The Formerly Best Undersampled Telescope in the Universe

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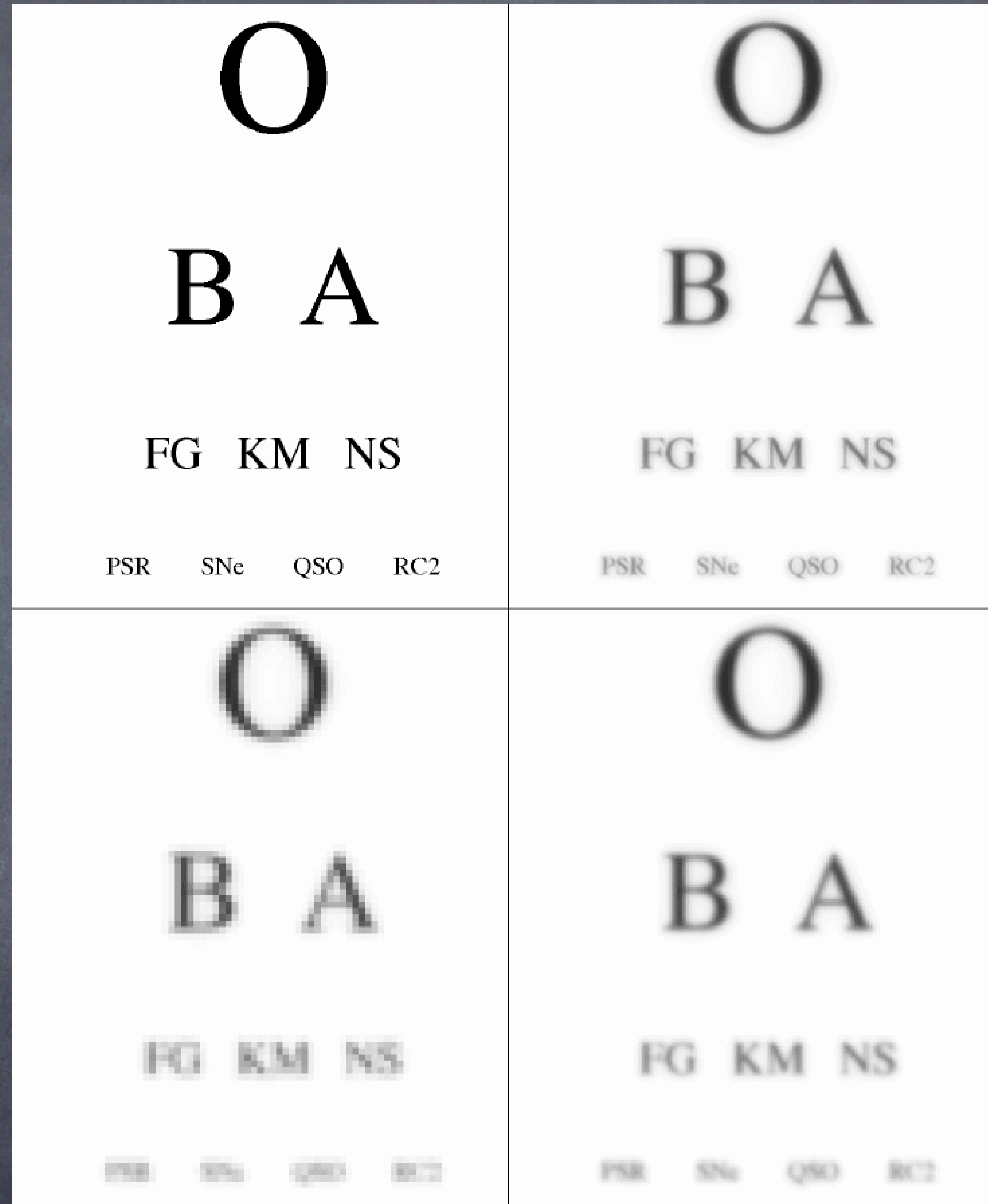
The Great Eye Chart in the Sky



What is an Image?

$$I = T \otimes O \otimes E$$

- I is final output image
- T is the true initial image
- O is the convolution with the optics
- E is electronic pixel
- THEN SAMPLED AT THE CENTERS OF THE PIXELS!!

Why are Telescopes Undersampled?

The speed of an astronomical survey scales as:

$$Speed = Area * Sensitivity^2$$

A larger pixel gives a larger area:

$$Area = N * P^2$$

Speed vs. Area

The area of a detector scales as P^2 but the instrumental PSF depends on the pixel scale as well:

$$I^2_{PSF} = O^2_{PSF} + \alpha P^2 + (\epsilon P)^2$$

$$I^2_{PSF} \approx O^2_{PSF} + (1 + \epsilon^2) P^2$$

The sky, and thus the noise, under the PSF will increase with a larger pixel.

Speed vs. Area (2)

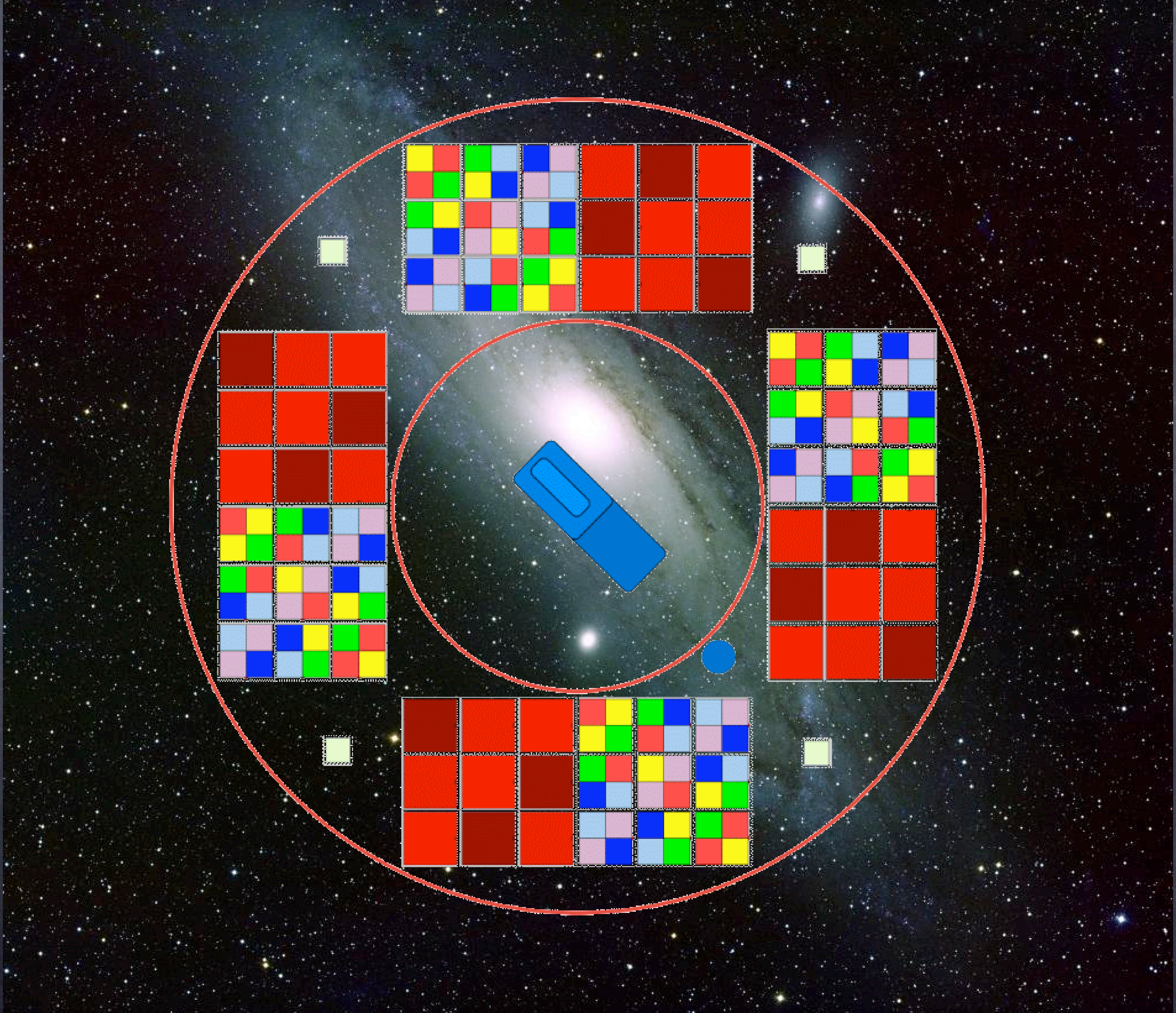
The amount of sky under a point source goes as I^2, so the signal-to-noise of a point source goes as I^{-1}. The total speed of the survey is then

$$S = \frac{P^2}{I^2_{PSF}}$$

or

$$S \approx \frac{P^2}{O^2_{PSF} + (1+\epsilon^2)P^2}$$

A SNAPshot of M31



Shift-and-Add: The Astronomical Default

- Input images are shifted over an output grid according to their dither displacement
- The ouput pixel scale is usually either equal to or one-half the input scale
- Area interpolation used

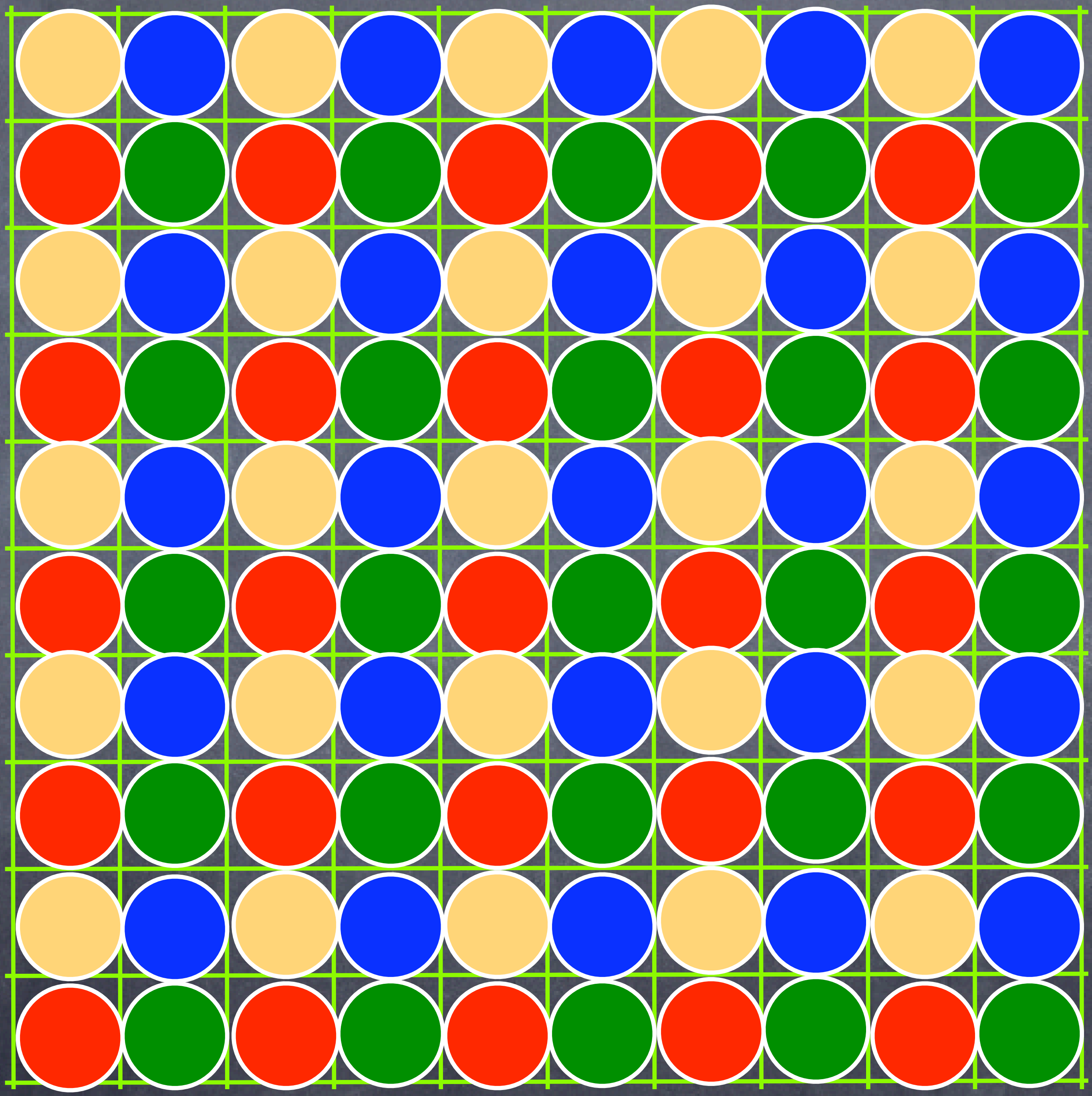
What is an Image after Shift-and-Add?

