Controlling unfairness in traffic networks

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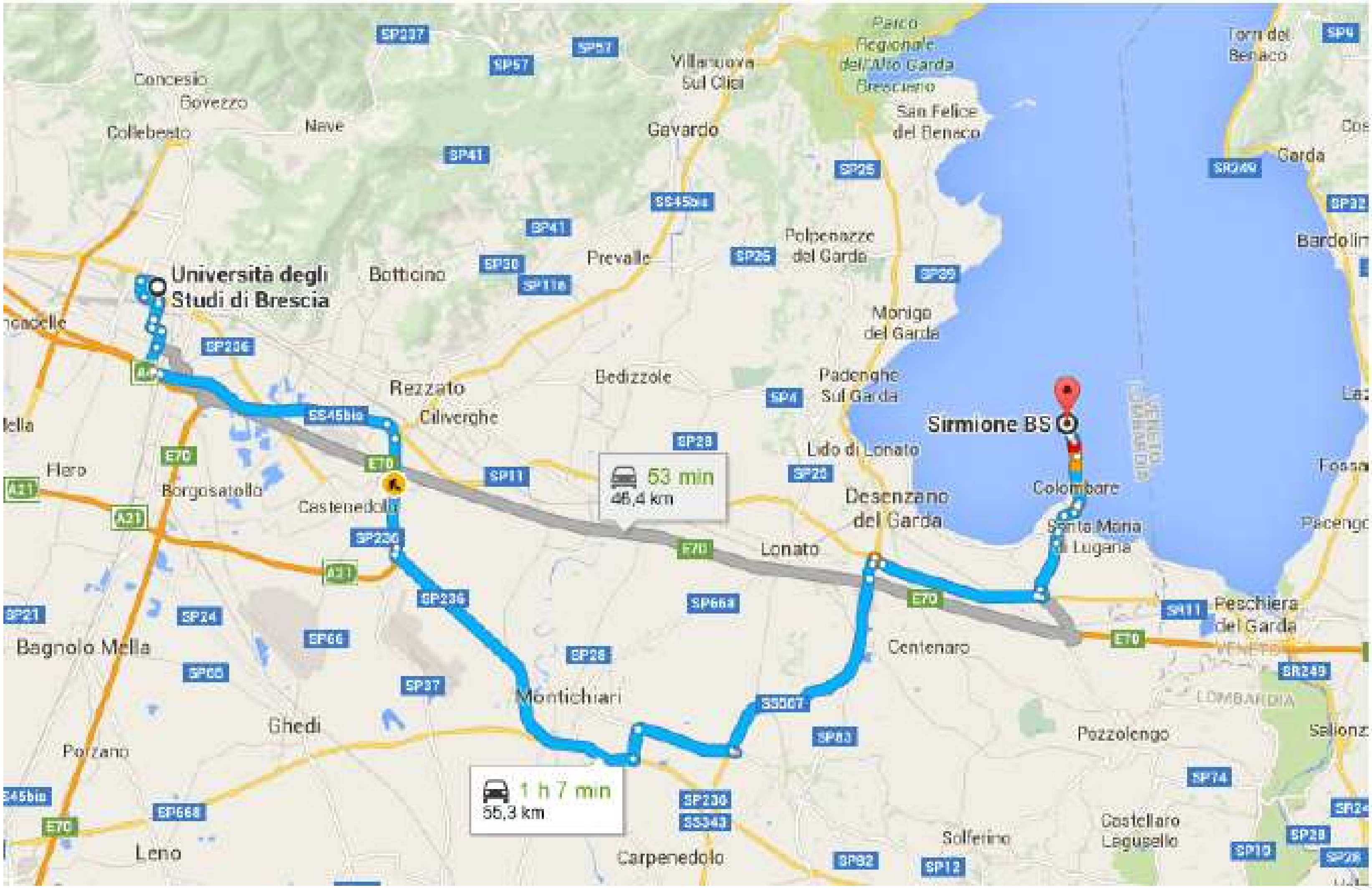
My major directions in people transportation

Car and ride sharing

On-demand services

Route guidance



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Alcatraz
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TURRO
CIMIANO
Viale Monza
SP11
CASORETTO
ISOLA
CENTRO DIREZIONALE
CHINATOWN
Politecnico di Milano
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CITY LIFE TRE TORRI
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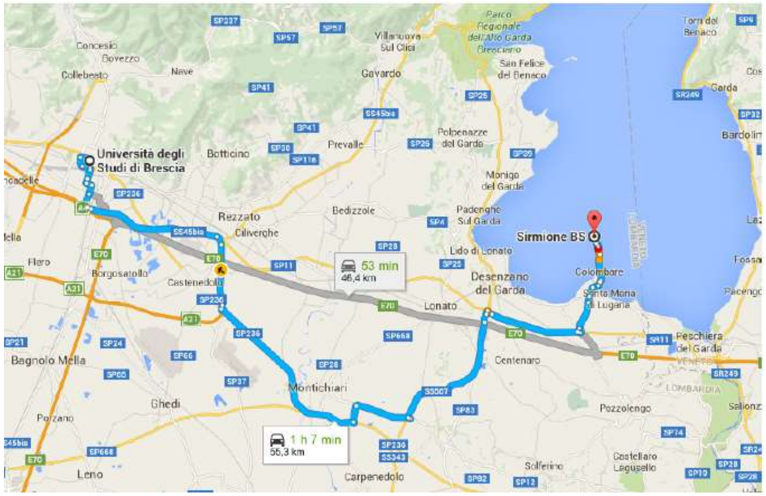
How to reach your destination….



