Quantum phase estimation in the language of digital signal processing

Sukin Sim (Dylan)

PsiQuantum

IPAM BNF2026 Workshop

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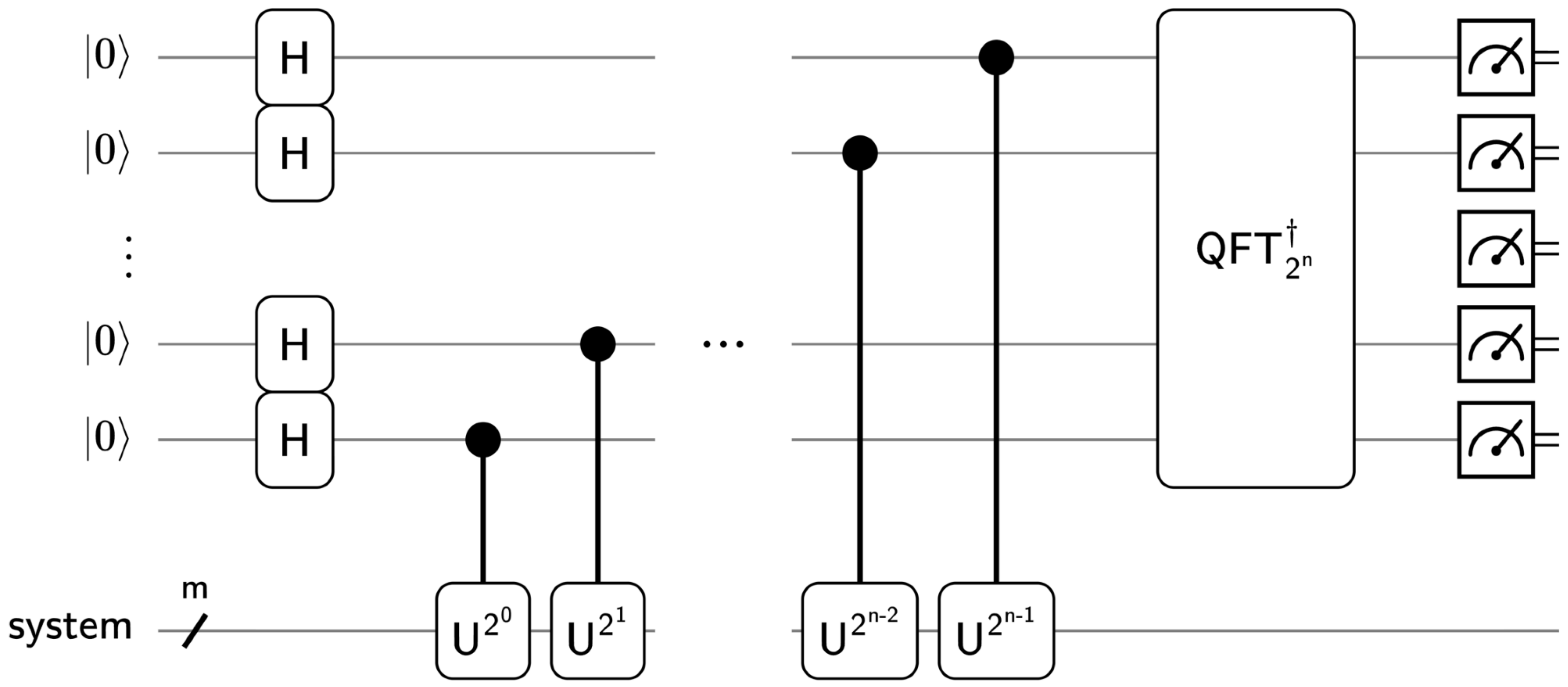
PsiQuantum

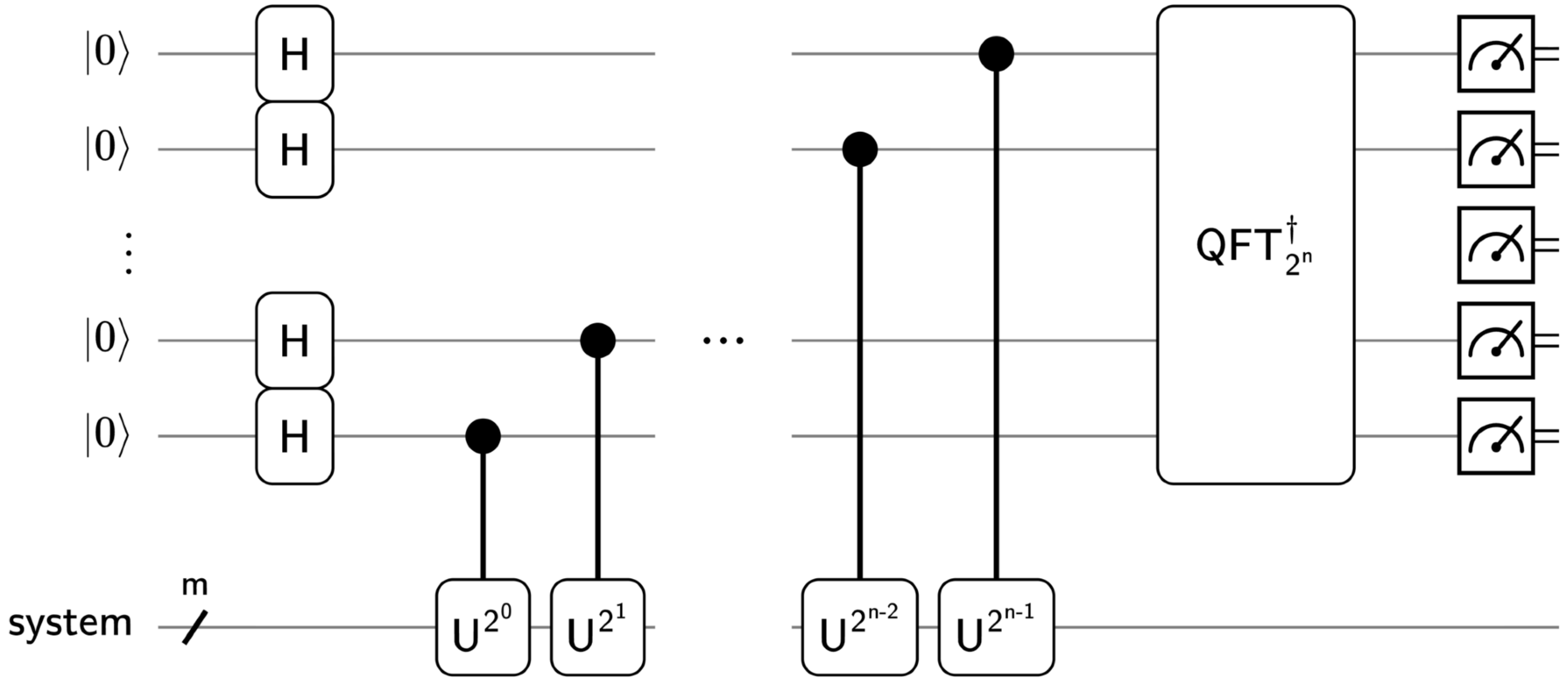
Quantum phase estimation (QPE)



$$U|\psi\rangle = e^{2\pi i\phi}|\psi\rangle, \quad \phi \in [0,1)$$

Kitaev, A. *arXiv* (1995) arxiv.quant-ph/9511026.

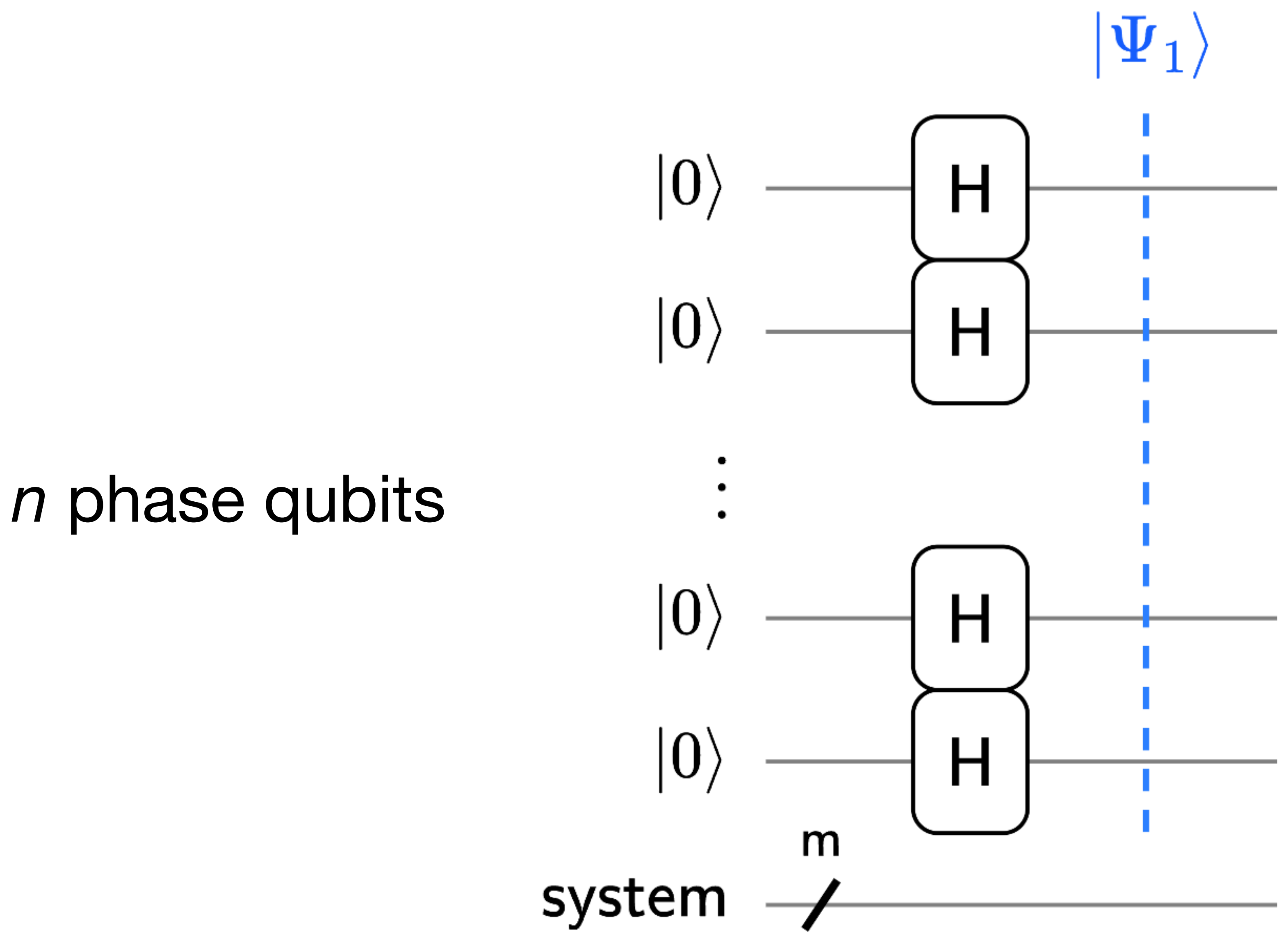
Quantum phase estimation (QPE)



- Used as a subroutine in many quantum algorithms
- Gap between how textbook QPE is introduced vs. improvements in literature

Kitaev, A. *arXiv* (1995) arxiv.quant-ph/9511026.

Walking through QPE

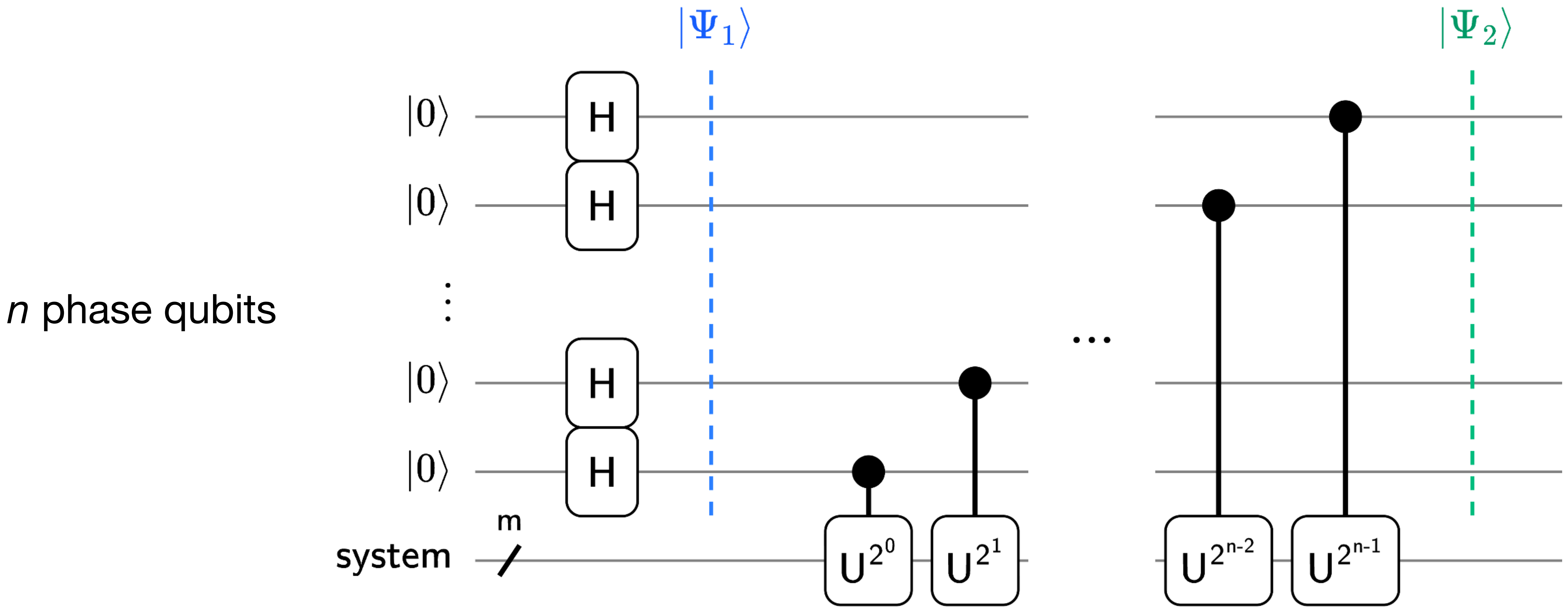


$$\frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} |k\rangle$$

Superposition

Assuming perfect eigenstate preparation

Walking through QPE

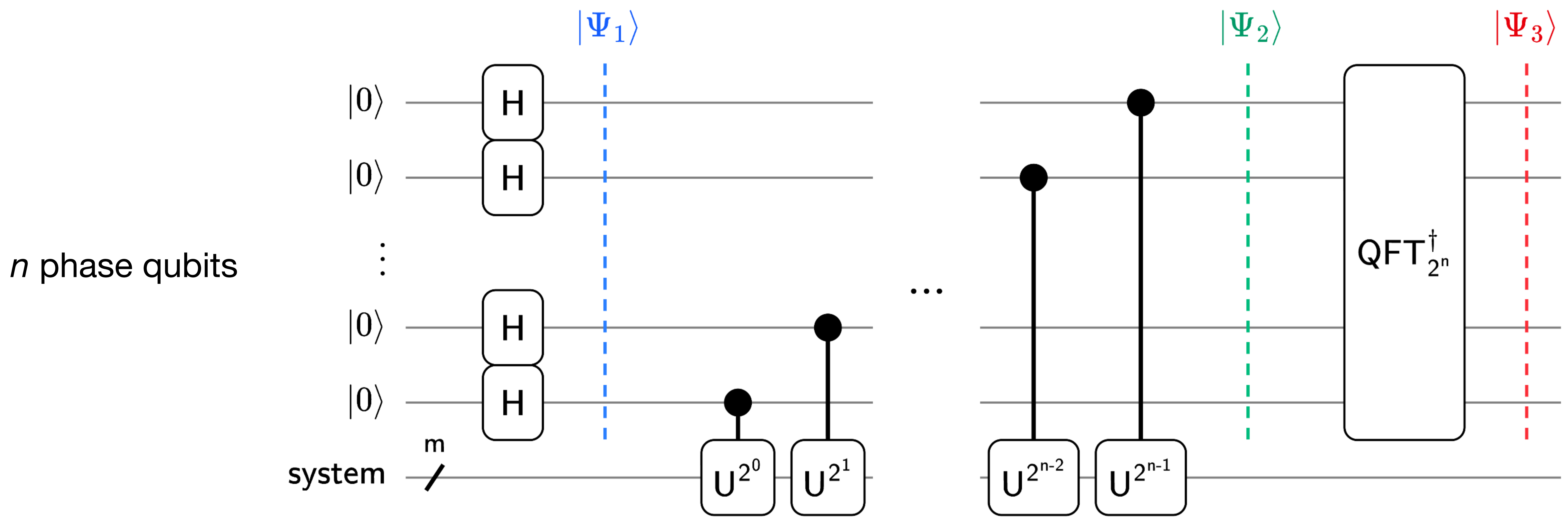


$$\frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{2\pi i k \phi} \,|\, k\rangle$$

Controlled powers of U

Assuming perfect eigenstate preparation

Walking through QPE



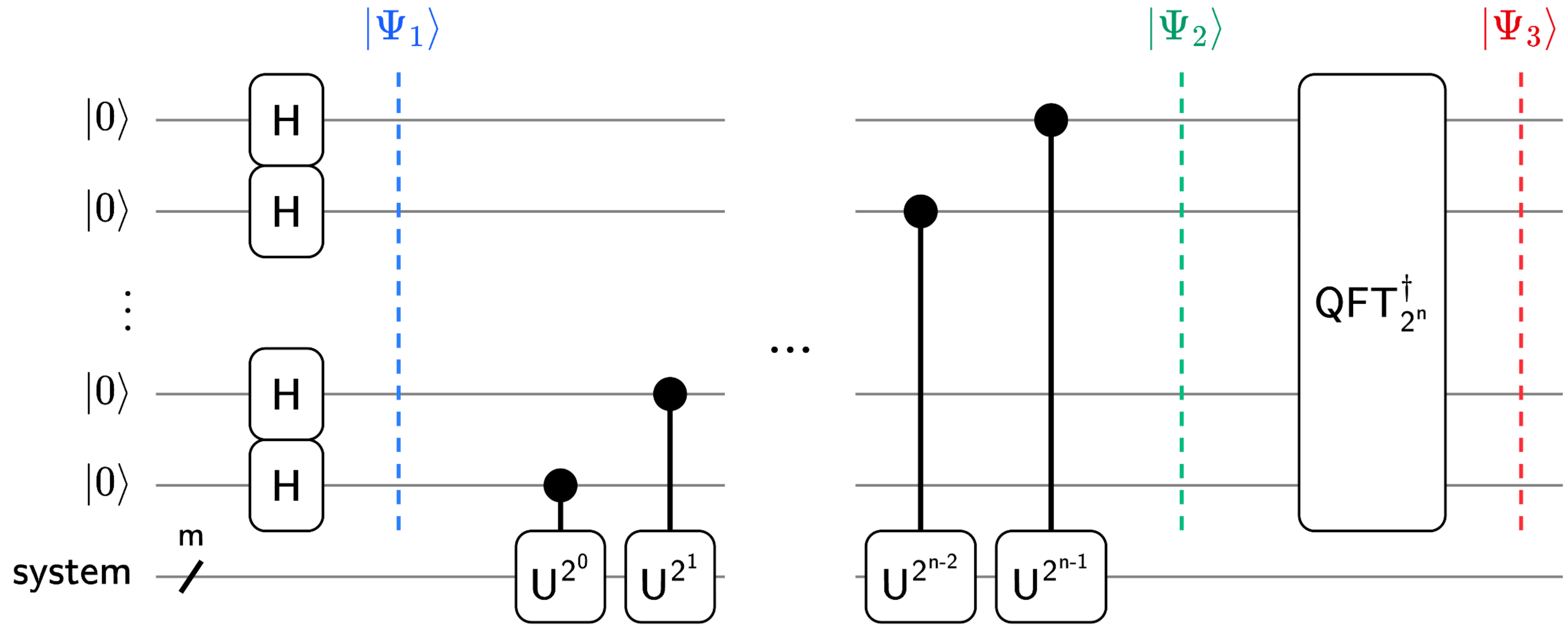
$$\frac{1}{2^n}\sum_{x=0}^{2^n-1}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi} e^{\frac{-2\pi i k x}{2^n}} |x\rangle$$

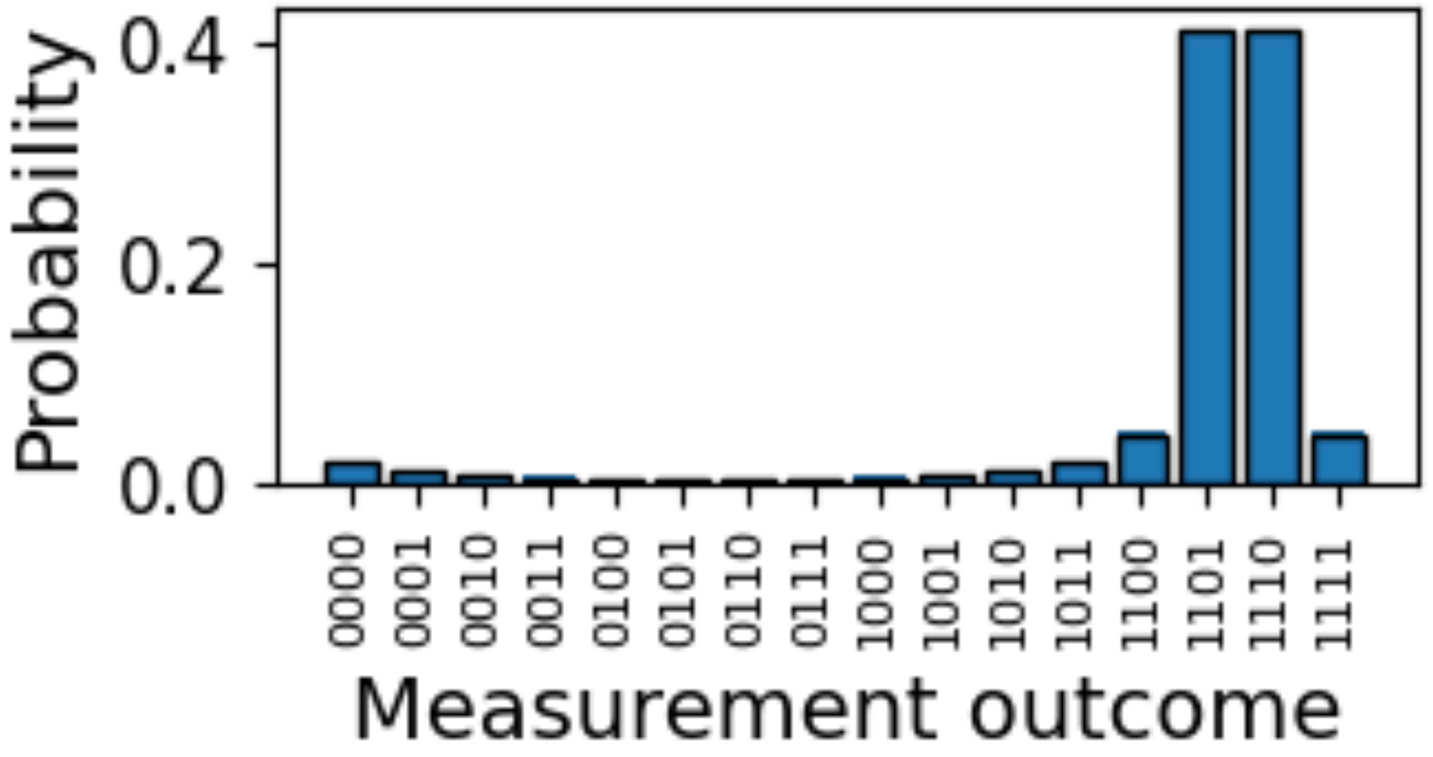
Inverse QFT

Assuming perfect eigenstate preparation

Walking through QPE

n phase qubits





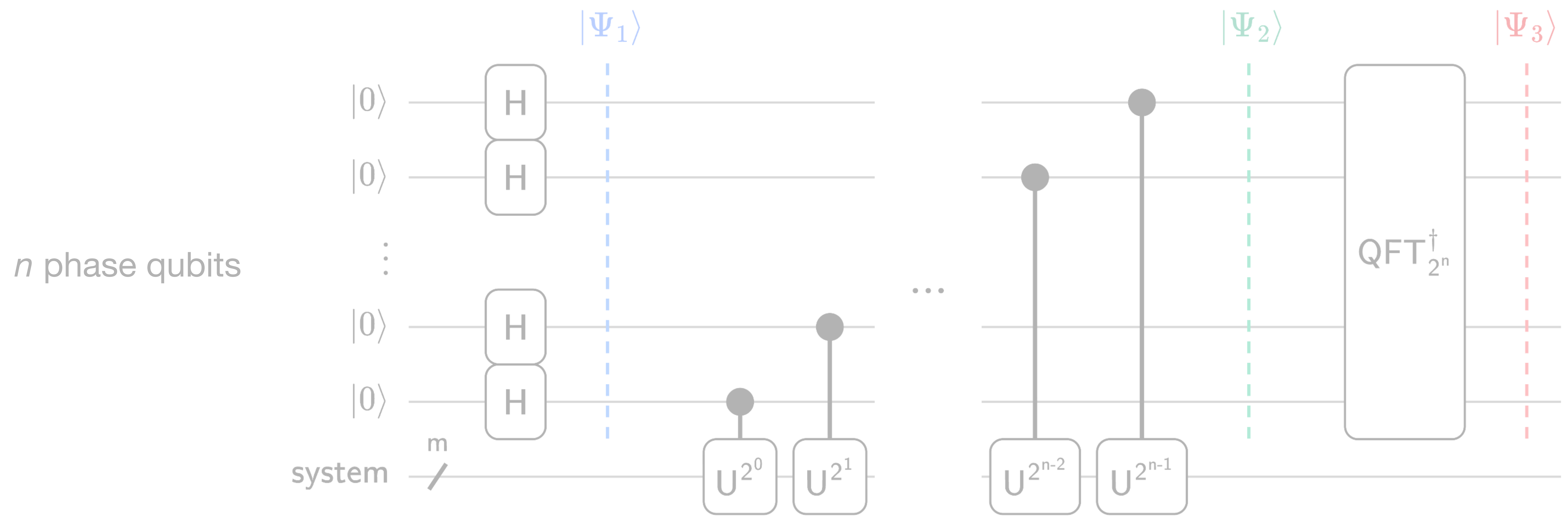
Outcome distribution

$$P(y) = \left| \frac{1}{2^n} \sum_{k=0}^{2^n-1} e^{-\frac{2\pi i k}{2^n}(y - 2^n \phi)} \right|^2$$

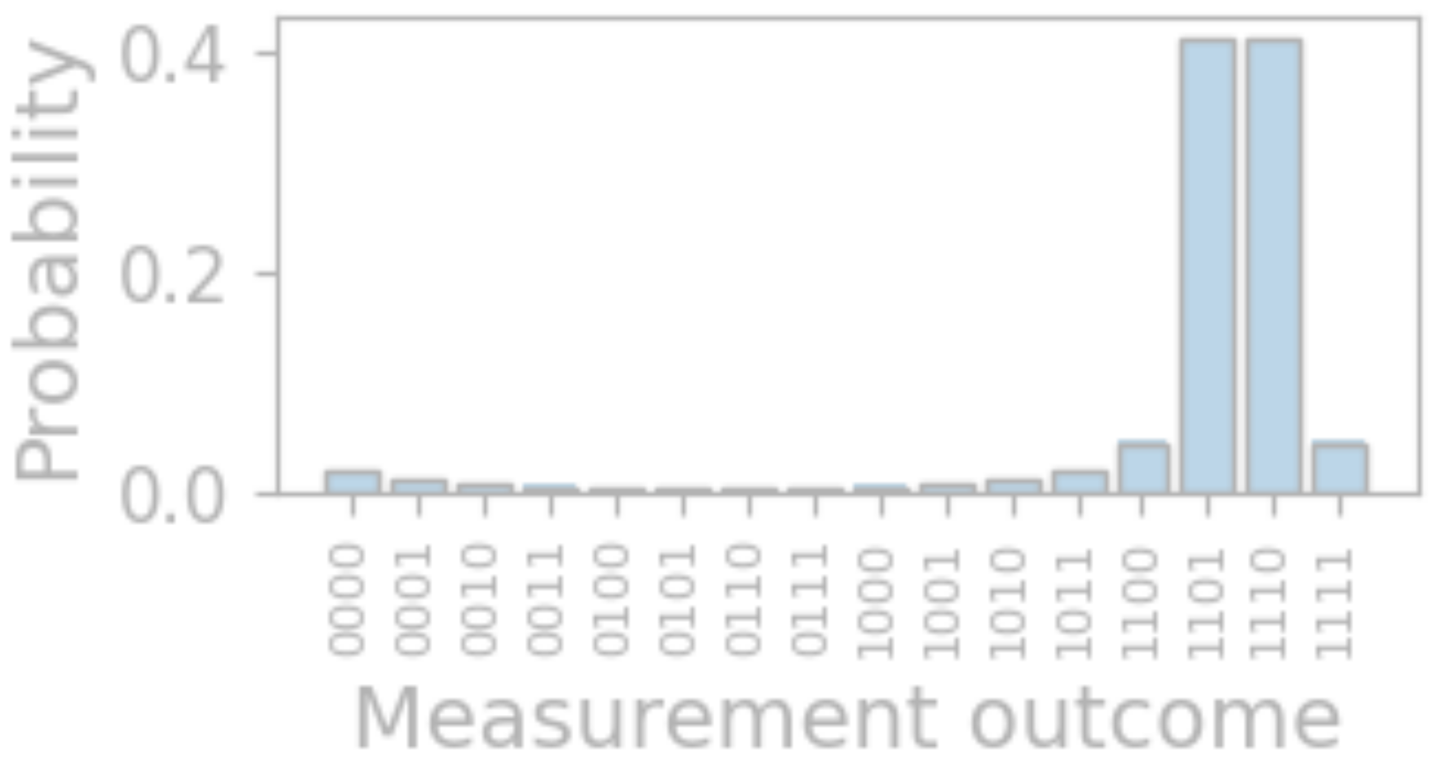
error, success probability

Assuming perfect eigenstate preparation

Walking through QPE



How should we understand (recent) improvements?



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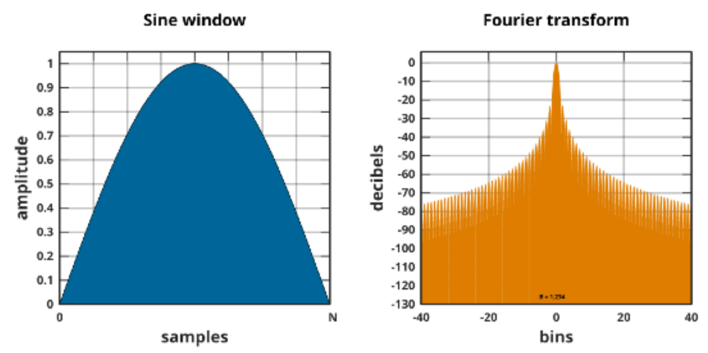
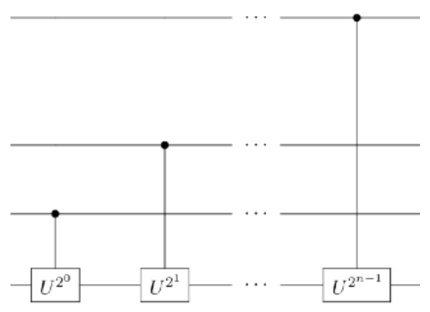
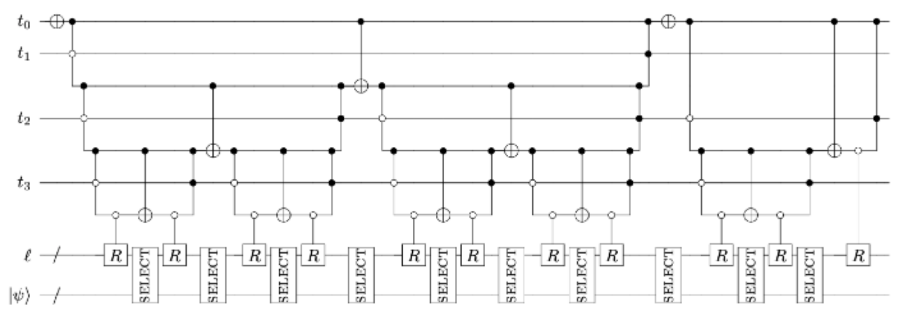
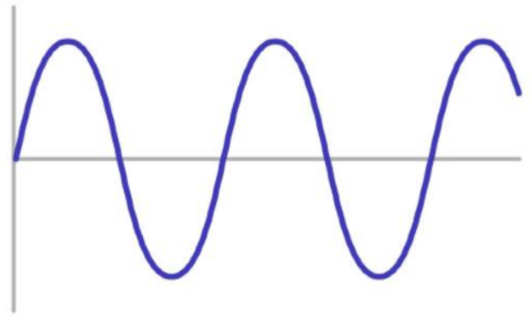
Assuming perfect eigenstate preparation

QPE features and improvements









Window/probe states

Cosine

Sine

Kaiser

Slepian

Gaussian

m-th B-spline

Control structure for phase kickback

Unary iteration

Optimized SELECT using temporary AND

Bidirectional phase kickback

Encoding spectral information

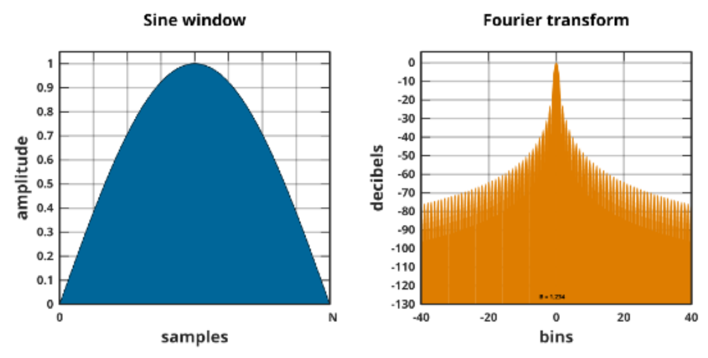
Qubitization

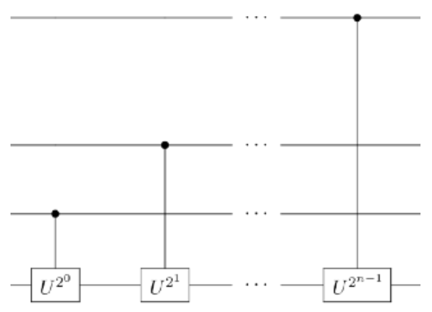
Trotterization

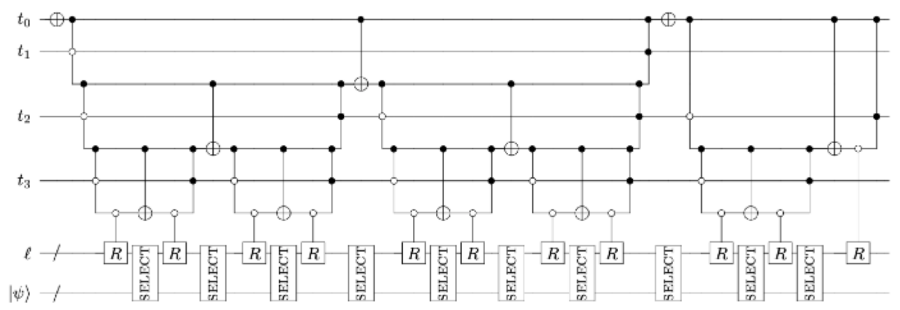
QSVT

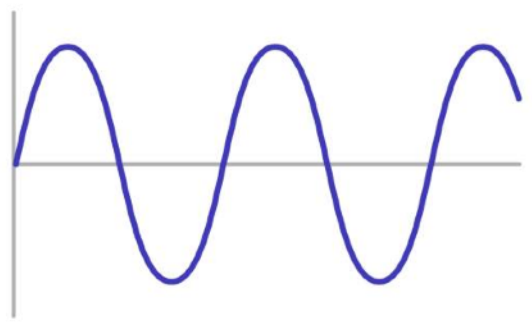
and more!

QPE features and improvements









Window/probe states

Cosine
Rendon (2022)

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Luis (1996), Babbush (2018), Navajo (2023)

Kaiser
Sanders (2020), Berry (2024), Kristjuhan (2026)

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Unary iteration
Lee (2021)

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Bidirectional phase kickback
Wecker (2015), Babbush (2018) , Kivlichan (2020), Simon (2025)

Encoding spectral information

Qubitization
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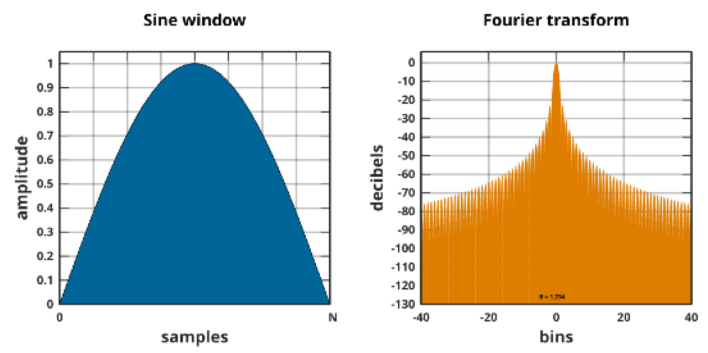
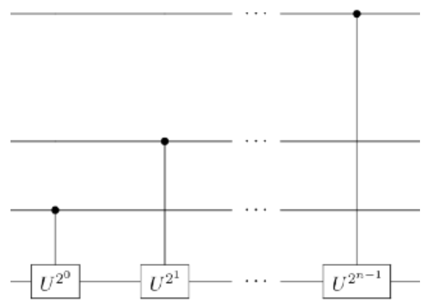
Gilyen (2019), Martyn (2021), Rall (2021)

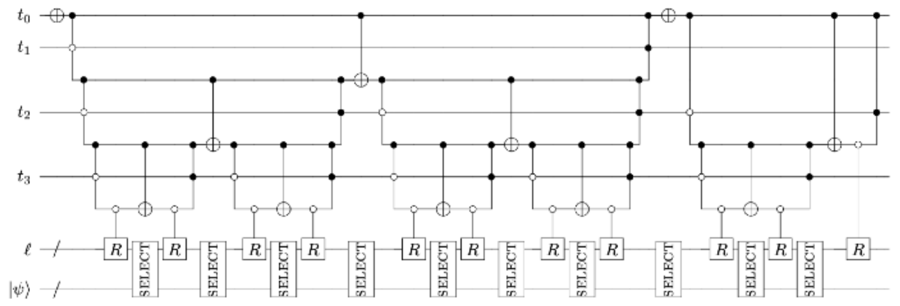
and more!

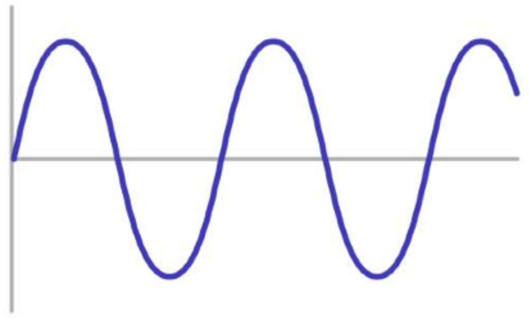
Citations on final slide

QPE features and improvements









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and more!

How should we understand (recent) improvements?

How to mix and match?

Citations on final slide

Programming QPE

class TextbookQPE

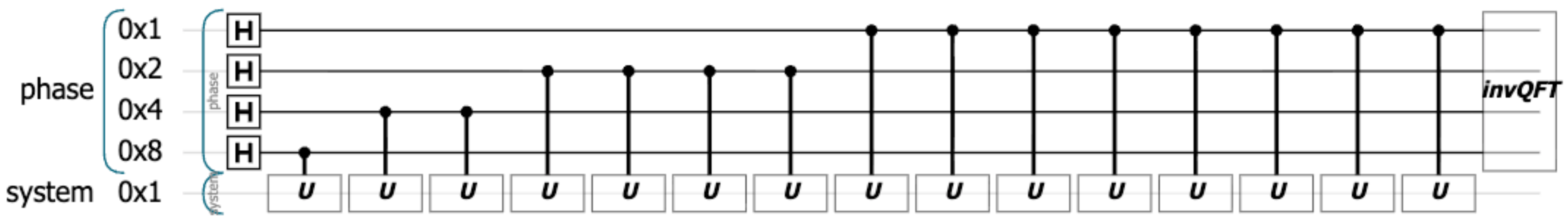
```
# Superposition
phase_reg.had()


# Controlled-(U^k) operations
for i in range(phase_reg.num_qubits):

    for k in range(2**i):
        unitary.compute(system_reg, ctrl=phase_reg[i])


# Inverse QFT
inv_QFT.compute(phase_reg)
```

Workbench code



(Circuit diagram from Workbench)

