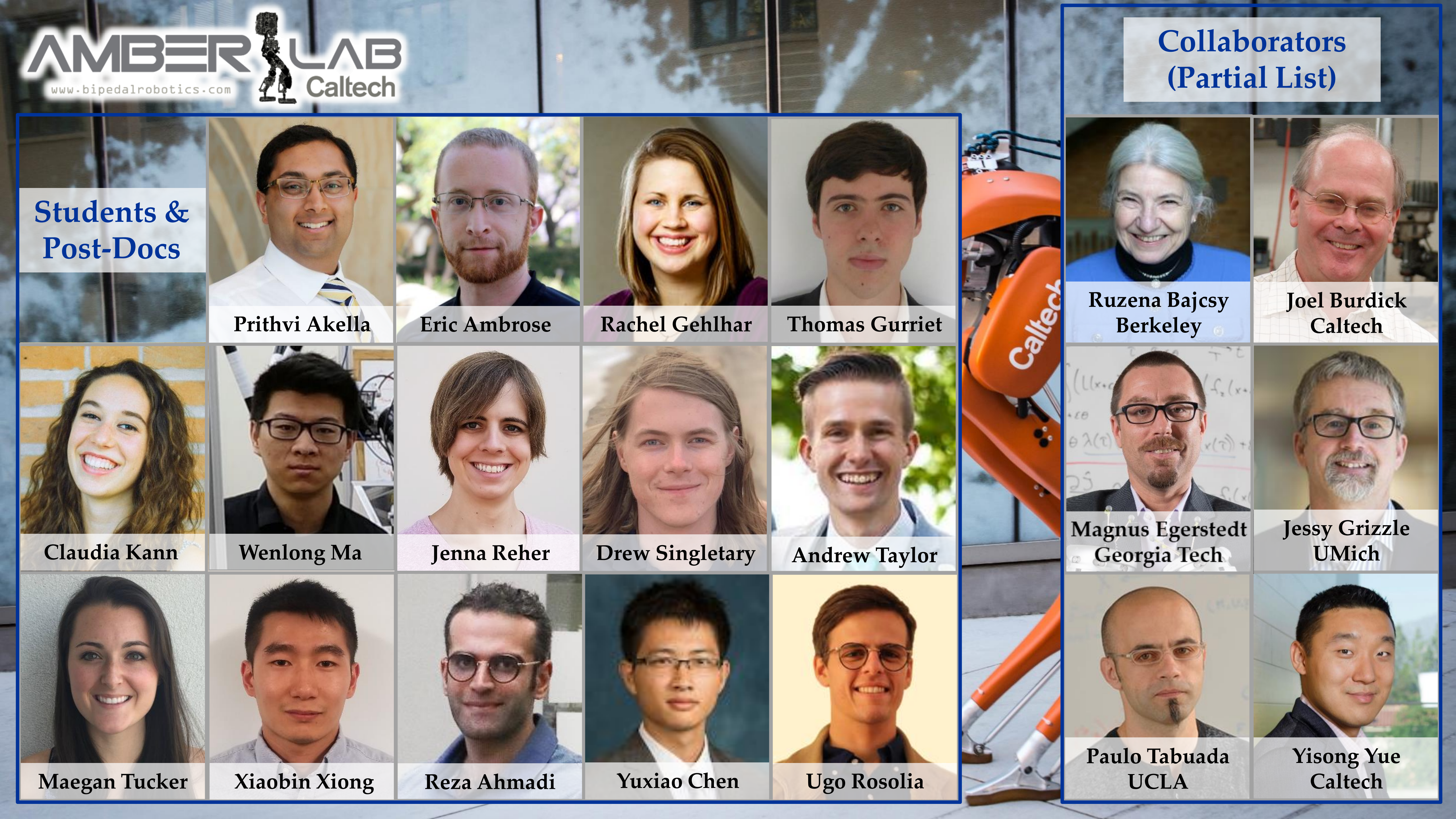
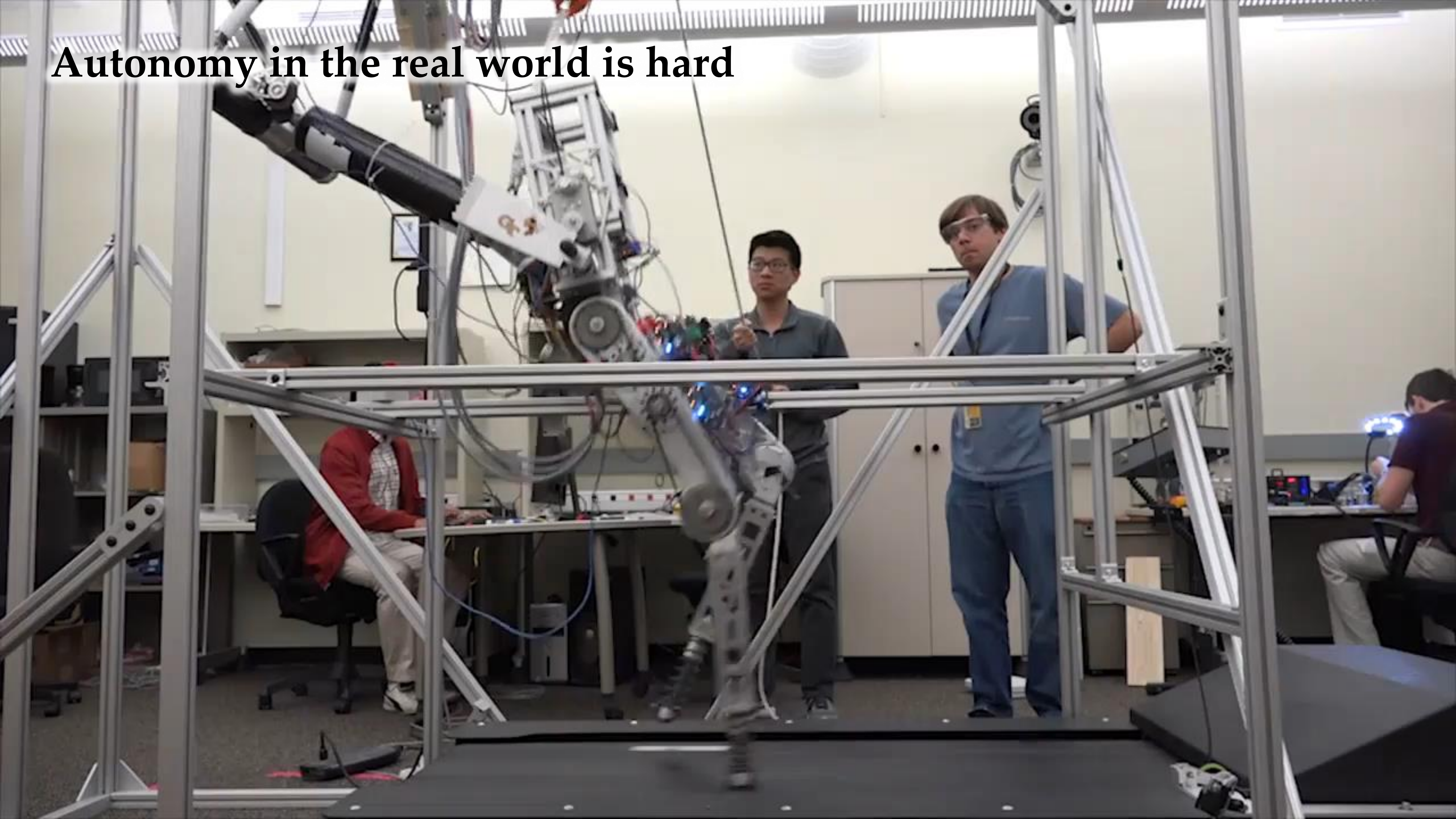
Safety-Critical Control of Autonomous Systems

Aaron D. Ames
Bren Professor
Mechanical and Civil Engineering
Control and Dynamical Systems
California Institute of Technology

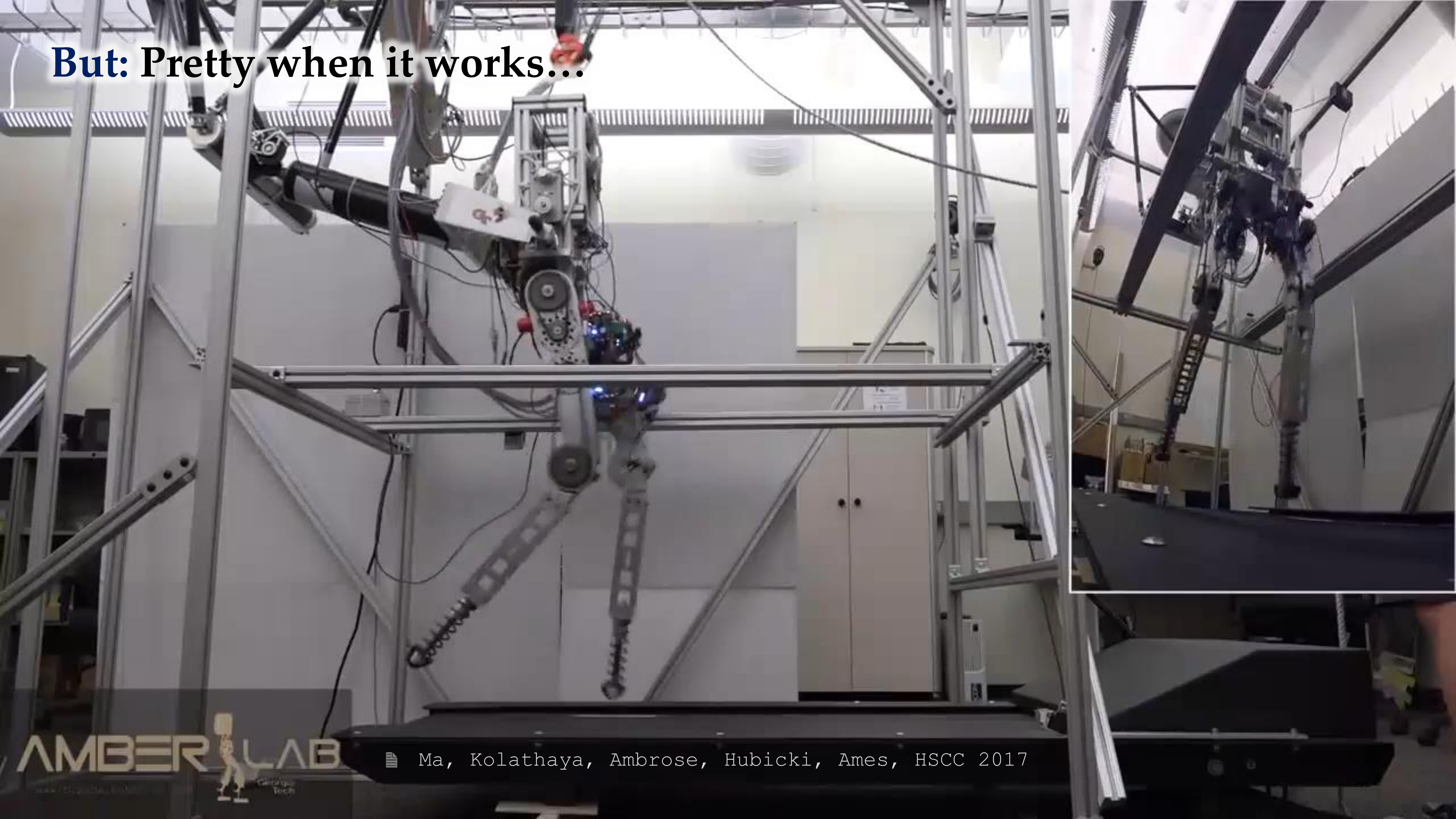
IPAM Workshop
Safe Operation of Connected and Autonomous Vehicle Fleets
October 25th, 2020

Caltech


AMBER LAB
www.bipedalrobotics.com
Caltech
Students & Post-Docs
Prithvi Akella
Eric Ambrose
Rachel Gehlhar
Thomas Gurriet
Claudia Kann
Wenlong Ma
Jenna Reher
Drew Singletary
Andrew Taylor
Maegan Tucker
Xiaobin Xiong
Reza Ahmadi
Yuxiao Chen
Ugo Rosolia
Collaborators (Partial List)
Ruzena Bajcsy Berkeley
Joel Burdick Caltech
Magnus Egerstedt Georgia Tech
Jessy Grizzle UMich
Paulo Tabuada UCLA
Yisong Yue Caltech
Caltech


Autonomy in the real world is hard

But: Pretty when it works…

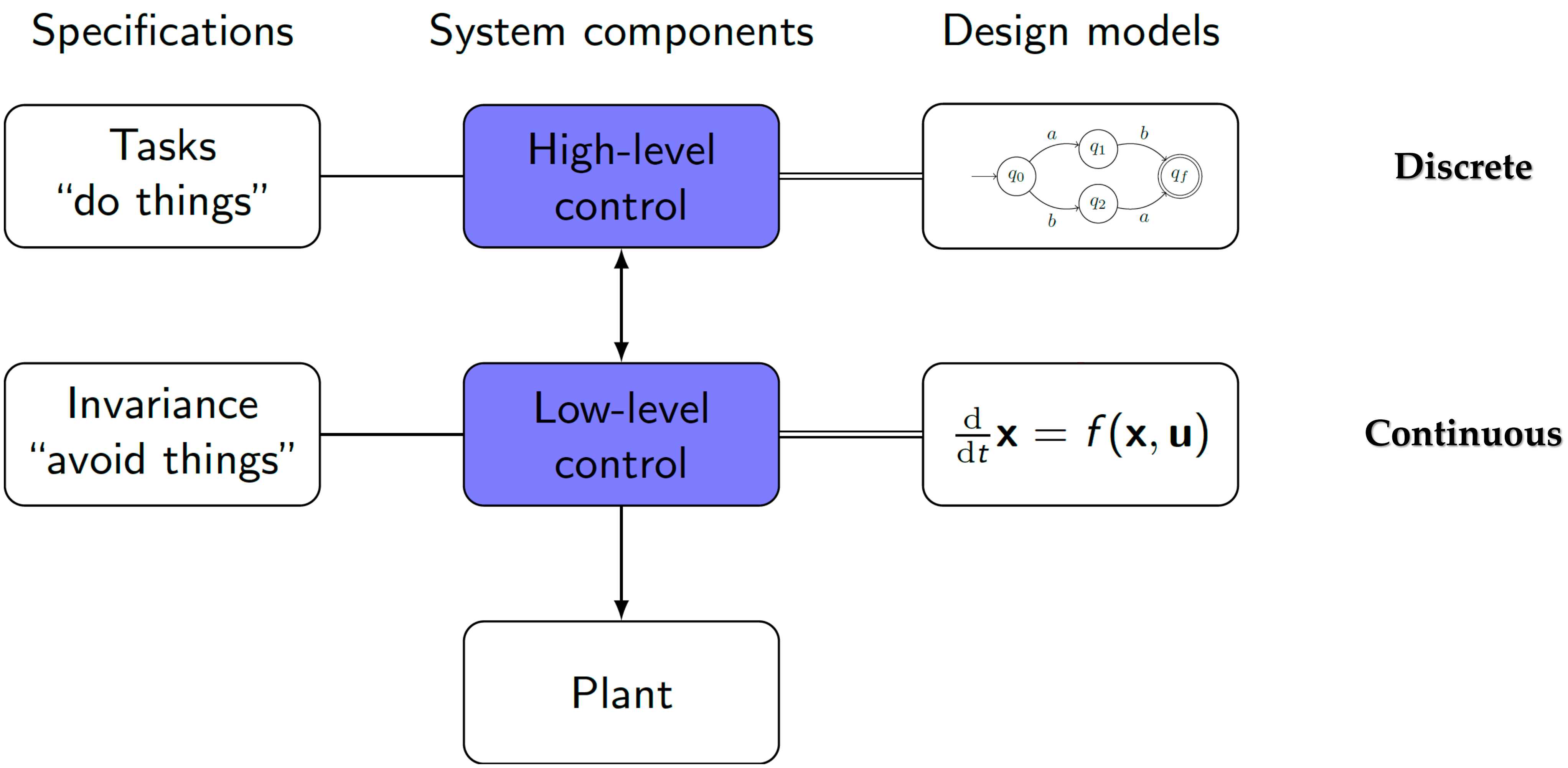


Ma, Kolathaya, Ambrose, Hubicki, Ames, HSCC 2017

Question: How do we make safety guarantees?

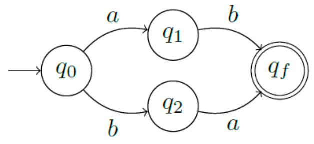
Autonomy: The Big Picture

Caltech



Specifications | System components | Design models

High-level control

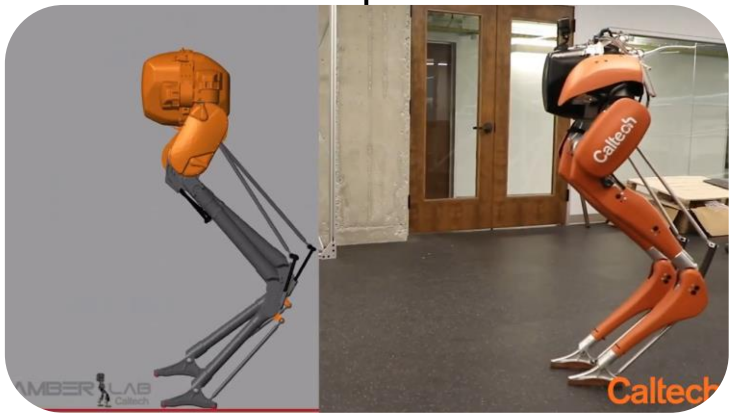


Discrete

Low-level control

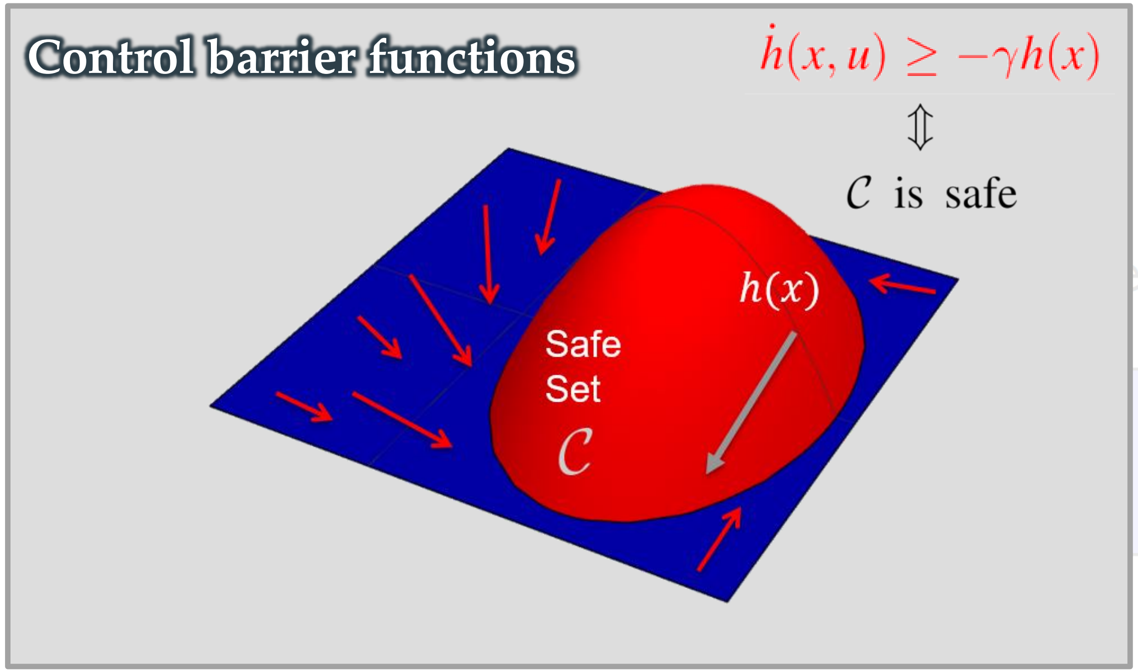
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = f(\mathbf{x}, \mathbf{u})$$

