Fundamentals of optimization
Part II

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Heuristics

Needed where exact methods inadequate

Designed for a specific problem

Require implementation effort

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The greedy

At each step the most attractive decision

No looking back

Myopic

Being greedy is never good

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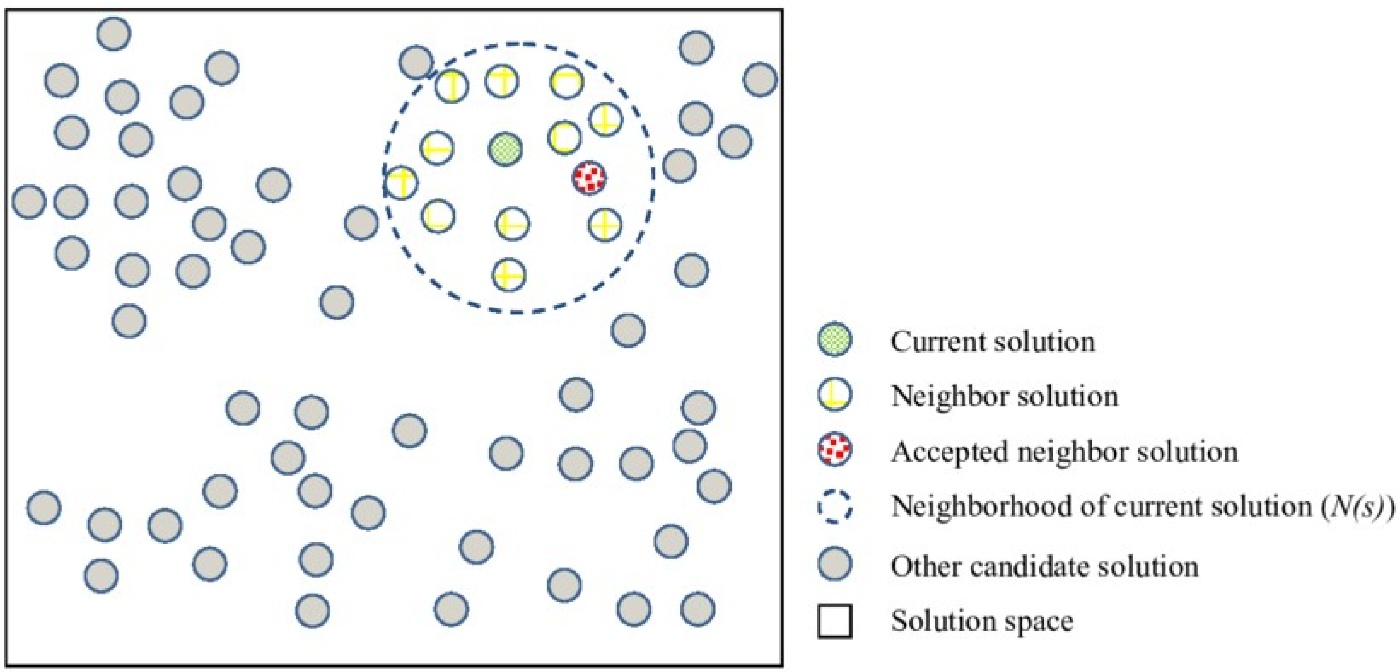
The concept of neighborhood



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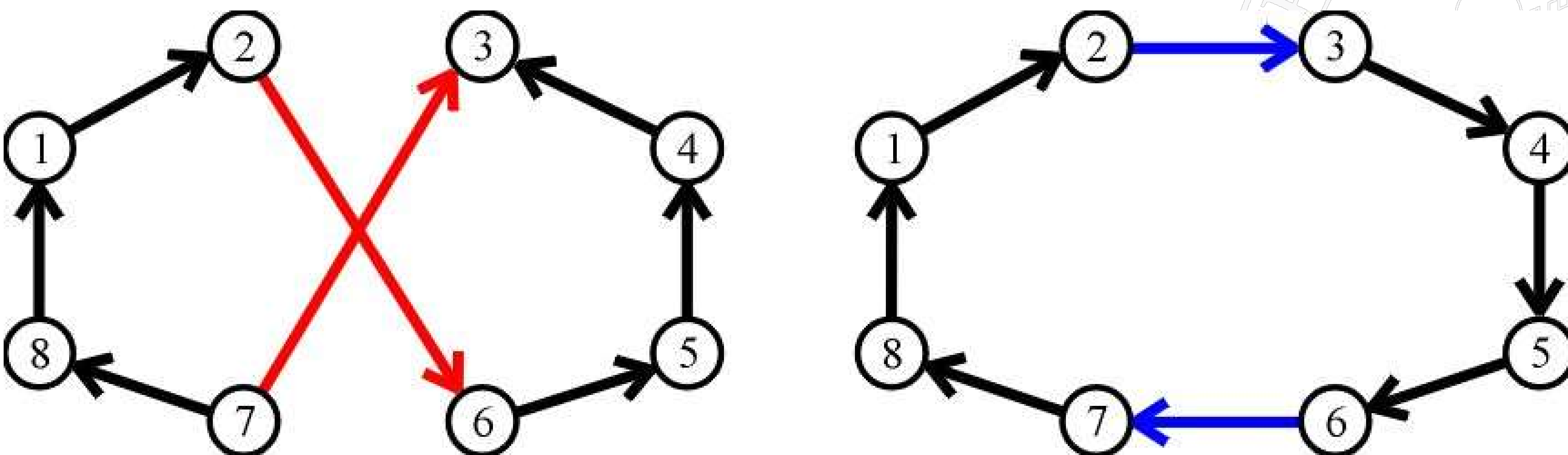
The concept of neighborhood



A 2-opt move

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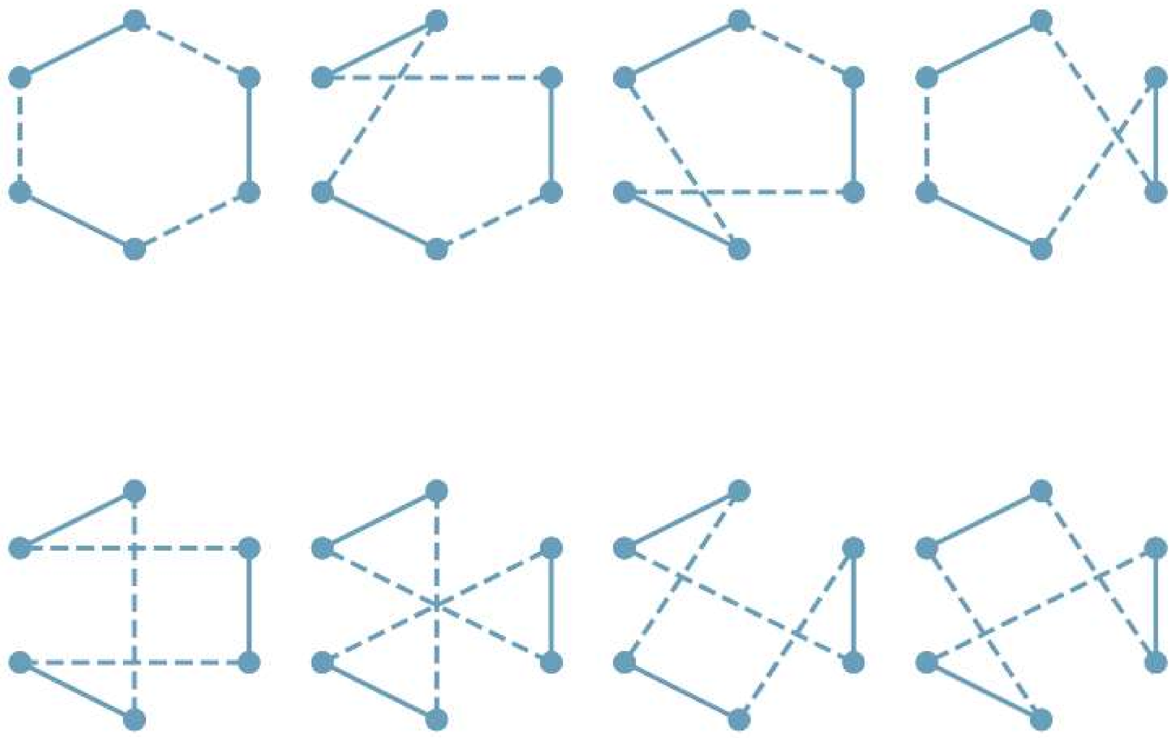
The concept of neighborhood

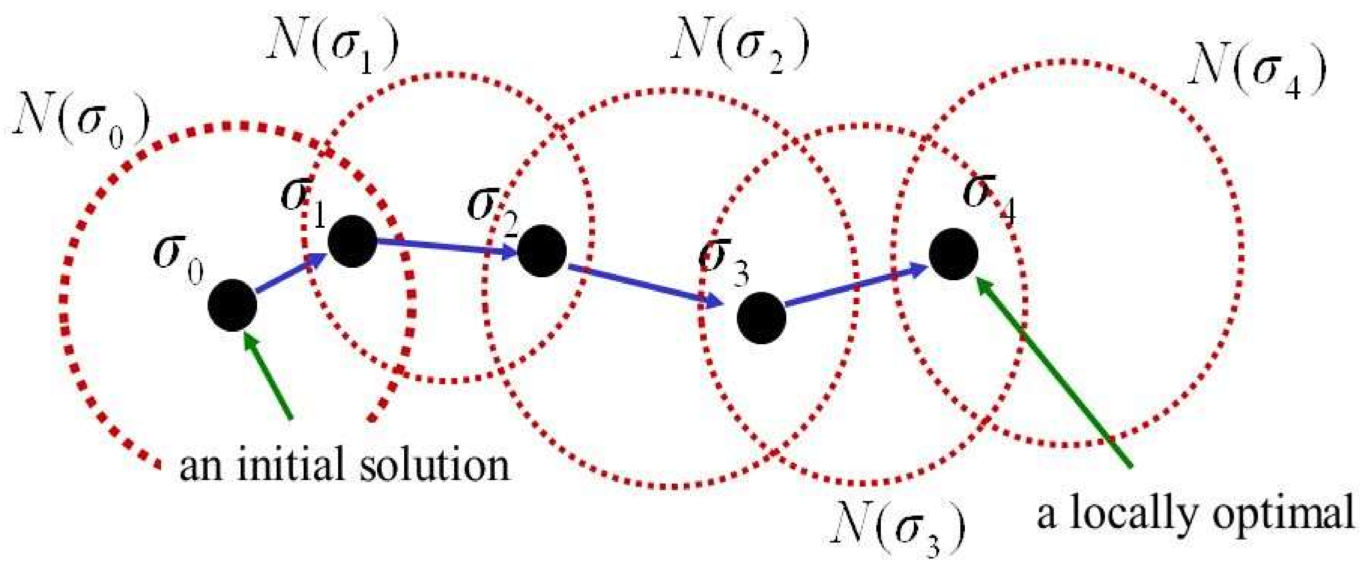


The 7 3-opt moves

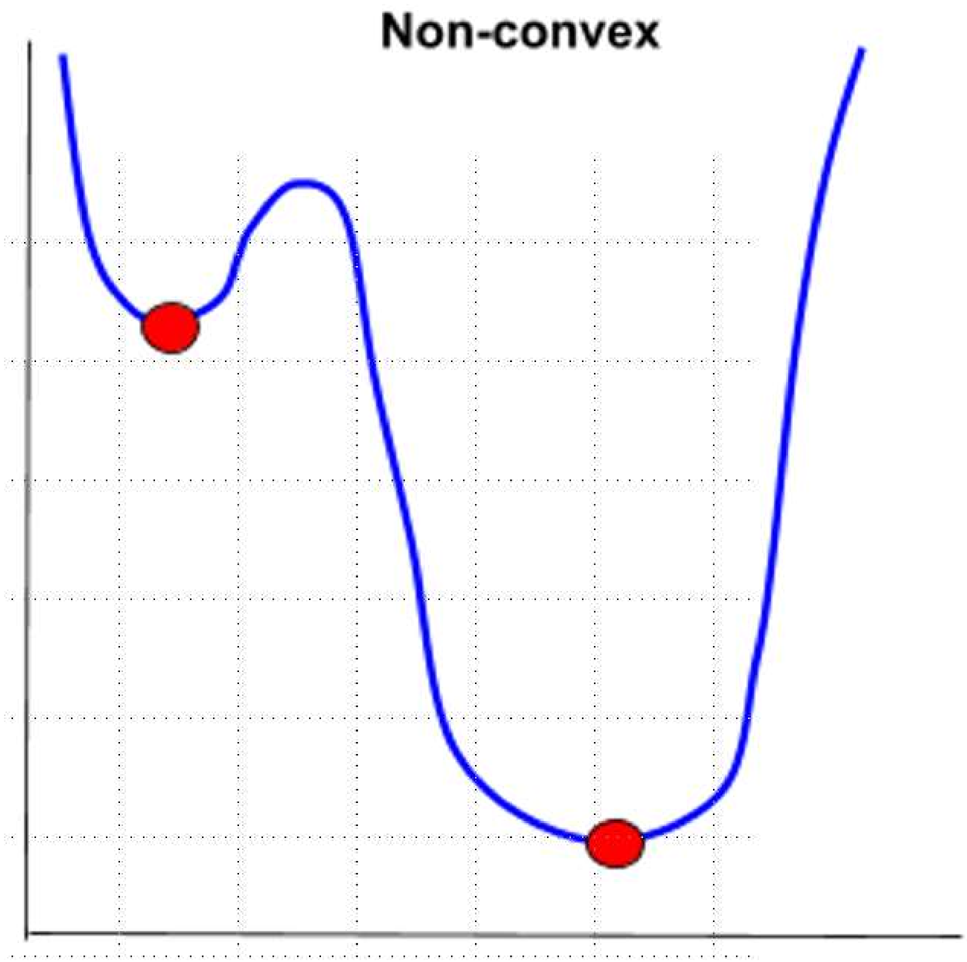
The local search (LS)

Avoids loops

LS repeats replacing σ with a better solution
In its neighborhood $N(\sigma)$



Global, not local!



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Heuristics

Metaheuristics

- Iterated local search
- Tabu search
- Simulated annealing
- Variable neighborhood search
- (Adaptive) large neighborhood search
- Ant colony optimization
- Genetic algorithms
- …