Sat-nav	Shortest path (time/distance)	Map and O/D
Sat-nav	First k shortest paths (time)	Map, traffic and O/D

Time to think about a new generation? Coordination

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Traffic assignment approaches

Long paths for some drivers ← Minimum total travel time

User-equilibrium vs System-optimum

Selfish behaviour

(Nash equilibrium,

no one can switch to a path of smaller length)

→ Congestion

(more than necessary)

Price of anarchy

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Route guidance

The goal:

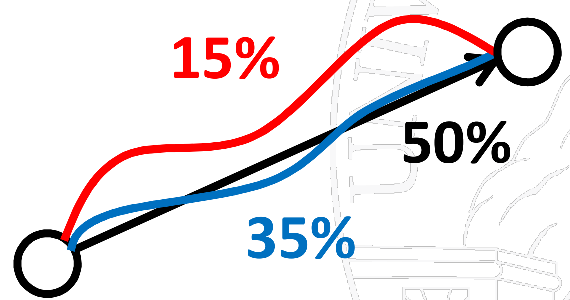
To find a traffic assignment
with controlled limited inconvenience,
and reduce the price of anarchy

Constrained approaches

Maximum allowed inconvenience

=

5% more than shortest path



The first constrained approach

Jahn, Möhring, Schulz, Stier-Moses, OR, 2005

Min Total travel time

on paths of limited length

The first constrained SO approach

$$t_{ij}^{FF}[1 + 0.15(\frac{x_{ij}}{u_{ij}})^4]$$

Travel time on arc (i,j) with flow x_{ij}

Min Total travel time

on paths of limited length

➡

Non linear optimization problem

on an exponential number of paths

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Linear constrained SO models

Angelelli, Morandi, Savelsbergh, Speranza, TR B, 2016	First linear model
Angelelli, Morandi, Speranza, C&OR, 2018	Heuristic
Angelelli, Morandi, Savelsbergh, Speranza, under revision	*Linearized classical model*
Angelelli, Morandi, Speranza, C&OR, 2019	CVaR objective
Angelelli, Morandi, Speranza, C&OR, to appear	Embedding paths

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Inconvenience (unfairness)

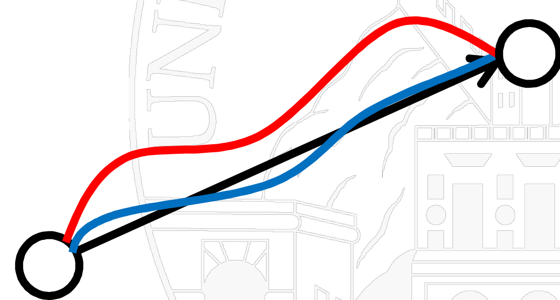
For an OD pair: concepts for inconvenience (unfairness)

Loaded inconvenience: ratio of the experienced travel time w.r.t. the fastest traveler, with current congestion level

Normal inconvenience: ratio of the length w.r.t. the shortest path

User equilibrium (UE) inconvenience: ratio of the experienced travel time w.r.t. to the travel time in a user equilibrium

Free-flow inconvenience: ratio of the experienced travel time w.r.t. the fastest path, with free-flow travel time



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Linearized classical constrained SO

$$\boxed{t_{ij}^{FF}[1 + 0.15(\frac{x_{ij}}{u_{ij}})^4]}$$

$$\min \sum_{(i,j)\in A} t_{ij}(x_{ij})x_{ij}$$

Eligible path

$$x_{ij} = \sum_{c\in C} \sum_{k\in K_c^\gamma} a_{ij}^{kc} y_{ck} \qquad \forall (i,j) \in A$$

$$d_c = \sum_{k\in K_c^\gamma} y_{ck} \qquad \forall c \in C$$

OD pair

$$x_{ij} \geq 0 \qquad \forall (i,j) \in A$$

$$y_{ck} \geq 0 \qquad \forall c \in C \quad \forall k \in K_c^\gamma.$$

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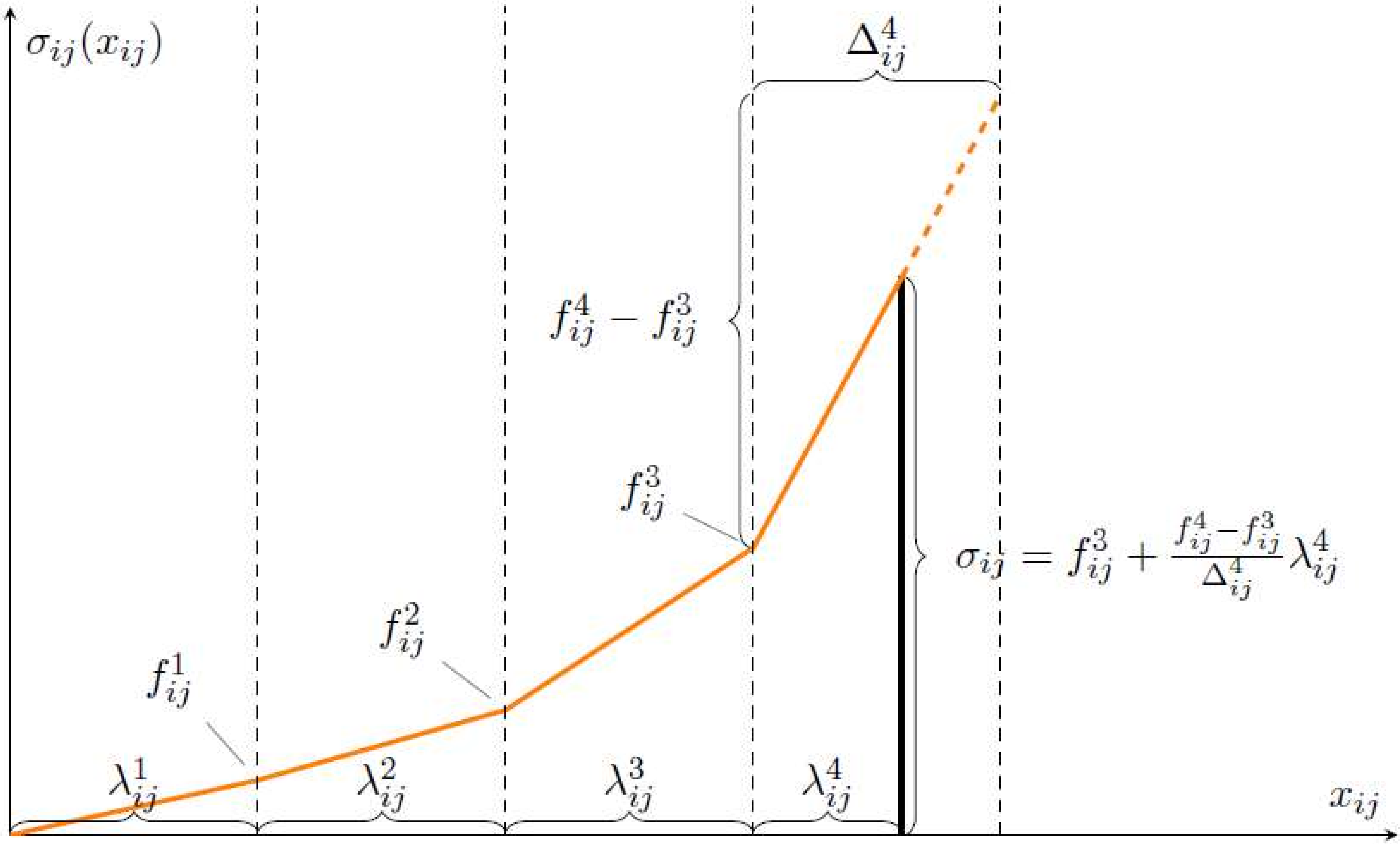
Linearized classical constrained SO



