Some Blind Deconvolution Techniques in Image Processing

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Joint work with
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Outline

Part I:

Phase Diversity-Based Blind Deconvolution

Part II:

High-resolution Image Reconstruction with Multi-sensors

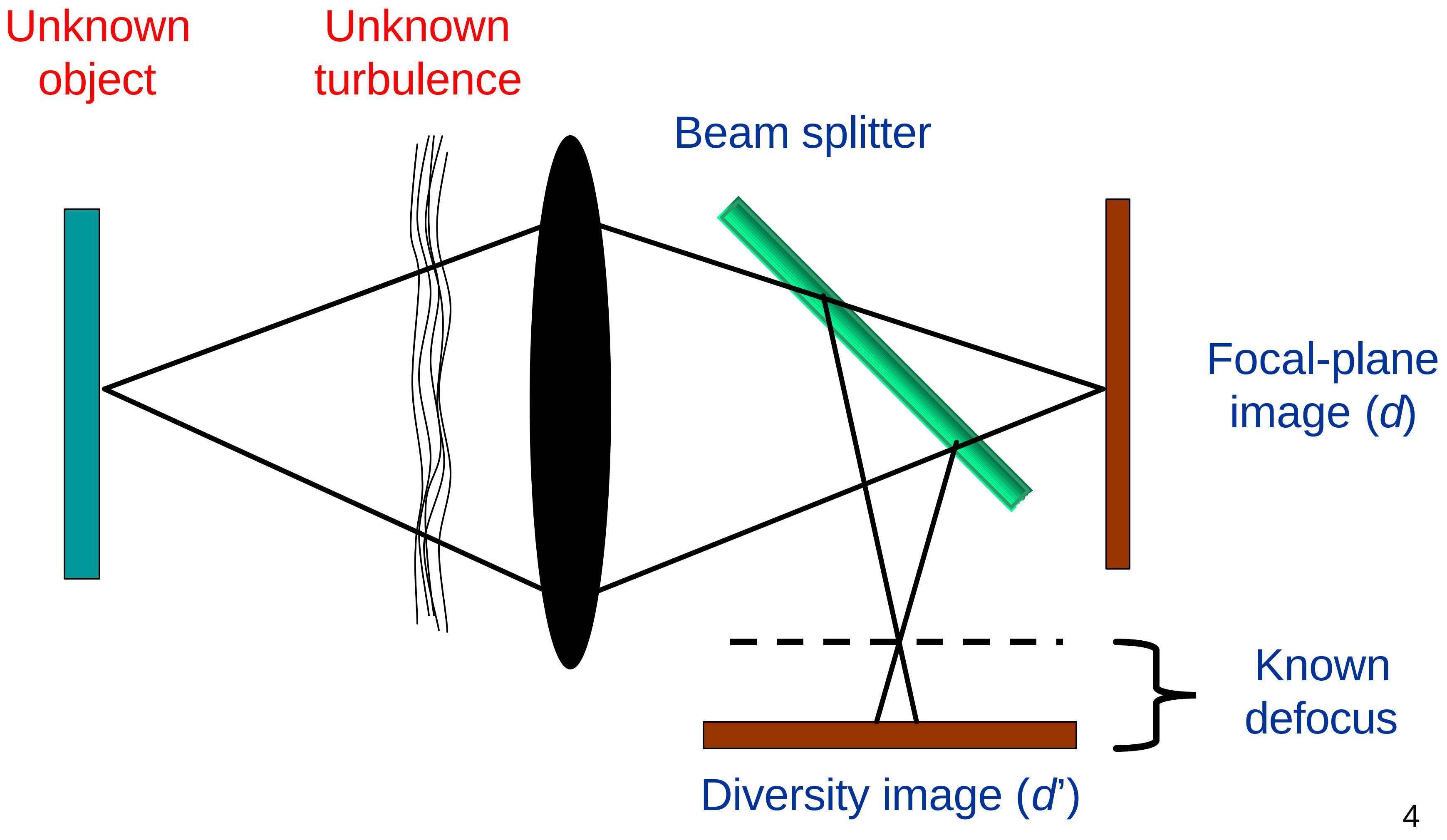
Part III:

Total Variation Blind Deconvolution

Part I:
Phase Diversity-Based Blind Deconvolution

Joint work with
Curt Vogel and
Robert Plemmons
SPIE, 1998

Conventional Phase-Diversity Imaging



Model for Image Formation

$$d = s * f + h$$

$$s[f](x, y) = \left| F^{-1}\left\{ p(x, y) e^{i f(x,y)} \right\} \right|^2$$

- *f* = unknown object
- *s* = unknown point spread function
- *h* = unknown noise
- *p* = aperture function (pre-determined by the telescope's primary mirror)
- *f* = phase characterizing the medium through which light travels
- *F* = Fourier transform operator

Object and Phase Reconstruction Model

- Measurements:

$$d = s[f] * f + h$$

$$d' = s[f + q] * f + h'$$

θ: known phase perturbation

- Model: (Vogel, C. and Plemmons ‘98; Gonsalves ’82)

$$\min_{f,f} \left\{ \left\| d - s[f] * f \right\|^2 + \left\| d' - s[f + q] * f \right\|^2 + \underbrace{g J_{object}[f]}_{\text{quadratic}} + \underbrace{a J_{phase}[f]}_{\text{A priori statistics}} \right\}$$

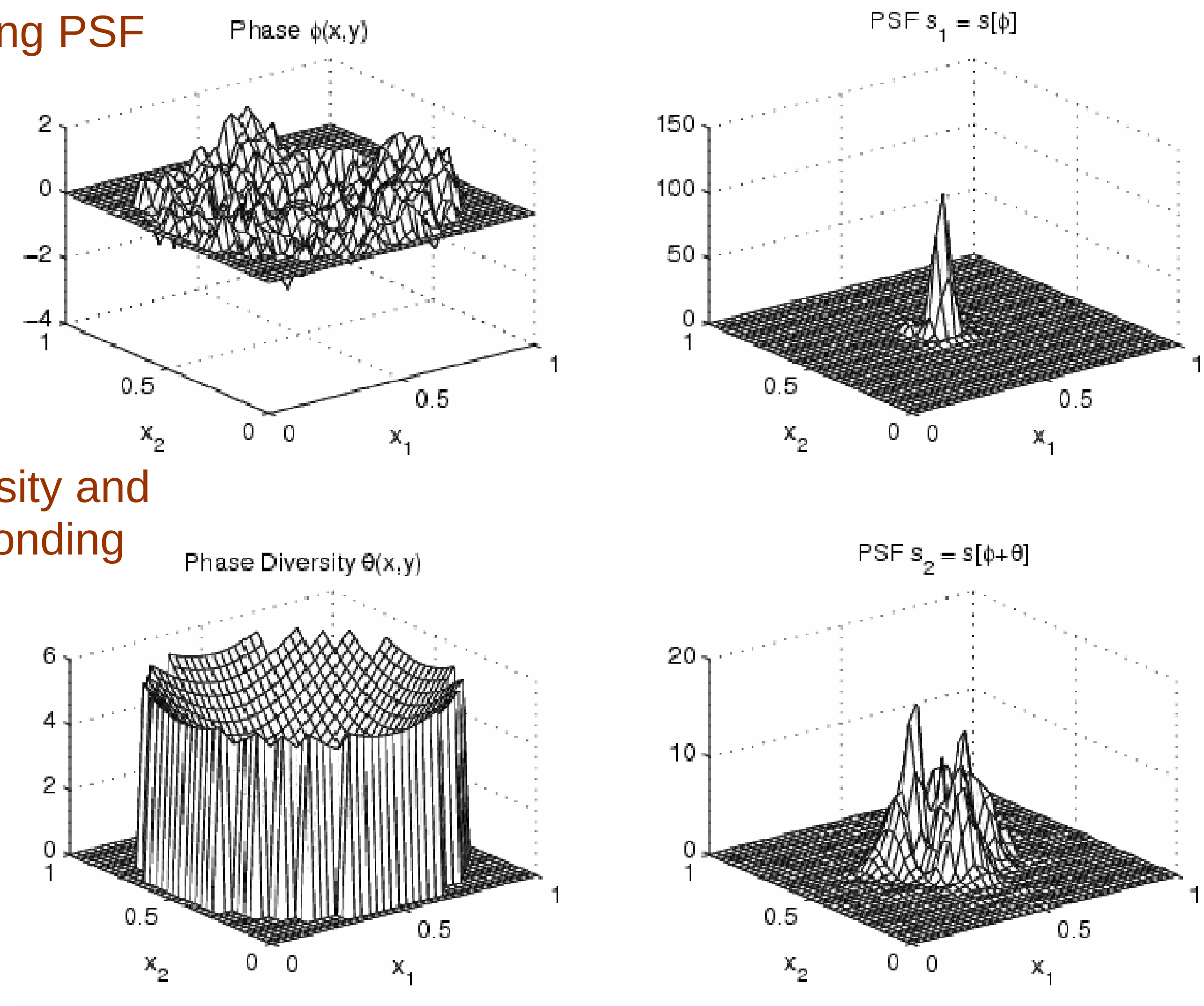
Numerical Methods

1. Reduce the objective $J[f,f]$ to $J'[f]=J[f,f[f]]$
   -- Possible because ***f*** is quadratic
2. Derive gradient and Hessian*vector for ***J***
   -- Involve FFTs and inverse FFTs
3. Minimize using:
   1. Finite difference Newton (quadratic convergence, need inversion of Hessian; needs good initial guess)
   2. Gauss-Newton + Trust Region + CG (for nonlinear least squares, robust; linear convergence, small #iterations)
   3. BFGS (needs only gradients; superlinear convergence; fast per step; often most efficient)

Simulations

Phase and its corresponding PSF

Phase diversity and its corresponding PSF



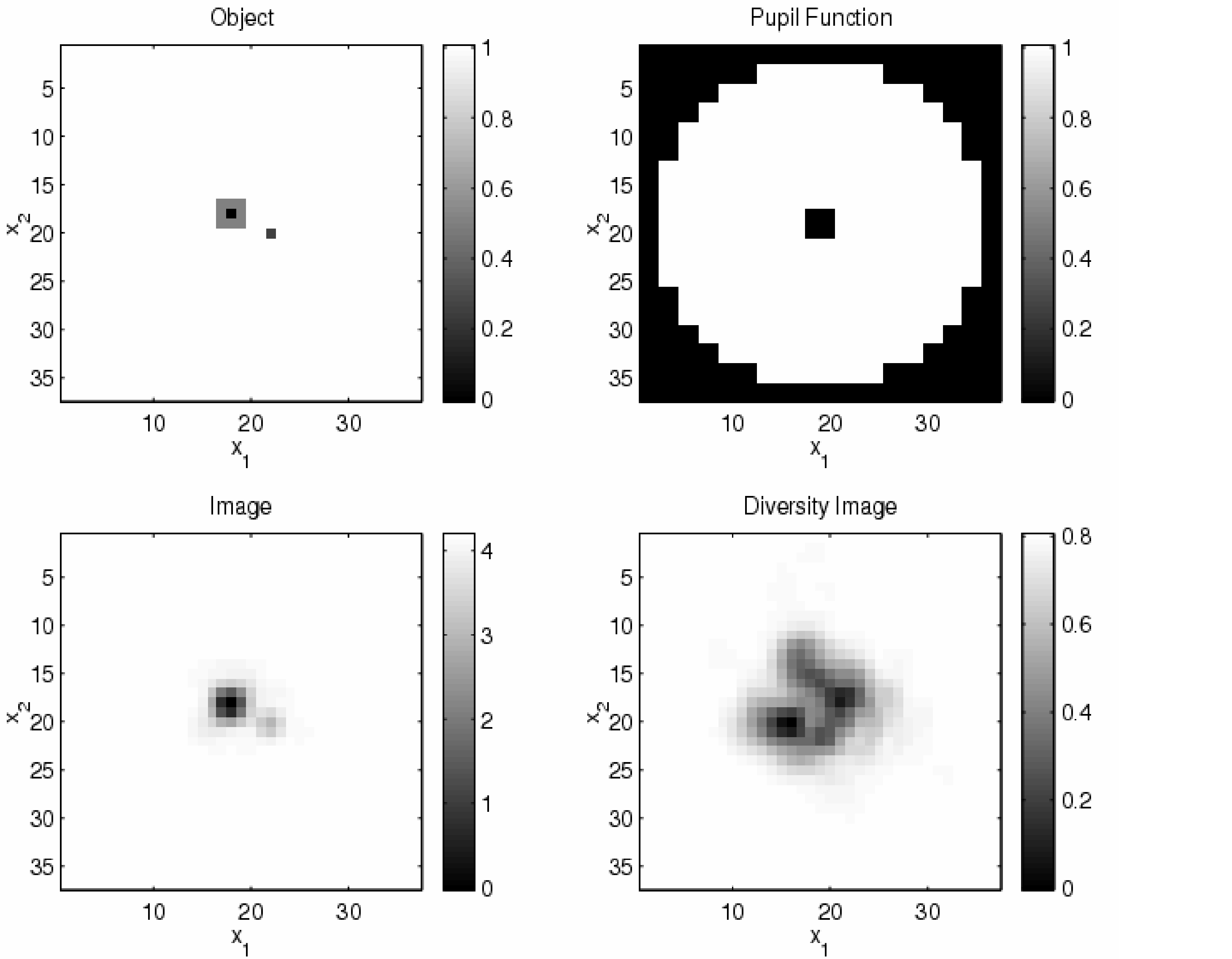


Figure 2. Negative grayscale plot of simulated object $f(x_1, x_2)$ at upper left; image $d_1 = s[\phi] \star f + \eta_1$ at lower left; and diversity image $d_2 = s[\phi + \theta] \star f + \eta_2$ at lower right. At the upper right is a grayscale plot of the pupil, or aperture, function.