Matheuristics

Not only for MILP

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Main challenge

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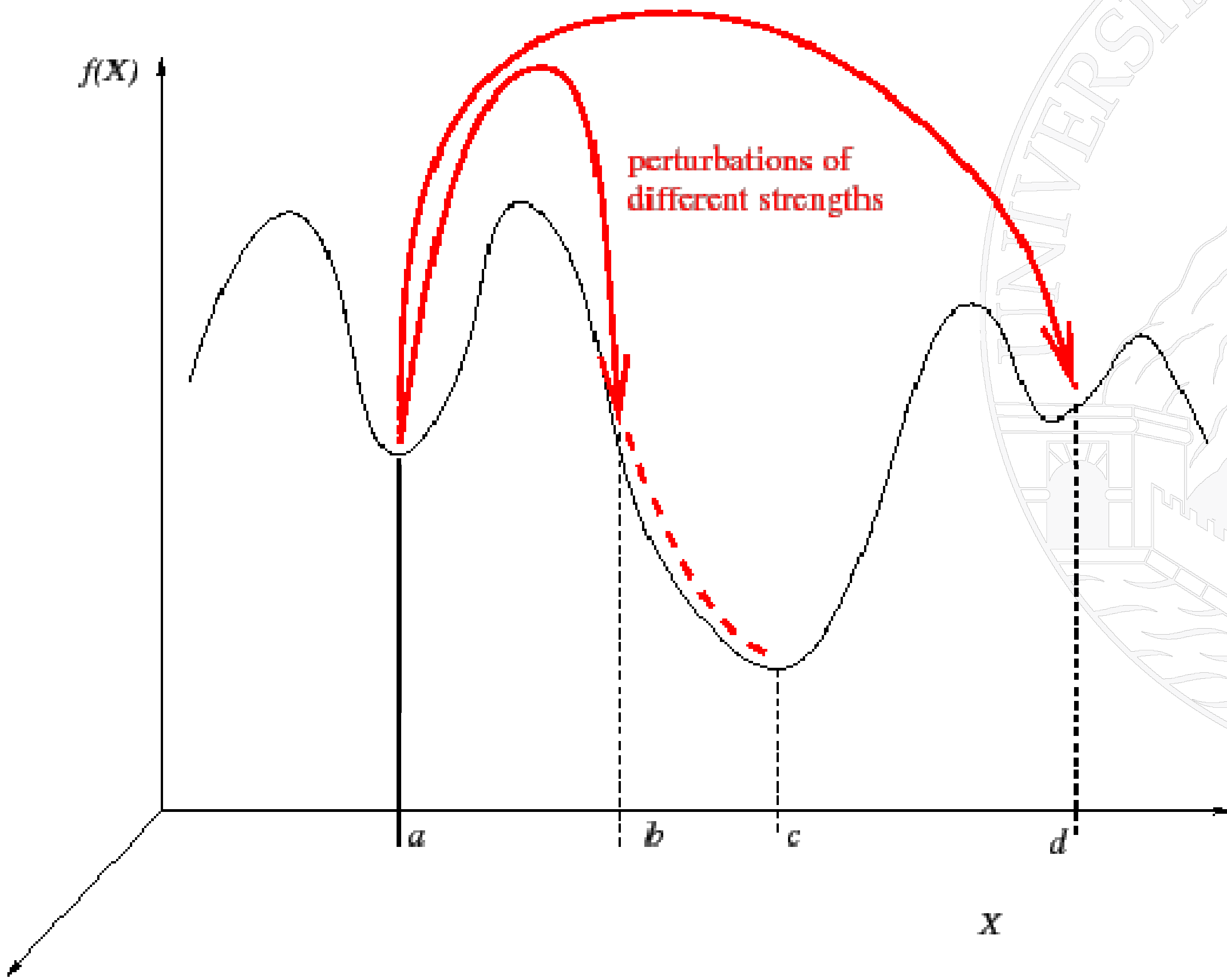
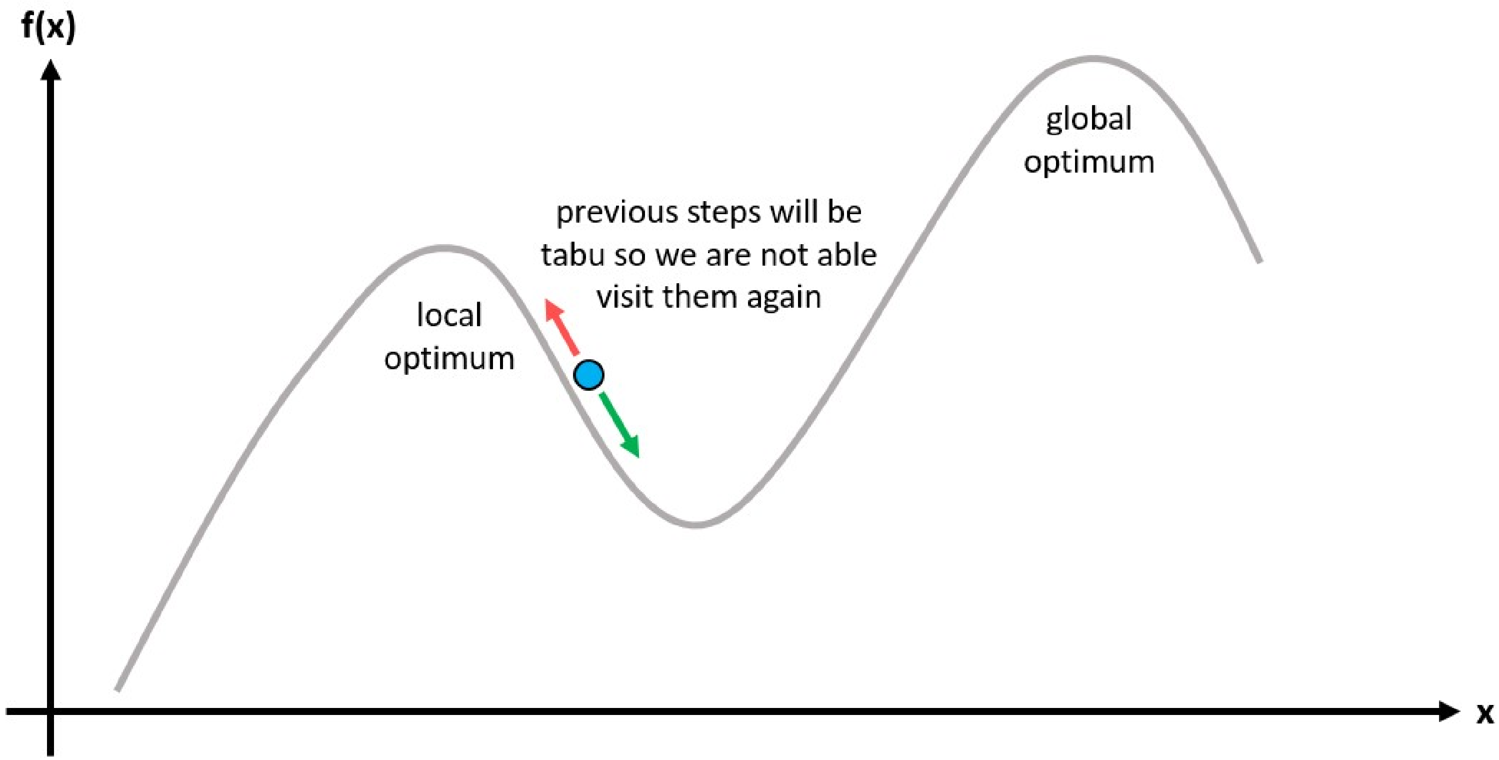
Iterated local search

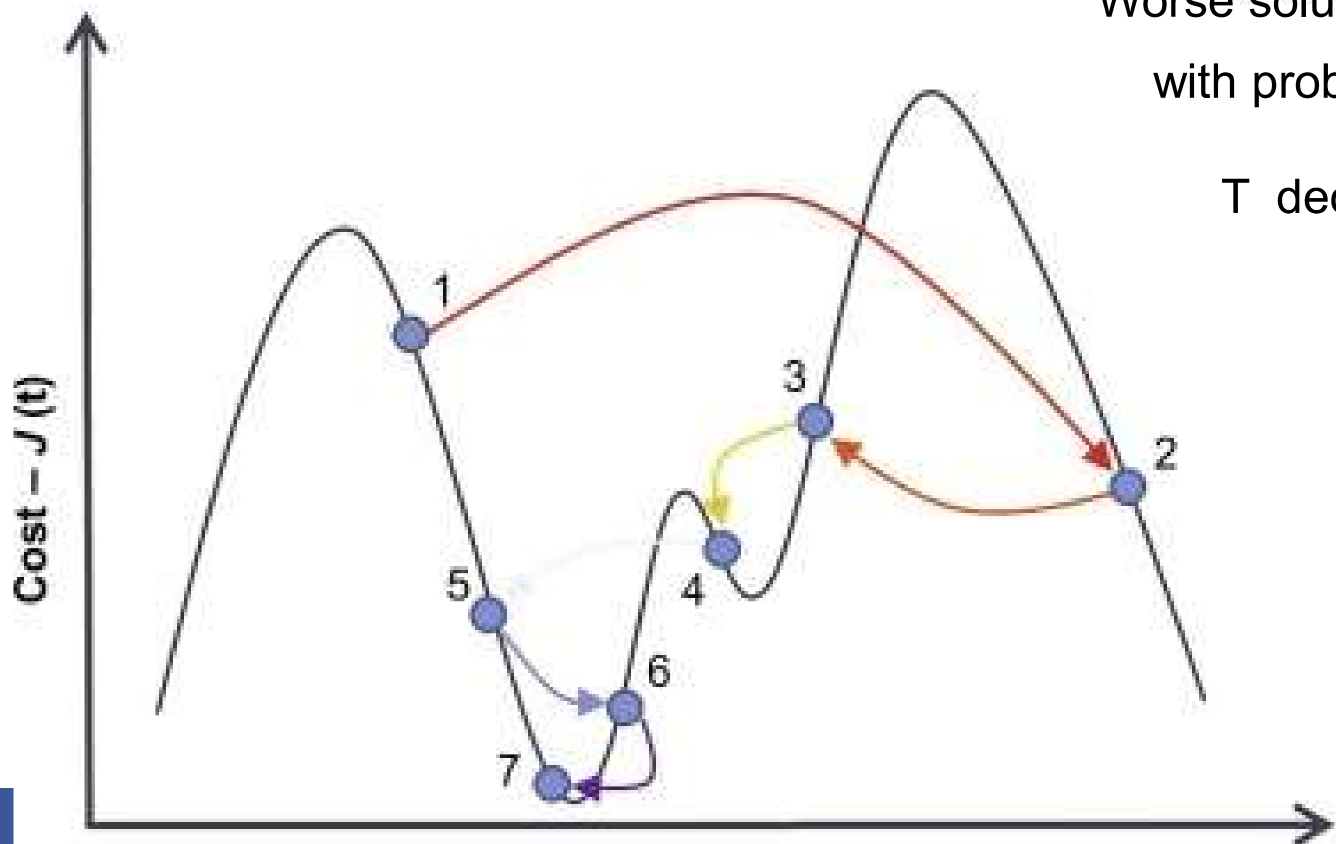


Tabu search



f(x)
x
local
optimum
previous steps will be
tabu so we are not able
visit them again
global
optimum

Simulated annealing



Worse solution accepted with probability $e^{-\frac{\Delta}{T}}$

T decreases

Large Neighborhood Search

$x \leftarrow x_0$

while stop criteria not met **do**

1. Find neighborhood $\mathcal{N}_x$ by *destroying* and *repairing* x
2. Find "best" solution in $\mathcal{N}_x$: x_{best}
3. $x \leftarrow x_{\text{best}}$

end while

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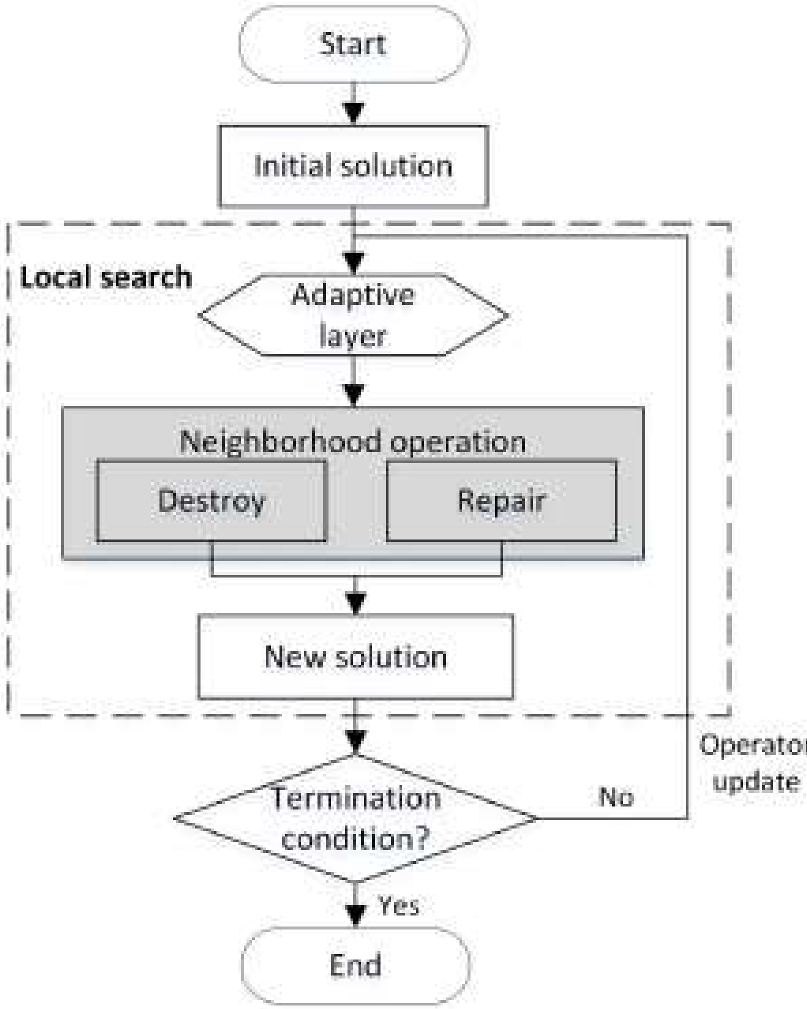
Adaptive LN Search



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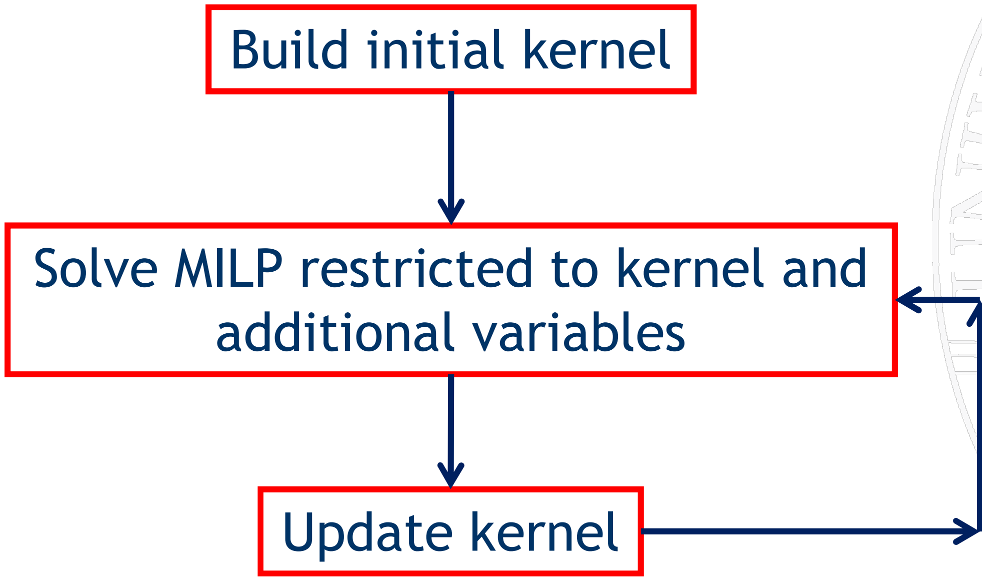
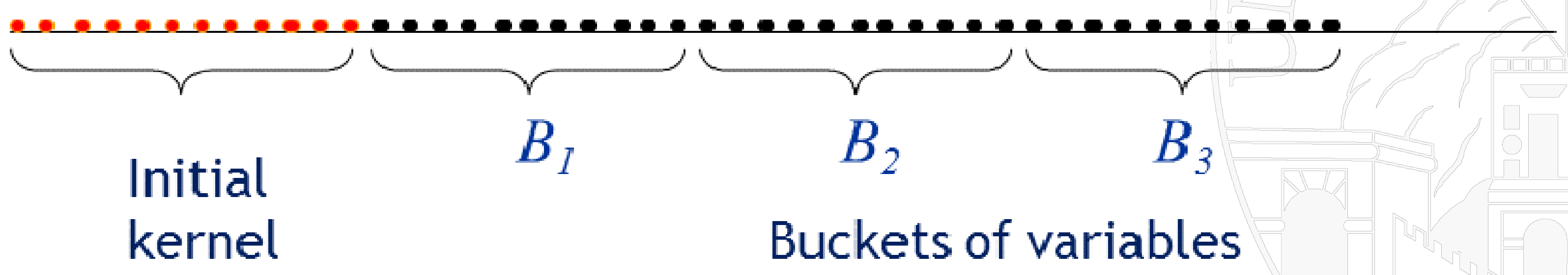
A matheuristic: Kernel search

