Safety-Critical Control of Autonomous Systems

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Safe Operation of Connected and Autonomous Vehicle Fleets
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Autonomy in the real world is hard.
But: Pretty when it works...
Question: How do we make safety guarantees?
Autonomy: The Big Picture

Specifications
- Tasks “do things”
- Invariance “avoid things”

System components
- High-level control
- Low-level control

Design models
- Discrete
- Continuous

\[ \frac{d}{dt} x = f(x, u) \]
Specifications: "Tasks: ‘do things’"

Invariance: "avoid things"

System components:
- High-level control
- Low-level control

Design models:
- Discrete: $a \rightarrow b \rightarrow c$ with $a$, $b$, and $c$
- Continuous: $\frac{dx}{dt} = f(x, u)$

Plant: Caltech
Control barrier functions: $\dot{h}(x, u) \geq -\gamma h(x)$ implies $C$ is safe.

Control Lyapunov Functions: $\dot{V}(x, u) \leq -\alpha V(x)$ implies stability.

Invariance: “avoid things”

Low-level control: $\frac{dx}{dt} = f(x, u)$

Plant
Control Lyapunov Functions

Lyapunov (1892)

PROBLÈME GÉNÉRAL

LA STABILITÉ DU MOUVEMENT,

PAR M. A. LIAPONOFF.

Sontag (1989)

A ‘universal’ construction of Artstein’s theorem on nonlinear stabilization

Eduardo D. SONTAG *
Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, U.S.A.
Received 7 March 1989

Dynamics: For \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \):

\[ \dot{x} = f(x) + g(x)u \]

Lyapunov: \( V : X \to \mathbb{R}_{\geq 0} \) satisfying:

\[ c_1 \|x\|^2 \leq V(x) \leq c_2 \|x\|^2 \]

\[ \inf_{u \in U} \dot{V}(x, u) \leq -\alpha V(x) \]

Main idea:

\[ \dot{V}(x, u) \leq -\alpha V(x) \Rightarrow V(x(t)) \leq e^{-\alpha t} V(x(0)) \]
Theorem

If there exists control Lyapunov function:

$$\inf_{u \in U} \left[ \dot{V}(x, u) + \alpha V(x) \right] \leq 0$$

then for all feedback controllers:

$$u(x) \in \{ u \in U : \dot{V}(x, u) \leq -\alpha V(x) \}$$

$$\downarrow \quad \dot{x} = f(x) + g(x)u(x)$$

$$x \to 0 \text{ Exponentially.}$$

Dynamics: For $$x \in \mathbb{R}^n$$ and $$u \in \mathbb{R}^m$$:

$$\dot{x} = f(x) + g(x)u$$

Lyapunov: $$V : X \to \mathbb{R}_{\geq 0}$$ satisfying:

$$c_1 \| x \|^2 \leq V(x) \leq c_2 \| x \|^2$$

$$\inf_{u \in U} \dot{V}(x, u) \leq -\alpha V(x)$$

Main idea:

$$\dot{V}(x, u) \leq -\alpha V(x) \quad \Rightarrow \quad V(x(t)) \leq e^{-\alpha t} V(x(0))$$
Human-Like Walking

Lyapunov Controller

\[ u^*(x) = \arg\min_{(u,\delta)\in U\times\mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

s.t. \[ \dot{V}(x, u) \leq -\alpha V(x) \]

+ Theorem \( \Rightarrow \) Stable Walking

AA, TAC 2014
AA, Galloway, Sreenath, Grizzle, TAC 2014
Reher, Hereid, Kolathaya, Hubicki, AA, WAFR 2016
First dynamic walking (without crutches) for paraplegics

Lyapunov Controller

\[ u^*(x) = \arg\min_{(u, \delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

s.t. \[ \dot{V}(x, u) \leq -\alpha V(x) \]

+ Theorem \( \Rightarrow \) Stable Walking

Gurriet, Finet, Boeris, Hereid, Harib, Masselin, Grizzle, AA, ICRA 2018
Lyapunov Controller

\[ u^*(x) = \arg\min_{(u, \delta) \in U \times \mathbb{R}} \| u - u_{des}(x) \|^2 \]

s.t. \[ \dot{V}(x, u) \leq -\alpha V(x) \]

+ Theorem \( \Rightarrow \) Stable Walking
Application to Quadrupeds

Lyapunov Controller

\[ u^*(x) = \arg\min_{(u, \delta) \in U \times \mathbb{R}} ||u - u_{\text{des}}(x)||^2 \]

s.t. \[ \dot{V}(x, u) \leq -\alpha V(x) \]

+ Theorem \( \Rightarrow \) Stable Walking
Theorems imply generality...