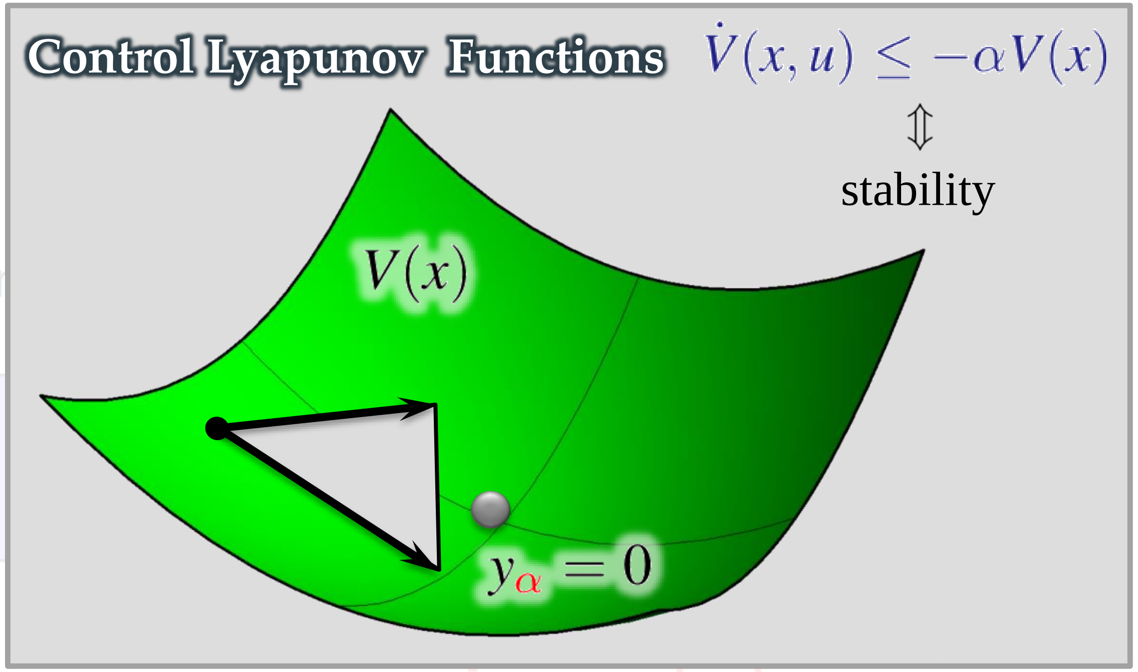
Continuous

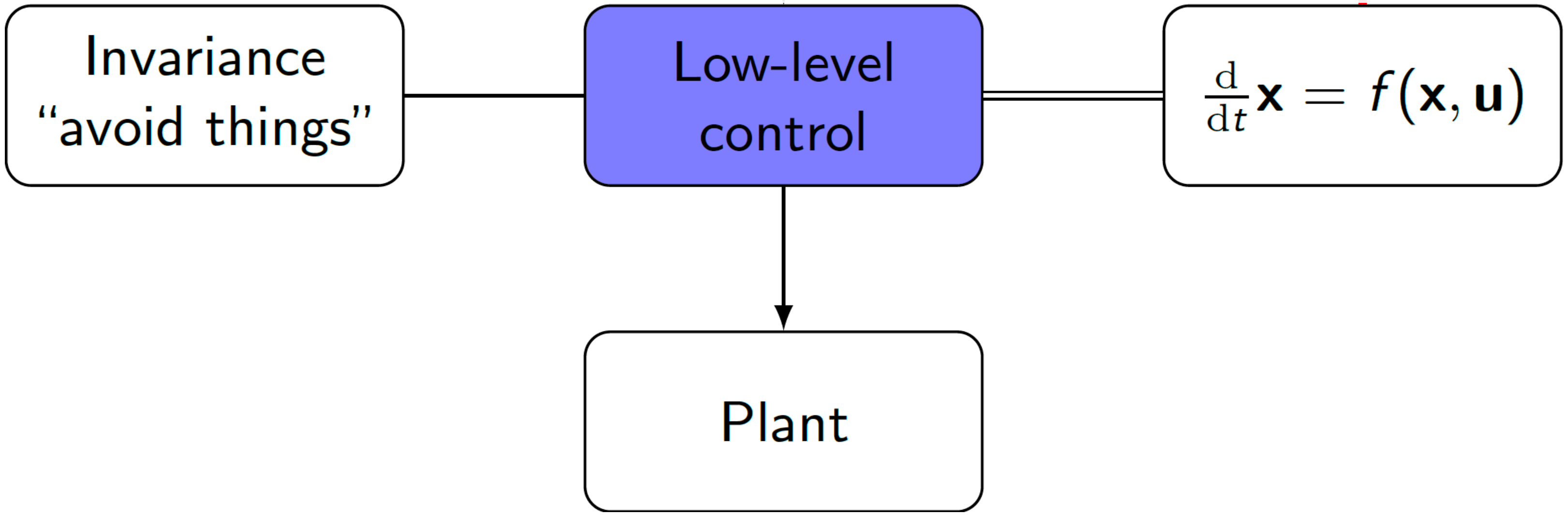


Plant

Reher, Ma, AA, ECC 2019


Control barrier functions
$\dot{h}(x,u) \geq -\gamma h(x)$
$\Updownarrow$
$\mathcal{C}$ is safe
$h(x)$
Safe Set
$\mathcal{C}$


Control Lyapunov Functions
$\dot{V}(x,u) \leq -\alpha V(x)$
$\Updownarrow$
stability
$V(x)$
$y_\alpha = 0$


Invariance
"avoid things"
Low-level control
$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = f(\mathbf{x},\mathbf{u})$
Plant

Control Lyapunov Functions

Lyapunov (1892)

PROBLÈME GÉNÉRAL

DE

LA STABILITÉ DU MOUVEMENT,

PAR M. A. LIAPOUNOFF.

Sontag (1989)

Systems & Control Letters 13 (1989) 117–123 117
North-Holland

A 'universal' construction of Artstein's theorem on nonlinear stabilization

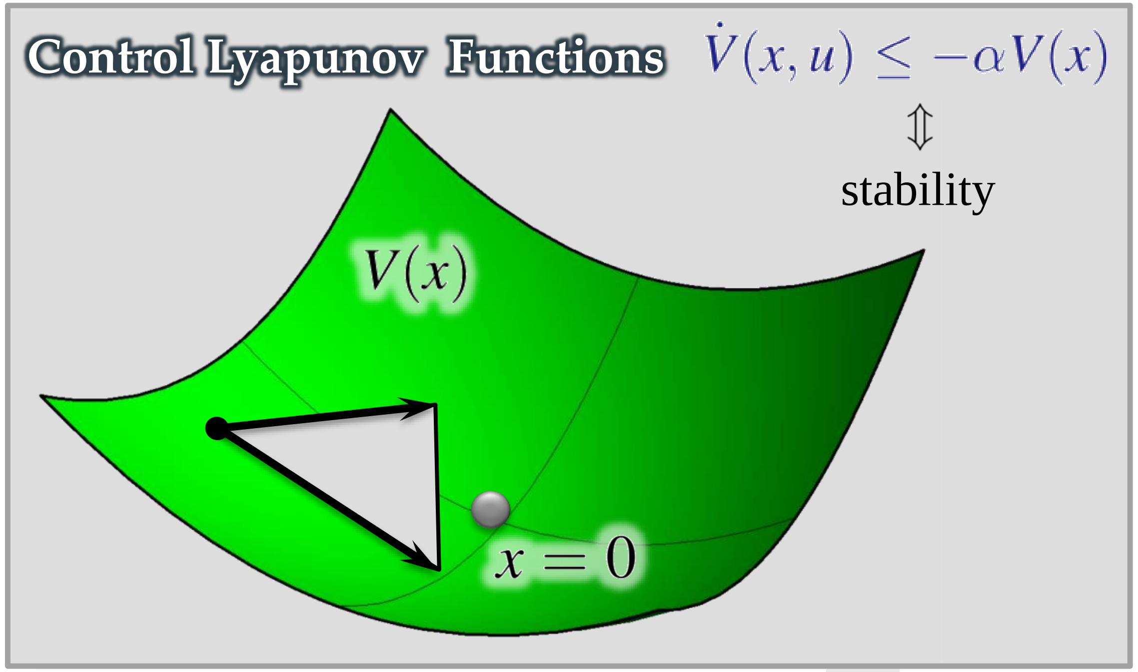
Eduardo D. SONTAG *

Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, U.S.A.

Received 7 March 1989

so that

$$\inf_{u \in \mathbb{R}^m} \{ L_f V(x) + u_1 L_{g_1} V(x) + \cdots + u_m L_{g_m} V(x) \} < 0 \qquad (2)$$



- Dynamics: For $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$:

$$\dot{x} = f(x) + g(x)u$$

- Lyapunov: $V : X \to \mathbb{R}_{\geq 0}$ satisfying:

$$c_1 \|x\|^2 \leq V(x) \leq c_2 \|x\|^2$$
$$\inf_{u \in U} \dot{V}(x,u) \leq -\alpha V(x)$$

- Main idea:

$$\dot{V}(x,u) \leq -\alpha V(x) \quad \Rightarrow \quad V(x(t)) \leq e^{-\alpha t} V(x(0))$$

PROBLÈME GÉNÉRAL

LA STABILITÉ DU MOUVEMENT,

Par M. A. LIAPOUNOFF.

Lyapunov Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \; \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{V}(x,u) \leq -\alpha V(x)$$

Theorem

If there exists control Lyapunov function:

$$\inf_{u\in U} \left[\dot{V}(x,u) + \alpha V(x)\right] \leq 0$$

then for all feedback controllers:

$$\begin{aligned} u(x) \quad &\in \quad \{u \in U : \dot{V}(x,u) \leq -\alpha V(x)\} \\ &\Downarrow \quad \dot{x} = f(x) + g(x)u(x) \\ x \quad &\to \quad 0 \;\; \textit{Exponentially.} \end{aligned}$$

Control Lyapunov Functions $\dot{V}(x,u) \leq -\alpha V(x)$

- Affine Dynamics: $\dot{x} = f(x) + g(x)u$
- Affine Constraint: The input u is affine in $\dot{V}$:

$$\dot{V}(x,u) = \frac{\partial V}{\partial x} f(x,z) + \frac{\partial V}{\partial x} g(x,z)u \leq -\alpha V(x)$$

- Synthesis: Closed form Controller:

$$m(x) = \begin{cases} -\frac{L_g V(x)^T (L_f V(x) + \alpha(V(x)))}{L_g V(x) L_g V(x)^T} & \text{if} \quad L_f V(x) > -\alpha(V(x)) \\ 0 & \text{if} \quad L_f V(x) \leq -\alpha(V(x)) \end{cases}$$

- Dynamics: For $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$:

$$\dot{x} = f(x) + g(x)u$$

- Lyapunov: $V : X \to \mathbb{R}_{\geq 0}$ satisfying:

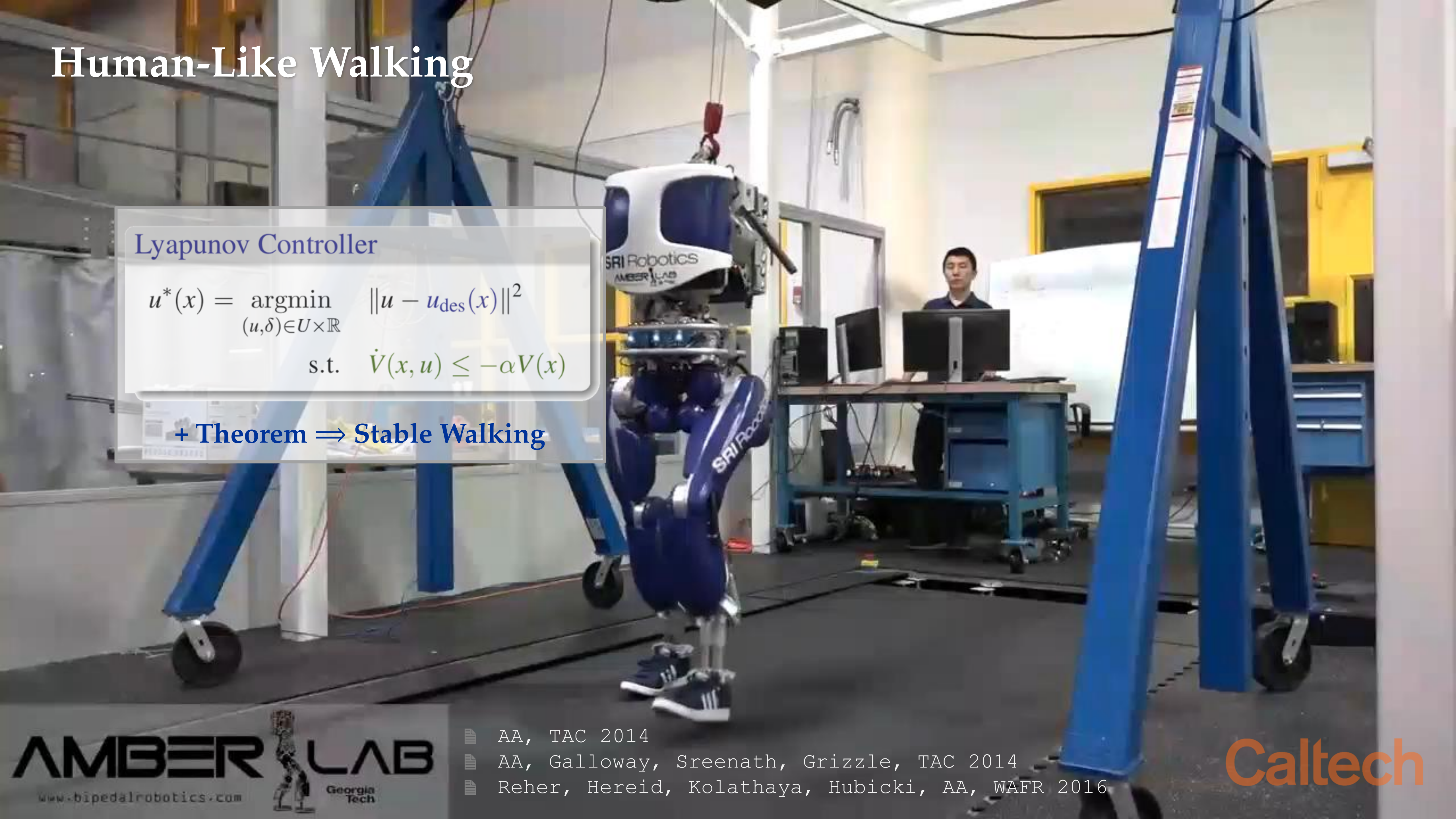
$$c_1\|x\|^2 \leq V(x) \leq c_2\|x\|^2$$

$$\inf_{u\in U} \dot{V}(x,u) \leq -\alpha V(x)$$

- Main idea:

$$\dot{V}(x,u) \leq -\alpha V(x) \quad \Rightarrow \quad V(x(t)) \leq e^{-\alpha t} V(x(0))$$

Human-Like Walking



Lyapunov Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\mathrm{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{V}(x,u) \leq -\alpha V(x)$$

+ Theorem $\Longrightarrow$ Stable Walking

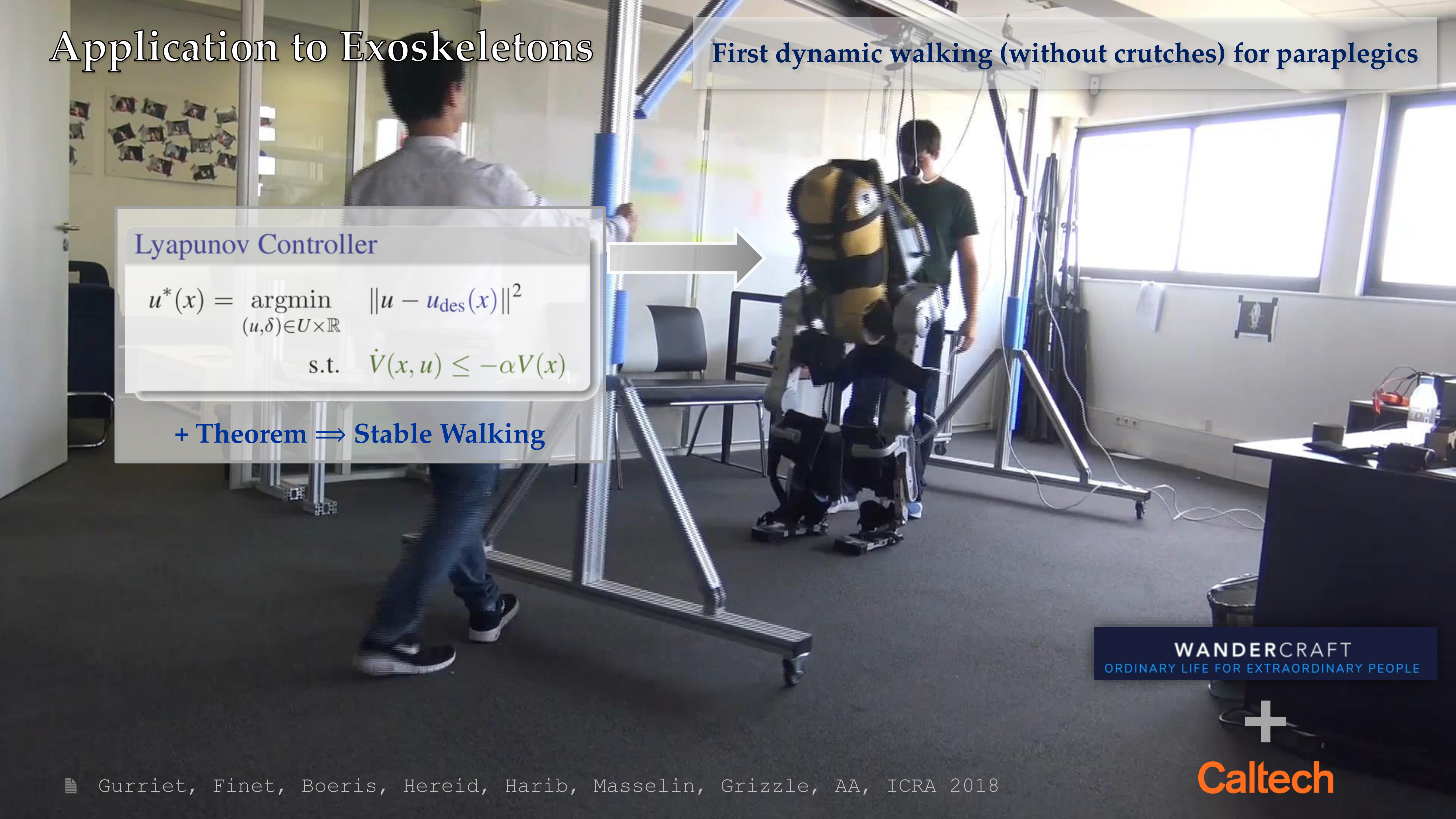
AMBER LAB www.bipedalrobotics.com Georgia Tech

- AA, TAC 2014
- AA, Galloway, Sreenath, Grizzle, TAC 2014
- Reher, Hereid, Kolathaya, Hubicki, AA, WAFR 2016

Caltech

Application to Exoskeletons

First dynamic walking (without crutches) for paraplegics



Lyapunov Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{V}(x,u) \leq -\alpha V(x)$$

+ Theorem $\Longrightarrow$ Stable Walking

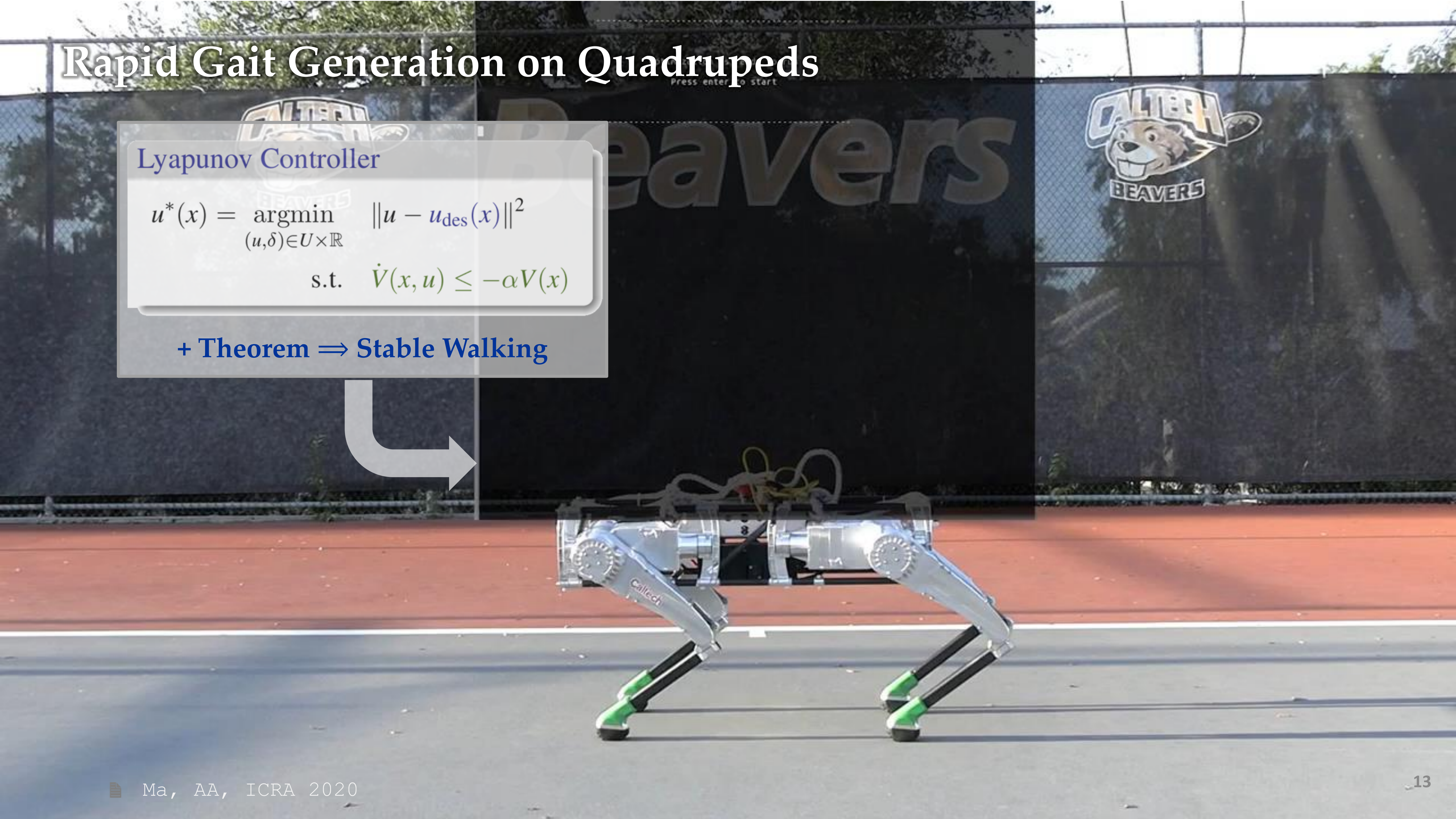
WANDERCRAFT
ORDINARY LIFE FOR EXTRAORDINARY PEOPLE

