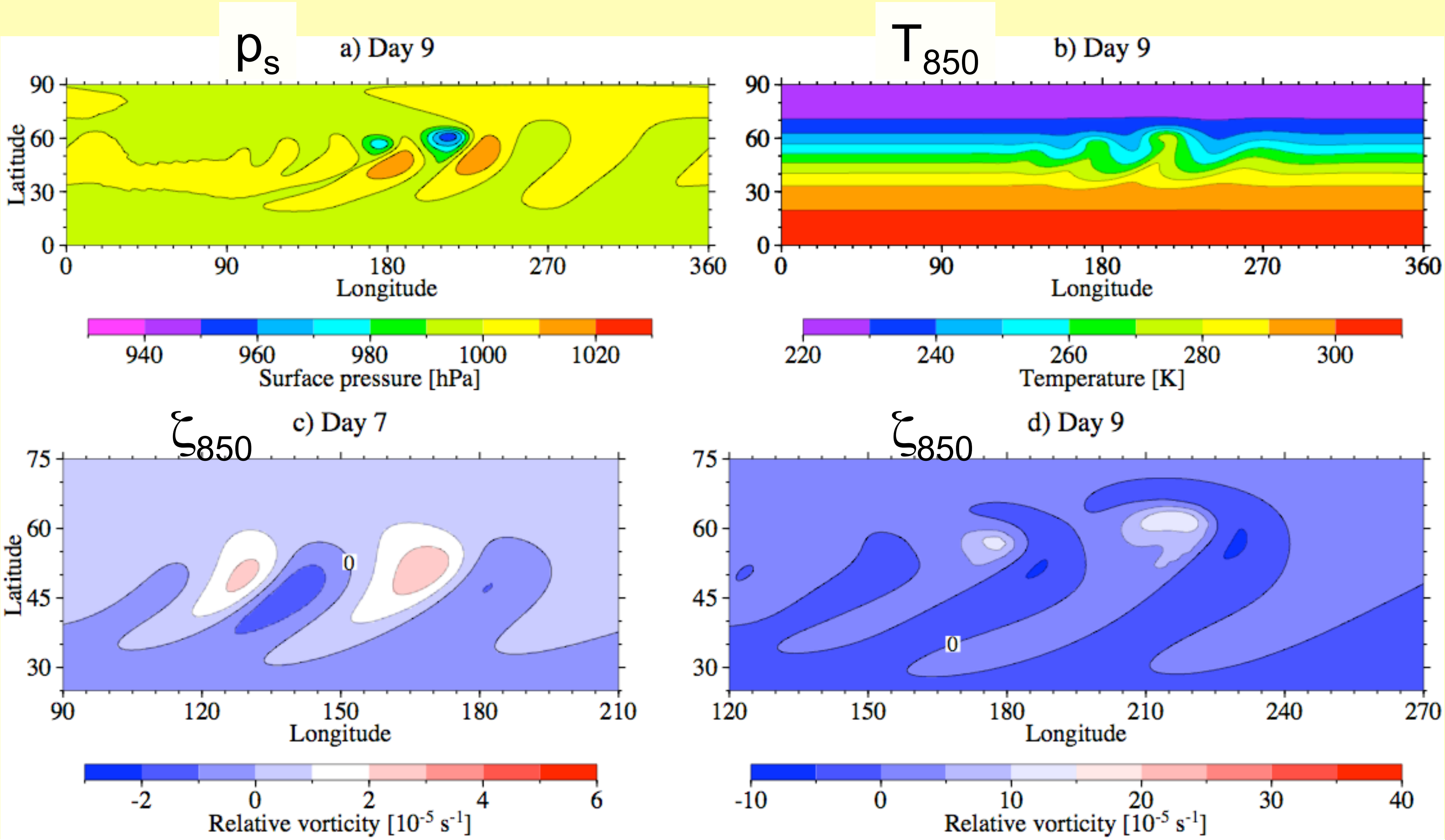
Impact of Explicit Diffusion: Baroclinic Waves

- EUL T85L26 with **standard** $K_4 = 10^{15}\ m^4/s$ diffusion coefficient
- Spectral noise (Gibb's oscillations)



Impact of Explicit Diffusion: Baroclinic Waves

- EUL T85L26 with K_4 **increased** by a factor of 10 (10^{16} m^4/s)
- No spectral noise, but **severe damping** of the circulation



Explicit Horizontal Diffusion: Smagorinsky-type for the velocities

- Consider a second-order diffusion mechanism applied to the horizontal momentum equations (here u) and allow the diffusion coefficient *K* to vary:

$$\frac{\partial u}{\partial t} = \cdots + \nabla \bullet \left(K \, \nabla u \right)$$

- Example of '**nonlinear**' diffusion
- The 'art' is to define the nonlinear coefficient
- Smagorinsky (1963) proposed (deformation-based):

$$K_h \propto \sqrt{(\partial_x u - \partial_y v)^2 + (\partial_x v + \partial_y u)^2}.$$

Problems in spherical geometry !

- Others proposals are (Andrews et al. 1983)

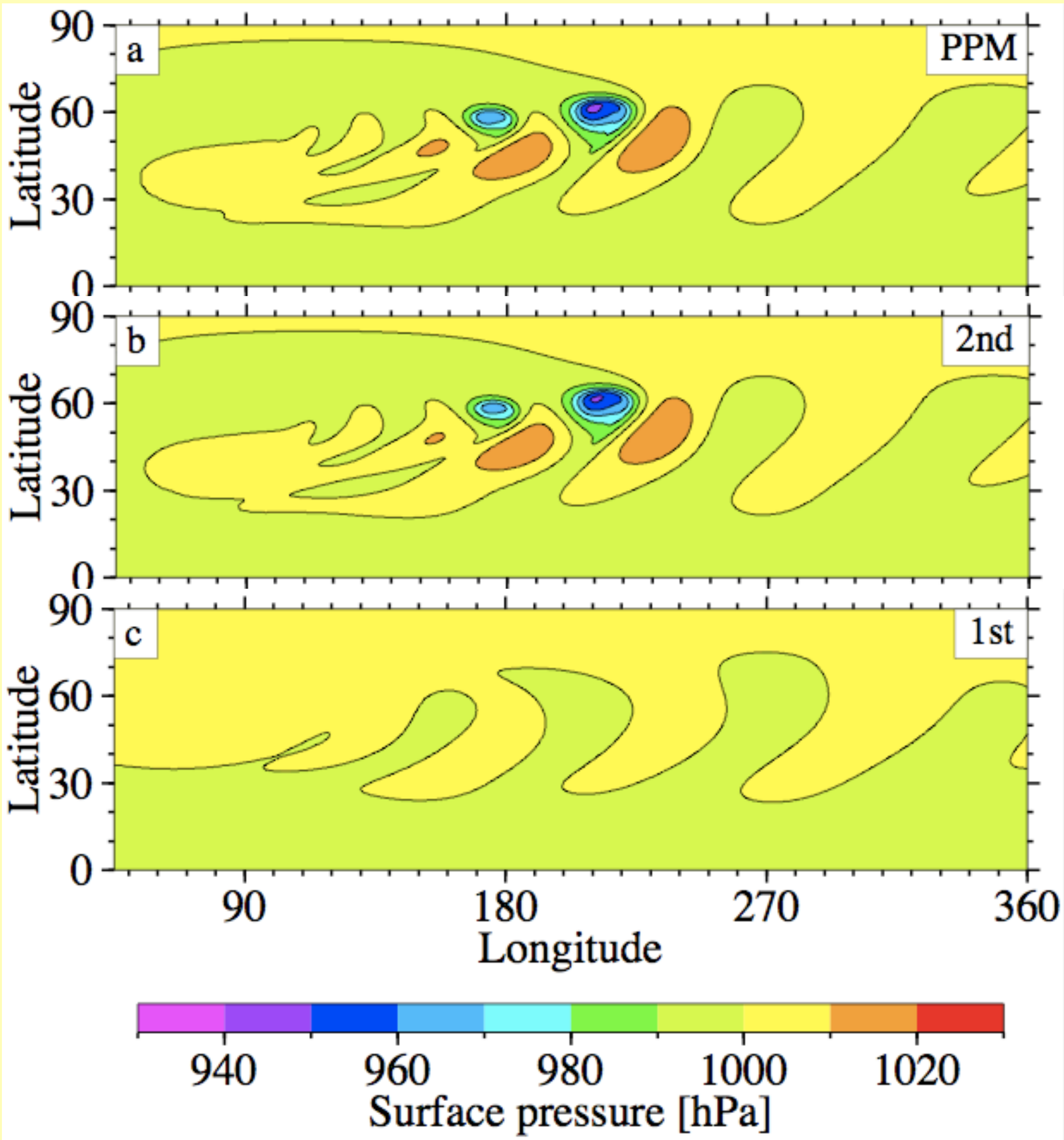
$$K_h \propto \sqrt{(|\partial_x u| + |\partial_y v|)^2 + (|\partial_x v| - |\partial_y u|)^2}$$

See Becker and Burkhardt (MWR, 2007) for overview

Implicit / Numerical Diffusion

- Implicit diffusion: diffusion that is inherent in the numerical scheme
- Sources of implicit / numerical diffusion:
  - Order of accuracy: 1st order, 2nd order, 3rd order, …, higher order schemes
  - The higher the order, the less diffusive
  - Monotonicity constraints
  - Decentering parameters in semi-implicit time-stepping schemes

