On Resilient Control for Secure Connected Vehicles

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Opportunities for Intelligent Transportation Systems

Connected Autonomous Vehicles¹,²,³,⁴:
• Expected to improve:
  • Traffic throughput.
  • Safety.
  • Fuel economy.
• V2V and V2I communications through Dedicated Short-Range Communication (DSRC).

Famous application is Cooperative Adaptive Cruise Control (CACC):
• Relies on V2V.
• Employs on-board sensors.
• Autonomous vehicles

Vulnerable to Cyber-Attacks

V2X Applications

Road Safety
- Collision Warning
- Collision Avoidance
- Road Hazard Warning
- Intersection Warning
- Remote diagnostic
- Emergency Vehicle, etc

Traffic Management
- Speed Management
- Traffic Information
- Routing Information
- Cooperative Navigation
- etc

Infotainment
- In-vehicle Internet
- Access, Video
- Streaming, Online gaming,
- Weather Information
- Points of Interests, etc

Autonomous Driving
- Electronic stability
- control, Automatic
- Braking, Adaptive Cruise
- Control, Cooperative
- awareness, etc

V2X Connectivity Challenges

V2X Security: Types of Attacks on V2X

V2X Threats and Attacks on:

- **Availability**
  - Denial of service
  - Jamming
  - Broadcast tampering
  - Spamming
  - Black hole attack
  .......

- **Authenticity and Identification**
  - Sybil attack
  - Impersonation
  - Masquerading
  - Replay attack
  - GPS Spoofing
  - Tunnelling
  - Key/Certificate replication
  - Message modification/alteration
  - Message Tampering
  .......

- **Confidentiality**
  - Eavesdropping
  - Information gathering
  - Bogus information sharing
  - Traffic analysis
  - Location spoofing
  .......

- **Integrity and Data Trust**
  - Message fabrication/suppression
  - Information forgery
  - Masquerade
  - Replay
  - Deletion
  - Man in the middle attack
  .......

- **Privacy**
  - Location Tracking
  - Identity disclosure
  .......

Review on V2X/CAV Security

Security threats have become more sophisticated, and cars already on the road require updated security mechanisms that address new risks.

Existing solutions

- Bit commitment and signature → Availability (e.g. DoS)
- Digital certification and zero knowledge → Identification and authenticity (e.g. Man in the middle, GPS spoofing replay attack)
- Trusted hardware → Identification and authenticity
- Group management system → Integrity and data trust (e.g. Message tampering)
- Encryption of data and the corresponding positioning and vehicle identification → Confidentiality and privacy (e.g. Traffic analysis)

- Security must not come at the expense of performance.
- In traffic safety scenarios, security verification must be performed in real time.
- Queueing of V2V messages is not an option.
- Finally, security systems must be certified. Certification of the complete solution assures safety.

Potential Ways to Lunch Cyber Attack

V2X Network Cyber Attack on Communication

Network flooding, affecting the availability of the network
Changing the content and integrity of the package

V2X Cyber Attack on In-Vehicle Network

Hacking the equipment, sensors and actuators
Affecting CAN bus readings, messages and high level controls
Previous Work

Denial of Service

\[
\begin{bmatrix}
\dot{d}_i(t) \\
\dot{v}_i(t) \\
\dot{a}_i(t)
\end{bmatrix}
= \begin{bmatrix}
0 & -1 & 0 \\
0 & \frac{k_p}{h} & 0 \\
\frac{1}{k_d} & 0 & \frac{1}{h}
\end{bmatrix}
\begin{bmatrix}
d_i(t) \\
v_i(t) \\
a_i(t)
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{k_d} \\
0 \\
0
\end{bmatrix}
v_{i-1}(t) + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}a_{i-1}(t - \tau)
\]

Packet drop out

\[
\chi(k) \in \{0, 1\}
\]

\[
\begin{aligned}
p(\chi(k) = 0) &= \lambda \\
p(\chi(k) = 1) &= 1 - \lambda
\end{aligned}
\]

\[
k < T_s \leq t < (k+1) \times T_s
\]

\[
\dot{u}_i = -\frac{1}{h} u_i + \frac{1}{h} (k_p e_i + k_d \dot{e}_i) + \frac{1}{h} \chi(k) \times u_{i-1}
\]

- Attacker provides several requests to access the network → communication network will be busy for real requests
Vehicle Platooning: Cooperative Adaptive Cruise Control (CACC)

Adaptive Cruise Control (ACC)
- Reference: \( d_{r_i} = r + h v_i \)
- Error: \( e_i = d_i - d_r \)
- String stability (shock waves attenuation)

Cooperative Adaptive Cruise Control (CACC)
- Shared information
- Closer inter-vehicle distance

