Fundamentals of optimization
Part II

M. Grazia Speranza
University of Brescia
Heuristics

Needed where exact methods inadequate

Designed for a specific problem

Require implementation effort
The greedy

At each step the most attractive decision
No looking back
Myopic

Being greedy is never good
The concept of neighborhood
The concept of neighborhood

A 2-opt move
The concept of neighborhood

The 7 3-opt moves
The local search (LS)

Avoids loops

LS repeats replacing $\sigma$ with a better solution
In its neighborhood $N(\sigma)$

$\sigma_0$ $\sigma_1$ $\sigma_2$ $\sigma_3$ $\sigma_4$

$N(\sigma_0)$ $N(\sigma_1)$ $N(\sigma_2)$ $N(\sigma_3)$ $N(\sigma_4)$

an initial solution

a locally optimal
Global, not local!
Heuristics

Metaheuristics

Iterated local search
Tabu search
Simulated annealing
Variable neighborhood search
(Adaptive) large neighborhood search
Ant colony optimization
Genetic algorithms
...

Matheuristics

Not only for MILP
Main challenge
Iterated local search
Tabu search

previous steps will be tabu so we are not able visit them again
Simulated annealing

Worse solution accepted with probability $e^{-\frac{\Delta}{T}}$ as $T$ decreases.
Large Neighborhood Search

\[ x \leftarrow x_0 \]

while stop criteria not met do
   1. Find neighborhood \( \mathcal{N}_x \) by *destroying* and *repairing* \( x \)
   2. Find ”best” solution in \( \mathcal{N}_x \): \( x_{\text{best}} \)
   3. \( x \leftarrow x_{\text{best}} \)
end while
Adaptive LN Search
A matheuristic: Kernel search

\[ \begin{align*}
\min & \ c'x \\
Ax &= b \\
x &\geq 0 \quad \text{Integer}
\end{align*} \]

- Build initial kernel
- Solve MILP restricted to kernel and additional variables
- Update kernel

Not only for MILP
Kernel search

Initial kernel

$B_1$

$B_2$

$B_3$

Buckets of variables
Kernel search

\[ B_1 \]

Restricted problem
Kernel search

\[ B_1 \]

Updated kernel
Kernel search

Restricted problem

$B_2$
Facility location
Facility location
Facility location

\[ \min z = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j y_j \]

s.t. \[ \sum_{i \in I} x_{ij} \leq s_j y_j \quad j \in J \]

\[ \sum_{j \in J} x_{ij} = d_i \quad i \in I \]

\[ x_{ij} \leq d_i \quad i \in I, j \in J \]

\[ x_{ij} \geq 0 \quad i \in I, j \in J \]

\[ y_j \in \{0,1\} \quad j \in J \]

Kernel search solves instances with 2000 customers and 4000 facilities
(max errors below 1%)
Graph

Nodes (vertices)

Edges/arcs

Properties

Directed/Undirected

Weighted

Colored

Data structure
The shortest path

Dijkstra’s algorithm
The shortest path

Easy:
Shortest path from one origin to all nodes
Shortest path from all origins
The shortest path

Efficiency:
Representation of road network
Efficiency of algorithm
The time-dependent shortest path

Transit time on an arc depends on the arrival time

We know a transit-time function $f(t)$ for each arc $(v, w)$ (where $t$ is the time to leave $v$).

What is the fastest route from $r$ to $s$?

A modified version of Dijkstra’s algorithm
The shortest path

The travel time on an arc depends on the traffic congestion and the weather. Transit function $f(t)$ often unknown a priori
Uncertainty

What and how can we forecast?

Probabilistic information about the future

Uncertainty (no distribution) over parameters

Stochastic

Robust
The stochastic shortest path problem

At each node, we must select a probability distribution over all possible successor nodes, out of a given set of probability distributions.

Standard results for the deterministic case extended (Markovian decision process)
The robust shortest path problem

In robust optimization one wants to hedge against (all) possible scenarios by considering, for example, the worst case for each solution.

The robust deviation shortest path problem:
To find among all paths from s to t the one that, over all scenarios, minimizes the maximum deviation of the path length from the optimal path length of the corresponding scenario.
Dynamic problems

The role of information (forecasting)

Information changes over time

More accurate if closer in time to decision
Dynamic problems

Info needed on transit times ahead
Dynamic problems

**Dynamic graph**: the cost (transit time) of a subset of arcs changes over time

Reoptimizing shortest paths on dynamic graphs consists in solving a sequence of shortest path problems, where each problem partially differs from the previous one.
Dynamic problems
Traveling salesman problem
Traveling salesman problem
Traveling salesman problem

\[
\begin{align*}
\min & \quad \sum_{i} \sum_{j} c_{ij} y_{ij} \\
\text{s.t} & \quad \sum_{j} y_{ij} = 1, \quad i = 0, 1, ..., n - 1 \\
& \quad \sum_{i} y_{ij} = 1, \quad j = 0, 1, ..., n - 1 \\
& \quad \sum_{i} \sum_{j} y_{ij} \leq |S| - 1 \quad S \subset V, 2 \leq |S| \leq n - 2 \\
& \quad y_{ij} \in \{0, 1\} \quad \forall i, j \in E
\end{align*}
\]
Traveling salesman problem
Traveling salesman problem
Traveling salesman problem
100000 points

Many good heuristics
Routing problems
Routing problems

Complete graph of shortest paths
($O(n^2)$ shortest paths)
Routing problems

A broad range of problems

Heuristics:

Tabu search
Large variable neighborhood search
Heuristic column generation
....
Routing problems
Routing problems