+

Caltech

Gurriet, Finet, Boeris, Hereid, Harib, Masselin, Grizzle, AA, ICRA 2018

Rapid Gait Generation on Quadrupeds



Lyapunov Controller

$$
\begin{aligned}
u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad & \|u - u_{\text{des}}(x)\|^2 \\
\text{s.t.} \quad & \dot{V}(x,u) \le -\alpha V(x)
\end{aligned}
$$

+ Theorem $\Longrightarrow$ Stable Walking

Ma, AA, ICRA 2020

Application to Quadrupeds

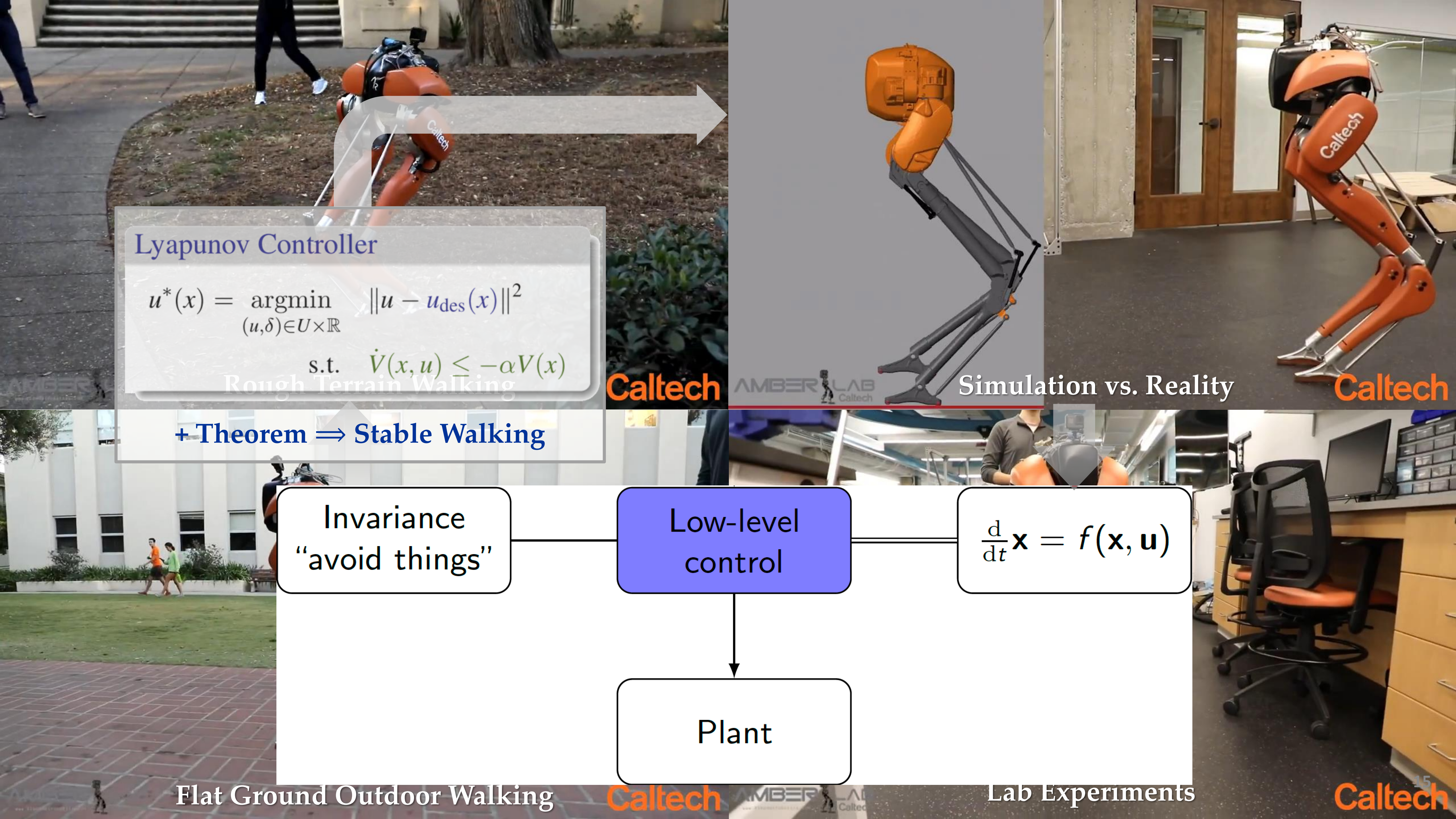
Lyapunov Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\text{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{V}(x,u) \leq -\alpha V(x)$$

+ Theorem $\Longrightarrow$ Stable Walking

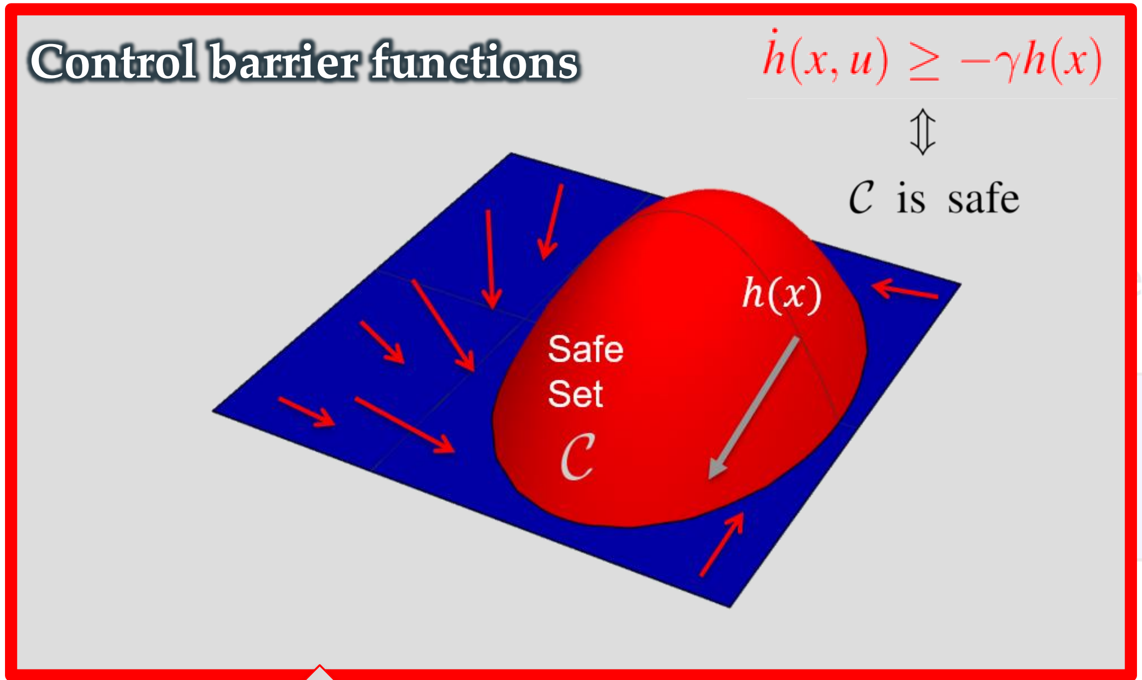
Ma, AA, ICRA 2020, CSL 2020

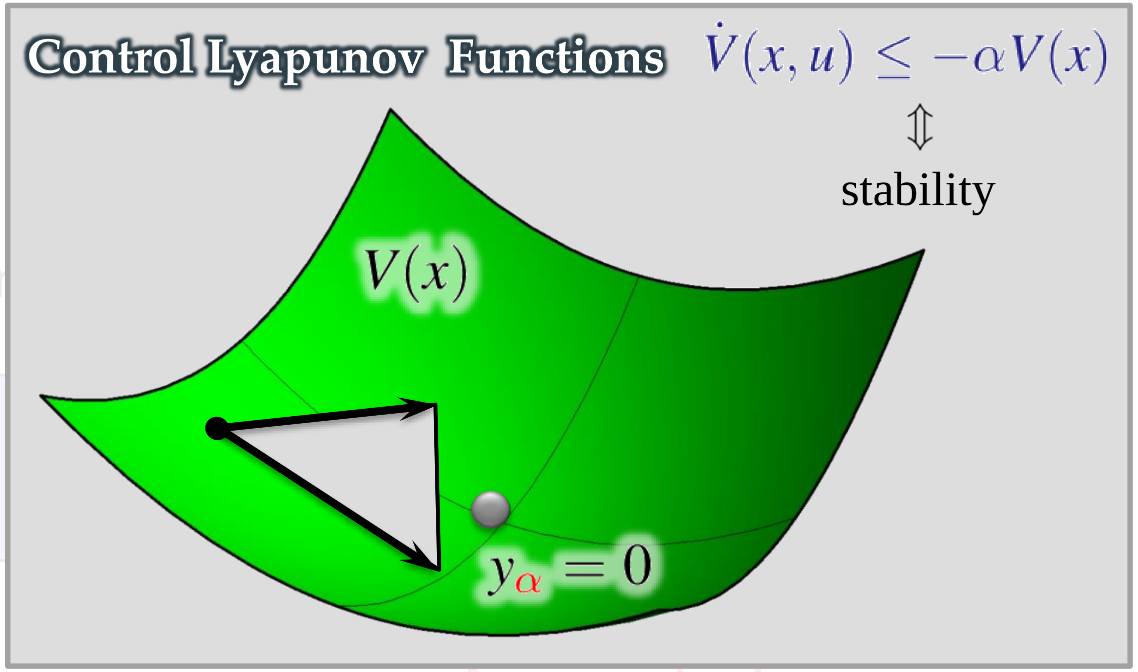


Lyapunov Controller
$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \; \|u - u_{\text{des}}(x)\|^2$
s.t. $\dot{V}(x,u) \le -\alpha V(x)$
+ Theorem ⟹ Stable Walking
Rough Terrain Walking
Caltech
AMBER LAB
Simulation vs. Reality
Caltech
Invariance
"avoid things"
Low-level control
$\frac{d}{dt}\mathbf{x} = f(\mathbf{x}, \mathbf{u})$
Plant
Flat Ground Outdoor Walking
Caltech
Lab Experiments
Caltech

Safety-Critical Walking

Reher, Ma, AA, ECC 2019





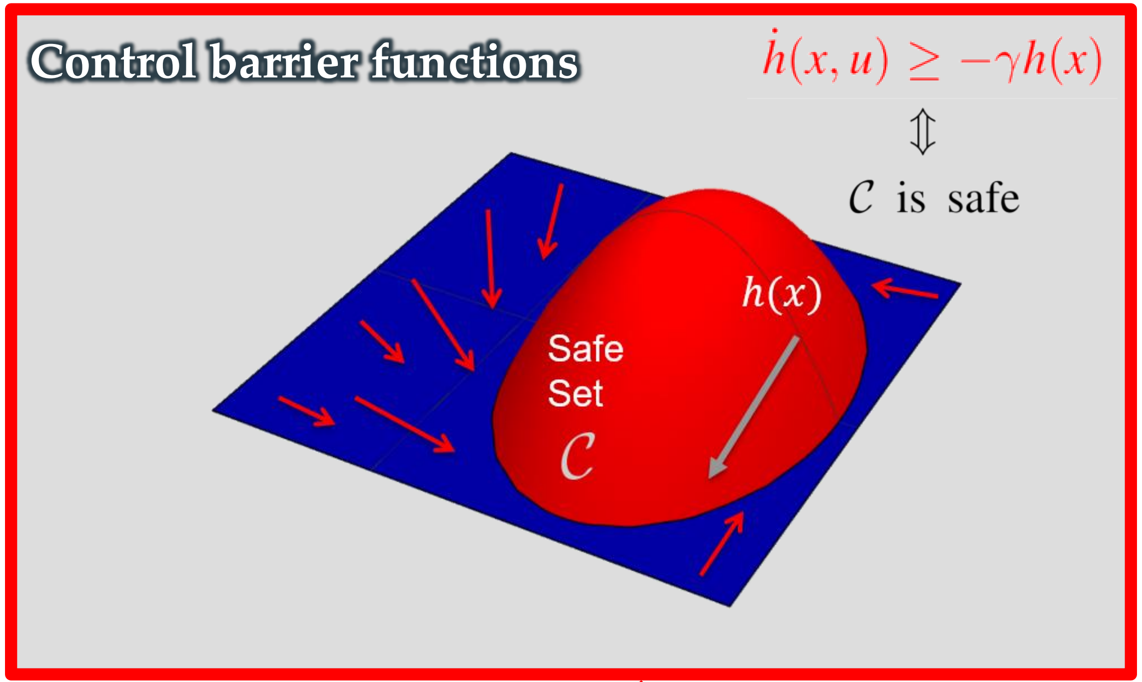
High-level control

Invariance "avoid things"

Low-level control

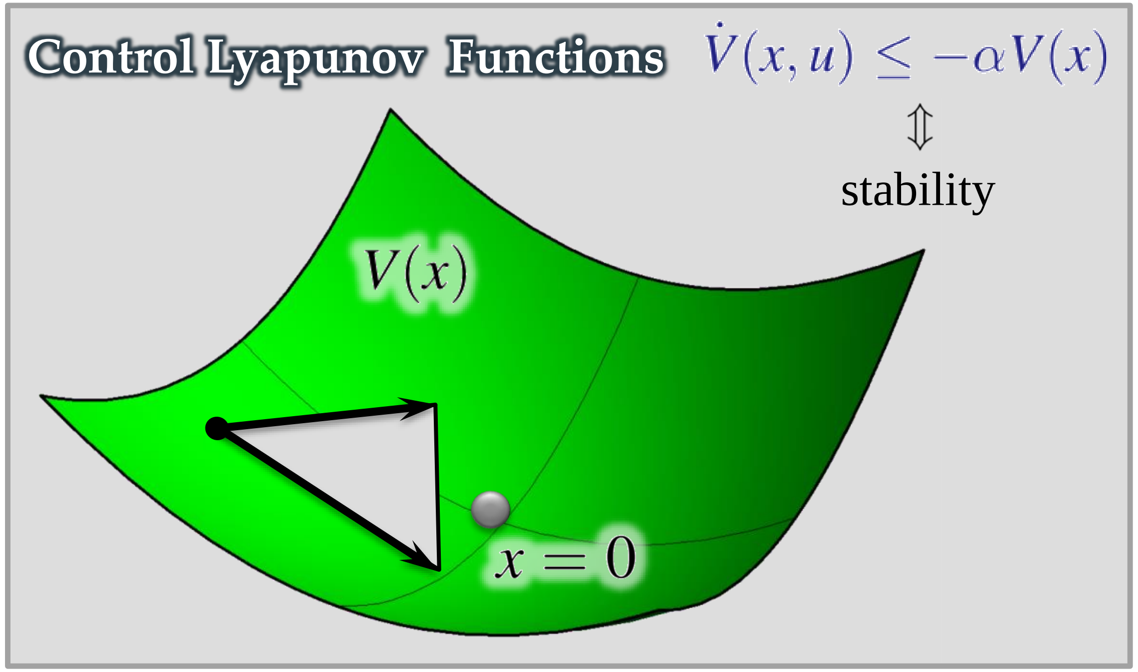
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = f(\mathbf{x}, \mathbf{u})$$

Plant

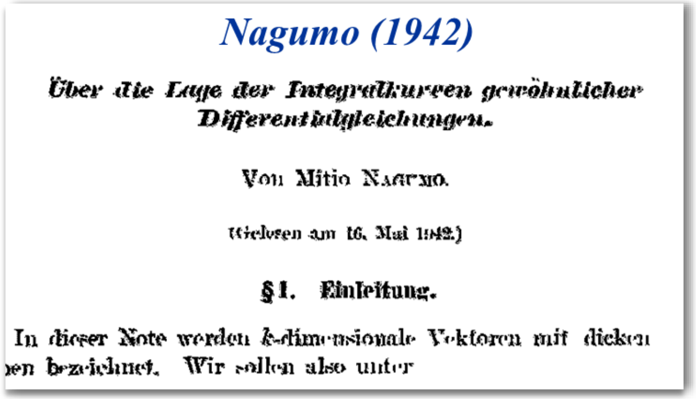


Embedding

$\hookleftarrow$



Safety-Critical Control $\Longrightarrow$ **Need something more general than Lyapunov**



Nagumo (1942)

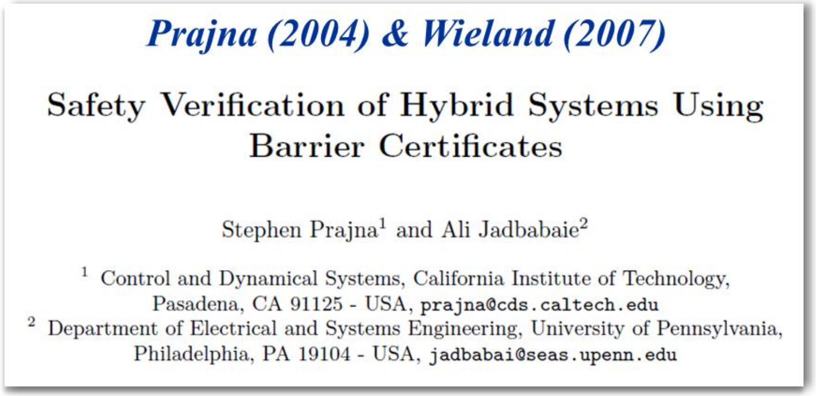
Über die Lage der Integralkurven gewöhnlicher Differentialgleichungen.

Von Mitio Nagumo.

(Gelesen am 16. Mai 1942.)

§ 1. Einleitung.

In dieser Note werden k-dimensionale Vektoren mit dicken ...en bezeichnet. Wir sollen also unter



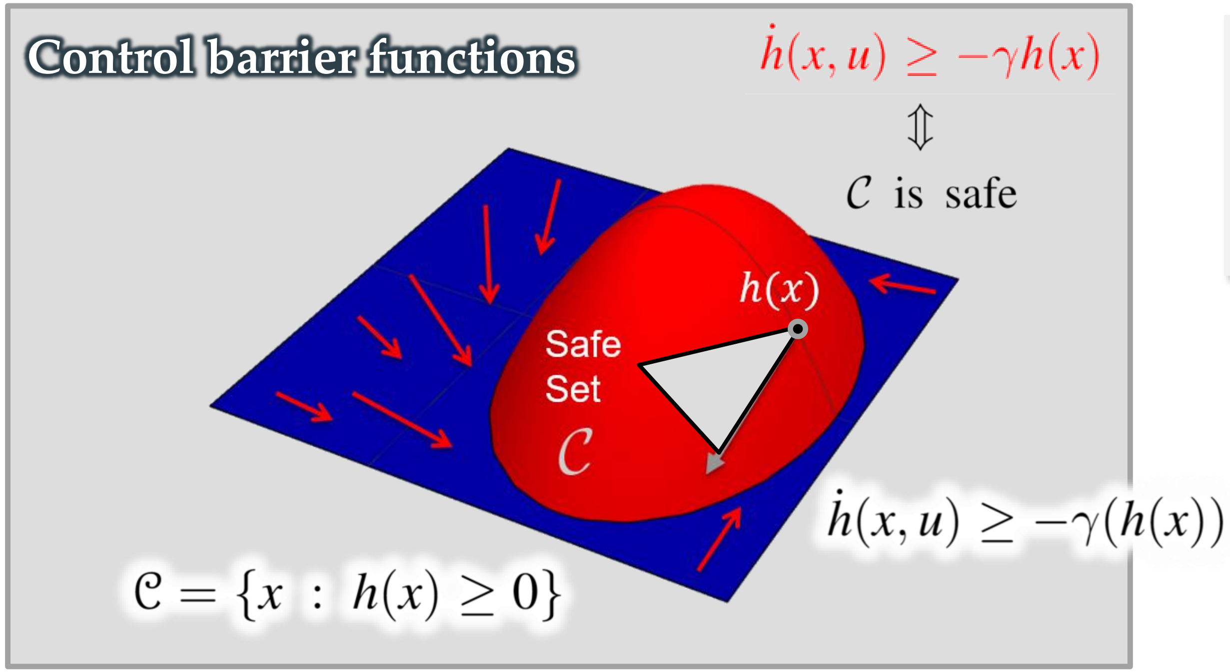
Prajna (2004) & Wieland (2007)

Safety Verification of Hybrid Systems Using Barrier Certificates

Stephen Prajna[1] and Ali Jadbabaie[2]

[1] Control and Dynamical Systems, California Institute of Technology, Pasadena, CA 91125 - USA, prajna@cds.caltech.edu
[2] Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, PA 19104 - USA, jadbabai@seas.upenn.edu

AA, Coogan, Egerstedt, Notomista, Sreenath, Tabuada, ECC 2019 (includes brief history)



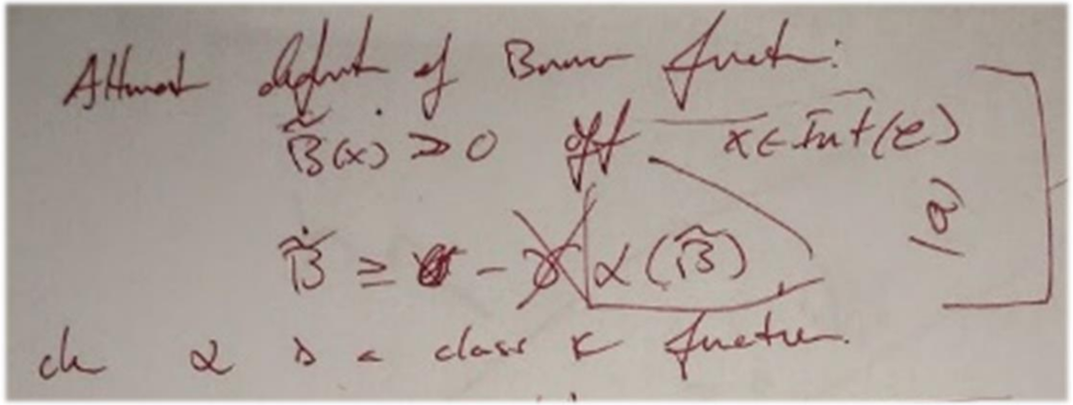
Control Barrier Functions

Provide necessary and sufficient conditions for set invariance, i.e., safety – ***on the entire safe set***

- **Dynamics:** $\dot{x} = f(x) + g(x)u$
- Safe set $\mathcal{C}$: defined by h:

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

Ames, Tabuada, Grizzle (2014)



Control Barrier Function

For all $x \in \mathcal{C}$, there exists $u \in \mathbb{R}^m$ such that:

$$\dot{h}(x,u) = \frac{\partial h}{\partial x}(x)(f(x) + g(x)u) \geq -\gamma(h(x))$$

⇕

$\mathcal{C}$ is safe

Here $\gamma : \mathbb{R} \to \mathbb{R}$ is an extended class $\mathcal{K}$ function (strictly increasing with $\gamma(0) = 0$).