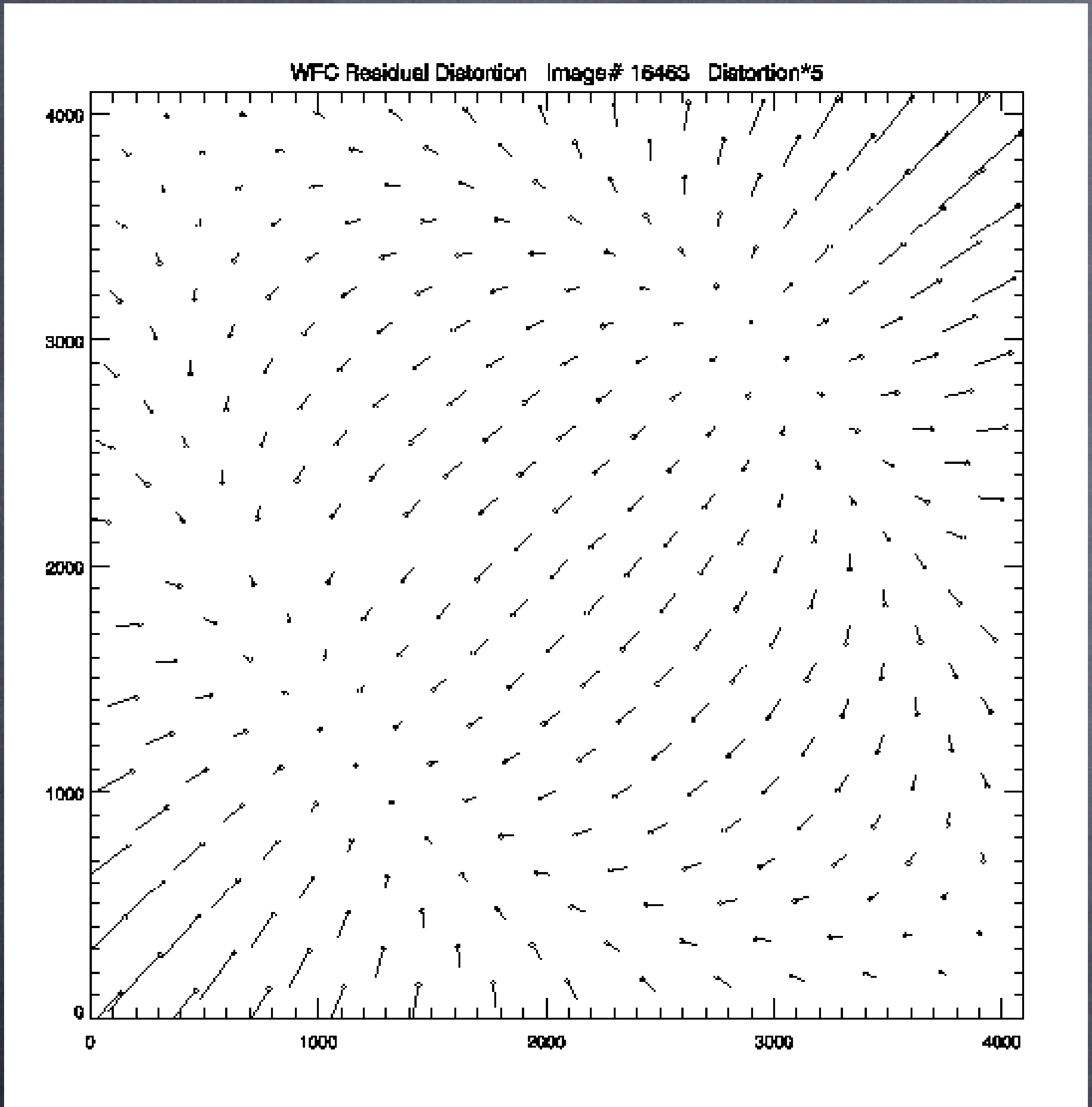
$$I = T \otimes O \otimes E \otimes \sout{P \otimes G}$$

- I, T, O, E as before
- P is the square, mathematical, input pixel
- G is the output grid pixel size
- Particularly bad if, as is frequently the case,

