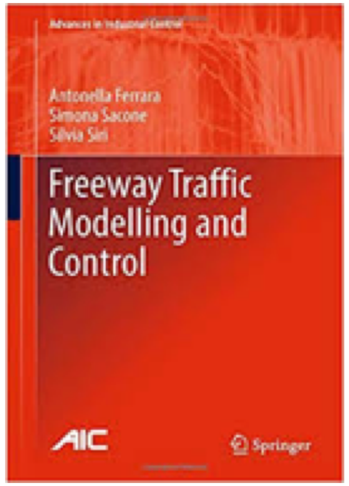
FROM CONNECTED AND AUTONOMOUS VEHICLES CONTROL TO VEHICULAR TRAFFIC CONTROL, A MULTI-SCALE PERSPECTIVE

Antonella Ferrara

University of Pavia

CREDITS



Annual Reviews in Control
Volume 48, 2019

Traffic control for freeway networks with sustainability-related objectives: Review and future challenges
C. Pasquale, S. Siri, S. Sacone, A. Ferrara

Transportation Research Part C: Emerging Technologies
Volume 97, 2018

A variable-length Cell Transmission Model for road traffic systems
C. Canudas-de-Wit, A. Ferrara

SIAM Journal on Applied Mathematics
Volume 80, 2020

A macroscopic model for platooning in highway traffic
G. Piacentini, P. Goatin, A. Ferrara

IEEE Control Systems Letters, to appear (Volume 5, April 2021)

Traffic Control Via Platoons of Intelligent Vehicles for Saving Fuel Consumption in Freeway Systems
G. Piacentini, P. Goatin and A. Ferrara

Other recent works with:
Kalle Johansson, Giulia Piacentini, Mladen Čičić

MOTIVATION

- **Efficient management of road traffic networks**: crucial in all the developed countries

- **Growth in the number of vehicles**:
  - **increase of congestion phenomena**
  - **increase of pollution and huge waste of time**

Milan, 7:21am on Sept 24th, 2019.
Photo Credit: Camilla Bastianon, my daughter

MOTIVATION

- **Direct road fatalities are strictly related to intense and not well regulated vehicular traffic**

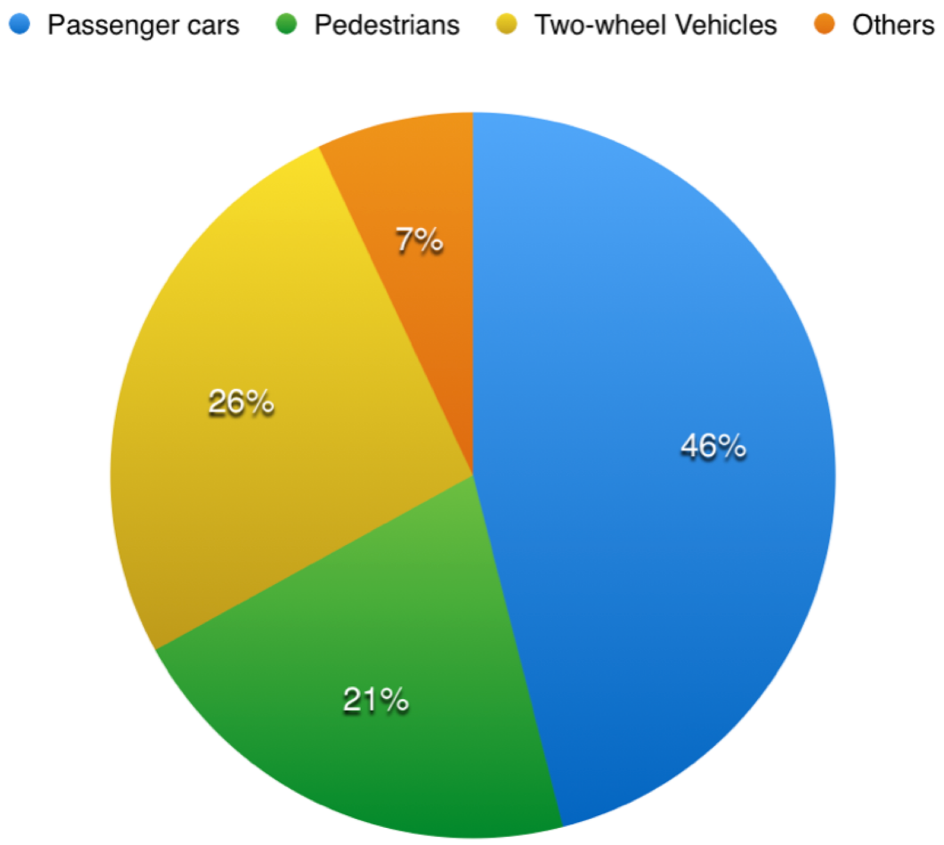
European Commission

European Commission - Fact Sheet

2018 Road safety statistics: what is behind the figures?

Brussels, 4 April 2019

The European Union has some of the safest roads in the world […]

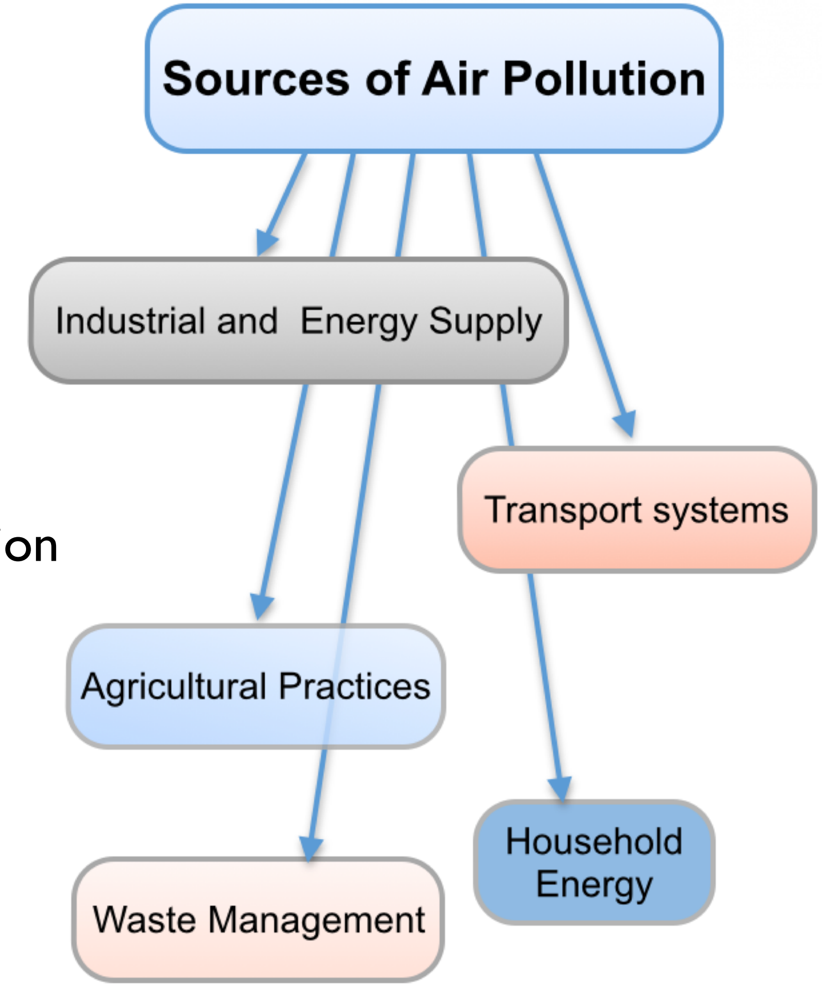


Road fatalities in the EU by transport mode in 2017
(Credits: Antonella Ferrara; Source of data: EC)

MOTIVATION

- **Indirect Fatalities**

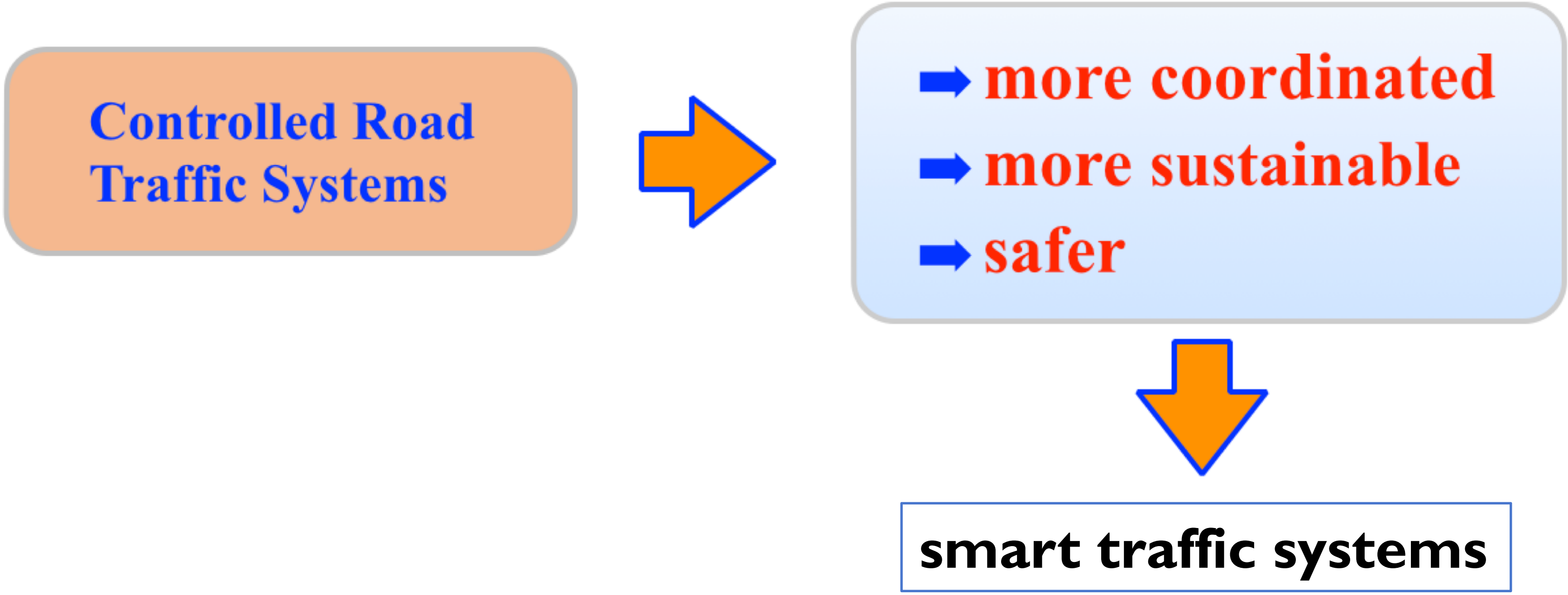
number of deaths which
can be attributed to air pollution



Source: 2018 World Health Organization (WHO) #UnitedNations' health agency

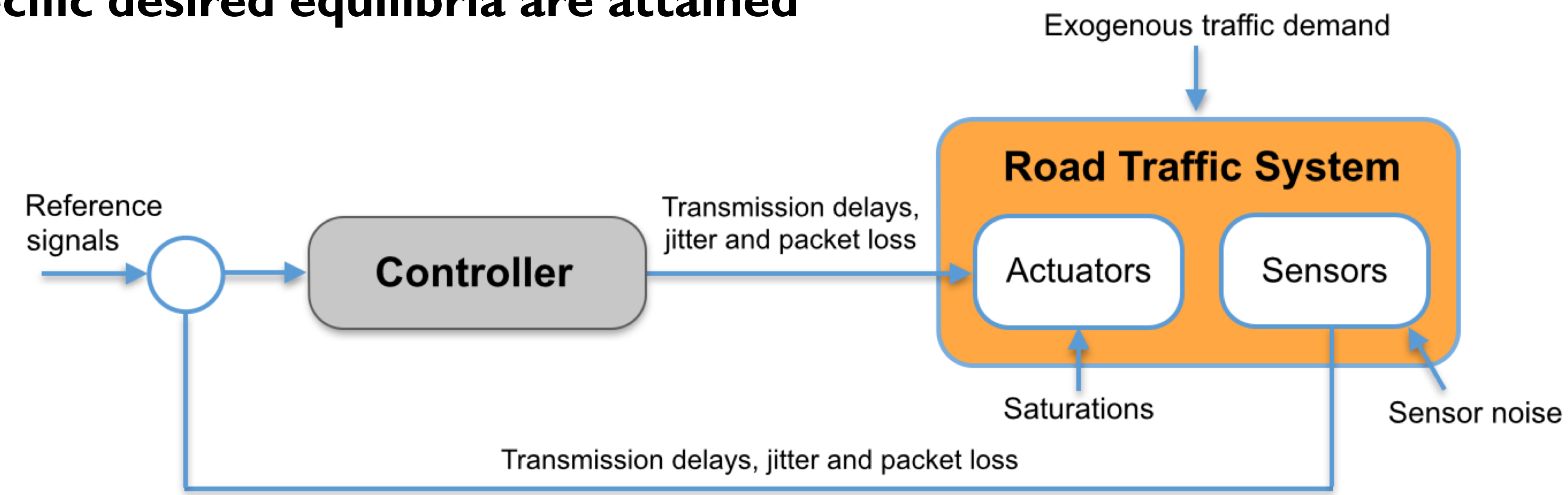
WHAT CAN CONTROL EXPERTS DO? THE ROLE OF CONTROL

- **New sensors, new actuators, new communication technologies**
- **Efficient resource-aware control strategies**



TRAFFIC CONTROL IN A NUTSHELL
WHAT DOES IT MEAN TO CONTROL ROAD TRAFFIC?

- **A road traffic system is a dynamical system (complex, large-scale, etc.)**
- **The traffic state evolution can be influenced by designed system inputs**
- **Specific desired equilibria are attained**



Credits: Antonella Ferrara

TRAFFIC CONTROL IN A NUTSHELL
WHICH ARE THE CONTROL OBJECTIVES?

- **Control Objectives**

  - Minimization of the **Total Time Spent [veh h] (TTS)** by vehicles in the road traffic network

  - Maximization of the **Total Traveled Distance [veh km] (TTD)** by vehicles in the road traffic network

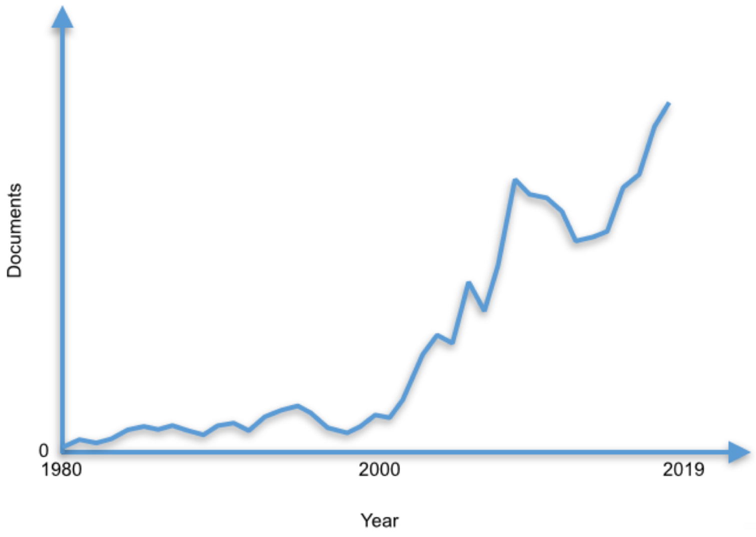
  - **Set-point Tracking** to maximize capacity exploitation, to penalize situations when the traffic density or the queue lengths at the on-ramps exceed given thresholds, etc.

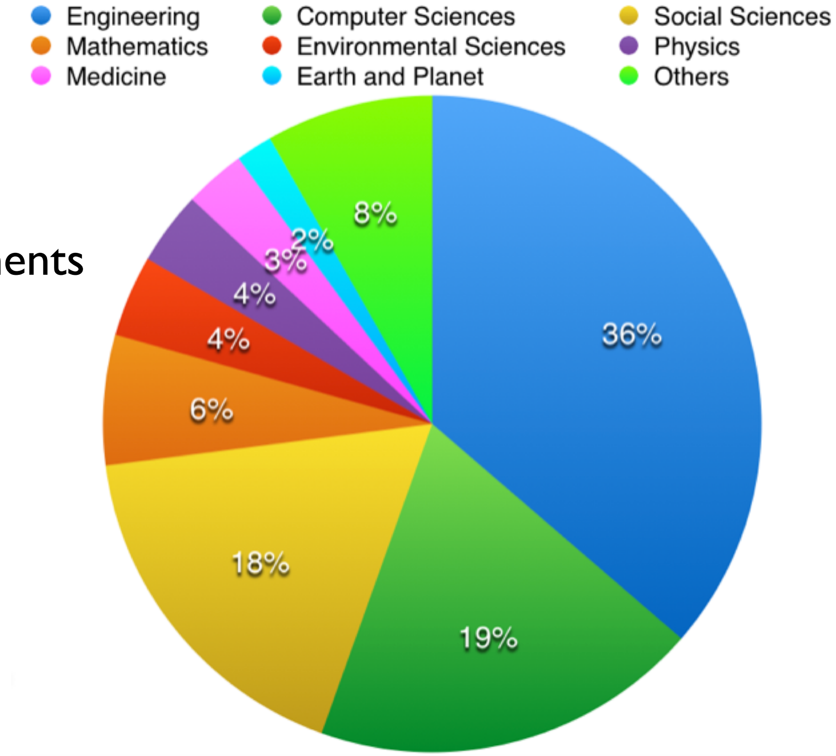
  - **Other objectives:** safety, emissions and noise reduction, energy efficiency, environmental risk mitigation, eco-driving, etc.