Reconstructed Phase and Object

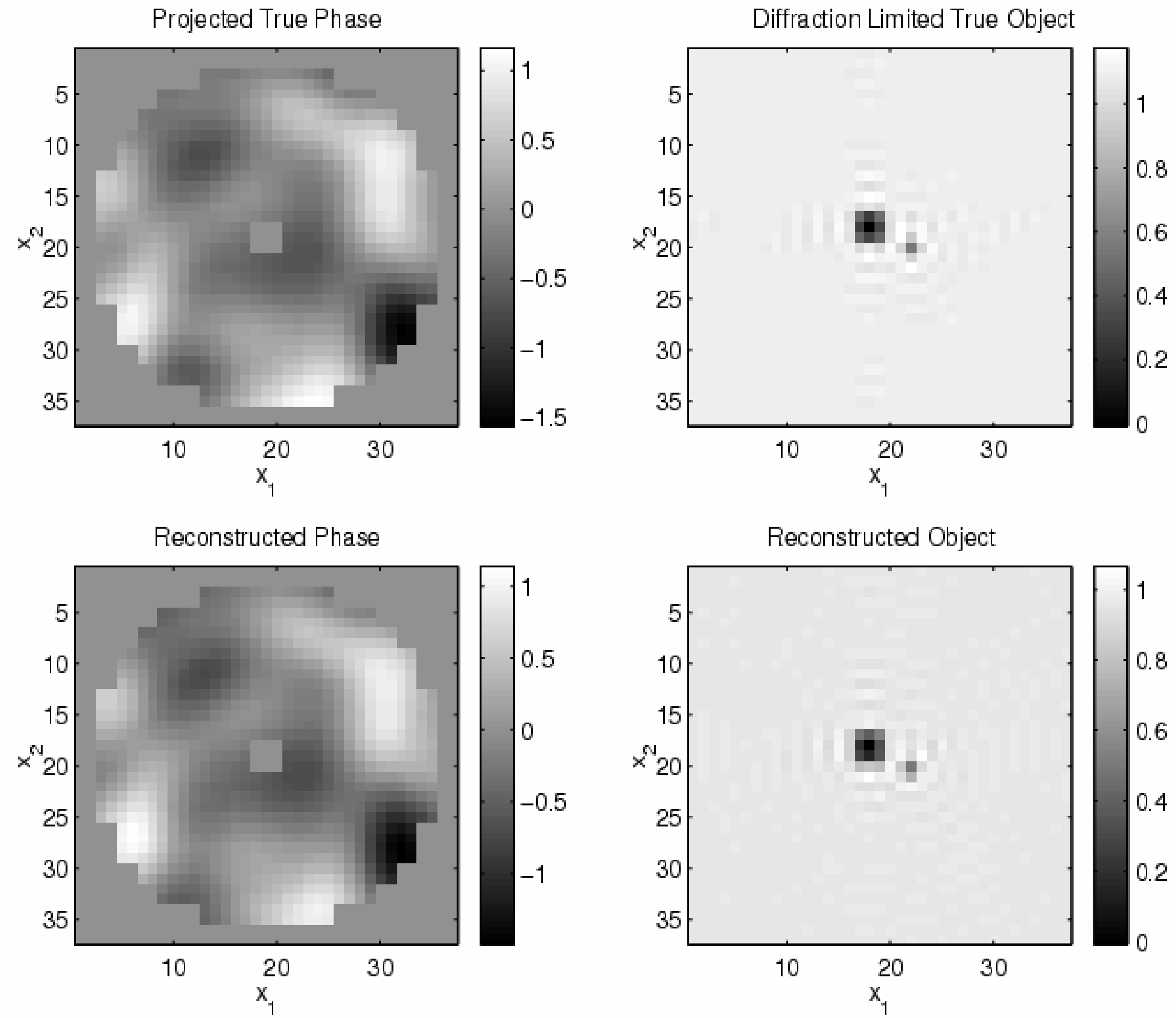
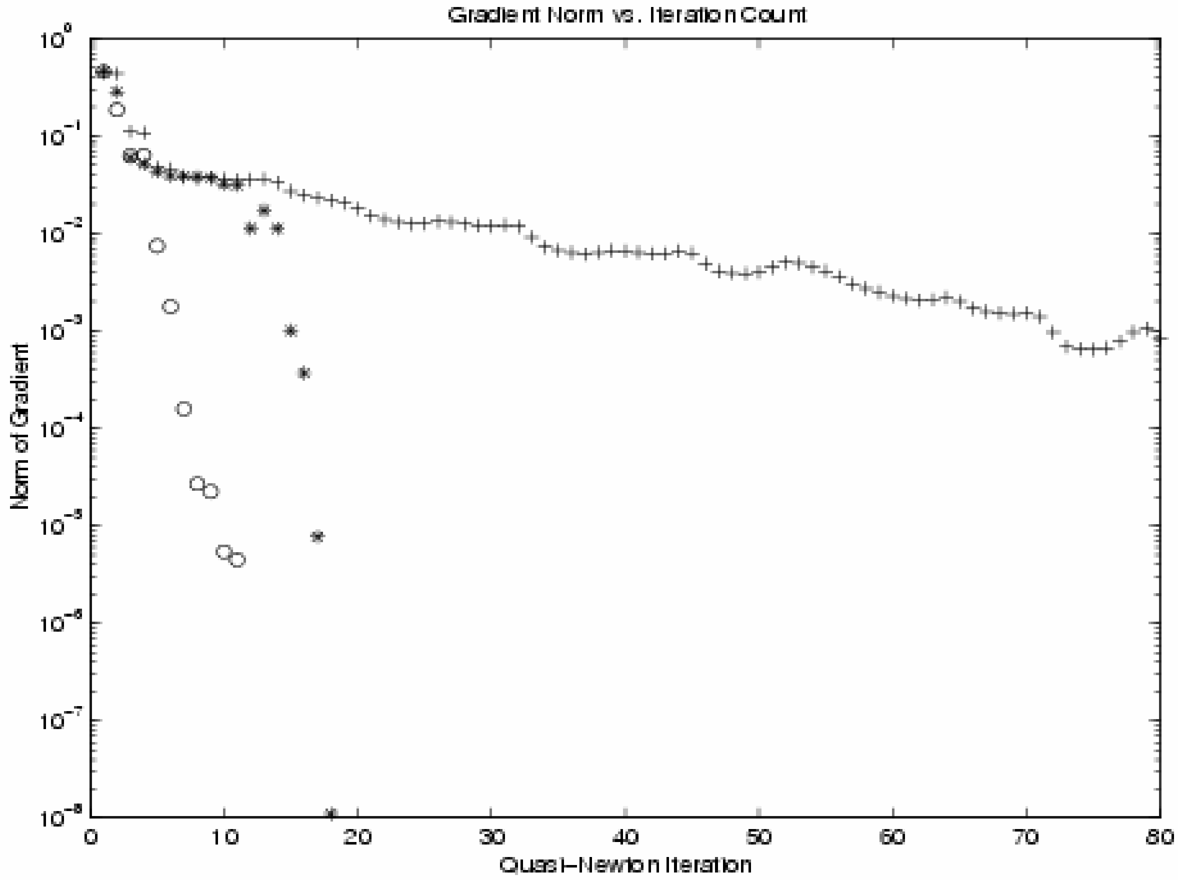


Figure 3. True and reconstructed phases and objects.

Performance of Various Methods for the Binary Star Test Problem



$\gamma = 10^{-7}$

$a = 10^{-5}$

o: Gauss-Newton method (linear convergence, robust to initial guess)

$*$: finite difference Newton's method with line search (quadratic convergence)

+: BFGS method with line search (linear convergence, lowest cost per iteration)

Remarks on Phase Diversity

- Multiple (>2) diversity images
  - Vogel et al; Löfdahl '02
  - Effect of #frames on image quality?
- Optimal choice of phase shift?
- Better optimization techniques designed for PD?

References

Phase Diversity-Based Image Reconstruction

1. C. R. Vogel, C. and R. Plemmons, *Fast Algorithms for Phase-Diversity-Based Blind Deconvolution*, SPIE Proc. Conference on Astronomical Imaging, vol. 3353, pp. 994-1005, March, 1998, Eds.: Domenico Bonaccini and Robert K. Tyson.
2. M. G. Löfdahl, *Multi-Frame Blind Deconvolution with Linear Equality Constraints*, SPIE Proc. Image Reconstruction from Incomplete Data II, vol. 4792, pp. 146-155, Seattle, WA, July 2002, Eds.: Bones, Fiddy & Millane.

Part II:
High-resolution Image Reconstruction with Multi-sensors

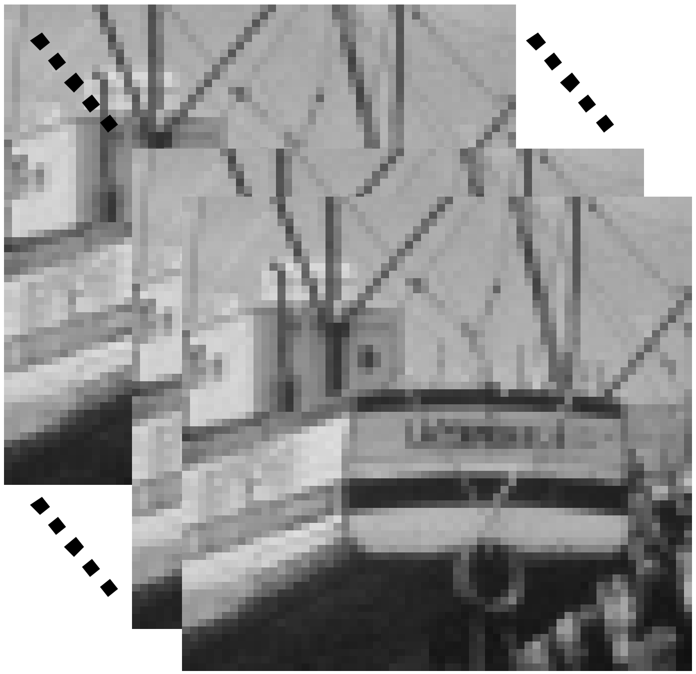
Joint work with Raymond Chan, Michael Ng, Wun-Cheung Tang, Chiu-Kwong Wong and Andy M. Yip ('98, 2000)

Images at Different Resolutions

Resolution = 64×64

Resolution = 256×256

The Reconstruction Process



$L \times L$ Low-Resolution Frames

($L \times L$ frames, each has $m \times m$ pixels, shifted by sub-pixel length)

→

Interlaced High-resolution

(A single image with $Lm \times Lm$ pixels)