Vehicle and Controller Dynamics\(^1\):
\[
\begin{align*}
\mathbb{V}_i &:= \begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i - h a_i \\ a_i \\ -\frac{1}{\tau_d} a_i + \frac{1}{\tau_d} u_i \end{pmatrix} \\
\dot{u}_i &= -\frac{1}{h} u_i + \frac{1}{h} \left( k_p e_i + k_d \dot{e}_i + u_{i-1} \right)
\end{align*}
\]

The presence of the network leads to network imperfections such as:\(^1\):

**Question:**
How do we deal with these phenomena in our model?

---

Controller Dynamics:

\[
\dot{u}_i = -\frac{1}{h} u_i + \frac{1}{h} \left( k_p e_i + k_d \dot{e}_i + u_{i-1} \right)
\]

Assumptions

1. Variable transmission intervals:
   \[
   \delta T \leq t_{k_{i-1}+1} - t_{k_{i-1}} < T_{mati}
   \]

2. Variable network delays
   \[
   0 \leq \tau_{d_{k_i}} \leq T_{mad} \leq T_{mati}
   \]

Ensures transmitted output is received by controller before next sample is sent.

The state of the error dynamics is modified such that the whole platoon is considered as a cascade of dynamical systems.

Benefits in achieving by design

- String stability
- Performance

Available at:
Each vehicle can be represented as linear system with jumps

\[
\begin{aligned}
    \dot{e}_i(t) &= \begin{bmatrix}
        0 & 1 & 0 \\
        \frac{k_p}{\tau_d} & -\frac{k_d}{\tau_d} & -\frac{1}{\tau_d}
    \end{bmatrix} \begin{bmatrix}
        0 \\
        0 \\
        \frac{1}{\tau_d}
    \end{bmatrix} u_{i-1}(t) + \begin{bmatrix}
        0 \\
        0 \\
        -\frac{1}{\tau_d}
    \end{bmatrix} \dot{u}_{i-1}(t) \\
    \dot{u}_{i-1}(t) &= -\frac{1}{h} u_{i-1}(t) + \frac{1}{h} \omega_{i-1}(t) \\
    \dot{\omega}_{i-1}(t) &= 0
    \end{aligned}
\]

∀t \neq t_{k_i-1} \land t_{k_i-1} \in \bigcup_{n \in \mathbb{N}} H_n

\[
\begin{aligned}
    e_i(t^+) &= e_i(t) \\
    u_{i-1}(t^+) &= u_{i-1}(t) \\
    \dot{u}_{i-1}(t^+) &= u_{i-1}(t)
    \end{aligned}
\]

∀t = t_{k_i-1} \land t_{k_i-1} \notin \bigcup_{n \in \mathbb{N}} H_n

\[
\begin{aligned}
    \omega_{i-1}(t) &= k_p e_{i-1}(t) + k_d \dot{e}_{i-1}(t) + u_{i-2}(t) \\
    \omega_i(t) &= k_p e_i(t) + k_d \dot{e}_i(t) + u_{i-1}(t)
    \end{aligned}
\]
Modelling Characteristics:

• Platooning as cascade of dynamical systems
• We follow the Hybrid Systems framework proposed by R. Goebel, R. Sanfelice, A. Teel
• State space of the model enlarged with the auxiliary variables subject to reset:
  • $\hat{u}_{i-1}$, $s_u$, $l$
  • $\tau_i - 1$ for time-based network trigger

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Control Requirements

String stability

\[ \frac{\|\omega_0\|_{\mathcal{L}_2}}{\|\omega_{i-1}\|_{\mathcal{L}_2}} \leq 1 \quad \Rightarrow \quad \mathcal{L}_2 \text{ stability of } \mathcal{V}_i \quad \frac{\|\omega_i\|_{\mathcal{L}_2}}{\|\omega_{i-1}\|_{\mathcal{L}_2}} \leq 1 \]

**Definition.** The vehicle platooning is said to be string stable if the systems \( \mathcal{V}_i \) in the platoon are \( \mathcal{L}_2 \)-stable from the input \( \omega_{i-1} \in \mathcal{L}_2 \) to the output \( \omega_i \in \mathcal{L}_2 \) with an \( \mathcal{L}_2 \)-gain less than or equal to one.