$$\min c'x$$
$$Ax = b$$
$$x \geq 0 \quad \text{Integer}$$

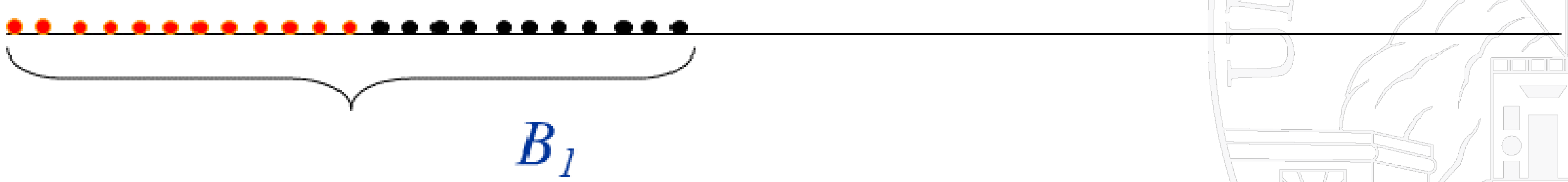


Not only for MILP

Kernel search

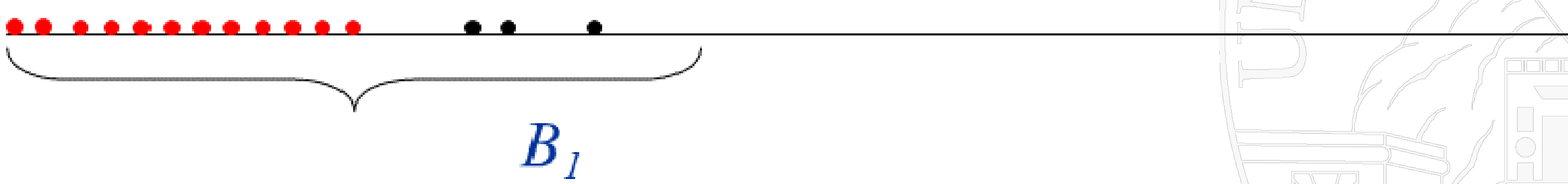


Kernel search



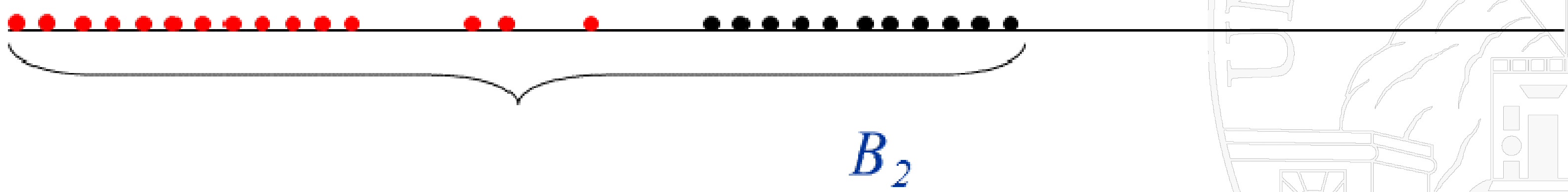
Restricted problem

Kernel search



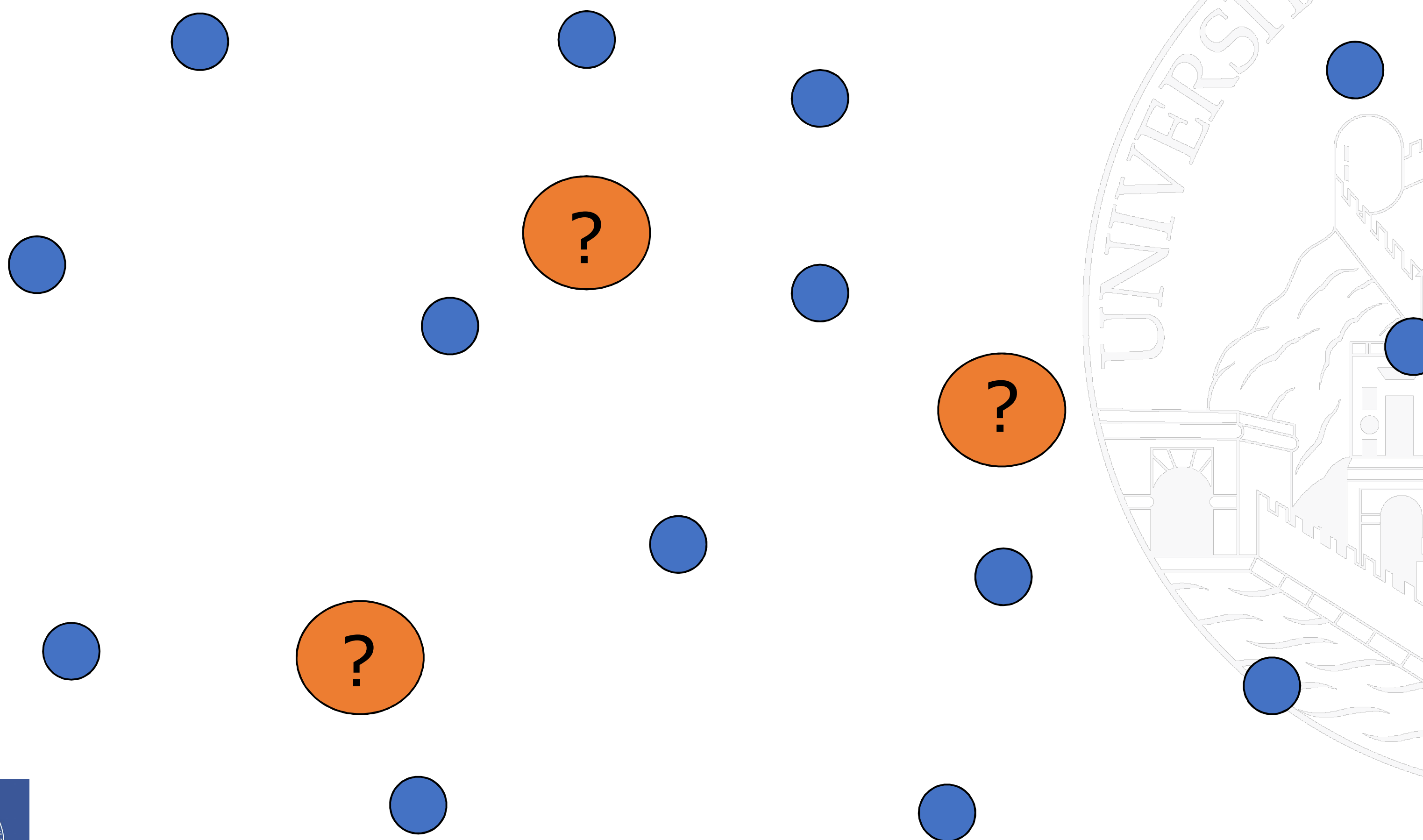
Updated kernel

Kernel search



Restricted problem

Facility location



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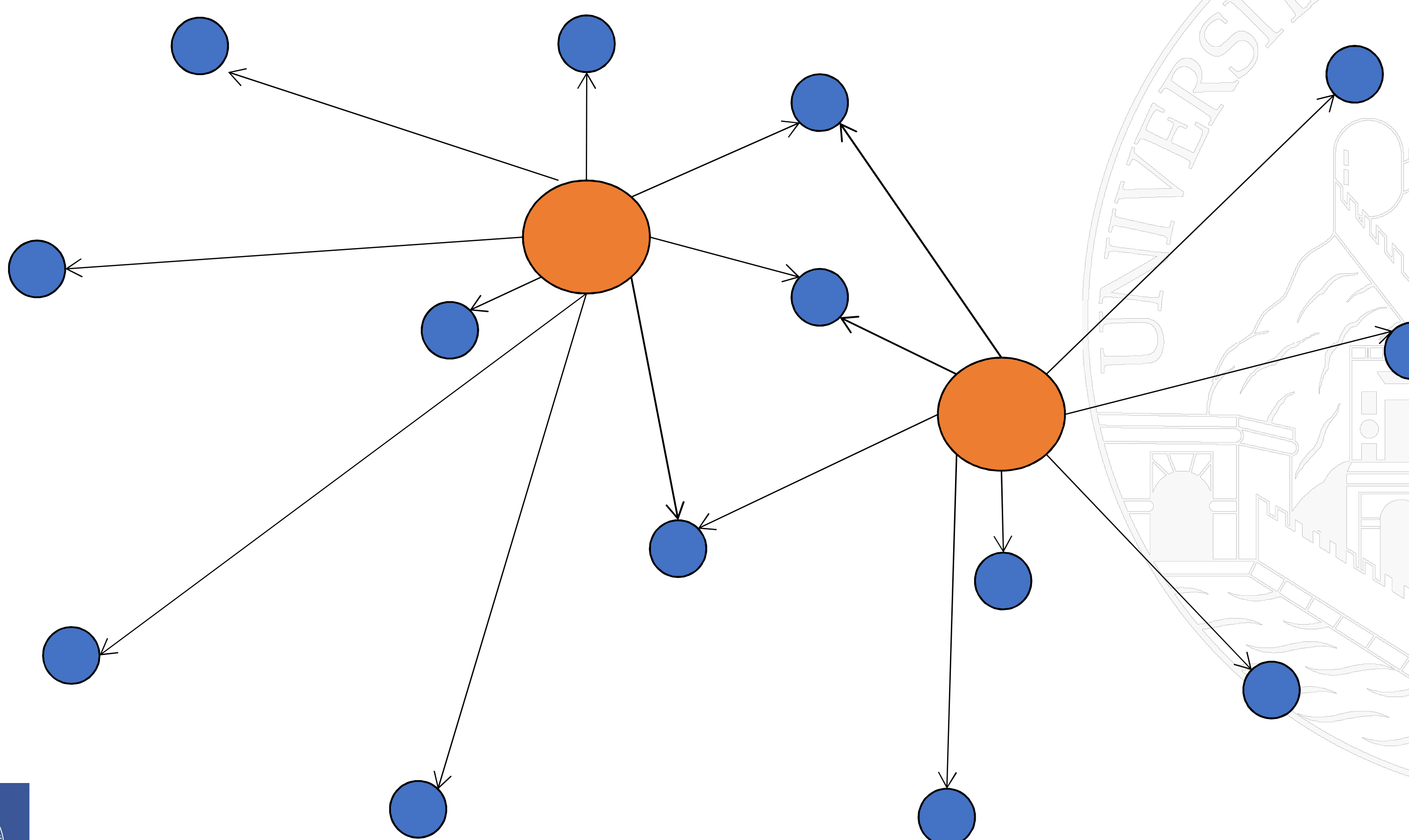
Facility location



Facility location

$$\min z = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j y_j$$

$$s.t. \quad \sum_{i \in I} x_{ij} \leq s_j y_j \qquad j \in J$$

$$\sum_{j \in J} x_{ij} = d_i \qquad i \in I$$

$$x_{ij} \leq d_i \qquad i \in I, j \in J$$

$$x_{ij} \geq 0 \qquad i \in I, j \in J$$

$$y_j \in \{0,1\} \qquad j \in J$$

NP-complete

Kernel search solves instances with 2000 customers and 4000 facilities (max errors below 1%)

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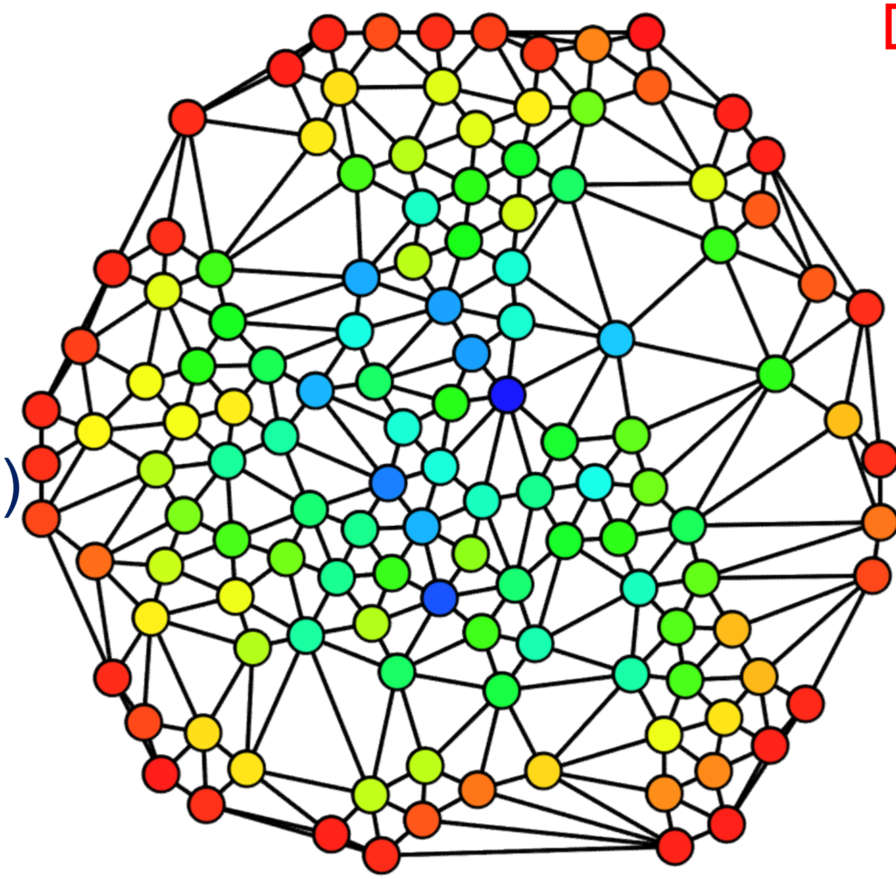
Graph

Data structure

Nodes (vertices)

Edges/arcs



Properties

Directed/Undirected

Weighted

Colored

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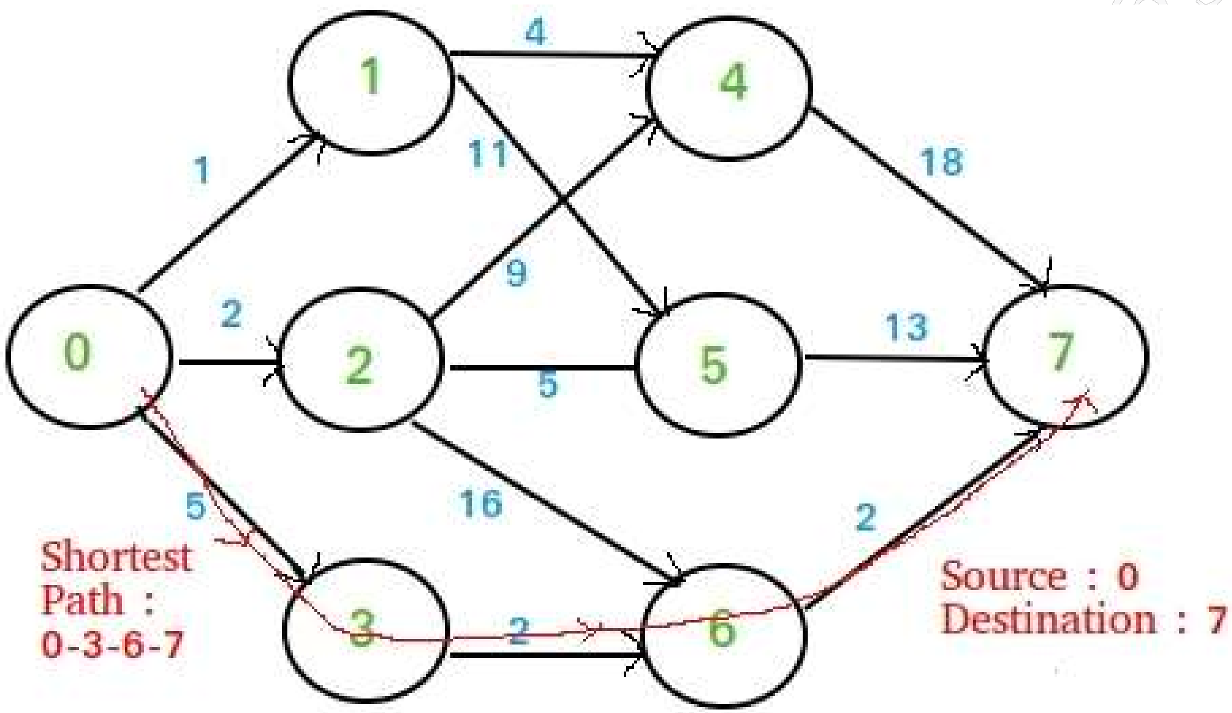
The shortest path



Dijkstra's algorithm

The shortest path

Easy:

Shortest path from one origin to all nodes

Shortest path from all origins

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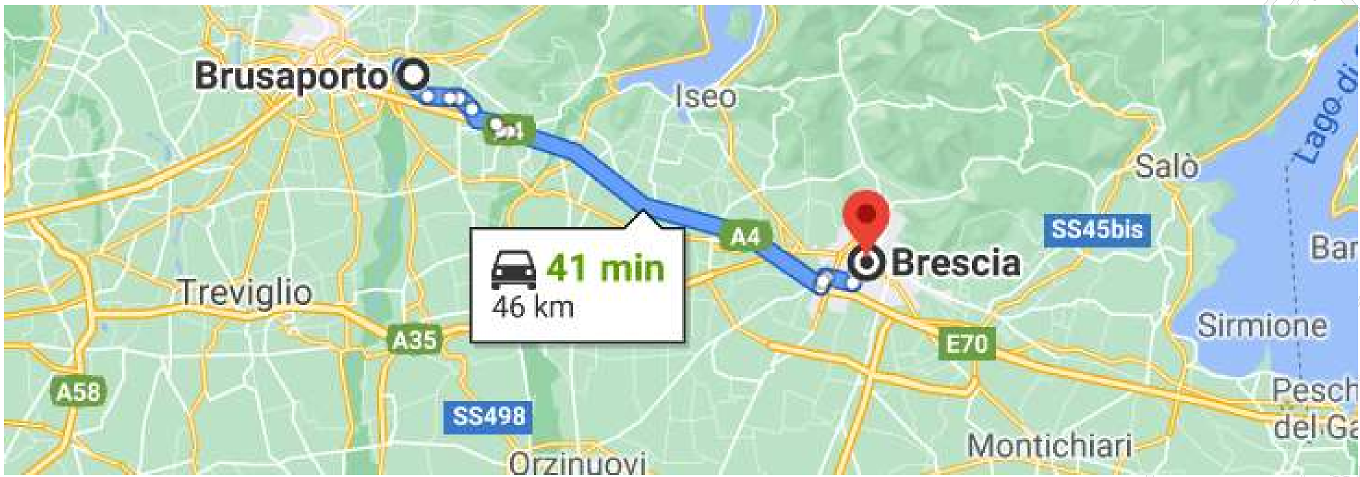
The shortest path