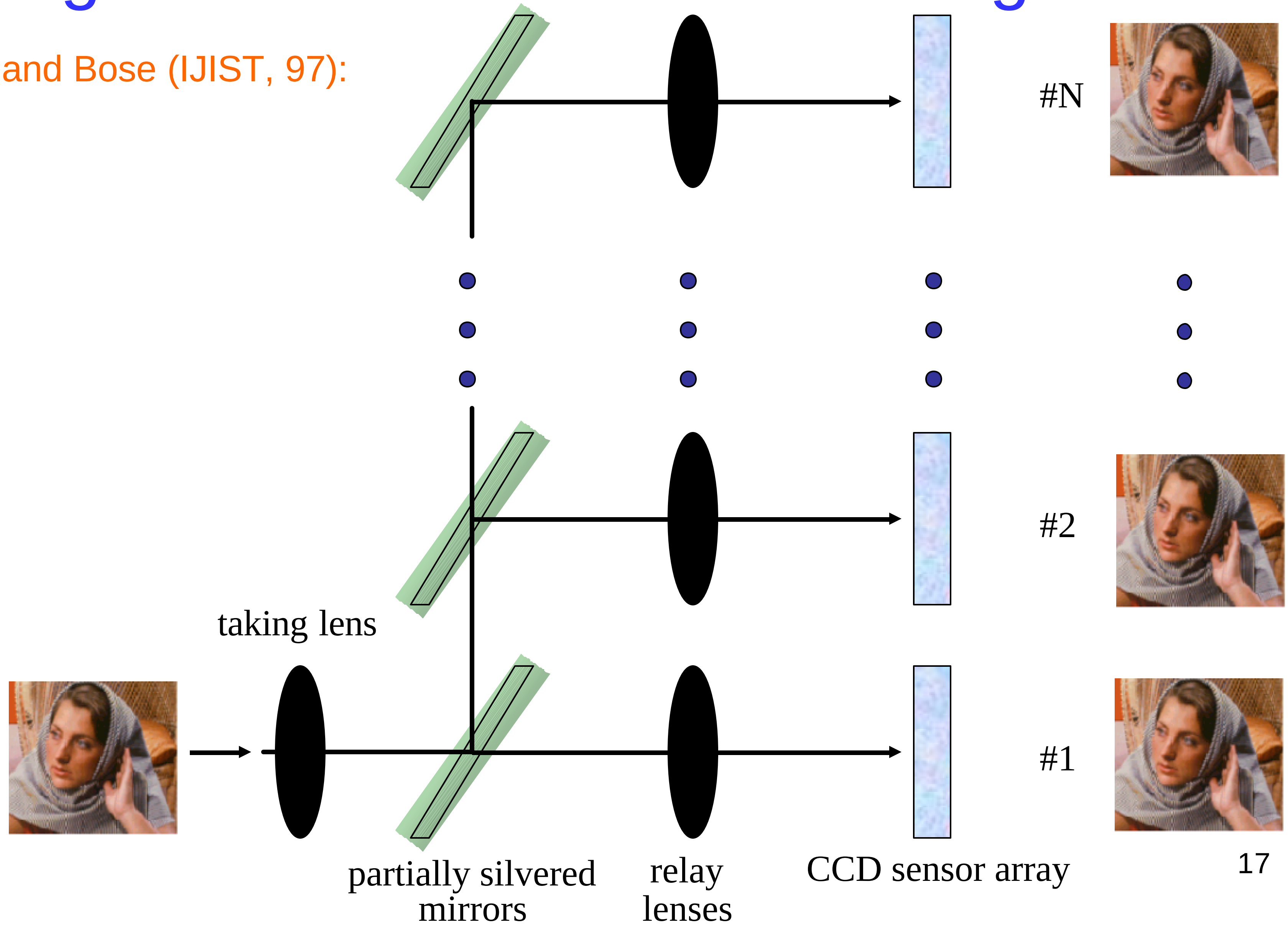
→

Reconstructed High-resolution

(A single image with $Lm \times Lm$ pixels)

High-Resolution Camera Configuration

Boo and Bose (IJIST, 97):



Construction of the Interlaced High Resolution Image

Four 2×2 images merged into one 4×4 image:

a_1	a_2
a_3	a_4

b_1	b_2
b_3	b_4

c_1	c_2
c_3	c_4

d_1	d_2
d_3	d_4

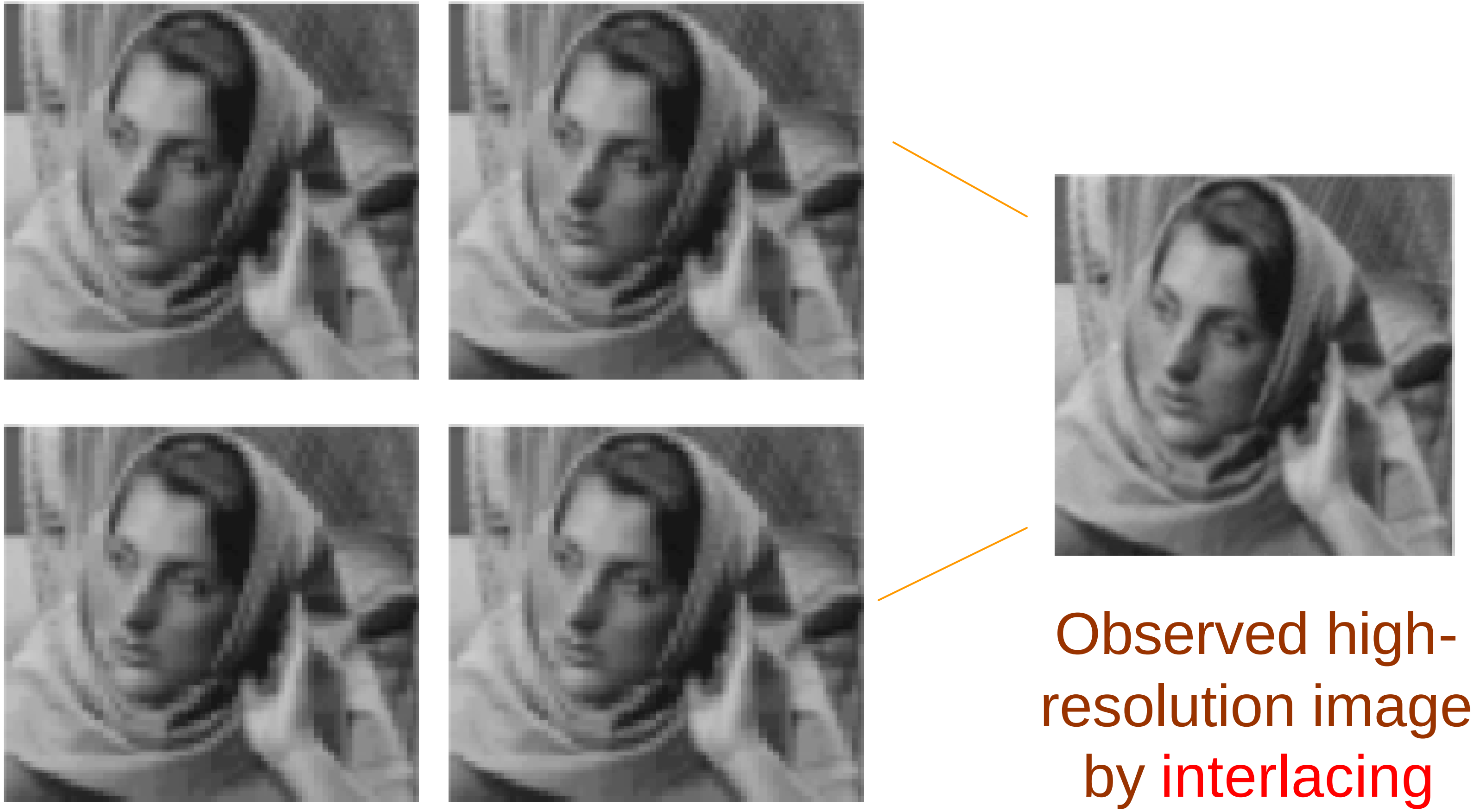
Four low resolution images

By interlacing

a_1	b_1	a_2	b_2
c_1	d_1	c_2	d_2
a_3	b_3	a_4	b_4
c_3	d_3	c_4	d_4

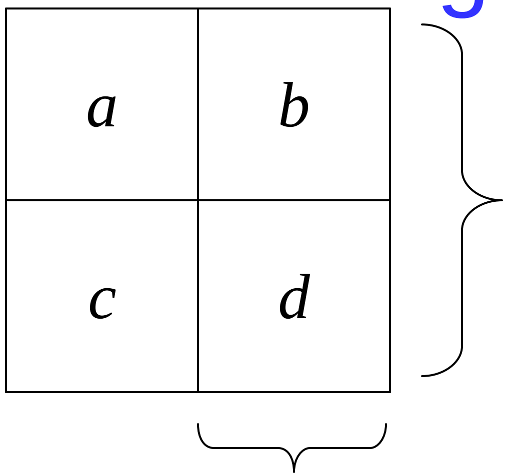
Interlaced high-resolution image

Four 64× 64 images merged into one by interlacing:



Observed high-resolution image by interlacing

Modeling of High Resolution Images

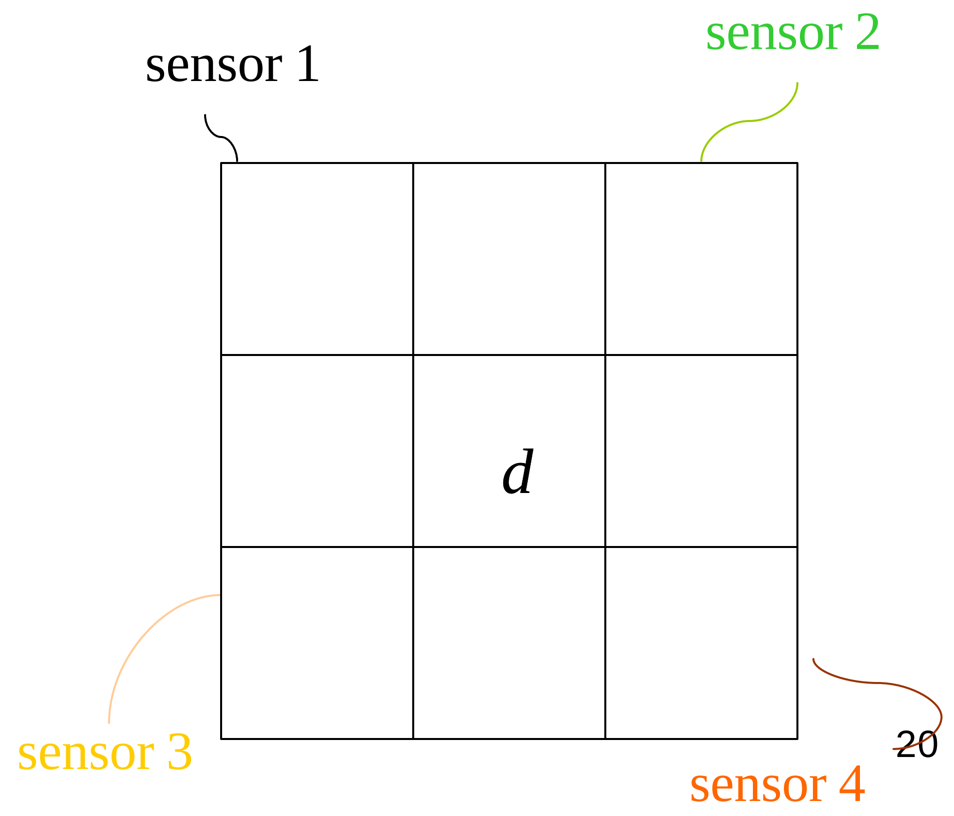


a low-resolution pixel

given intensity = (a+b+c+d)/4

a high-resolution pixel

4 low-resolution images
merge into 1

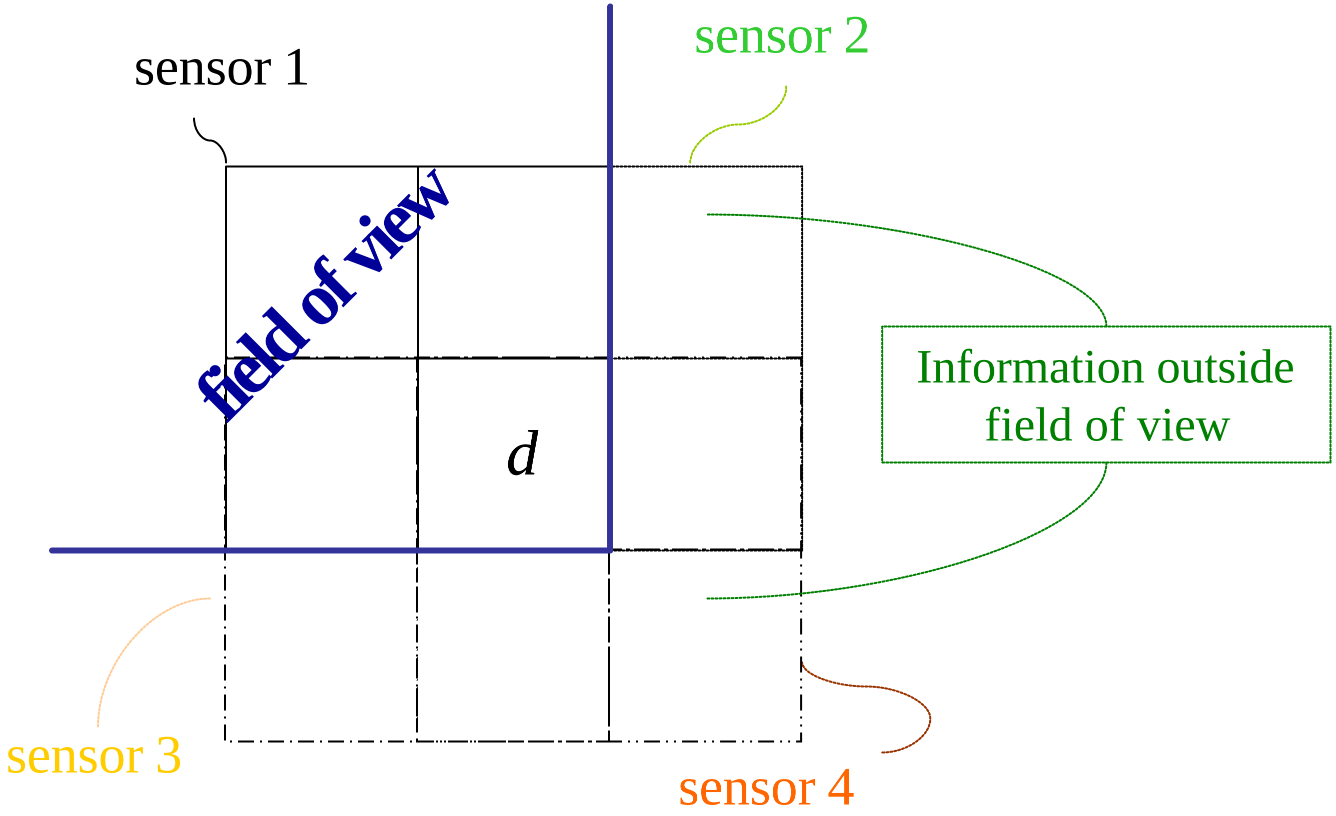


The Blurring Matrix

Let **f** be the true image, **g** the interlaced HR image, then

$$L\mathbf{f} = (L_x \otimes L_y)\mathbf{f} = \mathbf{g}.$$

f involves information of true image outside the field of view.



