

ACTIONS OF TENSOR CATEGORIES ON C^* -ALGEBRAS

IPAM

January 21-28, 2021

ACTIONS OF TENSOR CATEGORIES ON C^* -ALGEBRAS

UNITARY TENSOR CATEGORIES

- Group-like objects
- Provide framework for generalised symmetries

C^* -ALGEBRAS

- Operator algebras
- Non-commutative Hausdorff spaces

GENERALISE GROUP ACTIONS ON SPACES:

- Replace the space with an operator algebra
 - and the group by an action of a tensor category
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- Lots of activity relating to actions of unitary tensor categories on von Neumann algebras: today's language of subfactors mainly in terms of tensor categories.
 - Much less extensive in the C^* -setting.

GOALS

WORKSHOP GOALS

- Introduce key topics
- Connect researchers from both themes.
- Identify problems, and research directions.

SAMPLE PROBLEM

Which strongly amenable rigid C^ -tensor categories act on which classifiable C^* -algebras? When can such actions be reasonably classified?*

WHY NOW?

IN SHORT

C^* -algebras today are at the exciting cusp that von Neumann algebras were in the late '70s and early '80s

- Now is the time to be inspired by what came next.
- Importantly modern techniques in both the structure and classification of amenable C^* -algebras have strong parallels to von Neumann results (or use von Neumann results explicitly).

CLASSIFICATION OF AMENABLE ...

VON NEUMANN ALGEBRAS

Connes ('76), (last case Haagerup '87)

GROUP ACTIONS ON THE HYPERFINITE II_1 FACTOR R

- Finite cyclic groups [Connes]
- Finite groups [Jones]
- Amenable groups [Ocneanu]

CLASSIFIABLE SIMPLE NUCLEAR C^* -ALGEBRAS

- 1990-2015/21.

GROUP ACTIONS ON CLASSIFIABLE C^* -ALGEBRAS

- Currently seeing very rapid progress.
- Connes work paved the way for Jones' landmark results on subfactors in the '80s.

CLASSIFICATION OF SUBFACTORS

CLASSIFICATION OF SMALL INDEX SUBFACTORS

- by their **standard invariants** (Ocneanu, Popa '90s).

TODAY:

- view the standard invariant as a pair (\mathcal{C}, A) where \mathcal{C} is a **unitary tensor category** (UTC) and A is a certain **algebra object** in \mathcal{C} .
- UTCs act on an operator algebra N by tensor functors into some class of bimodules
$$F : \mathcal{C} \rightarrow \text{Bim}(N)$$
- Classification is possible when both \mathcal{C} and N (a von Neumann algebra) are amenable.
- This splits subfactor classification into two parts:
 - Functional Analytic: Classify actions of UTCs on factors
 - Categorical: Classify algebra objects in UTCs
- Very special case: actions of finite groups on N .

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WHAT HAPPENS WHEN A IS A CLASSIFIABLE C^* -ALGEBRA?

- When is there a fully faithful $F : \mathcal{C} \rightarrow \text{Bim}(A)$?
- What hypotheses make it suitably unique?

TODAY AND FRIDAY: SURVEY / EXPOSITORY LECTURES

- Courtney and Carrión: The abstract approach to classifying C^* -algebras
- Vaes: Outer actions of amenable groups on von Neumann algebras
- Szabó: Classification of group actions on C^* -algebras
- Yamashita: Unitary rigid tensor (and 2-)categories, and their actions on operator algebras

DISCUSSION SESSIONS: FRIDAY, MONDAY, TUESDAY

Collect topics in Friday's discussion session (and subsequent survey) for discussions on Monday, Tuesday.

- May commission mini expository talks for Monday, Tuesday in breakout rooms.

PROBLEM SESSION

Thursday: Collect a focused list of problems.