THE RESEARCH ON ROAD TRAFFIC CONTROL

- **SCOPUS Analytics**
  - KEY (road AND traffic AND control)
  - Selecting year range to analyze: 1980 to 2019

15.765 documents





Credits: Antonella Ferrara, Qualitative Plot and Pie Chart based on Scopus data

OUTLINE

- Traffic Modelling and control: classic methods
- Traffic Modelling and control: new methods
- Traffic models incorporating CAVs and their use for traffic control

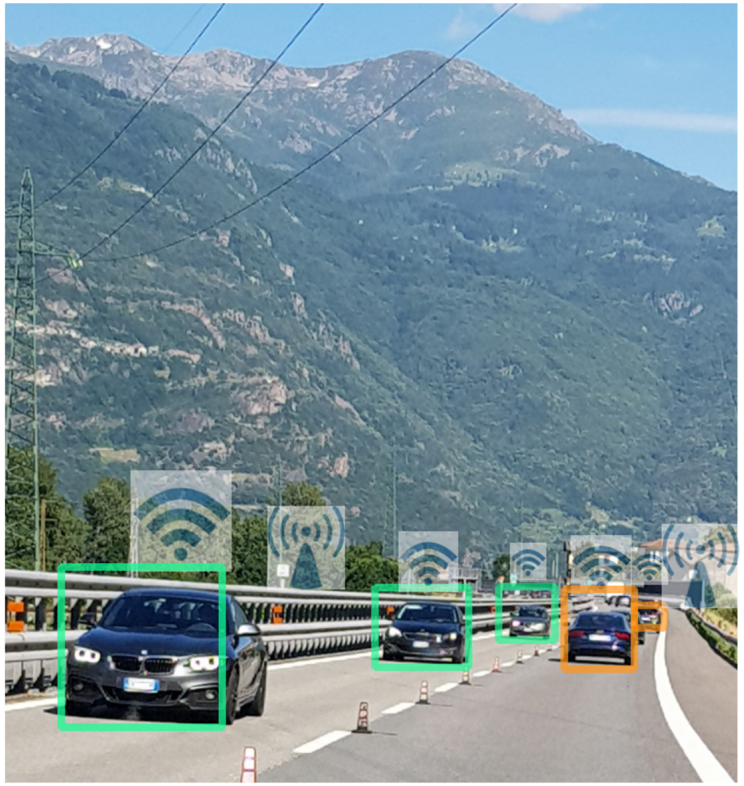


Photo Credit: Antonella Ferrara

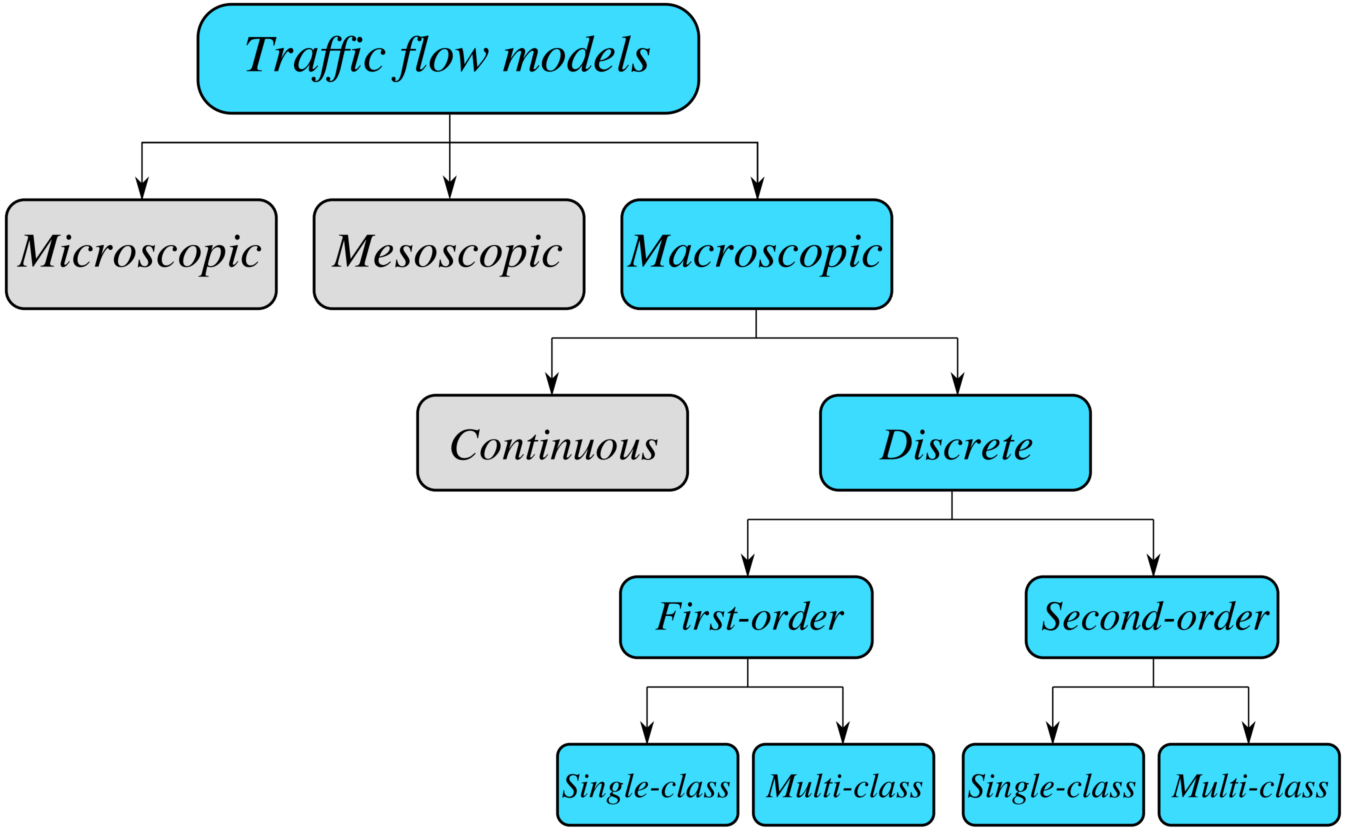
TRAFFIC MODELLING AND CONTROL: CLASSIC METHODS

DIFFERENT MODELS FOR DIFFERENT SCALES

- **Designing controllers**

↓

it is necessary to formulate a mathematical model of the process to control



C. Pasquale, S. Siri, S. Sacone, A. Ferrara, Traffic Control for Freeway Networks with Sustainability-Related Objectives: Review and Future Challenges, Annual Reviews in Control, 2019

DIFFERENT MODELS FOR DIFFERENT SCALES

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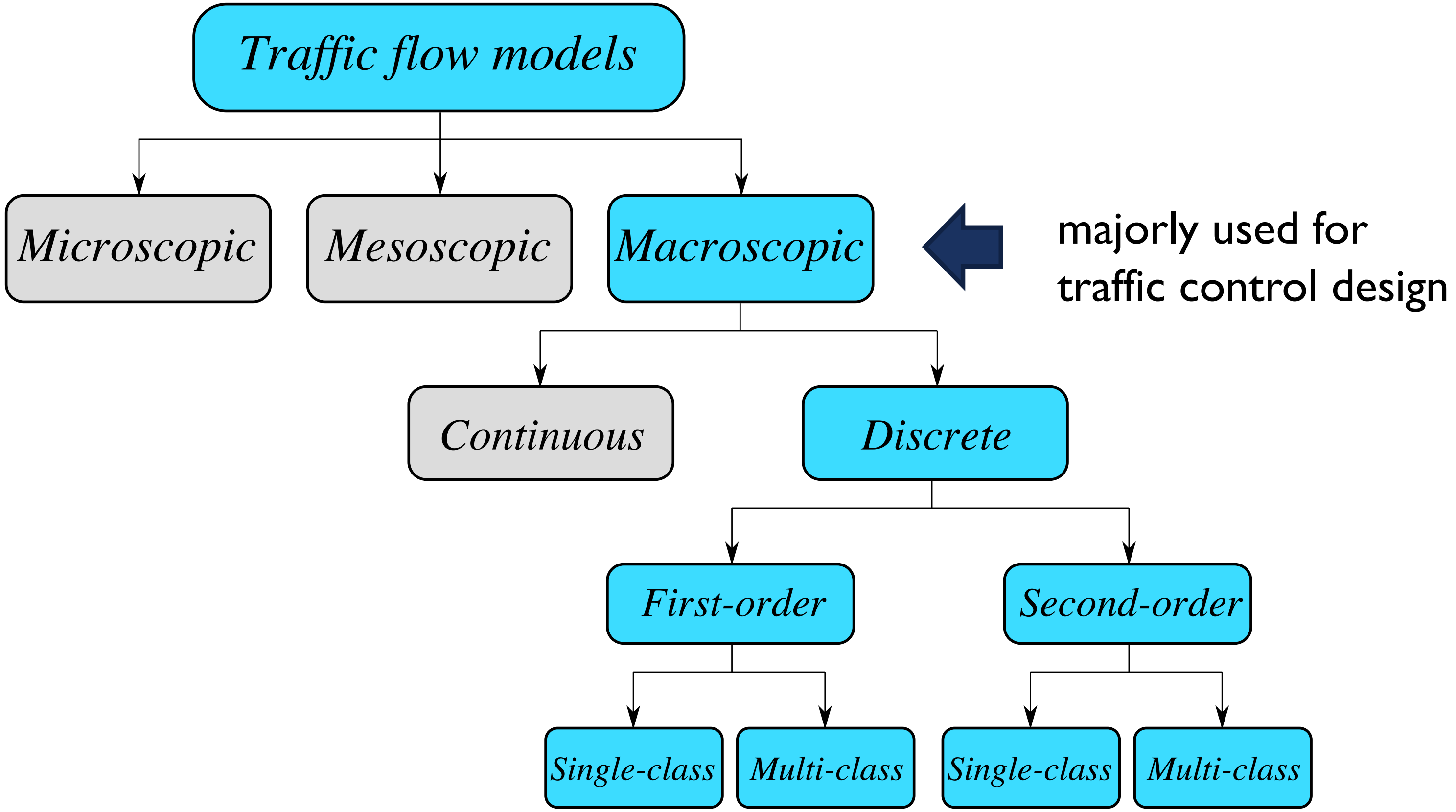


C. Pasquale, S. Siri, S. Sacone, A. Ferrara, Traffic Control for Freeway Networks with Sustainability-Related Objectives: Review and Future Challenges, Annual Reviews in Control, 2019

$$\frac{\partial \rho}{\partial t} + \frac{\partial f(\rho)}{\partial x} = 0$$

THE LIGHTHILL, WHITHAM AND RICHARDS MODEL (LWR MODEL)

Macroscopic variables:

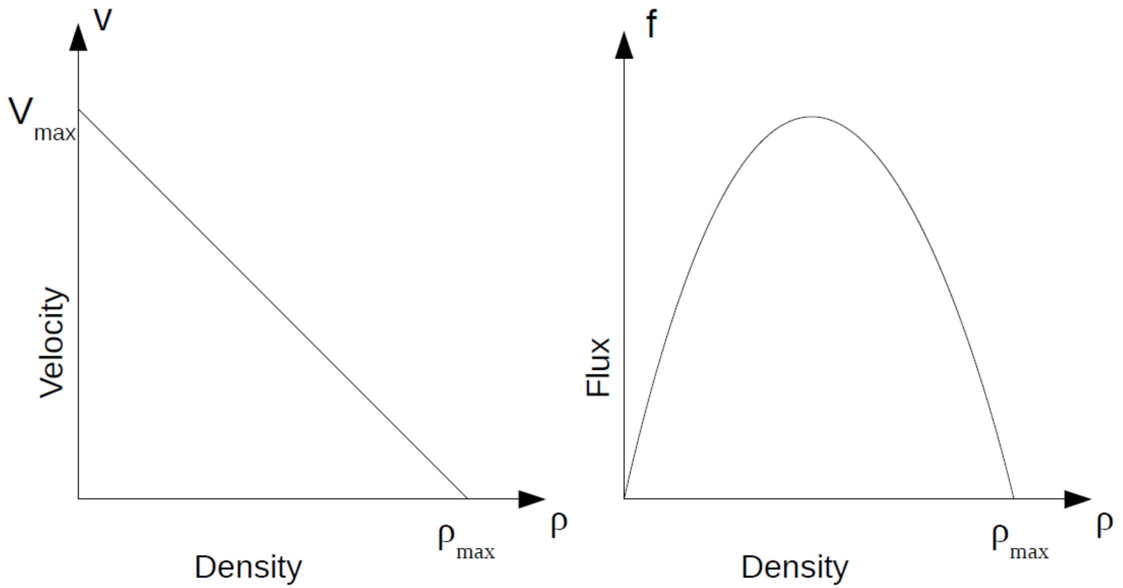
- ρ is the **density** of vehicles.
- **v** is the average **speed** of the flow.
- **f**=ρv is the **flow**.

- macroscopic model based on the equation of vehicles conservation
- Traffic Fundamental Diagram: the theoretical relation between flow and density in steady-state conditions

M.J. Lighthill, G.B. Whitham, On Kinematic Waves II: A theory of Traffic Flow on Long Crowded Roads, Proc. of The Royal Society A 229 (1955)

P.I. Richards, Shock Waves on the Highway, Operations Research 4 (1956)

Fundamental diagram

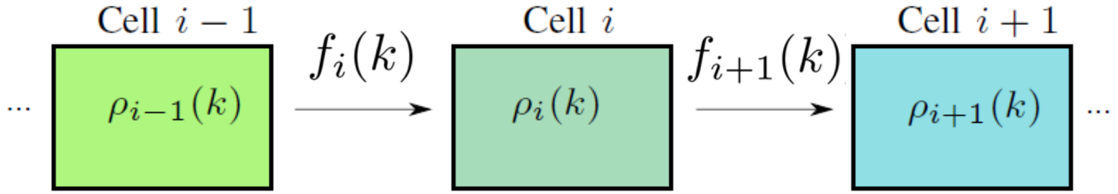


In the LWR Model $f(\rho(x,t))$ is a strictly concave C^2 function, $f(0) = 0$ and $f(\rho max) = 0$

THE CELL TRANSMISSION MODEL (CTM MODEL)

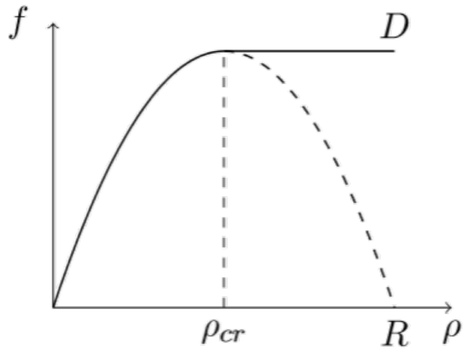
- Discretized version of the LWR model
- v is the free-flow speed
- ω is the congestion wave speed

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L}\left[f_i(k) - f_{i+1}(k)\right]$$

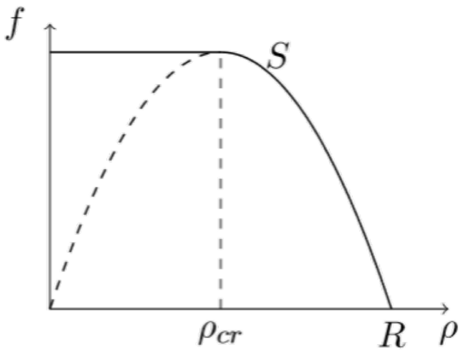


C.F. Daganzo, The Cell Transmission Model: A Dynamic Representation of Highway Traffic Consistent with the Hydrodynamic Theory, Transportation Research Part B 28 (1994)

Demand :



