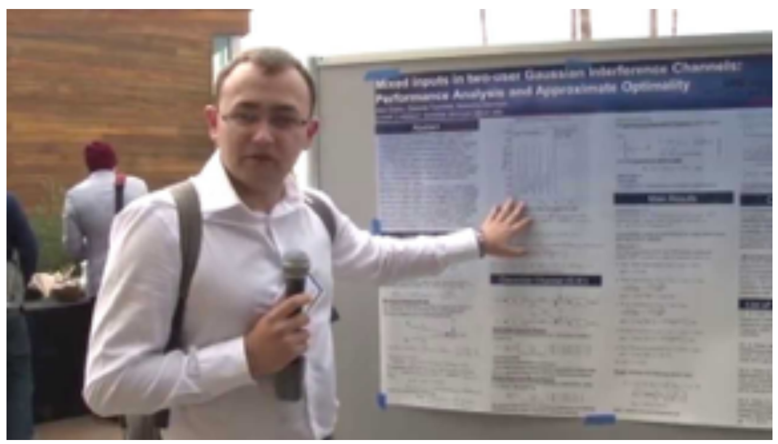
# Adding and removing assumptions in network information theory

Natasha Devroye




Alex Dysto

Daniela Tuninetti

portions based on his Ph.D. slides

***Natasha Devroye**, Associate Professor*

*Joint work with Alex Dytso, former Ph.D. student, postdoc at Princeton*

*Sara Shahi, current Ph.D. student*

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*Daniela Tuninetti, Professor*

ELECTRICAL AND COMPUTER ENGINEERING
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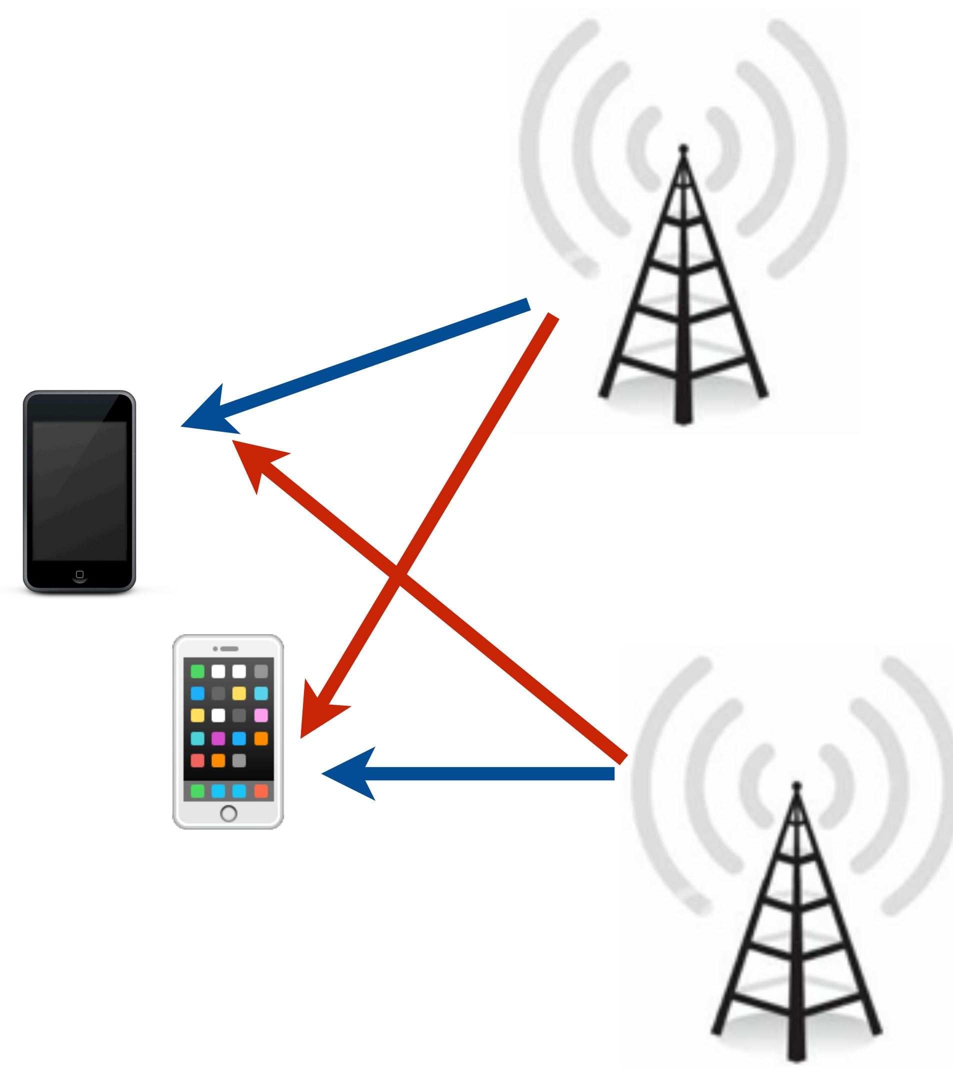
# Basic model for interfering wireless channels: the interference channel



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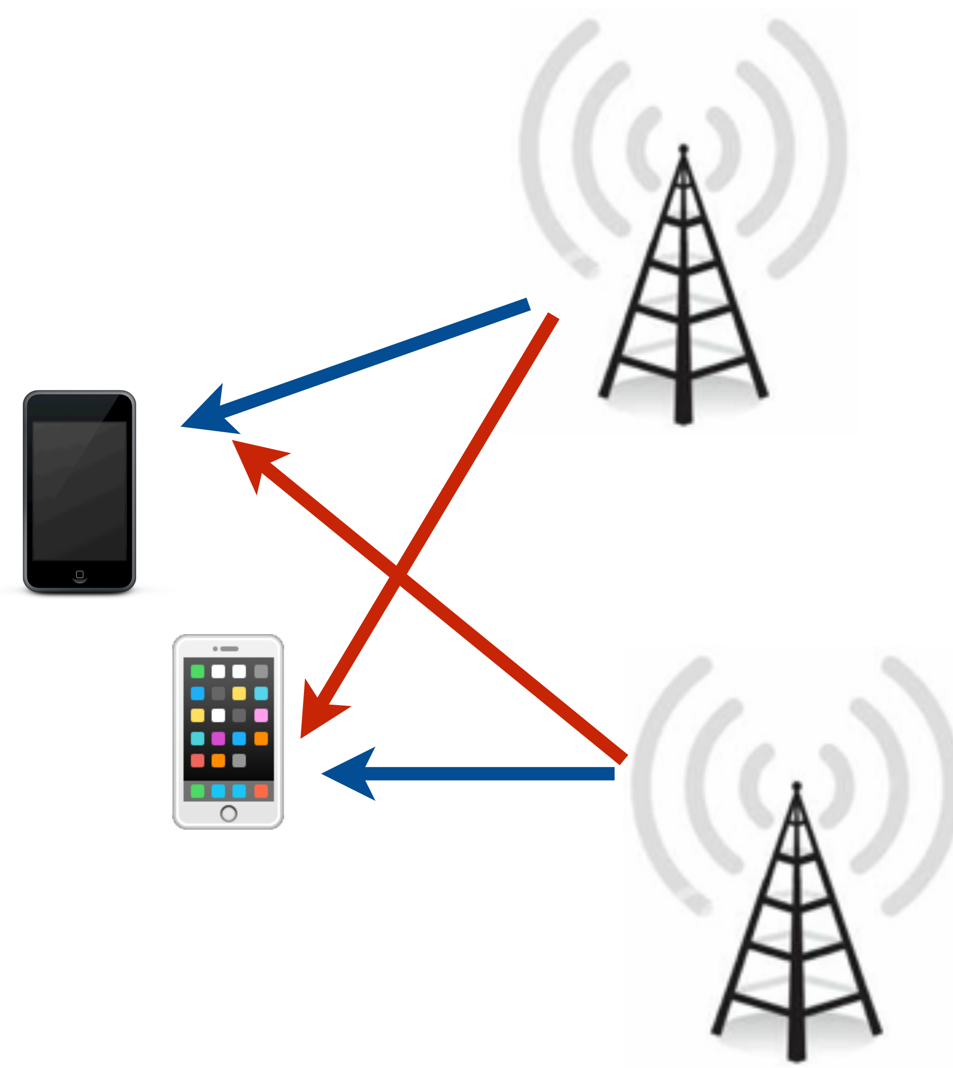

use this channel model to exemplify the effect (or lack thereof) of several commonly made network information theoretic assumptions

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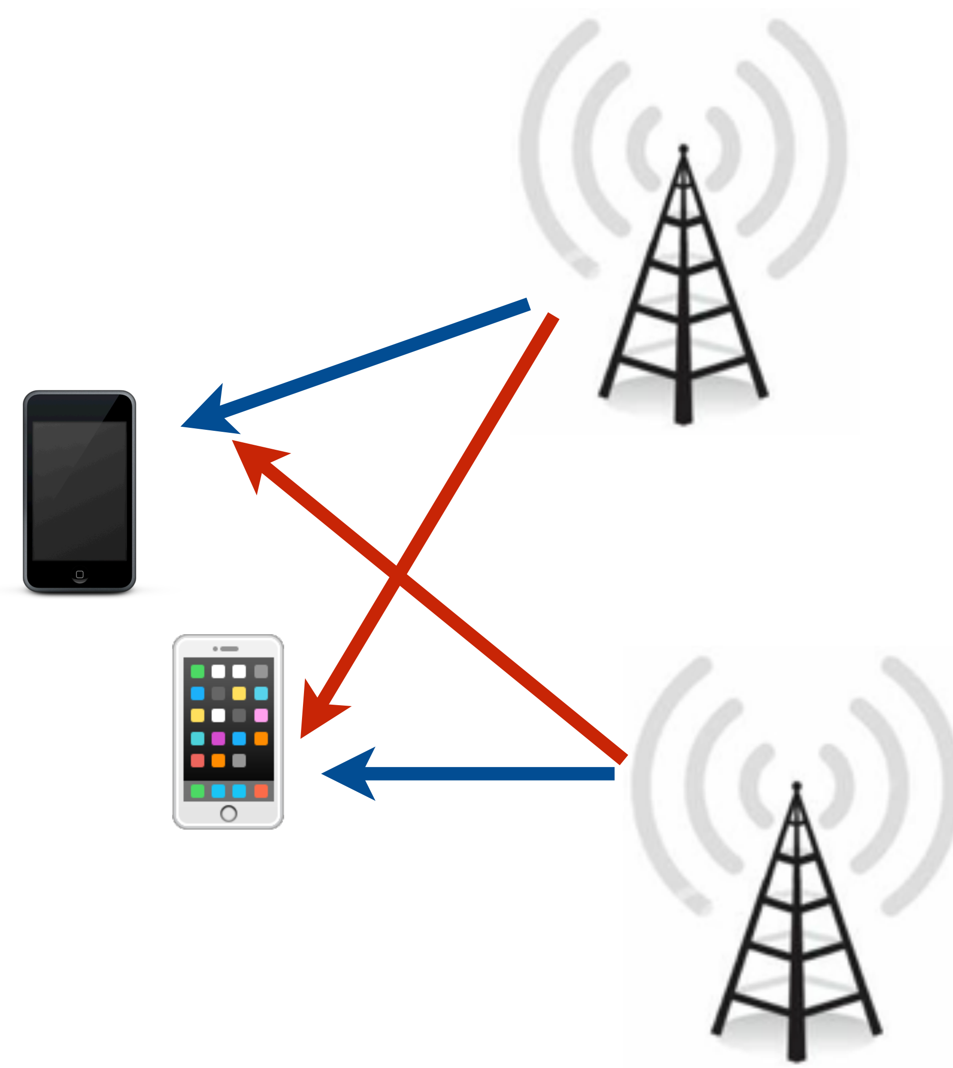