$$P \approx G \approx T \otimes O \otimes E$$

Interlacing in Action

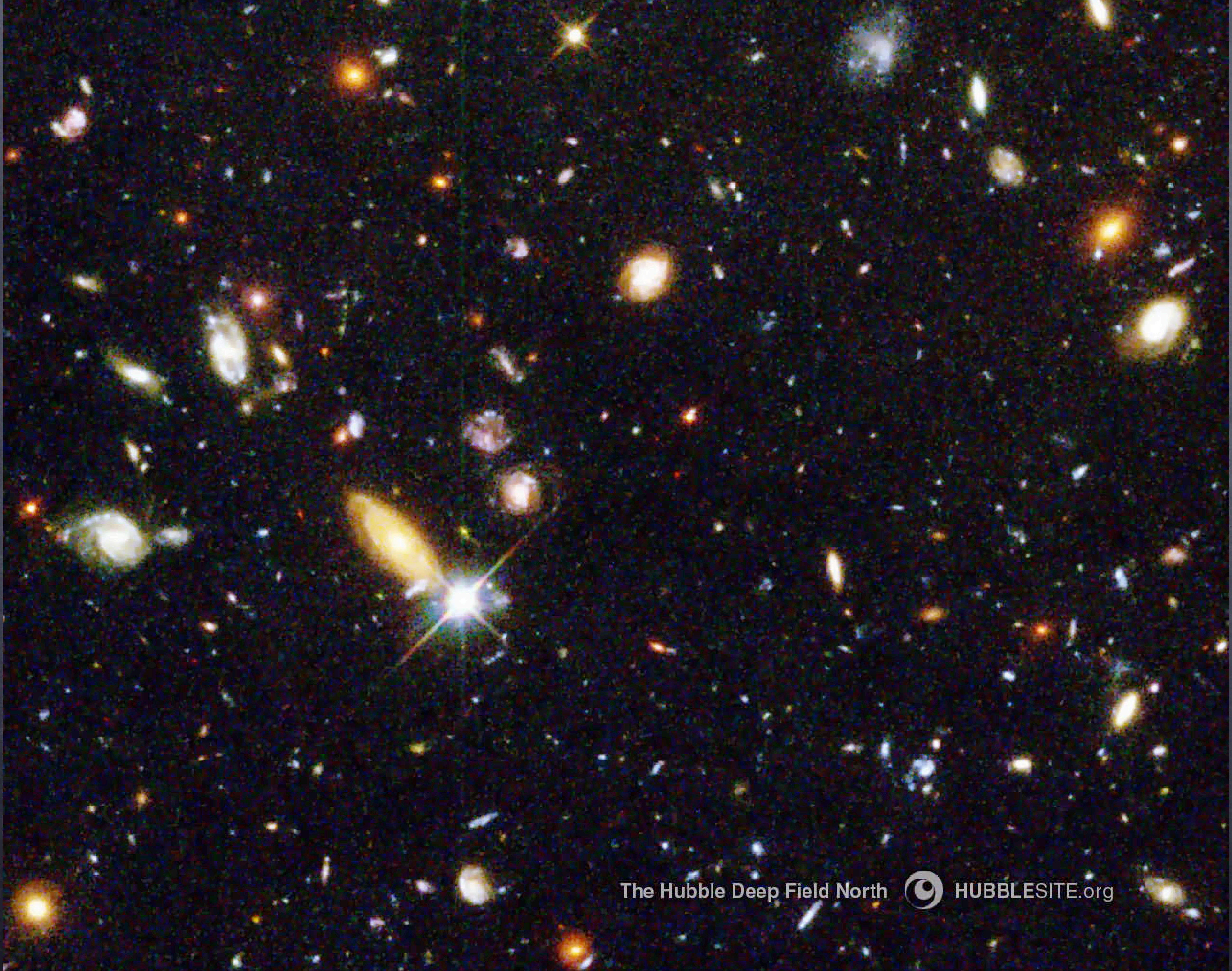




Interlacing requires accurate dithers

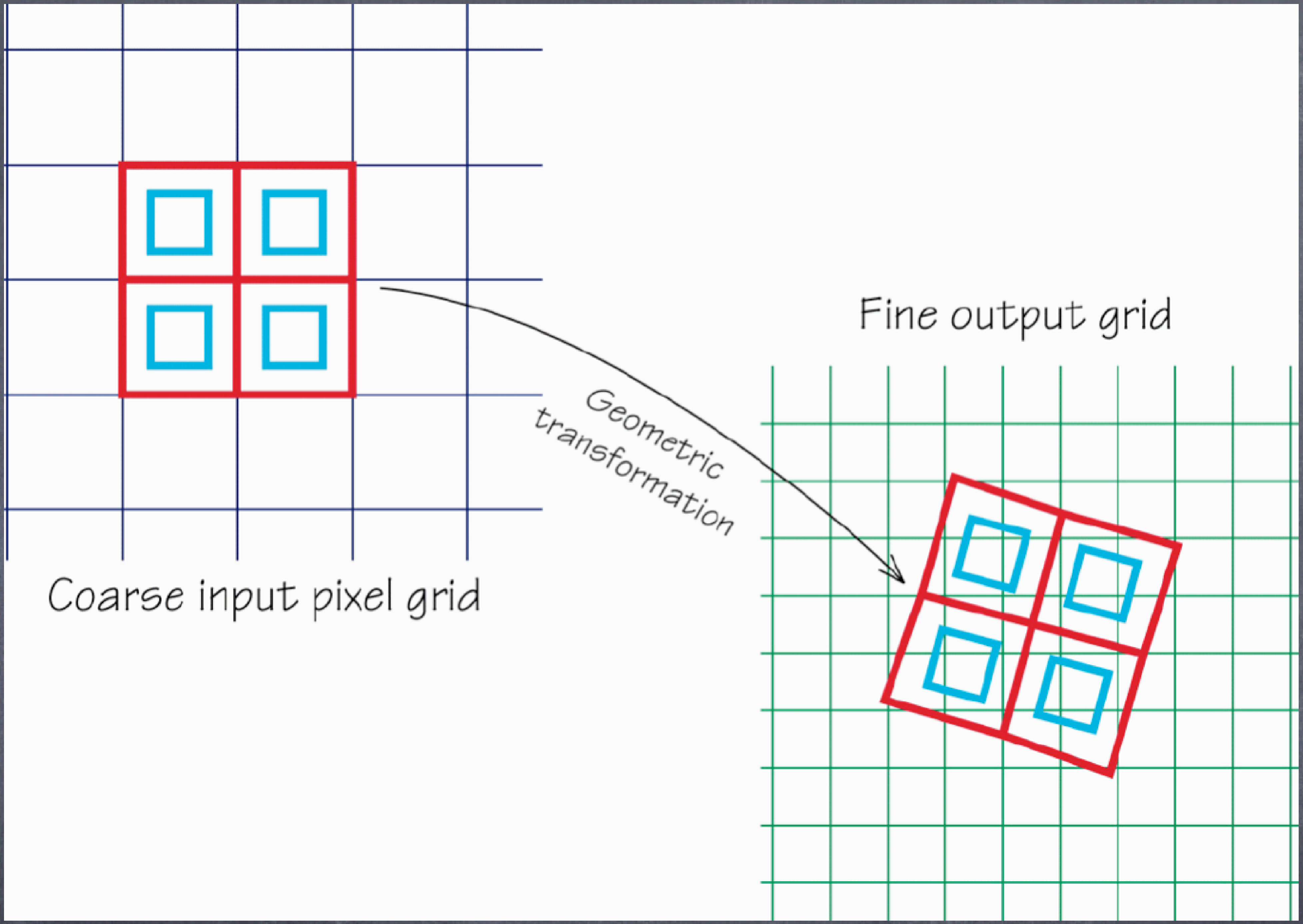
The ACS on HST has distortions of 10% and a FOV of 4000 pixels

Wanted by Hubble Deep Field:
a fast, robust method for combining shifted, rotated, distorted undersampled images. For use on 150 orbits of HST data on a single field. Should be well-suited to study of faint, marginally resolved objects. Method and all associated software must be ready in six months.


The Hubble Deep Field North
HUBBLESITE.org

Drizzle:

- Preserves photometry and resolution
- Produces optimally weighted output
- Can handle arbitrary shifts, rotations and geometric distortion
- Removes the effects of geometric distortion on photometry and astrometry
- Naturally handles missing data
- Provides a continuous set of functions between shift-and-add and interlacing

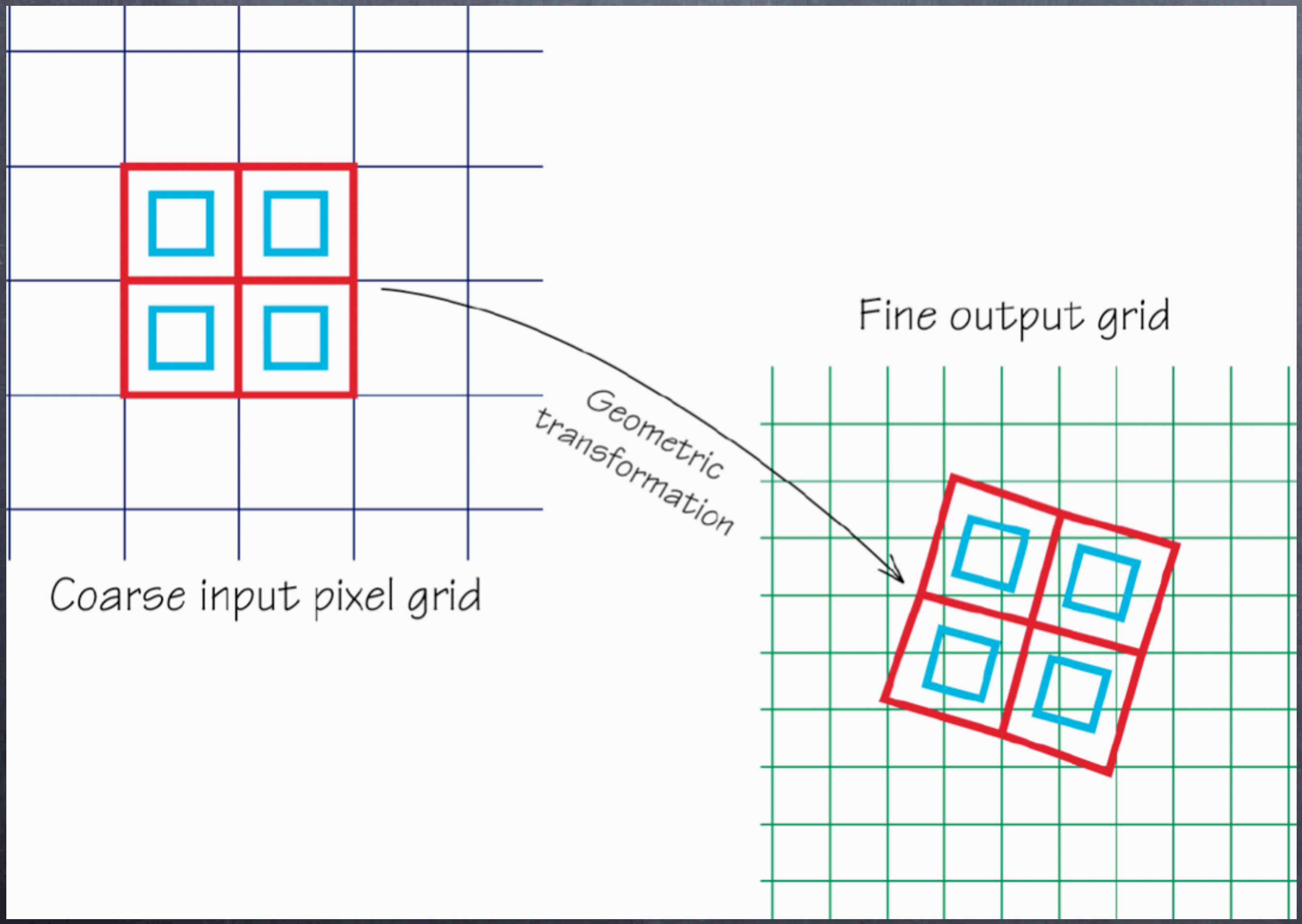


Drizzle: The Pictorial

The Basic Drizzle Equations

$$I'_{x_o y_o} = \frac{d_{x_i y_i} a_{x_i y_i x_o y_o} w_{x_i y_i} s^2 + I_{x_o y_o} W_{x_o y_o}}{W'_{x_o y_o}}$$

$$W'_{x_o y_o} = a_{x_i y_i x_o y_o} w_{x_i y_i} + W_{x_o y_o}$$



$$0 < a_{x_i y_i x_o y_o} < 1$$

$$0 < s < 1$$

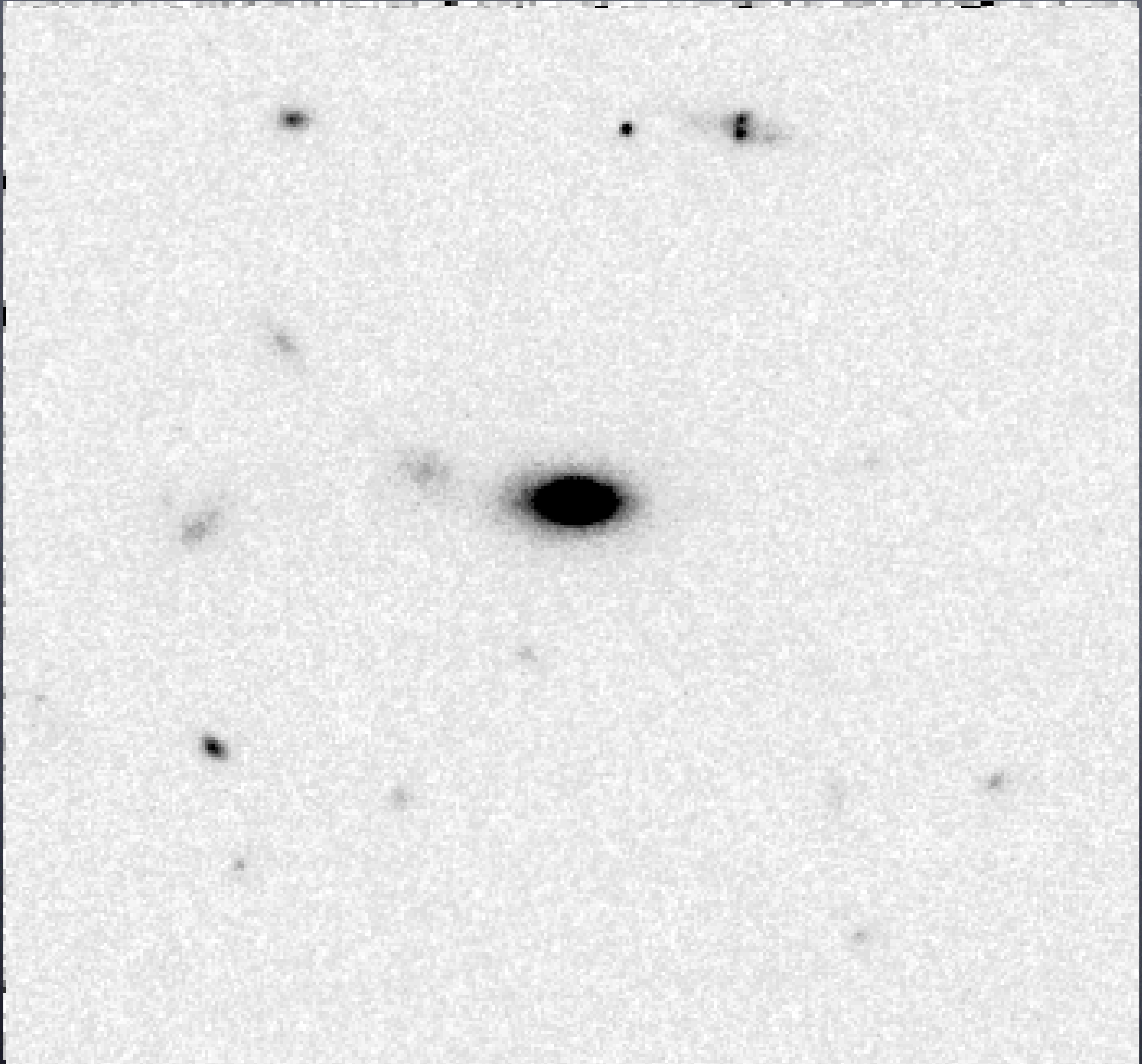
I_{xy}, W_{xy} are the iterative estimates of the image and weight