Boundary Conditions

Matrix is fat and long:

$$\begin{pmatrix} \quad & L & \quad \end{pmatrix} \begin{pmatrix} \mathbf{f} \end{pmatrix} = \begin{pmatrix} \mathbf{g} \end{pmatrix} .$$

Assume something about the image outside the field of view (boundary conditions).

After adding boundary condition:

$$\underset{N^2 \times N^2}{\begin{pmatrix} \quad & L & \quad \end{pmatrix}} \underset{N^2 \times 1}{\begin{pmatrix} \mathbf{f} \end{pmatrix}} = \underset{N^2 \times 1}{\begin{pmatrix} \mathbf{g} \end{pmatrix}} .$$

Periodic Boundary Condition (Gonzalez and Woods, 93)

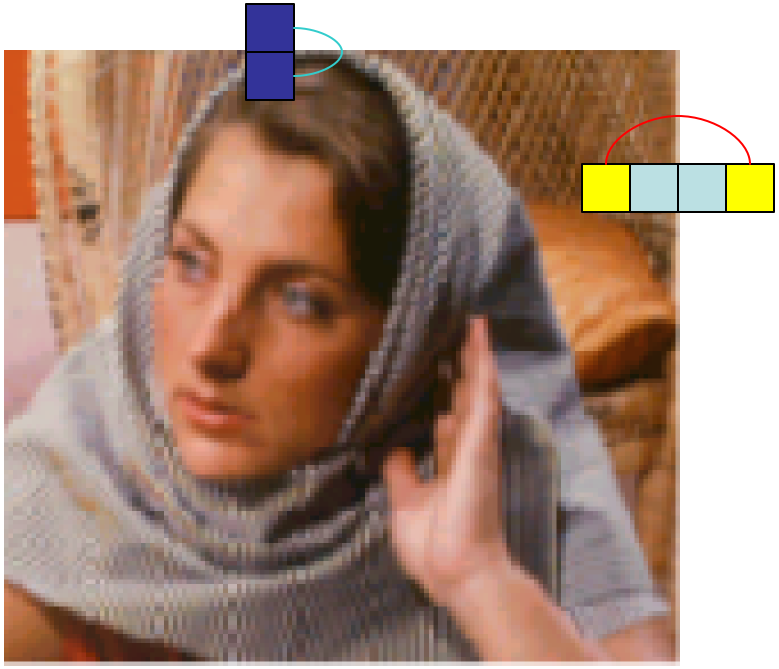
Assume data are periodic near the boundary.

Dirichlet (Zero) BC
(Boo and Bose, IJIST 97)

Assume data zeros outside boundary

Neumann Boundary Condition (Ng, Chan & Tang (SISC 00))

Assume data are reflective near boundary.



Boundary Conditions and Ringing Effects

original image

observed high-resolution image

Periodic B.C.

(ringing effect is prominent)

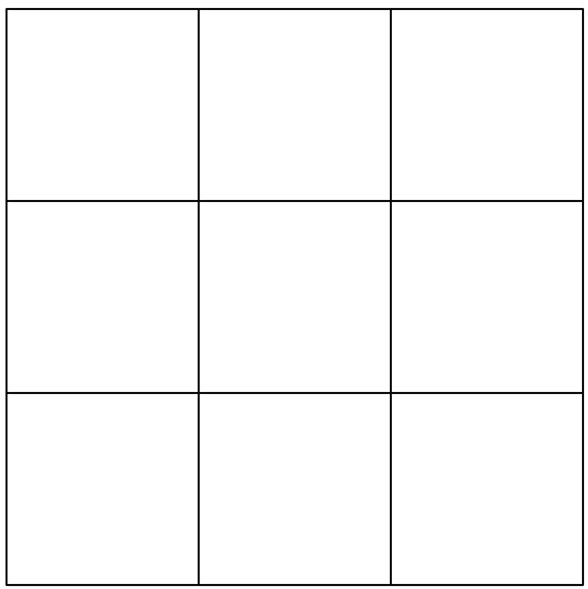
Dirichlet B.C.

(ringing effect is still prominent)

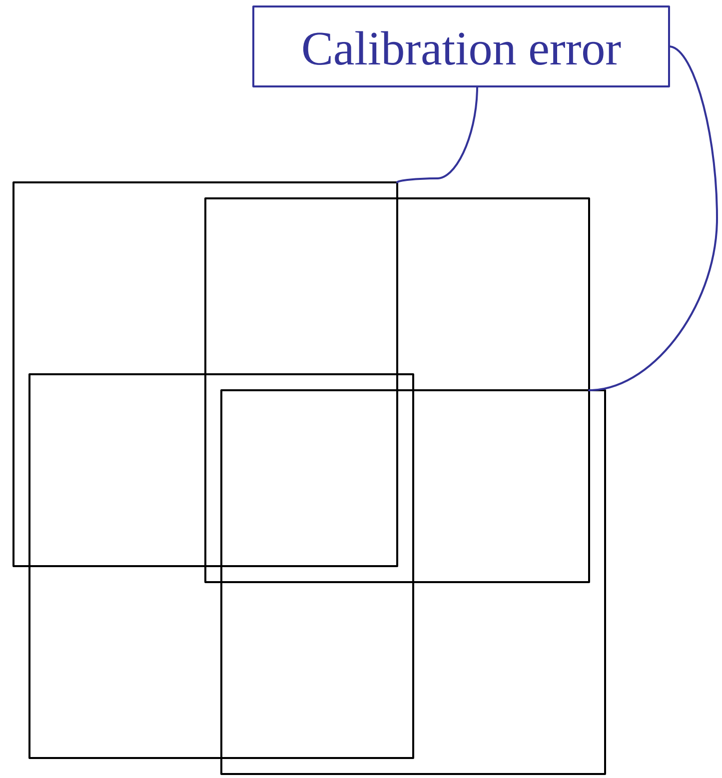
Neumann B.C.

(ringing effect is smaller)

Calibration Errors

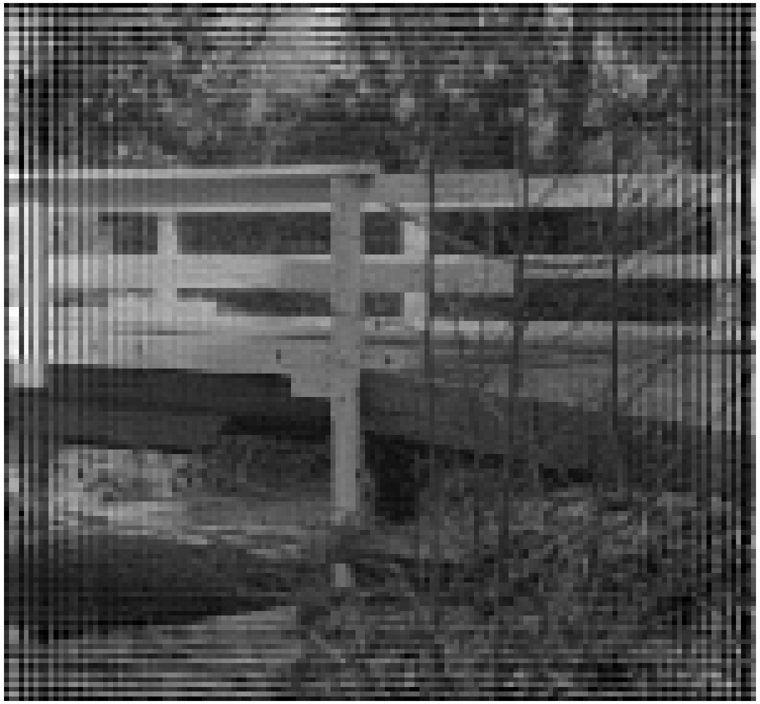


Ideal pixel positions

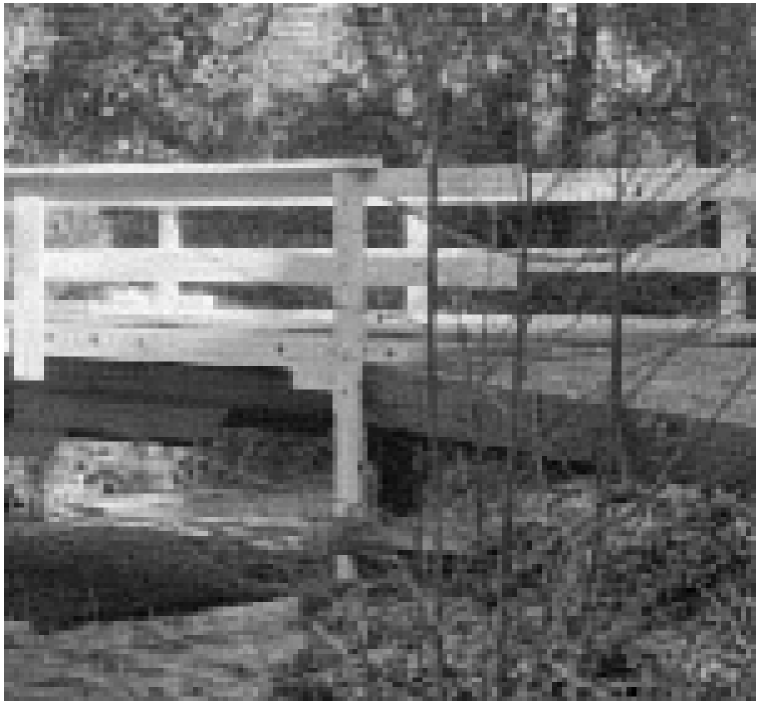


Pixels with displacement errors

Ringing effects are more prominent in the presence of calibration errors!



Dirichlet B.C.



Neumann B.C.

Regularization

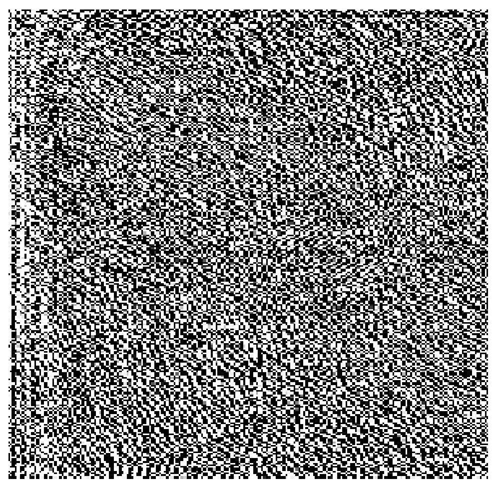
The problem *L* **f = g** is ill-conditioned.

Regularization is required:

$$(L^* L + b\ R)\mathbf{f} = L^* \mathbf{g}.$$

Here *R* can be *I*, Δ, or the TV norm operator.

