Direct Determination of Characteristic Fluctuation Frequencies from Phase Space Trajectories in Turbulent Plasmas

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Motivation

- Molecular scale effects are important in turbulence and it is important to sample on the appropriate time scales in order to observe them.
- If there is a unified theory of turbulence then it should be characterized in terms of an energy concept and standard parameters which can be related to the physics of the system.
- Recent evidence suggests that a phase transition model based on the concept of turbulent energy may be successful as a generic treatment of turbulent behavior*.

•*Williams, Podder, & Johnson III, Physics Letters A, Vol. 331, Issue (1-2),pp 70-76

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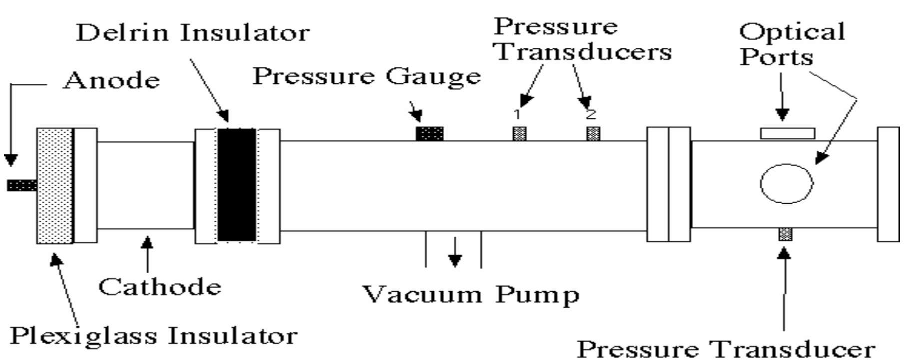
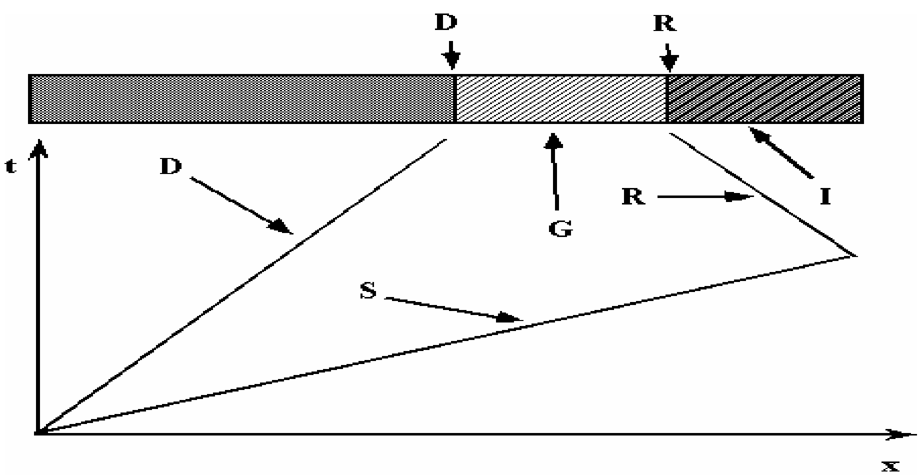
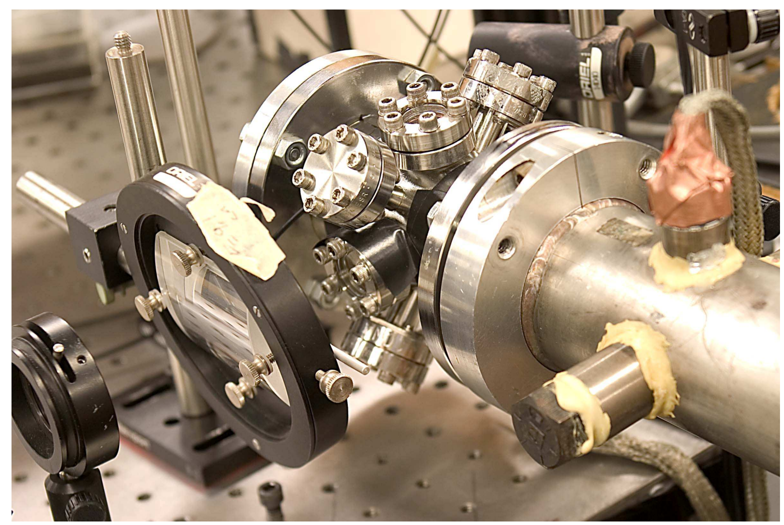
Arc Driven Shock Tube With x-t Diagram: Elements of Plasma Creation and Shock Wave Evolution





- **Discharge Voltage Between 18kV-28kV**
- **Medium is Argon for Shock Tube**
- **PMT Voltage @ -600V**
- **Shock Wave velocity> M 20**
- **Shock Tube Temperature ~3 to 4eV**
- **Shock Tube Density ~ 10^{22} m^{-3} (Primary), ~10^{23} m^{-3} (Reflected), α~0.6**
- **Diagnostics: Pressure Transducers and Laser Induced Fluorescence**



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Applying Ginzburg-Landau Theory to Turbulence

- **The transition to turbulence can be treated as a symmetry breaking disorder-to-order transition, producing a new symmetry breaking parameter (the order parameter, Γ) in the less symmetric (turbulent) phase.**

- **In the transition to turbulence, the state can change continuously through a regime where no distinction can be made between the laminar(non-turbulent) and turbulent state, the critical transition state.**

- **The thermodynamic regime, where a phase transition produces the discontinuous change in the transport parameters(Γ),defines the critical value of the total energy in the system for that thermodynamic state.**

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Applying Ginzburg-Landau Theory to Turbulence Cont'd

- **The order parameter (Γ) should then serve as an expansion parameter in the description of critical phenomena.**

- **The two key elements in determining a representation of transition to turbulence as a critical phenomenon are the determination of an order parameter and the determination of an appropriate field which will drive the relationship between the transport effects and a characterizing energy for the turbulent system.**