Programming QPE

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inv_QFT.compute(phase_reg)
```

Workbench code

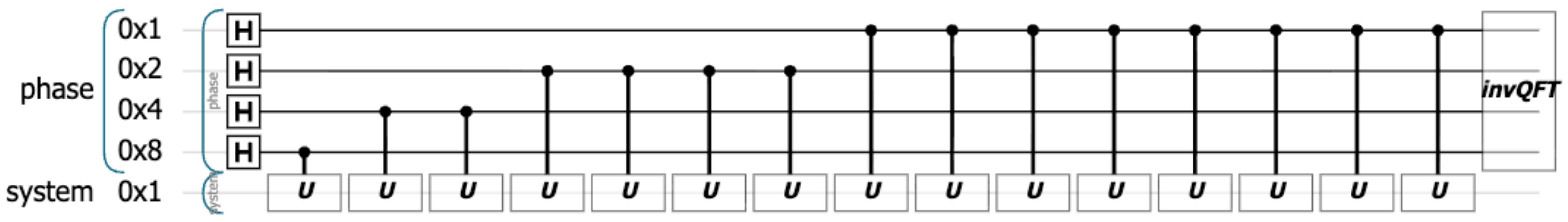
`class WindowedQPE`

`class BidirectionalKickbackQPE`

`class UnaryIterationQPE`

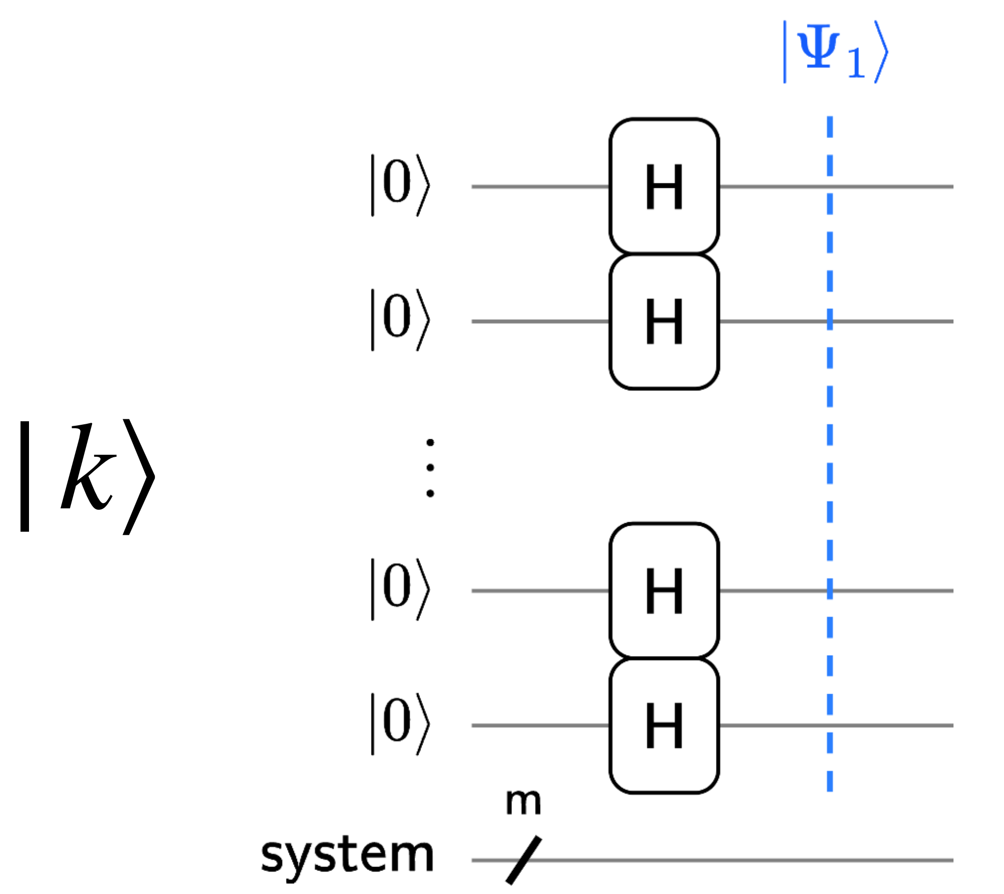
*Can lead to a lot of **code duplication** and difficult to mix and match improvements..*

Re-think QPE?



(Circuit diagram from Workbench)

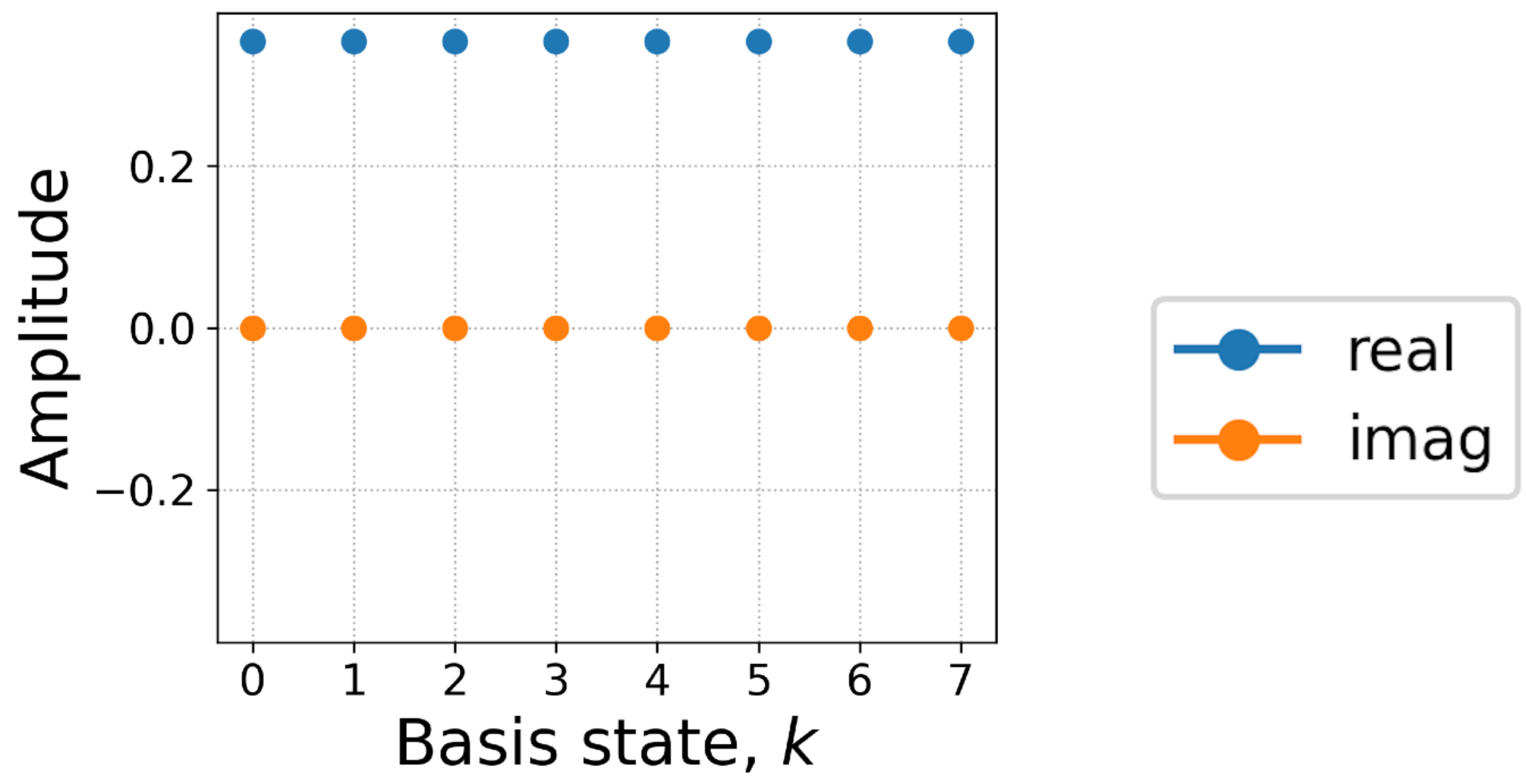
Revisiting QPE



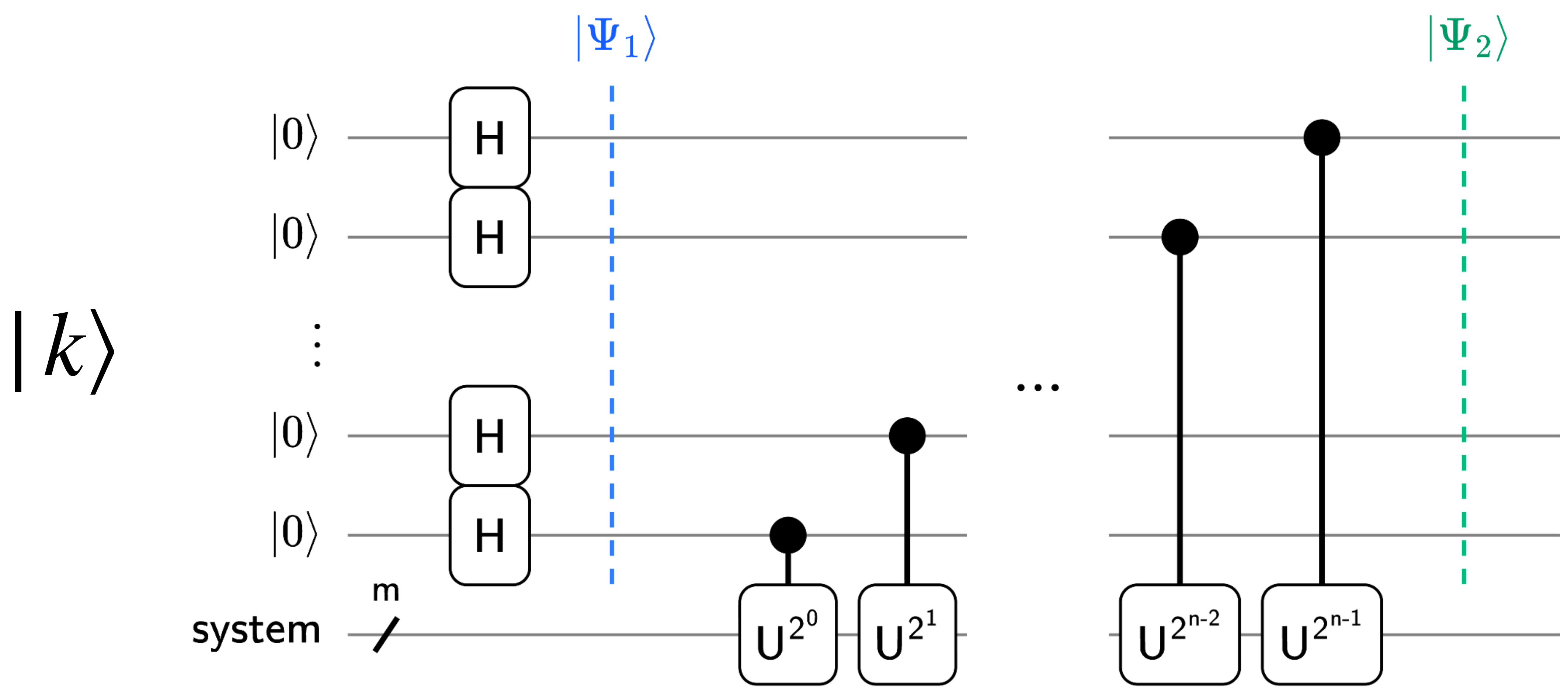
$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1}|k\rangle$$

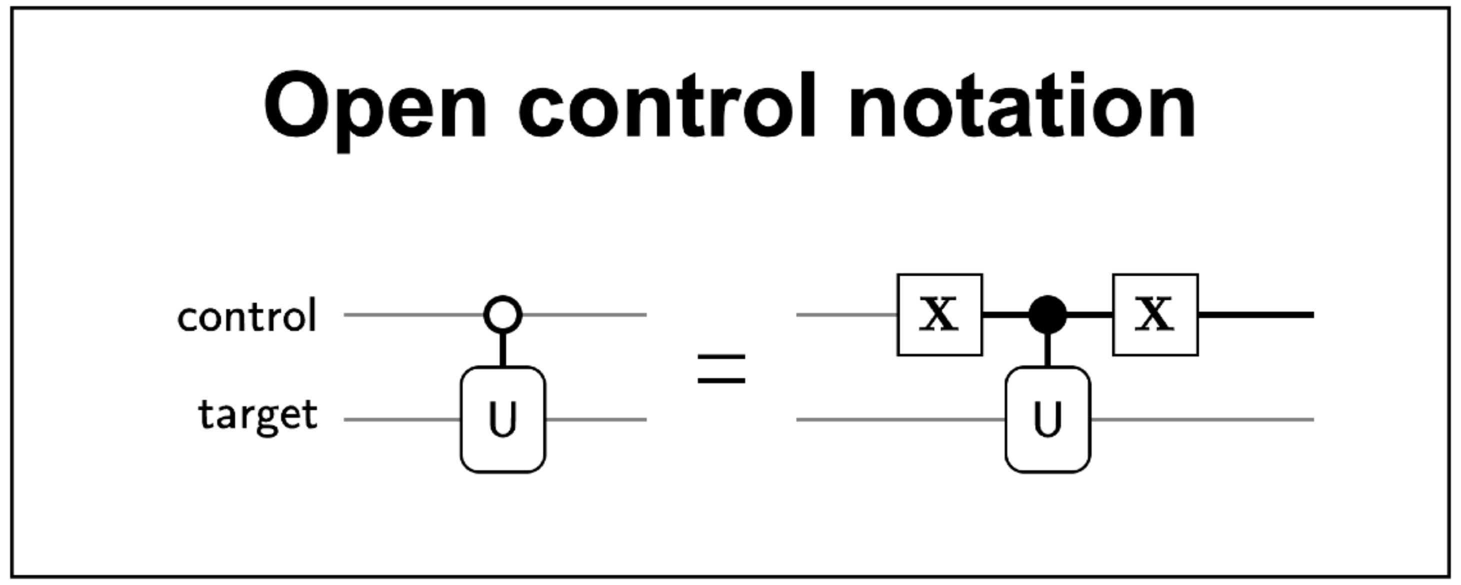
Initializing the register

(to nonzero amplitudes)

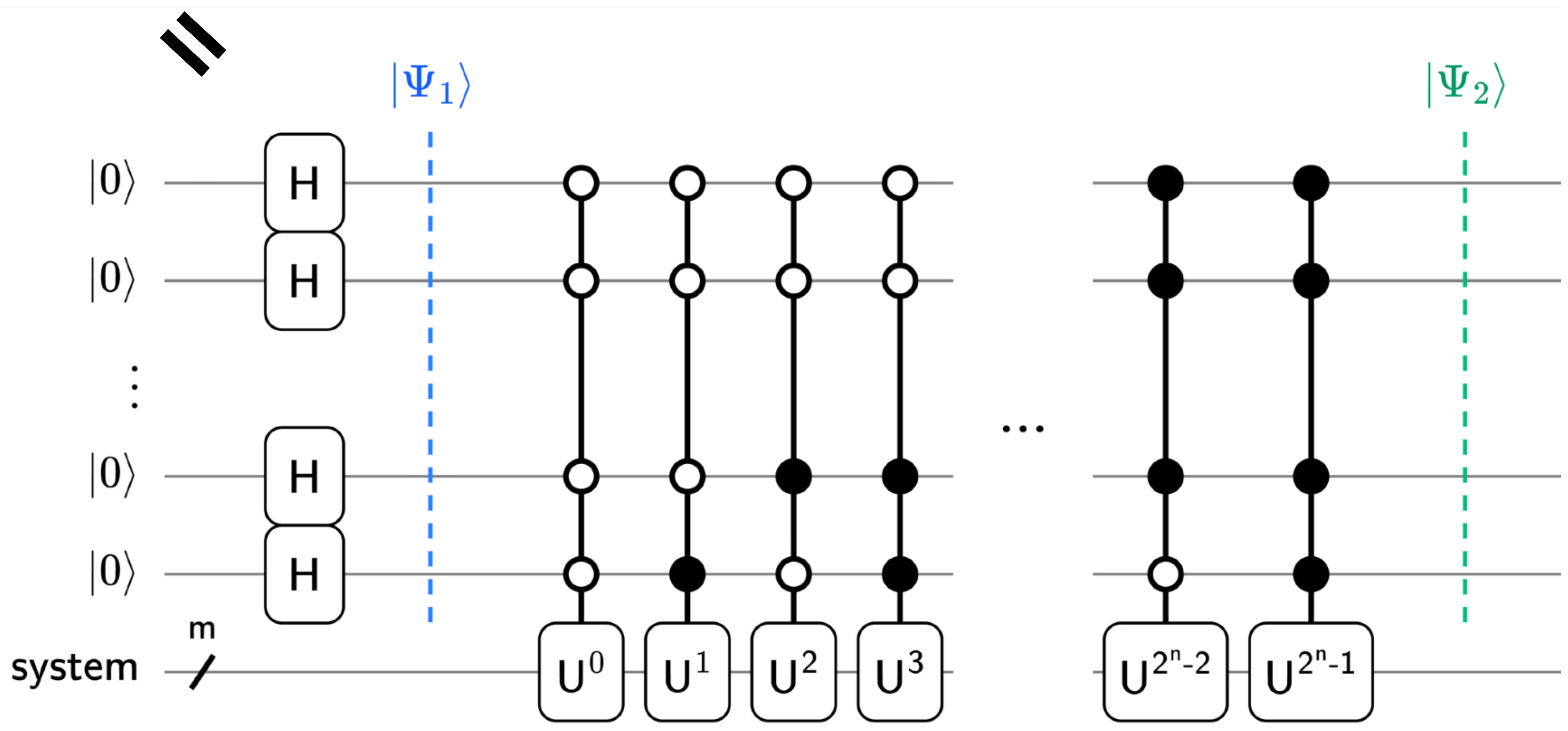


Revisiting QPE

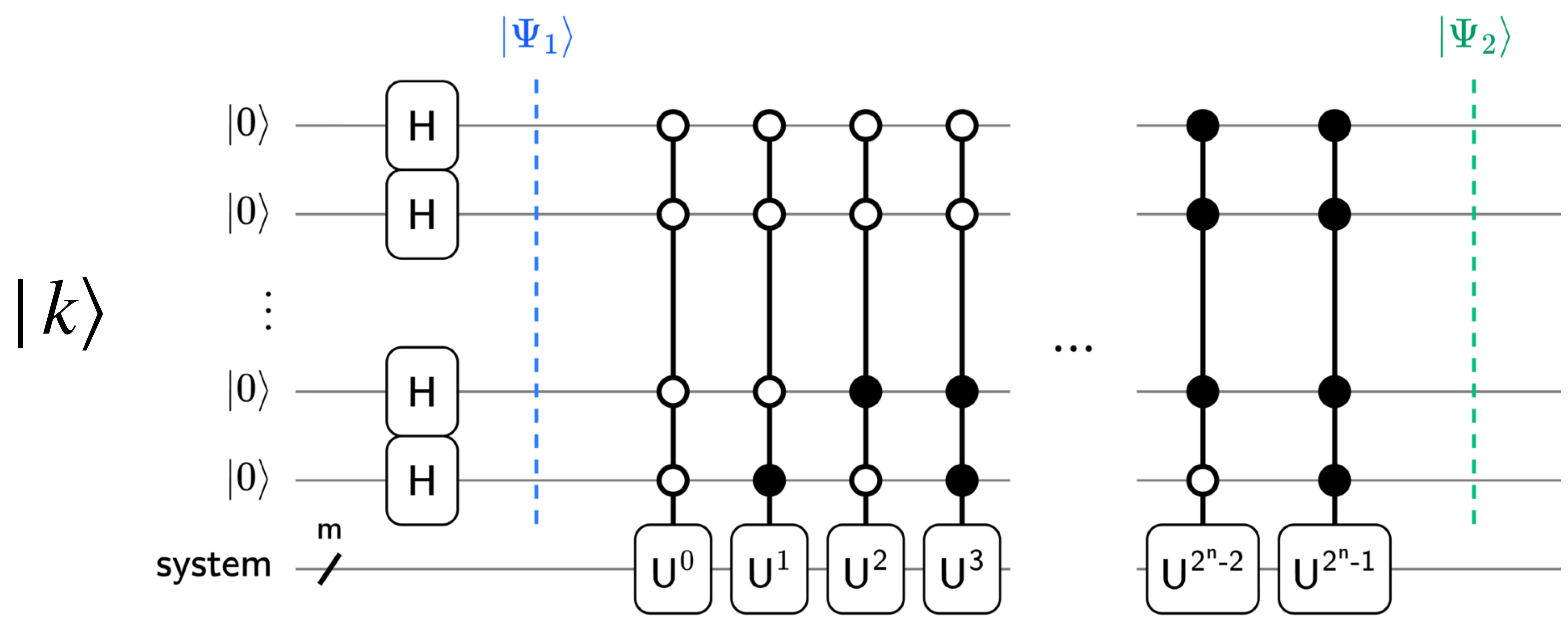




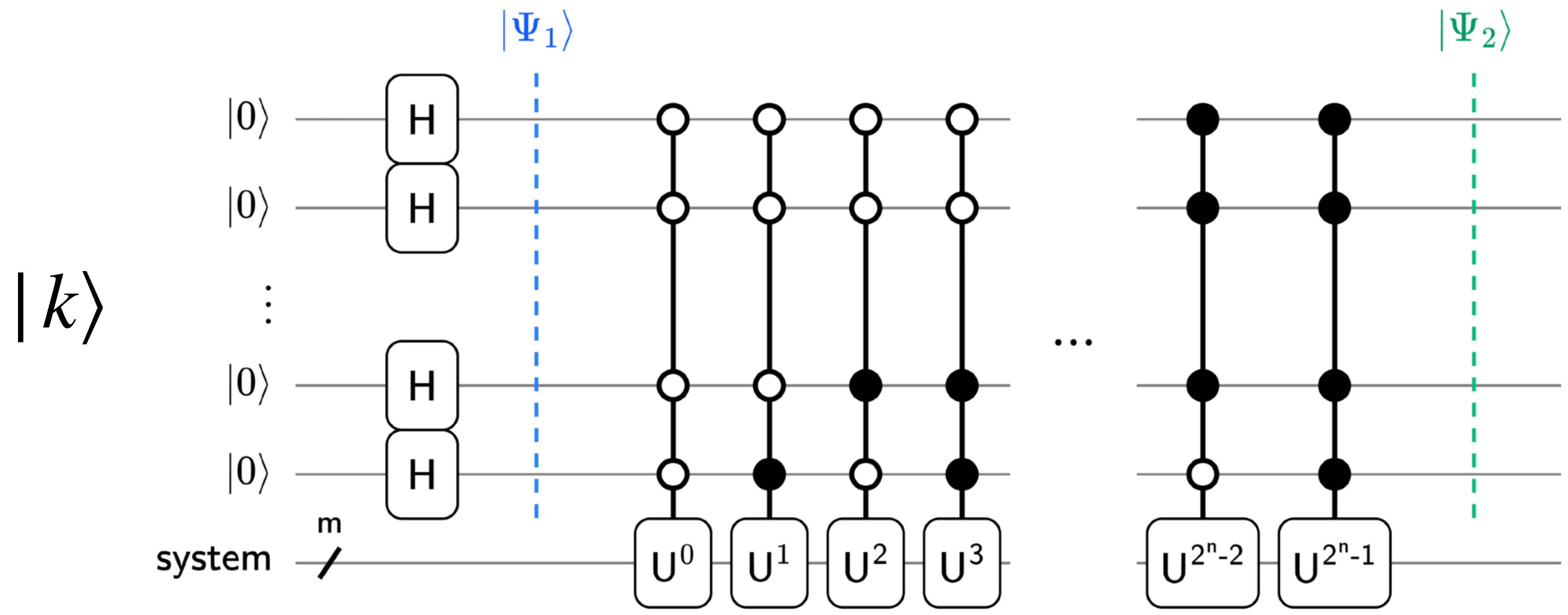
Efficient version of a more SELECT-like structure:



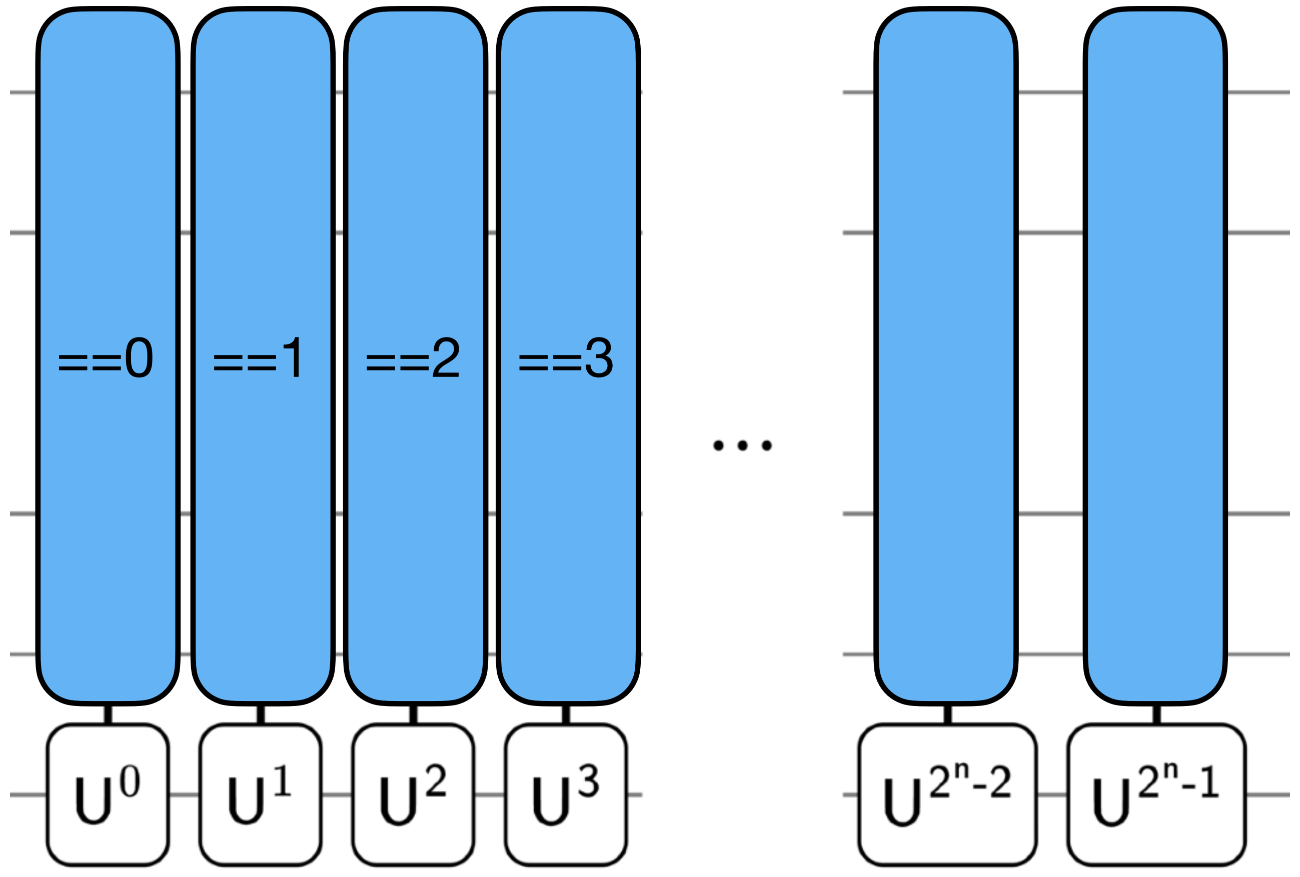
Revisiting QPE



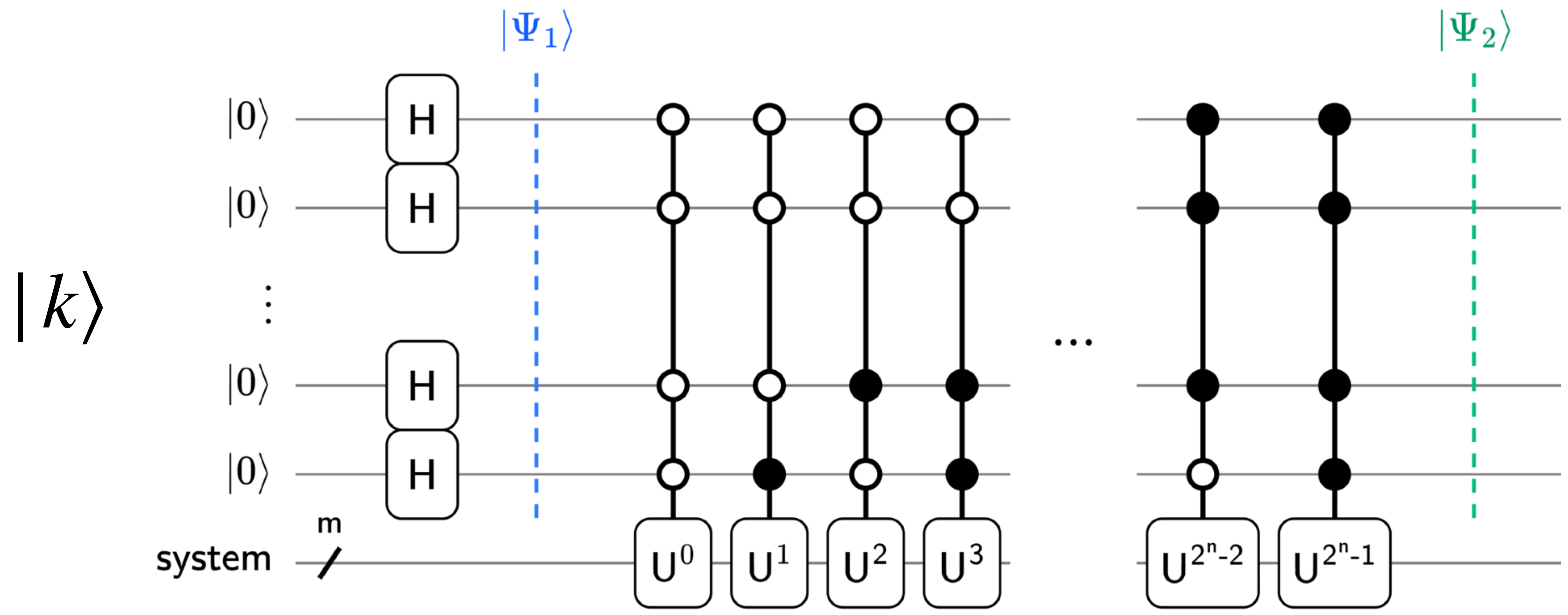
Revisiting QPE



Check the value of k and apply U^k



Revisiting QPE



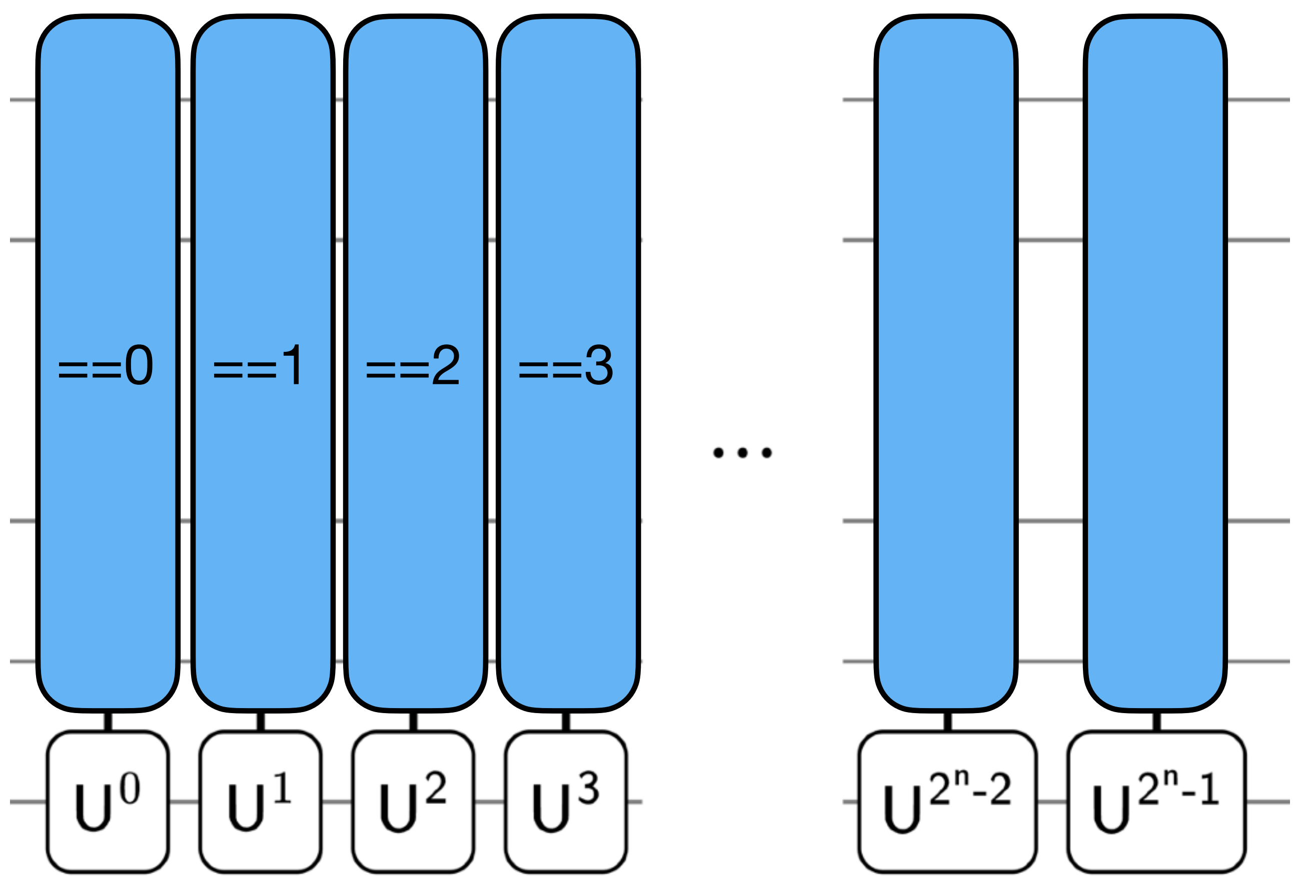
Suppose:

$U|\psi\rangle = e^{2\pi i\phi}|\psi\rangle$ where

$\phi = 0.1875$

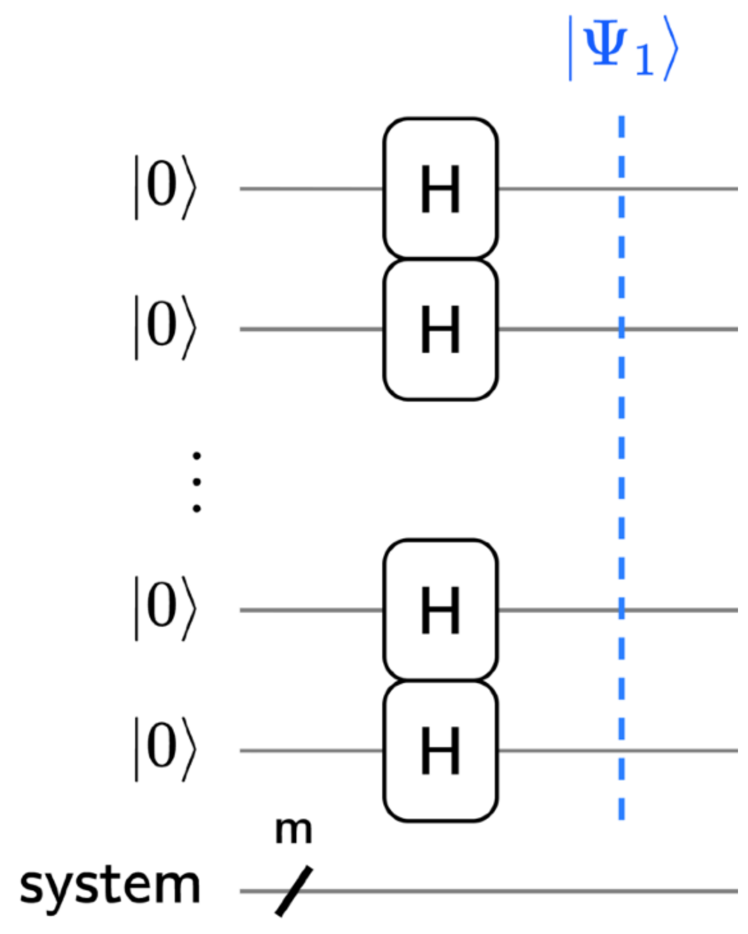
Check the value of

k and apply U^k



Revisiting QPE

$|k\rangle$



$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1}|k\rangle$$

Suppose:

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Initializing the register

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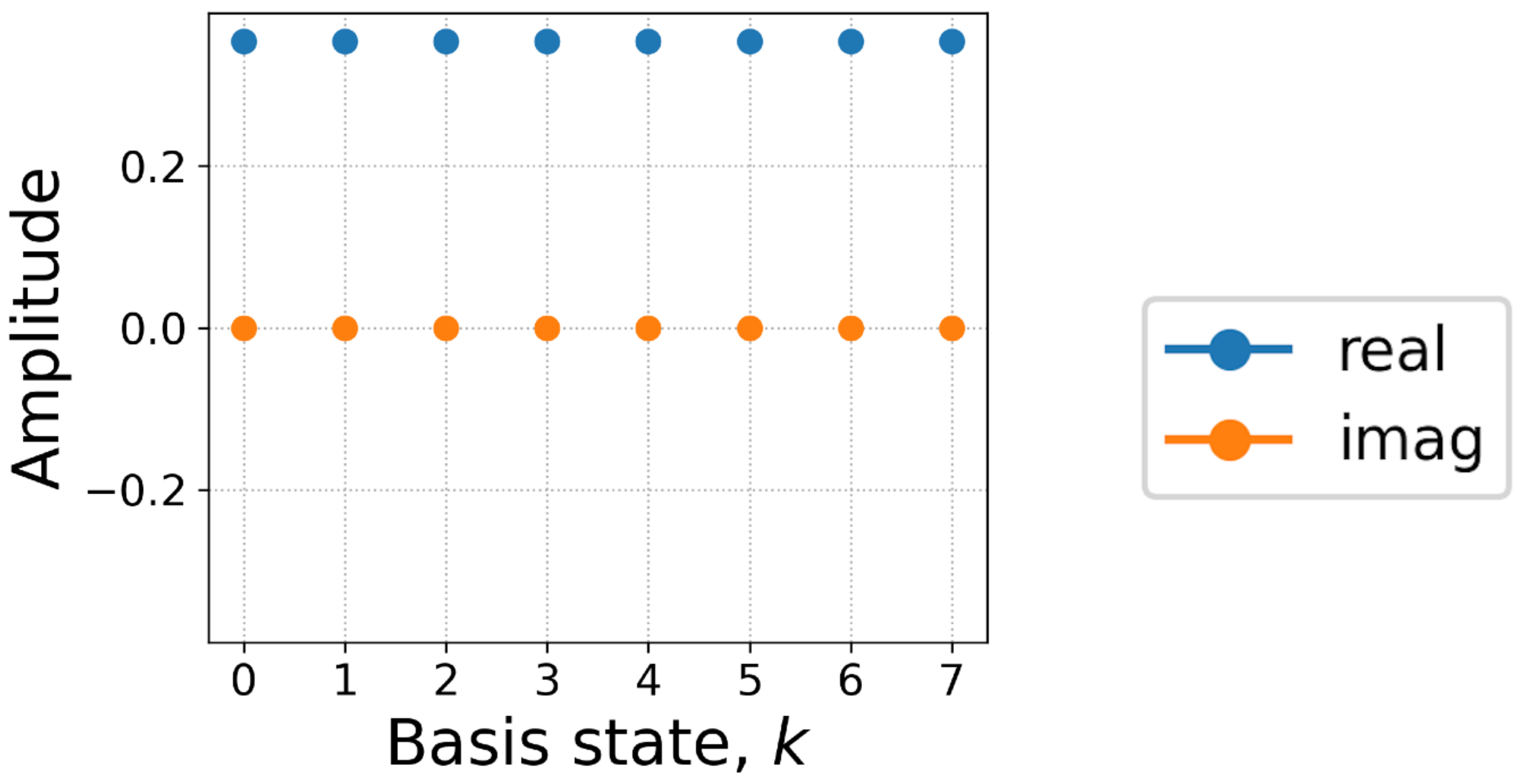
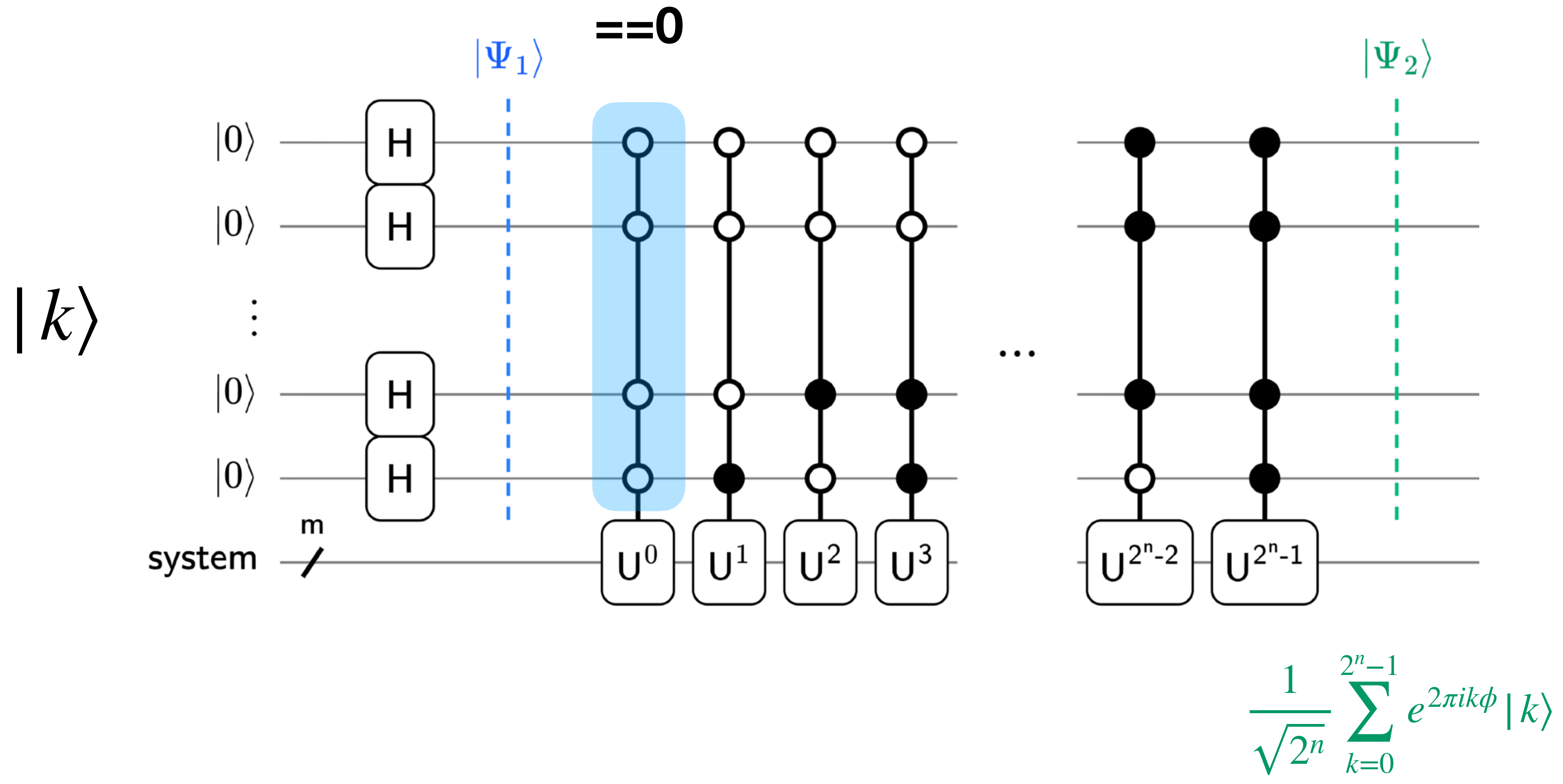


Revisiting QPE



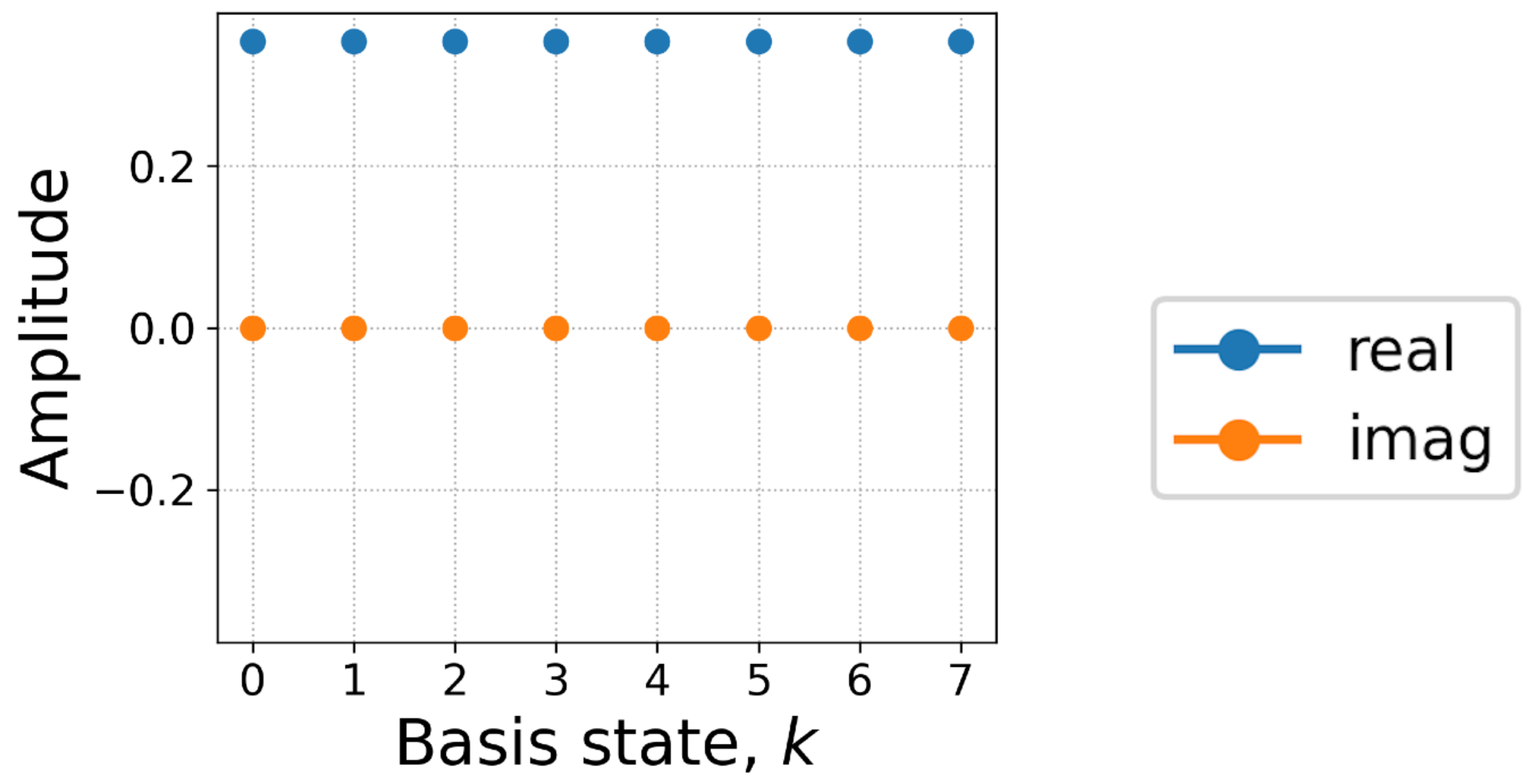
Suppose:

$U|\psi\rangle = e^{2\pi i\phi}|\psi\rangle$ where

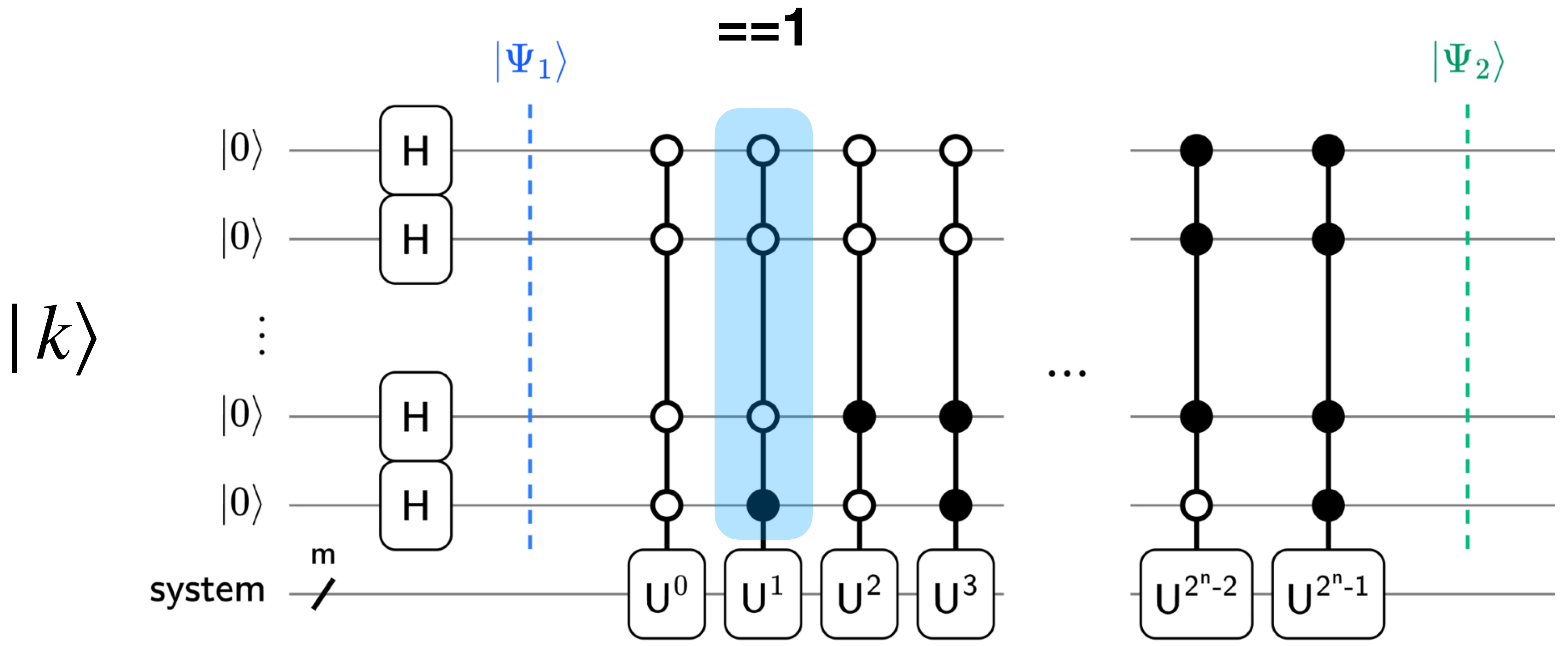
$\phi = 0.1875$

$k = 0$

(Identity)

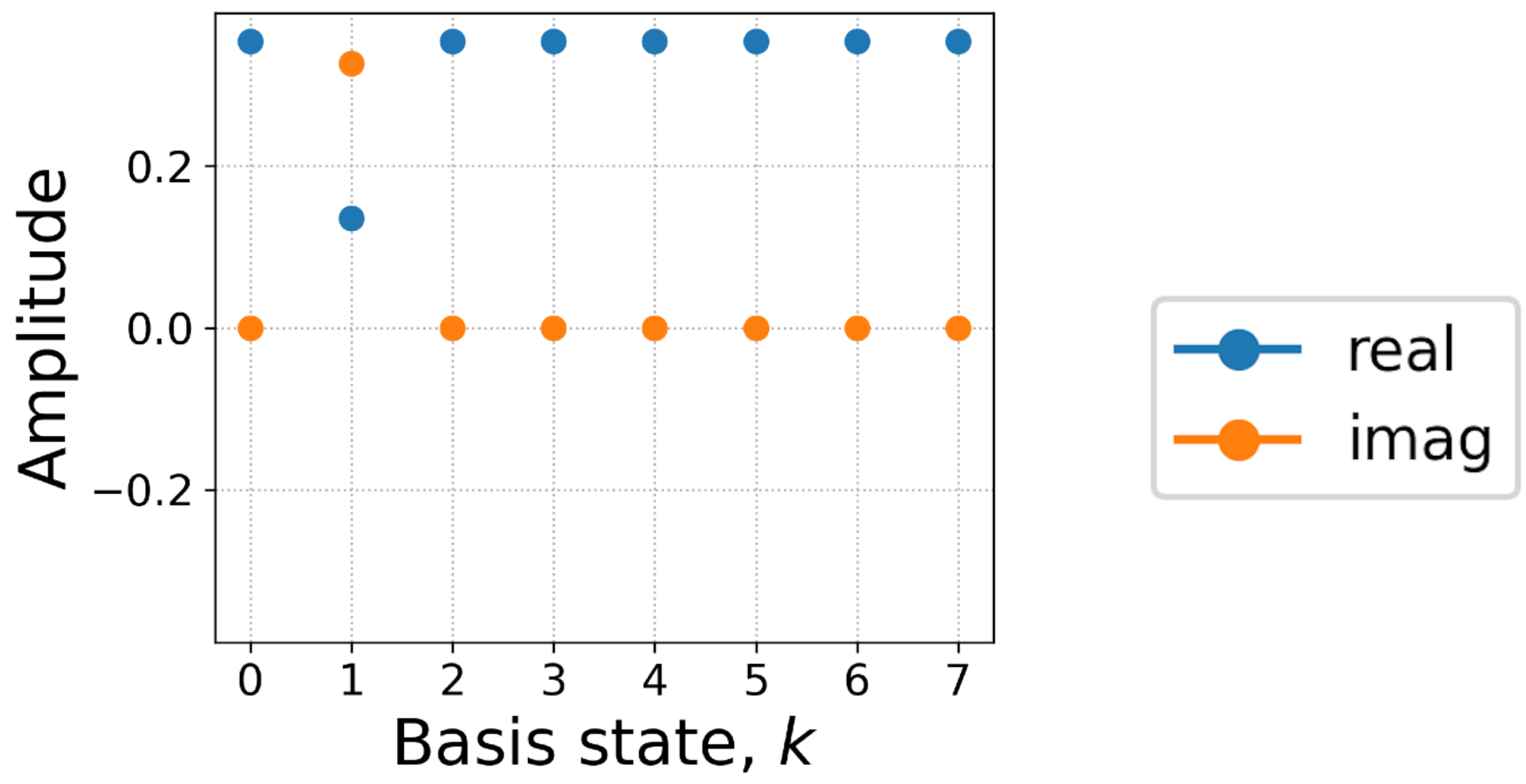


Revisiting QPE

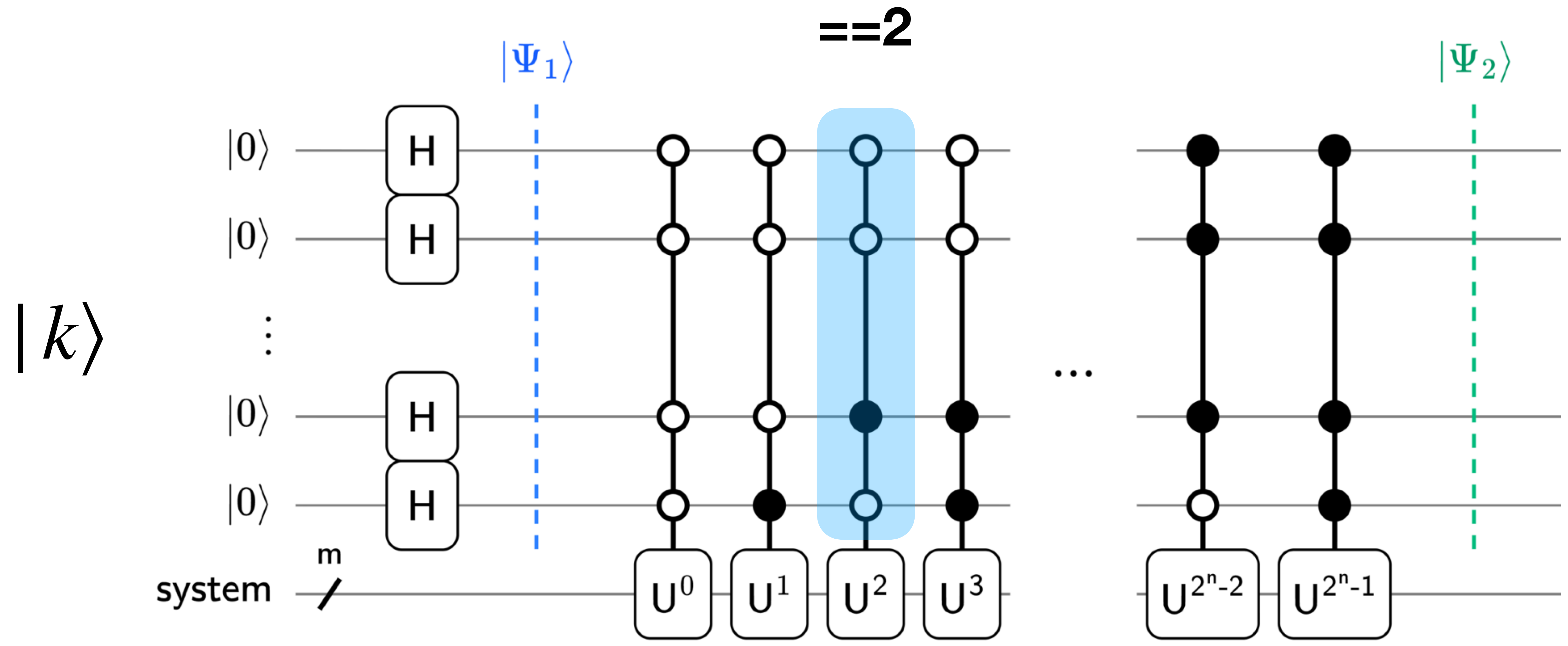


$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$k = 1$

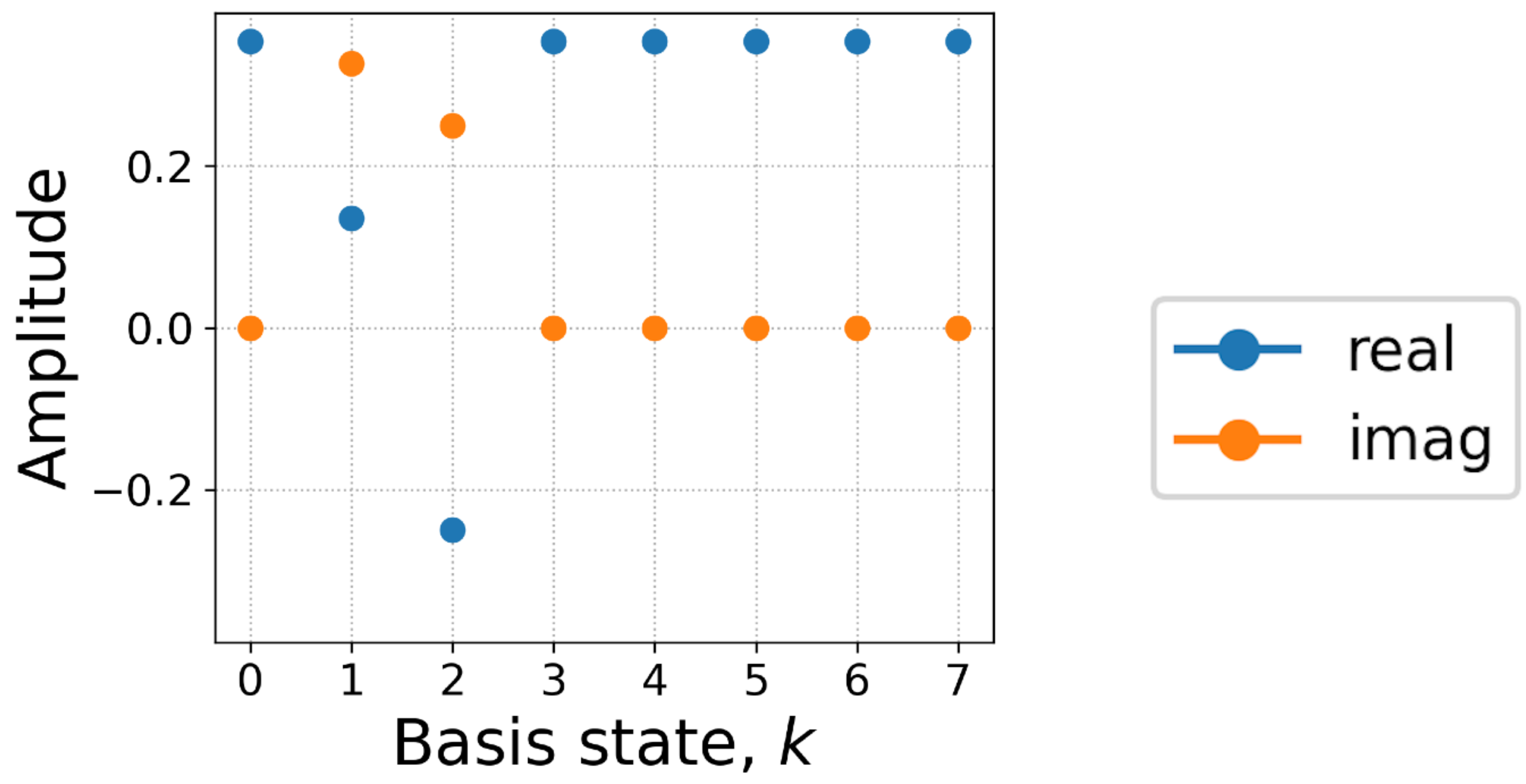


Revisiting QPE

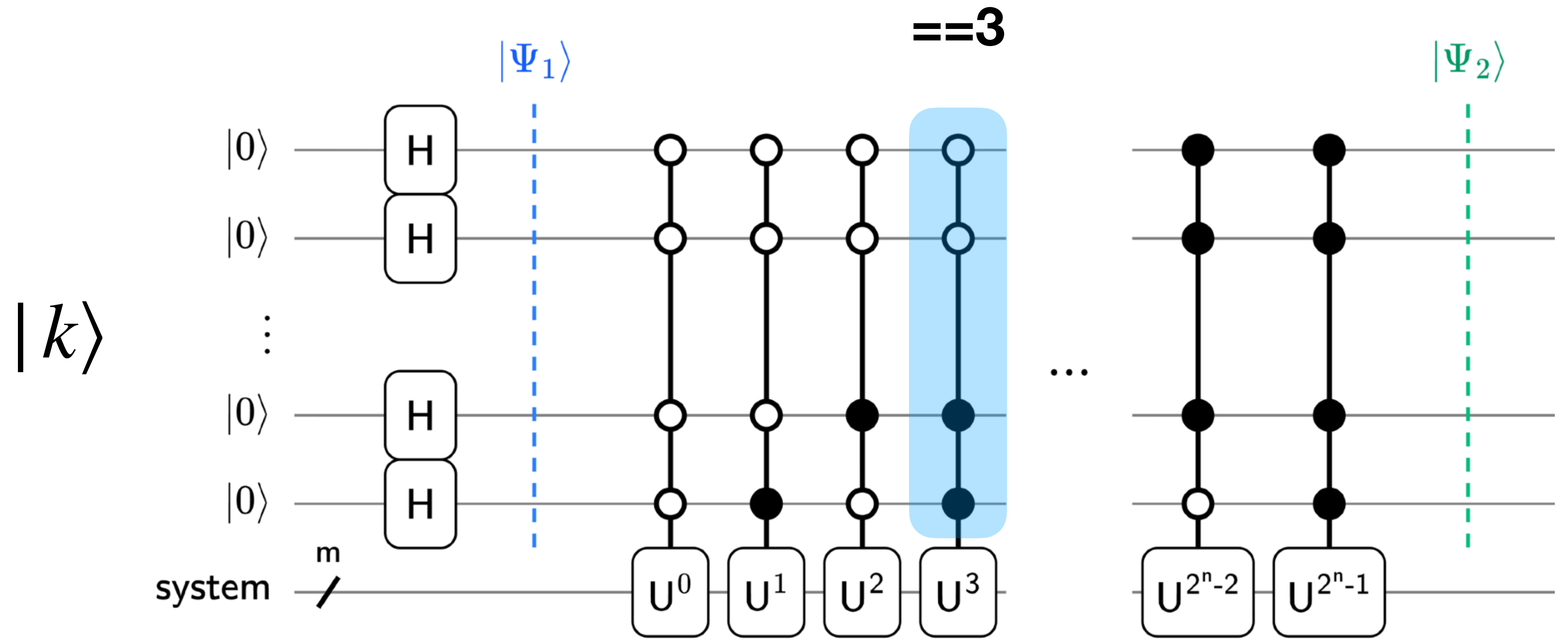


$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$k = 2$

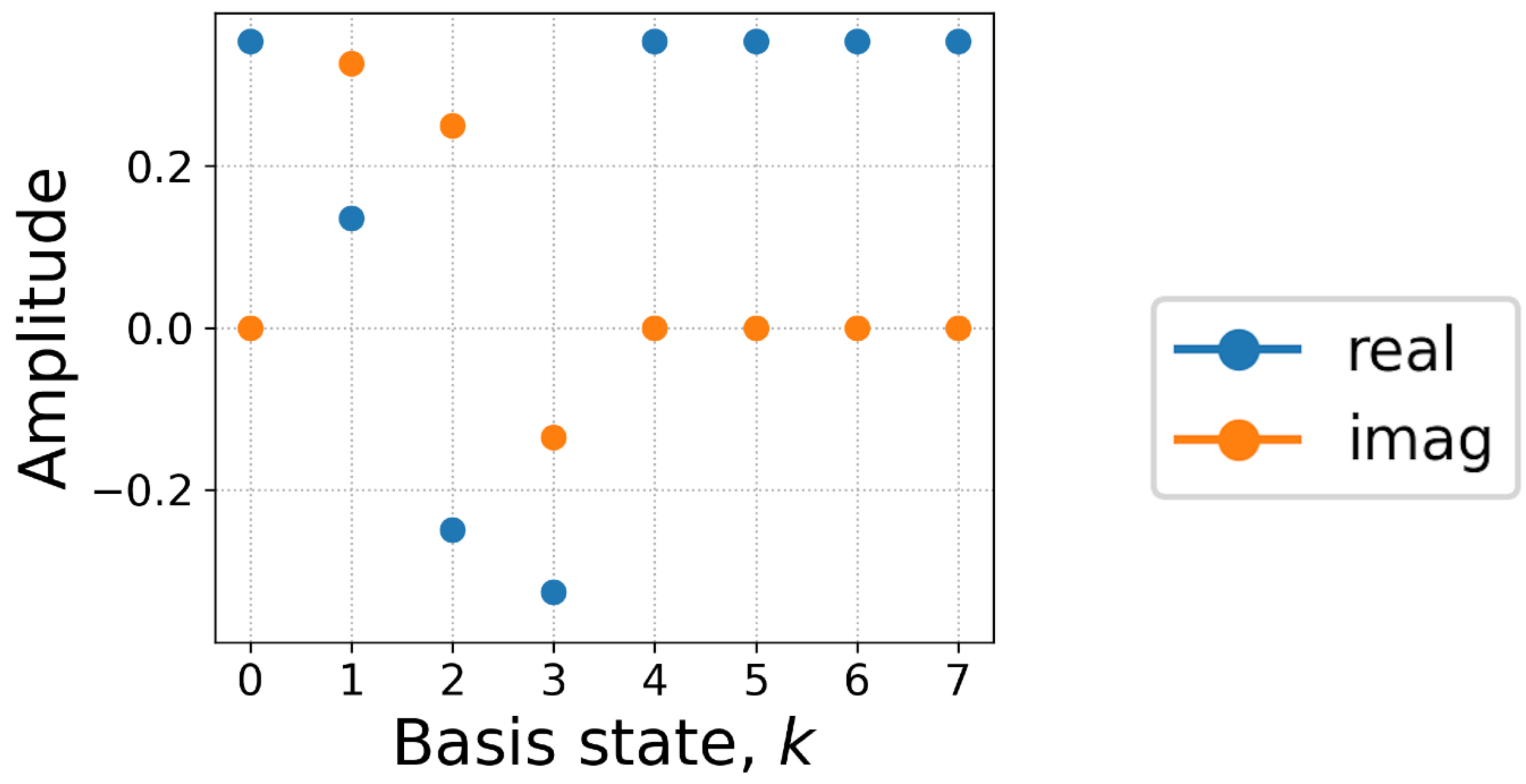


Revisiting QPE



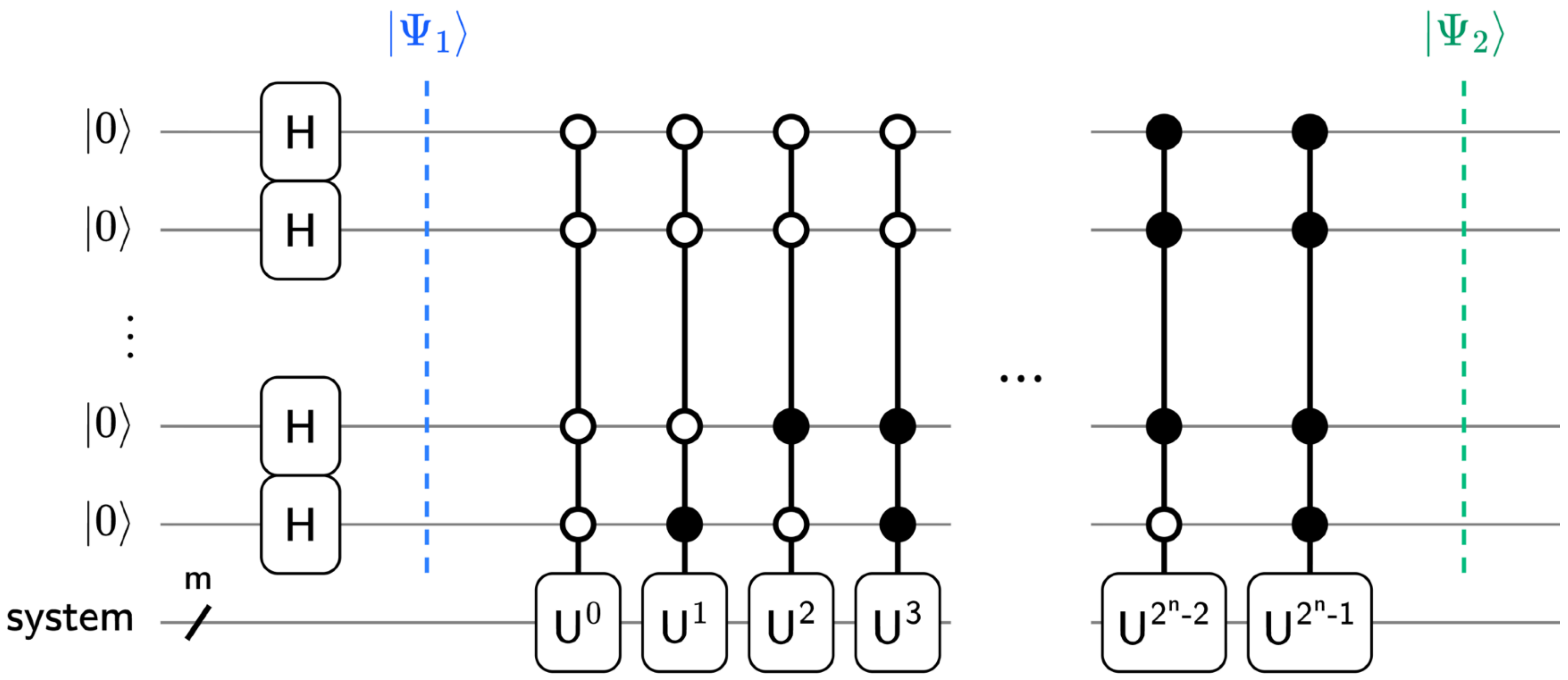
$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$k = 3$



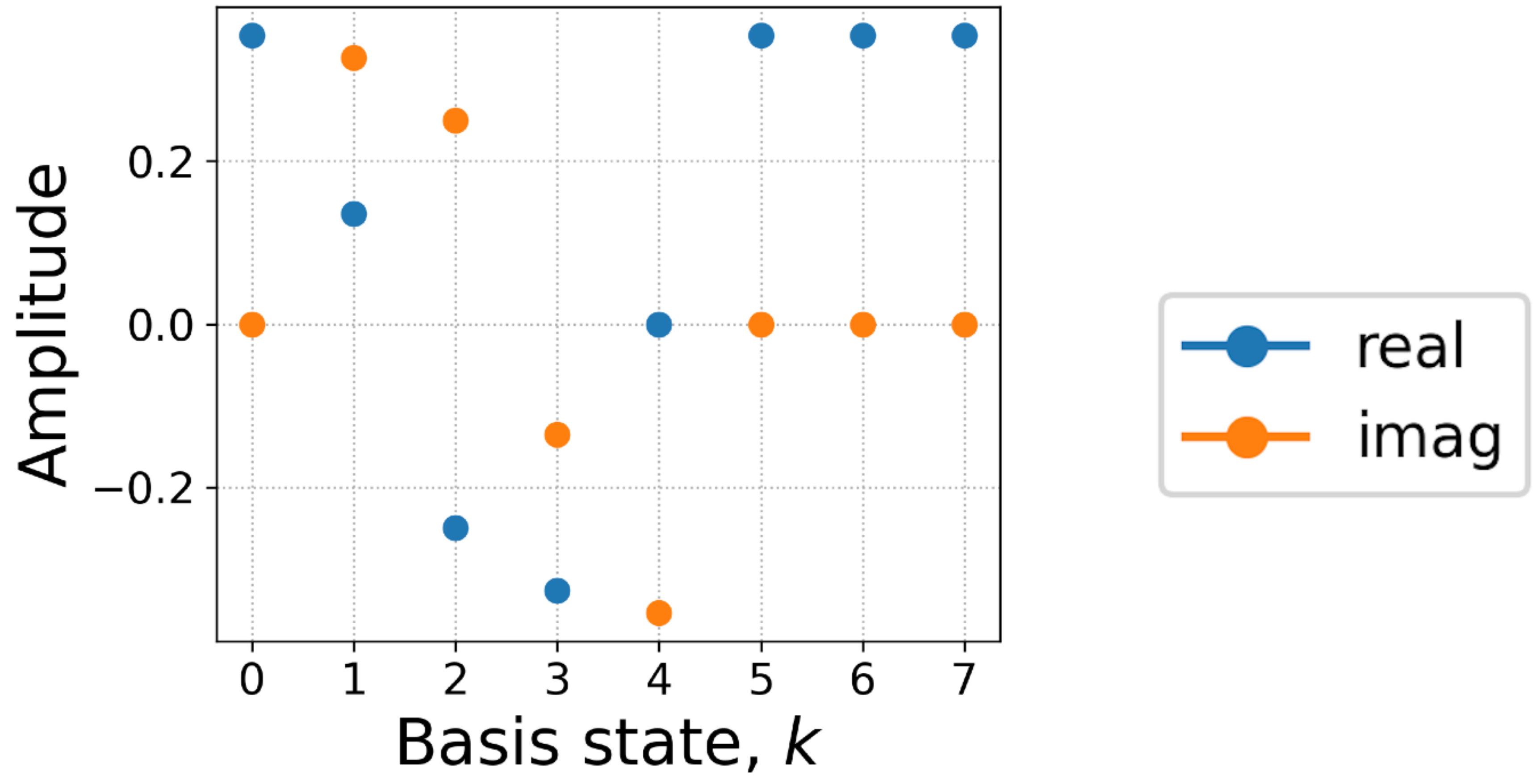
Revisiting QPE

$|k\rangle$



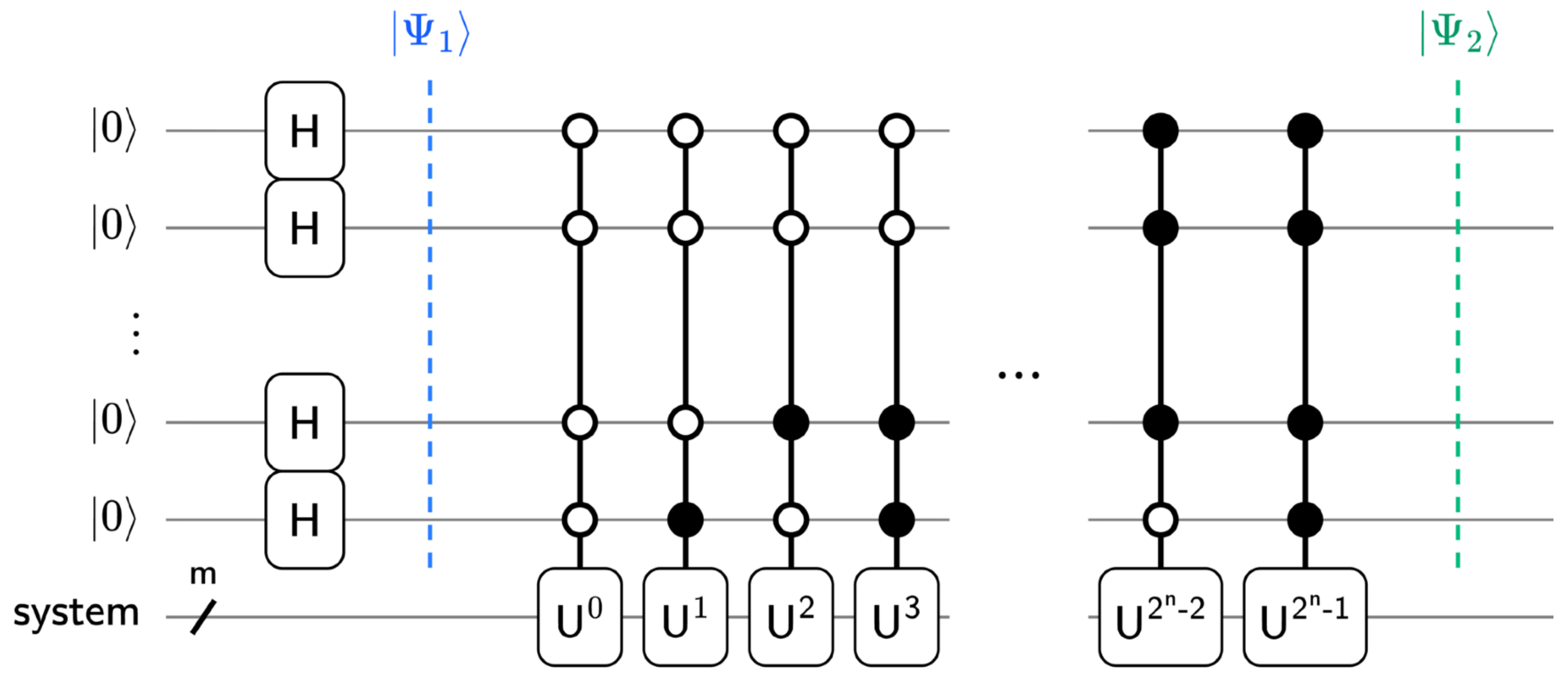
$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$k = 4$



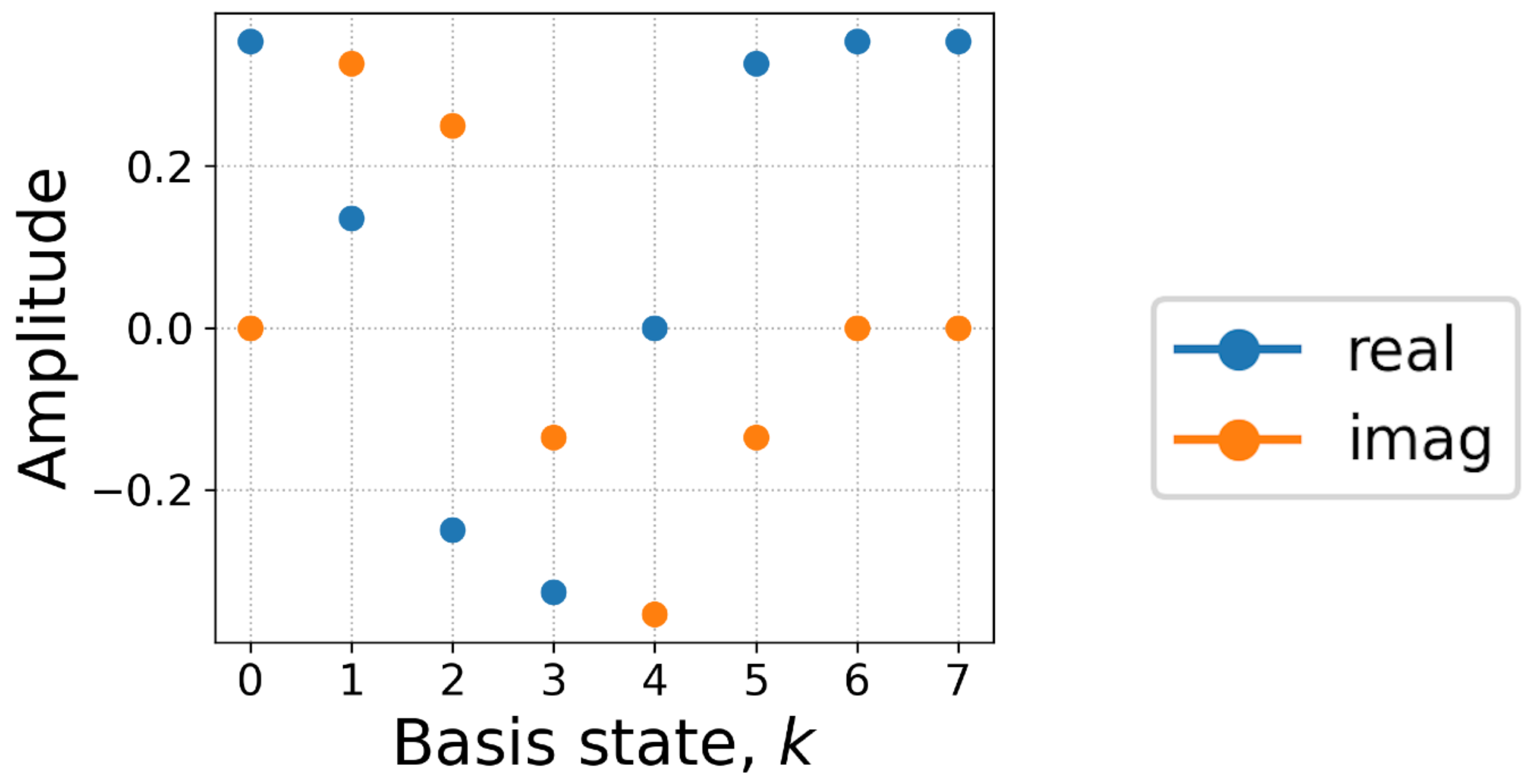
Revisiting QPE

$|k\rangle$



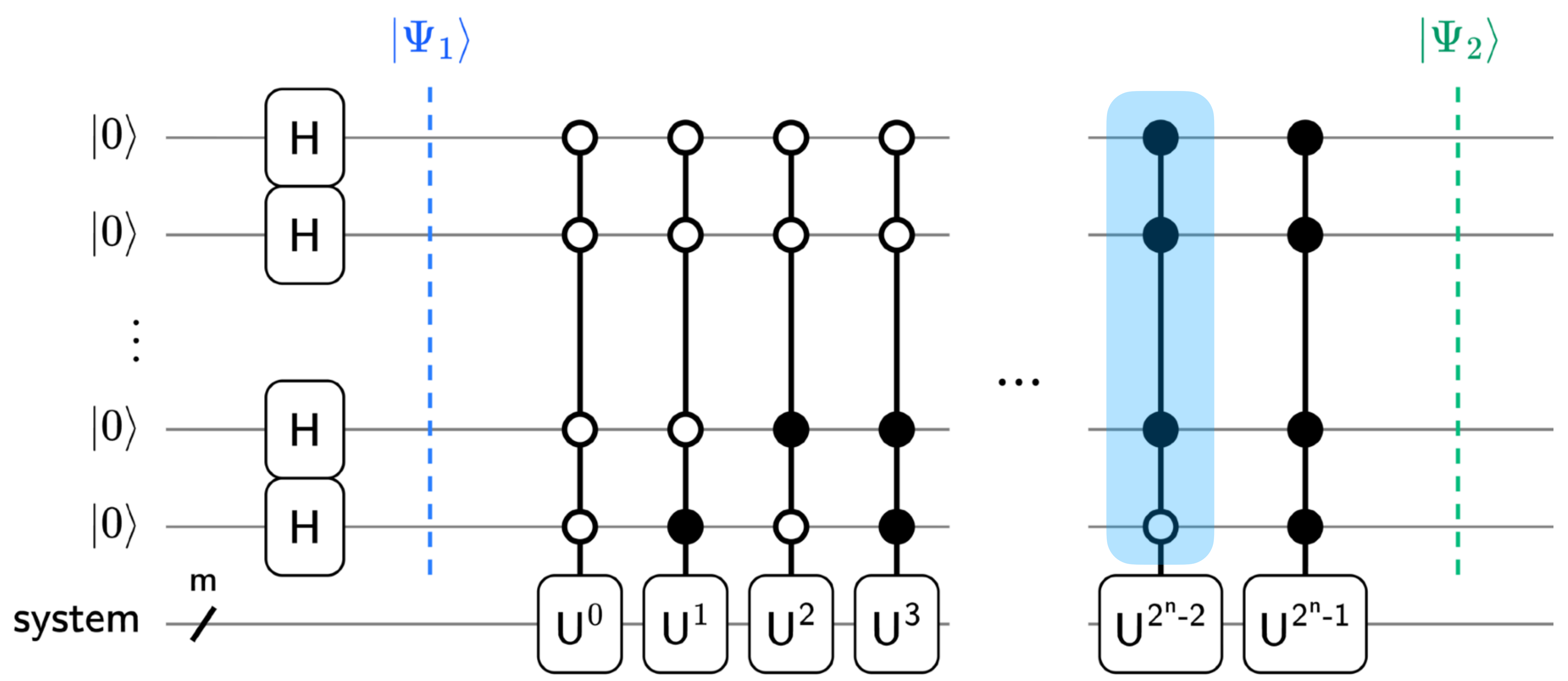
$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$k = 5$



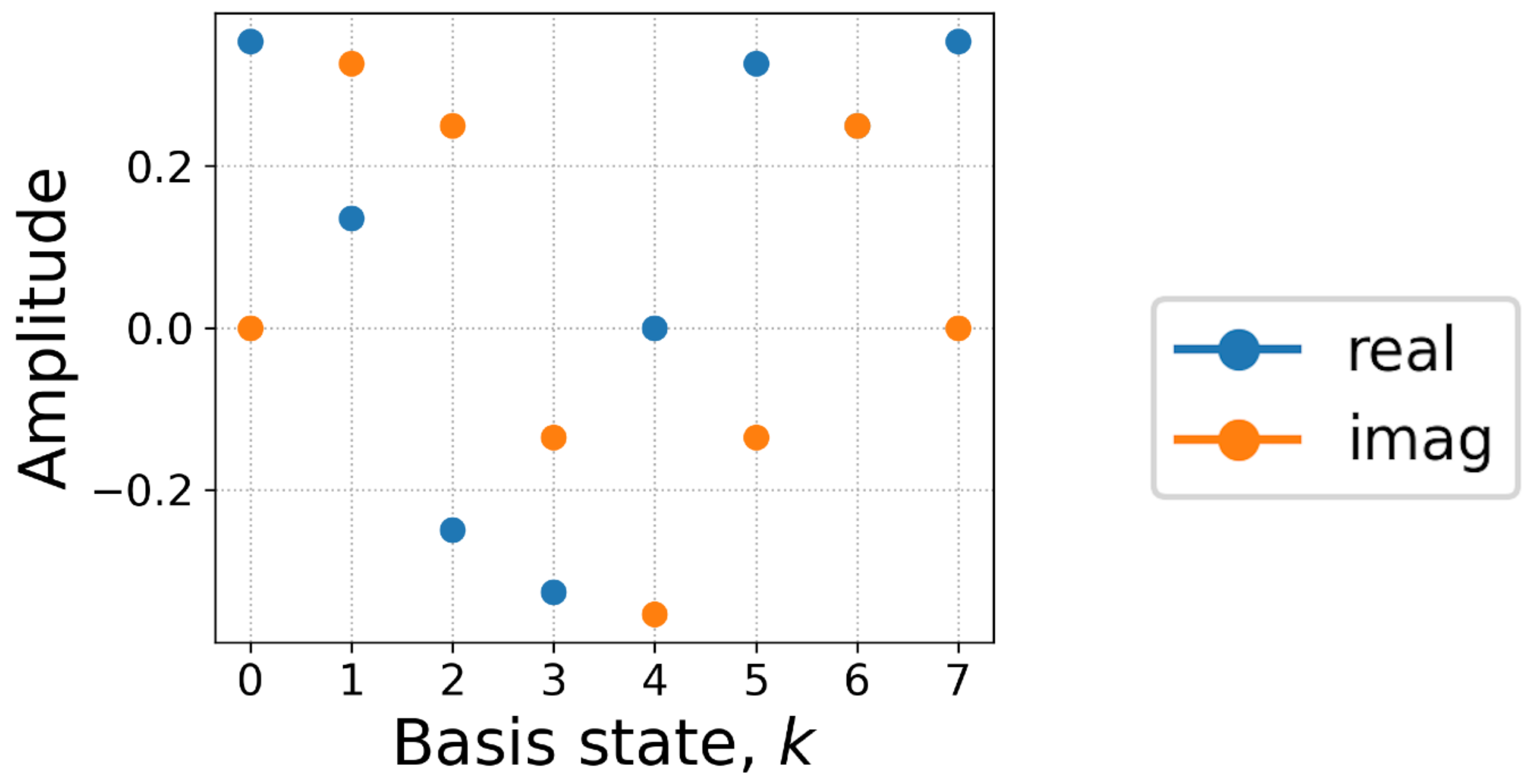
Revisiting QPE

$|k\rangle$



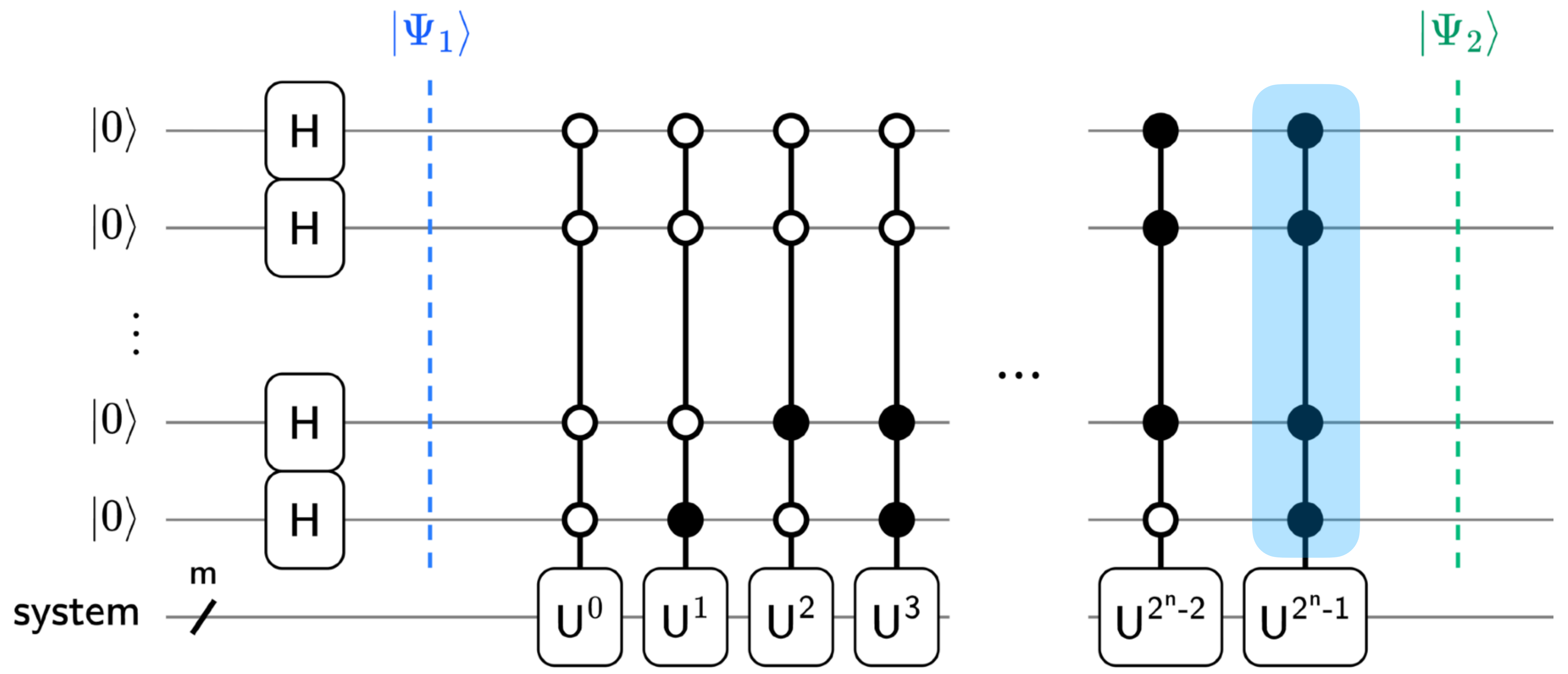