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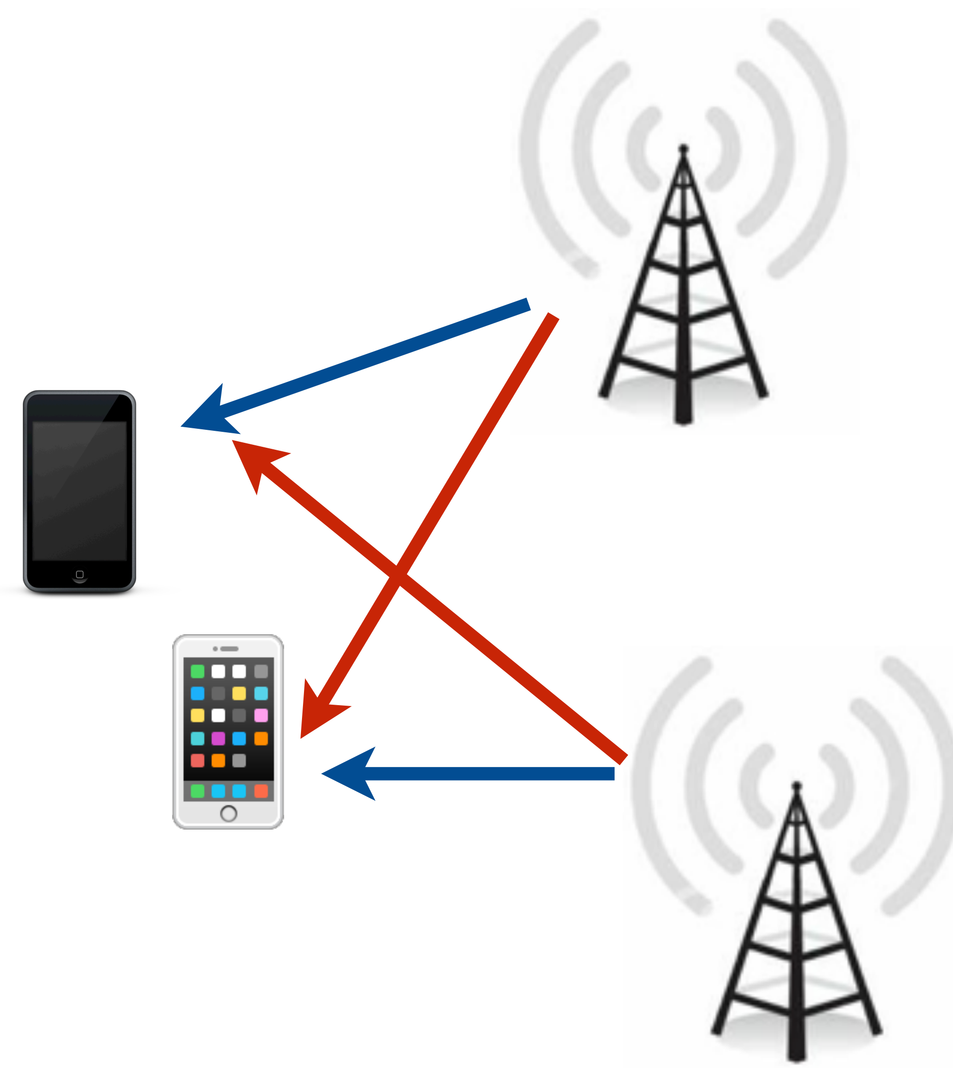

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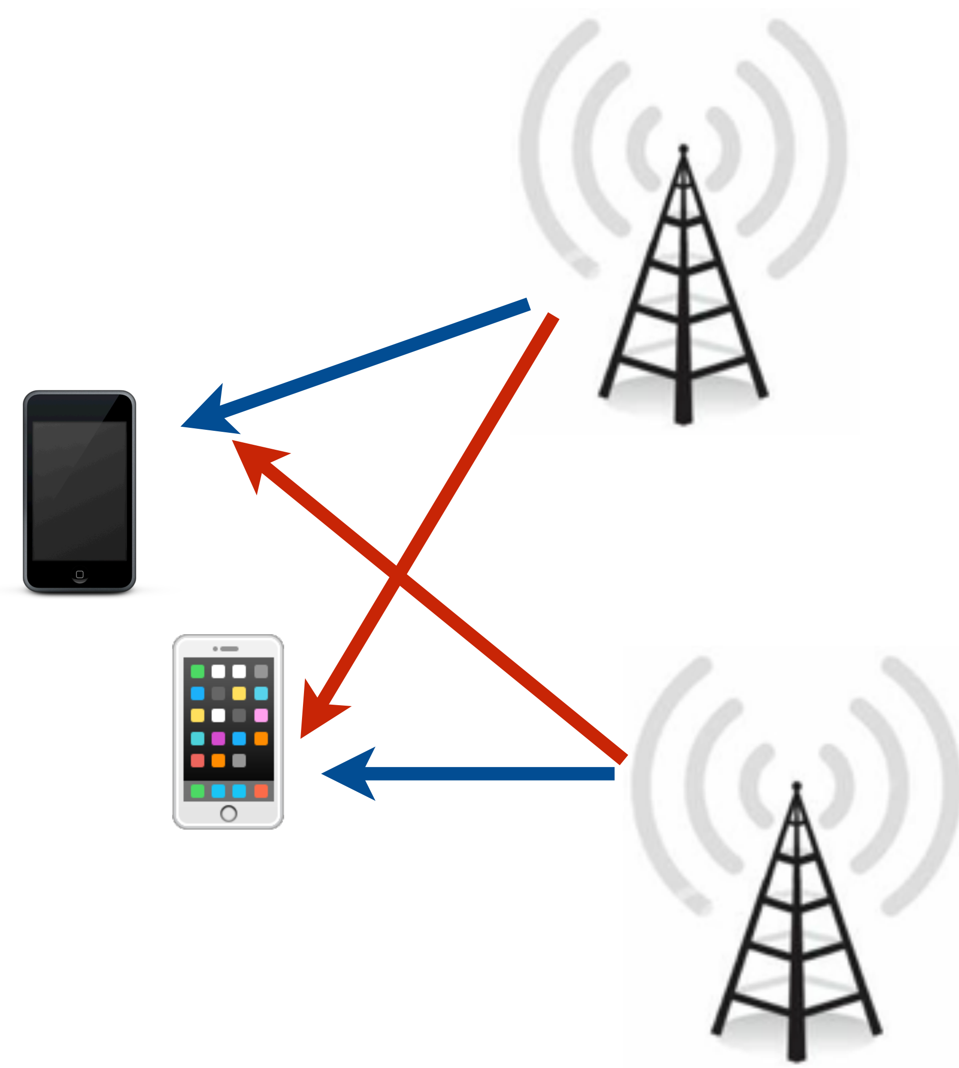

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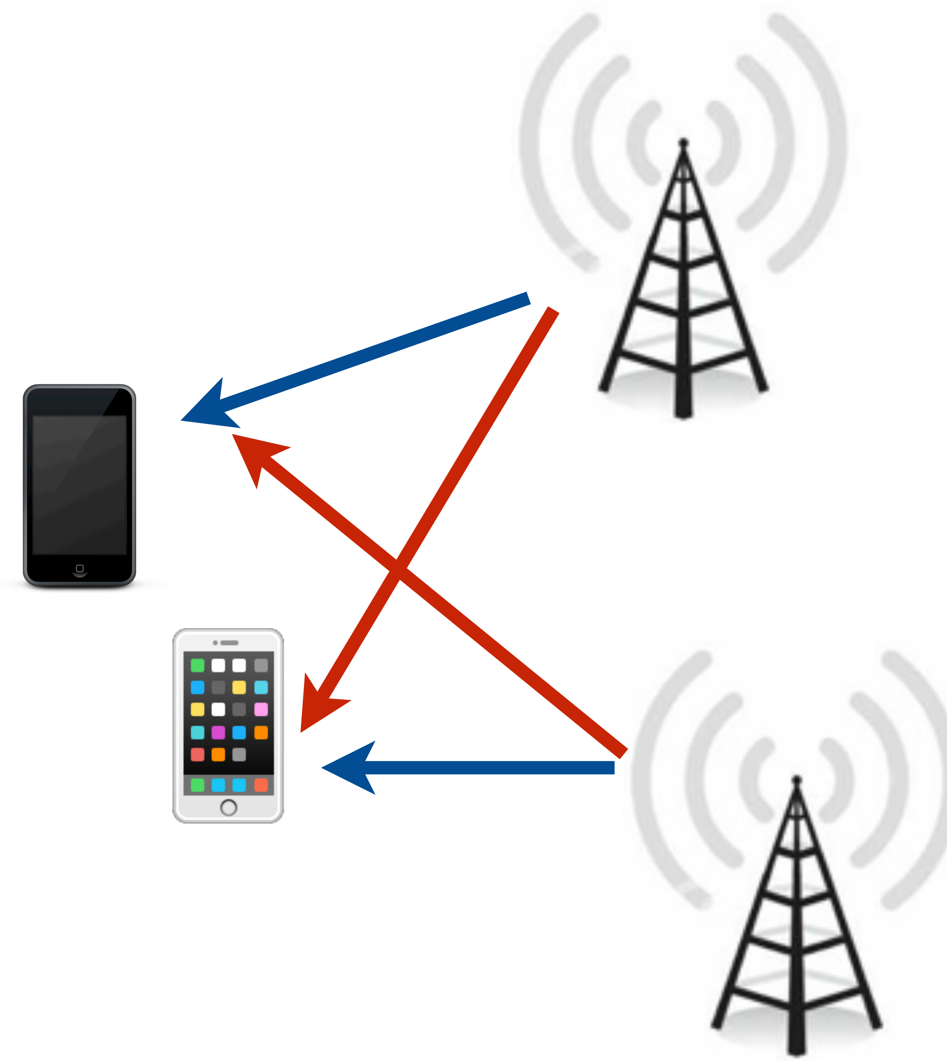