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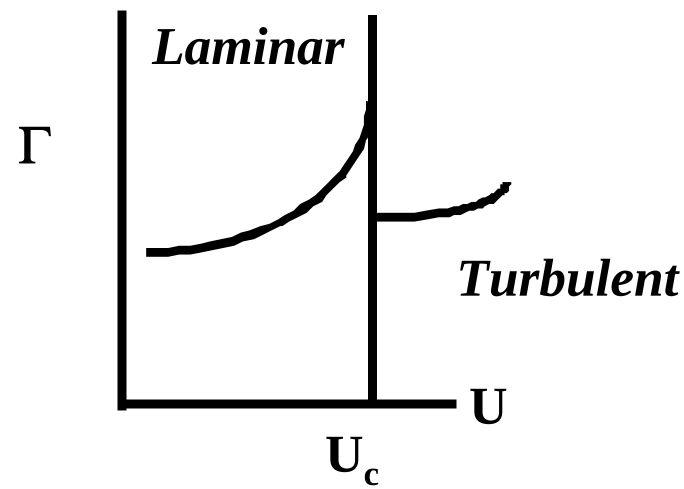
New Physics: Turbulent Systems from a 2nd Order G-L Phase Transformation

- **Discontinuous Γ -like changes should be found in principal transport parameters associated with a force-like constraint in the underlying dynamics.**

- **Significant changes in the degree of complexity are expected.**

- **A turbulent energy concept emerges with unique turbulence defining parameters: spectral index; characteristic frequency; chaotic dimension.**



There have now been many successes from this approach.

The Microscopic Approach

From the point of view of kinetic theory, consider the process that forces two molecules A and B to become, together, the turbulent quasimolecule P (with appropriate classical statistics) and generic particle distribution function as f(z) then

$$\Psi(z,\hat{z}) \equiv f^{II}(z,\hat{z}) - f(z)f(\hat{z})$$

$$\Psi(z,\hat{z}) \rightarrow 0$$

For laminar flow:

where the above condition for laminar flow also produces the Boltzmann equation and the standard equation for fluid dynamics. For turbulent transport parameter Γ, a change in the transport parameter takes the form:

$$\frac{(\Gamma_{App} - \Gamma)}{\Gamma} = \frac{\Psi_{AB}(z_A,\hat{z}_B)}{f_A f_B}$$

Transport parameter Γ is driven to turbulent behavior Γ_{App} by non-zero values of Ψ, ignoring quantum effects.

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The Macroscopic Approach

The turbulent free energy per unit volume takes the form:

$$\alpha_t(X,\varepsilon,\Gamma) = \alpha_l(X,\varepsilon) + \alpha_2(X,\varepsilon)|\Gamma|^2 + \alpha_4(X,\varepsilon)|\Gamma|^4 + \ldots$$

in which the order parameter Γ only appears in the less symmetric phase, the subscripts *t* and *l* refer to the turbulent (less symmetric) and laminar (symmetric) conditions, and ε is the force conjugate to the order parameter.

The Macroscopic Approach

$$\alpha_t(X,\varepsilon,\Gamma) = \alpha_l(X,\varepsilon) + \alpha_2(X,\varepsilon)|\Gamma|^2 + \alpha_4(X,\varepsilon)|\Gamma|^4 + \ldots$$

This form (with α_1=α_3=0) insures that the free energy can be minimized for Γ=0 below the transition and for $\Gamma\neq0$ above the transition. The form of α_2 is chosen to produce the critical value of X_c corresponding to the transition under circumstances where the two phases cannot be distinguished.

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Macroscopic Approach (cont'd)

Using the G-L thermodynamics of a phase transition, the transport parameter Γ is then driven to turbulent behavior Γ_{App} at values $X>X_c$ where

$$\Gamma_{App} - \Gamma(X) = \frac{X_c(\varepsilon)}{2}\frac{\alpha_2(X,\varepsilon)^2}{\alpha_4(X,\varepsilon)}\frac{1}{(X - X_c(\varepsilon))^2}$$

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Bringing Together the Macro and Micro Approaches

We can now produce a generic relationship, ignoring quantum effects, between the thermodynamic parameters for turbulence as a second order phase transition, the order parameter (which is now determined to be derivable from the intermolecular, i.e., microscopic forces and is a function of a complexity parameter) and the particle probability density functions as follows:

$$\frac{X_c(\varepsilon)}{2}\frac{\alpha_2(X,\varepsilon)^2}{\alpha_4(X,\varepsilon)}\frac{1}{(X-X_c(\varepsilon))^2}=\frac{\Psi_{AB}(z_A,\hat{z}_B)}{f_A\,\hat{f}_B}$$

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Bringing Together the Macro and Micro Approaches

$$\frac{X_c(\varepsilon)}{2}\frac{\alpha_2(X,\varepsilon)^2}{\alpha_4(X,\varepsilon)}\frac{1}{(X-X_c(\varepsilon))^2}=\frac{\Psi_{AB}(z_A,\hat{z}_B)}{f_A\,\hat{f}_B}$$

Notice that the chief shortcoming of this result is that the r.h.s. is entirely based upon classical kinetic theory. We are reminded that classical kinetic theory fails to predict the behavior of transport coefficients because of incorrect treatments of inherently quantum mechanical effects.

Chaotic Dimension (D_2)

- **A state vector X_j is constructed from the scalar time series x_j**

$$\vec{X}_j = \left(x_j, x_{j+l}, x_{j+2l}, \ldots\ldots x_{j+(m-1)l}\right)$$

- **Delay time**
- **Correlation Integral**

$$\tau = l\Delta t$$

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{j,k=1;\, j\neq k}^{N} H\left(r - \left|X_j - X_k\right|\right)$$

- **For small values of r the correlation integral behaves as a power of r.**

$$C(r) \propto r^{D_2}$$

- **Where D_2 is the chaotic dimension, an indicator of a system's complexity.**

- **Since the trajectory is cyclical the number of revolutions are also determined.**

