Using methods of kinetic theory of gases to treat fluctuations in particle beams

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Outline of the talk

- One-particle distribution function, Vlasov equation and fluctuations in particle beams
- Incoherent radiation
- FEL bunching factor
- Stochastic cooling
- Noise suppression in beams

Introduction

The kinetic (Vlasov) equation for the distribution function $f(\mathbf{r}, \mathbf{p}, t)$ is

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + e[\mathbf{E}(\mathbf{r}, t) + \frac{1}{c}\boldsymbol{v} \times \mathbf{B}(\mathbf{r}, t)] \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$f(\mathbf{r}, \mathbf{p}, t)$ is the distribution function averaged over small volumes of the phase space. In principle, the RHS can include effects of particle collisions, damping, diffusion, etc. The electric and magnetic fields are calculated self-consistently from Maxwell's equations with account of charges and currents generated by the beam

$$\rho(\mathbf{r}, t) = e \int d^3p \, f(\mathbf{r}, \mathbf{p}, t), \qquad \mathbf{j}(\mathbf{r}, t) = e \int d^3p \, \boldsymbol{v} \, f(\mathbf{r}, \mathbf{p}, t)$$

The Vlasov equation is widely used in accelerator physics for studies of collective effects, in theory of FELs, plasma wake field acceleration, etc. [Variables $(\mathbf{r}, \mathbf{p}, t)$ are usually replaced by more convenient variables.]

This distribution function treats the beam as a *fluid* evolving in 6D phase space under the influence of the self consistent EM fields.

In the standard interpretation of $f(\mathbf{r}, \mathbf{p}, t)$, it does not include fluctuations in the beam. In problems where the *shot noise* in the beam is important (start up of a SASE FEL, stochastic and collective cooling) it is usually introduced switching to the picture of discrete particles ($\sum_{n=1}^{N_e}$).

Summation over discrete particles

FEL textbooks:

$$I(t) = e \sum_{j=1}^{N} \delta(t - t_j) \quad \text{with } t, t_j \in [-T/2, T/2],$$

$$\left(\frac{\partial}{\partial z} + i\Delta\nu k_u \right) E_\nu(z) = -\frac{e k_1 K [\mathrm{JJ}]_h}{8\pi\sigma_x^2 \epsilon_0 \gamma_r} \frac{1}{2\pi} \sum_{j=1}^{N_e} e^{-i\nu\theta_j(z)}.$$

Stochasting cooling

$$\left(\frac{g}{N_s} \sum_s x_i \right)^2 = g^2 \left(\langle x \rangle_s \right)^2 \to \frac{g^2}{N_s} x_{rms}^2$$

Collective cooling

$$\mathcal{E}_{\text{total}}(\zeta) = E_o \mathrm{Im} \Big[X \sum_{i,\text{hadrons}} K(\zeta - \zeta_i) e^{ik(\zeta - \zeta_i)} - \sum_{j,\text{electrons}} K(\zeta - \zeta_j) e^{ik(\zeta - \zeta_j)} \Big].$$

There is nothing wrong with this approach, but...

Summation over discreet particles

- It is inconsistent with the fluid treatment of the beam media
- It becomes complicated in difficult problems and is prone to errors

Justification for this treatment comes from the so called Klimontovich microscopic distribution function

$$f_M(\mathbf{r},\mathbf{p},t) = \sum_{n=1}^{N_e} \delta(\mathbf{r}-\mathbf{r}_n(t))\delta(\mathbf{p}-\mathbf{p}_n(t))$$

Summation over macroparticles is natural in computer codes. It is less natural in theoretical work.

One can also treat fluctuations in the beam through the formalism of the higher-order distribution functions[1]:

$$f_2(\mathbf{r}_1,\mathbf{p}_1,\mathbf{r}_2,\mathbf{p}_2,t) = f_1(\mathbf{r}_1,\mathbf{p}_1,t)f_1(\mathbf{r}_2,\mathbf{p}_2,t) + g_2(\mathbf{r}_1,\mathbf{p}_1,\mathbf{r}_2,\mathbf{p}_2,t)$$

[1] O. A. Shevchenko and N. A. Vinokurov, NIM, **507A**, 84 (2003).