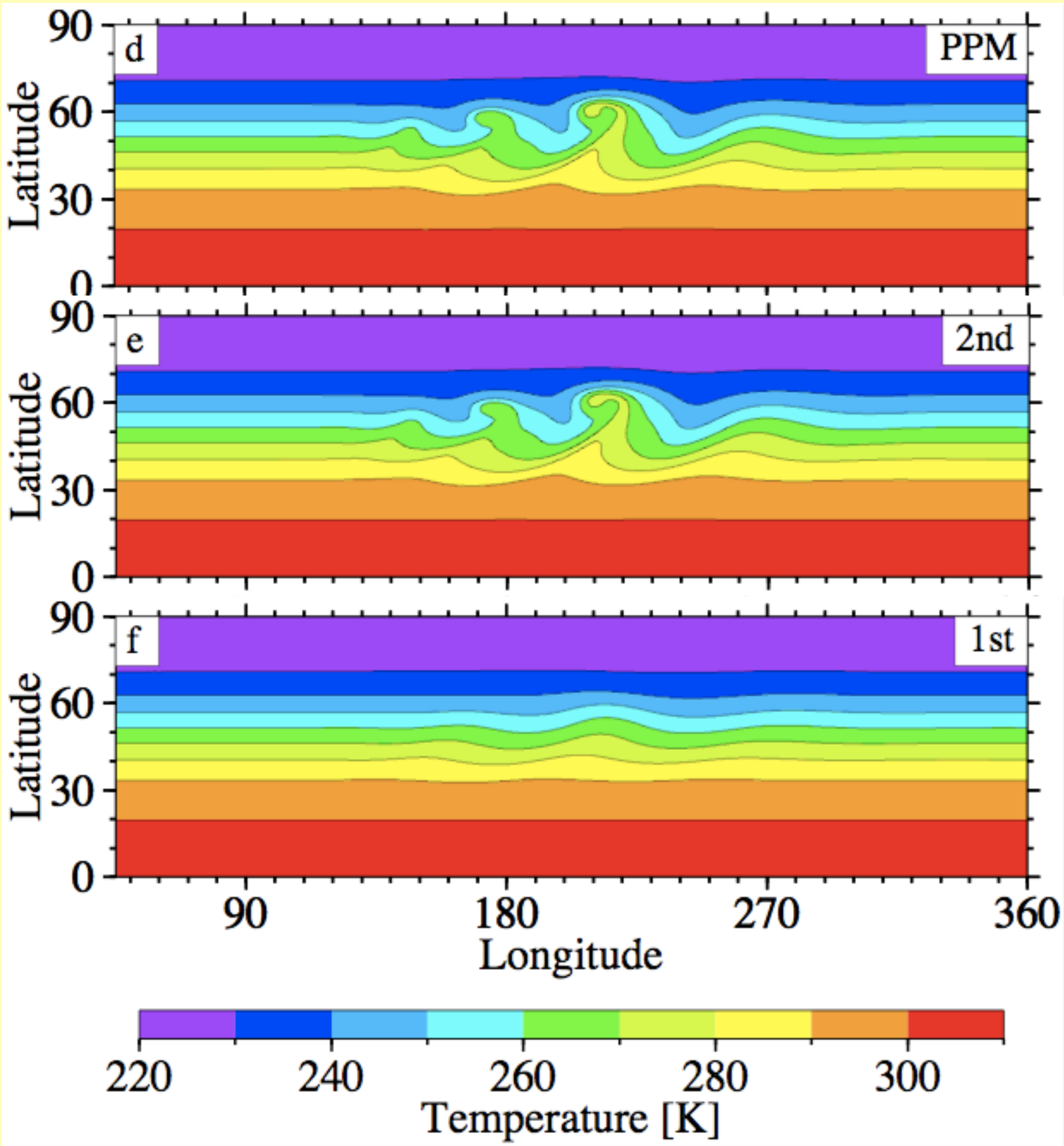
Implicit diffusion: Order of accuracy



- Third order (PPM)
- Second order (van Leer)
- First order upwind scheme

Baroclinic wave test 2
CAM FV 1°x 1.25° L26
p_s at day 9

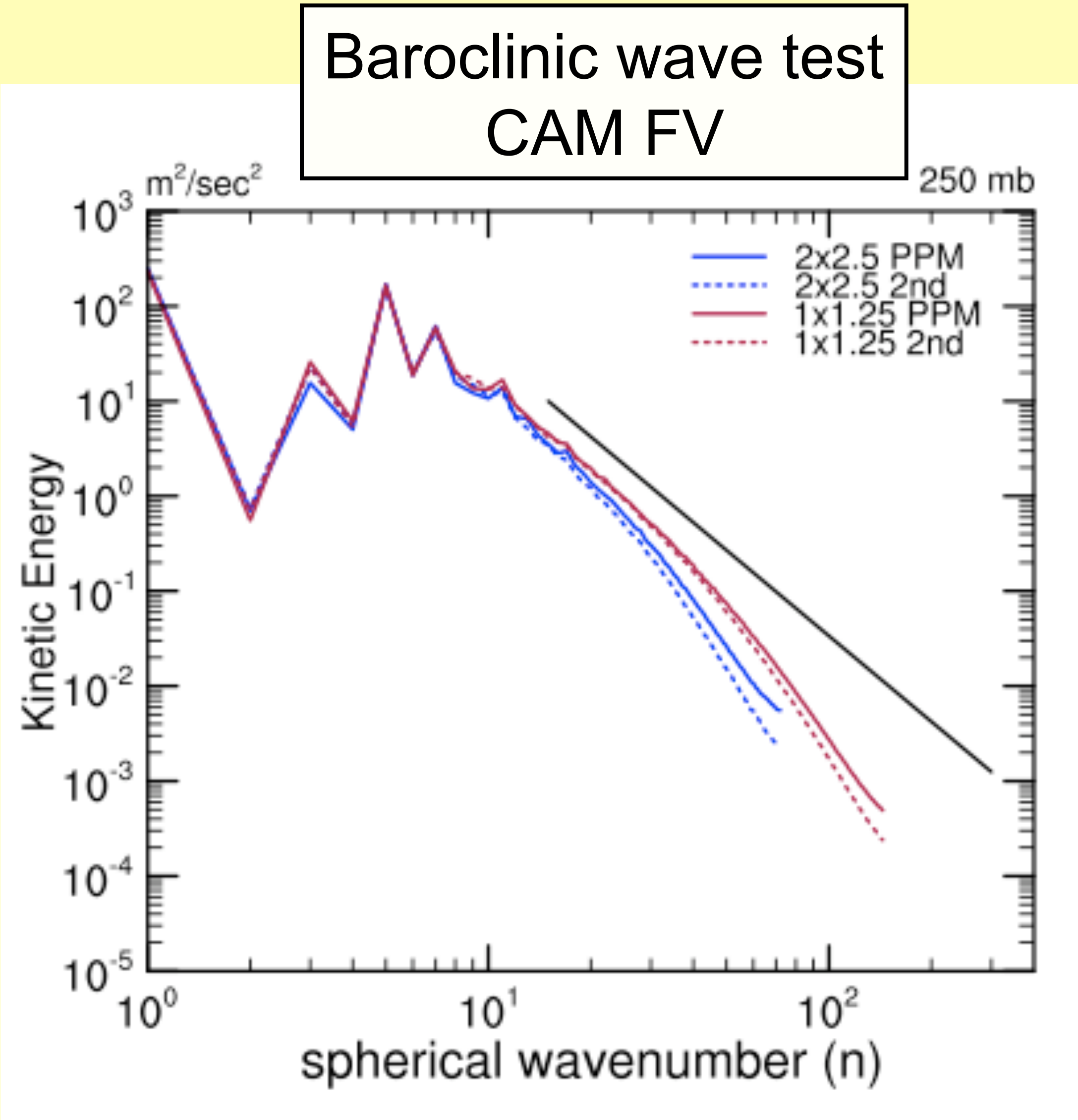
Implicit diffusion: Order of accuracy



- Third order (PPM)
- Second order (van Leer)
- First order upwind scheme

Baroclinic wave test 2
CAM FV 1°x 1.25° L26
$T_{850\ hPa}$ at day 9

Implicit diffusion: Order of accuracy



- Time-averaged kinetic energy spectrum at two different horizontal resolutions
- Third order (PPM)
- Second order (van Leer scheme)
- Tail of 2nd order scheme drops faster

provided by
D. Williamson (NCAR)

Implicit diffusion: Monotonicity constraints in Finite Volume Methods

- Linear subgrid distribution (van Leer scheme)

Reconstruction:

$$h(x,y) = \bar{h} + \Delta a^x\, x + \Delta a^y\, y$$

Slopes:

$$\begin{aligned} \Delta a^x &= \frac{1}{2}\left(h_{i+1,j} - h_{i-1,j}\right) \\ \Delta a^y &= \frac{1}{2}\left(h_{i,j+1} - h_{i,j-1}\right) \end{aligned}$$

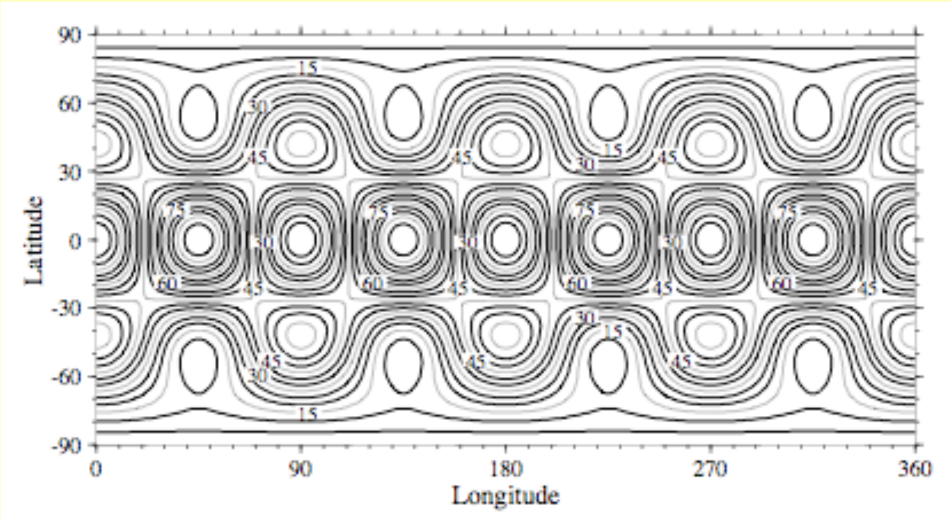
Slope limiter:

$$\begin{aligned} \Delta a^x &= min\left(|\Delta a^x|, 2|h_{i+1,j} - h_{i,j}|, 2|h_{i,j} - h_{i-1,j}|\right)\, sgn(\Delta a^x) \\ &\qquad \text{if } (h_{i+1,j} - h_{i,j})(h_{i,j} - h_{i-1,j}) > 0 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

- Parabolic subgrid distribution (PPM) with cross terms

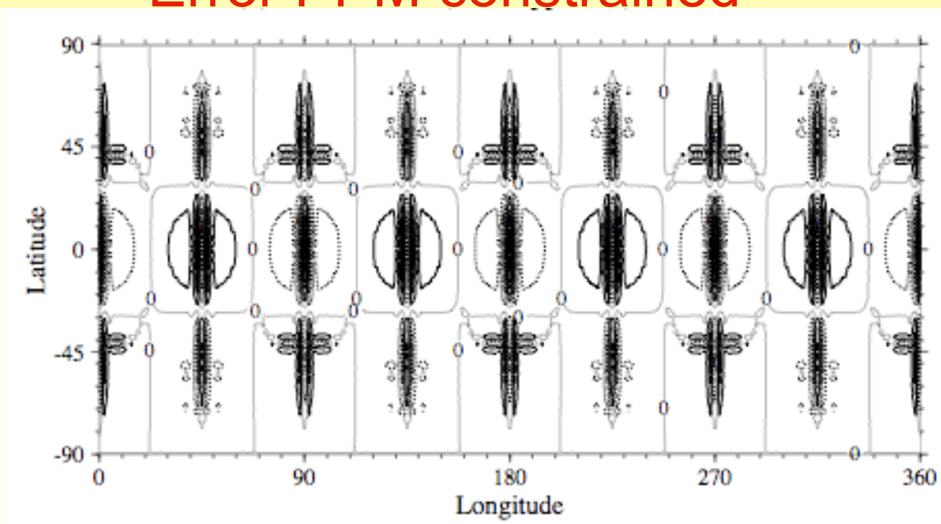
$$h(x,y) = \bar{h} + \delta a^x\, x + b^x\left(\frac{1}{12} - x^2\right) + \delta a^y\, y + b^y\left(\frac{1}{12} - y^2\right) + \frac{1}{2}(c^{xy} + c^{yx})\, x\, y$$

Implicit diffusion: Monotonicity constraint



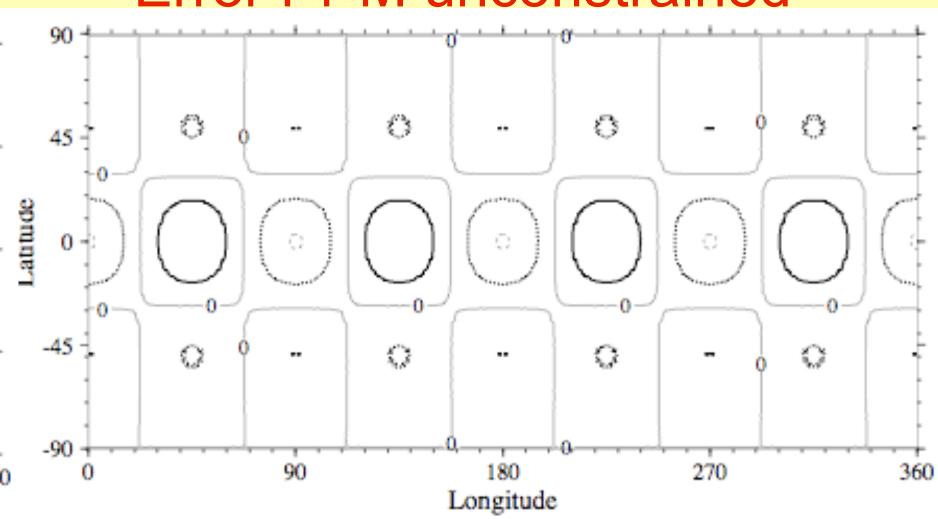
- 2D Rossby-Haurwitz wave
- Initial u field at 2°x 2.5°
- Split cells to 1°x 1.25° grid and interpolate via a PPM reconstruction, compare to analytical solution (error)

Error PPM constrained



Errors cluster near the extrema where the monotonicity constraint is strongest

Error PPM unconstrained



Errors are reduced, but over- or undershoots are possible

2D Divergence damping

- Example: 2D shallow water momentum equation

coefficient

Momentum equation:

$$\frac{\partial \vec{v}}{\partial t} = -\Omega_a \vec{k} \times \vec{v} - \nabla\left(\Phi + \mathcal{K} - c\,D\right)$$

