Understanding cultural norms via evolutionary game theory
A new approach

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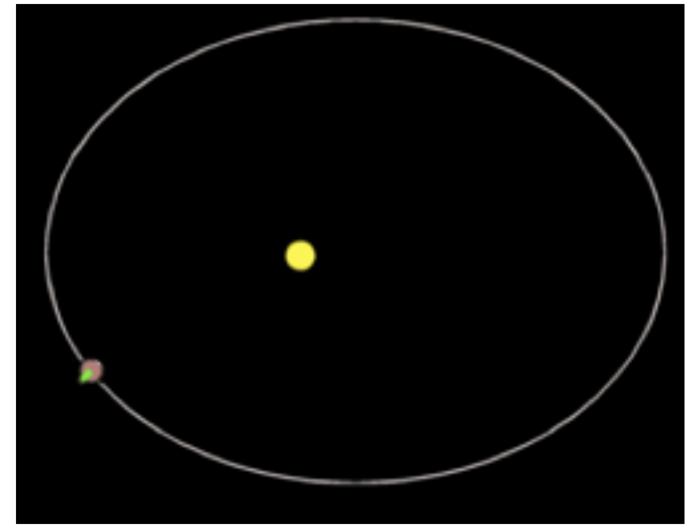

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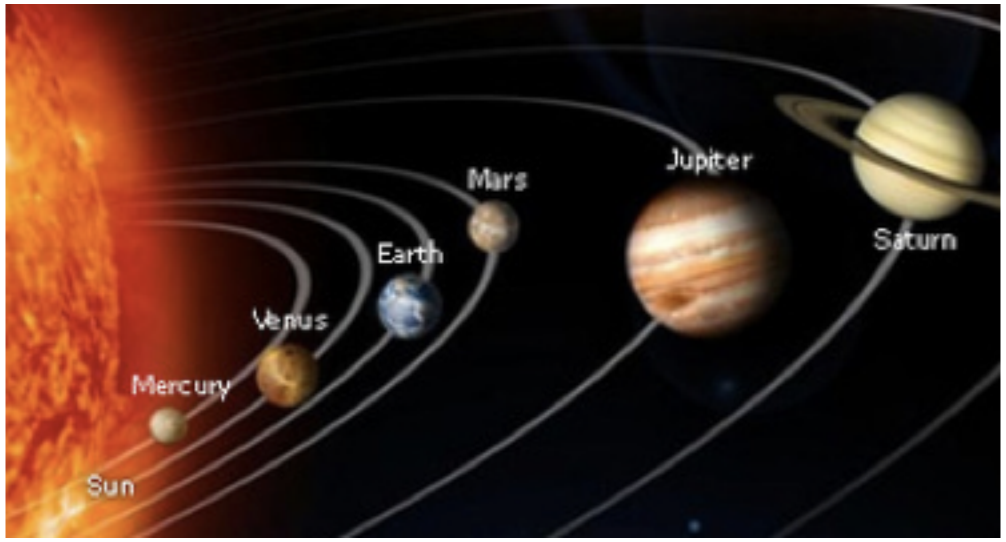

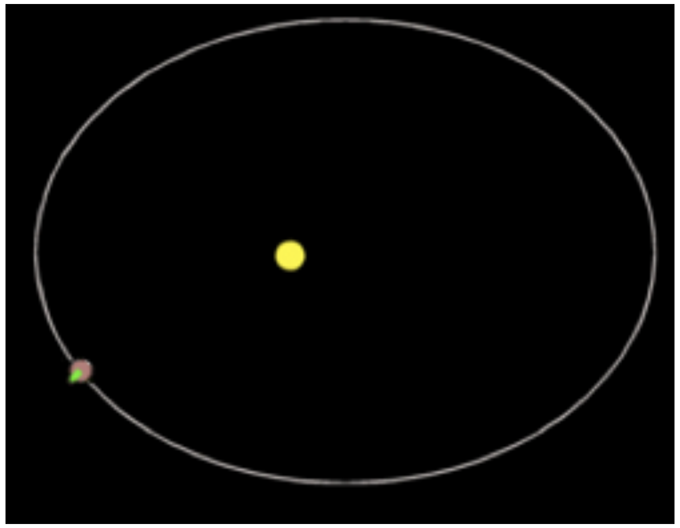

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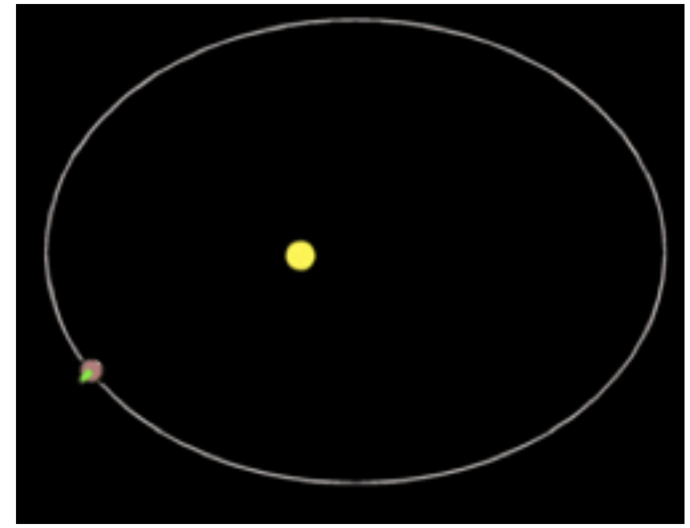

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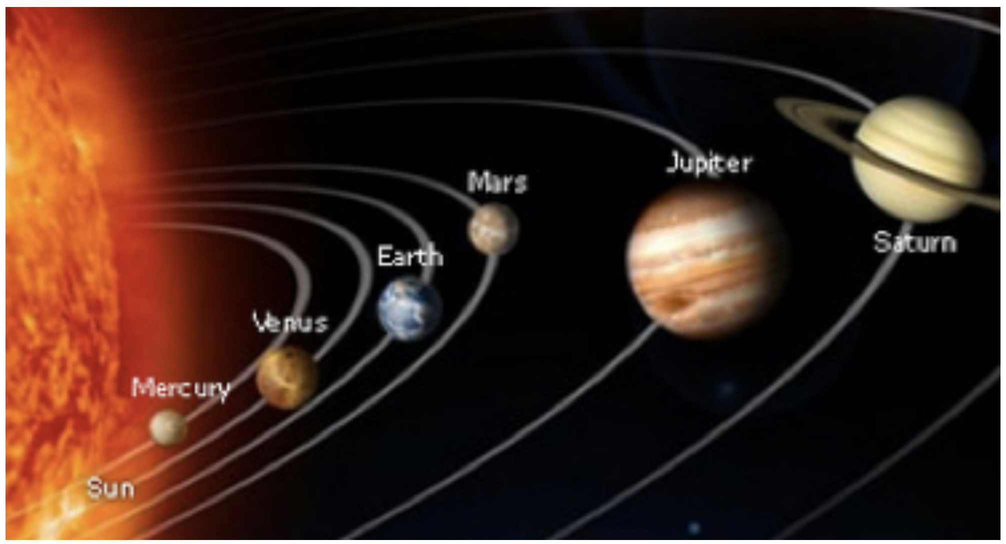

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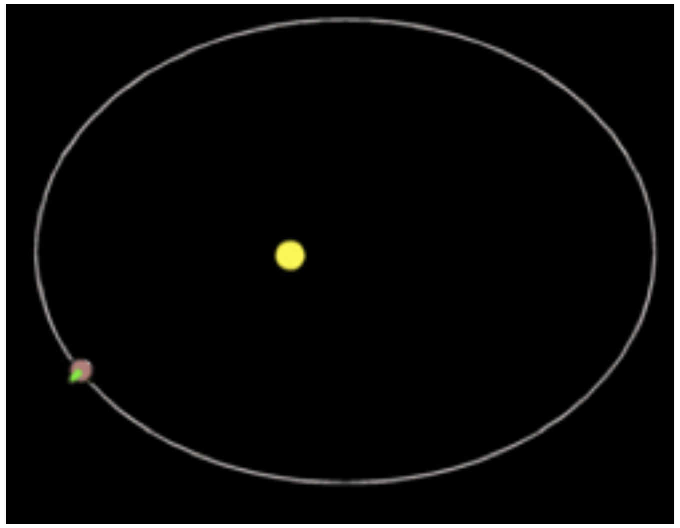

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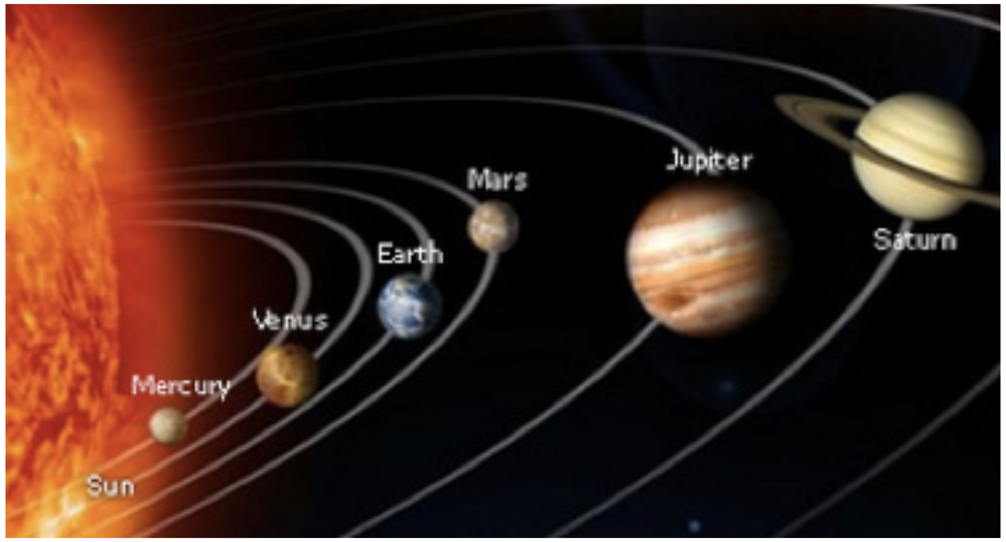

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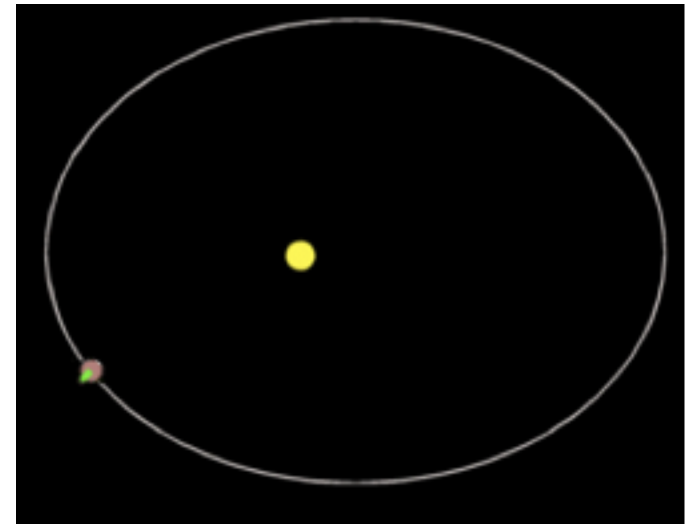

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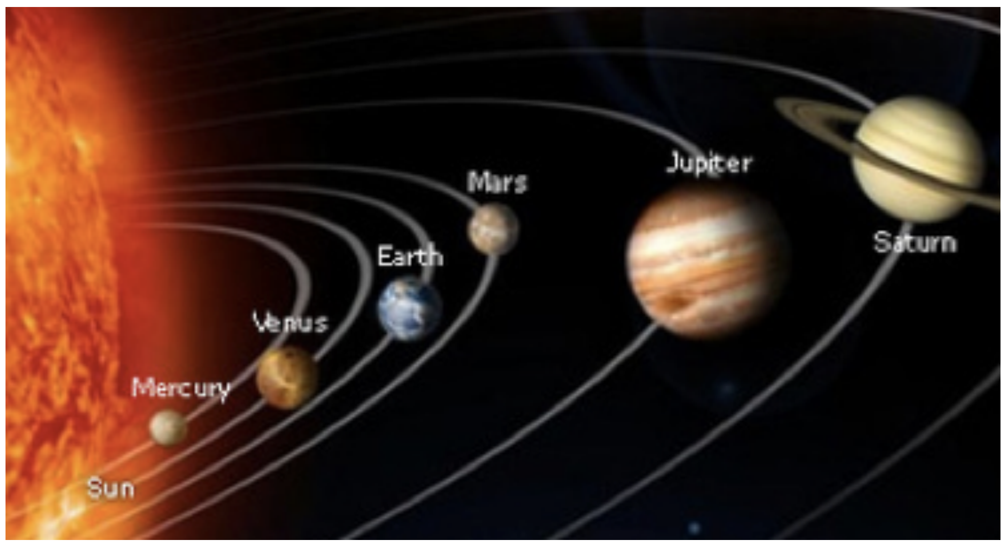

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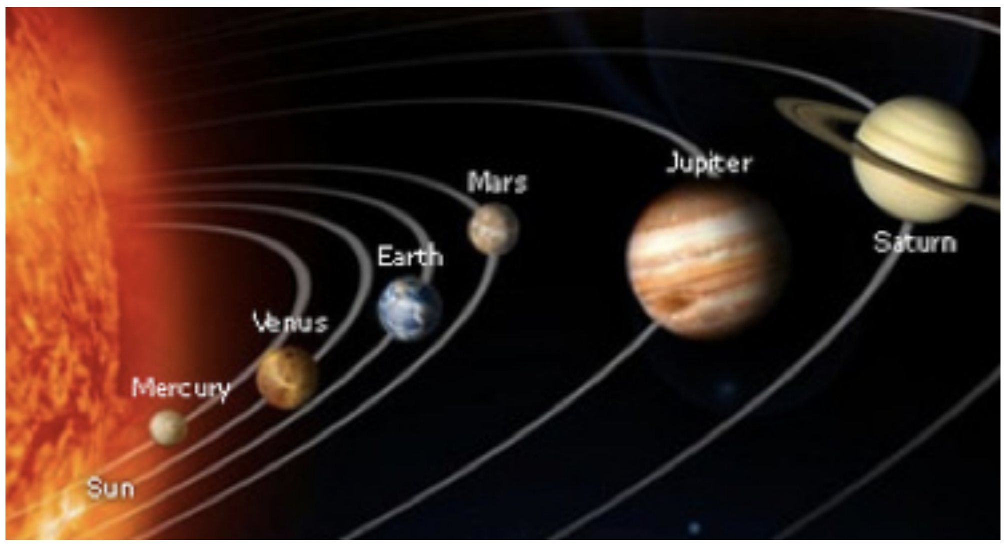

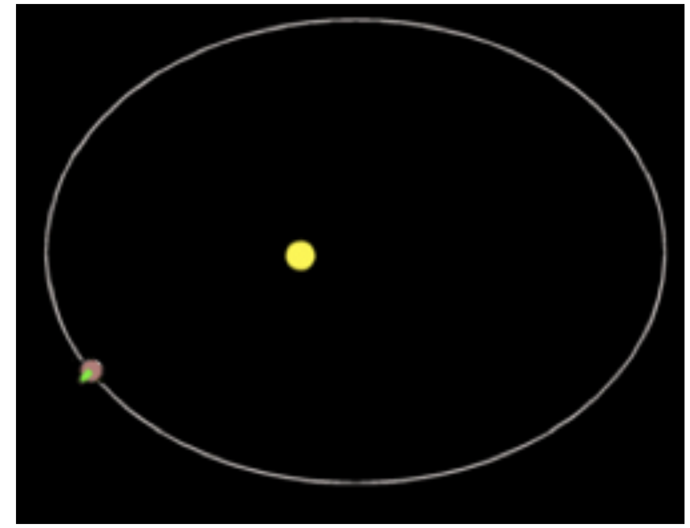

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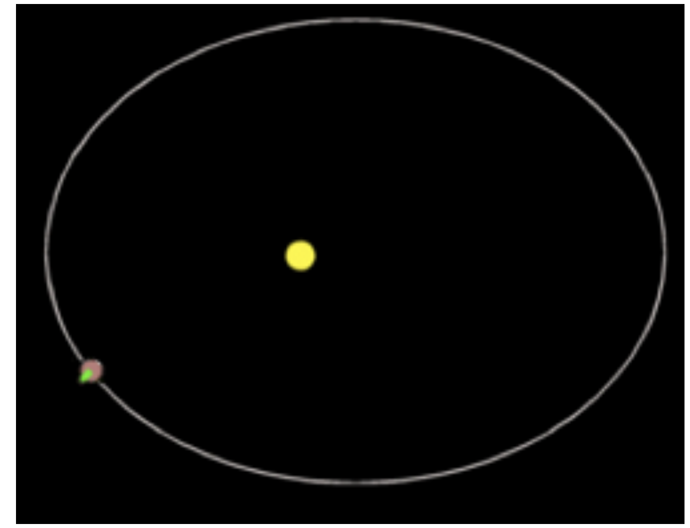

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But, what is needed?

Simple example: Ultimatum Game

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Evolutionary game theory.

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Data vs. Theory

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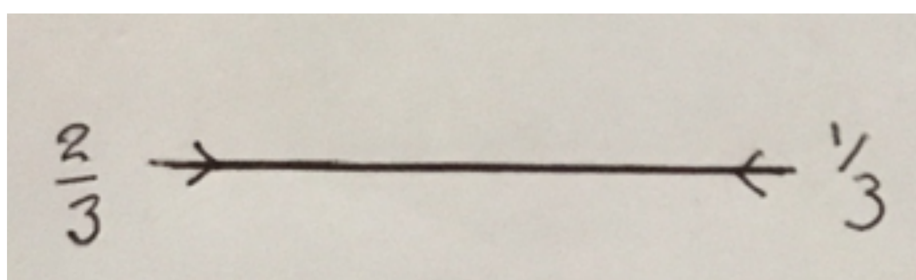

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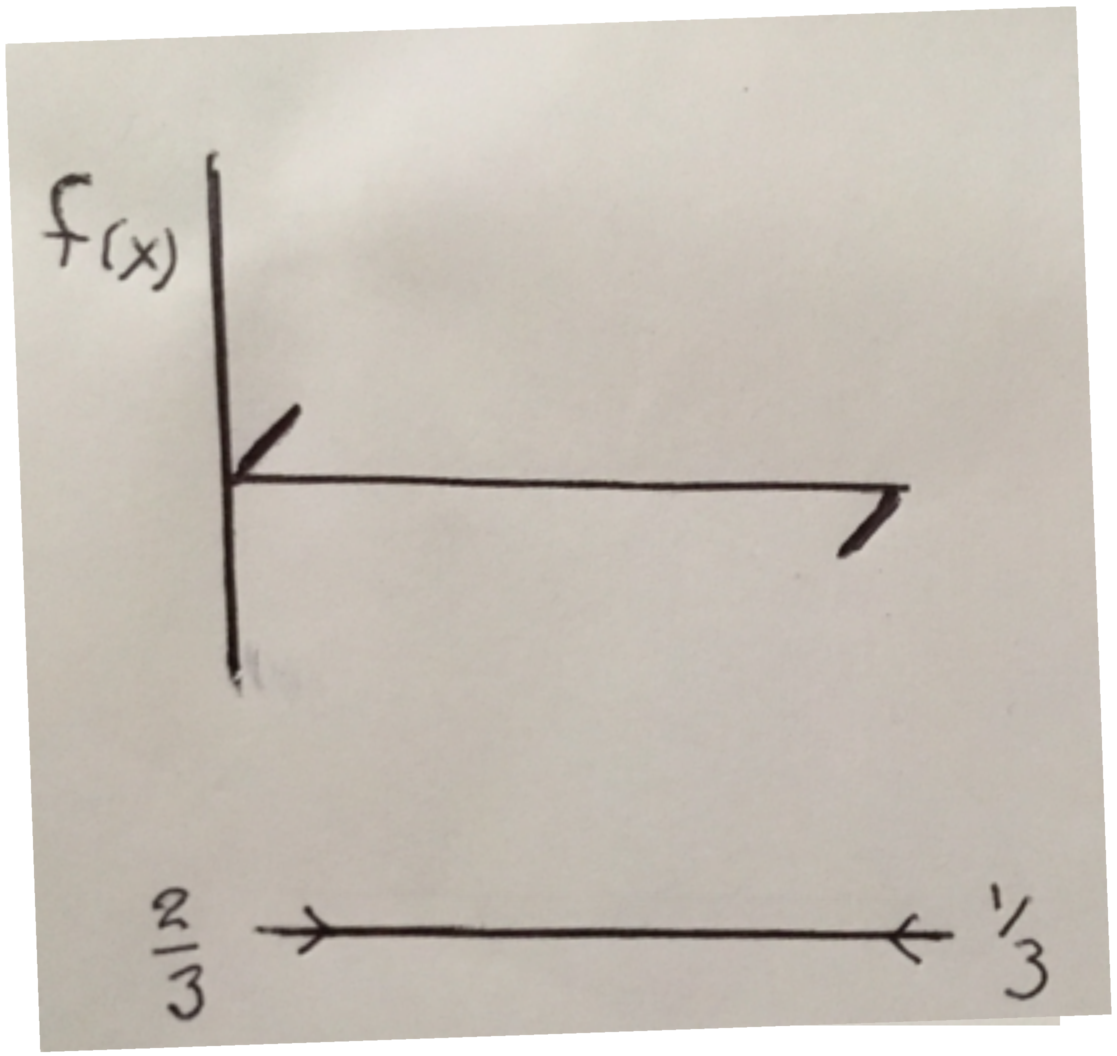