Supply:



$$D_i(k) = \begin{cases} f(\rho_i(k)) & \text{if } \rho_i(k) < \rho_{cr}, \\ f^{max} & \text{if } \rho_i(k) \geq \rho_{cr}, \end{cases} \qquad S_i(k) = \begin{cases} f^{max} & \text{if } \rho_i(k) < \rho_{cr}, \\ f(\rho_i(k)) & \text{if } \rho_i(k) \geq \rho_{cr}, \end{cases}$$

$$f_i(k) = \min\{D_{i-1}(k), S_i(k)\}$$

MERGE AND DIVERGE

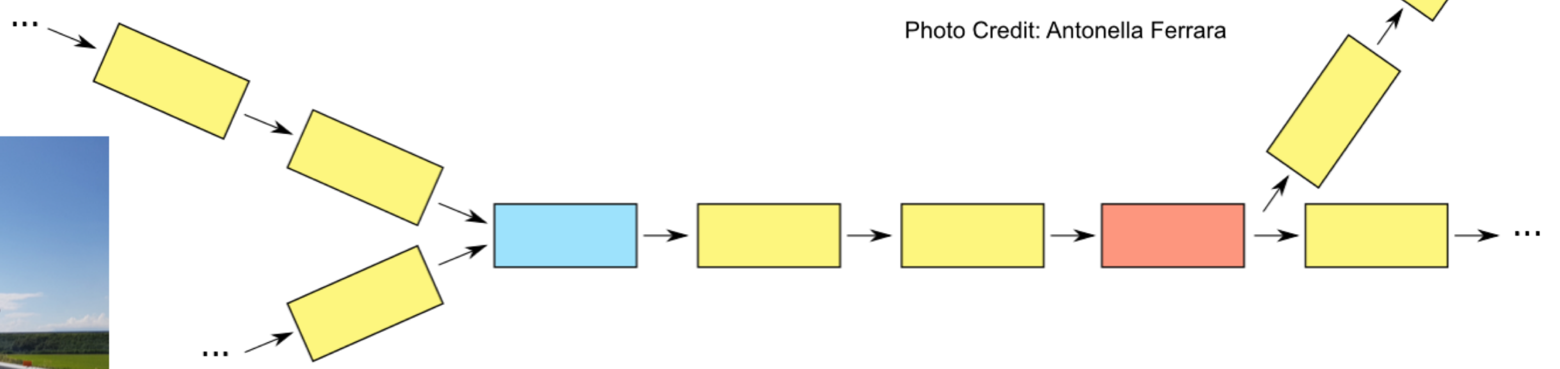


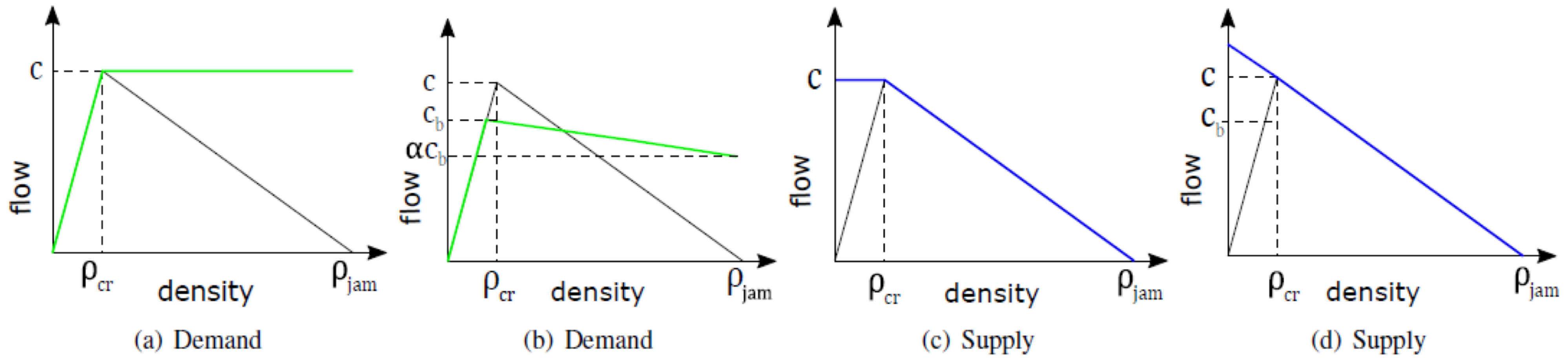
Photo Credit: Antonella Ferrara

Photo Credit: Antonella Ferrara

C.F. Daganzo, *The Cell Transmission Model, Part II: Network Traffic*, Transportation Research Part B 29 (1995)

CAPACITY DROP MODEL

- In the presence of a bottleneck the discharge flow is lower than the capacity of the bottleneck.
- The flow reduction is around the 5-20 %.
- The CTM is not able to capture such a phenomena. An extension to model the capacity drop:



Piacentini G., Ferrara A., Papamichail I., Papageorgiou M., 58th Conference on Decision and Control (CDC 2019), Nice, France, 2019

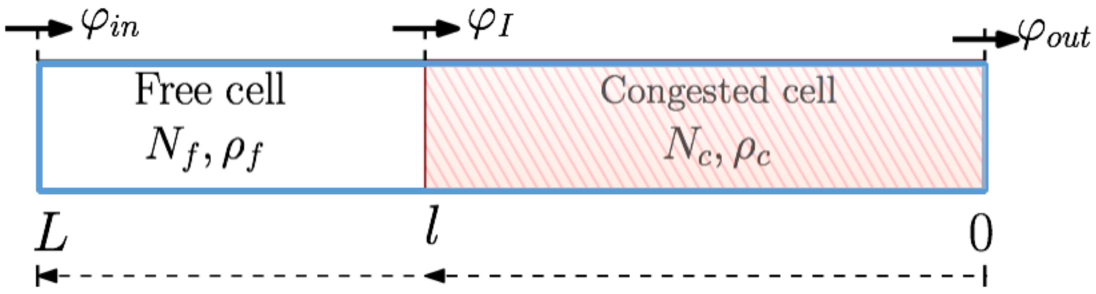
$$\dot{\rho}_f = \frac{1}{L-l}\left[\varphi_{in} - \Phi(\rho_f)\right]$$
$$\dot{\rho}_c = \frac{1}{l}\left[\Phi(\rho_c) - \varphi_{out}\right]$$
$$\dot{l} = \frac{\Phi(\rho_f) - \Phi(\rho_c)}{\rho_c - \rho_f}$$

C. Canudas de Wit, A. Ferrara, A Variable-Length Cell Transmission Model for Road Traffic Systems, Transportation Research Part C, Dec. 2018

THE VARIABLE LENGTH CELL TRANSMISSION MODEL (VLM)

Macroscopic variables:

- Continuous-time lumped variables model based on **three state variables** only.
- Simpler than classical continuous macroscopic models.
- it captures relevant phenomena of traffic dynamics such as **shock waves** and **rarefaction waves propagation.**



OTHER MACROSCOPIC MODELS

Asymmetric Cell Transmission Model (ACTM), different "merge" description

G. Gomes, R. Horowitz, Optimal Freeway Ramp Metering Using the Asymmetric Cell Transmission Model, Transportation Research Part C 14 (2006)

Multi-Class Versions of the CTM, different classes of vehicles (car/trucks)

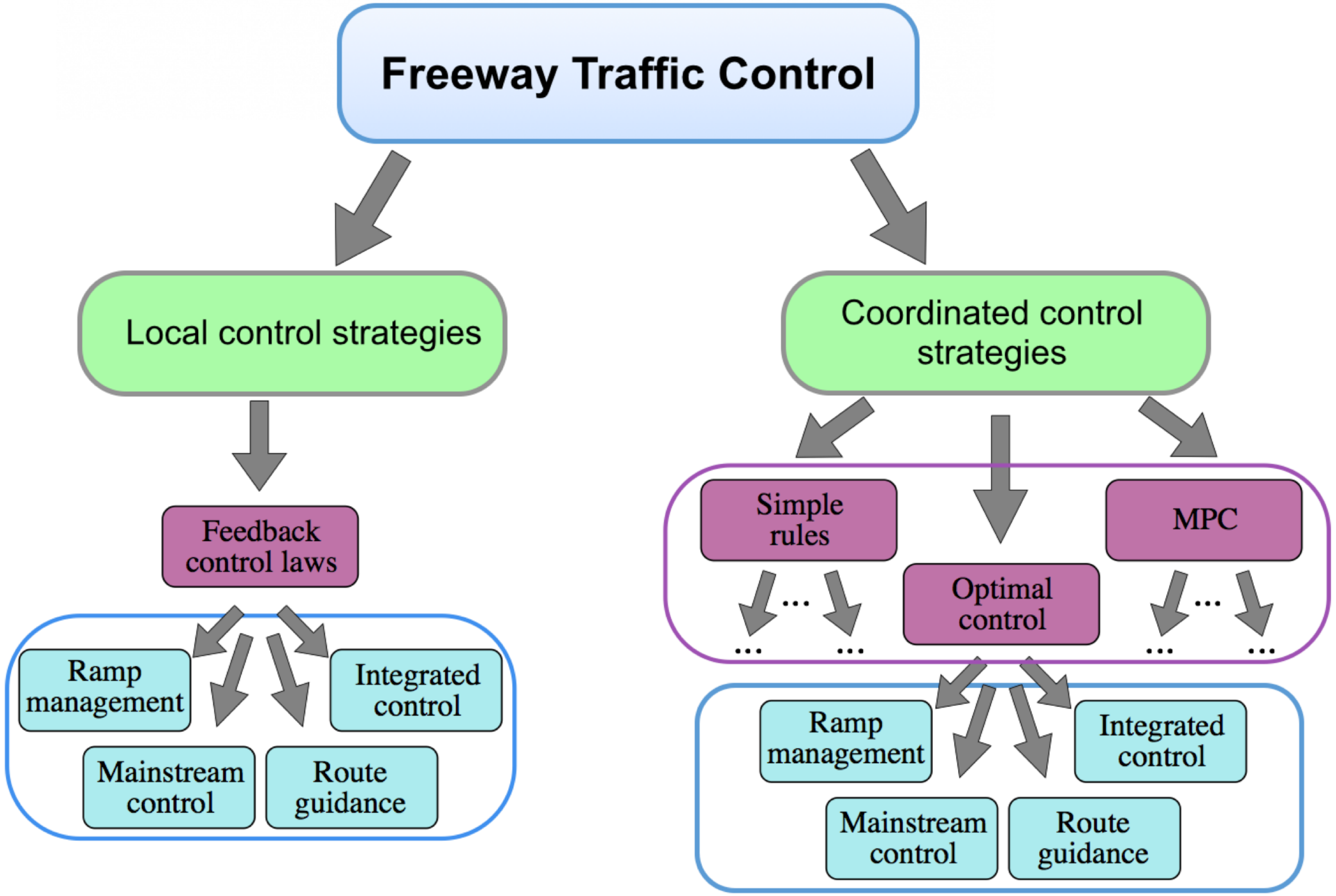
K. Tuerprasert, C. Aswakul, Multiclass Cell Transmission Model for Heterogeneous Mobility in General Topology of Road Network, Journal of Intelligent Transportation Systems 14 (2010)

METANET, discrete, second order

A. Messmer, M. Papageorgiou, *METANET: A Macroscopic Simulation Program for Motorway Networks*, Traffic Engineering & Control 31 (1990)

Ferrara A., Sacone S., Siri S. (2018), Freeway Traffic Modelling and Control, Part II Freeway Traffic Modelling. Advances in Industrial Control, Springer

CLASSICAL TRAFFIC CONTROL

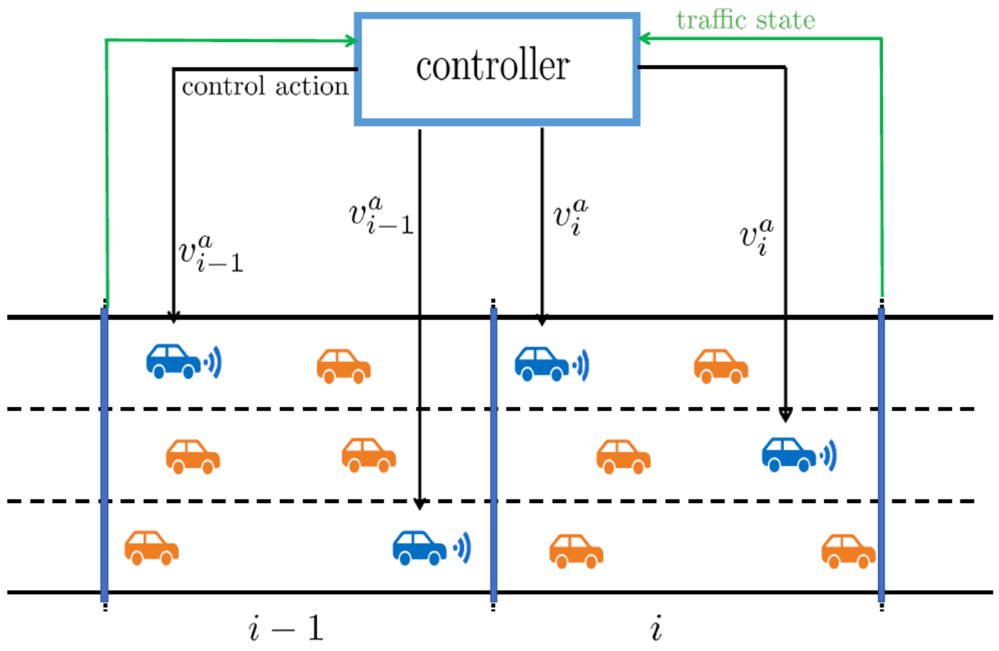


Ferrara A., Sacone S., Siri S. (2018), An Overview of Traffic Control Schemes for Freeway Systems. In: Freeway Traffic Modelling and Control. Springer

TRAFFIC MODELLING AND CONTROL: NEW METHODS

A PARADIGM SHIFT: CONNECTED AND AUTOMATED VEHICLES (CAVS) WHICH ROLE IN ROAD TRAFFIC CONTROL?

Given a traffic system with mixed traffic, different possibilities to control it by controlling CAVs:



- **Controlled as a macroscopic vehicular class***

*G. Piacentini, M. Čičić, A. Ferrara, K.H. Johansson, VACS Equipped Vehicles for Congestion Dissipation in Multi-Class CTM Framework, European Control Conference ECC 2019, Naples, Italy

CONNECTED AND AUTOMATED VEHICLES (CAVS): WHICH ROLE IN ROAD TRAFFIC CONTROL?

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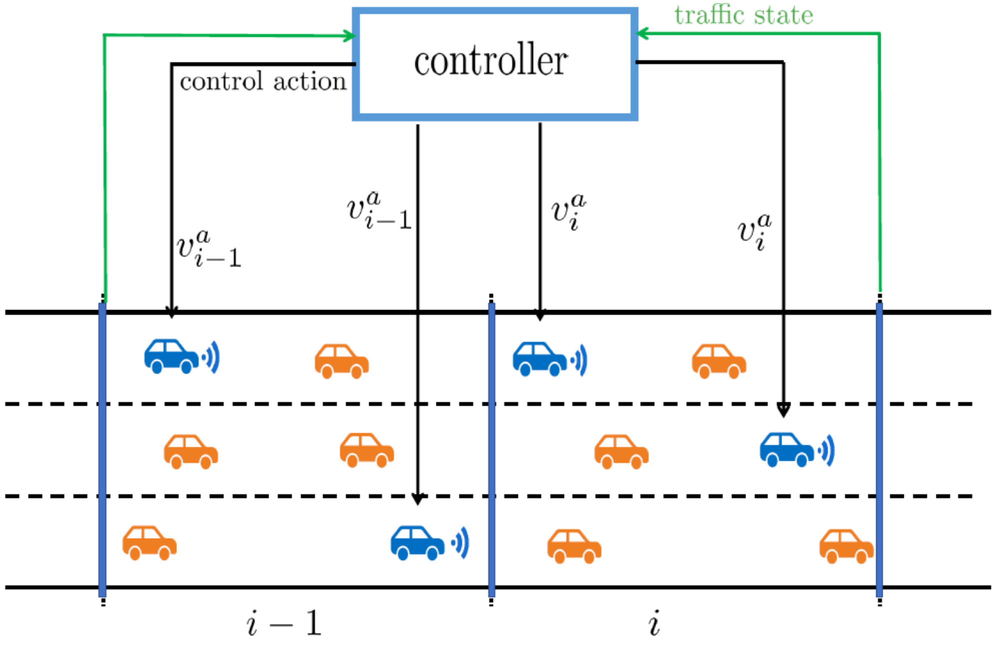

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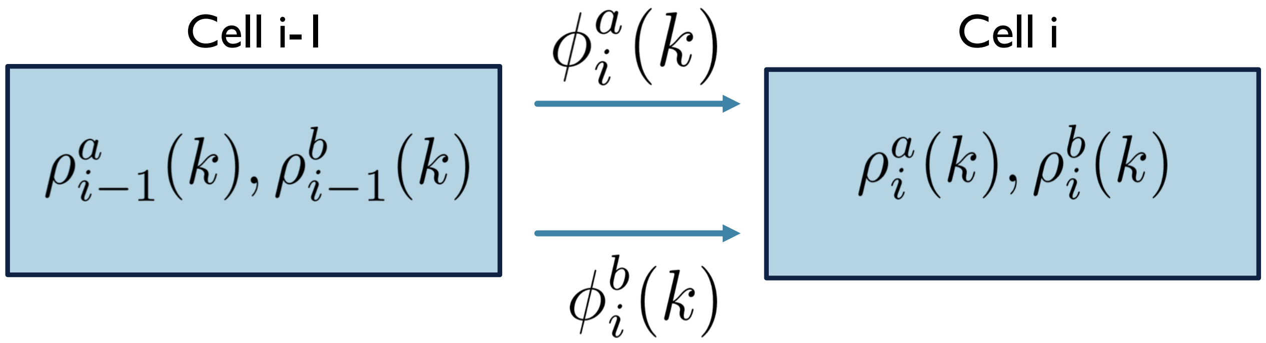

- Class a: **CAVs**, headway h_a
- Class b: **human-driven/conventional vehicles**, headway h_b