Horizontal divergence:

$$\begin{aligned} D &= \frac{1}{a\,\cos\varphi}\left[\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \varphi}(v\cos\varphi)\right] \\ &\approx \frac{1}{a\,\cos\varphi}\left[\frac{\Delta u}{\Delta \lambda} + \frac{\Delta(v\cos\varphi)}{\Delta \varphi}\right] \end{aligned}$$

Semi-discretized:

$$\frac{\partial \vec{v}}{\partial t} = -\Omega_a \vec{k} \times \vec{v} - \nabla\left(\Phi + \mathcal{K} - \left[c_u\,\Delta u + c_v\,\Delta(v\cos\varphi)\right]\right)$$

Divergence damping coefficients divided by metric terms, different in both directions

2D Divergence damping

- Divergence damping diffuses the divergent part of the flow

$$\frac{\partial \vec{v}_h}{\partial t} = -\Omega_a \vec{k} \times \vec{v}_h - \nabla\left(\Phi + K - cD\right) \qquad \text{(SW equation)}$$

$$\Rightarrow \frac{\partial \vec{v}_h}{\partial t} = \cdots + \nabla\left(cD\right)$$

$$\Rightarrow \nabla \bullet \frac{\partial \vec{v}_h}{\partial t} = \cdots + \nabla \bullet \nabla\left(cD\right) \qquad \text{Apply divergence operator}$$

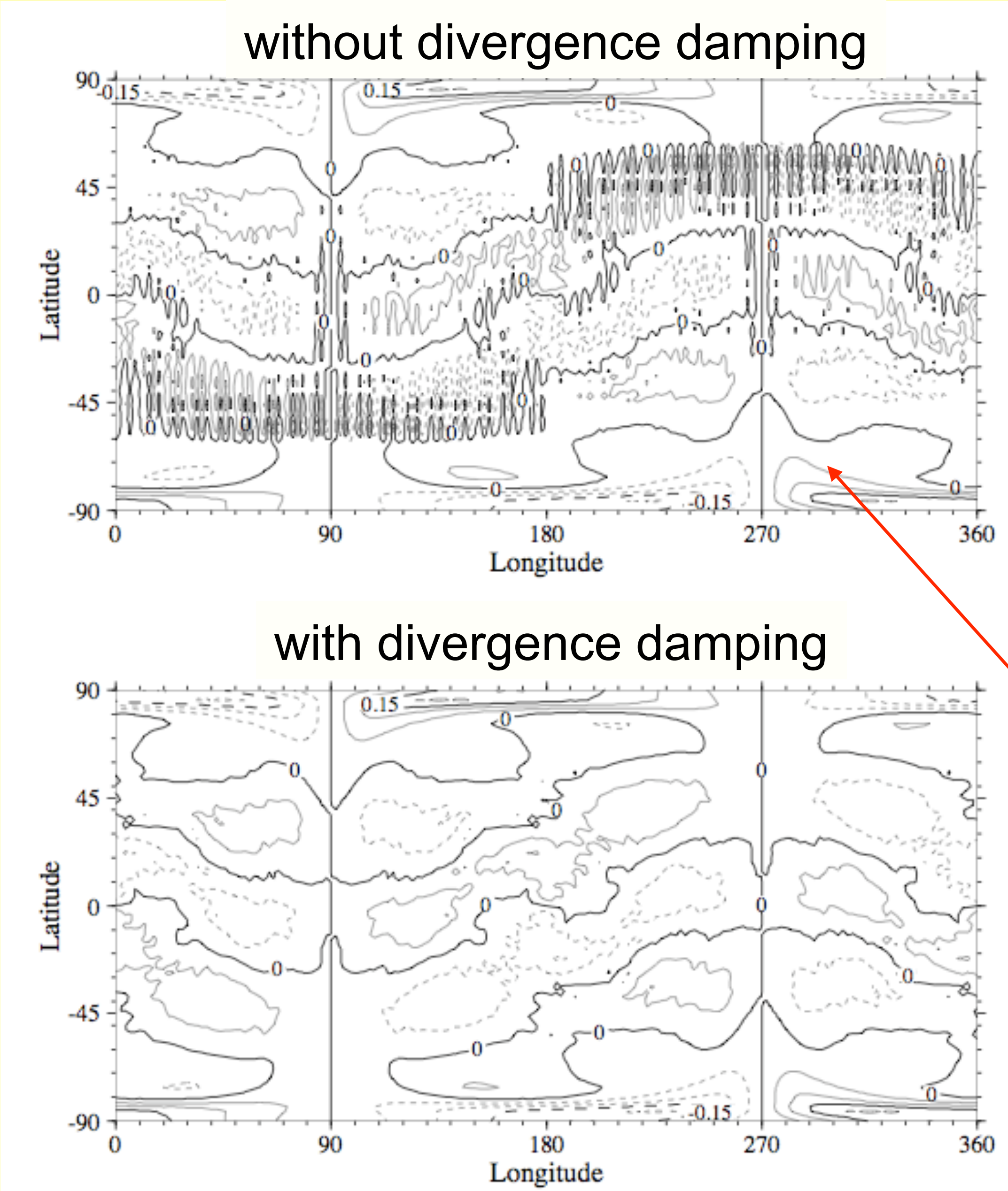
$$\Leftrightarrow \frac{\partial D}{\partial t} = \cdots + \nabla^2\left(cD\right) \qquad \text{with D : horizontal divergence}$$

2nd order diffusion

Spatially variant divergence damping coefficient, units m^2/s

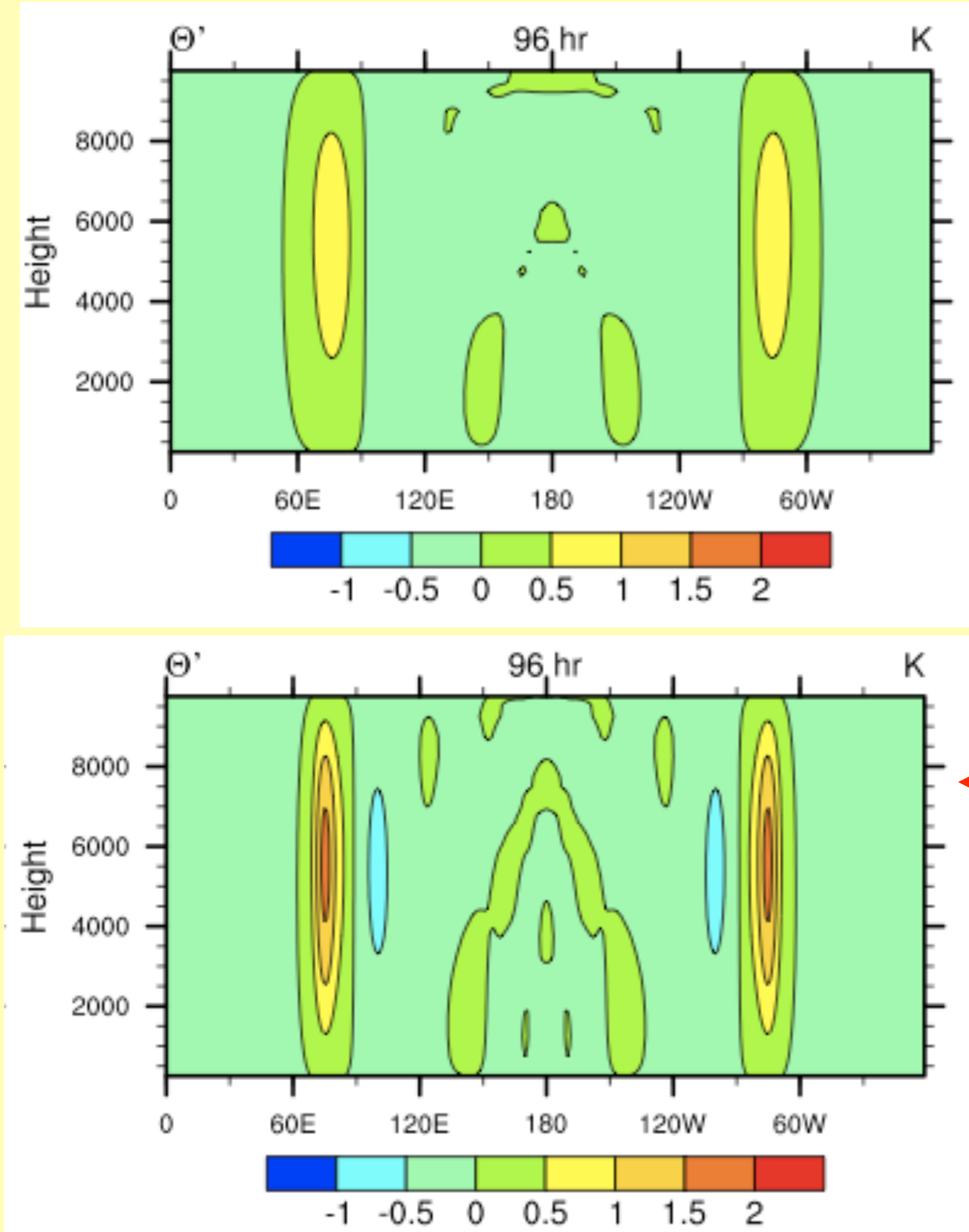
- Can you select any coefficient c? Selection criterion?

Divergence damping



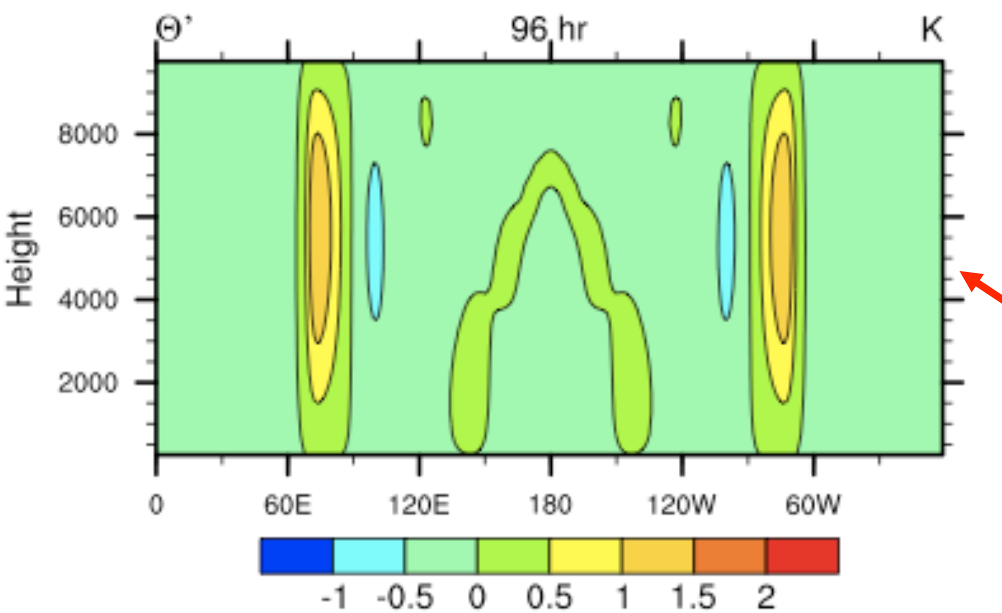
- Example: 2D SW steady state test case with α=90°, model FV
- Difference field at day 10 compared to analytical solution
- Contour interval is 0.05 m/s
- Why is the polar region always smooth? Because of other filters (here polar Fourier filter)