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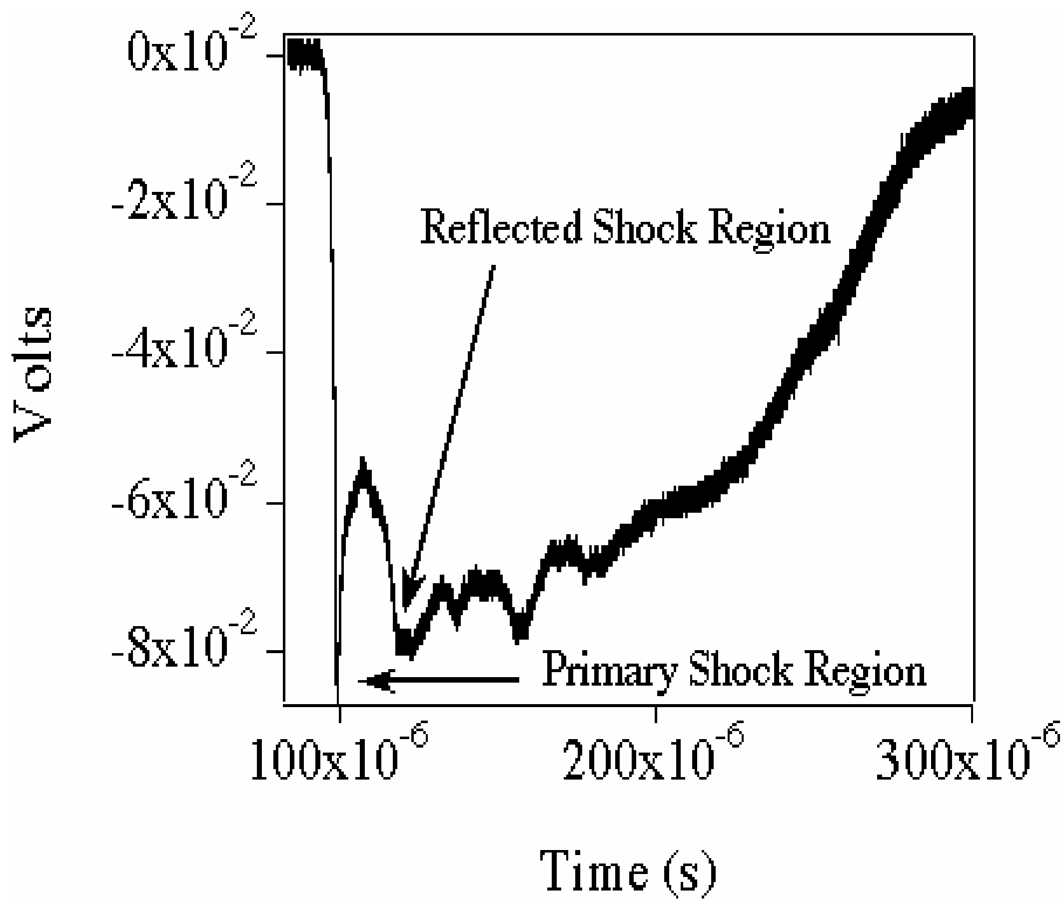
Sample Raw Data: Argon II (422.8nm) Emissions in ADST Plasma



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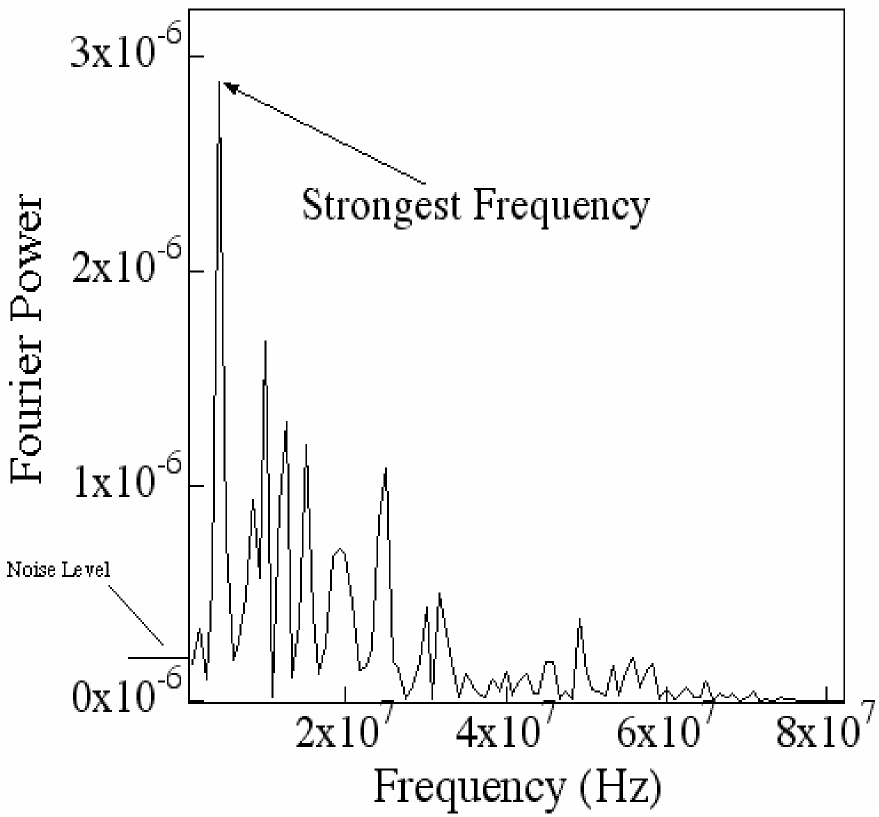
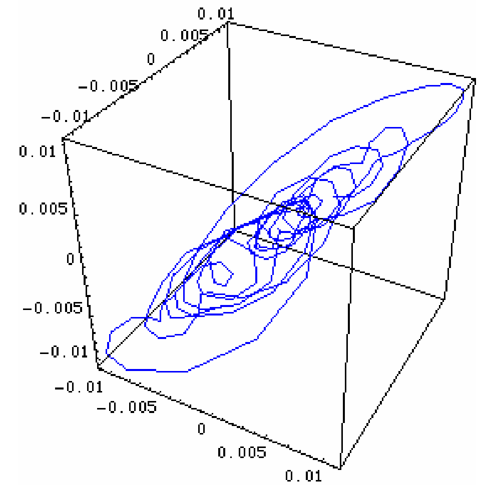
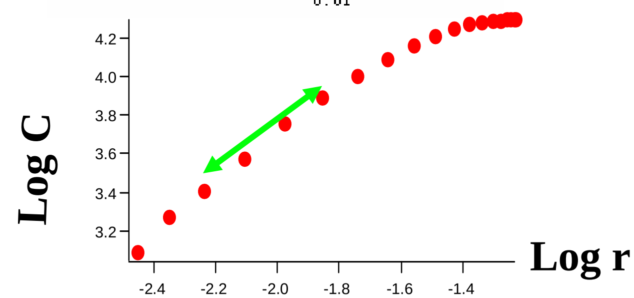
Determination of Key Parameters From Sample Data