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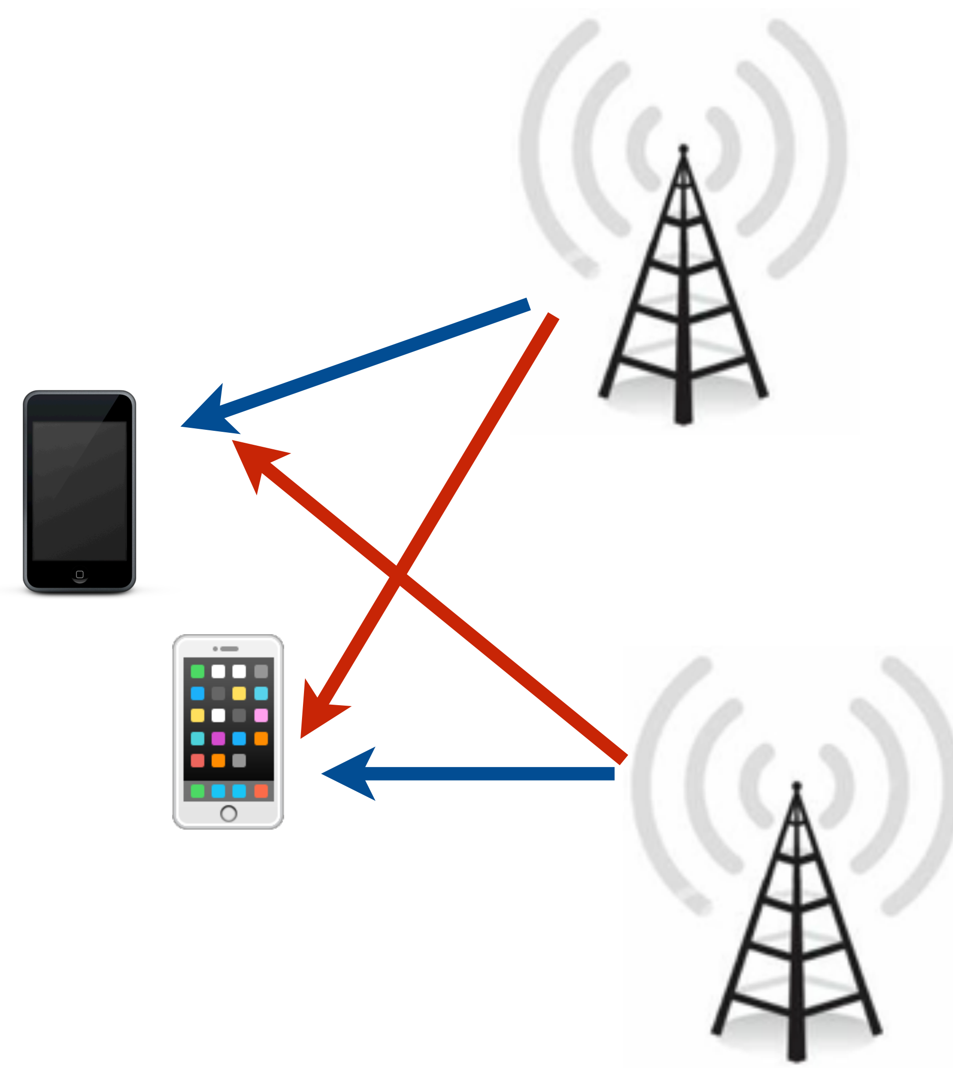

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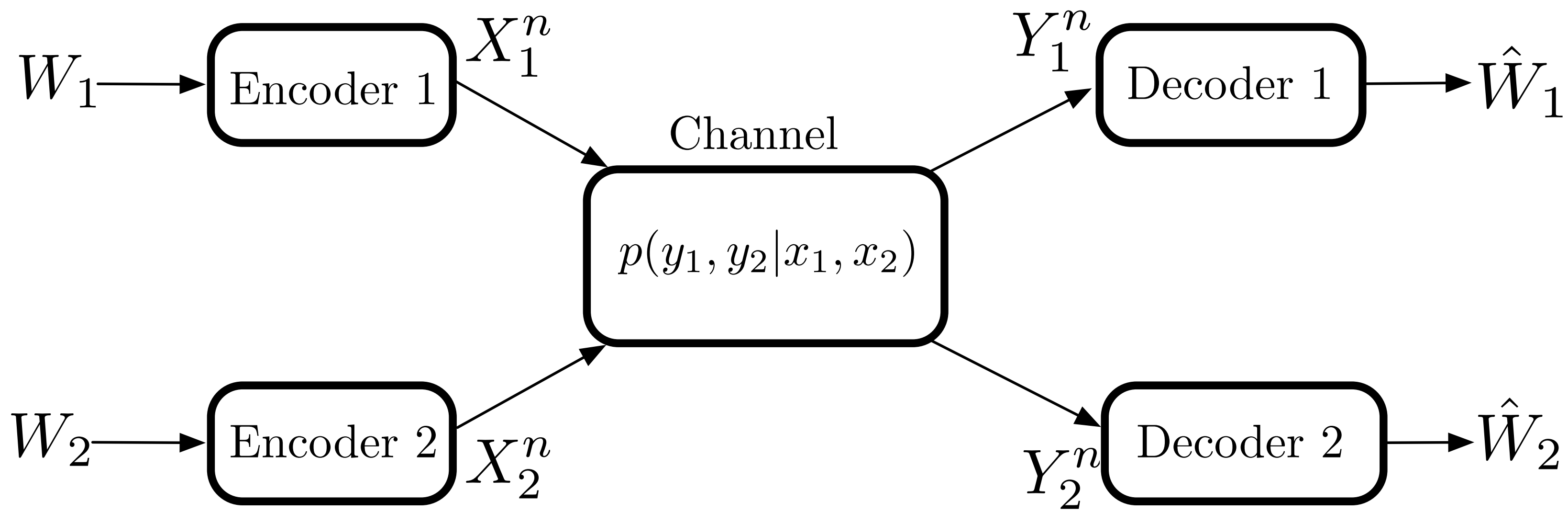
# Interference channel

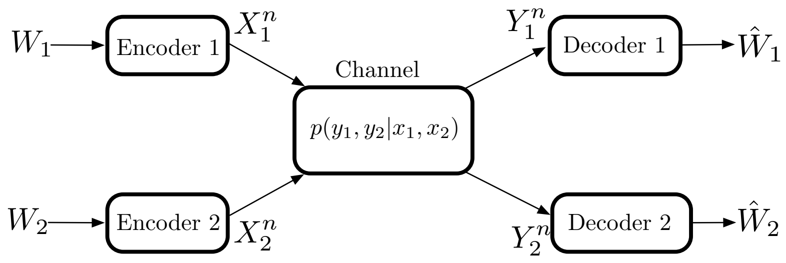



- a discrete memoryless interference channel (DM-IC) $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ consists of 4 finite sets/alphabets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$ and a collection of conditional pmfs $p(y_1, y_2|x_1, x_2)$

- sender $j = 1, 2$ sends an independent message $W_i$ to receiver $j$

- lower case $x$ is an instance of random variable $X$ in calligraphic alphabet $\mathcal{X}$

# Formal definition




- A $(2^{nR_1}, 2^{nR_2}, n)$ code for the IC consists of:
  1. Two message sets $[1 : 2^{nR_1}]$, and $[1 : 2^{nR_2}]$
  2. Two encoders:
  $$w_1 \in [1 : 2^{nR_1}] \to x_1^n(w_1)$$
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  $$y_1^n \to [1 : 2^{nR_1}] \cup \text{error}$$
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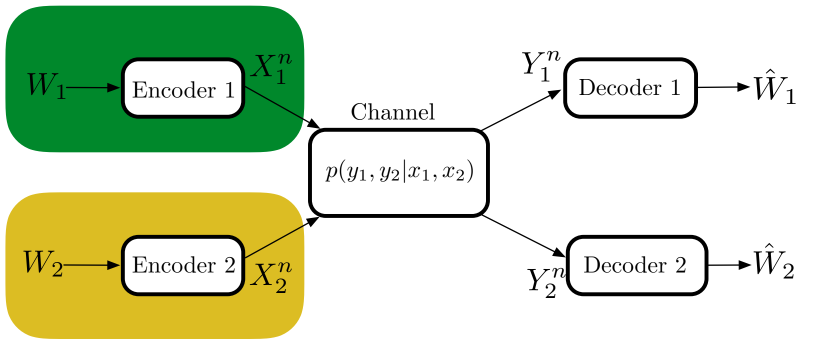


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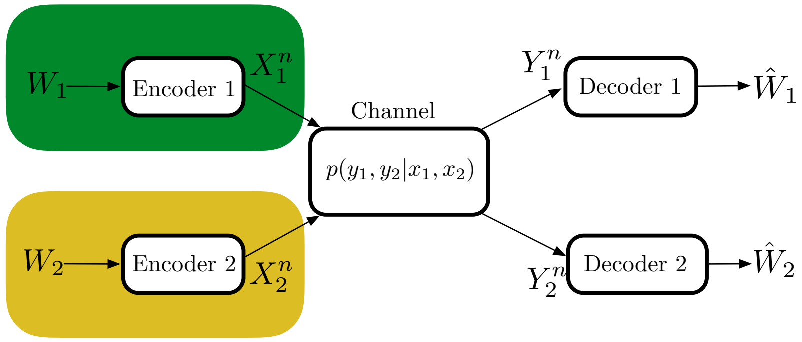


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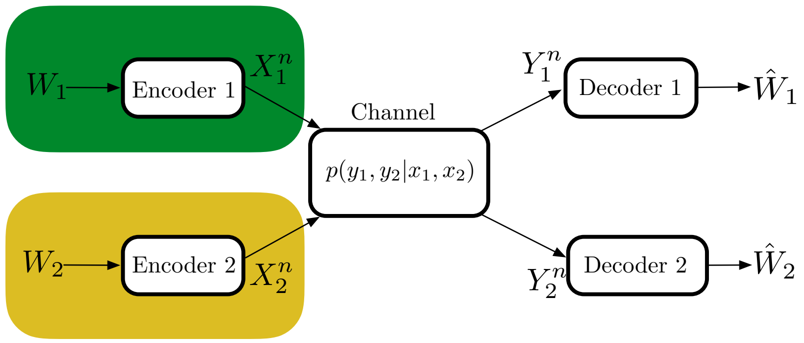


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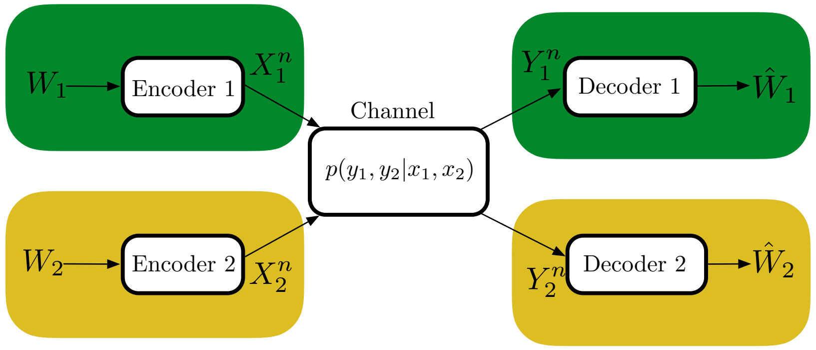


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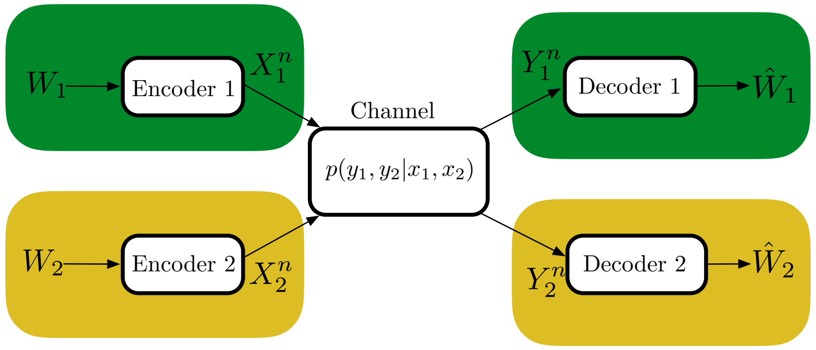