**Individual vehicle stability** \( (\lim_{t \to \infty} e_i(t) = 0) \)

“Networked-free” error dynamics with performance requirements

\[ \mathbb{P}(\lambda_M, \zeta_m) := \{ A \in \mathbb{R}^{n \times n} | \lambda_{\text{max}}(A) = \lambda_M, \zeta_{\text{min}}(A) \geq \zeta_m \} \]

\[ \lambda_M < 0, \zeta_m \in (0, 1] \]

\[ A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{k_p}{\tau_d} & -\frac{k_d}{\tau_d} & -\frac{1}{\tau_d} \end{bmatrix} \in \mathbb{P}(\lambda_M, \zeta_m) \]
**Problem.** Given the platooning parameters \( h \) and \( \tau_d \), and the performance requirements \( \mathbb{P} \), design gains \( k_p \) and \( k_d \) for the hybrid controller such that the vehicle platooning satisfies the following properties with the largest achievable value of \( T_{mati} \) and \( T_{mad} \):

(P1) Individual vehicle stability with performance \( \mathbb{P} \), i.e., \( A_e \in \mathbb{P} \).

(P2) String stability.

\[
\frac{\|\omega_i\|_{L_2}}{\|\omega_{i-1}\|_{L_2}} \leq 1
\]
Design Approach

1. Sufficient conditions for string stability and estimation of trade-off curves:
   - Based on [1], [2].
   - Turn into Matrix Inequalities where controller gains appear in a nonlinear fashion.

2. Necessary and sufficient conditions on controller gains to satisfy performance requirements

3. Tuning algorithm

Contribution

**Assumption 1.** There exist constants \( \gamma, \epsilon \in \mathbb{R}_{\geq 0}, \mu \in \mathbb{R}_{>0} \) and \( P = P^T > 0 \) such that

\[
\begin{bmatrix}
\text{He}(PA_{11}) + A_{21}^T A_{21} + \mu C^T C \\
PA_{12} + \mu C^T D \\
\mu D^T D - \gamma^2 \\
A_{21}^T A_{23} + PA_{13} \\
0 \\
A_{23}^T A_{23} - \mu(1 + \epsilon)
\end{bmatrix} < 0
\]

\( \gamma \downarrow \Rightarrow T_{mati} \uparrow, T_{mad} \uparrow \)

\( \mathcal{L}_2 \) gain from network imperfection to error dynamics

**Assumption 2.** There exists a pair \((T_{mati}, T_{mad})\) such that

\[
\begin{align*}
\gamma_1 \phi_1(\tau_{i-1}) & \geq \gamma_0 \phi_0(\tau_{i-1}), \quad \forall \tau_{i-1} \in [0, T_{mad}] \\
\gamma_0 \phi_0(\tau_{i-1}) & \geq \lambda^2 \gamma_1 \phi_1(0), \quad \forall \tau_{i-1} \in [0, T_{mati}]
\end{align*}
\]

with \( T_{mati} \geq T_{mad} \geq 0, \lambda \in (0, 1), \) constants \( \gamma_0 := \gamma \) and \( \gamma_1 := \frac{\gamma}{\lambda}, \) and where \( \phi_{l_{i-1}} : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}, l_{i-1} \in \{0, 1\} \) is the unique maximal solution to

\[
\dot{\phi}_{l_{i-1}} = -\gamma_{l_{i-1}}(\phi_{l_{i-1}}^2 + 1)
\]

with initial conditions \( \phi_1(0) \geq \phi_0(0) \geq \lambda^2 \phi_1(0), \phi_0(T_{mati}) \geq 0. \)

**Theorem.** Let Assumption 1 with \( \epsilon = 0 \) and Assumption 2 hold. Then, hybrid system \( \mathcal{V}_i \) is \( \mathcal{L}_2 \)-stable from the input \( \omega_{i-1} \) to the output \( \omega_i \) with an \( \mathcal{L}_2 \)-gain less than or equal to one.

Based on:

Suff. Cond. for string stability and estimation of trade-off curves are based on the solution of the matrix inequality:

\[
\begin{bmatrix}
\text{He}(PA_{11}) + A_2^T A_21 + \mu C^T C & PA_{12} + \mu C^T D & A_2^T A_{23} + PA_{13} \\
\vdots & \mu D^T D - \gamma^2 & 0 \\
\end{bmatrix} < 0
\]

Nonlinear in controller parameters. Proposed solution:

Condition (C1). The real eigenvalue is equal to \(\lambda_M\).

\[
k_d = f_{C1}(k_p) \\
k_p \in [k_{pC1}, \bar{k}_{pC1}]
\]

Condition (C2). Single couple of complex eigenvalues with real part equal to \(\lambda_M\).