Fluctuations in the kinetic equation

There are simple and powerful methods of treating fluctuations with distribution functions: Landau and Lifshitz, *Physical Kinetics*, vol. 10:

§ 19. Fluctuations of the distribution function in an equilibrium gas

The distribution function determined by the transport equation, denoted in §§ 19 and 20 by $\bar{f}$, gives the mean numbers of molecules in the phase volume element $d^3x\,d\Gamma$; for a gas in statistical equilibrium, $\bar{f}(\Gamma)$ is the Boltzmann distribution function f_0 (6.7), independent of time and (if there is no external field) of the coordinates $\mathbf{r}$. It is natural to consider the fluctuations of the exact microscopic distribution function $f(t, \mathbf{r}, \Gamma)$ as it varies with time in the motion of the gas particles under their exact equations of motion.‡

We define the *correlation function* of the fluctuations as

$$\langle \delta f(t_1, \mathbf{r}_1, \Gamma_1)\delta f(t_2, \mathbf{r}_2, \Gamma_2)\rangle, \tag{19.1}$$

†See K. Kawasaki and I. Oppenheim, *Physical Review* **139**, A1763, 1965.
‡This topic was first discussed by B. B. Kadomtsev (1957).

In many problems of the beam physics we need this technique applied to ideal gas only (an ensemble of non-interacting, non-correlated particles).

Correlation functions in 1D beam model

Denote by η the relative energy deviation of a particle in the beam, $\eta = \Delta E/E_0$. Consider a beam with the averaged energy distribution $h(\eta)$ that does not depend on z (an infinitely long beam). The fluctuating distribution function is

$$f(z,\eta) = n_0 h(\eta) + \delta f(z,\eta), \qquad \langle \delta f \rangle = 0$$

where $\int d\eta h(\eta) = 1$ and n_0 is the averaged 1D density of the beam. The fluctuational part $\delta f(z,\eta)$ can be Fourier expanded

$$\delta \hat{f}_k(\eta) = \int dz\, e^{-ikz} \delta f(z,\eta)$$

For the *shot noise*, in the ideal gas approximation, according to Landau and Lifshitz,

$$\boxed{\langle \delta f(z,\eta) \delta f(z',\eta') \rangle = n_0 h(\eta) \delta(z - z') \delta(\eta - \eta')}$$

This includes both the density fluctuations and the energy fluctuations. The Fourier transform gives

$$\boxed{\langle \delta \hat{f}_k(\eta) \delta \hat{f}_{k'}(\eta') \rangle = 2\pi n_0 h(\eta) \delta(k + k') \delta(\eta - \eta')}$$

Correlation functions in 1D beam model

The density fluctuation $\delta n(z)$ is

$$\delta n(z) = \int d\eta \, \delta f(z, \eta)$$

Integrating the correlation function over η and η' gives

$$\langle \delta n(z) \delta n(z') \rangle = n_0 \delta(z - z').$$

We can formally introduce the Fourier spectrum of $\delta n(z)$

$$\delta \hat{n}_k = \int_{-\infty}^{\infty} dz \, e^{-ikz} \delta n(z) = \int_{-\infty}^{\infty} d\eta \, \delta \hat{f}_k(\eta)$$

We have

$$\langle \delta \hat{n}_k \delta \hat{n}_{k'} \rangle = 2\pi n_0 \delta(k + k') \equiv |\delta n_k|^2 \delta(k + k')$$

Here $|\delta n_k|^2$ is the spectrum of the density fluctuations.

Other correlators

We can also consider fluctuations of the integrated energy (at a given location z)

$$\delta\bar{\eta}(z) = \int \eta\, \delta f(z, \eta) d\eta$$

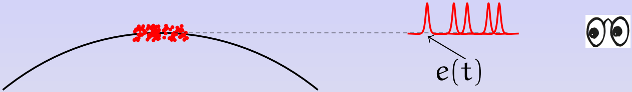
The correlation function

$$\langle \delta\bar{\eta}(z)\delta\bar{\eta}(z')\rangle = n_0 \delta(z - z')\sigma_\eta^2$$

where σ_η is the rms energy spread.