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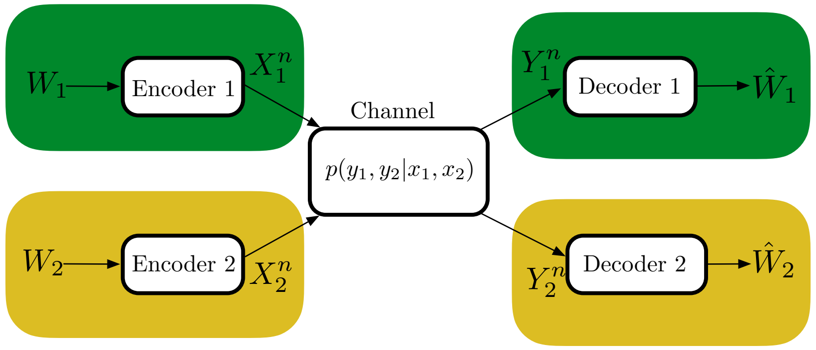


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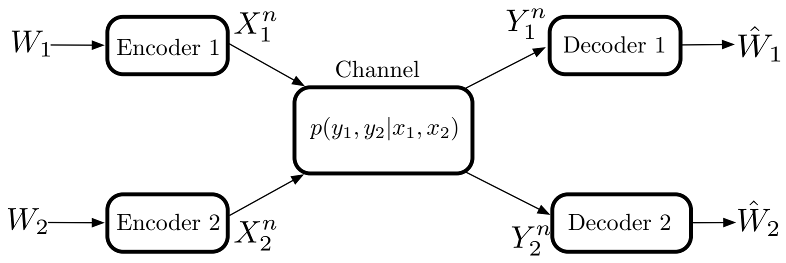


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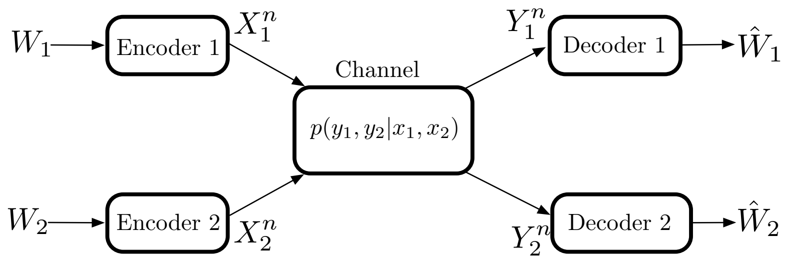


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- Rate pair $(R_1, R_2)$ is *achievable* if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes with $P_e^{(n)} \to 0$ as $n \to \infty$

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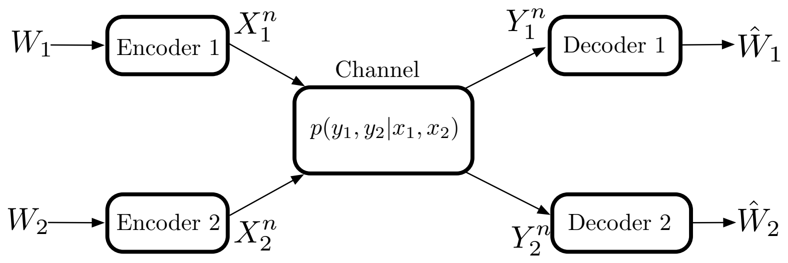


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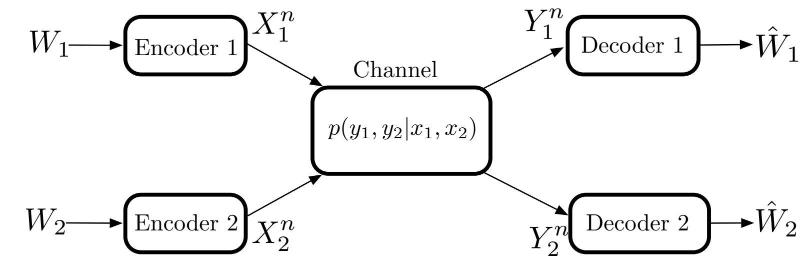


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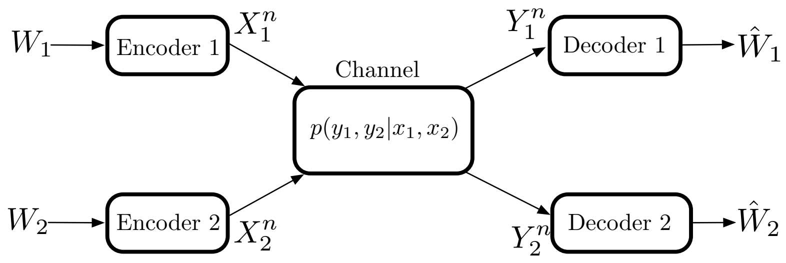


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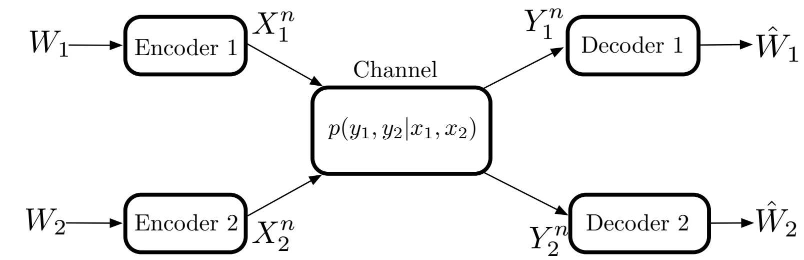
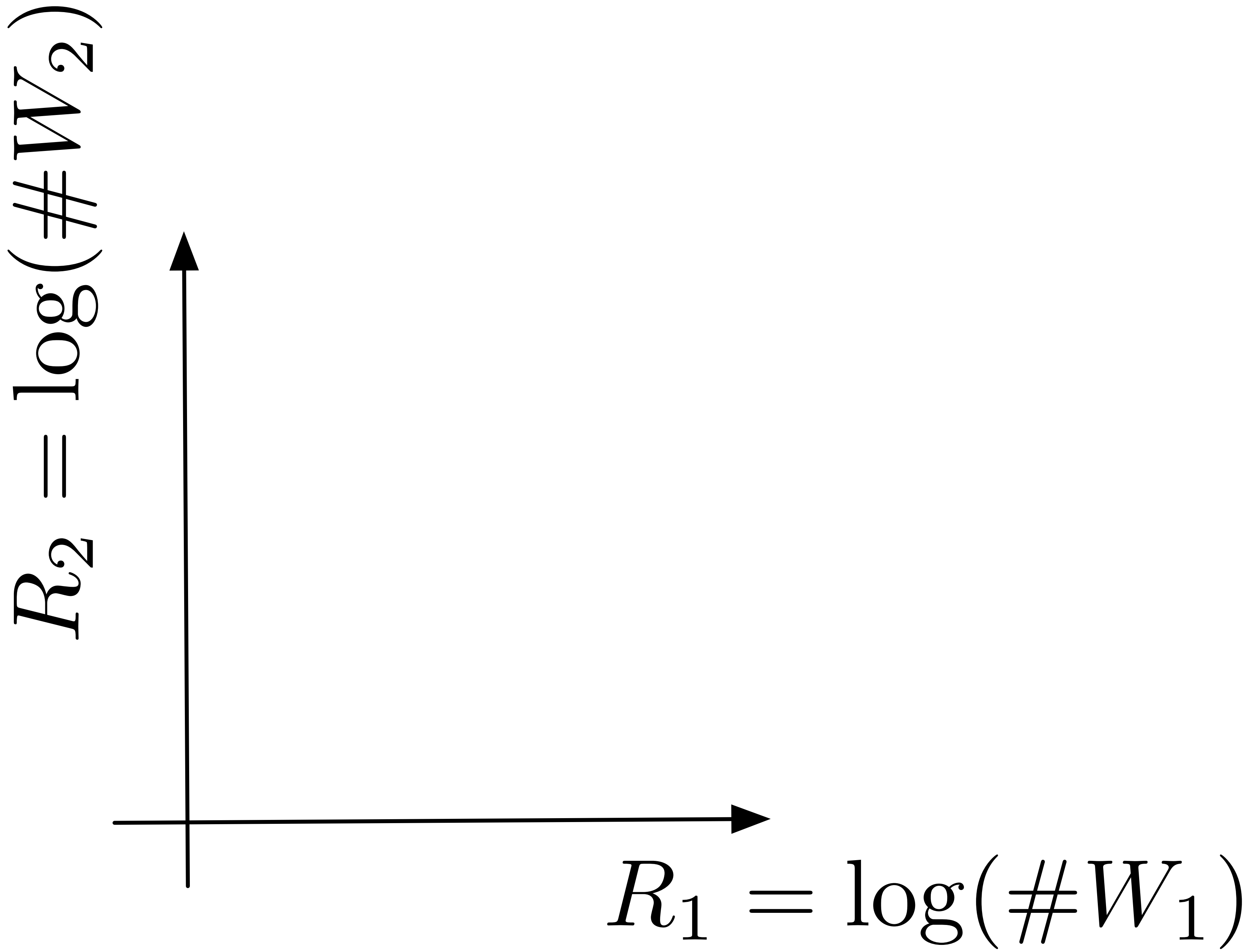
- Note: capacity region depends on $p(y_1, y_2|x_1, x_2)$ only through the marginals $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$

# Rate regions



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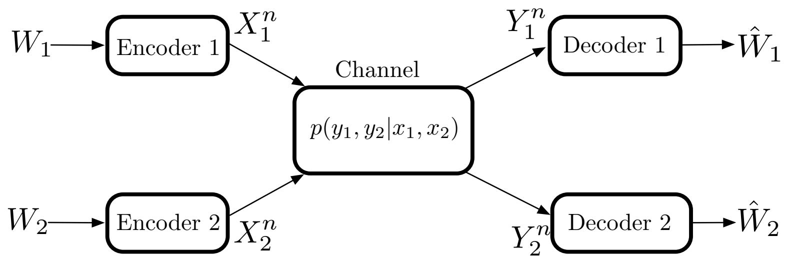