\[
k_d = f_{C2}(k_p) \\
k_p \in (k_{pC2}, \bar{k}_{pC2})
\]
Proposition 1 (N.S.C. for C1). Let $k_p, k_d \in \mathbb{R}$, $\lambda_M \in \mathbb{R}_{<0}$, and $\zeta_m \in \mathbb{R}_{>0}$. Then, C1 is satisfied if and only if the following conditions hold:

\[
\begin{align*}
  k_d &= f_{C1}(k_p) := -\frac{1}{\lambda_M} k_p - \lambda_M^2 \tau_d - \lambda_M \\
  k_p &\leq \frac{|\lambda_M| (\lambda_M \tau_d + 1)^2}{4 \tau_d \zeta_m^2} := \bar{k}_{PC1} \\
  k_p &\geq 2 \tau_d \lambda_M^3 + \lambda_M^2 := \underline{k}_{PC1} \\
  \lambda_M &> -\frac{1}{3 \tau_d}
\end{align*}
\]

Proposition 2 (N.S.C. for C2). Let $k_p, k_d \in \mathbb{R}$, $\lambda_M \in \mathbb{R}_{<0}$, and $\zeta_m \in (0, 1)$. Then, C2 holds if and only if the following conditions hold:

\[
\begin{align*}
  k_d &= f_{C2}(k_p) := \\
  &\quad \quad -8 \lambda_M^3 \tau_d^2 + 8 \lambda_M^2 \tau_d + 2 \lambda_M - \tau_d k_p \\
  &\quad \quad \quad \quad \quad \quad \quad 2 \lambda_M \tau_d + 1 \\
  k_p &\leq \frac{2 \tau_d \lambda_M^3 + \lambda_M^2}{\zeta_m^2} := \bar{k}_{PC2} \\
  k_p &> 2 \tau_d \lambda_M^3 + \lambda_M^2 := \underline{k}_{PC2} \\
  \lambda_M &> -\frac{1}{3 \tau_d}
\end{align*}
\]

Number of control variables is reduced
Perfomances are constrained by powertrain time constant
Controller Design: Tuning Algorithm

Design goal

Resiliency Metric \( \gamma \downarrow \Rightarrow T_{mati} \uparrow, T_{mad} \uparrow \)

Design Algorithm

\begin{align*}
\text{Case C1} & \quad \gamma_{C1} \quad k_d = f_{C1}(k_p) \\
& \quad k_p \in [k_{pC1}, \bar{k}_{pC1}] \\
\text{Find } k_p \text{ s.t. } \gamma \text{ is minimum in C1}
\end{align*}

\begin{align*}
\text{Case C2} & \quad \gamma_{C2} \quad k_d = f_{C2}(k_p) \\
& \quad k_p \in (k_{pC2}, \bar{k}_{pC2}] \\
\text{Find } k_p \text{ s.t. } \gamma \text{ is minimum in C2}
\end{align*}

\begin{align*}
\text{select } k_p : \\
\min \{\gamma_{C1}, \gamma_{C2}\}
\end{align*}

\gamma^*, k_p^*, k_d^*
Numerical Results

Tuning as in Ploeg at al.

Our approach

Network-resilient tuning

Numerical Results

Left lane, in red, network-resilient tuning

Right lane, in blue, non network-resilient tuning

Inside last vehicle in the left lane
network-resilient tuning

Inside last vehicle in the right lane
non network-resilient tuning
Resilient CACC Under Denial-of-Service Attacks
The attacker generates DoS with the purpose of disrupting the network for the longest time possible as possible.

**CACC Pros:**
- Closer inter-vehicle distance:
  - Traffic throughput
  - Fuel economy

**CACC Cons:**
- String Stability influenced by:
  - Network unreliability
  - Cyber Attacks

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Secure and Safe Control Strategies for Vehicle Platooning

Adaptive Cruise Control (ACC)
no communication

Estimation-based CACC\(^1\)
action is estimated by on-board sensors

Switching Strategy CACC/ACC\(^2\)
control strategy ACC and CACC switched over time based on communication status

Inter-vehicle time gaps without CACC are larger than inter-vehicle time gaps with CACC

Trade off between safety and performance is needed

Research questions:
1. What network unreliability and DoS attacks can we tolerate without the need to fall back to safe strategies?
2. How to design a CACC that maximize the resilience to network unreliability and DoS attack?


Vehicle Platooning with DoS attacks

Controller Dynamics:

\[ \dot{u}_i = -\frac{1}{h} u_i + \frac{1}{h} (k_p e_i + k_d \dot{e}_i + u_{i-1}) \]

Available in sporadic fashion: data shared with period \( T_s \)

DoS attacks as a sequence of intervals \( \{H_n\}_{n \in \mathbb{N}} \)

of limited length bounded by \( \Delta \in \mathbb{N}_0 \)

\( \Delta \in \mathbb{N}_0 \) is the maximum allowable number of successive packet dropouts (MANSD)

\( \Delta \) is the resiliency metric
**Problem.** Given the platooning parameters $h$ and $\tau_d$, and the performance requirements $\mathbb{P}$, design gains $k_p$ and $k_d$ for the hybrid controller such that the vehicle platooning satisfies the following properties with the largest achievable value of $\Delta$:

(P1) Individual vehicle stability with performance $\mathbb{P}$, i.e., $A_e \in \mathbb{P}$.