For the Non-Iteratively Inclined

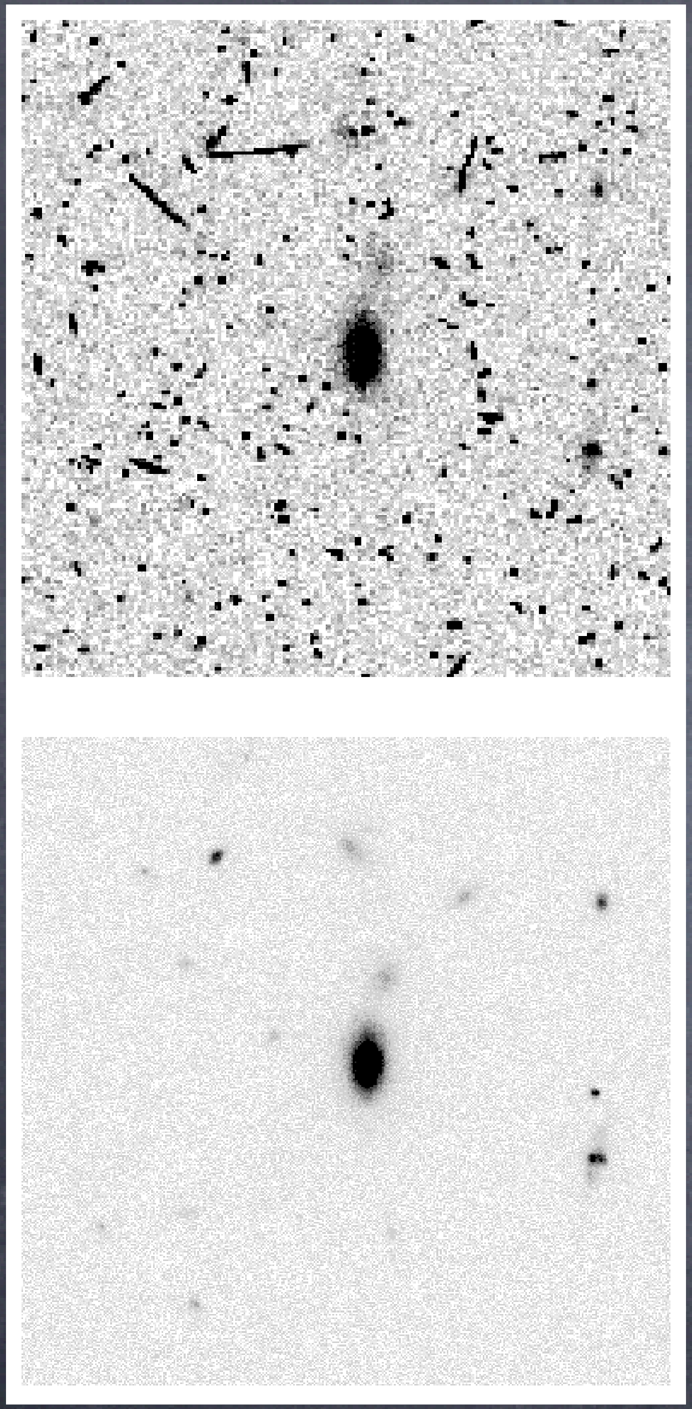
$$I_{x_o y_o} = \frac{d_{x_i y_i} a_{x_i y_i x_o y_o} w_{x_i y_i} s^2}{W_{x_o y_o}}$$

$$W_{x_o y_o} = a_{x_i y_i x_o y_o} w_{x_i y_i}$$

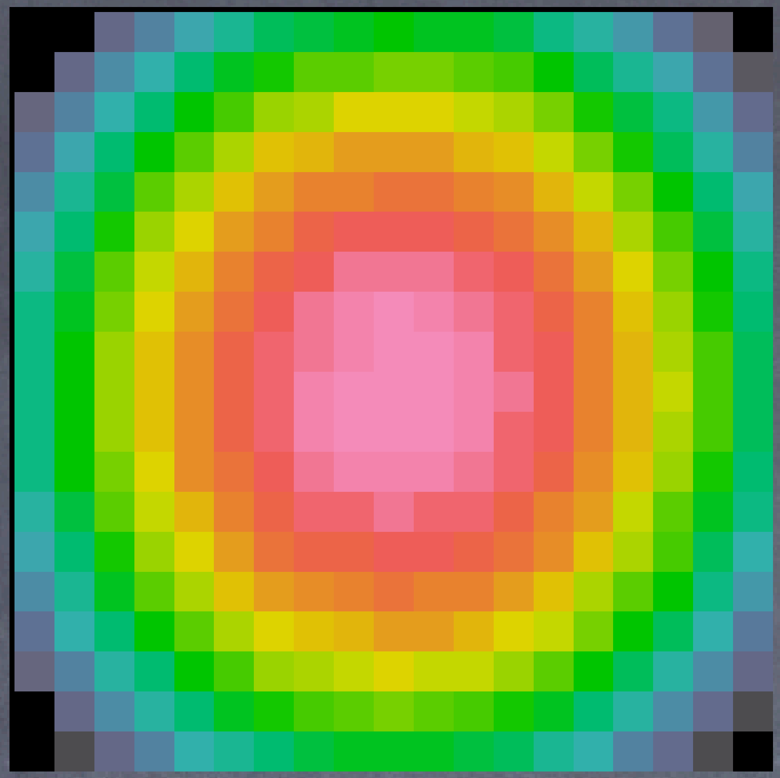
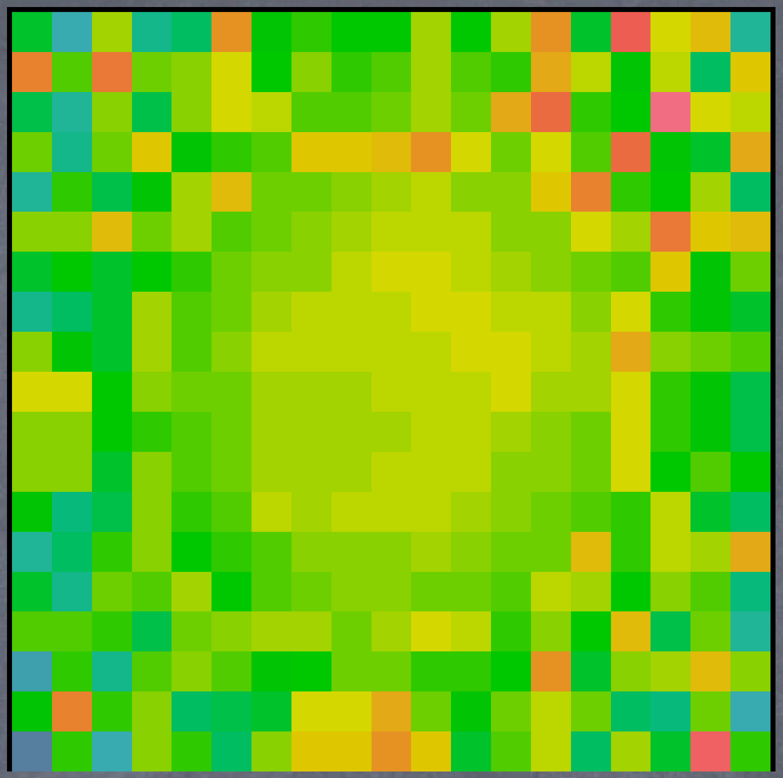
where the Einstein summation convention is used, summing x_i, y_i over all images