AA, Tabuada Grizzle, CDC 2014
AA, Xu, Tabuada Grizzle, TAC 2017

Control barrier functions

$\dot{h}(x,u) \geq -\gamma h(x)$

Synthesis: Let:

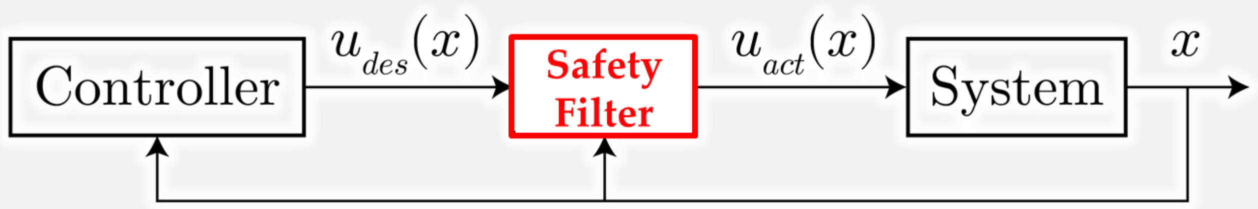
$$\varphi(x) \triangleq L_f h(x) + L_g h(x) u_{\text{des}}(x) + \alpha(h(x))$$

Closed form controller:

$$u^*(x) = u_{\text{des}}(x) - \begin{cases} \frac{\varphi(x) L_g h(x)^T}{L_g h(x) L_g h(x)^T} & \text{if} \quad \varphi(x) < 0 \\ 0 & \text{if} \quad \varphi(x) \geq 0 \end{cases}$$

Safety Filters

Filter a given control input to guarantee safety:



- Safe set $\mathcal{C}$: defined by h:

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$

Control Barrier Function

For all $x \in \mathcal{C}$, there exists $u \in \mathbb{R}^m$ such that:

$$\dot{h}(x,u) = \frac{\partial h}{\partial x}(x)(f(x) + g(x)u) \geq -\gamma(h(x))$$

$\Updownarrow$

$\mathcal{C}$ is safe

Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Existing (desired) controller

Safety (barrier function) constraint

AA, Tabuada Grizzle, CDC 2014

AA, Xu, Tabuada Grizzle, TAC 2017

Control barrier functions $\dot{h}(x,u) \geq -\gamma h(x)$

Synthesis: Let:

$$\varphi(x) \triangleq L_f h(x) + L_g h(x) u_{\text{des}}(x) + \alpha(h(x))$$

Closed form controller:

$$u^*(x) = u_{\text{des}}(x) - \begin{cases} \dfrac{\varphi(x) L_g h(x)^T}{L_g h(x) L_g h(x)^T} & \text{if} \quad \varphi(x) < 0 \\ 0 & \text{if} \quad \varphi(x) \geq 0 \end{cases}$$

Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta) \in U \times \mathbb{R}}{\text{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Existing (desired) controller

Safety (barrier function) constraint

AA, Tabuada Grizzle, CDC 2014

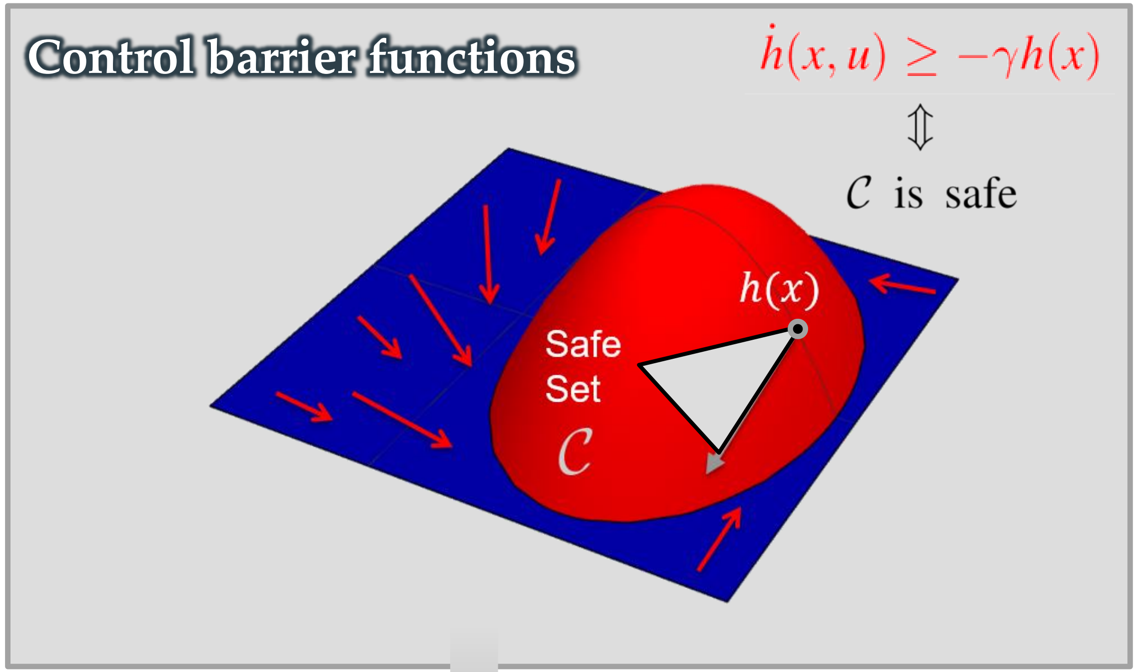
AA, Xu, Tabuada Grizzle, TAC 2017

Safety Filters

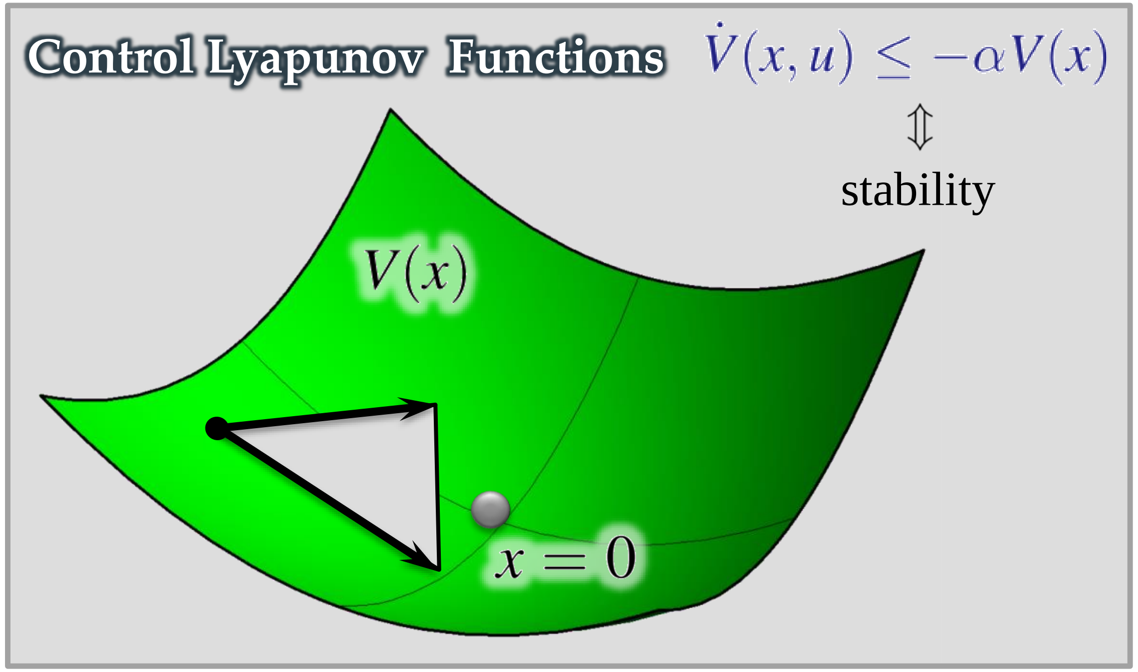
Filter a given control input to guarantee safety.

No Control Barrier function ⟹ Unsafe

With Control Barrier function ⟹ Safe



Embedding



Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Fundemental Properties:

Lemma. *Control barrier functions imply stability of the set* $\mathcal{C}$.

Control Barrier Function h: Yields a Lyapunov function for $\mathcal{C}$:

$$V_{\mathcal{C}}(x) \triangleq \begin{cases} 0 & \text{if} \quad x \in \mathcal{C} \\ -h(x) & \text{if} \quad x \in \overline{\mathcal{C}} = \mathbb{R}^n - \mathcal{C} \end{cases}$$

Theorem

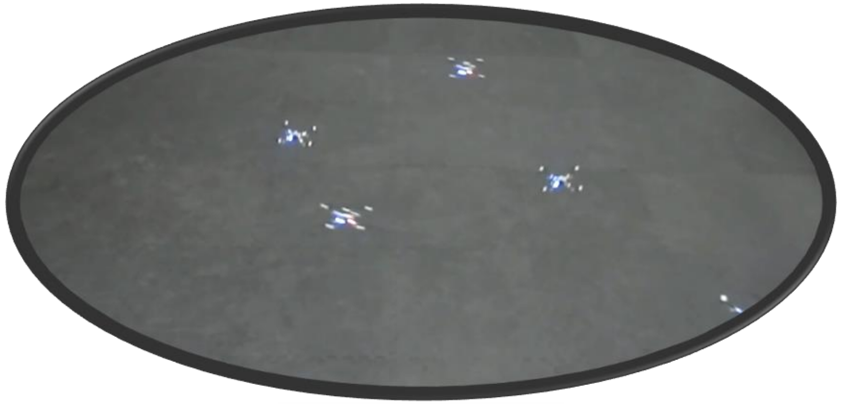
Lyapunov is the special case of barriers for $\mathcal{C} = \{0\}$.

AA, Xu, Tabuada Grizzle, TAC 2017

APPLICATIONS

Caltech

Walking Robots



Multi-Robot Systems

Automotive Systems

Collision Avoidance

Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

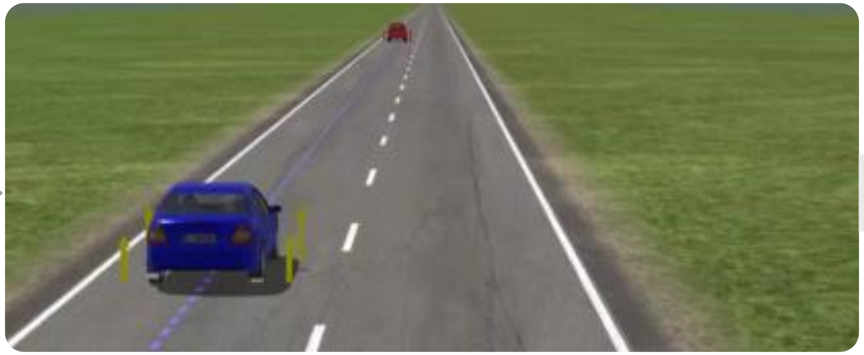
$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

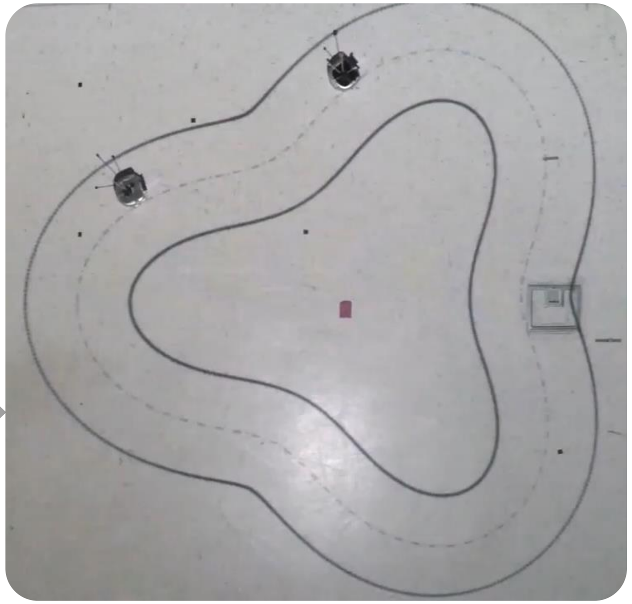
Joint with: Egersted (GaTech), Tabuada (UCLA), Grizzle (UMich), Feron (GaTech), Xu (UW), Wandercraft, Hutter (ETH), Orosz (UMich)

Application to Automotive Systems

Adaptive Cruise Control (ACC)

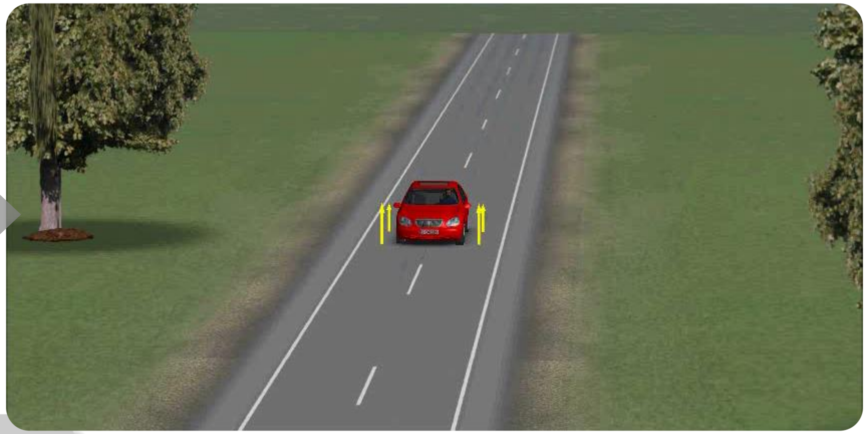
- Safety Constraints: “half the speedometer” following rule
- Control Objectives: Achieve a desired speed.

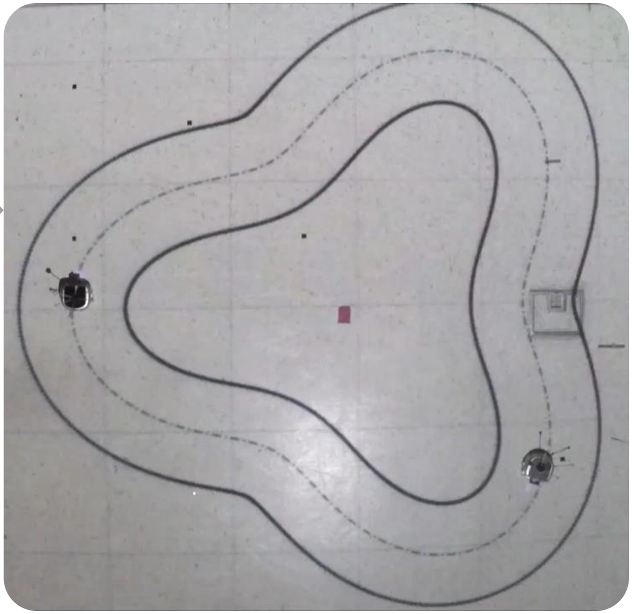




Lane Keeping

- Safety Constraints: Stay in the lane for all time
- Control Objectives: Achieve reference signal





Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\mathrm{des}}(x)\|^2$$

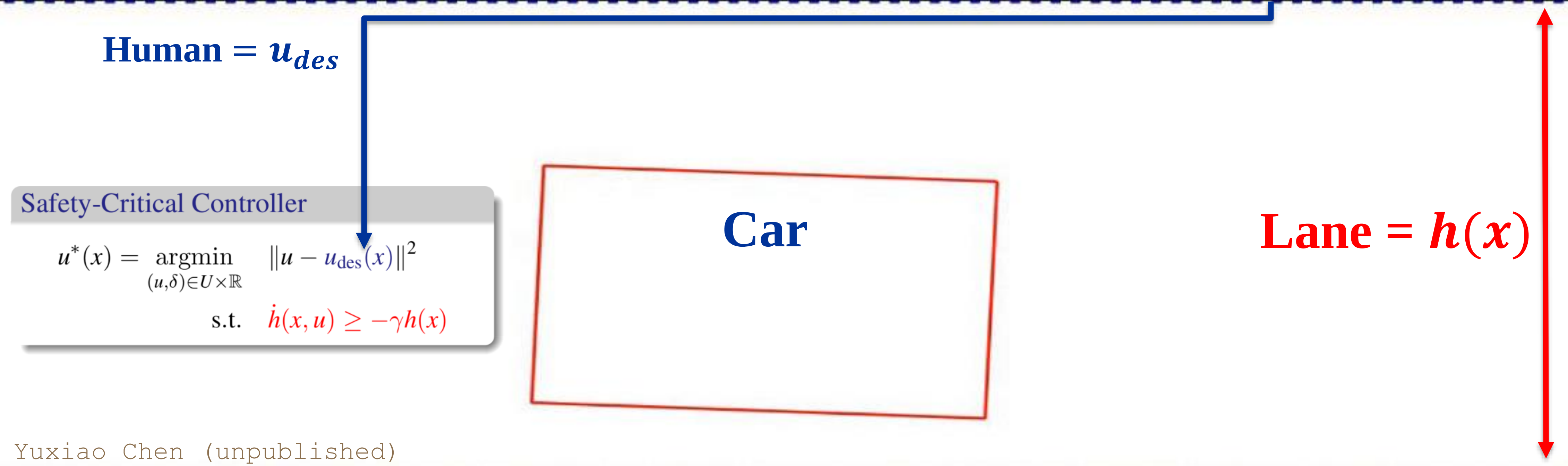
$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Existing (desired) controllers

Safety (Barrier function) constraint

Xu, Grizzle, Tabuada and AA, TASE 2018

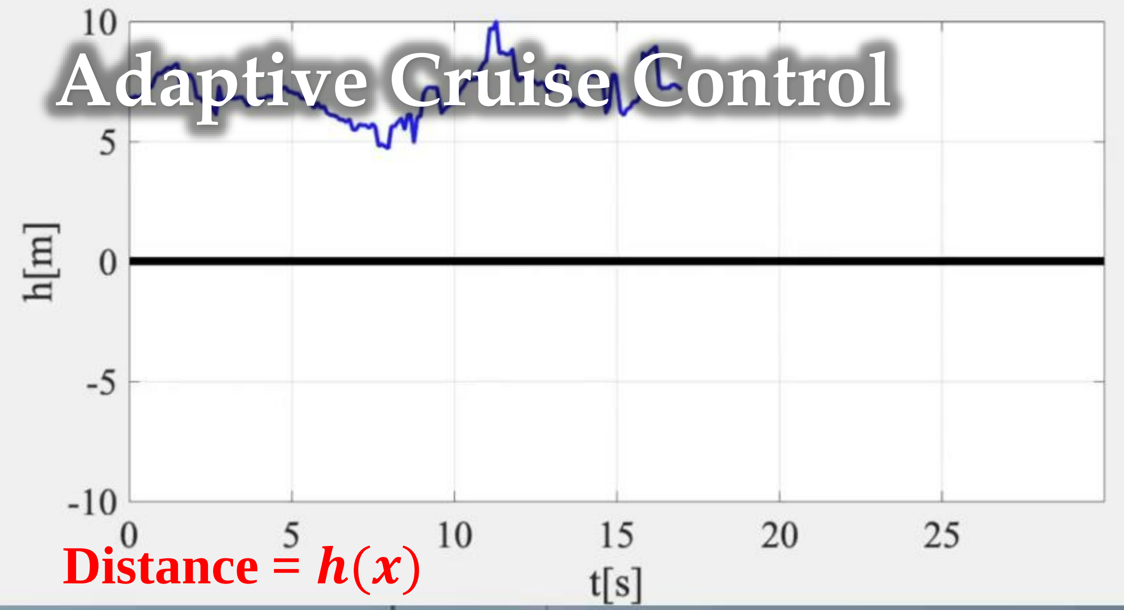
Lane Keeping



$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \|u - u_{\text{des}}(x)\|^2$$
$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Yuxiao Chen (unpublished)