Simulation vs. Reality
Lab Experiments
Flat Ground Outdoor Walking
Rough Terrain Walking

+ Theorem $\implies$ Stable Walking

Lyapunov Controller

$$u^*(x) = \arg\min_{(u,\delta)\in U \times \mathbb{R}} \|u - u_{des}(x)\|^2$$

s.t. $\dot{V}(x, u) \leq -\alpha V(x)$

Invariance “avoid things”

Low-level control

$$\frac{d}{dt} x = f(x, u)$$

Plant

Lab Experiments
Safety-Critical Walking
Control barrier functions: $\dot{h}(x, u) \geq -\gamma h(x)$

$\updownarrow$

Safe Set $C$, $C$ is safe

Control Lyapunov Functions: $\dot{V}(x, u) \leq -\alpha V(x)$

$\updownarrow$

stability

Invariance “avoid things”

Low-level control

$\frac{d}{dt} x = f(x, u)$

Plant

Caltech
Control barrier functions

$$\dot{h}(x, u) \geq -\gamma h(x)$$

⇓

C is safe

Safe Set

Control Lyapunov Functions

$$\dot{V}(x, u) \leq -\alpha V(x)$$

⇓

stability

Embedding

Safety-Critical Control

⇒

Need something more general than Lyapunov

Nagumo (1942)

Über die Lage der Integralkurven gewöhnlicher Differentialgleichungen.

Von Mitio Nagumo.


§1. Einleitung.

In dieser Note werden $n$-dimensionale Vektoren mit dicken

Caltech


Safety Verification of Hybrid Systems Using Barrier Certificates

Stephen Prajna$^1$ and Ali Jadbabaie$^2$

$^1$ Control and Dynamical Systems, California Institute of Technology, Pasadena, CA 91125 - USA, prajna@cs.caltech.edu

$^2$ Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, PA 19104 - USA, jadbabaie@seas.upenn.edu

AA, Coogan, Egerstedt, Notomista, Sreenath, Tabuada, ECC 2019
(includes brief history)
Control Barrier Functions

Provide necessary and sufficient conditions for set invariance, i.e., safety – on the entire safe set.

- **Dynamics:** \( \dot{x} = f(x) + g(x)u \)
- **Safe set** \( \mathcal{C} \): defined by \( h \):
  \[
  \mathcal{C} = \{ x \in \mathbb{R}^n : h(x) \geq 0 \}
  \]

Control Barrier Function

For all \( x \in \mathcal{C} \), there exists \( u \in \mathbb{R}^m \) such that:

\[
\dot{h}(x,u) = \frac{\partial h}{\partial x}(x)(f(x) + g(x)u) \geq -\gamma(h(x))
\]

\( \mathcal{C} \) is safe

Here \( \gamma : \mathbb{R} \to \mathbb{R} \) is an extended class \( \mathcal{K} \) function (strictly increasing with \( \gamma(0) = 0 \)).
Control barrier functions

\[ \dot{h}(x, u) \geq -\gamma h(x) \]

Synthesis: Let:

\[ \varphi(x) = L_f h(x) + L_g h(x)u_{\text{des}}(x) + \alpha(h(x)) \]

Closed form controller:

\[ u^*(x) = u_{\text{des}}(x) - \begin{cases} 
\frac{\varphi(x)L_g h(x)^T}{L_g h(x)L_g h(x)^T} & \text{if } \varphi(x) < 0 \\
0 & \text{if } \varphi(x) \geq 0 
\end{cases} \]

Safety Filters

Filter a given control input to guarantee safety:

\[ u_{\text{des}}(x) \xrightarrow{\text{Safety Filter}} u_{\text{act}}(x) \xrightarrow{\text{Controller}} x \]

