Mixed autonomy in ride-sharing networks

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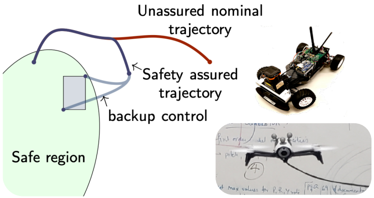
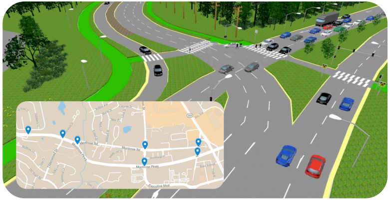
October 27, 2020

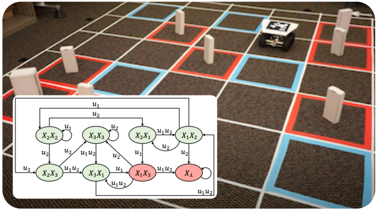
Formal methods and Autonomous Control of Transportation Systems (FACTS) Lab

Safe autonomy

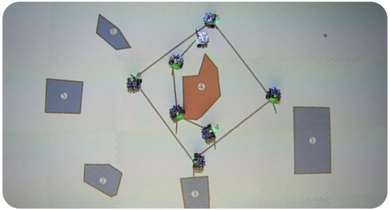


Analysis and control of flow networks





Formal methods in control systems



Multi-agent distributed control

Autonomous vehicles and ride-sharing

- Autonomous vehicles are likely to appear in ride-sharing/ride-hailing fleets first
- Anticipated benefits:
  - Decreased operating cost for same service (e.g., per trip)
  - Deployable to locations with increased demand
  - Do not require incentives to relocate (compared to, e.g., surge pricing)

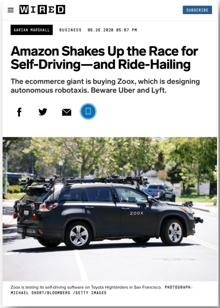

The Washington Post
Democracy Dies in Darkness

Technology

Waymo to launch fully driverless service to the public — a first just in time for the pandemic

The company said all rides on its Phoenix-area ride-hailing service would be driverless in the short term

Waymo said Thursday it is opening up driverless ride-hailing service to riders in the Phoenix metro area. (Caitlin O'Hara/Reuters) (Caitlin O'hara/Reuters)


WIRED
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AARIAN MARSHALL BUSINESS 06.26.2020 05:07 PM

Amazon Shakes Up the Race for Self-Driving—and Ride-Hailing

The ecommerce giant is buying Zoox, which is designing autonomous robotaxis. Beware Uber and Lyft.

Zoox is testing its self-driving software on Toyota Highlanders in San Francisco. PHOTOGRAPH: MICHAEL SHORT/BLOOMBERG /GETTY IMAGES


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TECH

Uber's self-driving cars are a key to its path to profitability

PUBLISHED TUE, JAN 28 2020·7:01 AM EST | UPDATED TUE, JAN 28 2020·2:21 PM EST

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Focus of this talk

To propose and analyze model for ride-sharing with autonomous vehicles (AVs) and human-driven vehicles (HVs)

- Multi-location network where operator/platform sets prices for riders, compensation to drivers of HVs, and operates AVs for fixed price
- Platform objective: profit maximization
- Focus on equilibrium condition

Q. Wei, R. Pedarsani, S. Coogan. "Mixed autonomy in ride-sharing networks". *IEEE Transactions on Control of Network Systems*, to appear.

Main findings

1. If both HV and AV are available to provide a ride, several reasonable resolutions are reasonable; surprisingly, all lead to same optimal profits
2. Transition from HV-only to AV-only can be abrupt or gradual, depending on network specifics
3. Transition occurs *strictly after* AVs reach operating cost parity; this is because there is additional overhead to operating AVs (*e.g.* must be dispatched to regions of high rider demand)

Outline of talk

1. Definition of ride-sharing model with mixed autonomy
2. Characterization of equilibria
3. Convex optimization problem for obtaining equilibria and implications
4. Specialization to star-to-complete network and example

The model—Network and riders

- Network of n equidistant locations; in each time-step, a ride is possible between any two locations
- At each location, mass θ_i of potential riders, $i \in \{1, \ldots, n\}$
- Column-stochastic, $n \times n$ routing matrix A; A_{ij} is fraction of riders at i traveling to j
- A and θ assumed constant

Our model extends the model proposed in [Bimpikis et. al][1]to include AVs.

[1]Bimpikis, Kostas, Ozan Candogan, and Daniela Saban. "Spatial pricing in ride-sharing networks." Operations Research 67.3 (2019): 744-769.