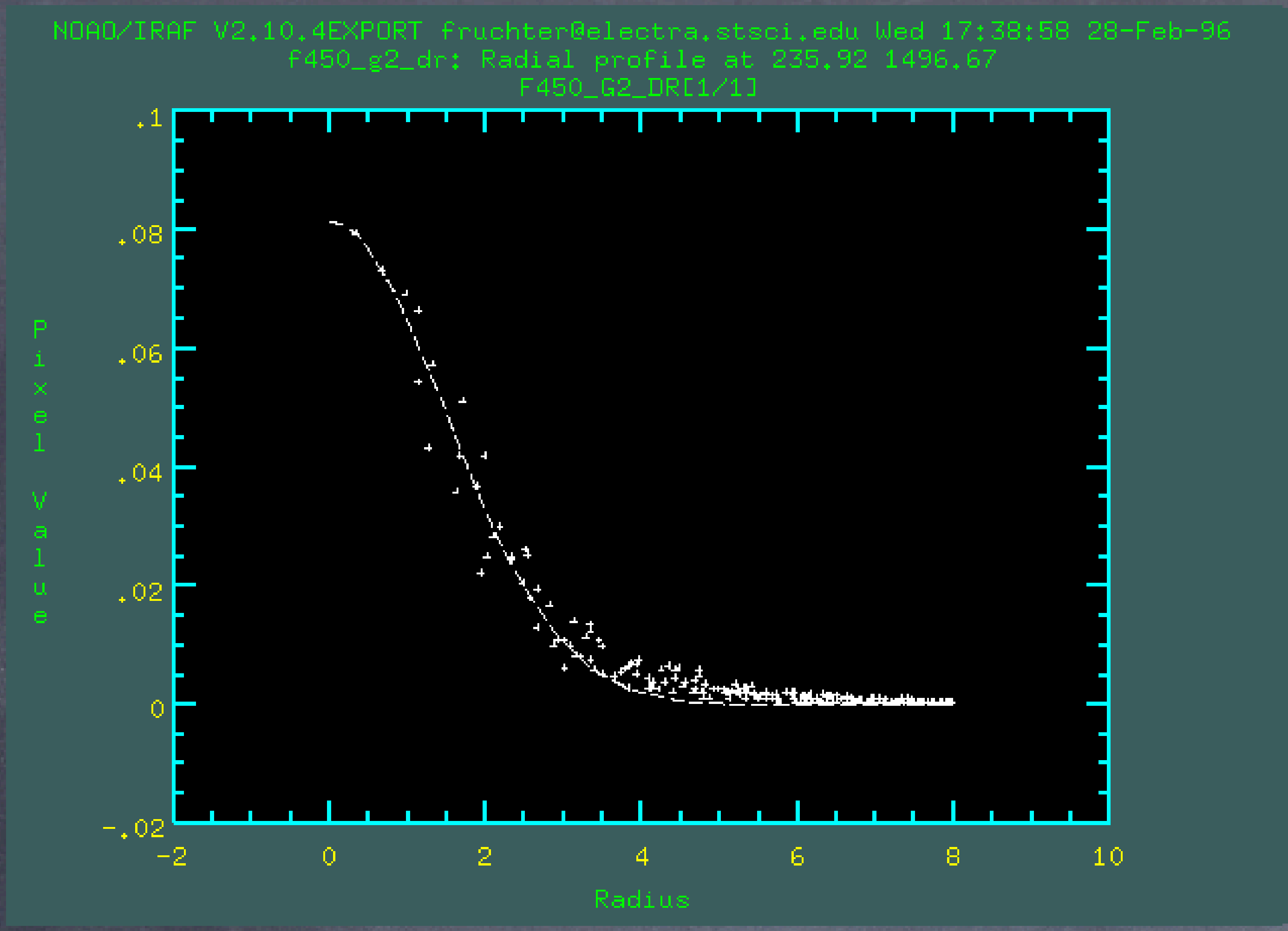
Cosmic Ray Removal Recipe

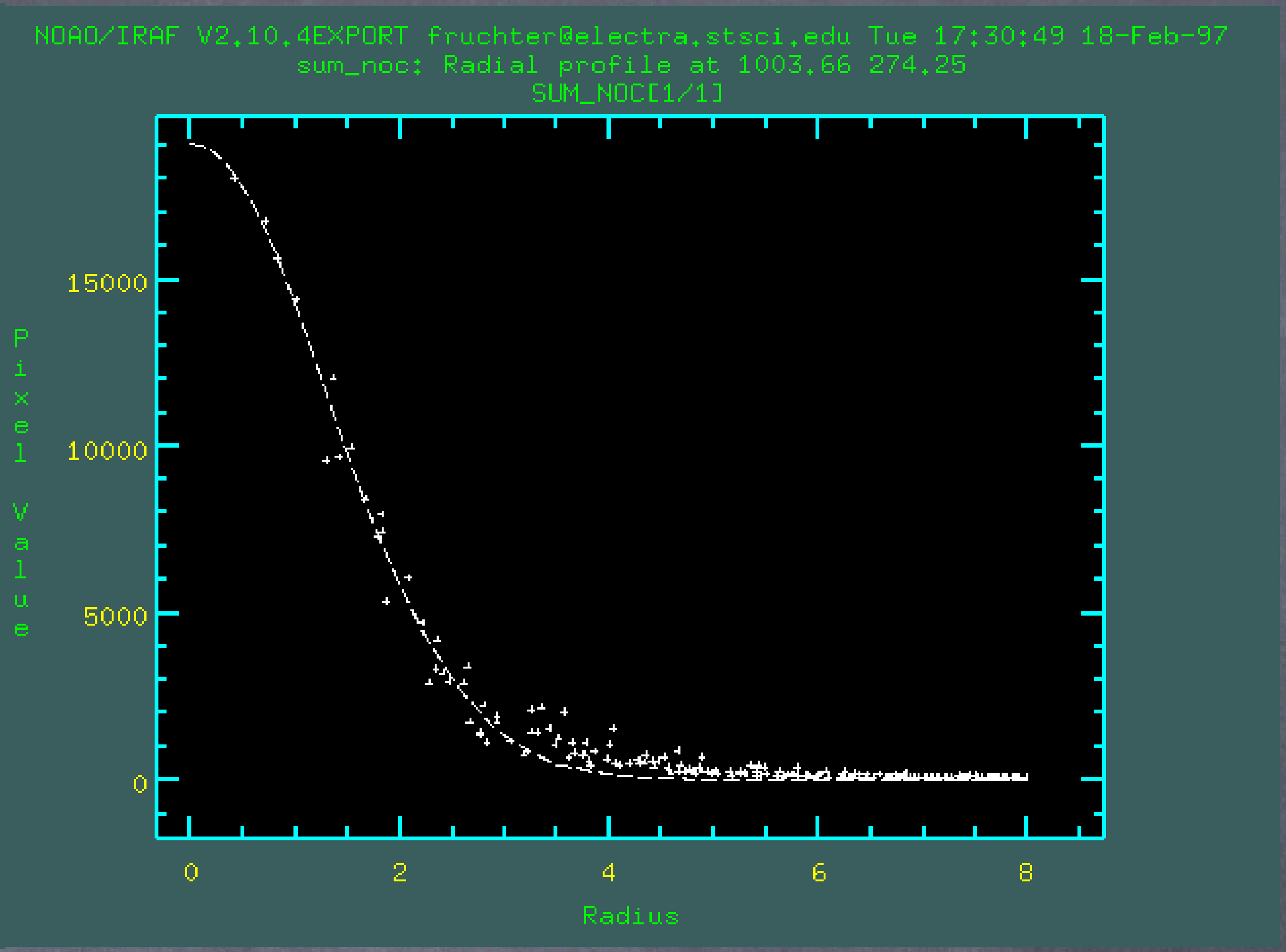


- Drizzle each image onto a separate sub-sampled output image using pixfrac=1.0
- Take the median of the resulting aligned drizzled images.
- Map (blot) the median image back to the input plane of each of the individual images
- Take the spatial derivative of each of the blotted output images.
- Compare each original image with the corresponding blotted image; mask pixels showing excessive differences
- Repeat on adjacent pixels with stricter criteria
- Drizzle all images onto a single output using cosmic-ray masks

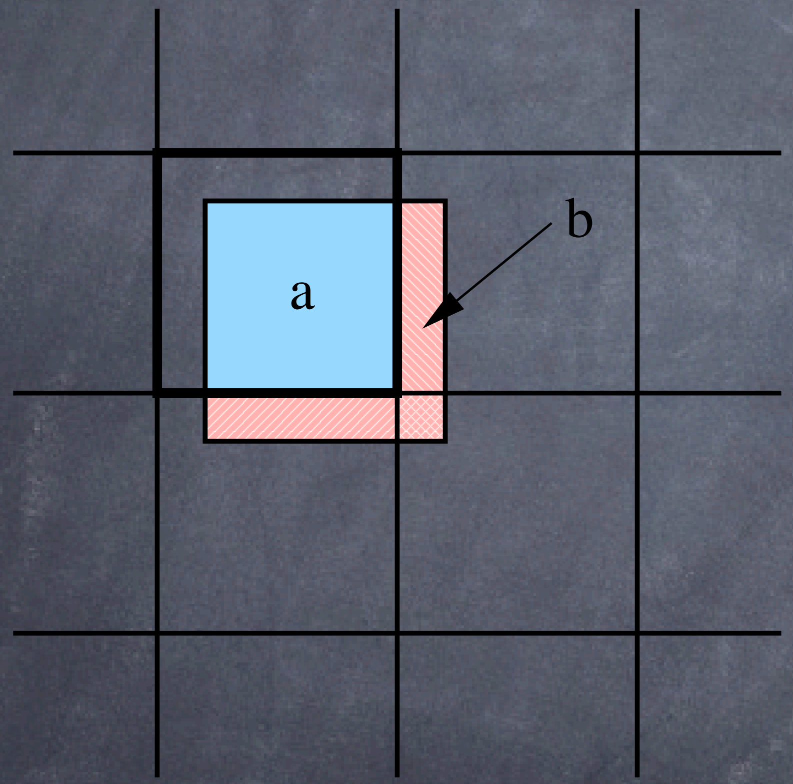
- With a distorted image one can flatten sky or stars, but not both
- With WFPC2 a flat sky produced stellar photometric changes of up to 4%
- Drizzle while removing distortion flattens both the stars and the sky





- Some real PSFs showed larger noise about expected shape than in simulations
- This problem is worse in the HDF-S than the HDF-N
- Primary cause appears to be charge transfer defects

Correlated Noise



Consider an input pixel drizzled onto a primary output pixel a and secondary pixels b_1, b_2, b_3.

Noise ϵ from input pixel is divided between a and three other pixels b_1, b_2, b_3.

The total noise in these four pixels

$$(a^2 + b_1^2 + b_2^2 + b_3^3)\epsilon^2 < \epsilon^2.$$

Correlated Noise (2)

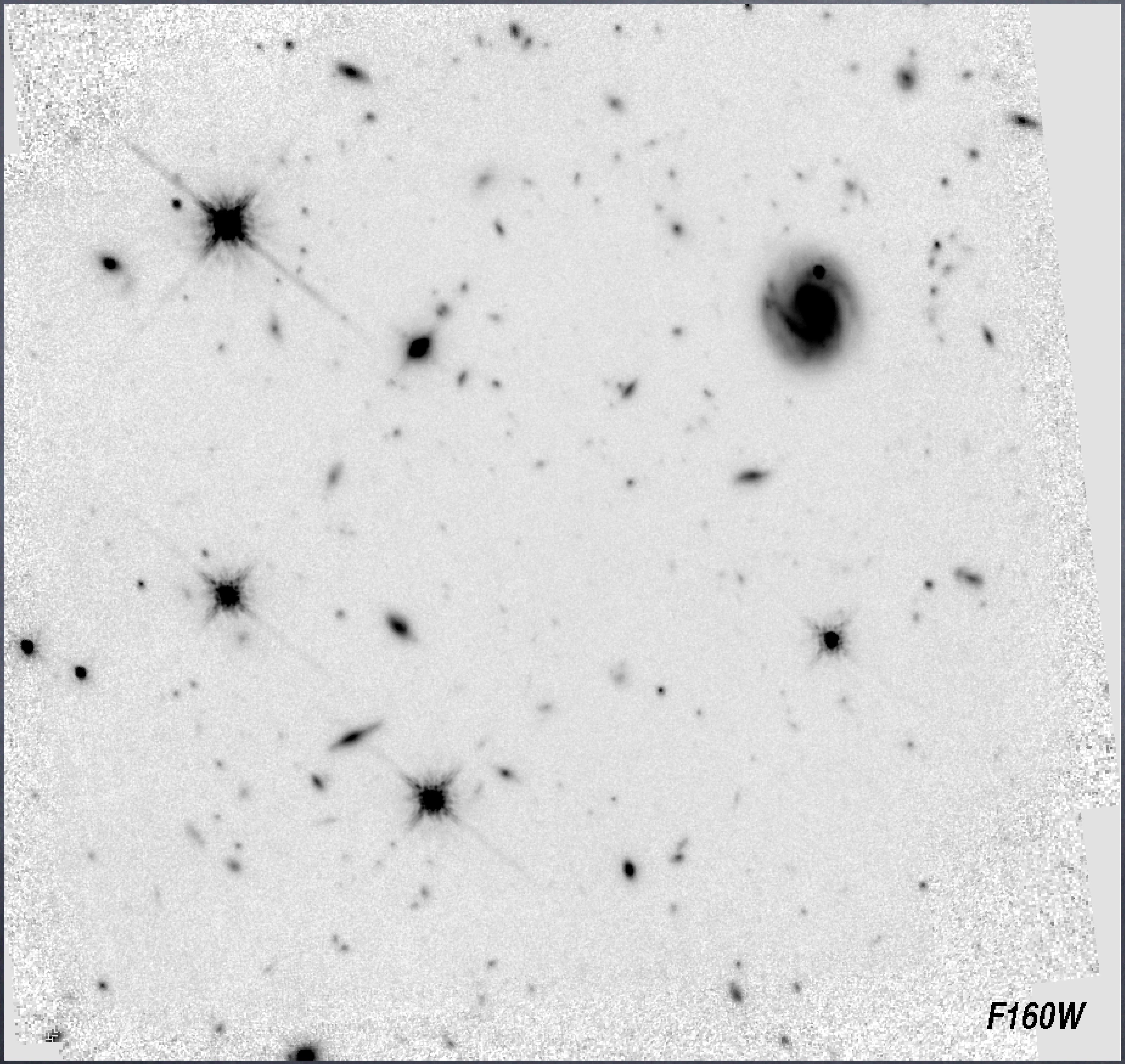
The low noise value is due to missing cross terms $ab_1, ab_2, b_1b_2....$ in single pixel noise estimate