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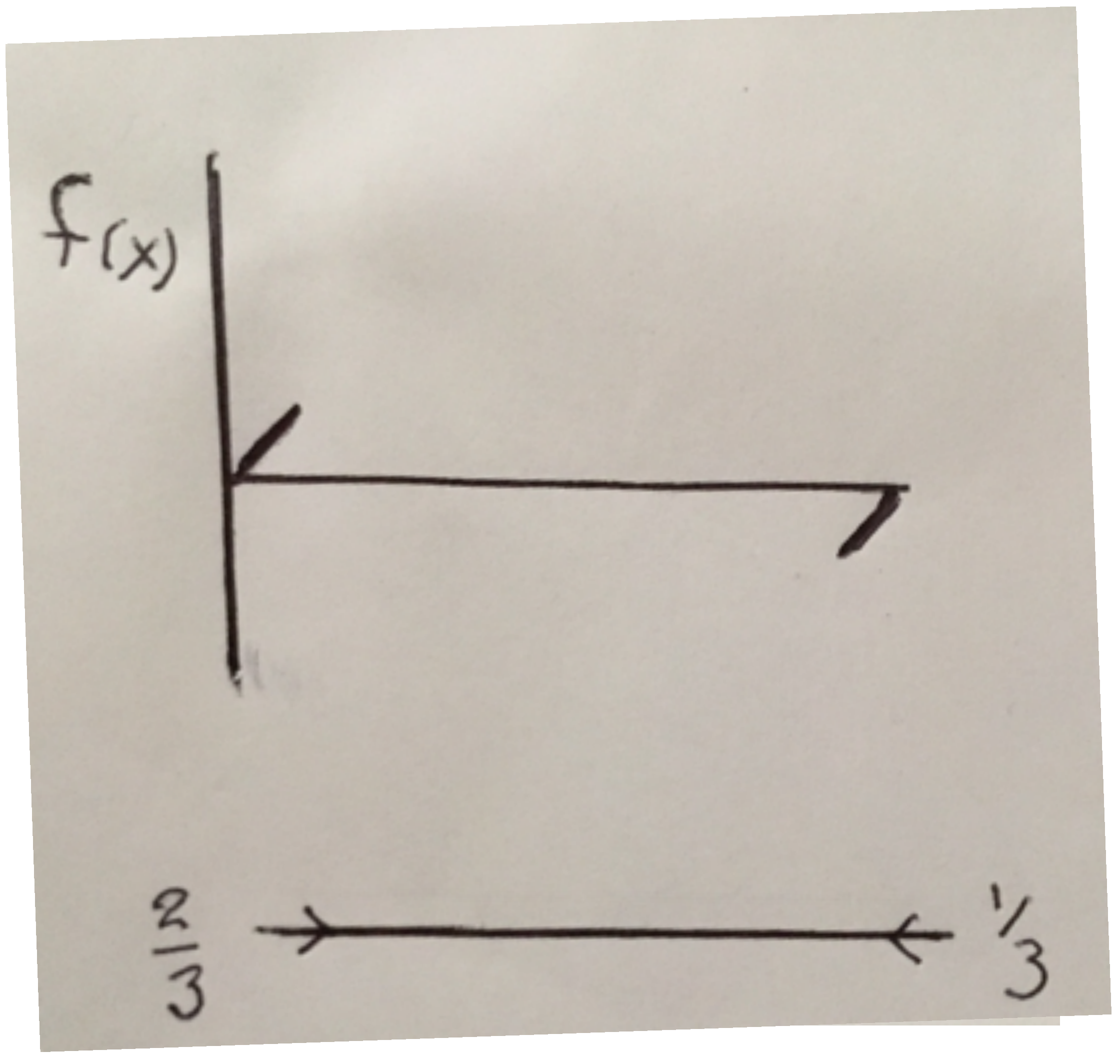

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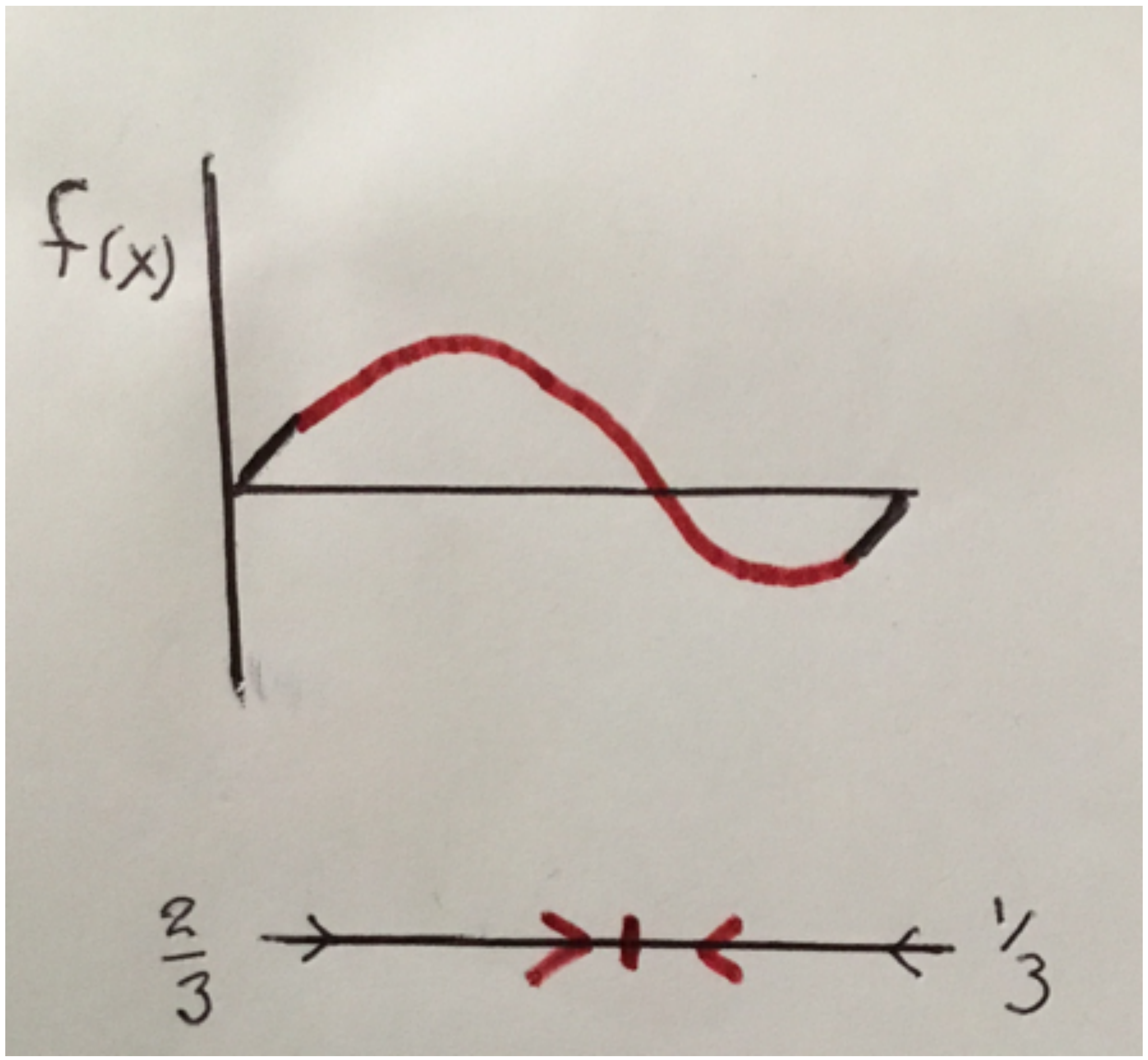

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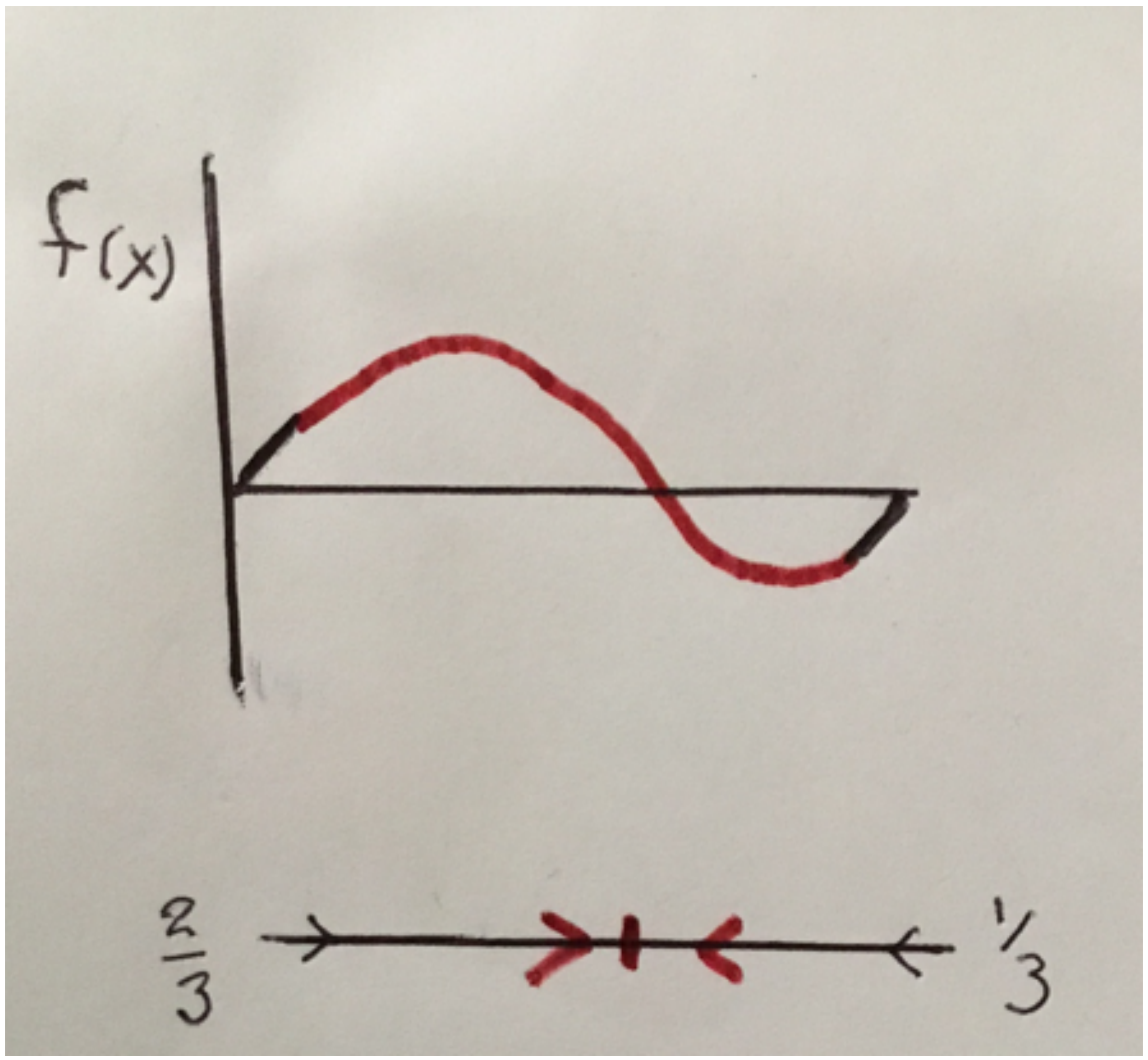

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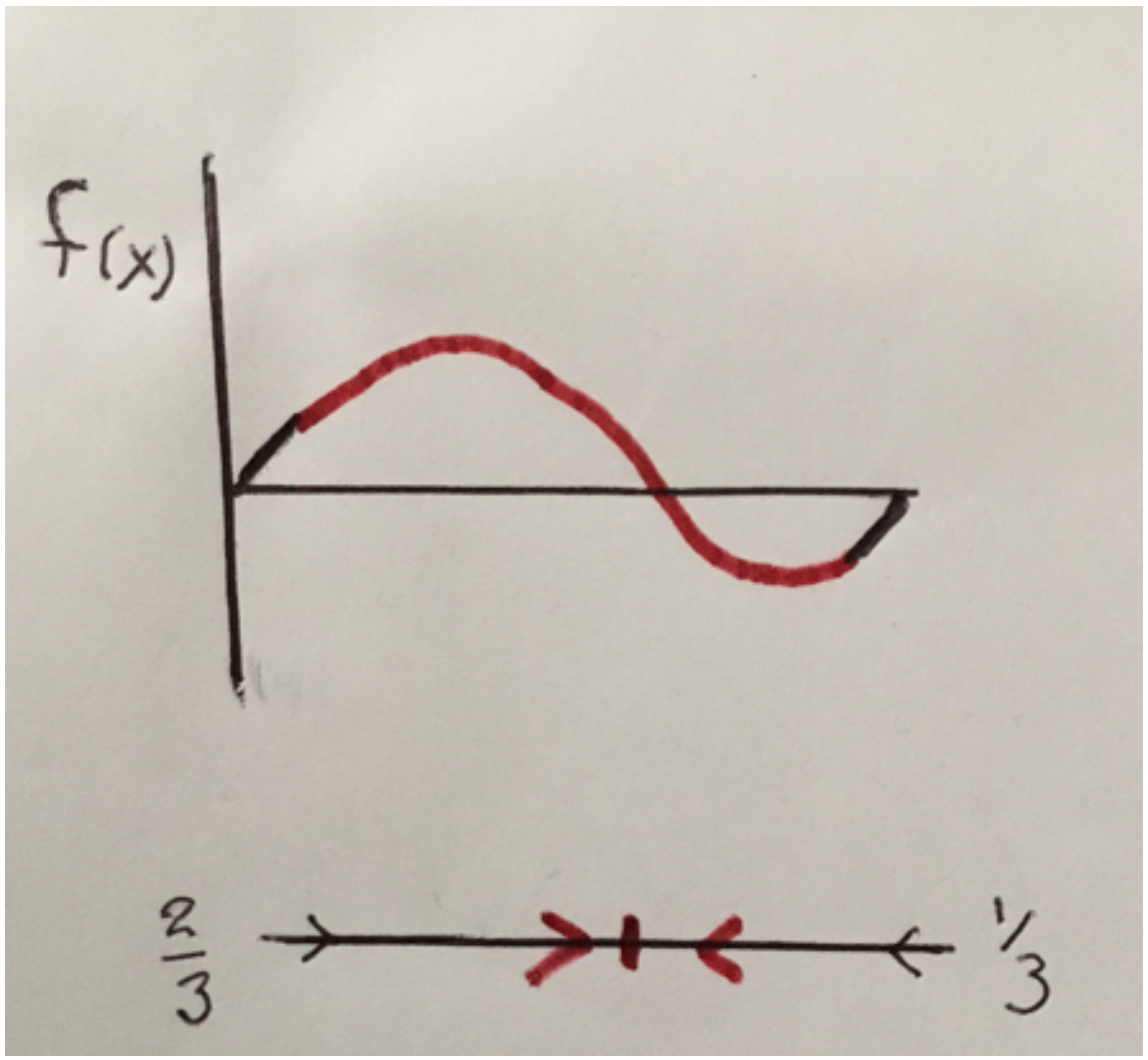

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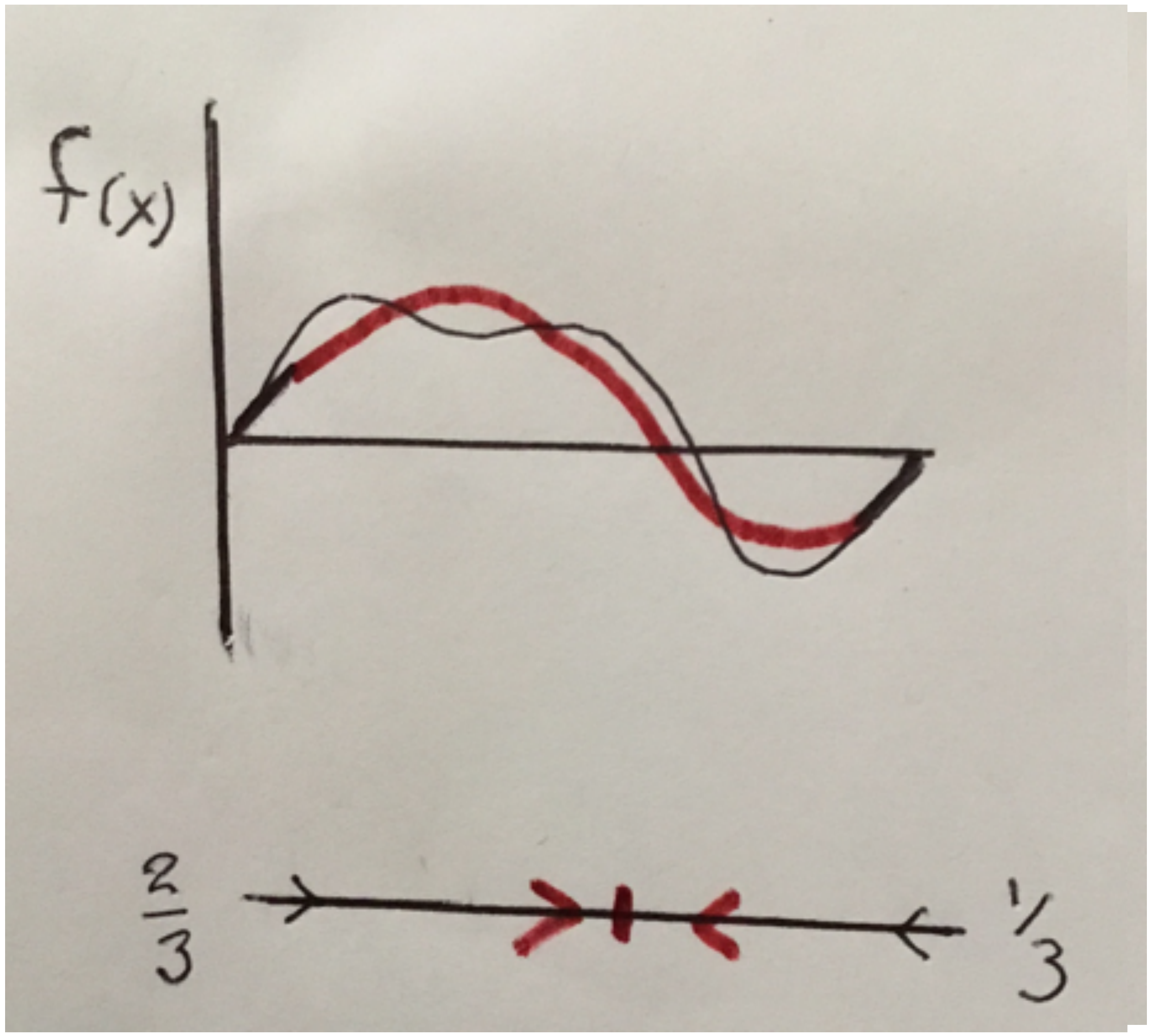

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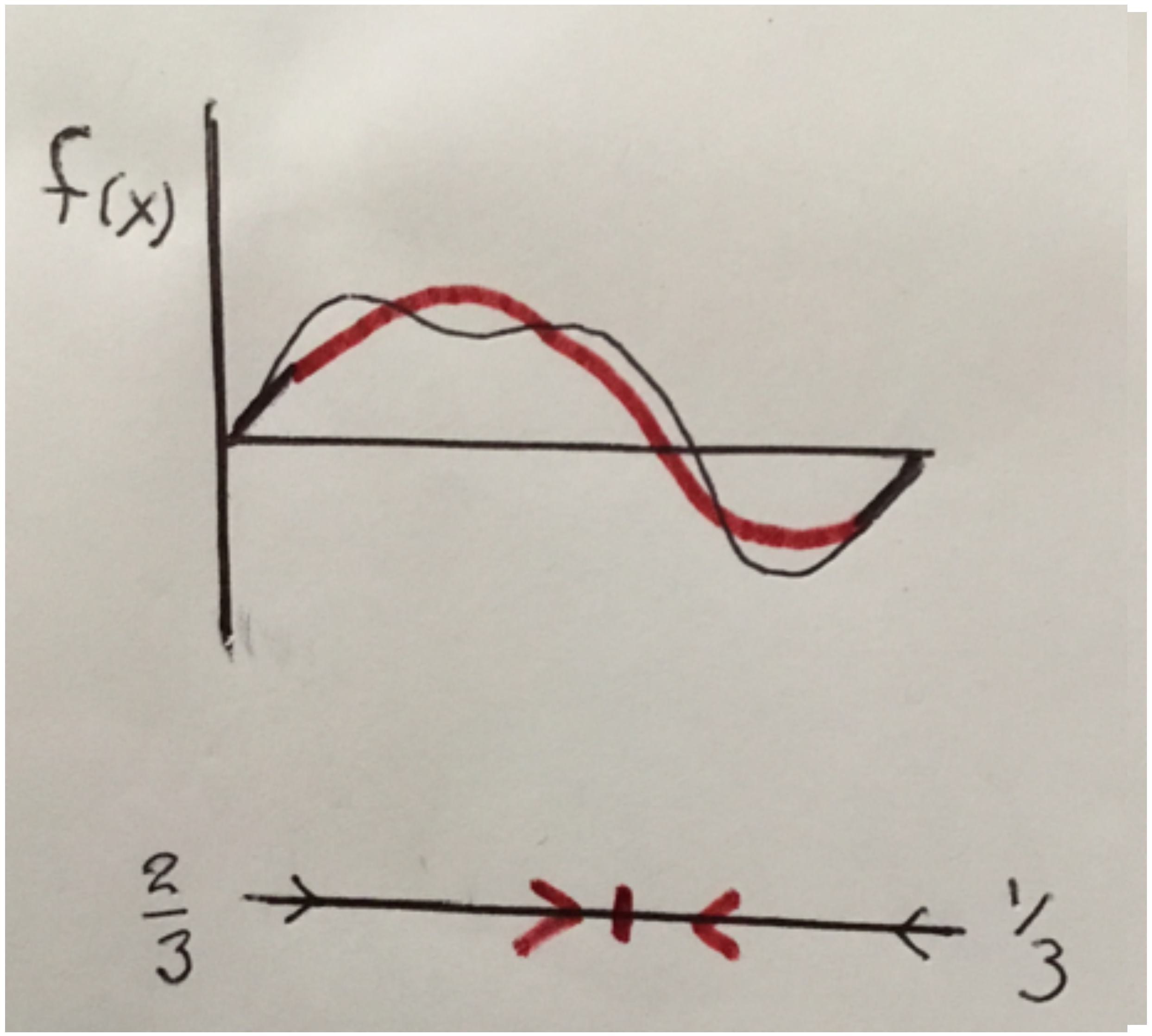

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Model is robust

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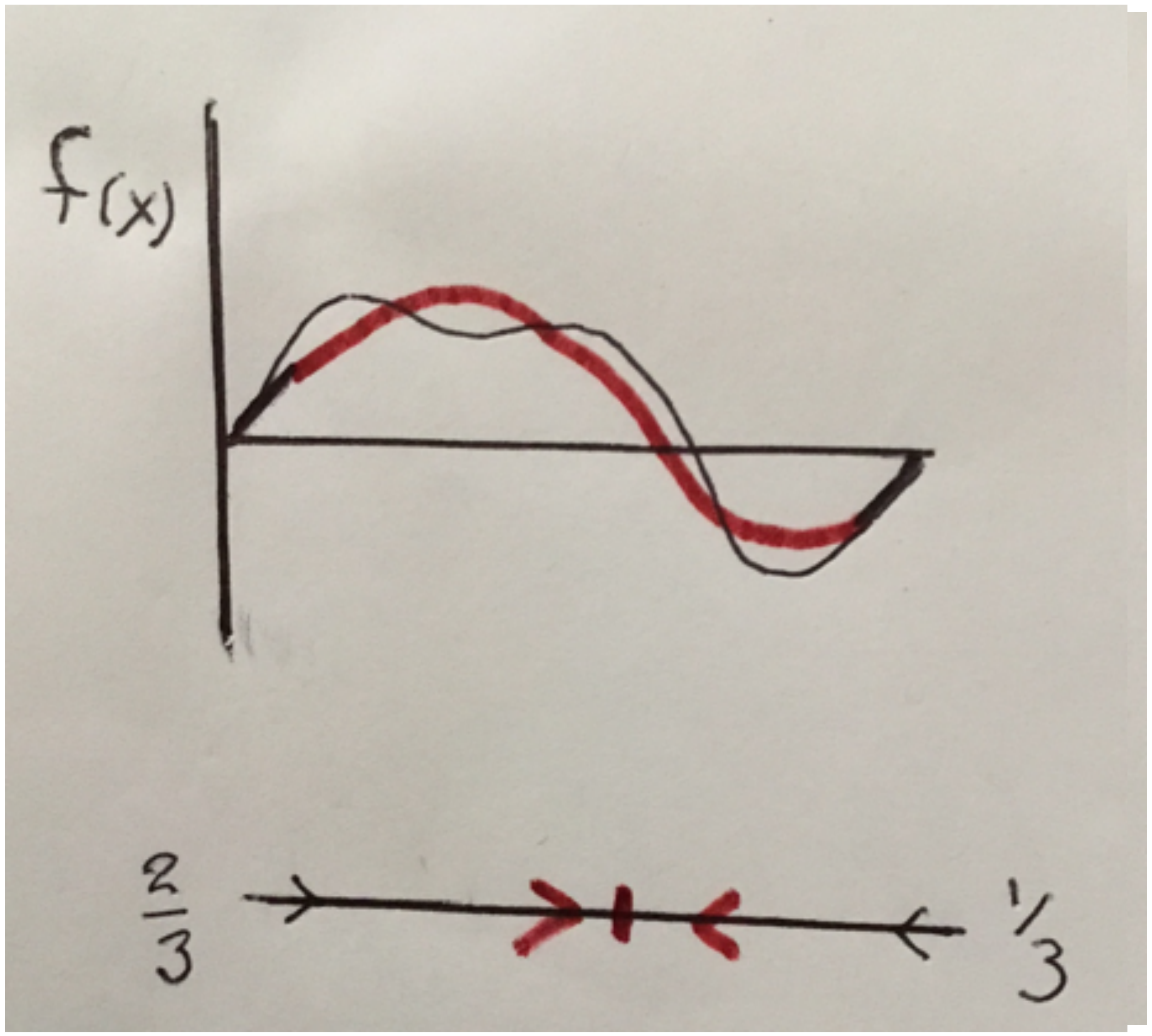
What can be done?