The model—Human-driven vehicles (HVs)

- After each time period, an HV exits the network with probability $(1-\beta)$; expected lifetime of HV in the network is therefore $(1-\beta)^{-1}$
- Each driver has outside earning option of ω, and only participates if expected compensation exceeds ω

The model—Autonomous vehicles (AVs)

- Platform can choose to operate AV for fixed cost of s each time-step
- AVs are in continual use and do not exit platform
- Introduce ratio

$$\kappa = \frac{s(1-\beta)^{-1}}{\omega} = \frac{\text{AV operating cost over expected HV lifetime}}{\text{outside earning option}}$$

The model–Platform

- Platform sets price p_i for a ride from location i
- Platform compensates driver with c_i for providing ride at location i
- Rider willingness-to-pay cumulative distribution $F(\cdot)$: when confronted with price p, fraction $1-F(p)$ accept price, remaining $F(p)$ balk without ride
- Then, effective demand at location i is $\theta_i(1-F(p_i))$

The model–Priority assignment

Number of riders willing to pay platform price may be $<$, $=$, or $>$ total HVs+AVs.

If riders $<$ total HVs+AVs, how should platform resolve this?

Three plausible options:

1. **HV priority:** Platform assigns rides to HVs first; keeps drivers happy and engaged
2. **AV priority:** Platform assigns rides to AVs first; platform views HVs as temporary supplement to full autonomy
3. **Weighted priority:** Rides assigned in proportion to availability of AVs and HVs

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Equilibrium conditions under HV priority—HVs

Introduce some variables:

- x_i = HVs at location i
- y_{ij} = HVs at i that decide to relocate to j without providing a ride

$$\sum_{j=1}^{n} y_{ij} = \underbrace{\max\left\{x_i - \underbrace{\theta_i(1-F(p_i))}_{\text{effective demand at } i}, 0\right\}}_{\text{HVs without rides at } i}. \tag{equil-1}$$

- δ_i = new drivers who enter platform at i; at equilibrium, from conservation of mass,

$$x_i = \beta\left[\underbrace{\sum_{j=1}^{n} A_{ji} \underbrace{\min\{x_j, \theta_j(1-F(p_j))\}}_{\text{demand served by HVs at } j}}_{\text{HVs at } i \text{ after completing rides}} + \sum_{j=1}^{n} y_{ji}\right] + \delta_i. \tag{equil-2}$$

Equilibrium conditions under HV priority—AVs

Introduce some variables:

- z_i = AVs at location i
- r_{ij} = AVs at i that relocate to j without providing a ride; at equilibrium,

$$z_i = \underbrace{\sum_{j=1}^{n} A_{ji} \underbrace{\min\left\{z_j, \max\left\{\theta_j(1-F(p_j)) - x_j, 0\right\}\right\}}_{\text{demand served by AVs at } j}}_{\text{AVs at } i \text{ after completing rides}} + \sum_{j=1}^{n} r_{ji}. \qquad \text{(equil-3)}$$

At equilibrium, relocating AVs satisfy

$$\sum_{j=1}^{n} r_{ij} = \max\left\{\underbrace{z_i - \max\left\{\theta_i(1-F(p_i)) - x_i, 0\right\}}_{\text{Excess AVs at } i \text{ with no ride}}, 0\right\}. \qquad \text{(equil-4)}$$

Equilibrium conditions under HV priority—Driver expected earnings

Let $V_i =$ expected future earnings of driver at location i; $\{V_i\}$ satisfies Bellman-type relationship:

$$V_i = \underbrace{\min\left\{\frac{\theta_i(1-F(p_i))}{x_i}, 1\right\}}_{\text{likelihood of providing a ride}} \underbrace{\left(c_i + \sum_{k=1}^{n} A_{ik}\beta V_k\right)}_{\text{earnings if giving ride}} + \underbrace{\left(1 - \min\left\{\frac{\theta_i(1-F(p_i))}{x_i}, 1\right\}\right)\beta \max_j V_j}_{\text{when no ride is given}} \tag{equil-5}$$

- Drivers only enter platform if $V_i \geq \omega$
- Platform can always reduce c_i to avoid $V_i > \omega$ and achieve $V_i = \omega$

Equilibrium under HV priority

For some prices and compensations $\{p_i, c_i\}_{i=1}^n$, the collection $\{\delta_i, x_i, y_{ij}, z_i, r_{ij}\}_{i,j=1}^n$ is *an equilibrium under* $\{p_i, c_i\}_{i=1}^n$ *for HV priority assignment* if

- (equil-1)–(equil-4) is satisfied; and
- V_i as defined in (equil-5) satisfies $V_i = \omega$

for all $i = 1, \ldots, n$.