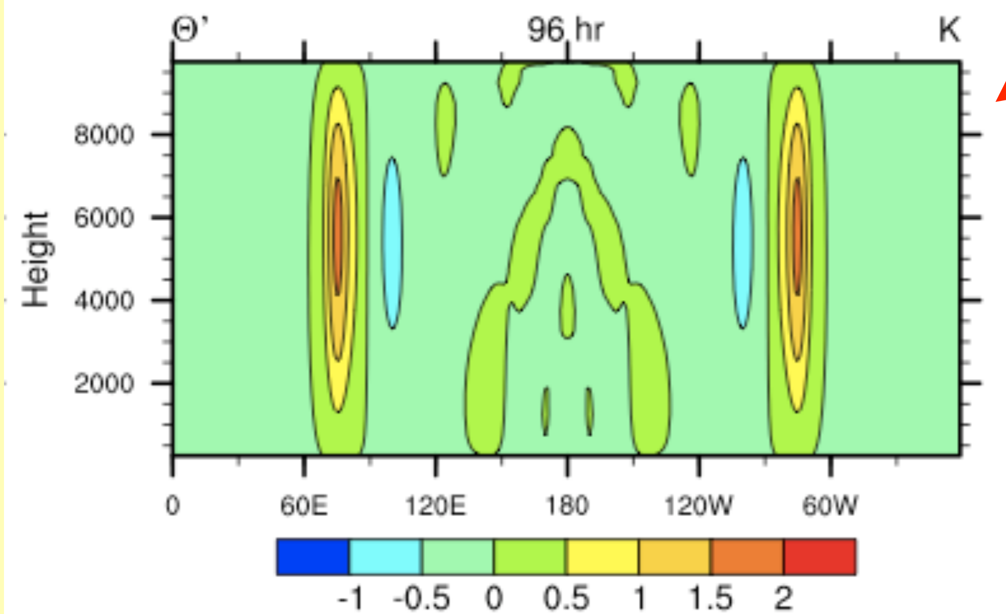
Divergence damping: Effects



- Example: 3D gravity wave test
- Model CAM FV 1°x 1° L20 at day 4, cross section at equator
- with standard divergence damping coefficients (top)
- without divergence damping (bottom)
- Clear difference in the amplitudes of the gravity wave

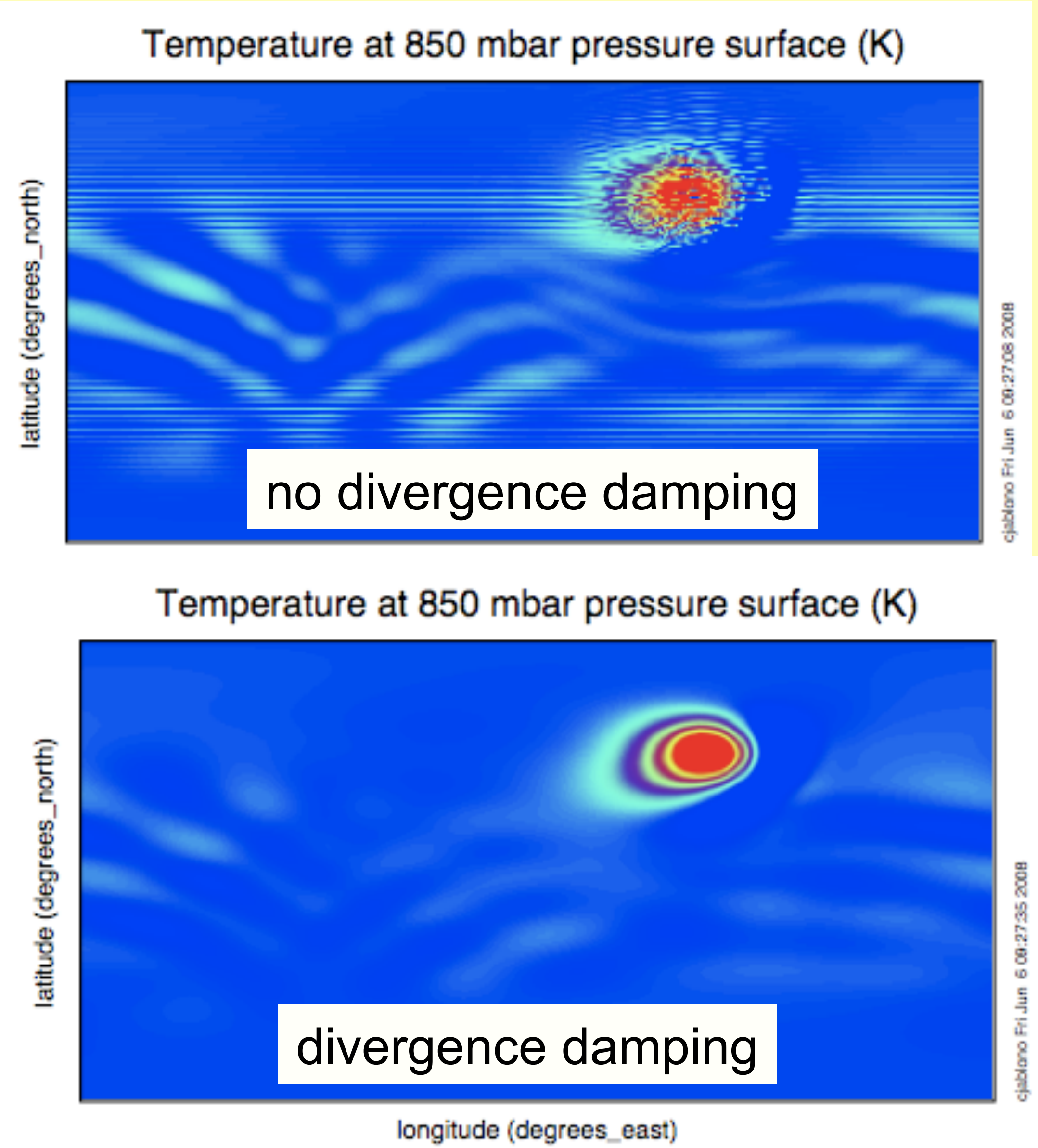
All types of diffusion change the solution





- Example: 3D gravity wave test, cross section at the equator at day 4
- Model CAM EUL T106L20 with explicit ∇^4 diffusion (top)
- Model CAM FV 1°x 1° L20, no divergence damping (bottom)
- Clear difference in the shape of the potential temperature perturbation
- Check sharpness of leading edge

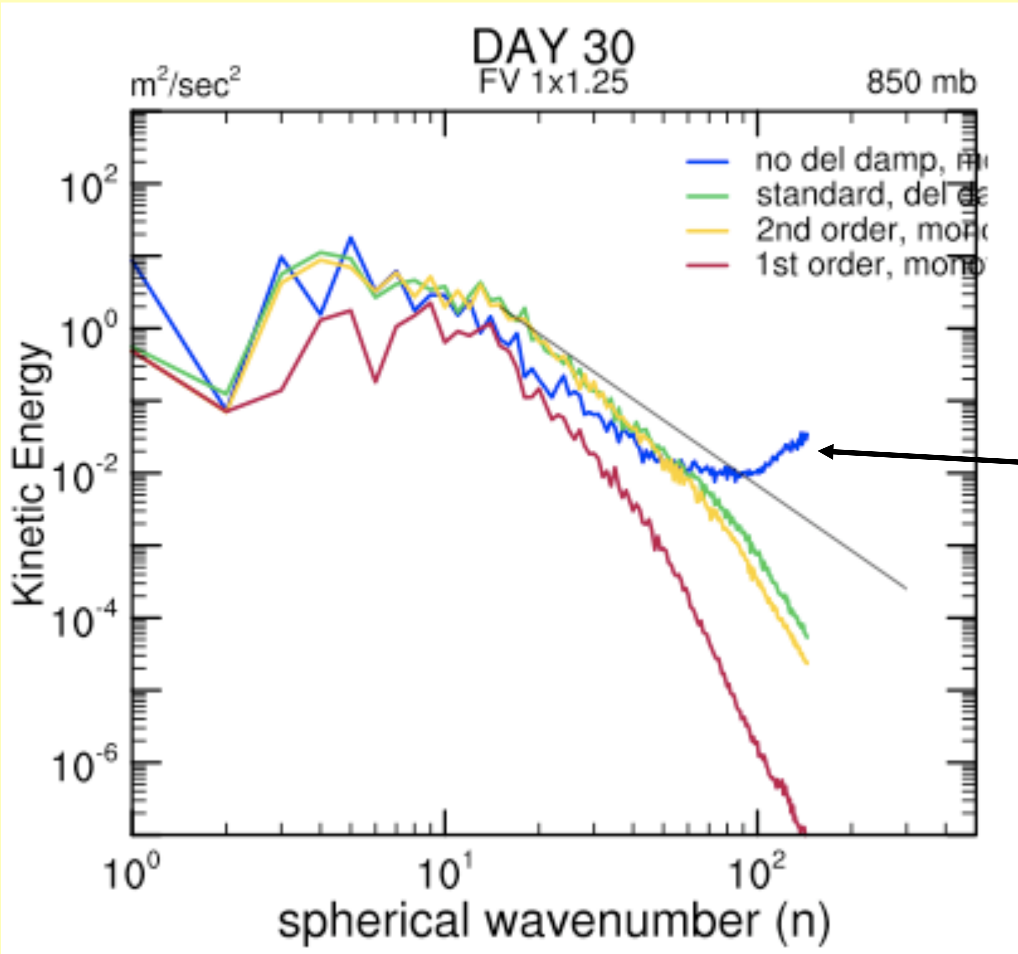
Divergence damping: Needed for stability?



- Example: alternative 3D inertio-gravity wave test with background flow
- Model CAM FV 1°x 1° L20 at day 5.5, lat-lon cross section at 850 hPa
- Numerical stability of CAM FV depends on the resolution- and time step dependent choice of the divergence damping coefficient c

Divergence Damping

- Effects of the divergence damping and order of accuracy on the Kinetic Energy spectrum (baroclinic wave)



Blue: PPM, **no** divergence damping

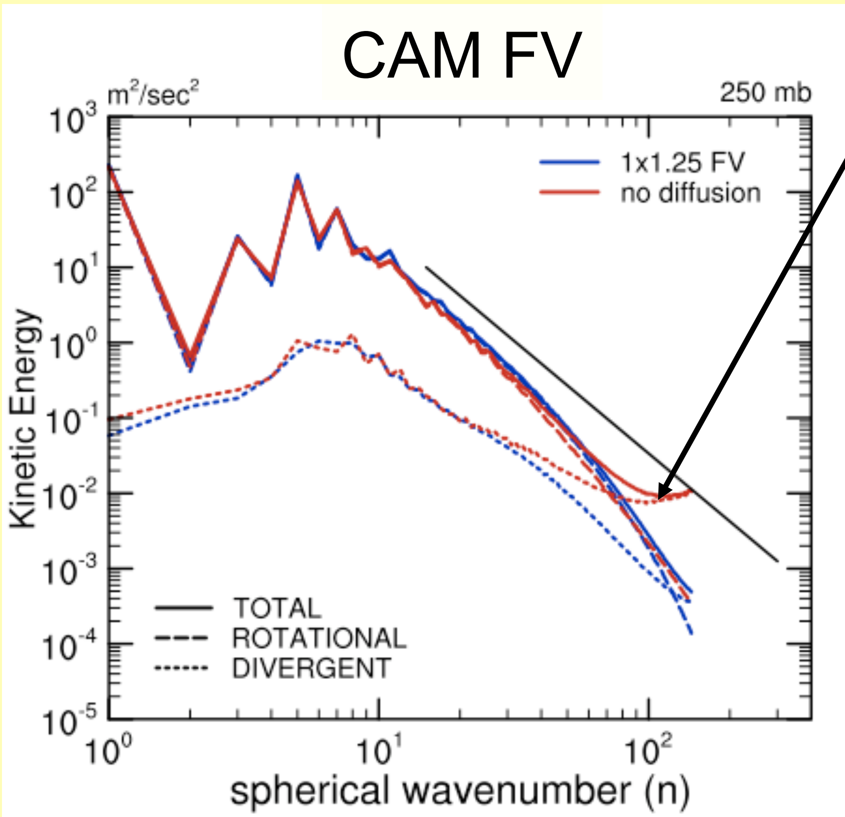
Green: PPM, standard divergence damping

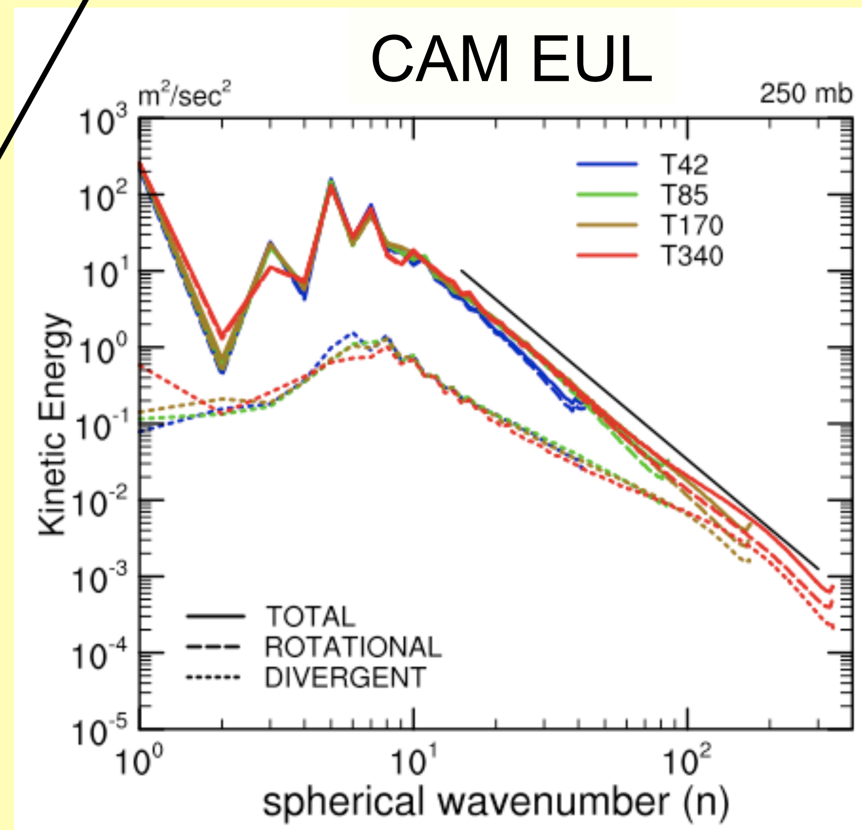
Very harmful: Accumulation of energy at small scales without divergence damping