$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$k = 6$



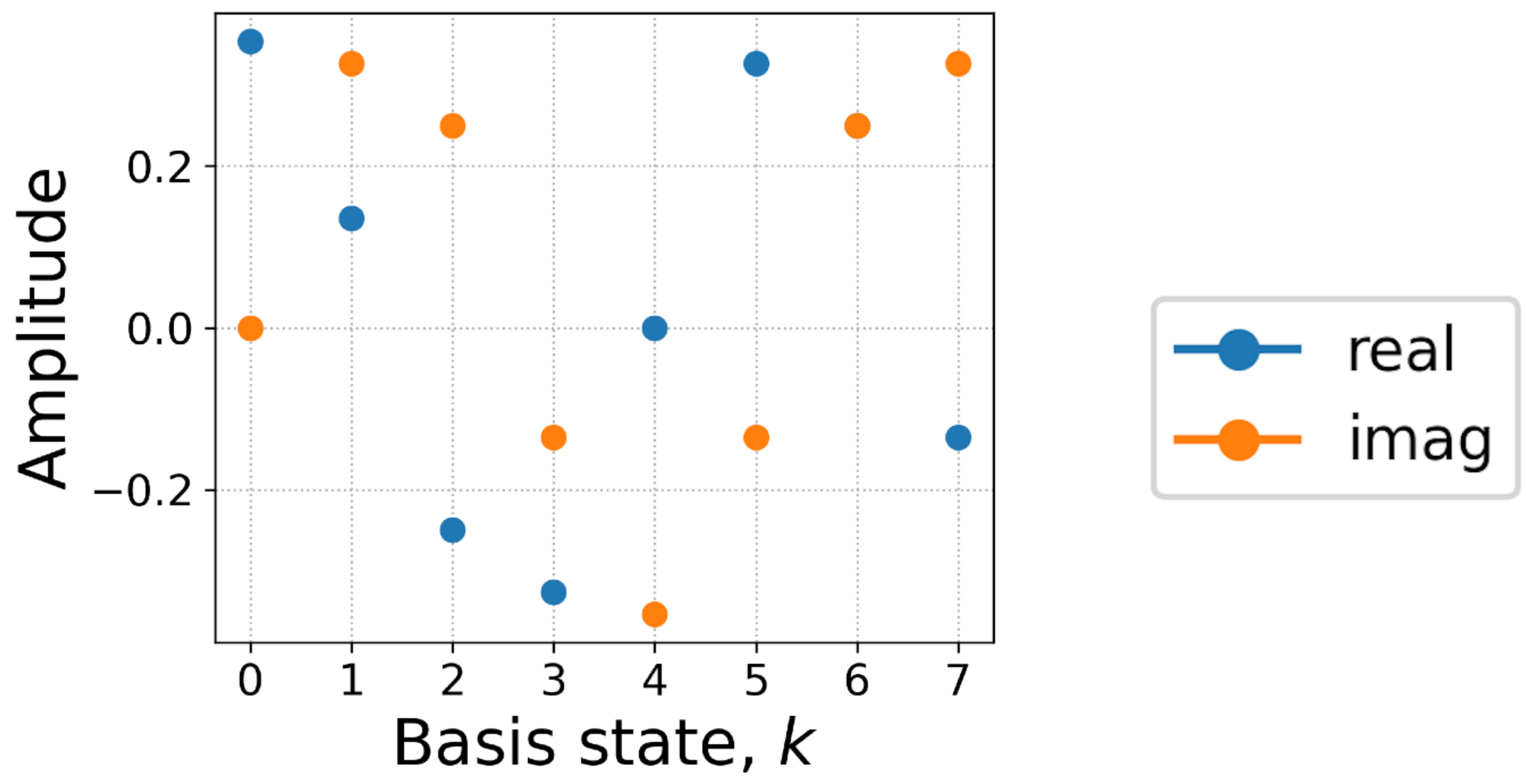
Revisiting QPE

$|k\rangle$



$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$k = 7$



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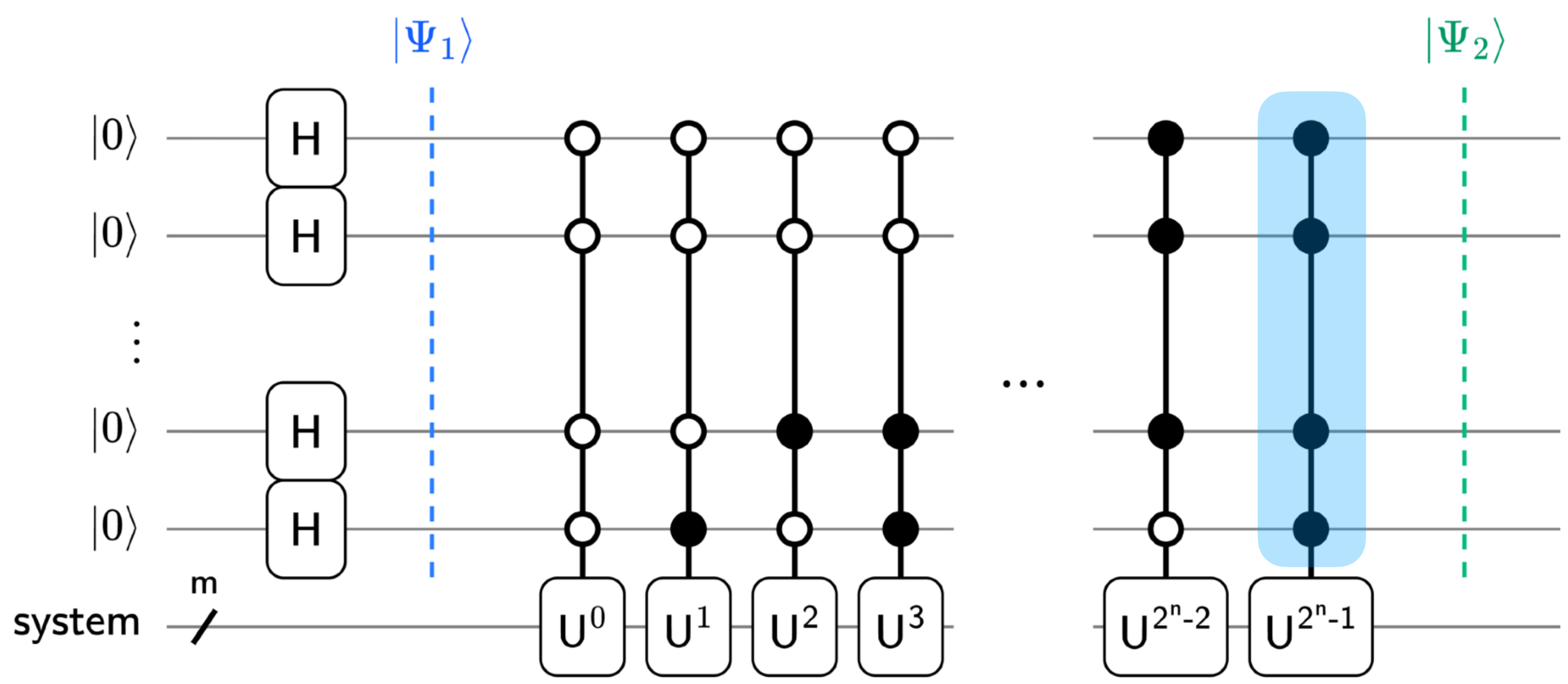

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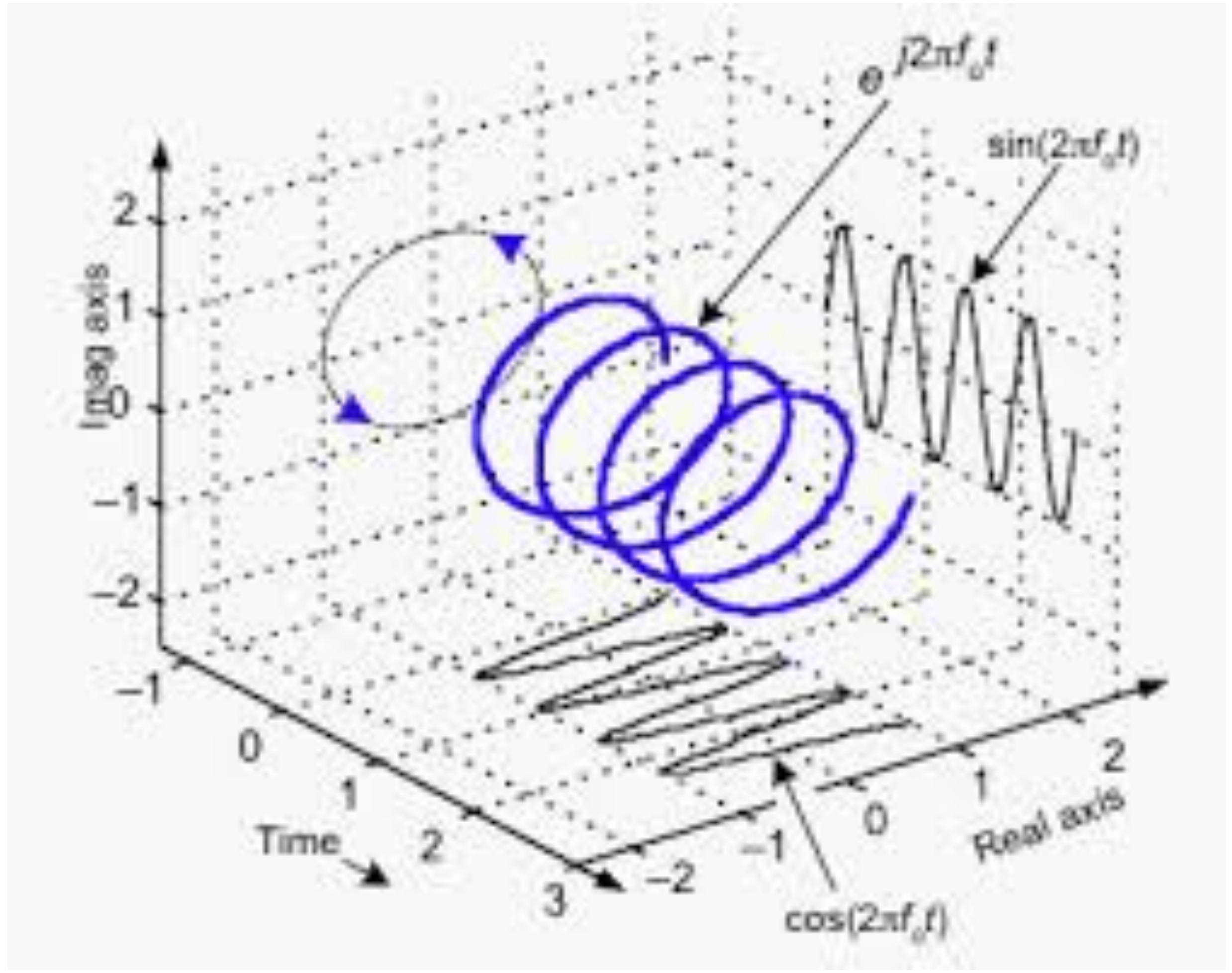


$k = 7$

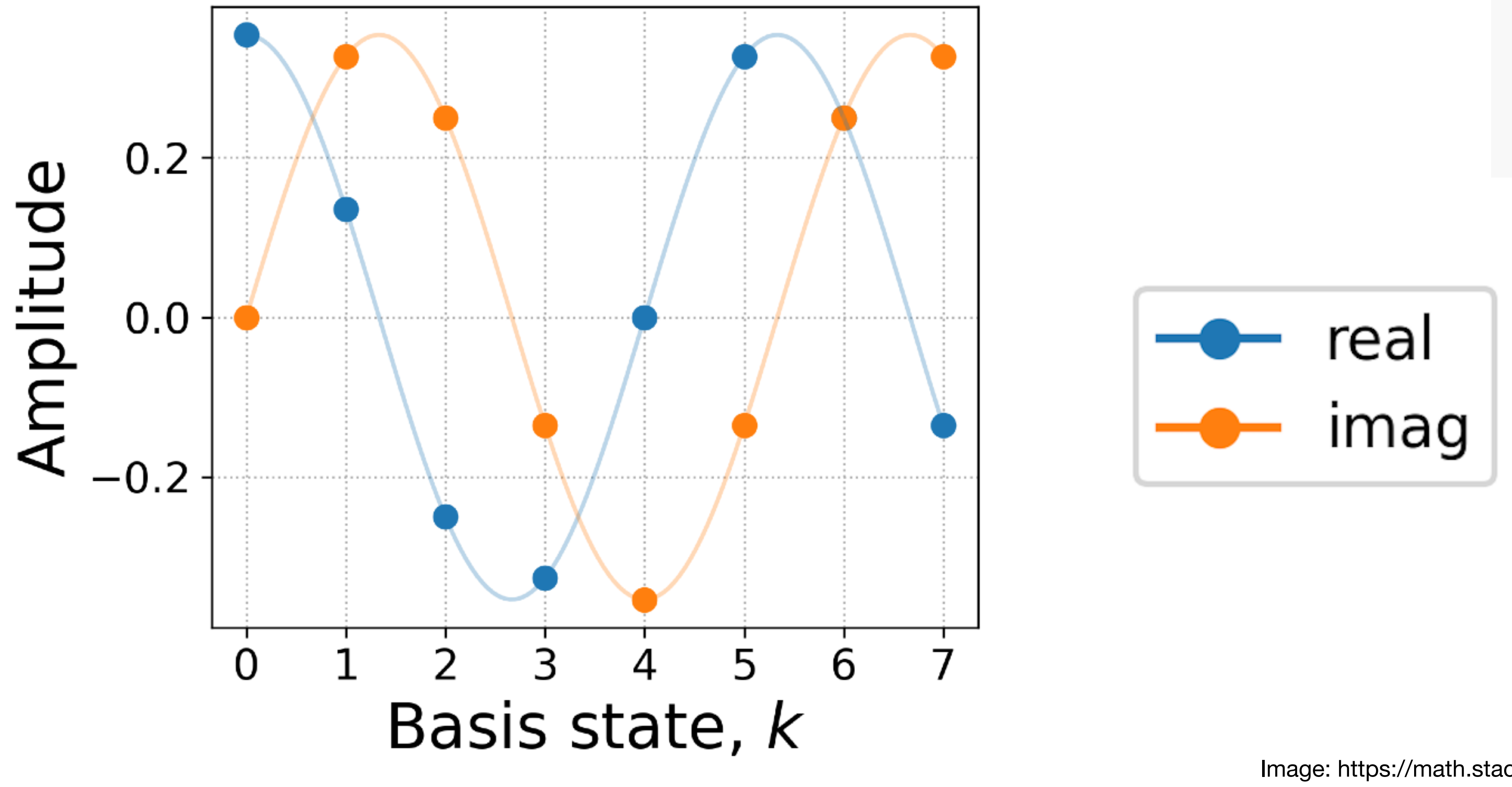
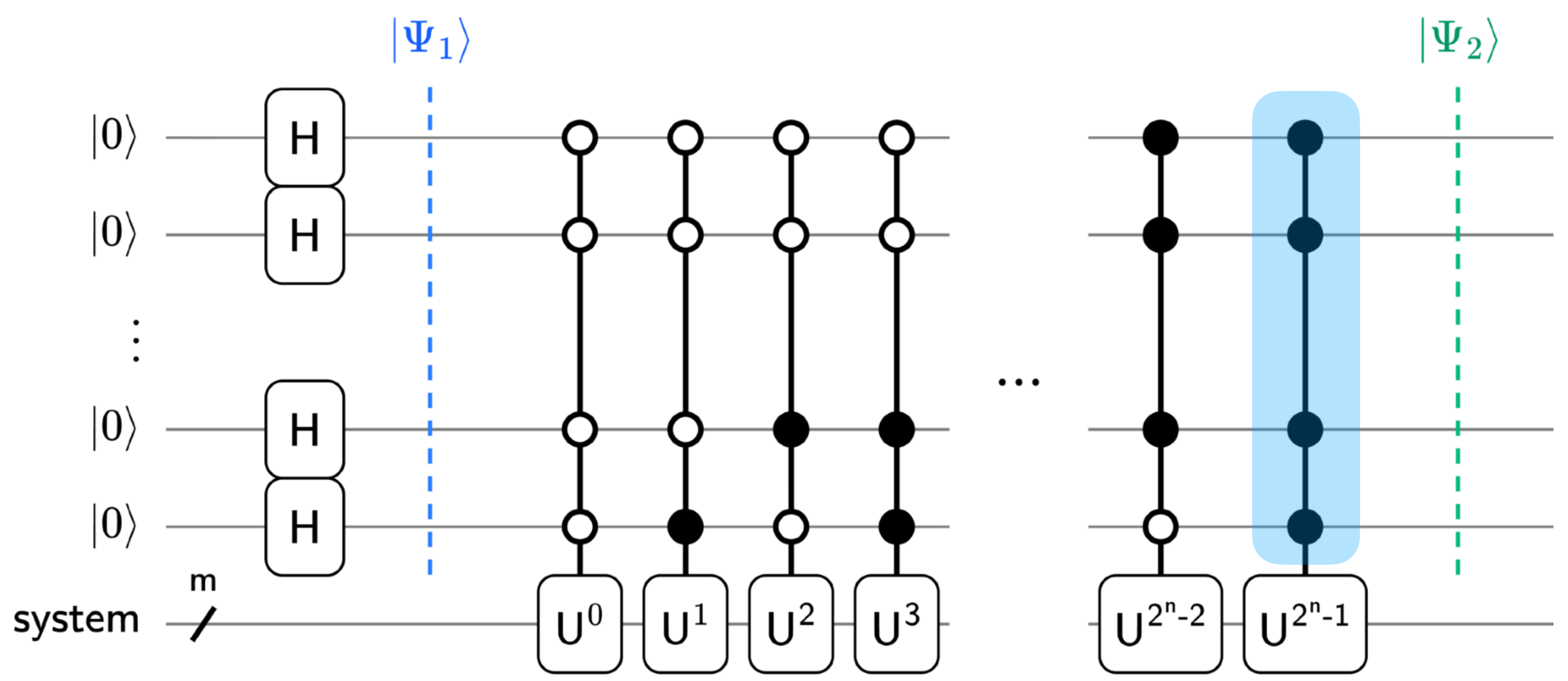


Image: https://math.stackexchange.com/questions/144268/is-there-a-name-for-this-type-of-plot-function-on-complex-plane-vs-time-shown

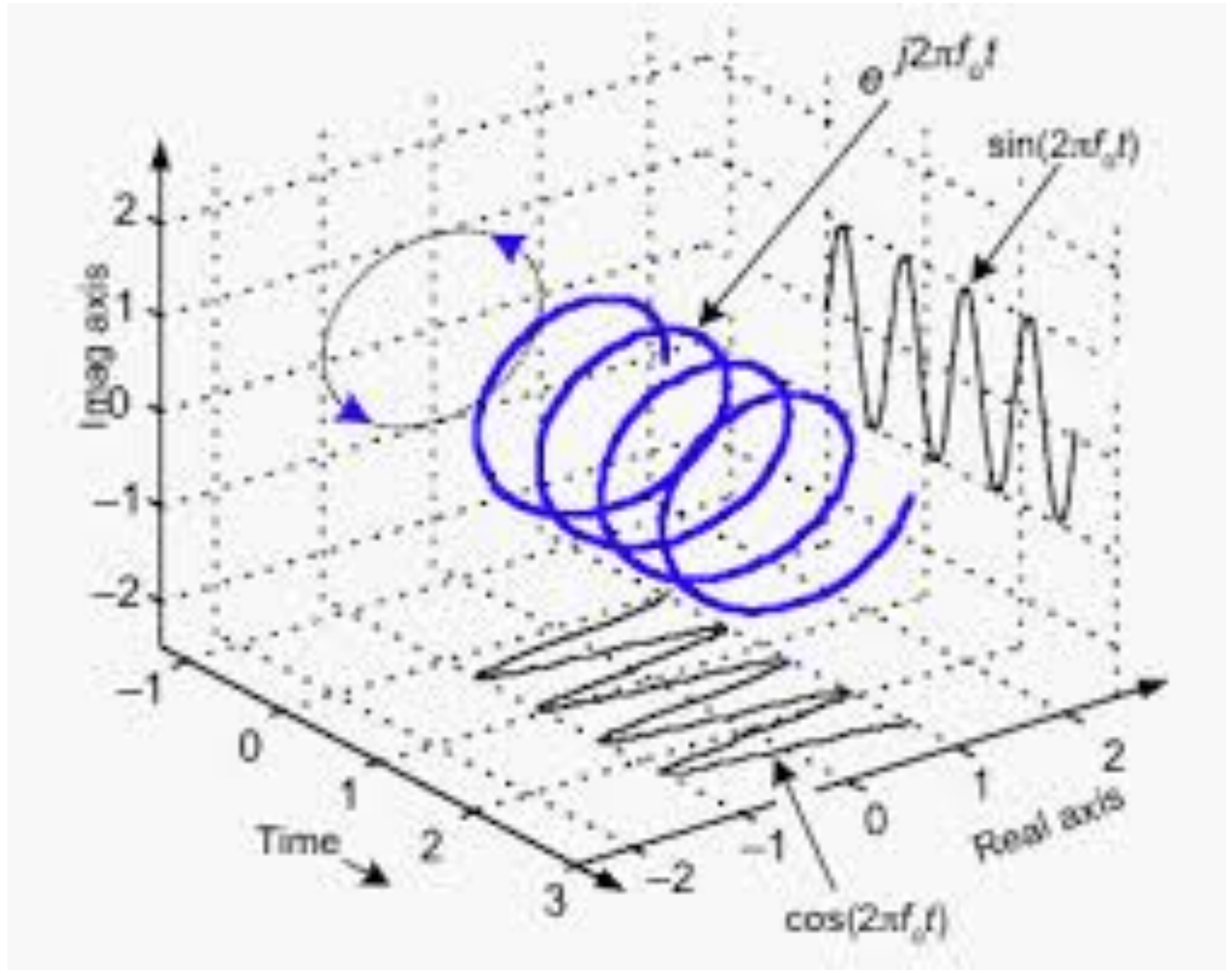
Revisiting QPE



$|k\rangle$

$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

Where have we seen this?



$k = 7$

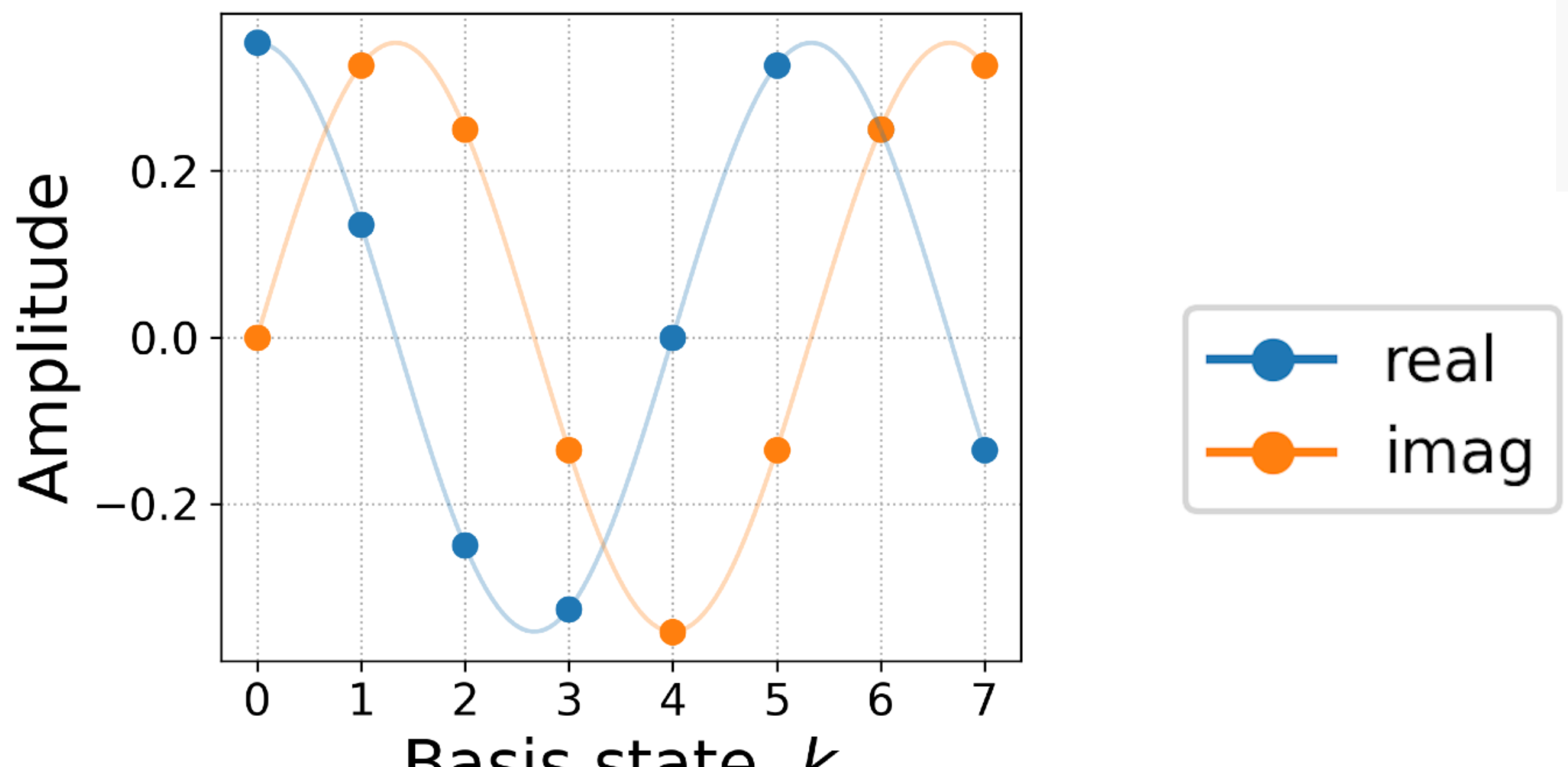
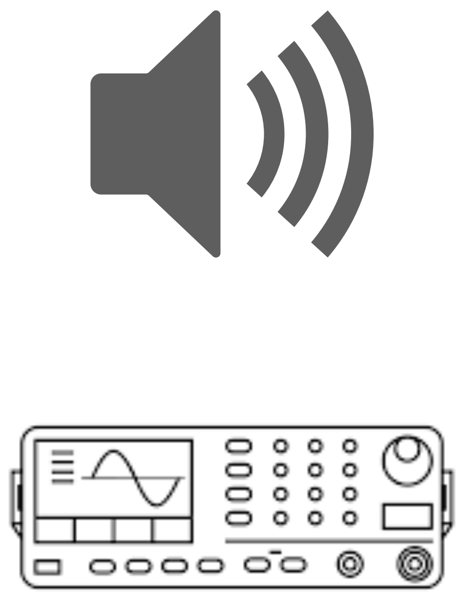
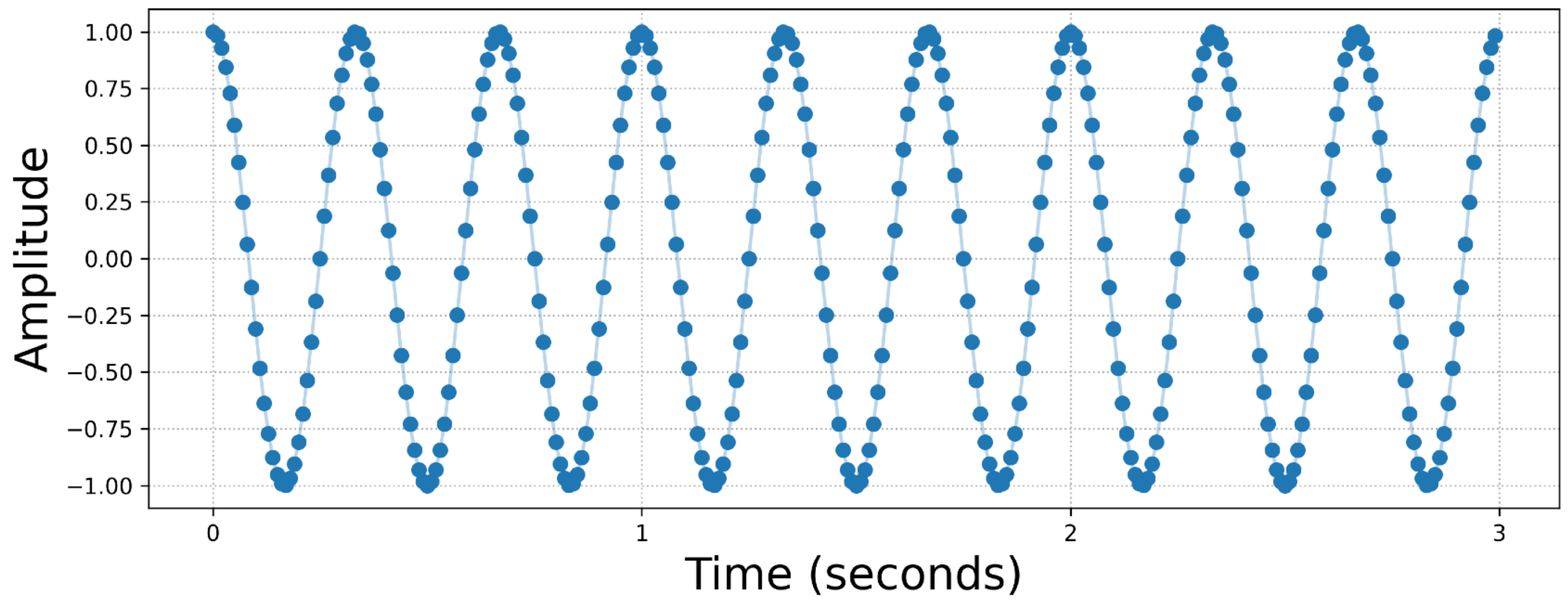


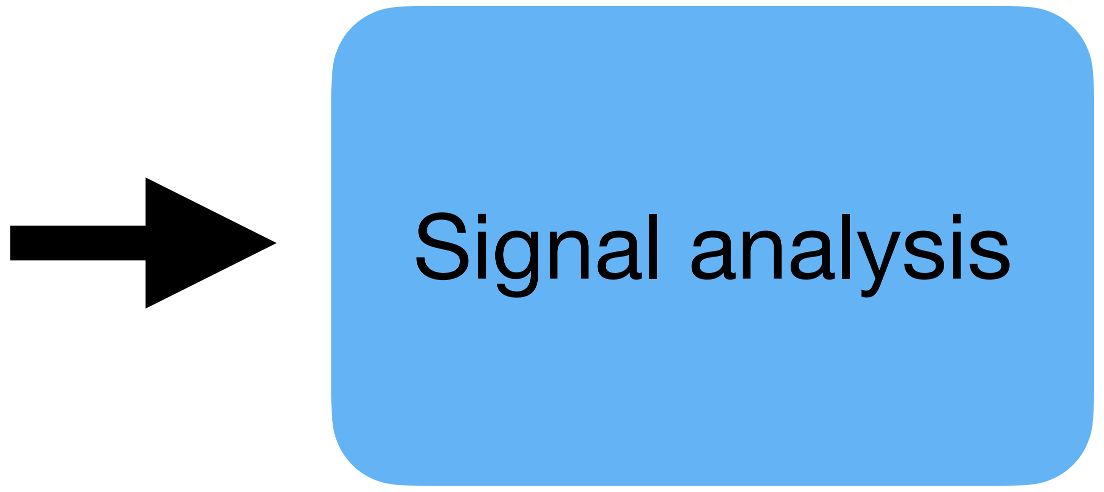
Image: https://math.stackexchange.com/questions/144268/is-there-a-name-for-this-type-of-plot-function-on-complex-plane-vs-time-shown

Digital signal processing (DSP)

Incoming signal







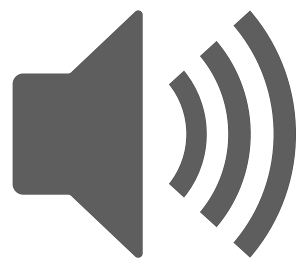
Sampling/recording

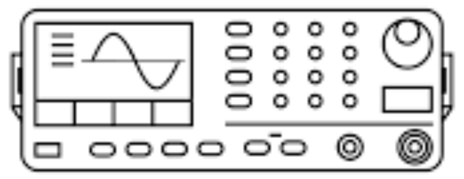
signal at discrete times

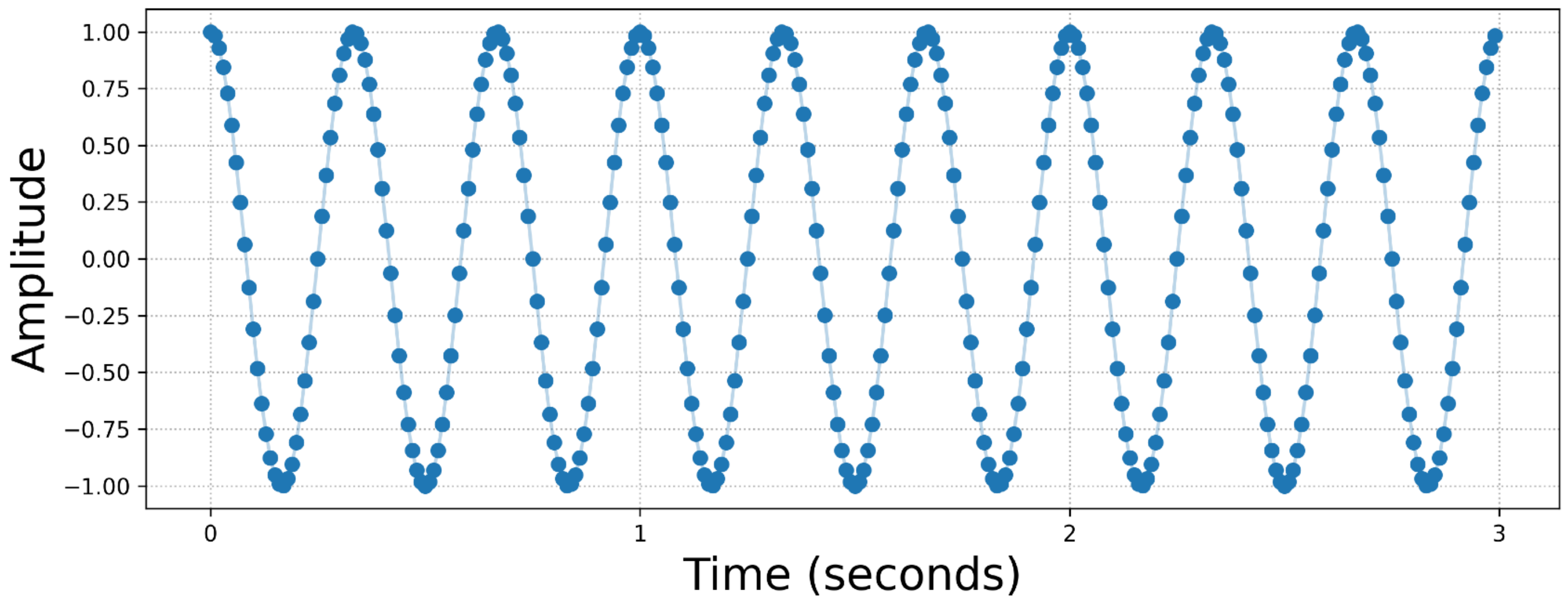
$$x[t_k] = Ae^{2\pi i f t_k}$$

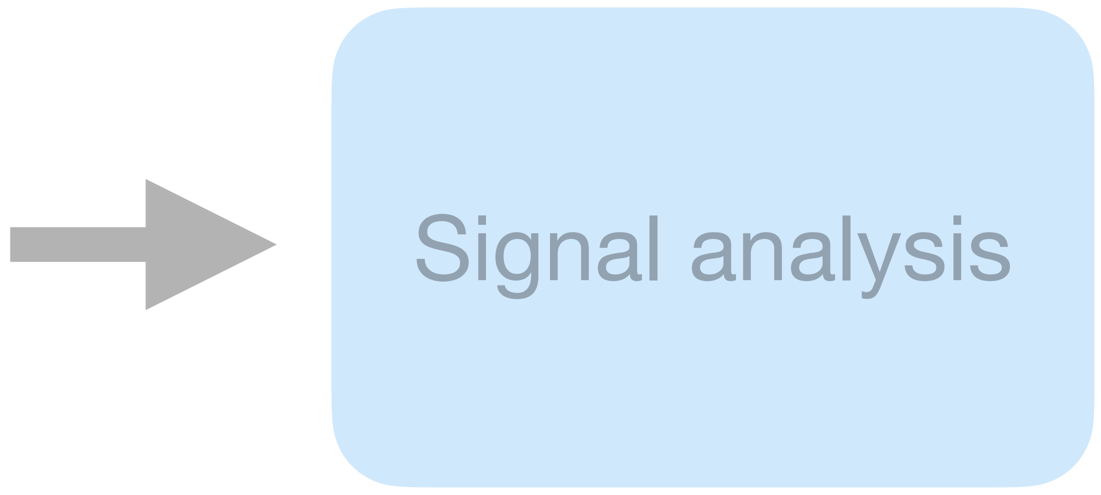
Digital signal processing (DSP)

Incoming signal

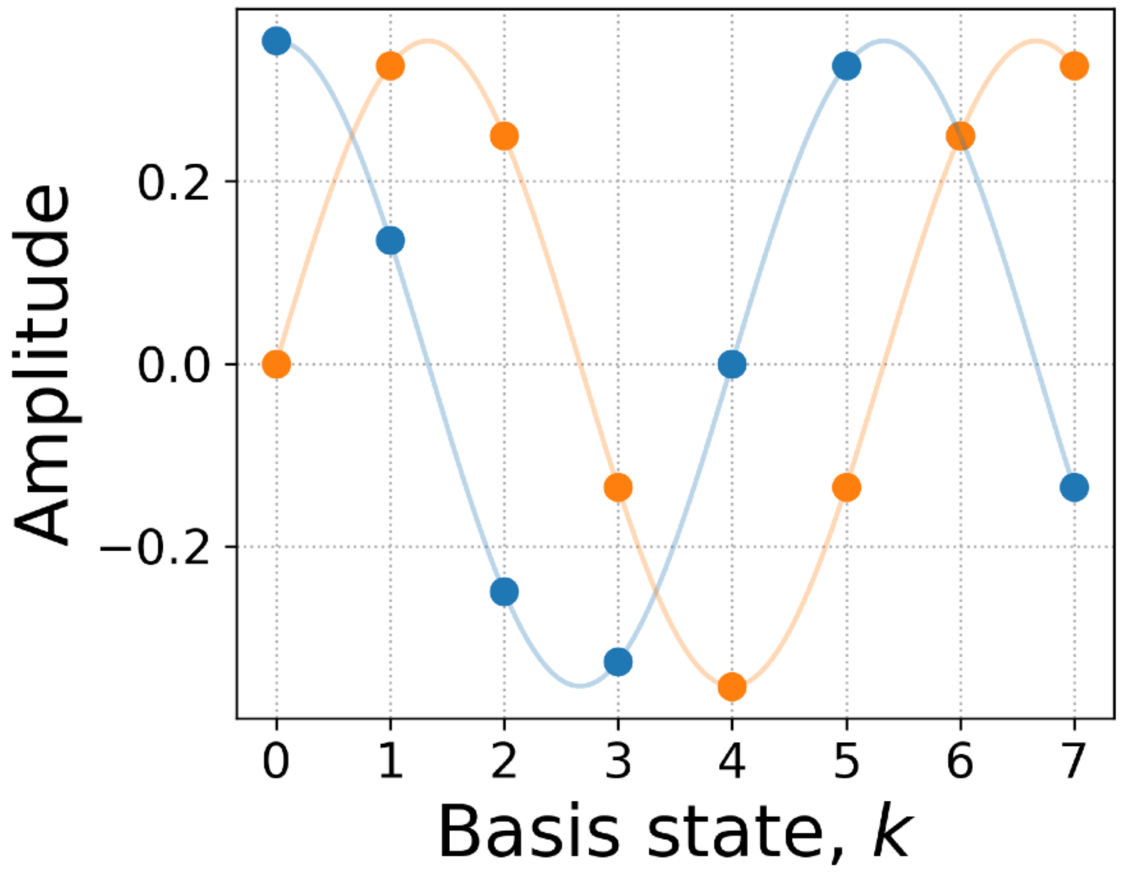






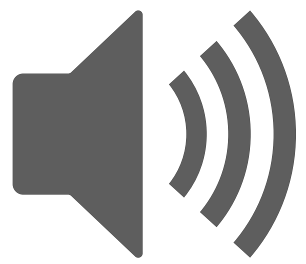


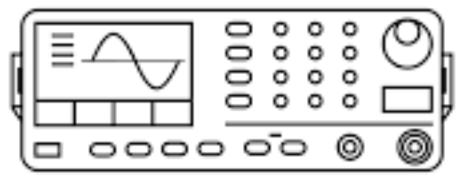
In QPE, k labels the time points at which we are sampling → time is a *quantum variable*

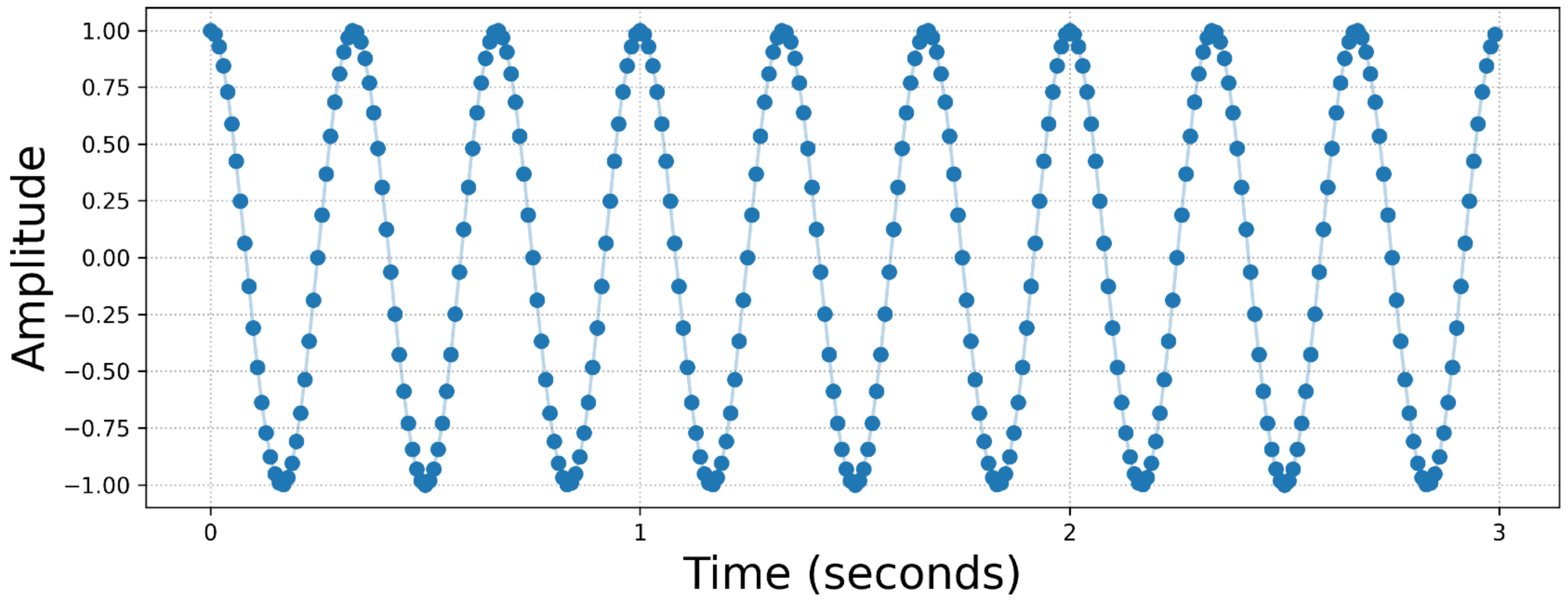


Digital signal processing (DSP)

Incoming signal

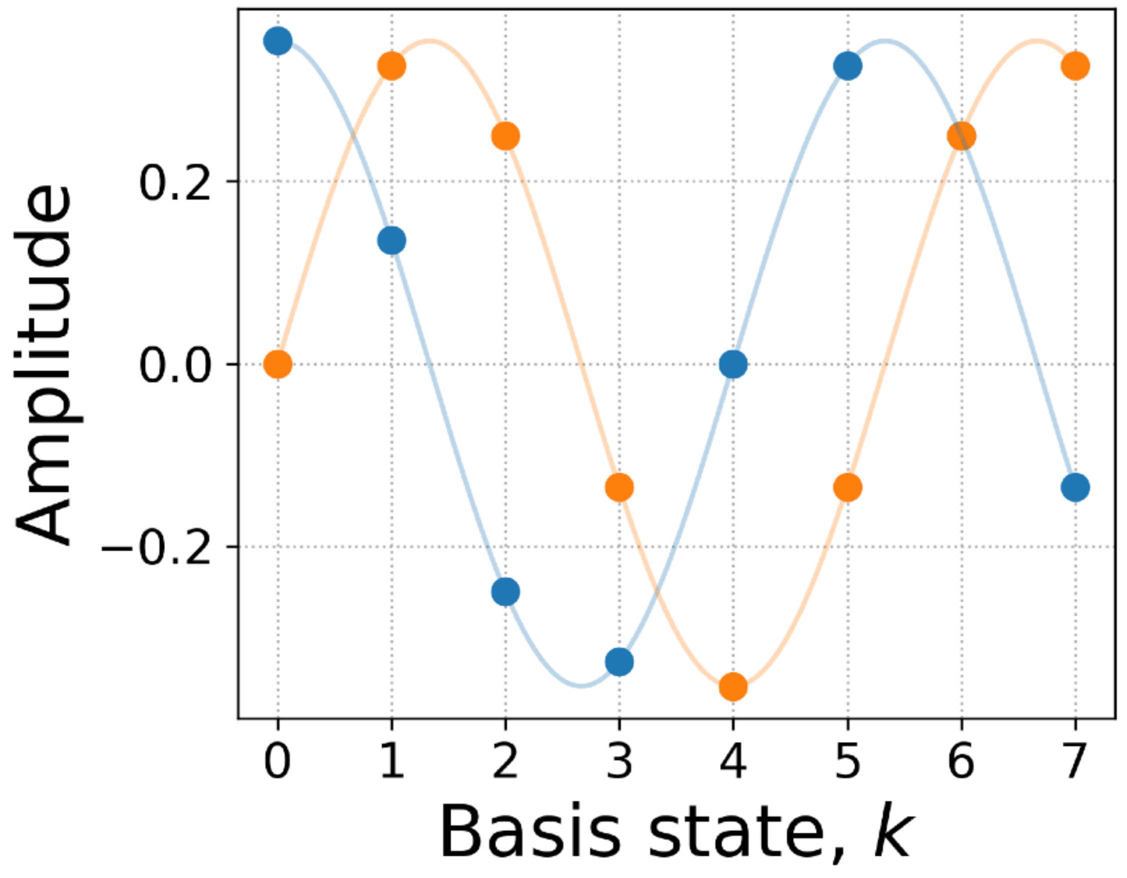


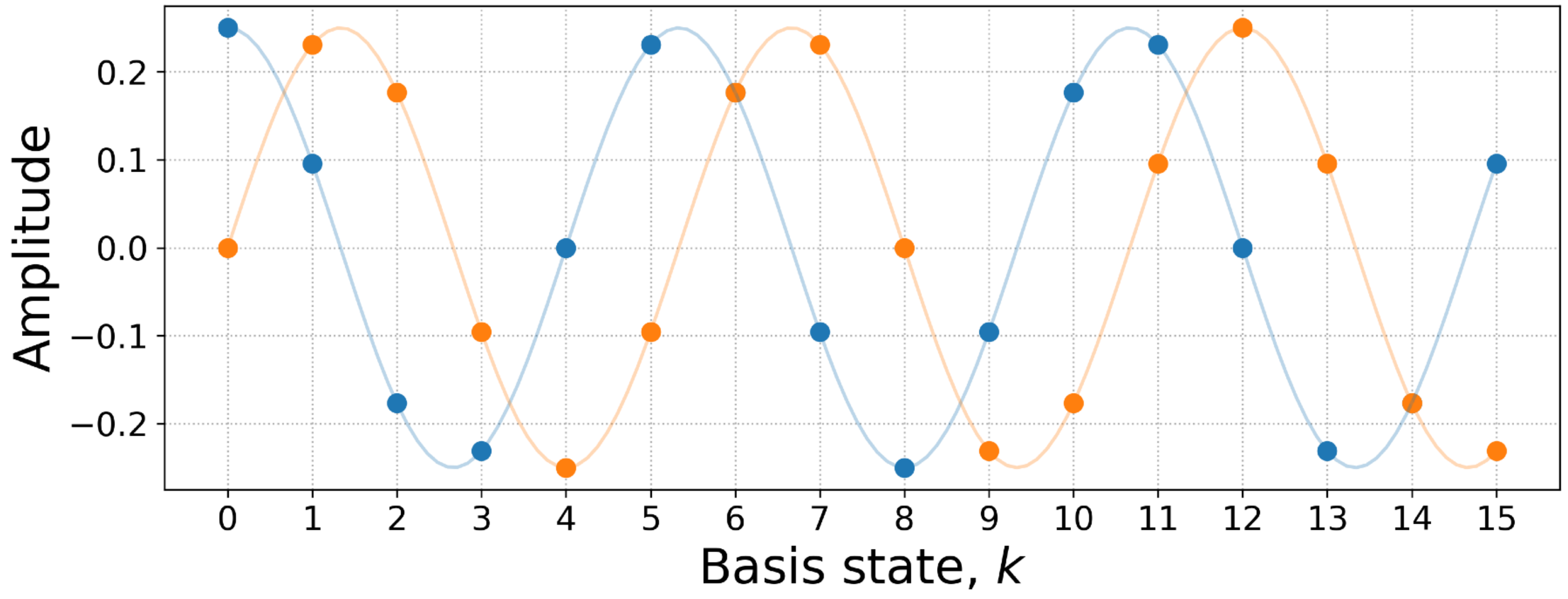




Signal analysis

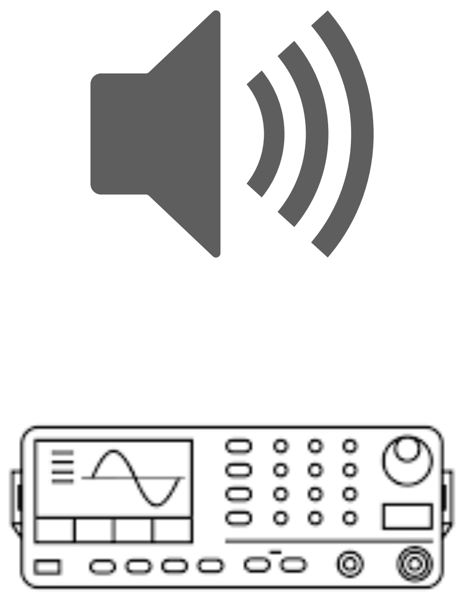
Increasing the number of phase qubits increases the sampling duration!

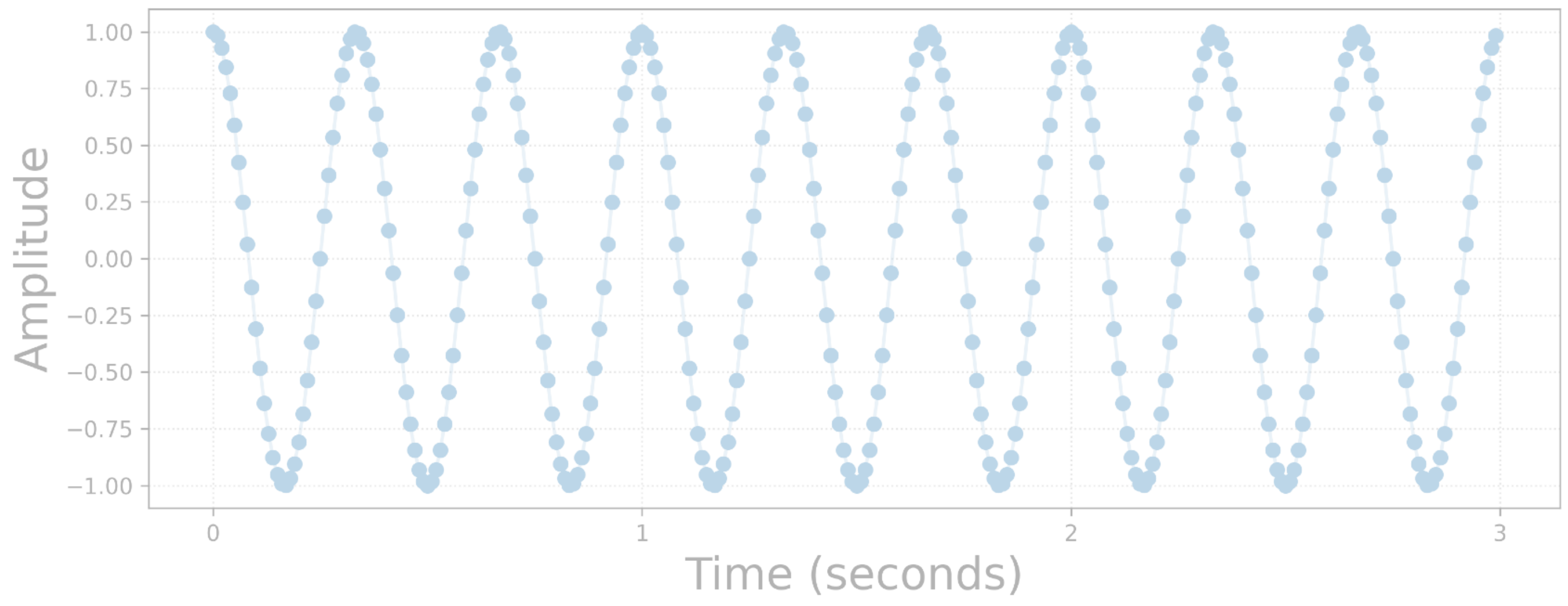




Digital signal processing (DSP)

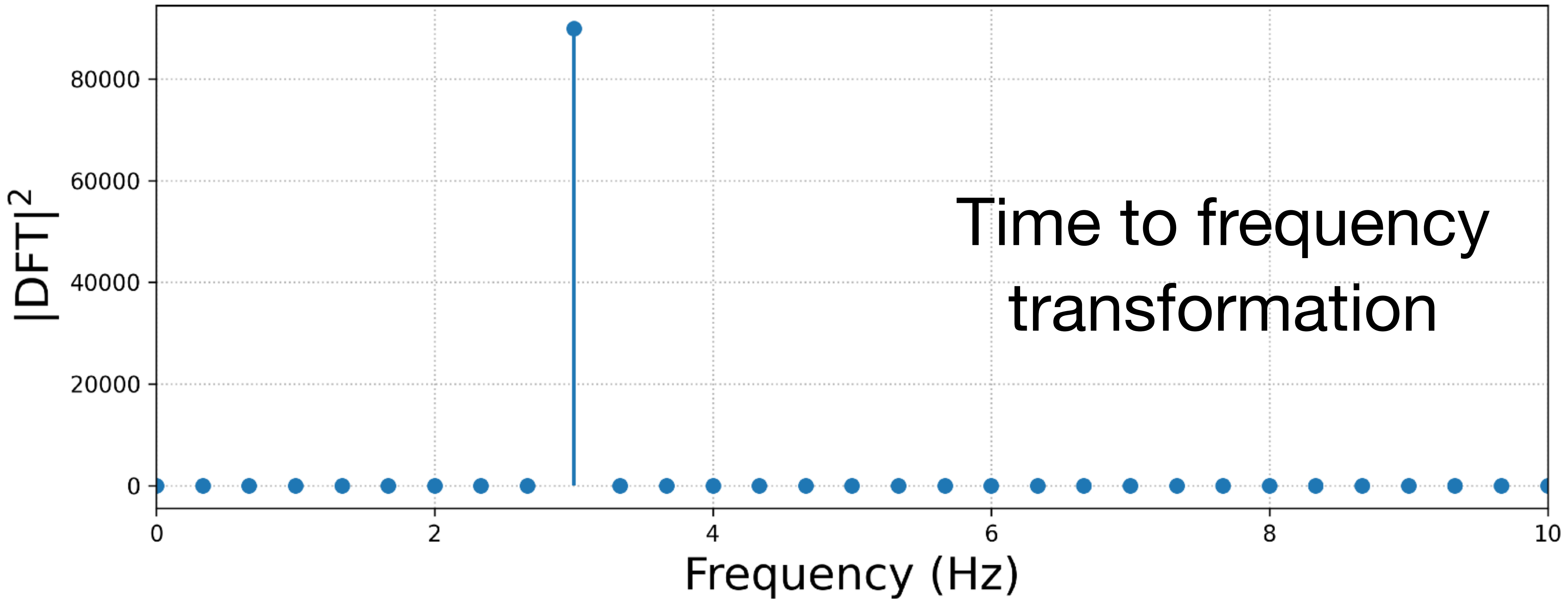
Incoming signal





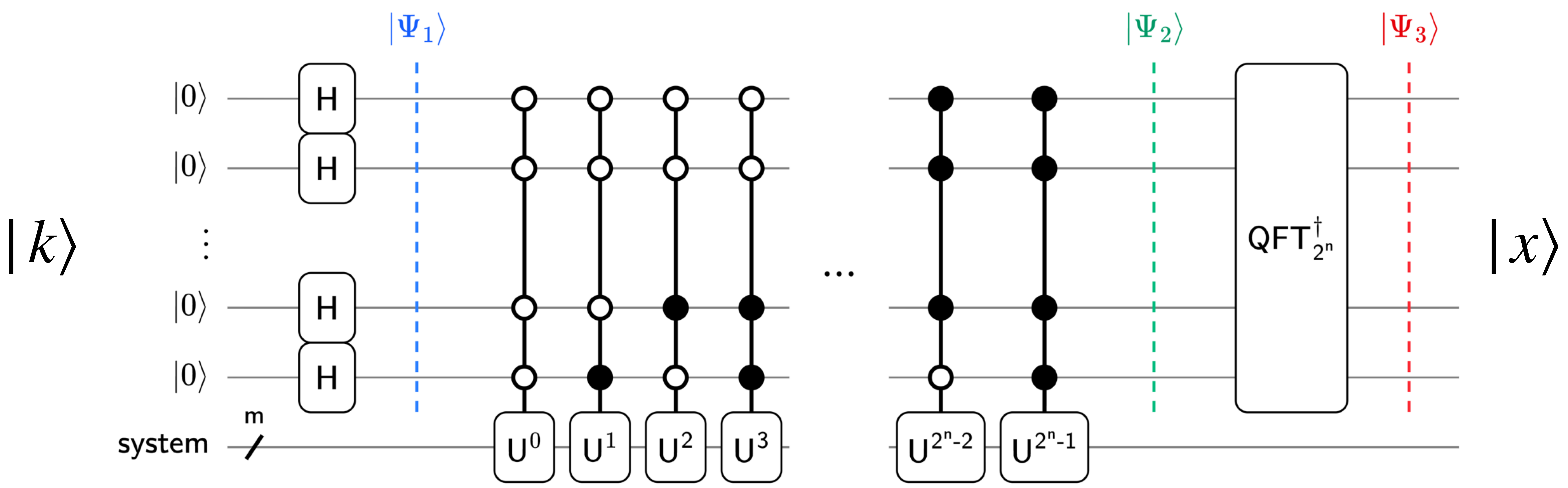
Signal analysis

Analyze through discrete Fourier Transform (DFT)



“Power spectrum”

Revisiting QPE

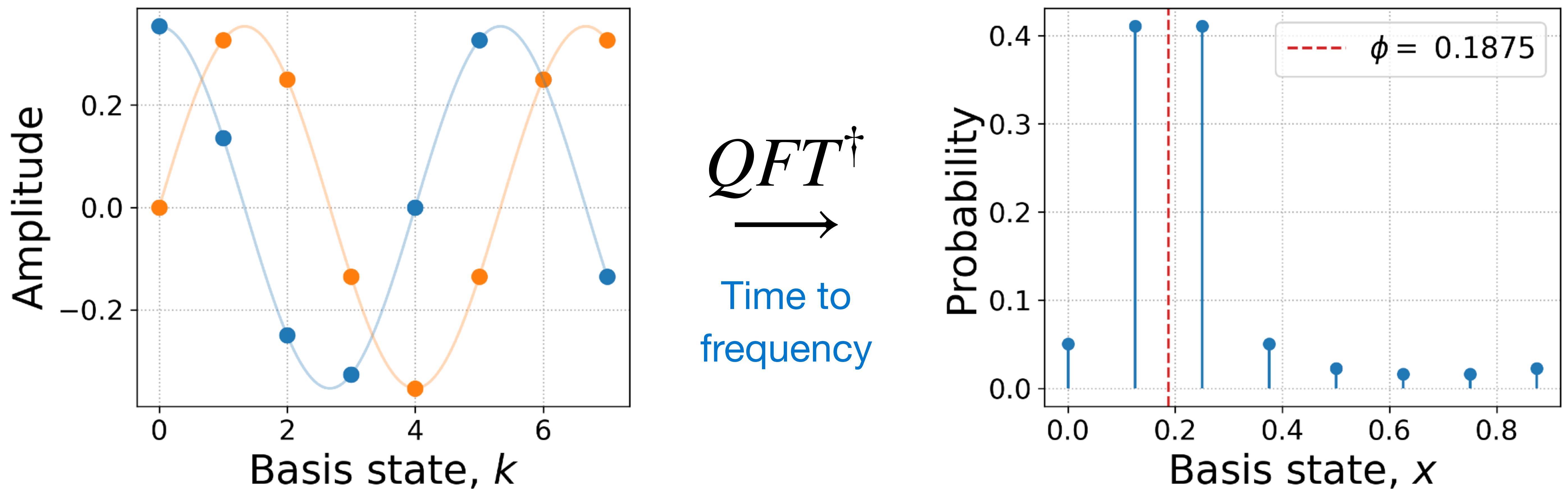


$$P(x) = \left| \frac{1}{2^n} \sum_{k=0}^{2^n-1} e^{-\frac{2\pi i k}{2^n}(x - 2^n\phi)} \right|^2$$

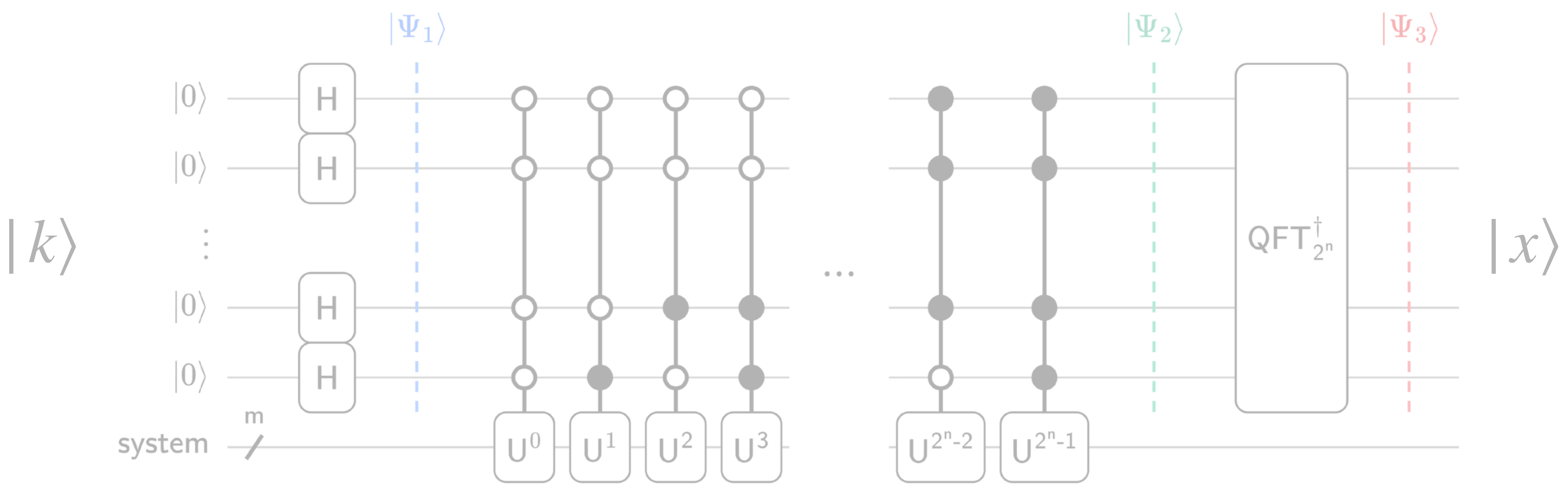
Power spectrum

(Squared magnitude)

Inverse QFT analogous to forward pass of DFT!



Revisiting QPE



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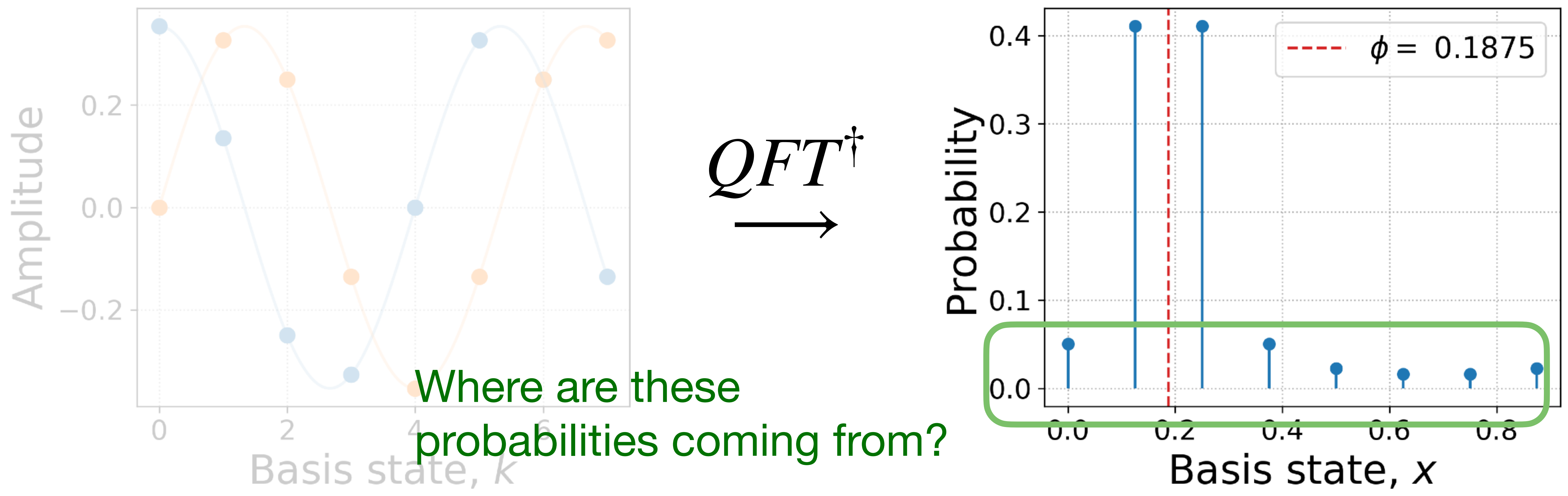
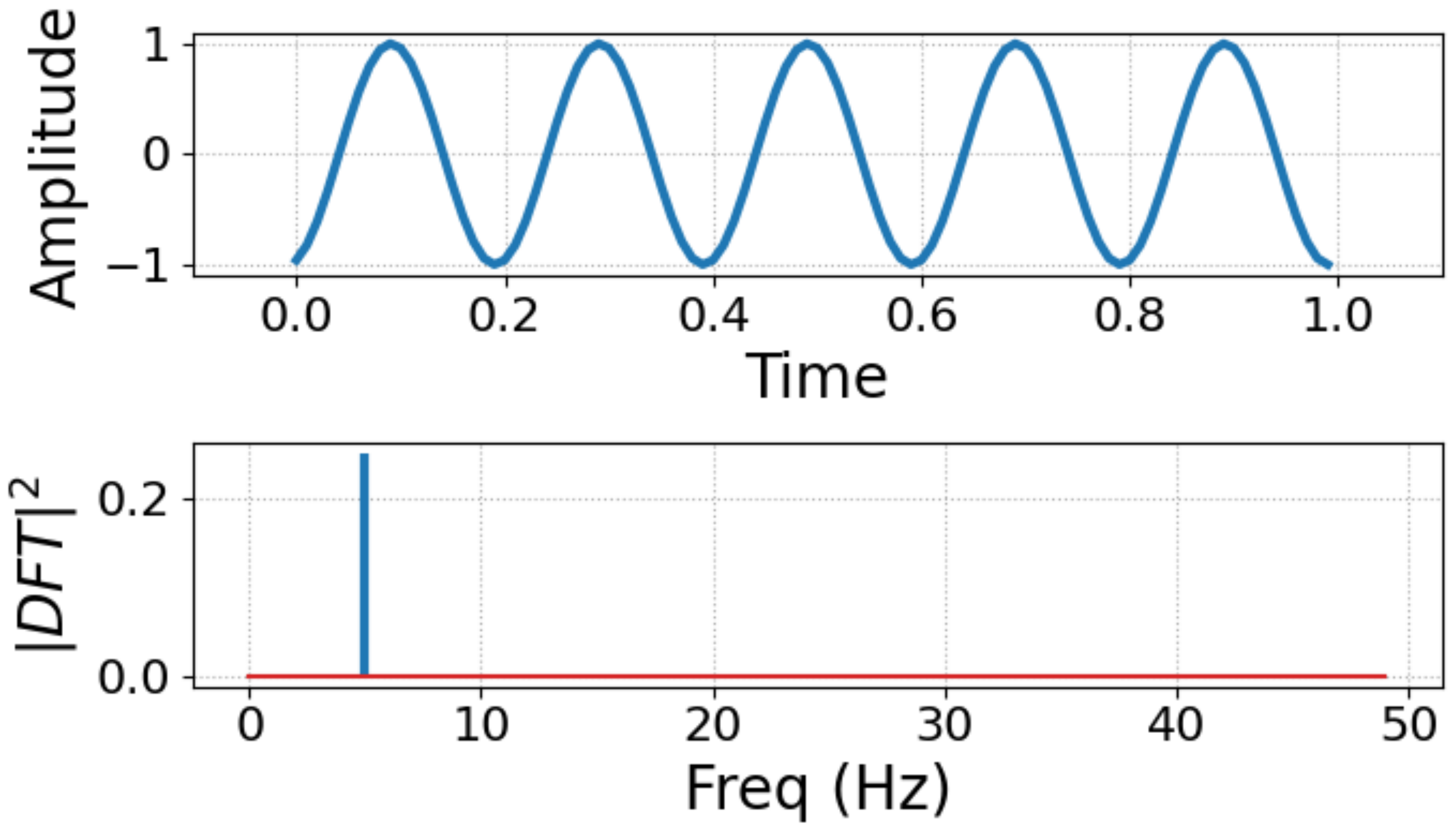


Time-limited sampling

Consider a **5 Hz sine wave**:

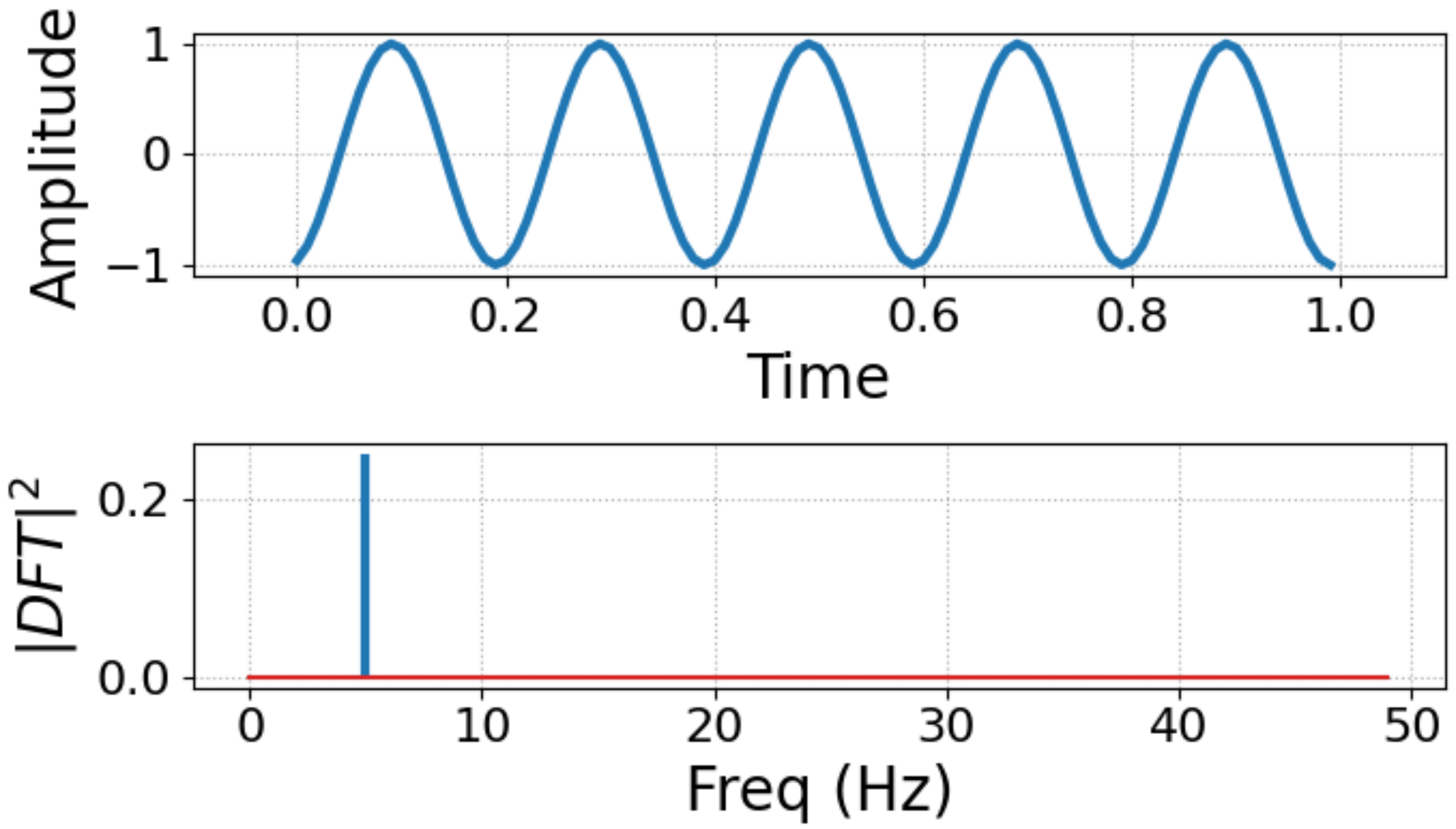


Sampled from t=0 to 1s

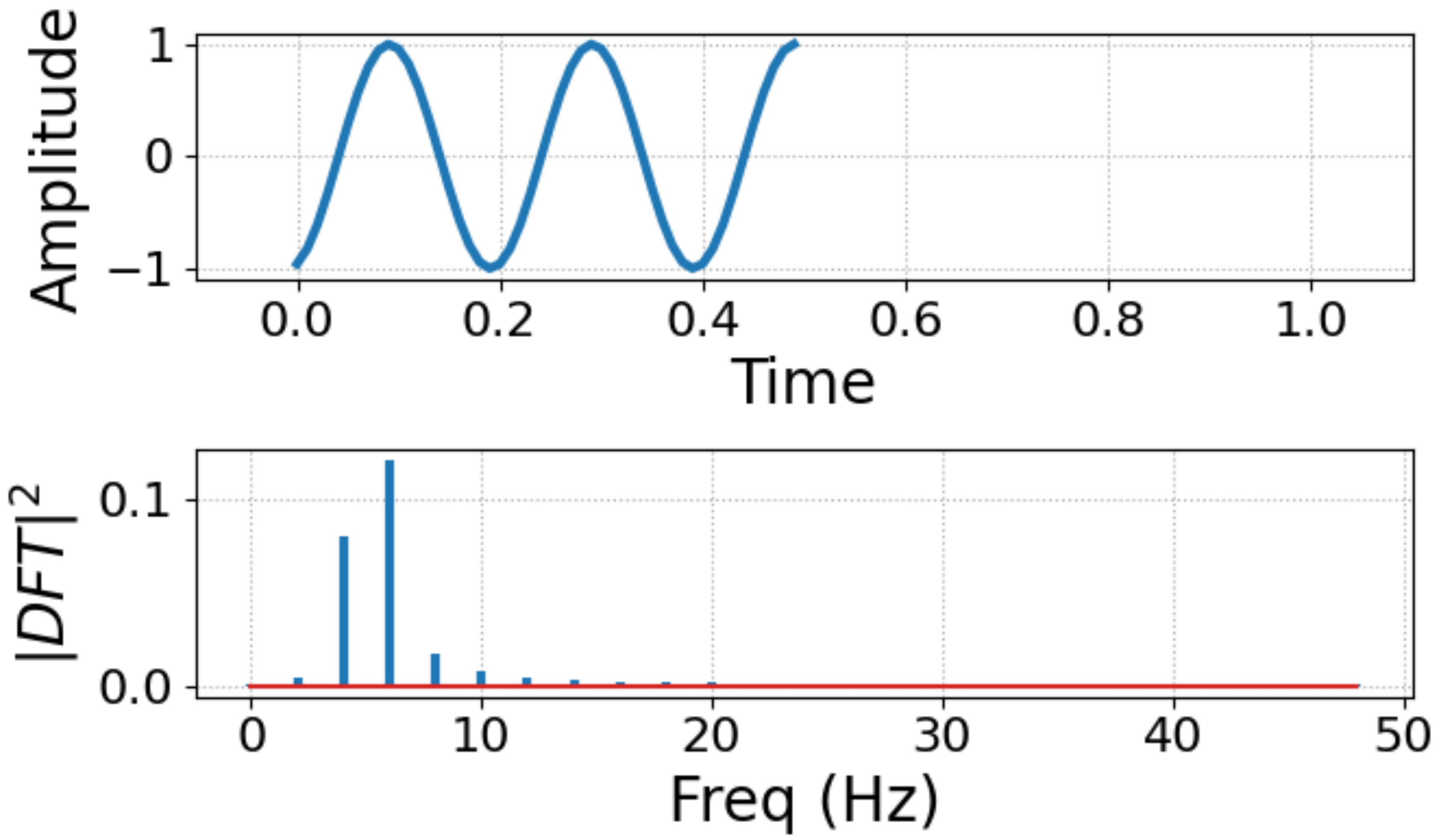
http://qingkaikong.blogspot.com/2016/10/signal-processing-why-do-we-need-taper.html

Time-limited sampling

Consider a **5 Hz sine wave**:



Sampled from t=0 to 1s

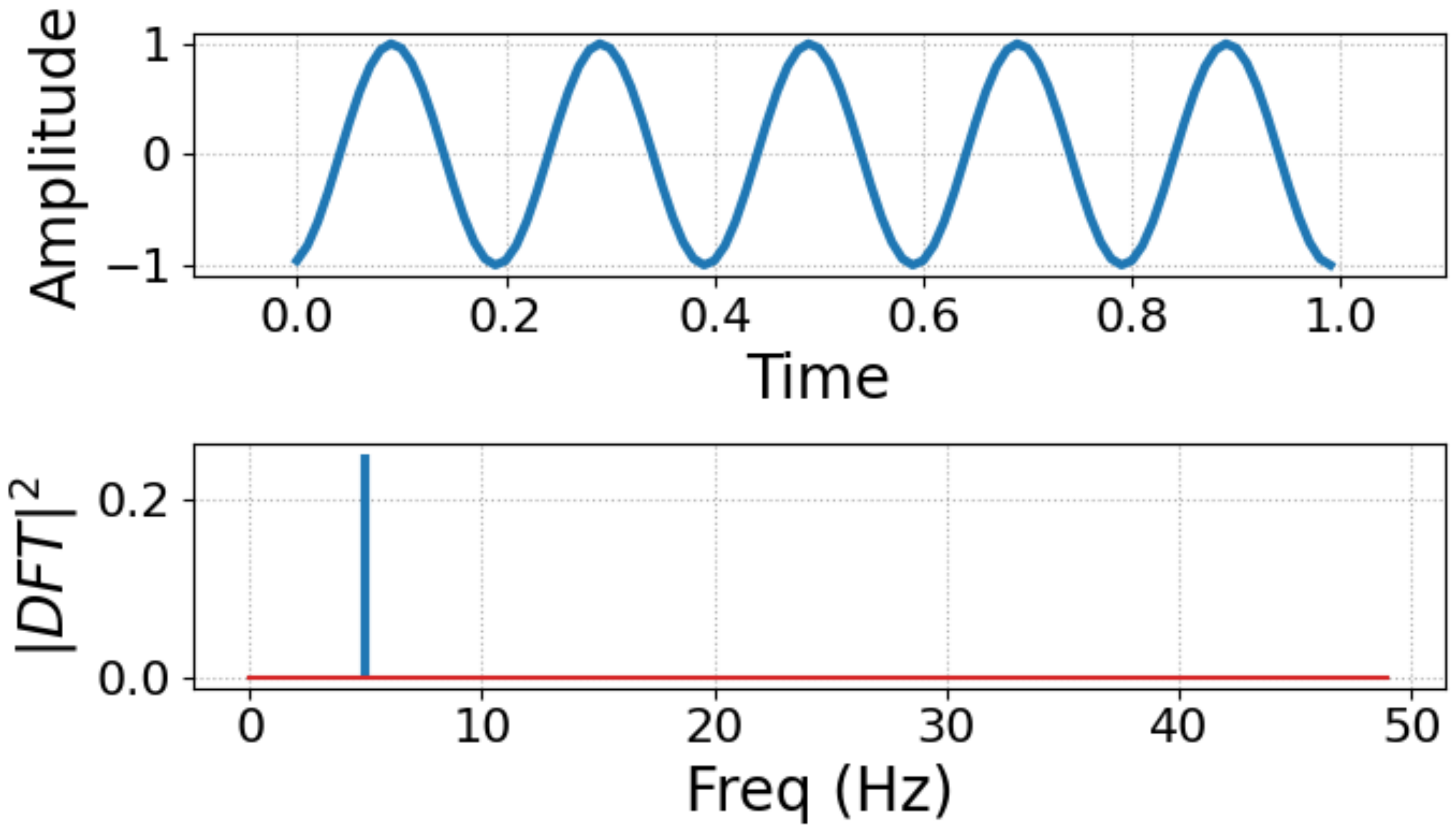


Sampled from t=0 to 0.5s

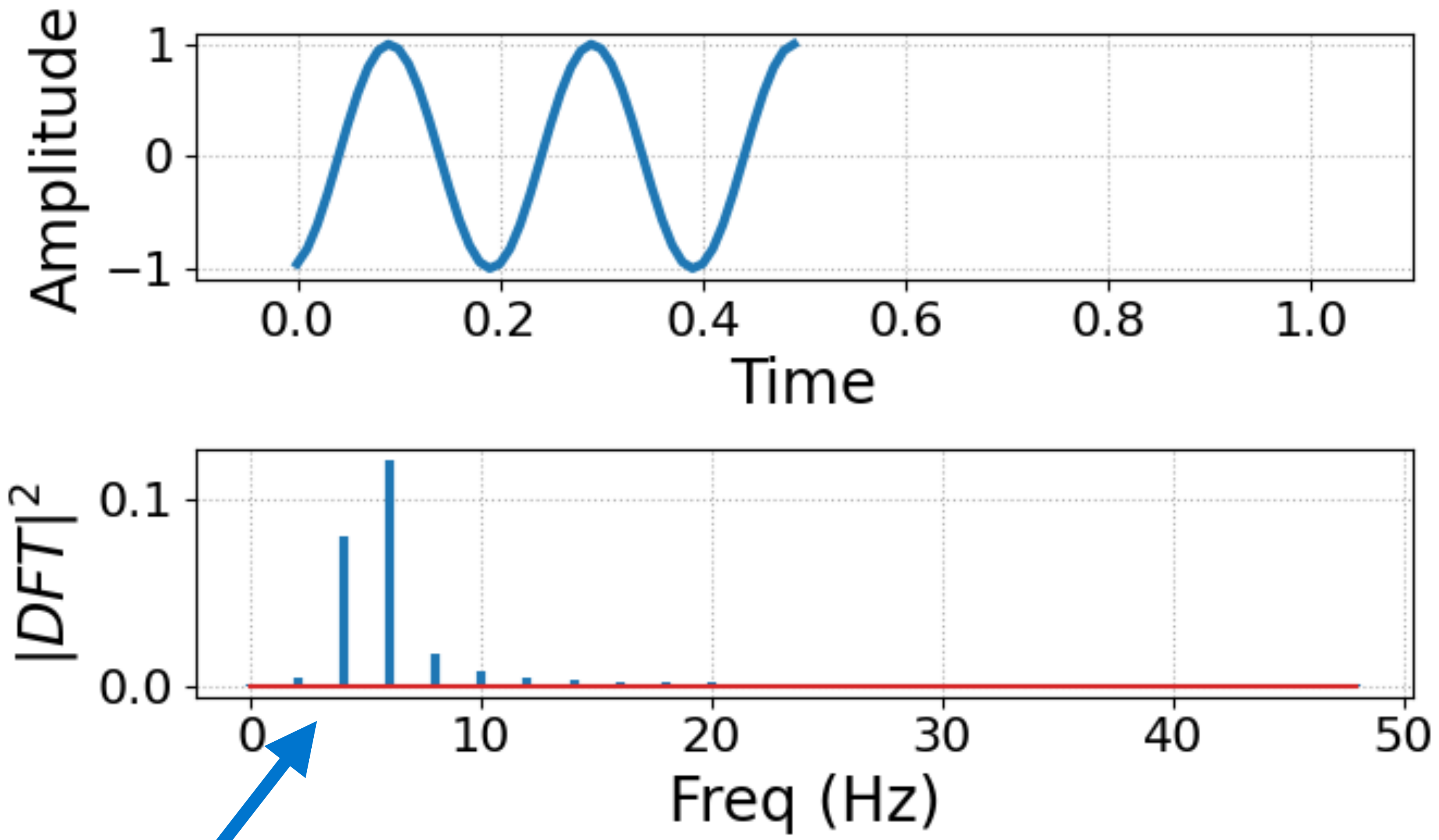
"Time-limited sample"

Time-limited sampling

Consider a **5 Hz sine wave**:



Sampled from t=0 to 1s



Sampled from t=0 to 0.5s

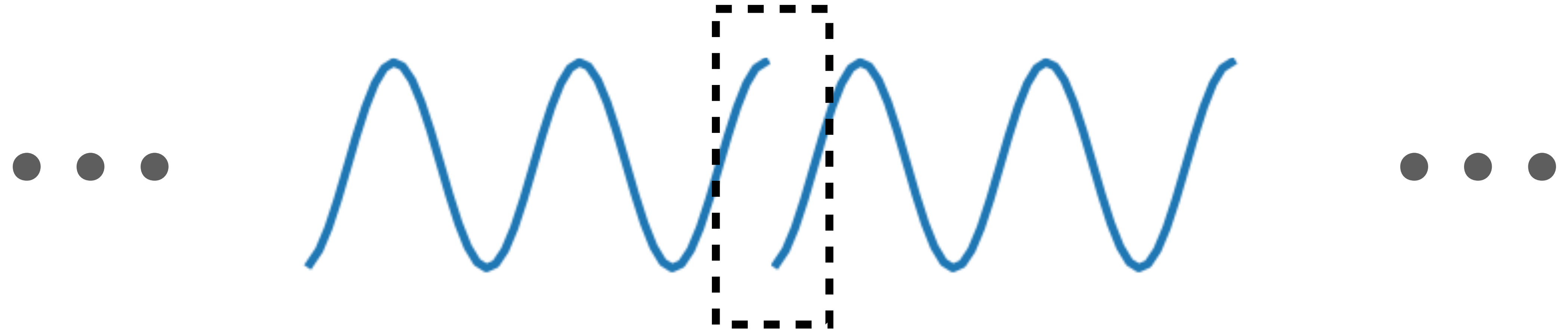
"Time-limited sample"

Why do we see this spread in frequencies?

Time-limited sampling

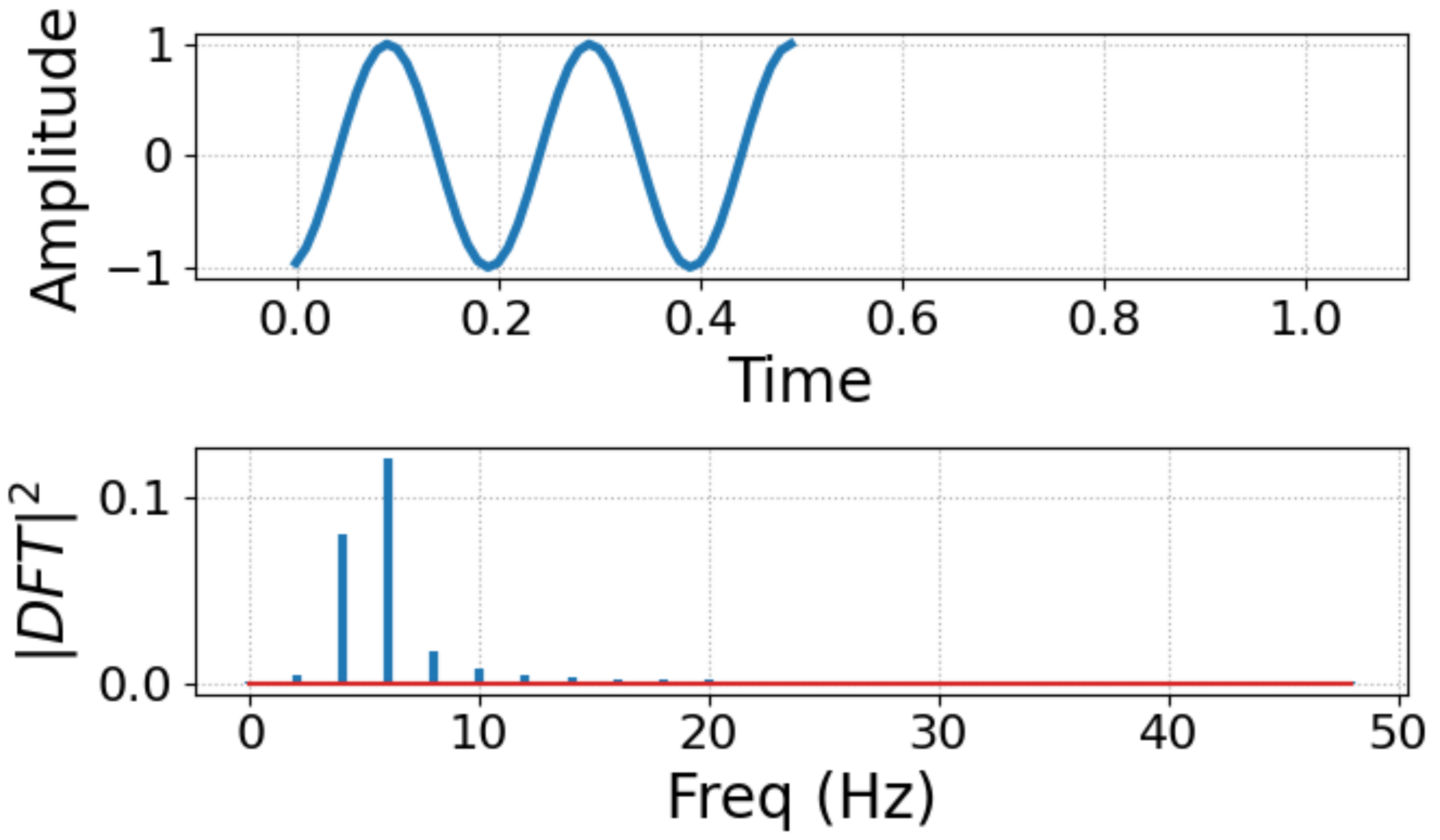
Consider a **5 Hz sine wave**:

Fourier transforming **assumes periodicity**



Discontinuity observed!
→ Contains artificial frequency components

We call this *“**spectral leakage**”*

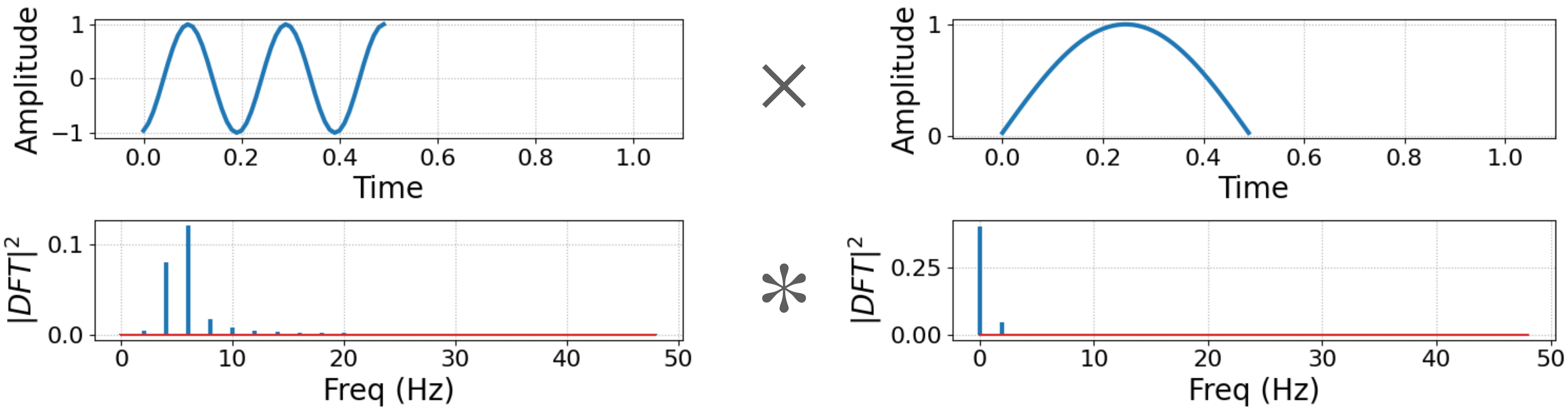


Sampled from t=0 to 0.5s

“Time-limited sample”

Treating spectral leakage

Can apply a *window function* (also called *tapering function*)!

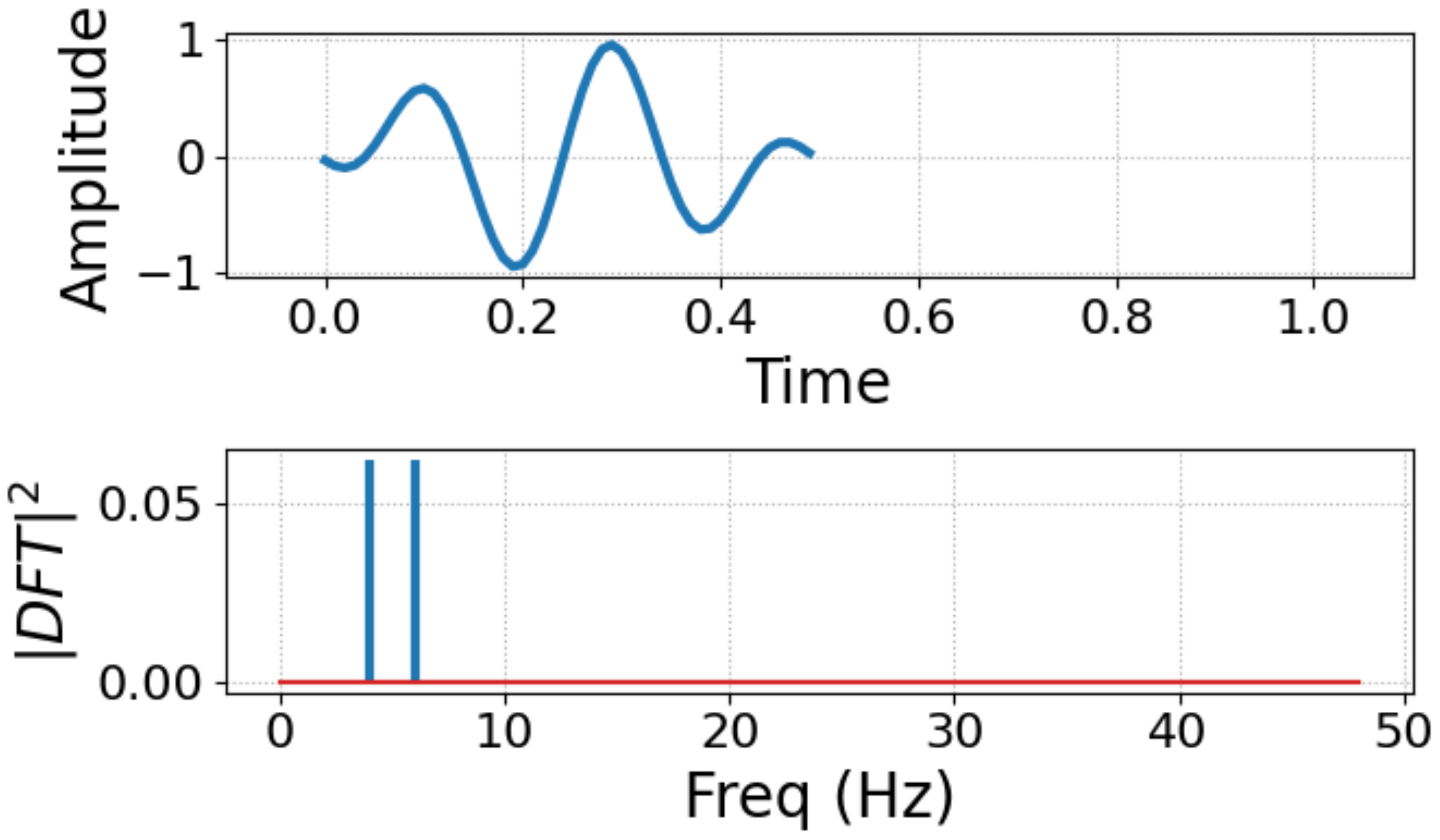


Time-limited signal

Cosine window

Treating spectral leakage

Can apply a *window function* (also called *tapering function*)!

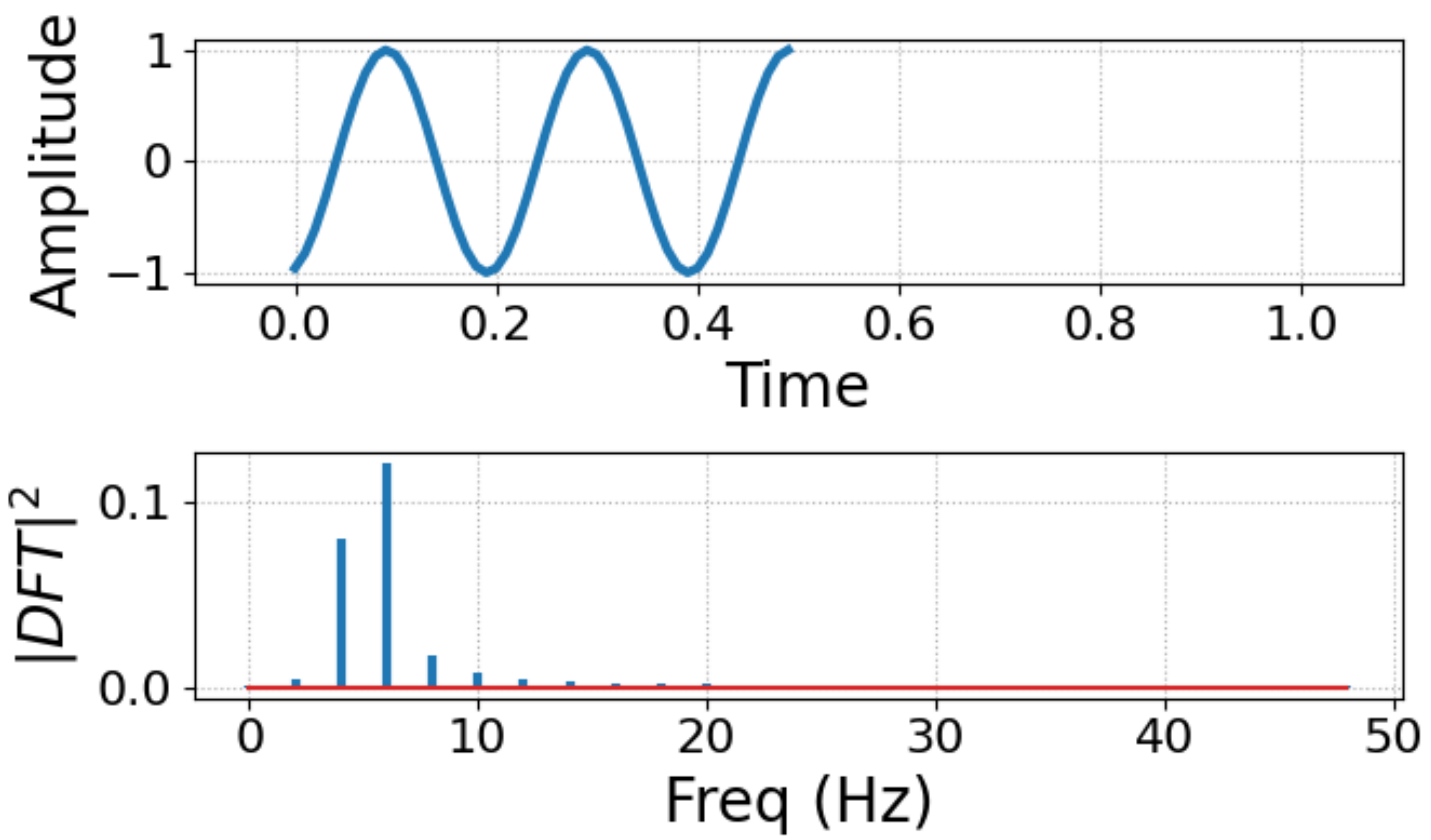


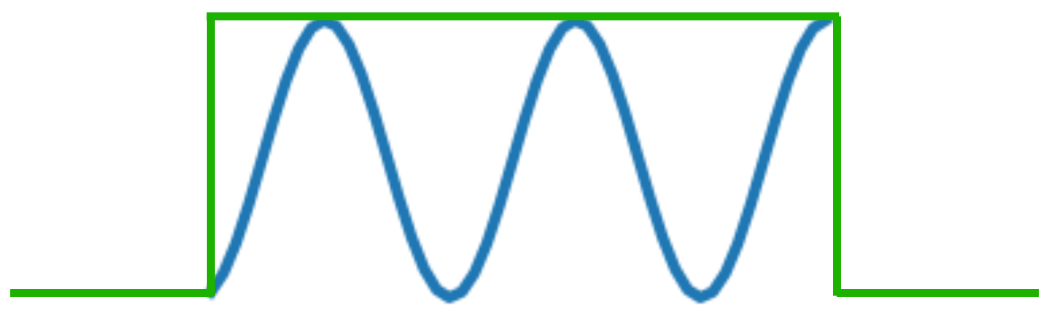
Now more periodic than before!

Frequency spectrum looks better (suppressed artificial frequencies from discontinuity)!

Treating spectral leakage

Can apply a *window function* (also called *tapering function*)!

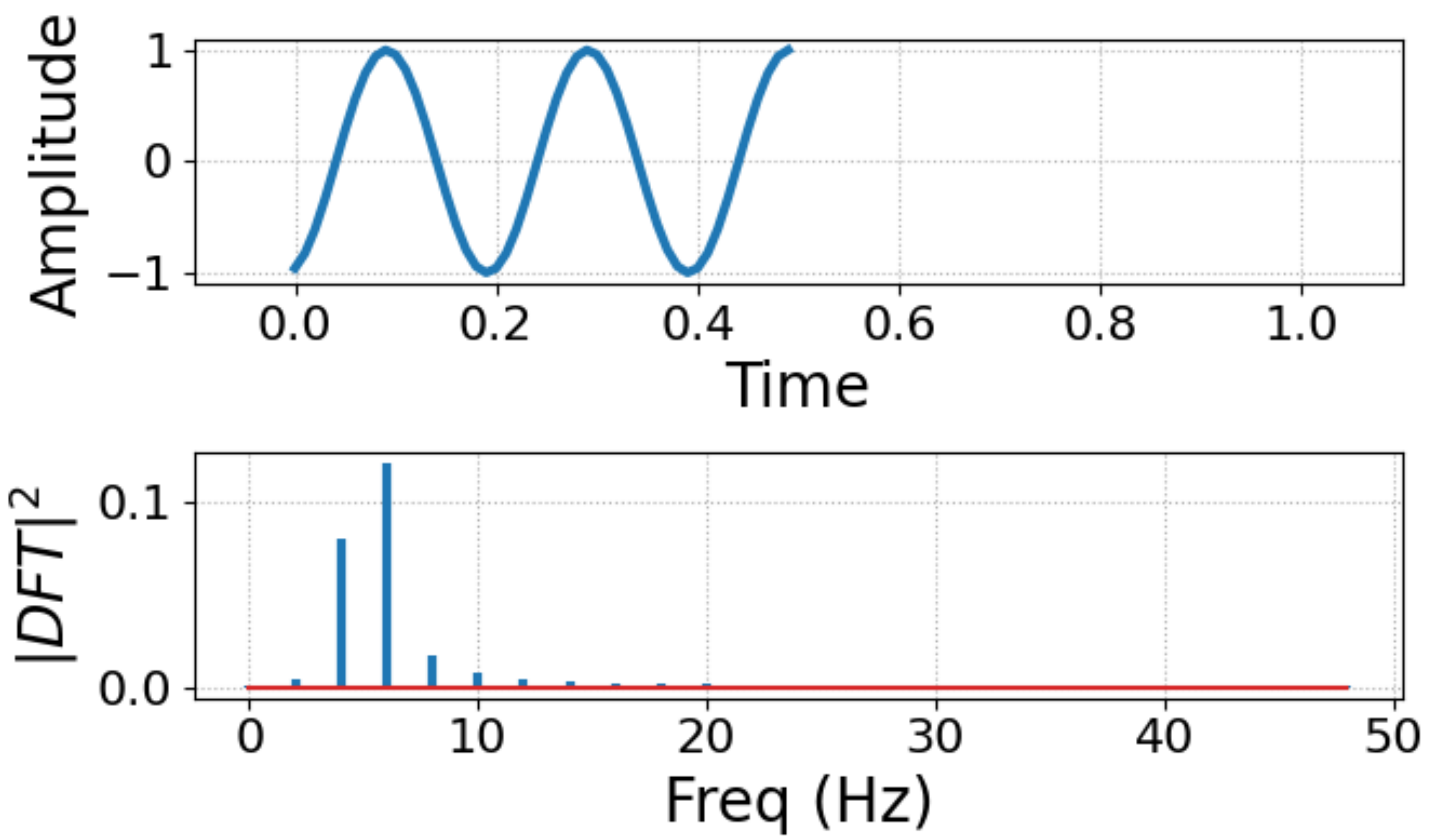


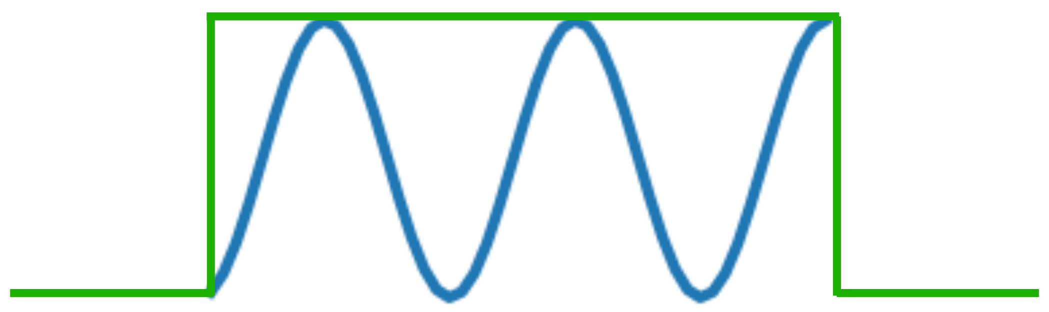


Note that this also assumed a window — a rectangular one

Treating spectral leakage

Can apply a *window function* (also called *tapering function*)!

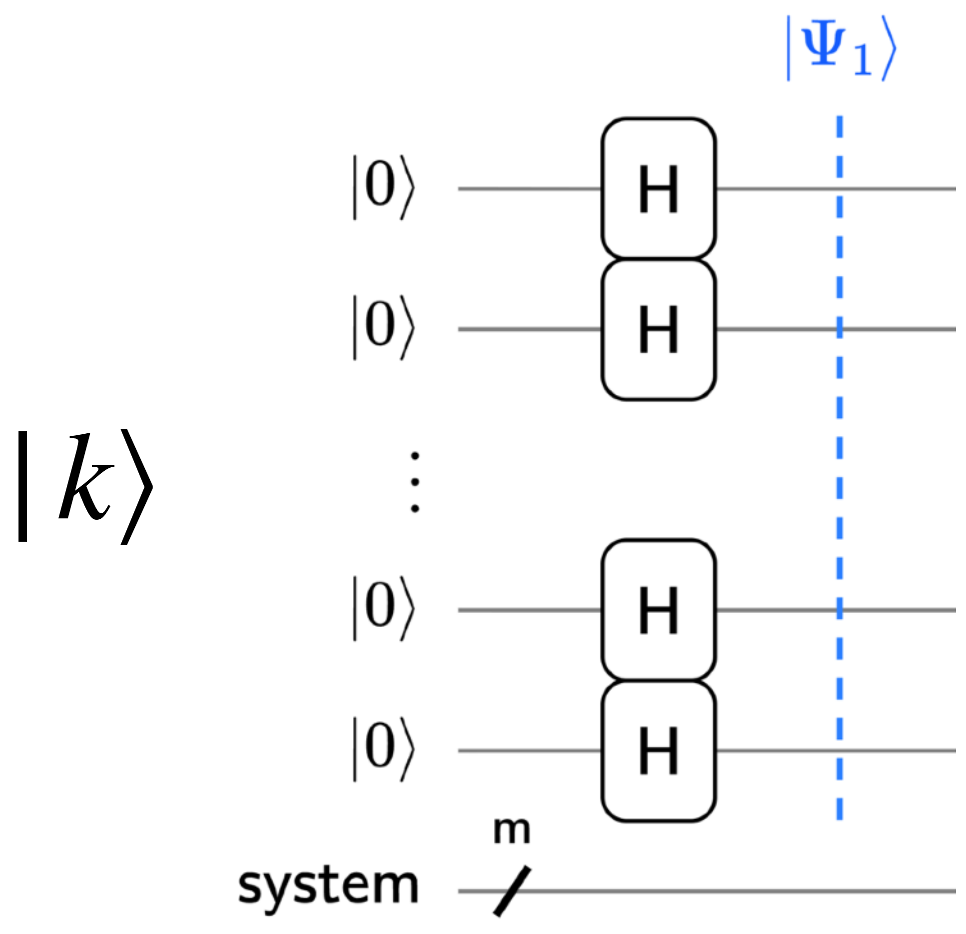




Note that this also assumed a window — a rectangular one

Can we bring it back to QPE?

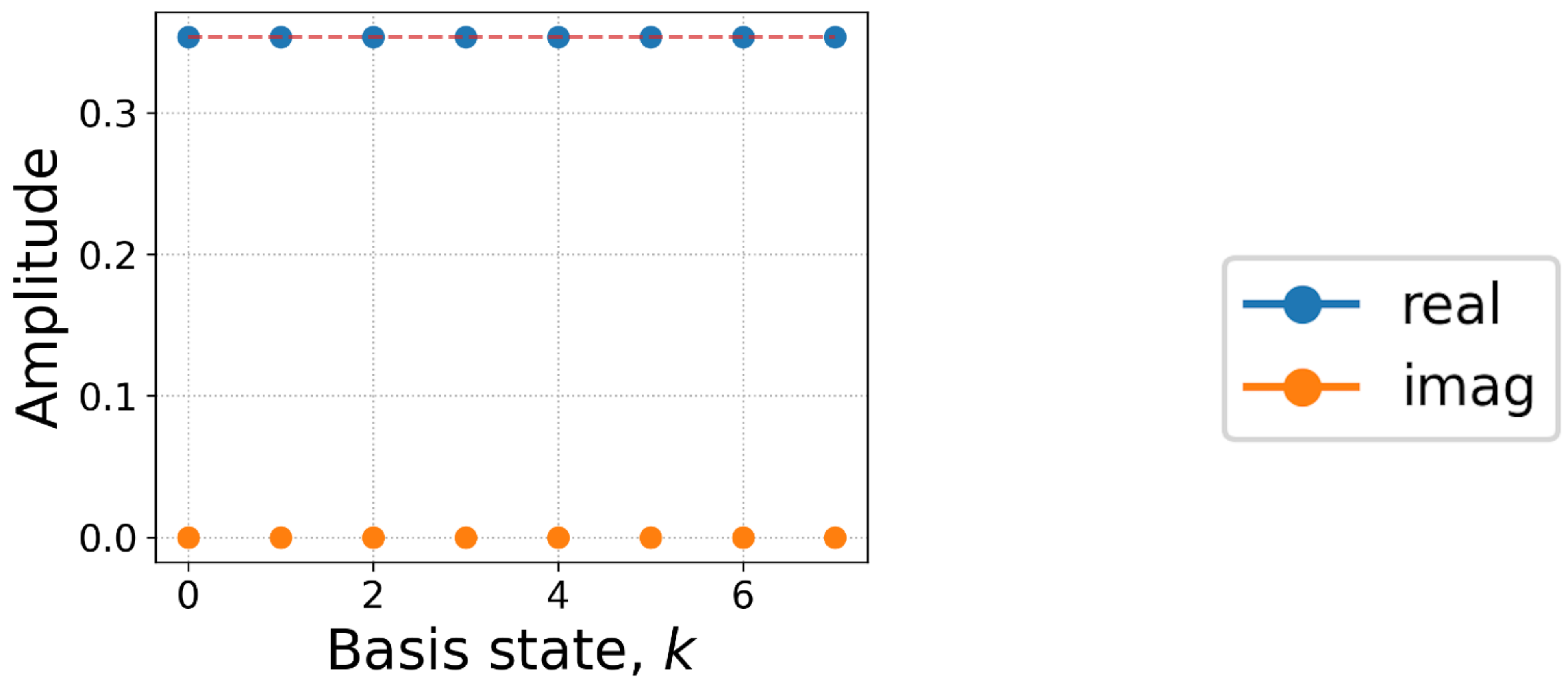
Revisiting QPE



$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1}|k\rangle$$

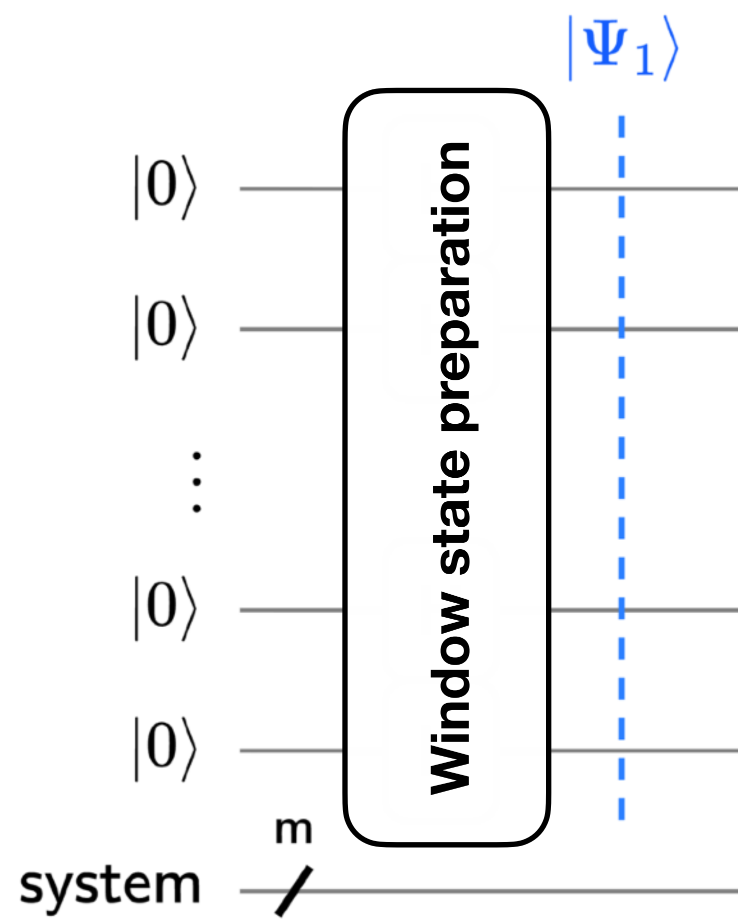
A rectangular window

by default!



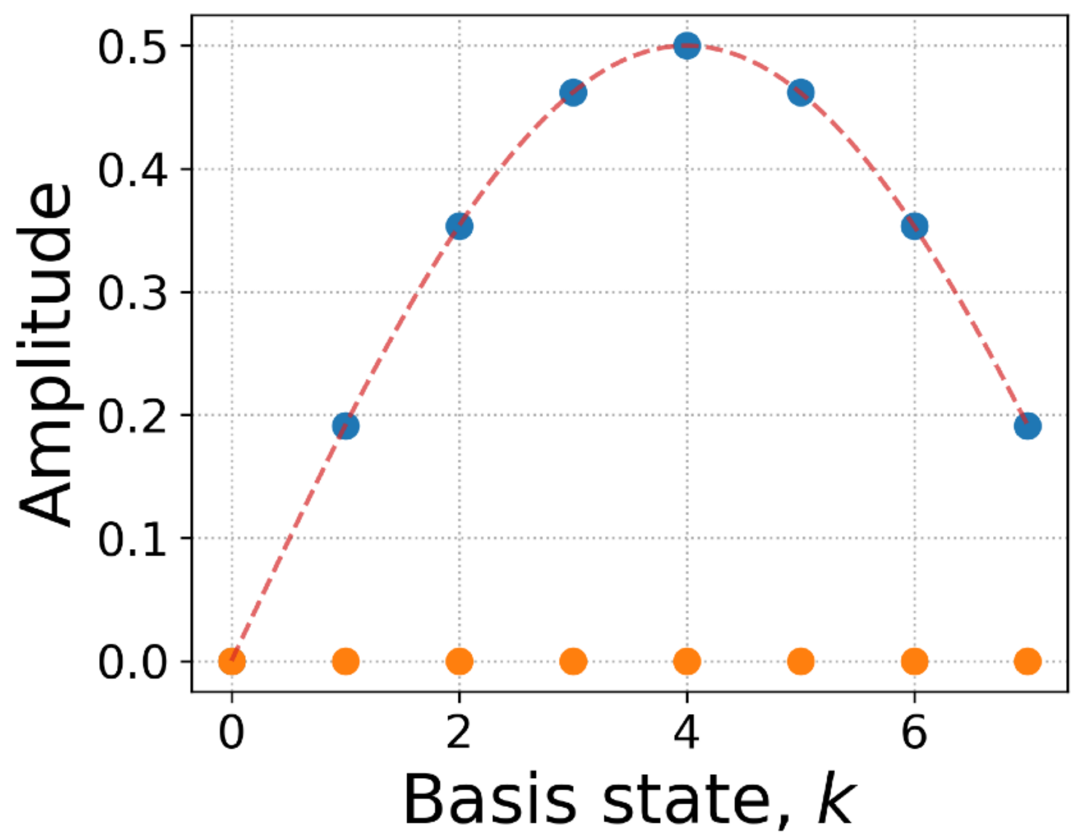
Revisiting QPE

$|k\rangle$

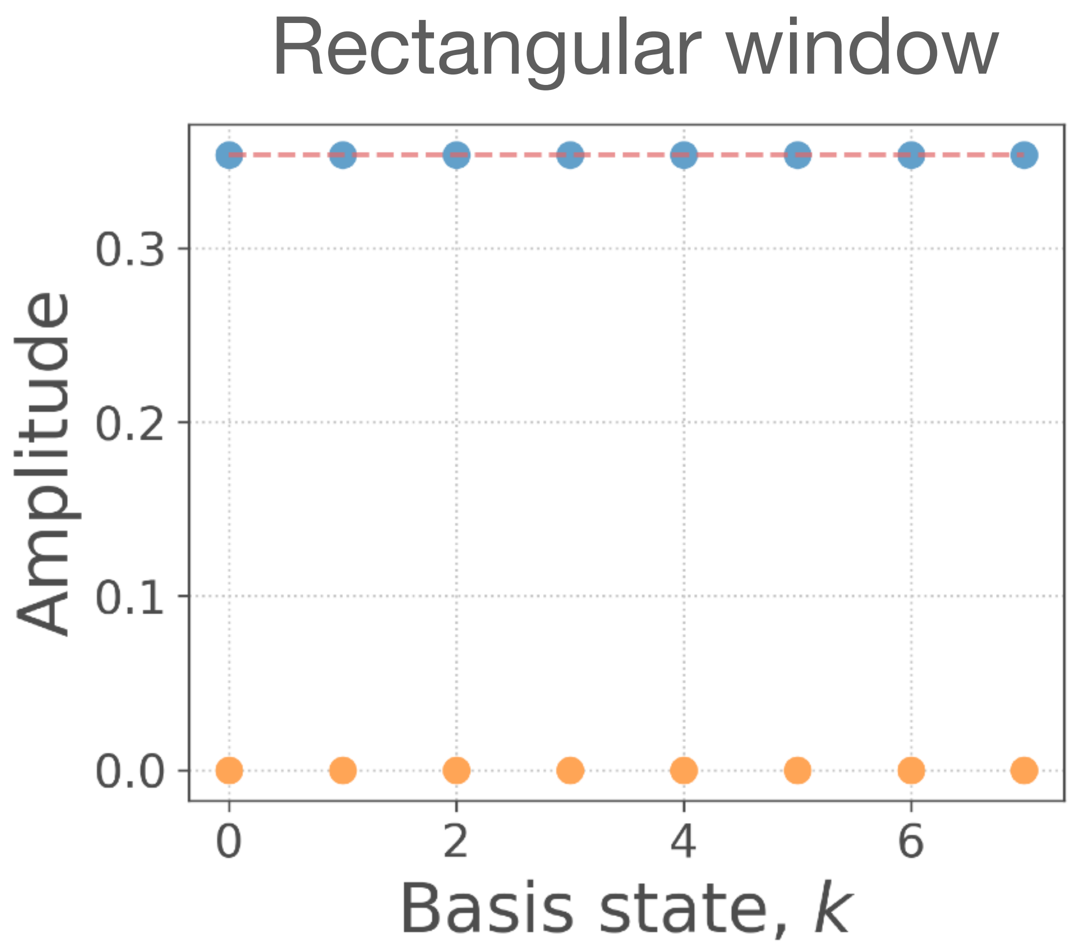


$$\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} W(k)\,|k\rangle$$

Using the cosine window [1]

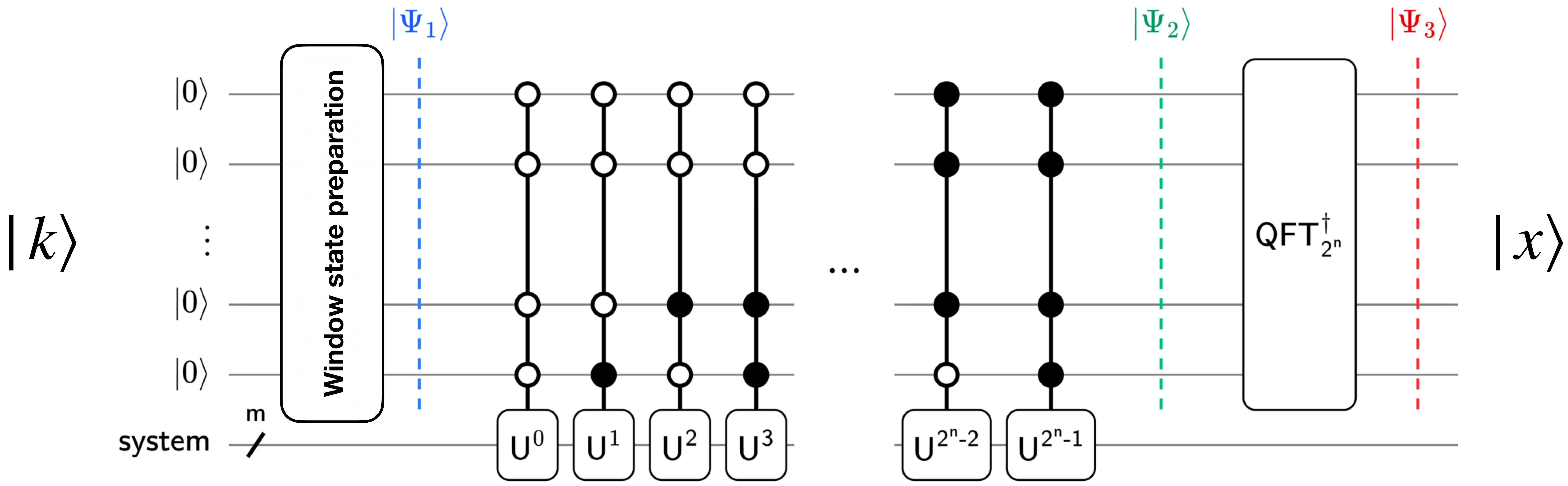


vs.

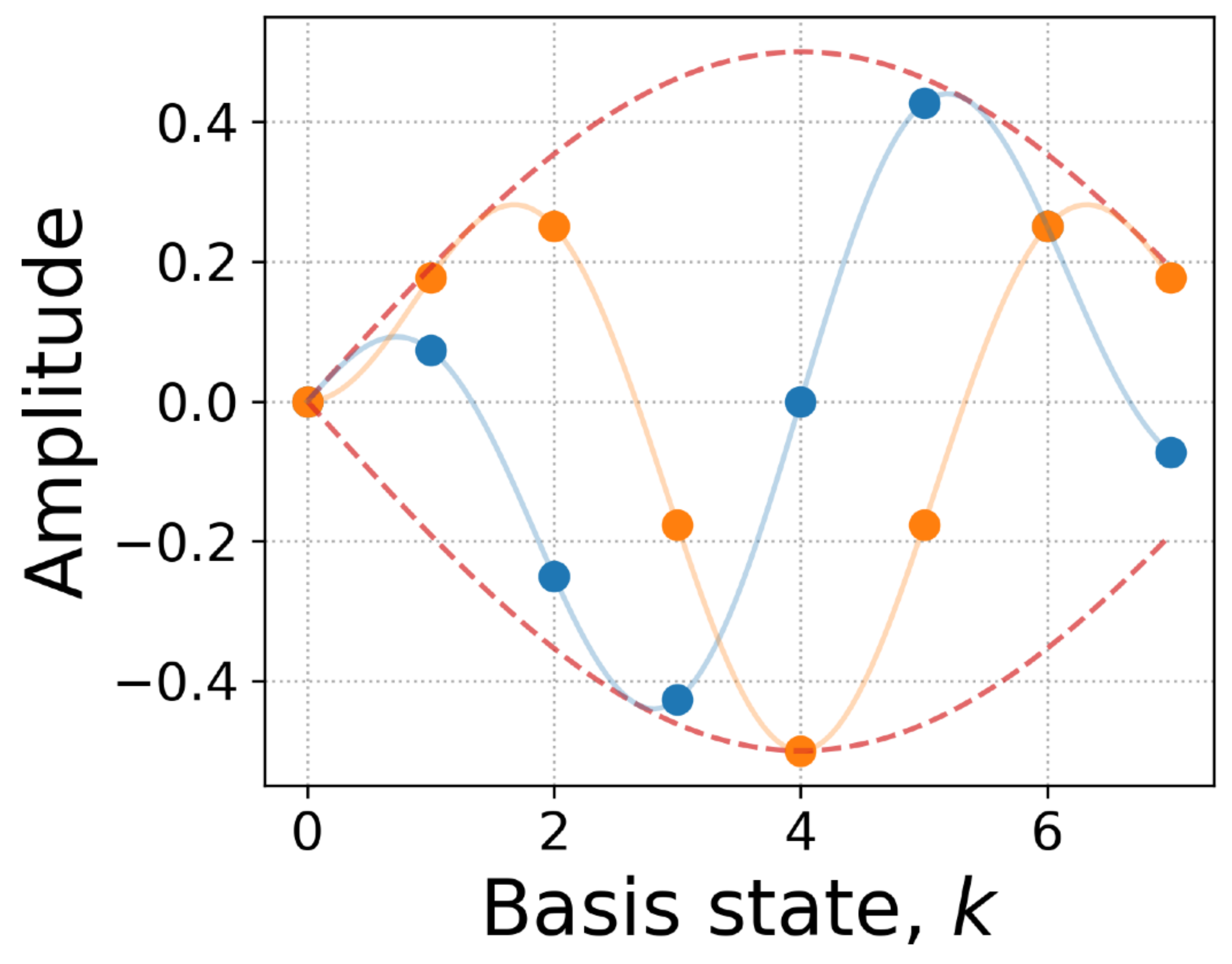


[1] Rendon, G., Izubuchi, T. & Kikuchi, Y. *Phys Rev D* **106**, 034503 (2022).

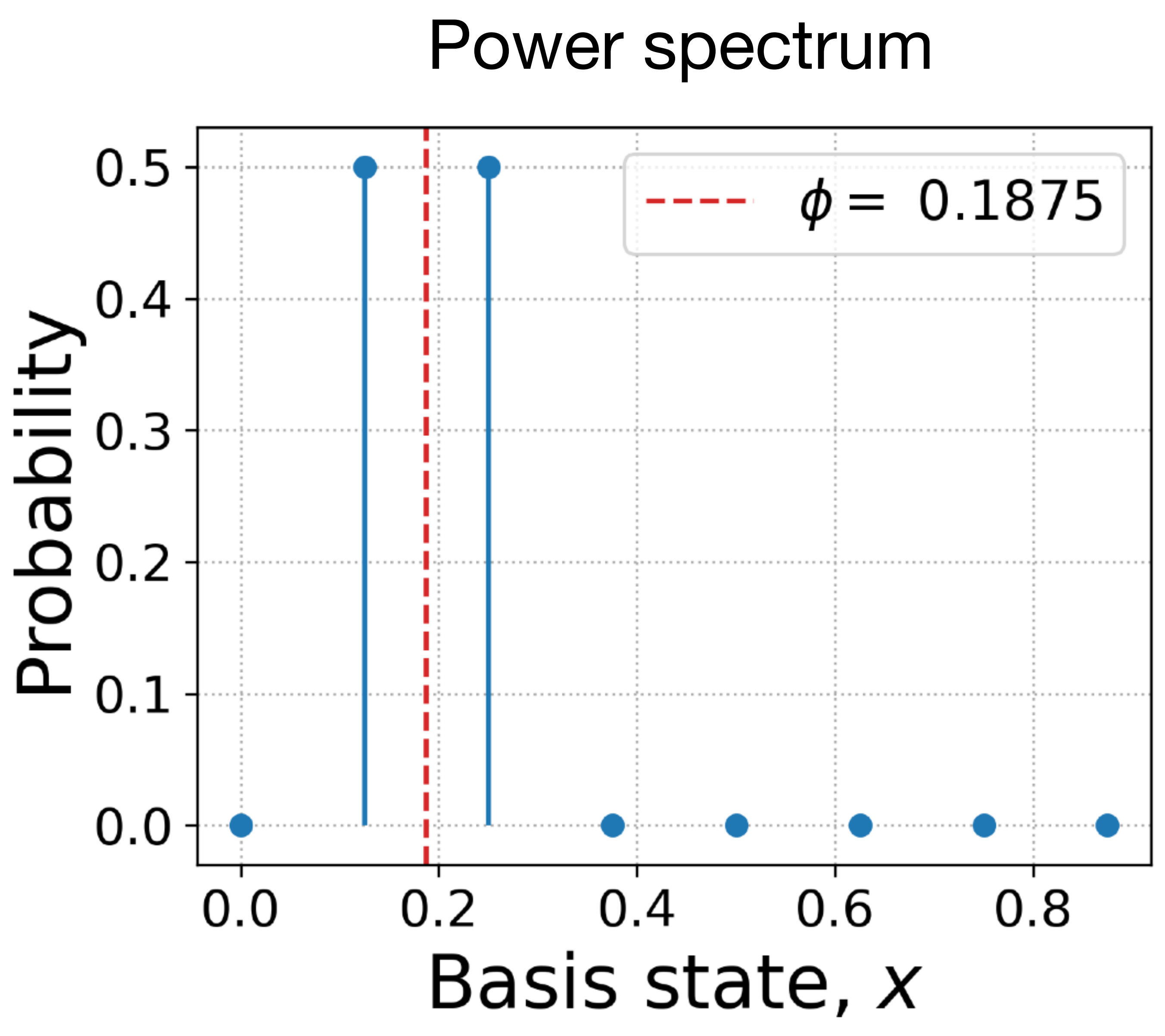
Revisiting QPE



$$P(x) = \left| \frac{1}{2^n} \sum_{k=0}^{2^n-1} e^{-\frac{2\pi i k}{2^n}(x - 2^n \phi)} \right|^2$$

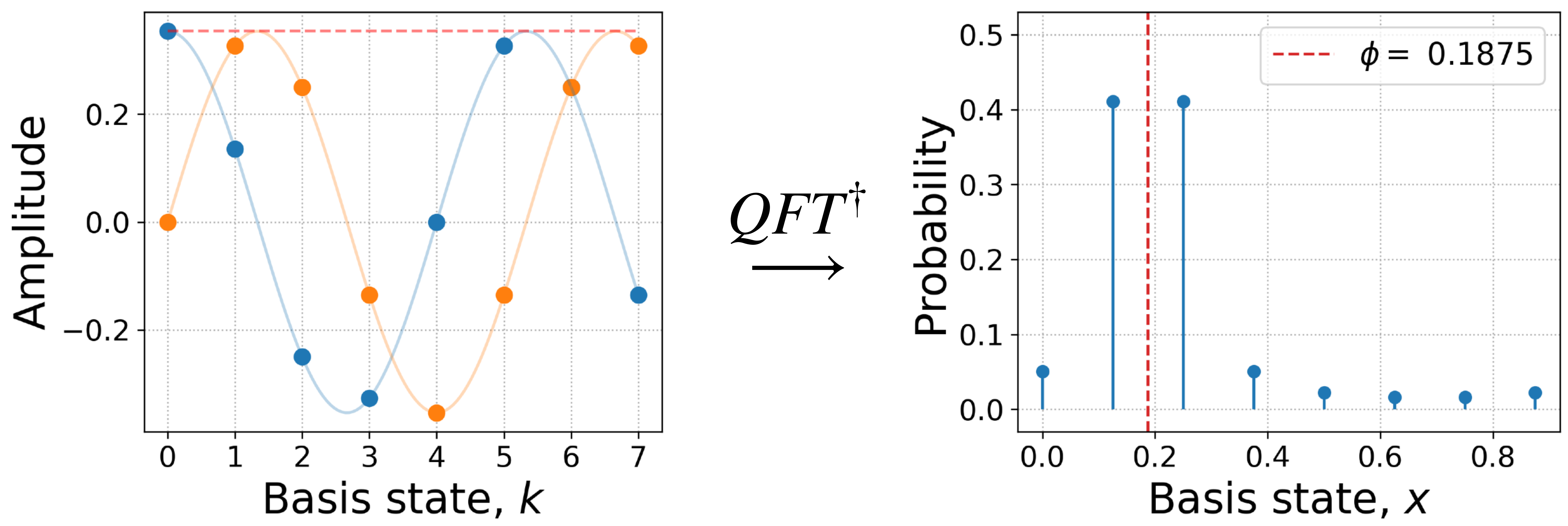


$\xrightarrow{QFT^\dagger}$