Safe set \( \mathcal{C} \): defined by \( h \):

\[ \mathcal{C} = \{ x \in \mathbb{R}^n : h(x) \geq 0 \} \]

Control Barrier Function

For all \( x \in \mathcal{C} \), there exists \( u \in \mathbb{R}^m \) such that:

\[ \dot{h}(x, u) = \frac{\partial h}{\partial x}(x)(f(x) + g(x)u) \geq -\gamma (h(x)) \]

\( \Downarrow \)

\( \mathcal{C} \) is safe

AA, Tabuada Grizzle, CDC 2014

AA, Xu, Tabuada Grizzle, TAC 2017
Control barrier functions

\[ \dot{h}(x, u) \geq -\gamma h(x) \]

Synthesis: Let:

\[ \varphi(x) \triangleq L_f h(x) + L_g h(x) u_{\text{des}}(x) + \alpha(h(x)) \]

Closed form controller:

\[ u^*(x) = u_{\text{des}}(x) - \begin{cases} 
\frac{\varphi(x)L_g h(x)^T}{L_g h(x)L_g h(x)^T} & \text{if } \varphi(x) < 0 \\
0 & \text{if } \varphi(x) \geq 0 
\end{cases} \]

Safety Filters

Filter a given control input to guarantee safety.

No Control Barrier function \( \Rightarrow \) Unsafe

With Control Barrier function \( \Rightarrow \) Safe

Safety-Critical Controller

\[ u^*(x) = \arg\min_{(u, \delta) \in U \times \mathbb{R}} \|u - u_{\text{des}}(x)\|^2 \]

s.t. \( \dot{h}(x, u) \geq -\gamma h(x) \)

\[ AA, \ Tabuada \ Grizzle, \ CDC \ 2014 \]
\[ AA, \ Xu, \ Tabuada \ Grizzle, \ TAC \ 2017 \]
Control barrier functions

\[ \dot{h}(x, u) \geq -\gamma h(x) \]

\[ \Downarrow \]

C is safe

Safety-Critical Controller

\[ u^*(x) = \arg\min_{(u, \delta) \in U \times R} \| u - u_{des}(x) \|^2 \]

s.t. \[ \dot{h}(x, u) \geq -\gamma h(x) \]

Control Lyapunov Functions

\[ \dot{V}(x, u) \leq -\alpha V(x) \]

\[ \Downarrow \]

stability

Embedding

Fundamental Properties:

**Lemma.** Control barrier functions imply stability of the set \( C \).

Control Barrier Function \( h \): Yields a Lyapunov function for \( C \):

\[ V_C(x) \triangleq \begin{cases} 0 & \text{if} \quad x \in C \\ -h(x) & \text{if} \quad x \in \overline{C} = \mathbb{R}^n - C \end{cases} \]

**Theorem**

Lyapunov is the special case of barriers for \( C = \{0\} \).
Walking Robots

Safety-Critical Controller

\[ u^*(x) = \arg\min_{(u,\delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

\[ \text{s.t. } \dot{h}(x, u) \geq -\gamma h(x) \]

Multi-Robot Systems

Collision Avoidance

Automotive Systems

APPLICATIONS

Joint with: Egersted (GaTech), Tabuada (UCLA), Grizzle (UMich), Feron (GaTech), Xu (UW), Wandercraft, Hutter (ETH), Orosz (UMich)
Application to Automotive Systems

Adaptive Cruise Control (ACC)
- Safety Constraints: “half the speedometer” following rule
- Control Objectives: Achieve a desired speed.

Lane Keeping
- Safety Constraints: Stay in the lane for all time
- Control Objectives: Achieve reference signal

Safety-Critical Controller

\[ u^*(x) = \arg\min_{(u,\delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

s.t. \[ \dot{h}(x, u) \geq -\gamma h(x) \]

