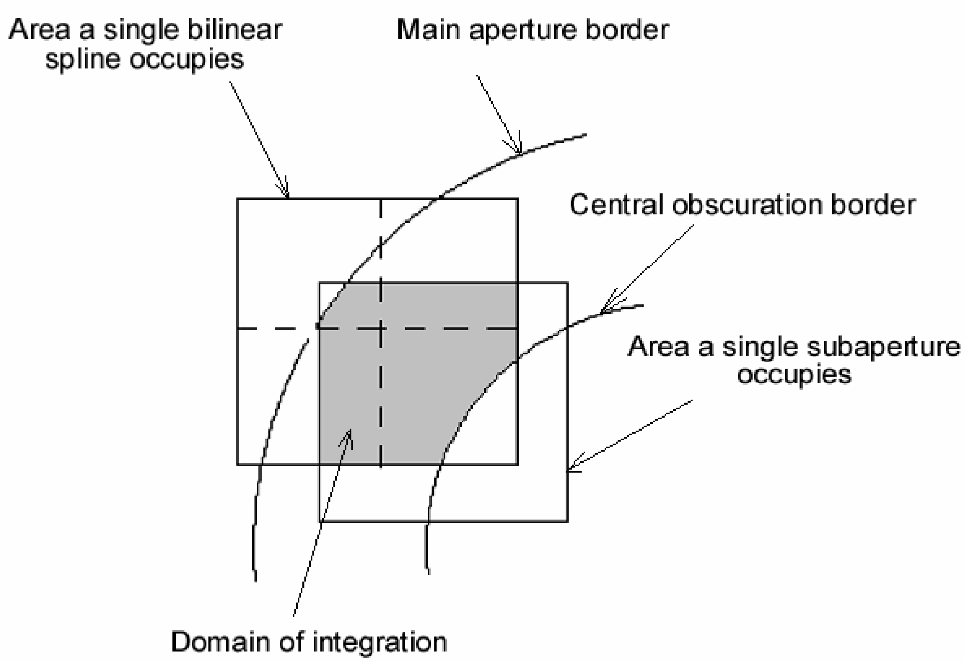
Phase-to-WFS influence matrix elements cross validation using alternative computational approaches

General geometrical setting for computation of a single influence matrix element



Mathematical expressions for phase-to-WFS matrix elements

$$(G)_{ij} = \frac{1}{s_{ij}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_a(x,y) W_o(x,y) W_{sa}^i(x,y) \nabla h(\frac{x-x_j}{\delta}, \frac{y-y_j}{\delta}) ds,$$

$$W_a(x,y) = \begin{cases} 1,\_(x,y)\_\in\_main\_aperture \\ 0,\_otherwise \end{cases}$$

$$W_o(x,y) = \begin{cases} 0,\_(x,y)\_\in\_obscuration\_circle \\ 1,\_otherwise \end{cases}$$

$$W_a(x,y) = \begin{cases} 1,\_(x,y)\_\in\_subaperture\_number\_i \\ 0,\_otherwise \end{cases}$$

Bilinear spline function

$$h(x) = \begin{cases} (1-\operatorname{sgn}(x)x)(1-\operatorname{sgn}(y)y),\_-1 \le x,y \le 1 \\ 0,\_otherwise \end{cases}$$

s_{ij} is the area of unobscured part of subaperture *i* that falls in the localization area of spline *j*.

More convenient form of G-matrix

$$(G)_{ij} = \frac{1}{s_{ij}} \int \int_{W_a \cap W_o \cap W_{sa}^i} \nabla h(\frac{x-x_j}{\delta}, \frac{y-y_j}{\delta}) ds$$

WAVEFRONT SENSING MODEL

Output from sub-aperture i

$$s_i = \frac{1}{s_{sa}} \iint_{W_{sa}^i} \nabla \varphi(x,y) ds + n,$$

s_{sa} is a sub-aperture area, W_{sa}^i is a region occupied by sub-aperture i.