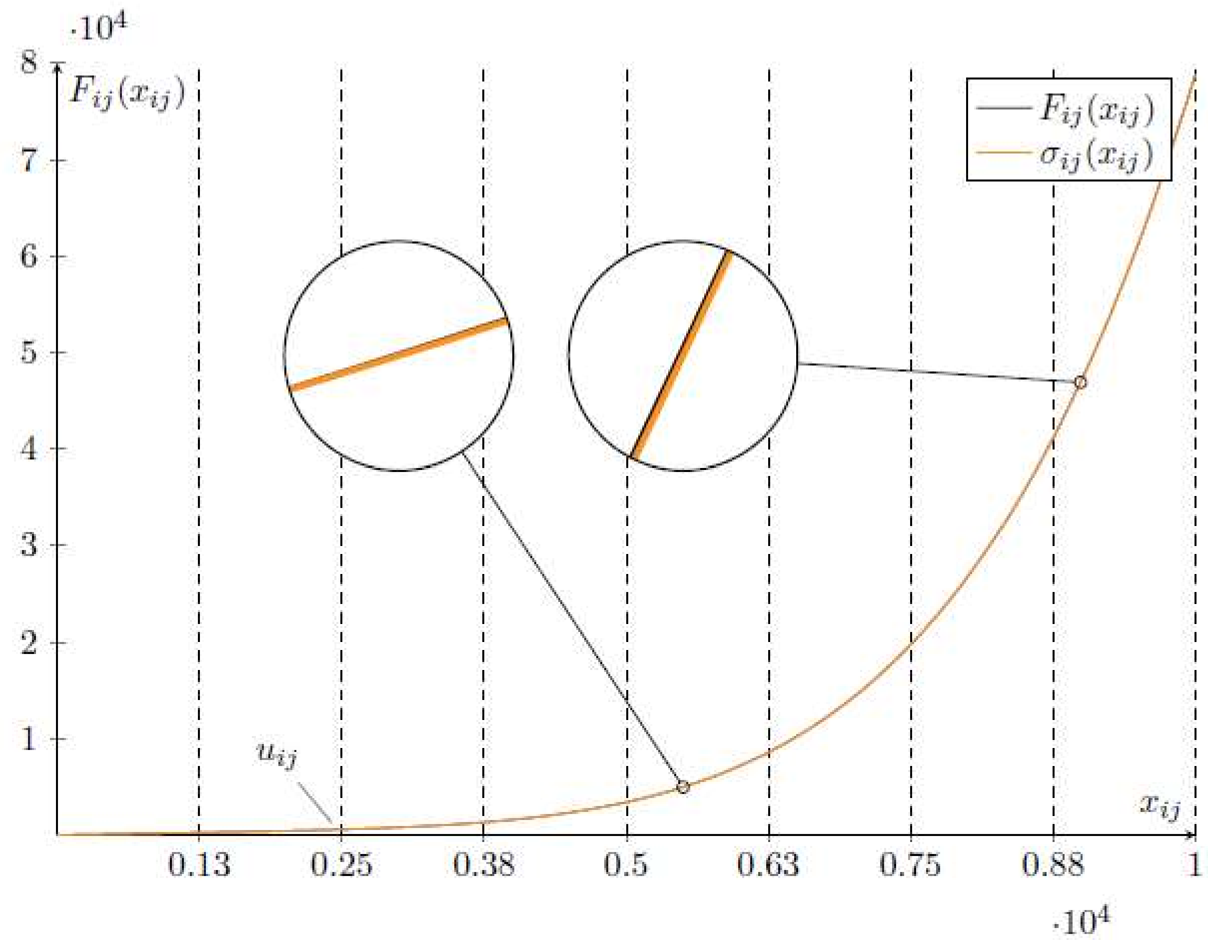
Linearized classical constrained SO

$$
\begin{aligned}
\min \quad & \sum_{(ij)\in A} \sigma_{ij} \\
& x_{ij} = \sum_{h=1}^{n} \lambda_{ij}^{h} && \forall (i,j) \in A \\
& \sigma_{ij} = \sum_{h=1}^{n} \frac{f_{ij}^{h} - f_{ij}^{h-1}}{\Delta_{ij}^{h}} \lambda_{ij}^{h} && \forall (i,j) \in A \\
& 0 \le \lambda_{ij}^{h} \le \Delta_{ij}^{h} && \forall (i,j) \in A \quad \forall h = 1, \dots, n \\
& x_{ij} = \sum_{c\in C} \sum_{k\in K_c^{\gamma}} a_{ij}^{kc} y_{ck} && \forall (i,j) \in A \\
& d_c = \sum_{k\in K_c^{\gamma}} y_{ck} && \forall c \in C \\
& x_{ij} \ge 0 && \forall (i,j) \in A \\
& y_{ck} \ge 0 && \forall c \in C \quad \forall k \in K_c^{\gamma}
\end{aligned}
$$