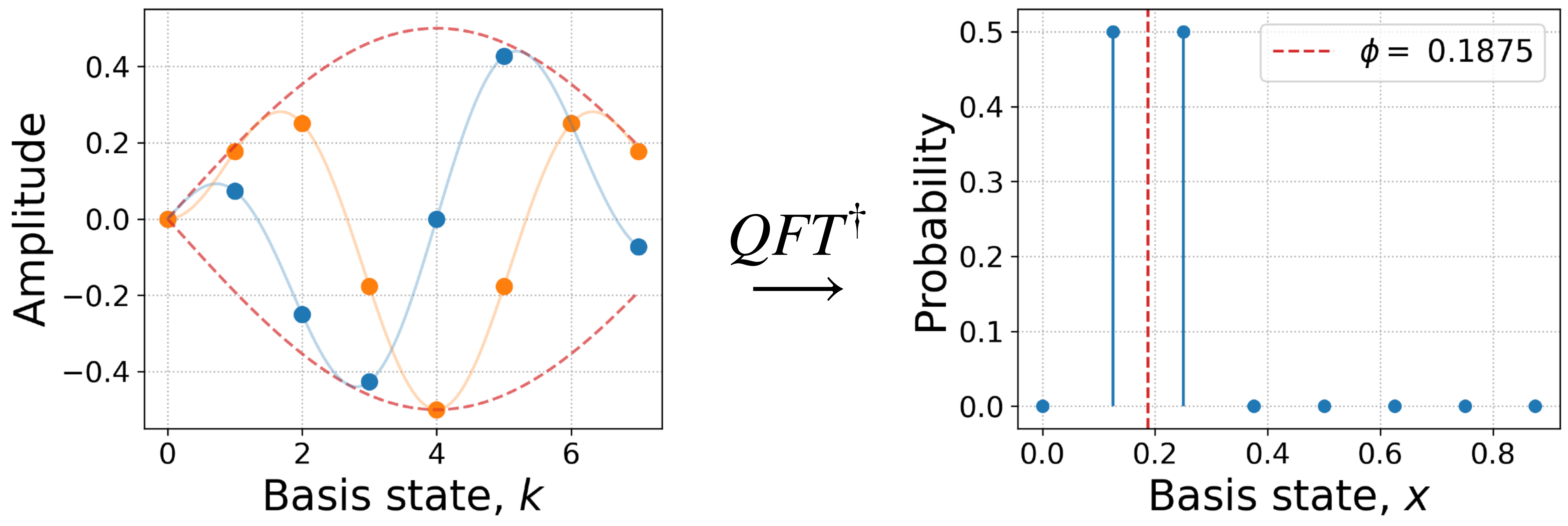


Revisiting QPE

Rectangular window



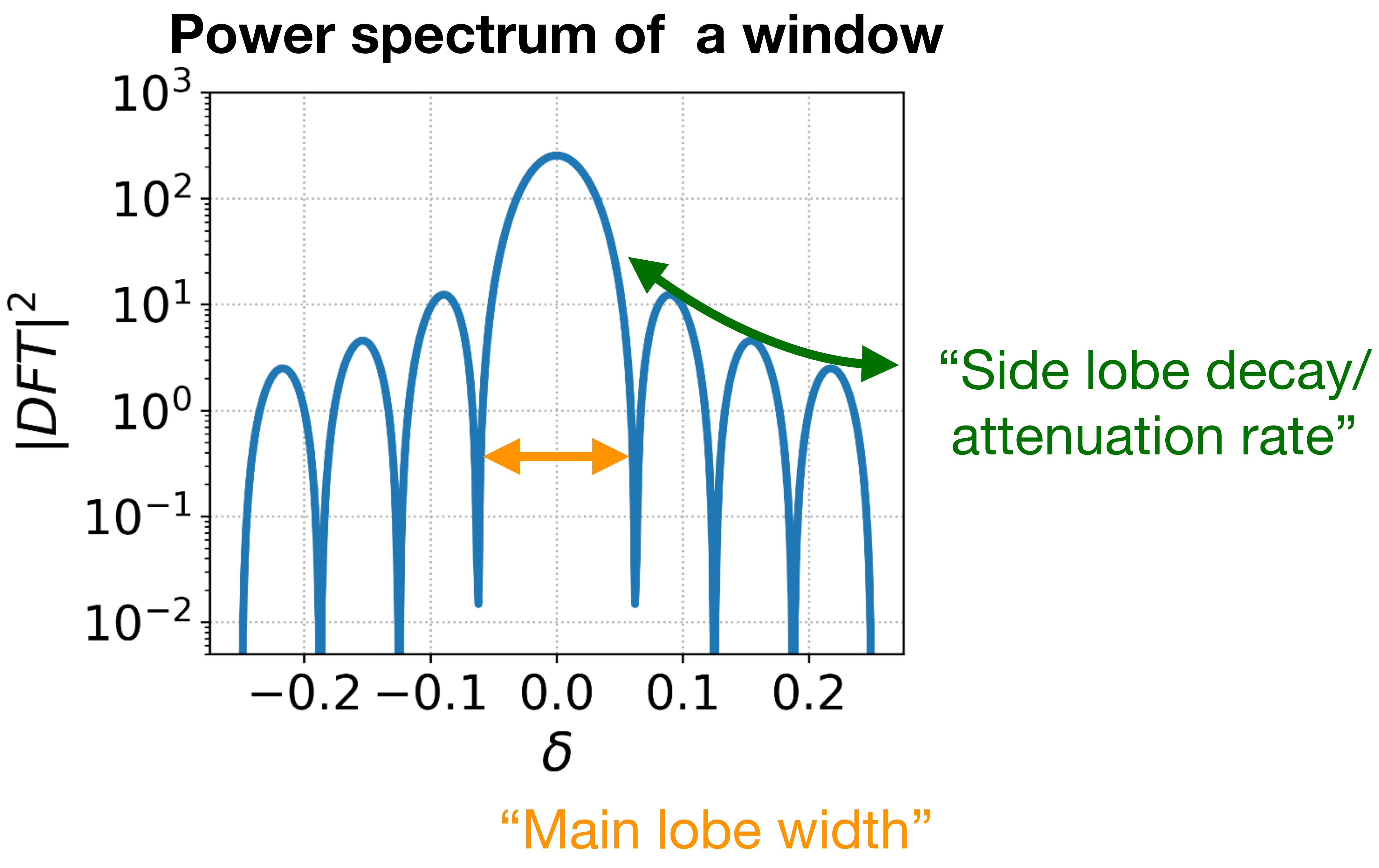
Cosine window



Windowing

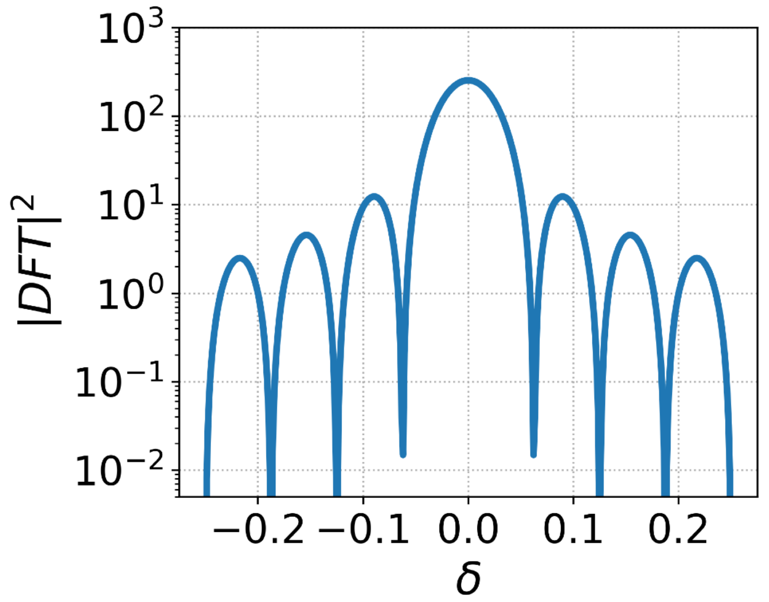
- Like in digital signal processing (DSP), applying a non-trivial window in QPE can reduce the spectral leakage or increase the success probability

- In DSP, there are many window types with trade-offs: width of dominant peak vs. rapid decay of side peaks



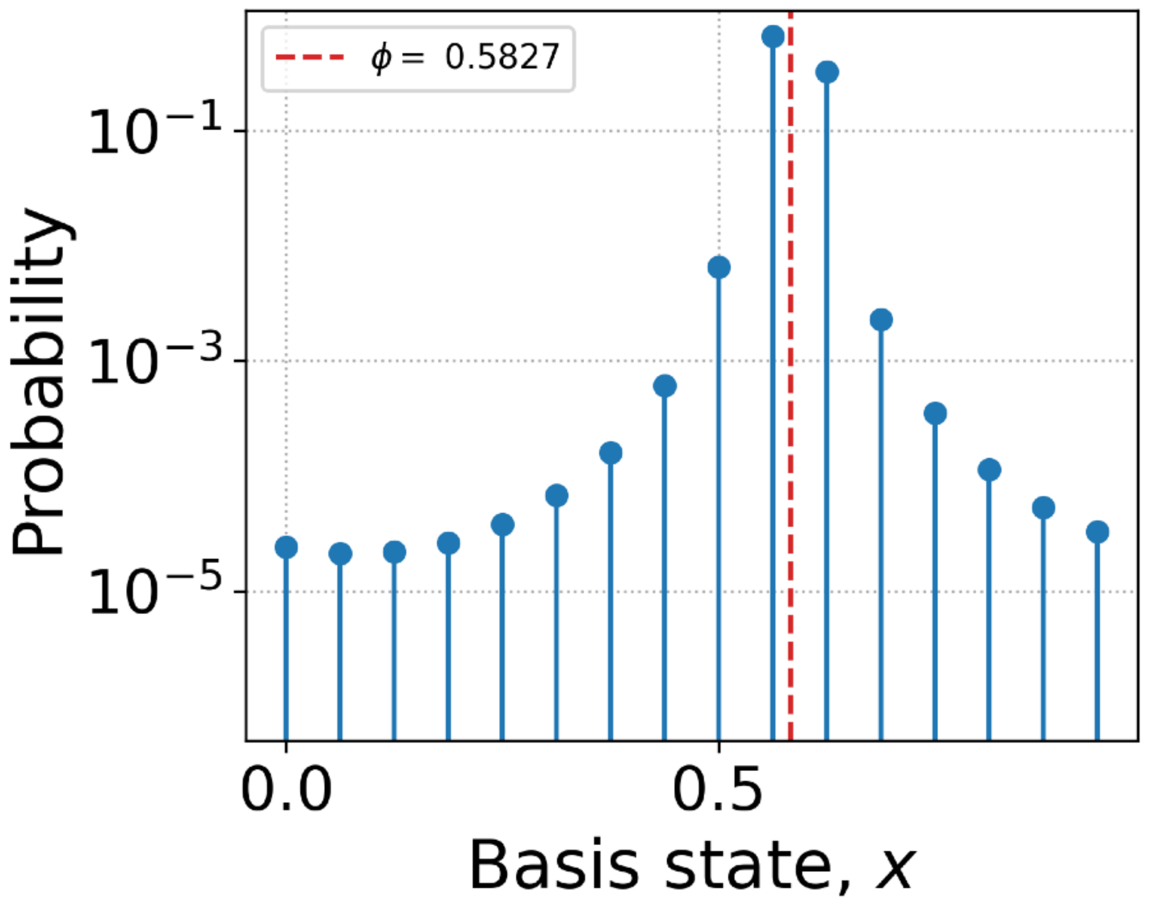
https://vru.vibrationresearch.com/lesson/windowing-process-data/

Power spectrum



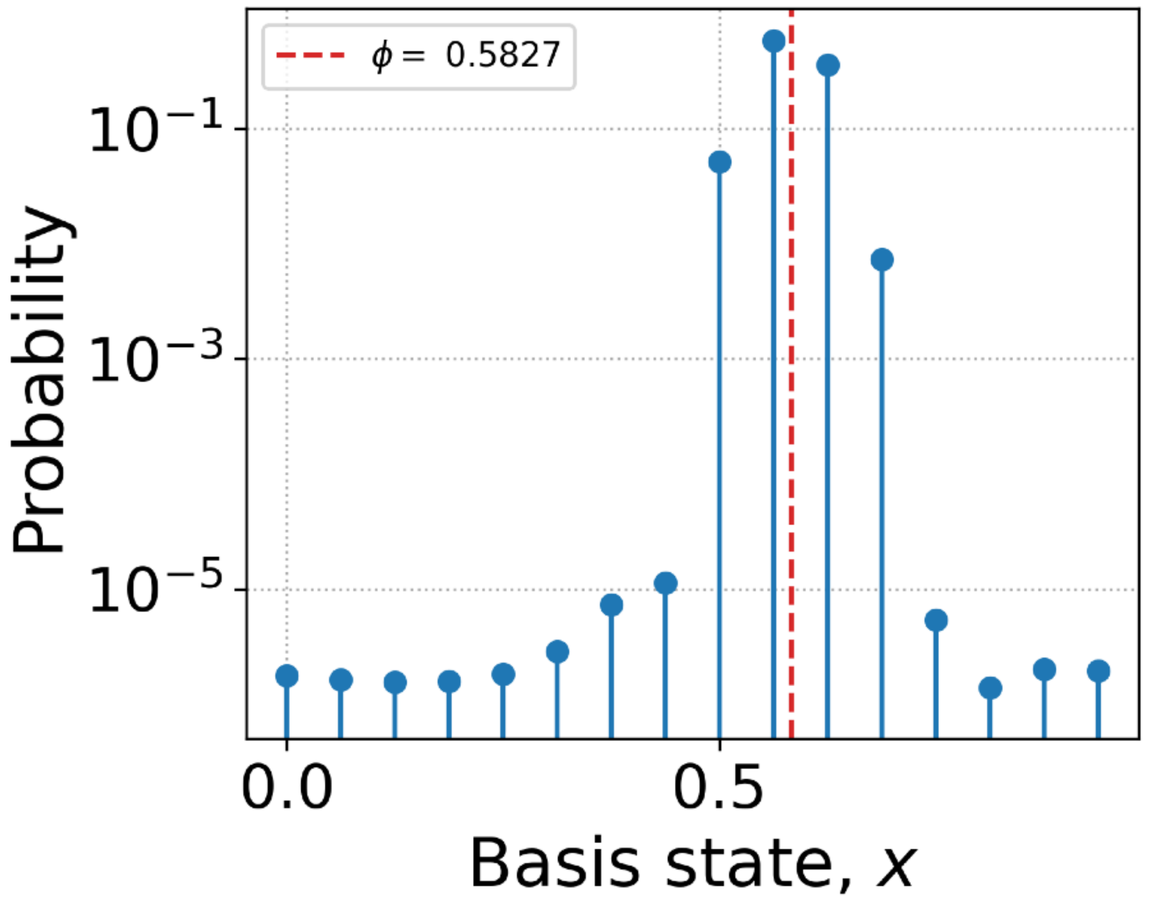
Windowing

Narrower main lobe, slowly decaying side lobes



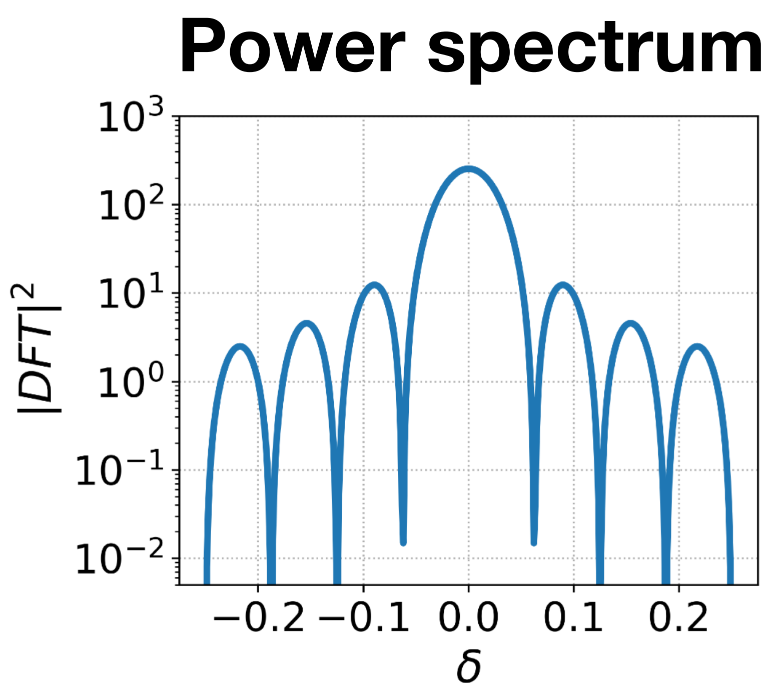
Power spectrum of
rectangular windowed QPE
(log scale)

Wider main lobe, rapidly decaying side lobes



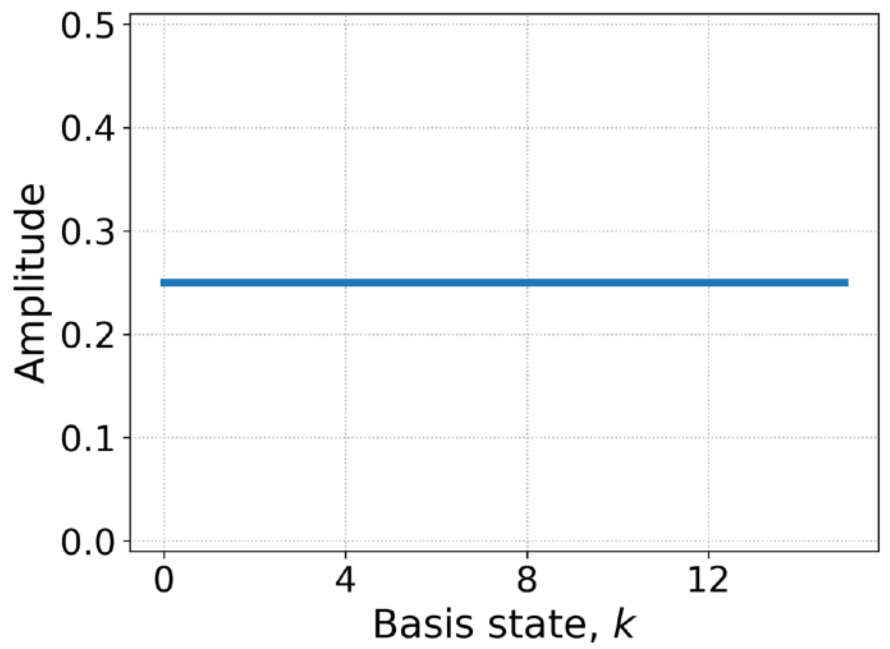
Power spectrum of
Kaiser windowed QPE [1-3]

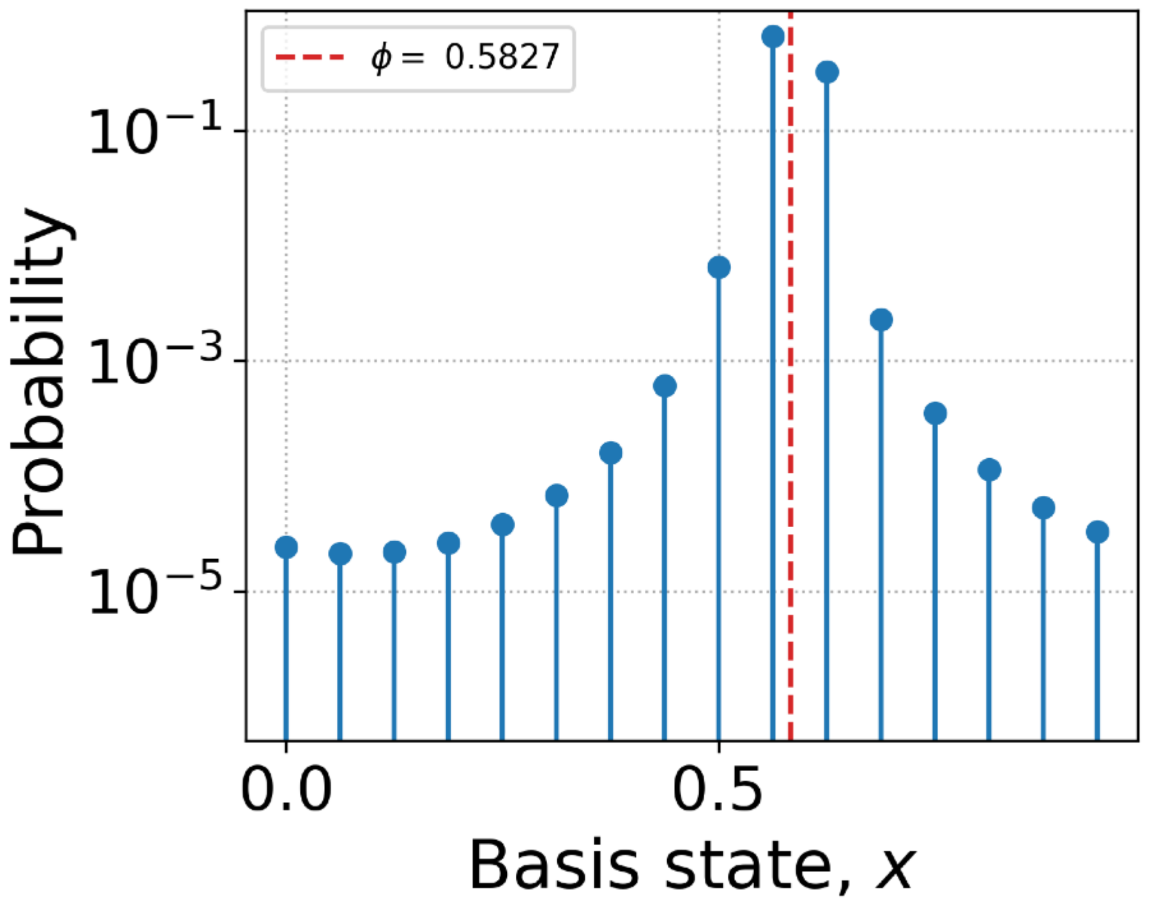
[1] Sanders, Y. R. *et al. PRX Quantum* **1**, (2020).
[2] Berry, D. W. *et al. arXiv* (2024) doi:10.48550/arxiv.2409.11748.
[3] Kristjuhan, K. & Berry, D. W. *arXiv* (2026) doi:10.48550/arxiv.2601.16474.



Windowing

Narrower main lobe, slowly decaying side lobes





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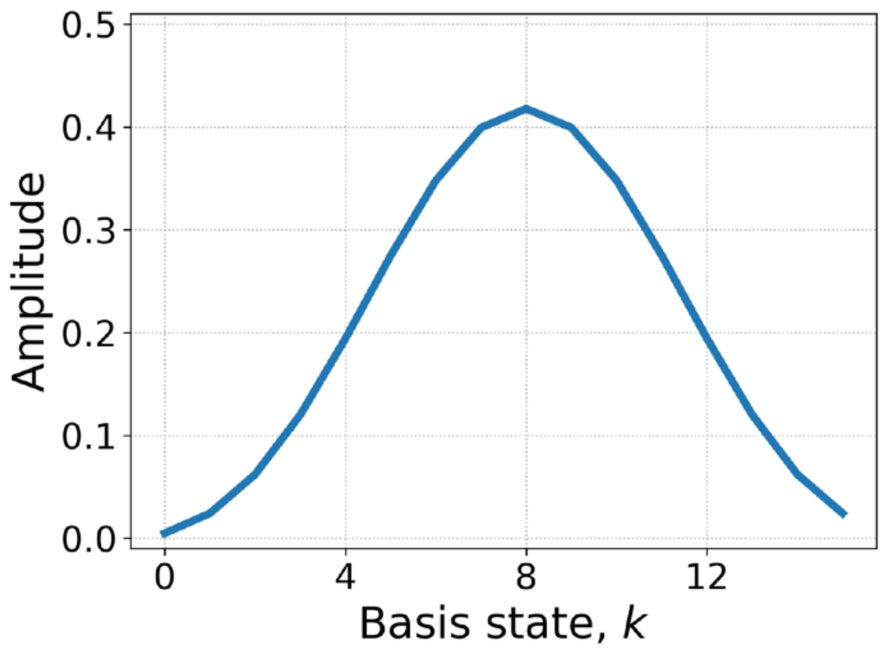

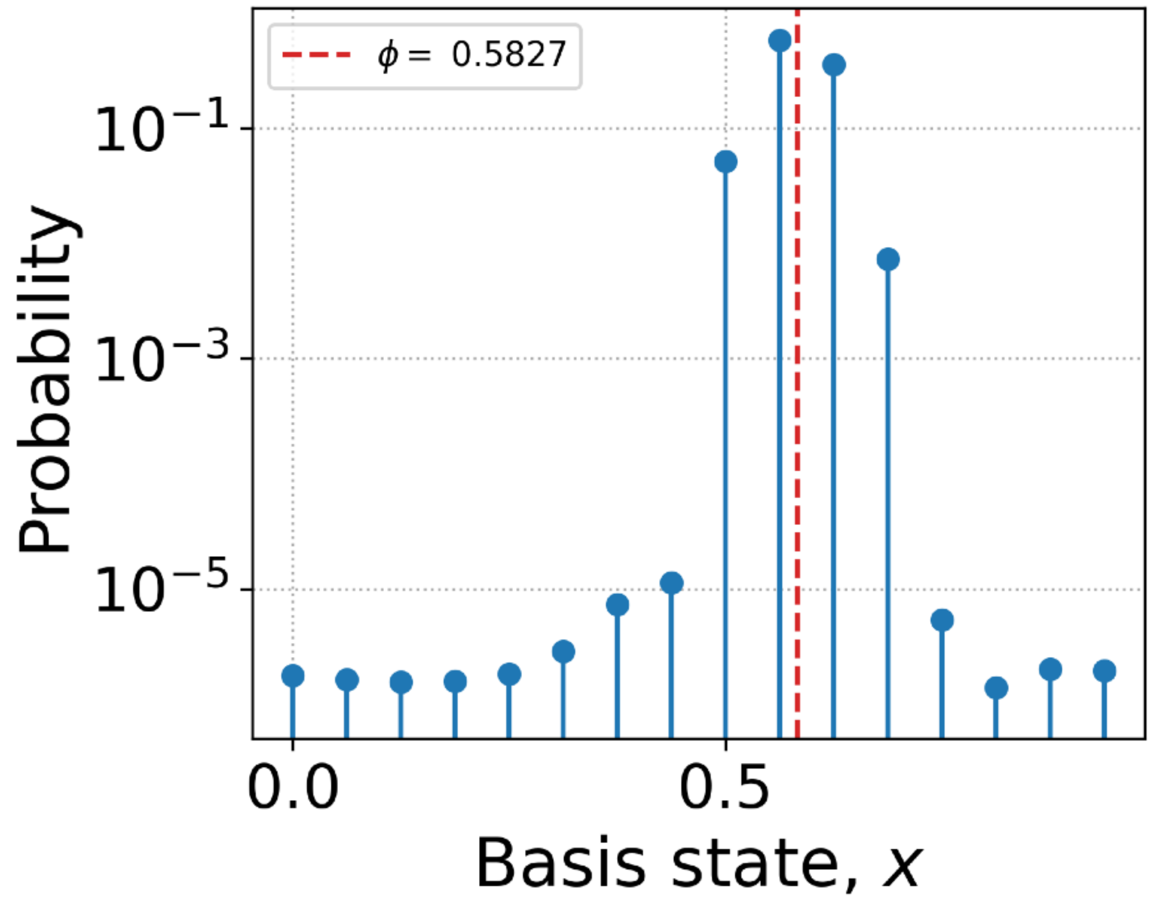

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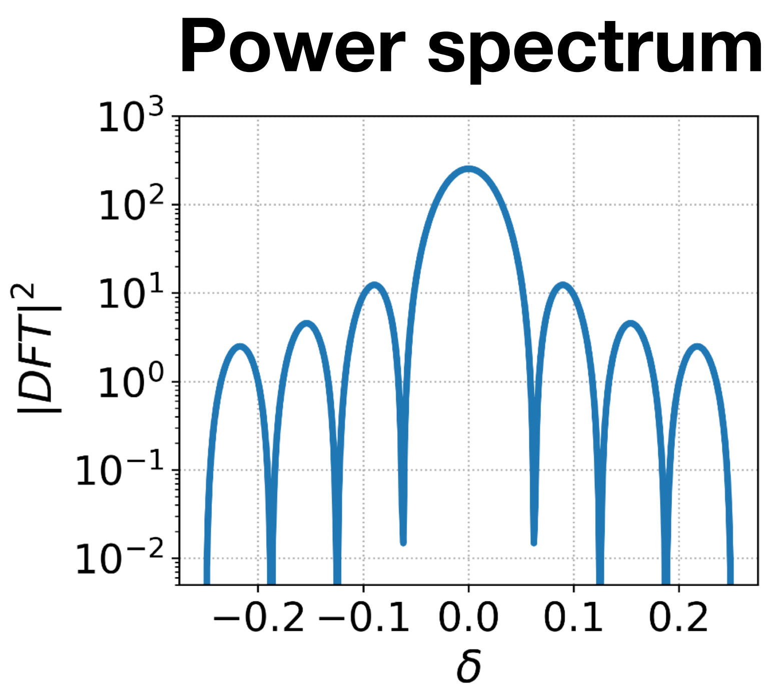
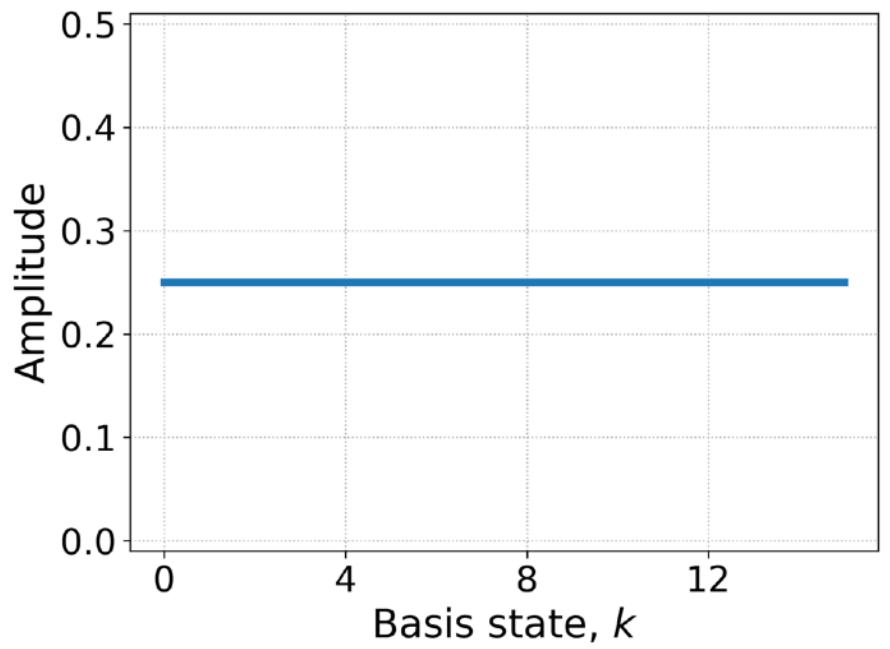
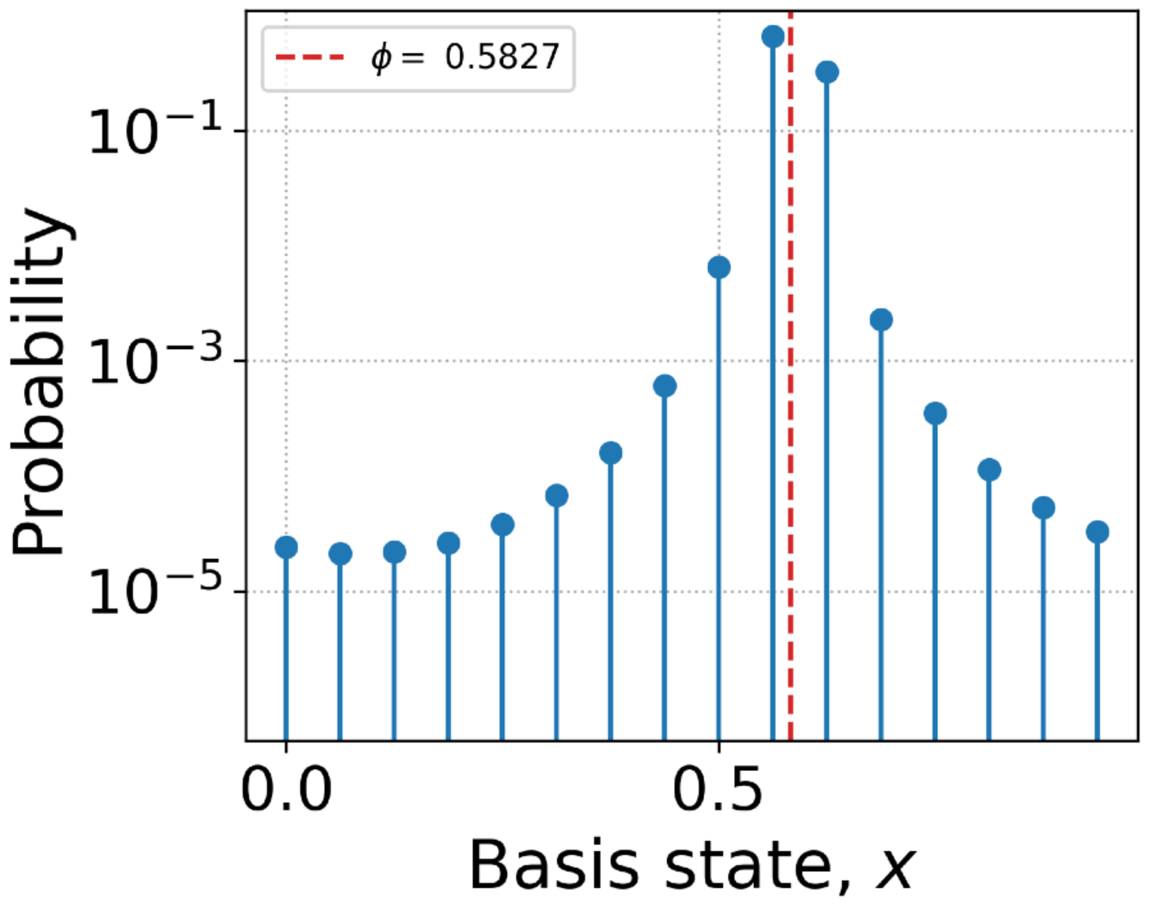
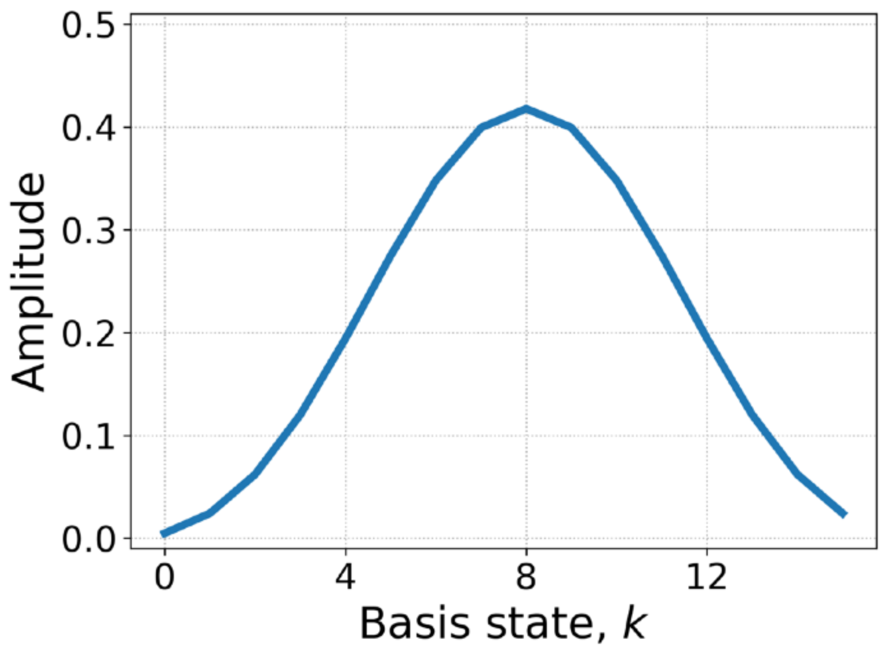


Windowing

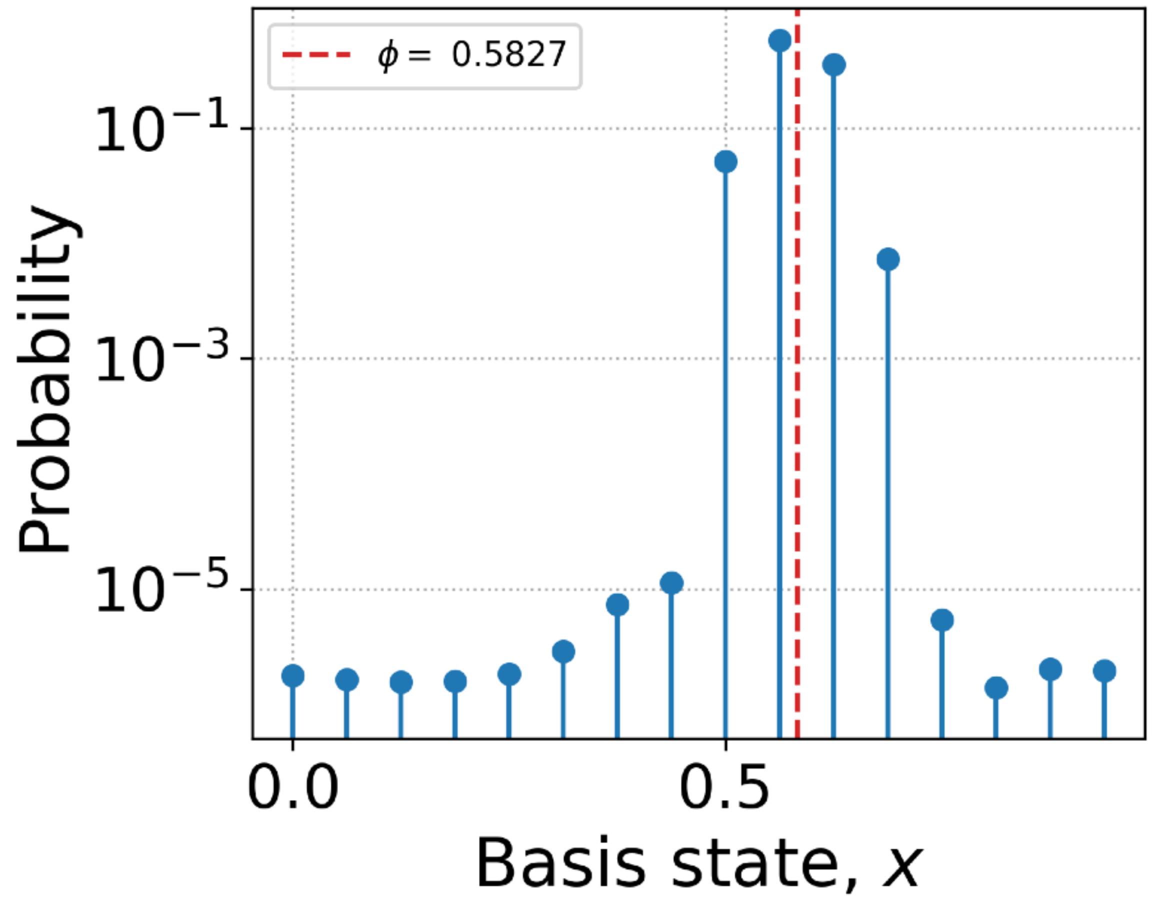


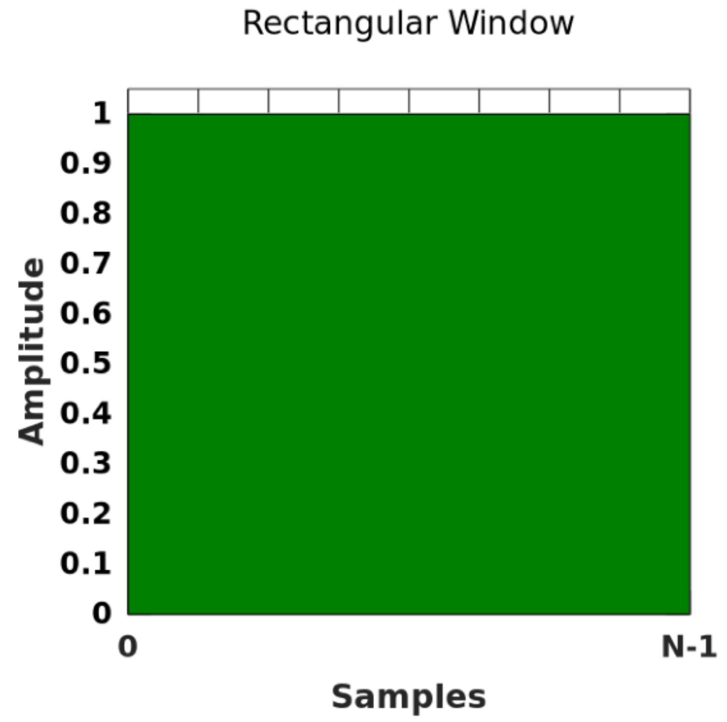
Narrower main lobe, slowly decaying side lobes

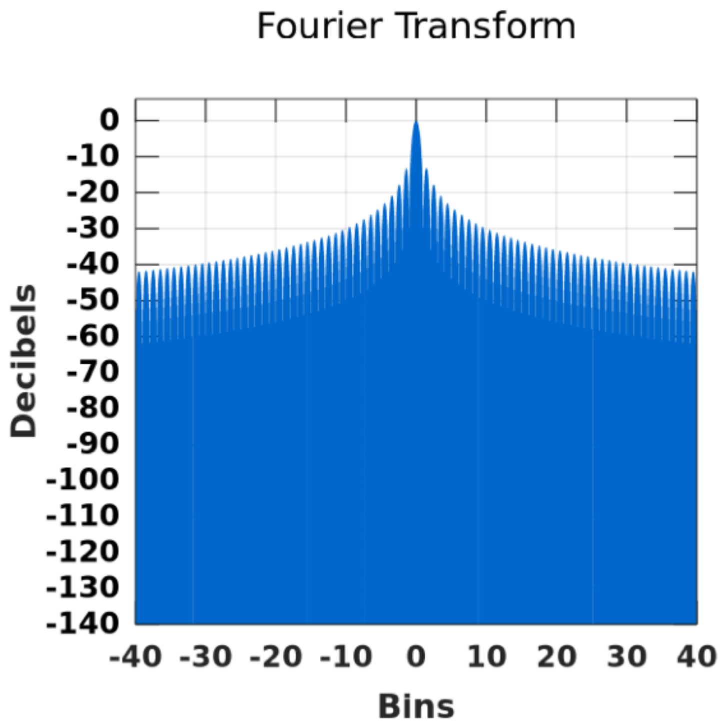


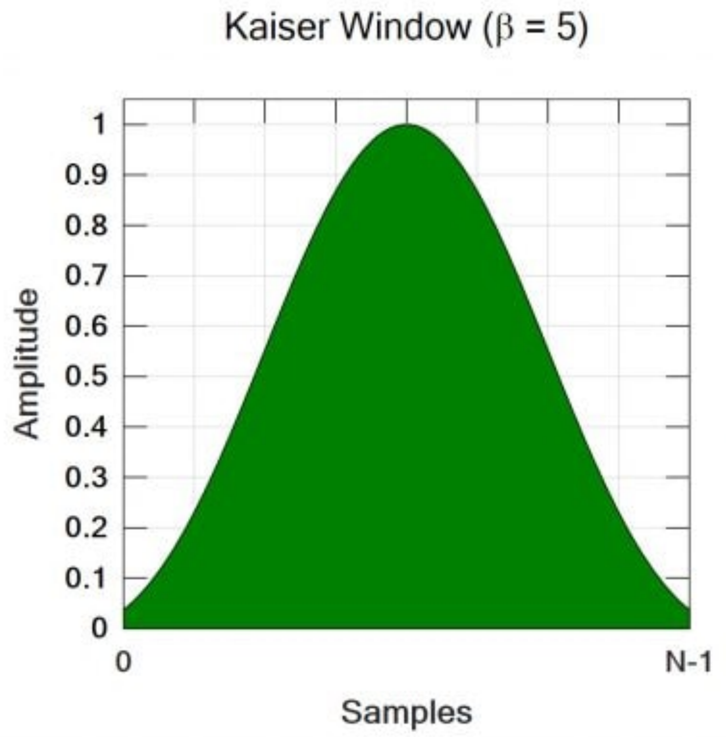


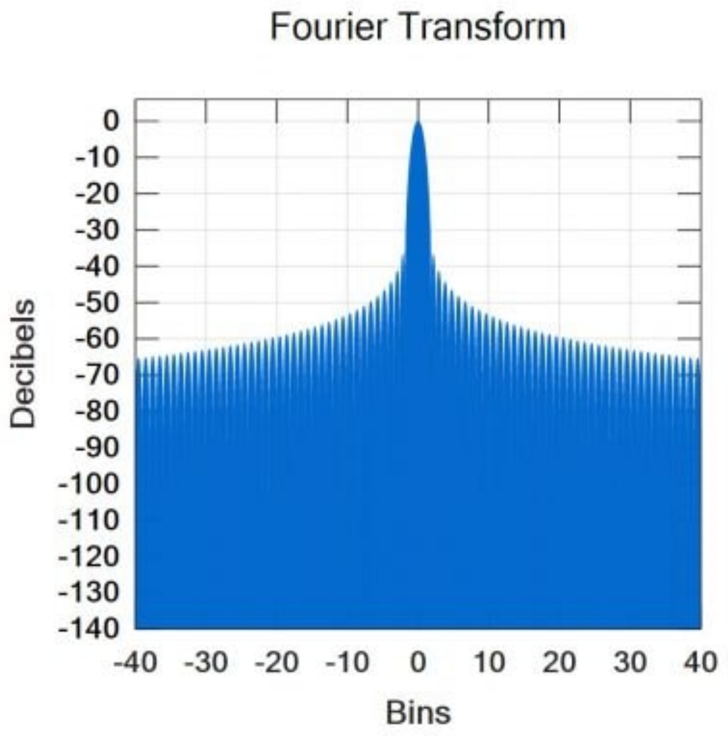
Wider main lobe, rapidly decaying side lobes



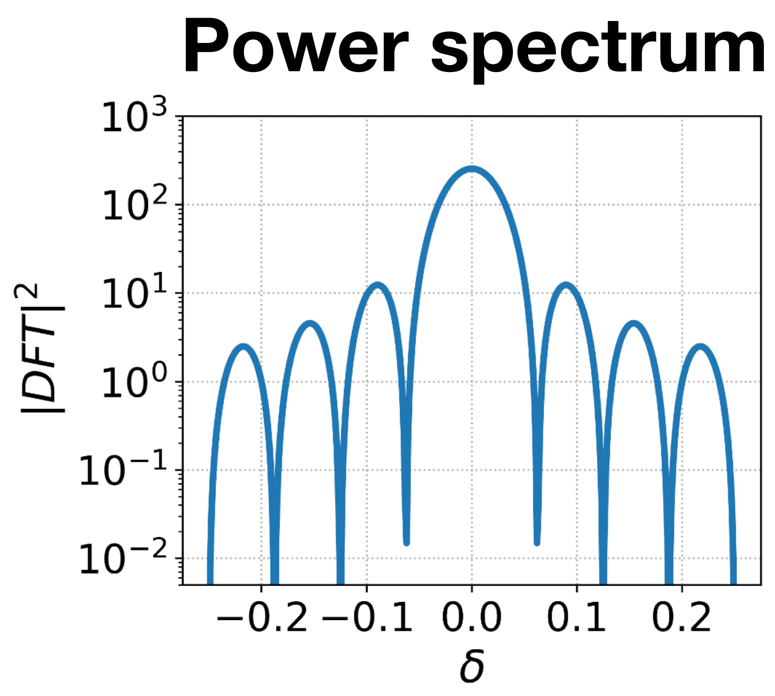






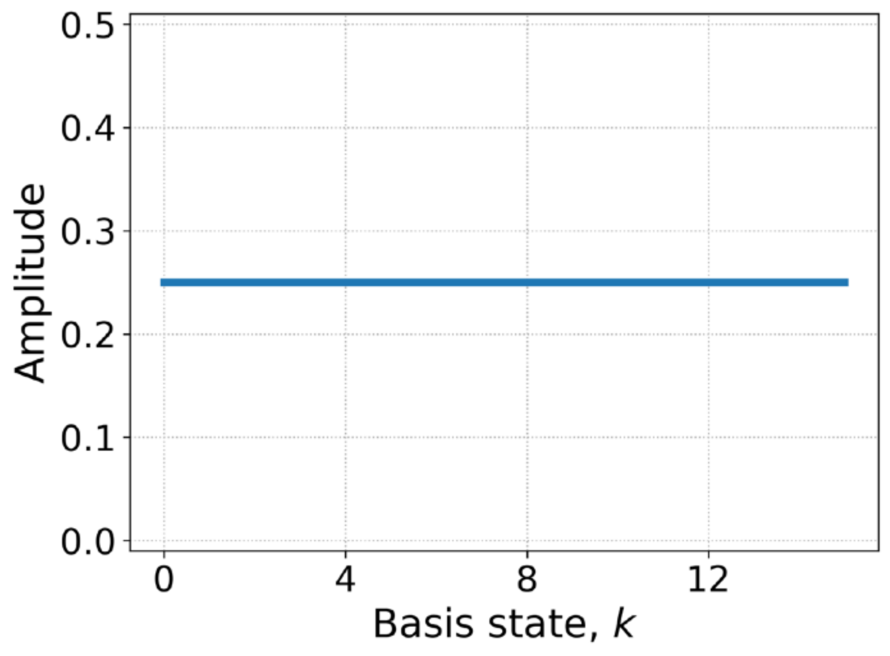


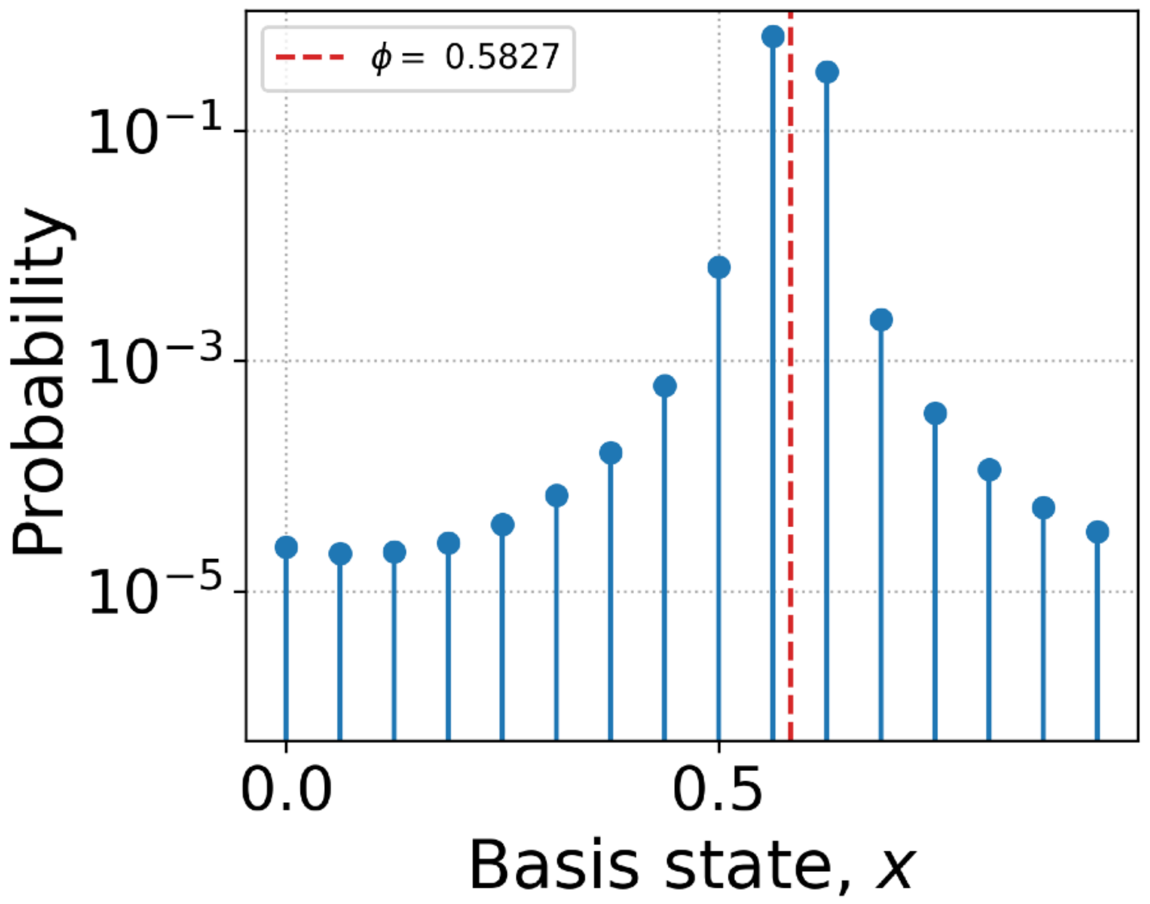
https://vru.vibrationresearch.com/lesson/windowing-process-data/

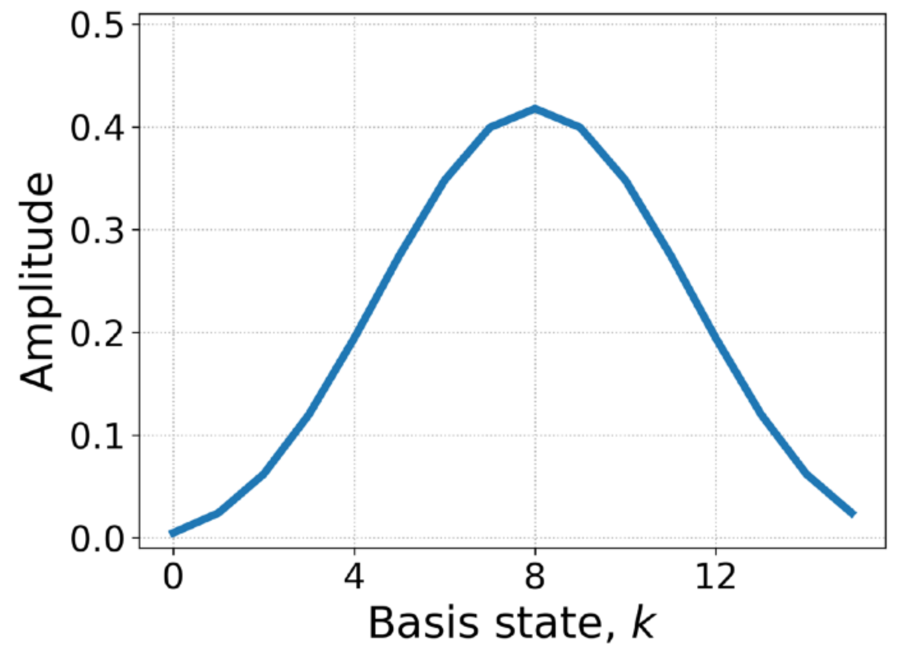


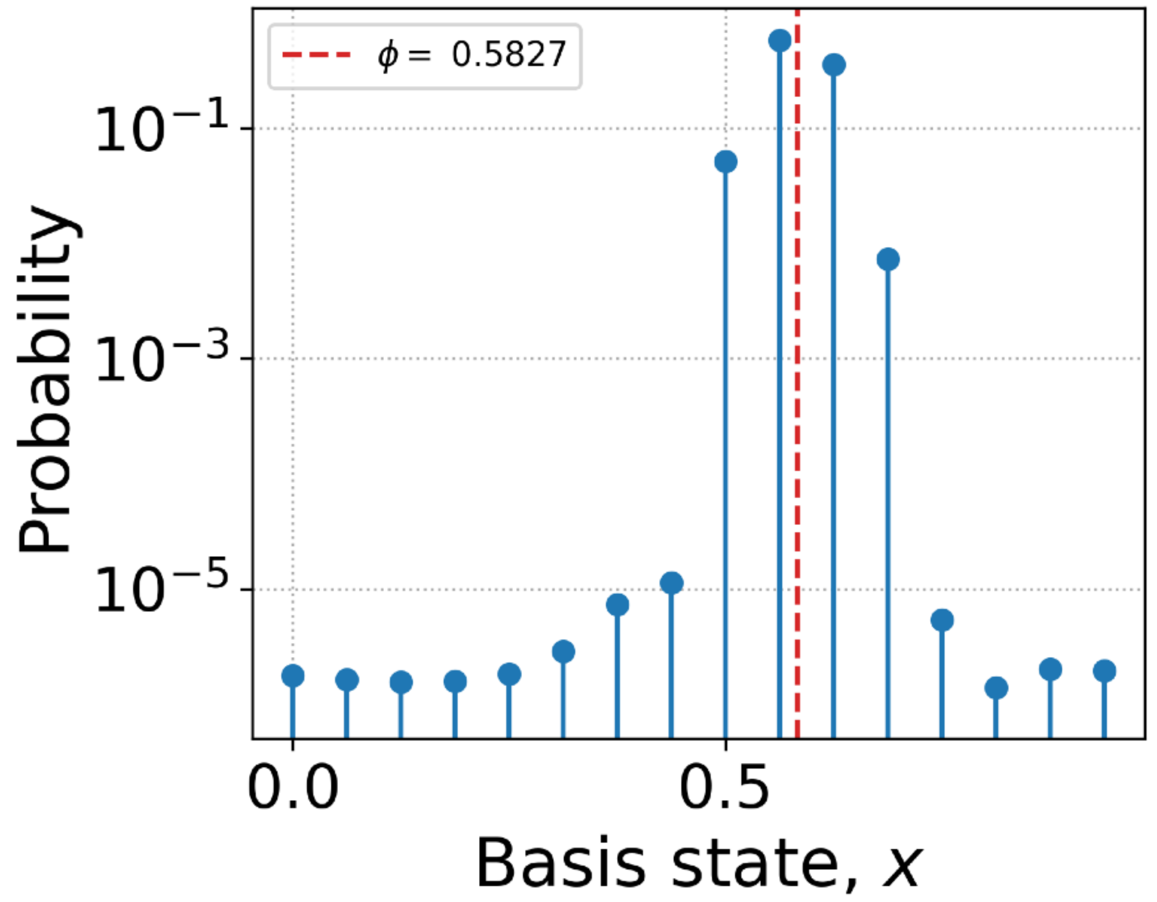
Windowing

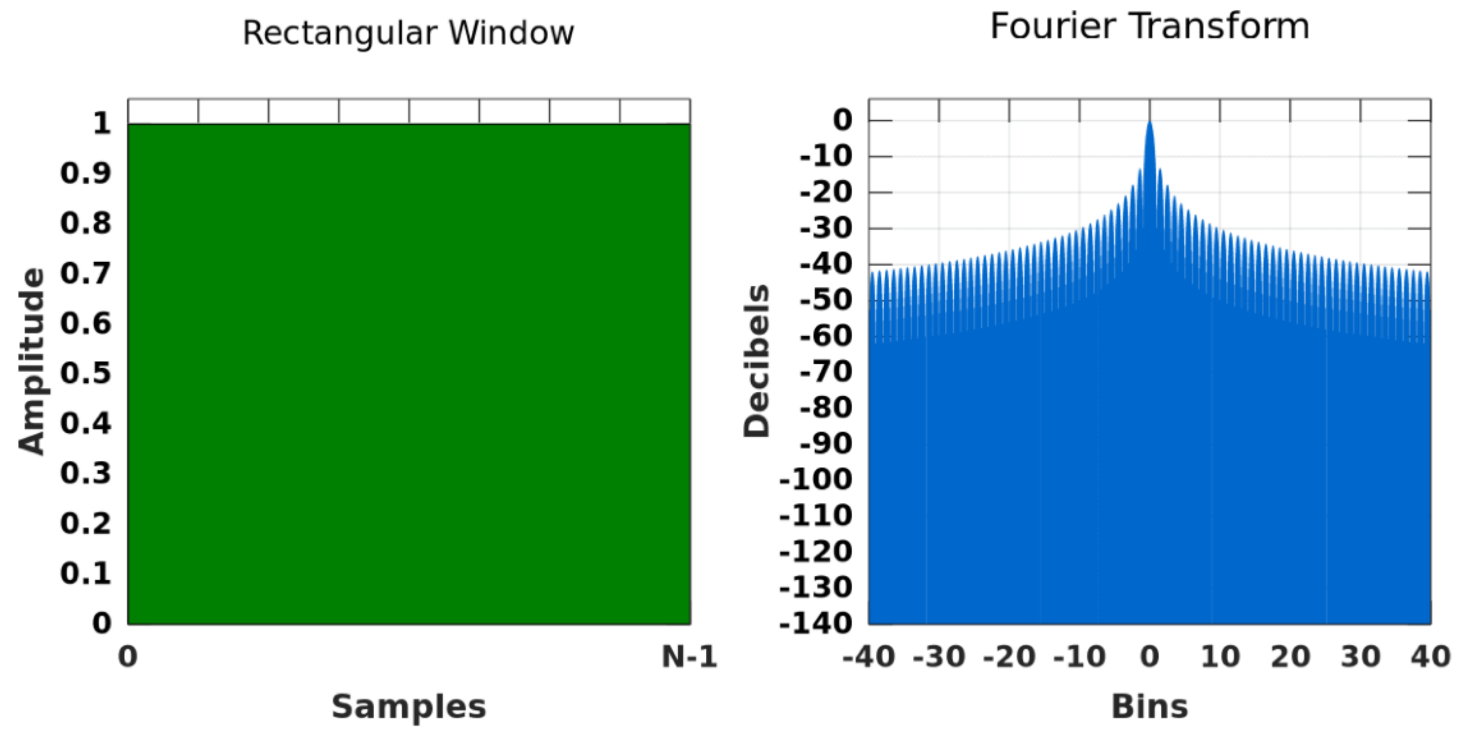
Open questions e.g. when should we apply particular window functions and why?

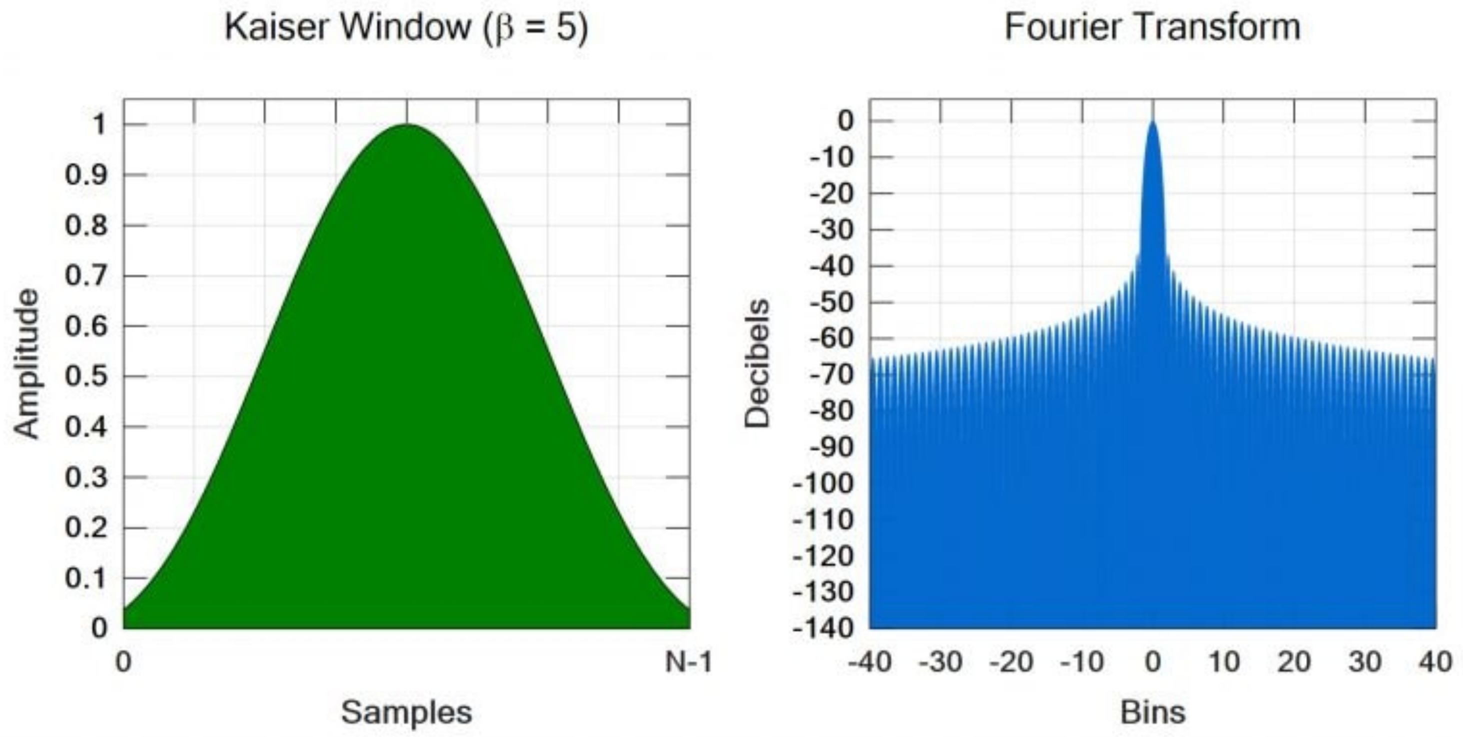






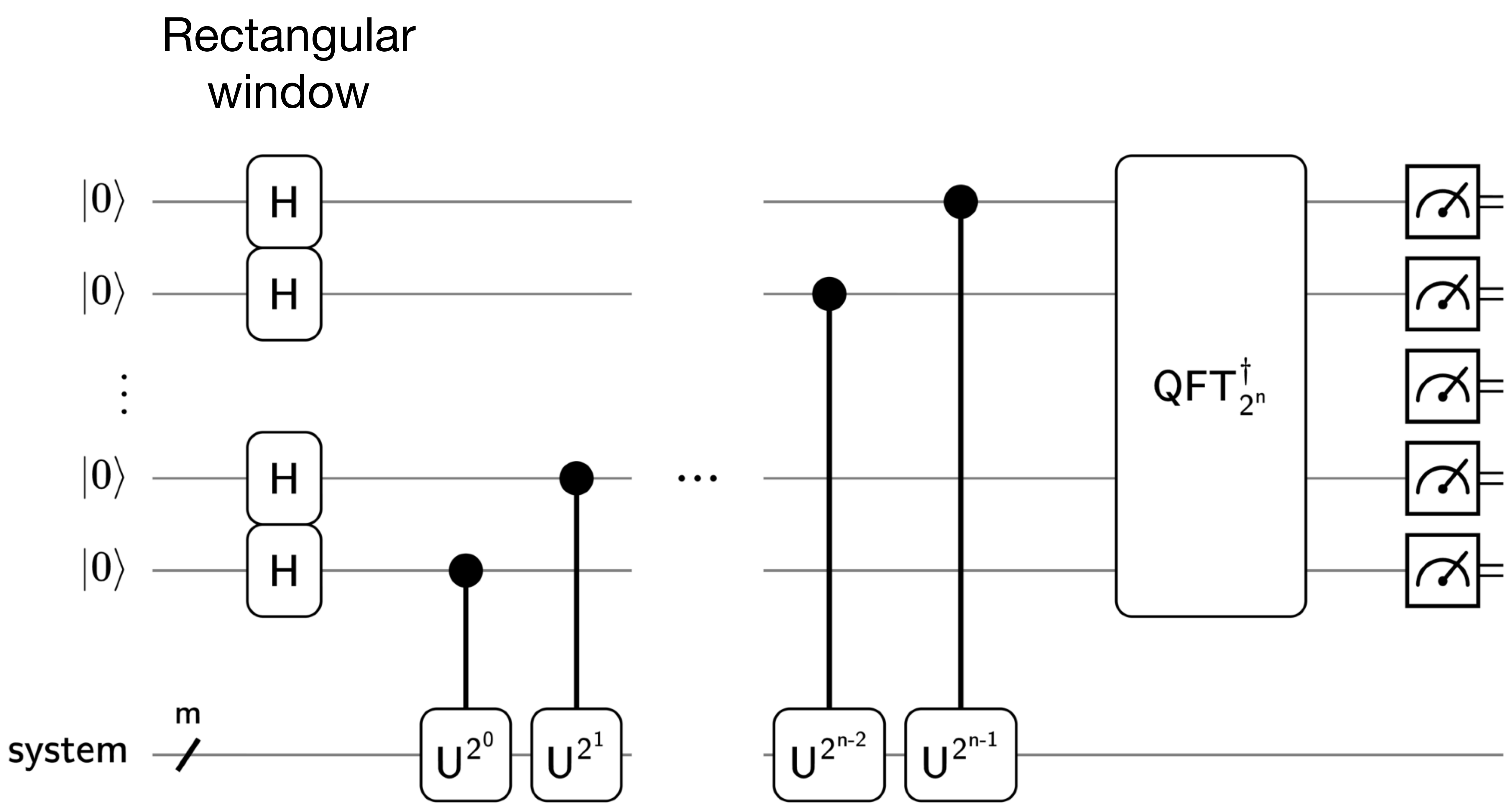




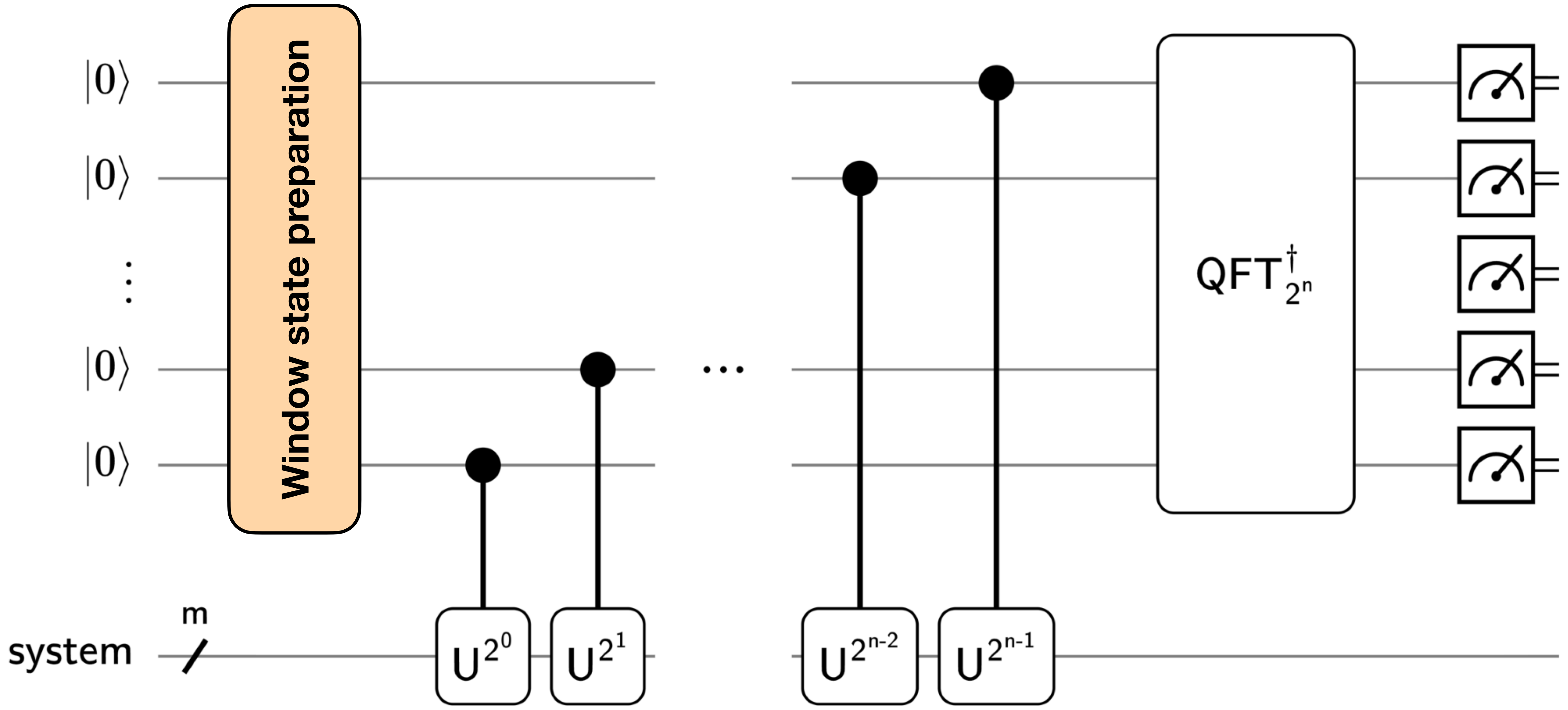


https://vru.vibrationresearch.com/lesson/windowing-process-data/

Updating our picture of QPE

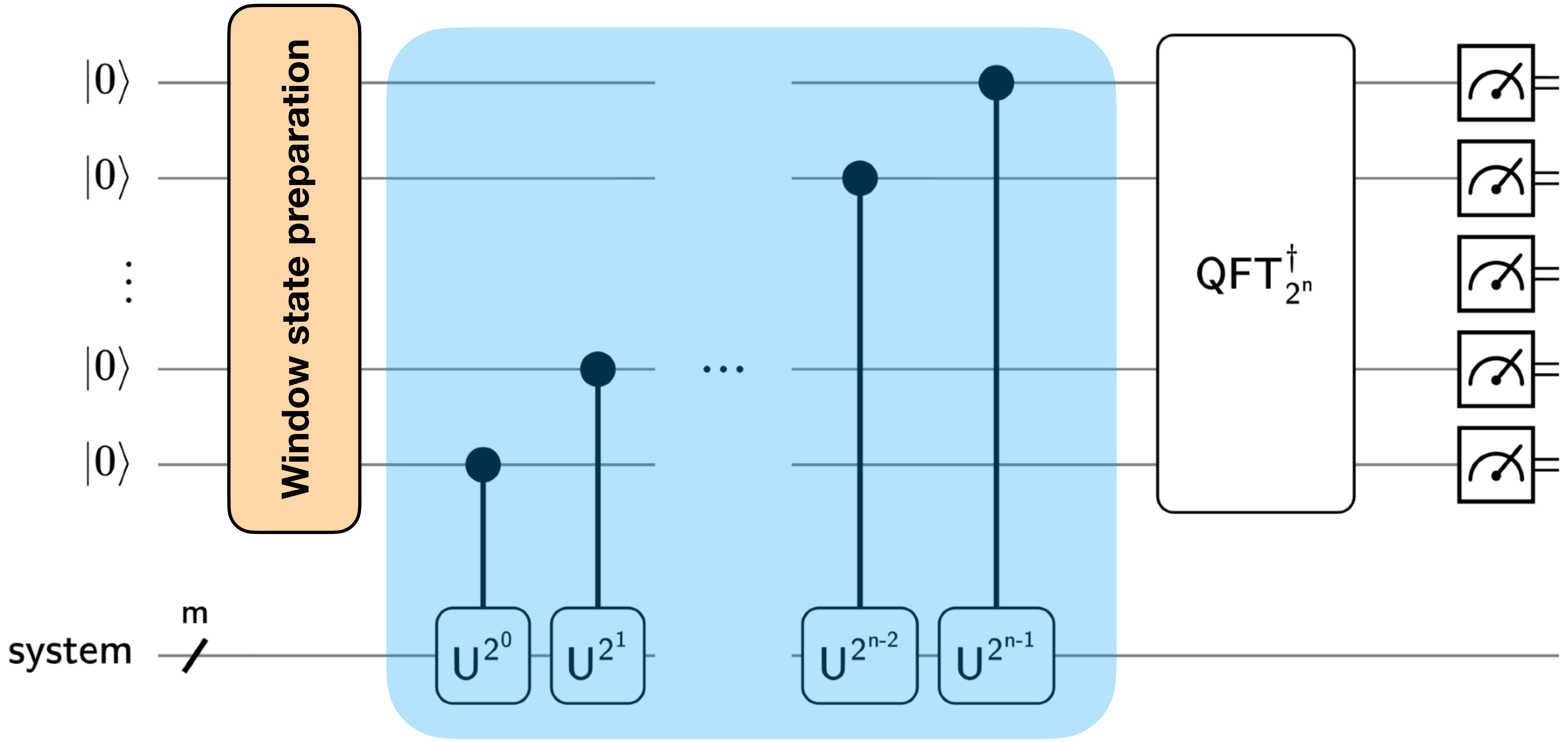


Updating our picture of QPE



Updating our picture of QPE

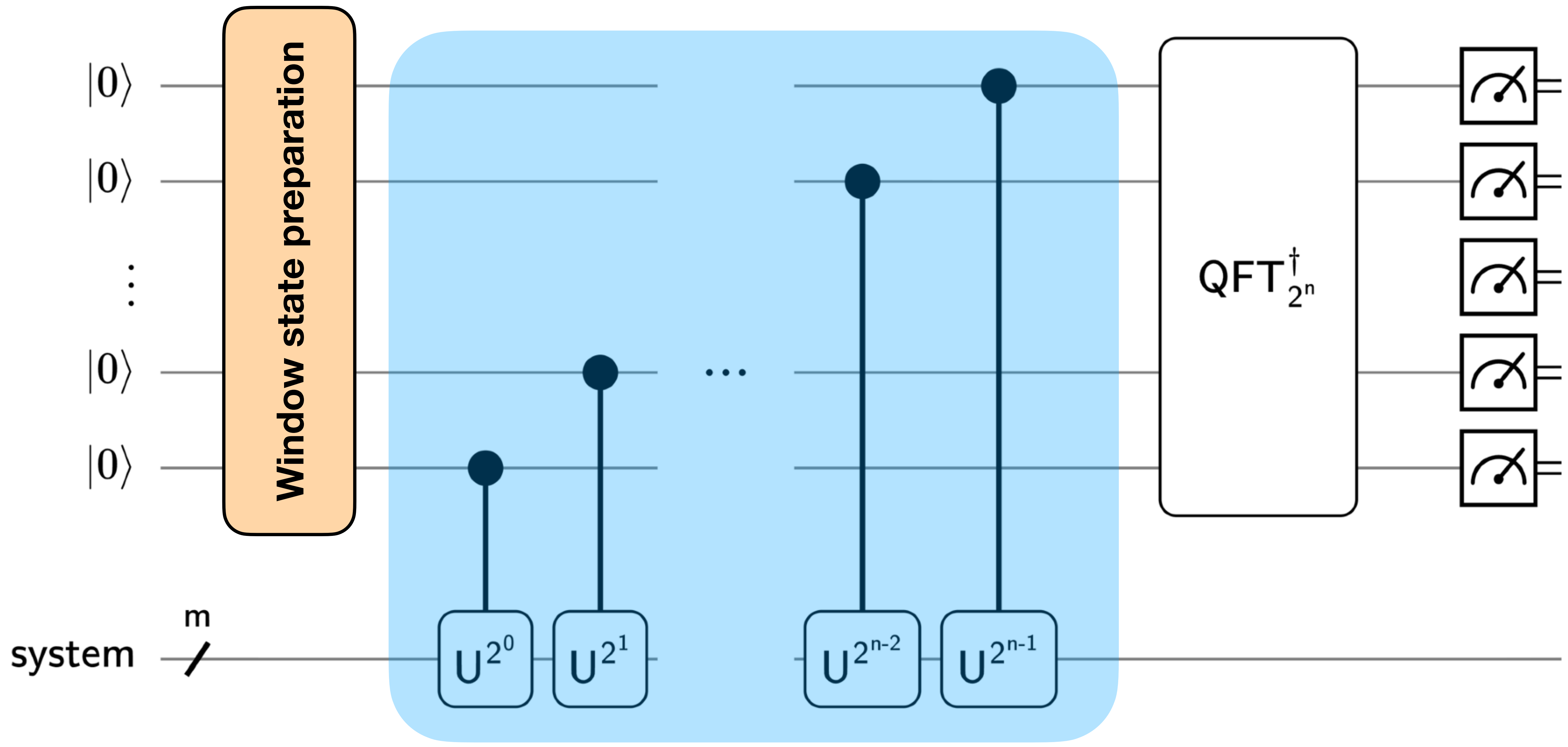
Where we "sample" or record the signal



Also the costliest part of the algorithm