Model: CAM FV, plot provided by D. Williamson (NCAR)

Divergence Damping

- Without diffusion (here divergence damping): divergent part of the flow responsible for the hook





plots provided by D. Williamson (NCAR)

3D Divergence damping

- Often used in non-hydrostatic models
- Highly specific to filtering the acoustic modes
- Same idea as the 2D divergence damping: add a term to each of momentum equations that is proportional to the gradient of the 3D divergence:

$$\frac{\partial u}{\partial t} = \ldots + \alpha \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\frac{\partial v}{\partial t} = \ldots + \alpha \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\frac{\partial w}{\partial t} = \ldots + \alpha \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

with coefficient

$$\alpha \propto C \frac{\Delta x \Delta y}{\Delta t}$$

- Introduces a 2^{nd}-order diffusion of the 3D divergence

Example: Skamarock and Klemp (MWR, 1992)

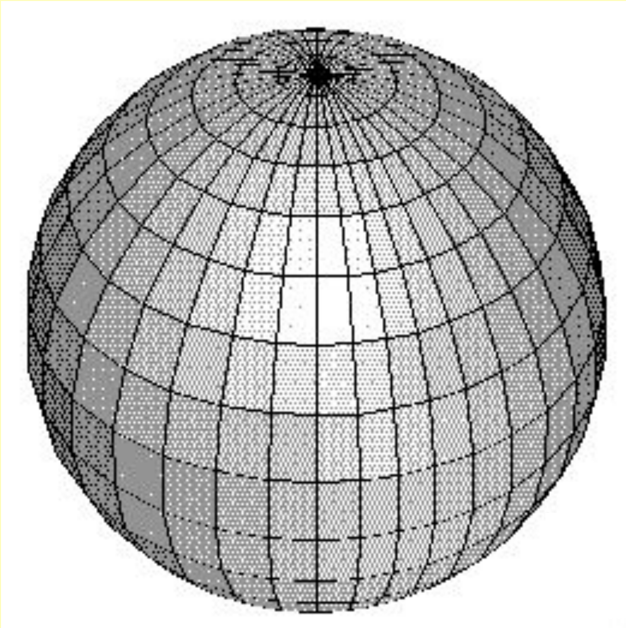
External mode damping

- In hydrostatic models the fastest propagating modes (waves) are so-called external (barotropic) modes: e.g. external gravity wave or the Lamb wave (horizontally propagating sound wave)
- Can be damped via a mechanism called external mode damping (rarely used)
- Similar to 2D divergence damping
- Damps the horizontal velocities via an integrated approach (vertical integral of the 2D divergence)

$$\frac{\partial \vec{v}_h}{\partial t} = \ldots + c\nabla \int_{z=0}^{z_{top}} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz$$

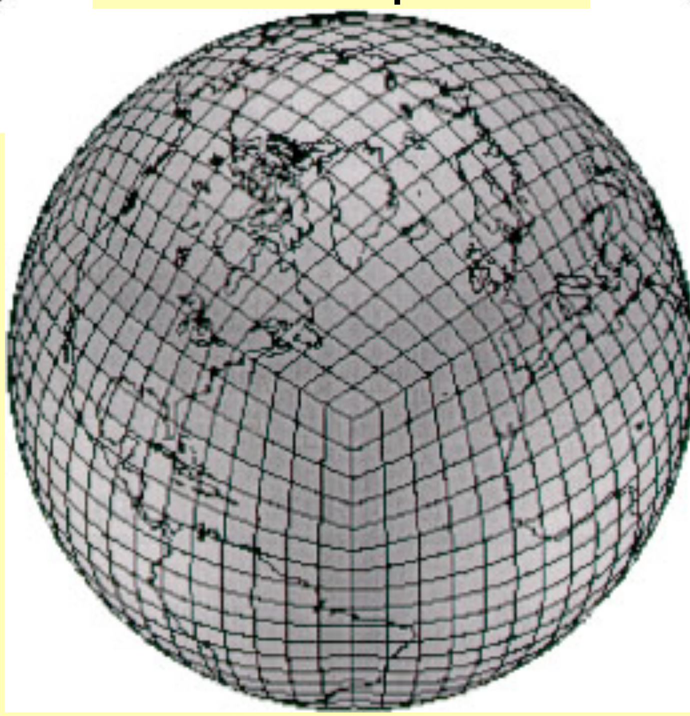
- The ‘art’ is to set the damping coefficient ’c’

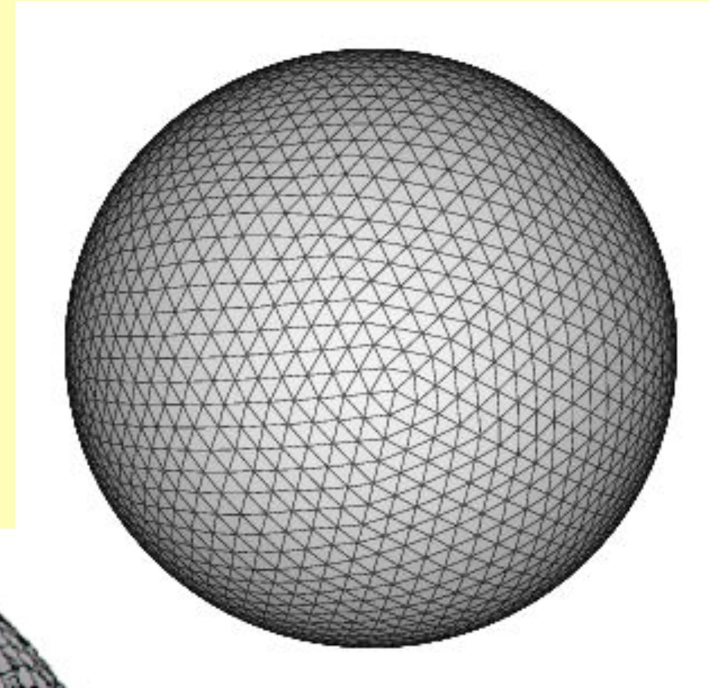
Computational grids (horizontal)



Latitude-longitude grid: needs polar filtering due to convergence of meridians

Cubed sphere





Icosahedral grid

No polar filter required

Spatial filters

- Most popular and most effective polar filter: 1D Fourier filter (spectral filter), used in the zonal (x) direction
- **Basic idea:**
  - Transform the grid point data into spectral space via Fourier transformations
  - Eliminate or damp high wave numbers (noise) by either setting the spectral coefficients to 0 or multiplying them with a damping coefficient $\in [0,1]$
  - Transform the field back from spectral space into grid point space: result is a filtered data set
- Filter strength is determined by the spectral damping coefficients, can be made very scale-selective and dependent on the latitude (e.g. less strong towards equator)
- Drawback: needs all data along latitude ring (poor scaling)

Polar Fourier Filters

- A Fourier filter application for all zonal wavenumbers k can be written as:

$$\hat{\phi}(k)_{filtered} = a(k)\hat{\phi}(k)$$

  where $\hat{\phi}(k)$ are the Fourier coefficients and *a(k)* are the filter coefficients
- The filter coefficients depend on latitude *ψ*, they are e.g. defined by (with n: # grid points in the zonal direction)

$$a(k) = \min\left[1., \max\left(0., \left(\frac{\cos\psi}{\cos\psi_o}\right)^2 \frac{1}{\sin^2(\pi k/n)}\right)\right]$$

- Coefficients become small (or zero) at high latitudes and for high wave numbers. Filter becomes inactive (*a(k)*=1) at latitude ψ_0 (often chosen to be between 30-45 degrees N/S).

Digital filters

- Digital or algebraic filters are local grid-point filters that only take neighboring grid points into account
- Examples are the Shapiro filters (Shapiro, 1975)
- 4th order (n=2) Shapiro filter (i is the grid point index):

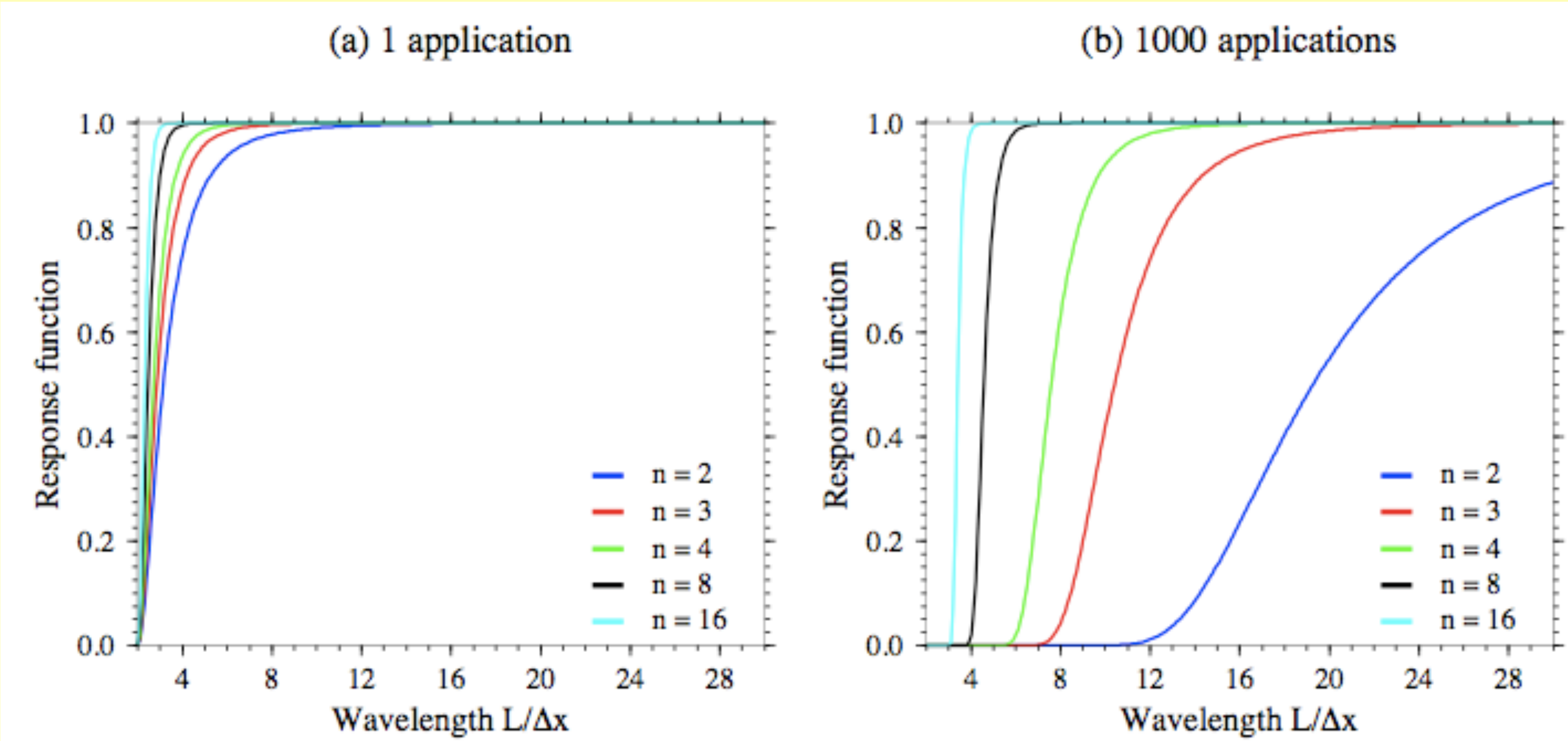
$$\bar{f}_i = \frac{1}{16}\left(- f_{i-2} + 4f_{i-1} + 10f_i + 4f_{i+1} - f_{i+2}\right)$$