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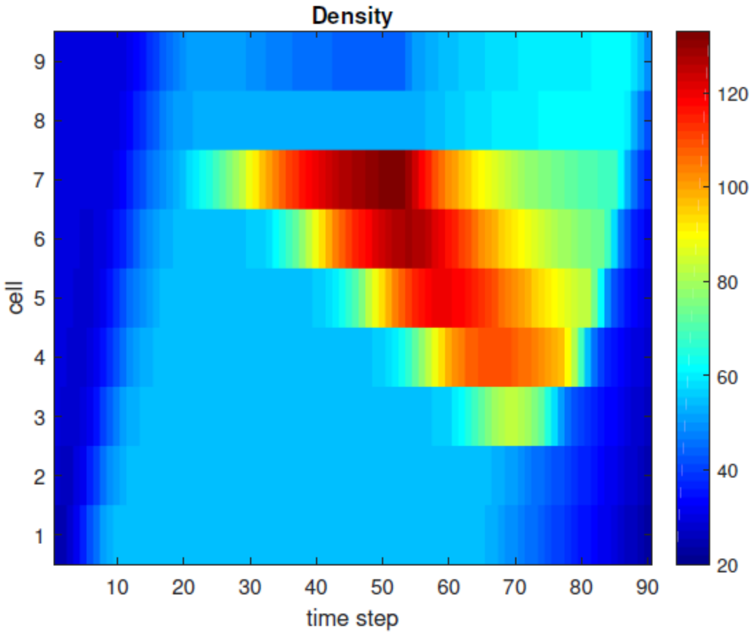
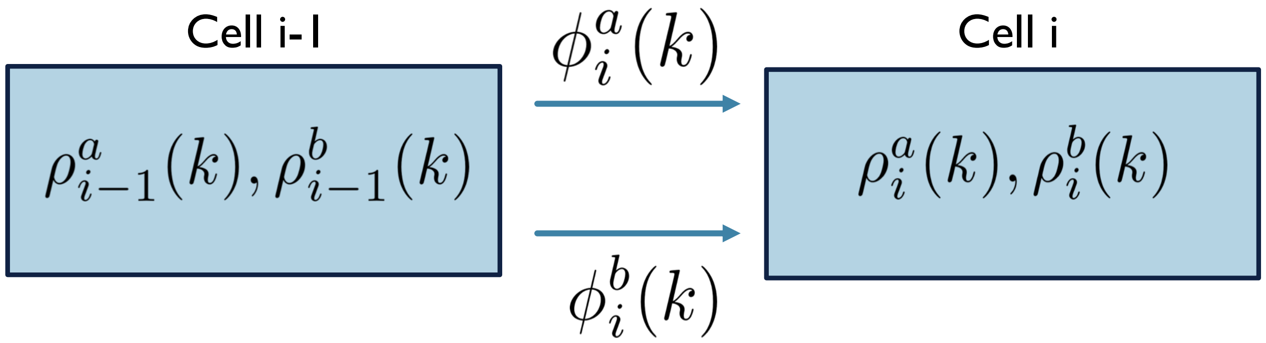


No control

- **Controlled as a macroscopic vehicular class***



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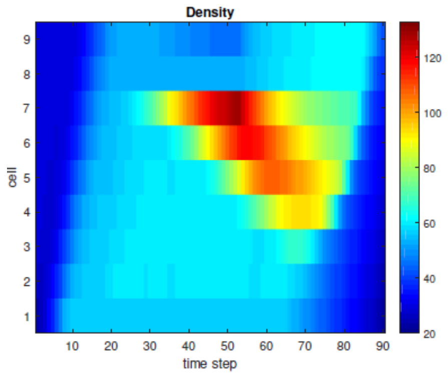
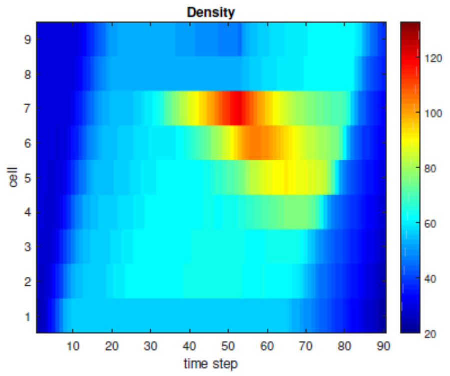
SIMULATION RESULTS

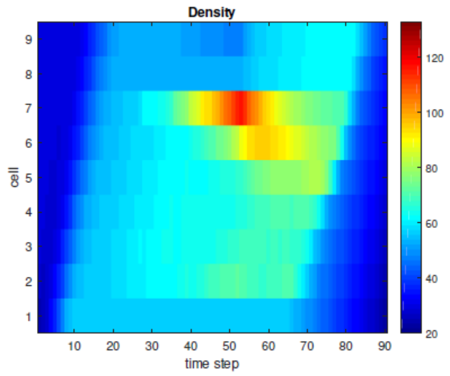
- Trend of the density with different penetration rate of CAVs.
- The effectiveness of the control increases if the penetration rate increases.



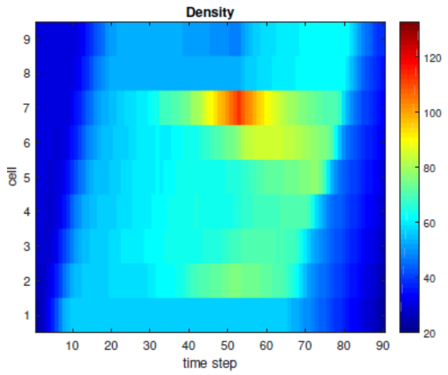
(a) penetration rate ~ 0.1



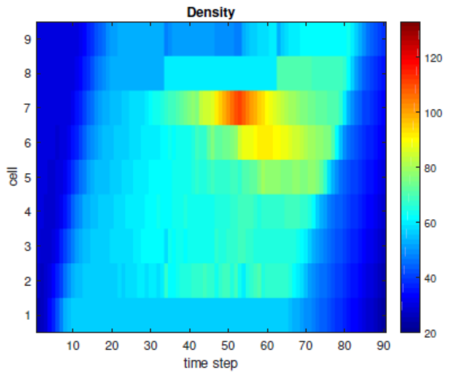
(b) penetration rate ~ 0.2



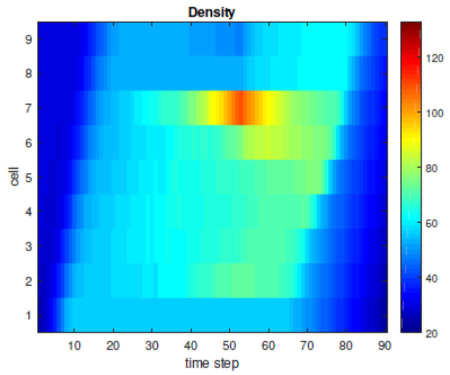
(c) penetration rate ~ 0.3



(d) penetration rate ~ 0.4



(e) penetration rate ~ 0.5

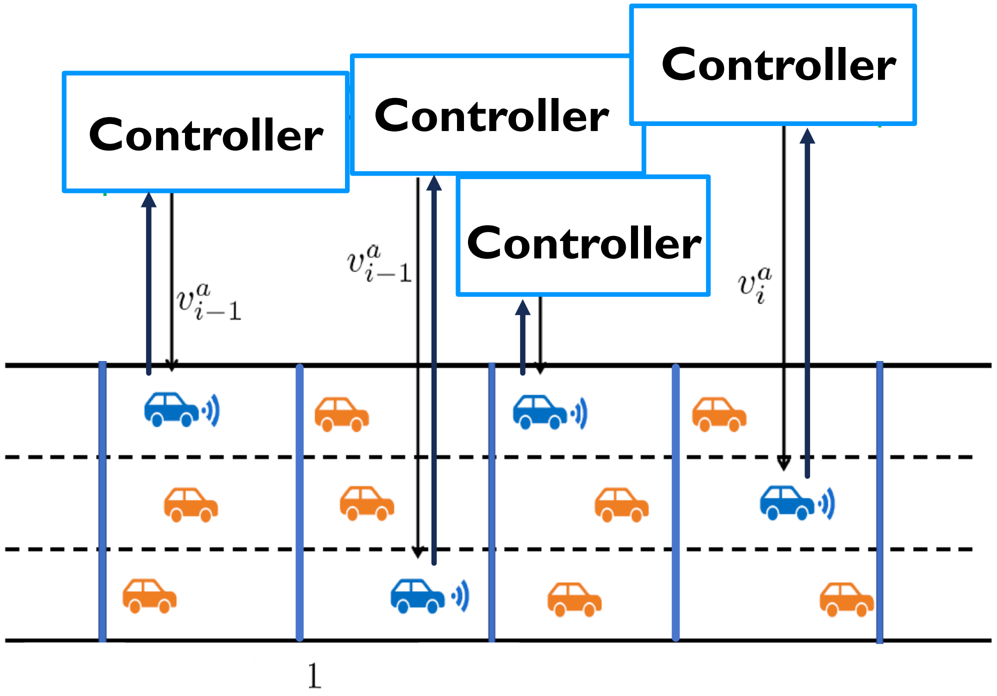


(f) penetration rate ~ 0.6

*G. Piacentini, M. Čičić, A. Ferrara, K.H. Johansson, VACS Equipped Vehicles for Congestion Dissipation in Multi-Class CTM Framework, European Control Conference ECC 2019, Naples, Italy

CONNECTED AND AUTOMATED VEHICLES (CAVS): WHICH ROLE IN ROAD TRAFFIC CONTROL?

Different possibilities to control the traffic system by controlling CAVs:



- Controlled as a macroscopic vehicular class
- **Controlled individually**

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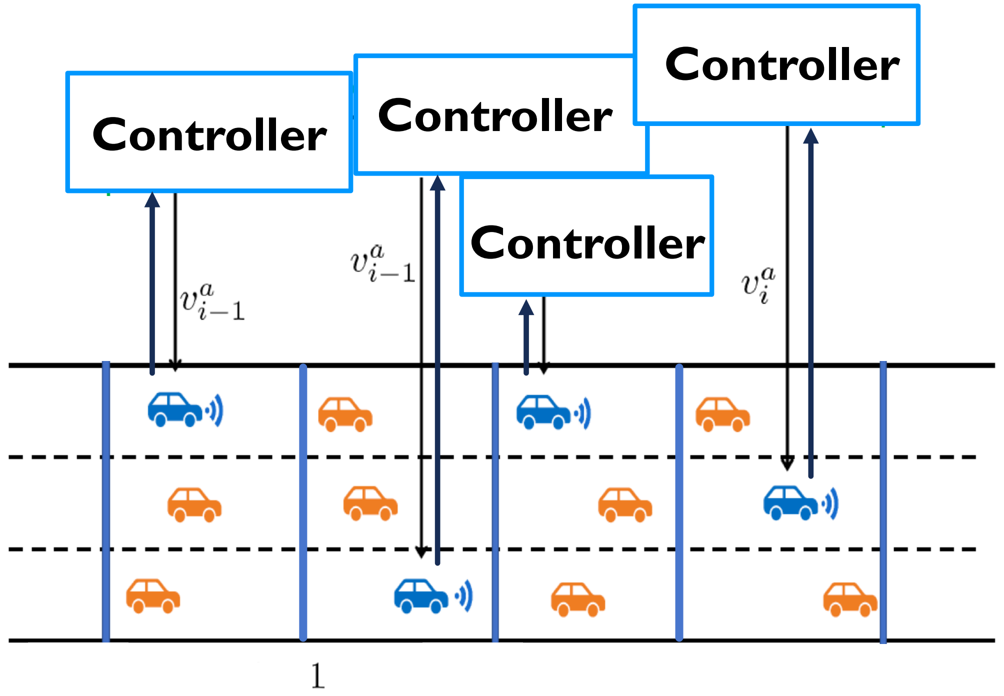

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Balance between selfish behaviours and social optimum

CONNECTED AND AUTOMATED VEHICLES (CAVS): WHICH ROLE IN ROAD TRAFFIC CONTROL?

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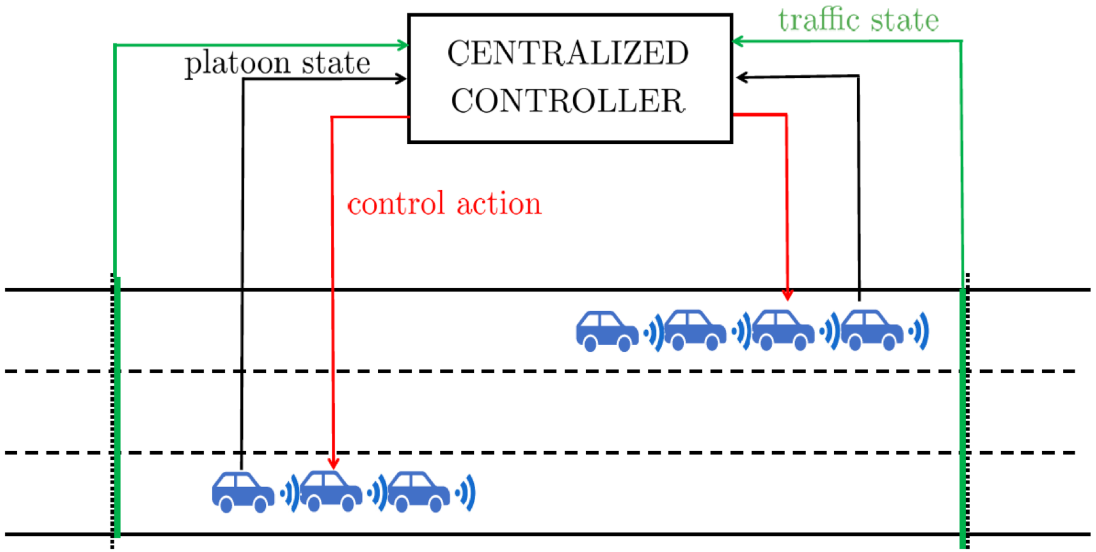

- Controlled as a macroscopic vehicular class
- Controlled individually
- **Controlled to create formations (e.g. platoons)**

CONNECTED AND AUTOMATED VEHICLES (CAVS): WHICH ROLE IN ROAD TRAFFIC CONTROL?

Different possibilities to control the traffic system by controlling CAVs:

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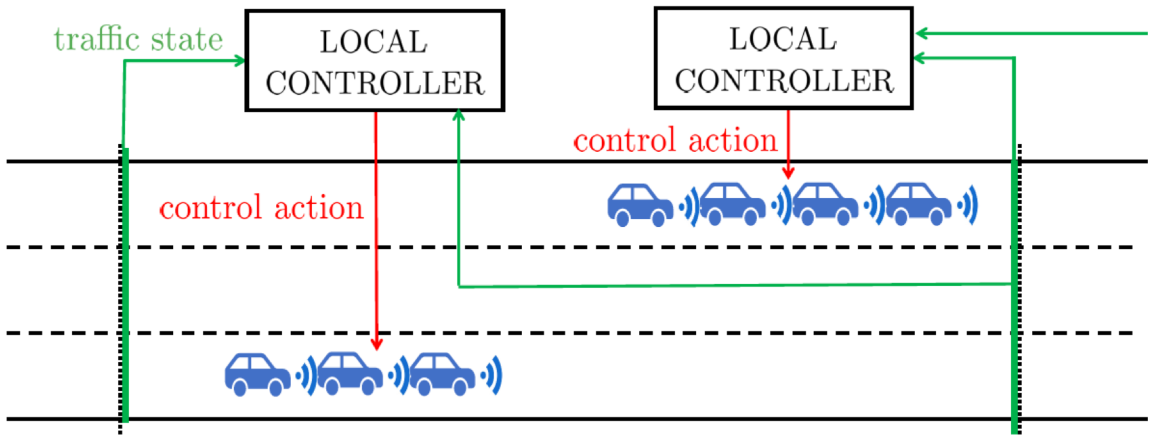
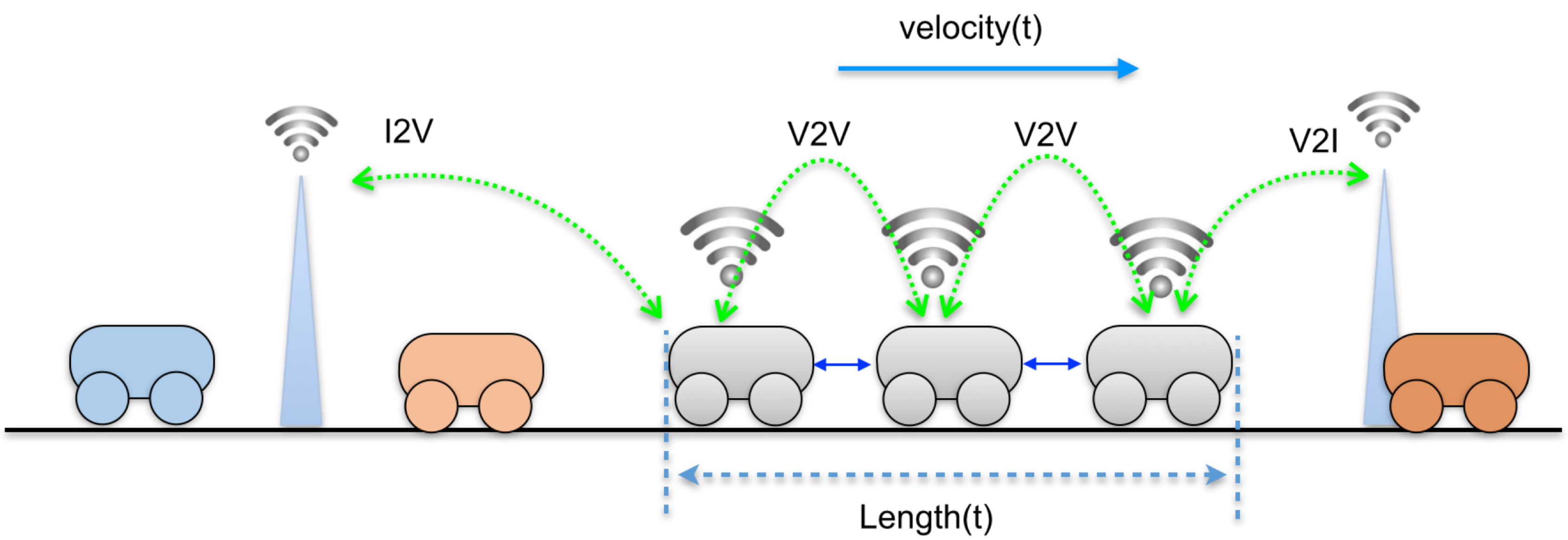
- Controlled individually
- **Controlled to create formations (e.g. platoons)**



CONNECTED AND AUTOMATED VEHICLES (CAVS): WHICH ROLE IN ROAD TRAFFIC CONTROL?