Examples of strongest (dominant or characteristic) frequency. Turbulent energy is estimated from $E_\tau = \sum_{i=1}^{noise} P_i$



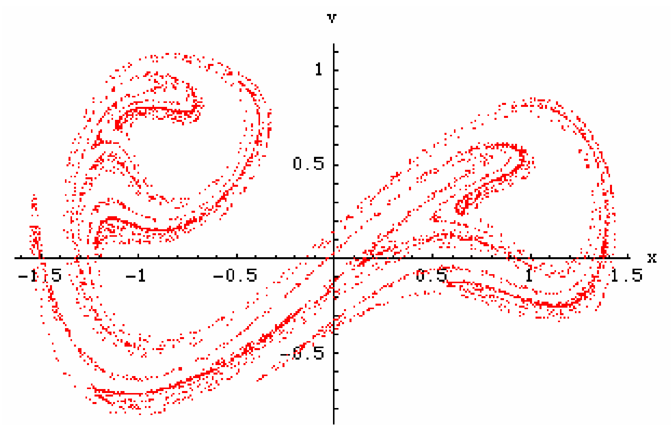


For the sample data above (200 points at 4 ns/sample), the phase trajectory shows regular behavior and D_2=0.941; the green arrow indicates the linear regime for fitting. Frequencies of evolution in phase space are determined by counting the number of cycles made in 200 points with the conversion one cycle corresponds to a frequency of 0.625MHz

Mapping

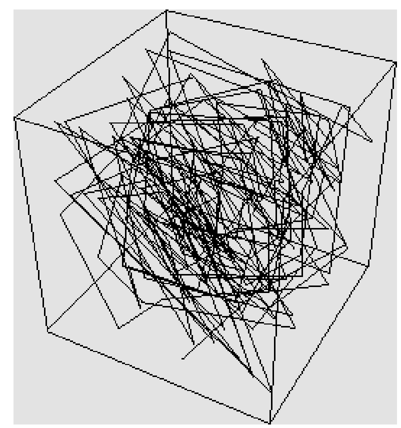
For a physical observable represented in time series data the progression of a nonlinear system at a given state *n* can be studied by investigating the dependence of the (*n*+1) state on the n^{th} state. For example, $x_{n+1}=(6x_n-4)^3$ is a **mapping** used to describe the system progression.

Poincaré section in the chaotic regime for Duffing equation



- **A state vector X_j is constructed from the scalar time series x_j.***

$$\vec{X}_j = \left(x_j, x_{j+l}, x_{j+2l}, \ldots\ldots x_{j+(m-1)l}\right)$$



*F. Takens et al, Lecture Notes in Mathematics, Vol.898 p.366 (Springer)

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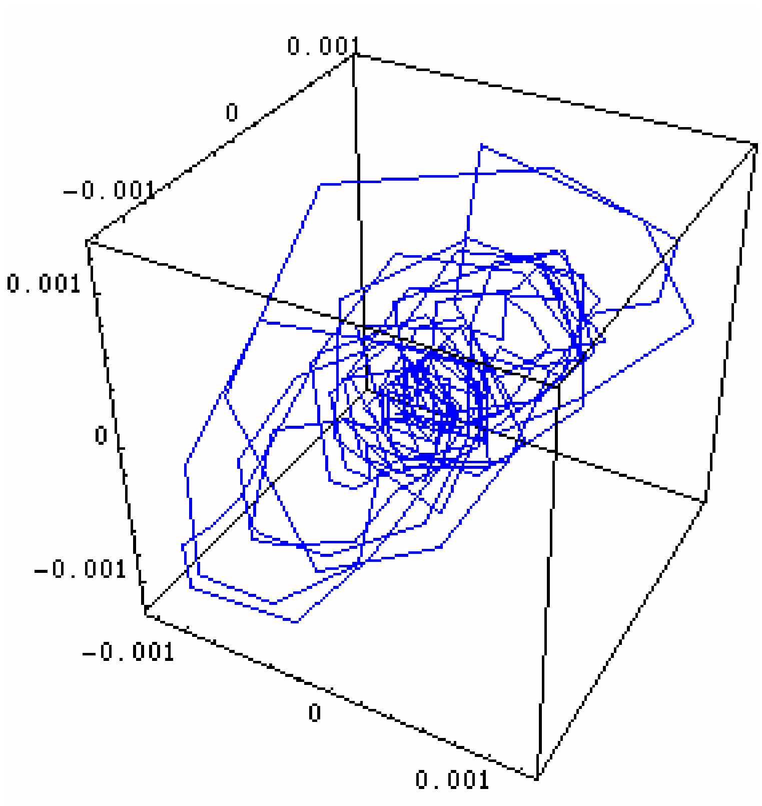
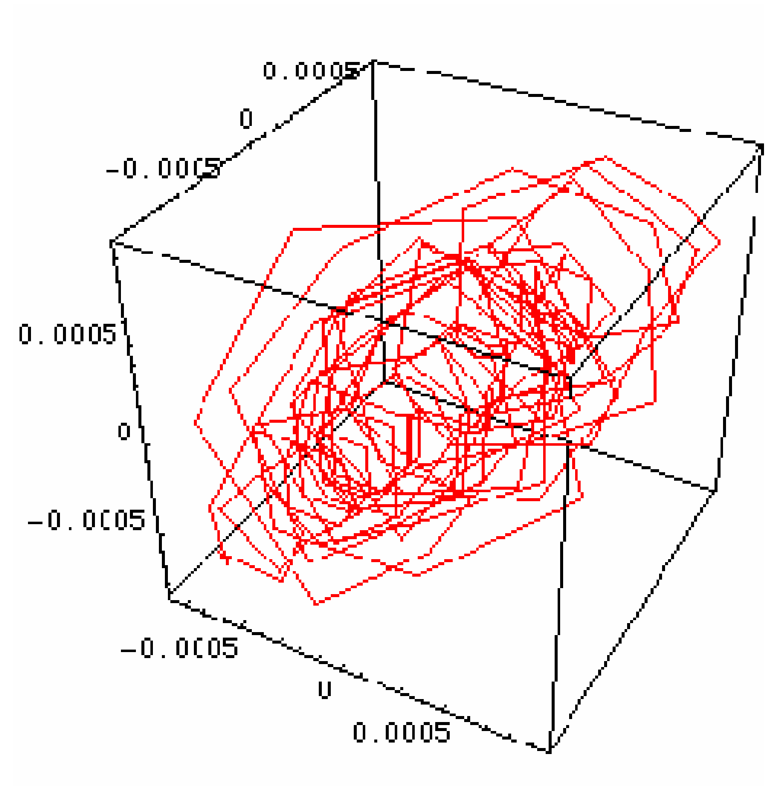
Chaotic Trajectories: Primary and Reflected Shock Regions of ADST
Making the Case for Universality of Turbulence