Efficiency:

Representation of road network

Efficiency of algorithm

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The time-dependent shortest path

Transit time on an arc depends on the arrival time

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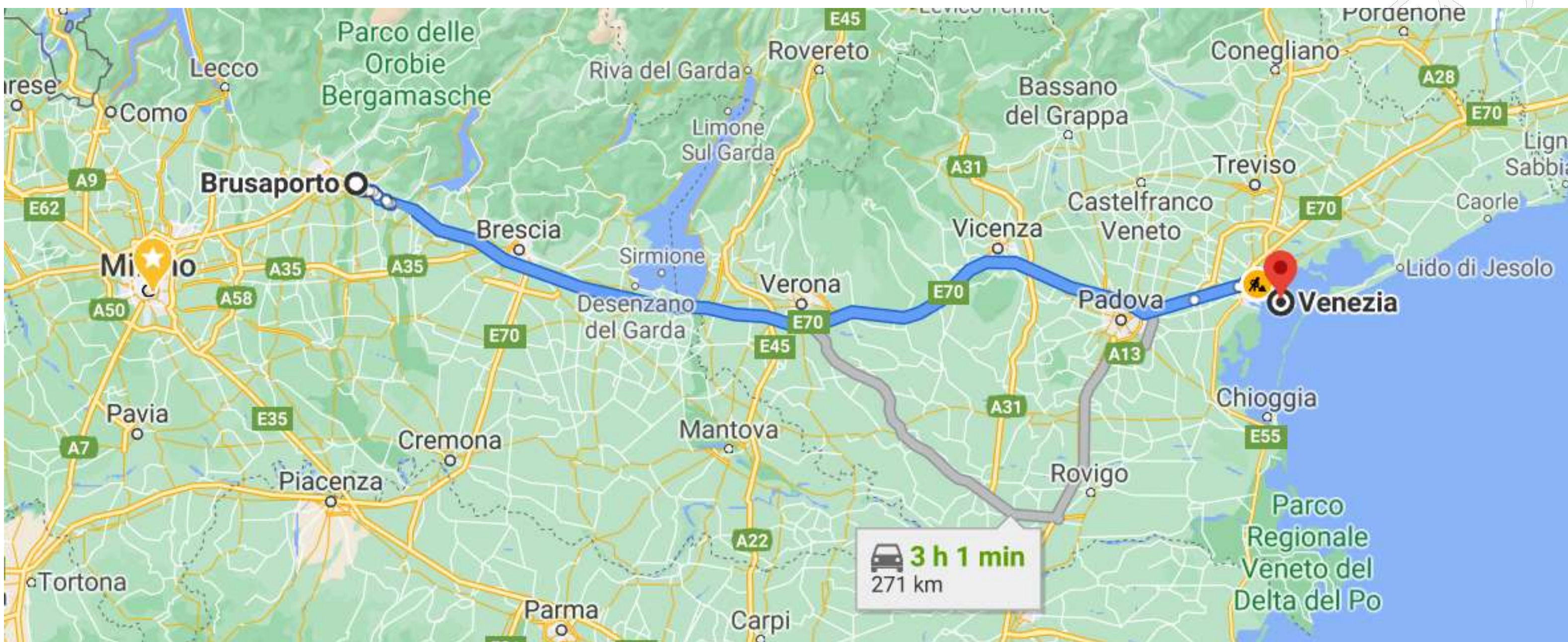
f(t)

We know a transit-time function f(t) for each arc (v, w) (where t is the time to leave v).

What is the fastest route from r to s?

A modified version of Dijkstra’s algorithm

The shortest path



tramite A4 — 2 h 18 min

Percorso più veloce, traffico regolare — 220 km

The travel time on an arc depends on the traffic congestion and the weather.

Transit function f(t) often unknown a priori

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Uncertainty

What and how can we forecast?

Probabilistic information about the future	Stochastic
Uncertainty (no distribution) over parameters	Robust

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The stochastic shortest path problem

At each node, we must select a probability distribution over all possible successor nodes, out of a given set of probability distributions.

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Standard results for the deterministic case extended (Markovian decision process)

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The robust shortest path problem

In robust optimization one wants to hedge against (all) possible scenarios by considering, for example, the worst case for each solution.

The robust deviation shortest path problem:
To find among all paths from s to t the one that, over all scenarios, minimizes the maximum deviation of the path length from the optimal path length of the corresponding scenario.

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Dynamic problems

The role of information (forecasting)

Information changes over time

More accurate if closer in time to decision

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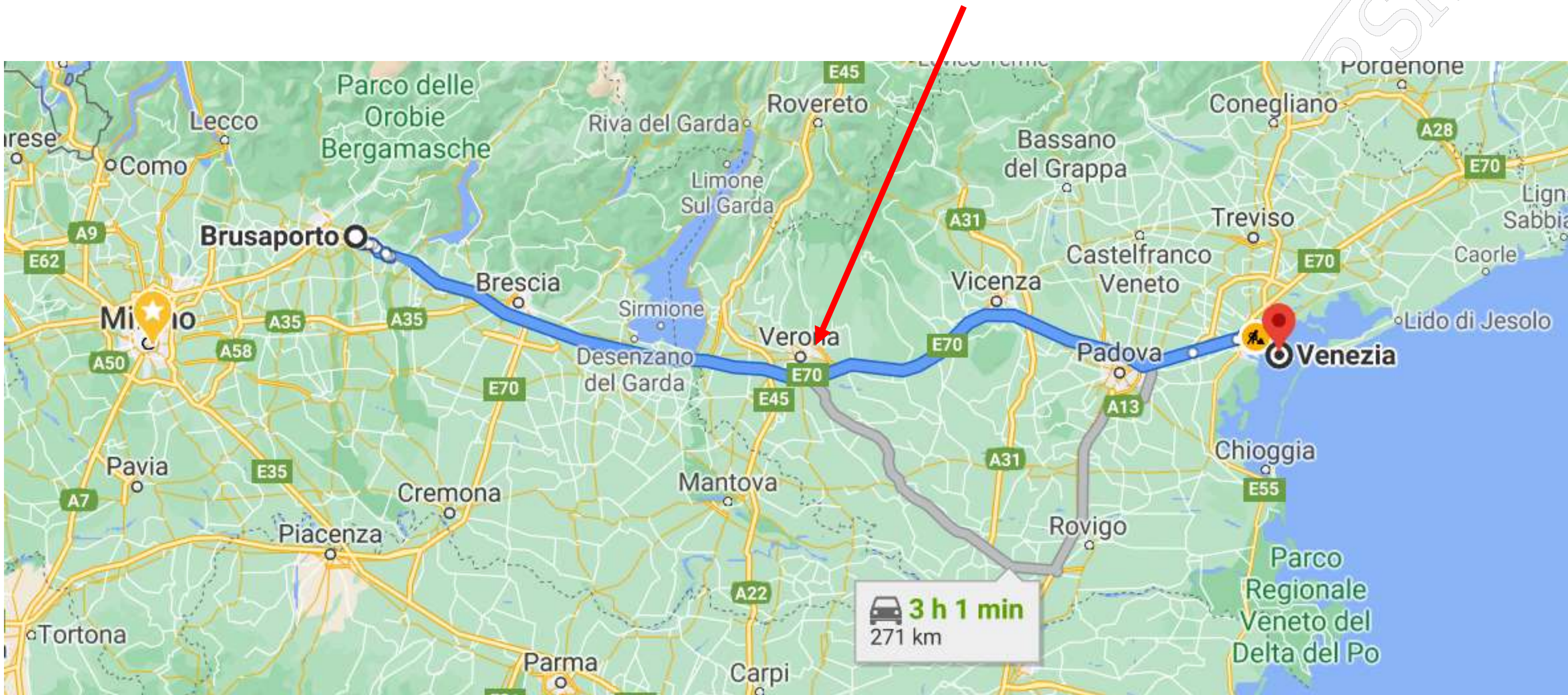
Dynamic problems

Info needed on transit times ahead



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Dynamic problems

Dynamic graph: the cost (transit time) of a subset of arcs changes over time

Reoptimizing shortest paths on dynamic graphs consists in solving a sequence of shortest path problems, where each problem partially differs from the previous one

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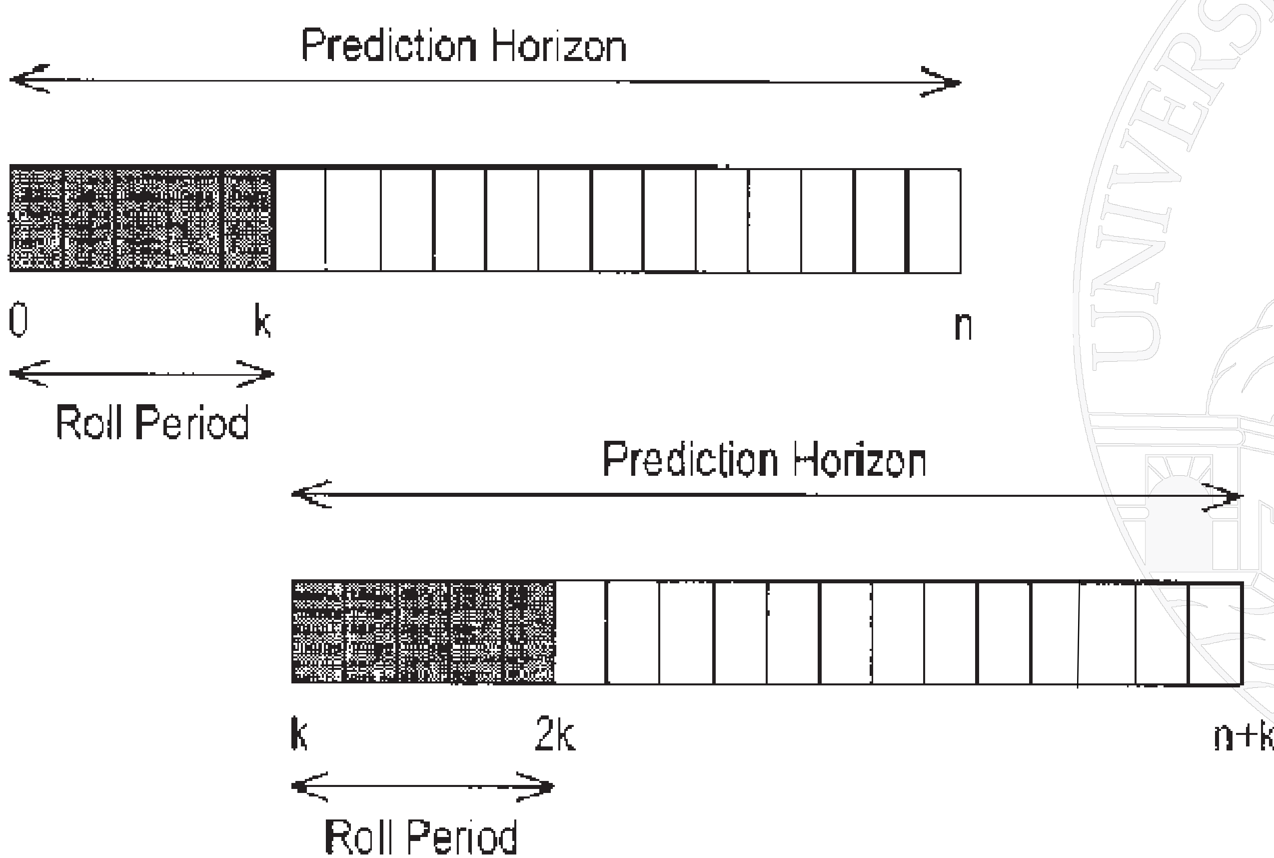
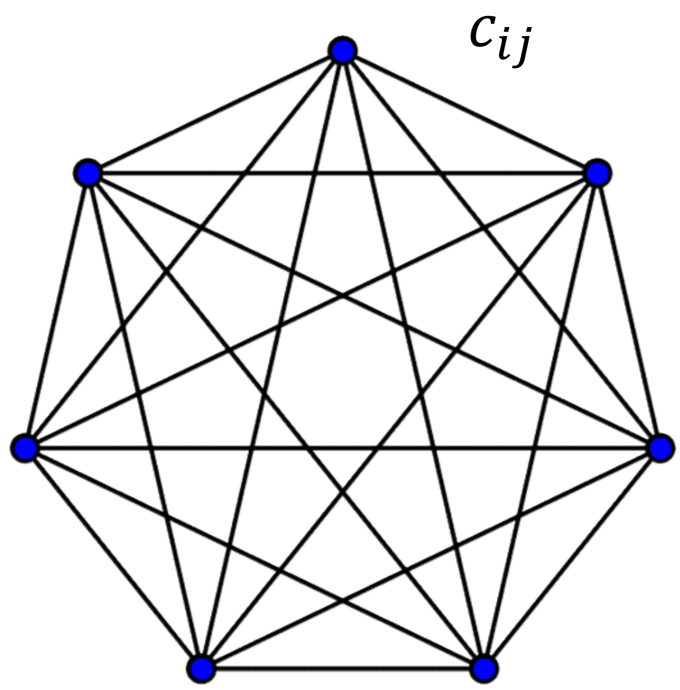
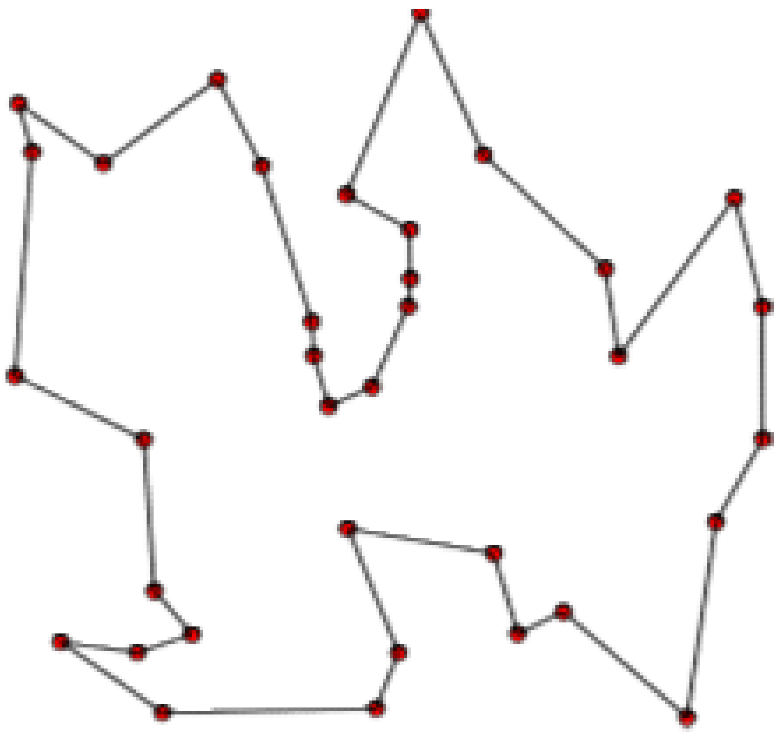
Dynamic problems



Traveling salesman problem



Traveling salesman problem



Traveling salesman problem

$$
\begin{aligned}
\min \quad & \sum_i \sum_j c_{ij} y_{ij} \\
\text{s.t} \quad & \sum_j y_{ij} = 1, \quad i = 0, 1, ..., n-1 \\
& \sum_i y_{ij} = 1, \quad j = 0, 1, ..., n-1 \\
& \sum_i \sum_j y_{ij} \leq |S| - 1 \quad S \subset V, 2 \leq |S| \leq n-2 \\
& y_{ij} \in \{0, 1\} \; \forall i, j \in E
\end{aligned}
$$