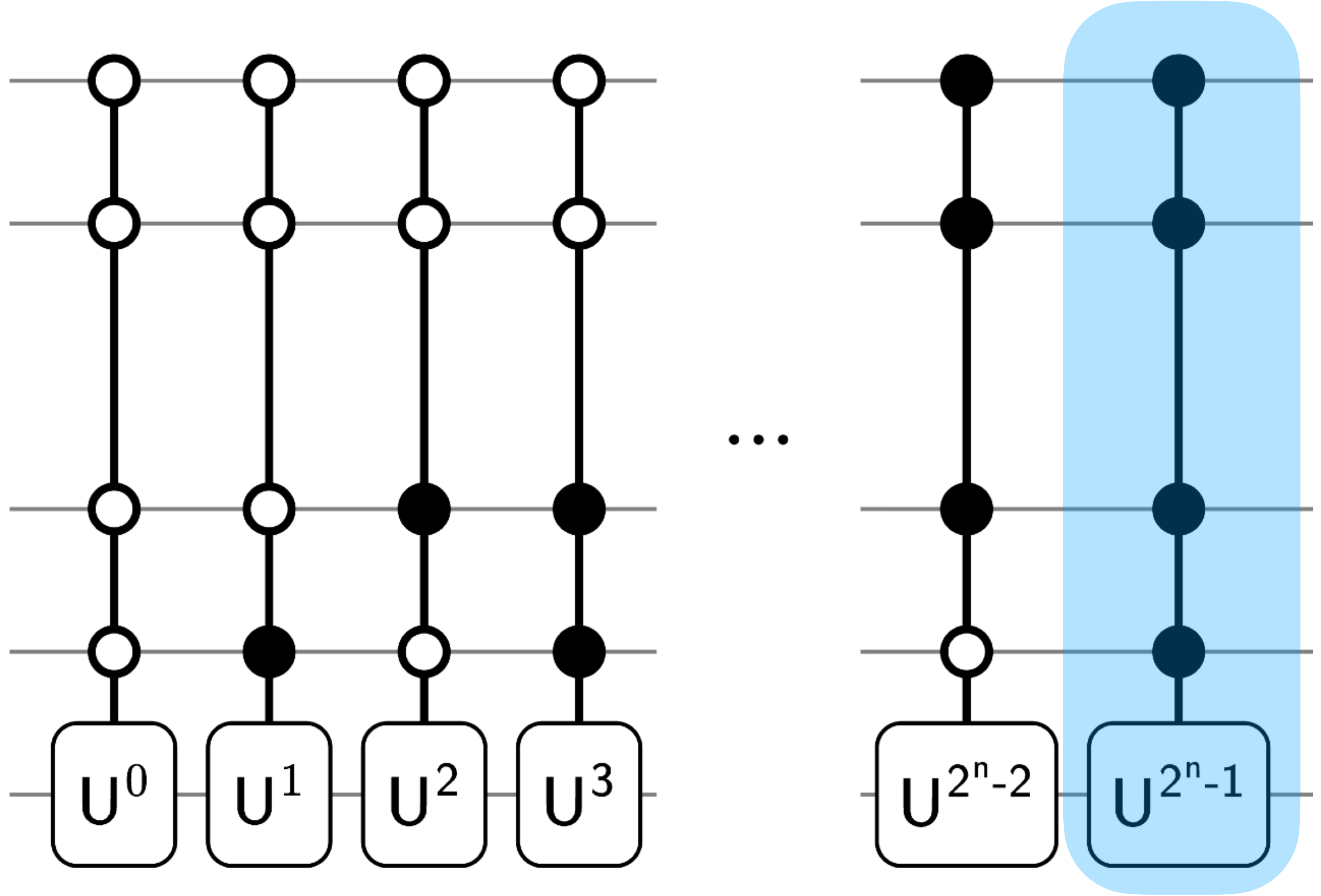
Cost of QPE

Where we "sample" or record the signal

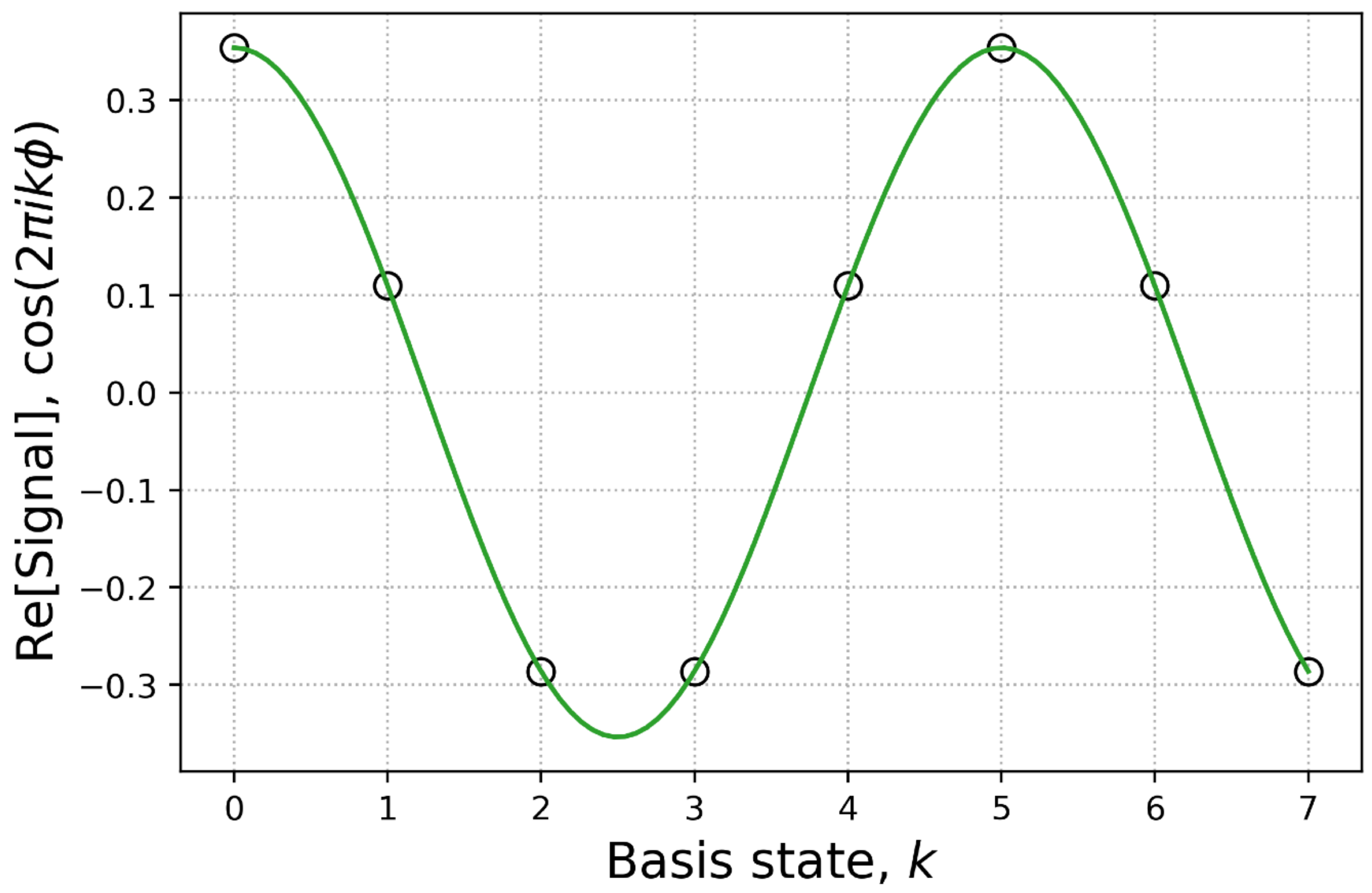


Also the costliest part of the algorithm

Cost comes largely from applying the maximum power of U



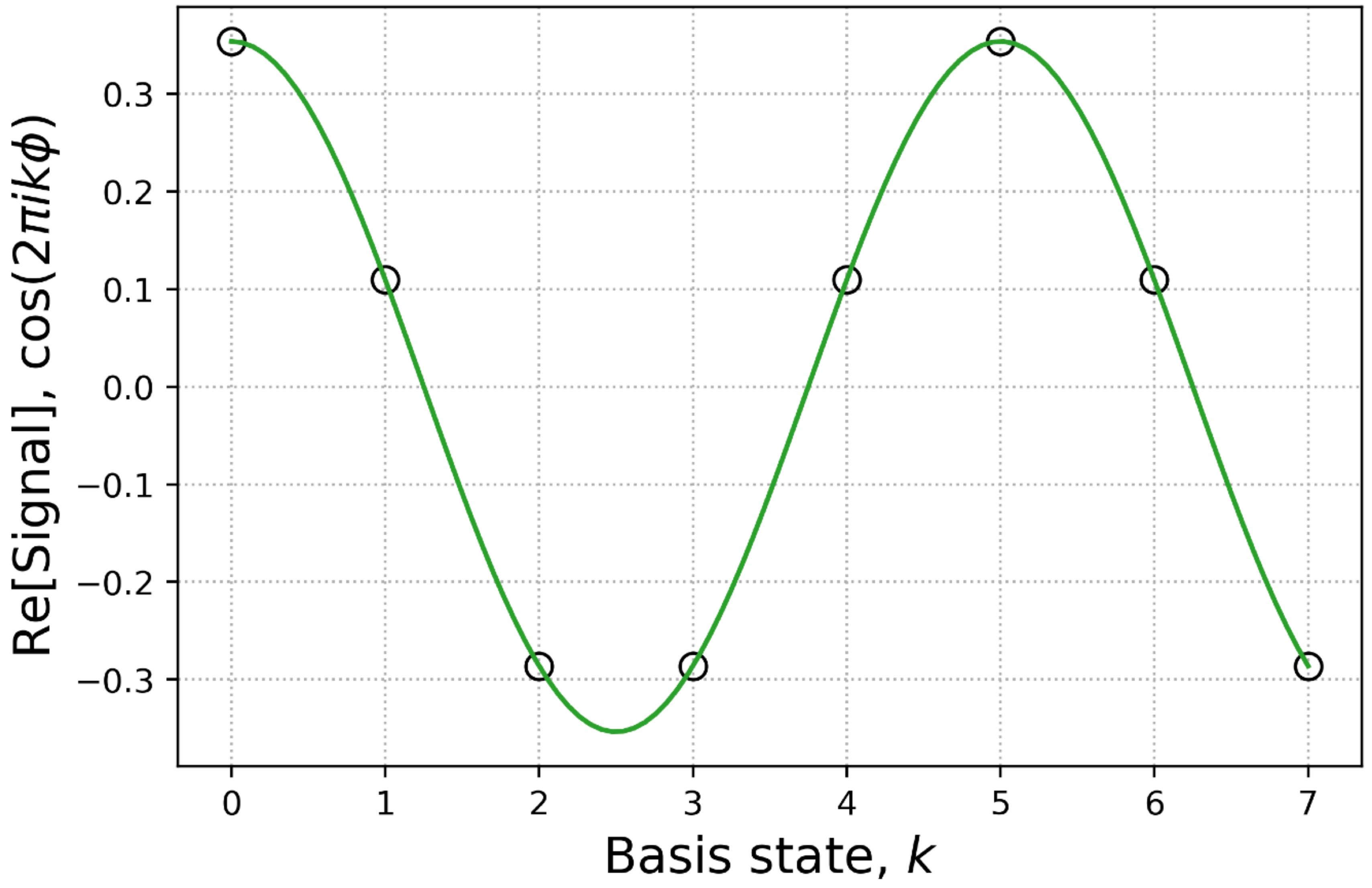
Going back to signal sampling/recording



Basis state, k	Signal unitary
0	U^0
1	U^1
2	U^2
3	U^3
4	U^4
5	U^5
6	U^6
7	U^7

Going back to signal sampling/recording

What if we shift the signal in time?



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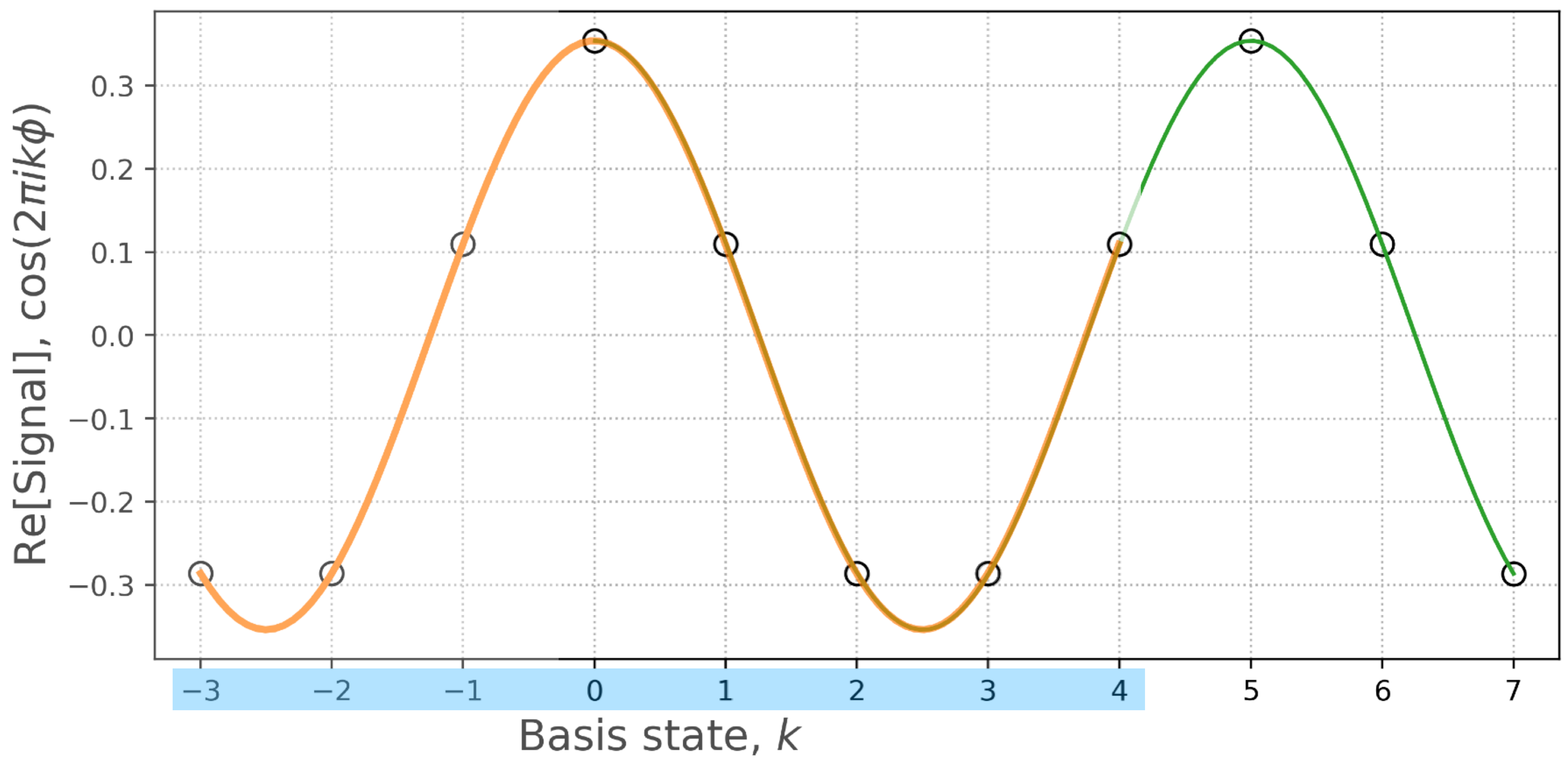


Basis state, k	Shifted	Unshifted
0	U^{-3}	U^{0}
1	U^{-2}	U^{1}
2	U^{-1}	U^{2}
3	U^{0}	U^{3}
4	U^{1}	U^{4}
5	U^{2}	U^{5}
6	U^{3}	U^{6}
7	$\boldsymbol{U^{4}}$	$\boldsymbol{U^{7}}$

Maximum (absolute value) of power of U approximately halved!

Shifting a signal in time (from signal processing lens)

Fourier transform of a time shifted signal *x*: $\mathcal{F}\left[x(t-t_0)\right] = e^{-i\omega t_0}\, X(\omega)$ where $X(\omega) = \mathcal{F}\left[x(t)\right]$

Global phase!

Will not change the power spectrum of the original signal, $|X[\omega]|^2$

Continuous case but discrete case should also follow.

Shifting a signal in time (from signal processing lens)

Fourier transform of a time shifted signal *x*: $\mathscr{F}\left[x(t-t_0)\right] = e^{-i\omega t_0}\, X(\omega)$ where $X(\omega) = \mathscr{F}\left[x(t)\right]$

Global phase!

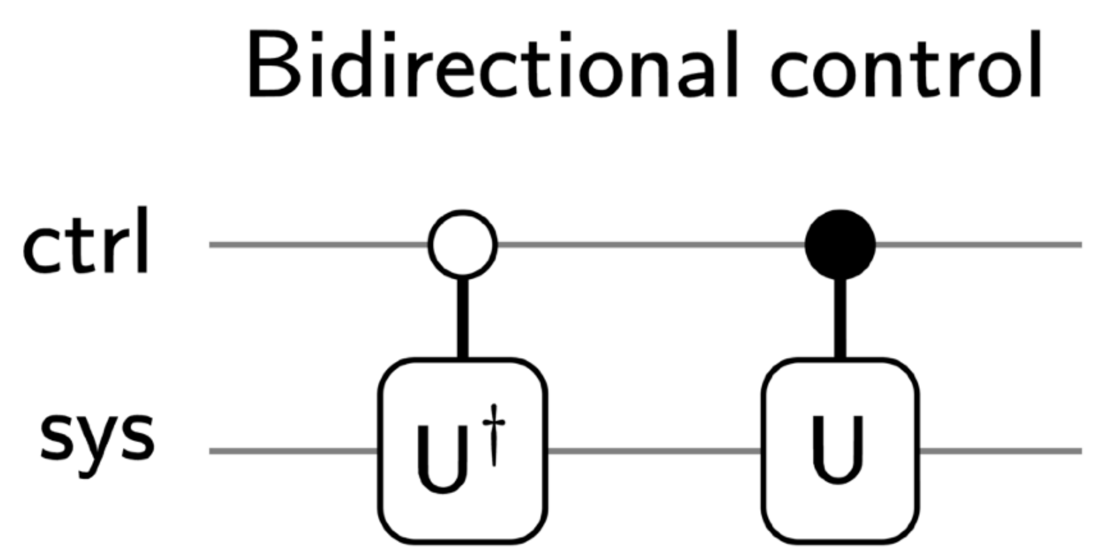
Will not change the power spectrum of the original signal, $|X[\omega]|^2$

Idea translates to what happens in QPE!

Shifting the signal in time amounts to a **global phase** in the phase register

Continuous case but discrete case should also follow.

Bidirectional phase kickback



$$|0\rangle|\psi\rangle \rightarrow e^{-2\pi i\phi}|0\rangle|\psi\rangle$$

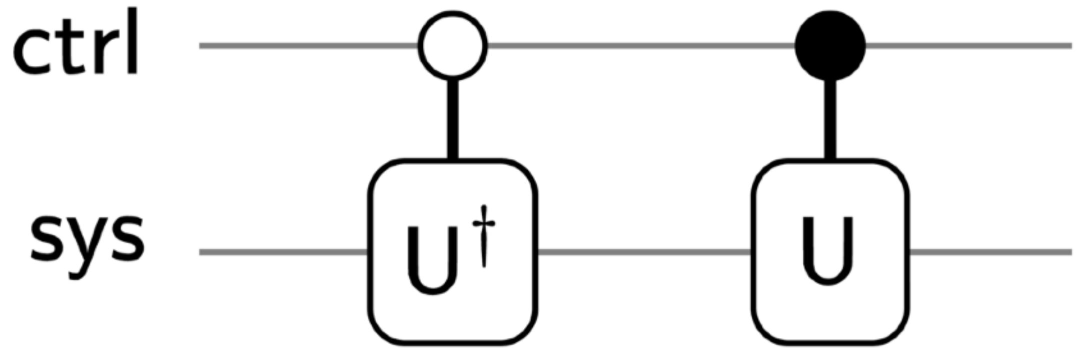
$$|1\rangle|\psi\rangle \rightarrow e^{+2\pi i\phi}|1\rangle|\psi\rangle$$

Applies a “bi-directional phase kickback”

Wecker, D. *et al.* *Phys. Rev. A* **92**, 062318 (2015).
Babbush, R. *et al.* *Phys Rev X* **8**, 041015 (2018).
Kivlichan, I. D. *et al.* *Quantum* **4**, 296 (2020).
Simon, W. A. & Love, P. J. *arXiv* (2025) doi:10.48550/arxiv.2511.13855.

Bidirectional phase kickback

Bidirectional control



$$|0\rangle|\psi\rangle \to e^{-2\pi i\phi}|0\rangle|\psi\rangle$$

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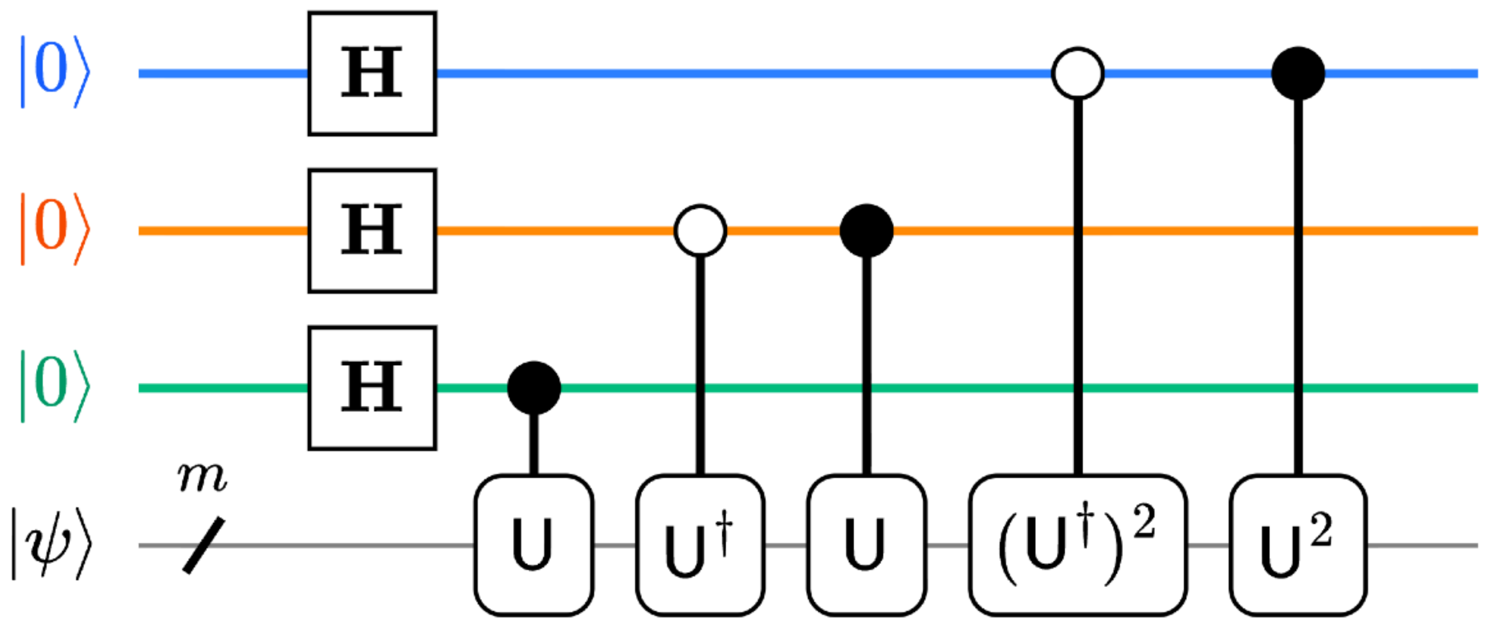
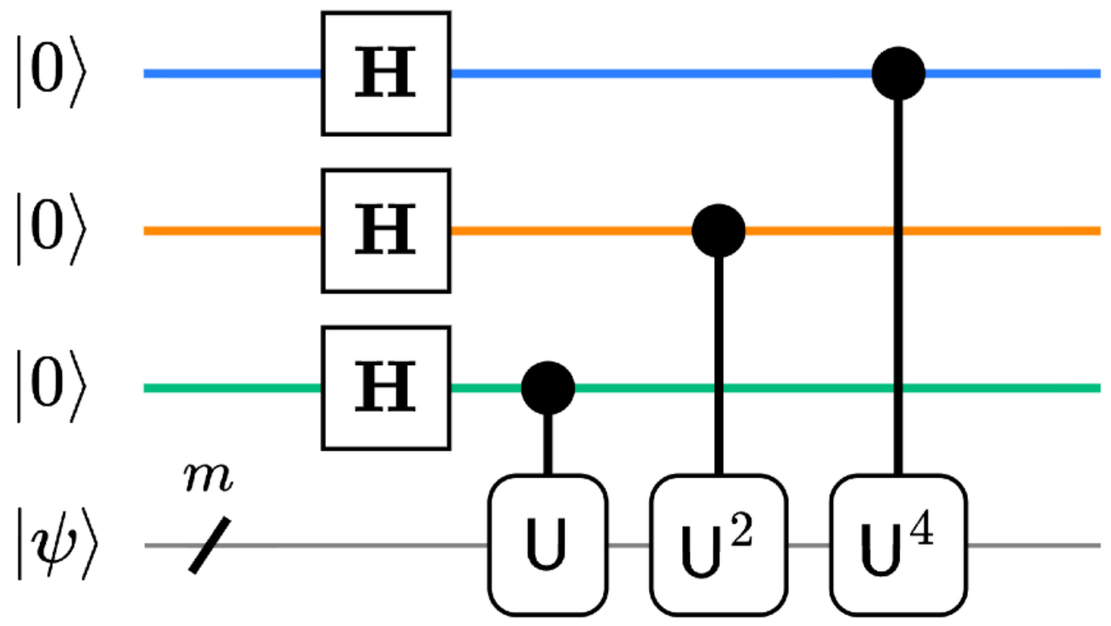
Shifted



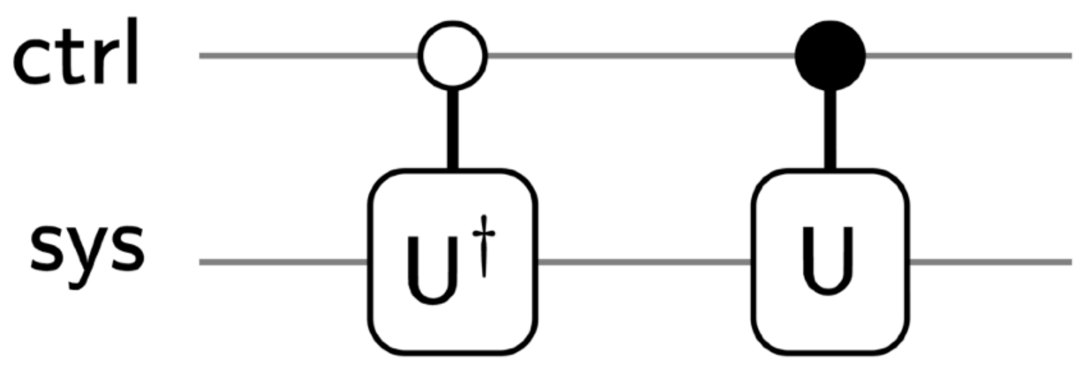
vs.

Unshifted



Bidirectional phase kickback

Bidirectional control



$$|0\rangle|\psi\rangle \rightarrow e^{-2\pi i\phi}|0\rangle|\psi\rangle$$