(P2) String stability.

\[ \frac{\|\omega_i\|_2}{\|\omega_{i-1}\|_2} \leq 1 \]
Design Approach

1. Necessary and Sufficient Conditions on controller gains to satisfy performance requirements.

2. S.C. for string stability and estimation of $\Delta$:
Matrix Inequalities where controller gains appear in a nonlinear fashion.

3. Tuning algorithm.
To satisfy performance

$$\mathbb{P}(\lambda_M, \zeta_m) := \{A \in \mathbb{R}^{n \times n} | \lambda_{\text{max}}(A) = \lambda_M, \ \zeta_{\text{min}}(A) \geq \zeta_m\}$$

$$A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{k_p}{\tau_d} & -\frac{k_d}{\tau_d} & -\frac{1}{\tau_d} \end{bmatrix} \in \mathbb{P}(\lambda_M, \zeta_m)$$

**Condition (C1).** The real eigenvalue is equal to $\lambda_M$ and, the other two eigenvalues have real part less than or equal to $\lambda_M$ with damping ratio greater than $\zeta_m$.

**Condition (C2).** The spectrum of $A_e$ is characterized by a single couple of complex eigenvalues with real part equal to $\lambda_M$ and damping ratio greater than $\zeta_m$, and the other real eigenvalue is less than $\lambda_M$. 
Lemma. Let $P_1 \in S_+^4$, $p_2$, $\delta$, $\tau_d$, $h$, and $T_s$ be given positive real number, $\Delta \in \mathbb{N}_0$, and $k_p$, and $k_d$ be given real numbers. For each $\tau_{i-1} \in [0, (\Delta + 1)T_s]$, define $\mathcal{M} : \tau_{i-1} \mapsto \mathcal{M}(\tau_{i-1})$. Then, $\text{rge} \mathcal{M} = \text{Co}\{\mathcal{M}(0), \mathcal{M}((\Delta + 1)T_s)\}$. Therefore, $\mathcal{M}(\tau_{i-1}) < 0$, $\forall \tau_{i-1} \in [0, (\Delta + 1)T_s]$ holds if and only if

$$\mathcal{M}(0) < 0, \quad \mathcal{M}((\Delta + 1)T_s) < 0$$

$$\mathcal{M}(\tau_{i-1}) = \begin{pmatrix}
\text{He}(P_1 A_{xx}) + C_\omega^T C_\omega & P_1 A_{x\eta} + C_\omega^T + e^{-\delta \tau_{i-1}} p_2 A_{\eta x}^T \delta p_2 e^{-\delta \tau_{i-1}} + 1
-\delta p_2 e^{-\delta \tau_{i-1}} + 1 & P_1 A_{xw} - e^{-\delta \tau_{i-1}} p_2 / h - 1
\end{pmatrix} < 0, \forall \tau_{i-1} \in [0, (\Delta + 1)T_s]$$

$k_p$, $k_d$, $\delta$ fixed.

$k_d$ unequivocally determined by the choice of $k_p$ via Conditions 1 and 2

$k_p$ gridding on an interval known a priori.
Design goal

\[
\begin{align*}
\text{maximize } & \quad \Delta \\
\text{subject to } & \quad A_e \in \mathbb{P}, \ M(0) < 0, \ M((\Delta + 1)T_s) < 0
\end{align*}
\]

Design Algorithm

- **Case C1**
  \[k_d = f_{C1}(k_p)\]
  \[k_p \in [k_{pC1}, \bar{k}_{pC1}]\]

- **Case C2**
  \[k_d = f_{C2}(k_p)\]
  \[k_p \in (\bar{k}_{pC2}, \bar{k}_{pC2}]\]

\[\Delta_i = 0, (k_p, k_d) \text{ given} \]

- Line search on \(\delta\) s.t. LMI feasible

- \[\Delta_i \leftarrow \Delta_i + 1\]

- LMI feasible?
  - Yes (Y)
    - \[\Delta_i \leftarrow \Delta_i - 1\]
  - No (N)
    - \[\Delta_{Ci} \leftarrow \Delta_i - 1\]

- Select \(k_p : \max\{\Delta_{C1}, \Delta_{C2}\}\)

- \(\Delta^*, k_p^*, k_d^*\)
Numerical Results

- Tuning as in [Ploeg et al CITS ‘11]\(^1\)

- Tuning as our approach

Available at:
Numerical Results – DoS

Left lane, in red, network-resilient tuning

Right lane, in blue, non network-resilient tuning

Inside last vehicle in the left lane
network-resilient tuning

Inside last vehicle in the right lane
non network-resilient tuning
Zero-Order Hold
\[
\begin{align*}
\dot{u}_{i-1}(t) &= 0 \quad \forall t \neq t_{k_{i-1}} \lor \text{NoDoS} \\
\dot{u}_{i-1}(t^+) &= u_{i-1}(t_{k_{i-1}}) \quad \forall t = t_{k_{i-1}} \land \text{DoS}
\end{align*}
\]

Can we enhance the ZOH by using continuous-time measurements in between communication updates?