Eligible path

OD pair

Linearized classical constrained SO



n=1000

Linearized classical constrained SO

Heuristic column generation

Through a procedure that works on a modified network

Generate new improving paths

Solve the restricted problem

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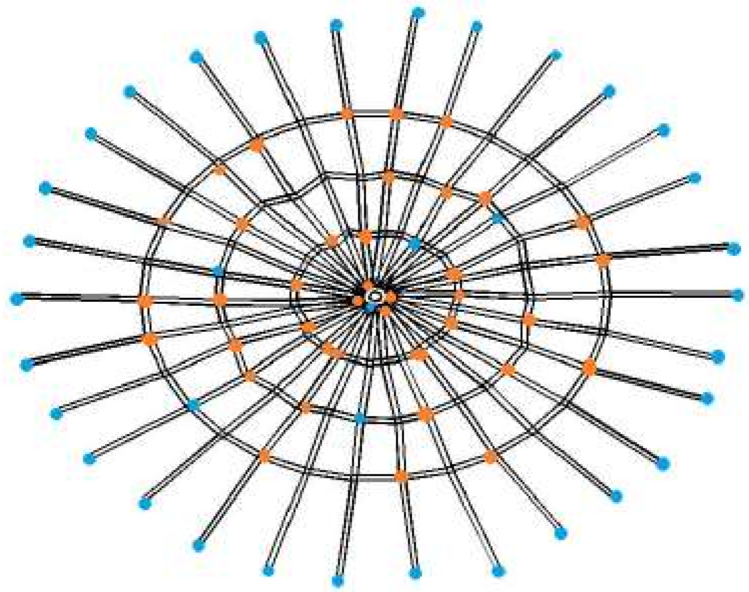
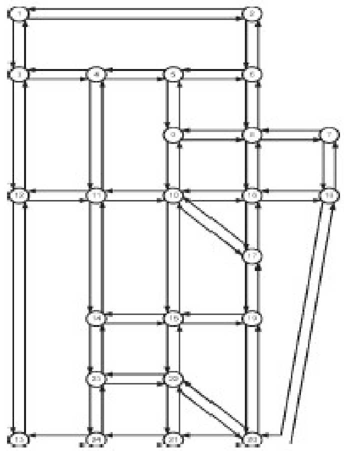
Instances

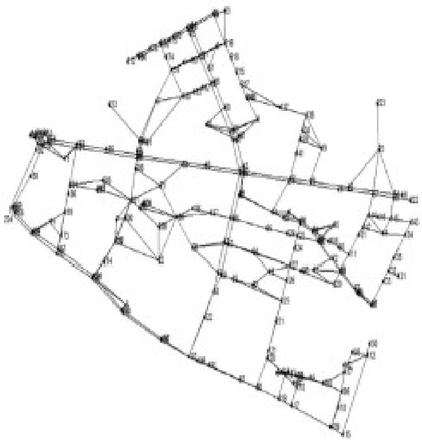


40 instances with 150 nodes and
8 instances with 330 nodes

Instances

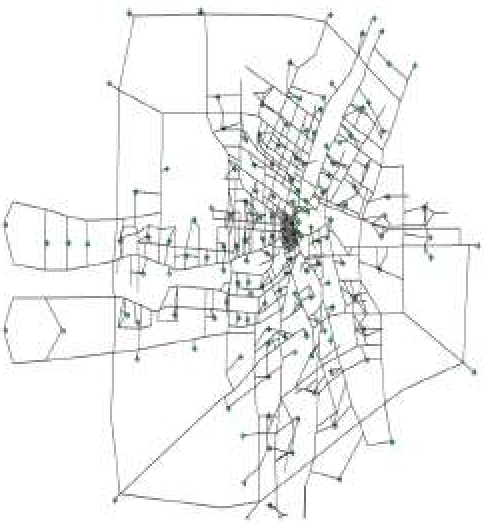


Sioux Falls

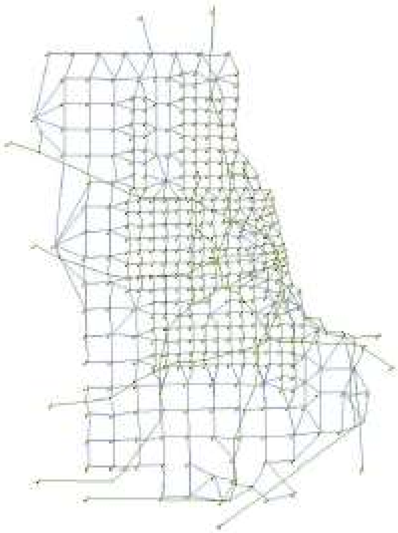


Berlin-
Friedrichshain

Winnipeg



Chicago



Instances

	|V|	|A|	|K|	|A||K|
Sioux Falls	24	76	528	40K
Berlin-Friedrichshain	224	523	506	265K
Winnipeg	1,067	2,975	4,344	13M
Chicago	933	2,950	83,113	245M

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Performance of the heuristic

Shortest paths on modified network

Dual variables

	γ (%)	SP-GEN l 100	SP-GEN l 1000	DUAL-GEN l 100	DUAL-GEN l 1000	LIN-C-SO n 1000
run time	0	0.3(0)	3.4(0)	0.3(0)	4.1(0)	16.63
	5	1.8(0.1)	16.4(0.1)	8.0(0.5)	20.6(2.2)	23.87
	10	1.8(0.1)	14.1(0.2)	12.5(0.7)	29.1(2.7)	30.65
	15	1.9(0.1)	13.3(0.3)	22.3(0.6)	42.2(2.4)	43.2
avg. gap	0	0.0	0.0	0.0	0.0	-
	5	0.41	0.41	0.02	0.01	-
	10	0.08	0.07	0.06	0.05	-
	15	0.07	0.05	0.09	0.08	-
max. gap	0	0.0	0.0	0.0		-
	5	1.88	1.84	0.11	0.09	-
	10	0.68	0.64	0.19	0.18	-
	15	0.51	0.45	0.31	0.28	-