Existing (desired) controllers

Safety (Barrier function) constraint

Xu, Grizzle, Tabuada and AA, TASE 2018
**Lane Keeping**

Human = $u_{\text{des}}$

Safety-Critical Controller

$$u^*(x) = \arg\min_{(u,\delta)\in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2$$

s.t. $\dot{h}(x, u) \geq -\gamma h(x)$

Yuxiao Chen (unpublished)
Distance $= h(x)$

Safety-Critical Controller

$$u^*(x) = \arg\min_{(u, \delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2$$

s.t. $h(x, u) \geq -\gamma h(x)$

Joint work with: Orosz (UMich)
Multi-Robot Systems

Desired Controller: Go straight

Safety-Critical Controller

\[ u^* (x) = \underset{(u, \delta) \in U \times \mathbb{R}}{\text{argmin}} \| u - u_{\text{des}} (x) \|^2 \]

s.t. \[ \dot{h}(x, u) \geq -\gamma h(x) \]

Joint work with: Egerstedt (GT)
Multi-Robot Systems

Human = \( u_{des} \)

Safety-Critical Controller

\[
u^*(x) = \arg\min_{(u,\delta) \in U \times \mathbb{R}} \|u - u_{des}(x)\|^2 \\
s.t. \quad \dot{h}(x,u) \geq -\gamma h(x)
\]
Obstacle Avoidance

Go to waypoint $= u_{des}$

Safety-Critical Controller

$$u^*(x) = \arg\min_{(u,\delta)\in U \times R} \| u - u_{des}(x) \|^2$$

s.t. $\dot{h}(x, u) \geq -\gamma h(x)$

Safe Set $h(x) \geq 0$

Joint work with: AeroVironment

Singletary, Klingebiel, Bourne, Browning, Tokumaru, AA,
Submitted to RAL/ICRA 2020
**Control Barrier Functions**

**Artificial Potential Fields**

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**Theorem**

Control Barrier Functions include Artificial Potential Fields as a special case.

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**Safety-Critical Controller**

\[ u^*(x) = \arg \min_{(u, \delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

s.t. \[ h(x, u) \geq -\gamma h(x) \]

\[ u_{\text{des}}(x) = \nabla U_{\text{att}}(x) \]

\[ h(x) = \frac{1}{1 + U_{\text{rep}}(x)} - \delta \]

---

**Repulsive Potential** \( U_{\text{rep}}(x) \): Blows up at obstacle:

\[ U_{\text{rep}}(x) \rightarrow \infty \quad \text{as} \quad \| x - x_{\text{obst}} \| \rightarrow D_{\text{obst}}. \]

**Attractive Potential** \( U_{\text{att}}(x) \): Positive definite about the goal:

\[ \varepsilon \| x - x_{\text{goal}} \|^2 \leq U_{\text{att}}(x) \leq \bar{\varepsilon} \| x - x_{\text{goal}} \|^2. \]

**Artificial Potential**:

\[ U(x) = U_{\text{rep}}(x) + U_{\text{att}}(x) \]

Yields:

\[ u(x) = \nabla U(x) = \nabla U_{\text{rep}}(x) + \nabla U_{\text{att}}(x) \]
Robotic Walking

Desired Controller: Stable Walking

Safety-Critical Controller

\[ u^*(x) = \arg\min_{(u, \delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

s.t. \[ \dot{h}(x, u) \geq -\gamma h(x) \]

Location of stepping stones = \( h(x) \)

Grandia, Taylor, AA and Hutter, RAL/ICRA 2020 (submitted)
Main Idea: Barrier Functions for assist-as-needed control on exos

Safety-Critical Controller

\[ u^*(x) = \arg\min_{(u,\delta) \in \mathcal{U} \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^p \]

s.t. \[ h(x, u) \geq -\gamma h(x) \]

Eight Subjects Tested

Human = \( u_{\text{des}} \)

Tubes around Trajectories = \( h(x) \)

Barriers on Exo

Metabolic Cost Evaluated
Back to the Big Picture

Need: Unify high and low level methods

Safety-Critical Controller

\[ u^*(x) = \arg\min_{(u, \delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

s.t. \[ \dot{h}(x, u) \geq -\gamma h(x) \]

Control Barrier Functions

\[ \frac{d}{dt} x = f(x, u) \]
Multi-Robot Coordination

Specifications

Tasks
“do things”

System components

High-level control

Design models

Need provably safe behavior

Nilsson and AA, CDC 2018
Nilsson, Haesaert, Vasile, Thakker, Agha, Murray, AA, RSS 2018
High Level Specifications

Specifications

Tasks “do things”

System components

High-level control

Design models

Safety-Critical POMDPs

\( b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0} \)