Platooning/vehicle formation generation to create **artificial moving bottlenecks** to be used as a kind of **special actuators** to control road traffic



Credits: Antonella Ferrara

M. Zambelli, P. Carulli, M. Steinberger, M. Horn, A. Ferrara. A Novel Formation Creation Algorithm for Heterogeneous Vehicles in Highway Scenarios: Assessment and Experimenta Validation, IFAC World Congress 2020

CONNECTED AND AUTOMATED VEHICLES (CAVS): WHICH MODELS FOR CONTROL DESIGN?

The different possibilities to control traffic systems by controlling CAVs require different types of models:

Single-scale modelling	⇐	▪ **CAVs controlled as a macroscopic vehicular class**
Multi-scale modelling	⇐	▪ **CAVs controlled individually**
Multi-scale modelling	⇐	▪ **CAVs controlled to create formations (e.g. platoons)**

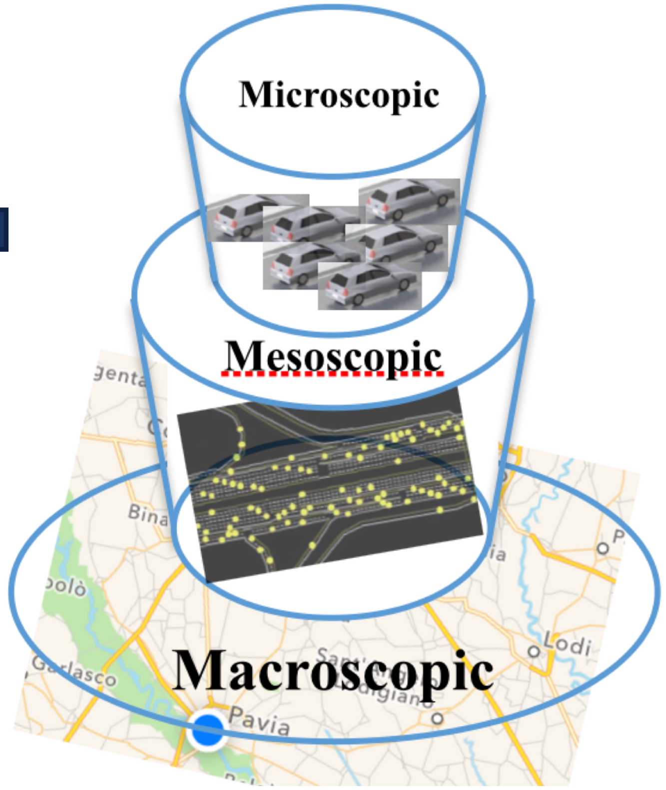
CONNECTED AND AUTOMATED VEHICLES (CAVS): WHICH MODELS FOR CONTROL DESIGN?

In case of multi-scale models:

Single CAV

Platoons of CAVs

Road traffic



MULTI-SCALE TRAFFIC MODELS INCORPORATING CAVs AND THEIR USE FOR TRAFFIC CONTROL

CTM FOR TRAFFIC WITH PLATOONS OF CAVS

EXTENSION OF THE CTM MODEL TO INCLUDE PLATOONS OF CAVS (I.E. MOVING BOTTLENECKS)

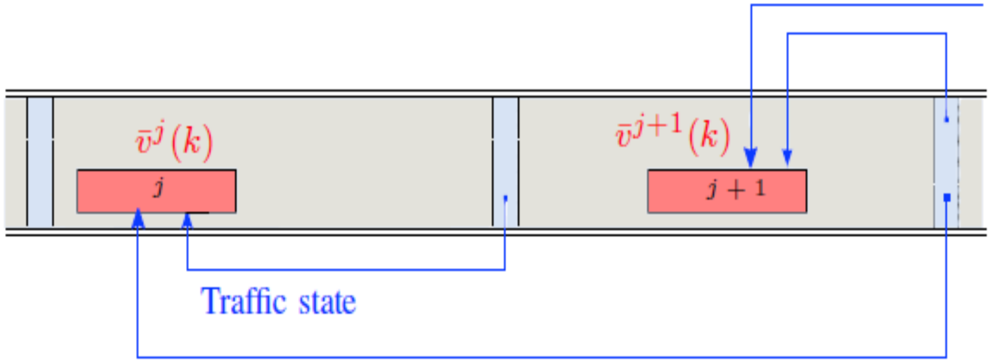
The dynamic state equation of ρ is extended in order to consider the presence of moving bottlenecks

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L}\left[f_i(k) - f_{i+1}(k)\right] + \sum_{j=1}^{J}\left[\delta_i^j(k)\frac{o^j}{L} - \delta_{i+1}^j(k)\frac{o^j}{L}\right]$$

where o^j is the **occupancy of the moving bottleneck** j=1,…, J,
δ is a **binary variable** adopted to indicate the **entrance of the moving bottlenecks in a cell**.

The presence of the platoon in cell modifies the **free flow speed** v:

$$v_i(k) = \begin{cases} f(\bar{v}^j(k)) & \text{if the moving bottleneck } j \text{ is in} \\ & \text{cell } i \text{ at time } k \\ v_i^{\text{free}} & \text{otherwise} \end{cases}$$



EXTENSION OF THE CTM MODEL TO INCLUDE PLATOONS OF CAVS

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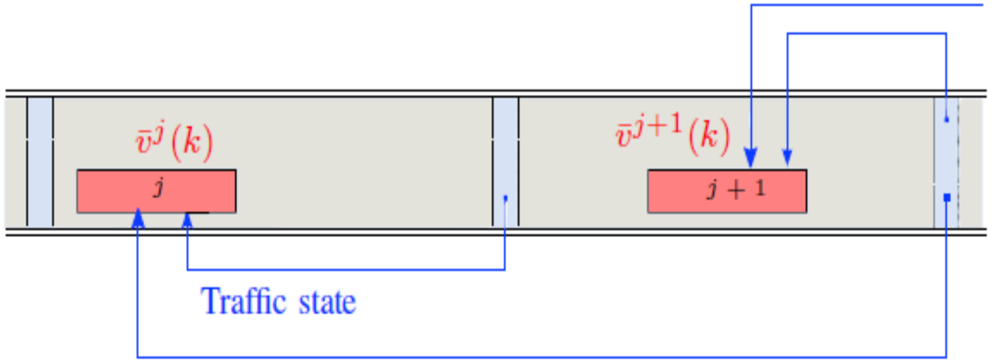


EXTENSION OF THE CTM MODEL TO INCLUDE PLATOONS OF CAVS

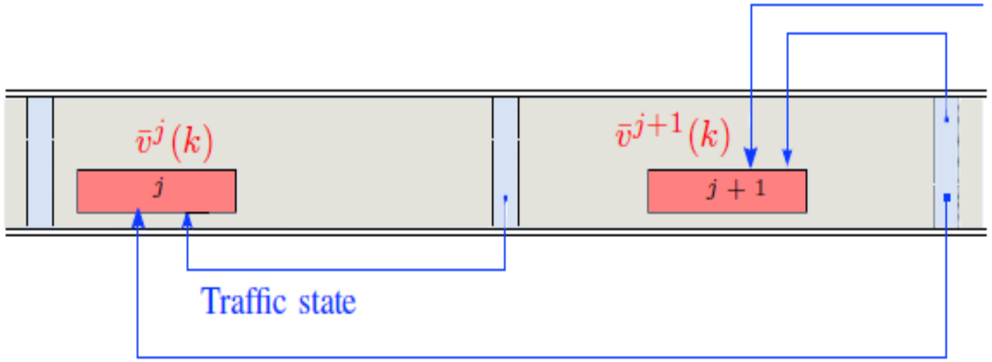
The dynamic state equation of ρ is extended in order to consider the presence of moving bottlenecks

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L}\left[f_i(k) - f_{i+1}(k)\right] + \sum_{j=1}^{J}\left[\delta_i^j(k)\frac{o^j}{L} - \delta_{i+1}^j(k)\frac{o^j}{L}\right]$$

where o^j is the **occupancy of the moving bottleneck** j=1,…, J,
δ is a **binary variable** adopted to indicate the **entrance of the moving bottlenecks in a cell**.

The presence of the platoon in cell modifies the **free flow speed** v:

$$v_i(k) = \begin{cases} f(\bar{v}^j(k)) & \text{if the moving bottleneck } j \text{ is in} \\ & \text{cell } i \text{ at time } k \\ v_i^{\text{free}} & \text{otherwise} \end{cases}$$



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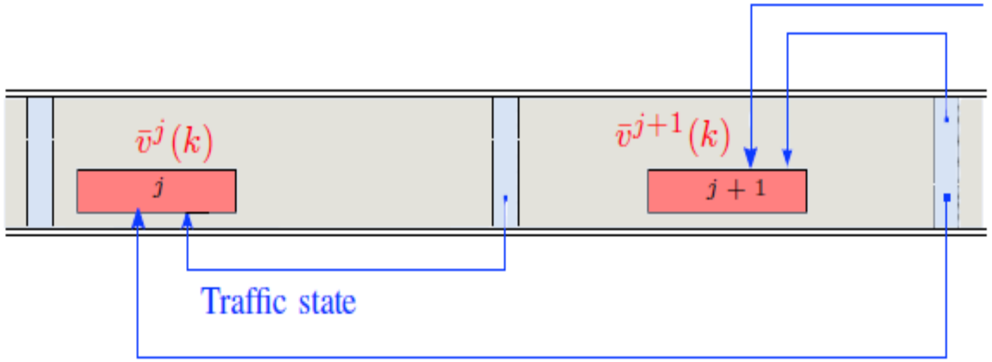
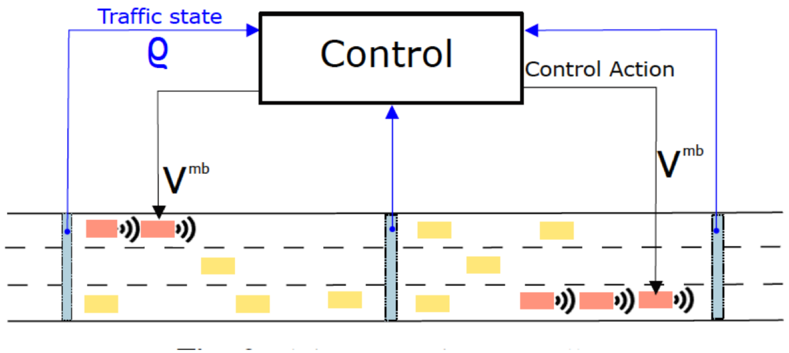


MPC CONTROL

Control variable: the speed of the platoons of CAVs traveling in the traffic system



Control Problem: find the optimal control sequence $\underline{u}(h), h = k \ldots k + K_p$ that minimizes the cost function:

$$C = \beta_1 T \sum_{h=k}^{k+K_p} \sum_{i=1}^{N} L_i \rho_i(k) - \beta_2 \sum_{h=k}^{k+K_p} \phi_{\bar{i}}(k) - \beta_3 \sum_{h=k}^{k+K_p} |\rho_{\bar{i}}(k) - \rho^{cr}|$$

Total Travel Time

Max the discharge flow from the bottleneck

Density error to keep the density below its critical value

Piacentini G., Ferrara A., Papamichail I., Papageorgiou M., 58th Conference on Decision and Control (CDC 2019), Nice, France, 2019

SIMULATION RESULTS

L	500 [m]
N	16
T	10 [s]
K	720
Simulation time	2 [h]
ρ_{cr}	70 [veh/km]
ρ_{jam}	320 [veh/km]
c	6000 [veh/h]
α	0.83
Bottleneck cell	13
Bottleneck capacity	5400 [veh/h]

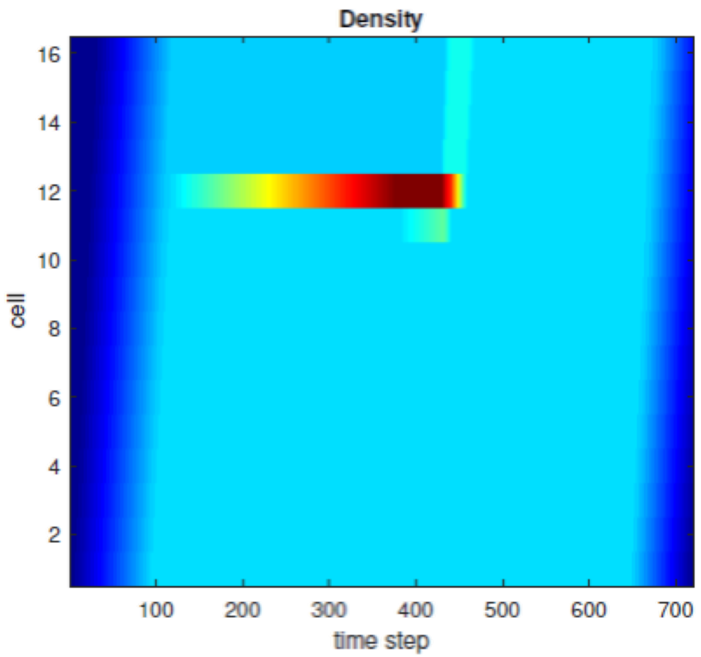
To create congestion, a **temporary physical bottleneck** is simulated in cell i = 13. It reduces the capacity to c_{13} = 5400 [veh/h] for k < 540, then it is restored to 6000 [veh/h].

For the MPC, the **prediction horizon** K_p is 20 time steps.

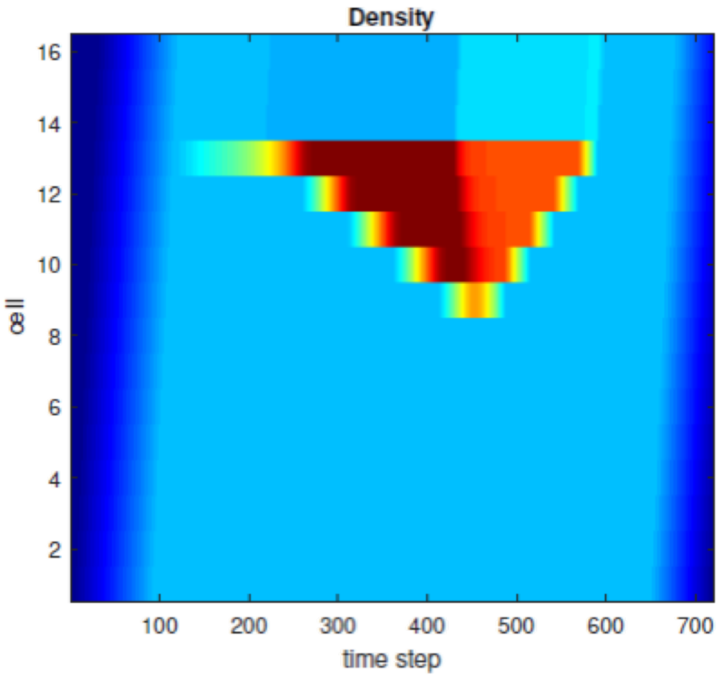
During the **2 hours of simulation** several CAVs enter the stretch and are controlled creating **13 platoons of 2 vehicles**.