Adaptive Cruise Control



Distance = $h(x)$

Human = u_{des}

Safety-Critical Controller

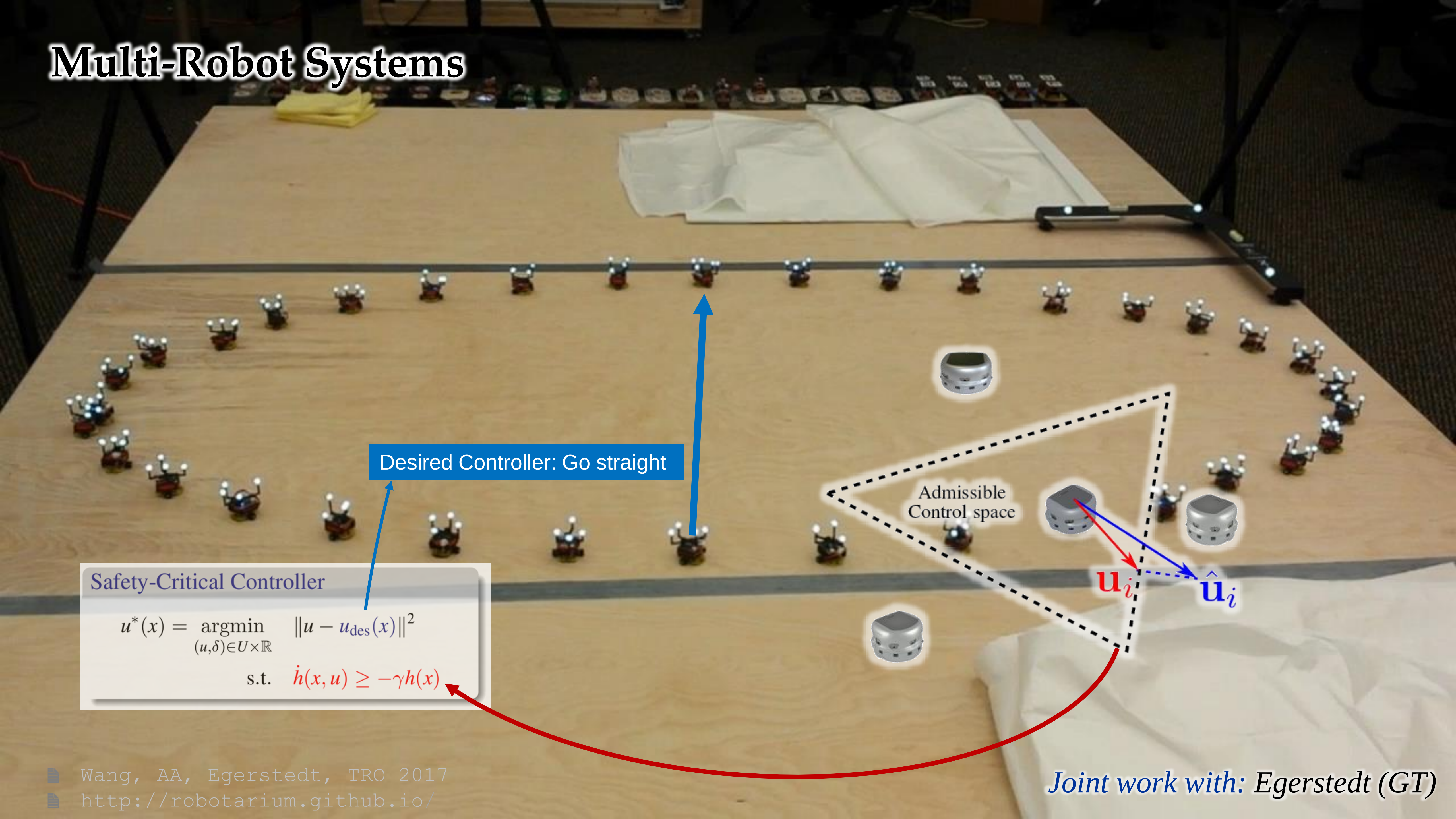
$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Alan, Taylor, He, Orosz and AA, 2020 (In Preparation)

Joint work with: Orosz (UMich)

Multi-Robot Systems



Safety-Critical Controller

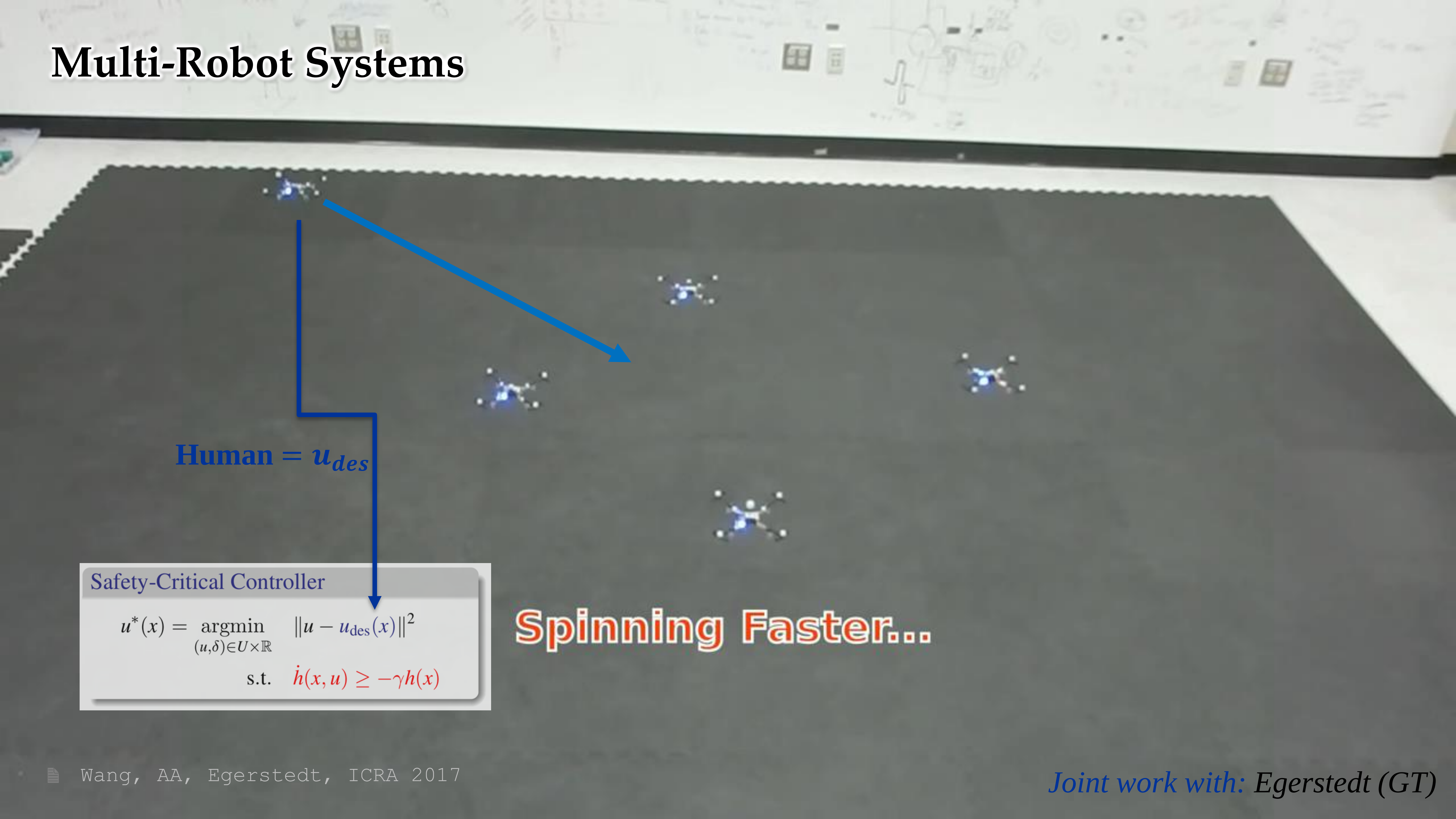
$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

- Wang, AA, Egerstedt, TRO 2017
- http://robotarium.github.io/

Joint work with: Egerstedt (GT)

Multi-Robot Systems



Human = u_{des}

Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

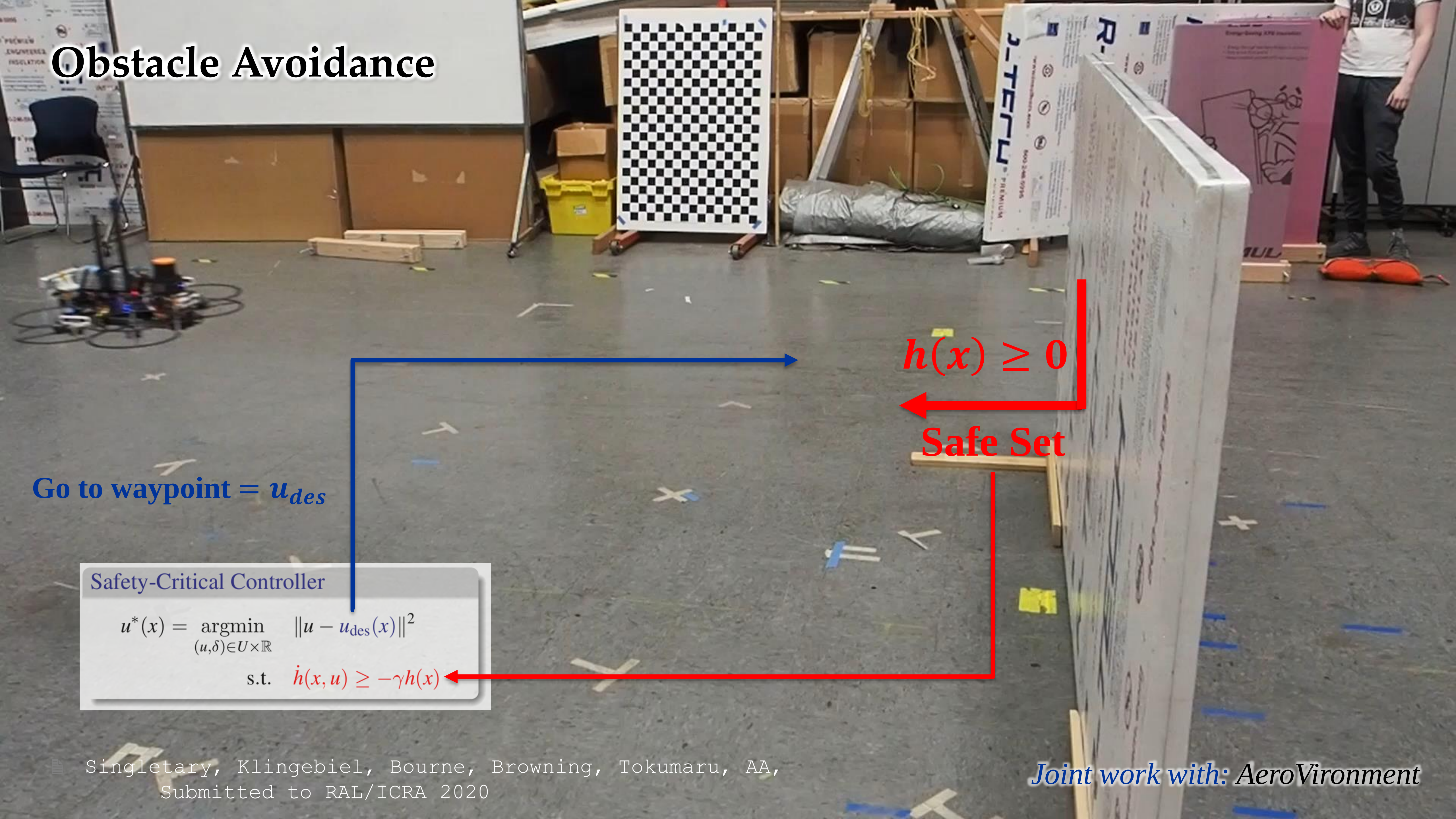
$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Spinning Faster...

Wang, AA, Egerstedt, ICRA 2017

Joint work with: Egerstedt (GT)

Obstacle Avoidance



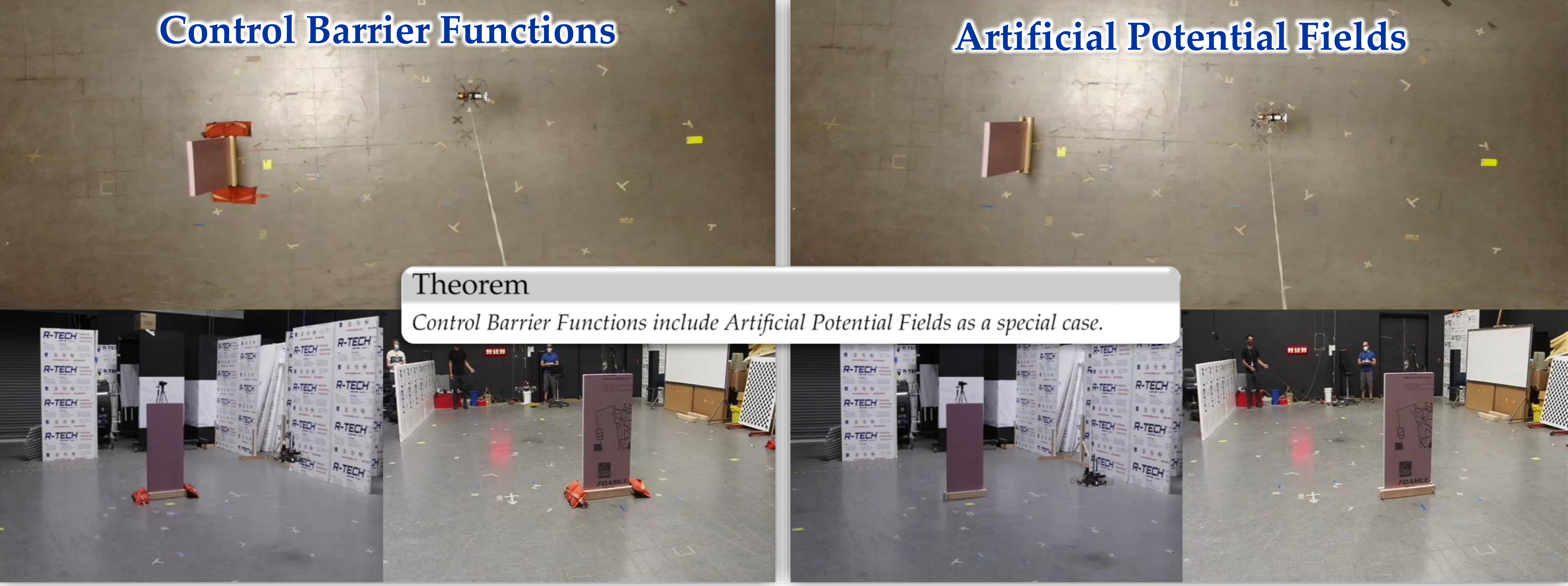
Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Singletary, Klingebiel, Bourne, Browning, Tokumaru, AA,
Submitted to RAL/ICRA 2020

Joint work with: AeroVironment



Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$
$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

$$u_{\text{des}}(x) = \nabla U_{\text{att}}(x)$$

$$h(x) = \frac{1}{1+U_{\text{rep}}(x)} - \delta$$

Repulsive Potential $U_{\text{rep}}(x)$**:** Blows up at obstacle:

$$U_{\text{rep}}(x) \to \infty \quad \text{as} \quad \|x - x_{\text{obst}}\| \to D_{\text{obst}}.$$

Attractive Potential $U_{\text{att}}(x)$**:** Positive definite about the goal:

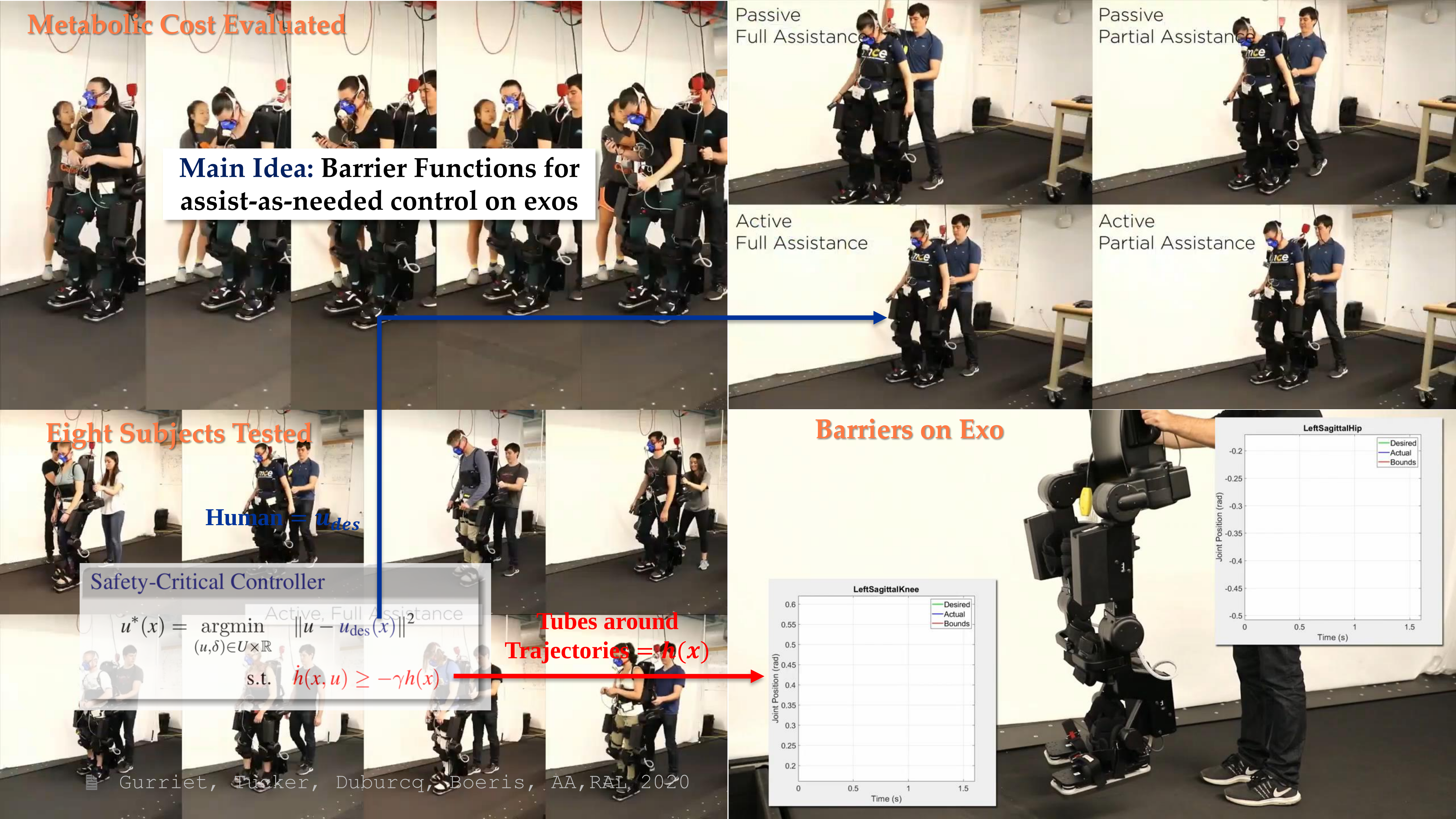
$$\underline{c}\|x - x_{\text{goal}}\|^2 \leq U_{\text{att}}(x) \leq \bar{c}\|x - x_{\text{goal}}\|^2.$$

Artificial Potential: $U(x) = U_{\text{rep}}(x) + U_{\text{att}}(x)$**:** Yields:

$$u(x) = \nabla U(x) = \nabla U_{\text{rep}}(x) + \nabla U_{\text{att}}(x)$$

Singletary, Klingebiel, Bourne, Browning, Tokumaru, AA,
Submitted to RAL/ICRA 2020

Robotic Walking
Desired Controller: Stable Walking
Safety-Critical Controller
$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$
$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$
Location of stepping stones = $h(x)$
Grandia, Taylor, AA and Hutter, RAL/ICRA 2020 (submitted)
Joint work with: Hutter (ETH)

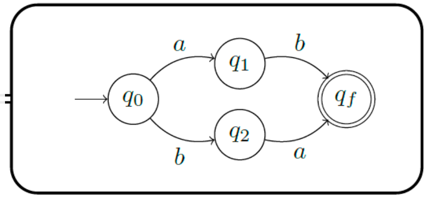

Metabolic Cost Evaluated
Main Idea: Barrier Functions for assist-as-needed control on exos
Passive
Full Assistance
Passive
Partial Assistance
Active
Full Assistance
Active
Partial Assistance
Eight Subjects Tested
Human = u_{des}
Safety-Critical Controller
Active, Full Assistance
$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \|u - u_{\text{des}}(x)\|^2$
s.t. $\dot{h}(x,u) \geq -\gamma h(x)$
Tubes around
Trajectories = $h(x)$
Gurriet, Tucker, Duburcq, Boeris, AA, RAL, 2020
Barriers on Exo
LeftSagittalKnee
Desired
Actual
Bounds
Joint Position (rad)
Time (s)
LeftSagittalHip
Desired
Actual
Bounds
Joint Position (rad)
Time (s)

Back to the Big Picture

Caltech

System components

Design models



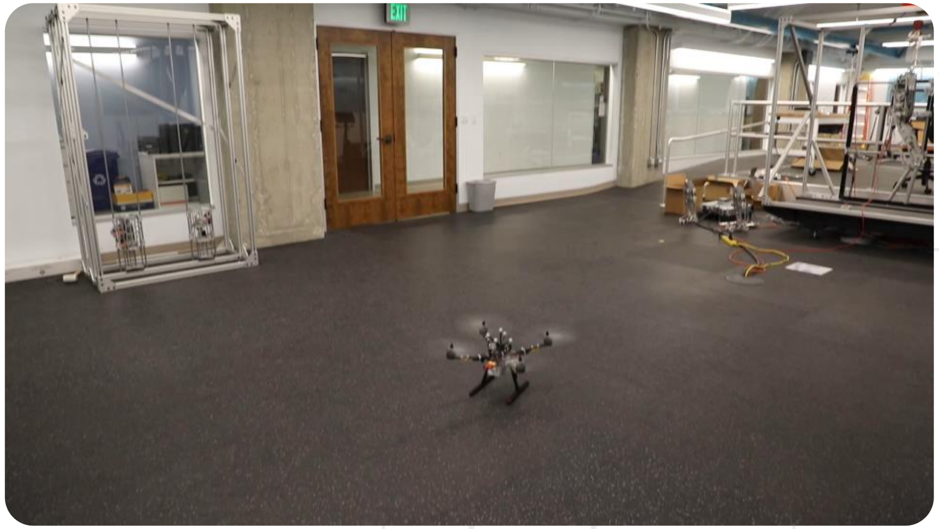
High-level control

Need: Unify high and low level methods

Control Barrier Functions

Low-level control

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = f(\mathbf{x}, \mathbf{u})$$