\( C := \{ b \in D \mid h(b) \geq 0 \} \)

\( \Diamond (\text{GOAL}) \land_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS}_i)) \)

Markov Decision Processes (MDP)

Plan Mission

Execute while \( \mathbb{P}(\varphi) > p_{\text{acc}} \)

or task finished

Explore

Plan Exploration

Plan

model update
Safe Multi-Robot Coordination

Specifications

Tasks
"do things"

System components

High-level control

Design models

\[ \frac{\text{d}}{\text{d}t} \mathbf{x} = f(\mathbf{x}, \mathbf{u}) \]

Control Barrier Functions

Safety-Critical POMDPs

\[ b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0} \]

\[ C := \{ b \in D \mid h(b) \geq 0 \} \]

\[ \Diamond (\text{GOAL}) \land_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS}_i)) \]
Safe Multi-Robot Coordination

Heterogeneous robot team safely exploring

Safe Multi-Robot Coordination

Heterogeneous robot team safely exploring

Safety-Critical POMDPs

Dynamics:
\[ b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0} \]

Safe Set:
\[ C := \{ b \in \mathcal{D} \mid h(b) \geq 0 \} \]

Specifications:
\[ \Diamond (\text{GOAL}) \land_{i=1}^{3} (P_{\leq 0.05}(\text{OBS})_i) \]

Design models

\[ \frac{d}{dt} x = f(x, u) \]

Control Barrier Functions

Ahmadi, Singletary, Burdick and AA, CDC 2019

Jones, Schwager, Belta, CDC 2013

Ahmadi, Jansen, Wu, Topcu, IEEE TAC 2020
Safe Multi-Robot Coordination: Discrete Time Barriers

Heterogeneous robot team safely exploring

Dynamics:
\[ b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0} \]

Safe Set:
\[ C := \{ b \in \mathcal{D} \mid h(b) \geq 0 \} \]

Specifications:
\[ \Diamond (\text{GOAL}) \land \bigwedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS}))_i \]

Theorem (Safety with Barrier Functions)
For the system \( b^{t+1} = f(b^t) \), the set \( C \) is safe (forward invariant) if and only if there exists a discrete-time barrier function.

Discrete Time Barrier Function (DTBF)
\[ h : \mathcal{D} \rightarrow \mathbb{R} \] is a discrete time barrier function for the set \( C \), if there exists \( \alpha \in \mathcal{K} \) satisfying \( \alpha(r) < r \) for all \( r > 0 \) with:
\[ h(b^{t+1}) - h(b^t) \geq -\alpha(h(b^t)), \quad \forall b \in \mathcal{D}. \]
Safe Multi-Robot Coordination: Composing Safe Sets

Heterogeneous robot team safely exploring

Safety-Critical POMDPs

Dynamics:
\[ b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0} \]

Safe Set:
\[ C := \{ b \in \mathcal{D} \mid h(b) \geq 0 \} \]

Specifications:
\[ \Diamond (\text{GOAL}) \land \exists_{i=1}^3 (\mathbb{P}_{\leq 0.05}(\text{OBS}_i)) \]

Intersection
\[ \mathcal{E}_i = \{ b \in \mathcal{D} \mid h_i(b) \geq 0 \}, \quad i = 1, \ldots, k. \]

Union
\[ \mathcal{E}_\cap = \bigcap_{i \in \{1, \ldots, k\}} \mathcal{E}_i \]
\[ \mathcal{E}_\cup = \bigcup_{i \in \{1, \ldots, k\}} \mathcal{E}_i \]

Proposition

If there exist an \( \alpha \in \mathcal{K} \) satisfying \( \alpha(r) < r \) for all \( r > 0 \) such that
\[ \min_{i=1, \ldots, k} h_i(b^{t+1}) - \min_{i=1, \ldots, k} h_i(b^t) \geq -\alpha \left( \min_{i=1, \ldots, k} h_i(b^t) \right) \]
then \( \mathcal{E}_\cap = \bigcap_{i} \mathcal{E}_i \) is forward invariant.