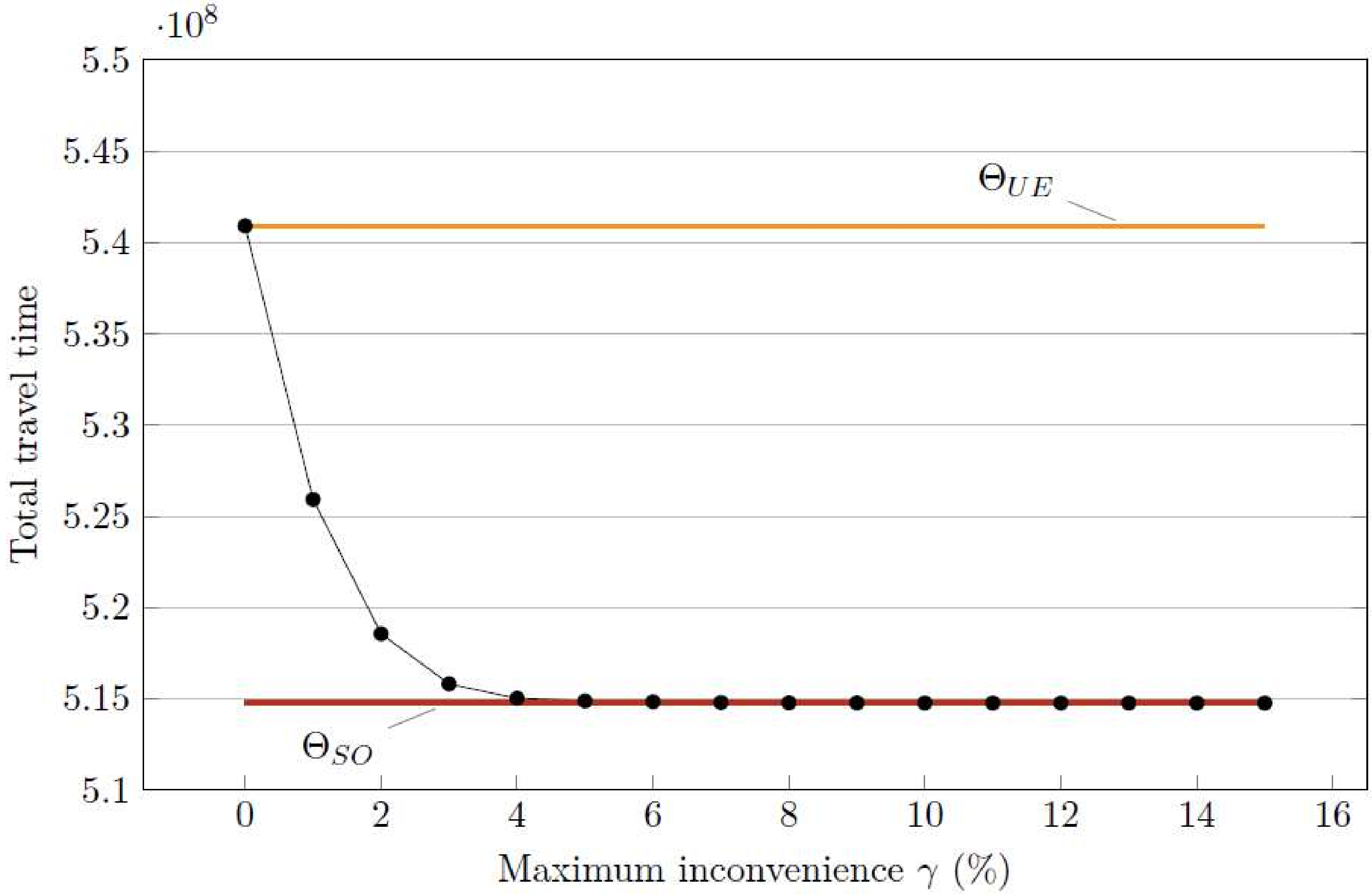
Average over 40 instances

Performance of the heuristic

		γ (%)			
		0	5	10	15
$n = 1000$	LIN-C-SO paths	1170	17128	45553	93559
SP-GEN $l = 100$	paths	1170	1540	1590	1604
	% LIN-C-SO paths	100	9.04	3.50	1.72
	iterations	4.5	4.4	4.1	4.0
SP-GEN $l = 1000$	paths	1170	1538	1585	1601
	% LIN-C-SO paths	100	9.03	3.49	1.71
	iterations	4.4	4.6	4.2	4.2

Average over 40 instances

Between UE and SO



Average over 40 instances

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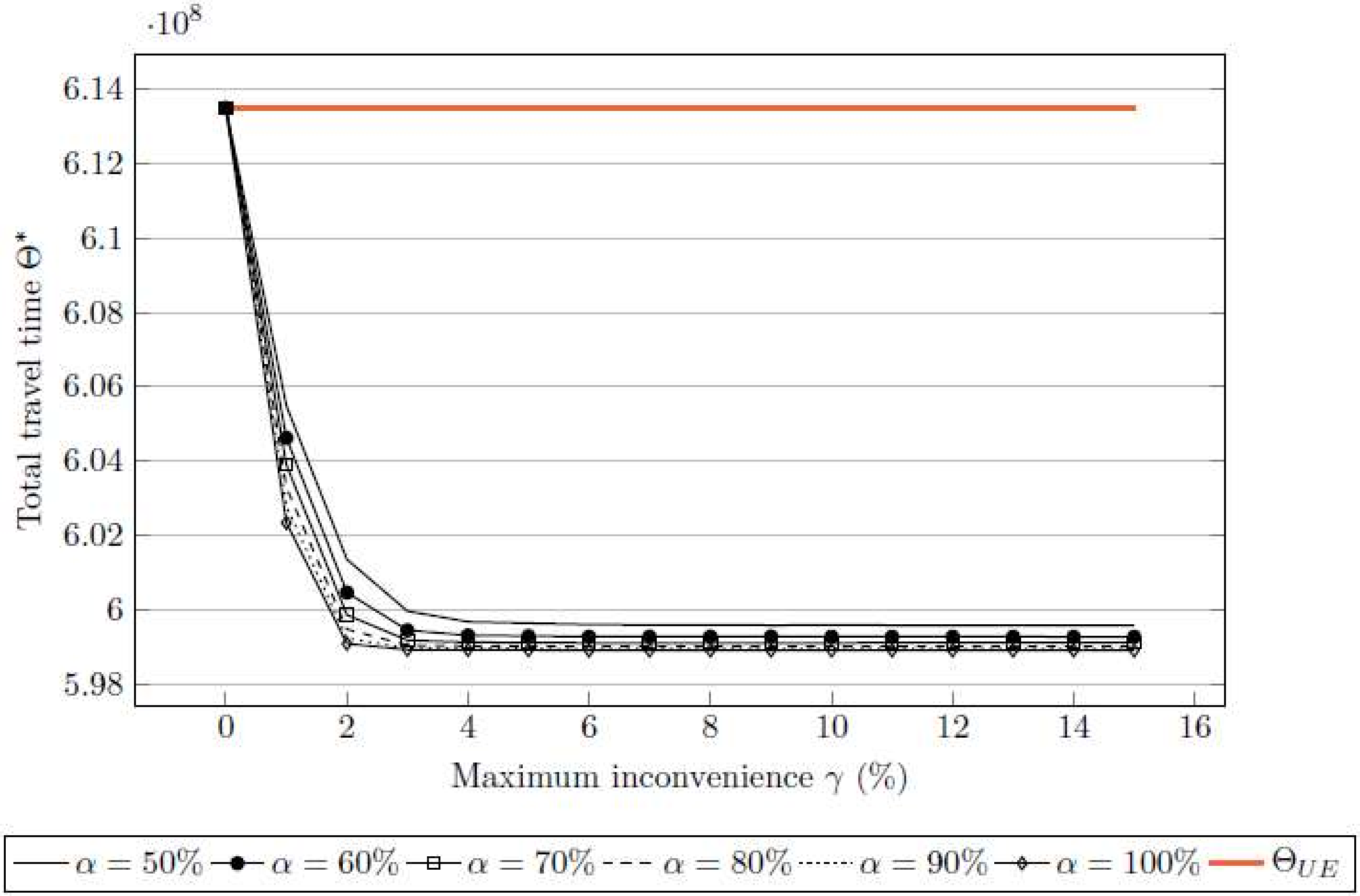
Impact of compliance



Instance: 150 nodes, 480 arcs, and 1170 OD pairs
Oligo-centric, in-peak, and high in-city traffic

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CVaR objective

Objective: Min Total travel time   Large travel time on some arcs

CVaR type objective:
Min Average arc congestion
over a given percentage of the most congested arcs

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CVaR objective

Average arc congestion
over the β*100% most congested arcs

$$\Gamma_\beta \equiv \min \omega + \frac{1}{\lceil \beta |A| \rceil} \sum_{(ij)\in A} \eta_{ij}$$

s.t.

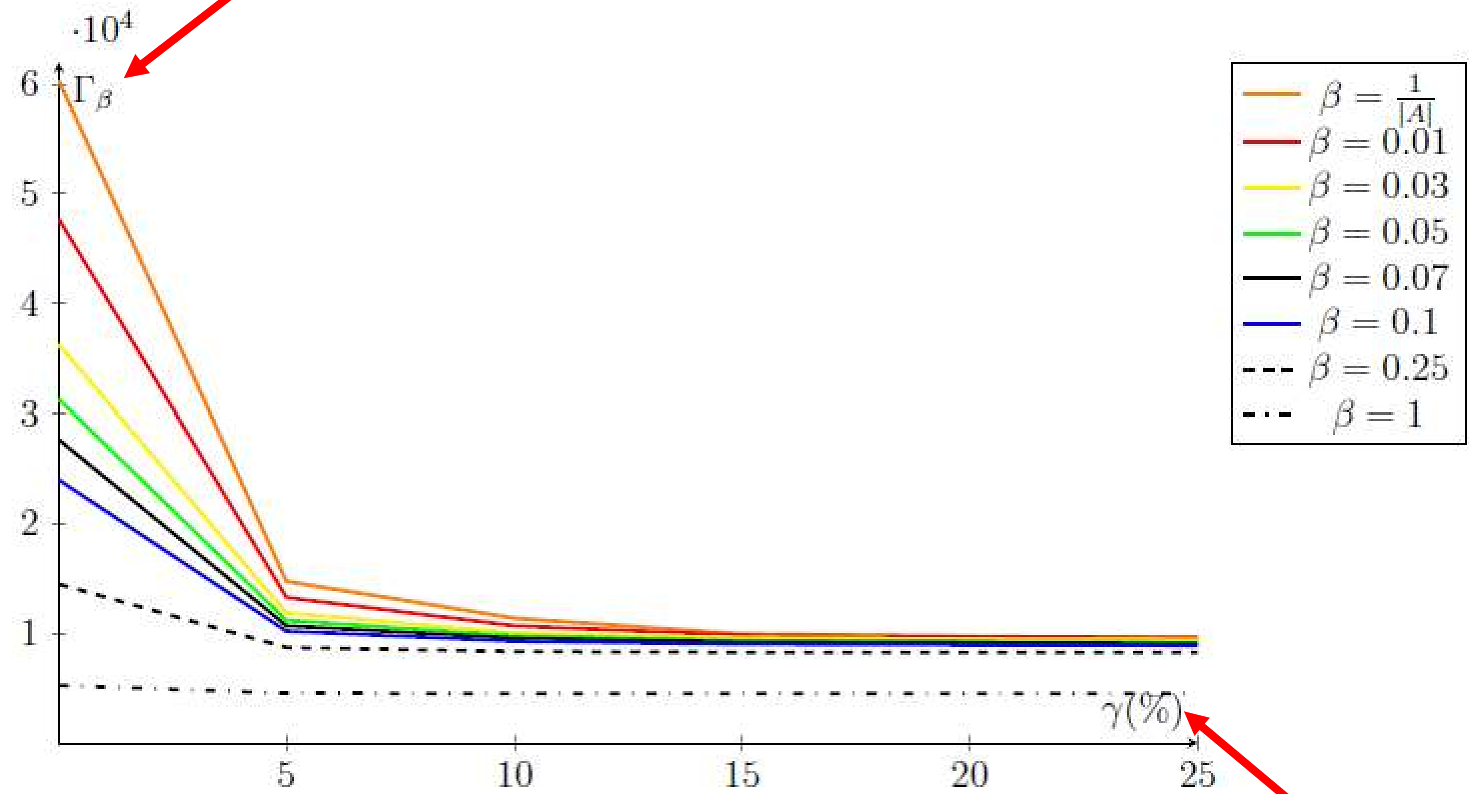
Eligible path

OD pair

$$
\begin{aligned}
& d_c = \sum_{k \in K_c^\gamma} y_{ck} && \forall c \in C \\
& x_{ij} = \sum_{c\in C} \sum_{k\in K_c^\gamma} a_{ij}^{ck} y_{ck} && \forall (i,j) \in A \\
& x_{ij} = \sum_{h=1}^{n} \lambda_{ij}^h && \forall (i,j) \in A \\
& 0 \le \lambda_{ij}^h \le \Delta_{ij}^h && \forall (i,j) \in A \quad \forall h = 1, ..., n \\
& \epsilon_{ij} = \sum_{h=1}^{n} \frac{\epsilon_{ij}^h - \epsilon_{ij}^{h-1}}{\Delta_{ij}^h} \lambda_{ij}^h && \forall (i,j) \in A \\
& \omega + \eta_{ij} \ge \epsilon_{ij} && \forall (i,j) \in A \\
& x_{ij} \ge 0 && \forall (i,j) \in A \\
& y_{ck} \ge 0 && \forall c \in C \quad \forall k \in K_c^\gamma \\
& \eta_{ij} \ge 0 && \forall (i,j) \in A.
\end{aligned}
$$

CVaR objective

Average arc congestion on the β *100% most congested arcs



Maximum inconvenience

Average over 40 instances

The constrained SO approaches

Generation of paths with limited inconvenience

Optimization model to assign traffic to the generated paths

Real inconvenience of users can be computed

The constrained SO approaches

Some paths could turn out to be more inconvenient than planned

(e.g., a priori inconvenience = 5%, a posteriori = 10%)

and some good paths could be missed

One step ahead…

Idea:

A model that embeds the path generation

A model embedding the path generation

Min Total travel time (piecewise approximation)

subject to

- Total flow routed for each O/D pair
- Travel time on a path does not exceed the travel time experienced on any other path by a given percentage φ

Any other path? **possibly unused** or **used**

FP-UC-SO and L-UC-SO

Exponential number of binary variables

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A complete model

FP-UC-SO

$$\min \boxed{\sum_{(ij)\in A} \sigma_{ij}}$$

Total travel time

$$x_{ij} = \sum_{c\in C}\sum_{k\in K_c} a_{ij}^{ck} y_{ck} \qquad \forall (i,j)\in A$$

$$d_c = \sum_{k\in K_c} y_{ck} \qquad \forall c\in C$$

$$x_{ij} = \sum_{h=1}^{n} \lambda_{ij}^h \qquad \forall (i,j)\in A$$

$$\sigma_{ij} = \sum_{h=1}^{n} \frac{f_{ij}^h - f_{ij}^{h-1}}{\Delta_{ij}^h}\lambda_{ij}^h \qquad \forall (i,j)\in A$$

$$\tau_{ij} = \sum_{h=1}^{n} \frac{t_{ij}(b_{ij}^h) - t_{ij}(b_{ij}^{h-1})}{\Delta_{ij}^h}\lambda_{ij}^h \qquad \forall (i,j)\in A$$

$$\tau_{ck} = \sum_{(i,j)\in A} a_{ij}^{ck}\tau_{ij} \qquad \forall c\in C \quad \forall k\in K_c$$

Limited inconvenience w.r.t. possibly unused paths

$$\boxed{\tau_{ck} \le (1+\phi)\tau_{ck'} + M(1-v_{ck})} \qquad \forall c\in C \quad \forall k\in K_c \quad \forall k'\in K_c\setminus\{k\}$$

$$y_{ck} \le d_c v_{ck} \qquad \forall c\in C \quad \forall k\in K_c$$

$$x_{ij} \ge 0 \qquad \forall (i,j)\in A$$

$$y_{ck} \ge 0 \qquad \forall c\in C \quad \forall k\in K_c$$

$$0 \le \lambda_{ij}^h \le \Delta_{ij}^h \qquad \forall (i,j)\in A \quad \forall h=1,\dots,n$$

$$\boxed{v_{ck} \in \{0,1\}} \qquad \forall c\in C \quad \forall k\in K_c.$$

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A complete model

L-UC-SO

$$\min \boxed{\sum_{(ij)\in A} \sigma_{ij}}$$

Total travel time

$$x_{ij} = \sum_{c\in C}\sum_{k\in K_c} a_{ij}^{ck} y_{ck} \qquad \forall (i,j)\in A$$

$$d_c = \sum_{k\in K_c} y_{ck} \qquad \forall c\in C$$

$$x_{ij} = \sum_{h=1}^{n} \lambda_{ij}^h \qquad \forall (i,j)\in A$$

$$\sigma_{ij} = \sum_{h=1}^{n} \frac{f_{ij}^h - f_{ij}^{h-1}}{\Delta_{ij}^h}\lambda_{ij}^h \qquad \forall (i,j)\in A$$

$$\tau_{ij} = \sum_{h=1}^{n} \frac{t_{ij}(b_{ij}^h) - t_{ij}(b_{ij}^{h-1})}{\Delta_{ij}^h}\lambda_{ij}^h \qquad \forall (i,j)\in A$$

$$\tau_{ck} = \sum_{(i,j)\in A} a_{ij}^{ck}\tau_{ij} \qquad \forall c\in C \quad \forall k\in K_c$$