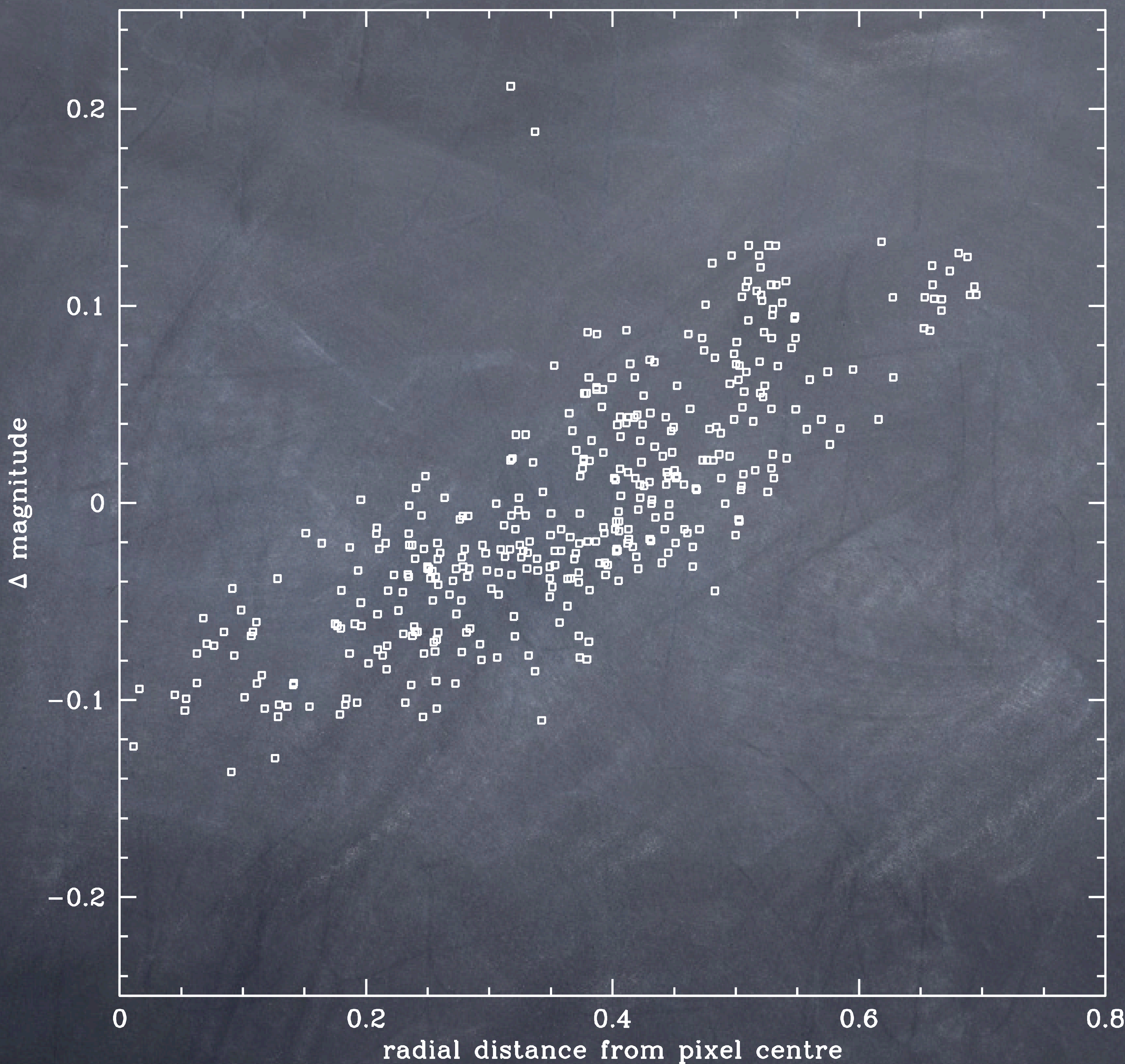
Ratio of true noise to apparent single-pixel noise $\mathcal{R}$ is, for $r = p/s \geq 1$,

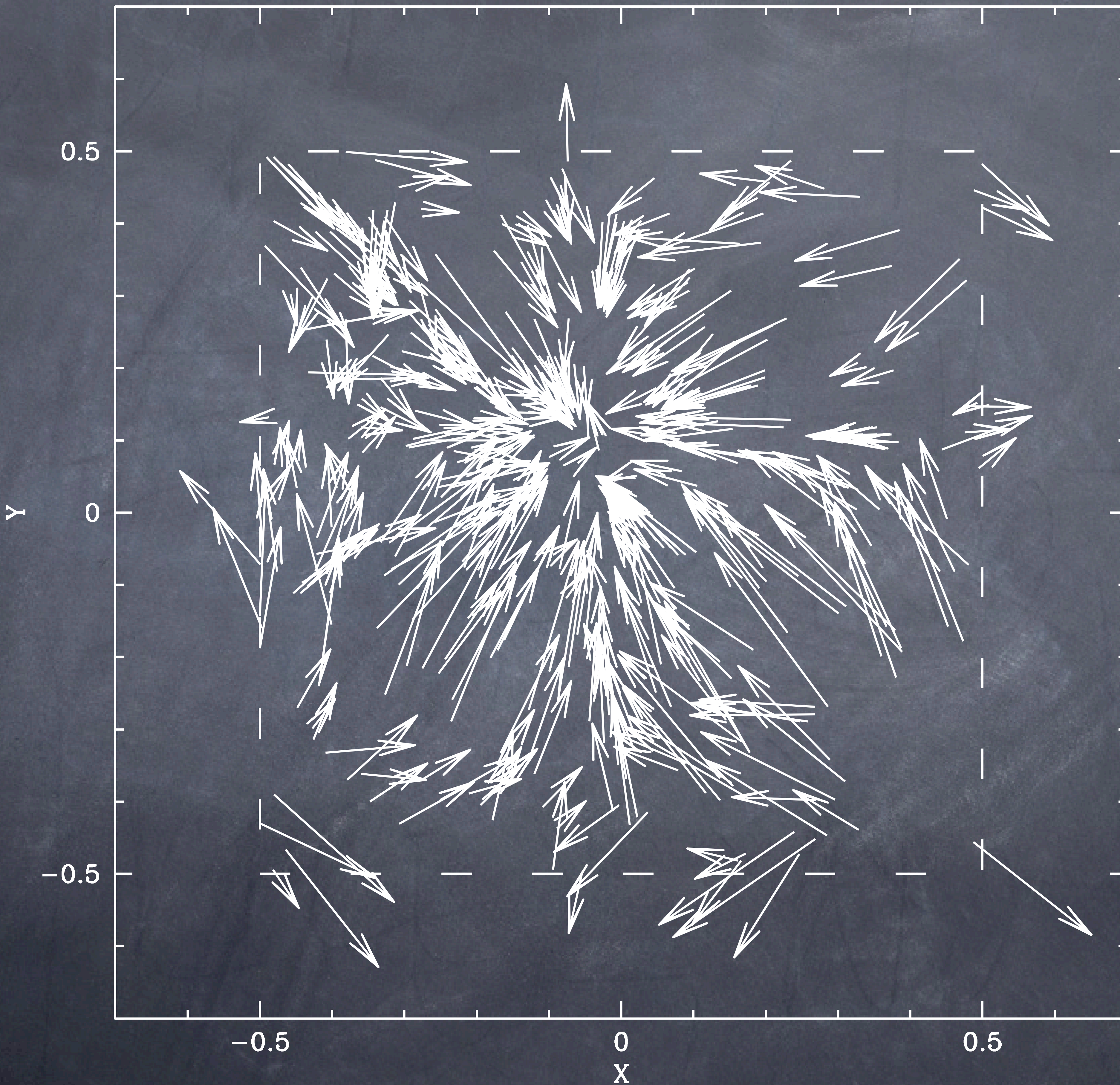
$$\mathcal{R} = \frac{r}{1 - \frac{1}{3r}}$$

and for $r < 1$

$$\mathcal{R} = \frac{1}{1 - \frac{r}{3}}$$


F160W


0.2
0.1
0
−0.1
−0.2
Δ magnitude
0
0.2
0.4
0.6
0.8
radial distance from pixel centre



0.5
0
−0.5
Y
−0.5
0
0.5
X

Future Developments

- Needle exchange program
- More general solution to the irregular sampling problem

Gröchenig and Strohmer (2001)