x' = f(x) (i.e., accepting "change")

f is continuous, but not known



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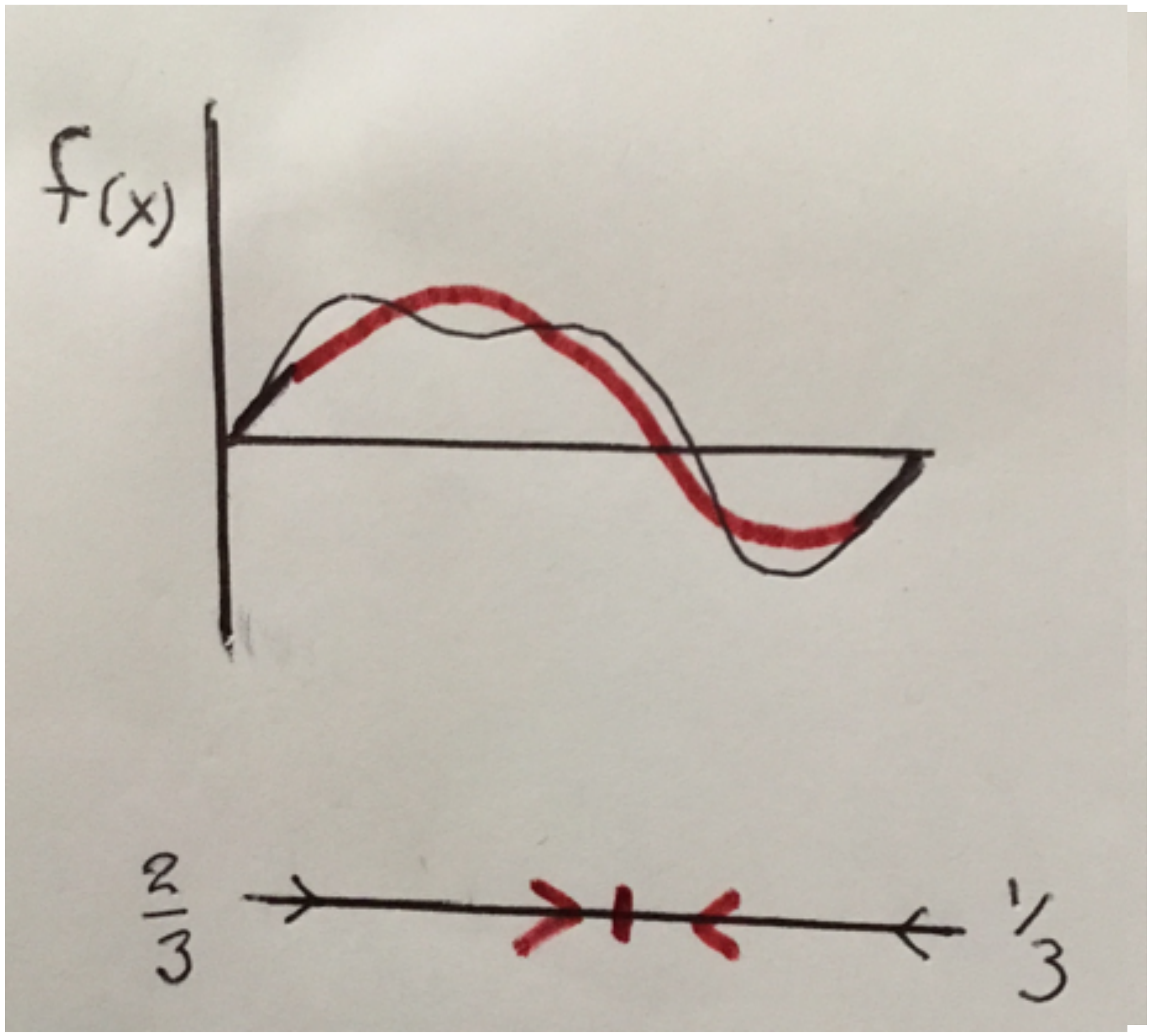

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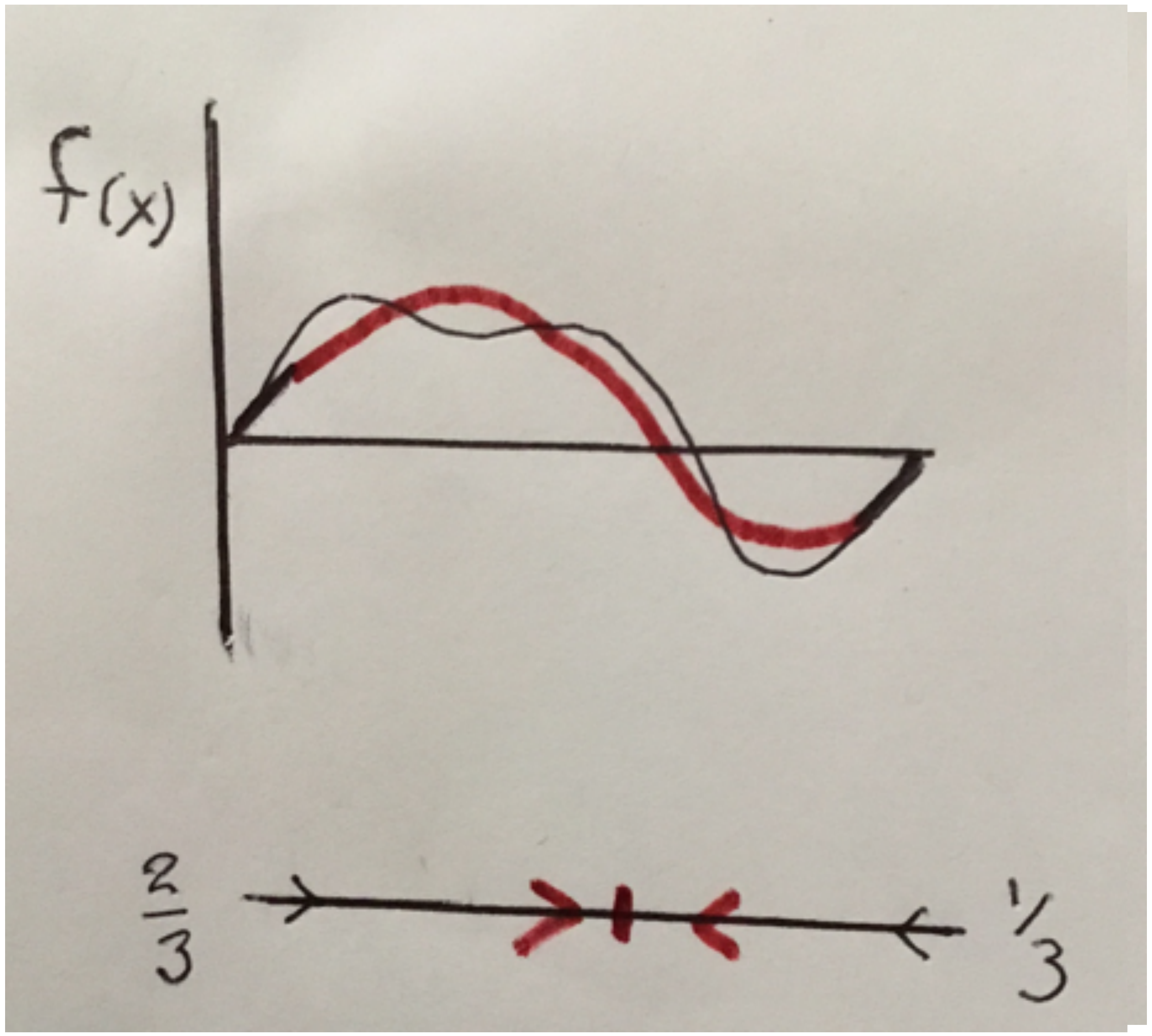

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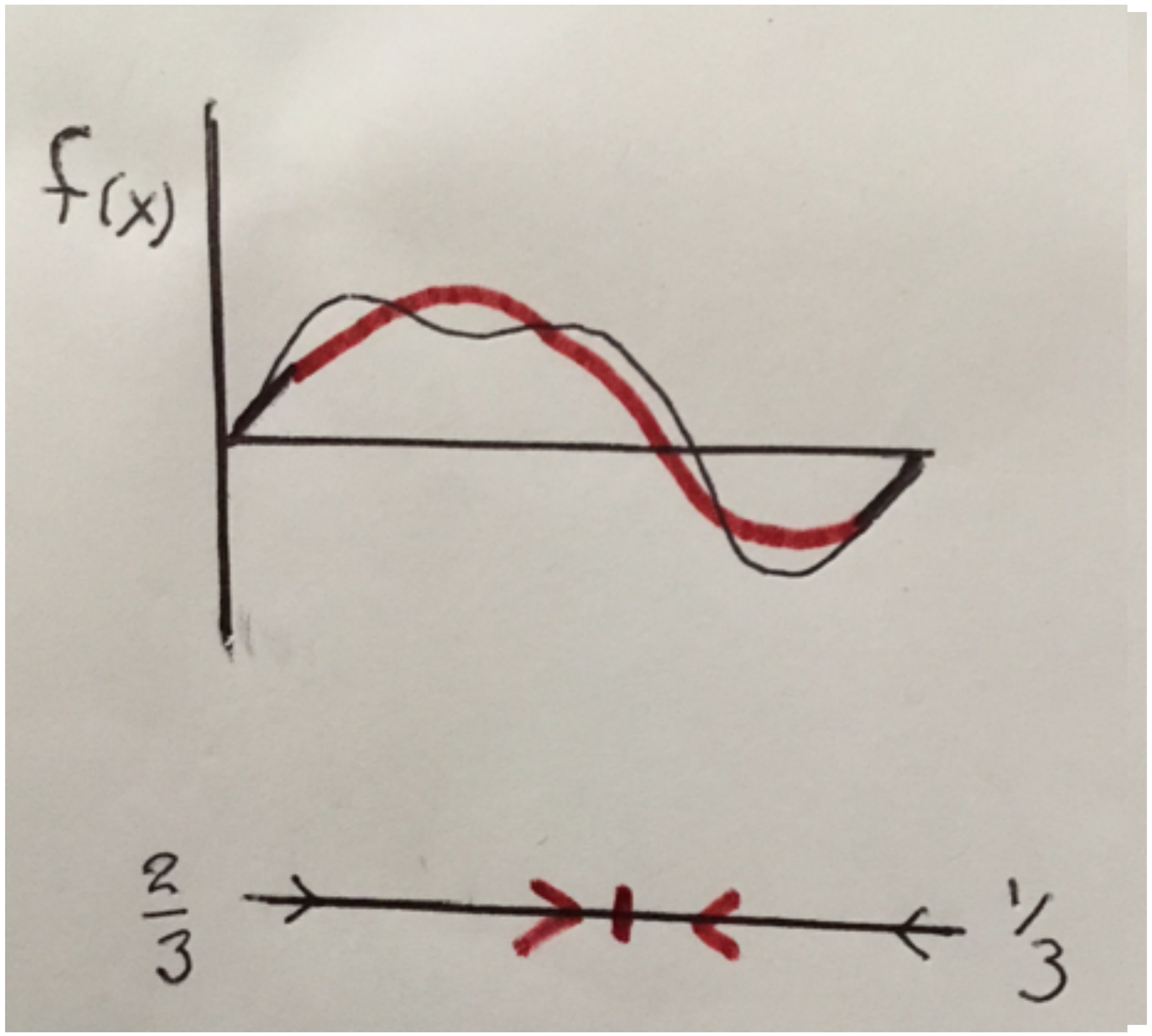

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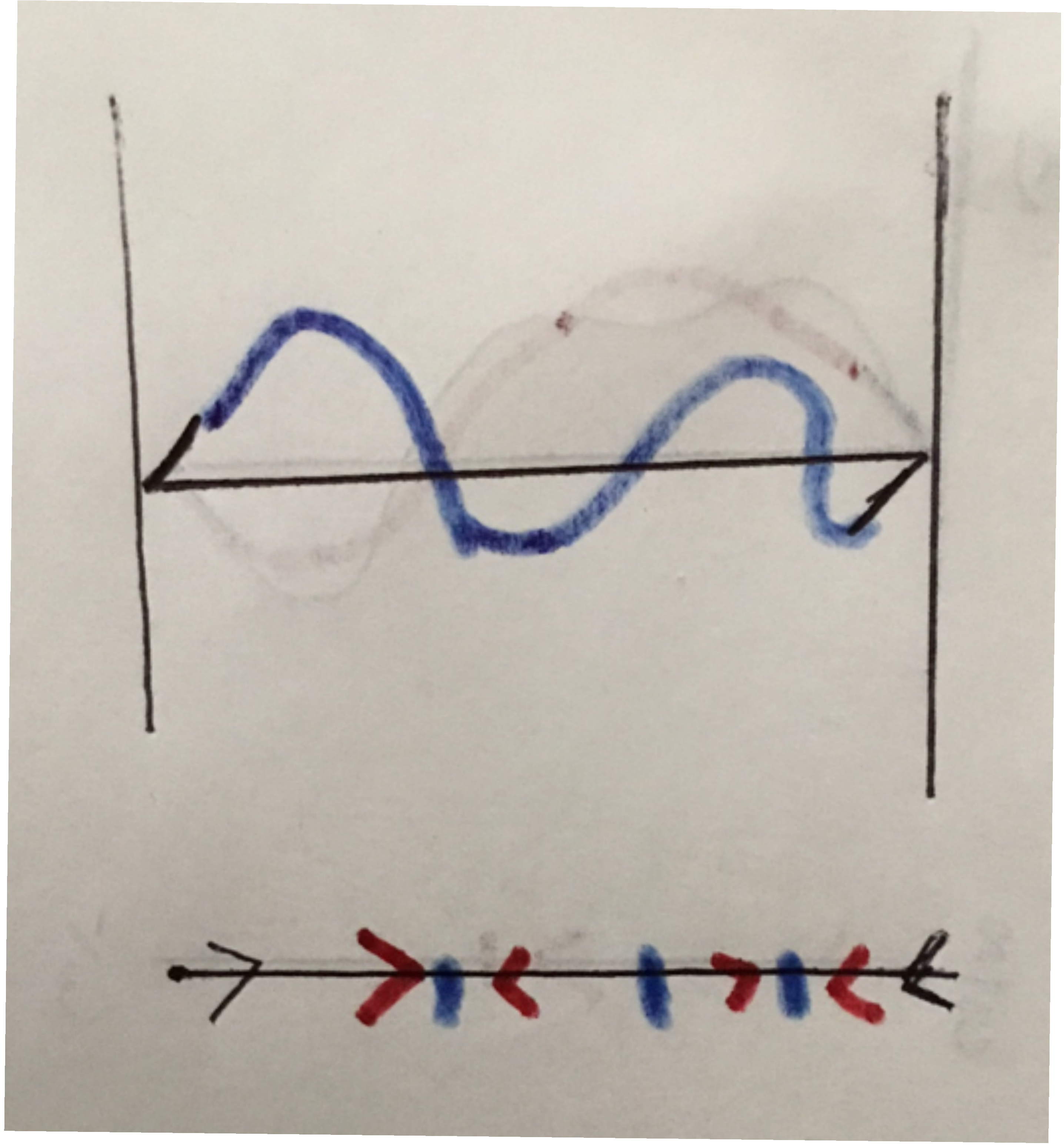

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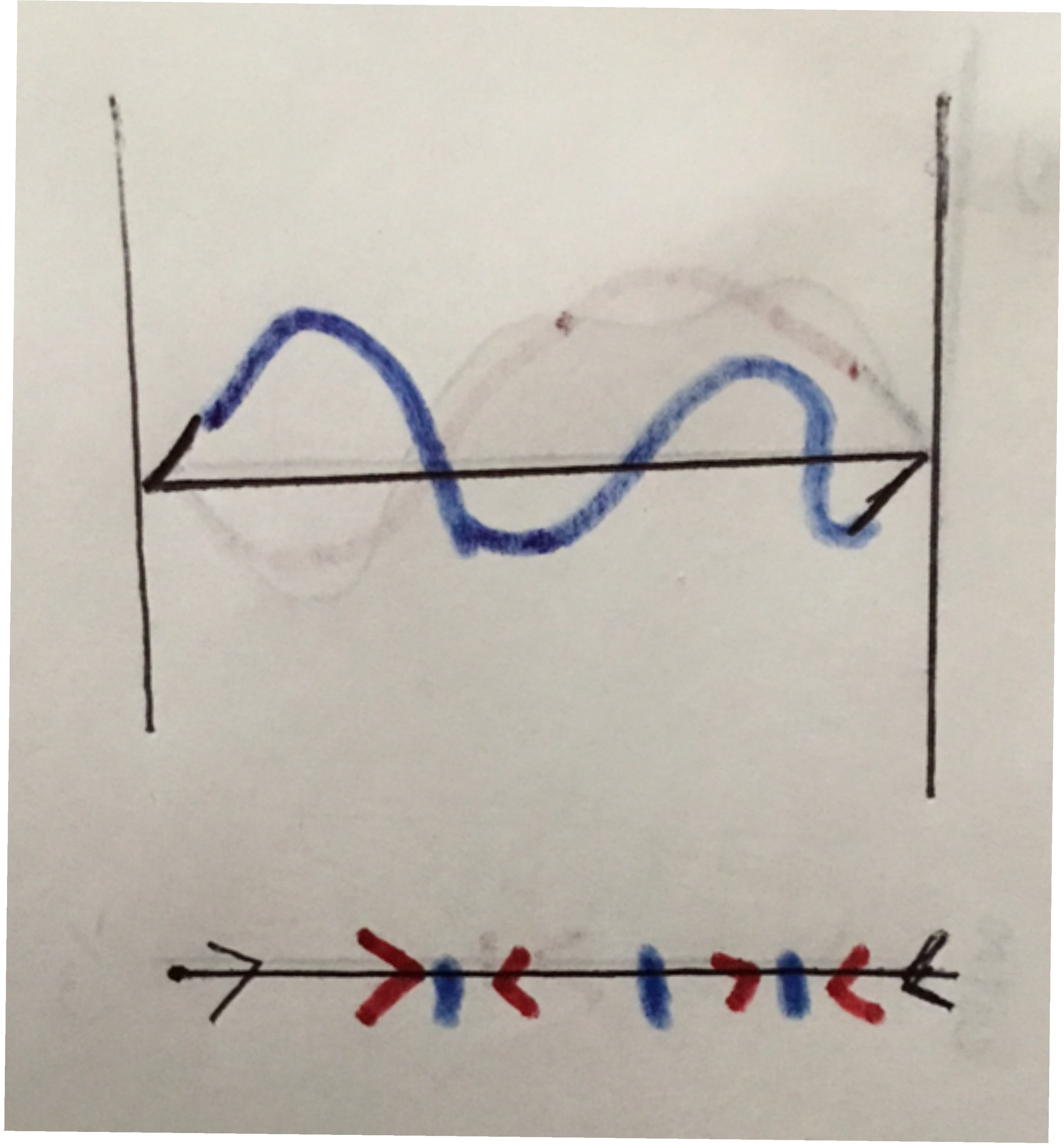

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Try next level of a model

It crosses the x-axis three times.

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{1}{3}x_1(1 - x_1 - 2x_1x_2) \\ \frac{dx_2}{dt} &= \frac{1}{3}x_2x_1(1 - 2x_2) \end{aligned}$$

Gangs!!