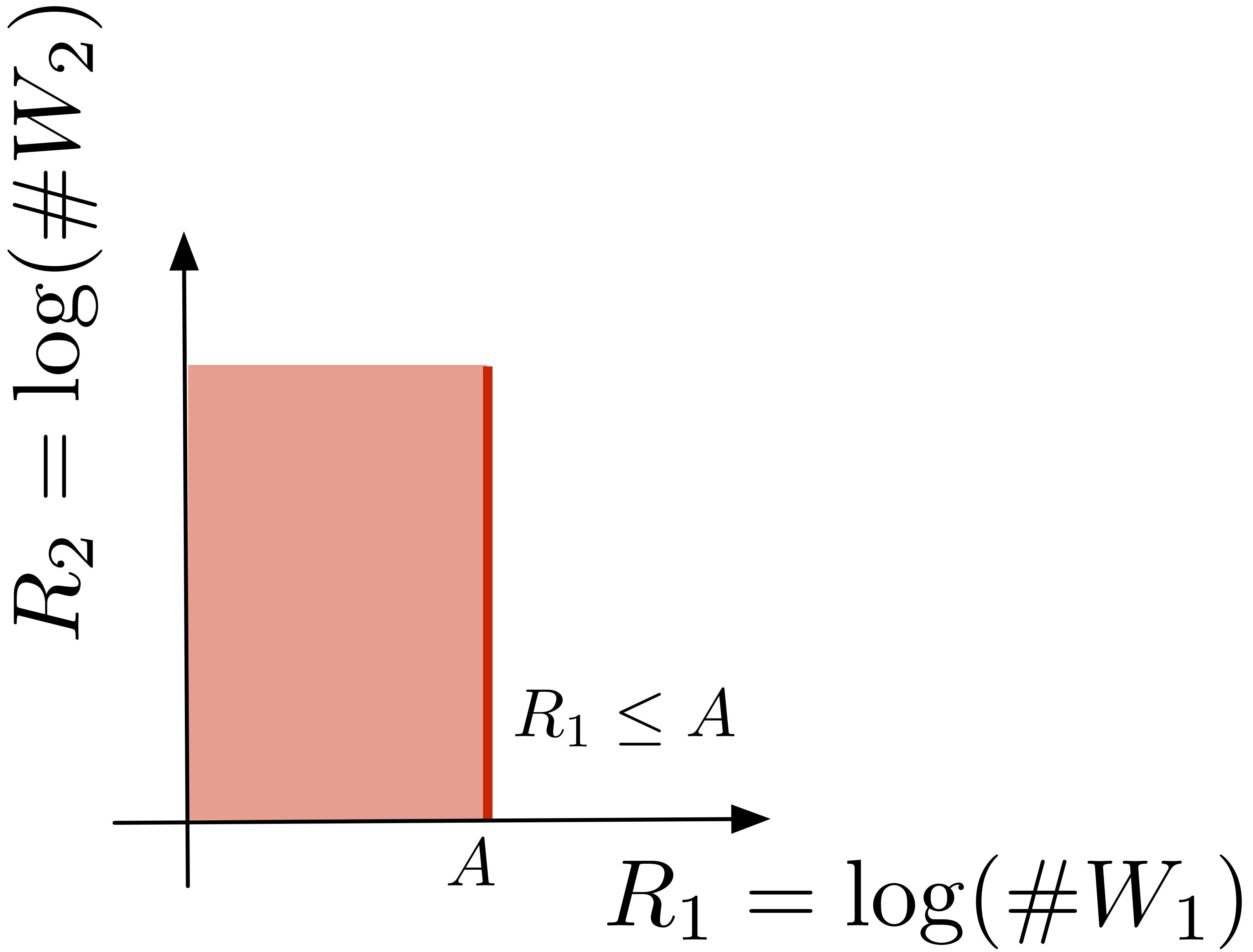

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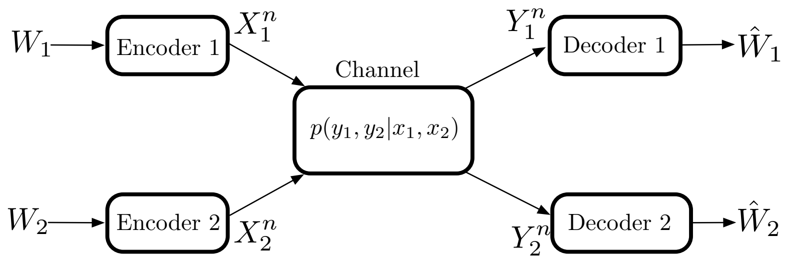


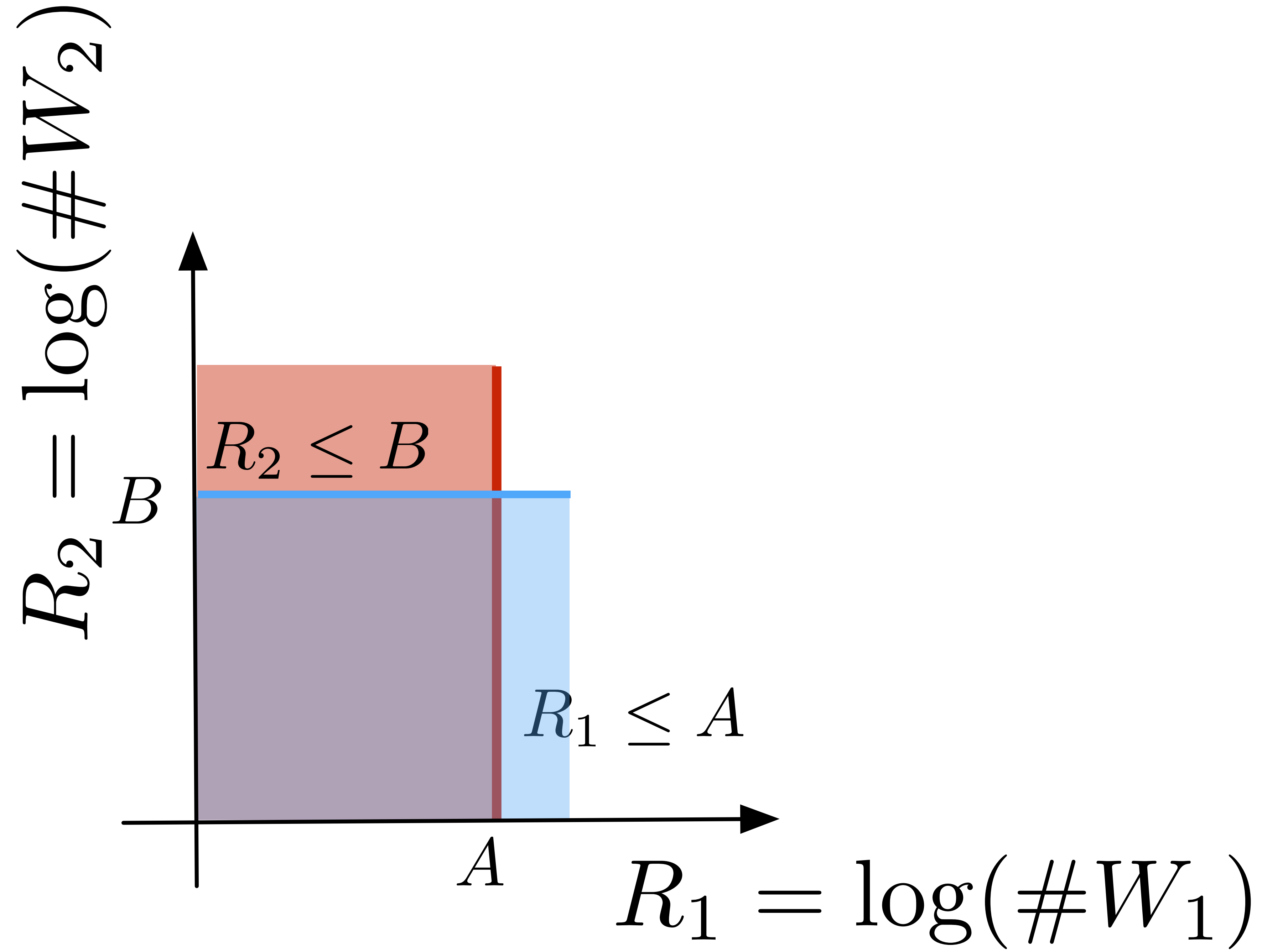

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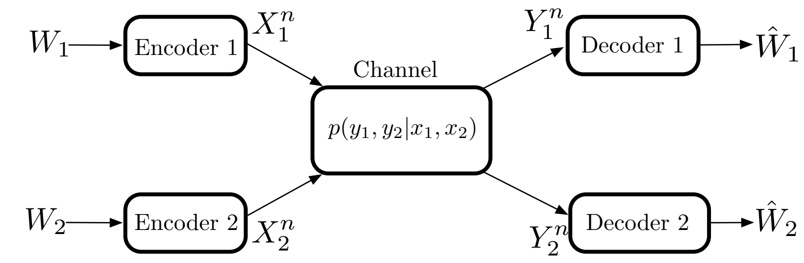


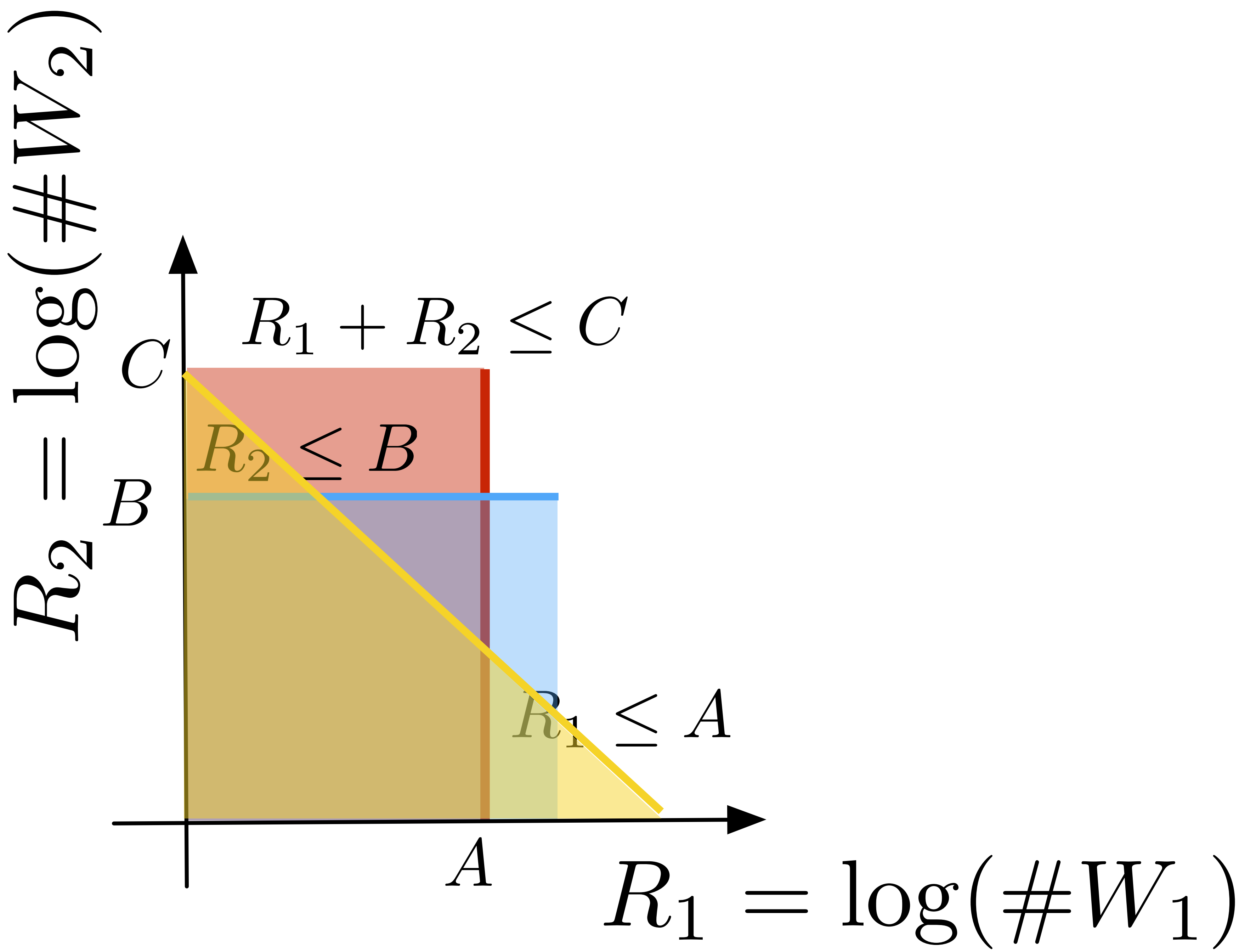

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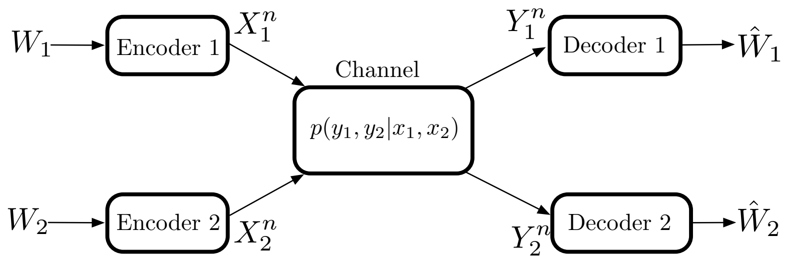