$\mathbf{g}$



$(L^* L)^{-1} L^* \mathbf{g}$

$(L^* L + b\ R)^{-1} L^* \mathbf{g}$

Regularization Systems

- No calibration errors

  $(L^*L + bR)\,\mathbf{f} = L^*\mathbf{g}$

  – Structured, easily invertible

- With calibration errors e

  $(L(e)^*L(e) + bR)\,\mathbf{f} = L(e)^*\mathbf{g}$

  – Spatially variant, difficult to invert

  – Preconditioning (fast transforms):

  $(L^*L + bR)^{-1}\,(L(e)^*L(e) + bR)\,\mathbf{f} = (L^*L + bR)^{-1}\,L(e)^*\mathbf{g}$

Reconstruction Results

Original

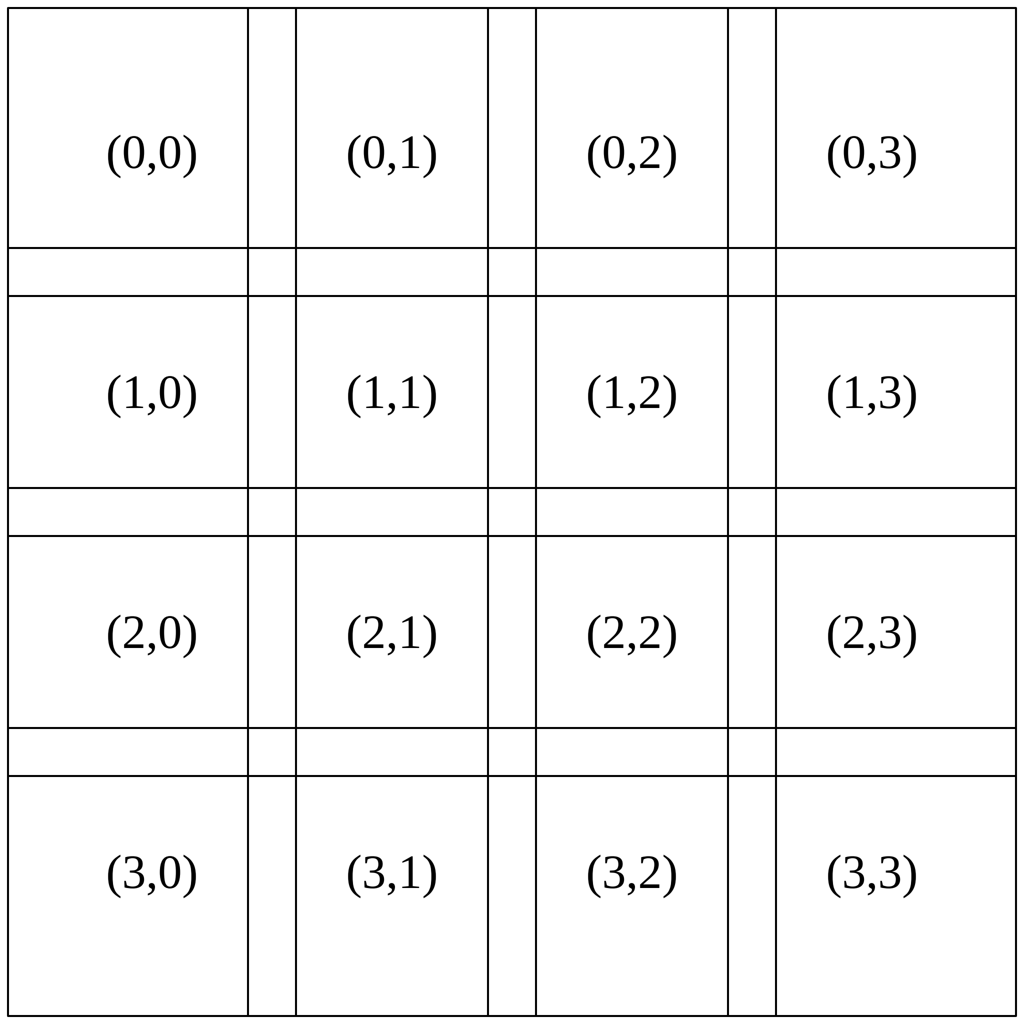
LR Frame

Interlaced HR

Reconstructed

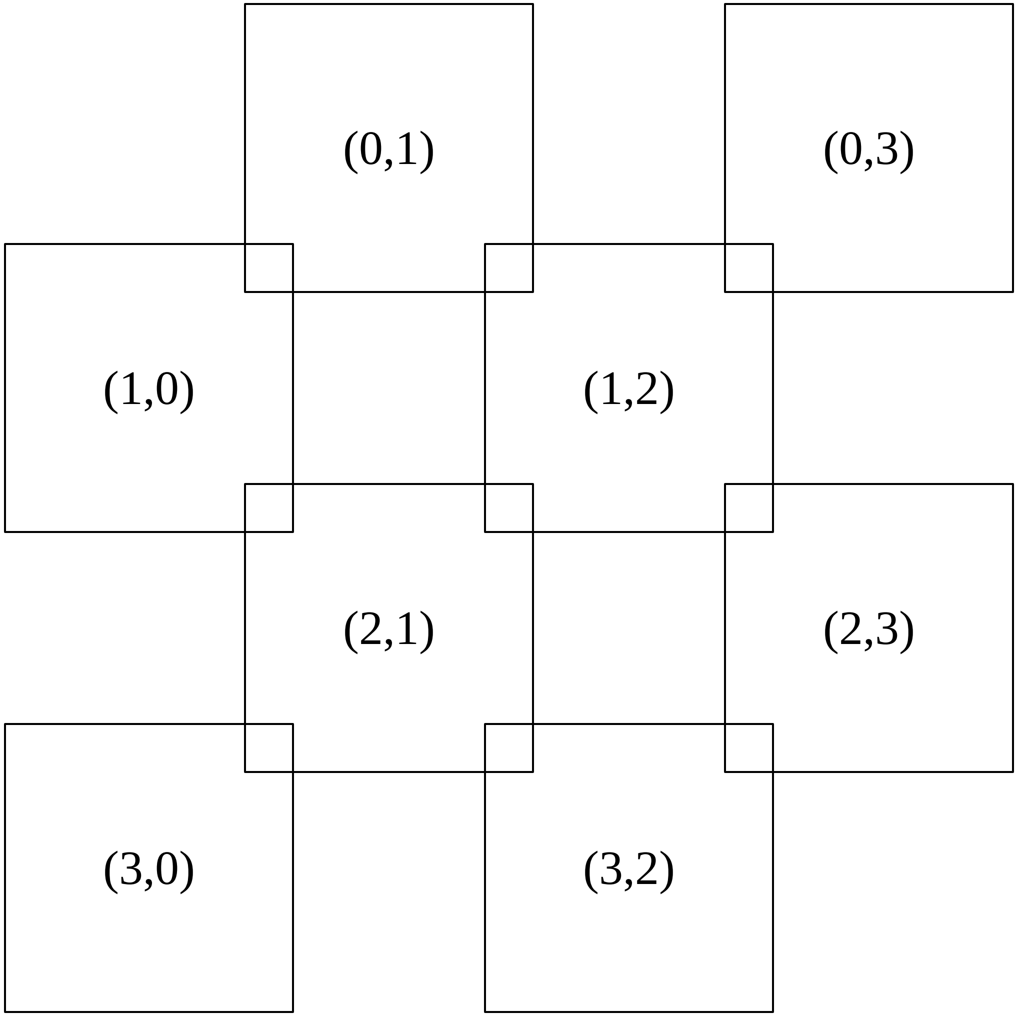
Super-Resolution: not enough frames

Example: 4×4 sensor with missing frames:



Super-Resolution: not enough frames

Example: 4×4 sensor with missing frames:



Super-Resolution

i. Apply an interpolatory subdivision scheme to obtain the missing frames.

ii. Generate the interlaced high-resolution image **w**.

iii. Solve for the high-resolution image **u**.

iv. From **u**, generate the missing low-resolution frames.

v. Then generate a new interlaced high-resolution image **g**.

vi. Solve for the final high-resolution image **f**.

Reconstruction Results

Observed LR

Final Solution

PSNR	Rel. Error
27.44	0.0787

References

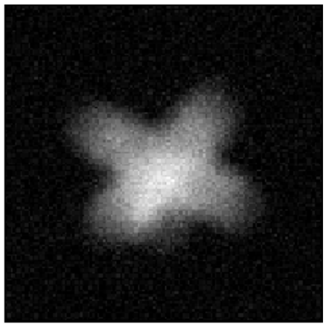
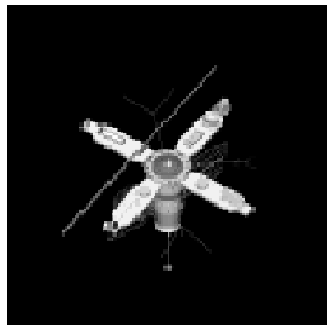
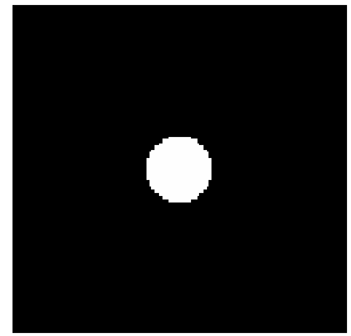
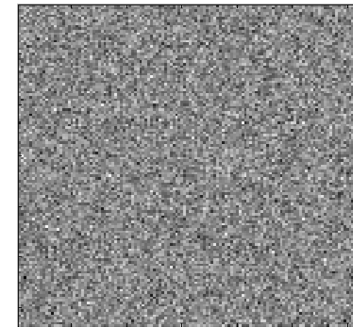
High-Resolution Image Reconstruction

1. N. K. Bose and K. J. Boo, *High-Resolution Image Reconstruction with Multisensors*, International Journal of Imaging Systems and Technology, 9, 1998, pp. 294-304.
2. R. H. Chan, C., M. K. Ng, W.-C. Tang and C. K. Wong, *Preconditioned Iterative Methods for High-Resolution Image Reconstruction with Multisensors*, Proc. to the SPIE Symposium on Advanced Signal Processing: Algorithms, Architectures, and Implementations, vol. 3461, San Diego, CA, July 1998, Ed.: F. Luk.
3. M. K. Ng, R. H. Chan, C. and A. M. Yip, *Cosine Transform Preconditioners for High Resolution Image Reconstruction*, Linear Algebra Appls., 316 (2000), pp. 89-104.
4. M. K. Ng and A. M. Yip, *A Fast MAP Algorithm for High-Resolution Image Reconstruction with Multisensors*, Multidimensional Systems and Signal Processing, 12, pp. 143-161, 2001.

Part III:
Total Variation Blind Deconvolution

Joint work with
Frederick Park,
Chiu-Kwong Wong
and Andy M. Yip

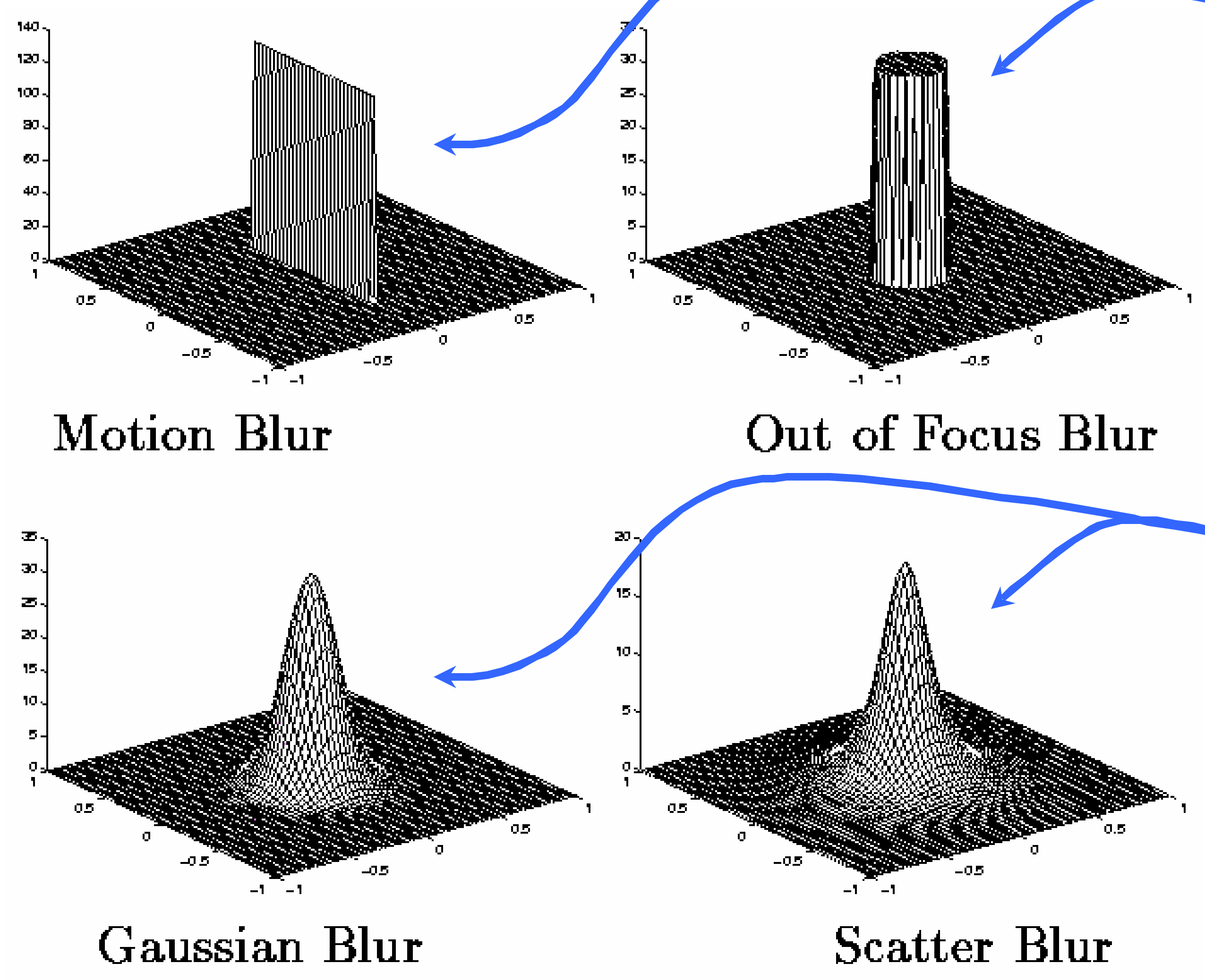
Blind Deconvolution Problem

 =  *  + 

Observed image u_{obs}	Unknown true image u_{orig}	Unknown point spread function k	Unknown noise h