- For AV priority, equilibrium conditions are entirely dual
- For weighted priority, equilibrium conditions are similar in spirit

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Profit-maximization program and convexification

Original optimization problem for profit maximization:

$$\max_{\{p_i,c_i\}_{i=1}^n} \quad \sum_{i=1}^{n} \Bigg[\underbrace{\min\{x_i+z_i,\theta_i(1-F(p_i))\}\cdot p_i}_{\text{revenue from rides}} - \underbrace{\min\{x_i,\theta_i(1-F(p_i))\}\cdot c_i}_{\text{compensation to drivers}} - \underbrace{z_i\cdot s}_{\text{cost of AVs}} \Bigg]$$

s.t. $\{\delta_i,x_i,y_{ij},z_i,r_{ij}\}_{i,j=1}^n$ is an equilibrium under $\{p_i,c_i\}_{i=1}^n$ for HV priority assignment.

Theorem. Under a convexity assumption on riders' willingness-to-pay $F(\cdot)$, the original profit maximization problem for HV priority assignment can be recast into an equivalent convex optimization such that a solution to the original problem is recovered from a solution to the equivalent problem, and vice-versa.

Role of priority assignment

Theorem. An equilibrium is optimal for one priority assignment if and only if it is optimal for all priority assignments. Therefore, all priority assignments achieve same optimal profits at equilibrium.

Explanation: At an optimal equilibrium, every location is served either by only HVs or only AVs regardless of priority scheme, and therefore the priority scheme becomes moot.

Sufficient condition for ruling out AVs

Proposition. If $\kappa = \dfrac{s(1-\beta)^{-1}}{\omega} > 1$, then it is optimal for the platform to only use HVs and no AVs.

- Above sufficient condition not very surprising
- It *is* surprising that the converse does not hold generally, i.e., it is not a necessary condition

Why not the converse? AVs and HVs do not operate equivalently; AVs must be routed to desired service locations. HVs can be motivated into service with compensations and leave service with rate β.

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Closed-form solution for star-to-complete networks

A **n-location star-to-complete network** parameterized by $\zeta \in [0,1]$ has adjacency matrix that is ζ-weighted average of complete equal-weight adjacency matrix and all-to-one star adjacency matrix

Theorem. For a star-to-complete network, two cases are possible, depending on relationship of ζ and β:

1. There exists $k_1 < 1$ such that: $k < k_1 \implies$ AV-only is optimal and $k > k_1 \implies$ HV-only is optimal (sudden change in optimality at $k = k_1$)
2. There exists $k_1 < k_2 < 1$ such that: $k < k_1 \implies$ AV-only is optimal; $k_1 < k < k_2 \implies$ mixed deployment is optimal; $k > k_2 \implies$ HV-only is optimal (gradual change in optimality)

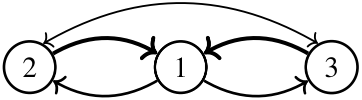
All parameters available in closed-form.

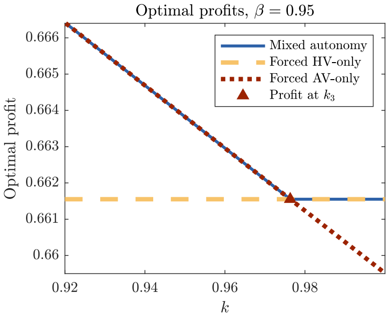
3-location example

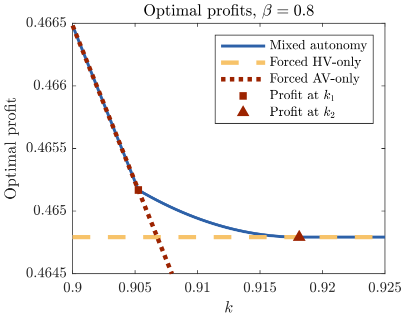
- 3-location star-to-complete network,

$$A = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.9 & 0 & 0.1 \\ 0.9 & 0.1 & 0 \end{bmatrix},$$

i.e., locations 2 and 3 have strong travel preference to location 1







Review of main findings

1. If both HV and AV are available to provide a ride, several reasonable resolutions are reasonable; surprisingly, all lead to same optimal profits
2. Transition from HV-only to AV-only can be abrupt or gradual, depending on network specifics
3. Transition occurs *after* reaching cost parity (i.e., occurs for $k < 1$); this is because there is additional overhead to operating AVs (*e.g.* must be dispatched to regions of high rider demand)

Thank You

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