Exponential number of constraints

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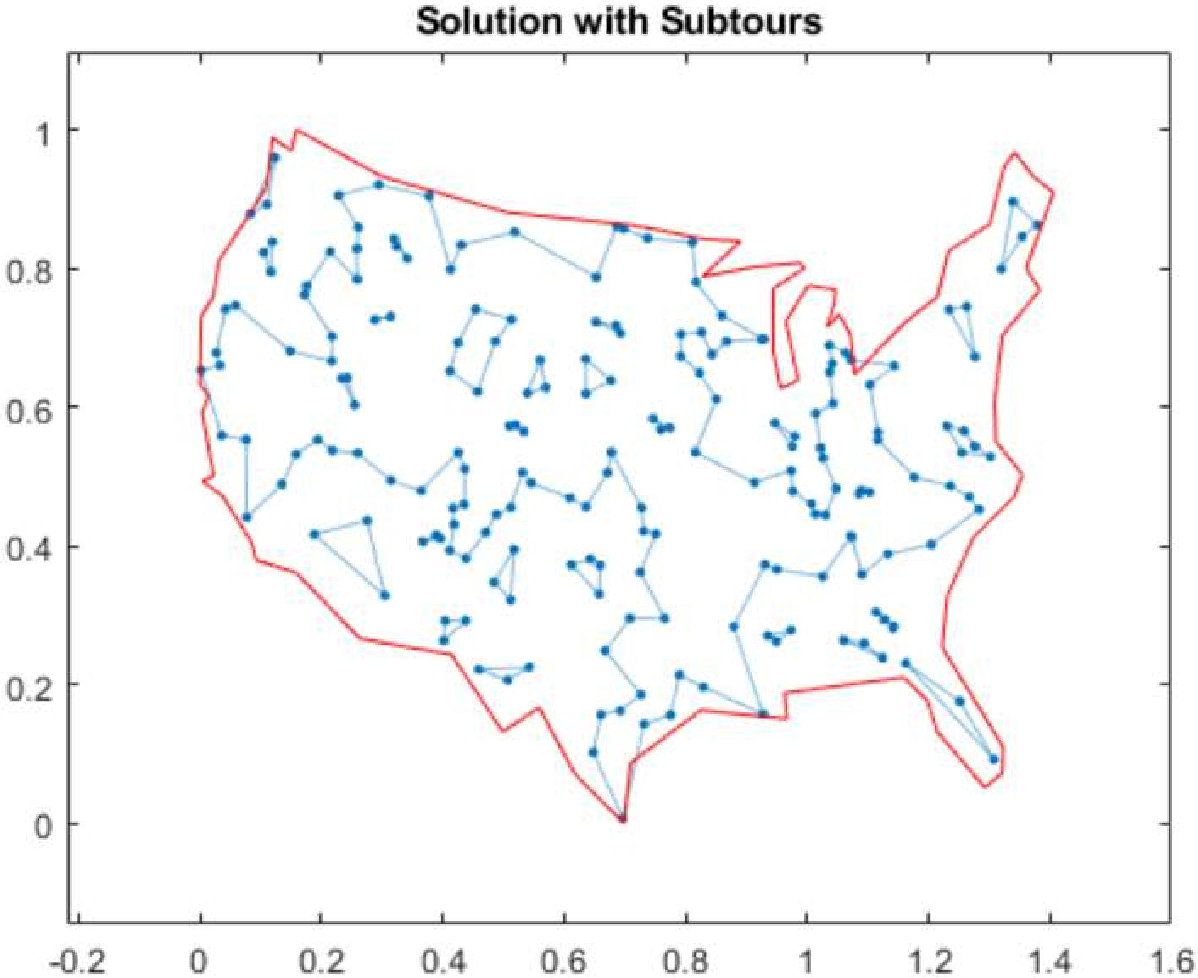
Traveling salesman problem



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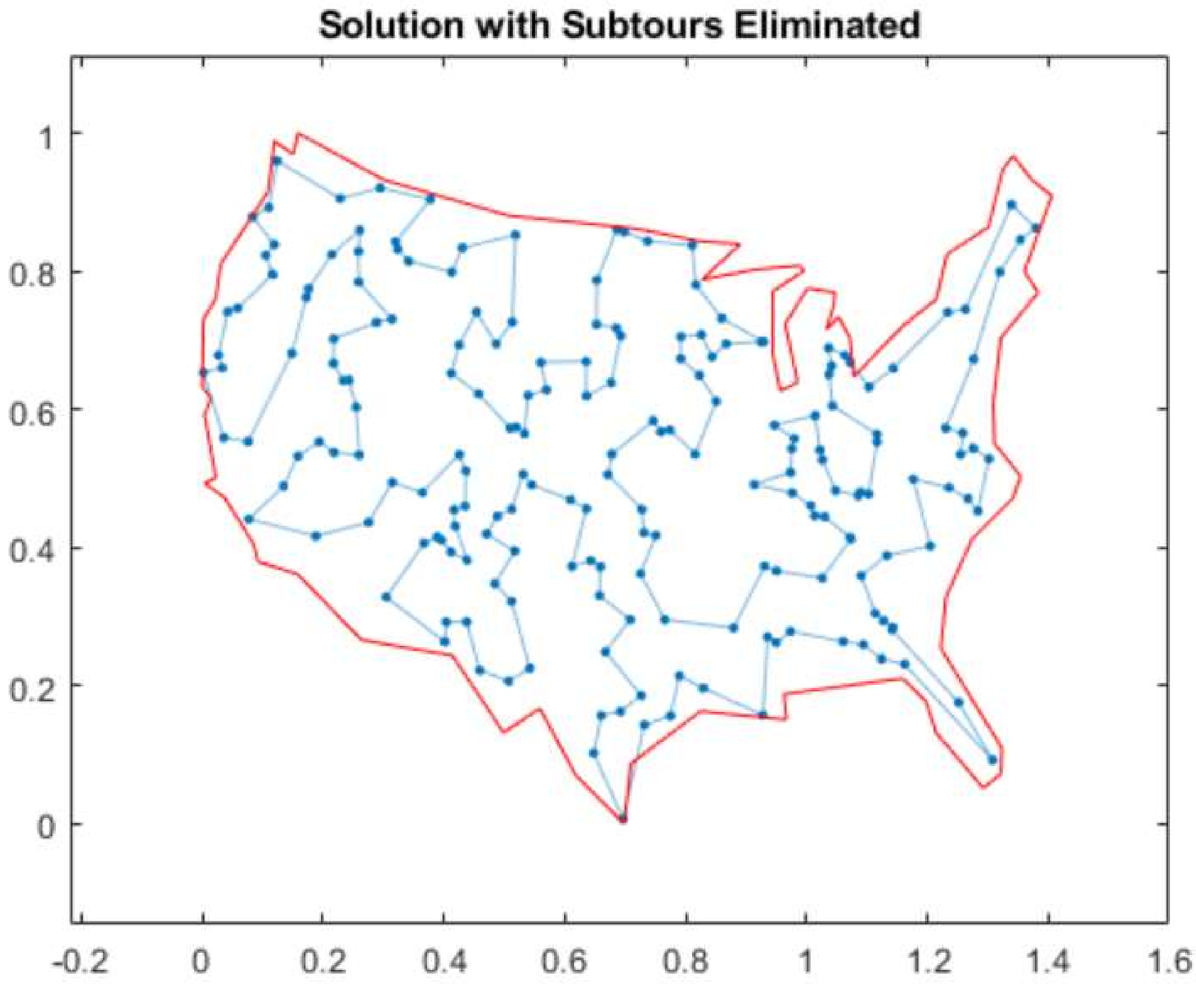
Traveling salesman problem



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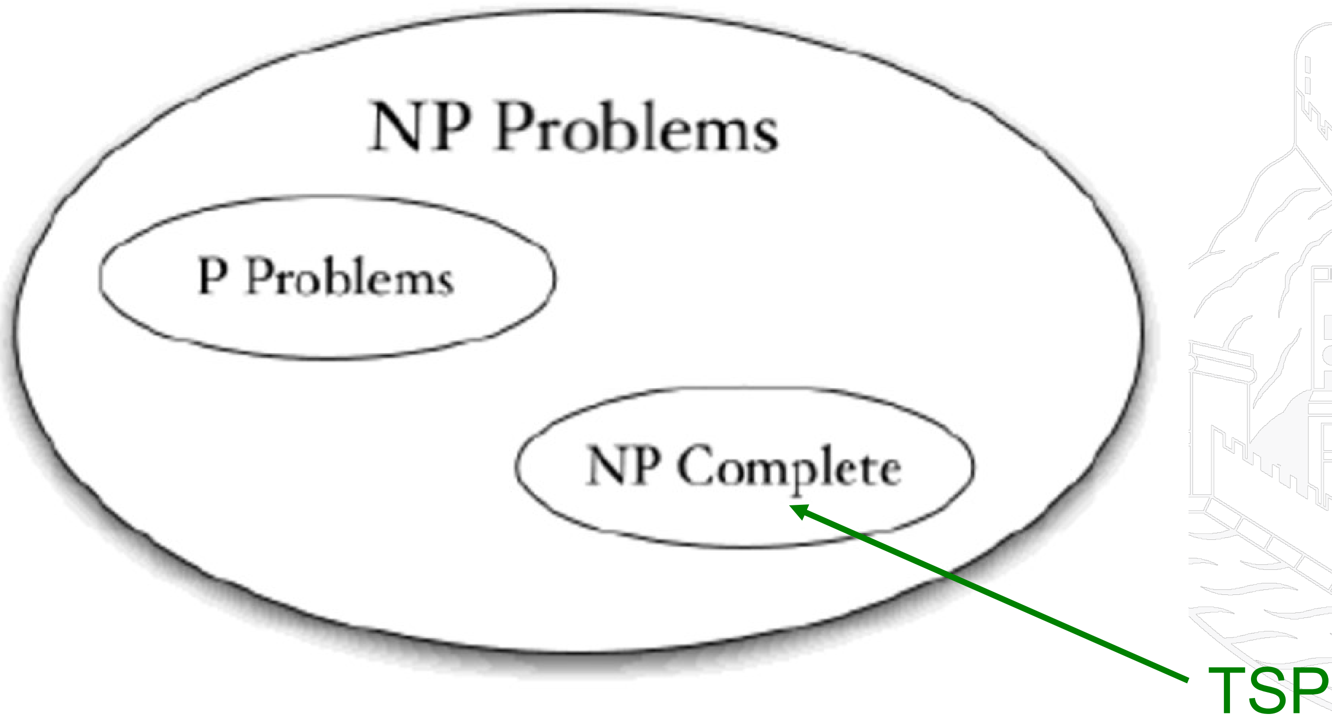
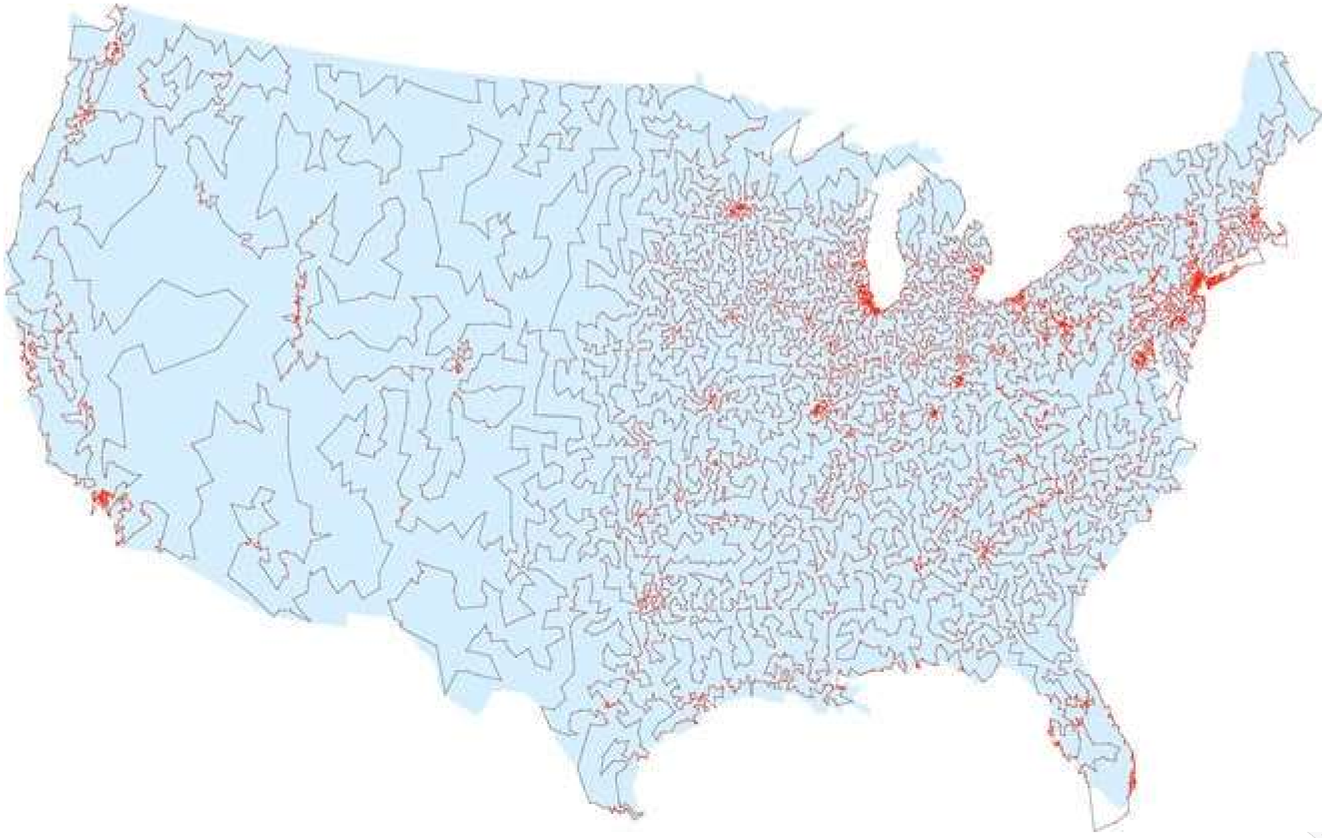
Traveling salesman problem





13509 cities

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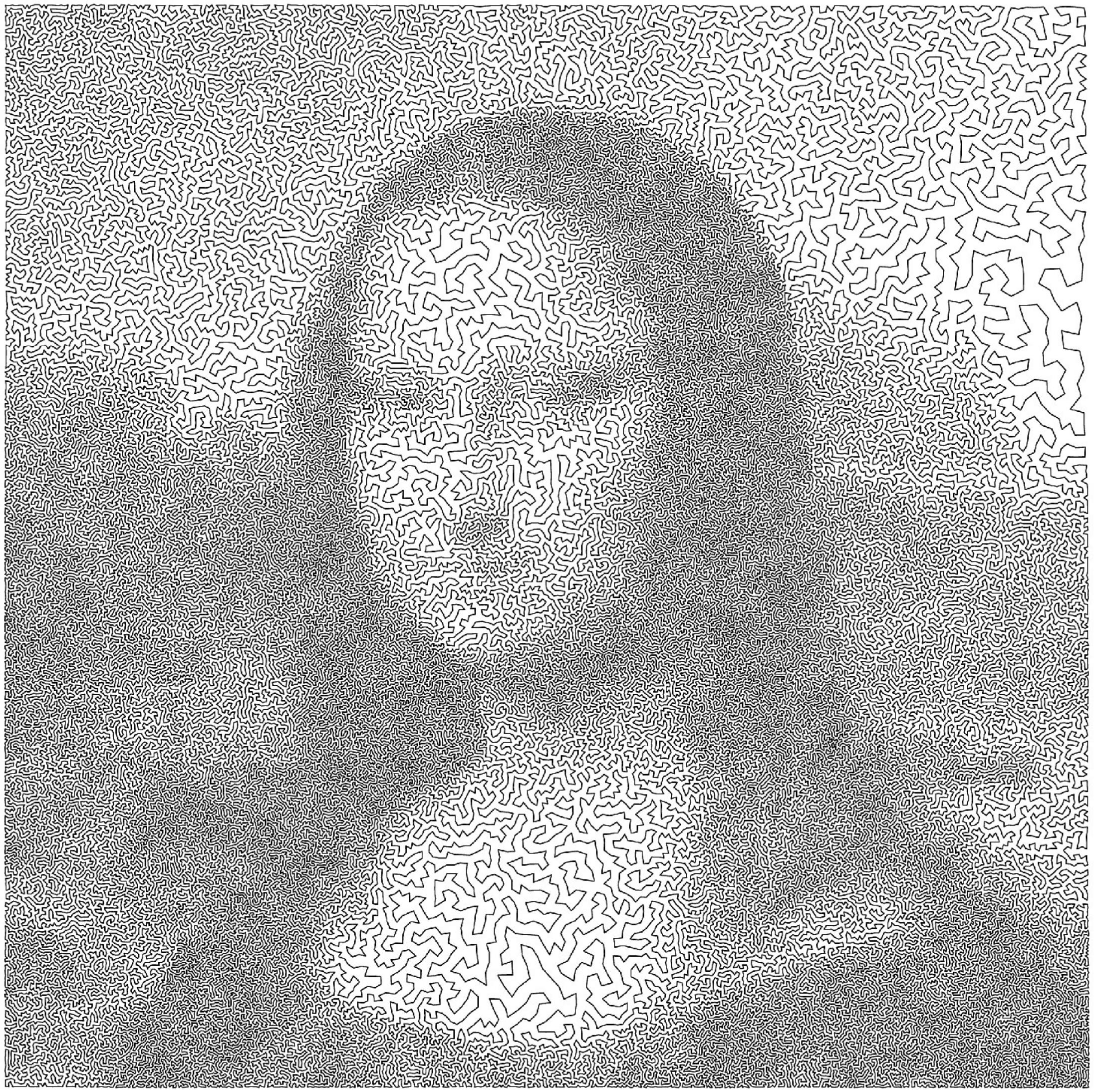