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$$U|\psi\rangle = e^{2\pi i\phi}|\psi\rangle, \quad \phi \in [0,1)$$

Basis state, k	Shifted	Unshifted
0 = 000	U^{-3}	U^{0}
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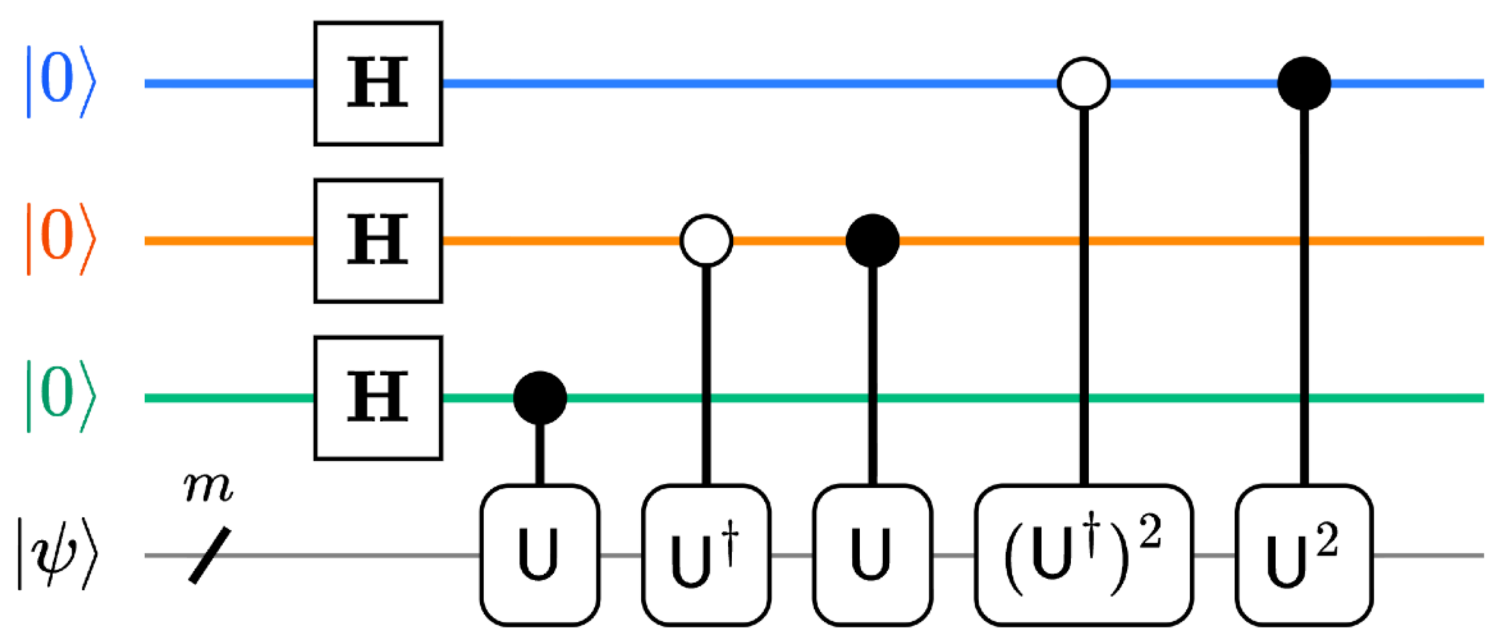
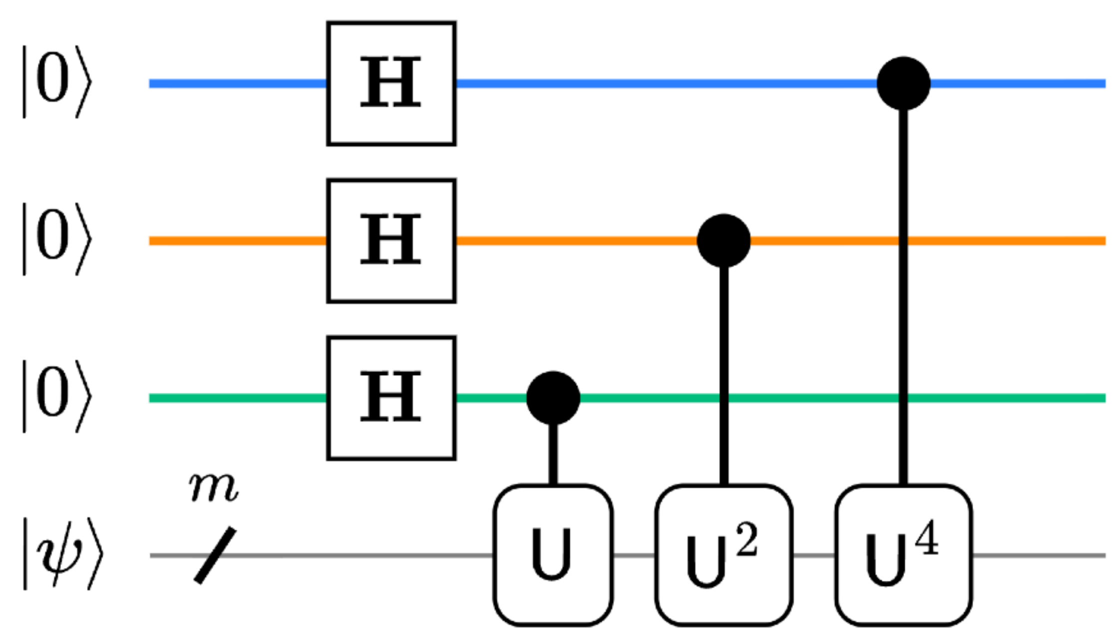
Shifted



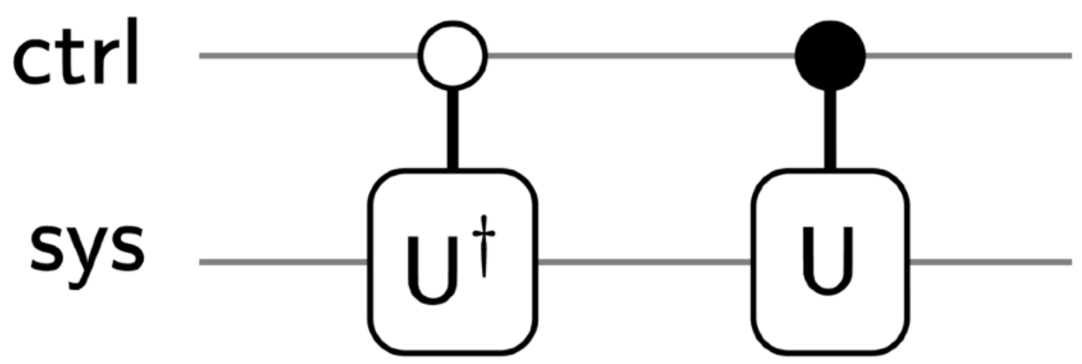
vs.

Unshifted



Bidirectional phase kickback

Bidirectional control



$$|0\rangle|\psi\rangle \rightarrow e^{-2\pi i\phi}|0\rangle|\psi\rangle$$

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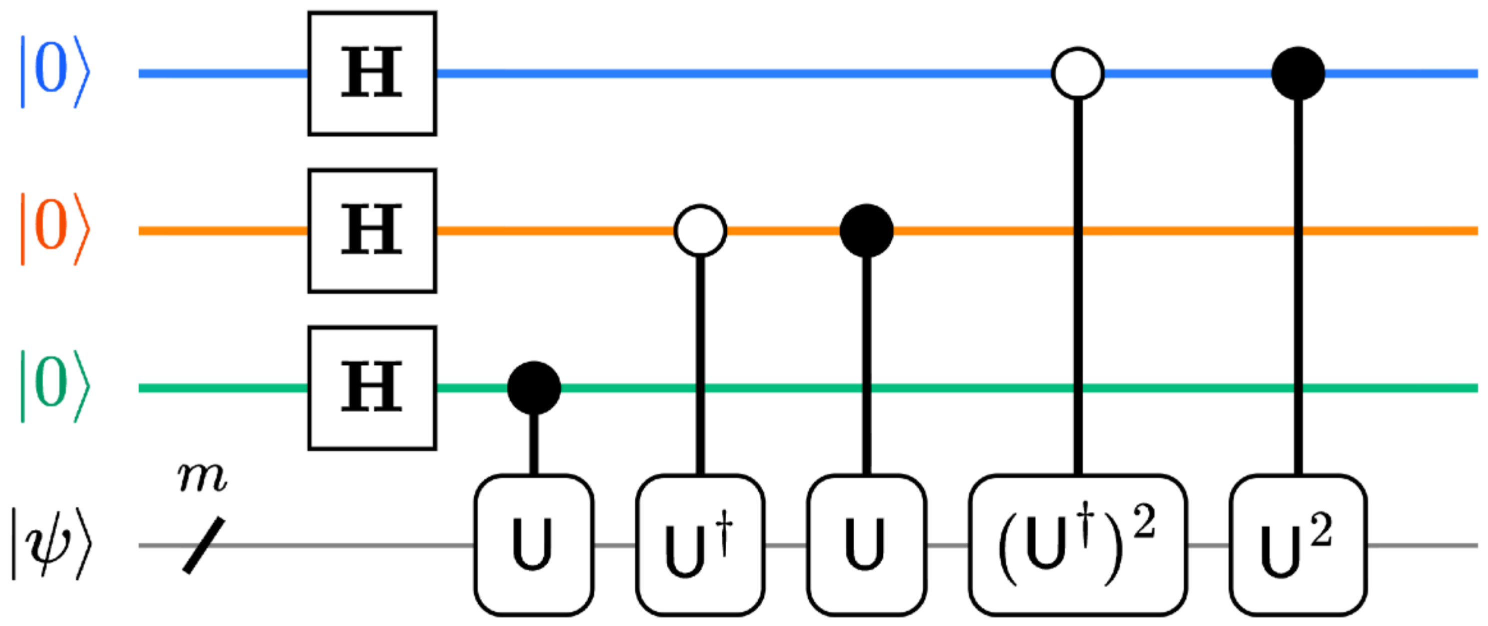
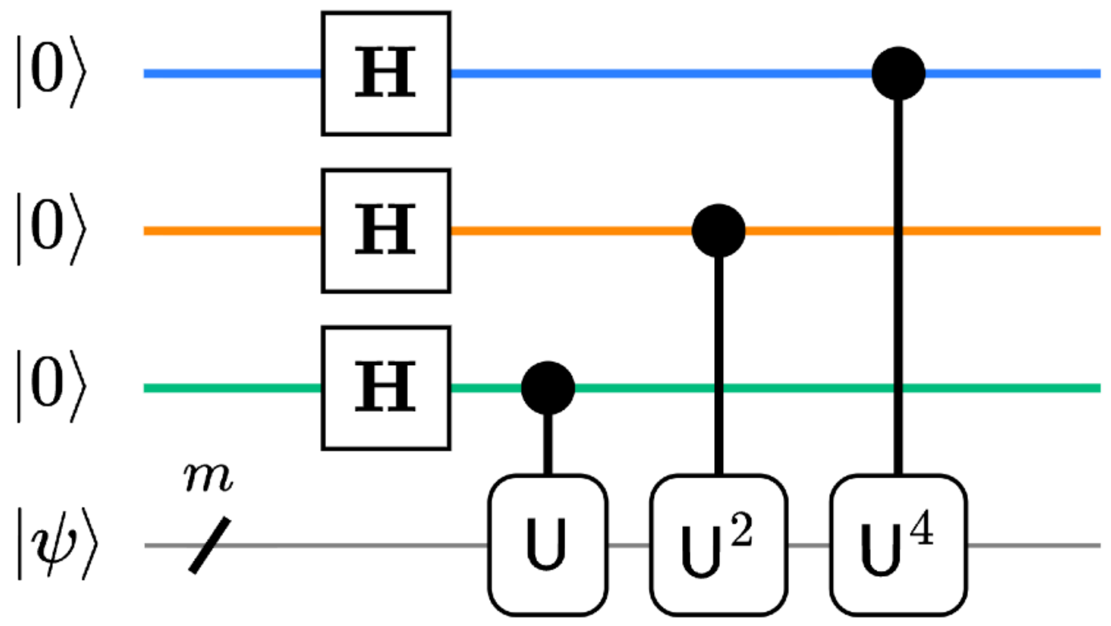
Shifted



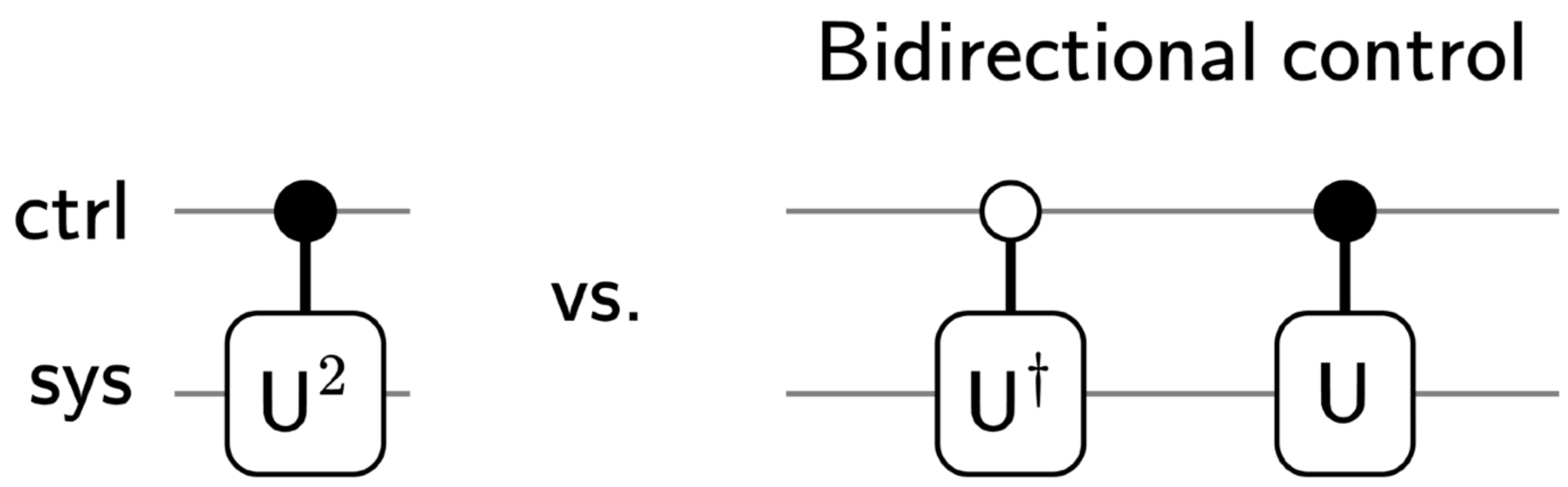
vs.

Unshifted

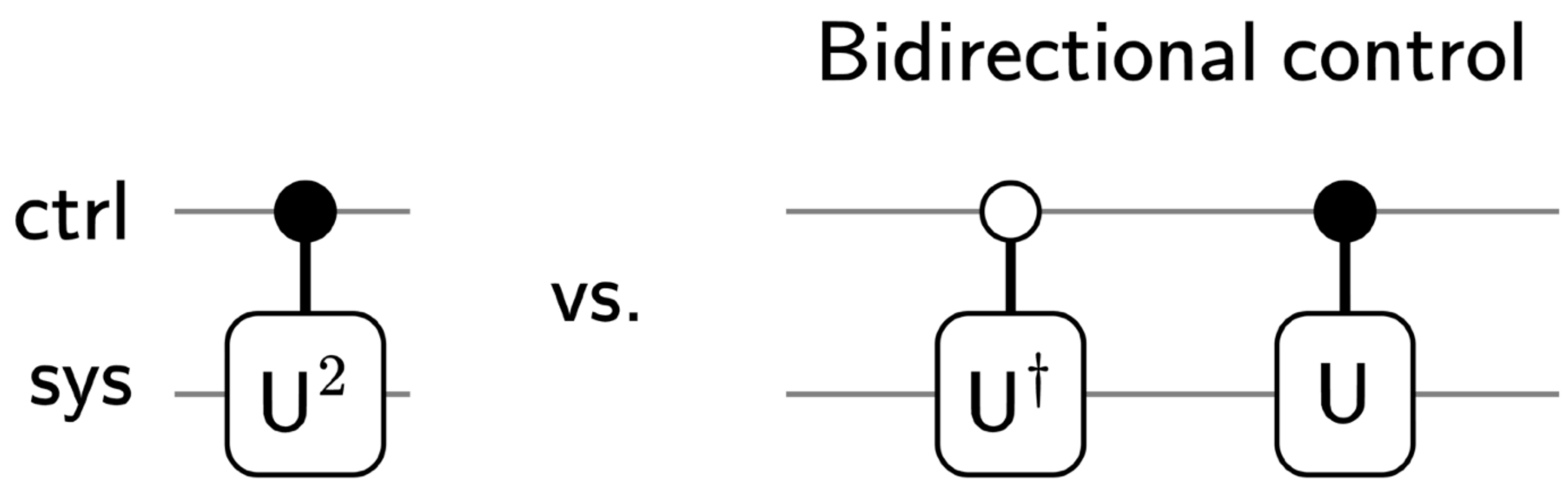


But looks like they're about the same cost..

Bidirectional phase kickback



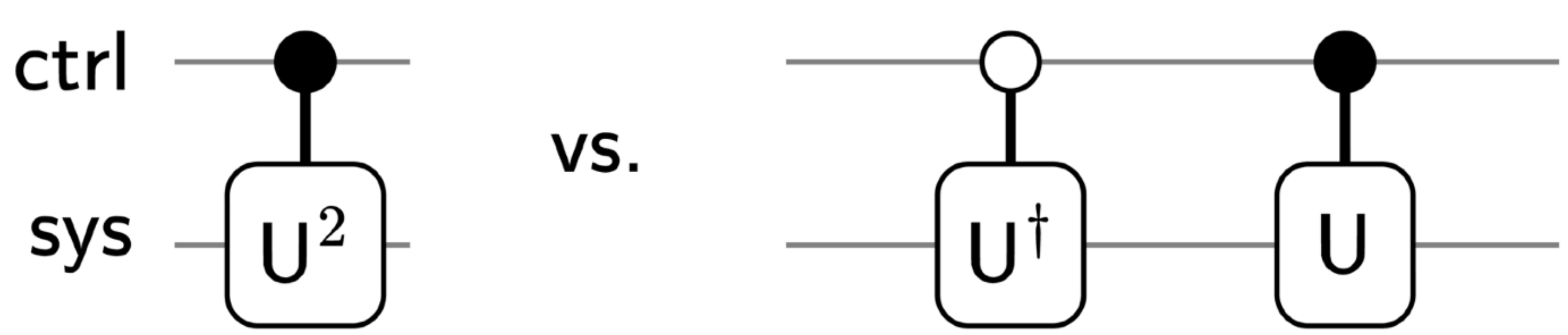
Bidirectional phase kickback



Cost can be nearly halved depending on U

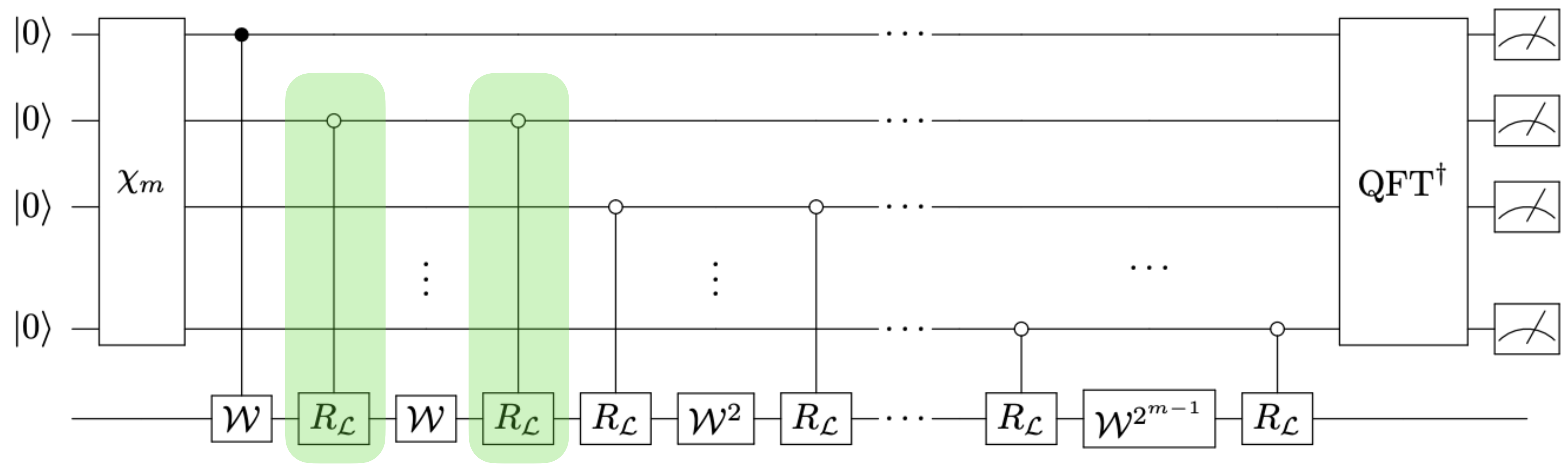
Bidirectional phase kickback

Bidirectional control



Cost can be nearly halved depending on U

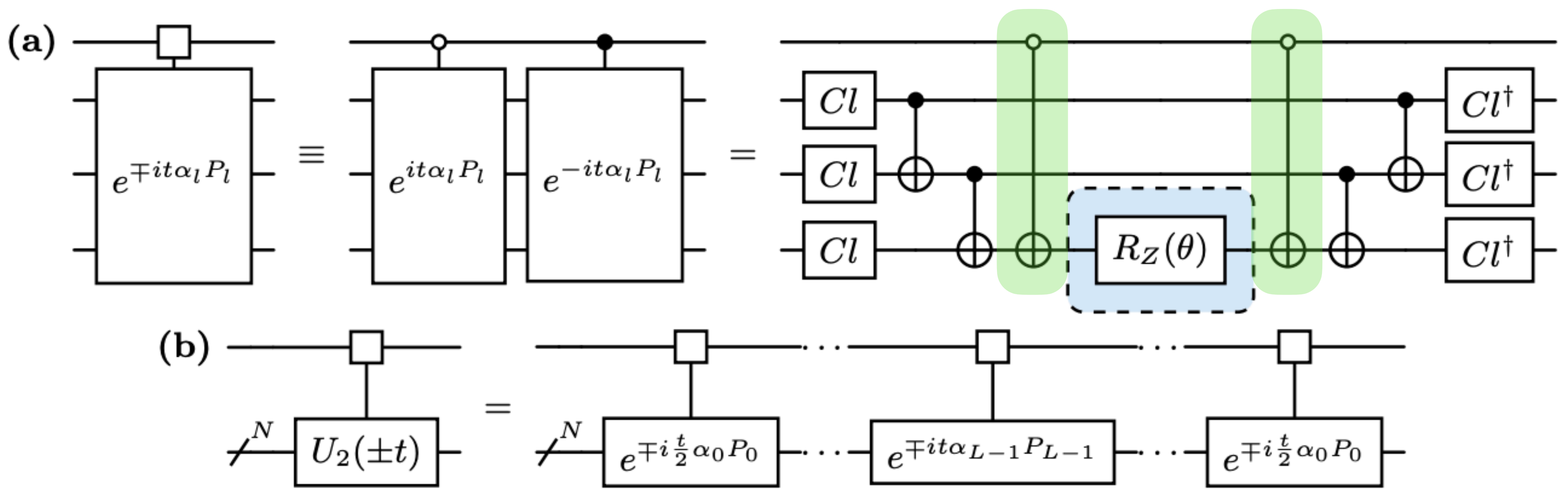
Qubitization



$$\mathcal{R}_\mathcal{L} \cdot \mathcal{W}^n \cdot \mathcal{R}_\mathcal{L} = {\mathcal{R}_\mathcal{L}}^2 \cdot (\text{SELECT} \cdot \mathcal{R}_\mathcal{L})^n = (\text{SELECT} \cdot \mathcal{R}_\mathcal{L})^n = (\mathcal{W}^\dagger)^n$$

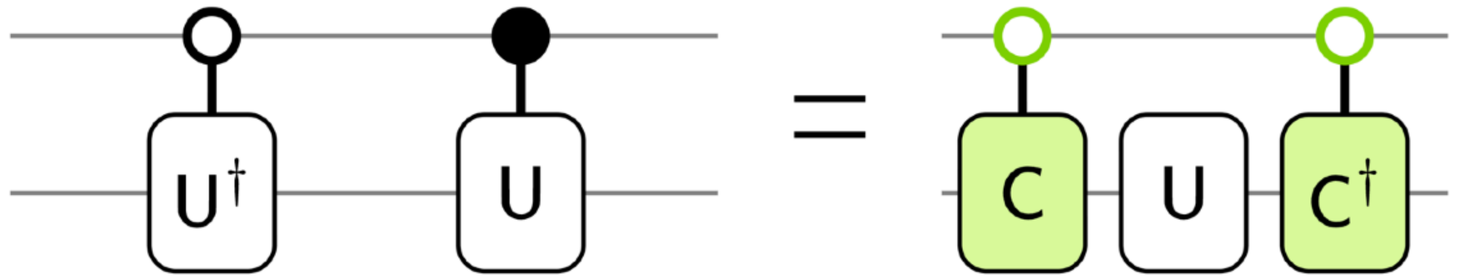
Phys Rev X **8**, 041015 (2018), *PRX Quantum* **2**, 030305 (2021)

Trotterization (symmetric)



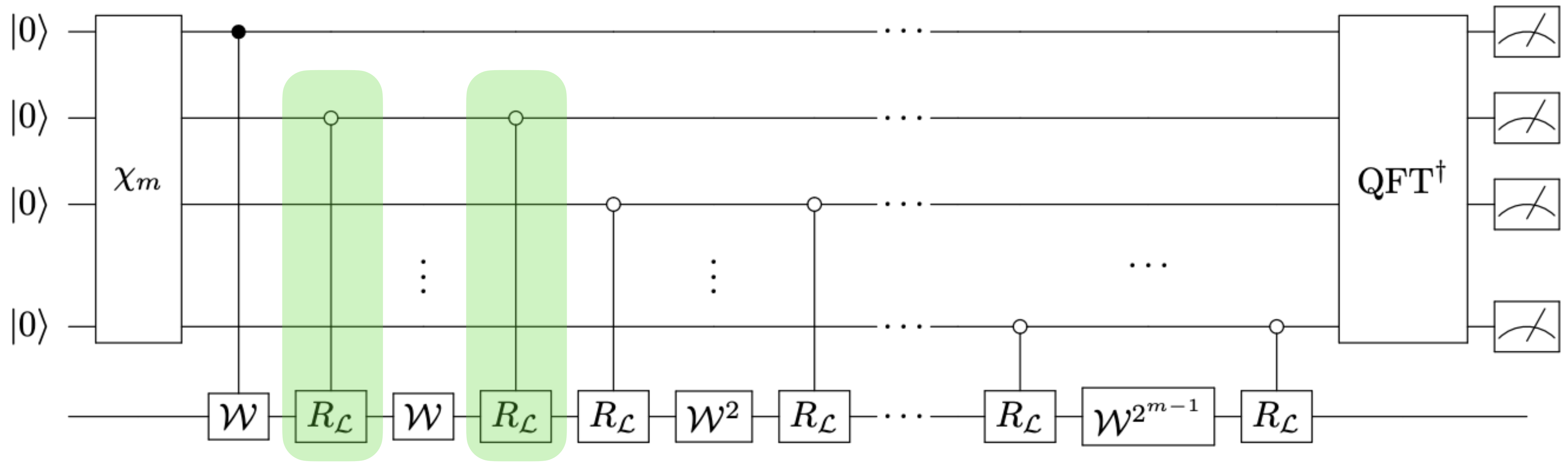
Phys. Rev. A **92**, 062318 (2015), *Quantum* **4**, 296 (2020), arxiv:2511.13855

Bidirectional phase kickback



where auxiliary operations (controlled-C) are significantly cheaper than U

Qubitization



$$\mathcal{R}_\mathcal{L} \cdot \mathcal{W}^n \cdot \mathcal{R}_\mathcal{L} = \mathcal{R}_\mathcal{L}{}^2 \cdot (\textsc{select} \cdot \mathcal{R}_\mathcal{L})^n = (\textsc{select} \cdot \mathcal{R}_\mathcal{L})^n = (\mathcal{W}^\dagger)^n$$

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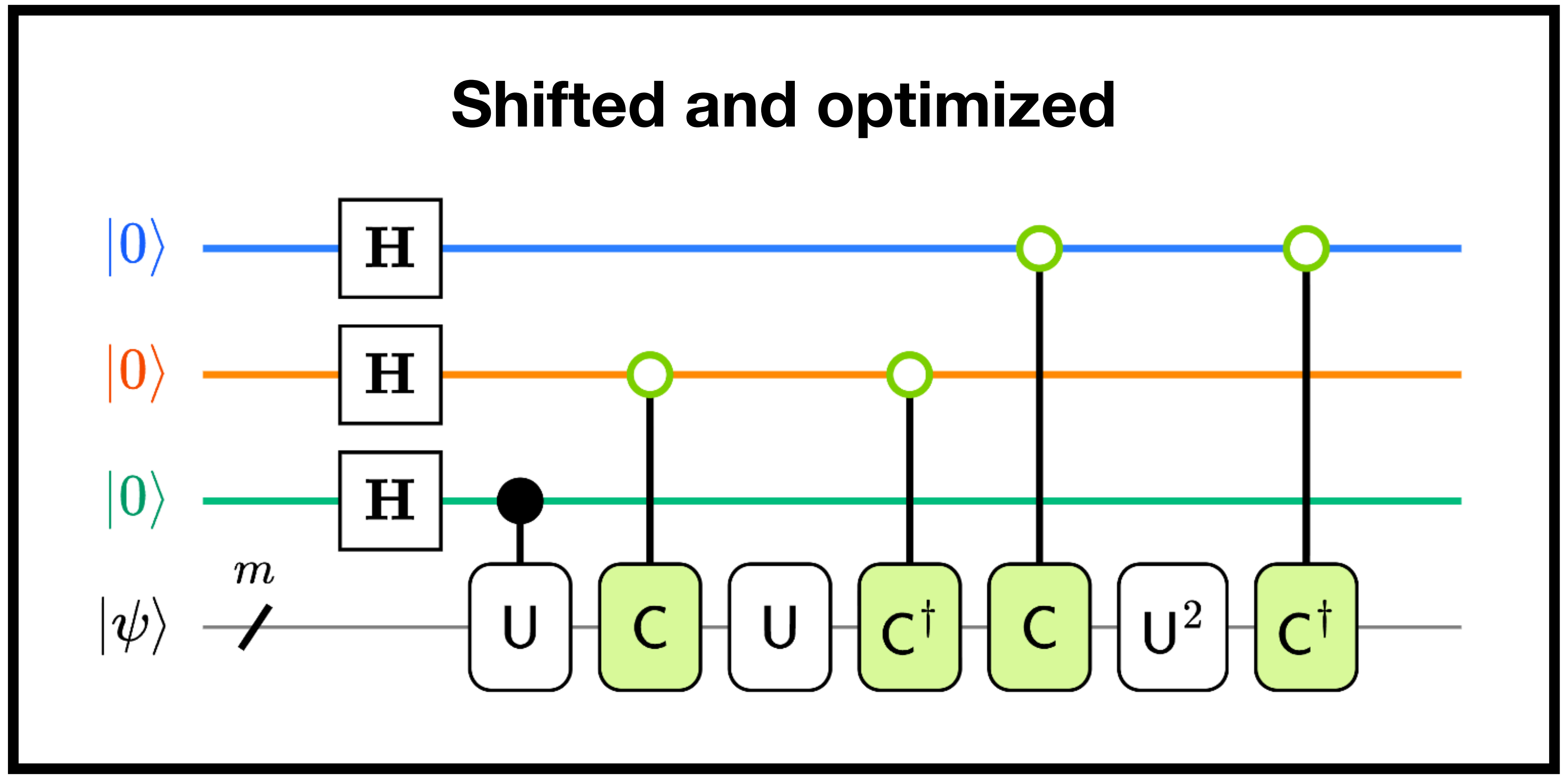
Bidirectional phase kickback



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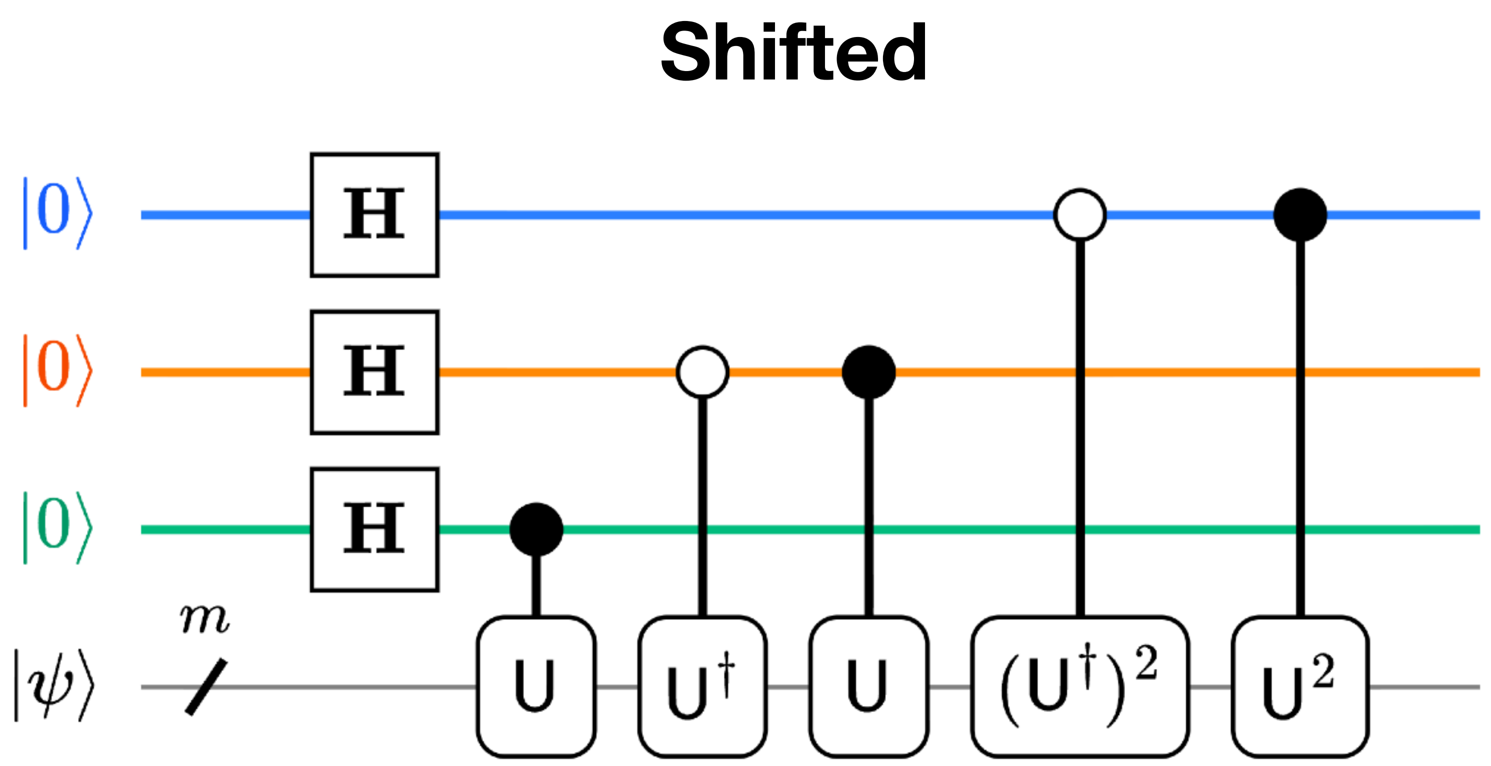
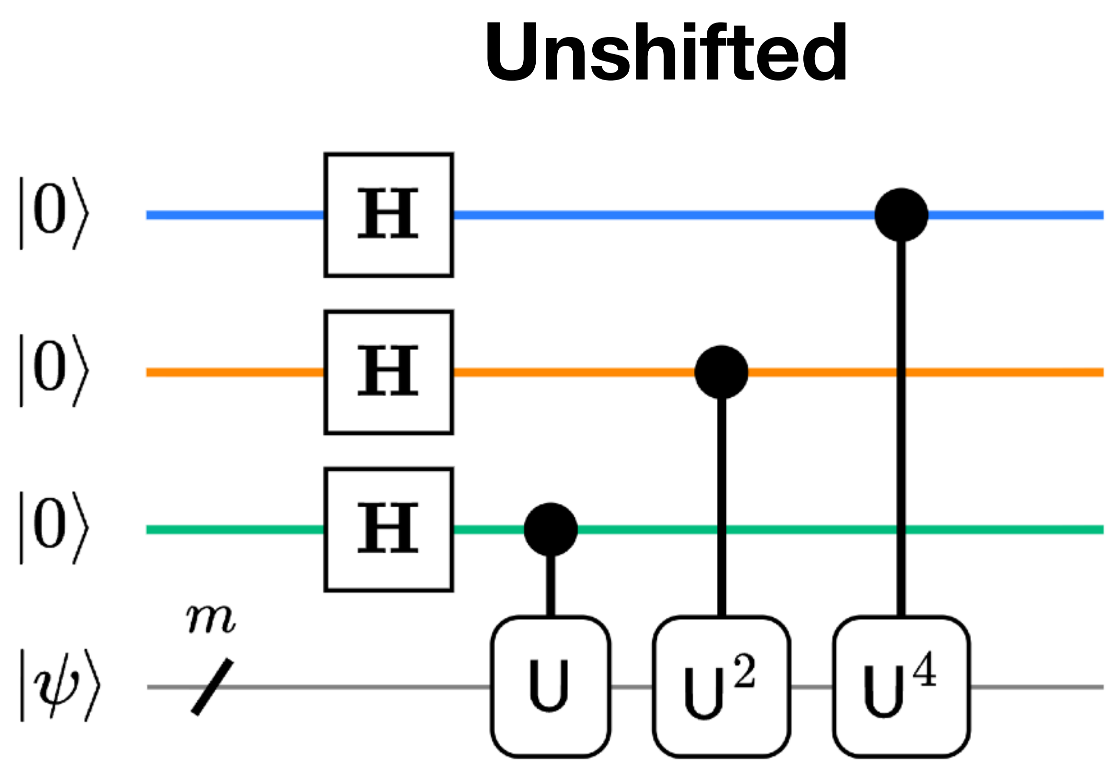


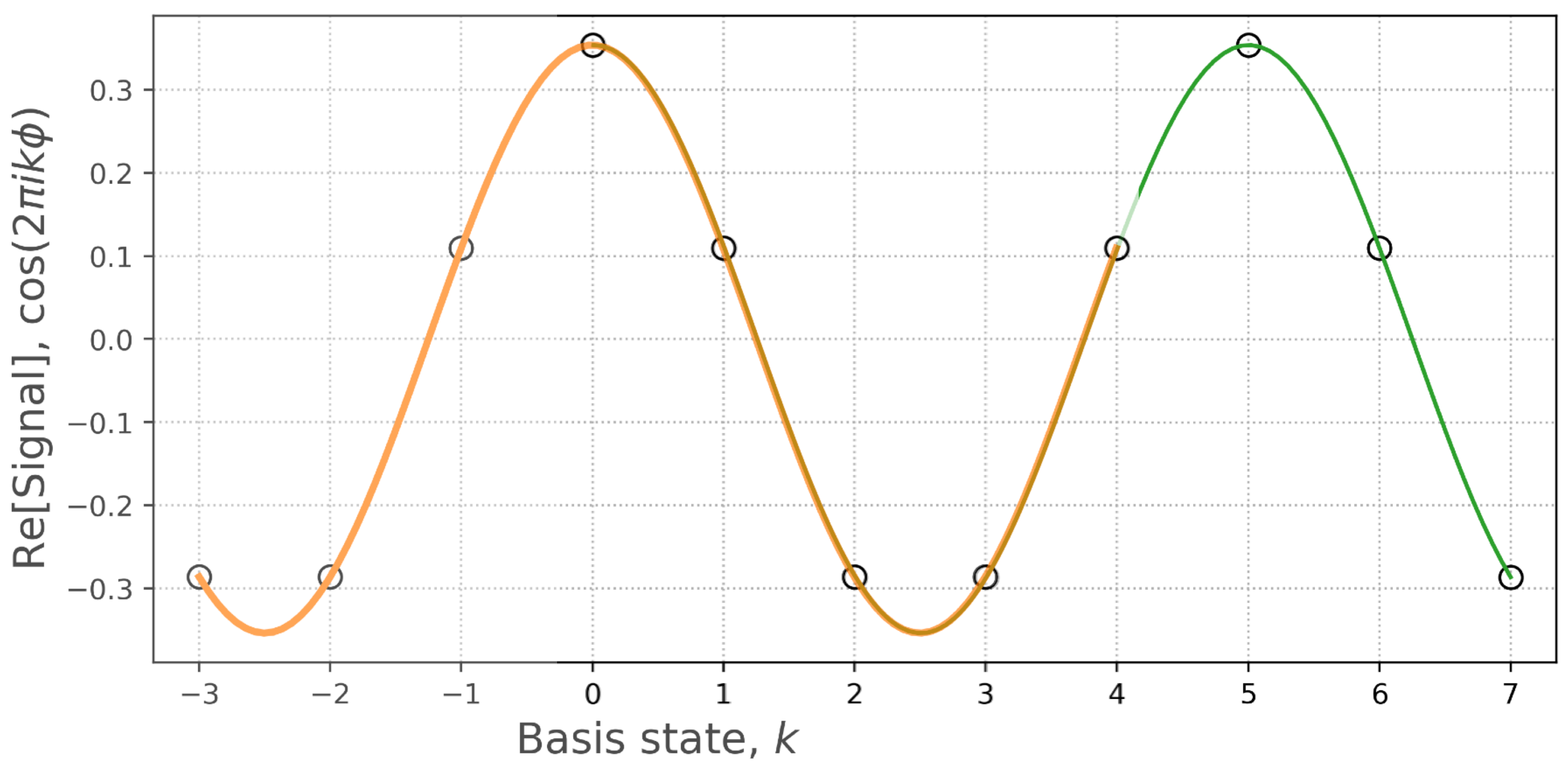
vs.



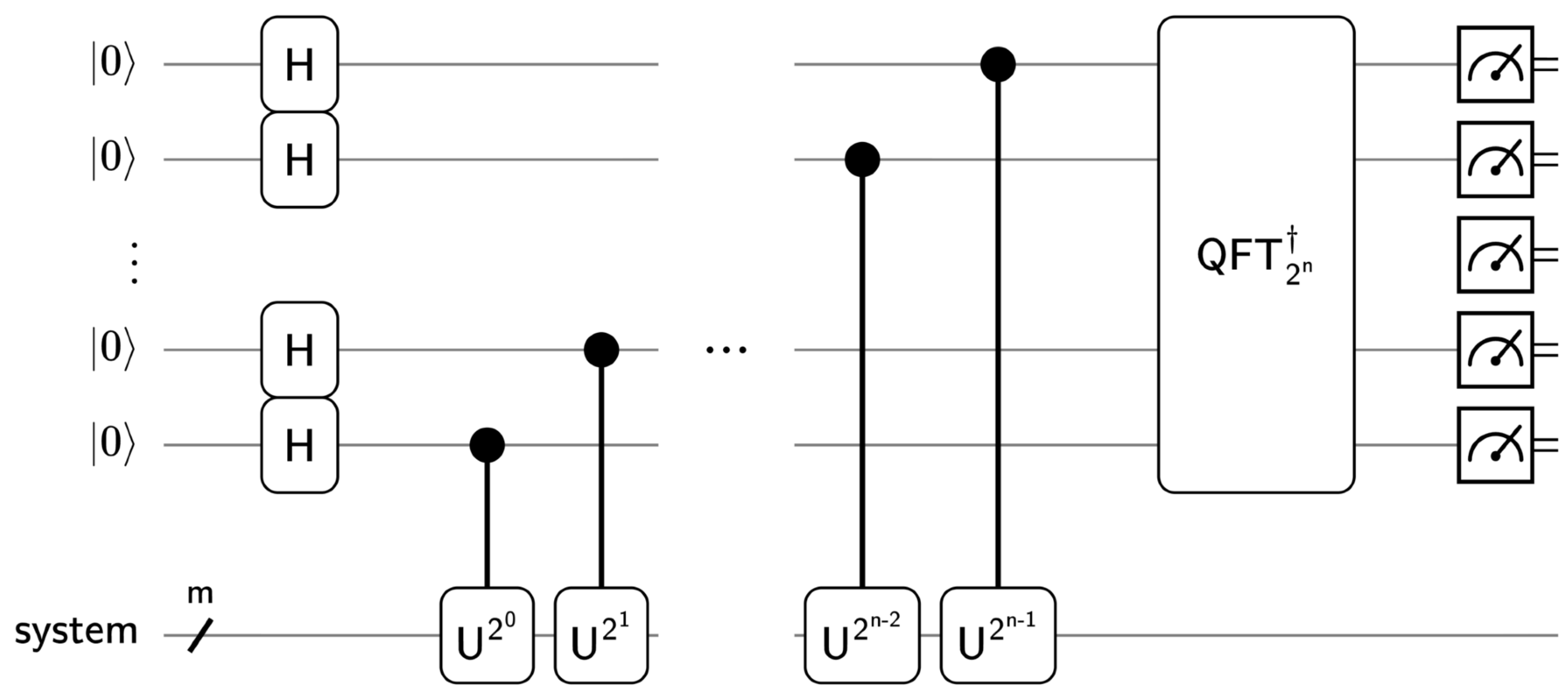
Bidirectional phase kickback

- Resource savings: can save almost half the number of queries required for a given precision

- Why does this work? This corresponds to shifting your signal in the time domain. This will not change your power spectrum.



Abstracting QPE



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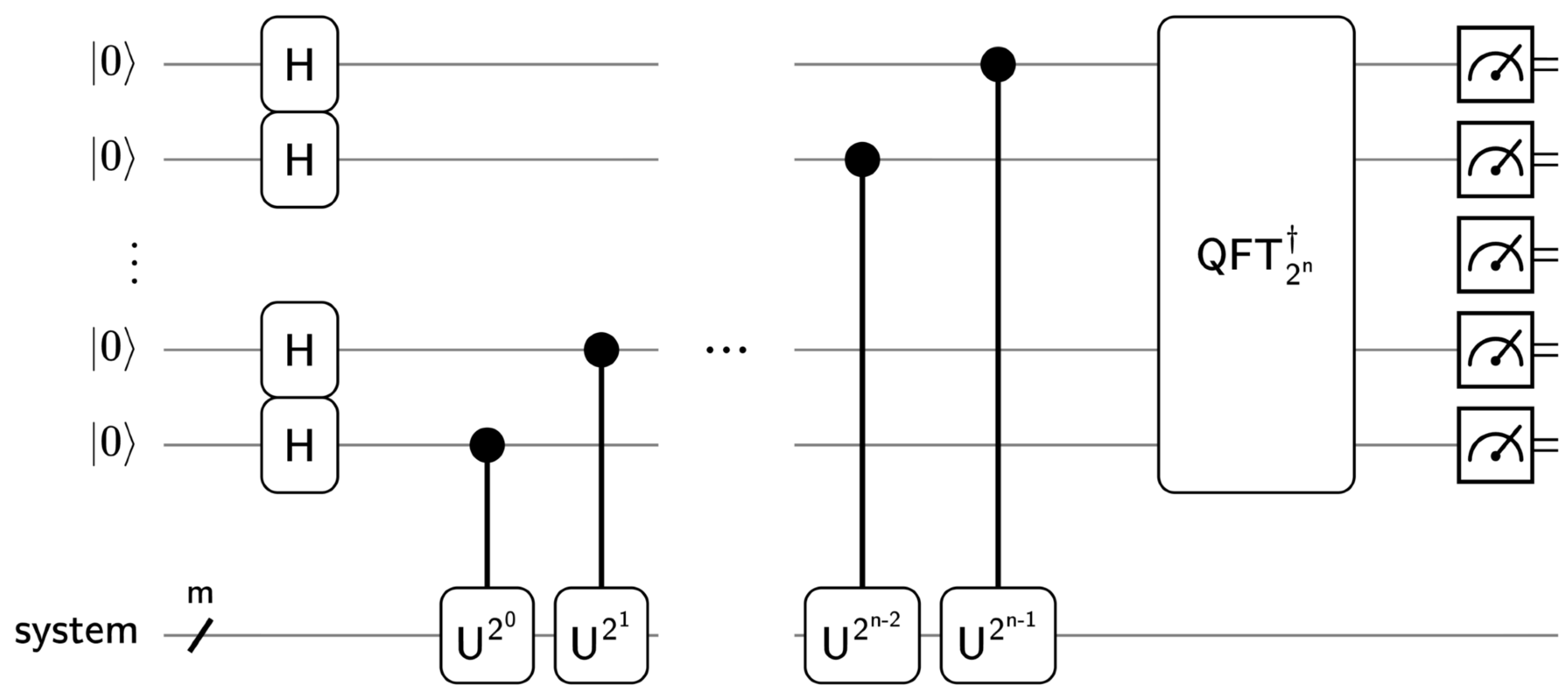


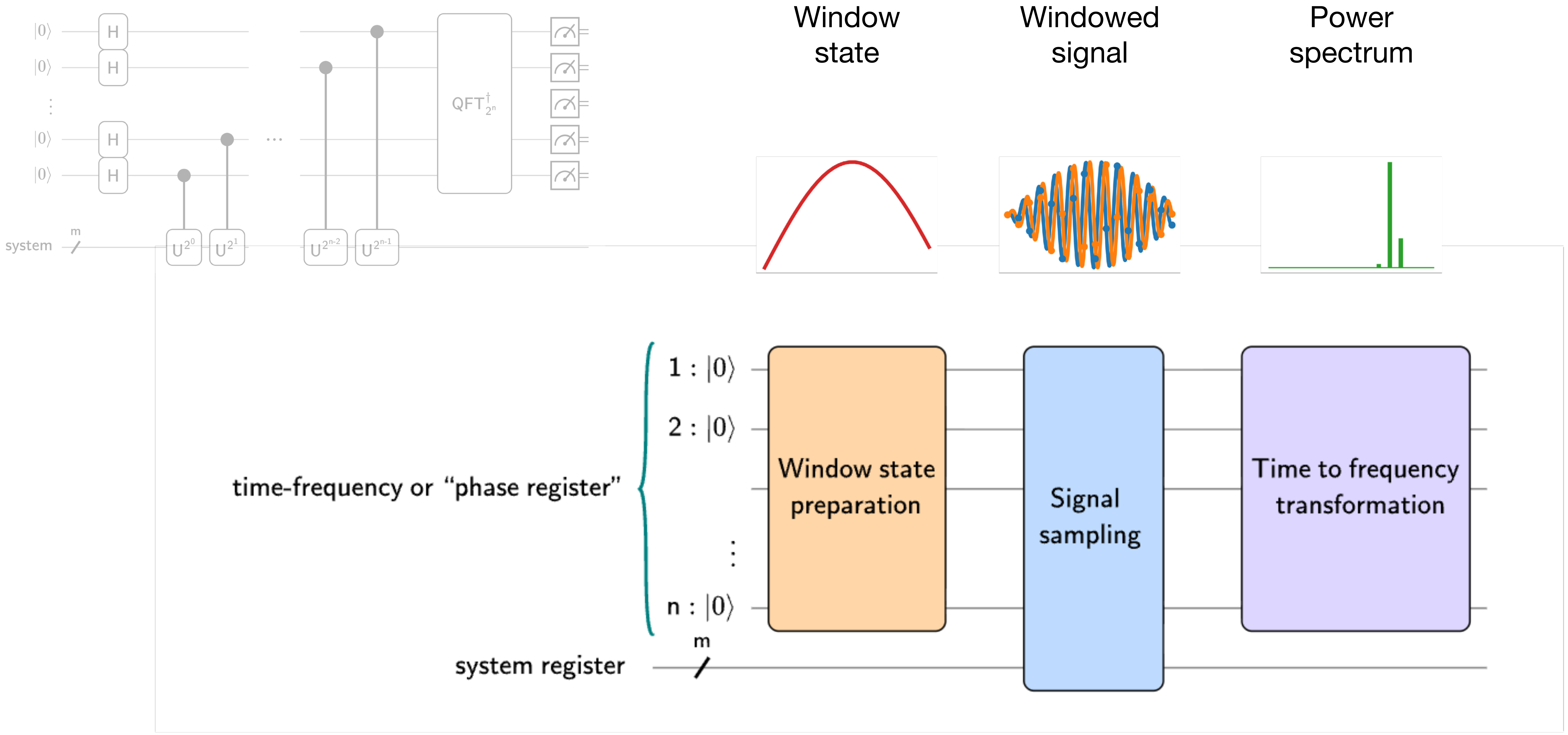