Particle interactions in relativistic beams are weak, and, for a *quiet beam*, it is usually assumed that the ideal gas approximation is a good one (at least in the range of high frequencies).

Incoherent radiation



Consider electromagnetic radiation of a particle beam. Each particle radiates its own pulse $e(t)$ of the electric field: $\int_{-\infty}^{\infty} dt\, e(t) = 0$.

What is the energy radiated by the beam?

$$W_{\mathrm{EM}} \propto \left[\sum_{n=1}^{N_e} e(t - t_n)\right]^2 \neq \sum_{n=1}^{N_e} e(t - t_n)^2$$

In general, the radiation pulses from different particles overlap, or interfere, and one cannot neglect terms $e(t - t_i)e(t - t_j)$.

The standard approach is to use the Fourier decomposition of $e(t)$ and argue that in the shot noise the phases of each harmonic from different electrons are random.

Incoherent radiation

An alternative approach: think about the beam as a charged fluid with density fluctuations. Particles are moving with velocity v; an electron located at z radiates at $t' = -z/v$

$$E(t) = \int d(vt')\,[n_0 + \delta n(vt')]e(t-t') = \int d(vt')\,\delta n(vt')e(t-t')$$

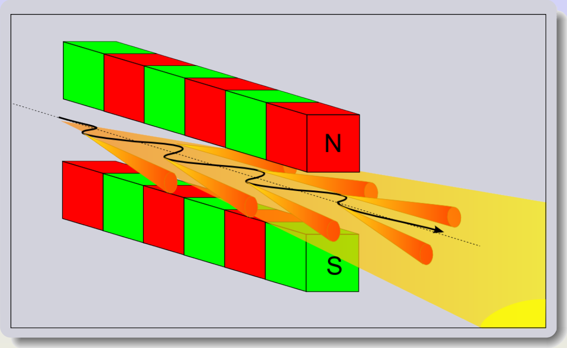
The averaged field is zero, $\langle E(t)\rangle = 0$. Calculate the averaged square $\langle E^2(t)\rangle$ using $\langle \delta n(z)\delta n(z')\rangle = n_0\delta(z-z')$,

$$\begin{aligned}\langle E^2(t)\rangle &= \int d(vt')\,d(vt'')e(t-t')e(t-t'')\langle \delta n(vt')\delta n(vt'')\rangle \\ &= vn_0 \int dt' e^2(t-t')\end{aligned}$$

The intensity of radiation is equal to the EM energy in one pulse multiplied by the number of electrons per unit time.

In FEL the correlation $\langle \delta n(z)\delta n(z')\rangle$ is different and the radiation is greatly amplified compared with the shot noise.

Bunching factor in 1D FEL



SASE FEL starts from the shot noise in the electron beam. What is the density fluctuations at the exit from the undulator of length ℓ?

Bunching factor in 1D FEL

In 1D cold-beam, linear FEL theory we calculate the amplification of an initial density modulation (assuming no initial electric field and no energy modulation)

$$\delta\hat{n}_k(\ell) = H(k)\delta\hat{n}_k^{(0)}$$

with

$$H(k) = \frac{1}{3}\exp\left(2\rho k_u \ell\left[\frac{\sqrt{3}}{2} + \frac{i}{2} - \frac{i}{3}\frac{q}{2\rho} - \frac{1}{9}\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)\left(\frac{q}{2\rho}\right)^2\right]\right)$$

where $q = (k - k_0)/k_0$, ρ is the Pierce parameter, ck_0 is the FEL fundamental frequency. We then have

$$\langle\delta\hat{n}_k(\ell)\delta\hat{n}_{k'}(\ell)\rangle = H(k)H(k')\langle\delta\hat{n}_k^{(0)}\delta\hat{n}_{k'}^{(0)}\rangle = 2\pi n_0\delta(k+k')|H(k)|^2$$

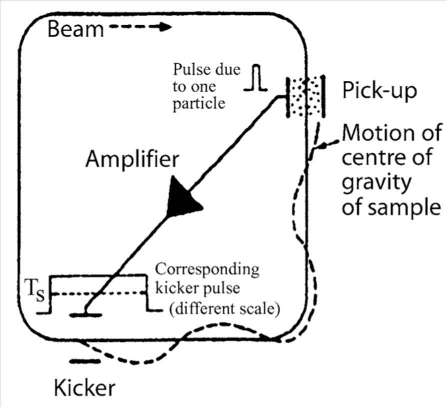
Hence $\langle\delta\hat{n}_k(\ell)\delta\hat{n}_{k'}(\ell)\rangle = |\delta n_k(\ell)|^2\delta(k+k')$ with

$$\frac{|\delta n_k(\ell)|^2}{2\pi n_0} = \boxed{\frac{1}{9}\exp\left\{2\sqrt{3}\rho k_u\ell\left[1 - \frac{2}{9}\left(\frac{k-k_0}{2\rho k_0}\right)^2\right]\right\}}$$

$|\delta n_k(\ell)|^2$ is the spectrum of density fluctuation. Its Fourier transform gives the correlation function of the fluctuations in the beam at distance ℓ.

Stochastic cooling

For this work, van der Meer was awarded the Nobel Prize in Physics in 1984.



In this problem we need to consider an ensemble of oscillators.

Stochastic cooling

The Hamiltonian

$$H(x, p) = \frac{p^2}{2} + \omega^2 \frac{x^2}{2}$$

We assume that all the oscillators have the same frequency ω. It is convenient to work with the action-angle variables I, ϕ instead of x, p,

$$x = \sqrt{\frac{2I}{\omega}} \cos\phi, \qquad p = -\sqrt{2I\omega} \sin\phi$$

Oscillator dynamics: $I = \text{const}$ and $\phi = \omega t + \phi_0$.

Stochastic cooling

We need to know the dynamics of the distribution function of the beam entering the pick-up.
The distribution function

$$f(\phi, I, t) = f_0(I) + \delta f(\phi, I, t)$$

where the averaged distribution function $f_0(I)$ satisfies

$$2\pi \int_0^\infty dI \, f_0(I) = 1$$

We assume that when the beam enters pick up, it is represented by an ensemble of non-interacting, uncorrelated particles (ideal gas). For stochastic cooling one needs the correlator at different times

$$\langle \delta f(\phi, I, t) \delta f(\phi', I', t') \rangle = \frac{1}{N} \delta(I - I') \delta[\phi - \phi' - \omega(t - t')] f_0(I)$$

N is the number of particles in the beam

Stochastic cooling

First calculate the fluctuating offset of the beam Δx at time t

$$\Delta x = \int dI\, d\phi\, x\, \delta f(I, \phi, t) = \int dI\, d\phi \sqrt{\frac{2I}{\omega}} \cos(\phi) \delta f(I, \phi, t)$$

This offset is measured, amplified and then applied at time $t_1 > t$ to the whole beam as a kick Δp

$$\Delta p = g\, \omega \Delta x$$

where g is the amplification factor. Without the kick the distribution function at time t_1 would be $f_0(I) + \delta f(\phi, I, t_1)$; with the kick it is shifted along p by Δp:

$$f_0(x, p) + \delta f(x, p, t_1) \to f_0(x, p - \Delta p) + \delta f(x, p - \Delta p, t_1)$$

Assume that Δp is small and use the Taylor expansion in Δp.

Stochastic cooling

Consider the fluctuating part,

$$\Delta f_1 = -\frac{\partial \delta f}{\partial \phi}\frac{\partial \phi}{\partial p}\Delta p - \frac{\partial \delta f}{\partial I}\frac{\partial I}{\partial p}\Delta p$$

Note that $\langle \Delta f_1 \rangle \neq 0$.
Using the correlation function $\langle \delta f(\phi, I, t)\delta f(\phi', I', t')\rangle$ one can find

$$\langle \Delta f_1 \rangle = \frac{1}{N} g \frac{\partial}{\partial I} I f_0(I)$$

Assuming that the cooling events are repeated with the period T we can write the kinetic equation for the evolution of the averaged distribution function $f_0(I, t)$

$$\frac{\partial f_0}{\partial t} = \frac{1}{T\,N} g \frac{\partial}{\partial I} I f_0$$

Evolution of the average action (emittance) $\bar{I} = \int I\, f_0(I, t) dI$

$$\frac{d\bar{I}}{dt} = -\frac{g}{T\,N}\bar{I}$$

Stochastic cooling

There is another contribution to Δf from the non-fluctuating part of the distribution function. It comes from the second order term in the averaged distribution function (the linear term vanishes after the averaging):

$$\Delta f_2 = \frac{1}{2}\left[\frac{\partial^2 f_0}{\partial I^2}\left(\frac{\partial I}{\partial p}\right)^2 + \frac{\partial f_0}{\partial I}\frac{\partial^2 I}{\partial p^2}\right](\Delta p)^2$$

The result is

$$\langle \Delta f_2 \rangle = \frac{1}{2N} g^2 \bar{I}$$

When we combine with $\langle \Delta f_1 \rangle$ we find the following equation for $\bar{I}$

$$\frac{d\bar{I}}{dt} = -\frac{1}{T\,N}\left(g - \frac{g^2}{2}\right)\bar{I}$$

For cooling, the amplification factor g should be smaller than 2.

Noise reduction in relativistic beams

For the shot noise we have

$$|\delta n_k|^2 = 2\pi n_0$$

Can we make

$$|\delta n_k|^2 < 2\pi n_0$$

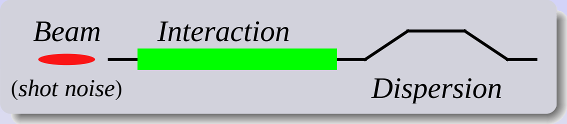
in some parts of the spectrum?

The general idea is known since 1950s from RF sources—after a quarter of plasma oscillations in an electron beam the shot noise is reduced. Noise suppression for relativistic beams was proposed by Gover and Dyunin[2] in 2009.

Possible applications include: suppression of the fundamental harmonic in favor of higher harmonics in FELs; noise suppression for seeding; application in collective electron cooling[3].

[2] Gover and Dyunin, PRL, **102**, 154801, (2009).
[3] Litvinenko, paper TUOB05 FEL 2009.

Shot noise suppression in electron beams



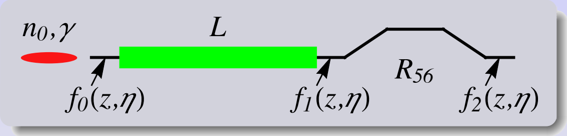
In the interaction region, the density modulation changes particles' energy through the *longitudinal wake* $w(z)$. The energy change $\Delta E(z)$ of particles at point z after passage of the interaction region is

$$\Delta E(z) = e^2 \int_z^\infty w(z' - z)\delta n(z')dz'$$

The longitudinal *impedance* is proportional to the Fourier transform of the wake, $Z(k) = \hat{w}_k/c$. In terms of $Z(k)$

$$\Delta\eta(z) = \frac{\Delta E(z)}{E_0} = \frac{r_e c}{2\pi\gamma} \int_{-\infty}^{\infty} dk Z(k)\delta\hat{n}_k e^{ikz}$$

Calculation of noise suppression



Let $f_0(z, \eta) = h(\eta) + \delta f$ be the distribution function before the interaction. After the interaction,

$$f_1(z, \eta) = f_0(z, \eta - \Delta\eta(z))$$

Sending the beam through the chicane with the dispersive strength R_{56} shift the particles longitudinally $\Delta z = R_{56}\eta$. We change the distribution function again

$$f_2(z, \eta) = f_1(z - R_{56}\eta, \eta)$$

We consider $\Delta\eta$ as a small quantity and use the Taylor expansion

$$f_2(z, \eta) = n_0 h(\eta) - n_0 \Delta\eta(z - R_{56}\eta) h'(\eta) + \delta f(z - R_{56}\eta, \eta)$$

Noise suppression

Our goal is to compute the spectum $|\delta n_k|^2$. If $|\delta n_k|^2$ becomes smaller than $|\delta n_k^{(0)}|^2 = 2\pi n_0$ then the noise is suppressed.