The cyclic behavior in the two graphs indicate a deterministic process and as well as frequency driven behavior.

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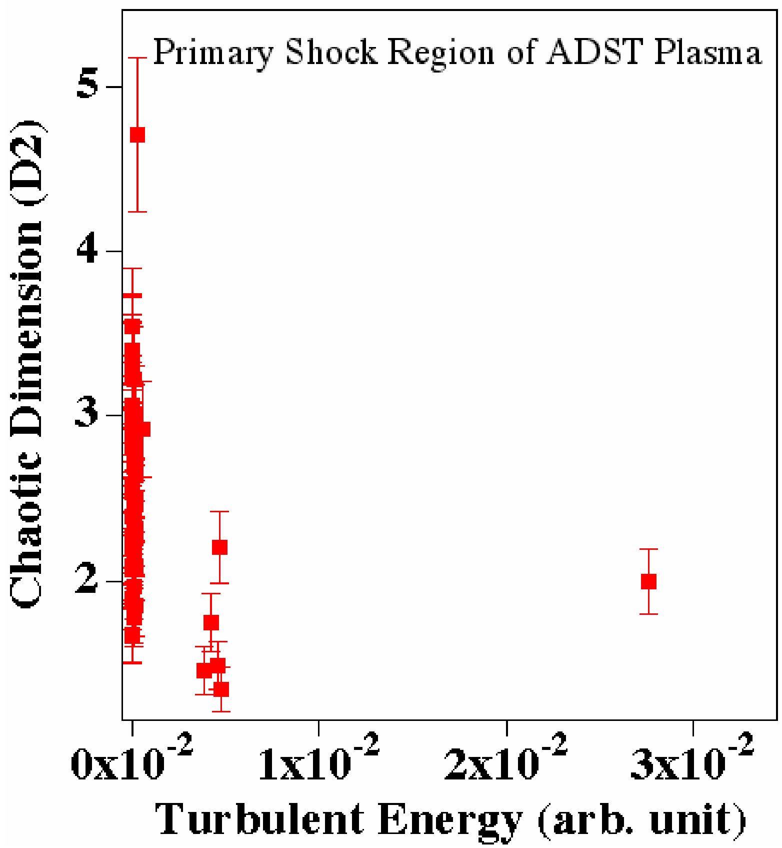
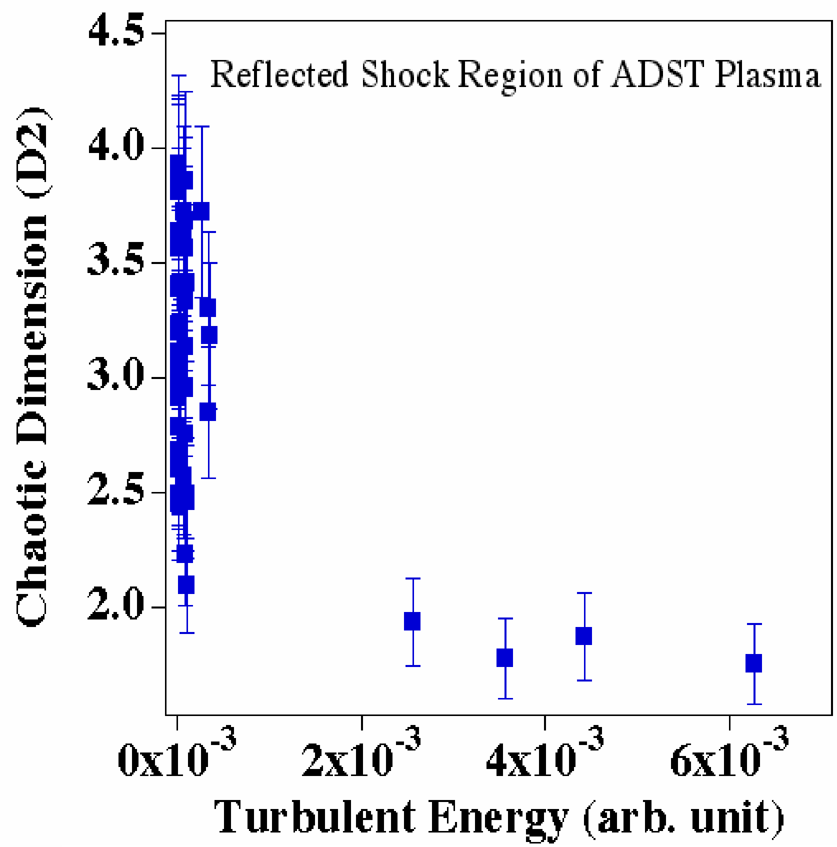
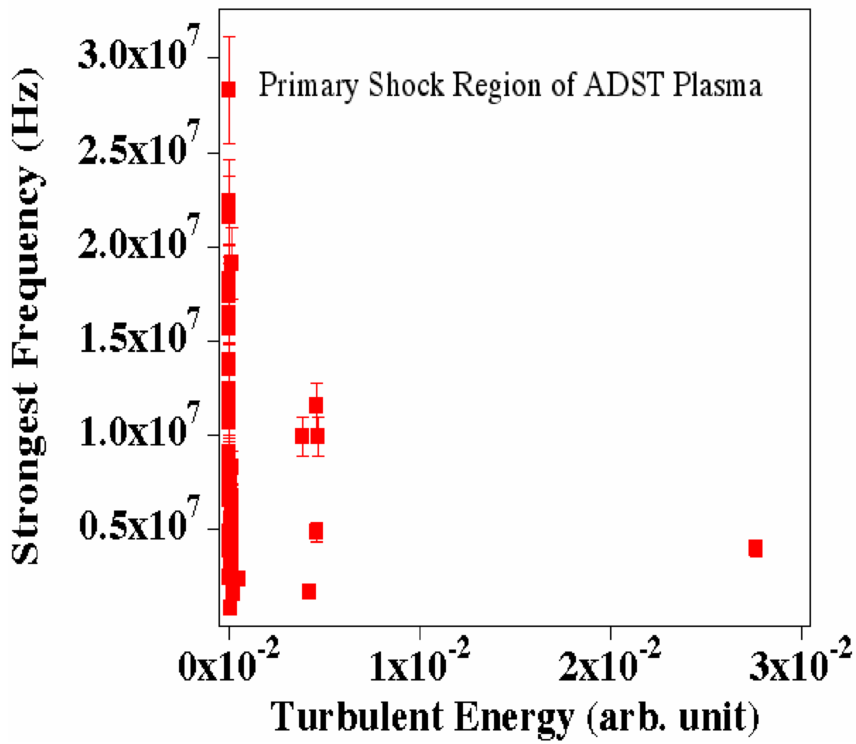
Overall Complexity in the Ionizing Shock Waves Plasmas

