Understanding cultural norms via evolutionary game theory
A new approach

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Common comment during this workshop:
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Common comment during this workshop:
“How do we make the social sciences into a science?”
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We are at IPAM, so how to we incorporate mathematics?
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Much of mathematics reflects symbiotic relationship between
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Toward being able to get sharper equations and make predictions — models are evidence based
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Problem: no such symbiotic relationship currently exists for mathematics and the social/behavioral sciences
Must be created — qualitative!
But, what is needed?
Simple example: Ultimatum Game
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Two players; neither knows the identity of the other but both know everything else
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1. You are given $1000, with the following condition:
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If X=1, the other player gets at least one dollar, so X=1
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   How do we develop a tentative theory to explain?
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Evolutionary game theory.
Standard approach
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Two types, $x_1$ always wants $2/3$ of what is offered, $x_2$ wants $1/3$
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\begin{align*}
\frac{dx_1}{dt} &= \frac{1}{3} x_1 (1 - x_1 - 2x_1 x_2) \\
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\[x' = f(x)\]
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So, this approach is posing the *unknown* behavior $f(x)$ to discover the *unknown* behavior

Serious part of goal toward creating a science is to learn how to discover an appropriate $f(x)$, i.e., the unknown behavior
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Data vs. Theory
What can be done?
What can be done?

\[ x' = f(x) \text{ (i.e., accepting “change”) } \]
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\[ f \text{ is continuous, but not known} \]
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So, if local information points inward at ends, then the simplest model has a stable equilibrium.
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\[ \text{Location of stable point? Comes from field and data.} \]

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If evidence proves simplest model not appropriate?

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Try next level of a model

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It crosses the x-axis three times.

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Gangs!!
$x' = f(x)$

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- Local information:
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E.g., let A be Apple and B be Microsoft.
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Pocket of co-existence, stability.
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Location is based on data, evidence.
If evidence shows not applicable, try next level
E.g., let A be Apple and B be Microsoft
Pocket of co-existence, stability.
Predictions are consistent with models—without difficulties
Back to the Ultimatum game
Back to the Ultimatum game

$x' = f(x)$
Back to the Ultimatum game

\[ x' = f(x) \]

f is continuous, but not known
Back to the Ultimatum game

\[ x' = f(x) \]

f is continuous, but not known
Three types: 2/3, 1/3 plus 1/2
Back to the Ultimatum game

\[ x' = f(x) \]

f is continuous, but not known

Three types: 2/3, 1/3 plus 1/2

What will happen?
Back to the Ultimatum game

\[ x' = f(x) \]
\[ f \text{ is continuous, but not known} \]
Three types: 2/3, 1/3 plus 1/2
What will happen?

\[
\begin{align*}
\frac{dx_1}{dt} &= x_1 \left[ \frac{1}{3} (1 - x_1 - 2x_1x_2) - \frac{1}{2} x_3 (1 - x_2) \right] \\
\frac{dx_2}{dt} &= x_2 \left[ \frac{1}{3} x_1 (1 - 2x_2) - \frac{1}{2} x_3 (1 - x_2) \right] \\
\frac{dx_3}{dt} &= x_3 \left[ \frac{1}{3} (-x_1 - 2x_1x_2) + \frac{1}{2} (1 - x_3)(1 - x_2) \right]
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f is continuous, but not known

Three types: 2/3, 1/3 plus 1/2

What will happen?

Same simple graph approach does not work
Back to the Ultimatum game

$x' = f(x)$

$f$ is continuous, but not known

Three types: $2/3, 1/3$ plus $1/2$

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What will happen?

Same simple graph approach does not work

Local indices add up to four
Back to the Ultimatum game

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Local indices add up to four

Sum of local indices equals global index
Back to the Ultimatum game

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The type:

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Local indices add up to four

Sum of local indices equals global index

Global index equal 3

One more equilibrium of index -1
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One more equilibrium of index -1
What if the middle equilibrium differed?
What if the middle equilibrium differed?
What if the middle equilibrium differed?
What if the middle equilibrium differed?

Key is wedding between local information, basic assumption of change ($x' = f(x)$) and data, data, data leading to predictions.
What if the middle equilibrium differed?

Key is wedding between local information, basic assumption of change (\(x' = f(x)\)) and data, data, data leading to predictions

One approach, but provides new insights and conclusions
What if the middle equilibrium differed?

Key is wedding between local information, basic assumption of change ($x'=f(x)$) and data, data, data leading to predictions.

One approach, but provides new insights and conclusions offers way to narrow down on choice of $f(x)$. 