The complete graph of the shortest paths cannot be computed in advance

1. The complete graph of the shortest paths has to be re-computed (computational challenges)

2. Work on the graph of the map (computational challenges)
Towards sustainable transportation

- Better use of the road network
- Less congestion
- Better use of the capacity of vehicles
- Less kilometers traveled
- Less vehicles on the road network

Reduction of emissions

- Less space occupied by traveling and parked vehicles

M.Grazia Speranza
Freight
Booking of loading/unloading areas
Booking of loading/unloading areas
Booking of loading/unloading areas

Each vehicle makes a reservation of the L/U areas

Windows of availability for the following vehicles

Variant of TSP with multiple time windows

Mor, Speranza, Viegas, TR E, to appear
Booking of loading/unloading areas
Booking of loading/unloading areas

Better use of the road network

M.Grazia Speranza
Collaboration
Collaboration
### Individual solutions (without collaboration) always optimal

Reduction of emissions: 21%

**Fernandes, Roca-Riu, Speranza, EJOR, 2018**

<table>
<thead>
<tr>
<th>Instance</th>
<th>- %</th>
<th>-%A</th>
<th>-%B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.9</td>
<td>35.5</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>8.9</td>
<td>17.9</td>
<td>2.7</td>
</tr>
<tr>
<td>3</td>
<td>16.7</td>
<td>1.8</td>
<td>31.6</td>
</tr>
<tr>
<td>4</td>
<td>14.2</td>
<td>14.5</td>
<td>10.8</td>
</tr>
<tr>
<td>5</td>
<td>10.8</td>
<td>10.3</td>
<td>13.3</td>
</tr>
<tr>
<td>6</td>
<td>10.1</td>
<td>17.6</td>
<td>2.9</td>
</tr>
<tr>
<td>7</td>
<td>8.6</td>
<td>0</td>
<td>12.9</td>
</tr>
<tr>
<td>8</td>
<td>17.8</td>
<td>1.6</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>25.2</td>
<td>33.3</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>14.9</td>
<td>18.1</td>
<td>6.5</td>
</tr>
<tr>
<td>11</td>
<td>9.4</td>
<td>22.4</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>6.5</td>
<td>0</td>
<td>8.9</td>
</tr>
</tbody>
</table>
Freight and people
Occasional drivers
Occasional drivers
Occasional drivers

Without occasional drivers
Occasional drivers

With occasional drivers:
Compensation scheme proportional to detour
<table>
<thead>
<tr>
<th>Compensation scheme</th>
<th>% cost reduction w.r.t. VRP</th>
<th>% routes reduction w.r.t. VRP</th>
<th>%OD used w.r.t. total cost</th>
<th>% OD cost w.r.t. total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>C101</td>
<td>43.85</td>
<td>71.67</td>
<td>85.50</td>
<td>35.36</td>
</tr>
<tr>
<td>C201</td>
<td>20.49</td>
<td>50.00</td>
<td>66.42</td>
<td>17.04</td>
</tr>
<tr>
<td>R101</td>
<td>40.79</td>
<td>64.17</td>
<td>74.92</td>
<td>21.27</td>
</tr>
<tr>
<td>R201</td>
<td>33.70</td>
<td>50.00</td>
<td>71.32</td>
<td>20.58</td>
</tr>
<tr>
<td>RC101</td>
<td>33.47</td>
<td>52.96</td>
<td>64.20</td>
<td>14.70</td>
</tr>
<tr>
<td>RC201</td>
<td>30.05</td>
<td>50.00</td>
<td>61.56</td>
<td>14.28</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>= 50</td>
<td>26.85</td>
<td>48.07</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>= 100</td>
<td>40.60</td>
<td>64.86</td>
</tr>
<tr>
<td>ς=1.1</td>
<td>31.66</td>
<td>54.58</td>
<td>67.58</td>
<td>14.51</td>
</tr>
<tr>
<td>ς=1.2</td>
<td>33.16</td>
<td>56.10</td>
<td>69.40</td>
<td>17.11</td>
</tr>
<tr>
<td>ς=1.3</td>
<td>34.27</td>
<td>56.87</td>
<td>71.60</td>
<td>23.09</td>
</tr>
<tr>
<td>ς=1.4</td>
<td>34.74</td>
<td>57.56</td>
<td>73.10</td>
<td>23.82</td>
</tr>
<tr>
<td>ς=1.5</td>
<td>34.80</td>
<td>57.21</td>
<td>72.40</td>
<td>24.16</td>
</tr>
<tr>
<td>ρ=1.2</td>
<td>34.86</td>
<td>56.70</td>
<td>72.67</td>
<td>20.48</td>
</tr>
<tr>
<td>ρ=1.4</td>
<td>33.69</td>
<td>57.12</td>
<td>72.72</td>
<td>20.53</td>
</tr>
<tr>
<td>ρ=1.6</td>
<td>32.63</td>
<td>55.58</td>
<td>63.70</td>
<td>20.60</td>
</tr>
<tr>
<td>Average</td>
<td><strong>33.72</strong></td>
<td><strong>56.47</strong></td>
<td><strong>70.75</strong></td>
<td><strong>20.54</strong></td>
</tr>
</tbody>
</table>

Reduction of emissions: 33.72%
People
Shared taxi

Emissions and occupation of space

M.Grazia Speranza
Shared taxi

- Public transportation
- Personal car
- Shared taxi
Shared taxi

A simulation model

Input:
- Origins
- Destinations
- Request time
- Desired departure time
- Flexibility factor
Shared taxi

Reduction of emissions: more than 50%

M.Grazia Speranza
Final question

Old or new problems with autonomous vehicles?