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Local information:

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Local information:

If sufficiently dominant, each gang will eliminate the other one

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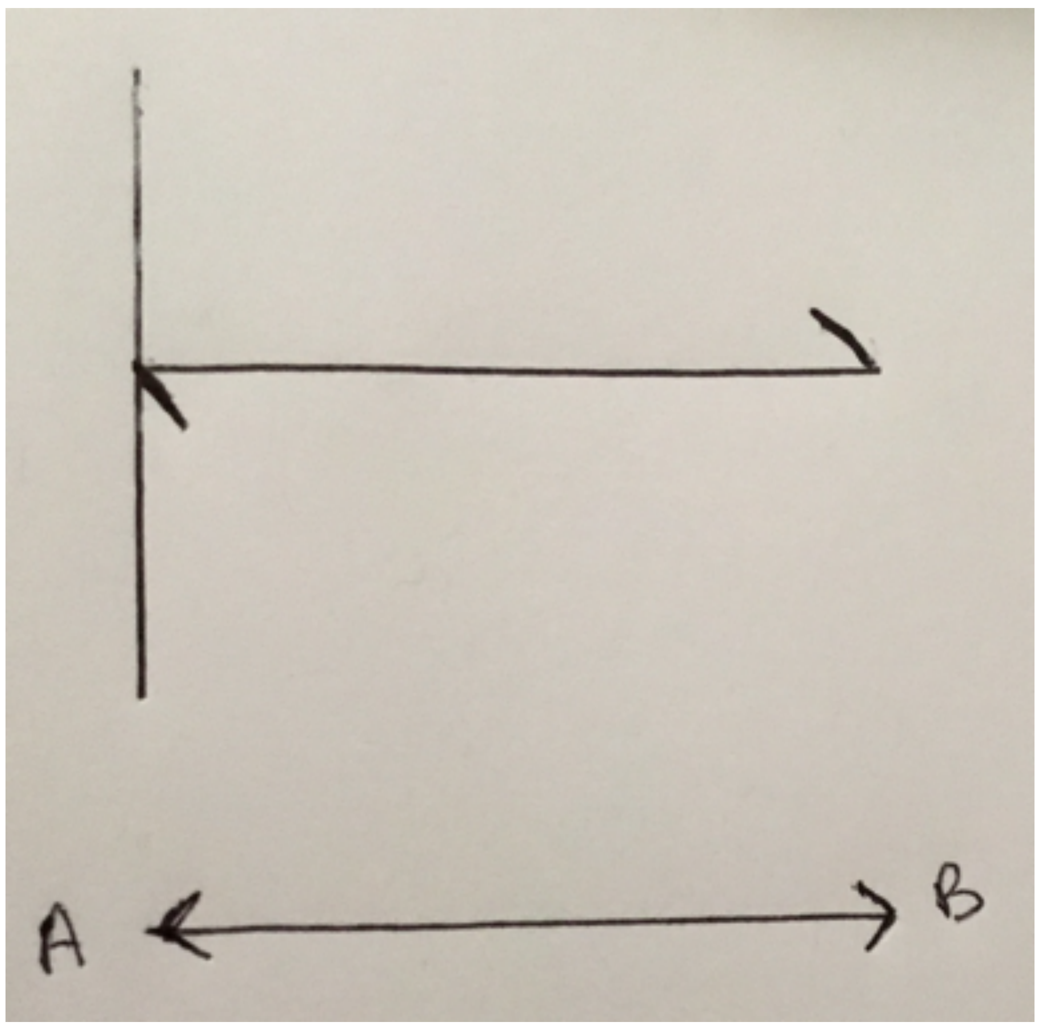

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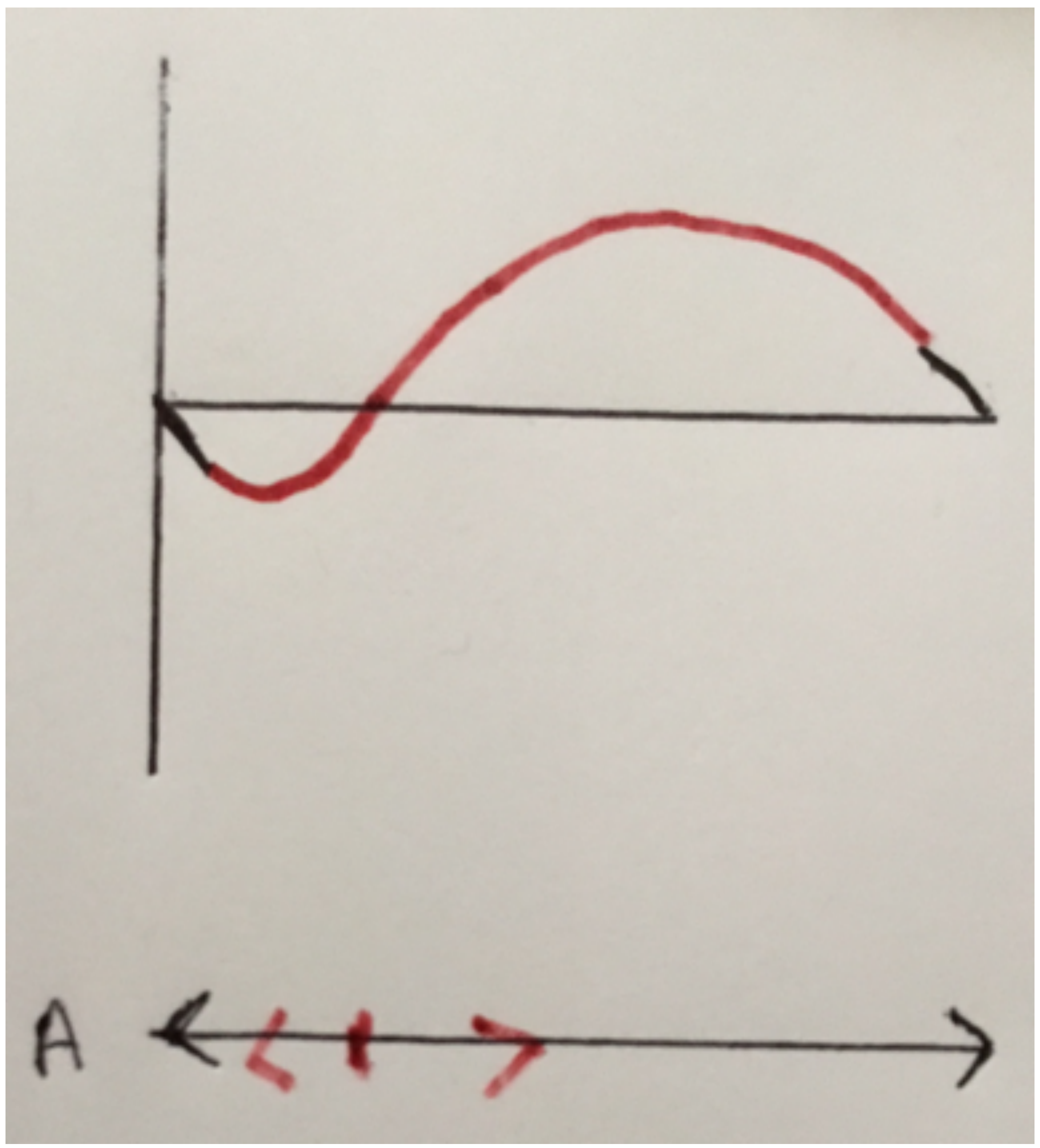

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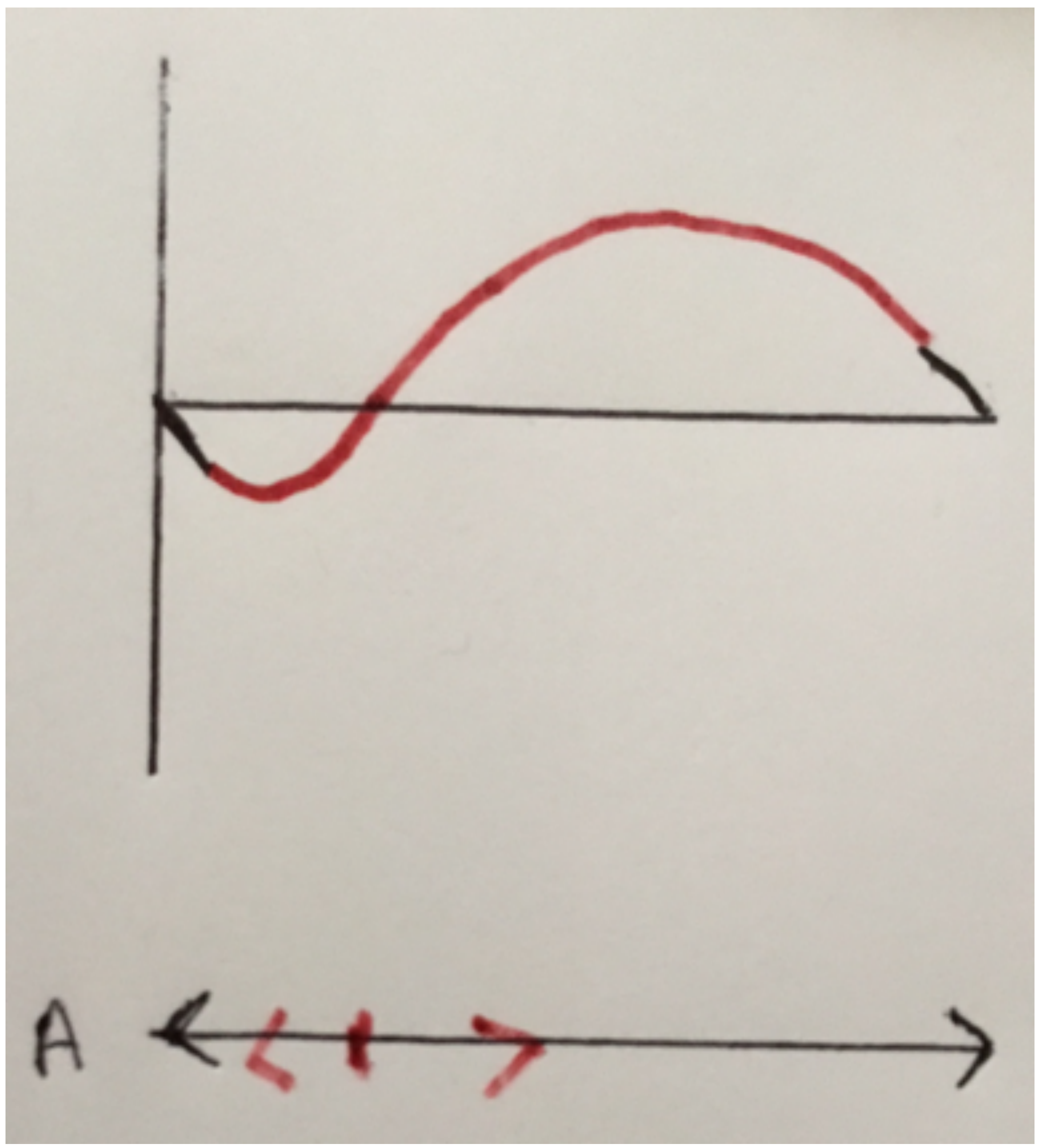

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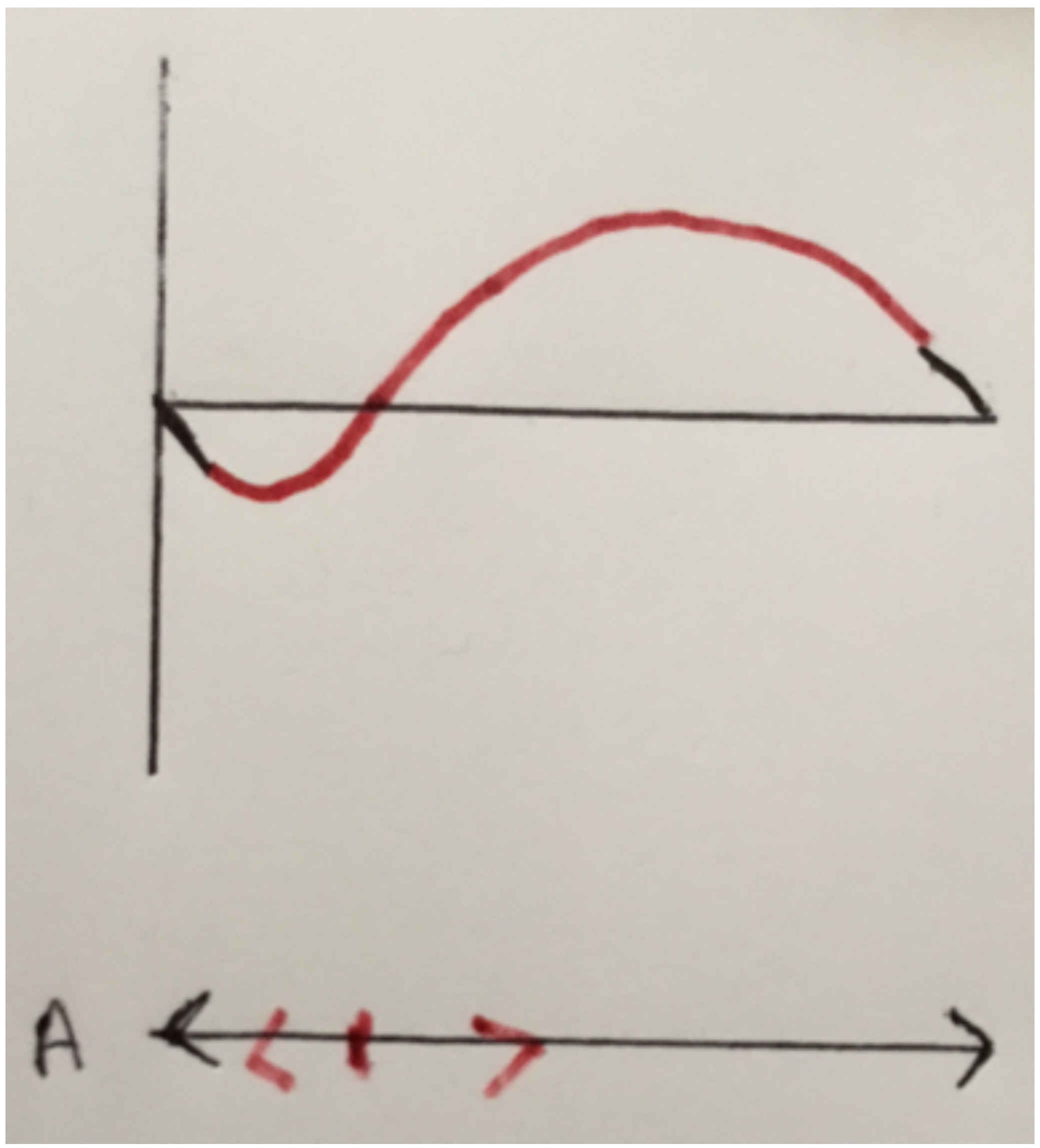

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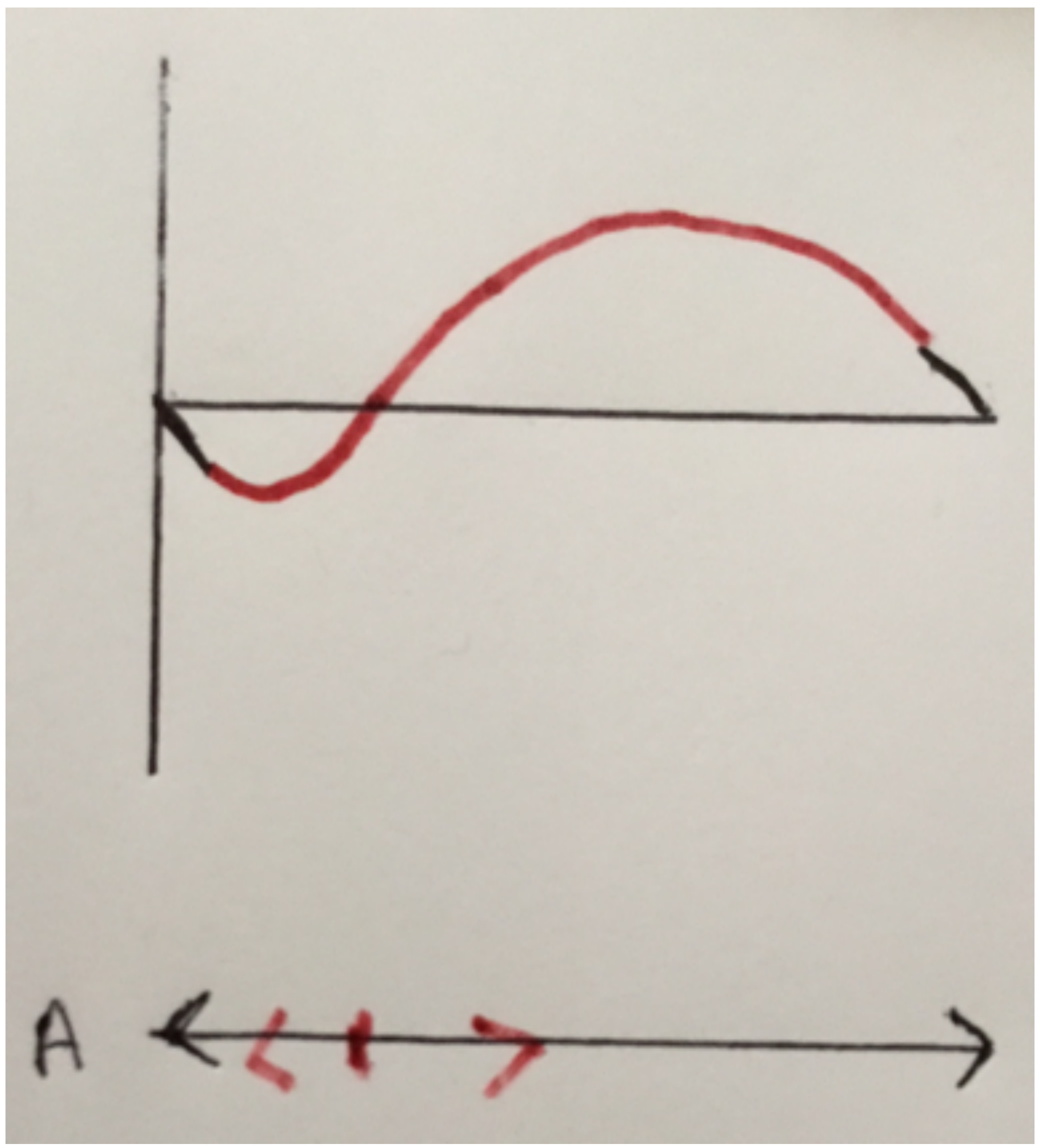

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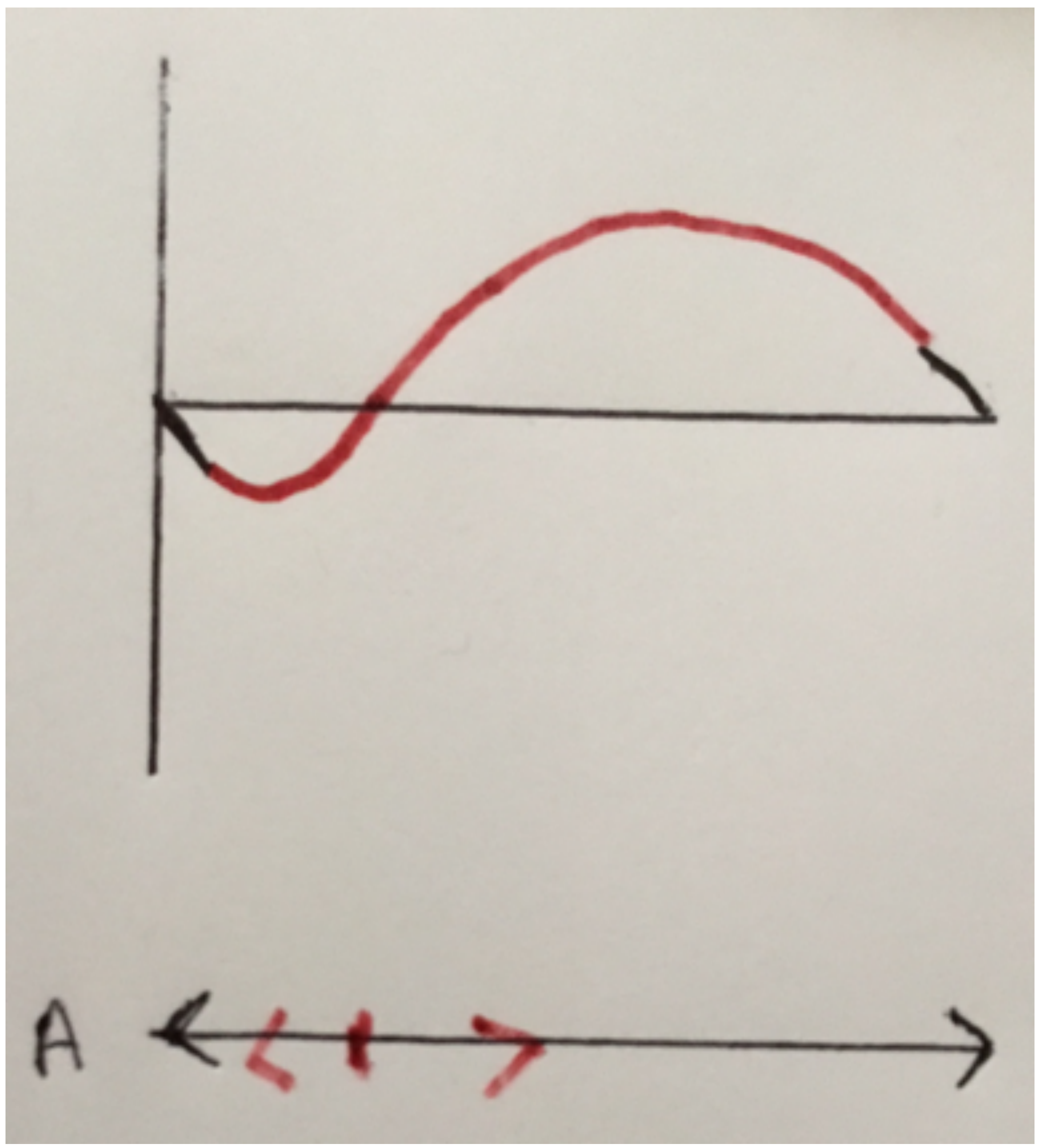

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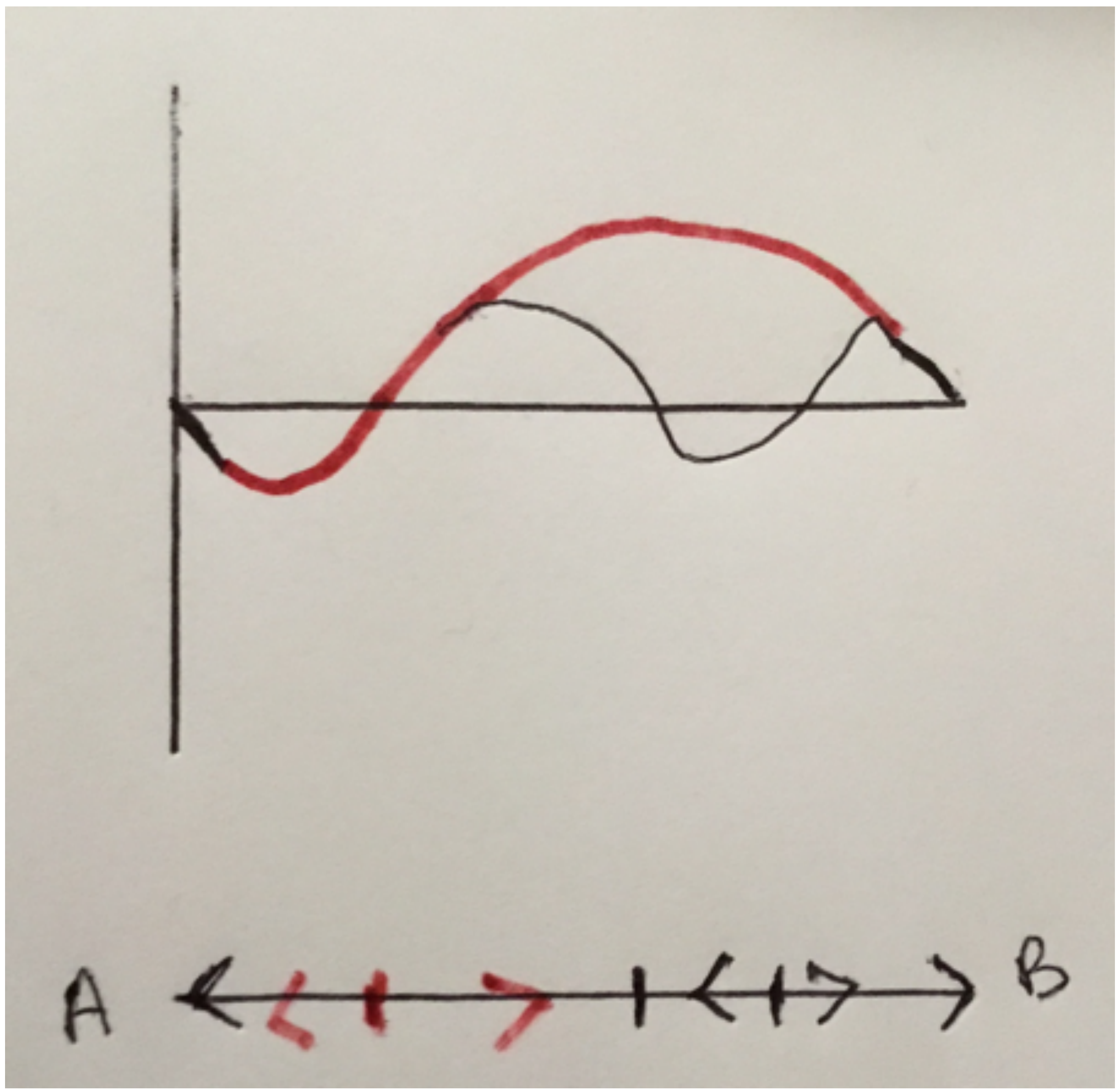