Goal: Given u_{obs}, recover both u_{orig} and k

Typical PSFs



Blur w/ sharp edges

Blur w/ smooth transitions

H^1 Blind Deconvolution Model

Objective: (You and Kaveh '98)

$$\min_{u,k} F(u,k)$$

$$F(u,k) = \|u * k - u_{obs}\|^2 + a_1\|\nabla u\|^2 + a_2\|\nabla k\|^2$$

- Simultaneous recovery of both *u* and *k*
- ill-posed --- the need of regularization on *u* and *k*
- Sharp edges in *u* and *k* are smeared as H^1-norm penalizes against discontinuities

Total Variation Regularization

- Image deconvolution problems are ill-posed
- Total variation Regularization:

$$TV(u) = \int \left|\nabla u(x)\right| dx \qquad TV(k) = \int \left|\nabla k(x)\right| dx$$

- TV measures jump * length of level sets of ***u***
- TV norm can capture sharp edges as observed in **motion** or **out-of-focus** blurs
- TV norm does not penalize smooth transitions as observed in **Gaussian** or **scatter** blurs
- Same properties are true for the recovery of images

(Rudin, Osher, Fatemi '93; C & Wong '98)

TV Blind Deconvolution Model

Objective: (C. and Wong (IEEE TIP, 1998))

$$\min_{u,k} F(u,k)$$

$$F(u,k) = \|u * k - u_{obs}\|^2 + a_1 \|u\|_{TV} + a_2 \|k\|_{TV}$$

Subject to:

$$u, k \geq 0$$

$$\int k(x,y)\,dxdy = 1$$

$$k(x,y) = k(-x,-y)$$

Alternating Minimization Algorithm

- Variational Model:

$$\min_{u,k} \{ F(u,k) = \|u * k - u_{obs}\|^2 + \alpha_1 \|u\|_{TV} + \alpha_2 \|k\|_{TV} \}$$

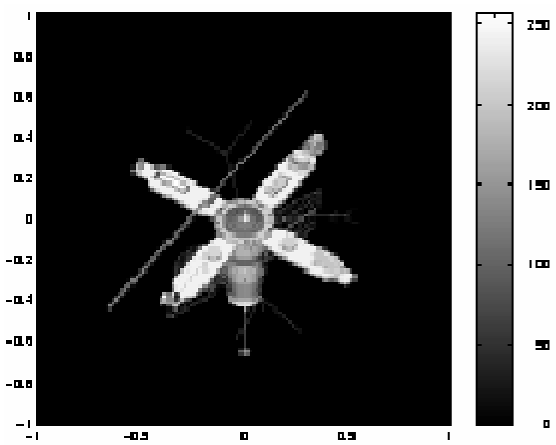
- Alternating Minimization Algorithm: (You and Kaveh '98)

$$F(u^{n+1}, k^n) = \min_u F(u, k^n)$$

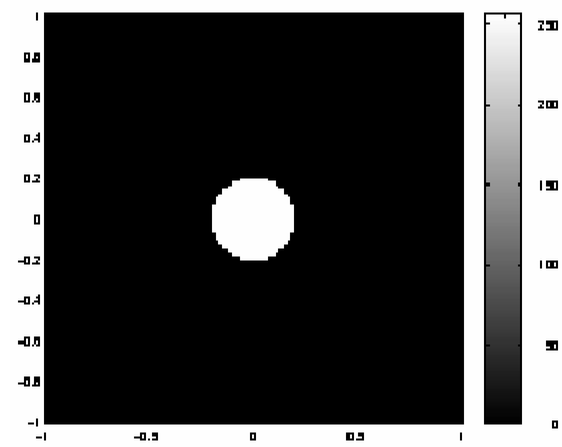
$$F(u^{n+1}, k^{n+1}) = \min_k F(u^{n+1}, k)$$

- a_1 determined by signal-to-noise ratio
- a_2 determined by focal length
- Globally convergent with H-1 regularization (C & Wong '00).

Data for Simulations

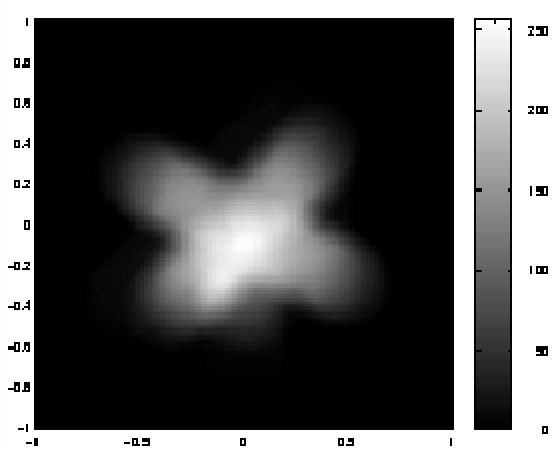


Satellite Image

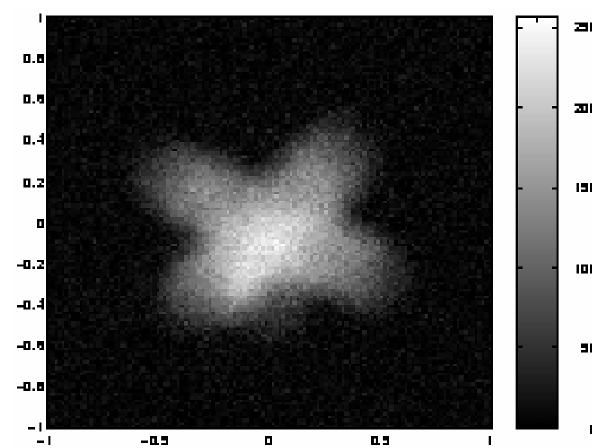


Out-of-focus Blur

127-by-127
Pixels



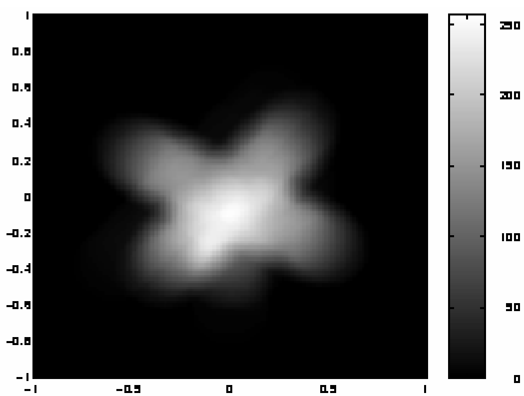
Blurred Image



Blurred and Noisy
Image

Blind v.s. non-Blind Deconvolution

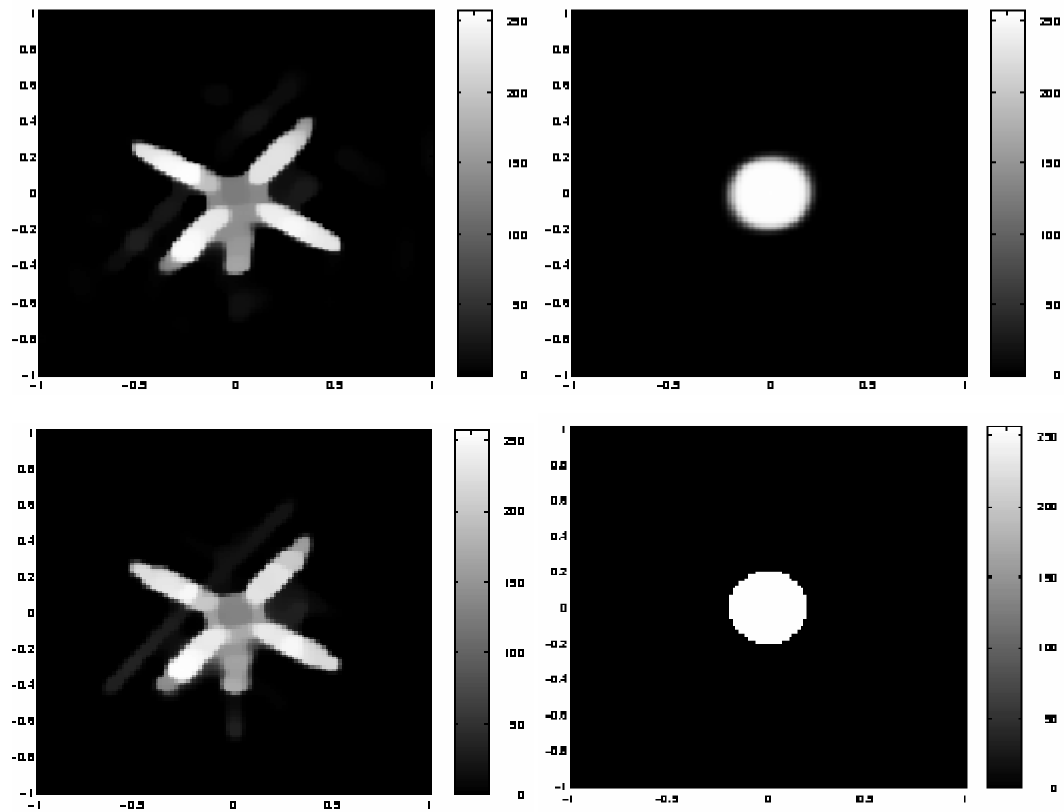
Observed Image



SNR = ¥ dB

Recovered Image PSF

Blind

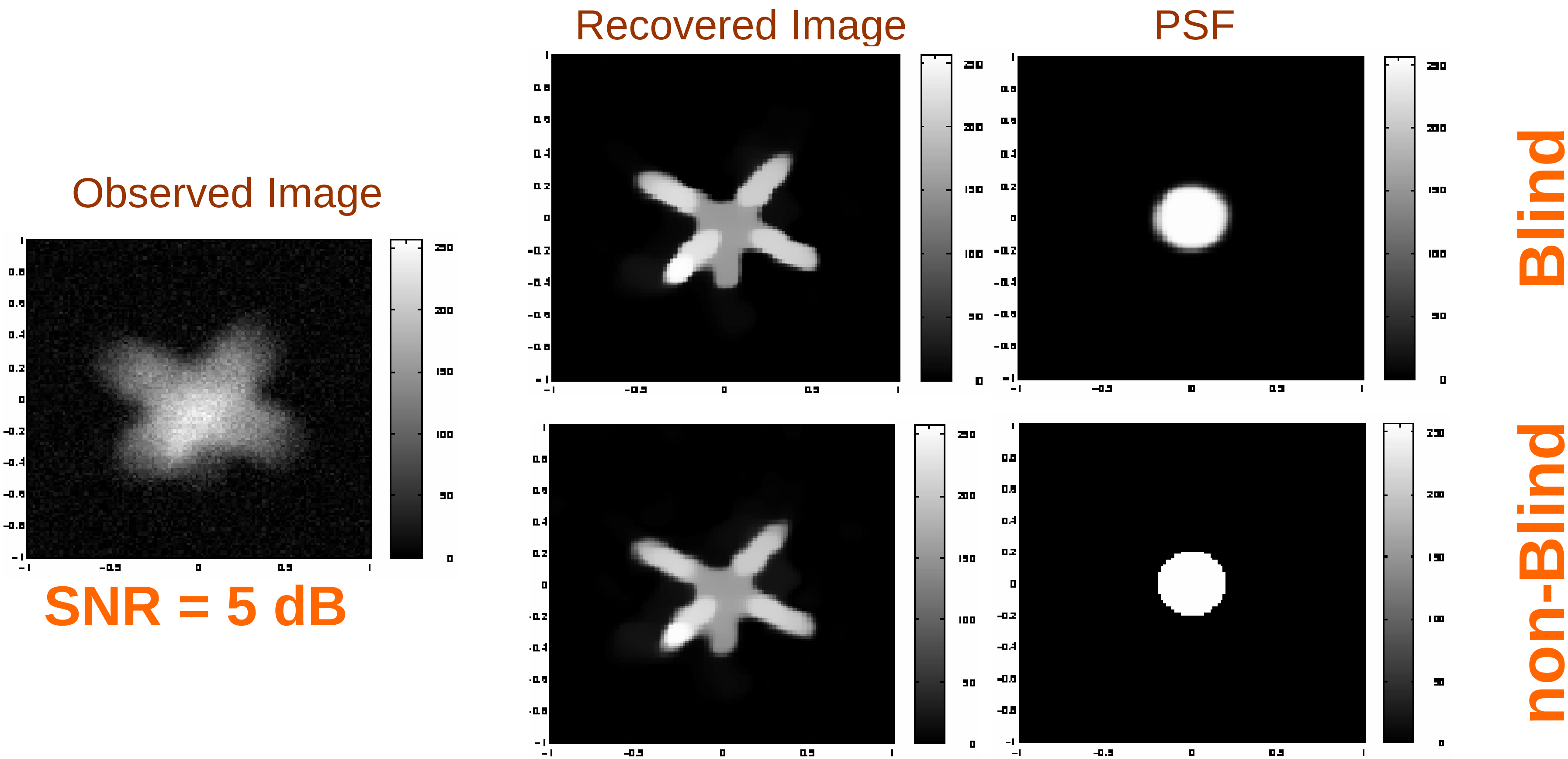


non-Blind

$\alpha_1 = 2\times10^{-6}$, $\alpha_2 = 1.5\times10^{-5}$

- Recovered images from blind deconvolution are almost as good as those recovered with the exact PSF
- Edges in both image and PSF are well-recovered