100000 points

Many good heuristics

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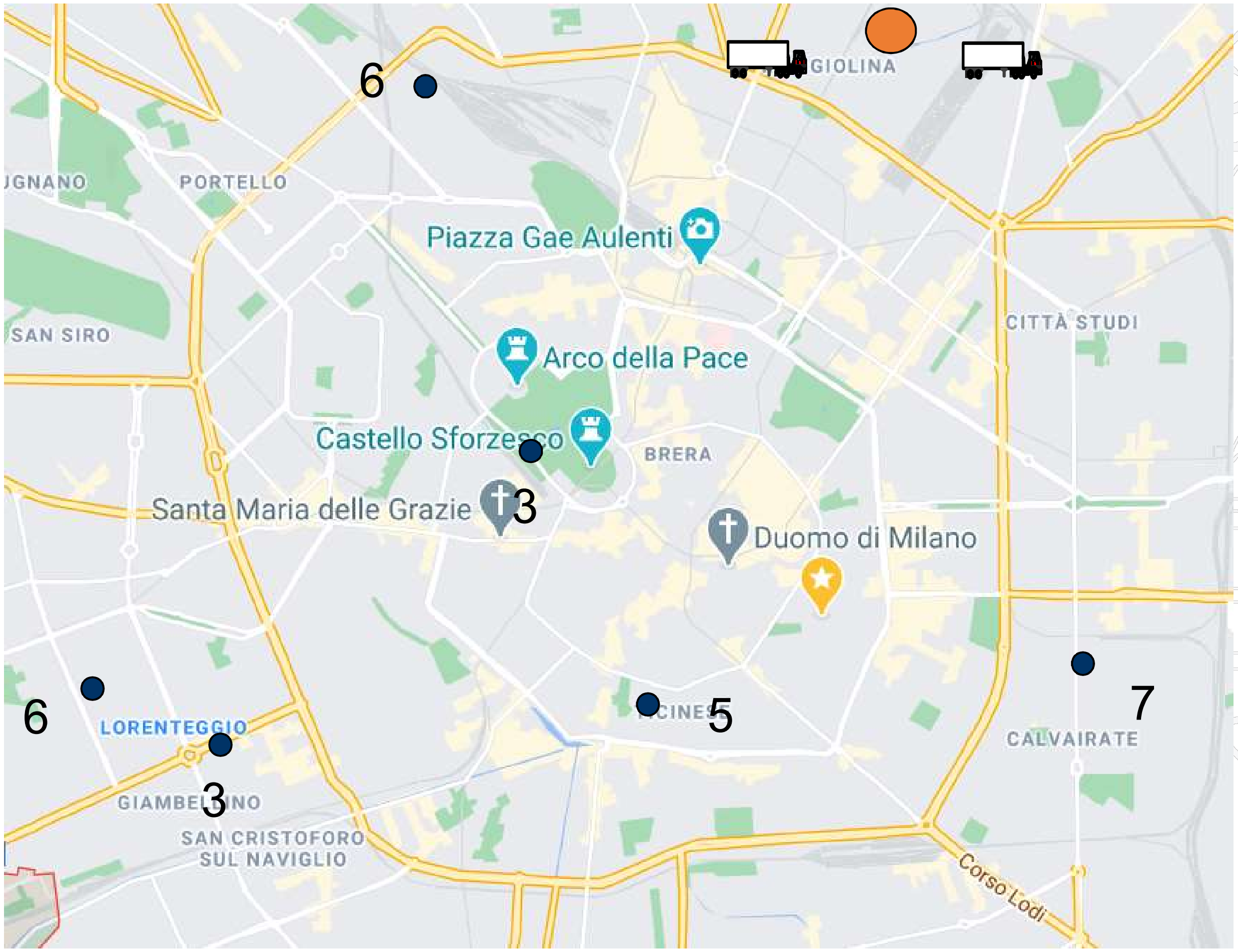
Routing problems



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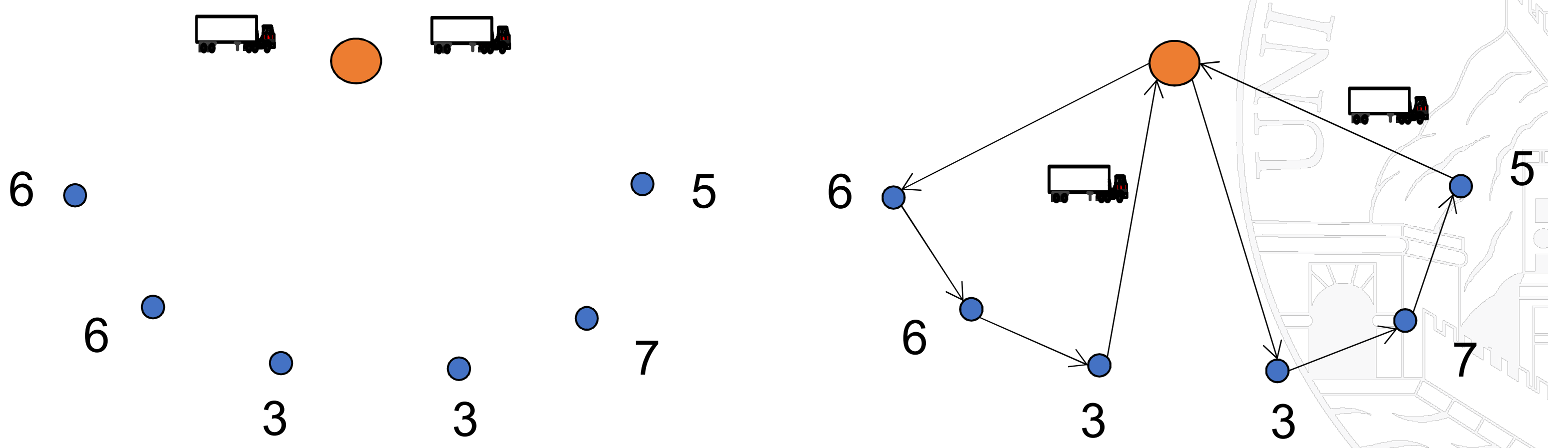
Routing problems

Complete graph of shortest paths
($O(n^2)$ shortest paths)



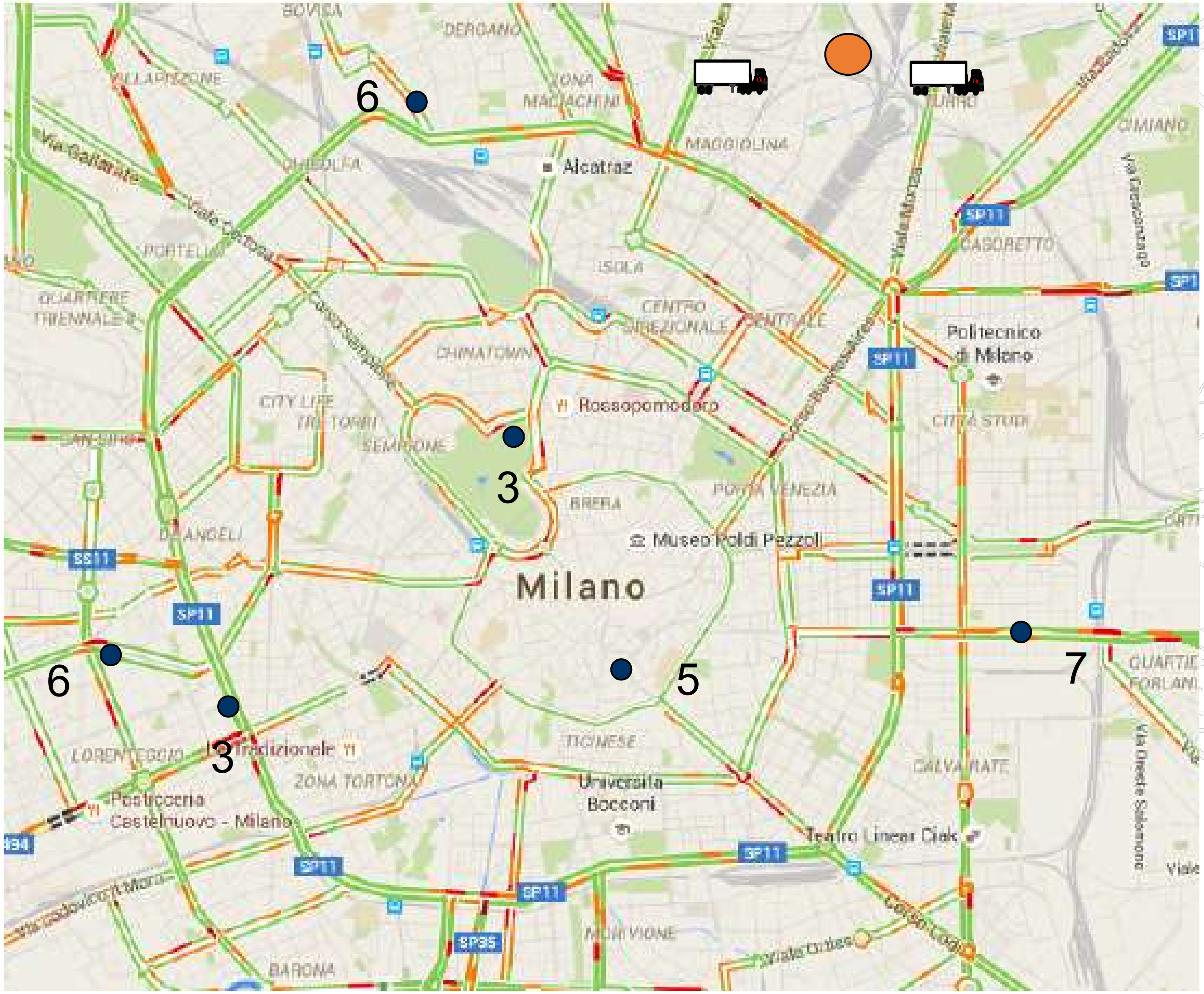
Routing problems

A broad range of problems

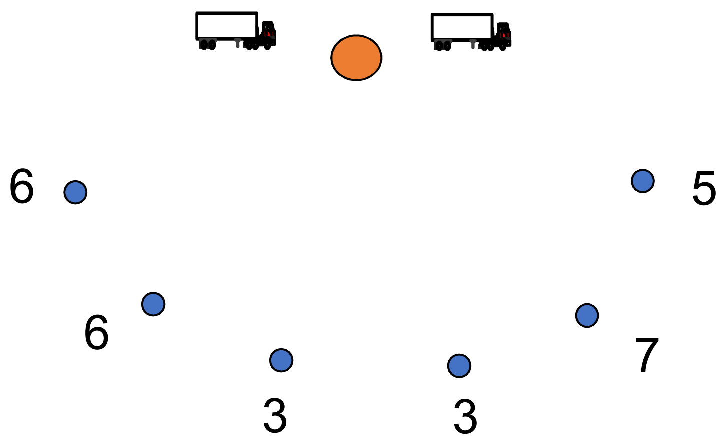
Heuristics:

- Tabu search
- Large variable neighborhood search
- Heuristic column generation
- ….

Routing problems



Routing problems



The complete graph of the shortest paths cannot be computed in advance

1. The complete graph of the shortest paths has to be re-computed (computational challenges)

2. Work on the graph of the map (computational challenges)

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Towards sustainable transportation

Better use of the road network

Better use of the capacity of vehicles

Less kilometers traveled

Less congestion

Less vehicles on the road network

Reduction of emissions

Less space occupied by traveling and parked vehicles

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Freight

Booking of loading/unloading areas

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Booking of loading/unloading areas

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Booking of loading/unloading areas

Each vehicle makes a reservation of the L/U areas

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Windows of availability for the following vehicles

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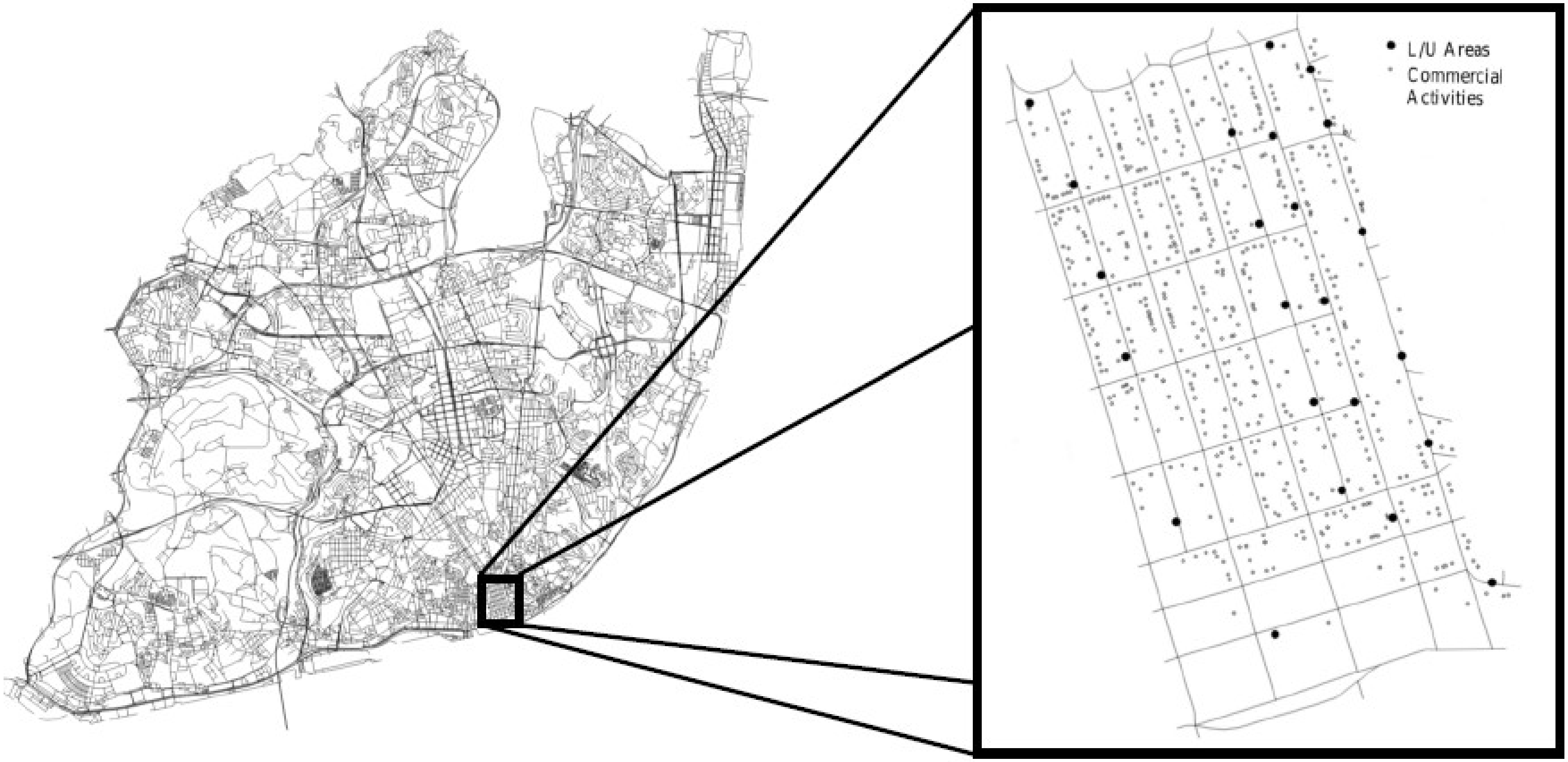
Variant of TSP with multiple time windows