Piacentini G., Ferrara A., Papamichail I., Papageorgiou M., 58th Conference on Decision and Control (CDC 2019), Nice, France, 2019

SIMULATION RESULTS



CTM no capacity drop, no control



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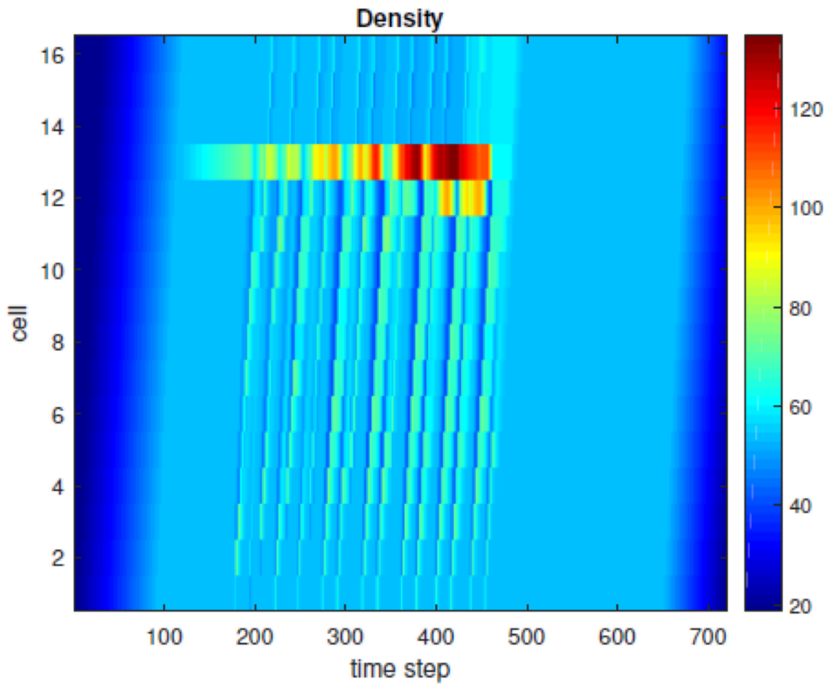
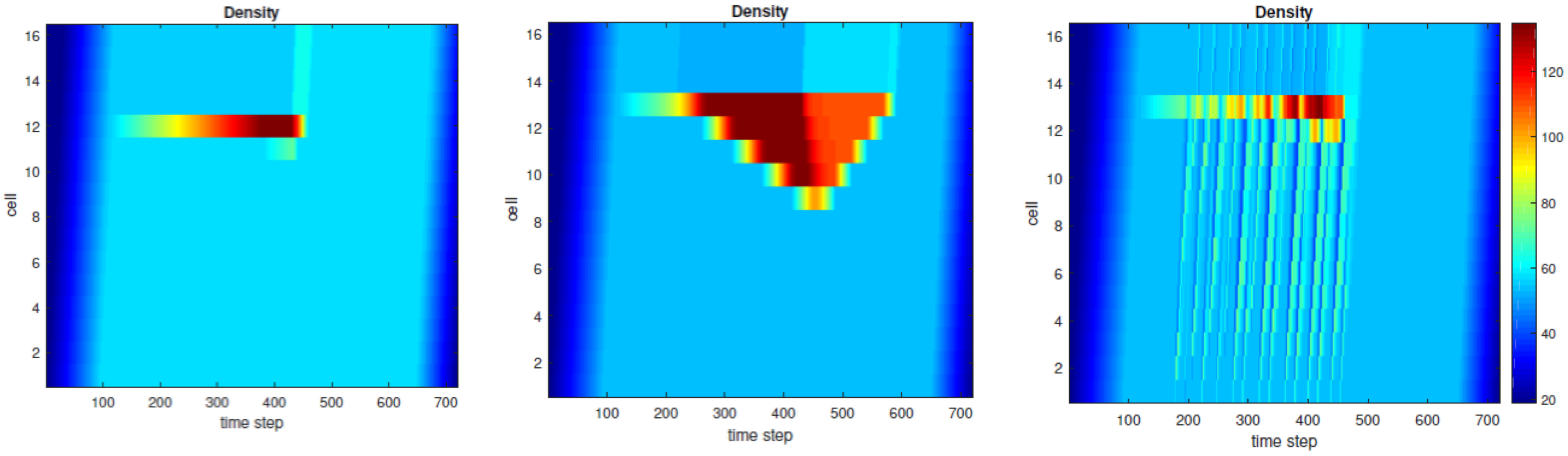


CTM with capacity drop, control

Piacentini G., Ferrara A., Papamichail I., Papageorgiou M., 58th Conference on Decision and Control (CDC 2019), Nice, France, 2019

SIMULATION RESULTS



Even with a small number of CAVs the capacity drop effect has been strongly reduced (a higher discharge flow from the temporary physical bottleneck has been observed) ➡ **Congestion is mitigated and travel times reduced**

Piacentini G., Ferrara A., Papamichail I., Papageorgiou M., 58th Conference on Decision and Control (CDC 2019), Nice, France, 2019

COUPLED PDE-ODE MODELS FOR TRAFFIC WITH CAVS (MOVING BOTTLENECKS)

A. FERRARA - IPAM WORKSHOP: SAFE OPERATION OF CONNECTED AND AUTONOMOUS VEHICLE FLEETS, OCT. 2020

A FIRST COUPLED PDE-ODE MODEL TO TAKE INTO ACCOUNT CAVS AS MOVING BOTTLENECKS IN A MACROSCOPIC TRAFFIC FLOW

$$
\begin{cases}
\frac{\partial \rho}{\partial t} + \frac{\partial f(\rho)}{\partial x} = 0 & (t,x) \in \mathbb{R}^+ \times \mathbb{R} \\
\rho(0,x) = \rho_0(x), & x \in \mathbb{R} \\
f(\rho(t,y(t))) - \dot{y}(t)\rho(t,y(t)) \leq \frac{\alpha \rho_{max}}{4V}(V - \dot{y}(t))^2 & \\
\dot{y}(t) = \omega(\rho(t,y(t)+)) & t \in \mathbb{R}^+ \\
y(0) = y_0 &
\end{cases}
$$

- ρ is the density of vehicles.
- v is the average speed of the flow.
- f = ρv is the flow.
- ω is the speed law of the MB.
- y is the position of the MB.

- The first equation of the system is the classical equation of conservation of vehicles of the LWR model.
- The MB is expressed as a **constraint on the flow** at the MB position in the third equation.
- α is MB occupancy ratio, i.e. the number lanes occupied by the MB.
- The last two equation describes the trajectory of the MB.

Delle Monache, M.L. and Goatin, P. Scalar conservation laws with moving constraints arising in traffic flow modeling: An existence result. Journal of Differentials Equations, 257, 2014

Piacentini, G., Goatin, P. & Ferrara, A.(2018). Traffic control via moving bottleneck of coordinated vehicles. *Proceedings of 15th IFAC Symposium on Control in Transportation Systems CTS 2018, 51*(9), 13–18

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$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial f(\rho)}{\partial x} = 0 & (t,x) \in \mathbb{R}^+ \times \mathbb{R} \\ \rho(0,x) = \rho_0(x), & x \in \mathbb{R} \\ f(\rho(t,y(t))) - \dot{y}(t)\rho(t,y(t)) \le \frac{\alpha \rho_{max}}{4V}(V - \dot{y}(t))^2 & \\ \dot{y}(t) = \omega(\rho(t,y(t)+)) & t \in \mathbb{R}^+ \\ y(0) = y_0 & \end{cases}$$

- ρ is the density of vehicles.
- v is the average speed of the flow.
- f = ρv is the flow.
- ω is the speed law of the MB.
- y is the position of the MB.

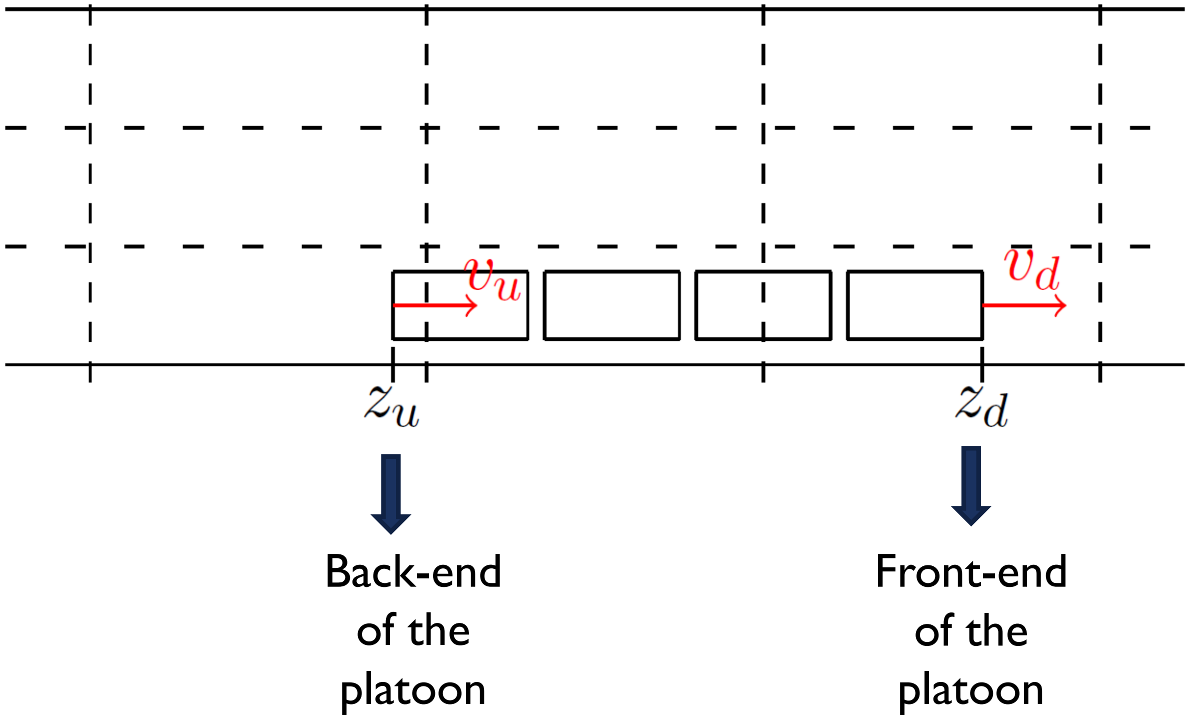
➡ The **MBs speeds: controlled** by means of a Model Predictive Control (**MPC**) to reduce the overall travel time.

Delle Monache, M.L. and Goatin, P. Scalar conservation laws with moving constraints arising in traffic flow modeling: An existence result. Journal of Differentials Equations, 257, 2014

Piacentini, G., Goatin, P. & Ferrara, A.(2018). Traffic control via moving bottleneck of coordinated vehicles. *Proceedings of 15th IFAC Symposium on Control in Transportation Systems CTS 2018, 51*(9), 13–18

A SECOND COUPLED PDE-ODE MODEL

A **second coupled PDE-ODE model** has been proposed **as an extension of the previous coupled PDE-ODE model for moving bottlenecks** with the aim of capturing the presence of platoons of CAVs (their length can vary in time).



Piacentini G., Goatin P., Ferrara A,. "A macroscopic model for platooning in highway traffic", SIAM Journal on Applied Mathematics, 2020.

A SECOND COUPLED PDE-ODE MODEL

$$\partial_t \rho + \partial_x F(t,x,\rho) = 0,$$

The macroscopic traffic fow is described by means of the LWR model.

The **flow F is discontinuous** due to the presence of the platoon

$$\rho(0,x) = \rho_0(x),$$

$$\dot{z}_u(t) = v_u(t,\rho(t,z_u(t)+)),$$

$$z_u(0) = z_u^0,$$

$$\dot{z}_d(t) = v_d(t,\rho(t,z_d(t)+)),$$

$$z_d(0) = z_d^0,$$

ODEs describing the **trajectories of the initial and final points (front-end and back-end)**

The **length of the platoon** is allowed to vary depending on the number of vehicles joining the platoon and their spacing

$$\dot{L}(t) = \dot{z}_d(t) - \dot{z}_u(t) = v_d(\rho(t, z_d(t)+)) - v_u(\rho(t, z_u(t)+)).$$

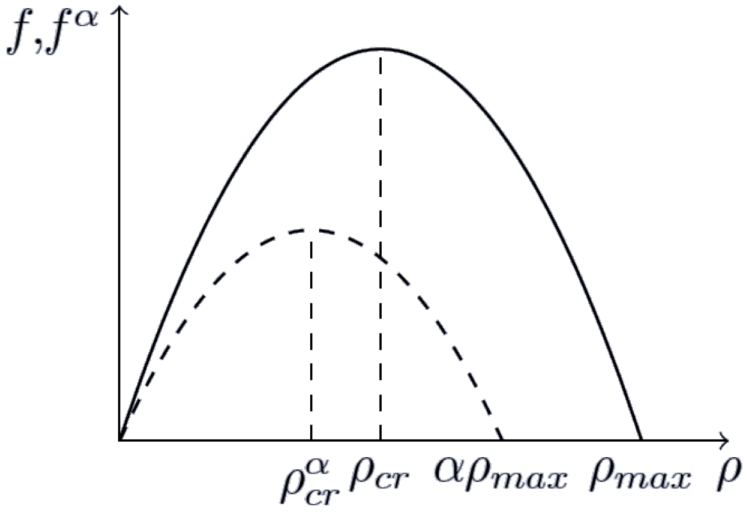
Piacentini G., Goatin P., Ferrara A,. "A macroscopic model for platooning in highway traffic", SIAM Journal on Applied Mathematics, 2020.

THE DISCONTINUOUS FLUX FUNCTION

The platoon occupies a portion of the road, acting as a **flux constraint in the interval [$z_u(t), z_d(t)$]**

$$F(t,x,\rho) := \begin{cases} f(\rho) & \text{if } x \notin [z_u(t), z_d(t)], \\ f_\alpha(\rho) := \alpha f(\rho/\alpha) & \text{if } x \in [z_u(t), z_d(t)]. \end{cases}$$

The flux is reduced in correspondence of the platoon



$$f(\rho) = V\rho\left(1 - \frac{\rho}{R}\right),$$

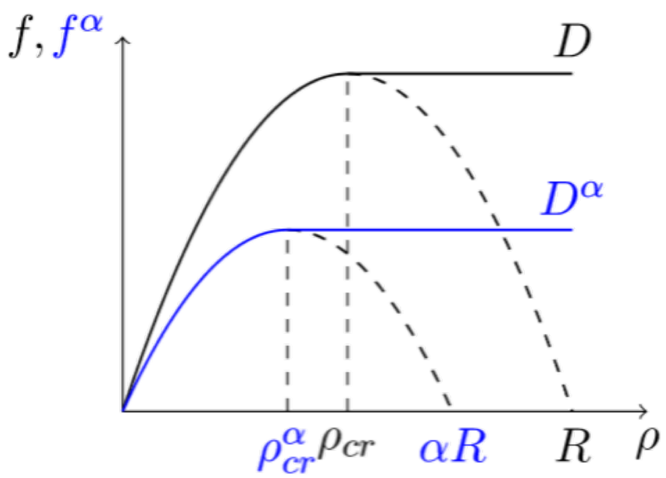
$$f_\alpha(\rho) = V\rho\left(1 - \frac{\rho}{\alpha R}\right).$$

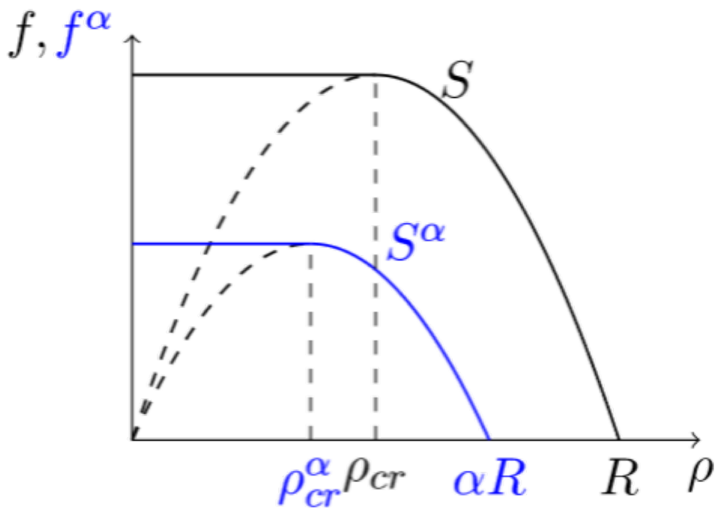
Piacentini G., Goatin P., Ferrara A.,."A macroscopic model for platooning in highway traffic", SIAM Journal on Applied Mathematics, 2020.

HOW TO SOLVE THE CONSERVATION LAW WITH DISCONTINUOUS FLOWS?