Blind v.s. non-Blind Deconvolution

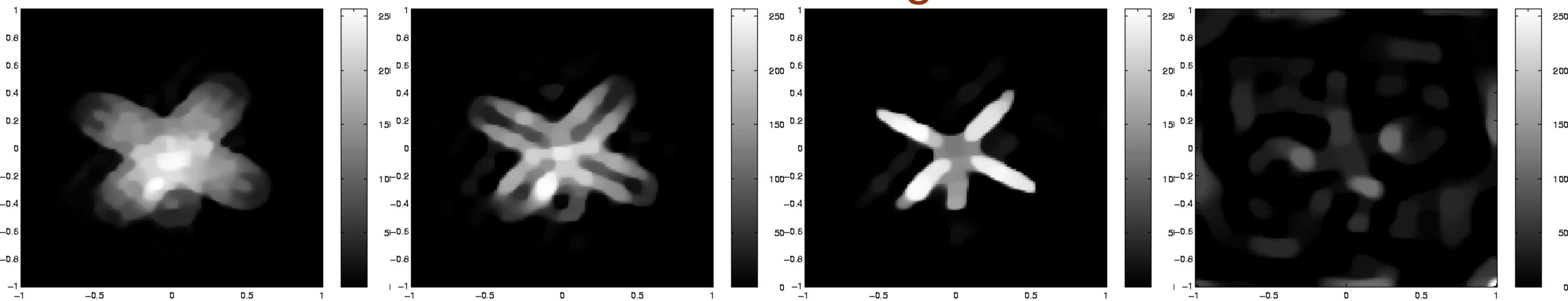


$\alpha_1 = 2\times10^{-5}$, $\alpha_2 = 1.5\times10^{-5}$

- Even in the presence of *high noise level*, recovered images from blind deconvolution are almost as good as those recovered with the exact PSF

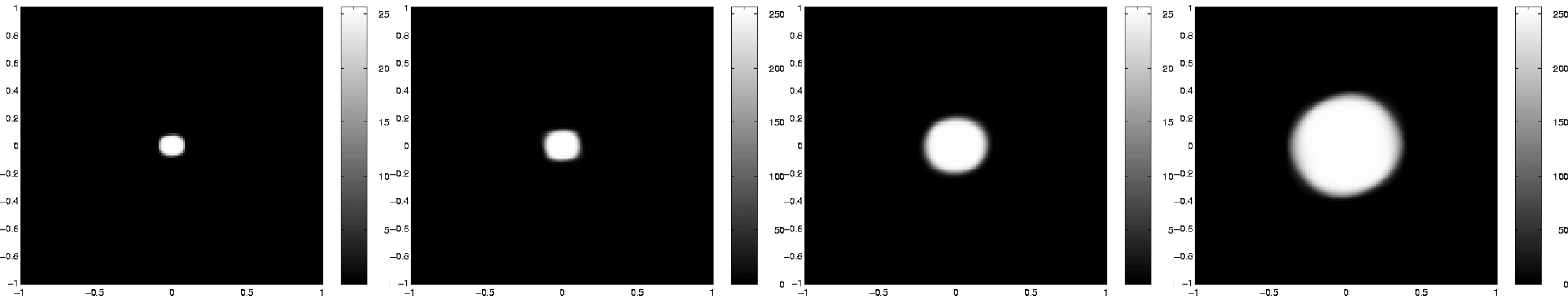
Controlling Focal-Length

Recovered Images



Recovered Blurring Functions

($\alpha_1 = 2\times10^{-6}$)

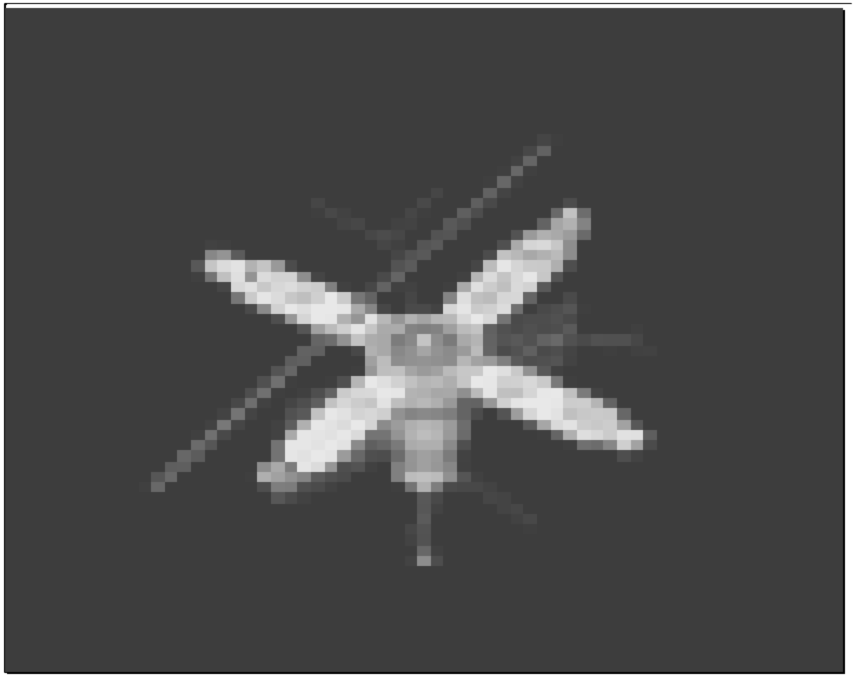


α_2: 0 | 1×10^{-7} | 1×10^{-5} | 1×10^{-4}

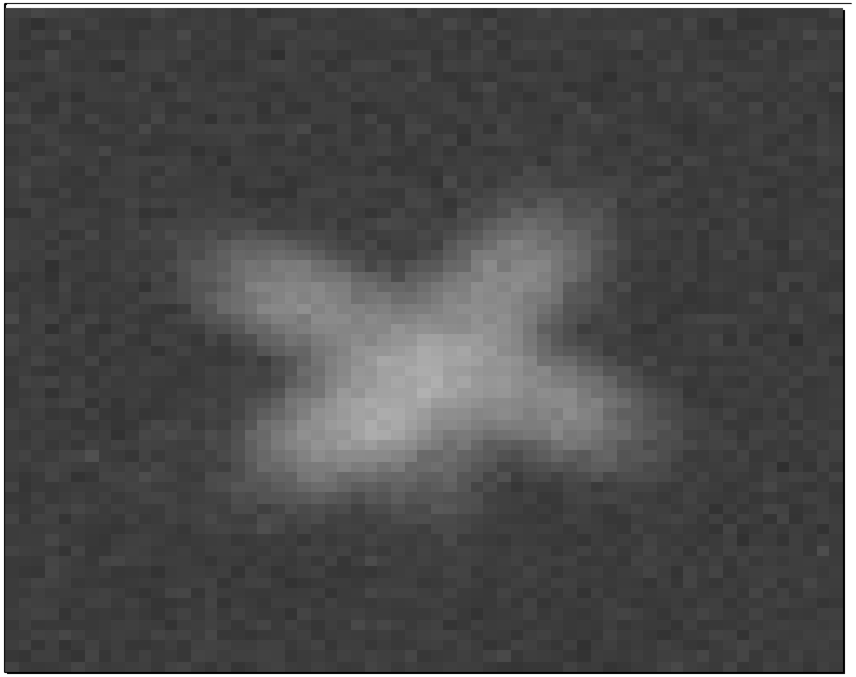
The parameter α_2 controls the focal-length

Auto-Focusing (on-going research)

Data for Simulations:



Clean image

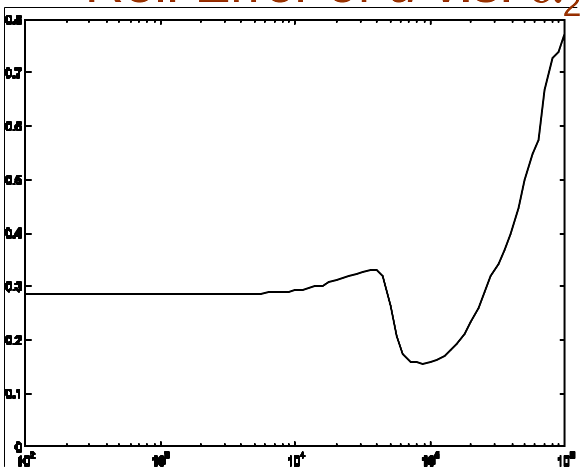


Noisy image (SNR=15dB)

63-by-63 pixels

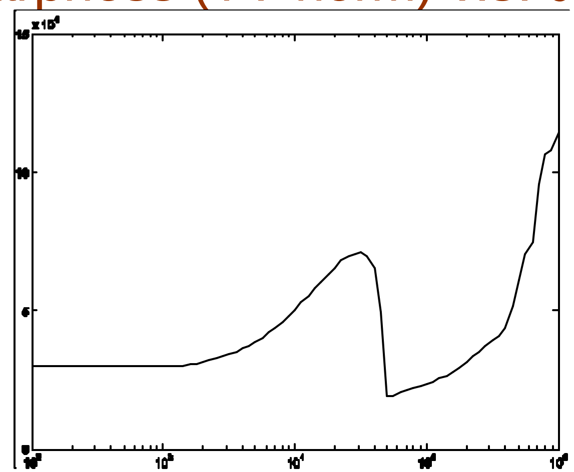
Measuring Image Sharpness

Rel. Error of *u* v.s. α_2

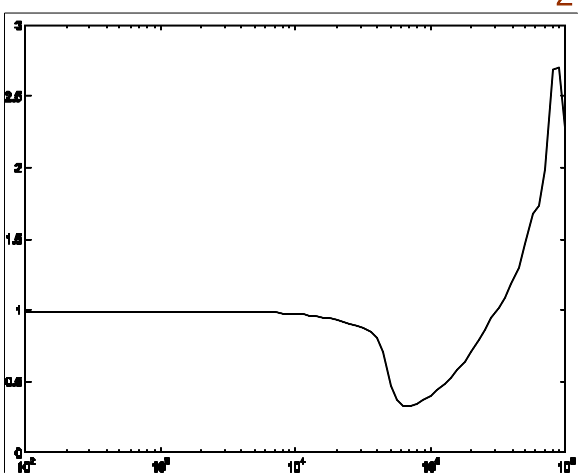


$a_1 = 0.2$ (optimal)

Sharpness (TV-norm) v.s. α_2

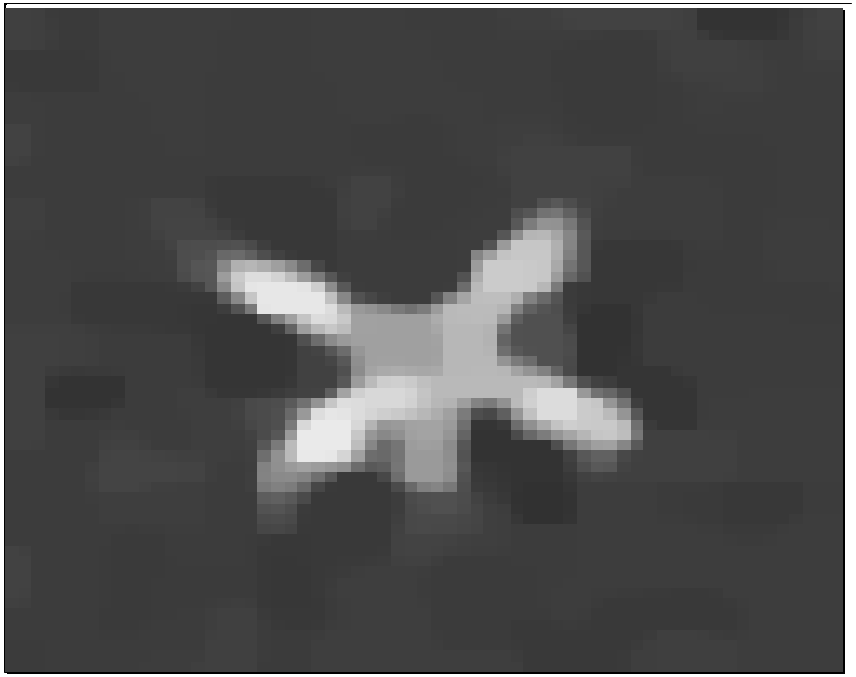


Rel. Error of *k* v.s. α_2



The minimum of the sharpness function agrees with that of the rel. errors of *u* and *k*

Optimal Restored Image
(minimizer of rel. error in u)



Auto-focused Image
(minimizer of sharpness func.)