Mor, Speranza, Viegas, TR E, to appear

Booking of loading/unloading areas



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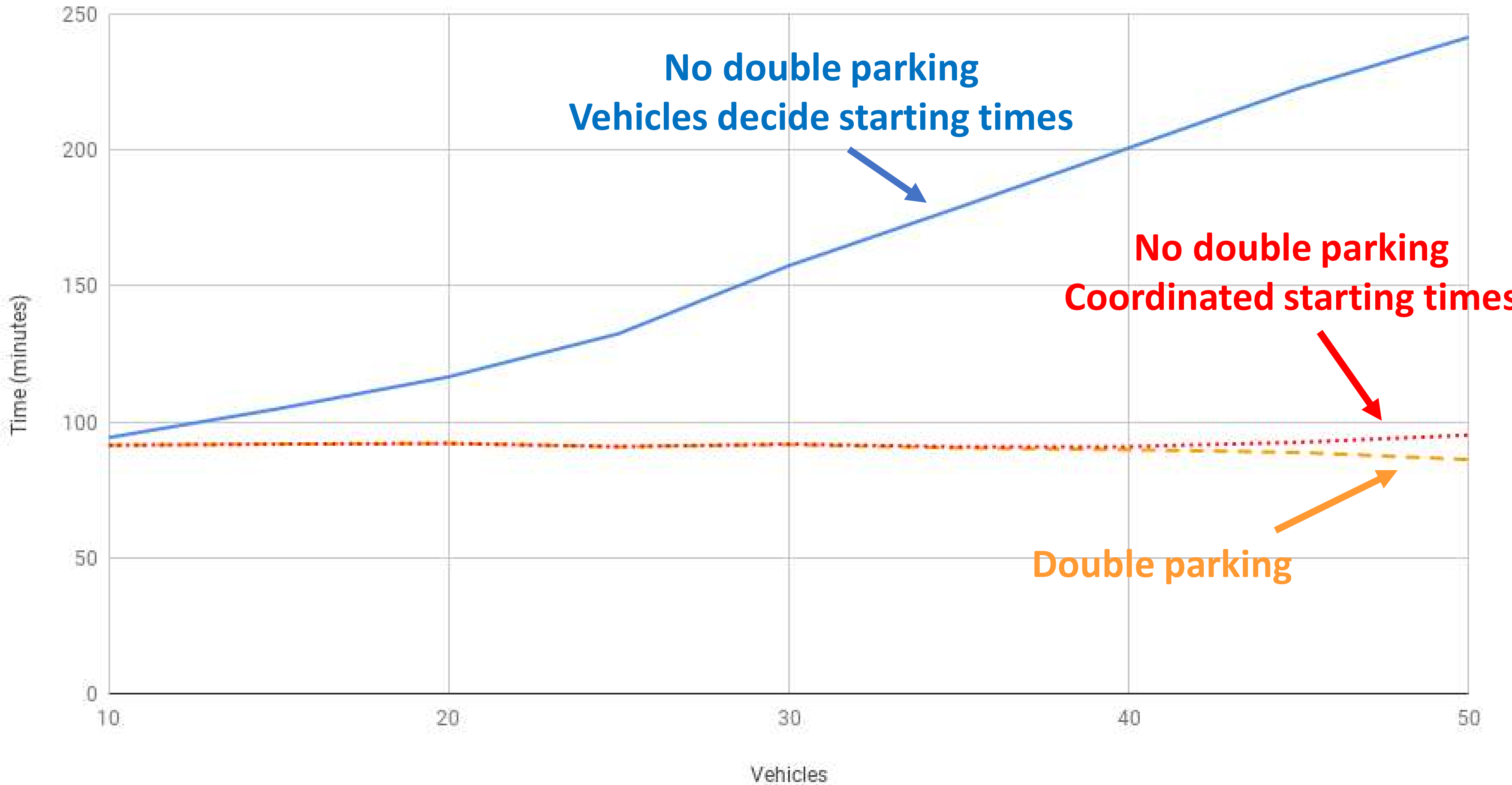
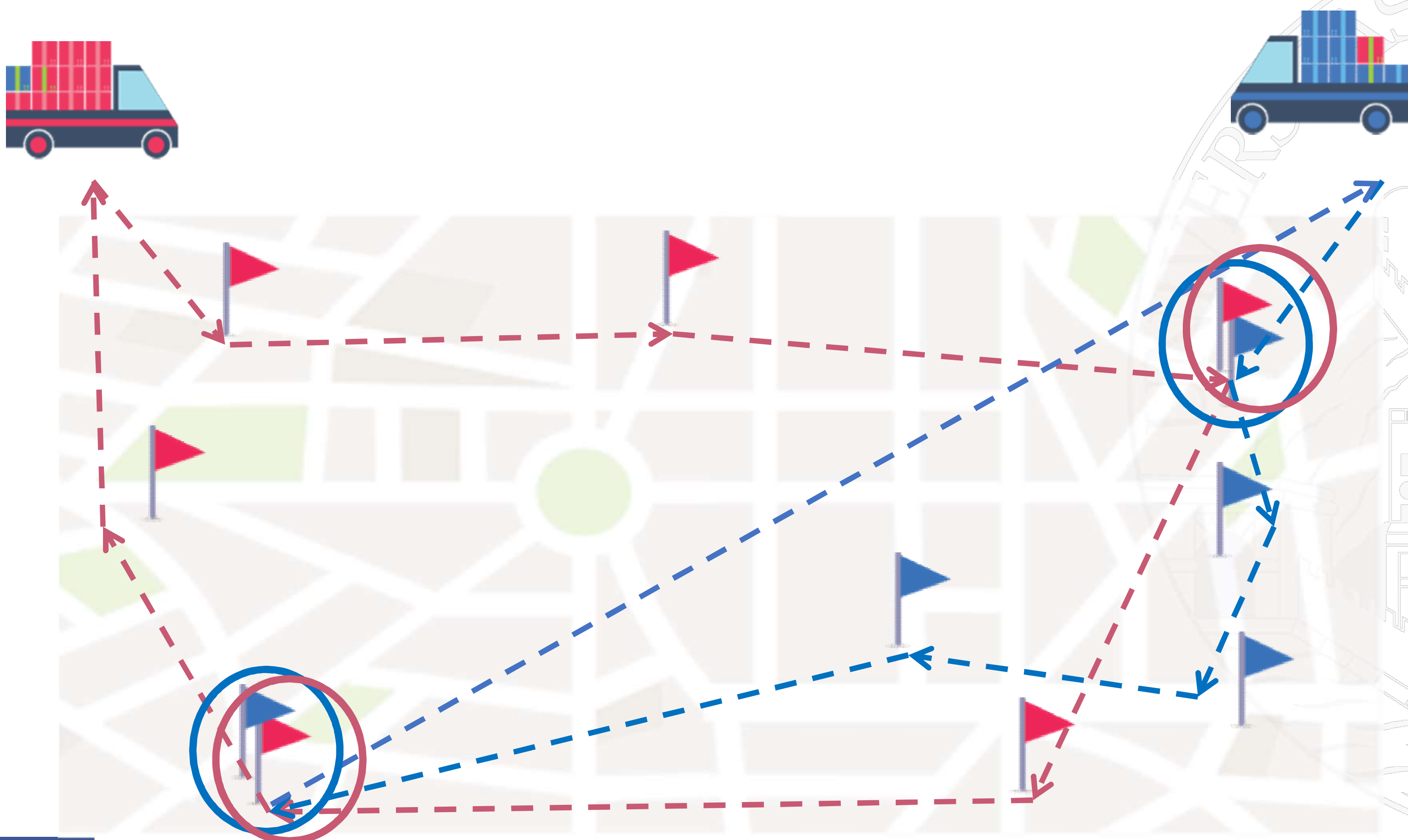
Booking of loading/unloading areas

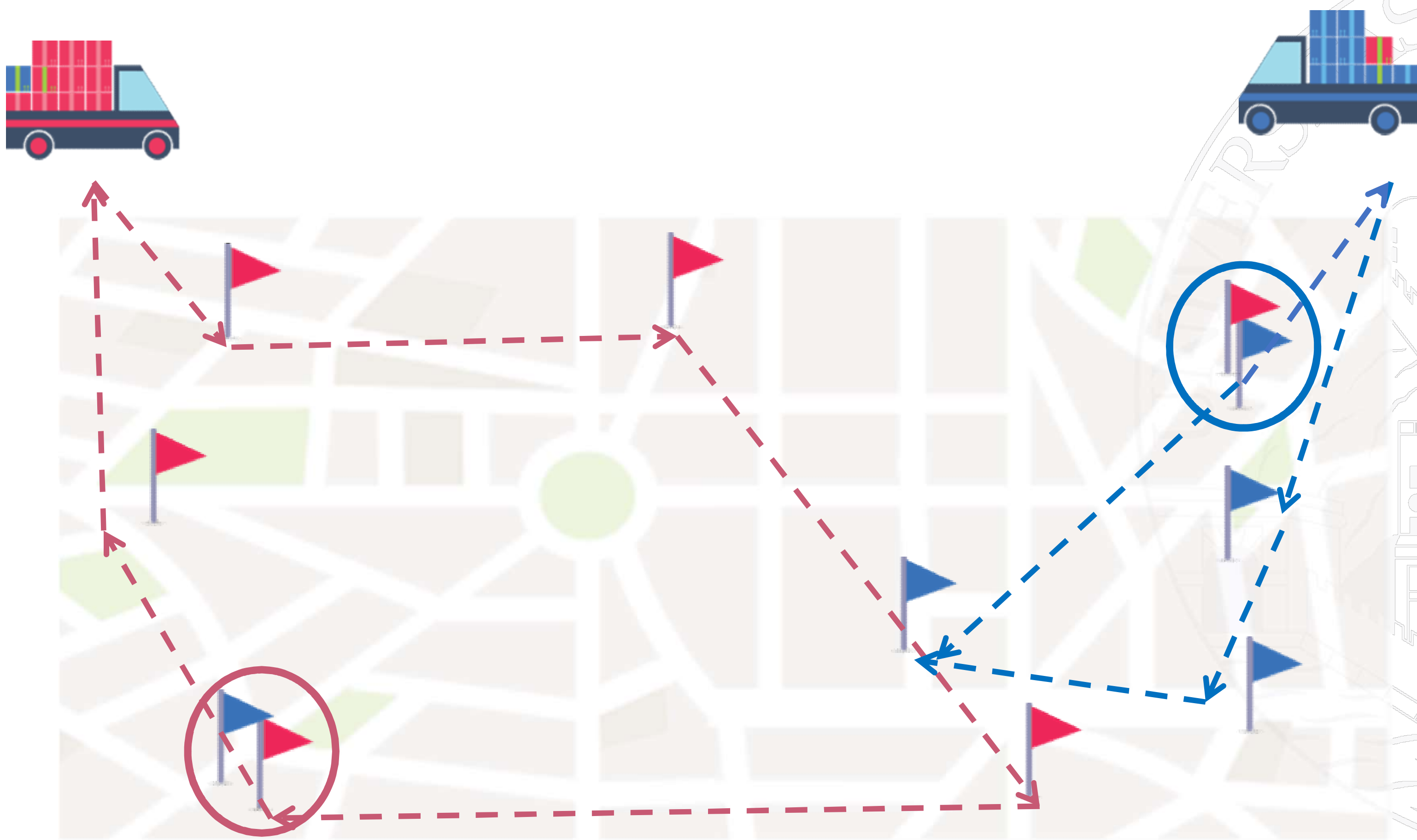


Better use of the road network

Collaboration



Collaboration



Collaboration

S1	Savings		
Instance	- %	$-\%_A$	$-\%_B$
1	23.9	35.5	2.9
2	8.9	17.9	2.7
3	16.7	1.8	31.6
4	14.2	14.5	10.8
5	10.8	10.3	13.3
6	10.1	17.6	2.9
7	8.6	0	12.9
8	17.8	1.6	25
9	25.2	33.3	0
10	14.9	18.1	6.5
11	9.4	22.4	0
12	6.5	0	8.9

Individual solutions
(without collaboration)
always optimal

Reduction of emissions: 21%

Fernandes, Roca-Riu, Speranza, EJOR, 2018

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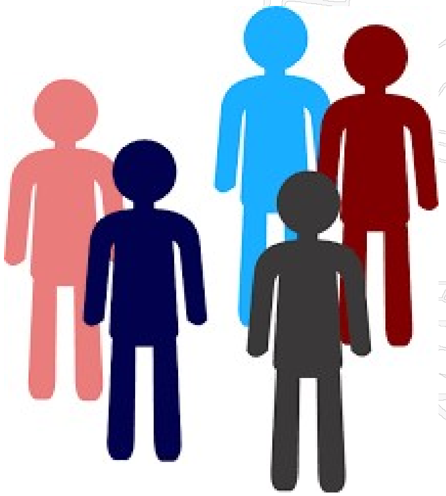
Freight and people



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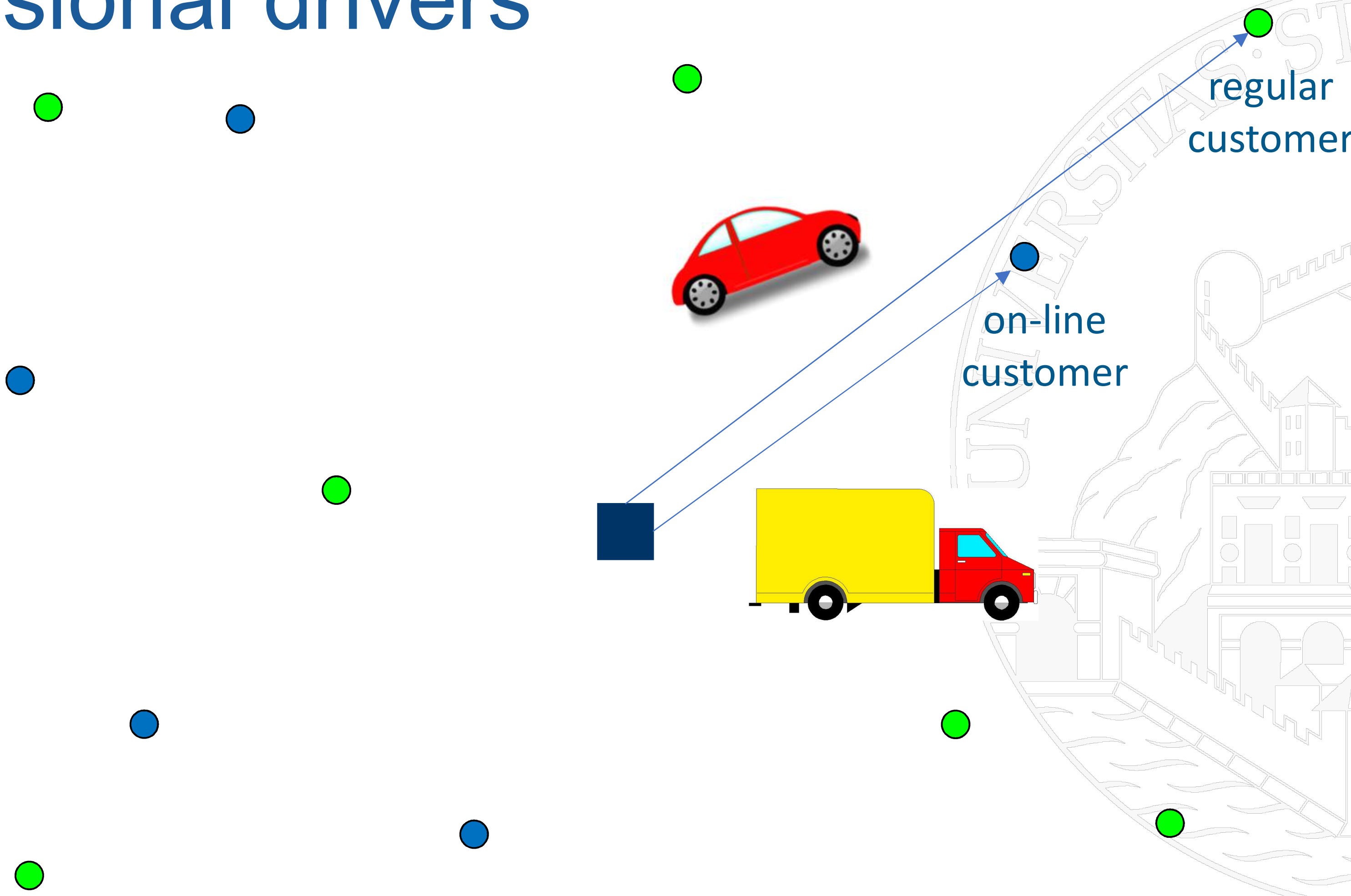
Occasional drivers