- The literature on conservation laws having discontinuous fux functions is vast.
- Most of the works focus only on discontinuities at fixed points in space (e.g. Andreianov, Hvistendahl Karlsen, Risebro).
- Only a few face the issue of **time dependent discontinuities (our case).**

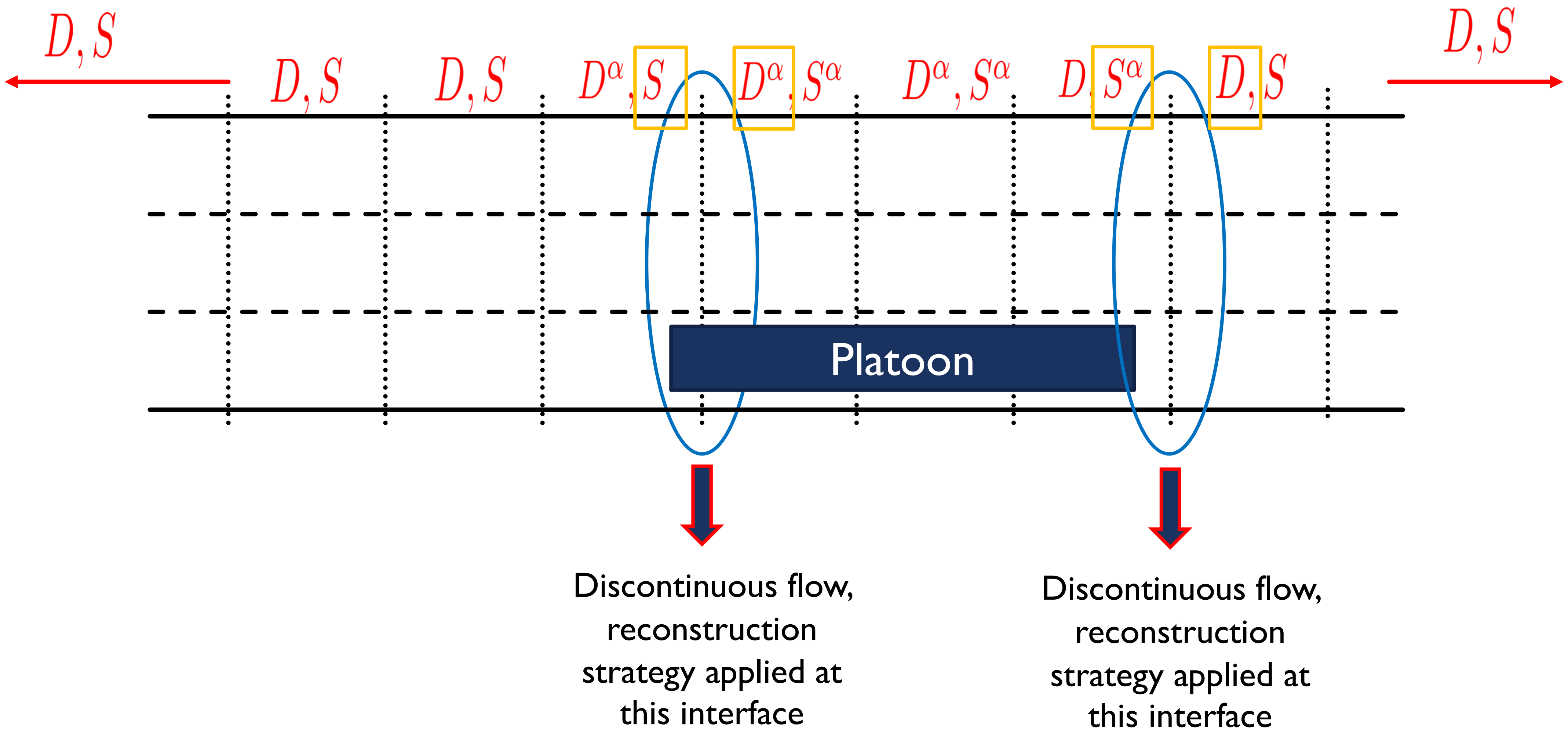
Modified demand and supply





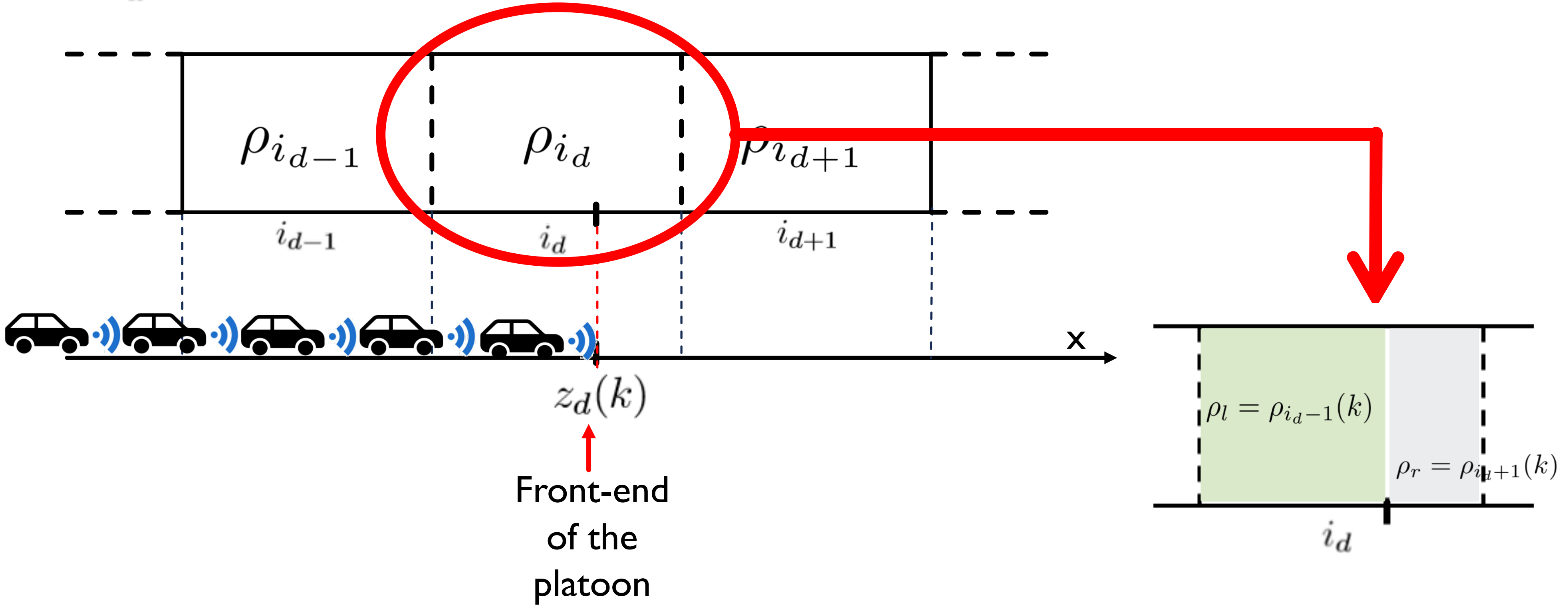
Piacentini G., Goatin P., Ferrara A,. "A macroscopic model for platooning in highway traffic", SIAM Journal on Applied Mathematics, 2020.

- For each cell demand and supply are defined.
- Interfaces with discontinuous flow need a special treatment



RECONSTRUCTION STRATEGY (E.G.THE FRONT END CASE)

1. Consider the cell i_d in which the front-end point of the platoon lies.



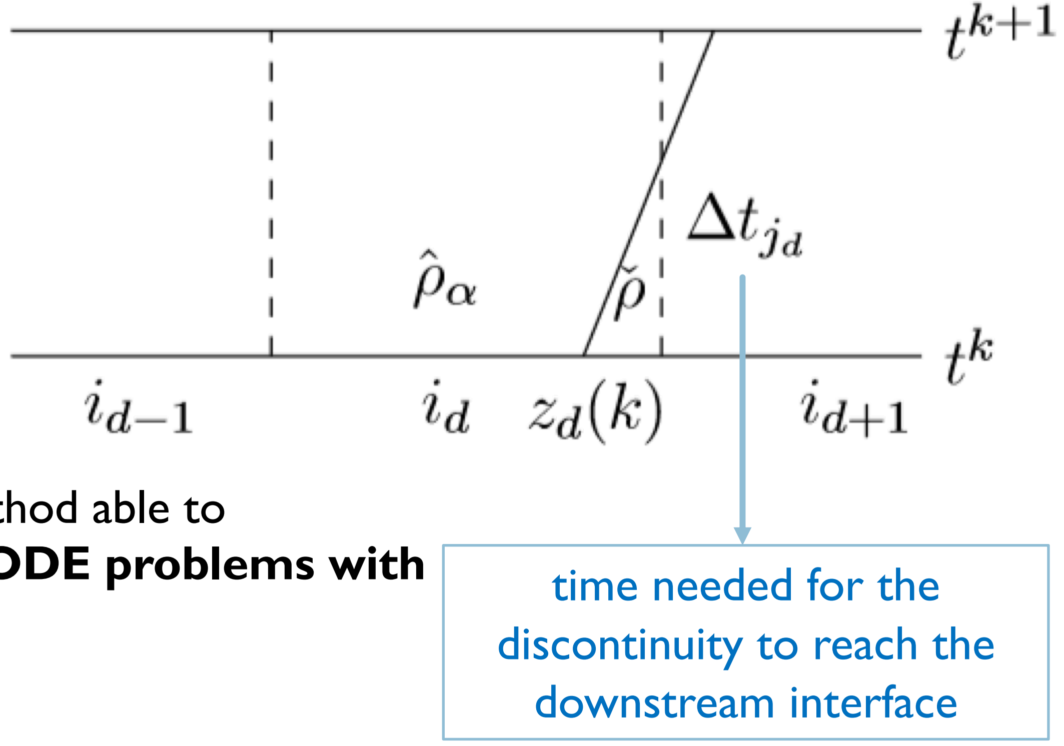
2. At each time stept k the density in the platoon cell i_d is approximated by considering the value $\rho_l = \rho_{i_d-1}(k)$ upstream, and $\rho_r = \rho_{i_d+1}(k)$ downstream the platoon position $z_d(k)$

SOLUTION OF THE RIEMANN PROBLEMS

3. The following Riemann problem is then solved

$$\begin{cases} \partial_t \rho + \partial_x F(t,x,\rho) = 0, \\ \rho(0,x) = \rho_0(x) = \begin{cases} \rho_l & \text{if } x < z_d^0, \\ \rho_r & \text{if } x \geq z_d^0, \end{cases} \end{cases}$$

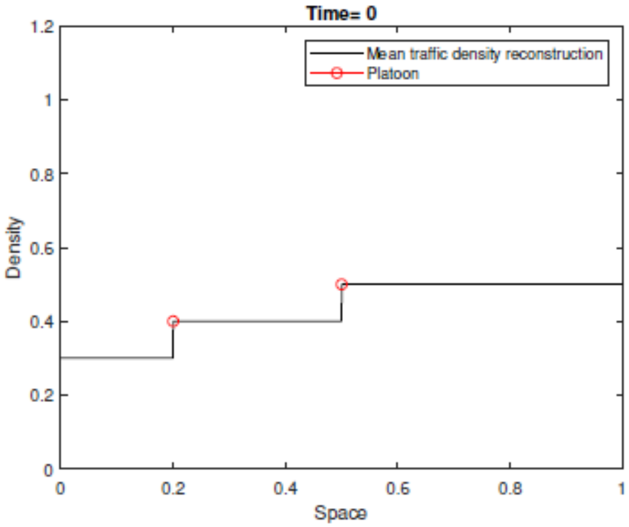
To get the solutions $\hat{\rho}^{\alpha}, \check{\rho}$



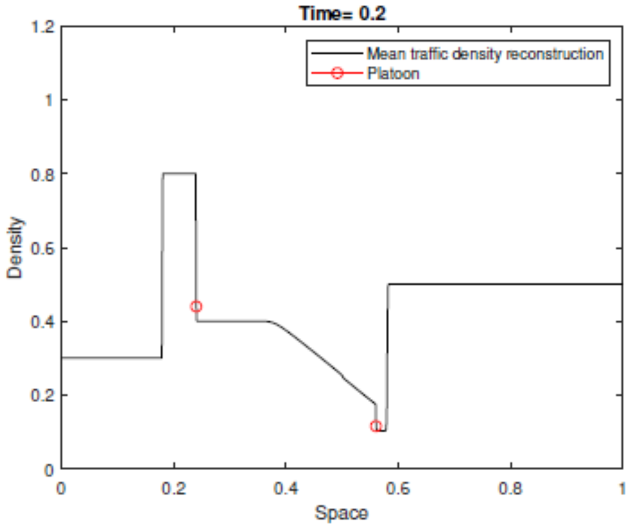
4. The flux at the interface is then reconstructed via a numerical method able to numerically capture **non-classical shocks for the coupled PDE-ODE problems with moving constraints**[*]

[*]Piacentini G., Goatin P., Ferrara A.,."A macroscopic model for platooning in highway traffic", SIAM Journal on Applied Mathematics, 2020.

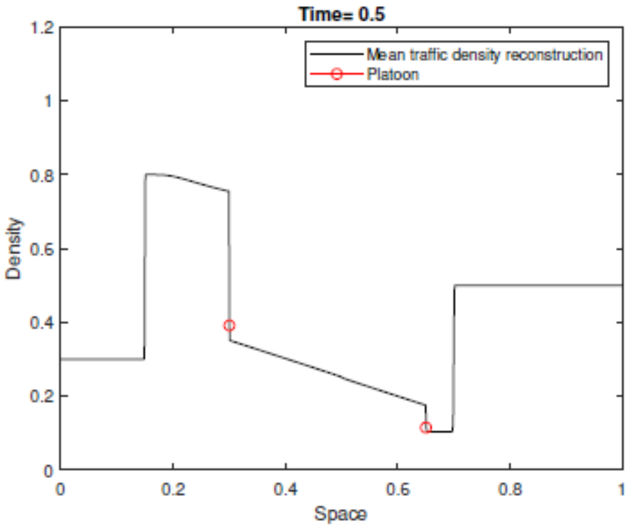
EVOLUTION IN TIME OF THE DENSITY



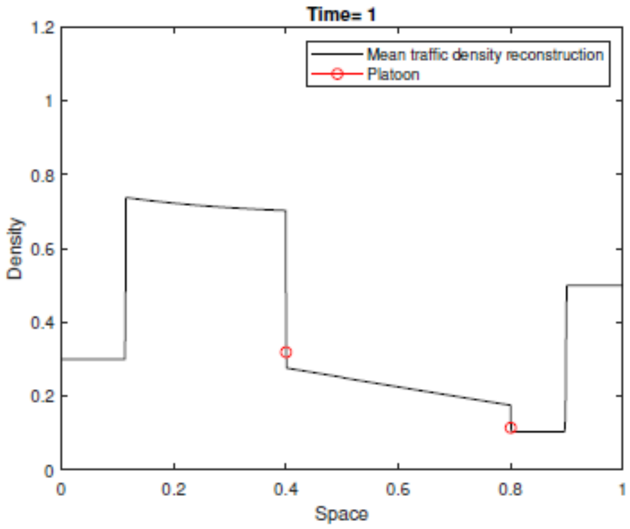
(a)



(b)



(c)



(d)

FROM THE NEW MODEL A NEW CONTROL APPROACH

- **Idea:** control the speeds of the platoon downstream and upstream end-points

⬇

This results in controlling both the SPEED and the LENGTH of the platoon

$$\dot{L}(t) = \dot{z}_d(t) - \dot{z}_u(t) = v_d(\rho(t, z_d(t)+)) - v_u(\rho(t, z_u(t)+)).$$

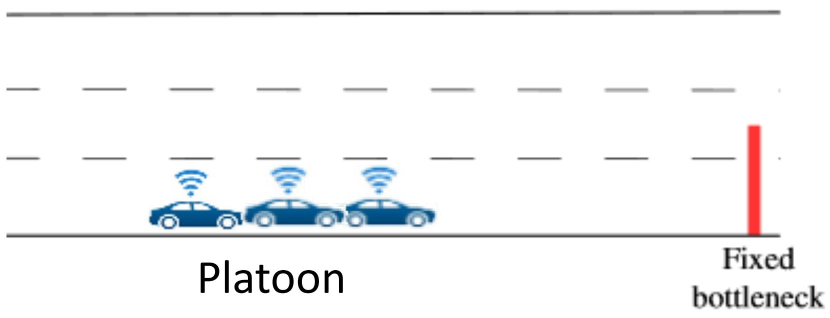
G. Piacentini, P. Goatin and A. Ferrara, "Traffic Control Via Platoons of Intelligent Vehicles for Saving Fuel Consumption in Freeway Systems," to appear in **IEEE Control Systems Letters**, April 2021 (available online)

CONTROL PROBLEM: MIMIMIZE THE TOTAL FUEL CONSUMPTION

- MPC control approach.
- At each iteration k, the input $\underline{u}(k) = [\underline{V_d}(k)\underline{V_u}(k)]$ is solution to:

$$\min_{\underline{u}} \sum_{h=k}^{k+K_p} \sum_{i=1}^{N} TFC(\rho_i(h))\Delta x \Delta t,$$

$$\begin{aligned} & L_{min} \leq L(h) \leq L_{max}, \\ & V_d^{min} \leq V_d(h) \leq V^{max} \\ & |V_d(h) - V_u(h)| \leq c. \end{aligned} \qquad \text{for } h = k, \ldots, k + K_p$$



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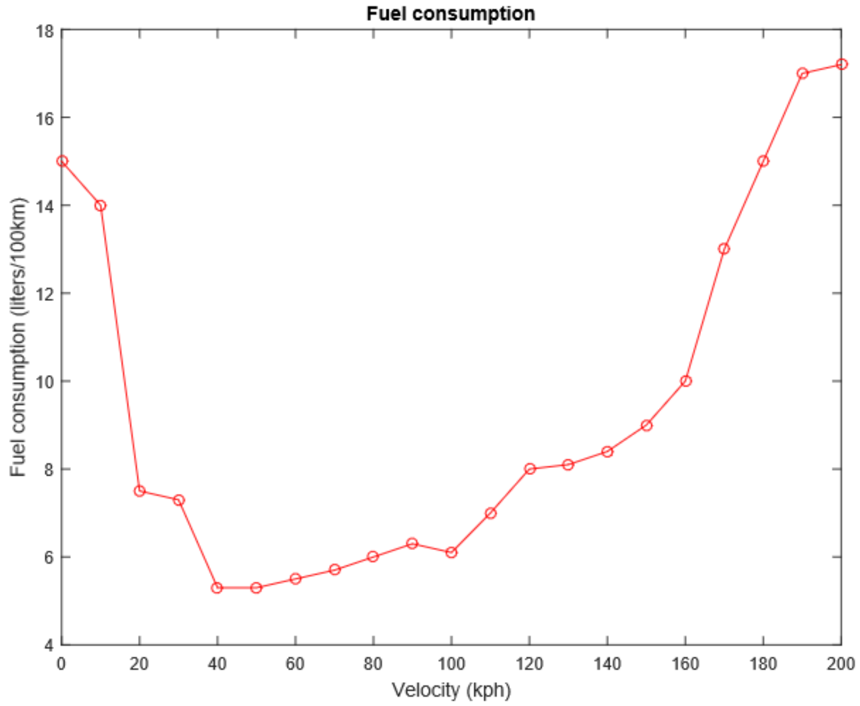
AVERAGE SPEED FUEL CONSUMPTION MODEL