Aerial Robots

Invariance

"avoid things"

Safety-Critical Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

$$\text{s.t.} \quad \dot{h}(x,u) \geq -\gamma h(x)$$

Ground Robots

Multi-Robot Coordination

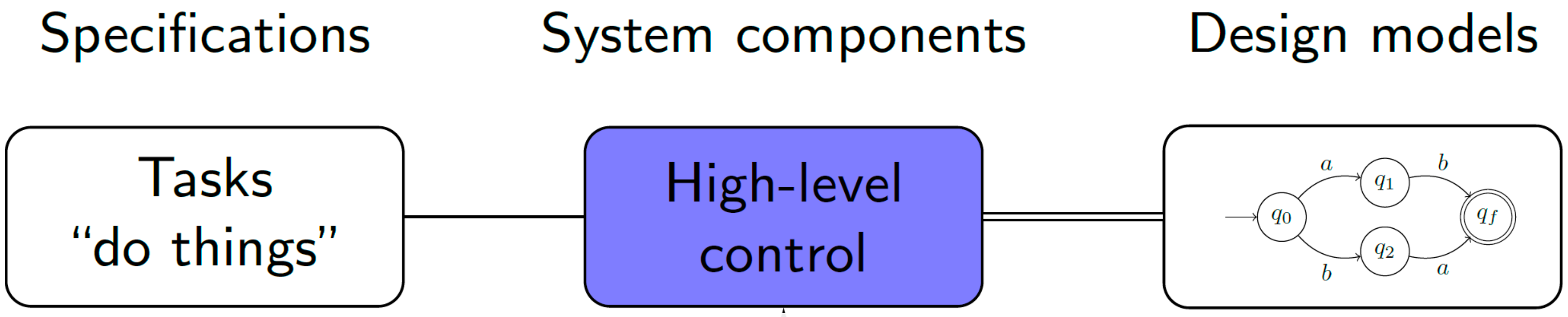
Caltech

Specifications | System components | Design models

Nilsson and AA, CDC 2018

Nilsson, Haesaert, Vasile, Thakker, Agha, Murray, AA, RSS 2018

High Level Specifications

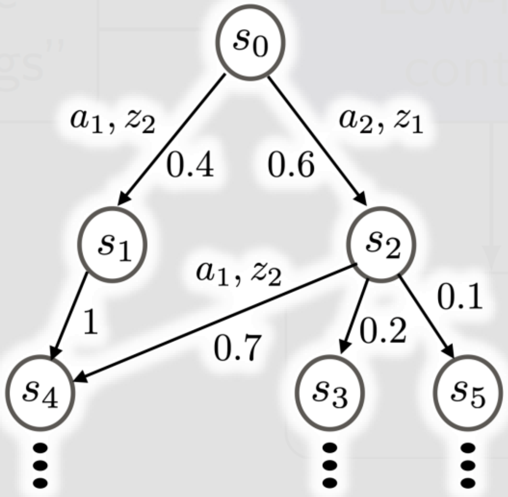


Safety-Critical POMDPs

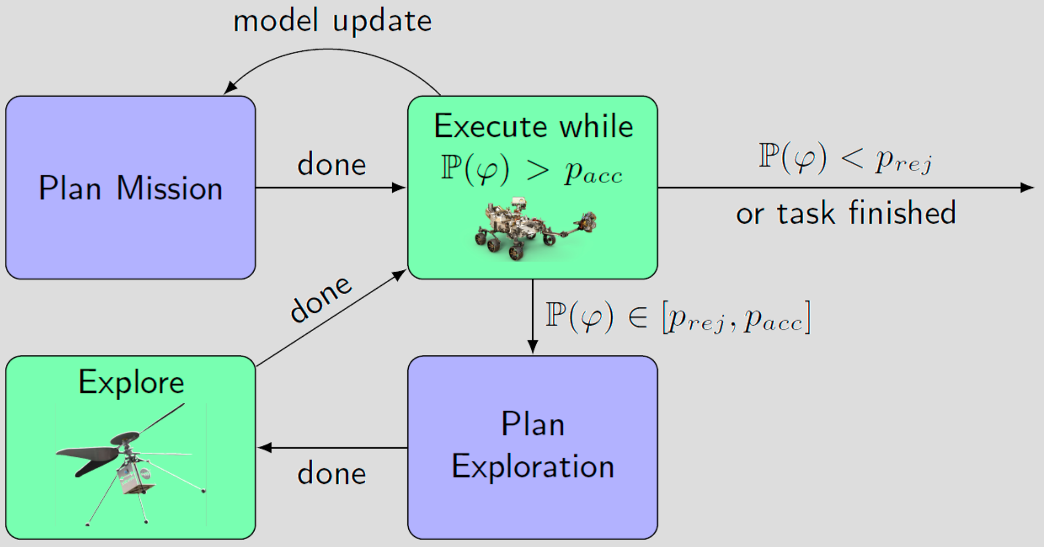
$$b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0}$$

$$\mathcal{C} := \{b \in \mathcal{D} \mid h(b) \geq 0\}$$

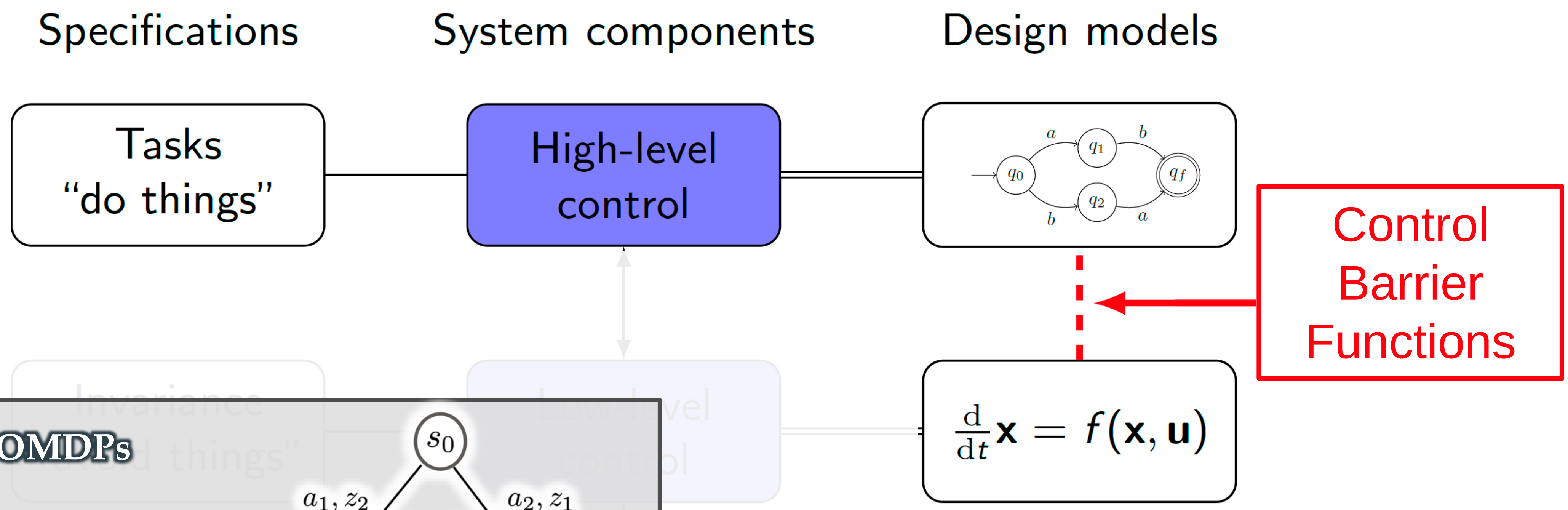
$$\Diamond(\text{GOAL}) \wedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS})_i)$$



Markov Decision Processes (MDP)



Safe Multi-Robot Coordination

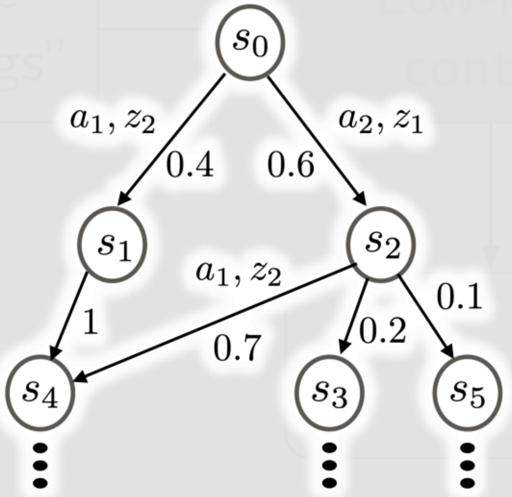


Safety-Critical POMDPs

$$b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0}$$

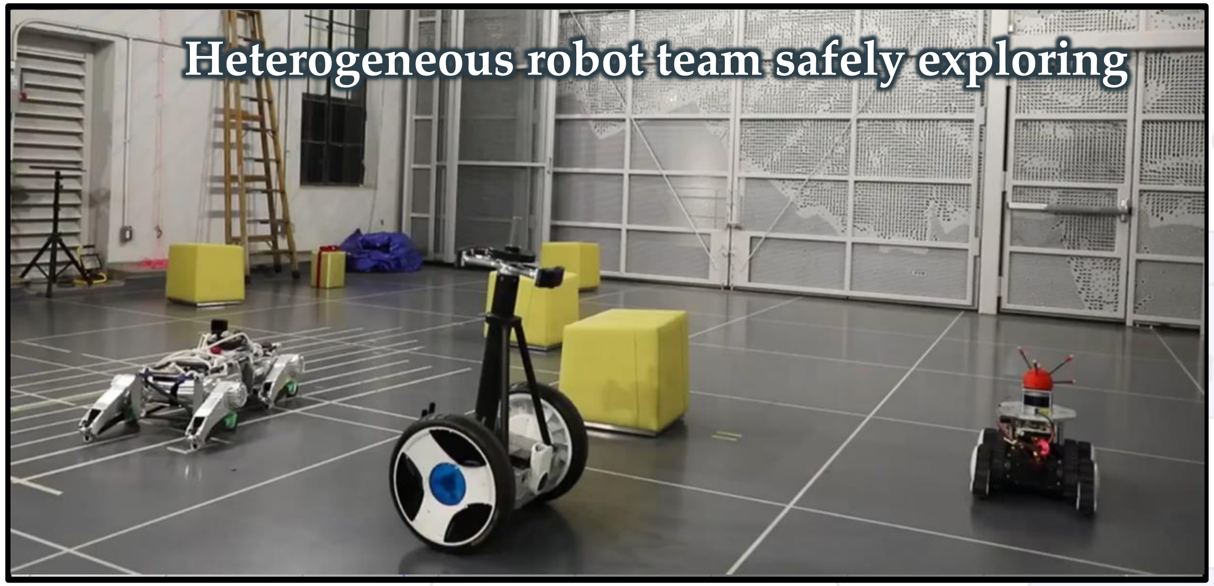
$$\mathcal{C} := \{b \in \mathcal{D} \mid h(b) \geq 0\}$$

$$\Diamond(\text{GOAL}) \wedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS})_i)$$



Safe Multi-Robot Coordination

Caltech



Safety-Critical POMDPs

Dynamics:

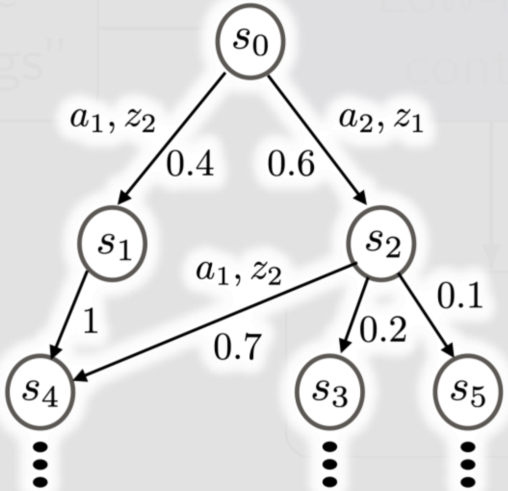
$$b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0}$$

Safe Set:

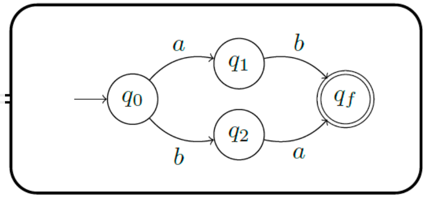
$$\mathcal{C} := \{b \in \mathcal{D} \mid h(b) \geq 0\}$$

Specifications:

$$\Diamond(\text{GOAL}) \wedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS})_i)$$



Design models



Control Barrier Functions

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = f(\mathbf{x}, \mathbf{u})$$

$$b^{t+1} := f(b^t) = \frac{O(q', a^t, z^{t+1}) \sum_{q \in Q} T(q, a^t, q') b^t(q)}{\sum_{q' \in Q} O(q', a^t, z^{t+1}) \sum_{q \in Q} T(q, a^t, q') b^t(q)}$$

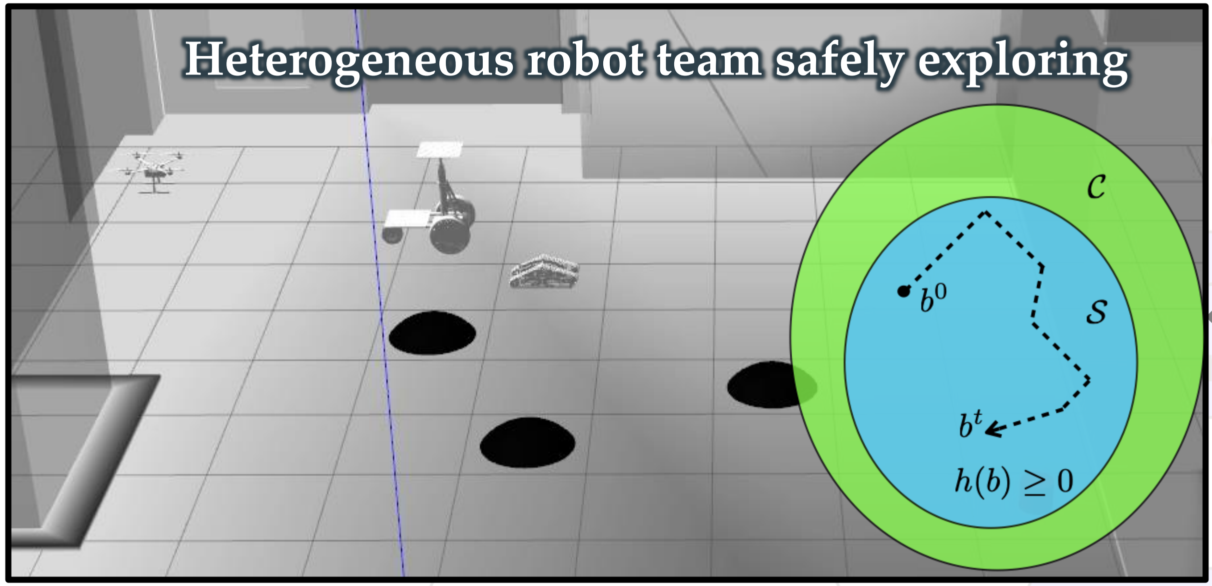
Jones, Schwager, Belta, CDC 2013

Ahmadi, Jansen, Wu, Topcu, IEEE TAC 2020

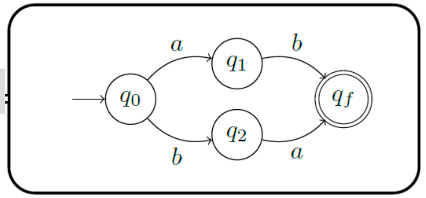
Ahmadi, Singletary, Burdick and AA, CDC 2019

Safe Multi-Robot Coordination: Discrete Time Barriers

Caltech



Design models



Control Barrier Functions

Safety-Critical POMDPs

Dynamics:

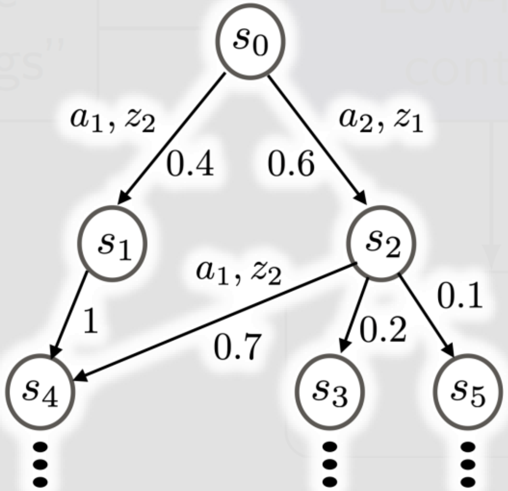
$$b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0}$$

Safe Set:

$$\mathcal{C} := \{b \in \mathcal{D} \mid h(b) \geq 0\}$$

Specifications:

$$\Diamond(\text{GOAL}) \wedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS})_i)$$



Theorem (Safety with Barrier Functions)

For the system $b^{t+1} = f(b^t)$, the set $\mathcal{C}$ is safe (forward invariant) if and only if there exists a discrete-time barrier function.

Discrete Time Barrier Function (DTBF)

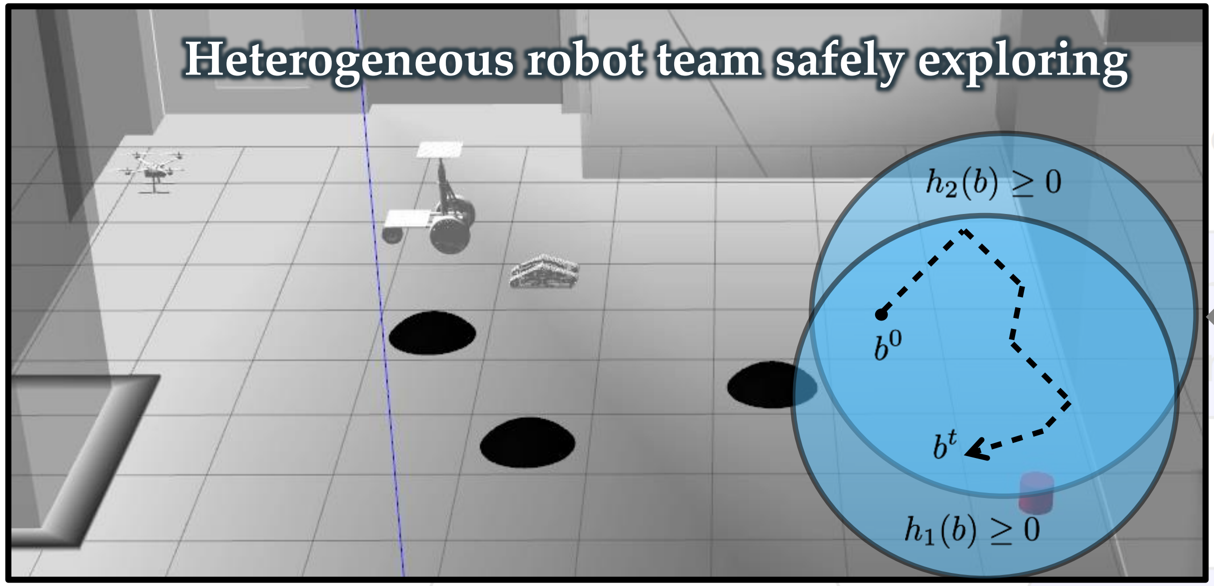
$h : \mathcal{D} \to \mathbb{R}$ is a discrete time barrier function for the set $\mathcal{C}$, if there exists $\alpha \in \mathcal{K}$ satisfying $\alpha(r) < r$ for all $r > 0$ with:

$$h(b^{t+1}) - h(b^t) \geq -\alpha(h(b^t)), \qquad \forall\, b \in \mathcal{D}.$$

Ahmadi, Singletary, Burdick and AA, CDC 2019

Safe Multi-Robot Coordination: Composing Safe Sets

Caltech



$$\mathcal{C}_i = \{b \in \mathcal{D} \mid h_i(b) \geq 0\}, \quad i = 1, \ldots, k.$$

$$\mathcal{C}_\cap = \bigcap_{i \in \{1,\ldots,k\}} \mathcal{C}_i \qquad \mathcal{C}_\cup = \bigcup_{i \in \{1,\ldots,k\}} \mathcal{C}_i$$

Intersection

Union

Safety-Critical POMDPs

Dynamics:

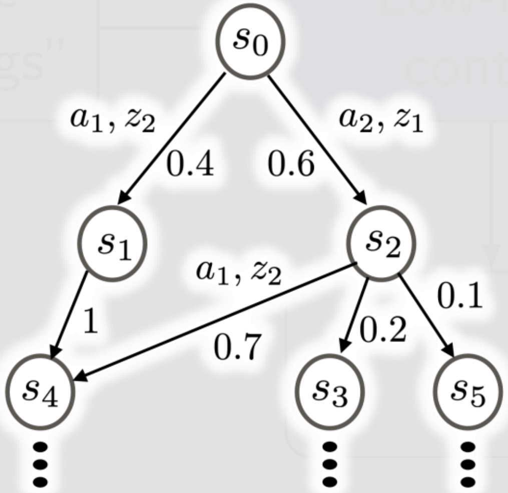
$$b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0}$$

Safe Set:

$$\mathcal{C} := \{b \in \mathcal{D} \mid h(b) \geq 0\}$$

Specifications:

$$\Diamond(\text{GOAL}) \wedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS})_i)$$



Proposition

If there exist an $\alpha \in \mathcal{K}$ satisfying $\alpha(r) < r$ for all $r > 0$ such that

$$\min_{i=1,\ldots,k} h_i(b^{t+1}) - \min_{i=1,\ldots,k} h_i(b^t) \geq -\alpha\left(\min_{i=1,\ldots,k} h_i(b^t)\right)$$

then $\mathcal{C}_\cap = \bigcap_i \mathcal{C}_i$ is forward invariant.