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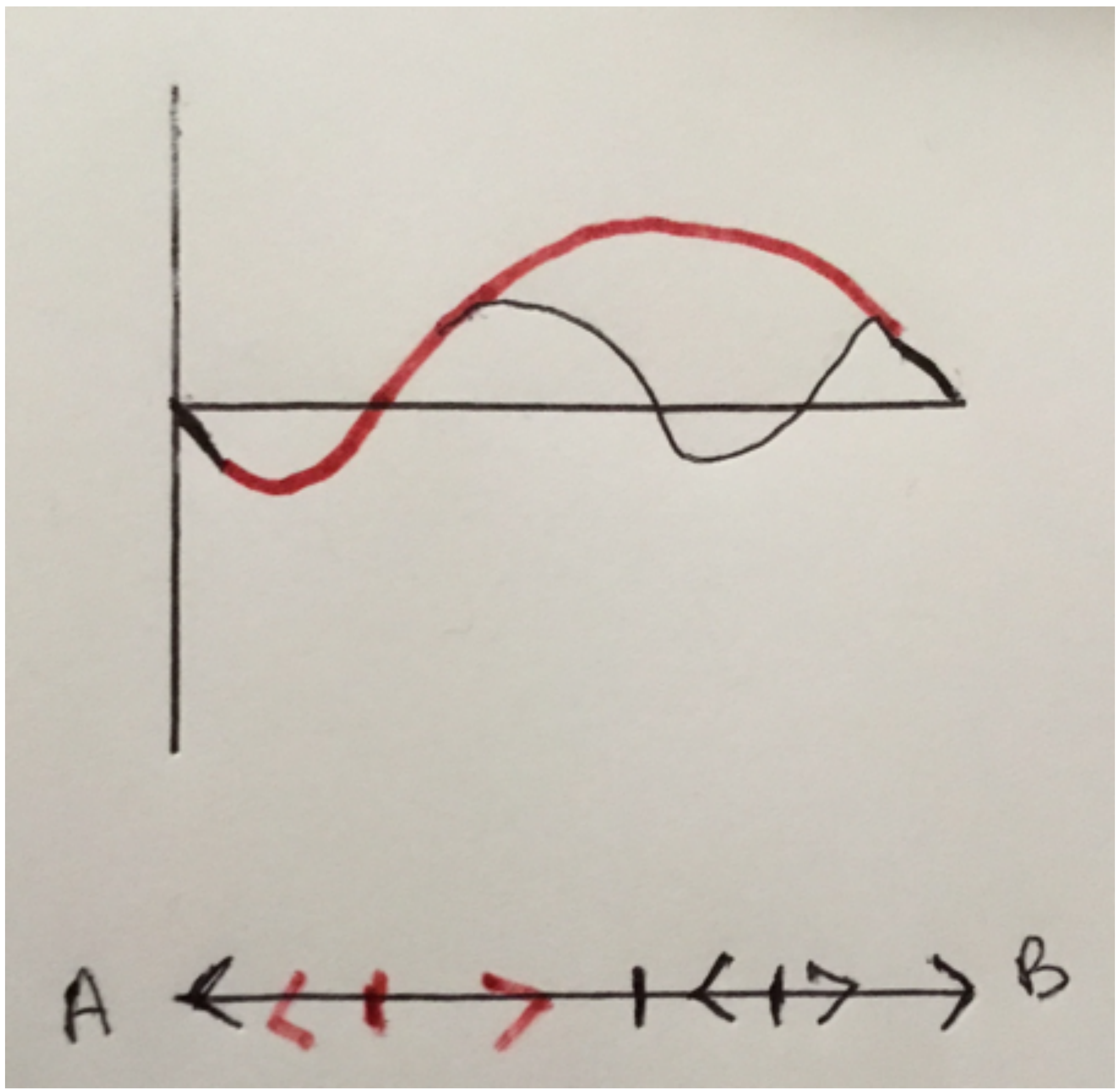

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Pocket of co-existence, stability.

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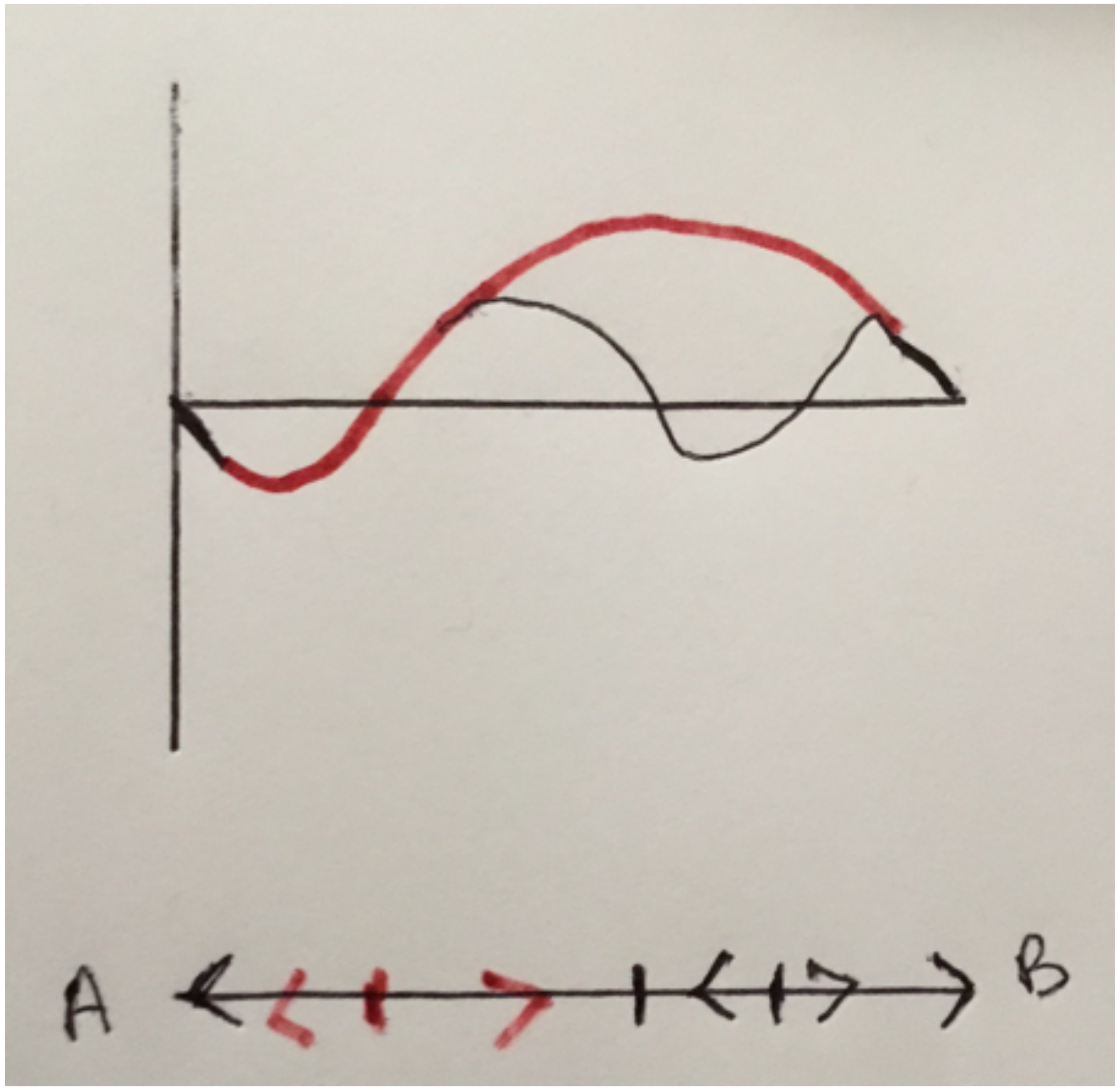

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Pocket of co-existence, stability.
Predictions are consistent with models—without difficulties

Back to the Ultimatum game

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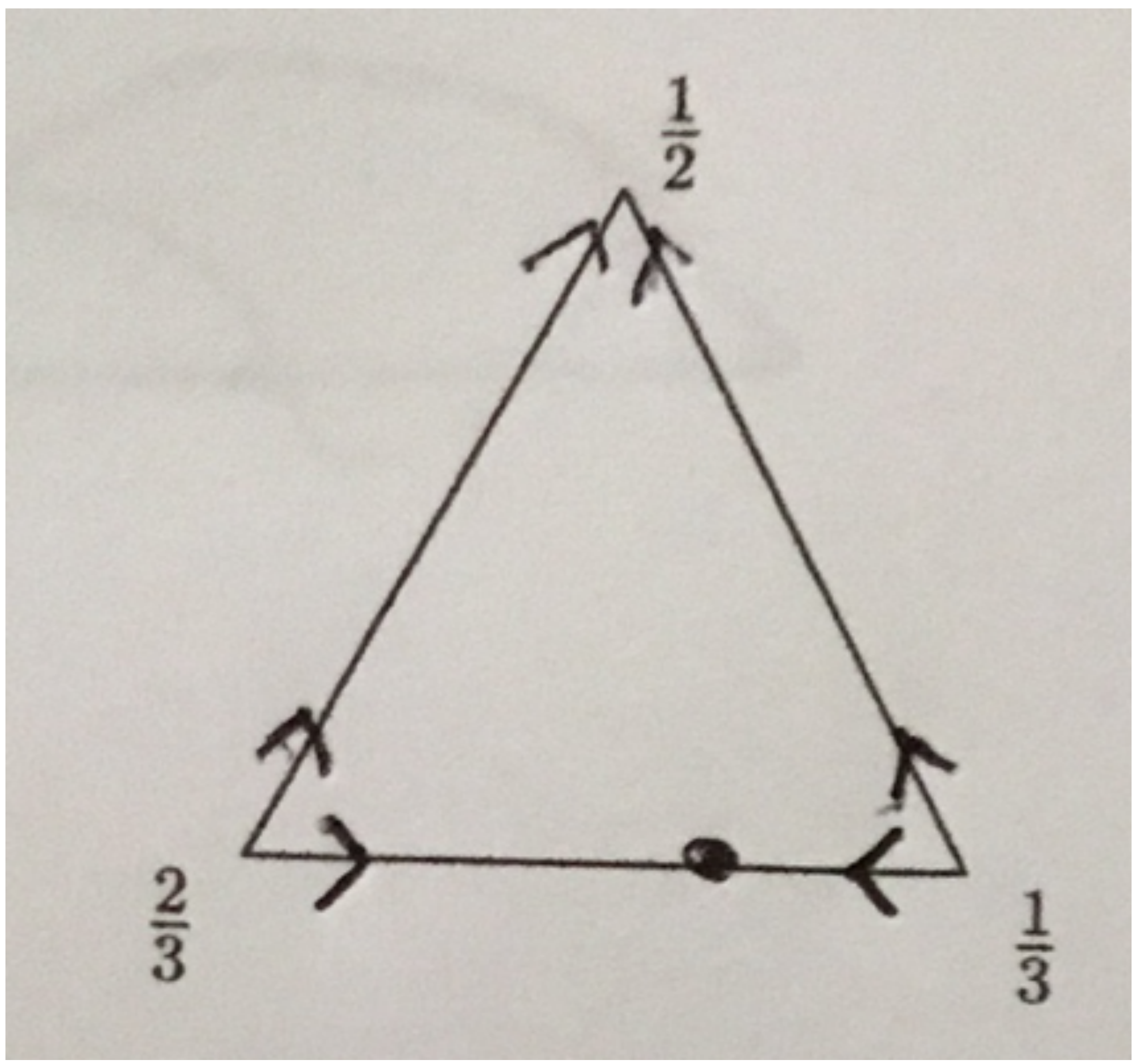

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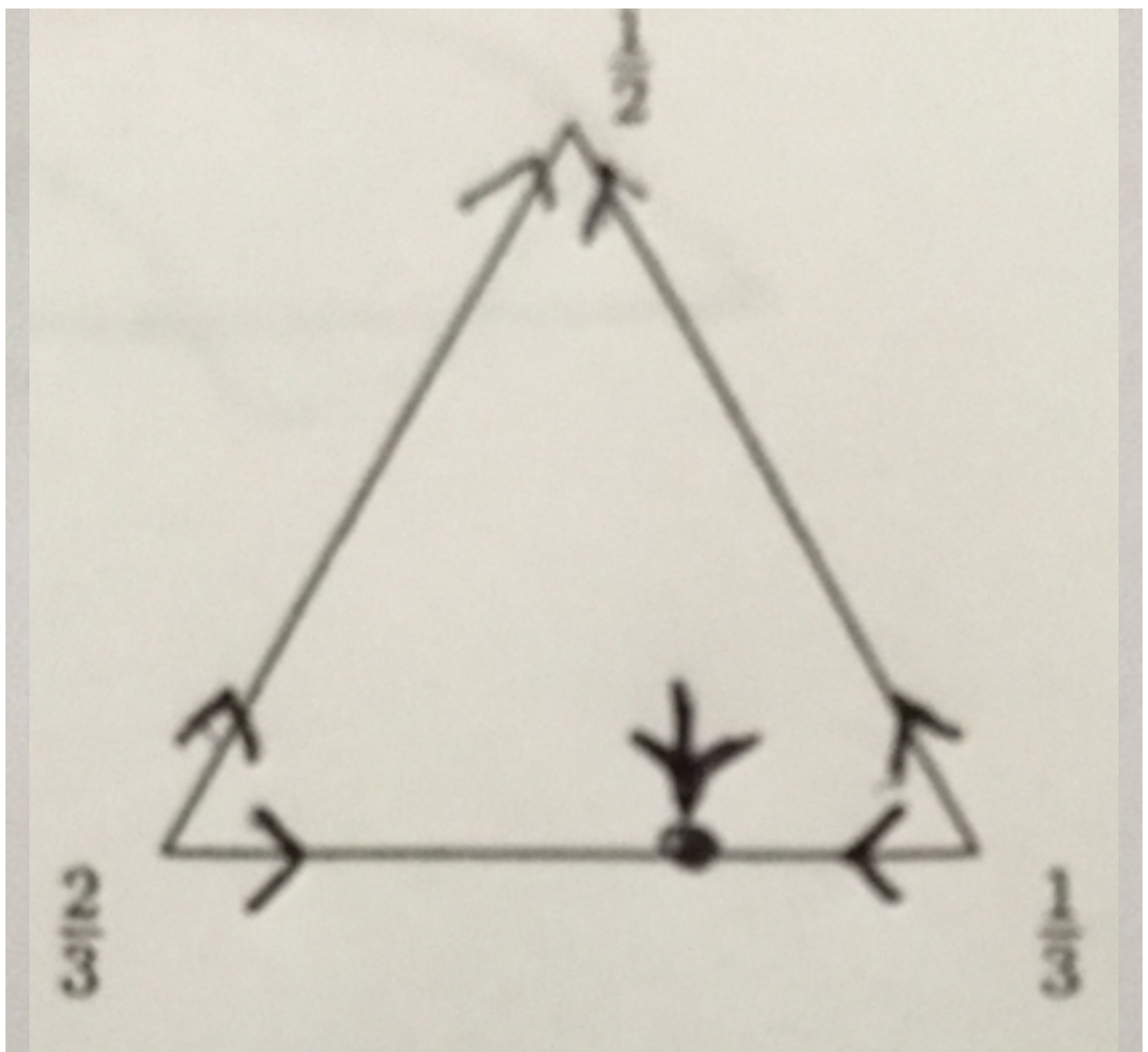

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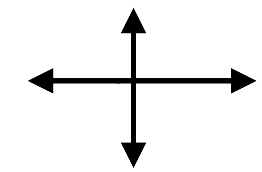

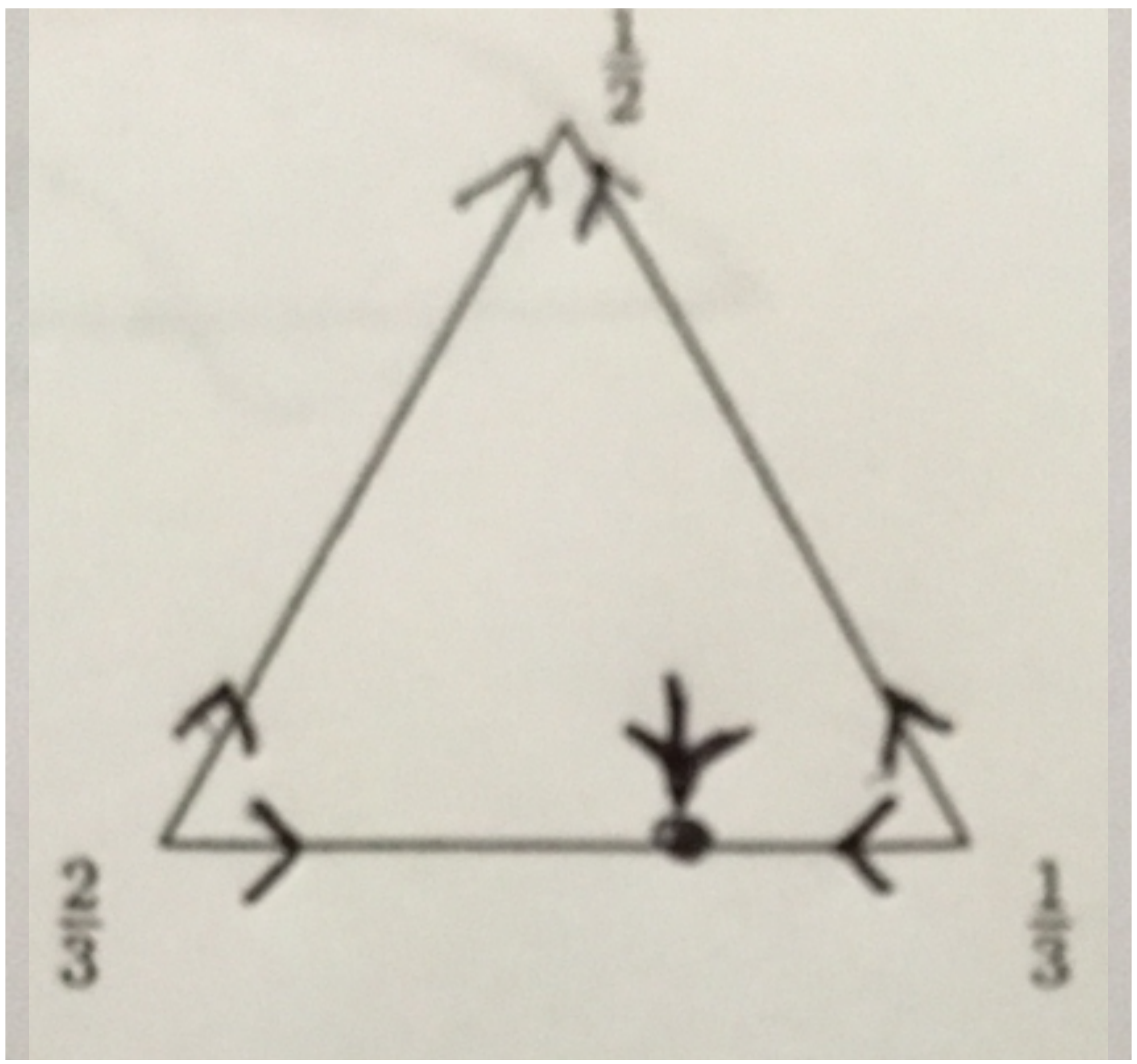

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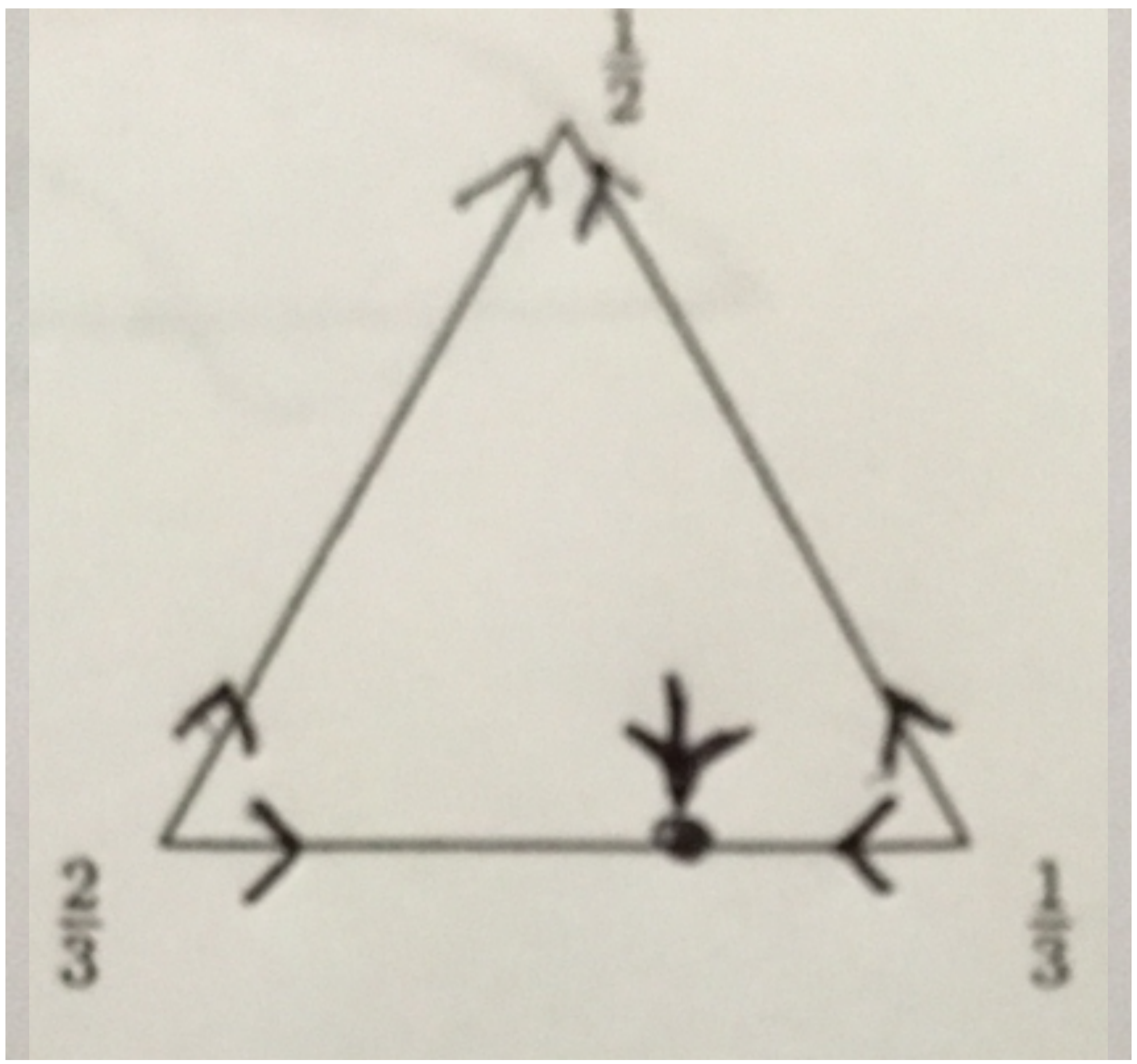