Intersample Dynamics
\[
\begin{align*}
\dot{u}_{i-1}(t) &= \gamma_p e_i + \gamma_d \dot{e}_i \quad \forall t \neq t_{k_{i-1}} \lor \text{NoDoS} \\
\dot{u}_{i-1}(t^+) &= u_{i-1}(t_{k_{i-1}}) \quad \forall t = t_{k_{i-1}} \land \text{DoS}
\end{align*}
\]
Same design approach.

LMI conditions become design tool for intersample dynamics.

\[
\mathcal{M}(\tau_i-1) = \begin{pmatrix}
\text{He}(P_1 A_{xx}) + C_\omega^T C_\omega & A_{x\eta} + C_\omega^T e^{-\delta \tau_i-1} & \gamma_p p_2 & \gamma_d p_2 & 0 & p_2/h \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{pmatrix}^T 
\begin{pmatrix}
P_1 A_{x\omega} \\
p_2 e^{-\delta \tau_i-1}/h \\
-(1 + \epsilon)
\end{pmatrix}
\]

Change of variables

\[
\gamma_p = \psi_p/p_2 \\
\gamma_d = \psi_d/p_2
\]

\[
\tilde{\mathcal{M}}(\tau_i-1) = \begin{pmatrix}
\text{He}(P_1 A_{xx}) + C_\omega^T C_\omega & A_{x\eta} + C_\omega^T e^{-\delta \tau_i-1} & \psi_p & \psi_d & 0 & p_2/h \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{pmatrix}^T 
\begin{pmatrix}
P_1 A_{x\omega} \\
p_2 e^{-\delta \tau_i-1}/h \\
-(1 + \epsilon)
\end{pmatrix}
\]
Numerical Results

- Tuning as in [Ploeg et al CITS ‘11] (with ZOH)
- Tuning as our approach with ZOH
- Tuning as our approach with Intersample
Remarks:
- Increasing time gaps, resilience increases.
- CACC with intersample always guarantee higher resilience.
- Resilience gained by increasing derivative action.
Adding Network Delays to the Resiliency Metric

Assumption:

\[ 0 \leq \tau_{d_{k_i}} \leq T_{mad} \leq T_s \]

communication period

maximum allowable delay

- Packet disorder phenomena – forced dropouts for outdated information
- Packet dropouts and network delays captured in a simple and unified model
- Vehicle networks: communication frequency 10 to 20 Hz
Modeling Network Delays

General results for NCS available at:
Sketch of the proof: based on Lyapunov theory for Hybrid Systems

\[ V(x_i) = \tilde{x}_i^T P_1 \tilde{x}_i + p_{2,i} \eta_{i-1}^2 e^{-\delta \tau_i} + p_{3,i} \sigma_{i-1}^2 e^{-\delta \tau_i} \]

\[ \begin{align*}
V_1(\tilde{x}_i) & + V_2(\eta_{i-1}, \tau_{i-1}, l_{i-1}) \\
& + V_3(\sigma_{i-1}, \tau_{i-1}, l_{i-1})
\end{align*} \]

Decreases at jumps

\[ p_{2,1} - e^{-\delta_2 (1+\Delta) T_s} p_{2,0} \leq 0 \]

\[ p_{2,0} + p_{3,0} - p_{3,1} \leq 0 \]

Decreases while flows

\[ M(\tau_{i-1}, l_{i-1}) < 0, \]
\[ \forall \tau_{i-1} \in [0, (\Delta + 1) T_s], l_{i-1} \in \{0, 1\} \]

\[ M(\tau_{i-1}, l_{i-1}) = \]
\[ \begin{pmatrix}
1 e(p_1 A_{xx}) + C_w^T C_w & P_1 A_{x\eta} + C_w^T + e^{-\delta \tau_{i-1}} p_{2,l_{i-1}} A_{\eta x}^T & e^{-\delta \tau_{i-1}} p_{3,l_{i-1}} A_{\eta x}^T & P_1 A_{xw} \\
0 & -e^{-\delta \tau_{i-1}} p_{2,l_{i-1}} & e^{-\delta \tau_{i-1}} p_{3,l_{i-1}} & -e^{-\delta \tau_{i-1}} p_{3,l_{i-1}} \\
\delta & 0 & \delta & 0 \\
\delta & 0 & \delta & 0
\end{pmatrix} \]

\[ \mathcal{L}_2\text{-stability} \]