Phase distribution approximation

$$\varphi(x,y) = \sum_{j=1}^{N} \varphi_j h_j(x,y),$$

$h_j(x,y)$ are basis functions.

$$s_i = \sum_{j=1}^{N} G_{ij} \varphi_j,$$

$$G_{ij} = \frac{1}{s_{sa}^i} \iint_{W_{sa}^i} \nabla h_j(x,y) ds.$$

POSSIBLE WAYS TO CHECK ELEMENTS OF G-MATRIX

1. Piston is in null space of G, i.e.

$$\sum_j G_{ij} = 0, \_\_ \forall i.$$

2. Pattern of zero/nonzero elements in G. Can be found analytically.
3. Sign pattern of nonzero elements. In most cases can be found analytically.
4. **Comparison of the results obtained with the aid of two different methods.**

TWO COMPUTATIONAL APPROACHES

1. **Use the Green's Theorem to transform the double integral into a linear one**

$$(G)_{ij} = \frac{1}{s_{ij}} \oint_{\partial(W_a \cap W_o \cap W_{sa}^i)} h\left(\frac{x(t)-x_j}{\delta}, \frac{y(t)-y_j}{\delta}\right) \begin{bmatrix} dy/dt \\ -dx/dt \end{bmatrix} dt \,,$$

This is useful for reference calculations by hand.

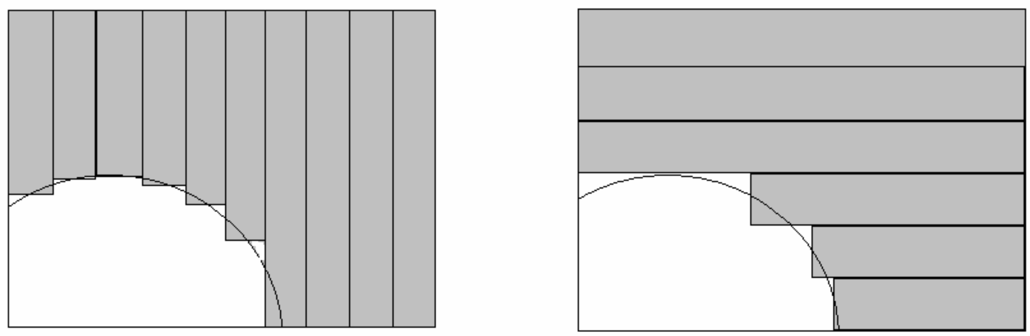
2. **Direct numerical evaluation of the double integral taking advantage of the simple form of the integrand.**

In fact,

$$\nabla h(x,y) = \begin{cases} -\operatorname{sgn}(x)(1-\operatorname{sgn}(y)y)\hat{x} - \operatorname{sgn}(y)(1-\operatorname{sgn}(x)x)\hat{y}, \_ -1 \le x, y \le 1 \\ 0, \_ otherwise \end{cases}$$

is a piecewise linear function and its integral over rectangle can be easily found analytically.

NUMERICAL INTEGRATION PROCESS



X-component of gradient. Integrand changes along y-direction.

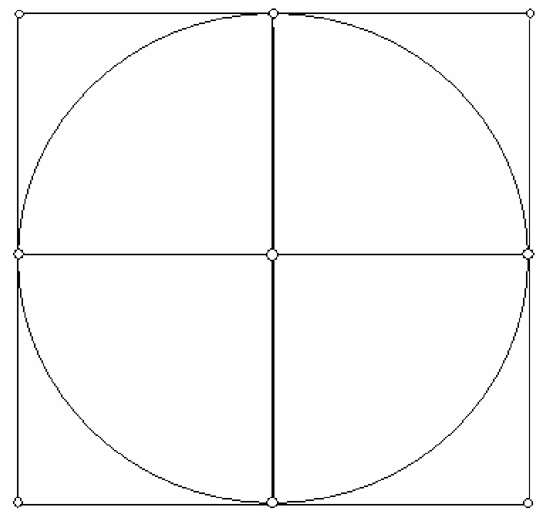
Y-component of gradient. Integrand changes along x-direction.