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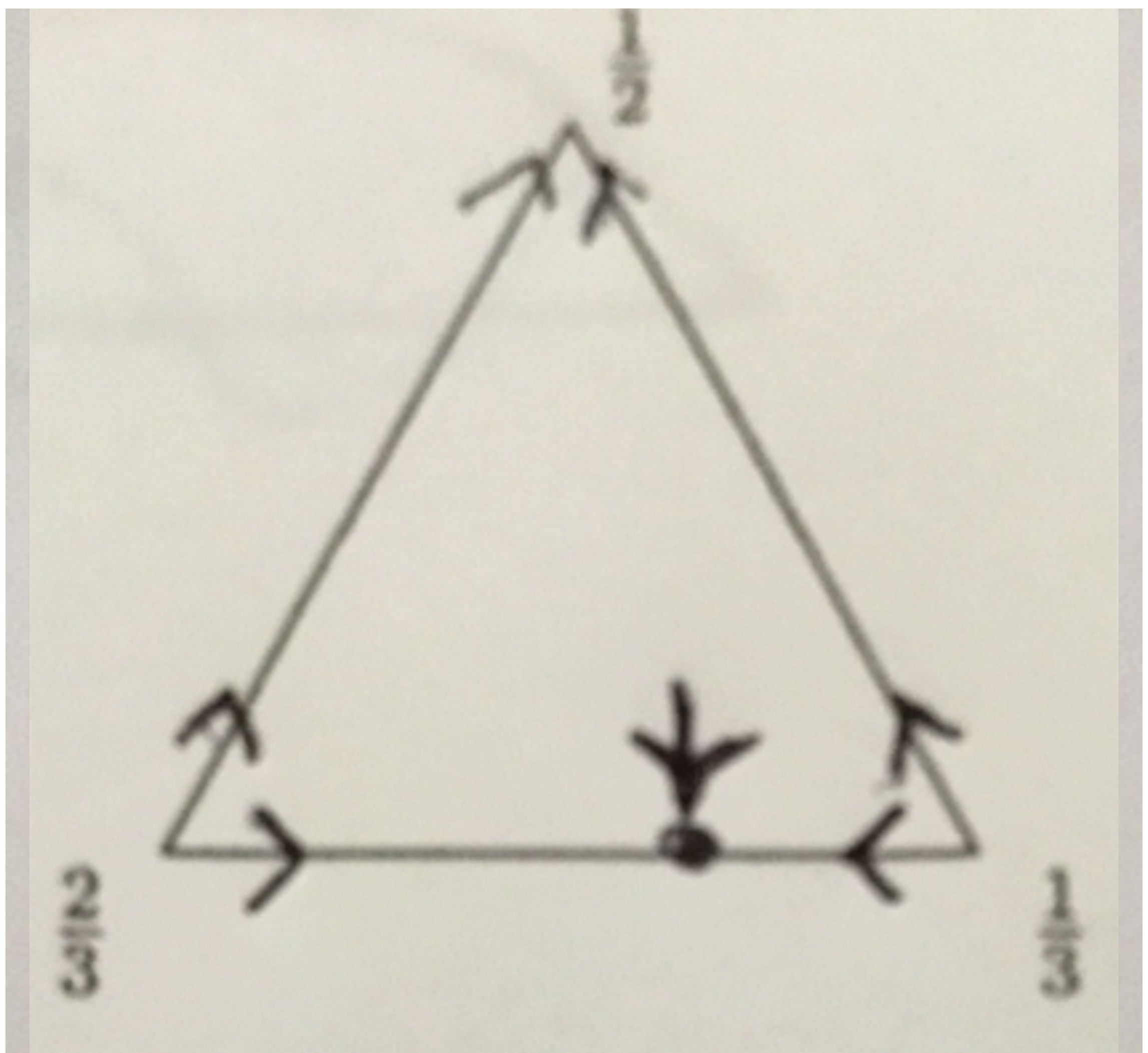
+1



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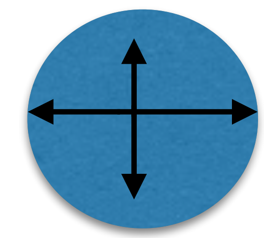

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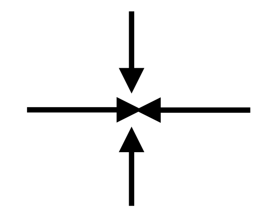

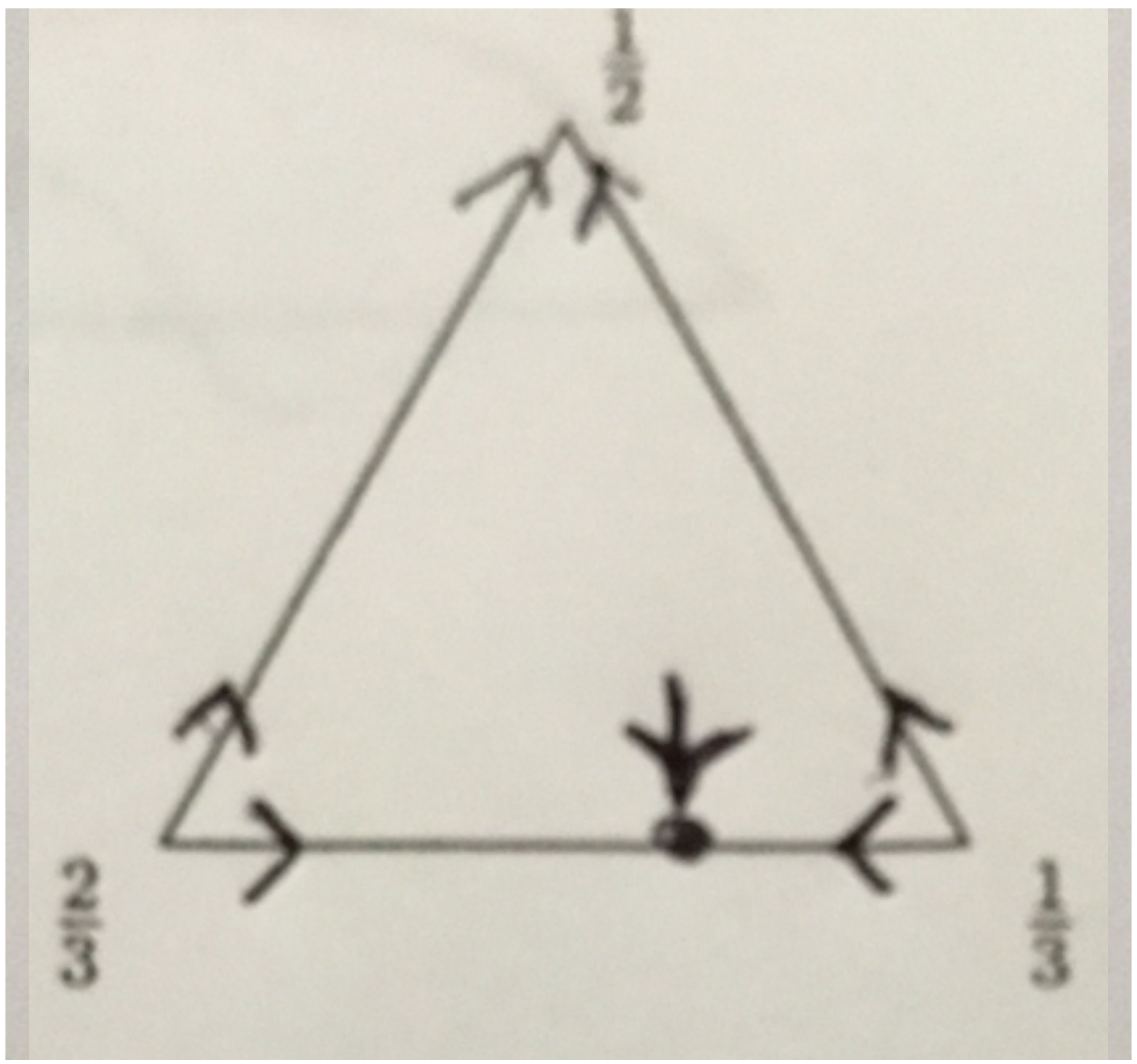

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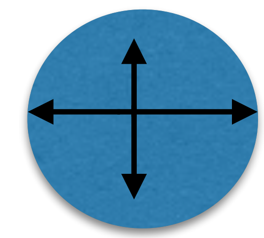

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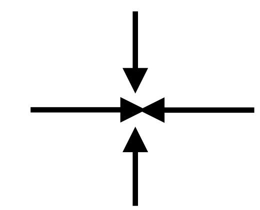

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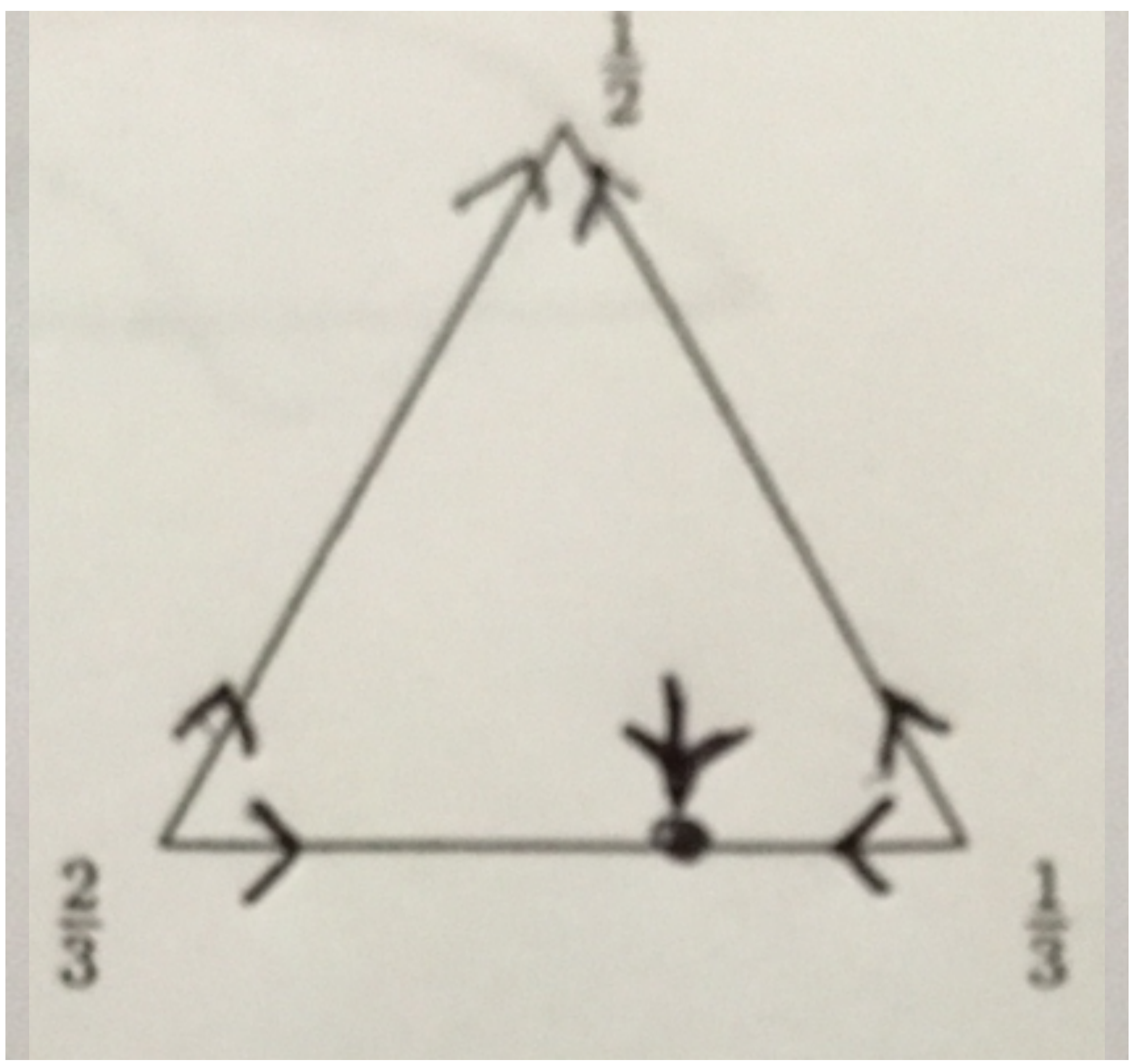

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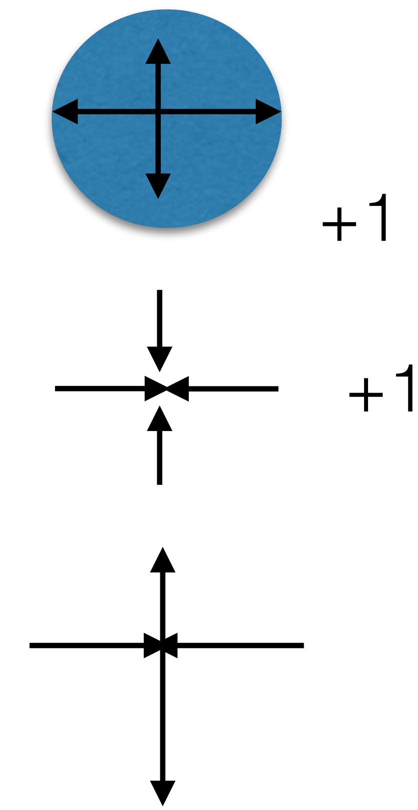

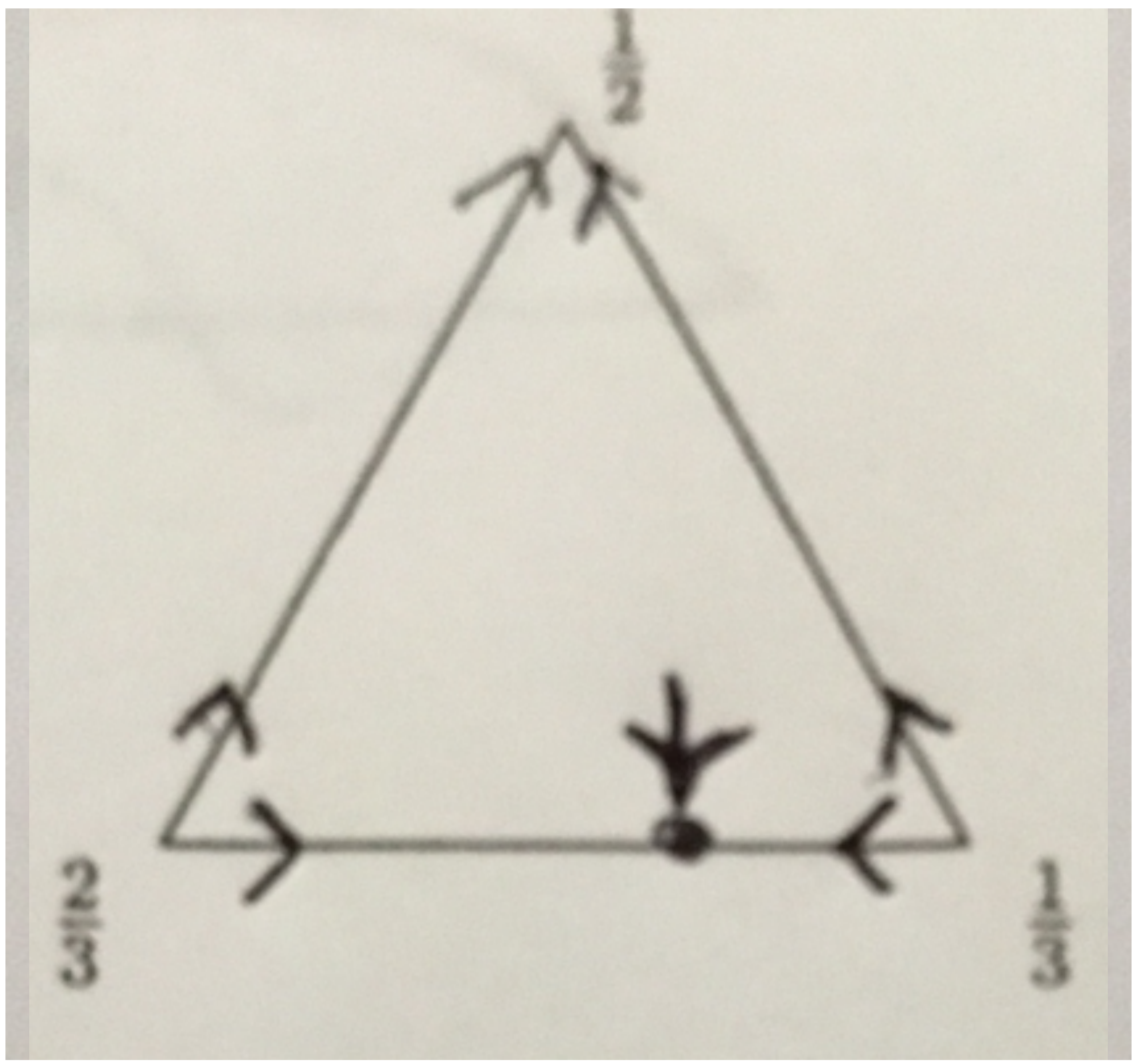

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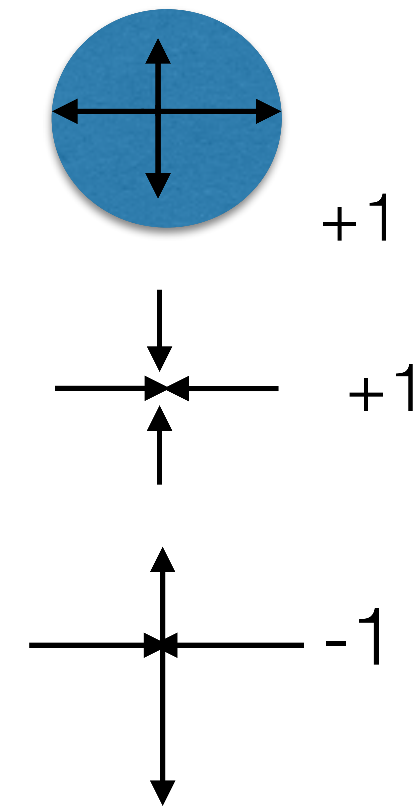

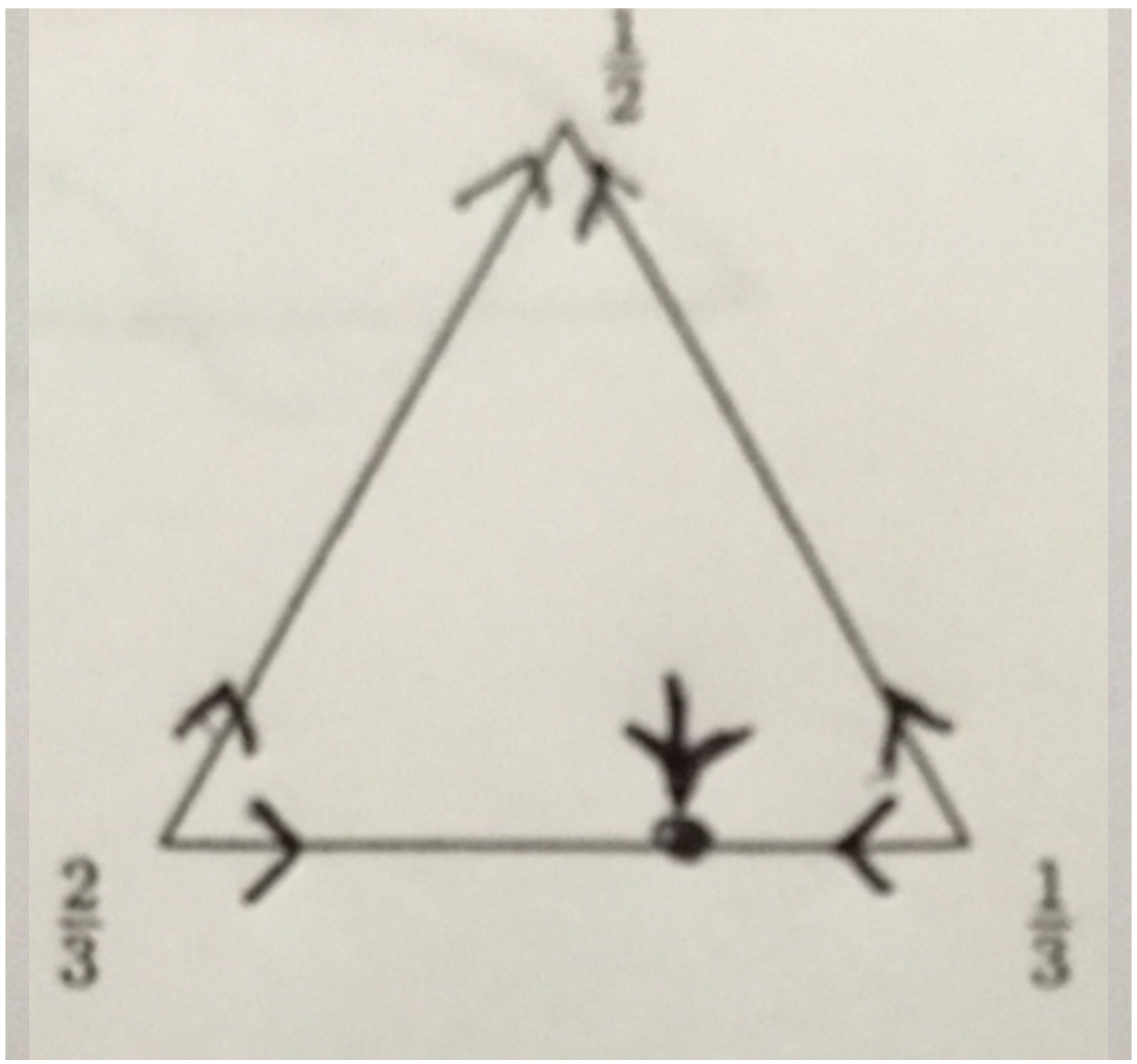

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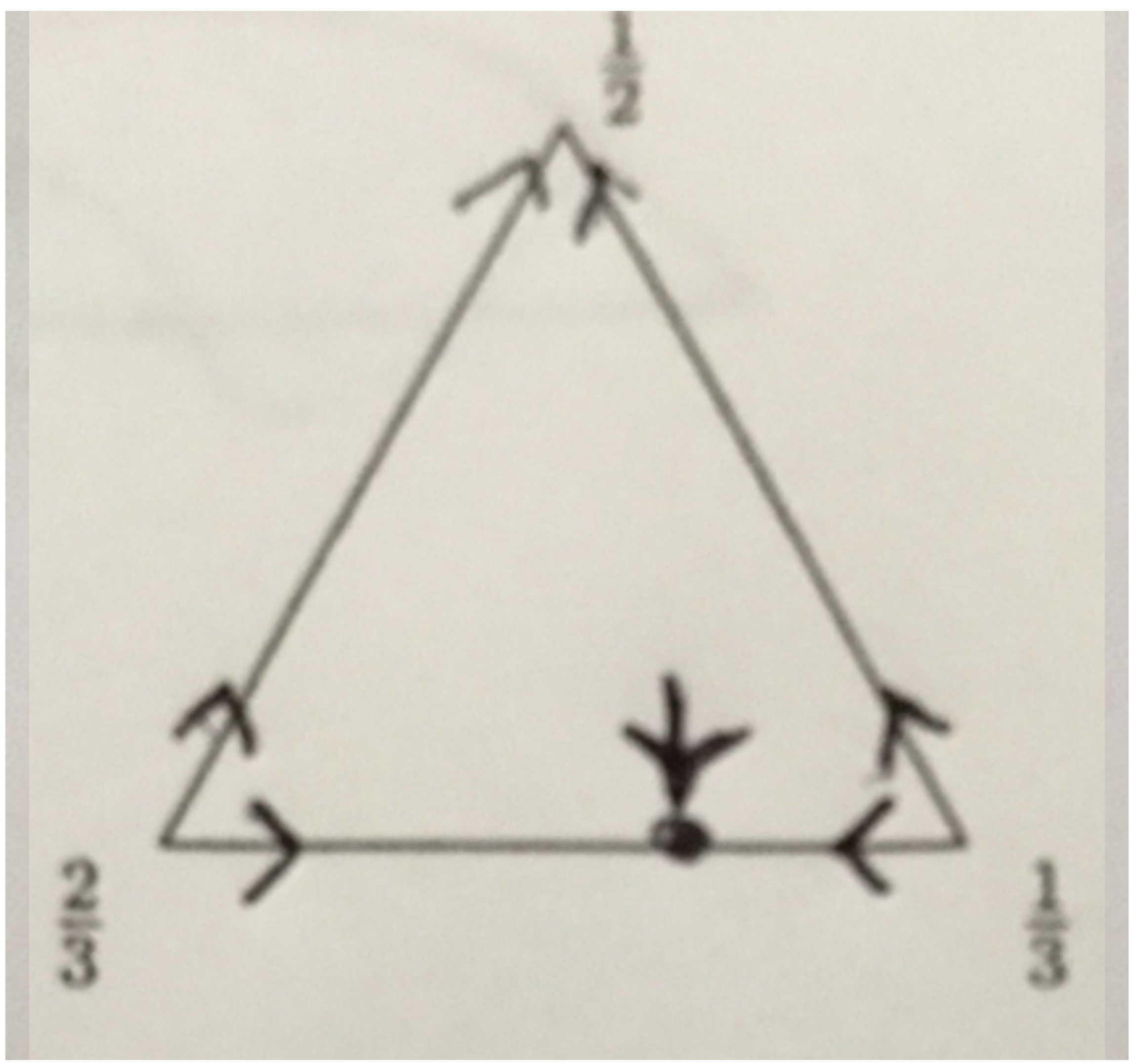