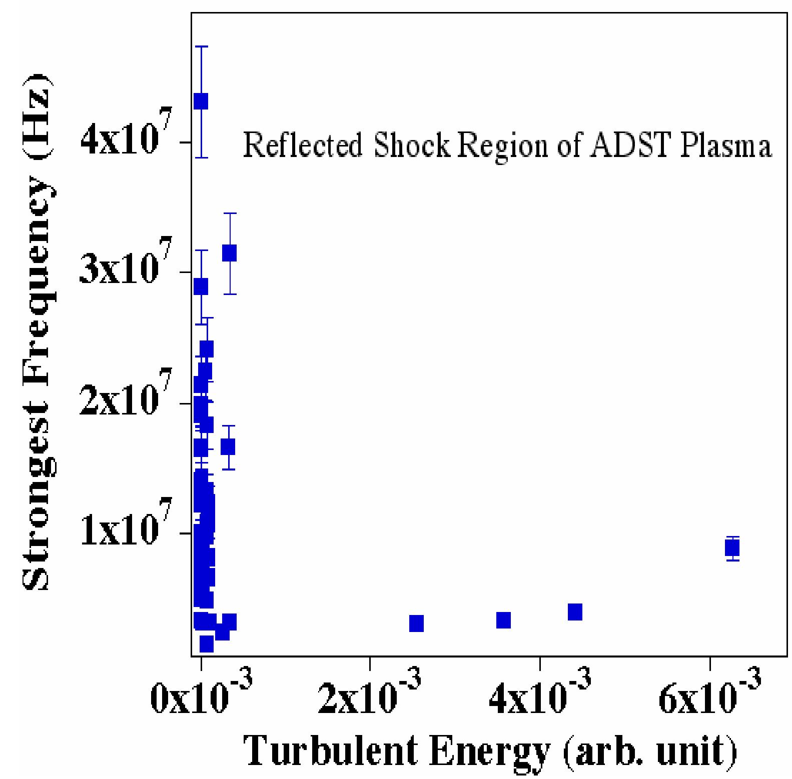




The chaotic dimension D_2 shows a Γ –like behavior with increasing turbulent energy.

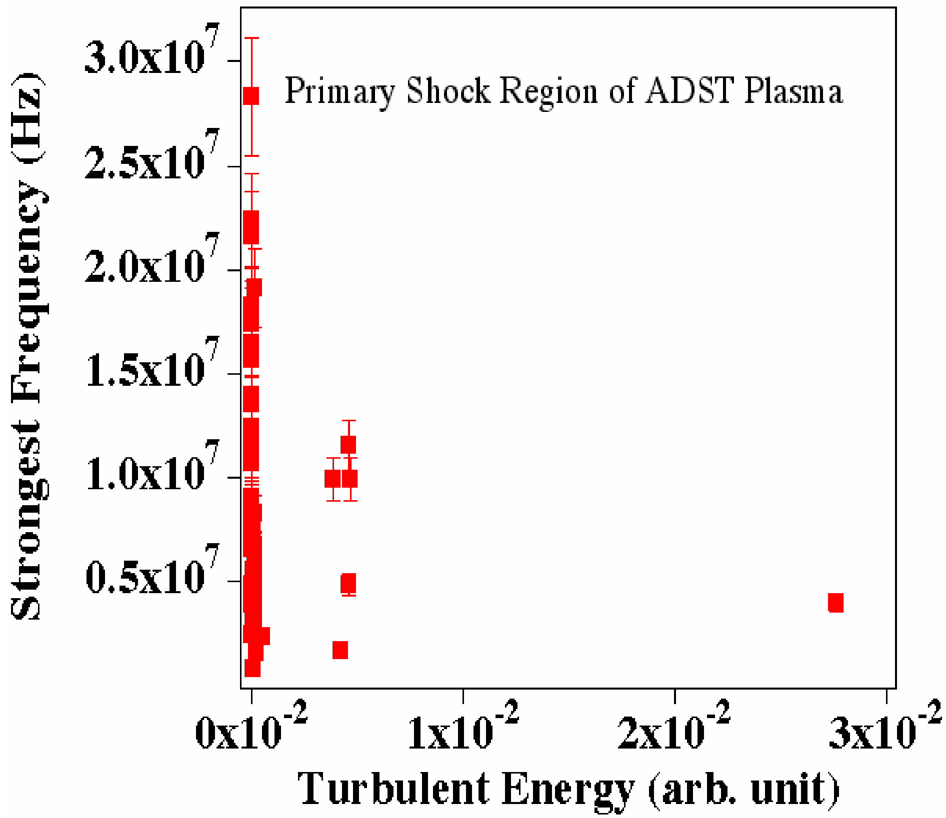
Characteristic Freq vs. Turbulent Energy