If there exist an $\alpha \in \mathcal{K}$ satisfying $\alpha(r) < r$ for all $r > 0$ such that

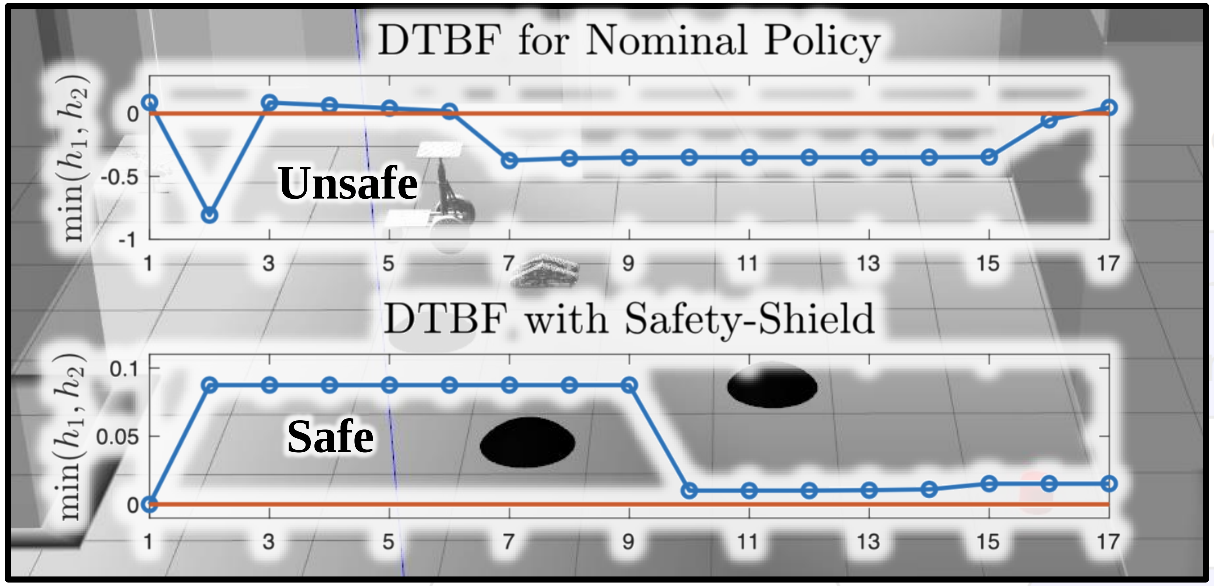
$$\max_{i=1,\ldots,k} h_i(b^{t+1}) - \max_{i=1,\ldots,k} h_i(b^t) \geq -\alpha\left(\max_{i=1,\ldots,k} h_i(b^t)\right)$$

then $\mathcal{C}_\cup = \bigcup_i \mathcal{C}_i$ is forward invariant.

Ahmadi, Singletary, Burdick and AA, CDC 2019

Safe Multi-Robot Coordination: Safety Specifications

Caltech



LDTL Specification	DTBF Implementation
$\omega^i \models A$	$h(b^i) = \sum_{q \in A} b^i(q) - 1$
$\omega^i \models \neg A$	$h(b^i) = \sum_{q \in Q \setminus A} b^i(q) - 1$
$\omega^i \models f$	$h(b^i) = -f(b^i) + \delta$
$\omega^i \models \neg f$	$h(b^i) = f(b^i)$
$\omega^i \models \phi_1 \wedge \phi_2$	$h(b^i) = \min\{h_1(b^i), h_2(b^i)\}$
$\omega^i \models \phi_1 \vee \phi_2$	$h(b^i) = \max\{h_1(b^i), h_2(b^i)\}$
$\omega^i \models \bigcirc \phi$	$h(b^{i+1}) = h_\phi(b)$
$\omega^i \models \phi_1 \mathfrak{U} \phi_2$	$h_2(b^j) < 0 \implies h = h_1(b^j), \forall j \geq i$
$\omega^i \models \Diamond \phi$	$h(b^j) = \tilde{h}(b^j), \ i \leq j \leq t^*$
$\omega^i \models \Box \phi$	$h(b^j) = h_\phi(b^j), \ \forall j \geq i$

Safety-Critical POMDPs

Dynamics:

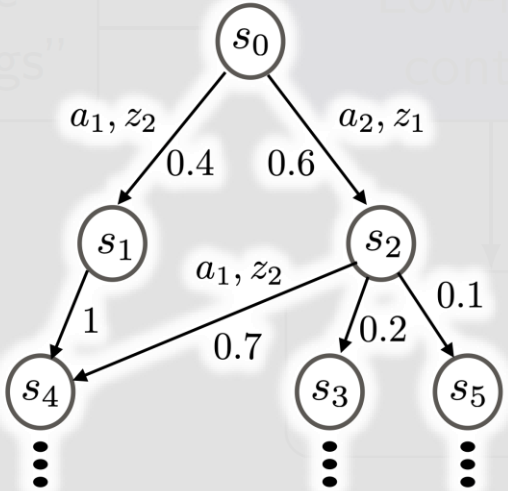
$$b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0}$$

Safe Set:

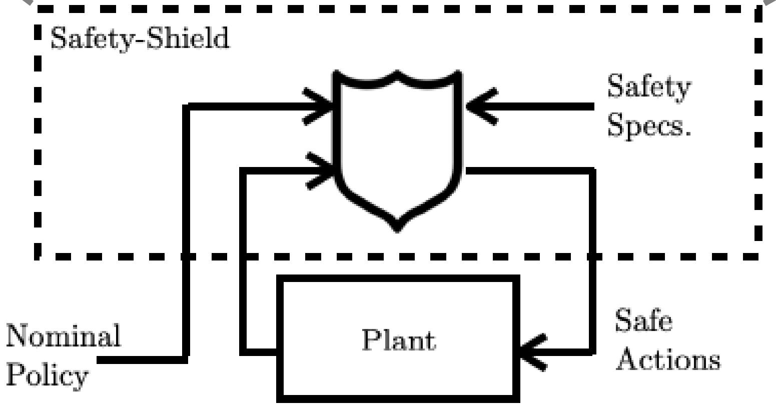
$$\mathcal{C} := \{b \in \mathcal{D} \mid h(b) \geq 0\}$$

Specifications:

$$\Diamond(\text{GOAL}) \wedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS})_i)$$



Safety Specifications



Ahmadi, Singletary, Burdick and AA, CDC 2019

Safe Multi-Robot Coordination

Safety-Critical POMDPs

Dynamics:

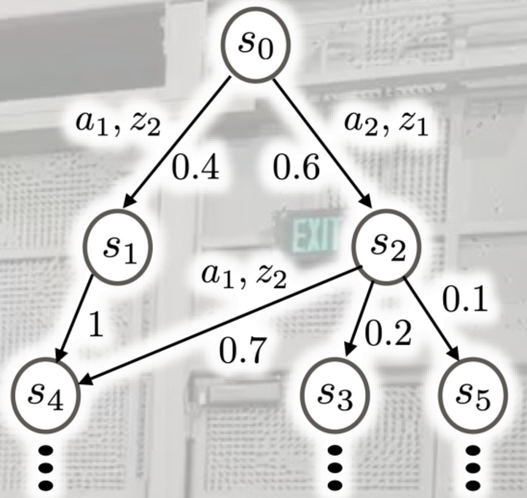
$$b^{t+1} = f(b^t), \quad t \in \mathbb{N}_{\geq 0}$$

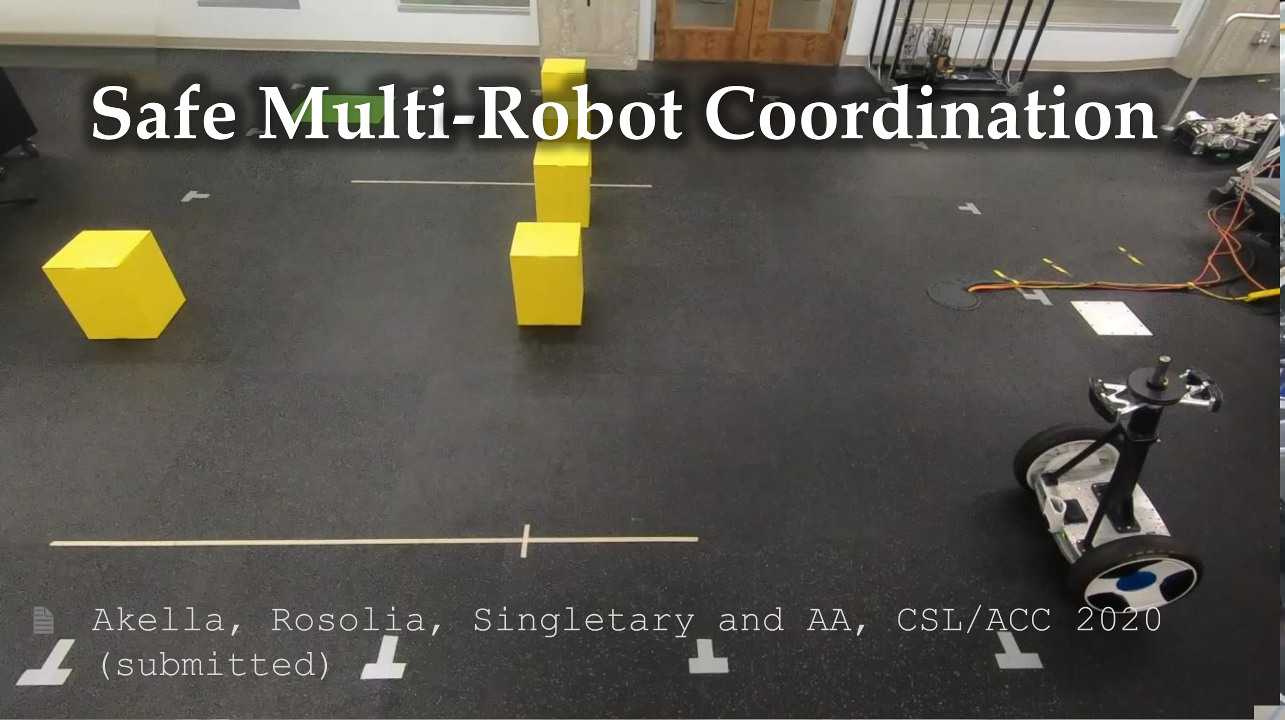
Safe Set:

$$\mathcal{C} := \{b \in \mathcal{D} \mid h(b) \geq 0\}$$

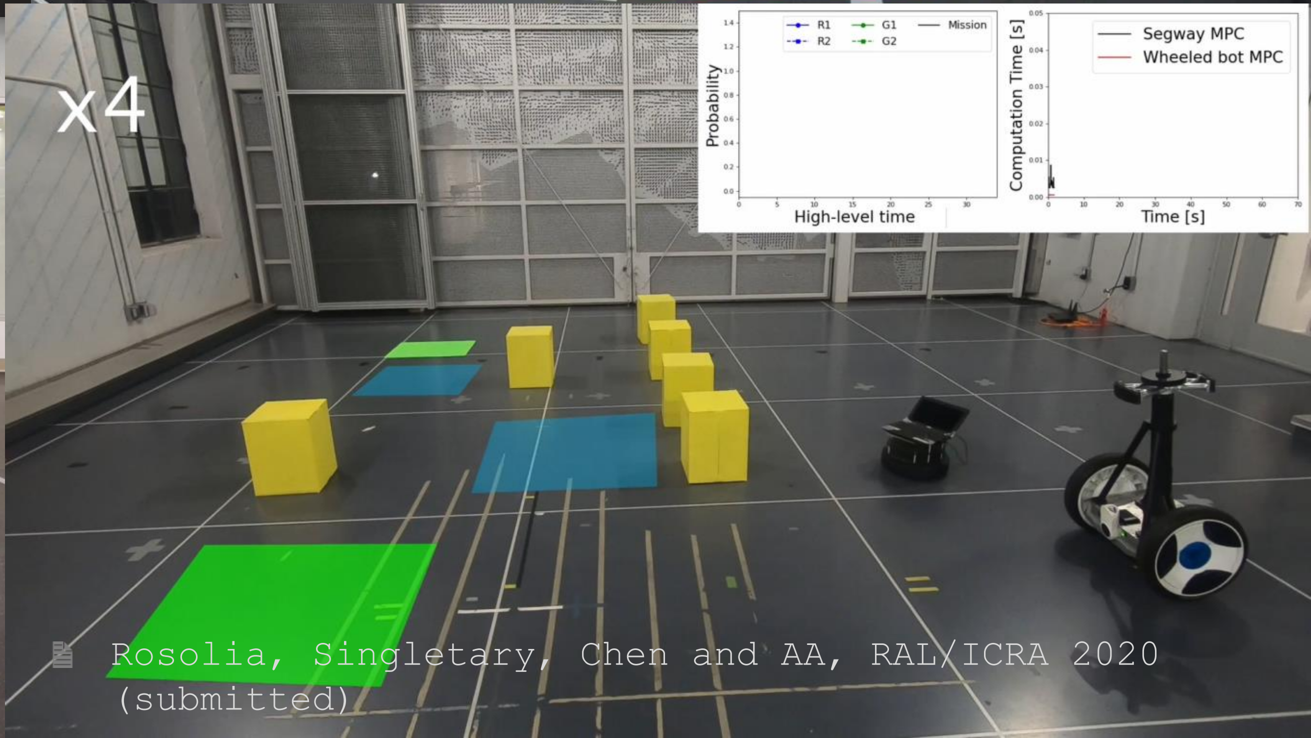
Specifications:

$$\Diamond(\text{GOAL}) \wedge_{i=1}^{3} (\mathbb{P}_{\leq 0.05}(\text{OBS})_i)$$

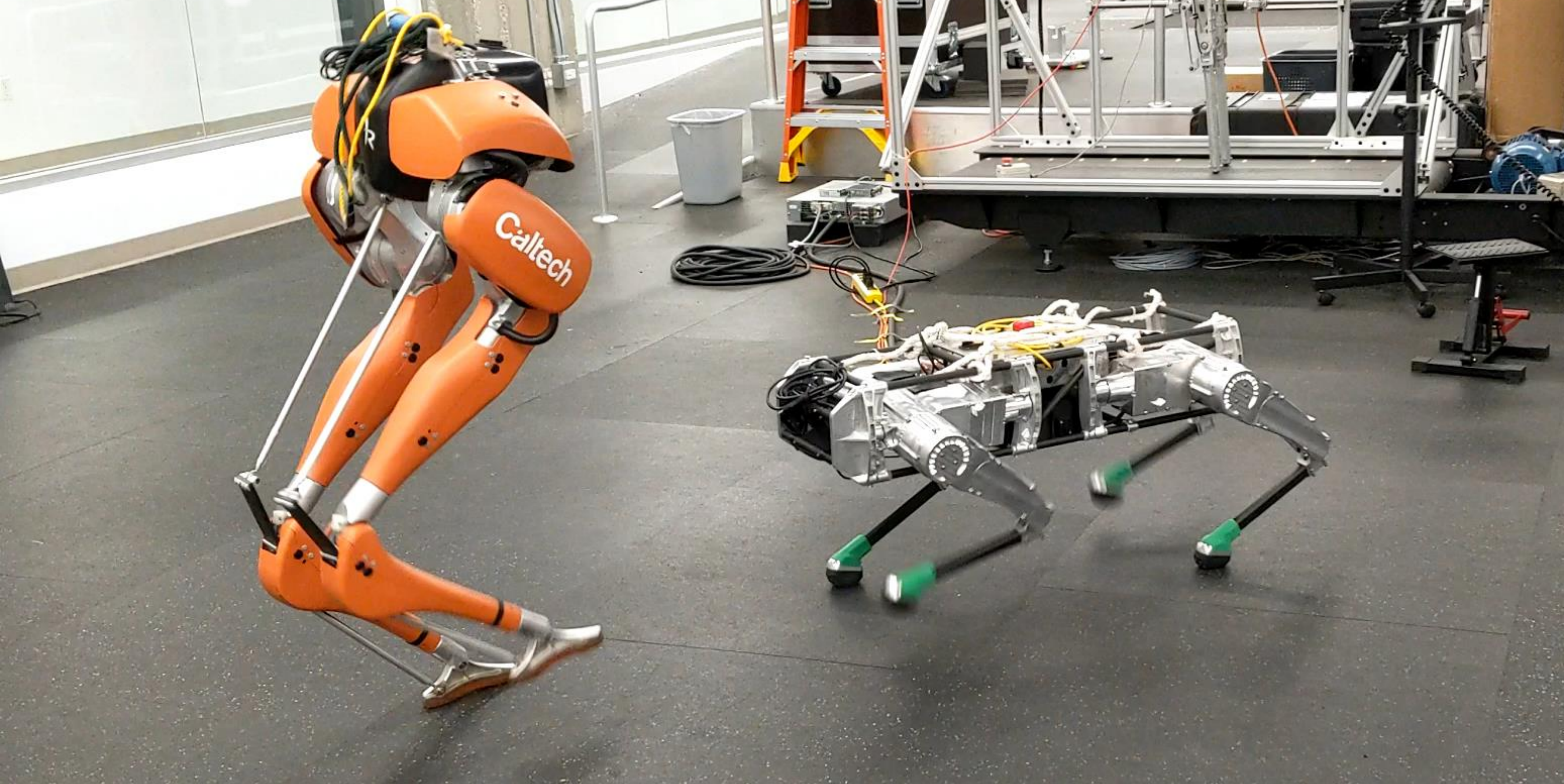



Safe Multi-Robot Coordination
Akella, Rosolia, Singletary and AA, CSL/ACC 2020
(submitted)

Both agents assigned to the task to increase
the expected reward
Chen, Rosolia and AA, RAL/ICRA 2020 (submitted)

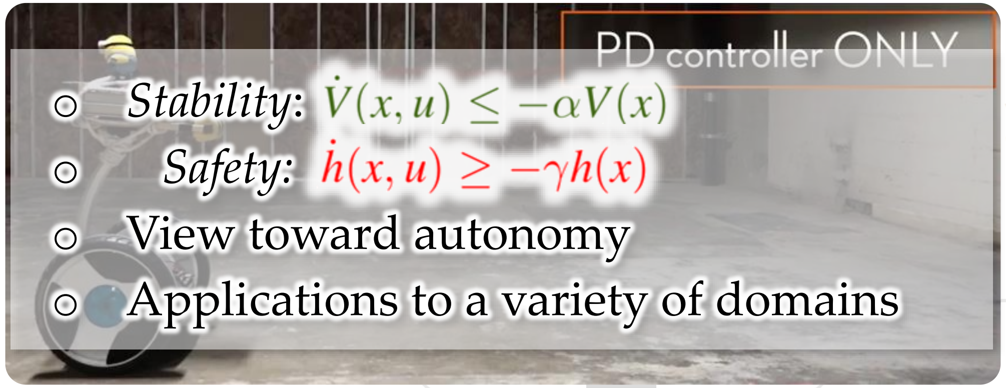
Rosolia and AA, CSL/CDC 2020

x4
R1
R2
G1
G2
Mission
Probability
High-level time
Computation Time [s]
Segway MPC
Wheeled bot MPC
Time [s]
Rosolia, Singletary, Chen and AA, RAL/ICRA 2020
(submitted)

Next Steps: Safe Multi-Robot Coordination

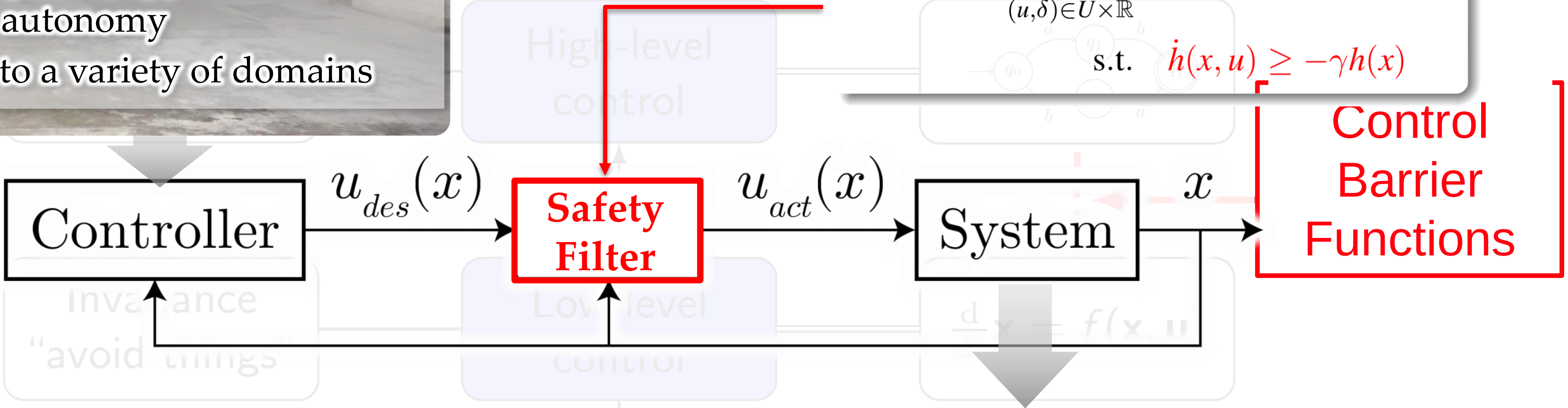


Conclusion: Safety-Critical Autonomy

Caltech



- *Stability:* $\dot{V}(x,u) \le -\alpha V(x)$
- *Safety:* $\dot{h}(x,u) \ge -\gamma h(x)$
- View toward autonomy
- Applications to a variety of domains