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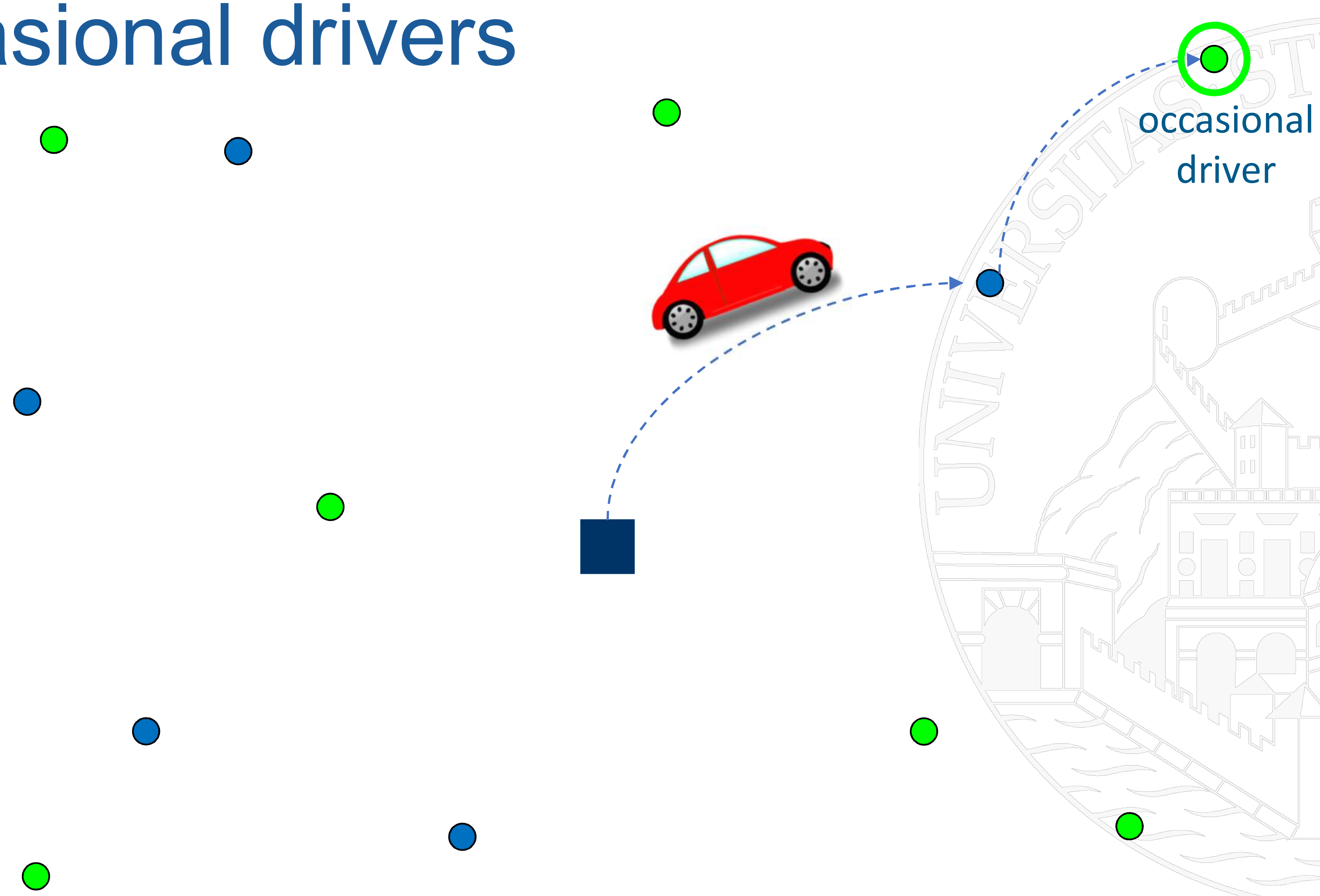
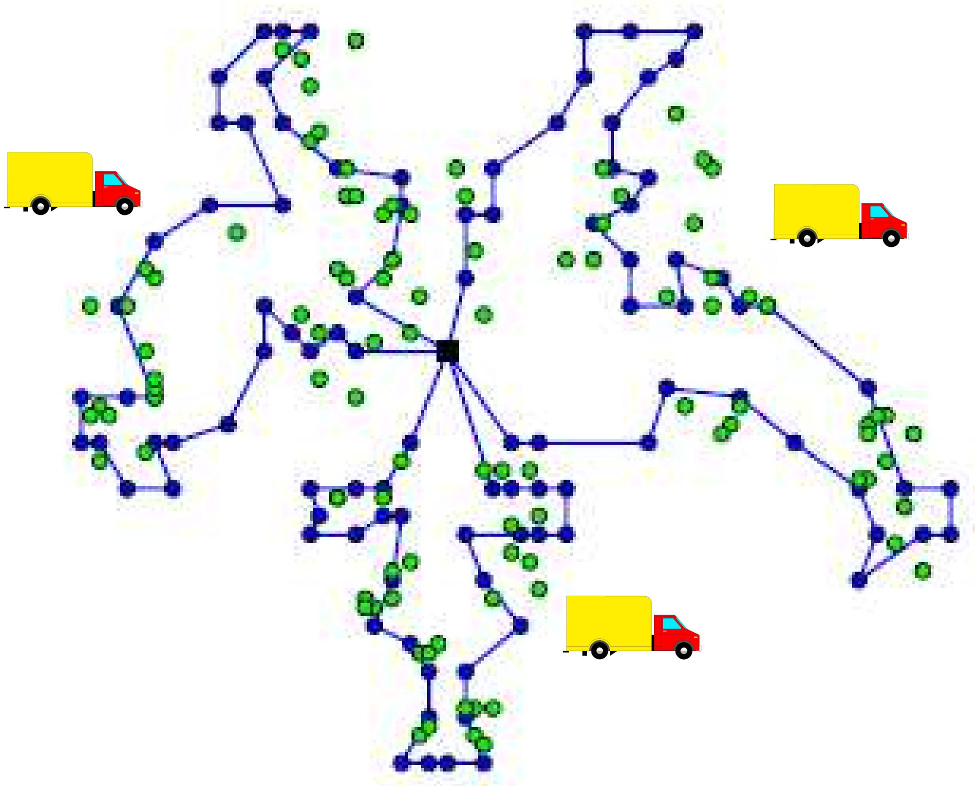
Occasional drivers



Occasional drivers

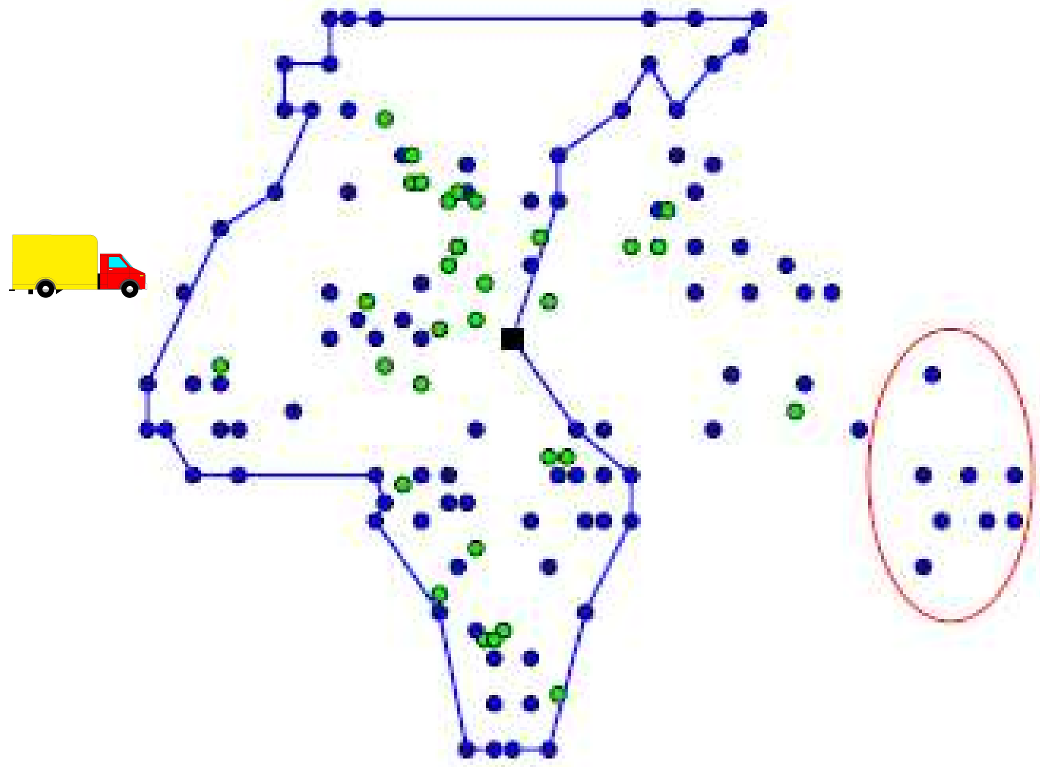
Without occasional drivers



Occasional drivers

With occasional drivers:

Compensation scheme proportional to detour



Savings

Compensation scheme proportional to detour

	% cost reduction w.r.t. VRP	% routes reduction w.r.t. VRP	%OD used	% OD cost w.r.t. total cost
C101	43.85	71.67	85.50	35.36
C201	20.49	50.00	66.42	17.04
R101	40.79	64.17	74.92	21.27
R201	33.70	50.00	71.32	20.58
RC101	33.47	52.96	64.20	14.70
RC201	30.05	50.00	61.56	14.28
$\vert K\vert=50$	26.85	48.07	80.73	12.31
$\vert K\vert=100$	40.60	64.86	63.77	28.77
$\varsigma=1.1$	31.66	54.58	67.58	14.51
$\varsigma=1.2$	33.16	56.10	69.40	17.11
$\varsigma=1.3$	34.27	56.87	71.60	23.09
$\varsigma=1.4$	34.74	57.56	73.10	23.82
$\varsigma=1.5$	34.80	57.21	72.40	24.16
$\rho=1.2$	34.86	56.70	72.67	20.48
$\rho=1.4$	33.69	57.12	72.72	20.53
$\rho=1.6$	32.63	55.58	63.70	20.60
Average	**33.72**	**56.47**	**70.75**	**20.54**

Reduction of emissions: 33.72%

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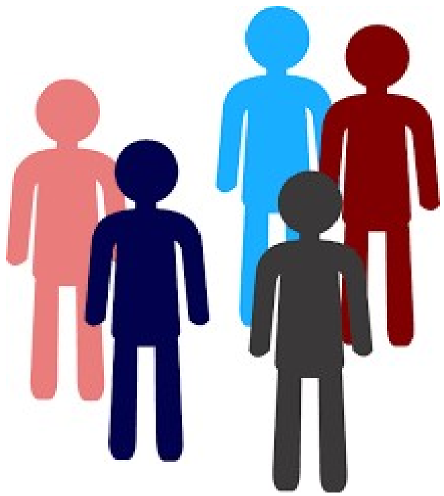
People



Shared taxi

Emissions

and occupation of space

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Shared taxi

- Public transportation

- Personal car

- Shared taxi

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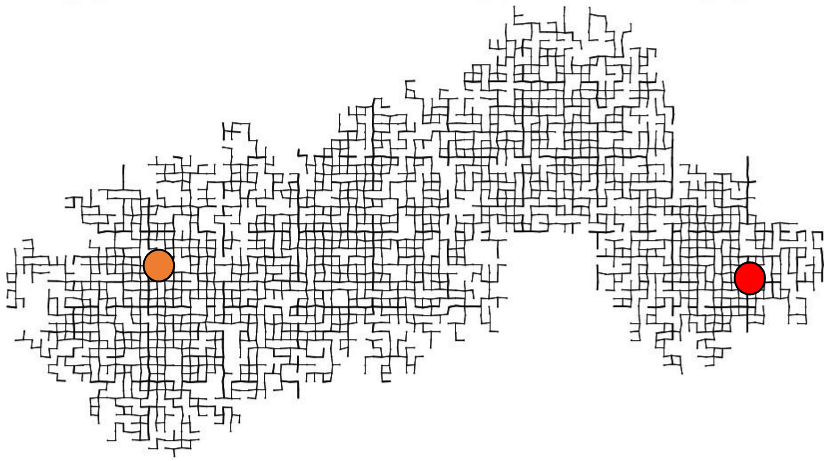
Shared taxi

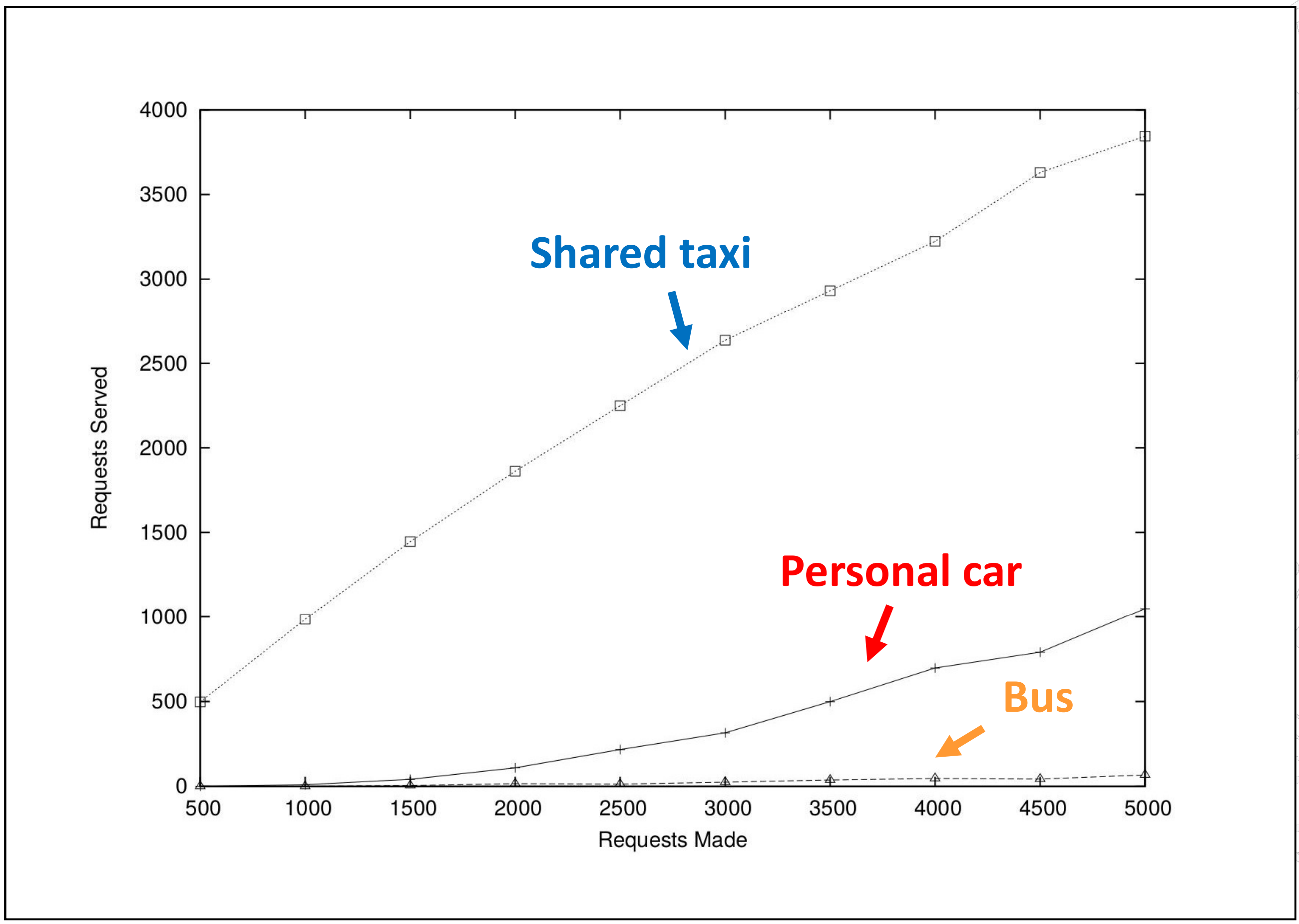
A simulation model

Input:

- Origins
- Destinations
- Request time
- Desired departure time
- Flexibility factor



Shared taxi



Reduction of emissions: more than 50%

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Final question

Old or new problems with autonomous vehicles?

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