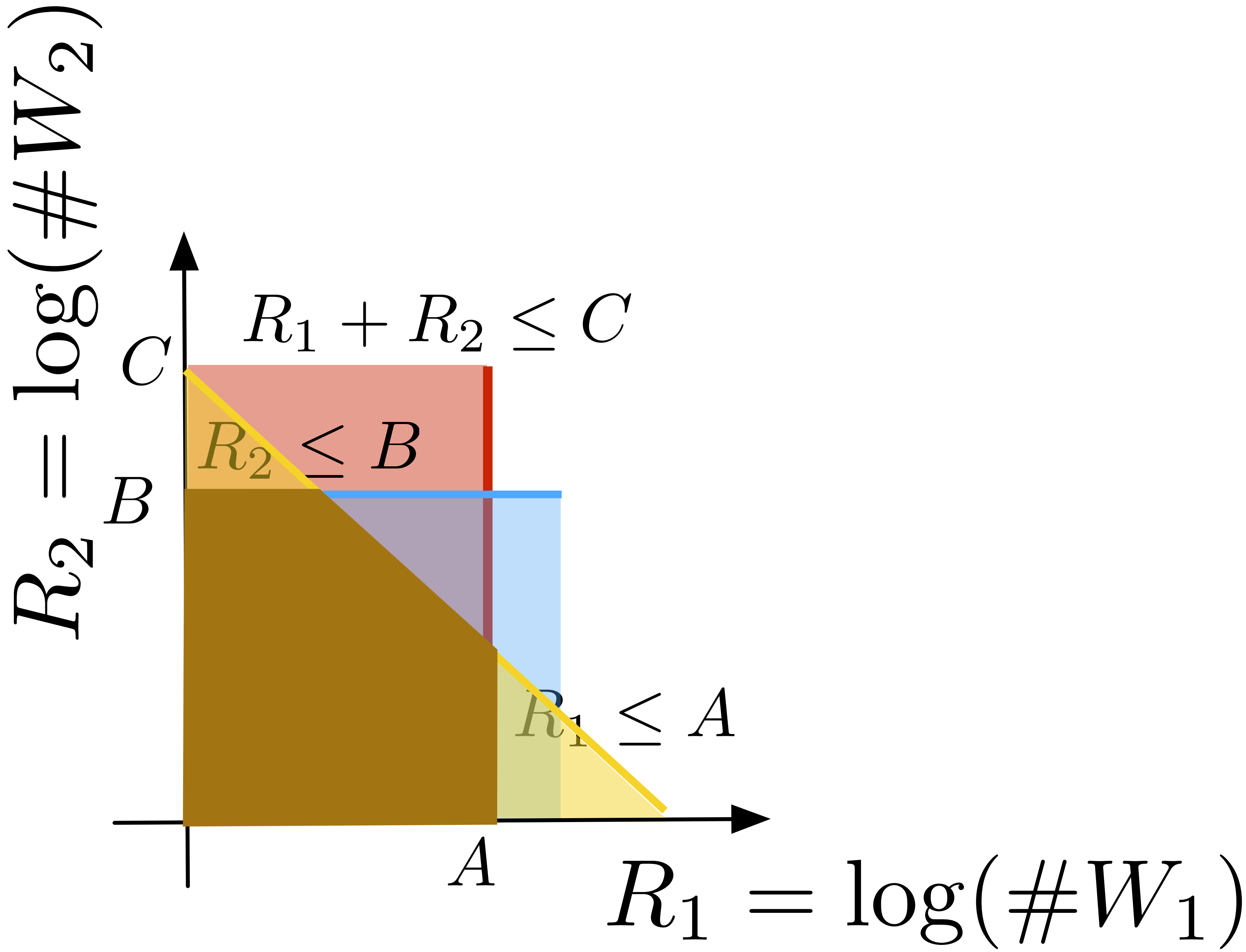

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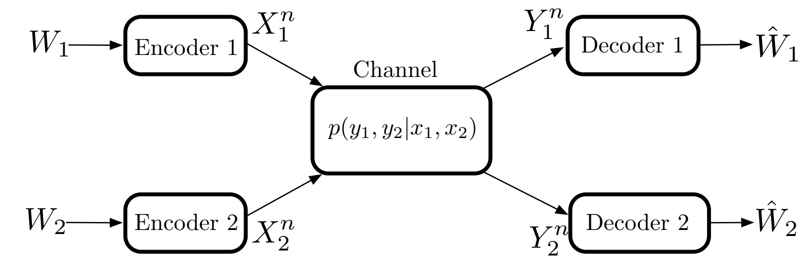
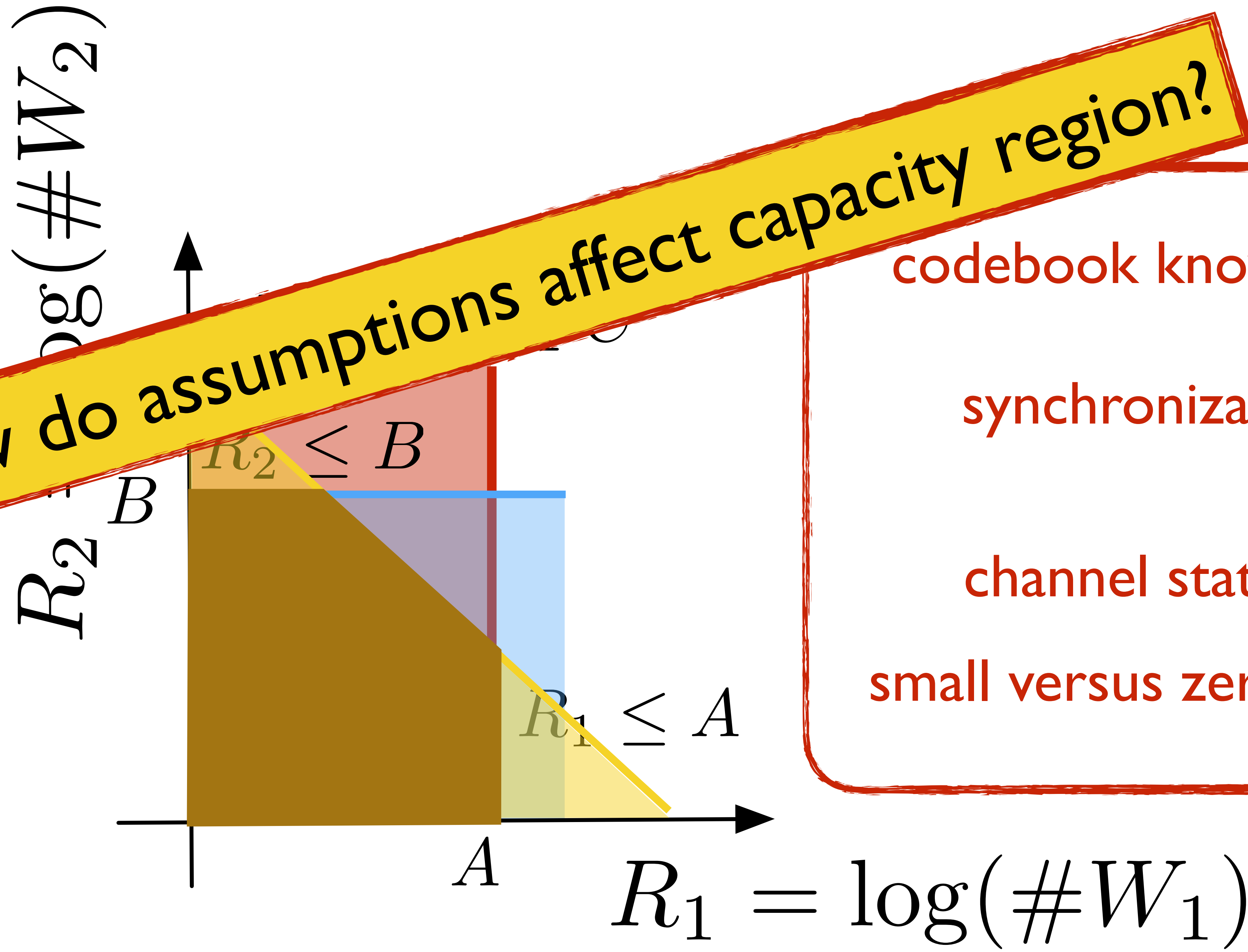



Q: how do assumptions affect capacity region?