(a) Lena

(b) 39 % missing samples: SNR = 4dB

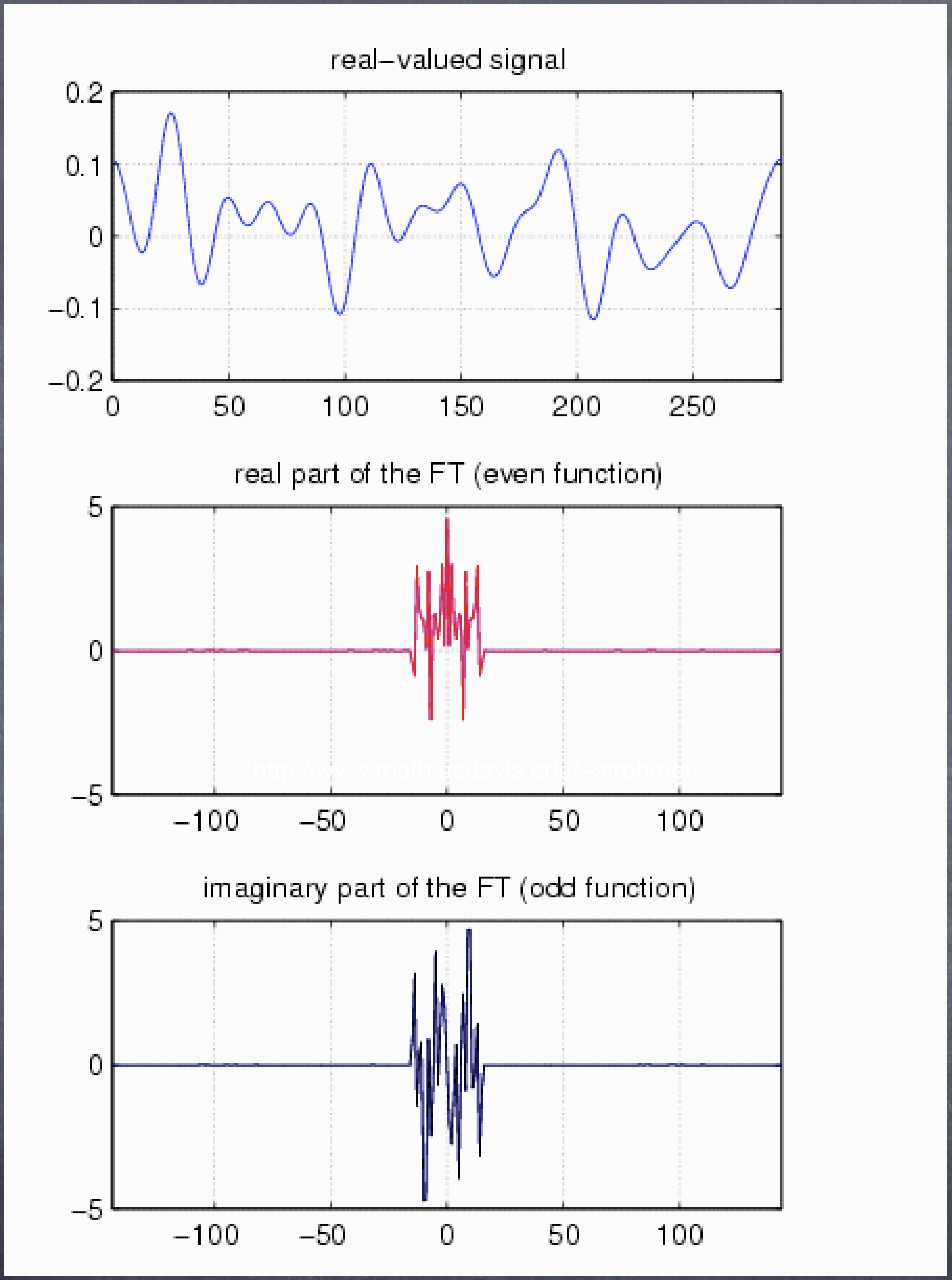
(c) Marvasti/Frame: SNR = 27.3dB

(d) Adaptive weights: SNR=29.4dB

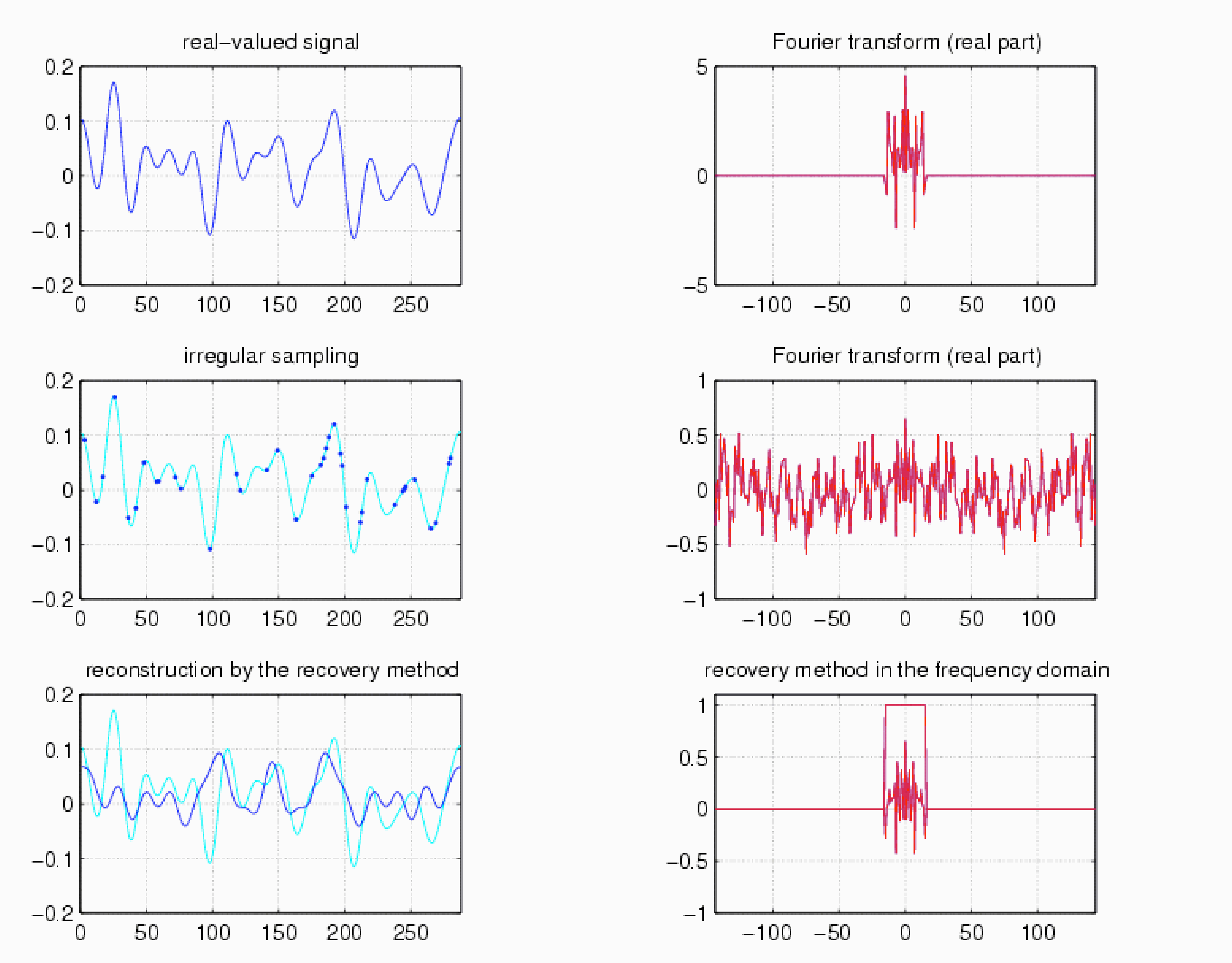
(e) ACT, no weights: SNR = 30.1dB

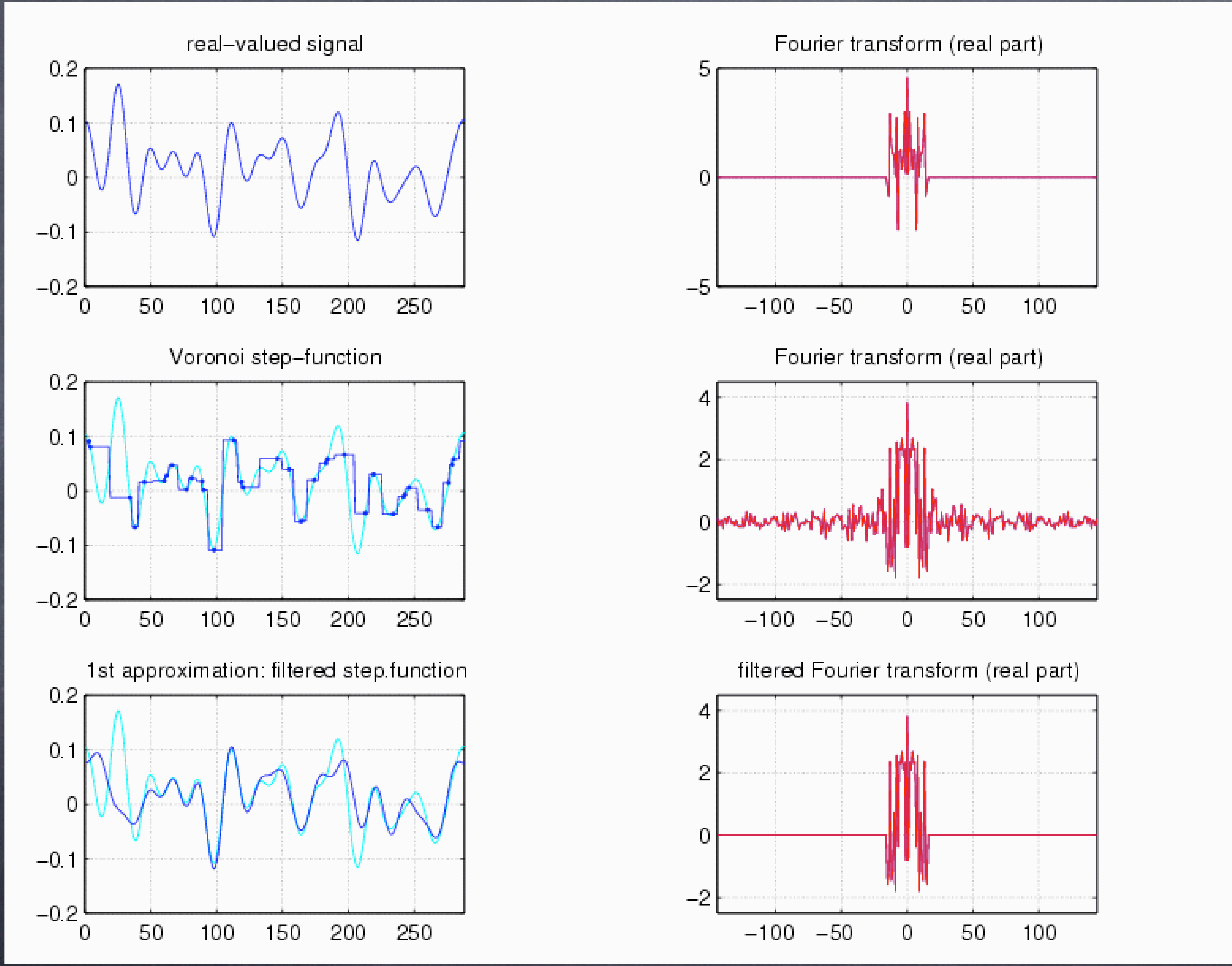
(f) ACT, with weights: SNR = 30.1dB

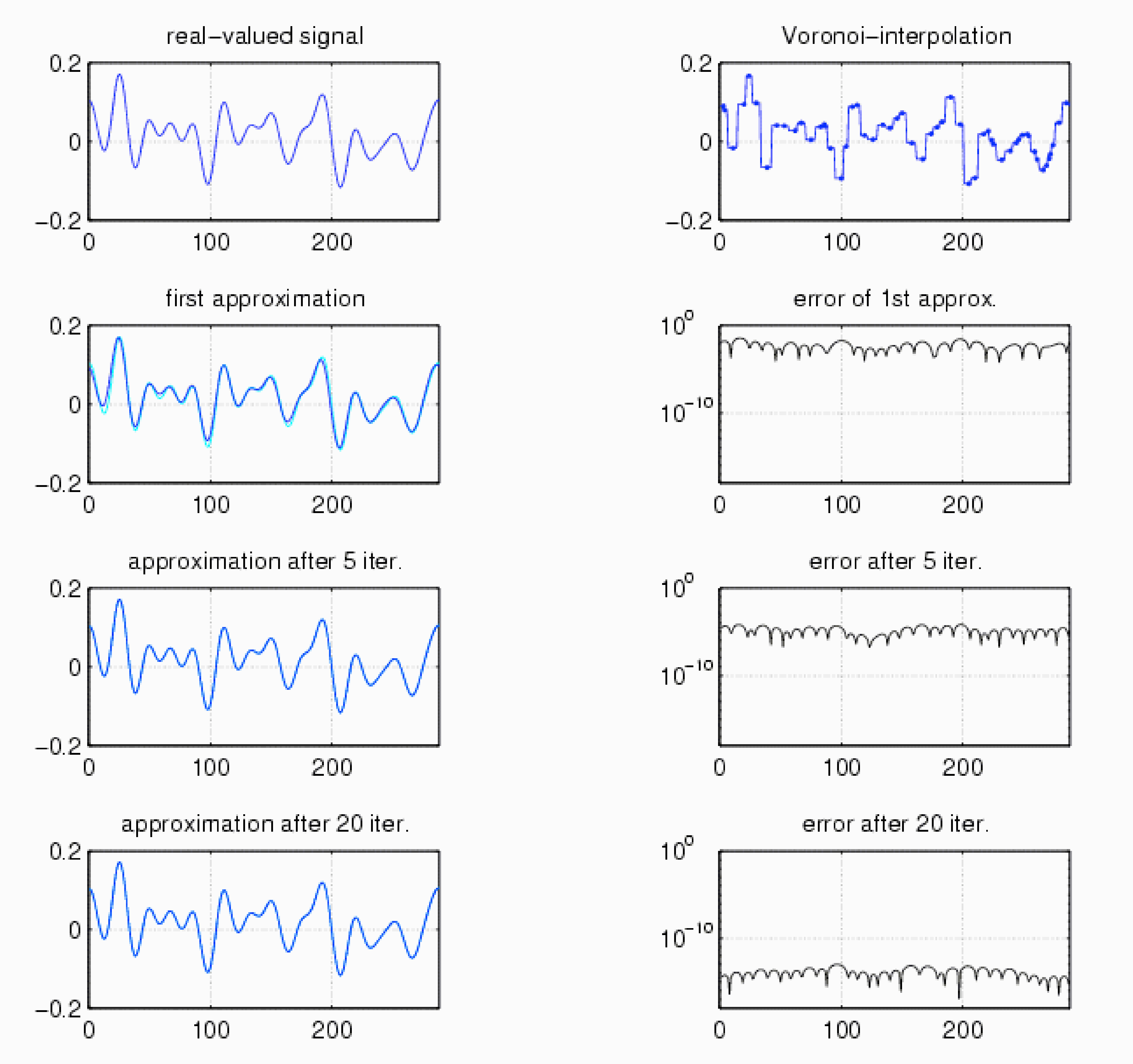
Figure 7: Nonuniformly sampled Lena and reconstructions after 15 iterations.