Future Work:

- More underlying theory and synthesis
- Continue to apply experimentally
- Translate to real-world systems

Research supported by: NSF CPS, NSF NRI, AFOSR, DARPA, Wandercraft, Disney, JPL

Conclusions – Next Steps

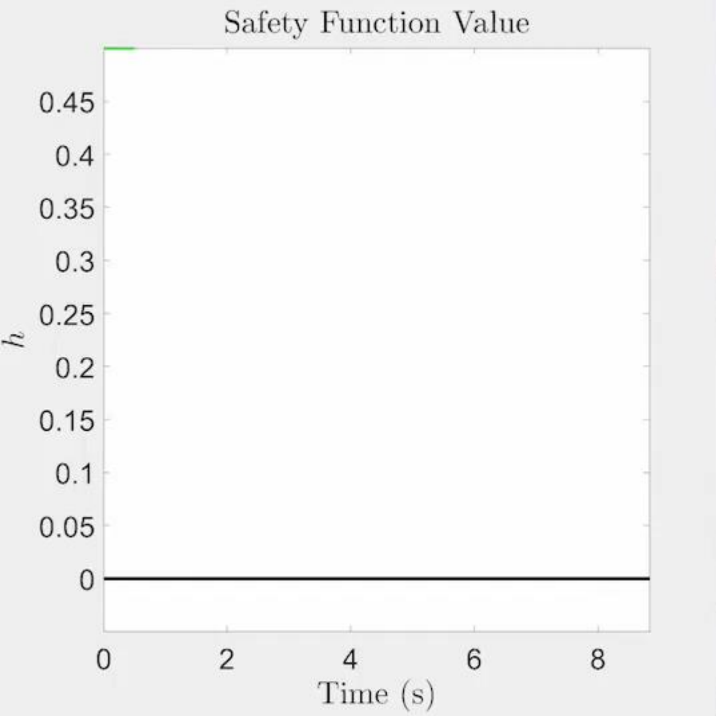
Multi-Robot Systems

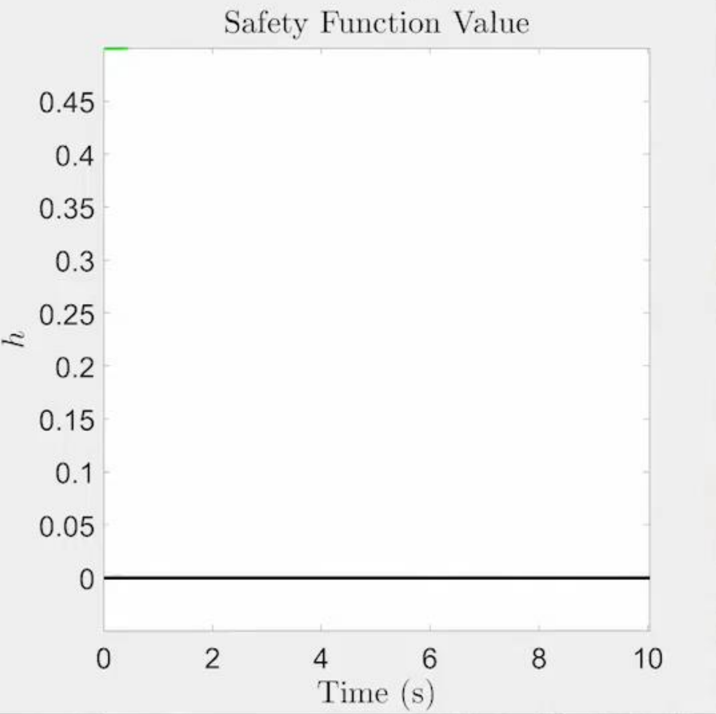
Walking Robots

Collision Avoidance

Robotic Assistance

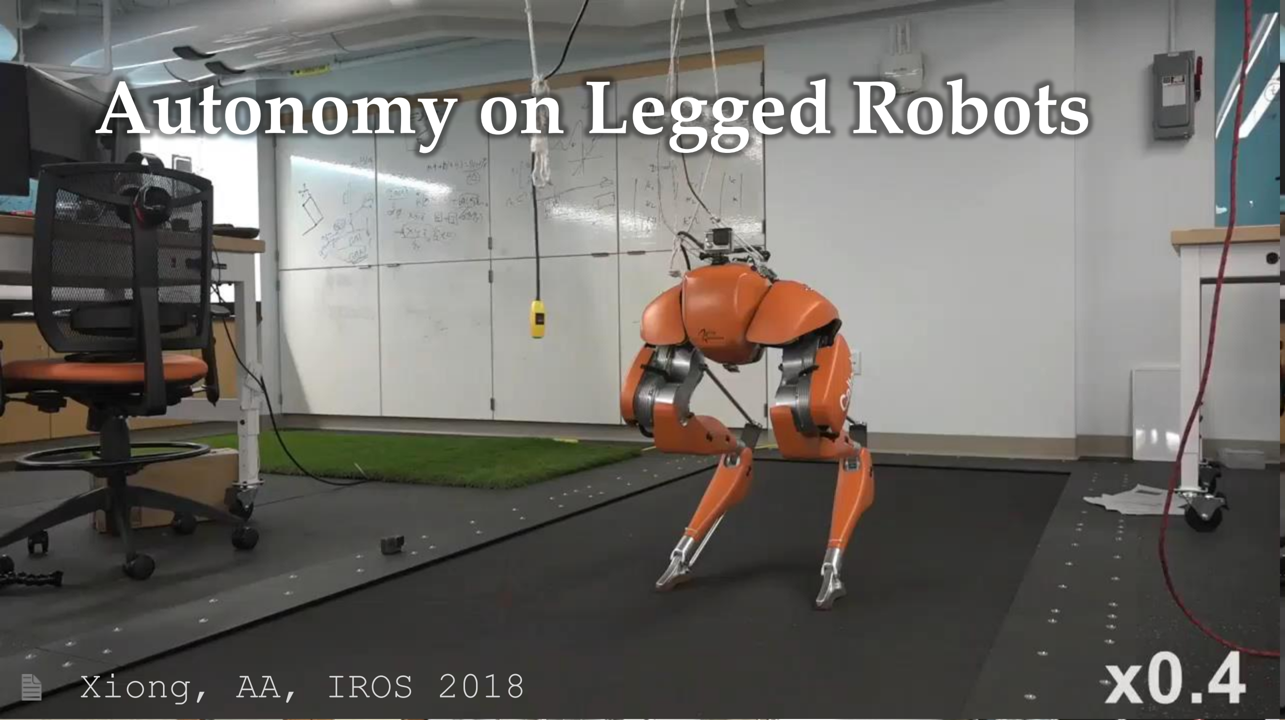
Goal: Robust Safety

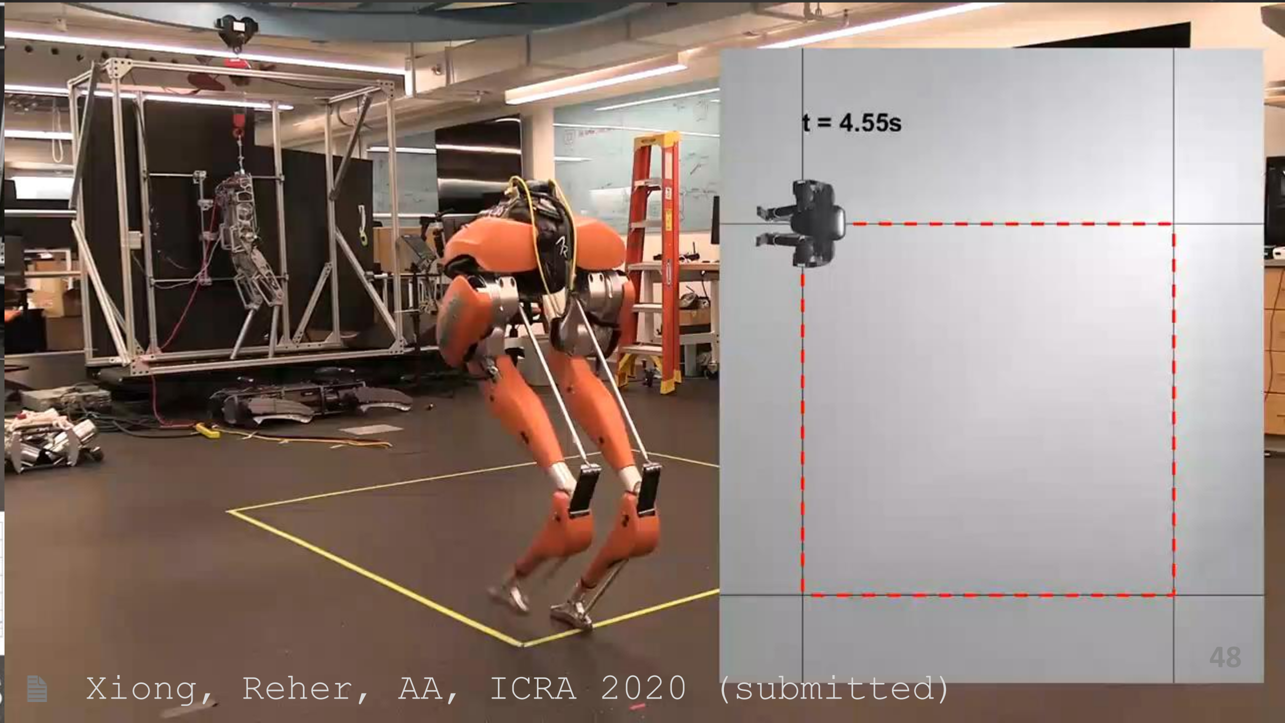


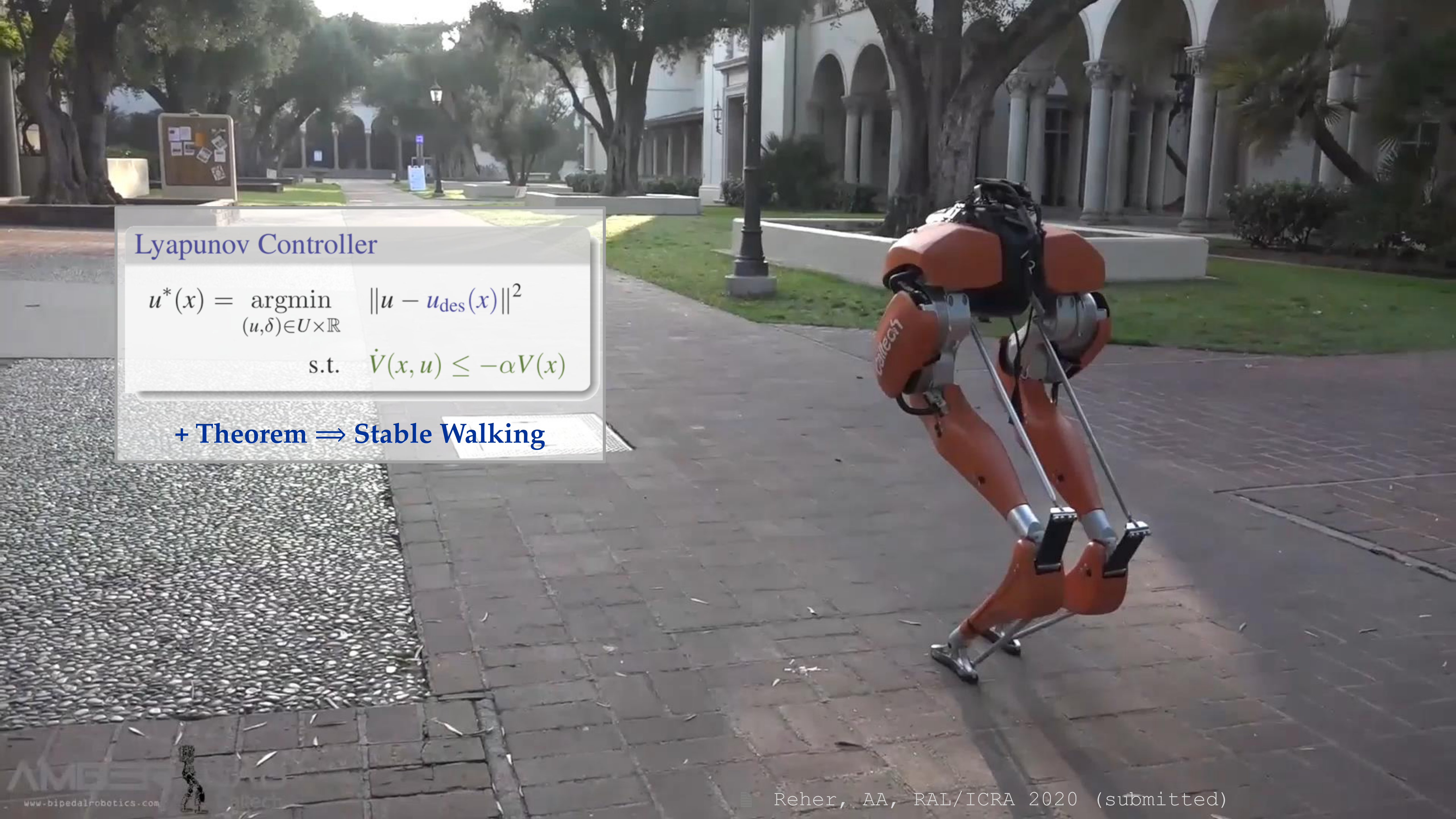


Taylor, Singletary, Yue, AA, L4DC 2020

Autonomy on Legged Robots







Lyapunov Controller

$$u^*(x) = \underset{(u,\delta)\in U\times\mathbb{R}}{\operatorname{argmin}} \quad \|u - u_{\text{des}}(x)\|^2$$

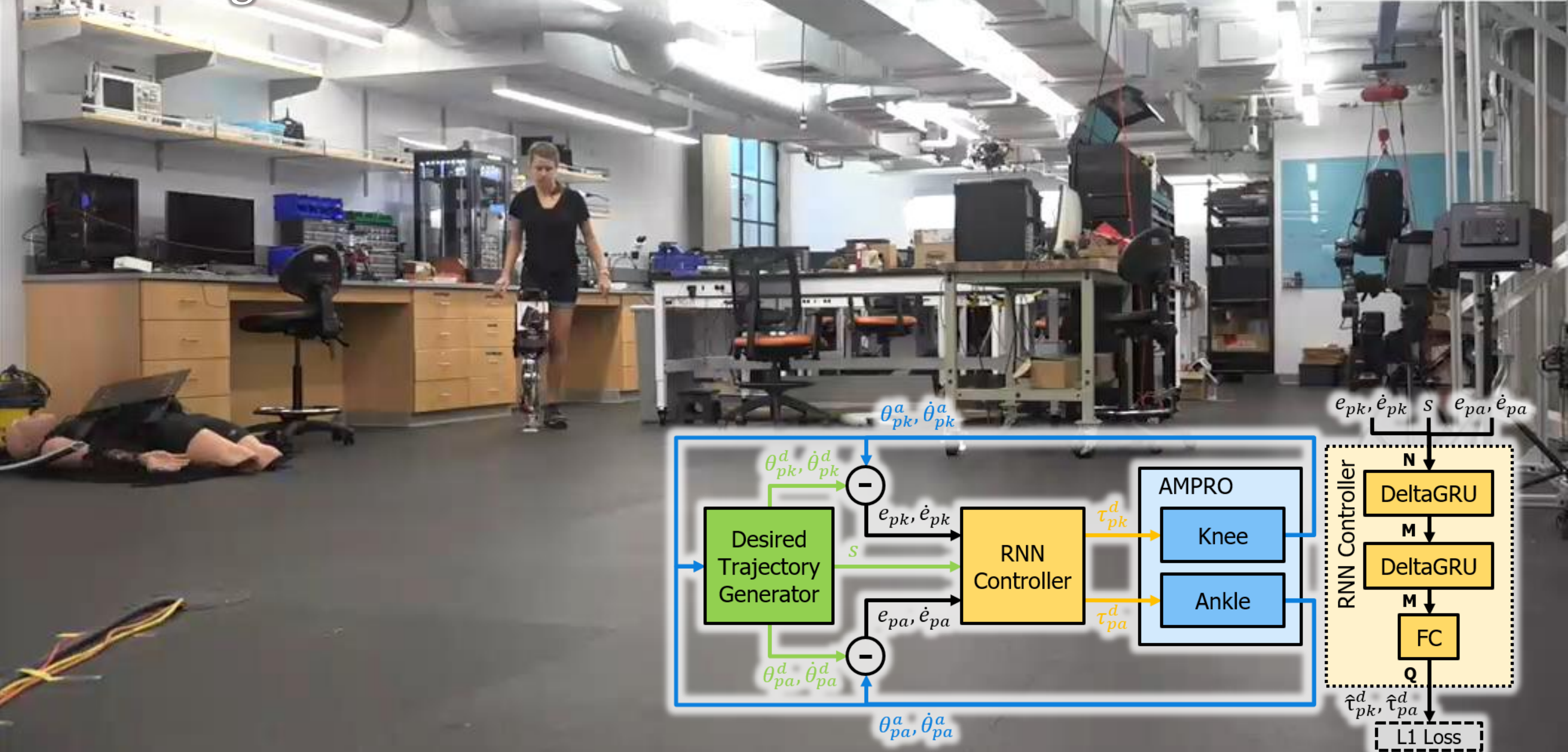
$$\text{s.t.} \quad \dot{V}(x,u) \leq -\alpha V(x)$$

+ Theorem ⟹ Stable Walking

Reher, AA, RAL/ICRA 2020 (submitted)

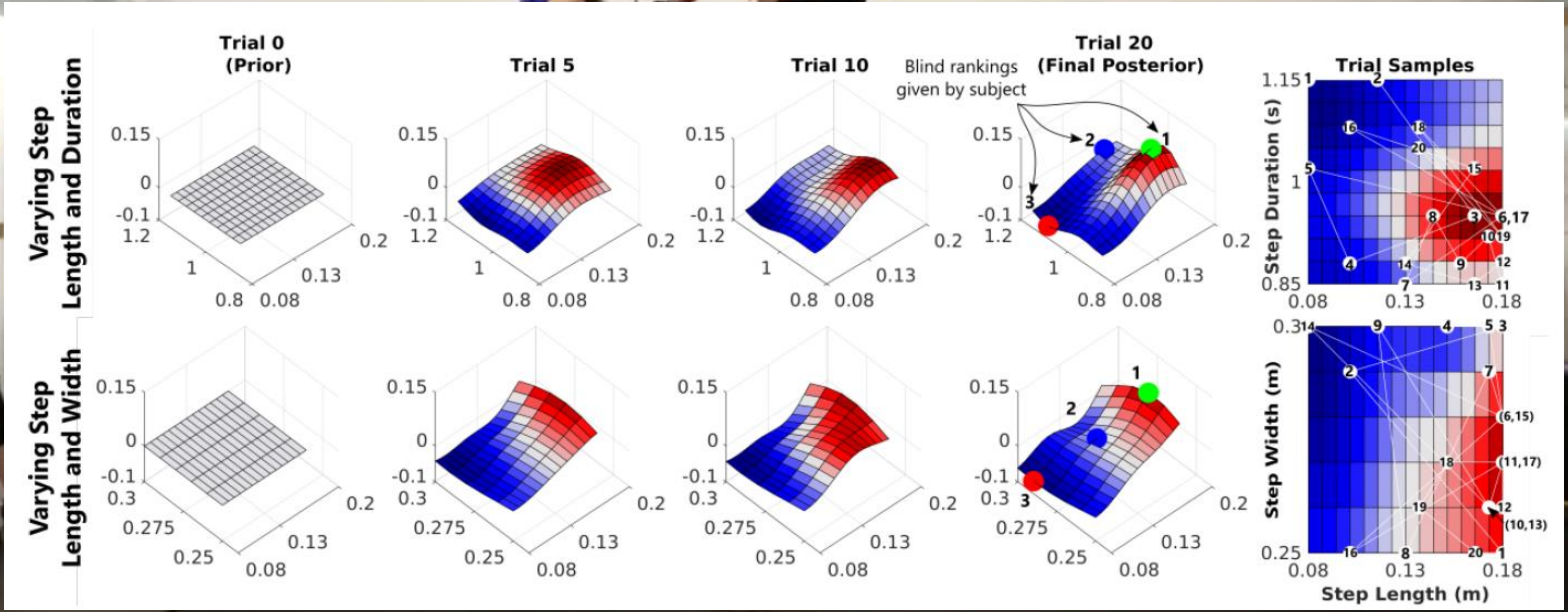

Robotic Assistive Devices
RoAMS Initiative
www.roams.caltech.edu
Aaron Ames / AMBER Lab
Caltech

Learning + Control of Assistive Devices



Gao, Gehlhar, AA, Liu, Delbruck, ICRA 2020

Learning + Control of Assistive Devices




Restoring Mobility
Caltech
WANDERCRAFT
ORDINARY LIFE FOR EXTRAORDINARY PEOPLE


Goal: Safe Real World Autonomy
Thank You