Limited inconvenience w.r.t. used paths

$$\boxed{\tau_{ck} \le (1+\phi)\tau_{ck'} + M(2 - v_{ck} - v_{ck'}) \qquad \forall c\in C \quad \forall k\in K_c \quad \forall k'\in K_c\setminus\{k\}}$$

$$y_{ck} \le d_c v_{ck} \qquad \forall c\in C \quad \forall k\in K_c$$

$$x_{ij} \ge 0 \qquad \forall (i,j)\in A$$

$$y_{ck} \ge 0 \qquad \forall c\in C \quad \forall k\in K_c$$

$$0 \le \lambda_{ij}^h \le \Delta_{ij}^h \qquad \forall (i,j)\in A \quad \forall h = 1,\dots,n$$

$$\boxed{v_{ck} \in \{0,1\}} \qquad \forall c\in C \quad \forall k\in K_c.$$

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A heuristic column-generation

Generate new improving paths

Solve restricted optimization problem

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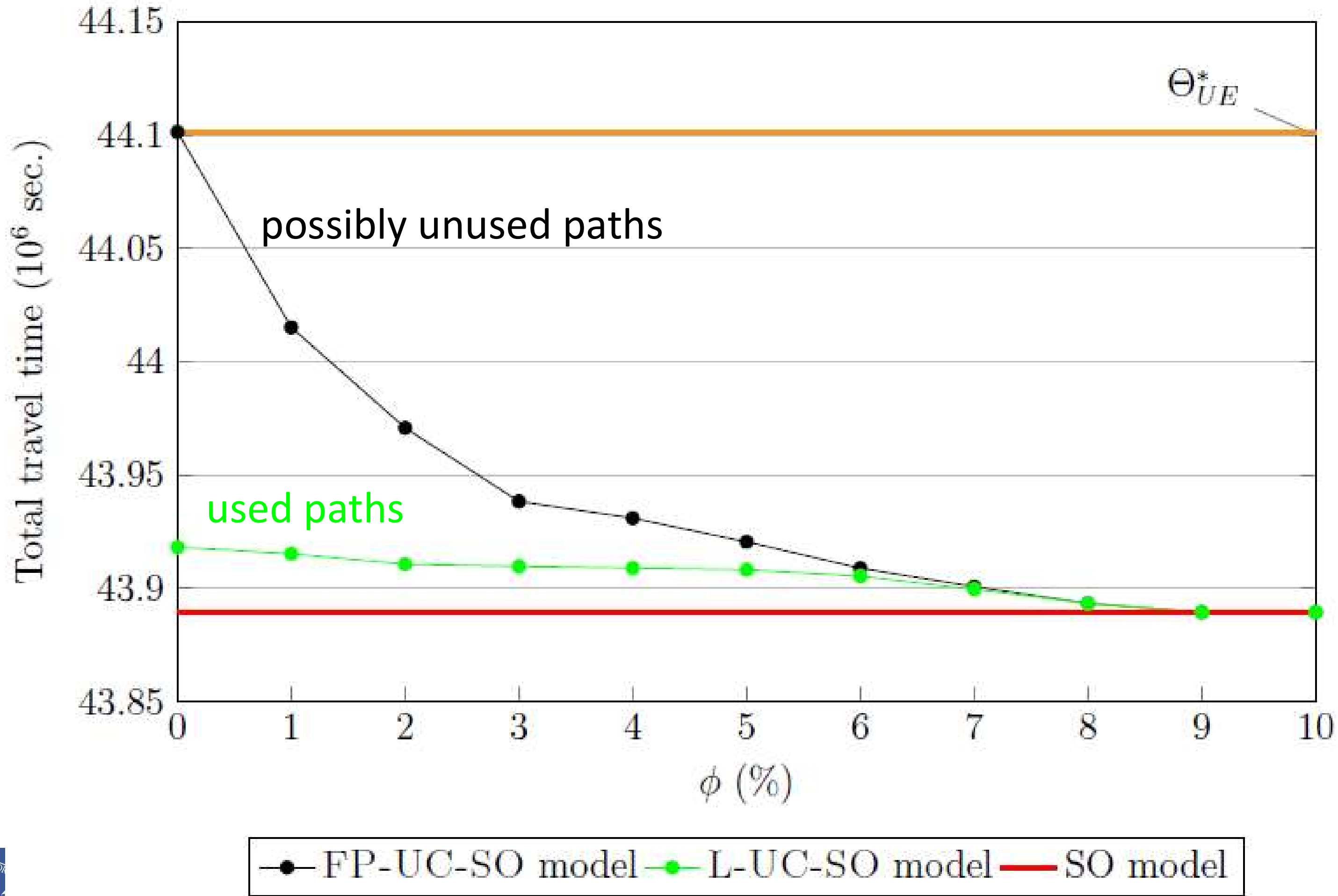
Quality of the heuristic

ϕ	Run time (sec) FP-UC-SO	PC-M	Θ Avg. gap (%)	Max. gap (%)
1	563	1.61	0.22	0.63
2	321	1.39	0.24	1.55
3	176	1.30	0.30	1.30
4	153	1.27	0.25	1.00
5	151	1.30	0.22	0.96
6	132	1.28	0.21	0.88
7	115	1.30	0.20	0.90
8	106	1.28	0.19	0.90
9	99	1.28	0.19	0.90
10	101	1.30	0.19	0.90

Comparison with UE and SO



Comments

How to distribute vehicles over the different paths?

Incentives

Time dependent network

Dynamic problem

Decentralized optimization

Thank you for your attention!

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