The value of double integral over each sub-rectangle is

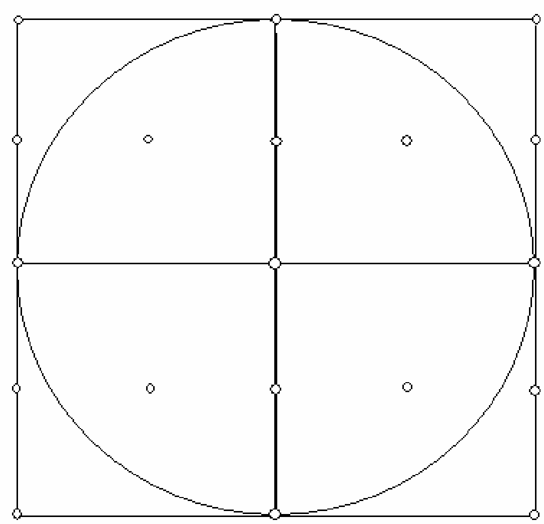
$$\nabla h(x_{cg} - x_j, y_{cg} - y_j) * area\_of\_a\_sub-rectangle,$$

where ($x_{cg}, \_ y_{cg}$) are the coordinates of the center of gravity of a sub-rectangle.

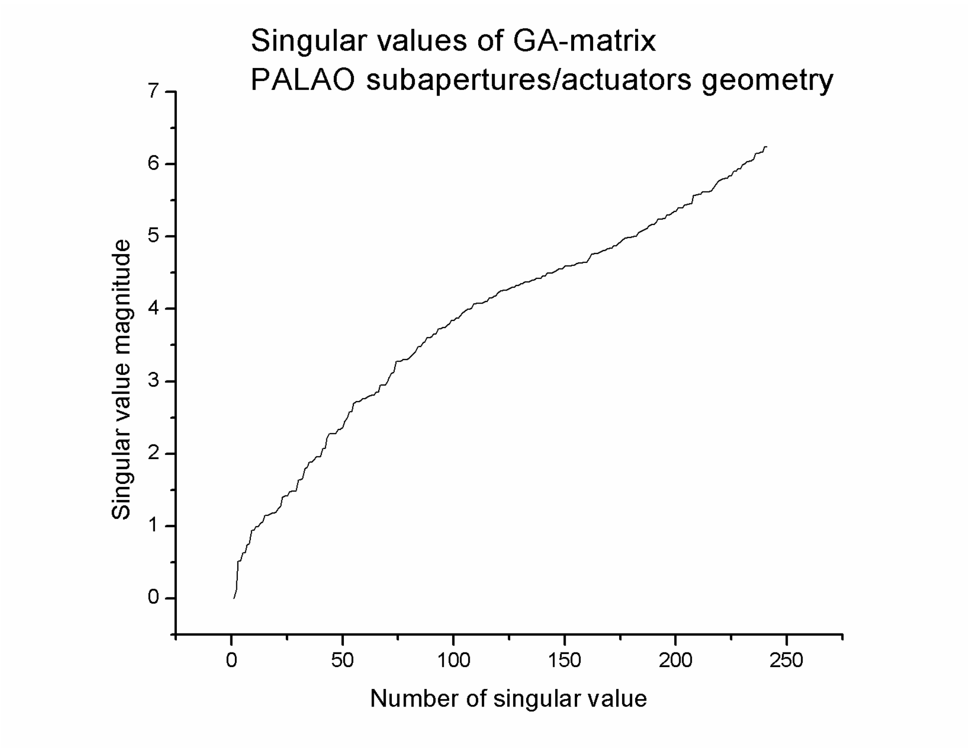
GEOMETRY FOR DM-to-WFS 8x9 MATRIX REFERENCE CALCULATION