If there exist an \( \alpha \in \mathcal{K} \) satisfying \( \alpha(r) < r \) for all \( r > 0 \) such that
\[ \max_{i=1, \ldots, k} h_i(b^{t+1}) - \max_{i=1, \ldots, k} h_i(b^t) \geq -\alpha \left( \max_{i=1, \ldots, k} h_i(b^t) \right) \]
then \( \mathcal{E}_\cup = \bigcup_{i} \mathcal{E}_i \) is forward invariant.
Safe Multi-Robot Coordination: Safety Specifications

Safety-Critical POMDPs

Dynamics:
\[ b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0} \]

Safe Set:
\[ C := \{ b \in \mathcal{D} \mid h(b) \geq 0 \} \]

Specifications:
\[ \Diamond (\text{GOAL}) \land \prod_{i=1}^{3} \left( P_{\leq 0.05}(\text{OBS})_i \right) \]

LDTL Specification | DTBF Implementation
--- | ---
\[ \omega^i \models A \] | \[ h(b^i) = \sum_{q \in A} b^i(q) - 1 \]
\[ \omega^i \models \neg A \] | \[ h(b^i) = \sum_{q \in Q \setminus A} b^i(q) - 1 \]
\[ \omega^i \models f \] | \[ h(b^i) = -f(b^i) + \delta \]
\[ \omega^i \models \neg f \] | \[ h(b^i) = f(b^i) \]
\[ \omega^i \models \phi_1 \land \phi_2 \] | \[ h(b^i) = \min \{ h_1(b^i), h_2(b^i) \} \]
\[ \omega^i \models \phi_1 \lor \phi_2 \] | \[ h(b^i) = \max \{ h_1(b^i), h_2(b^i) \} \]
\[ \omega^i \models \Box \phi \] | \[ h_{\Box}(b^i) = h_{\Box}(b) \]
\[ \omega^i \models \phi_1 \lor \phi_2 \] | \[ h_{\phi_1}(b^i) < 0 \implies h = h_1(b^i), \forall j \geq i \]
\[ \omega^i \models \Diamond \phi \] | \[ h(b^i) = h(b^i), \quad i \leq j \leq t^* \]
\[ \omega^i \models \Box \phi \] | \[ h(b^i) = h_{\phi}(b^i), \forall j \geq i \]
Safe Multi-Robot Coordination

Safety-Critical POMDPs

Dynamics:
\[ b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0} \]

Safe Set:
\[ C := \{ b \in \mathcal{D} \mid h(b) \geq 0 \} \]

Specifications:
\[ \Diamond (\text{GOAL}) \land \exists \sum_{i=1}^{3} (\mathbb{P}_{<0.05}(\text{OBS}_i)) \]
Safe Multi-Robot Coordination

Both agents assigned to the task to increase the expected reward
Next Steps: Safe Multi-Robot Coordination
**Conclusion:** Safety-Critical Autonomy

- **Stability:** $\dot{V}(x,u) \leq -\alpha V(x)$
- **Safety:** $\dot{h}(x,u) \geq -\gamma h(x)$
- View toward autonomy
- Applications to a variety of domains

**Future Work:**
- More underlying theory and synthesis
- Continue to apply experimentally
- Translate to real-world systems

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Conclusions – Next Steps

Walking Robots

Multi-Robot Systems

Collision Avoidance

Robotic Assistance
Goal: Robust Safety

Robust CBFs with uncertainty

Gurriet, Nilsson, Singletary, AA, ACC 2019, Access (submitted)
Learning + CBFs

With Learning

Exploratory Control

Experiment Data

Process Data

Learn $h_r$

Augment Controller

No Learning
Autonomy on Legged Robots

Xiong, AA, IROS 2018

Top Speed: 1.04 m/s

Xiong, Reher, AA, ICRA 2020 (submitted)
Lyapunov Controller

\[ u^*(x) = \arg\min_{(u,\delta) \in U \times \mathbb{R}} \| u - u_{\text{des}}(x) \|^2 \]

s.t. \[ \dot{V}(x, u) \leq -\alpha V(x) \]

+ Theorem \implies \text{Stable Walking}
Learning + Control of Assistive Devices

Gao, Gehlhar, AA, Liu, Delbruck, ICRA 2020
Learning + Control of Assistive Devices
Restoring Mobility
Goal: Safe Real World Autonomy

Thank You