General results for NCS available at:
Numerical Results

Tuning as [Ploeg et al CITS ’11]

Tuning as our approach with ZOH

Tuning as our approach with Intersample
From Vehicle Platooning to a Class of Networked Control Systems

- Modeling of Vehicle Platooning
  - Network-Aware CACC design
  - DOS-Resilient CACC design
- Modeling of a class of NCSs
  - Stability Analysis of NCSs
  - Design of controllers for NCSs
From Vehicle Platooning to a Class of Networked Control Systems

Vehicle Platooning model

Generic Networked Control System

Applications based on remote networked sensors:
• Drones
• Smart Grids
• Process control
• …
Majority of existing design approaches:

1. Does not consider dynamic output feedback controllers

\[
\begin{align*}
K & \begin{cases}
\dot{x}_c = A_c x_c + B_c \hat{y} \\
u = C_c x_c + D_c \hat{y}
\end{cases}
\end{align*}
\]

More suitable to account for performance

For example: $\mathcal{H}_\infty$ robust controllers

2. Are not suitable for the evaluation of trade-off curves as shown earlier

When NCS are not modeled as hybrid systems, network delays and packet dropping are commonly not considered together

3. Use Zero-Order Hold (ZOH) as holding device

\[
\begin{align*}
\text{ZOH} & \begin{cases}
\dot{y}(t) = 0 & \forall t \neq t_k + \tau_{d_k} \\
y(t^+) = y(t_k) & \forall t = t_k + \tau_{d_k}
\end{cases}
\end{align*}
\]

Suitable for small transmission intervals, but potentially not for long period without network updates

4. Does not explicitly account for network unreliability metrics in the stability analysis

Network resilience does not appear directly in design conditions
Problem Statements

To address research gaps we solve:

Problem #1 (Compute Trade-off Curves):  
Assume controller and holding device given. Find trade-off curves between transmission intervals and network delays such that the NCS is input-output stable.

Problem #2 (Controller Design):  
Design controller and holding device such that the NCS is input-output stable with the largest transmission intervals.
Challenges for Problem #1 (Compute Tradeoff Curves) and Problem #2 (Controller Design):

1. Choice of clock-dependent Lyapunov function (related to Problem #1 and #2)
   • Lyapunov function for Hybrid Dynamical Systems must be properly designed:
     • Decreases during flows
     • Decreases as jumps
     • Conservatism of the trade-off curves

2. Linearization of matrix inequalities
   • Linearization of stability conditions (related to Problem #1 and #2)
   • Controller parameters linearization (related to Problem #2)
Networked Control System

Hybrid dynamical system for stability analysis

Challenge:
Find the Clock-Dependent Lyapunov function

\[ V(x) = \begin{cases} \hat{x}^T P_1 \hat{x} + e^{-\delta \tau} \eta^T P_2 \eta \quad & V_1(\hat{x}) \\ e^{-\delta \tau} \sigma^T P_3 \sigma \quad & V_2(\eta, \tau, l) \end{cases} \]

Decreases at jumps

\[ P_{2,1} - e^{-\delta (T_{mati})} P_{2,0} \leq 0 \]
\[ P_{2,0} + P_{3,0} - P_{3,1} \leq 0 \]

Decreases during flows

\[ M(\tau, l) < 0, \quad \forall \tau \in [0, T_{mati}], l \in \{0, 1\} \]
Problem #1: Compute Trade-off Curves

Clock-Dependent Lyapunov function

\[
V(x) = \underbrace{\tilde{x}^\top P_1 \tilde{x}}_{V_1(\tilde{x})} + \underbrace{e^{-\delta \tau} \eta^\top P_{2,l} \eta}_{V_2(\eta, \tau, l)} + \underbrace{e^{-\delta \tau} \sigma^\top P_{3,l} \sigma}_{V_3(\sigma, \tau, l)}
\]

Decreases at jumps
\[
P_{2,1} - e^{-\delta(T_{mati})} P_{2,0} \leq 0 \\
P_{2,0} + P_{3,0} - P_{3,1} \leq 0
\]

Decreases during flows
\[
\mathcal{M}(\tau, l) < 0, \\
\forall \tau \in [0, T_{mati}], l \in \{0, 1\}
\]

Challenges:
- Variables appear in a nonlinear fashion
- Infinite conditions to be satisfies

Stability conditions expose trade-off curves parameters:
- Stability at a network operating point can be directly checked
- Trade-off curves can be obtained by an iterative algorithm by changing \( T_{mad}, T_{mati} \)