codebook knowledge

synchronization

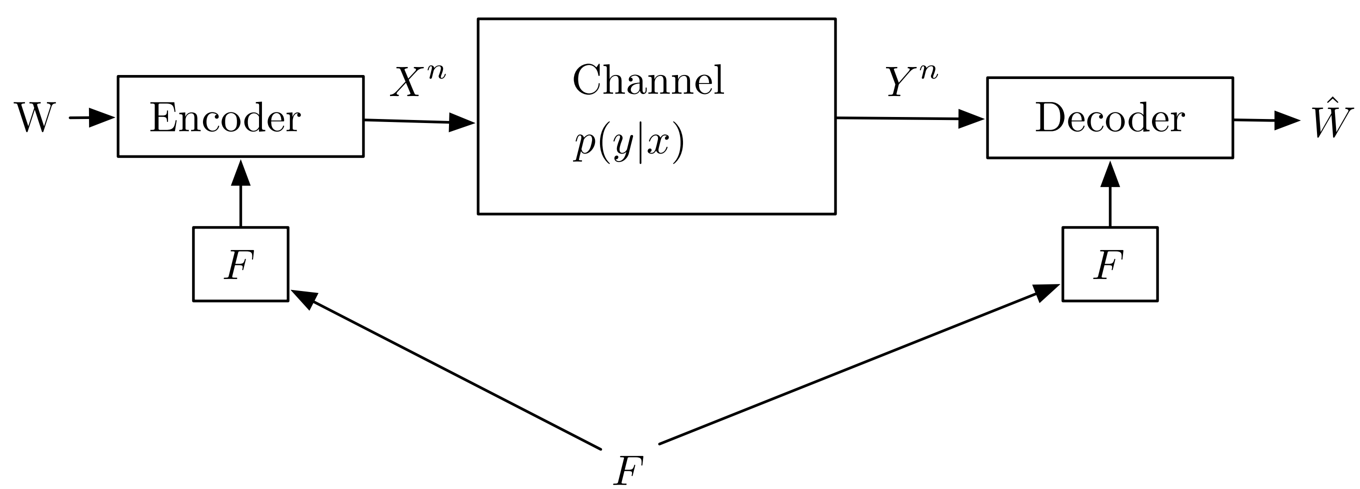
channel state

small versus zero error

# Codebook knowledge

# Point to point codebook knowledge

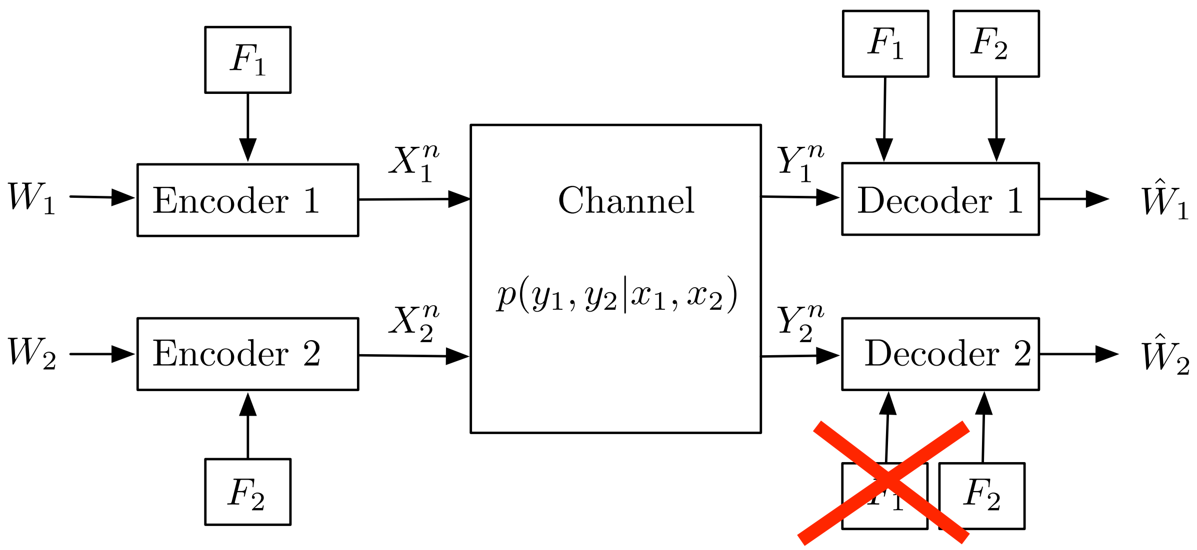
"F" is the codebook, known to the Tx,Rx




$$\left(\begin{array}{l} W \to X^n \\ \hline 1 \to X_1, X_2, \ldots, X_n \\ 2 \to X_1, X_2, \ldots, X_n \\ . \\ . \\ . \\ |W| \to X_1, X_2, \ldots, X_n \end{array}\right)$$

# ICs with lack of codebook knowledge

*Our motivation:*




**IC with one oblivious Rx**

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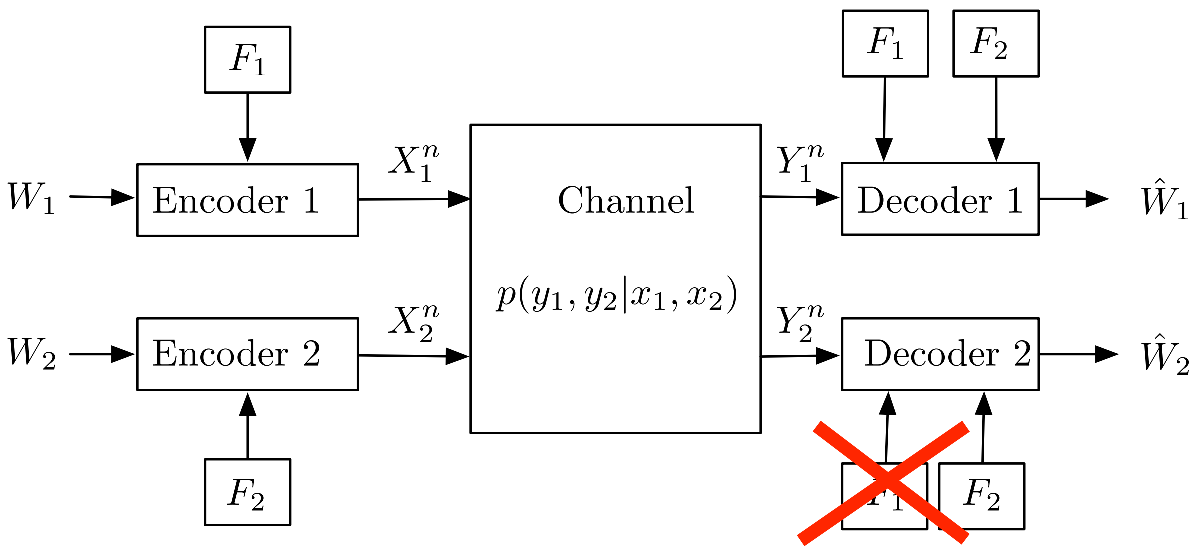


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"primary user" in a cognitive setup

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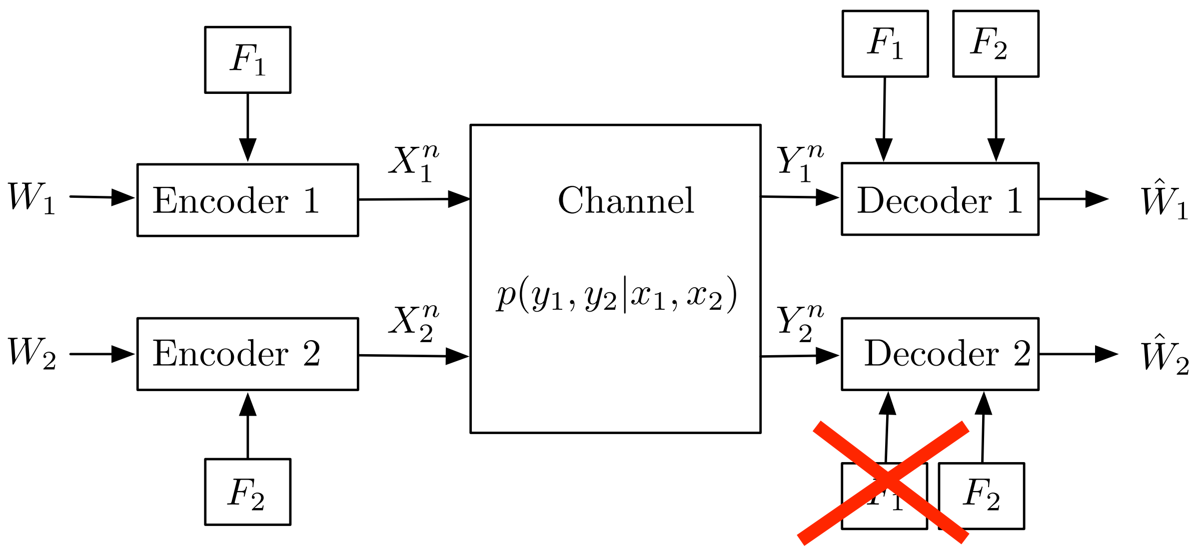


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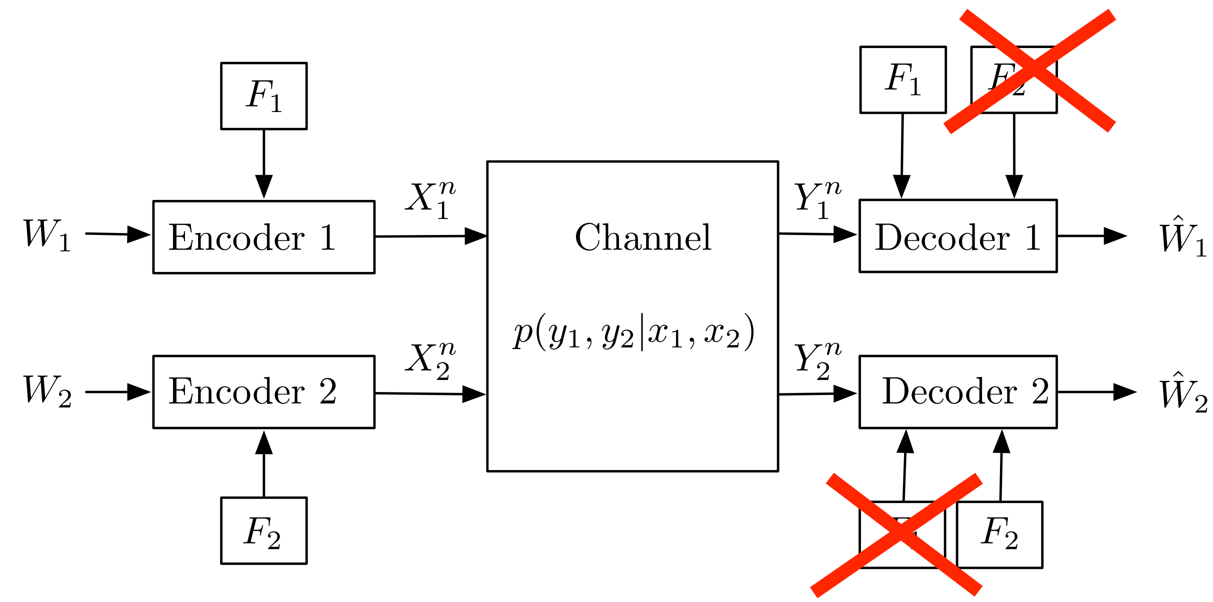