- The filter response/damping function is (Shapiro, 1971)

$$\begin{aligned} \rho_n(k) &= 1 - sin^{2n}\left(k\frac{\Delta x}{2}\right) \\ &= 1 - sin^{2n}\left(\pi\frac{\Delta x}{L}\right) \end{aligned}$$

2n: order

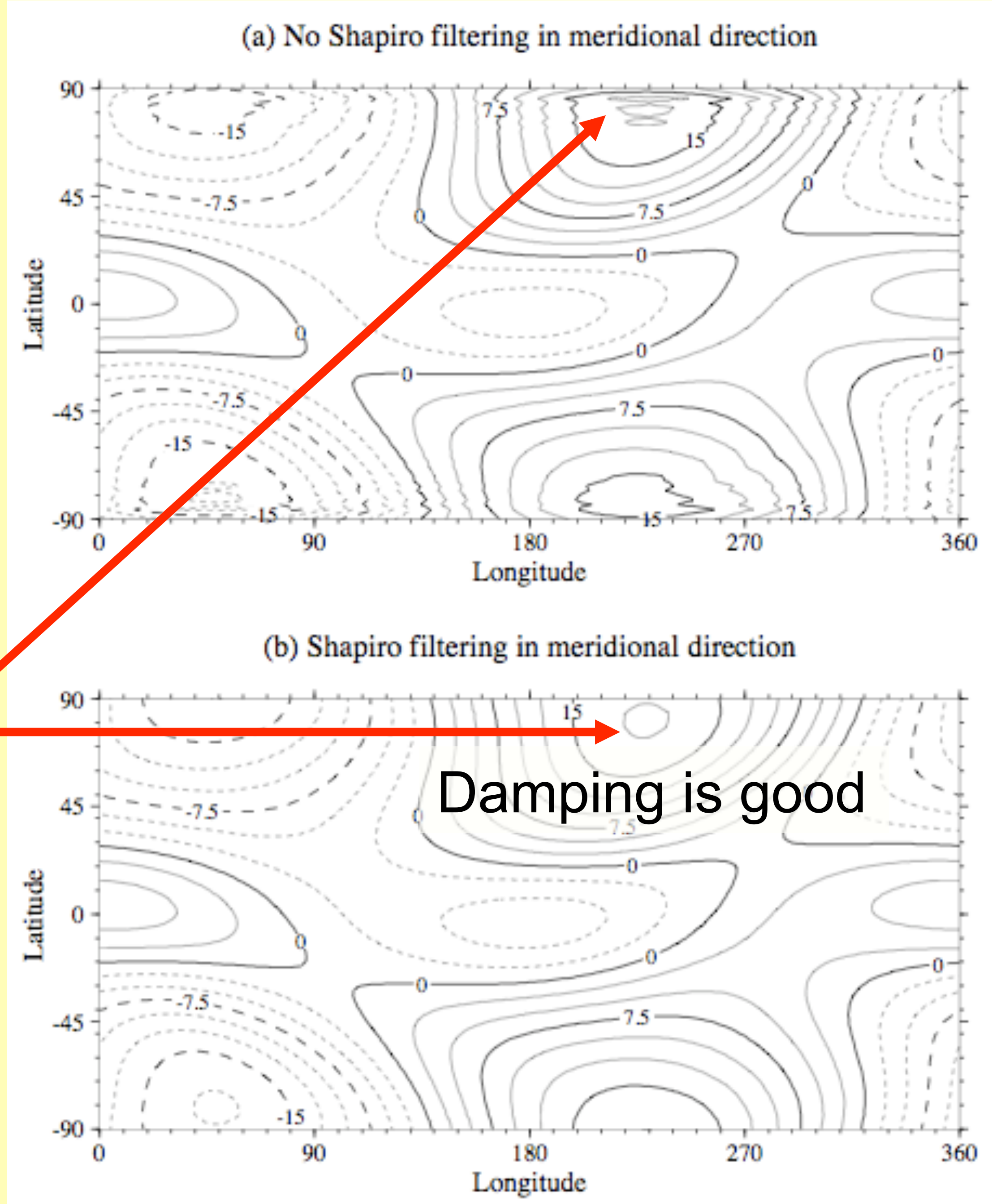
Digital filters: Response function



- Response function of different Shapiro filters after (a) 1 application and (b) 1000 applications. 2*n* indicates the order of the Shapiro filter. Higher orders need more data points.

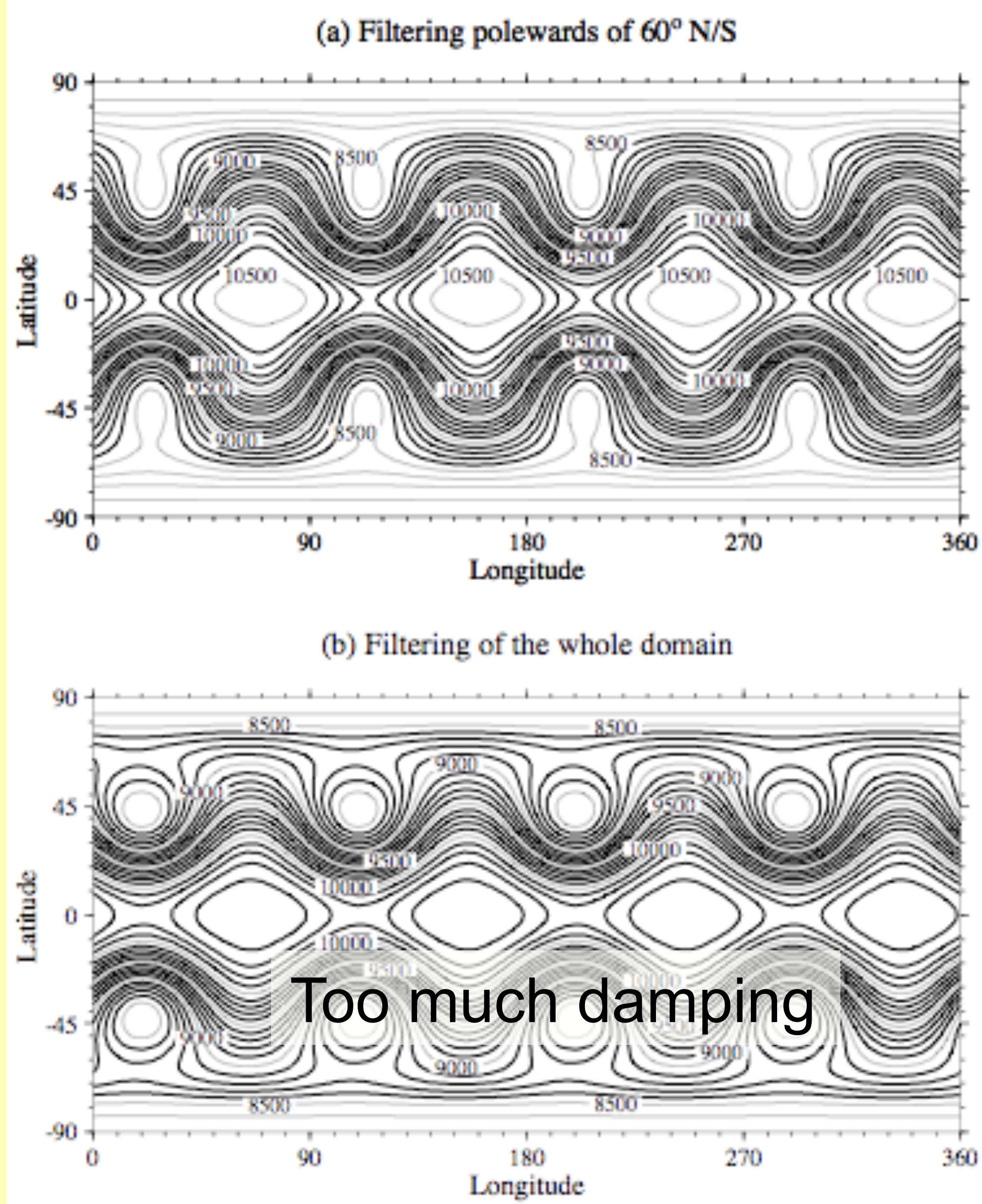
Digital filters

- Can provide a strong damping effect
- Use very selectively
- Example: SW simulation, digital filtering in y-direction applied near the pole points



Spatial Filters

- Can provide a strong damping effect
- Example: Rossby-Haurwitz wave in SW FV model, height at day 14
- (a) Fourier (90º-75º N/S) and digital Shapiro filtering (75º-60º N/S)
- (b) Digital Shapiro filter also applied between 60ºN - 60ºS, very diffusive, not suitable



Time filters

- Used in models with 3-time level schemes (e.g. Leapfrog)
- Most often used: Robert-Asselin filter (Asselin, 1972)
- Avoids that the even and odd time steps separate
- Basic idea: Second-order diffusion in time
- Example with time levels n-1, n, n+1:

$$\overline{\psi}^{n} = \psi^{n} + \alpha \left(\overline{\psi}^{n-1} - 2\psi^{n} + \psi^{n+1} \right)$$

- Filter strength is determined by the coefficient α
- Often used $\alpha \approx 0.05$

Sponge layers and Rayleigh friction

- Often desired: a wave-absorbing layer near the top of a GCM
- Prevents wave reflections of upward traveling waves that would normally leave the domain
- Some upper boundary conditions, e.g. that the model top is placed at a fixed height and w=0 m/s, are perfect reflectors, undesirable
- Practical approaches: Sponge layer near the model top, needs to be deep (at least 1-2 scale heights)
- Examples are enhanced 2nd-order diffusion or Rayleigh friction, e.g. of the types:

$$\frac{\partial u}{\partial t} = \ldots - \tau u \qquad \text{or} \qquad \frac{\partial u}{\partial t} = \ldots - \tau\left(u - \overline{u}(z)\right)$$

Attention: no physical justification

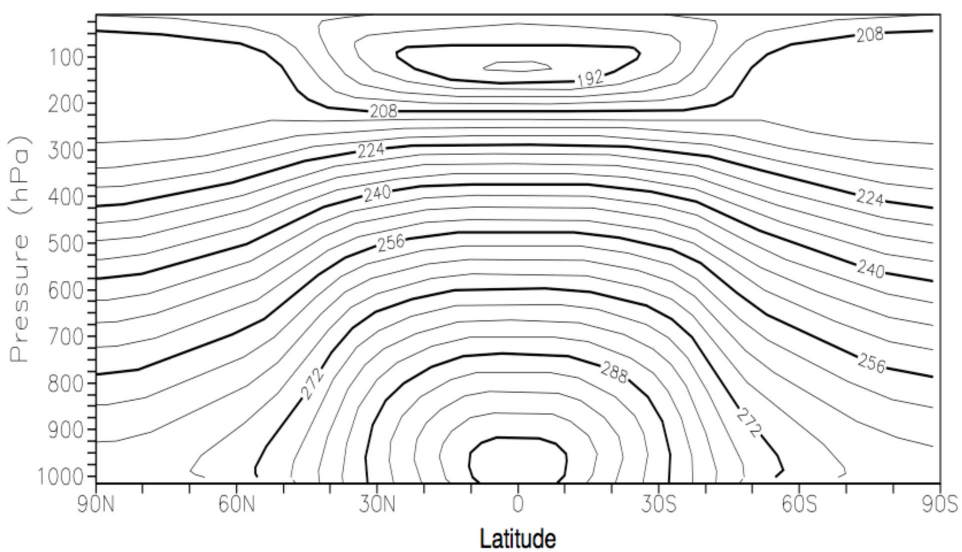
Conservation of Mass: Mass fixers

- Some dynamical cores are not mass-conserving by design
- But: Conservation of mass is needed in long-term climate simulations, less important in short weather prediction runs
- These models might apply an *a posteriori* mass fixer
- Basic idea behind the mass fixer: adjust the mean value of p_s after each time step, adjustment modifies all grid points at the surface
- This technique does not (!) alter the pressure gradients which are the driving force in the momentum equations
- Sounds okay? Let’s see (next slide):