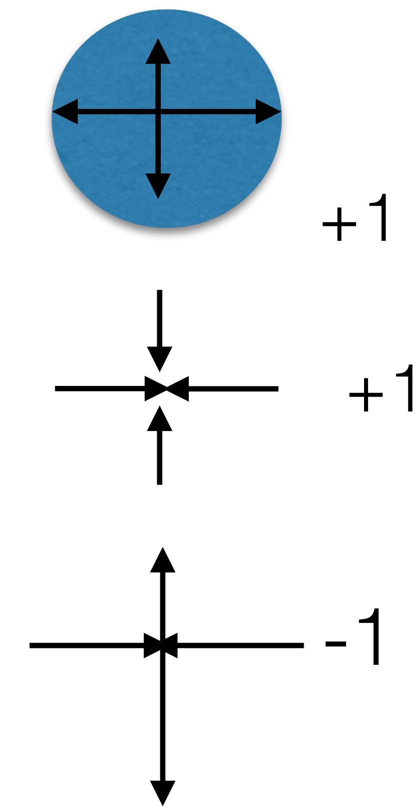

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Back to the Ultimatum game

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f is continuous, but not known

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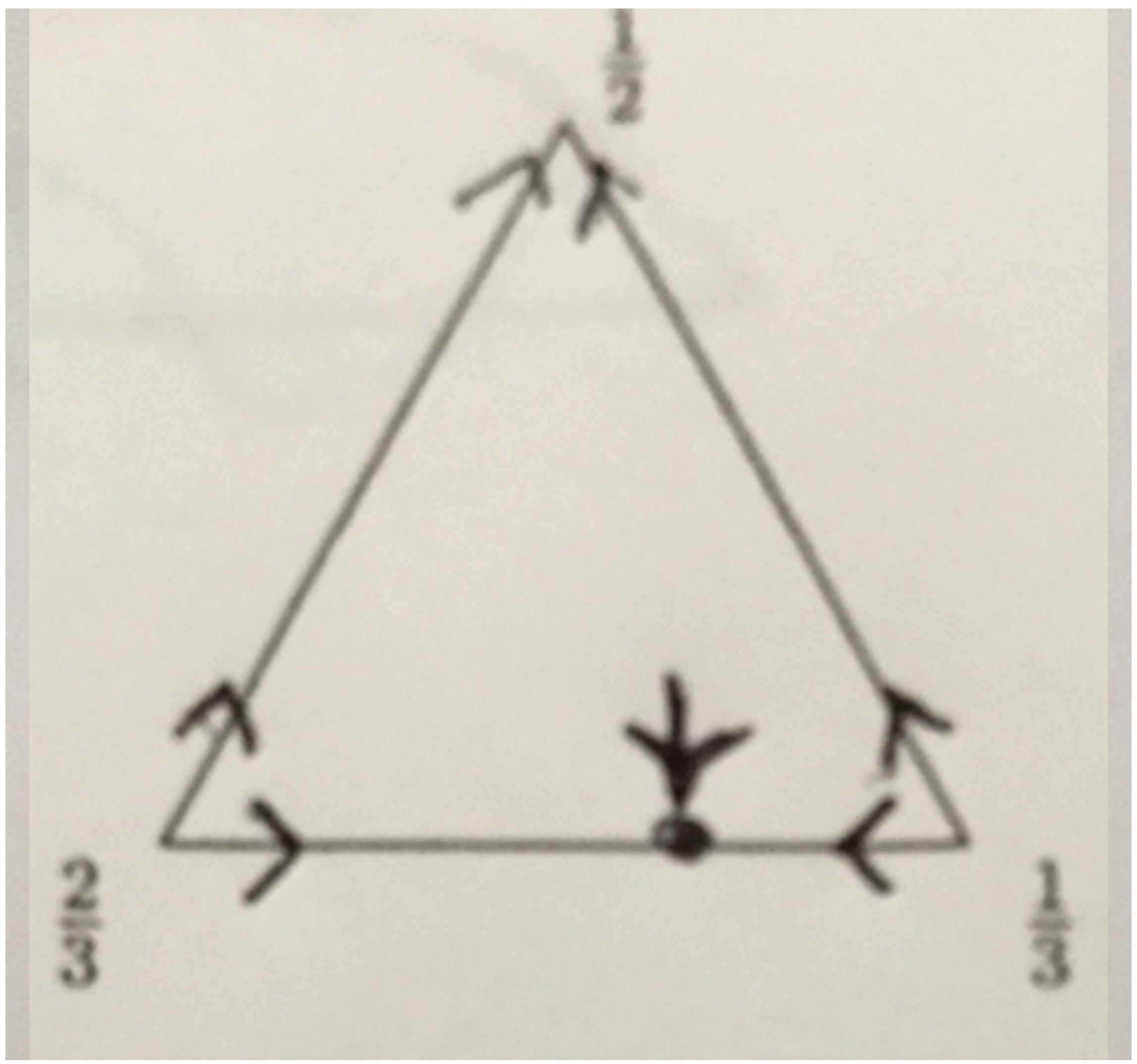

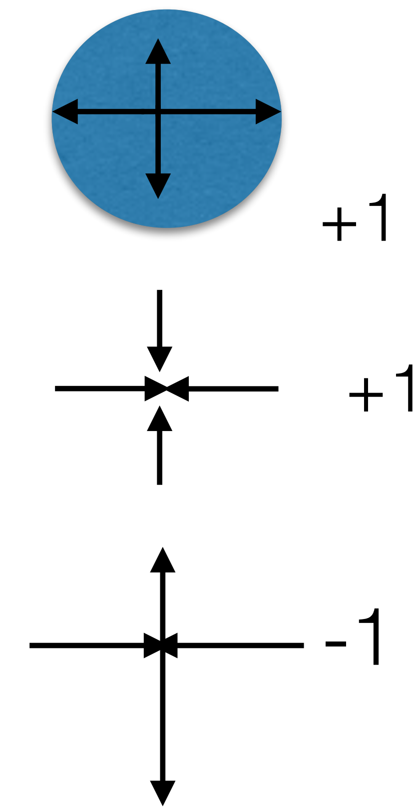

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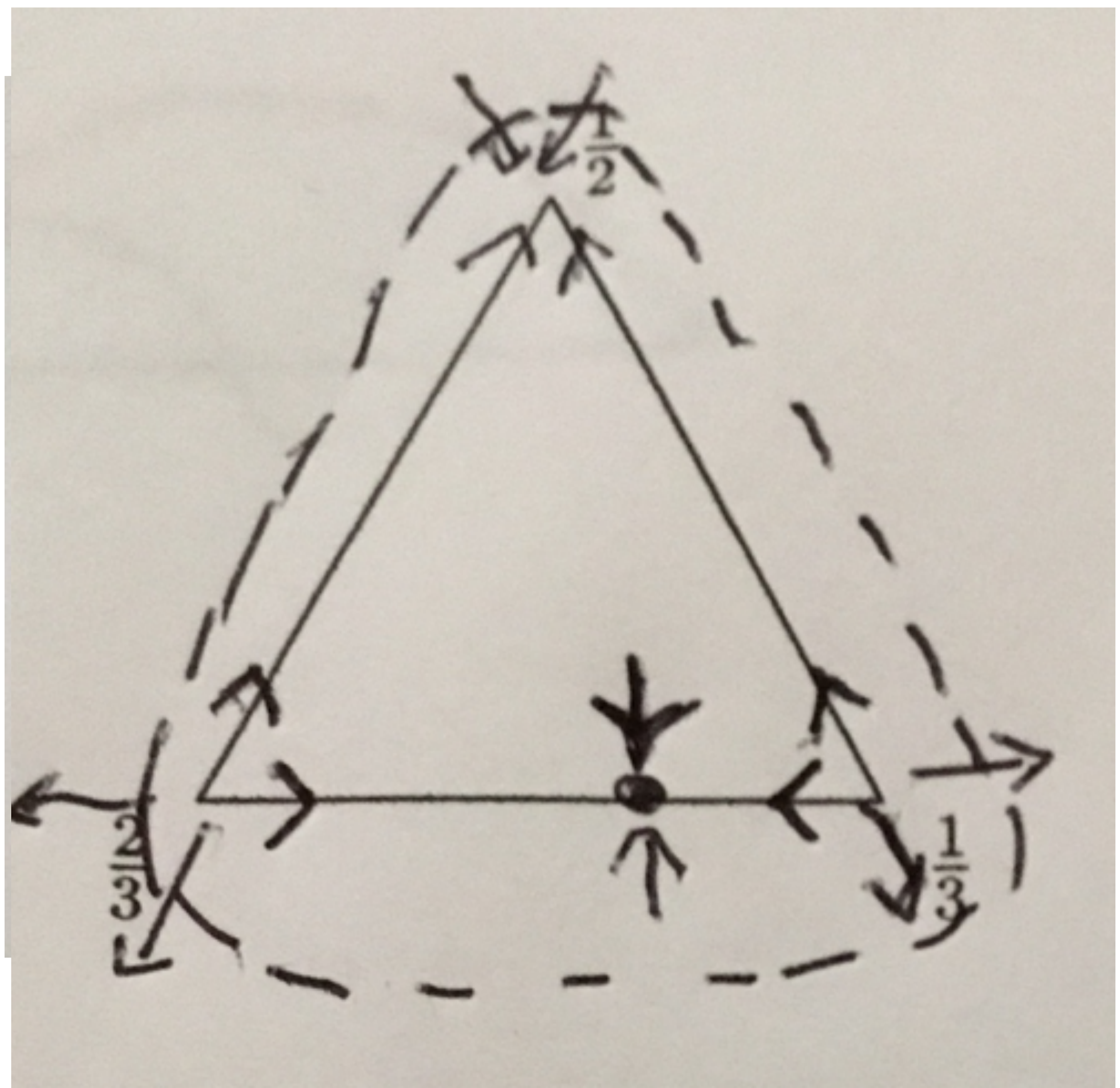

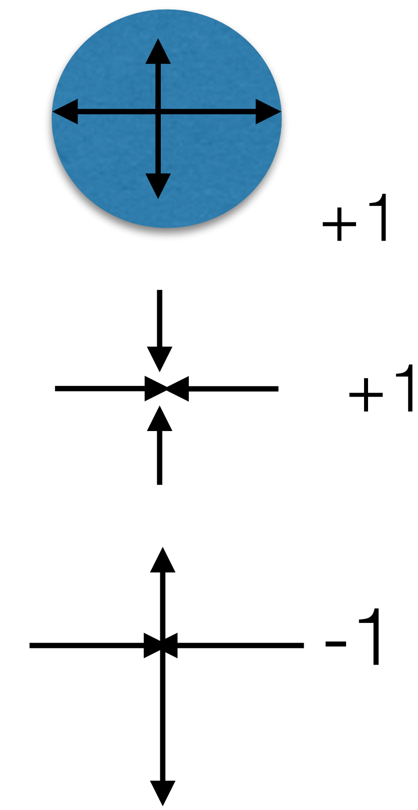

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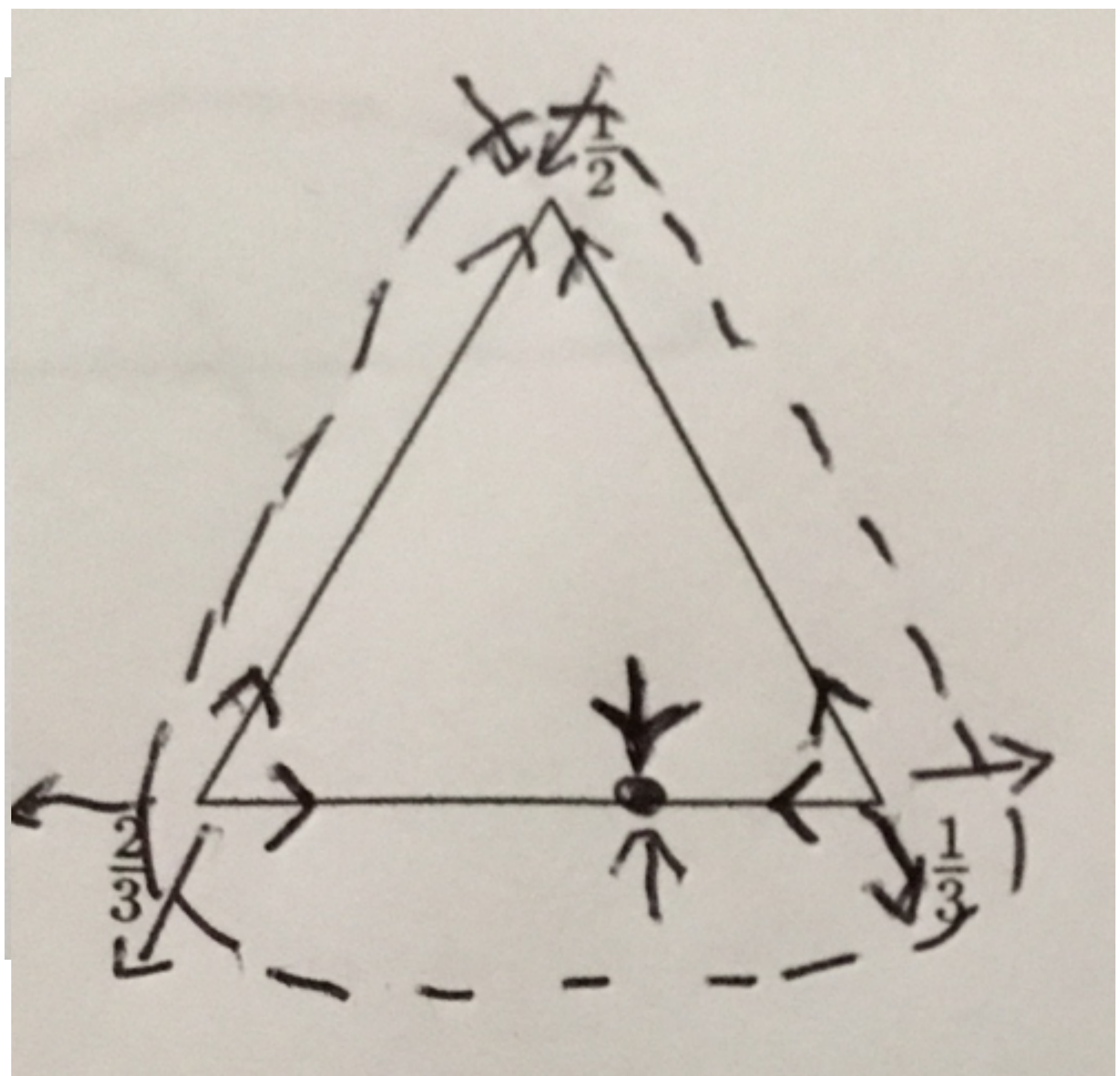
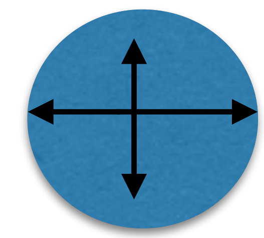
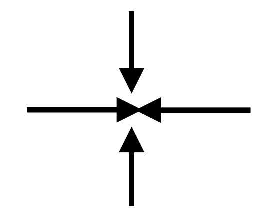




+1



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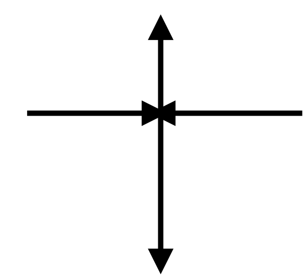

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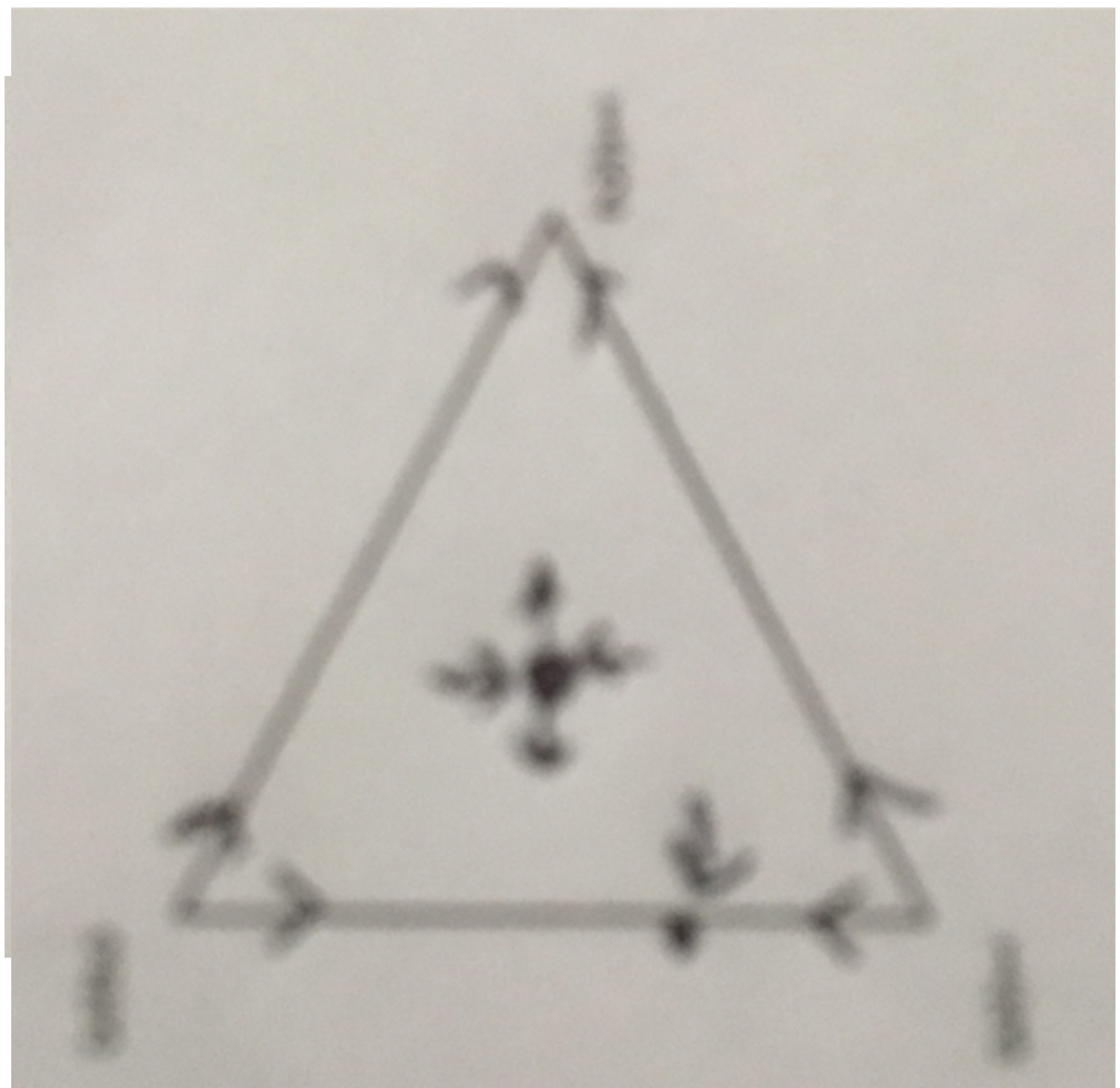

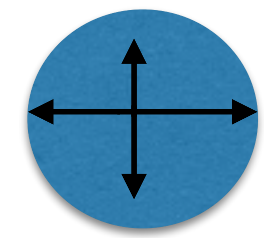

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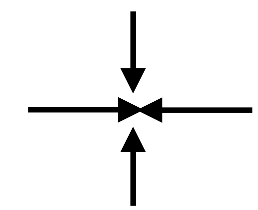

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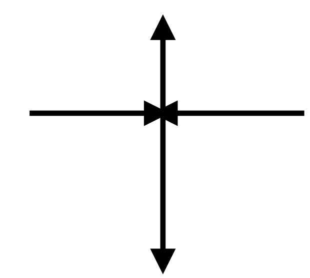