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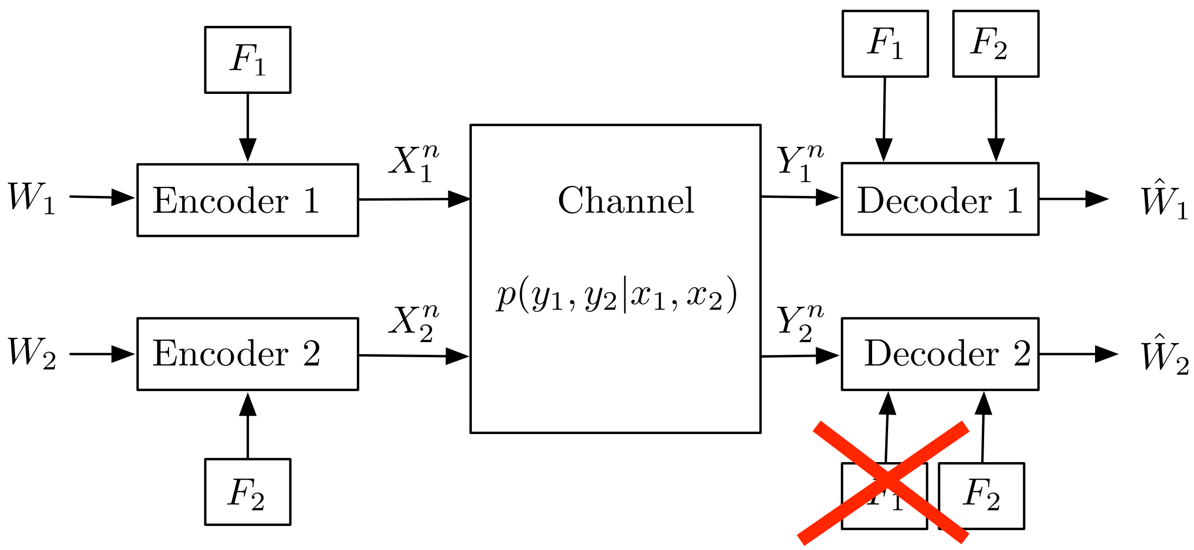


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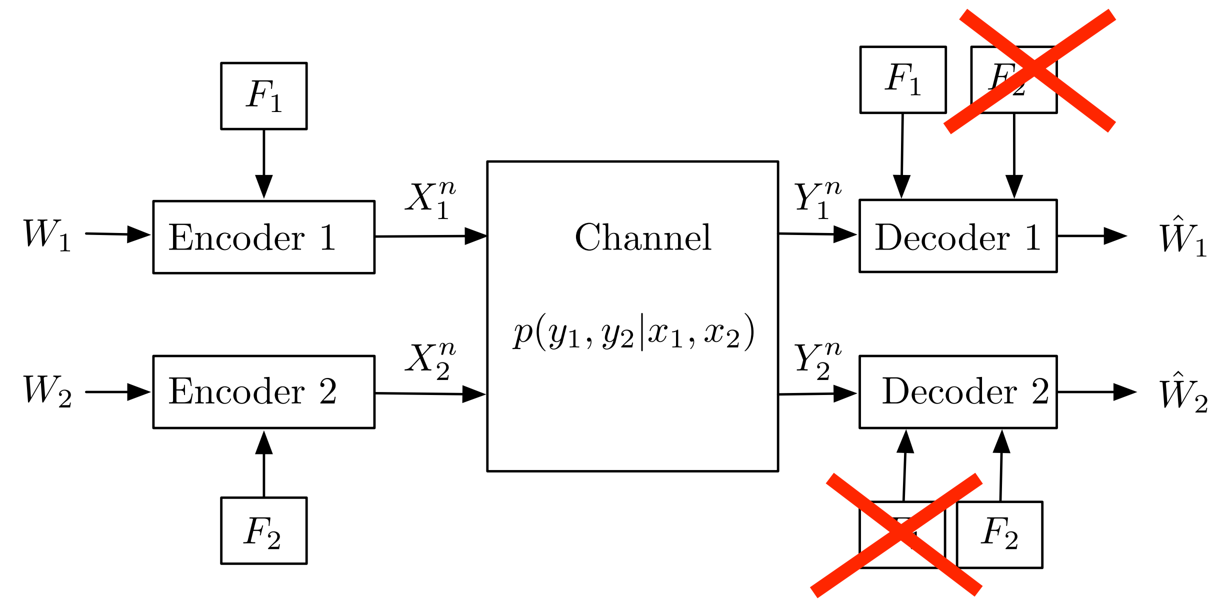


**IC with two oblivious Rx**

heterogeneous networks

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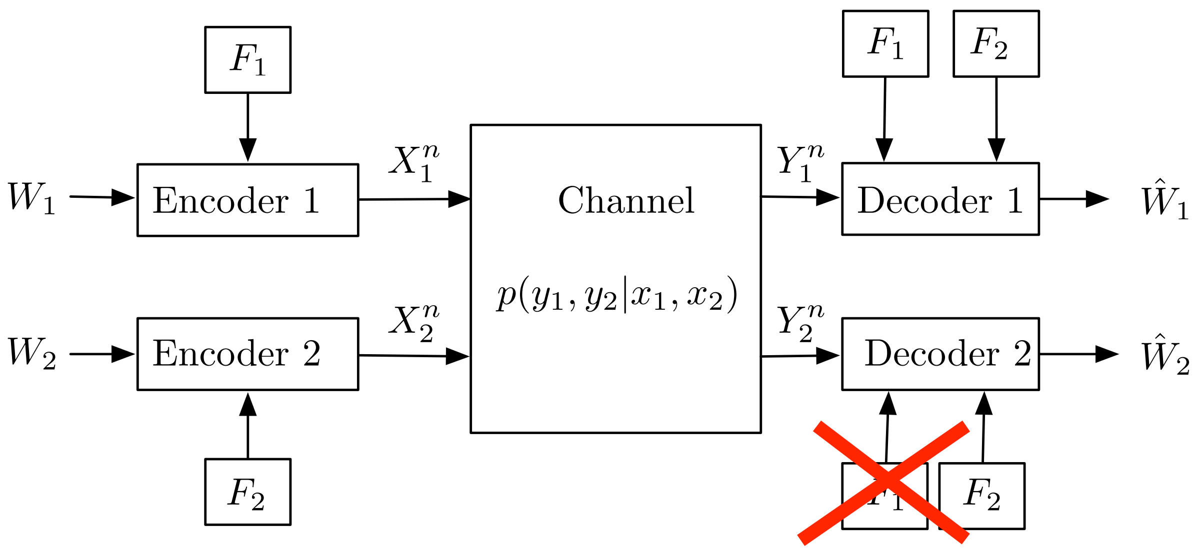
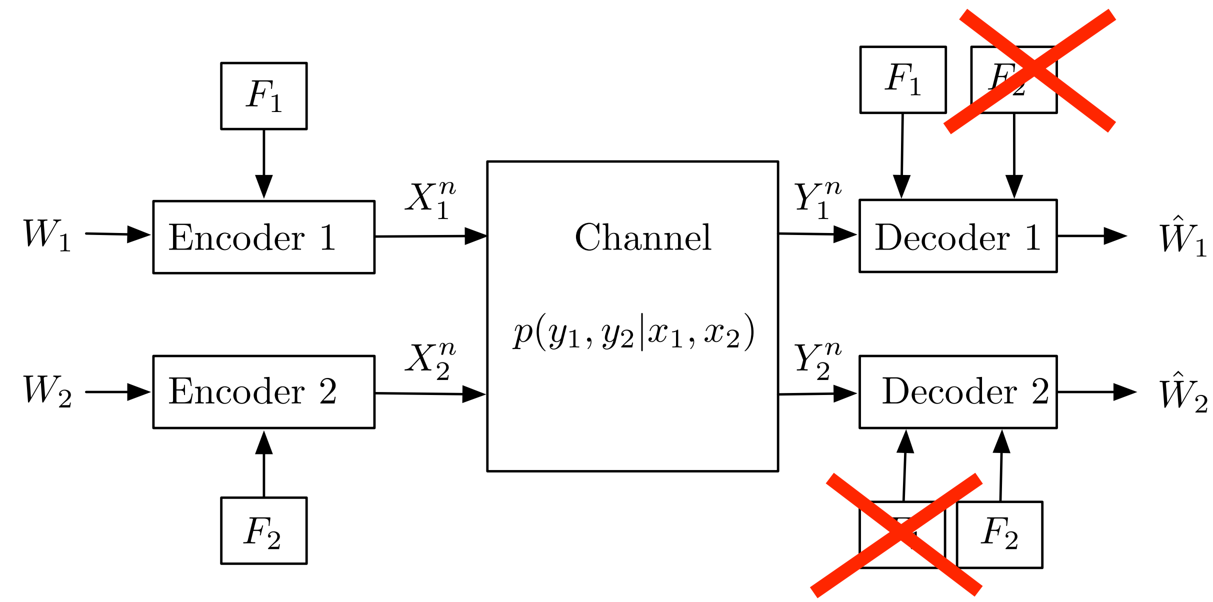



**IC with one oblivious Rx**

"primary user" in a cognitive setup




**IC with two oblivious Rx**

heterogeneous networks

A. Dytso, N. Devroye, and D. Tuninetti, "*On the capacity of interference channels with partial codebook knowledge*," ISIT 2013

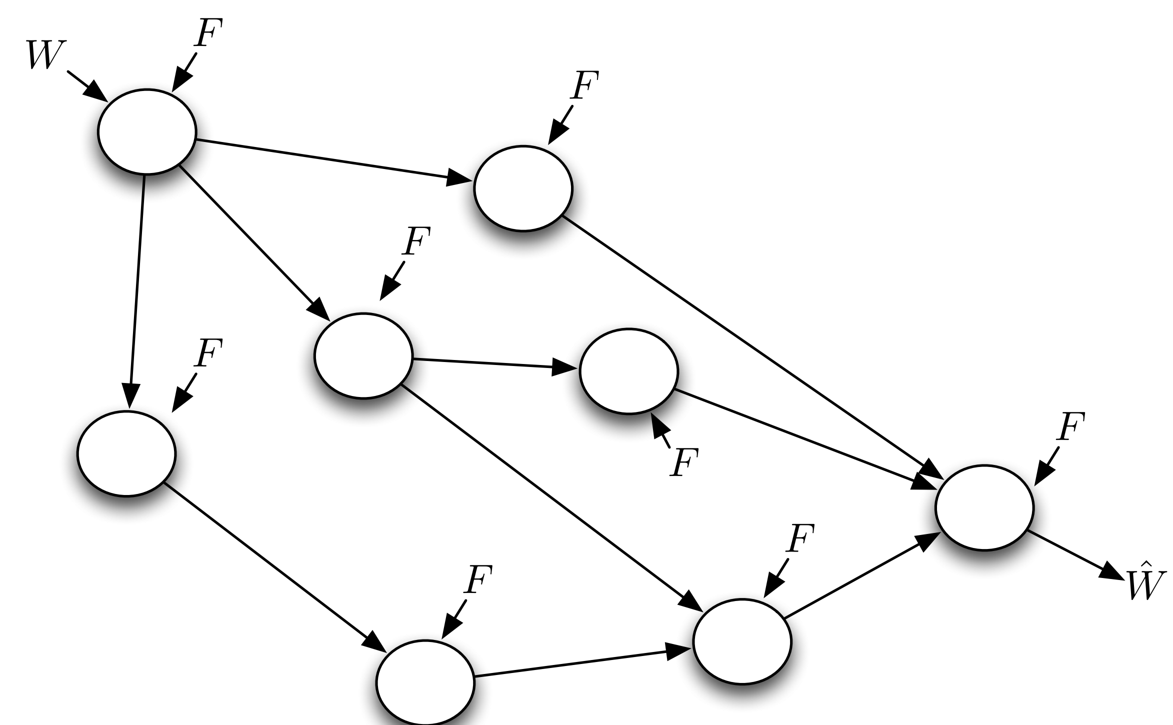
A. Dytso, D. Tuninetti and N. Devroye, ``On the Two-User Interference Channel With Lack of Knowledge of the Interference Codebook at One Receiver," IEEE Trans. on Infor. Theory, March 2015.

A. