UCLA, CA, JANUARY 23-27

THREE BEAM DYNAMICS PROBLEMS

received a

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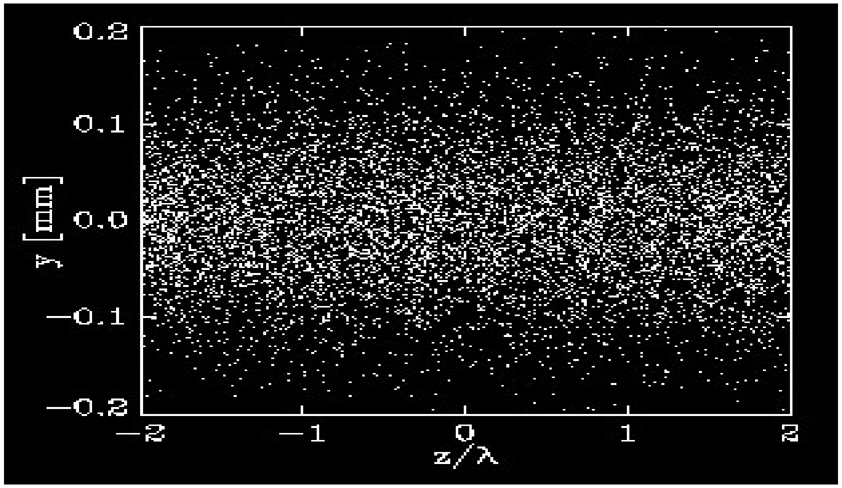
University of Chicago & ANL

1. KLIMONTOVICH DENSITY AND 1D PLASMA OSCILLATION
2. BEAM DESIGN FOR AN X-RAY FREE ELECTRON LASER OSCILLATOR
3. PROPAGATION OF PARTIALLY COHERENT X-RAY BEAM

BEAM OF DISCRETE PARTICLES



- Physical phenomena in a beam of discrete particles exhibit both collective and individual particle behavior.
- Klimontovich density, in which discrete particles are represented by delta functions, has been fruitful in discussing these phenomena.
- The case of the1D plasma oscillation is presented as a simple, illustrative example
- The FEL case will be discussed by Zhirong Huang.
- The mathematical validity of Klimontovich density will be considered by Jim Ellison
- Another approach will be presented by G. Stupakov

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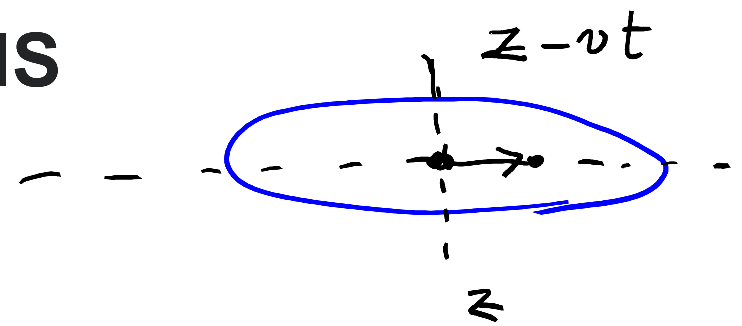
VARIABLES AND EQUATIONS



- Variables:
  - "Time" : *z*
  - Position: $\zeta = z - v_0 t$
  - Momentum: $\Delta\beta \equiv d\zeta / dz = 1 - \beta_0 / \beta = (\Delta\gamma / \gamma)(1/\beta\gamma^2)$
- Electron motion:

$$d\zeta / dz = \Delta\beta, \quad d\Delta\beta / dz = eE / (mc^2 \beta\gamma^3)$$

- $\boxed{\text{Klimontovich density: } f(\zeta, \Delta\beta; z) = \sum_i \delta(\zeta - \zeta_i(z))\delta(\Delta\beta - \Delta\beta_i(z))}$
- Continuity: $$\frac{\partial f}{\partial z} + \Delta\beta \frac{\partial f}{\partial \zeta} + \frac{eE}{mc\beta\gamma^3}\frac{\partial f}{\partial \Delta\beta} = 0$$
- Maxwell (Gauss-Poisson) equation for the longitudinal electric field *E*

$$\left.\frac{\partial E}{\partial z}\right|_t = \cancel{\frac{\partial E}{\partial z}} + \frac{\partial E}{\partial \zeta} = \frac{e}{\varepsilon_0 \Sigma_A}\int d\Delta\beta f$$

Small

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PERTURBATION SCHEME

- Decompose *f* into smooth background and the rest:
  - $f = f_0 + \hat{f}$
  - f_0: smooth background, treat as the zeroth order:

$$f_0(\Delta\beta) = n_0 g(\Delta\beta) \qquad g(\Delta\beta) = \exp(-\Delta\beta^2 / 2\sigma_{\Delta\beta}^2) / \sqrt{2\pi}\sigma_{\Delta\beta}$$

  - $\hat{f}$: high frequency part, regarded as the first order
  - Source of *E* is $\int d\Delta\beta \hat{f}(\Delta\beta; z)$ → thus *E* is of the first order
- K-M equations become linear in E and $\hat{f}$:

$$\frac{\partial\hat{f}}{\partial z} + \Delta\beta \frac{\partial\hat{f}}{\partial\zeta} + \frac{eE}{mc\beta\gamma^3}\frac{df_0}{d\Delta\beta} = 0; \quad \frac{\partial E}{\partial\zeta} = \frac{e}{\varepsilon_0\Sigma_A}\int d\Delta\beta \hat{f}$$

- Introduce Fourier transform in ζ and Laplace transform in z:

$$\hat{f}_{\omega,k}(\Delta\beta) = \int_0^\infty dz e^{i\omega z}\int_{-\infty}^{\infty} d\zeta e^{-ik\zeta}\hat{f}(\zeta,\Delta\beta,z), \quad E_{\omega,k} = \int_0^\infty dz e^{i\omega z}\int_{-\infty}^{\infty} d\zeta e^{-ik\zeta}E(\zeta,z)$$

- K-M equations become algebraic, containing the initial conditions $\hat{f}_k(\Delta\beta; 0)$. Solve them and perform the inverse Laplace transform.
- *These steps are identical to the perturbation analysis of Vlasov equations!*

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THE BUNCHING FACTOR:

- Bunching factor: $b_k(z) = \frac{1}{N_e}\int d\zeta e^{-ik}\int d\Delta\beta \hat{f}\,(\zeta, \Delta\beta; z) = \frac{1}{N_e}\sum_j \; e^{-ik\zeta_j(z)}$
- Solution: $b_k(z) = \frac{i}{2\pi N_e}\underbrace{\int_{\boldsymbol{L}} \; d\omega e^{-i\omega z}}_{\text{I. }\mathcal{L}} \underbrace{\frac{1}{\varepsilon(k,\omega)}\sum_j \frac{e^{-ik\zeta^0{}_j}}{\omega - k\Delta\beta^0{}_j}}_{b\,\omega,k}$

$$\varepsilon(k,\omega) = 1 + \Omega_P^2 \int d\Delta\beta \frac{g'(\Delta\beta)}{\omega - k\Delta\beta}$$

$$\Omega_P = \sqrt{e^2 n_0 / \varepsilon_0 m \beta \gamma^3}$$

(relativistic plasma frequency)

- The integral is the sum of residues of all the singularities.
- ***There are two classes of poles in ω:***
  - **Collective: $\boldsymbol{\omega = \omega_q;\ \varepsilon\left(k, \omega_q\right) = 0}$**
  - **Individual: $\boldsymbol{\omega = k\Delta\beta^0{}_i}$, *i* = 1,2,…*N*$_e$**

THE COLLECTIVE AND INDIVIDUAL PARTS

- We have the decomposition: $b_k(\mathrm{z}) = b_k{}^C(z) + b_k{}^I(z)$,
- Collective part: $\boldsymbol{b_k{}^C(z)} = \sum_q \boldsymbol{e^{-i\omega_q z}} \frac{\mathbf{1}}{\boldsymbol{\varepsilon'(k,\omega_q)}} \frac{\mathbf{1}}{\boldsymbol{N_e}} \sum_i \frac{\boldsymbol{e^{-ik\zeta_i^0}}}{\boldsymbol{\omega_q - k\Delta\beta^0{}_i}}$
- Individual part: $\boldsymbol{b_k{}^I(z)} = \frac{\mathbf{1}}{\boldsymbol{N_e}} \sum_i \frac{\boldsymbol{e^{-i\left(\zeta_i^0 + \Delta\beta^0{}_i z\right)}}}{\boldsymbol{\varepsilon(k, k\Delta\beta^0{}_i)}}$

- *This is the decomposition of plasma fluctuation into the collective and individual parts, discussed in the classic paper by Bohm and Pines**
- *The method here is similar as in the 1D high-gain analysis including SASE (self-amplified spontaneous emission)***

* D. Bohm and A. Pais, Phys. Rev., 85, 338 (1952)
**KJK, Nucl. Instr. Methods, A250, 396 (1986)

INCOHERENT FLUCTUATIONS DUE TO INDIVIDUAL MOTION

- $\boldsymbol{b_k}^{\,I}(\boldsymbol{z}) = \frac{\mathbf{1}}{\boldsymbol{N_e}} \sum_{\boldsymbol{i}} \frac{e^{-ik(\zeta_i^0+\Delta\beta_i^0\ \ z)}}{\varepsilon(k,k\Delta\ \ {}^{0}{}_{i})}$
- This corresponds to free motion of Debye-shielded particles (See, for example, Nicholson)
- The magnitude of the incoherent term:
- $\langle |b_k^I|^2 \rangle = \left\langle \frac{\mathbf{1}}{N_e^2} \sum_{\boldsymbol{j}} \frac{1}{|\varepsilon(k,k\Delta\beta_j^0|^2} \right\rangle + \left\langle \frac{\mathbf{1}}{N_e^2} \sum_{\boldsymbol{j \neq m}} \frac{e^{-ik(\zeta_j^0+\Delta\beta_j^0 z-\zeta_j^0-\Delta\beta_m^0 z)}}{\varepsilon(k,k\Delta\beta_j^0)\varepsilon^*(k,k\Delta\beta_m)} \right\rangle$
- The second term vanishes invoking random phase approximation and the first term can be written as

$$\langle |\boldsymbol{b_k^I}|^2 \rangle = \frac{\mathbf{1}}{\boldsymbol{N_e}} \int \boldsymbol{d\Delta\beta}\ \frac{\boldsymbol{g(\Delta\beta)}}{|\boldsymbol{\varepsilon(k,k\Delta\beta)}|^2}$$

- This can be evaluated exactly* for a Gaussian *g(Δβ)*= $const\ \exp(-\Delta\beta^2/2\sigma_{\Delta\beta}^2)$:

$$\langle |\boldsymbol{b_k^I}|^2 \rangle = \frac{\mathbf{1}}{\boldsymbol{N_e}} \frac{(\boldsymbol{k\lambda_D})^2}{1+(\boldsymbol{k\lambda_D})^2}\ ,\ \boldsymbol{\lambda_D} = \boldsymbol{\sigma_{\Delta\beta}/\Omega_p}$$ =Debye length

- *The part is not subject to plasma oscillation, and is large when* $\boldsymbol{k\lambda_D} \geq \mathbf{1}$

*N. Rostoker, Nucl. Fusion, **1**, 101 (1961)

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THE COLLECTIVE PART: PLASMA OSCILLATION

- Assume $k\lambda_D$ <<1 → $\varepsilon(k,\omega) \approx 1 - \Omega_P{}^2/\omega^2$. Then fluctuation is determined by the collective plasma oscillation:

$$\rightarrow b_k^C(z) = \frac{1}{2N_e}\sum_j e^{-ik\zeta_j^0}\left(e^{-i\Omega_P z}\frac{\Omega_P}{\Omega_P - k\Delta\beta_j} + e^{i\Omega_P z}\frac{\Omega_P}{\Omega_P + k\Delta\beta_j}\right)$$

$$\approx b_k(0)\cos(\Omega_P z) - i\frac{k}{\Omega_P}p_k(0)\sin(\Omega_P z)$$

- We also find the “collective momentum”

$$p_k^C(z) = \frac{\mathbf{1}}{N_e}\int d\Delta\beta\Delta\beta\,\hat{f}_k\,(\Delta\beta, z) = \frac{1}{N_e}\sum_j \Delta\beta_j\, e^{-ik\zeta_j(z)}$$

$$\approx -i\frac{\Omega_P}{k}b_k(0)\sin(\Omega_P z) + p_k(0)\cos(\Omega_P z)$$

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RELEVANCE TO X-RAY FEL

- The bunching factor is a seed for quasi-coherent SASE.
- However, it is also a noisy background in the harmonic generation process for producing fully coherent x-rays
- Thus there was a proposal to reduce the bunching factor taking advantage of the plasma oscillation*. (as was done for microwave device)
- Note b_k^C vanishes at z=(1/4) plasma wavelength if p_k is negligible.
- However, we find $\boldsymbol{k}\lambda_{\mathbf{D}}$ is too large for the scheme to be effective in the x-ray regime.

*A.Gover and E. Dyunin, Phys. Rev. Letts., 102, 154801 (2009); IEEE-JQE, **46**, 1511, 2010

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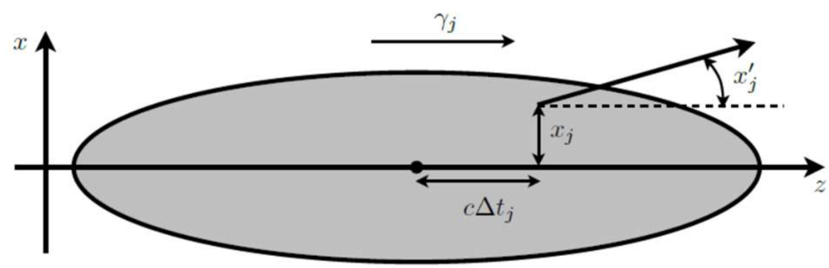
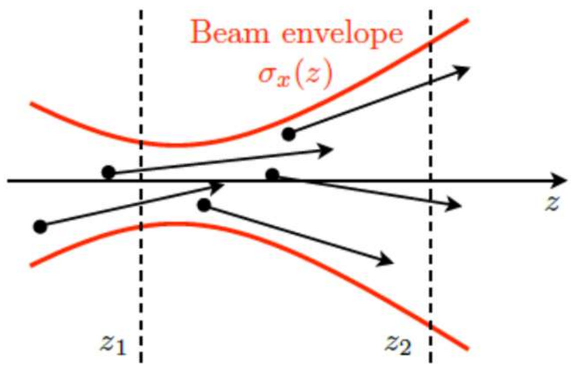
BEAM BASICS: E-BEAM





$$\sigma_x^2(z) = \langle x^2 \rangle = \frac{1}{N_e} \sum_j x_j^2.$$

$$\sigma_{x'}^2(z) = \left\langle x'^2 \right\rangle = \frac{1}{N_e} \sum_j {x'_j}^2.$$

$$\langle x\, x' \rangle = \frac{1}{N_e} \sum_j x_j x'_j$$

$$\boxed{\sigma_x(z) = \sqrt{\varepsilon_x \beta_x(z)} = \sqrt{\varepsilon_x \left(\beta_x^* + \frac{z^2}{\beta_x^*} \right)}}$$

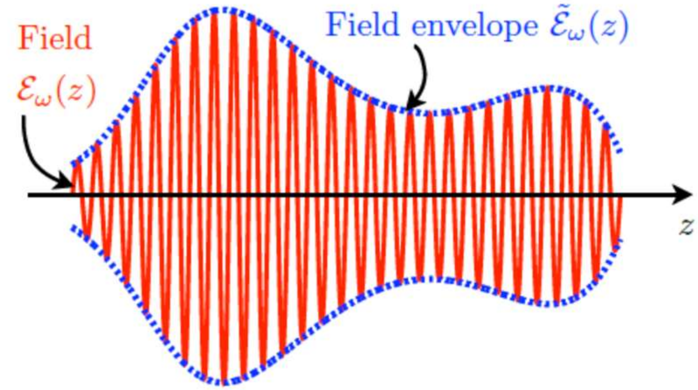
emittance

$$\boxed{\varepsilon_x \equiv \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}}$$

$$\beta_x = \frac{\langle x^2 \rangle}{\varepsilon_x} \qquad \gamma_x = \frac{\langle x'^2 \rangle}{\varepsilon_x} \qquad \alpha_x = -\frac{\langle x x' \rangle}{\varepsilon_x}$$

$$\beta_x \gamma_x - \alpha_x^2 = 1.$$

BEAM BASICS: RADIATION



$$\mathcal{E}_\omega(\boldsymbol{\phi}; z) = \frac{1}{\lambda^2} \int d\boldsymbol{x}\, e^{-ik\boldsymbol{\phi}\cdot\boldsymbol{x}} E_\omega(\boldsymbol{x}; z)$$

$$E_\omega(\boldsymbol{x}; z) = \int d\boldsymbol{\phi}\, e^{ik\boldsymbol{\phi}\cdot\boldsymbol{x}} \mathcal{E}_\omega(\boldsymbol{\phi}; z).$$

$$E(x;0) = E_0 \exp\left(-\frac{x^2}{4\sigma_r^2}\right) \qquad \mathcal{E}(\phi;0) = \mathcal{E}_0 \exp\left(-\frac{\phi^2}{4\sigma_{r'}^2}\right)$$

$$\sigma_r^2(z) = \langle \boldsymbol{x}^2 \rangle = \frac{\int d\boldsymbol{x}\, \boldsymbol{x}^2 \left|E_\omega(\boldsymbol{x}; z)\right|^2}{\int d\boldsymbol{x}\, \left|E_\omega(\boldsymbol{x}; z)\right|^2} \qquad \sigma_{r'}^2(z) = \langle \boldsymbol{\phi}^2 \rangle = \frac{\int d\boldsymbol{\phi}\, \boldsymbol{\phi}^2 \left|\mathcal{E}_\omega(\boldsymbol{\phi}; z)\right|^2}{\int d\boldsymbol{\phi}\, \left|\mathcal{E}_\omega(\boldsymbol{\phi}; z)\right|^2}$$

Radiation emittance

$$\boxed{\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi} \equiv \varepsilon_r.}$$

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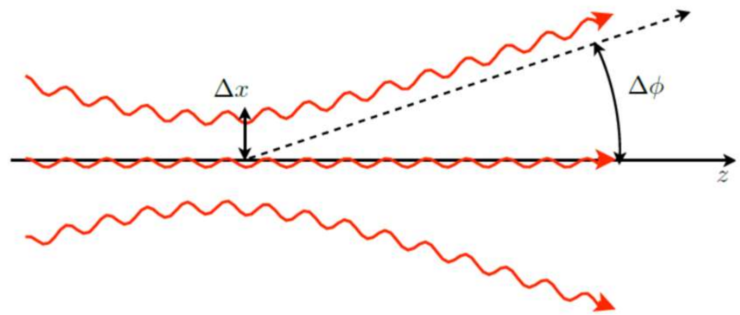
E-BEAM AND RADIATION CORRESPONDENCE



$$E_\omega(\boldsymbol{x};z) = \int d\boldsymbol{\phi}\; e^{ik\boldsymbol{\phi}\cdot\boldsymbol{x}} e^{ik(1-\phi^2/2)z} \mathcal{E}_\omega(\boldsymbol{\phi};0)$$
$$= \frac{ke^{ikz}}{2\pi i z}\int d\boldsymbol{x}'\; e^{ik(\boldsymbol{x}-\boldsymbol{x}')^2/2z} E_\omega(\boldsymbol{x}';0)$$

$$E(x;z) = \frac{E_0}{\sqrt{1+i\sigma_{r'}z/\sigma_r}} \exp\left[-\frac{x^2}{4\sigma_r^2(1+i\sigma_{r'}z/\sigma_r)}\right]$$
$$= \frac{E_0}{(1+z^2/Z_R^2)^{1/4}} \exp\left[-\frac{x^2(1-iz/Z_R)}{4\sigma_r^2(1+z^2/Z_R^2)} - \frac{i}{2}\tan^{-1}\left(\frac{z}{Z_R}\right)\right]$$

$$\boxed{\sigma_r(z) = \sqrt{\frac{\lambda}{4\pi}\left(Z_R + \frac{z^2}{Z_R^2}\right)}}$$

$$\boxed{\varepsilon_x \leftrightarrow \frac{\lambda}{4\pi} \qquad \beta_x^* \leftrightarrow Z_R}$$

For an efficient x-ray production, the e-beam emittance should satisfy $\varepsilon_x \le \lambda / 4\pi$

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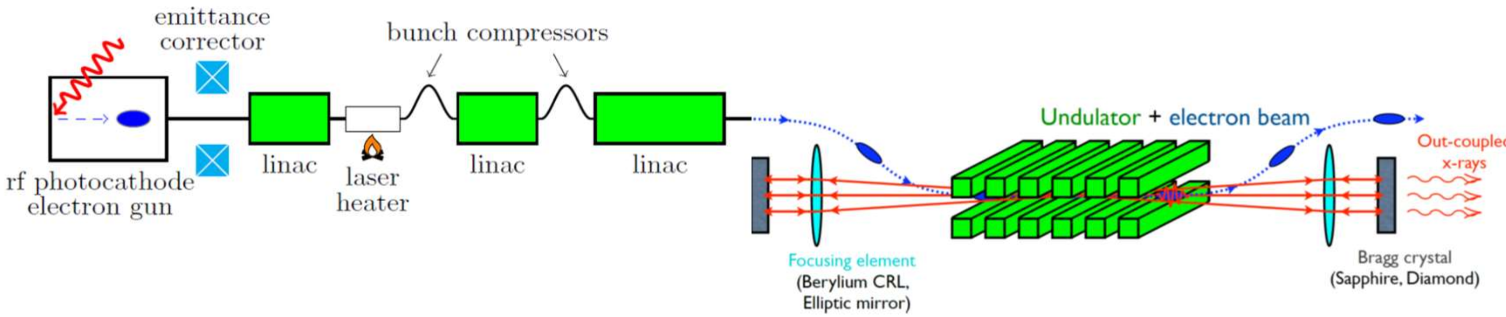
BEAM DESIGN FOR AN XFELO



- An x-ray FEL oscillator (talk by R. Lindberg) is a different device from a high-gain amplifier producing self-amplified spontaneous emission (talk by Z. Huang)
- A hard x-ray ($\lambda \leq 1$ Å) XFELO needs a superconducting accelerator producing a constant stream (~ MHz) of electron bunches with
  - Energy > 5 GeV
  - Low-emittance: $\gamma\varepsilon_x$ ~0.2 mm-mr
  - Small energy spread ($\delta E/E$ ~ 0.01% at > 4 GeV)
  - Peak current ~ 100 A (for sufficient gain)
  - Bunch length ~ 0.5 ps
  - *Energy flatness over the length of the bunch: $\Delta E/E$ ~ 0.01%*
- An XFELO out put is fully coherent, with ultra-fine spectral resolution
- ***The energy flatness requires a special beam design for an XFELO***

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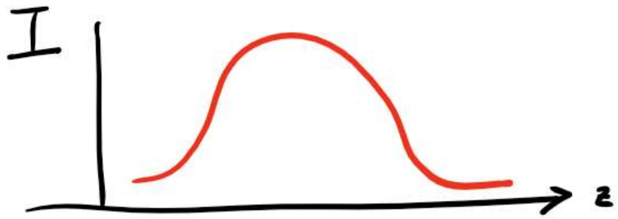
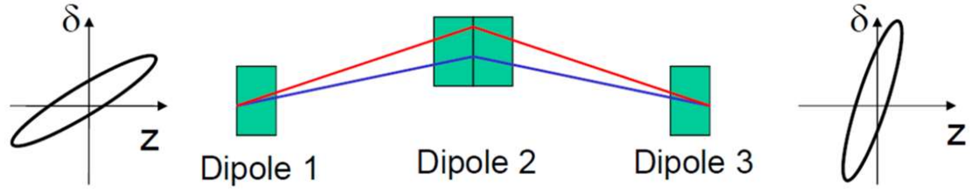
EFFECTS THAT NEED TO BE TAKEN ACCOUNT IN BEAM DESIGN

- The pulse shape of the laser driving the photocathode, thus the current profile

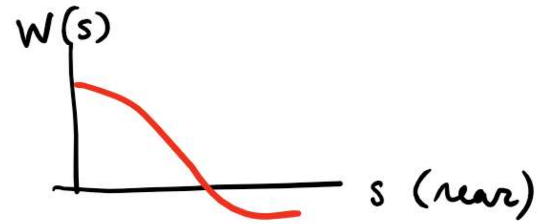


- The bunch compression process with remaining energy chirp



- The short-range wakefield in accelerator column impresses an energy variation that depends on the beam current profile (s small is head)

$$\delta(s) = \frac{\delta E(s)}{E} = -\frac{eN_e L_{acc}}{Ec} \int_{-\infty}^{s} W_L(s-s')I(s')ds'$$



- Coherent synchrotron radiation,…

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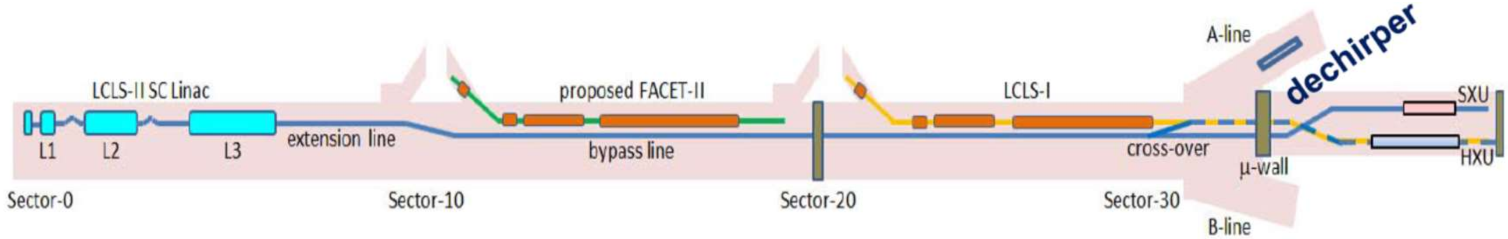
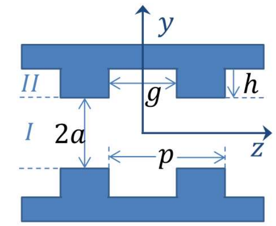
XFELO BEAM DESIGN*





- Anticipate that the current profile just before L2 should be linearly ramped**
- Assuming a parabolic profile of current at the cathode, optimize the injector (gun to L2 entrance) parameters (gun & linac phases, bunch compressor settings, etc) to obtain a current profile approximately increasing linearly toward tail
- Optimize the parameters of the L3 linac and transport to the A-line (the presumed XFELO line)
- A de-chirper, a beam pipe with a corrugated wall, to adjust the energy flatness
- Requires many beam simulation runs--LiTrack for a quick evaluation and *ELEGANT* for more accurate calculation
- Involve a large number of parameters→Use MOGA (Multi-Objective Genetic Algorithm)***.

* Weilun Qin, K. Bane, et al., IPAC 2016

** M. Cornacchia, S. Di Mitri, G. Penco, and A. A. Zholents, PRSTAB, 9, 120701 (2006)

*** First introduced for accelerator optimization by I. Bazarov

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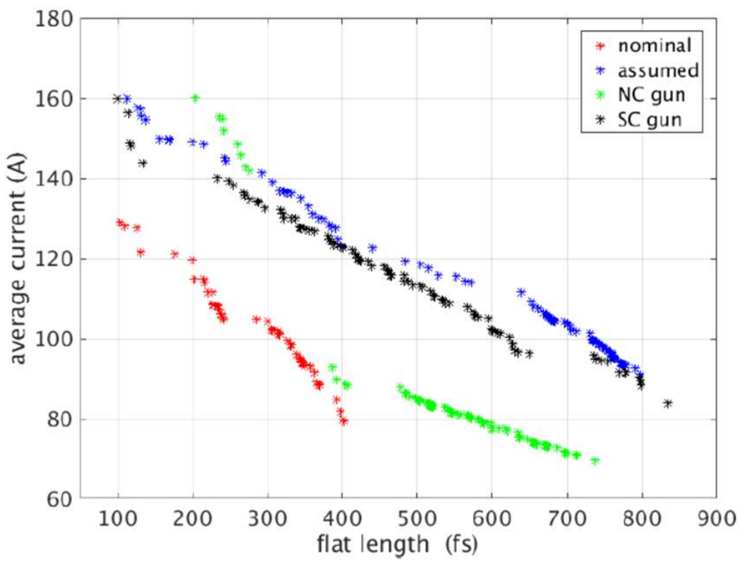
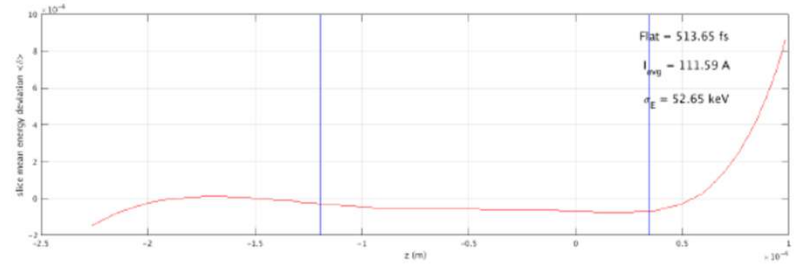
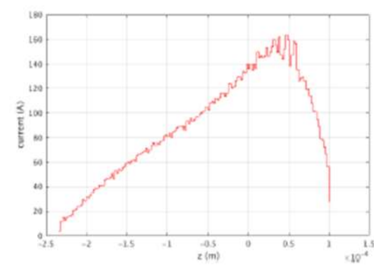
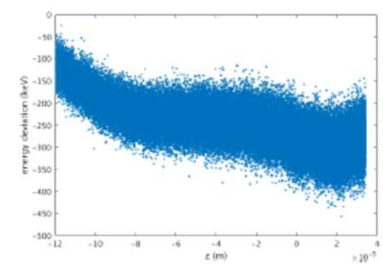
RESULTS AND QUESTION









- The energy flatness current slope meet specs for 400 fs long bunch
- Work in progress: The injector part and the main accelerator part are optimized separately
- Phase space manipulations for other applications will be discussed by Y. Sun.
- ***Question: Can we do some approximate "reverse" tracking ?***

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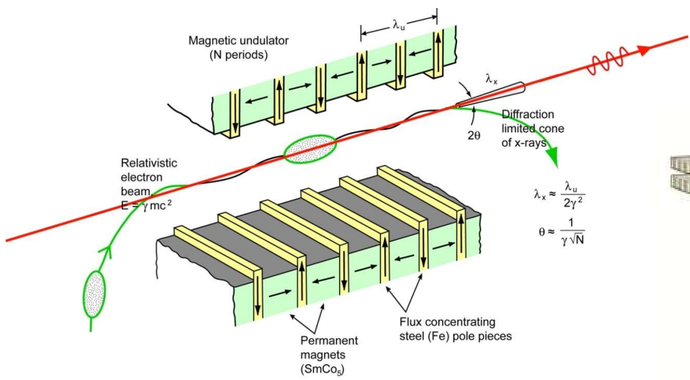
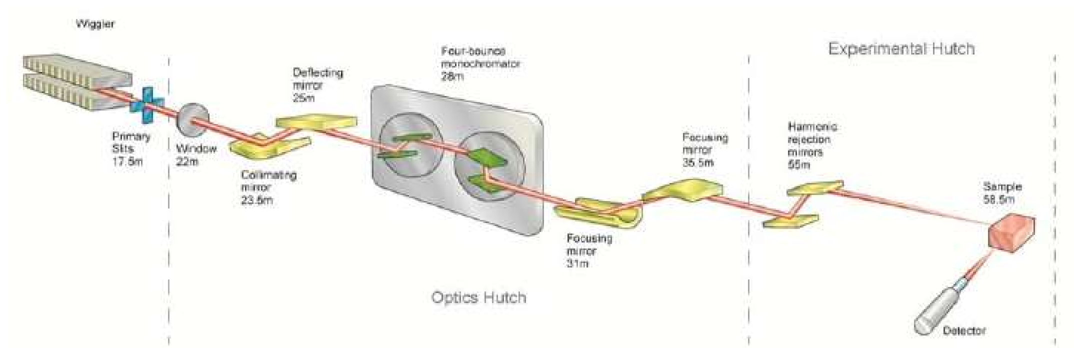
PROPAGATION OF PARTIALLY COHERENT X-RAY BEAMS

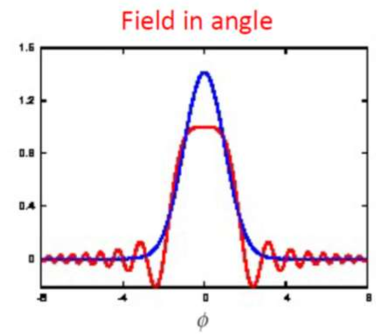


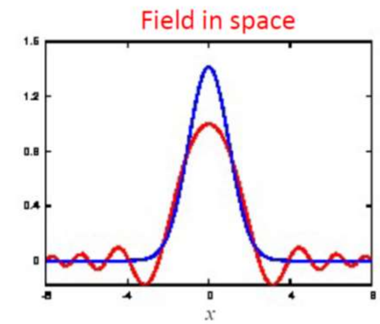


- X-ray beams from undulators are transported to samples via a set of grazing incidence flat or curved mirrors, apertures, and crystal monochromators
- Geometric optics (ray-tracing) is not adequate for partially coherent beams

UNDULATOR RADIATION

- Undulator radiation can be approximated by a coherent Gaussian radiation *after some negotiation* (R. Lindberg)



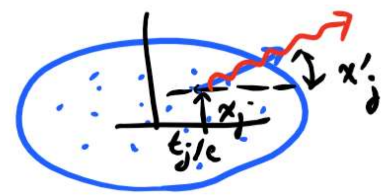


$$\sigma_{r'} = \frac{1}{2}\sqrt{\frac{\lambda_r}{L_u}} \qquad \sigma_r = \frac{\sqrt{\lambda_r L_u}}{2\pi} \qquad Z_R = \frac{\sigma_r}{\sigma_{r'}} = \frac{L_u}{\pi}$$

- Radiation from a beam is sum over contributions from each electron:

$$\mathcal{E}_\omega^{tot}(\phi) = \sum_j e^{i\omega t_j} e^{-ik\phi x_j} \mathcal{E}_\omega^{0}(\phi - x'_j)$$

$$E_\omega^{tot}(x) = \sum_j e^{i\omega t_j} e^{ik(x-x_j)x'_j} E_\omega^{0}(\phi - x'_j)$$



- The phase space density is then the convolution of the electron beam density and the radiation beam density
- The radiation beam is *incoherent*, *partially coherent, or fully coherent* if the electron beam emittance is much larger, similar, or much less than the radiation emittance $\lambda/4\pi$

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PARTIALLY COHERENT X-RAY BEAM

- Correlation function

$$\Gamma(\phi_1,\phi_2;z) = \left\langle \mathcal{E}(\phi_1)\mathcal{E}^*(\phi_2)\right\rangle_{ensemble} \equiv \int dx dx' f(x,x') e^{-i\phi_1 x}\mathcal{E}(\phi_1 - x') e^{i\phi_2 x}\mathcal{E}^*(\phi_2 - x')$$

- Brightness (Wigner density corresponding to field)

$$\mathcal{B}(x,\phi;z) = \int d\xi e^{ikx\xi}\Gamma(\phi - \xi/2, \phi + \xi/2; z)$$

- **For a linear medium -- free space and ideal lenses, the brightness transforms as in geometrical optics** (“ABCD law”)
- **A. Dragt has shown that the equivalence with particle beam transport breaks down when nonlinear elements/aberrations are present***
- Γ can be expanded in terms of a complete set of orthonormal modes

$$\Gamma(\phi_1,\phi_2;0) = \sum_n \beta_n \psi_n(\phi_1)\psi^*_n(\phi_2); \quad \int d\phi_2 \Gamma(\phi_1,\phi_2;0)\psi^*_n(\phi_2) = \beta_n \psi_n(\phi_1)$$

- The problem of transporting partially coherent beam is then solved if the propagation of coherent modes is determined.

* A. Dragt, Lie Algebraic methods for rays and wave optics, UM preprint 1994, also in 18th Advanced ICFA Beam Dynamics Workshop, Capri, Oct, 2000

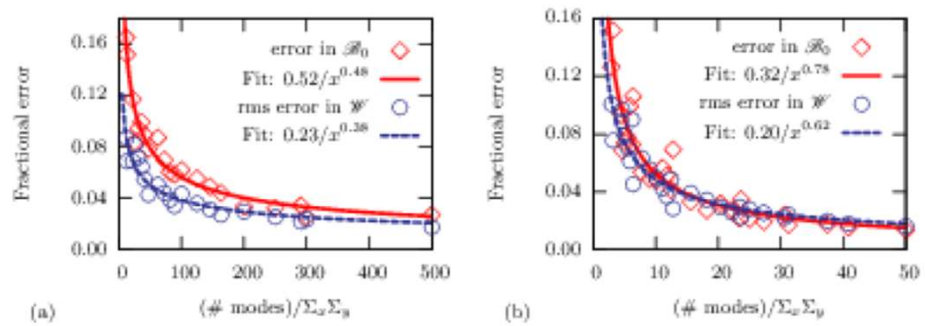
EXPANSION WITH NON-ORTHOGONAL MODES*

- However, finding the eigenvalues β_n is numerically intensive.
- However, we may interpret the discrete sum approximation in Γ as a non-orthogonal mode expansion:

$$\Gamma(\phi_1,\phi_2;z)=\left\langle \mathcal{E}(\phi_1)\mathcal{E}^*(\phi_2)\right\rangle \equiv \int dx dx' f(x,x') e^{-i\phi_1 x}\mathcal{E}(\phi_1-x')e^{i\phi_2 x}\mathcal{E}^*(\phi_2-x')$$

$$\approx \sum_n w_n \underbrace{f(x_n,x'_n)}_{\beta_n} \underbrace{e^{-i\phi_1 x_n}\mathcal{E}(\phi_1-x'_n)}_{\psi_n(\phi_1)} \underbrace{e^{i\phi_2 x_n}\mathcal{E}^*(\phi_2-x'_n)}_{\psi_n^*(\phi_1)}$$

- With a suitable sampling scheme of x_n (Halton sequence), relatively fast computation is feasible (a: random sequence, b: Halton sequence)



* R. Lindberg and KJK, PRSTAB, 18, 090702 (2015)

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PROPAGATION OF A COHERENT MODE IN THE PRESENCE OF GRAZING INCIDENCE MIRROR

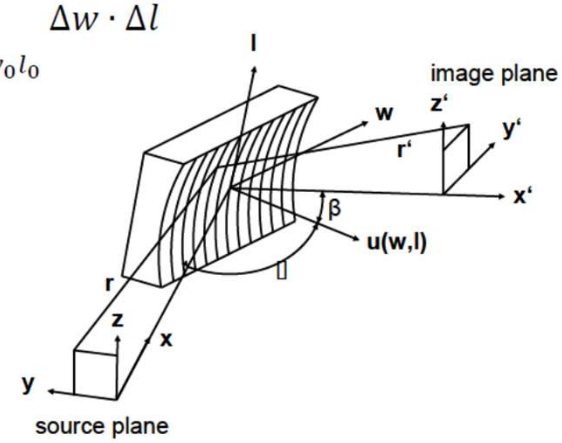
- Reflection from mirrors can be treated by Fresnel-Kirchhoff integral:

$$E(y',z') = \frac{\sqrt{\cos(\alpha)}\sqrt{\cos(\beta)}}{\lambda^2}\iint E(y,z)\left\{\iint \frac{1}{r\cdot r'}e^{ikPL}dw\cdot dl\right\}\left|\frac{\partial(y,z)}{\partial(dy',dz')}\right| ddy'\cdot ddz'$$

(Here PL= path length. Note, the integration over the source plane (y, z) is transformed to integration over the final angles (dy’, dz’))

- J. Bahrdt introduced a stationary phase approximation (SPA) in which the integration within the brackets { } is obtained by finding the point (w_0,l_0) about which the first order variations of PL vanish:

$$PL(w_0+\Delta w, l_0+\Delta l) = PL_{w_0 l_0} + \frac{1}{2}\frac{\partial^2 PL}{\partial w^2}\bigg|_{w_0 l_0}\Delta w^2 + \frac{1}{2}\frac{\partial^2 PL}{\partial l^2}\bigg|_{w_0 l_0}\Delta l^2 + \frac{\partial^2 PL}{\partial w\cdot\partial l}\bigg|_{w_0 l_0}\Delta w\cdot\Delta l$$



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ROUGH MIRRORS WITH ABERRATION

- The SPA works well when the mirror surface is smooth so that the path length PL can be represented by polynomial expansion—Bahrdt and collaborators developed computer programs dealing with up to 8th order polynomials
- Mirrors with roughness(or slope errors) are difficult to deal with. Some authors proposed to add extra phase determined with the ray tracing program
- A. Dragt reformulated the propagation through a transmitting medium via “quantal” Lie algebraic method.
- **Question: Can the Dragt formalism be extended to deal with grazing-incidence, rough mirrors?**