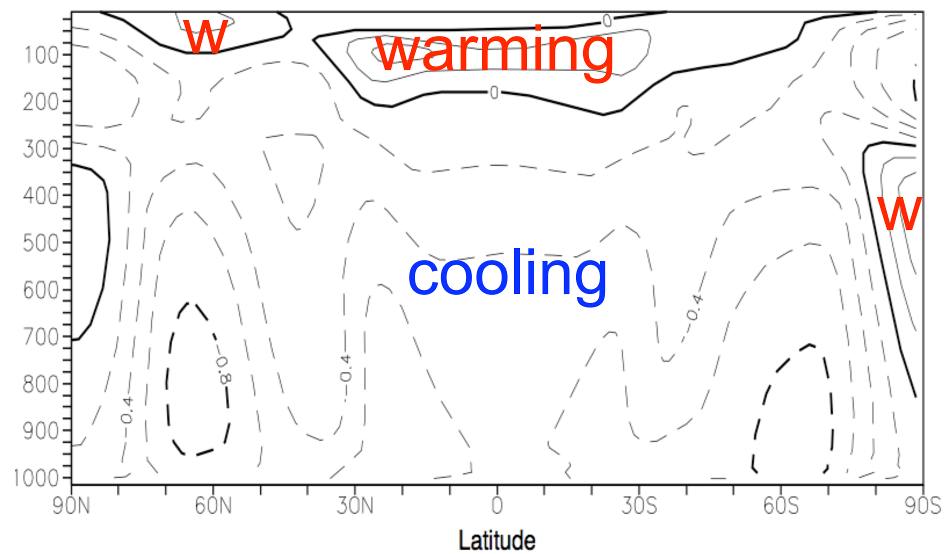
Conservation of Mass:
The potential impact of mass fixers

- Weather forecast model IFS run with the Held-Suarez test
- Compare the time-mean zonal-mean temperature of a run with and without mass fixer

Temperature
(without mass fixer)



Temperature difference
(with mass fixer - without)



Conservation of Total Energy

- There are many forms of the Total Energy (TE or E) Equation that depend on the choice of the fluid dynamics equations and the vertical coordinate (see Appendix F)
- An example for hydrostatic models with the height coordinate z in the vertical direction is (with A: horizontal integral)

$$\begin{aligned} E &= \int_A \int_{z_{top}}^{z_s} \left(\frac{\mathbf{v}^2}{2} + c_v T + g z \right) \rho \, dz \, dA \\ &\approx \int_A \left[\sum_{k=1}^{K_{max}} \left(\frac{u_k^2 + v_k^2}{2} + c_v T_k + g z_k \right) \rho_k \, \Delta z_k \right] dA. \end{aligned}$$

- In general: The TE equation is a global integral of the kinetic, thermal and potential energy in the model.
- The global integral is conserved in the continuous equations.

Conservation of Total Energy

- The question is whether TE is a conserved quantity in a dynamical core with numerical discretizations.
- Should we care?
  – in Weather Prediction Models
    - The answer is 'not necessarily'
  – in Climate Models
    - The answer is 'yes'
- When running for long times the violation of the total energy conservation leads to artificial drifts in the climate system (e.g. ocean heat fluxes)

Total Energy Fixer

- In nature:
  - conservation of total energy
  - energy lost by molecular diffusion provides heat
- In atmospheric models:
  - Energy is lost due to explicit or implicit (numerical) diffusion processes
  - Molecular diffusion is not represented on the model grid (spatial scale in models in way too big)
  - Numerical scheme might also lead to increase in total energy
- Therefore: some models provide an *a posteriori* energy fixer that restores the conservation of total energy by modifying the temperature

A posteriori Total Energy Fixer

- Goal: Total energy at each time step should be constant
- Compute the residual: $RES = \hat{E}^{+} - E^{-}$

- Compute the total energy before (-) and after (+) each time step

$$
\begin{aligned}
\hat{E}^{+} &= \int_A \left\{ \left[\sum_{k=1}^{K} \left(\frac{(\hat{\mathbf{v}}_k^{+})^2}{2} + c_p \hat{T}_k^{+} \right) (p_0 \, \Delta A_k + \hat{p}_s^{+} \, \Delta B_k) \right] + \Phi_s \hat{p}_s^{+} \right\} dA \\
E^{-} &= \int_A \left\{ \left[\sum_{k=1}^{K} \left(\frac{(\mathbf{v}_k^{-})^2}{2} + c_p T_k^{-} \right) (p_0 \, \Delta A_k + p_s^{-} \, \Delta B_k) \right] + \Phi_s p_s^{-} \right\} dA
\end{aligned}
$$

A posteriori Total Energy Fixer

- **Idea:** Correct the temperature field to achieve the conservation of total energy (mimics heating by molecular diffusion)
- Option: **Fixer 1**, correction proportional to the magnitude of the local change in T at that time step

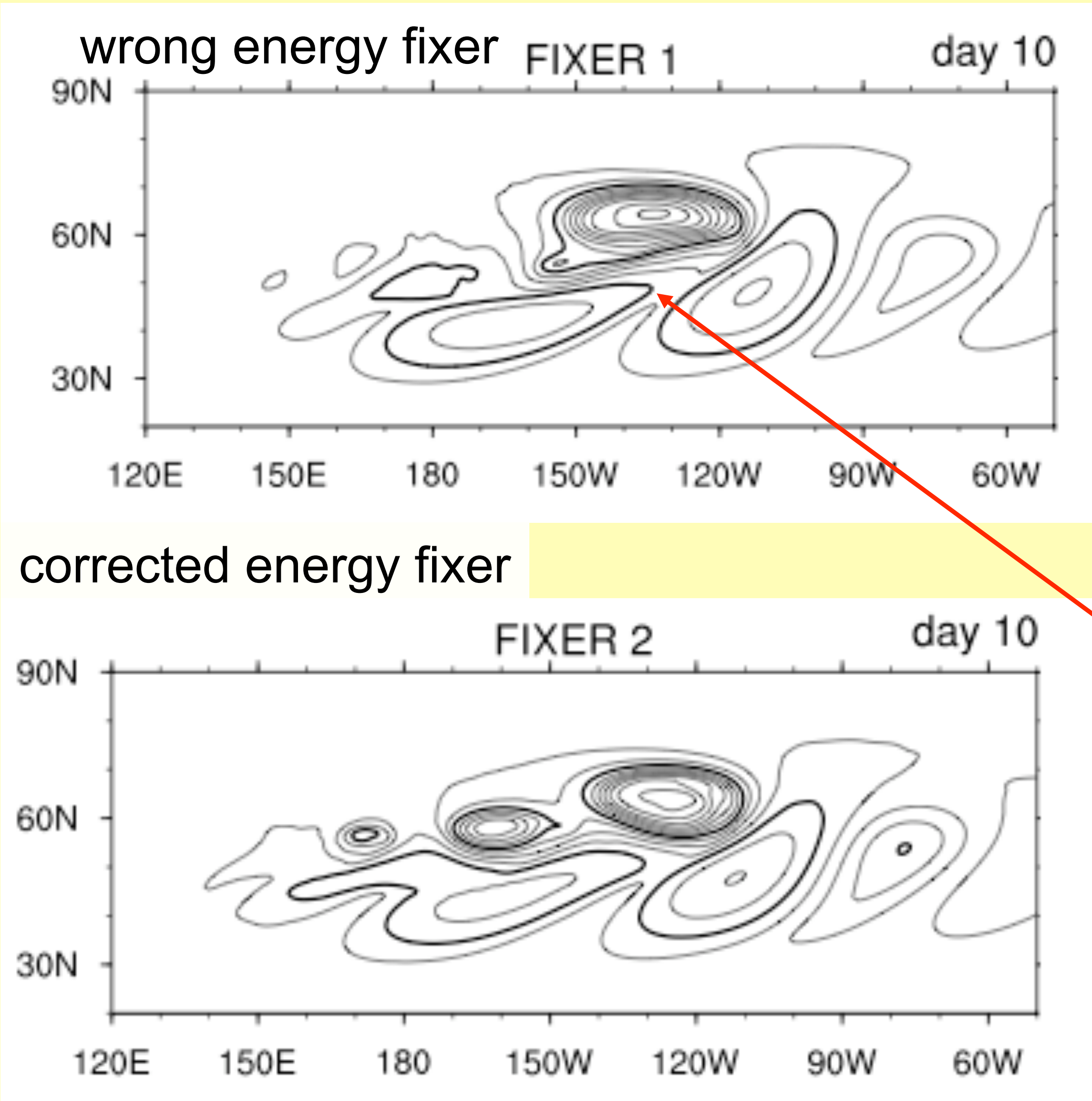
$$T^{+}(\lambda,\varphi,\eta) = \hat{T}^{+}(\lambda,\varphi,\eta) + \beta_1 |\hat{T}^{+}(\lambda,\varphi,\eta) - T^{-}(\lambda,\varphi,\eta)|$$

- Option: **Fixer 2**, correction is constant everywhere

$$T^{+}(\lambda,\varphi,\eta) = \hat{T}^{+}(\lambda,\varphi,\eta) + \beta_2$$

- Fixer 1 looks physical, but leads to wrong results

Energy Fixer: Surprising Consequences

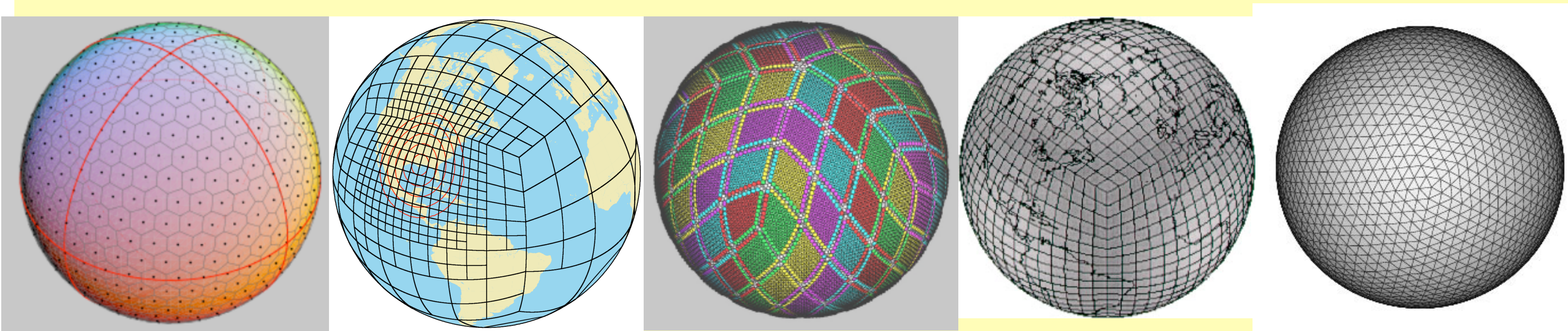


- Baroclinic wave, p_s at day 10
- CAM SLD with a 'wrong' and 'corrected' choice of an energy fixer
- Wrong choice leads to wrong circulation pattern

Williamson, Olson & Jablonowski, (MWR, 2009)

Trends in dynamical core design

- Alternative computational meshes (maybe with nested grids or AMR with good scaling characteristics on parallel computers