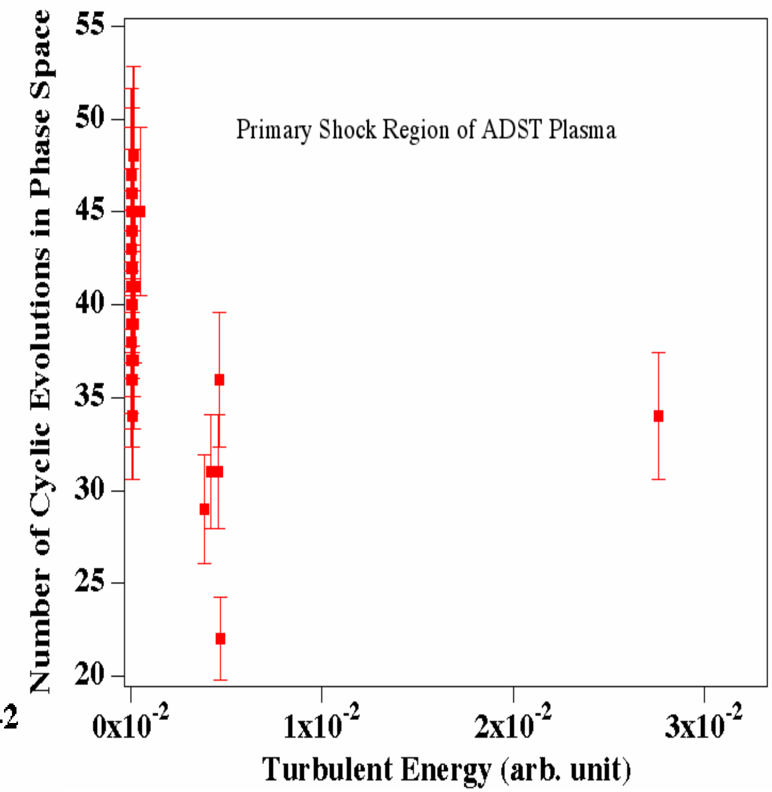




The strongest frequency, defined as the characteristic frequency, shows a Γ –like behavior with increasing turbulent energy.

Sample Comparisons: Frequency in the Fourier Spectrum with Frequency from Phase Trajectory vs. Turbulent Energy in the Primary Shock Region





The overall trends in the characteristic frequency are mirrored in direct measurements of frequency from the phase space trajectory for the moving plasma.

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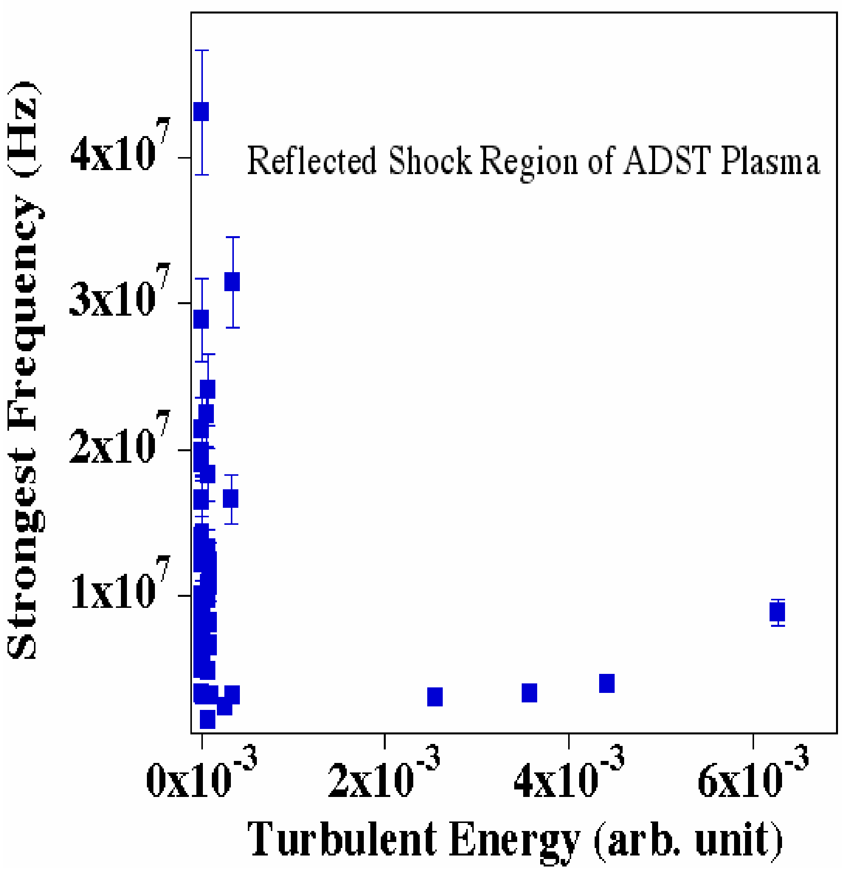
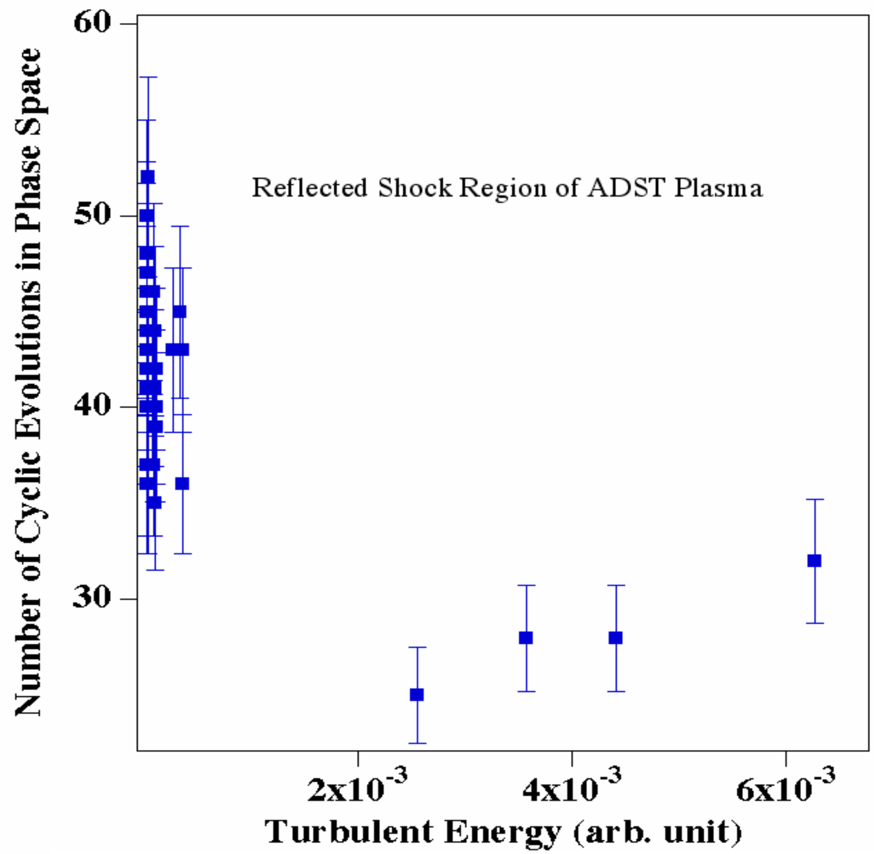
Sample Comparisons: Frequency in the Fourier Spectrum with Frequencies from Phase Trajectories vs. Turbulent Energy in the Reflected Shock Region