Tutorial by Tobias Werther (NUHAG)
see http://www.math.ucdavis.edu/~strohmer/


real–valued signal
Fourier transform (real part)
irregular sampling
Fourier transform (real part)
reconstruction by the recovery method
recovery method in the frequency domain


real–valued signal
Fourier transform (real part)
Voronoi step–function
Fourier transform (real part)
1st approximation: filtered step.function
filtered Fourier transform (real part)
0.2
0.1
0
–0.1
–0.2
0 50 100 150 200 250
5
0
–5
–100 –50 0 50 100
4
2
0
–2


real–valued signal
Voronoi–interpolation
first approximation
error of 1st approx.
approximation after 5 iter.
error after 5 iter.
approximation after 20 iter.
error after 20 iter.
0.2
0
–0.2
0
100
200
10^0
10^–10

Proposed Plan

- Use Drizzle instead of Voronoi approximation
- Iteration should effectively remove convolution with pixfrac and scale
- Comparison step allows introduction of sub-pixel response function

Are you frying when you should be drizzling?

Just because your olive oil is extra virgin doesn't mean it's multi-purpose. In fact, quite the opposite. Depending on where they're grown and how they're produced, some olive oils are better for drizzling, others for sautéing, marinating and so on. Tuscan olive oils (or California oils made in the Tuscan style) are especially good for drizzling on bruschetta, panini or other sandwiches. Deep green/gold in color with a classic peppery finish, they also pair well with pasta dishes and grilled meats. At Eddie's, you'll find a selection of over 46 olive oils, as well as pointers and ideas on which goes best with what. After all, we think the more you know about food, the more you'll appreciate Eddie's.

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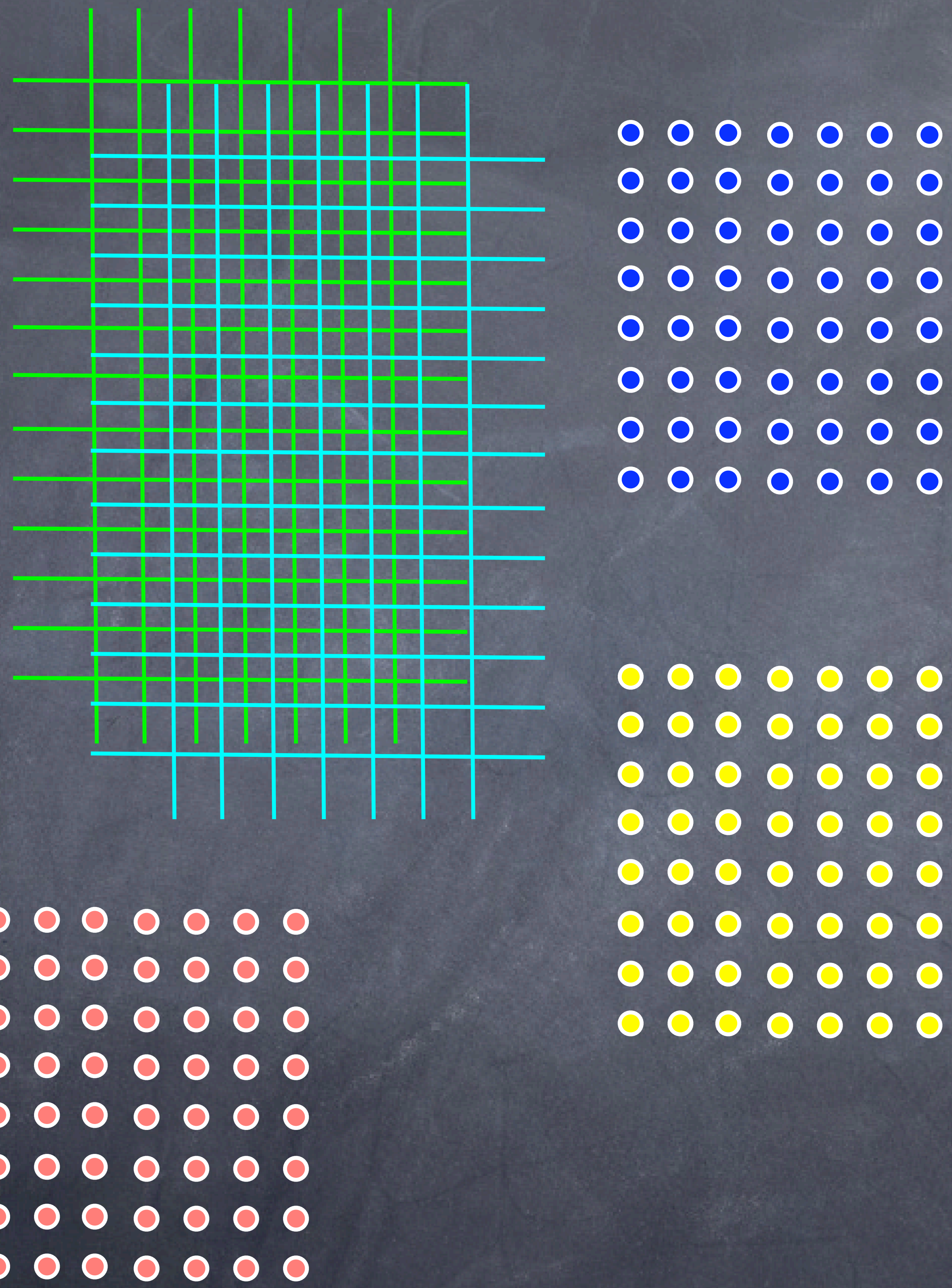

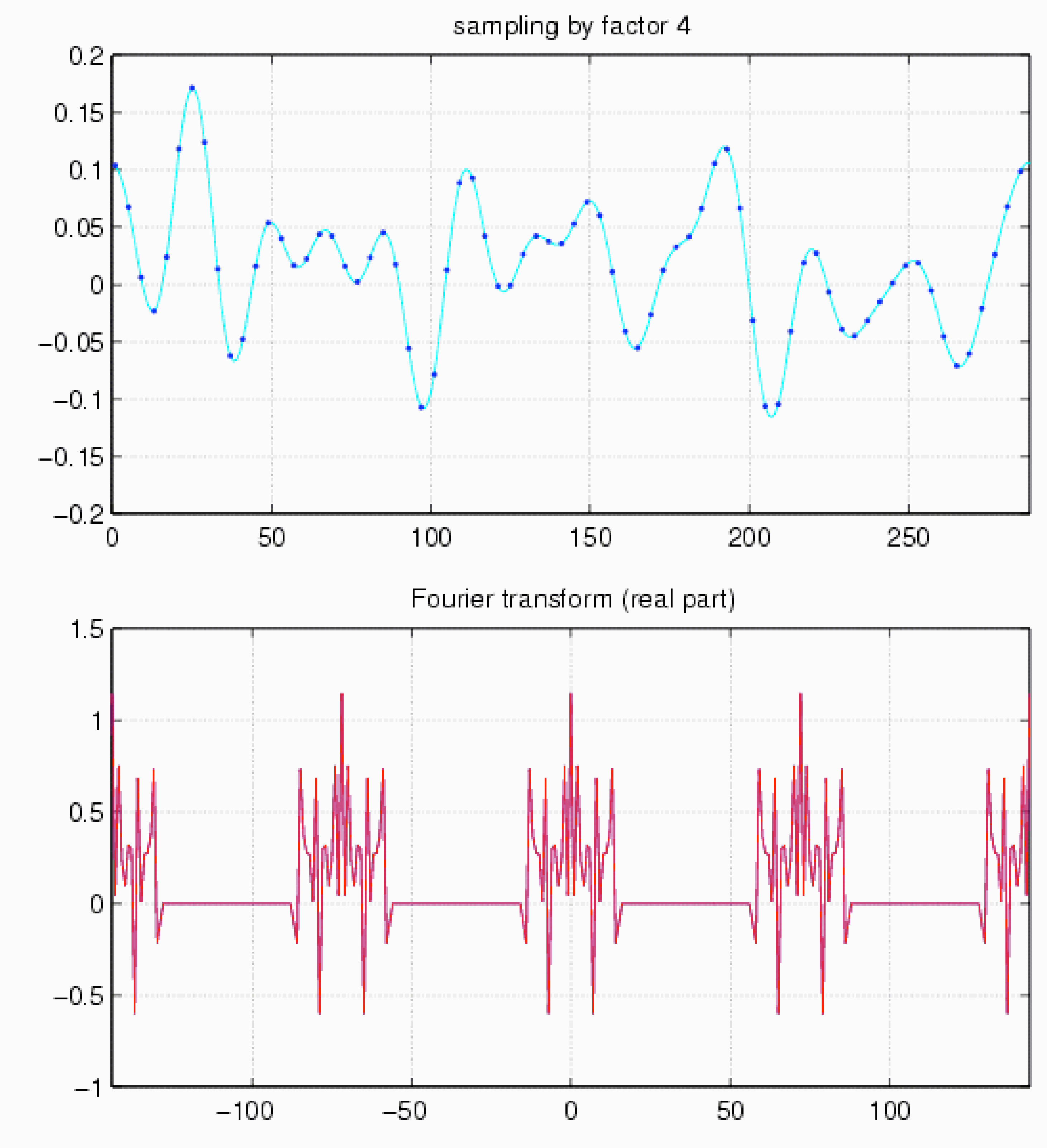

sampling by factor 4
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
0
50
100
150
200
250
Fourier transform (real part)
1.5
1
0.5
0
−0.5
−1
−100
−50
0
50
100