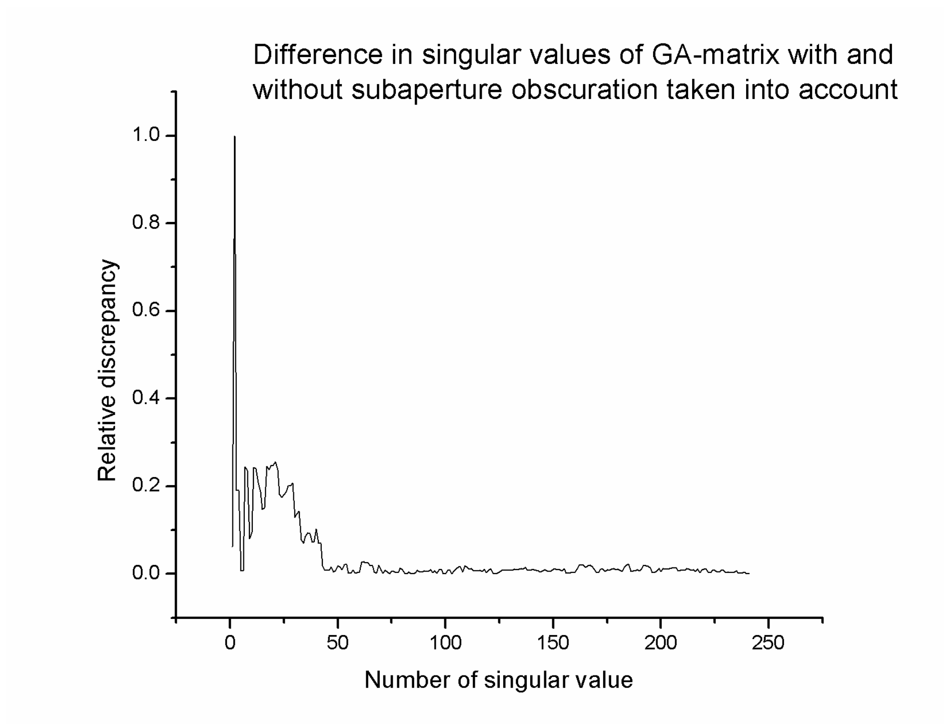


GEOMETRY FOR PHASE SCREEN-to-WFS 8x25 MATRIX REFERENCE CALCULATION

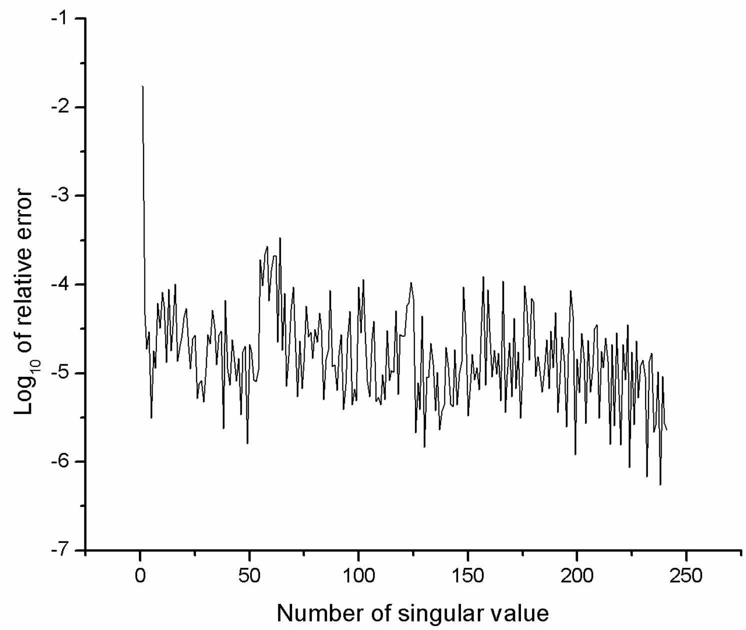


SINGULAR VALUES OF DM-to-WFS 408x241 INFLUENCE MATRIX FOR PALAO GEOMETRY





DIFFERENCE IN SINGULAR VALUES OF PALAO DM-to-WFS MATRIX COMPUTED WITH 100 AND 10000 SUBDIVISIONS



DIFFERENCE IN SINGULAR VECTOR NORMS OF PALAO DM-to-WFS MATRIX COMPUTED WITH 100 AND 10000 SUBDIVISIONS