The result of the calculations is

$$F \equiv \frac{|\delta n_k|^2}{2\pi n_0} = 1 + 2T \operatorname{Im} Q + |Q|^2 T,$$

where

$$Q(k) = R_{56} n_0 \frac{r_e c}{\gamma} k Z(k).$$

and, for a Gaussian distribution,

$$T(k) = e^{-(kR_{56}\sigma_\eta)^2}$$

For the noise suppression we need $\operatorname{Im} Q < 0$; for $R_{56} > 0$ this means $\operatorname{Im} Z(k) < 0$. Also, T should not be small (need small energy spread, not very large k).

Noise suppression

In case of the Coulomb interaction the impedance is

$$Z_{sc}(k) = -\frac{4\pi i L}{Skc}$$

where S is the beam cross section area. This formula is valid for $\sqrt{S} \gg \gamma/k$.

For a cold beam ($T = 1$) the formfactor F is[4]

$$F = (1 - \Upsilon)^2$$

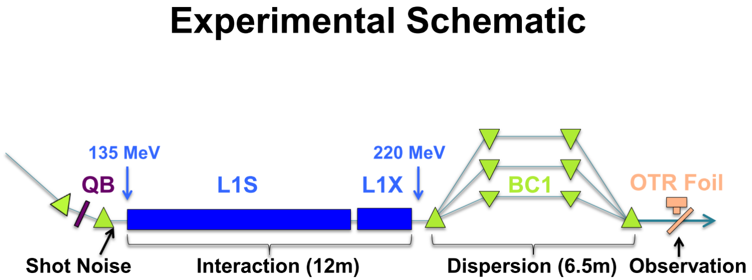
with

$$\Upsilon = n_0 R_{56} \frac{4\pi r_e L}{S\gamma}$$

[4] Ratner, Huang and Stupakov, PRSTAB, **14**, 060710, 2011.

Experiment at LCLS

Noise suppression experiment has been carried out at LCLS[5].

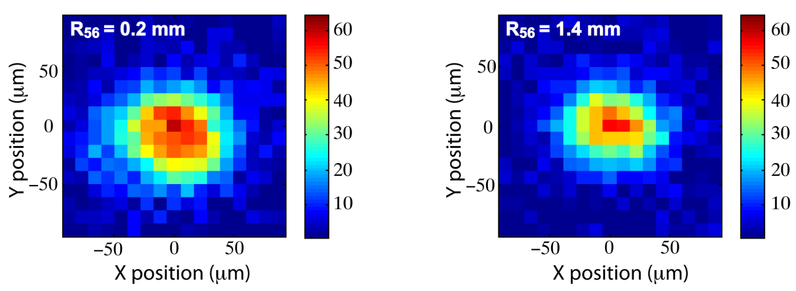


The OTR signal was observed after the BC1 chicane (no energy chirp was introduced in the beam). The intensity was measured as a function of R_{56} of the chicane. We expect a quadratic dependence of intensity of OTR versus R_{56}.
It is important to establish uncorrelated noise condition in the beam at the entrance to the system.

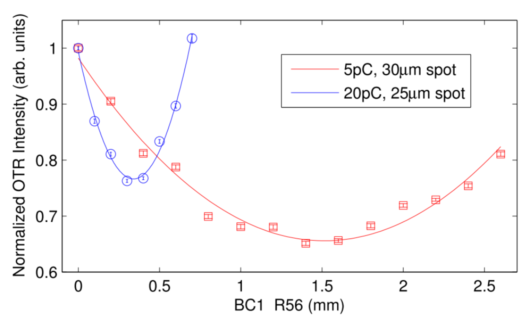
[5] Ratner, Stupakov, PRL **109**, 034801 (2012).

Noise suppression and experiment at LCLS

Beam image without noise suppression (left) and with (right).



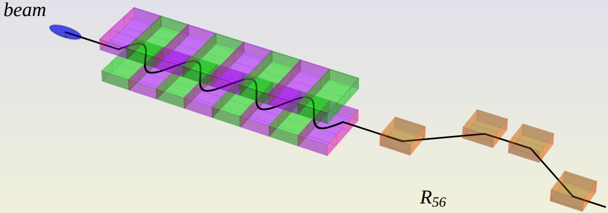
Noise suppression and experiment at LCLS



From theory we expect the minimal point to be located at $R_{56} \propto 1/Q_{beam}$ ($\Upsilon = n_0 R_{56} \frac{4\pi r_e L}{S\gamma}$). An incomplete suppression is explained by relatively large collection angle of the optics of the CCD camera and finite transverse size of the beam.

1D undulator interaction

Increasing the interaction length of the drift to make $\operatorname{Im} Z(k)$ larger would eventually violate the requirement of particles being frozen, $L \ll \pi c/2\omega_p$. Un undulator can generate a larger $\operatorname{Im} Z(k)$ than a drift, but in a limited range of k. Noise suppression with an undulator was propose in[6]



Electrons passing through an undulator interact with each other through emitted electromagnetic field (similar to CSR wake). 1D model is valid if $S \gg L_u/k$.

[6] Stupakov, Sessler, Zolotorev, Proceedings of COOL13 Conference, Mürren, Switzerland, 2013.

1D undulator interaction

Consider a helical undulator with N_u periods and the undulator parameter $K = eB/mc^2k_u$. 1D wake oscillates with the undulator radiation wavelength, $\lambda_0 = \lambda_u(1 + K^2)/2\gamma^2$:

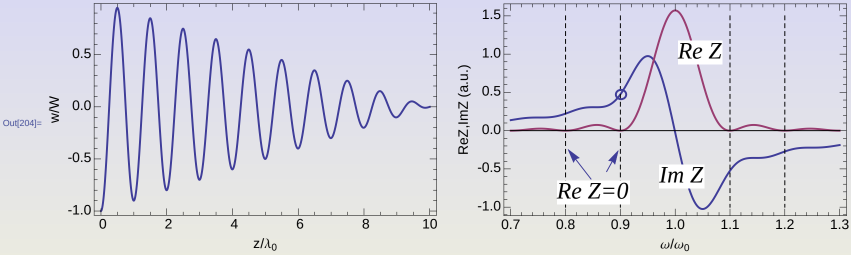
$$w_u(z) = \begin{cases} -W[1 - z/(N_u\lambda_0)] \cos k_0 z, & 0 < z < N_u\lambda_0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\lambda_0 = 2\pi/k_0$ is the wavelength of the undulator radiation,

$$W = 8\pi \frac{N_u\lambda_0\gamma^2}{S} \frac{K^2}{(1 + K^2)^2} \,.$$

1D undulator interaction

Plot of the wake and impedance for $N_u = 10$.



The maximal imaginary part of Z_u is at $\omega/\omega_0 = 1 \pm N_u^{-1}$

$$\mathrm{Im}\, Z_u \approx \pm \frac{WN_u}{2ck_0}\,.$$

1D undulator interaction

In the limit $K \gtrsim 1$, the undulator impedance is $N_u/2$ times larger than Z_{sc}. But the real part of Z_u vanishes only at particular values of ω.

Assuming for simplicity $K = 1$ we obtain for the noise factor

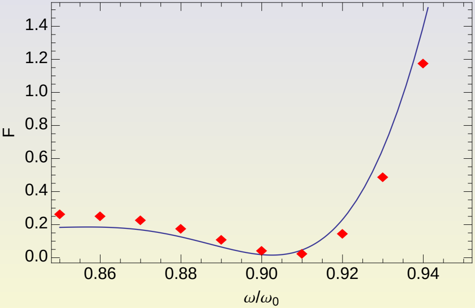
$$\min F = \left(\frac{I_A}{I} \frac{2S\sigma_\eta}{N_u^2 \lambda_0^2 \gamma} \right)^2$$

As a numerical example we consider the case: beam energy 1 GeV, $I = 1$ kA, $S = 100\ \mu\text{m} \times 100\ \mu\text{m}$, $\sigma_\eta = 10^{-4}$, $\lambda = 10$ nm, $N_u = 30$. These parameters give $\min F = 0.04$.

Increasing N_u would narrow the suppression band and lead to the FEL effects in the undulator.

Simulation

10^5 particles interact through the 1D undulator wakefield, $\Delta\eta_i = (e^2/\gamma mc^2)\sum_j w(z_i - z_j)$, and then shifted $\Delta z_i = R_{56}\eta_i$. The final bunching factor b_f is calculated as a function of frequency ω, $F = |b_f(\omega)|^2/b_0^2$.



Suppressing fluctuations driving FEL for warm beam

For and FEL with a warm beam the FEL starts not only from density, but also energy. Can we suppress this driver? The question was raised in[7]. The driver is

$$J = |\mu_0|^2 \int d\eta d\eta' \frac{\langle \delta\hat{f}_k(\eta)\delta\hat{f}^*_{k'}(\eta')\rangle}{(\mu_0-\eta)(\mu_0^*-\eta')}$$

where μ_0 is the complex frequency of the fastest growing FEL mode normalized by $2k_u c$. Note that in the limit of a cold beam, $\sigma_\eta \to 0$, the quantity J reduces to $\langle \delta\hat{n}_k \delta\hat{n}^*_{k'}\rangle$, as expected.

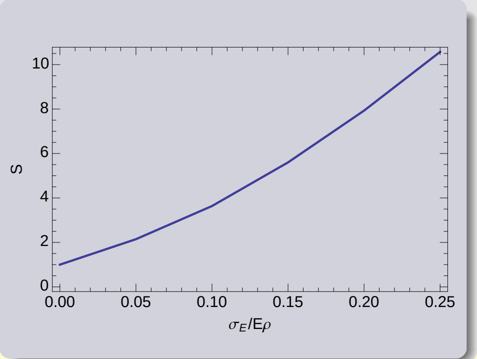
[7] K.-J.Kim, R. Lindberg, FEL 2011, (2011), p. 160.

Suppressing fluctuations driving FEL for warm beam

It turns out that suppression of the quantity J is not as effective as suppression of the density fluctuations. Define the suppression factor $F_n = J/J_0$ where J_0 is the value of J before the suppression. We find

$$\min F_n \equiv \frac{J}{J_0} \approx k^2 R_{56}^2 \sigma_\eta^2 \cdot S\left(kR_{56}\sigma_\eta, \frac{\sigma_\eta}{\rho}\right)$$

where the function S is greater than one.
Plot of S as a function of the relative energy spread for $kR_{56}\sigma_\eta = 0.1$.



Other areas where this method works

- Noise amplification in FEL seeding[8]
- Optical stochastic cooling[9]
- Coherent cooling[10]
- Other...

[8] Stupakov, FEL 2010; Stupakov, Huang, and Ratner, FEL 2010.

[9] Mikhailichenko and Zolotorev, PRL, **71**, 4146 (1993); Zolotorev and Zholents, Phys. Rev. E, **50** 3087 (1994).

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Conclusions

- Fluctuations in particle beams can be treated using the formalism of fluctuating, one-particle distribution functions. It is relatively simple, universal, and can be applied to variety of problems. It is naturally connected to the kinetic (Vlasov) equation, and allows one to take into account the interaction between the particles, collisions, radiation, etc.
- How far can it be extended?
  - 3D problems?
  - Higher-order (say, fourth order, needed for intensity correlators) correlations?
  - Extension beyond the Taylor expansion technique (stochastic cooling, large amplification g)?