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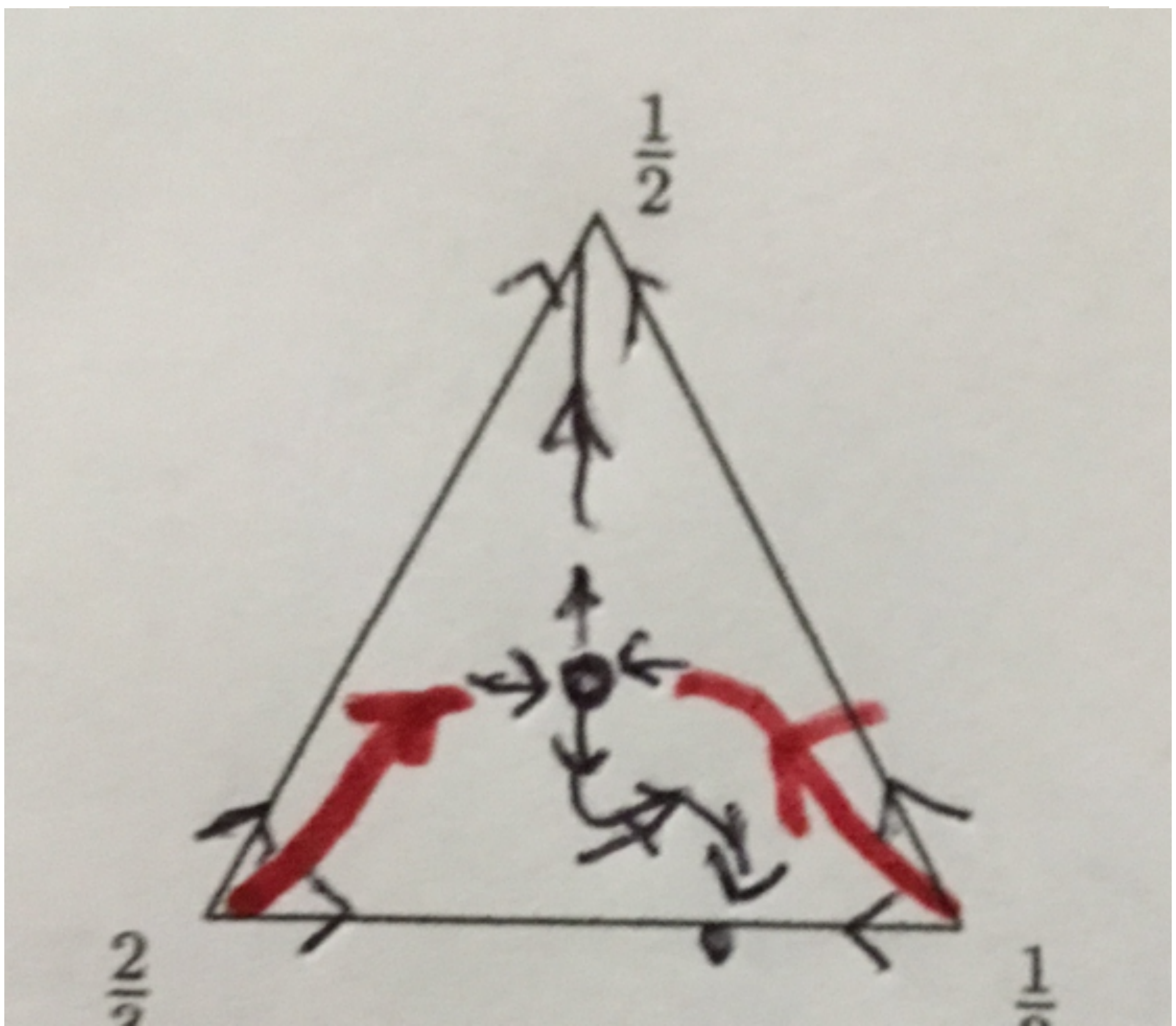

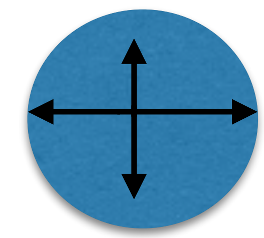

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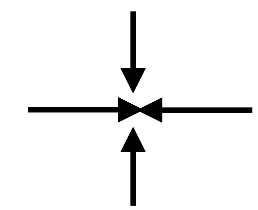

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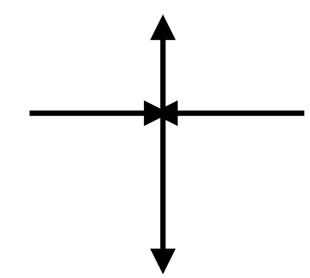

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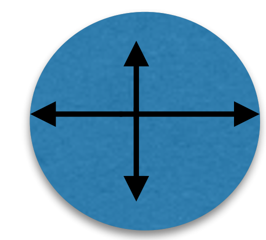

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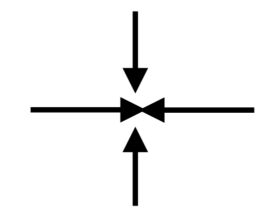

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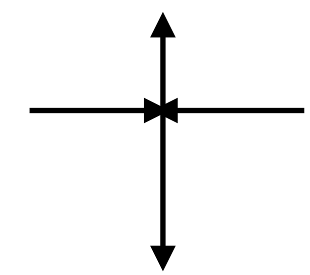

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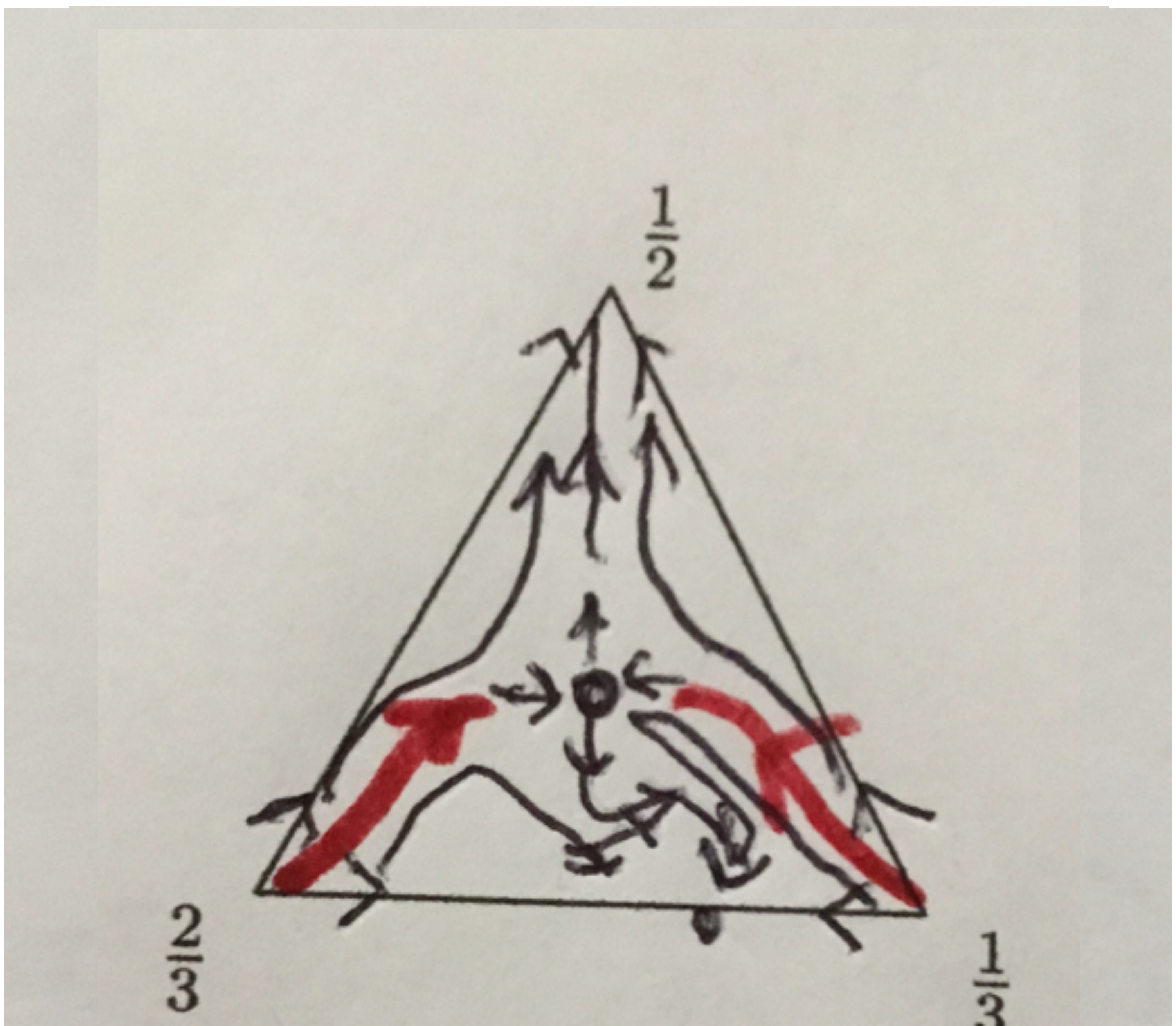

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What if the middle equilibrium differed?

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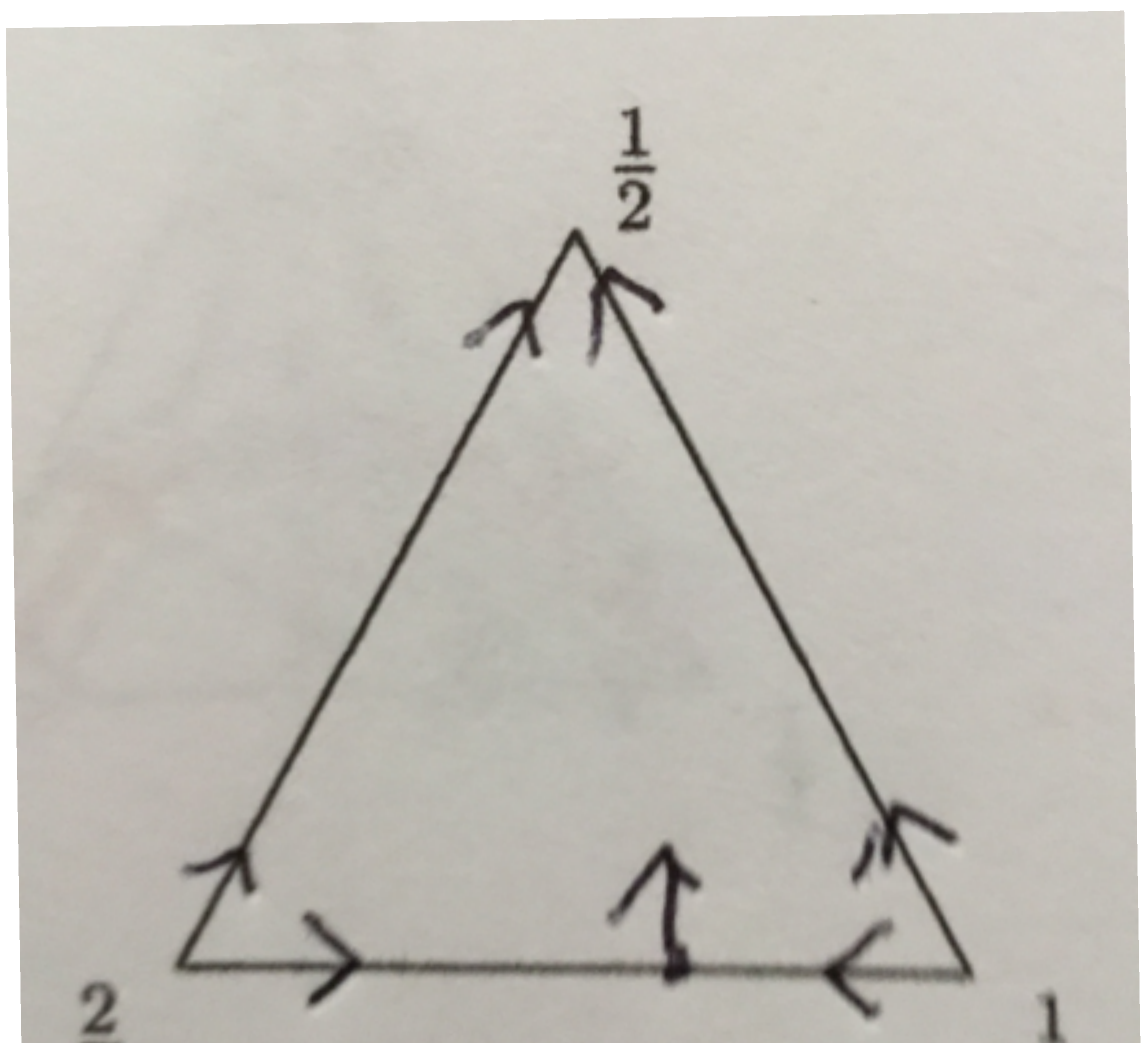

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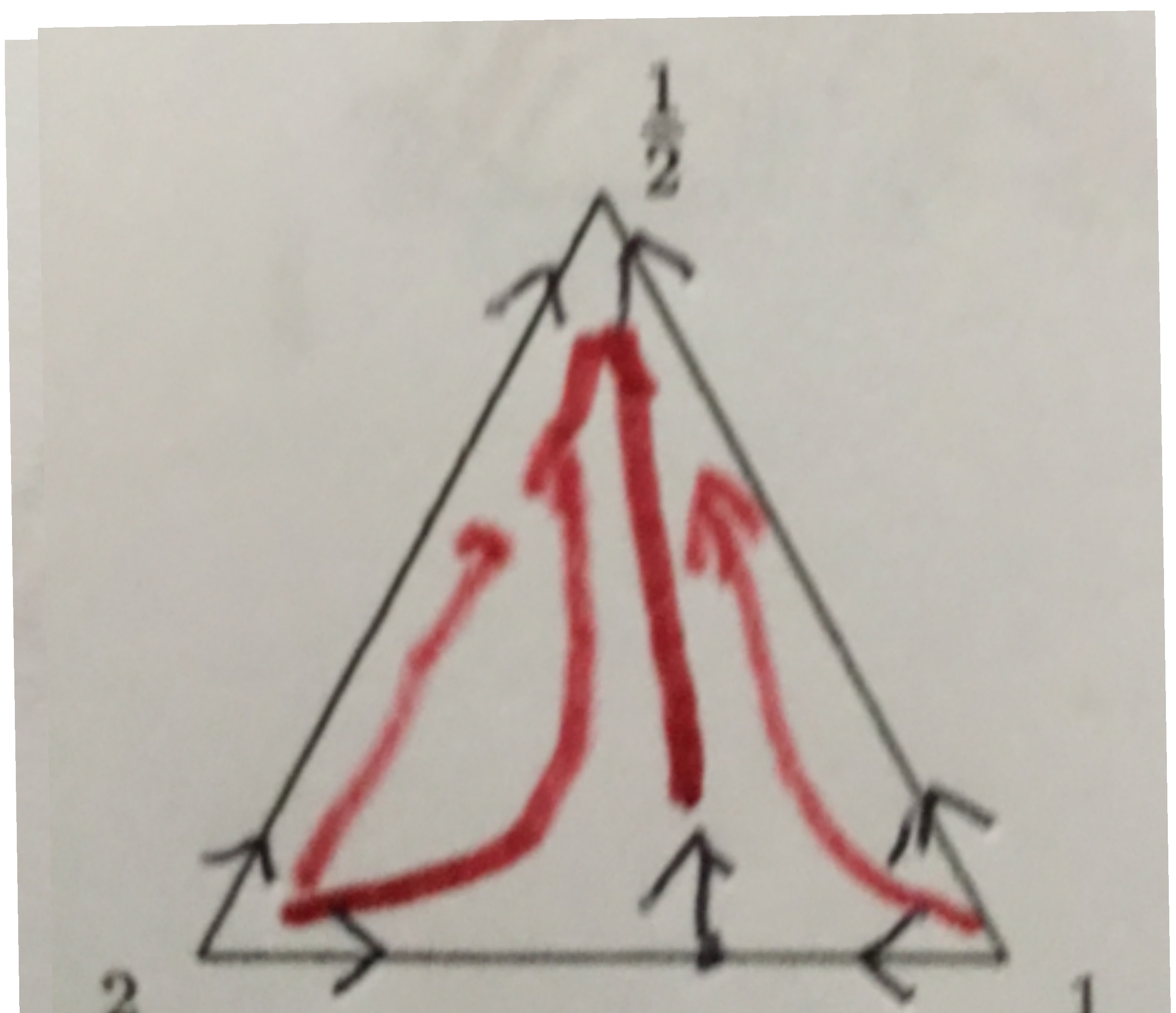

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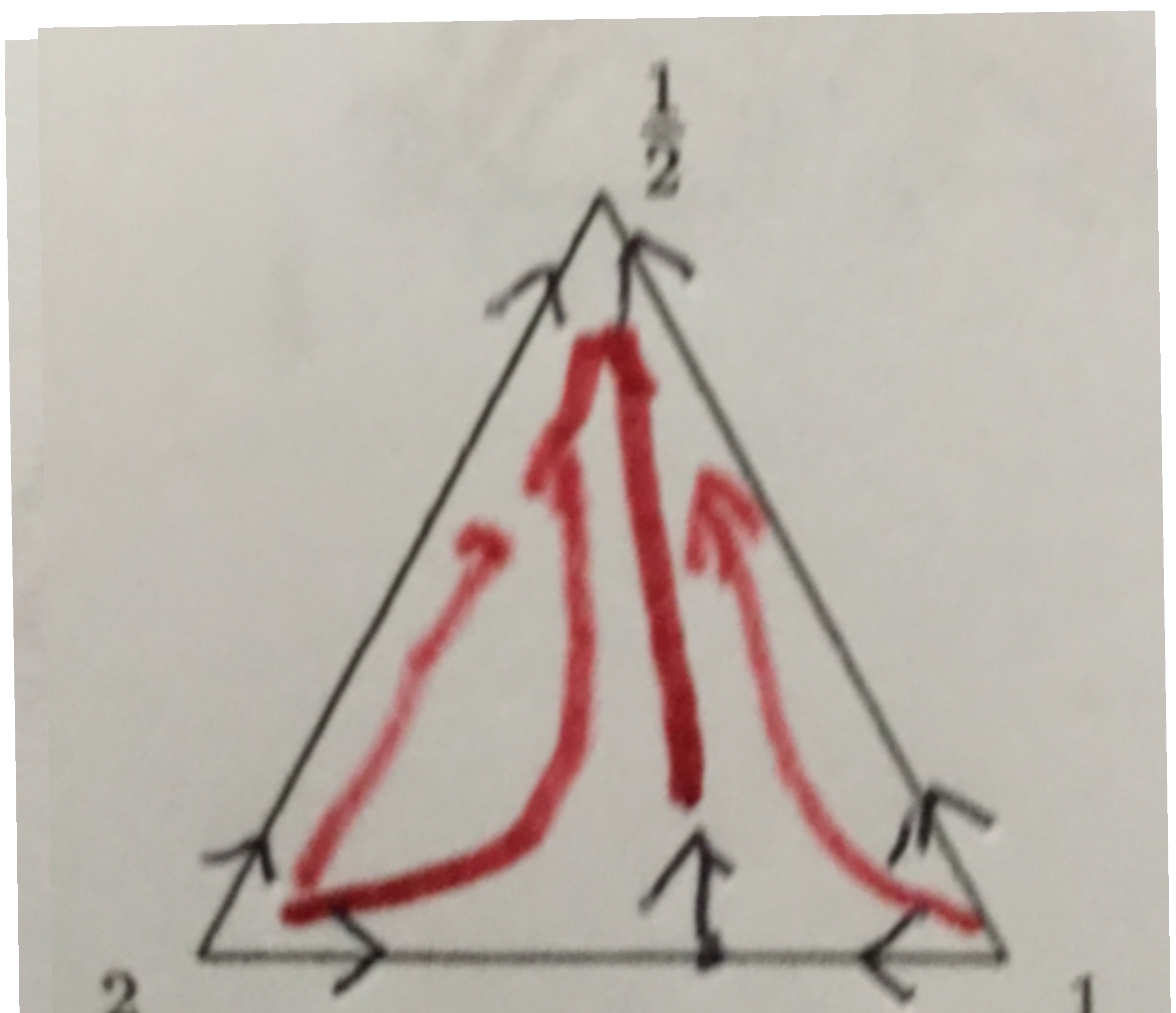
Key is wedding between local information, basic assumption of change ($x'=f(x)$) and data, data, data leading to predictions



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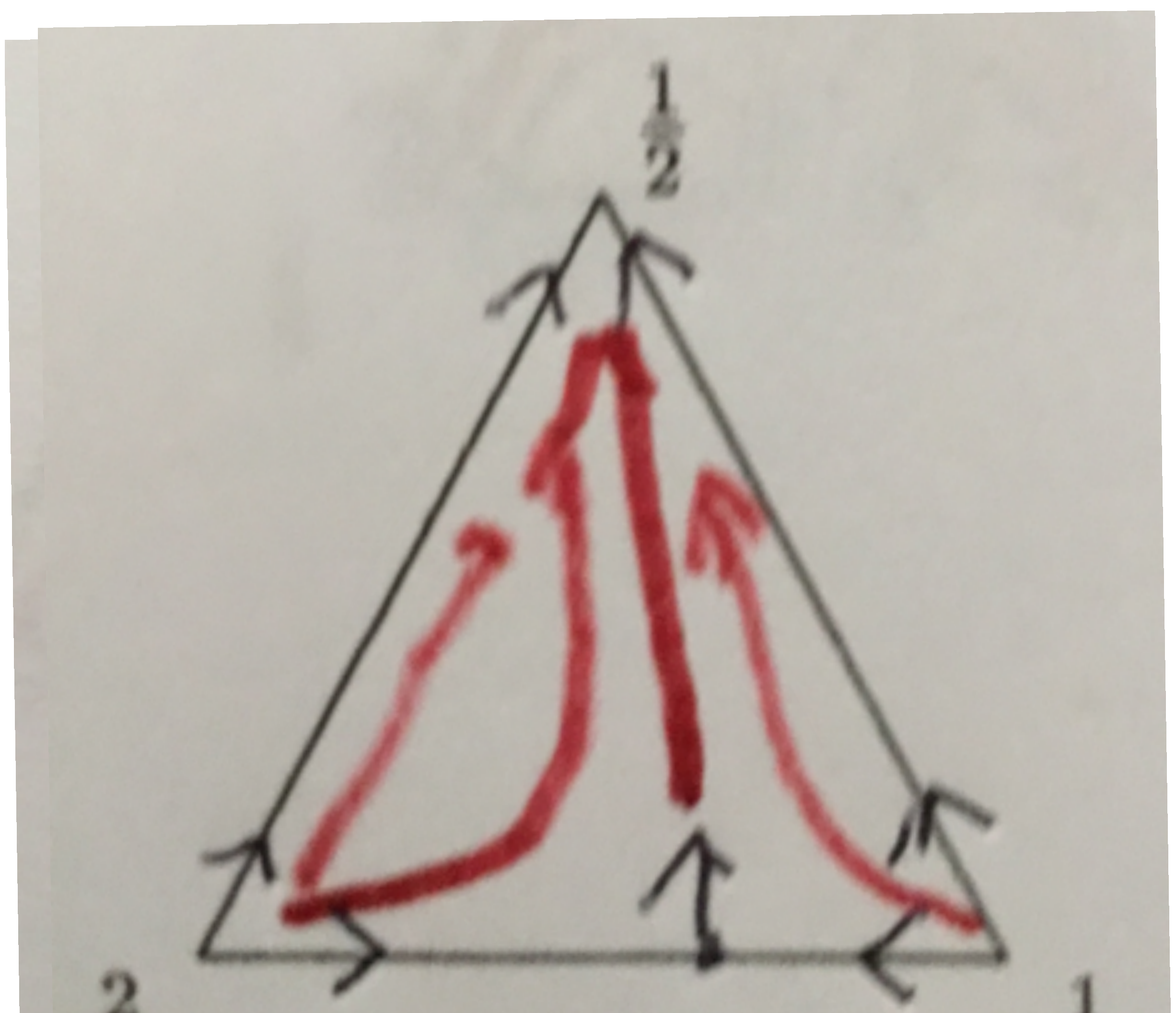

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One approach, but provides new insights and conclusions offers way to narrow down on choice of f(x)