- Fuel–speed curves of different vehicles are weighted and approximated via a 6° order polynomial:

$$K(v) = 5.7 \cdot 10^{-12} \cdot v^6 - 3.6 \cdot 10^{-9} \cdot v^5 + 7.6 \cdot 10^{-7} \cdot v^4 - 6.1 \cdot 10^{-5} \cdot v^3 + 1.9 \cdot 10^{-3} \cdot v^2 + 1.6 \cdot 10^{-2} \cdot v + 0.99.$$

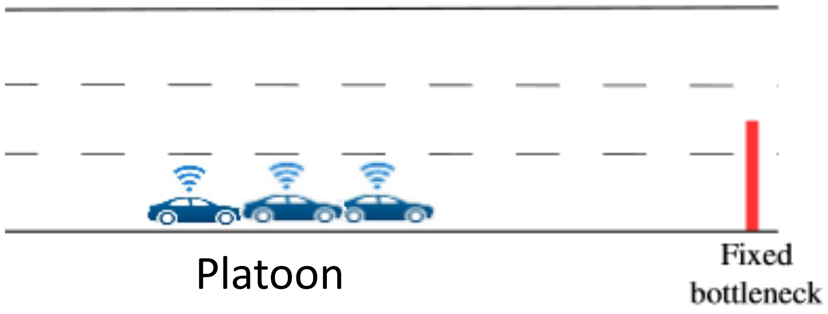


- $K(v)$ is re-parametrized in terms of density and multiplied by the density to get the Total Fuel Consumption (TFC): $\mathcal{F}(\rho) = \rho\mathcal{K}(\rho)$

- Control Problem: $\min_{\underline{u}} \sum_{h=k}^{k+K_p} \sum_{i=1}^{N} \mathcal{F}(\rho_i(h))\Delta x \Delta t,$

SIMULATION SCENARIO

Parameter	Value
Number of cells	200
Lenght of each cell	250 m
Maximum speed	140 km/h
Maximum density	400 veh/km
Capacity	14000 veh/h
Sampling time	5.76 s
Occupancy rate	0.6
Simulation time	1 hour

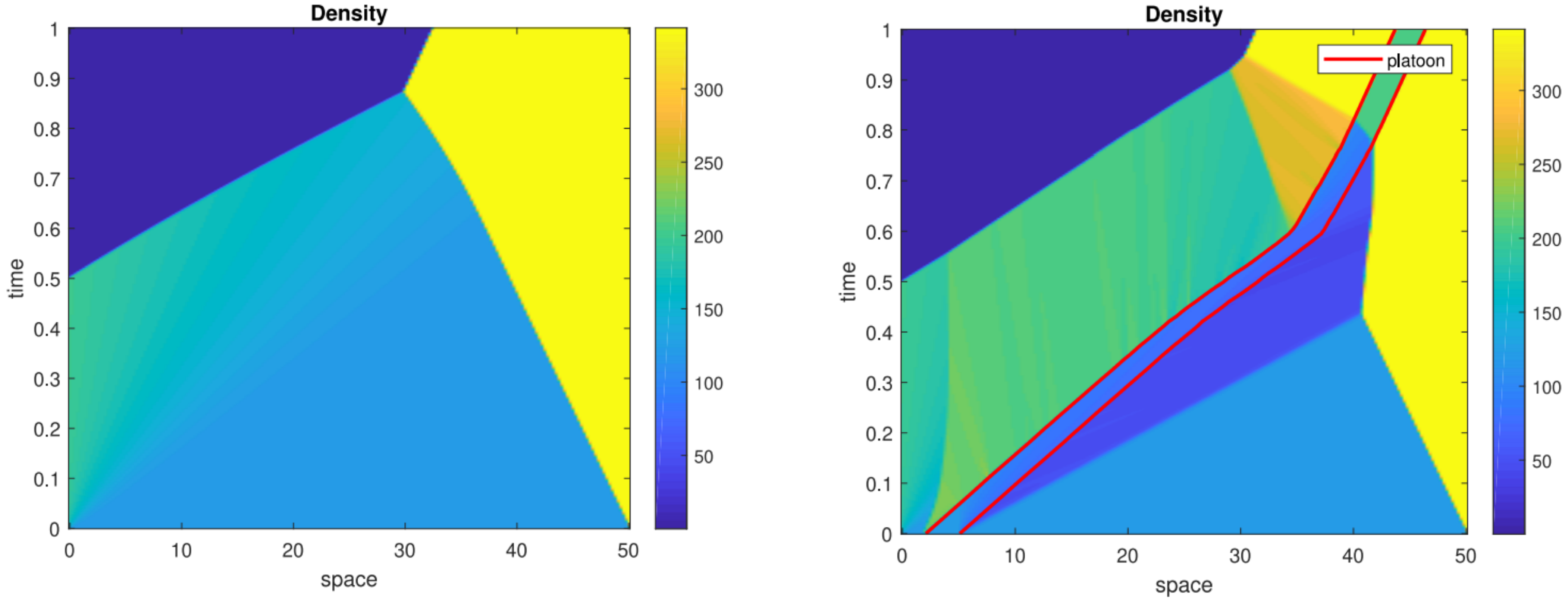


The arriving demand, the in-fow, is equal to the capacity for the first half of the simulation, while it is zero in the second half:

$$f_{in} = \begin{cases} f_{max} & if\ t < 0.5 \cdot Tf \\ 0 & if\ t > 0.5 \cdot Tf \end{cases}$$

$$f_{out} = 0.5 \cdot f_{max} \qquad \forall t \in [0, T_f]$$

SIMULATION RESULTS: DENSITY VS TIME AND SPACE



(a) no control (b) MPC control

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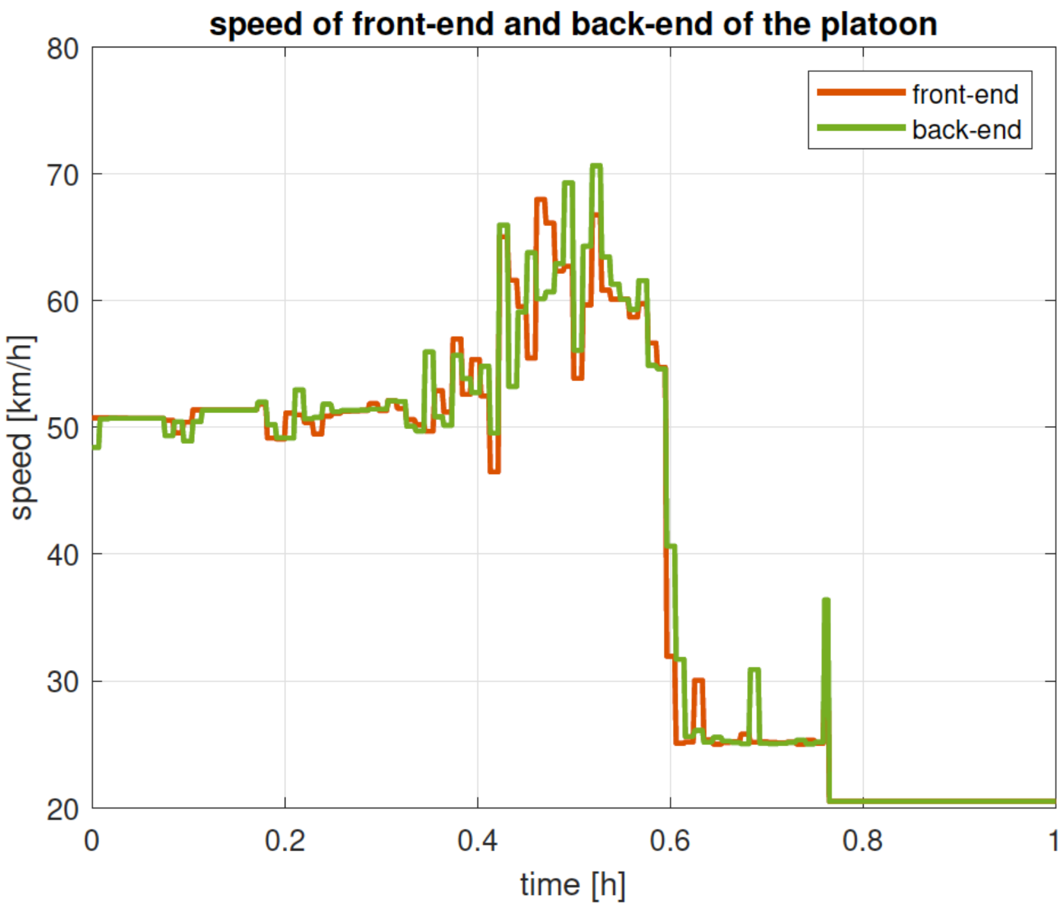
SIMULATION RESULTS: CONTROL VARIABLES VS TIME

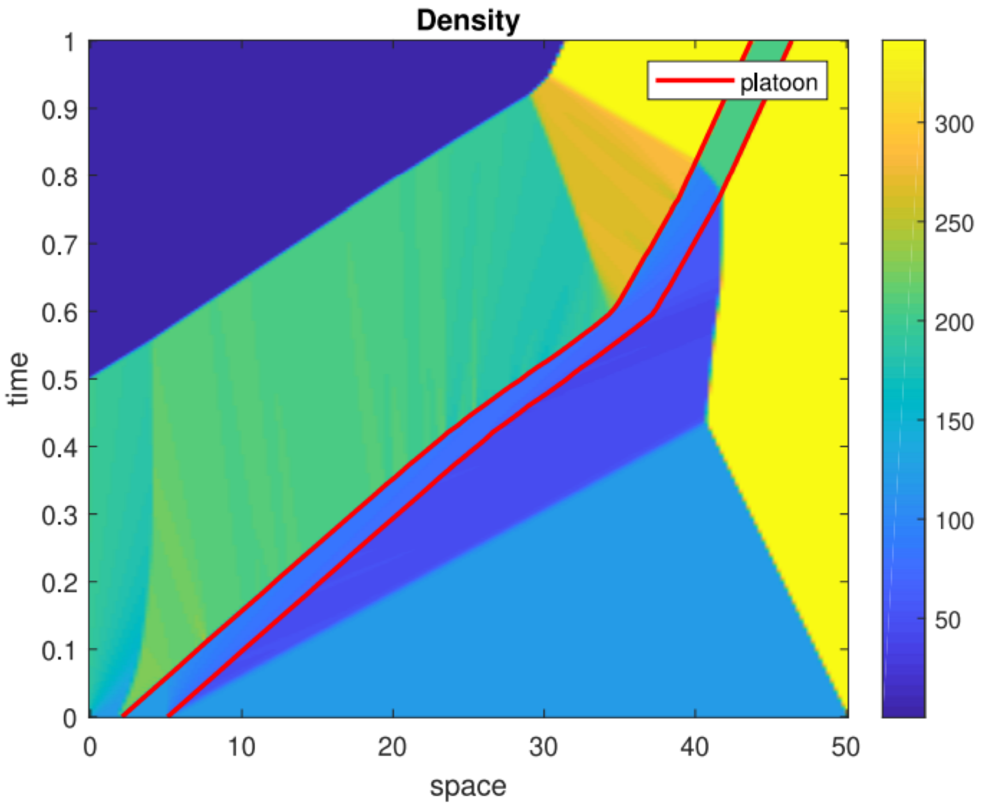


(a) Control speed

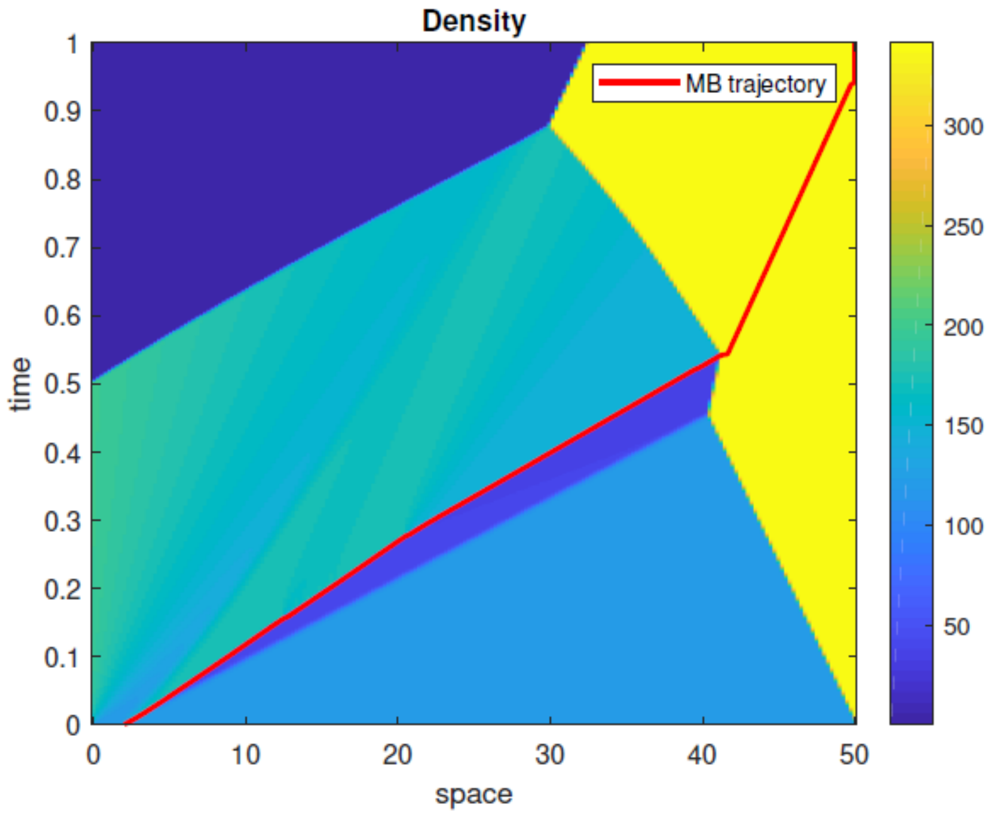
TFC reduction (vs. no control case): 2.6%

Fuel consumption of the overall traffic flow from 27629 liters to 26903 liters, that represents **a saving of 726 liters of fuel** (in this small scale example)

COMPARISON WITH THE SIMPLE MB CONTROL (FIRST PDE-ODE MODEL)



(b) MPC control



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CONCLUSIONS

- Traffic Modelling and control: from classic methods to new methods taking into account CAVs
- Multi-scale traffic models incorporating CAVs are needed (in this talk CTM with CAVs, and coupled PDE-ODEs)
- Their use for traffic control seems promising and worth of further investigation

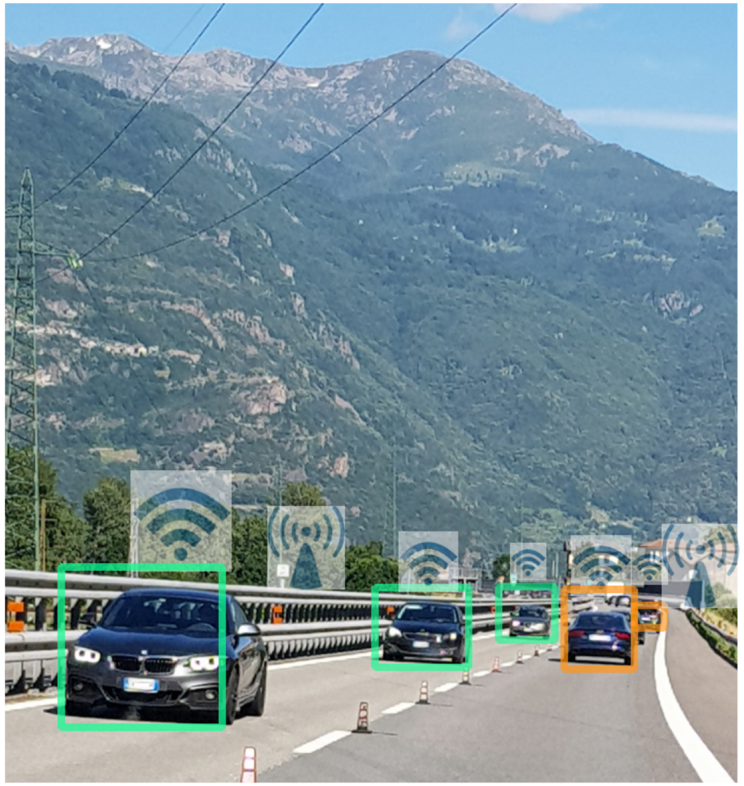

Photo Credit: Antonella Ferrara

THANKS TO MY CO-AUTHORS AND COLLABORATORS

THANK YOU FOR YOUR ATTENTION!