Preliminary TV-based sharpness func. yields
reasonably focused results

Generalizations to Multi-Channel Images

- E.g. Color images compose of red, blue and green channels
- In practice, inter-channel interference often exists
  - Total blur = intra-channel blur + inter-channel blur
  - Multi-channel TV regularization for both the image and PSF is used

Model for the Multi-channel Case

Color image (Katsaggelos et al, SPIE 1994):

$$H_k u + noise = u_{obs}$$

$$\begin{pmatrix} \frac{5}{7}H_{k_1} & \frac{1}{7}H_{k_2} & \frac{1}{7}H_{k_2} \\ \frac{1}{7}H_{k_2} & \frac{5}{7}H_{k_1} & \frac{1}{7}H_{k_2} \\ \frac{1}{7}H_{k_2} & \frac{1}{7}H_{k_2} & \frac{5}{7}H_{k_1} \end{pmatrix} \begin{pmatrix} u^R \\ u^G \\ u^B \end{pmatrix} + \text{noise} = \begin{pmatrix} u^R_{obs} \\ u^G_{obs} \\ u^B_{obs} \end{pmatrix}$$

k_1: within channel blur

k_2: between channel blur

Restoration Model

$$\min_{u,k} \left\| H_k u - u_{obs} \right\|^2 + a_1 TV_m(u) + a_2 TV_{m^2}(k)$$

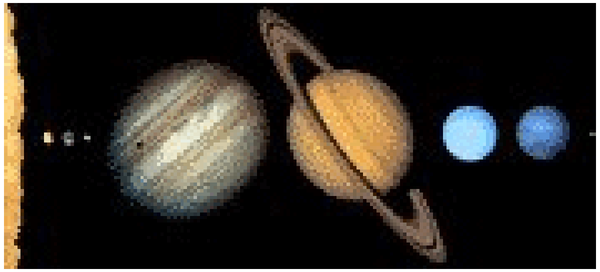
m-channel TV-norm

$$TV_m(u) = \sqrt{\sum_{i=1}^{m} \left\| u_i \right\|_{TV}^2}$$

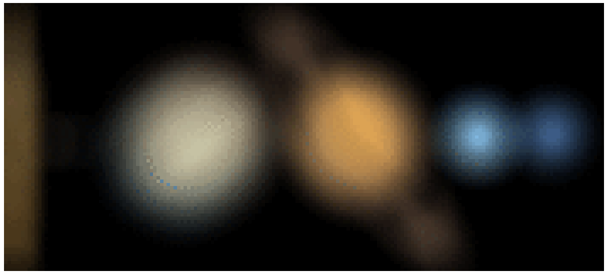
Examples of Multi-Channel Blind Deconvolution

(C. and Wong (SPIE, 1997))

Original image

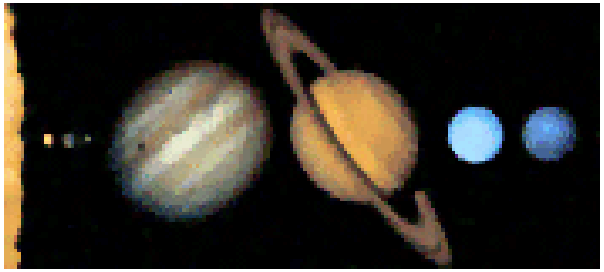


Out-of-focus blurred

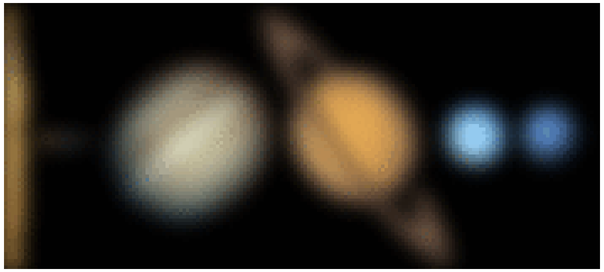


blind

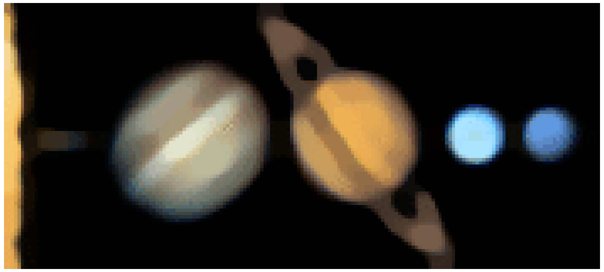
non-blind



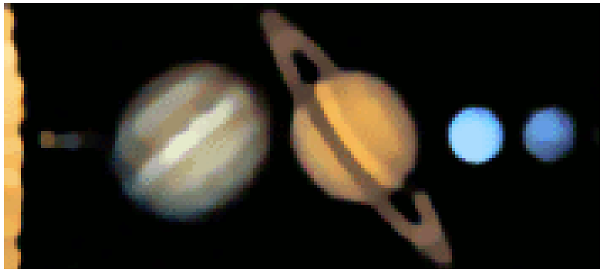
Gaussian blurred



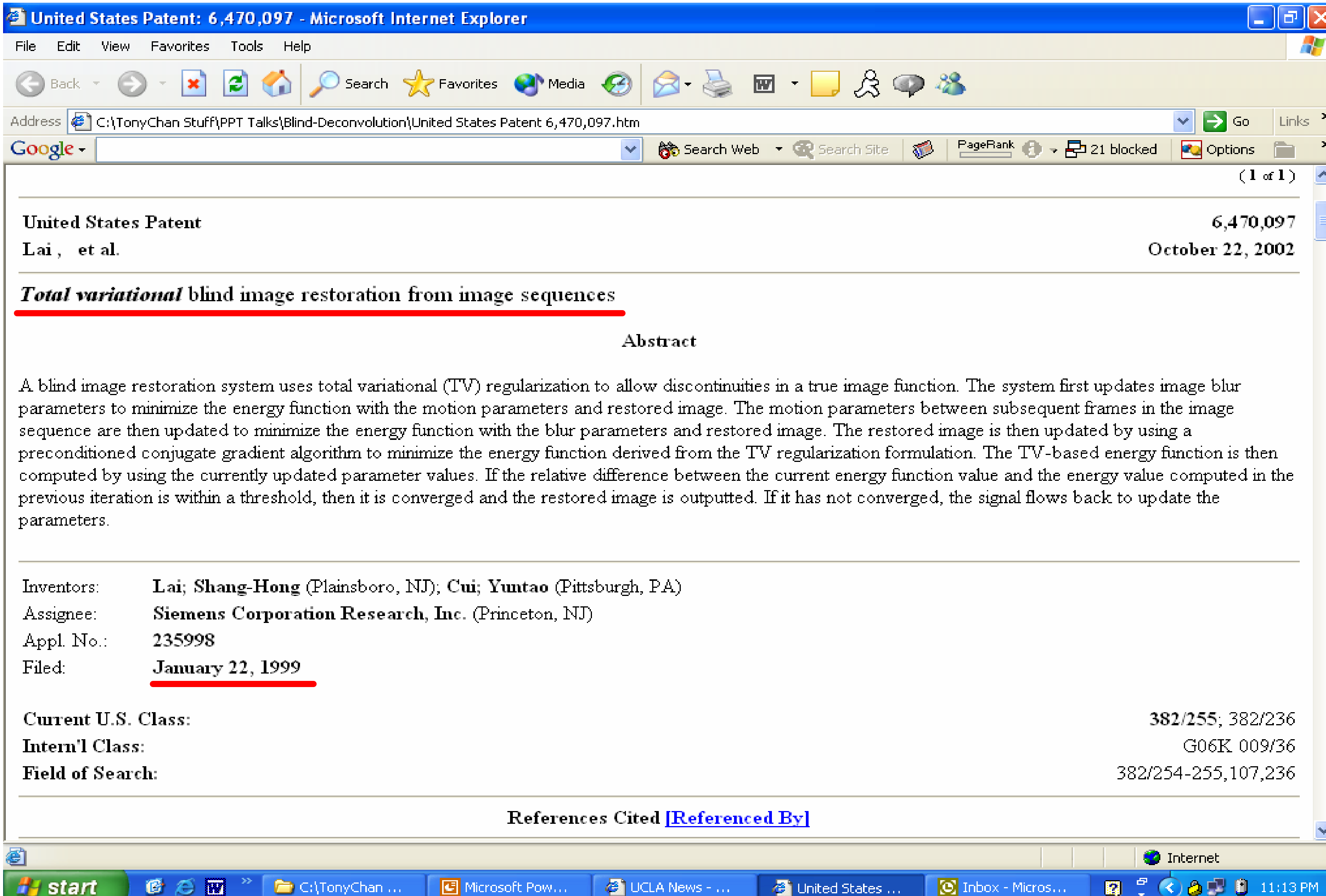
blind



non-blind



TV Blind Deconvolution Patented!



United States Patent — **6,470,097**

Lai , et al. — **October 22, 2002**

Total variational **blind image restoration from image sequences**

Abstract

A blind image restoration system uses total variational (TV) regularization to allow discontinuities in a true image function. The system first updates image blur parameters to minimize the energy function with the motion parameters and restored image. The motion parameters between subsequent frames in the image sequence are then updated to minimize the energy function with the blur parameters and restored image. The restored image is then updated by using a preconditioned conjugate gradient algorithm to minimize the energy function derived from the TV regularization formulation. The TV-based energy function is then computed by using the currently updated parameter values. If the relative difference between the current energy function value and the energy value computed in the previous iteration is within a threshold, then it is converged and the restored image is outputted. If it has not converged, the signal flows back to update the parameters.

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Assignee:	**Siemens Corporation Research, Inc.** (Princeton, NJ)
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Filed:	**January 22, 1999**

Current U.S. Class:	**382/255**; 382/236
Intern'l Class:	G06K 009/36
Field of Search:	382/254-255,107,236

References Cited [Referenced By]

United States Patent: 6,470,097 - Microsoft Internet Explorer

Address C:\TonyChan Stuff\PPT Talks\Blind-Deconvolution\United States Patent 6,470,097.htm

You and Kaveh's regularization formulation used an H.sup.1 norm for the smoothness constraint on the image and the blur. This smoothness constraint prohibits discontinuities in the solution. Unfortunately, there usually exist sharp discontinuities in the true images. The desirable details of the true image can be lost due to the smoothed discontinuities. Although You and Kaveh proposed a weighted regularization to alleviate this problem, this weighting scheme is somehow ad hoc and the parameters involved in this scheme may need to be tuned in a case by case basis.

A new regularization approach that employed the total variation (TV) norm in stead of the standard H.sup.1 norm for the image constraint was proposed by L. Rudin, S. Osher, and E. Fatemi in "Nonlinear Total Variation Based Noise Removal Algorithms", Physica D., Vol. 60, pp. 259-268, 1992 for the image denoising problem. The TV regularization has been proved to be capable of preserving discontinuities while imposing smoothness constraints and it is effective for recovering blocky images. T. F. Chan and C. K. Wong in "Total Variation Blind Deconvolution", IEEE Trans. Image Processing, Vol. 7, No. 3, pp. 370-375, 1998 modified You and Kaveh's regularization formulation by using the TV norm for the smoothness constraint on the image as well as the blur function instead of the H.sup.1 norm, thus preserving the discontinuities in the recovered image function.

SUMMARY OF THE INVENTION

The present invention provides a new formulation for blind image restoration from an image sequence. Total variational (TV) regularization is employed to allow discontinuities in a true image function. An iterative alternating algorithm using quasi-Newton iterations is provided to solve an image-blur coupled nonlinear optimization problem. This formulation is then extended to the blind image restoration from an image sequence by introducing motion parameters into a multi-frame data constraint.

The input to the blind image restoration system of the present invention contains an image sequence and initial values for image blur and motion parameters. Within an image blur parameters updater, the system first updates the image blur parameters by using Quasi-Newton iterations to minimize the energy function with the motion parameters and restored image, fixed with their current values. After that, the signal flows to a motion parameters updater where the motion parameters between subsequent frames in the image sequence are updated by using Newton iterations to minimize the energy function with the blur parameters and restored image, fixed with their current values. The restored image is then updated in a restored image updater by using a preconditioned conjugate gradient algorithm to minimize the energy function derived from the total variational (TV) regularization formulation. The TV-based energy function is then computed in a TV-based energy function computer by using the currently updated parameter values. Within a converged decider, if the relative difference between the current energy function value and the energy value computed in the previous iteration is within a threshold, then it is converged and the restored image is outputted. If it has not converged, the signal flows back to the image blur parameters updater to update the parameters.

BRIEF DESCRIPTION OF THE DRAWINGS

References

Total Variation Blind Deconvolution

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3. C. and C. K. Wong, *Convergence of the Alternating Minimization Algorithm for Blind Deconvolution*, LAA, 316(1-3), 259-286, 2000.
4. R. H. Chan, C. and C. K. Wong, *Cosine Transform Based Preconditioners for Total Variation Deblurring*, IEEE Trans. Image Proc., 8 (1999), pp. 1472-1478

Potential Applications to Astronomical Imaging

- Phase Diversity
  - Computational speed important?
  - Optimal phase shift?
- High/Super Resolution
  - Applicable in astronomy?
  - Auto calibrate?
- TV Blind Deconvolution
  - TV/Sharp edges useful?
  - Spatially varying blur?
  - Auto-focus: appropriate objective function?
  - How to incorporate a priori knowledge?