Proposed solutions:
- Line search on \( \delta \)
- Employ convexity of \( \mathcal{M}(\tau, l) \)

\[
\mathcal{M}(0, 1) < 0, \mathcal{M}(T_{mad}, 1) < 0, \\
\mathcal{M}(T_{mad}, 0) < 0, \mathcal{M}(T_{mati}, 0) < 0
\]
Batch reactor: output feedback control system

$\mathcal{L}_2$ stability

Blue: $\mathcal{L}_2$-gain $\leq 2.5$
Red: $\mathcal{L}_2$-gain $\leq 5$
Dashed lines - proposed approach
Solid lines - approach in Heemels

For this control system:
- Approach in Heemels et al. is less conservative for transmission intervals
- Proposed approach is less conservative for network delays
- Proposed approach accounts for intersample dynamics

Problem #2: Dynamic Output Feedback Controller Design

**Networked Control System**

- Approach in Scherer et al. introduced for $H_\infty$ control design, does not work in this case.

**Challenge:**
- Find the Clock-Dependent Lyapunov function such that stability conditions turn into linear matrix inequality.
- Parameters of controller and holding device can be computed by employing semidefinite programming tools.

**Hybrid dynamical system for stability analysis**

\[
\begin{align*}
\dot{x}_p &= (A_p + B_p D_c C_p)x_p + B_p C_c x_c - B_p D_c \eta \\
\dot{x}_c &= A_c x_c + B_c C_p x_p - B_c \eta \\
\dot{\eta} &= (C_p A_p - H C_p) x_p + E x_c + H \eta \\
\tau &= -1 \\
x^+_p &= x_p \\
x^+_c &= x_c \\
\eta^+ &= 0 \\
\tau^+ &\in [T_1, T_2]
\end{align*}
\]

\[
\mathcal{H} \left\{ (x_p, x_c, \eta, \tau) \in C \right\}
\]

\[
\mathcal{D} \left\{ (x_p, x_c, \eta, \tau) \in D \right\}
\]

**Problem #2: Dynamic Output Feedback Controller Design**

**Proposed solutions:**
Approach reminiscent of an “input-to-state stability small gain” philosophy.

\[
\Sigma_{x_{cl}}: \begin{bmatrix} \dot{x}_p \\ \dot{x}_c \end{bmatrix} = A \begin{bmatrix} x_p \\ x_c \end{bmatrix} + B \eta
\]

\[
\Sigma_\eta: \begin{cases} \begin{bmatrix} \dot{\eta} \\ \dot{\tau} \\ \eta^+ \end{bmatrix} = \begin{bmatrix} H\eta + J\dot{x} \\ -1 \\ 0 \end{bmatrix} & \text{if } \tau \in [0, T_2] \\ \begin{bmatrix} \eta \\ \tau \end{bmatrix} \in \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} & \text{if } \tau = 0 
\end{cases}
\]

\[
A := \begin{bmatrix} \frac{A_p + B_pD_cC_p}{B_cC_p} & \frac{B_pC_c}{A_c} \\ \frac{B_pD_c}{B_c} & \frac{B_pC_c}{A_c} \end{bmatrix}, \quad B := -\begin{bmatrix} \frac{B_pD_c}{B_c} \\ \frac{B_pC_c}{A_c} \end{bmatrix}
\]

\[
J := \begin{bmatrix} C_pA_p - HC_p & -E \end{bmatrix}
\]
Proposed solutions:
Approach reminiscent of an “input-to-state stability small gain” philosophy.

STEP 1
Continuous-time system

\[ \sum x_{cl} \]

\[ x_{cl} \]

\[ y_o \]

Hybrid system

\[ \sum \eta \]

\[ \omega \]

\[ \eta \]

STEP 2
Holding Device structured with intersample dynamics to make stability conditions suitable for controller design

Set conditions on Lyapunov function

\[ V_1(x_{cl}) := x_{cl}^TP_1x_{cl} \]

Overall system is stable

Set conditions on Lyapunov function

\[ V_2(\eta, \tau) := e^{\delta \tau} \eta^TP_2\eta \]
Problem #2: Dynamic Output Feedback Controller Design – Example

$L_2$ stability of a double integrator with $L_2$ gain less than or equal to 5. A value of $T_{mati} = 0.39 \text{ s}$ is obtained by design.

This material is based upon work supported by the National Science Foundation (NSF) under grant No. CNS-1544910. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Thanks to my former PhD student

Roberto Merco,

And collaborators:

Prof. Francesco Ferrante, Gipsa-Lab & University of Grenoble Alpes, France

Prof. Ricardo Sanfelice, University of California, Santa Cruz, CA

THANK YOU FOR YOUR ATTENTION!

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