Optics Design and Measurements

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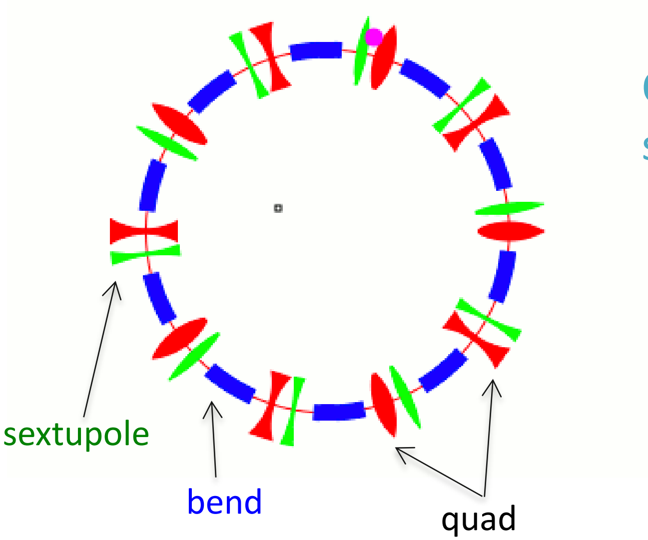
Sextupole design

How do accelerator physicists design sextupole distributions with adequate dynamic aperture? What is the process?

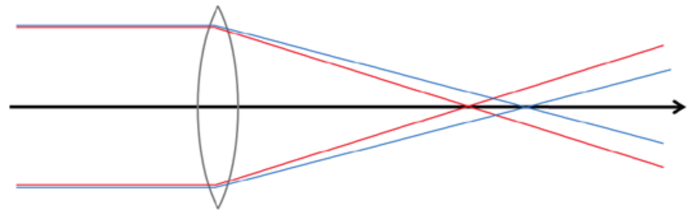
How do we characterize the distribution?

How do we measure dynamic aperture?

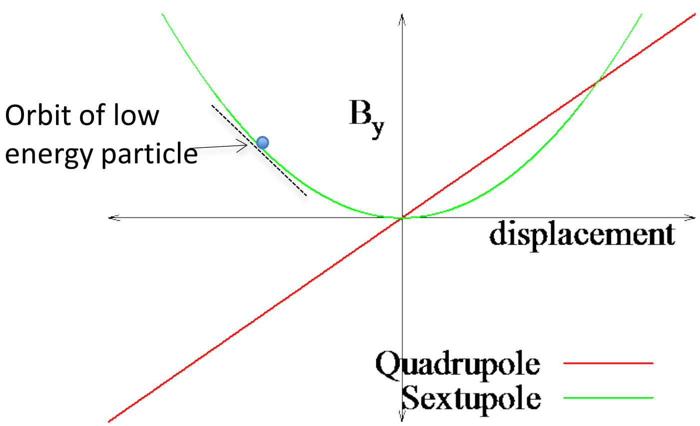
Sextupoles compensate chromaticity



Quadrupoles (linear) Alternating focus-defocus =>
stable motion : independent of amplitude



Energy dependent focusing



Sextupoles (nonlinear) compensate energy dependence of quads

$$F(s) = k_1(s)x + k_2(s)x^2$$

quadrupoles   sextupoles

Dynamic aperture

Begin with linear dynamics
Compensation of energy spread (chromaticity) with sextupoles
=> nonlinearity
Compensation of Q_x and Q_y requires just 2 sextupole values.

- Typically there are roughly the same number of sextupoles as quads
- In CESR there are 100 quadrupoles and 78 sextupoles
- All quads and all sextupoles are independently powered

There is a large number of degrees of freedom to distribution of sextupole moments

With goal of maximizing dynamic aperture.

We consider the upgrade of CESR for CHESS

Linear optics

Low emittance => bright x-ray source

What is the connection between emittance and dynamic aperture?

Radiation excitation

In the absence of any disturbance, a particle on the closed orbit will remain there and the single particle emittance is zero.

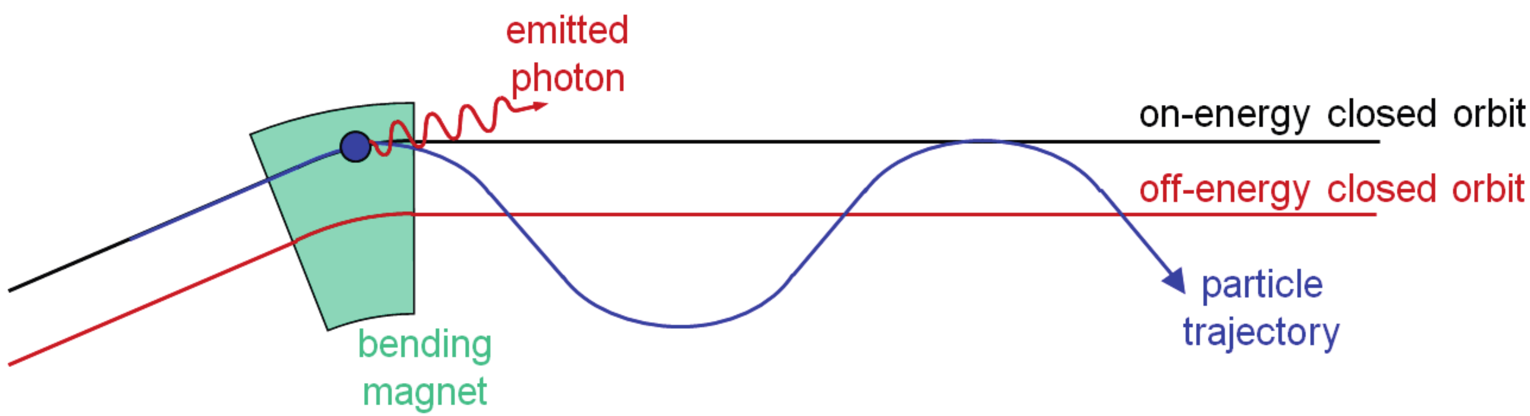
Electrons emit photons due to synchrotron radiation with some probability depending on energy and local B-field.

Photons are emitted very nearly tangent to particle trajectory $\vec{p}_\gamma \approx \vec{p}_e$

To first order, only the energy of the electron is changed. No change to transverse momentum or position. But the electron is abruptly displaced from the appropriate closed orbit by

$$\Delta\vec{x} = \frac{\delta E}{E}\vec{\eta} \quad \text{where} \quad \vec{\eta} = \frac{d\vec{x}_c}{d\delta}$$

The electron begins to oscillate about its *new* closed orbit with amplitude that depends on local η and β


emitted
photon
on-energy closed orbit
off-energy closed orbit
particle
trajectory
bending
magnet

Radiation damping

CESR parameters

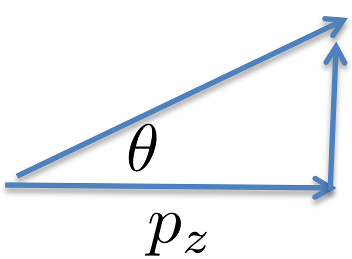
5.3 GeV Beam energy

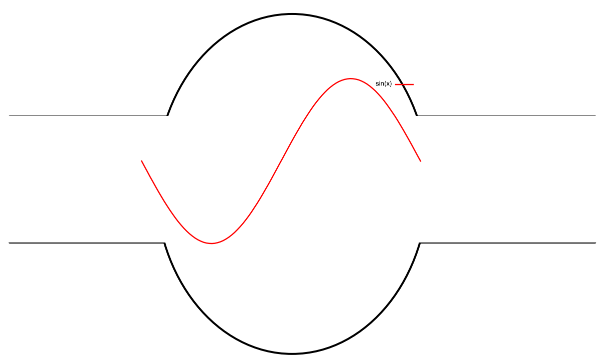
~ 800 photons are emitted/electron/turn

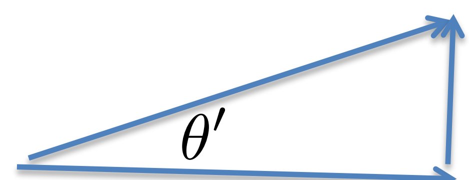
corresponding to ~1 MeV

Energy is restored by RF cavities

$$p_z = p_z + q\langle V_z \rangle$$







$$\theta' = \theta(1 - \frac{\Delta p_z}{p_z})$$

Radiation excitation

- Electrons and positrons radiate photons with some statistical distribution of energy
- In CESR ~ 1 photon/electron/meter
- Imagine particle on a closed orbit peculiar to its energy
  - With emission of a photon and energy change, it suddenly finds itself on the "wrong" orbit and begins to oscillate about the closed orbit of its new energy . . . and so on. The volume of the phase space occupied by the particle continues to grow

Damping

  - Photon, emitted tangential to particle trajectory, carries away transverse as well as longitudinal momentum
  - Only the longitudinal is restored from RF accelerating field => damping of the transverse motion

The equilibrium phase space area is the "emittance"

Equilibrium

The radiation damping time corresponds to the number of turns to radiate all of the energy – CESR at 5.3 GeV => 5300 turns (~15 ms)

Equilibrium of radiation excitation due to photon emission and radiation damping which depends on the average energy loss per turn => emittance

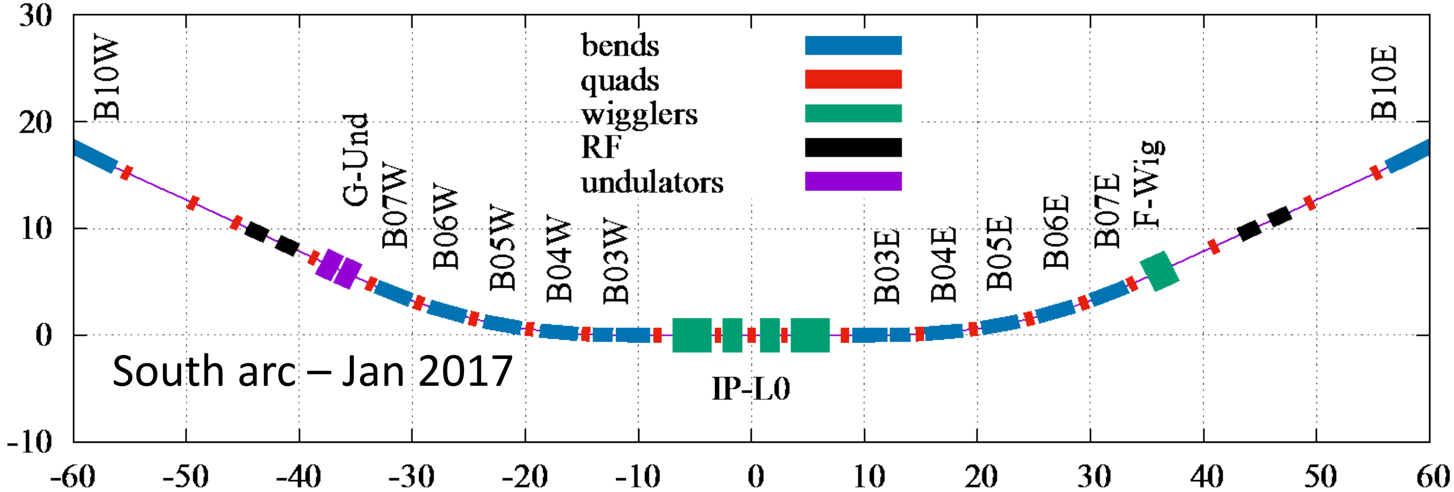
Equilibrium horizontal emittance depends on

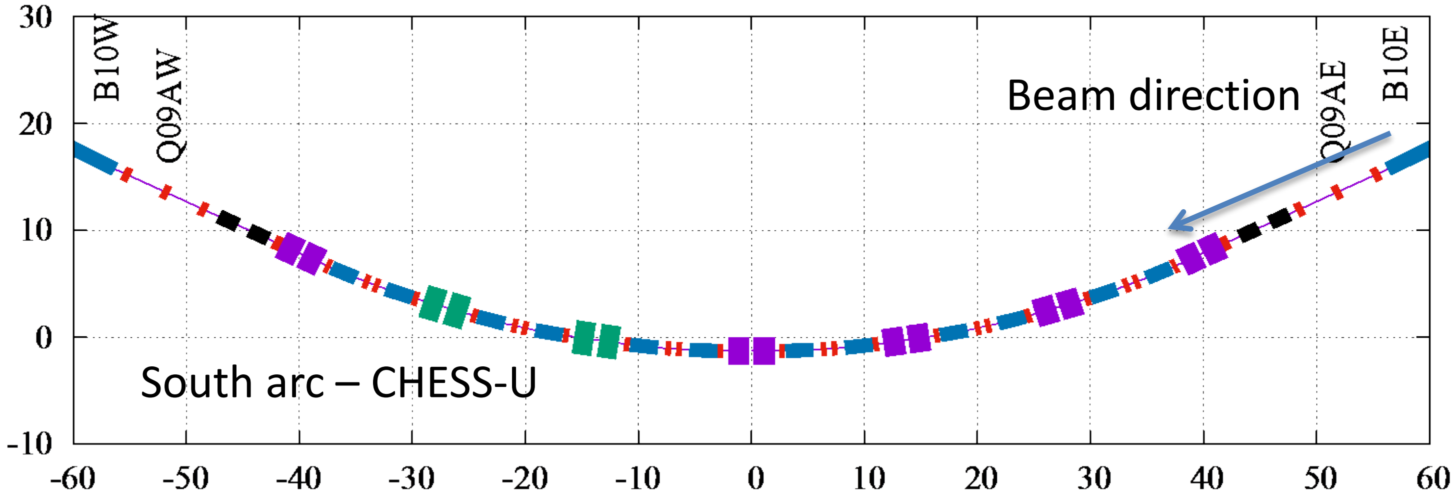
- Beam energy (number and energy of radiated photons ~ γ^2)
- Dispersion function
- Energy loss/turn

For a fixed energy and bending radius, minimize emittance by minimizing dispersion

=> very strong focusing

An example design exercise
CESR for CHESS-U

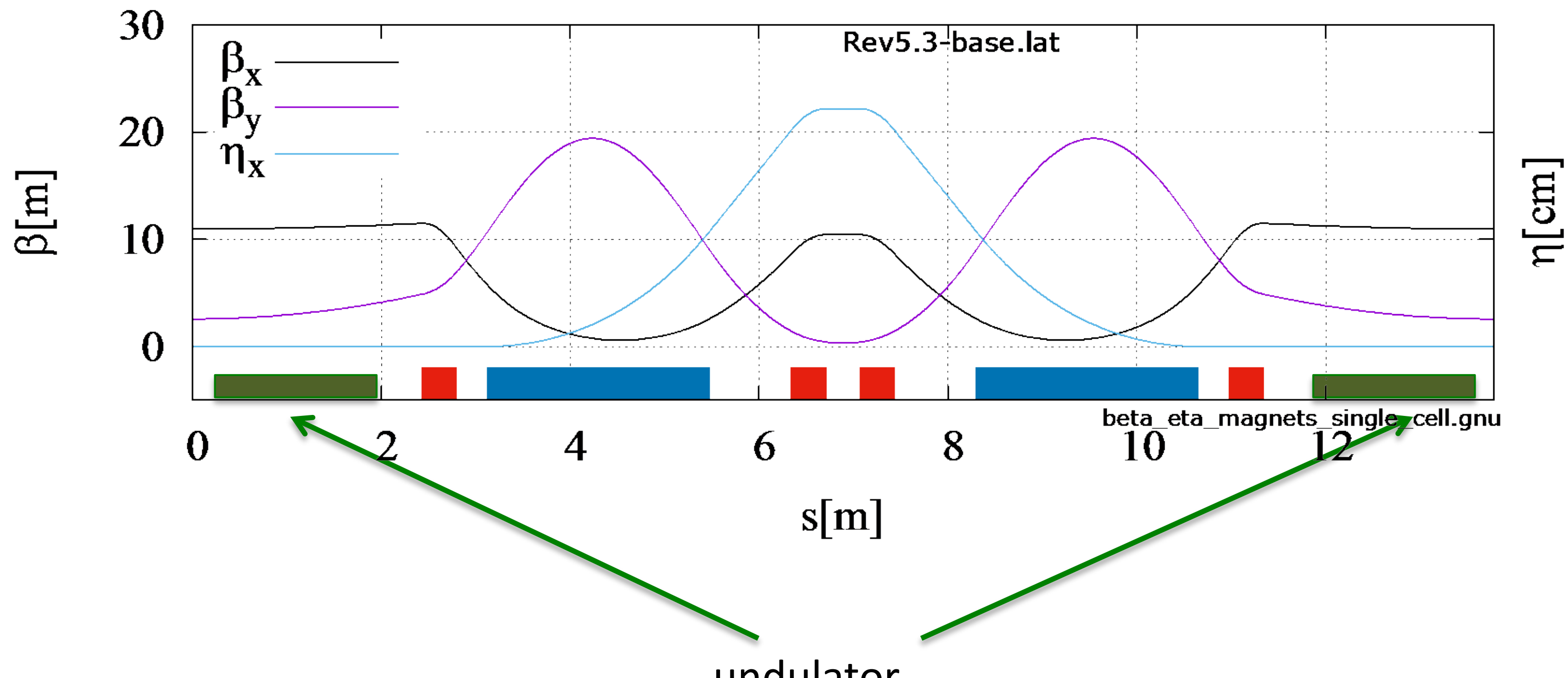




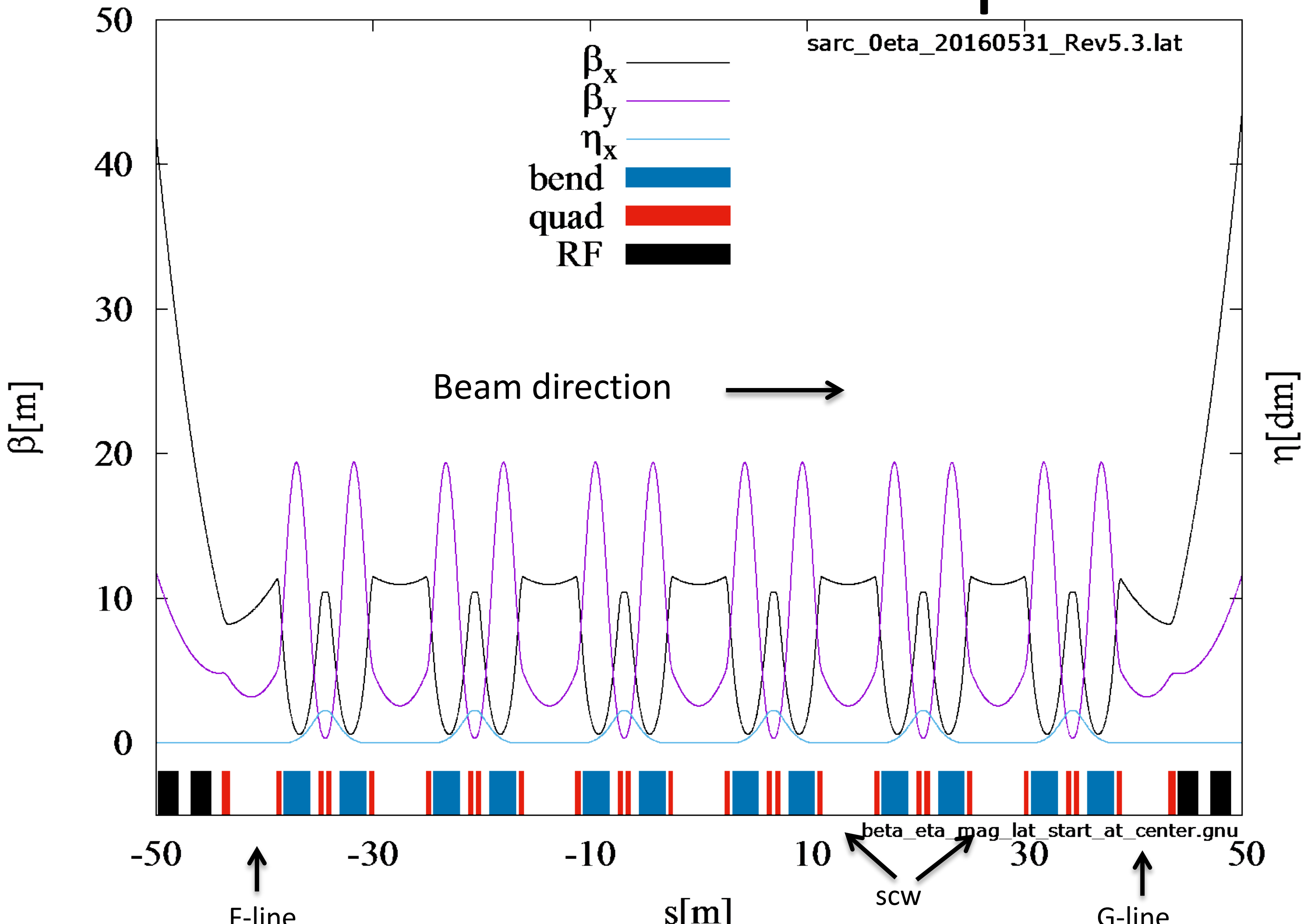
Basic cell – double bend achromat

- 4 horizontally focusing quadrupoles
- 2 combined function, vertically focusing bends
- 4.4 m zero dispersion straight

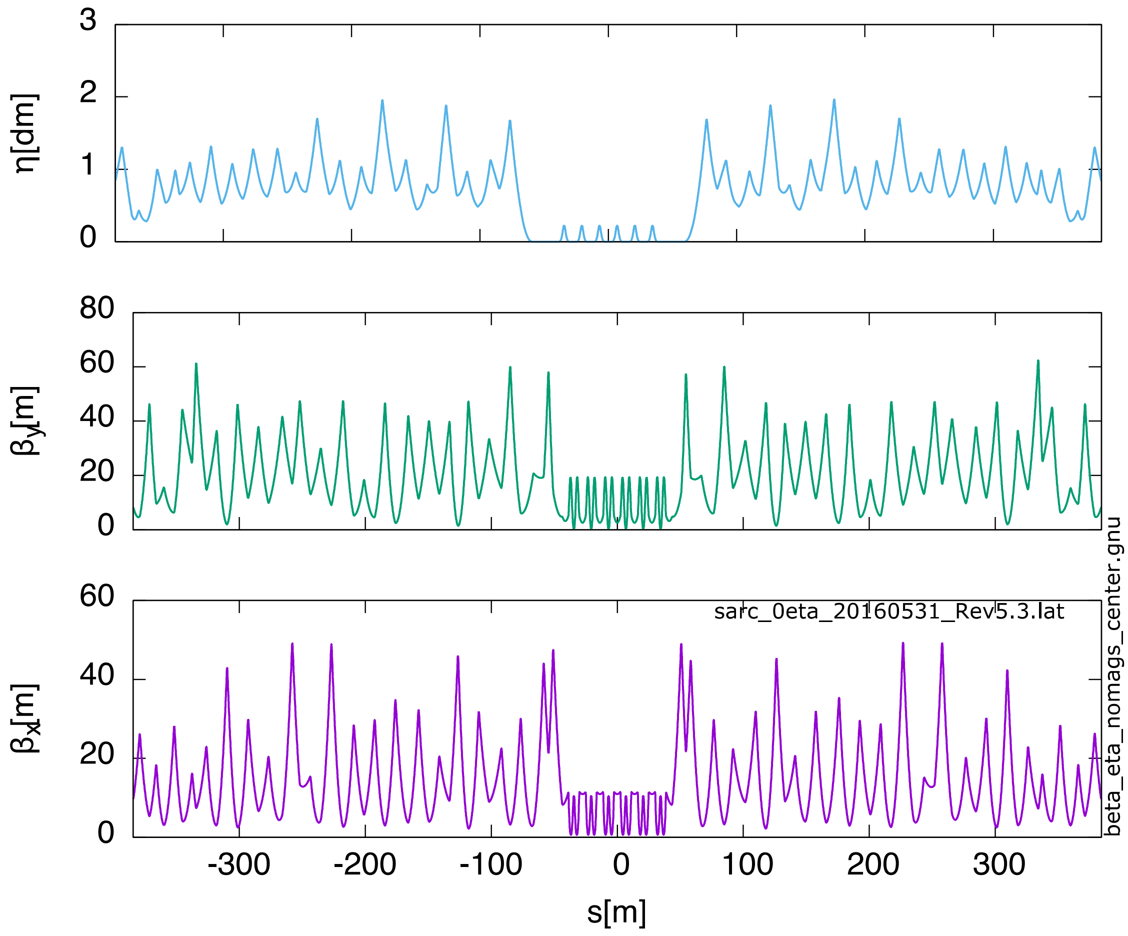
South arc (East RF to West RF) replaced by 6 DBA's



South arc CHESS-U optics



CHESS-U south arc matched into arcs

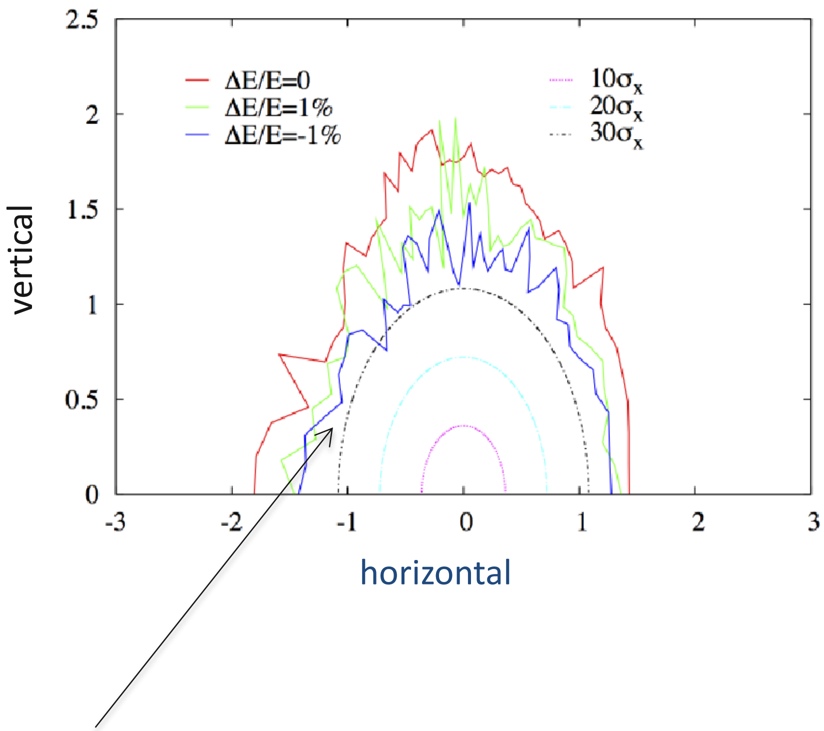


High dispersion in arcs

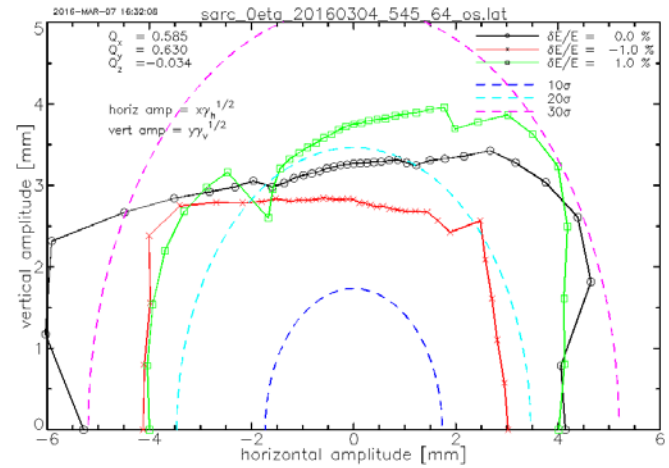
- Limits energy aperture
- Provides leverage for sextupoles
- Generates high momentum compaction

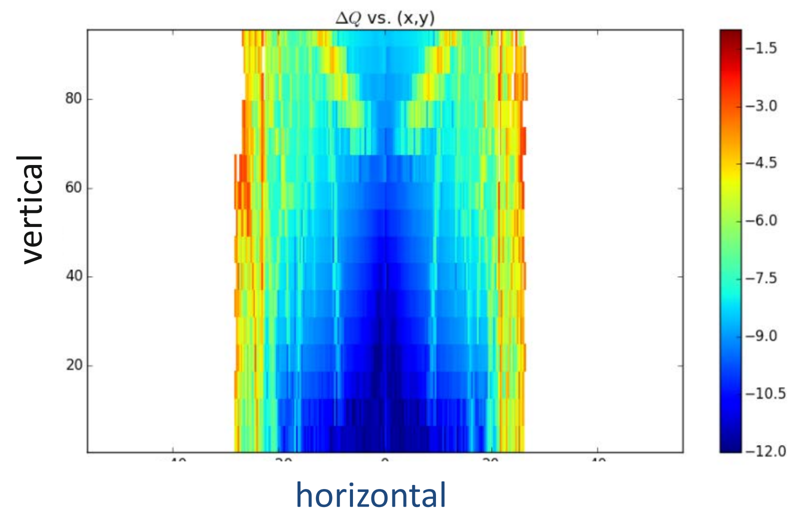
Dynamic aperture

Calculate DA by tracking



Boundary between stable and unstable motion





Tune shift vs position

Strategies for optimizing dynamic aperture

Lie Algebra or differential algebra or perturbation theory => resonance driving terms

- Minimize selection of terms
- Adjust operating point

Arrange sextupoles so that there is some cancellation, for example by spacing by ½ betatron wavelength

Compute 'distortion' functions and minimize

Compute and minimize energy dependence of β and amplitude dependence of tune

Or – compute dynamic aperture by tracking many turns (brute force) – and minimize perhaps by using genetic algorithms

Sextupole design strategy for CHESS U

Step 1. Compensate chromaticity with two families of sextupoles

We find that dynamic aperture is poor

Step 2. Maximize linear aperture

- Compute Jacobian numerically with finite $\Delta x^j_{in}(\delta)$ by mapping 5 independent phase space vectors through one turn

$$\tilde{J}_{ij}(\delta) = \frac{\Delta x^i_{out}(\delta)}{\Delta x^j_{in}(\delta)}$$

- If we manage to preserve linearity, while compensating chromaticity, then $\tilde{J}_{ij}(\delta)$ is symplectic.
- Minimize deviation from symplecticity
- Track dynamic aperture – significantly better but not good enough

Step 3. Minimize select set of resonance driving terms computed perturbatively

- Track again

How do we measure dynamic aperture?
How do we know that our model (basis of tracking) is faithful to the real accelerator?

Can we reproduce through measurement the frequency map, or the simpler maximum aperture map?

Let's begin with the more basic task of measuring the linear lattice parameters

Measurement of linear lattice

Magnet layout and magnetic fields determine linear lattice functions

- Lattice elements represented as matrices,
- Product of matrices => 1 turn linear map

$$M(s, s+C) = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\frac{1+\alpha^2}{\beta}\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

- And we extract tune, $2\pi Q = \mu,\ \beta,\ \alpha = -\frac{1}{2}\frac{d\beta}{ds}$

- Tune gives precision measurement of guide field

$$\Delta Q = \frac{1}{4\pi}\beta(s)\Delta k ds \sim \Delta k$$

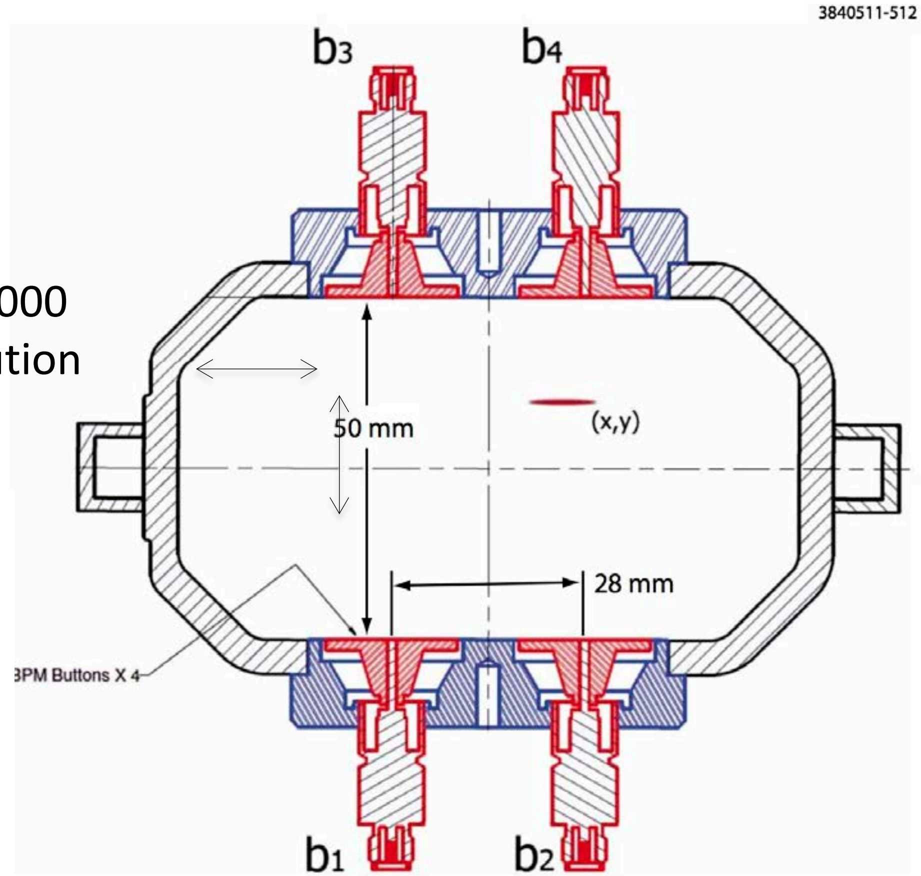
- How do we measure tune ?

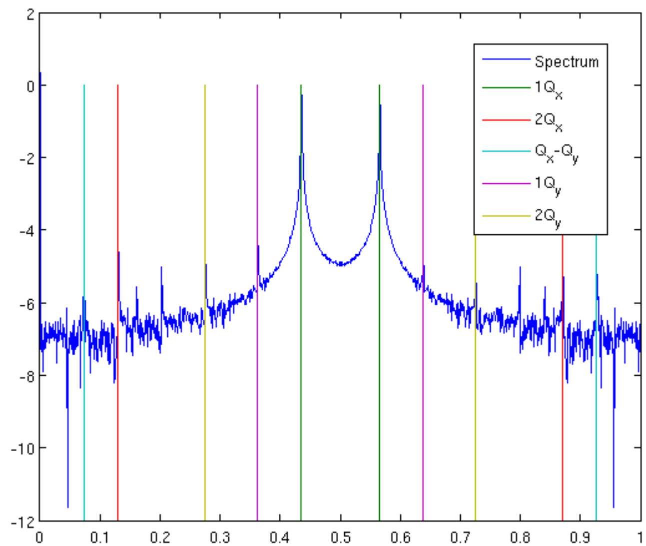
$$\beta(s) = f(k(s))$$

Measurment

Beam position monitor

Measure tune with resolution of ~1/10000
=> measure gradient with same resolution





FFT of turn by turn horizontal position

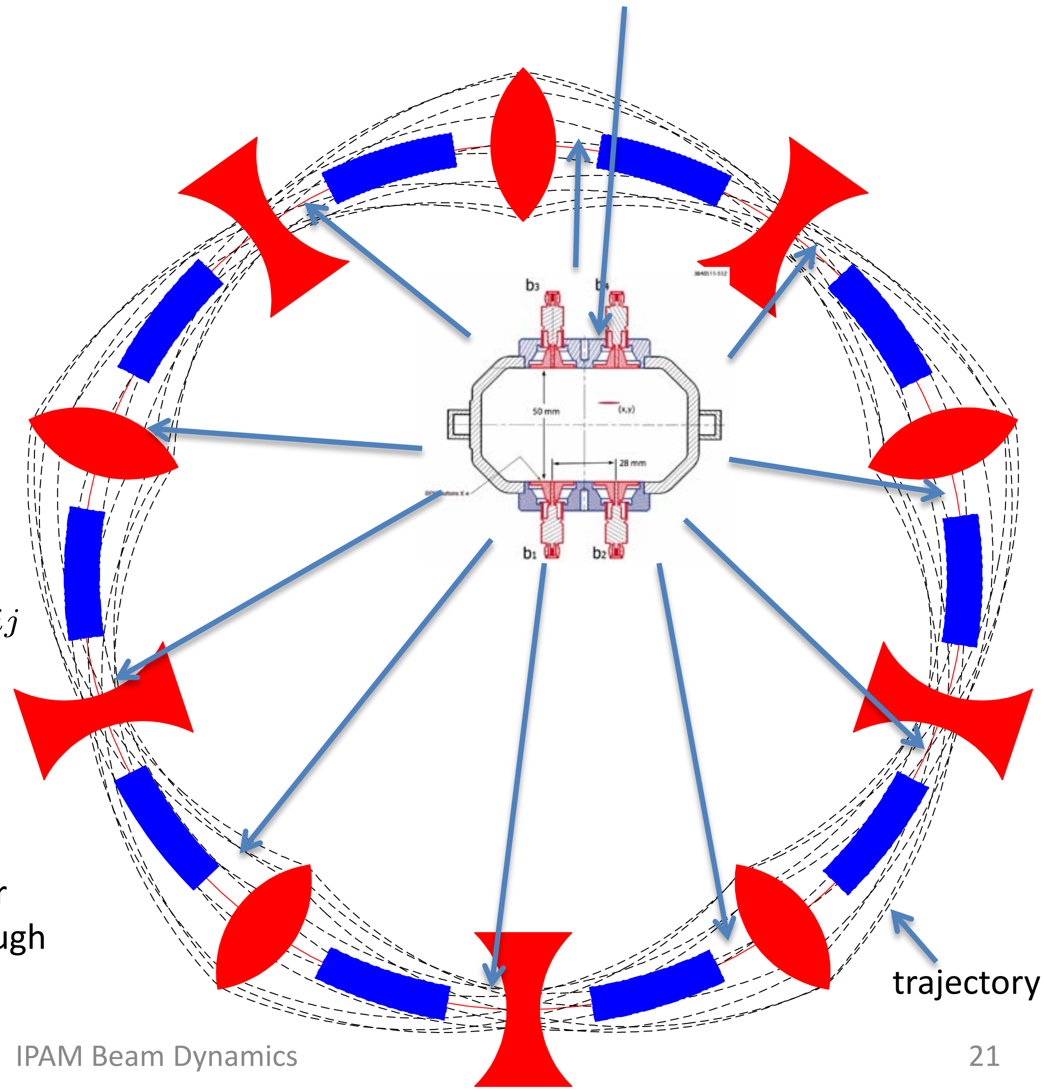
Tune is global – same at all BPMs
How do we learn about focusing locally

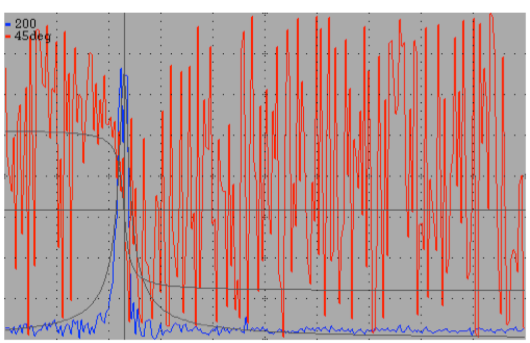
There is a Beam position monitor adjacent to every quadrupole (100 in CESR)

$$x(s) = \sqrt{a\beta(s)}\cos(\phi(s) - \phi_0)$$

$$\phi(s) = \int^s \frac{ds'}{\beta(s')}$$

- Drive beam at resonant frequencies (Q_x,Q_y) with a tune tracker – (phase locked loop)
- Measure amplitude and phase at frequency Q at each BPM
- Relative phase/amplitude => $\beta_i, \Delta\phi_{ij}$





Tune tracker sweep through resonance

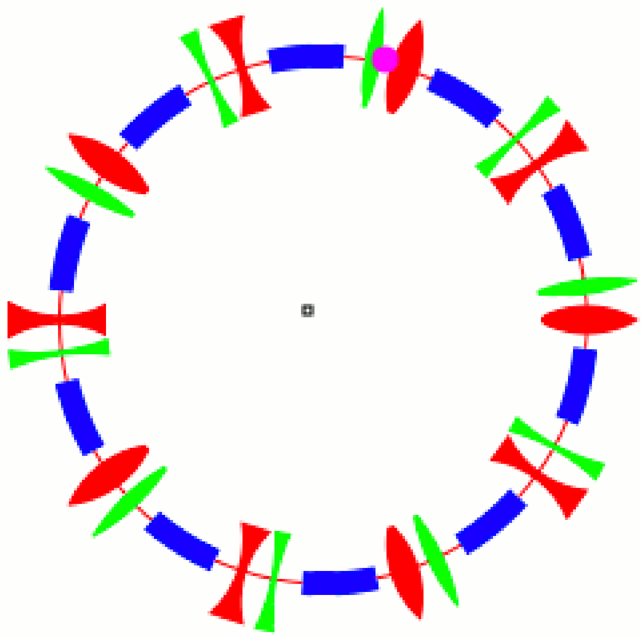
Another strategy

- Measure orbit vs dipole kickers
- Measurement of many orbits yields Orbit Response Matrix

$$R_{ij} = \frac{\Delta x_i}{\Delta k_j}$$

Sextupole moment

Energy dependence of tune (chromaticity) is a measure of global sextupole moment



$$x_0(s,n) = \sqrt{a\beta(s)}\cos(\mu n + \phi(s) - \phi_0)$$

$$\Delta x'(s,n) = \frac{1}{2}k_2 L x_0^2(s,n) = K\cos(2\mu n + 2(\phi(s) - \phi_0))$$

Sum over all previous kicks

$$z(s,n) = \sum_{i=-\infty}^{n} \Delta x'(s,i) R_{21}(n-i)$$

$$= \sum_{i=-\infty}^{n} \Delta x'(s,i)\sin(\mu(n-i))$$

$$z(s,n) = \frac{K}{4}\left[\frac{\cos(2\phi_n + 3\mu/2)}{\sin(3\mu/2)} - \frac{\cos(2\phi_n + \mu/2)}{\sin(\mu/2)}\right]$$

$$\phi_n = \mu n + \phi(s) - \phi_0$$

$$z(s,n) = A(s)\cos(2\mu n + \theta(s))$$

Measurement of amplitude and phase at twice the normal mode frequency => distribution of sextupoles

Measuring dynamic aperture

Kick beam so that it oscillates with some amplitude

- Measure tune vs amplitude
- Measure lost particles vs amplitude

Measure Injection efficiency

Measure phase space using BPM's space $\pi/2$

Complicated by decoherence, collective effects - wakes

- We measure the bunch, and not single particles

Summary

Calculating dynamic aperture is conceptually simple, computationally tedious

– Is there a better way?

Measuring dynamic aperture is difficult

- Is there a robust measureable figure of merit?

Measuring linear lattice functions is straightforward

We have a candidate technique for measuring sextupole moments