```
# Superposition
phase_reg.had()


# Controlled-(U^k) operations
for i in range(phase_reg.num_qubits):

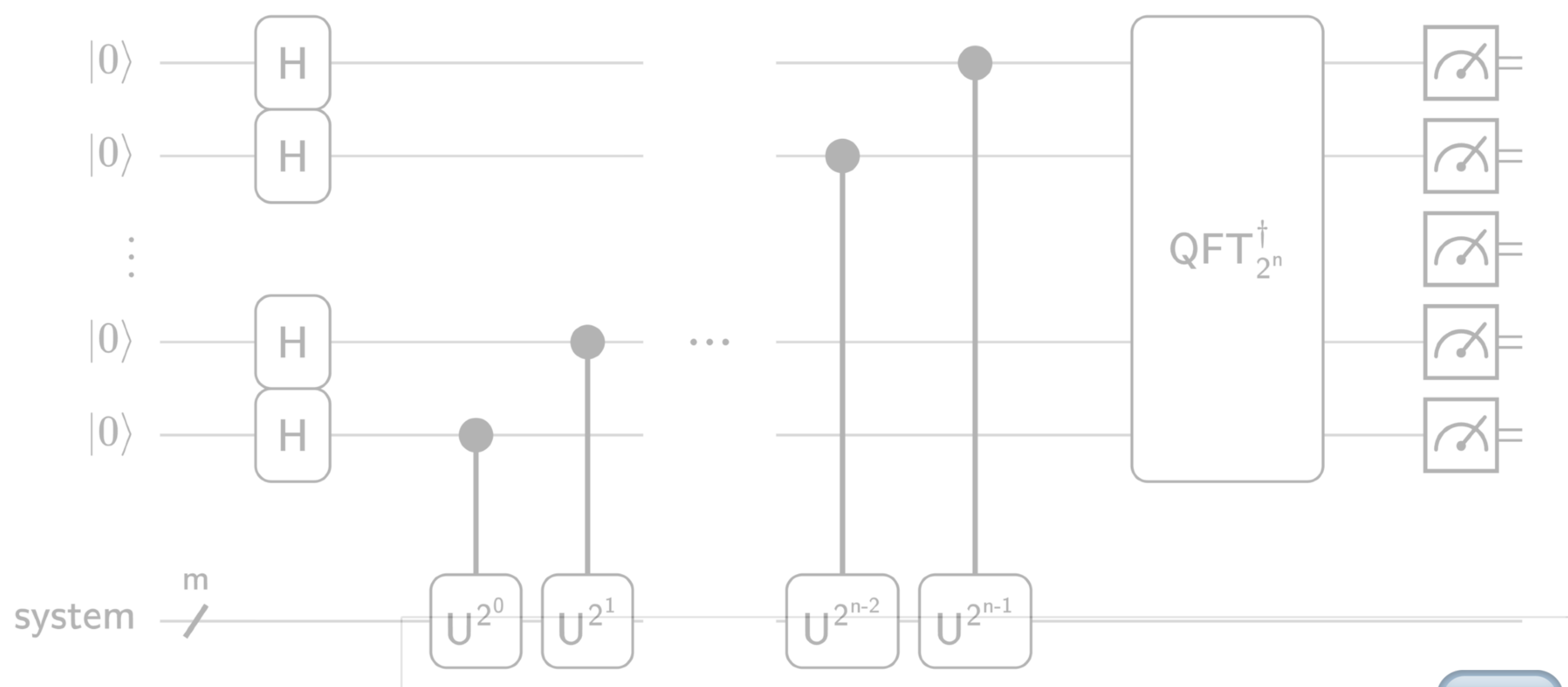
    for k in range(2**i):
        unitary.compute(system_reg, ctrl=phase_reg[i])

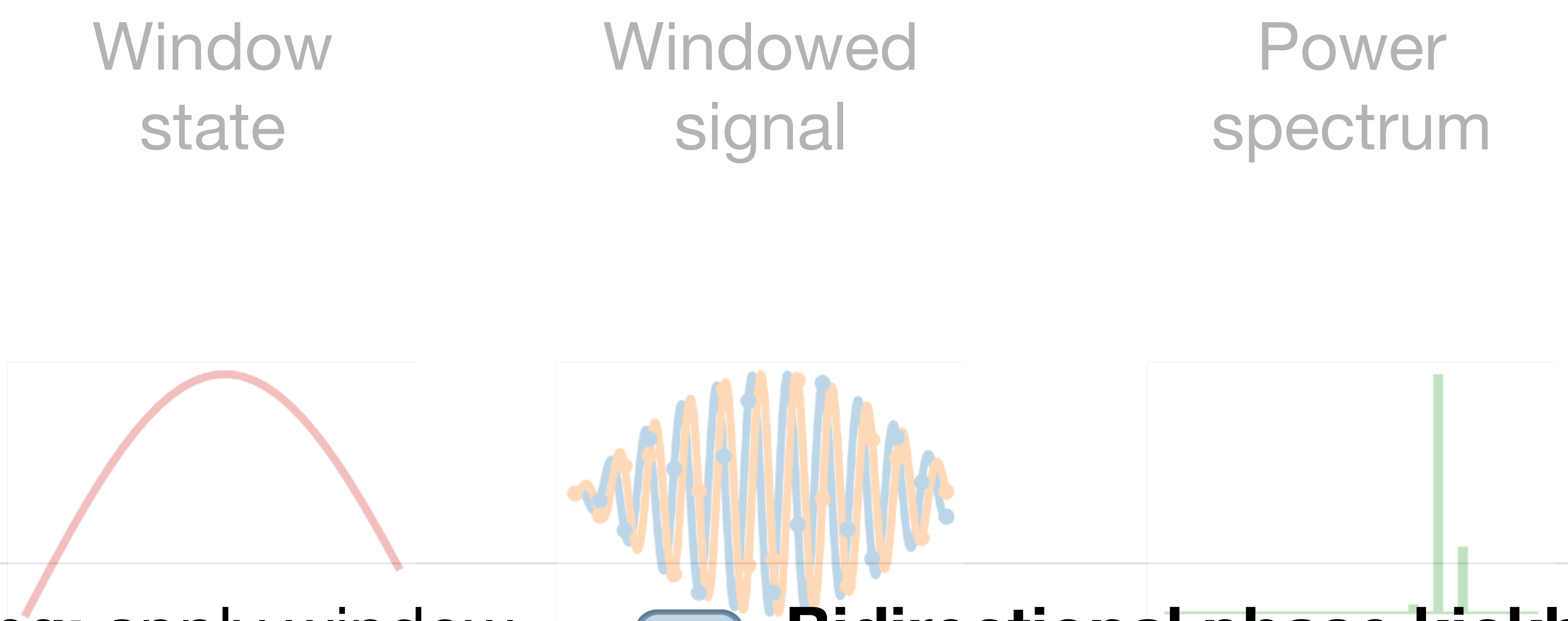

# Inverse QFT
inv_QFT.compute(phase_reg)
```

Abstracting QPE



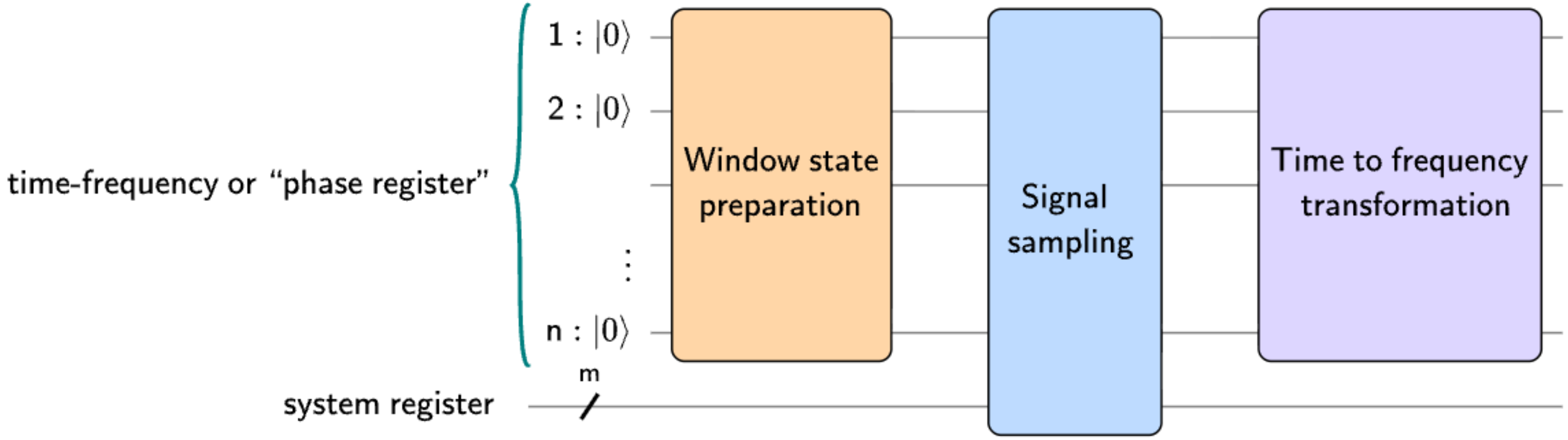
Abstracting QPE



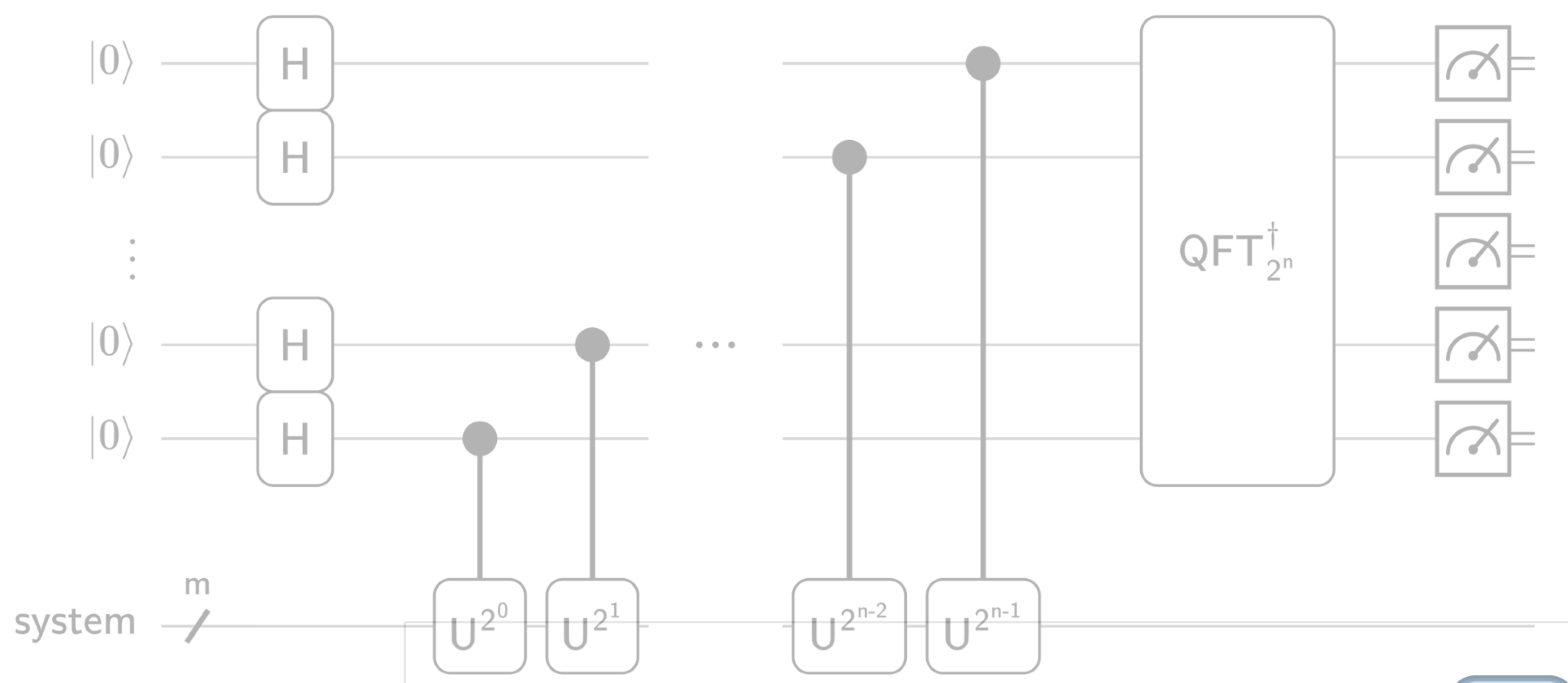


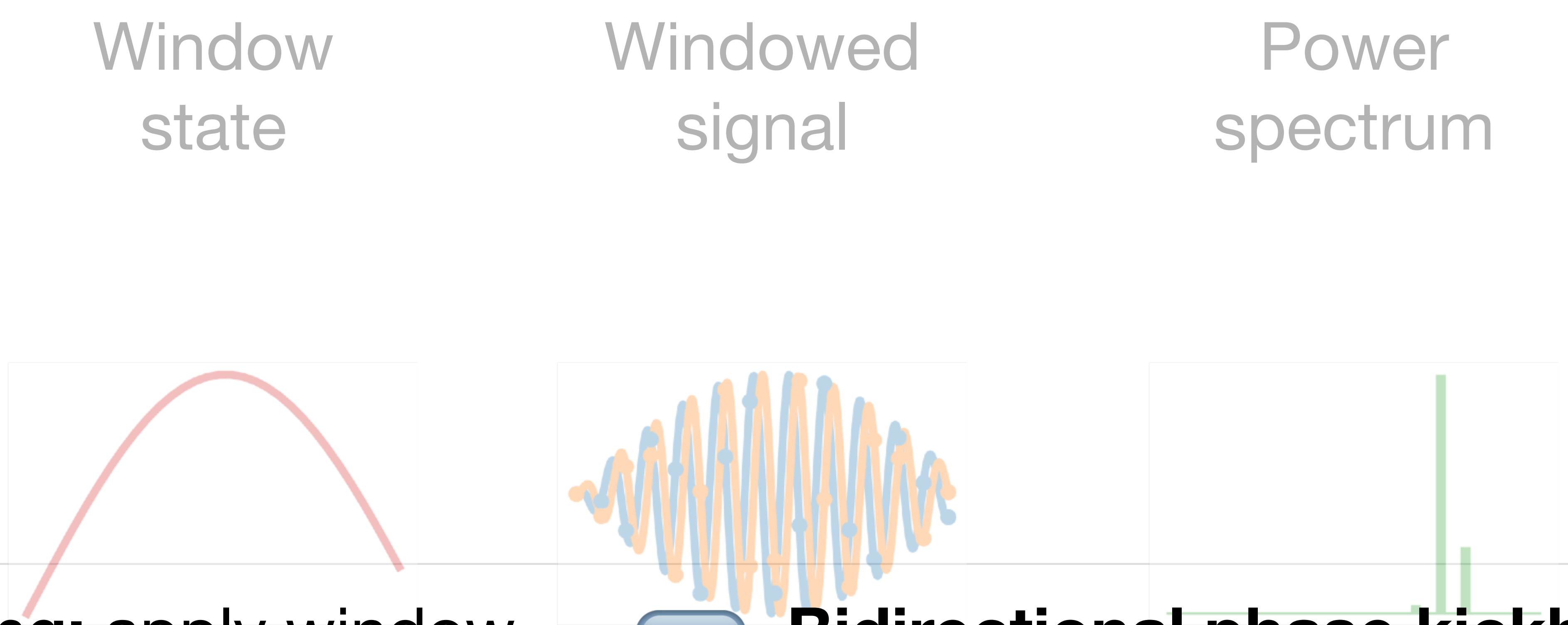
1 **Windowing:** apply window to reduce spectral leakage

2 **Bidirectional phase kickback:** time-shift your signal to save on number of queries



Abstracting QPE





1 **Windowing:** apply window to reduce spectral leakage

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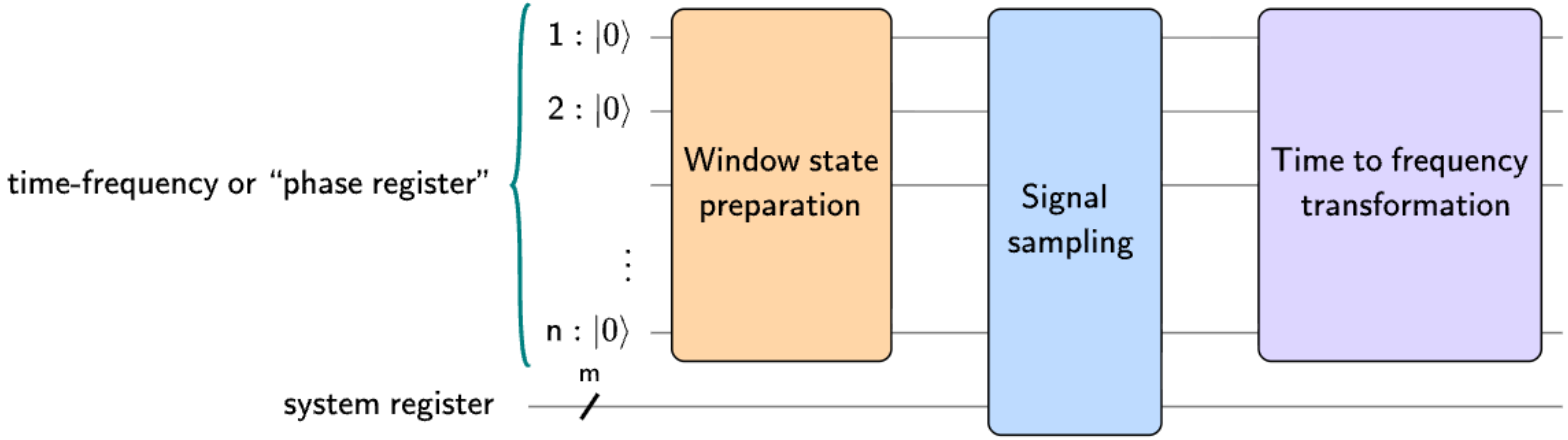

Many more connections to be made!

Can we further leverage ideas from DSP to understand and improve QPE?

Summary

- Re-interpreting QPE using the lens of DSP can help unify algorithmic features and improvements
    - This can help us build more extensible implementations of QPE
    - It can also serve as a pedagogical tool for (re)introducing QPE
    - Further adapt ideas from DSP to improve QPE?

Summary

- Re-interpreting QPE using the lens of DSP can help unify algorithmic features and improvements
  - This can help us build more extensible implementations of QPE
  - It can also serve as a pedagogical tool for (re)introducing QPE
  - Further adapt ideas from DSP to improve QPE?

- Beyond QPE: reframing quantum algorithms may be helpful for programming (and eventual deployment)

Thank you!

A practical guide to Quantum Phase Estimation: revisiting features and improvements in the language of digital signal processing

William A. Simon[1,2], Sean Greenaway[1], and Sukin Sim[1]

Connecting qubitization, zero-padded signal, aliasing …

Coming soon!

email: ssim@psiquantum.com

PsiQuantum

Citations for QPE features/improvements

Cosine

- Rendon, G., Izubuchi, T. & Kikuchi, Y. *Phys Rev D* **106**, 034503 (2022).

Sine

- Luis, A. & Peřina, J. *Phys. Rev. A* **54**, 4564–4570 (1996).
- Babbush, R. *et al. Phys Rev X* **8**, 041015 (2018).
- Najafi, P., Costa, P. C. S. & Berry, D. W. *arXiv* (2023) doi:10.48550/arxiv.2303.12503.

Kaiser

- Sanders, Y. R. *et al. PRX Quantum* **1**, (2020).
- Berry, D. W. *et al. arXiv* (2024) doi:10.48550/arxiv.2409.11748.
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- Patel, D., Tan, S. J. S., Subasi, Y. & Sornborger, A. T. *arXiv* (2024) doi:10.48550/arxiv.2403.18927.
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m-th B-spline

- O'Brien, O. & Sünderhauf, C. *Quantum* **9**, 1786 (2025).

Gaussian

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Unary iteration

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- Wecker, D. *et al. Phys. Rev. A* **92**, 062318 (2015).
- Babbush, R. *et al. Phys Rev X* **8**, 041015 (2018).
- Kivlichan, I. D. *et al. Quantum* **4**, 296 (2020).
- Simon, W. A. & Love, P. J. *arXiv* (2025) doi:10.48550/arxiv.2511.13855.

Trotterization

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Qubitization

- Poulin, D., Kitaev, A., Steiger, D. S., Hastings, M. B. & Troyer, M. *Phys. Rev. Lett.* **121**, 010501 (2018).
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QSVT

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Shifting a signal in time (from signal processing lens)

Fourier transform of a time shifted signal *x*: $\mathscr{F}\left[x(t-t_0)\right] = e^{-i\omega t_0}\, X(\omega)$ where $X(\omega) = \mathscr{F}\left[x(t)\right]$

$$\begin{aligned}\mathscr{F}\left[x(t-t_0)\right] &= \int_{-\infty}^{\infty} x(t-t_0)\; e^{-i\omega t} dt \\ &= \int_{-\infty}^{\infty} x(h)\; e^{-i\omega(h+t_0)} dh \\ &= e^{-i\omega t_0}\; X(\omega)\end{aligned}$$

Change of variable

$$h = t - t_0$$

Continuous case but discrete case should be very similar.

Bidirectional phase kickback

After applying controlled unitaries:

After applying inverse QFT:

Textbook QPE

$$\frac{1}{2^{n/2}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle$$

$$\frac{1}{2^n}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}\sum_{x=0}^{2^n-1} e^{-2\pi i kx/2^n}|x\rangle$$

Bidirectional phase kickback

$$\frac{1}{2^{n/2}}\sum_{k=0}^{2^n-1} e^{2\pi i(\overbrace{k-2^{n-1}+1}^{k'})\phi}|k\rangle$$

$$= e^{2\pi i(-2^{n-1}+1)\phi}\left(\frac{1}{2^{n/2}}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}|k\rangle\right)$$

$$k = [0,...,2^n - 1]$$

$$k' = [-2^{n-1},\ldots,2^{n-1} - 1]$$

$$\frac{1}{2^n}\sum_{k=0}^{2^n-1} e^{2\pi i(k-2^{n-1}+1)\phi}\sum_{x=0}^{2^n-1} e^{-2\pi i kx/2^n}|x\rangle$$

$$= e^{2\pi i(-2^{n-1}+1)\phi}\left(\frac{1}{2^n}\sum_{k=0}^{2^n-1} e^{2\pi i k\phi}\sum_{x=0}^{2^n-1} e^{-2\pi i kx/2^n}|x\rangle\right)$$

Global phase