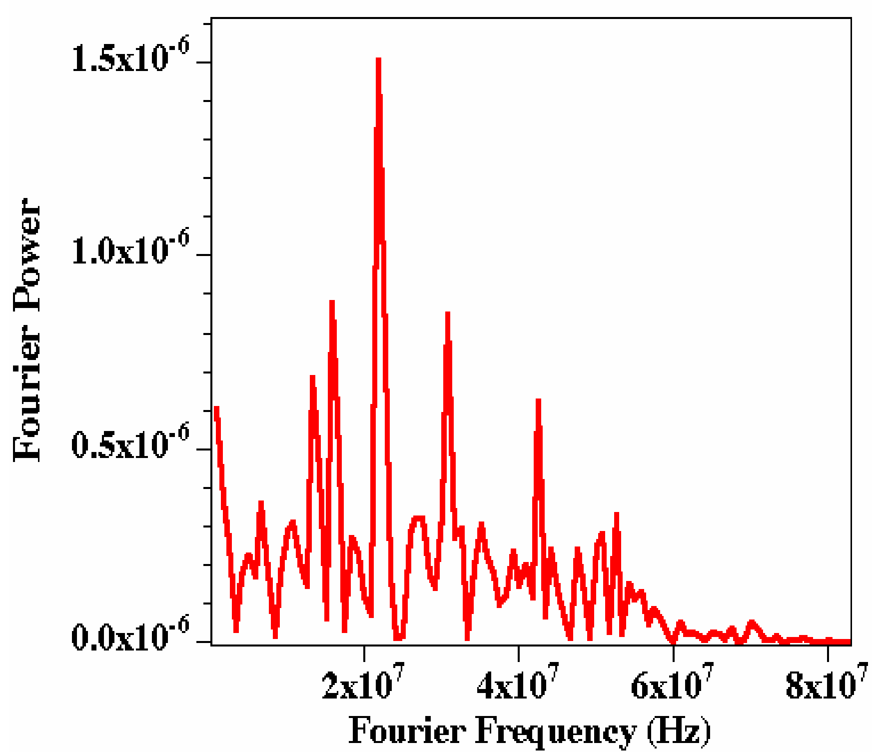


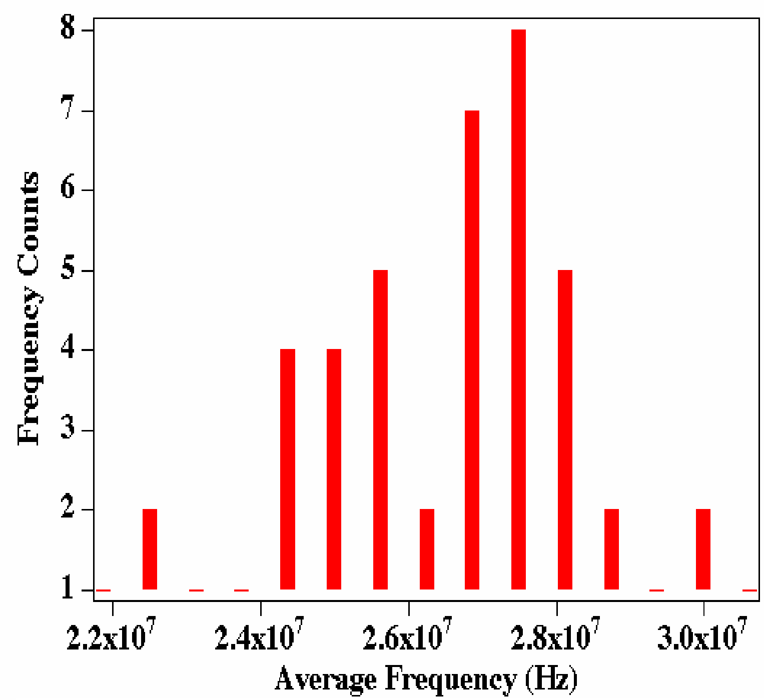


The overall trends in the characteristic frequency are mirrored in direct measurements of frequency from the phase space trajectory for the plasma at rest*.

***K. Belay,J. Valentine,R.L.Williams, J.A. Johnson, J. Appl. Phys. V 81,Iss 3 (1997)**

Sample: Overall Correlation of Frequency Determinations





On the left is a Fourier Power Spectrum from the turbulent plasma behind an ionizing shock wave. On the right, for the same data, is a histogram of frequencies measured using the phase space trajectories. The results are comparable.

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Summary

•The data indicate that the descriptors of turbulence (turbulent energy, characteristic frequency, and chaotic dimension (D2)) successfully show turbulence in both stationary and moving plasmas with the Γ-like behavior associated with a second order phase transformation, justifying a search for a force-like constraint.

•A direct correlation is found between the cyclic regular behaviors in the phase space trajectories and the characteristic fluctuation frequencies in both moving and stationary plasmas.

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Summary

- **Since these regular behaviors in phase space evolutions are generally thought to be a consequence of hidden underlying determinism, this consequence might also now be associated with the characteristic frequencies in our data as the basis for continuing studies of deterministic dynamical underpinnings in turbulence physics.**

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