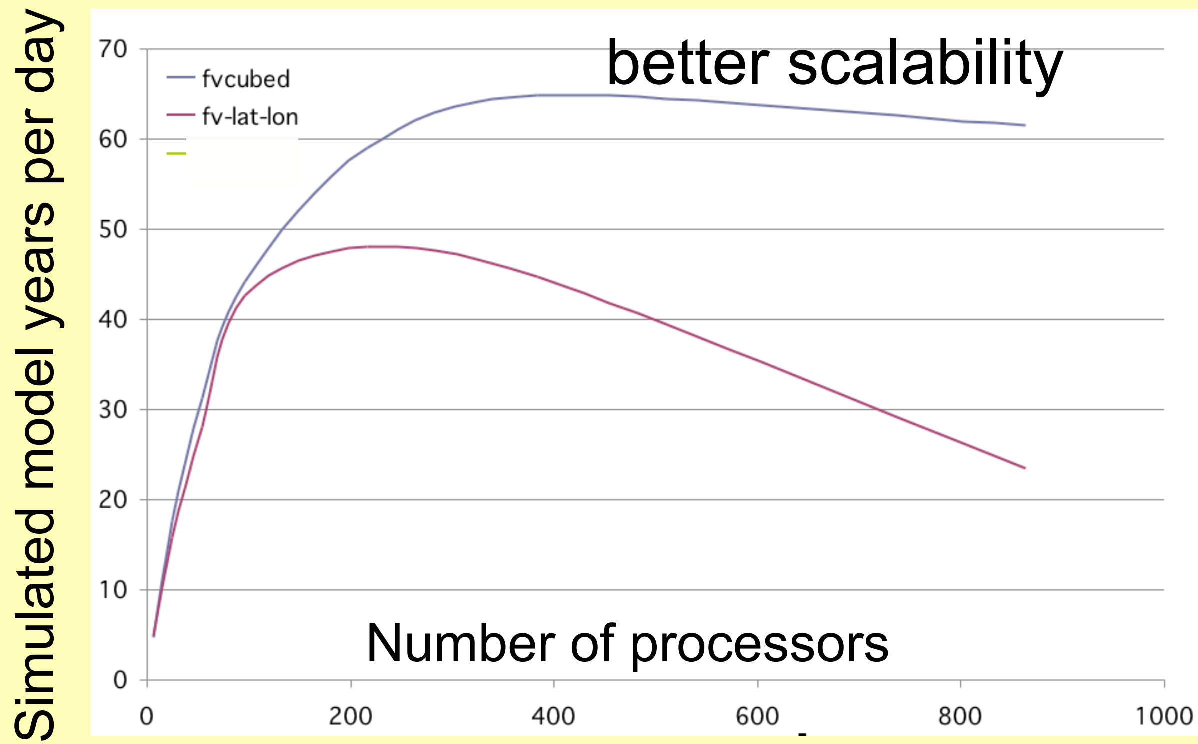


Colors represent
processor allocation (CSU model)

- Built-in conservation laws (especially mass)
- Slightly higher order (maybe 3rd or 4th), no need to go to really high order (e.g. 8th) since the dynamics-physics coupling is 1st order
- Higher physical consistencies: better tracer advection

Computational Performance

- The computational and parallel performance is a decisive factor in the design
- Motivates the push towards new grids
- Example: parallel scalability of the FV lat-lon and FV cubed-sphere dynamical core (NCAR, NASA)



Analysis on the DoE Cray machine *Jaguar*

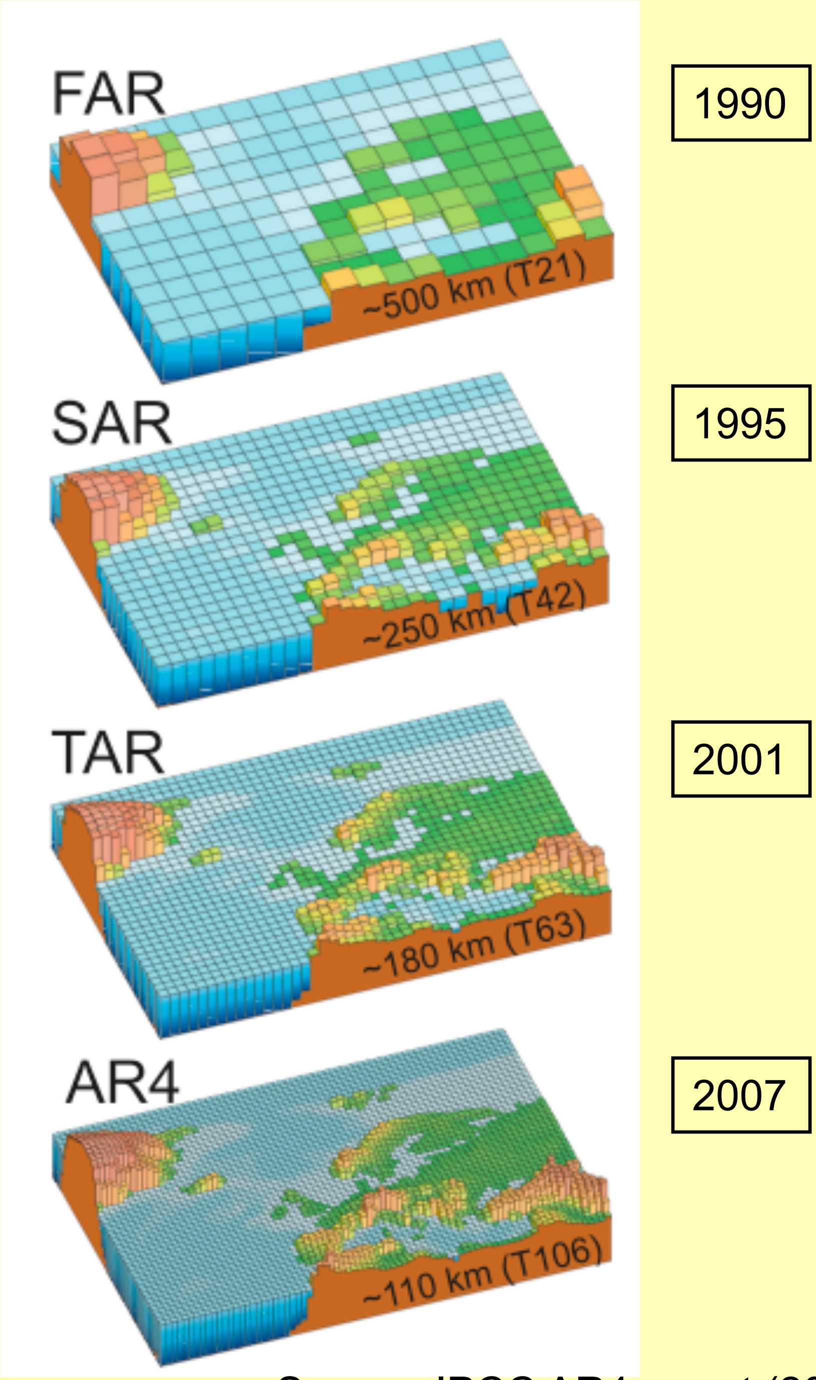
by Art Mirin (summer '09)

Trends: increasing horizontal resolutions of GCMs

Soon climate model
are likely to run with
grid spacings of about 50 km

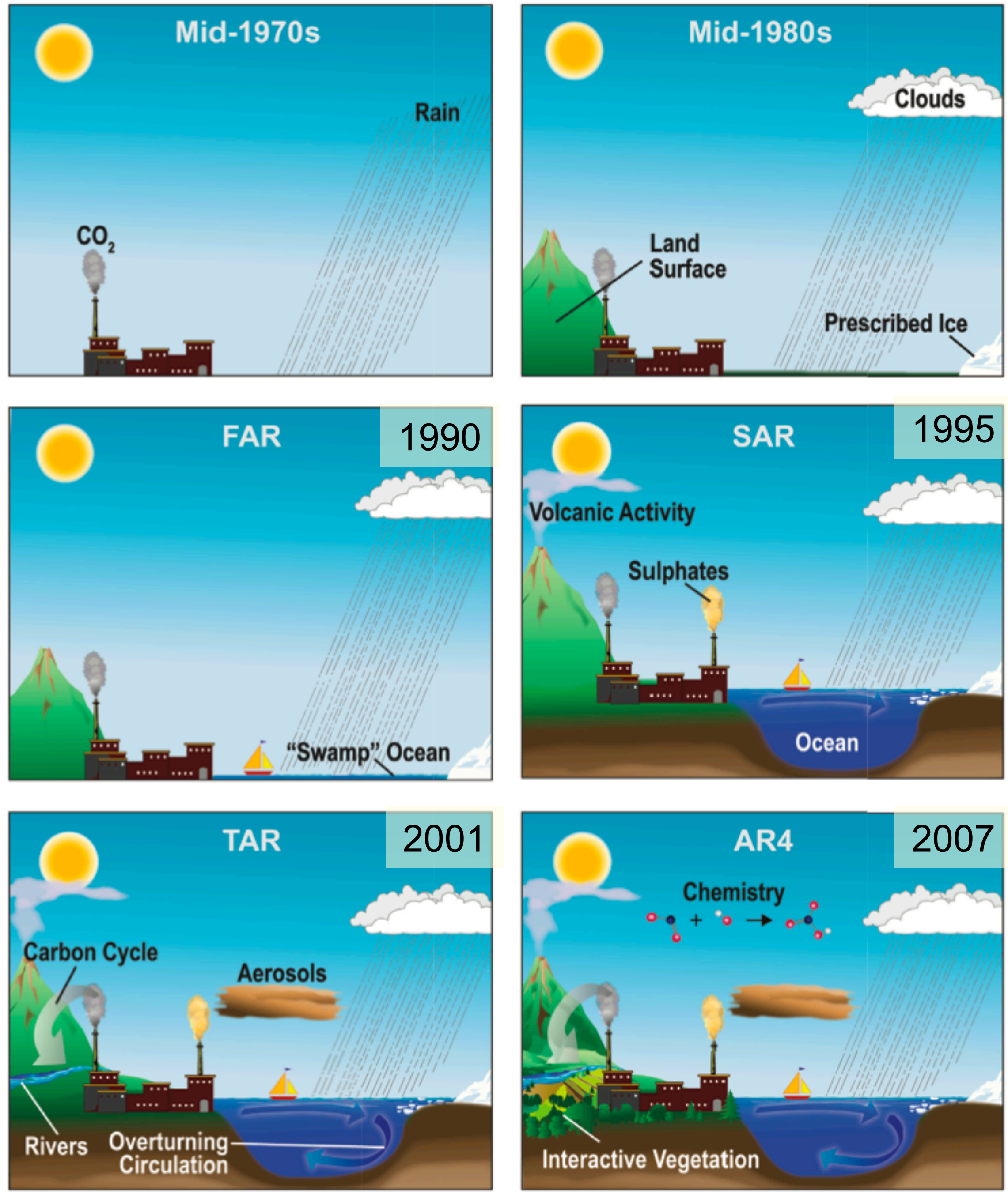
Global weather models
are likely to run with
grid spacings of about 10 km

But be careful: resolution
is not identical to grid spacing!
The scale of resolved features
is about 7 x Δx.



Source: IPCC AR4 report (2007)

Trends: Increased complexity in climate models



Source: IPCC AR4 report, WG 1 (2007)

Summary (I)

- Physical arguments need to drive your choices in the design of a dynamical core
- Set your priorities: mass conservation, consistent and monotonic tracer advection, etc.
- Designing a dynamical core is a **co-design** task: balance the scientific, numerical and computational constraints
- These constraints pull the developer in different directions, e.g.
  - a highly accurate dynamical core is maybe too expensive to run (so nobody will use it)
  - a cheap dynamical core is maybe too diffusive to be useful (so nobody will use it)

Summary (II)

- Diffusion and filters help maintain the numerical stability
- Some **diffusion** (either explicit or implicit) is **always needed** to prevent an accumulation of energy at the smallest scale (due to truncated energy cascade)
- But: Use the techniques selectively and know their consequences.
- Test and intercompare as much as possible
- Word of caution: It is very easy to compute nice-looking smooth, highly diffusive, but very inaccurate solutions to the equations of motion.
- But remember: **No sweat, no fun!** Tackling the challenges is rewarding. Just be persistent.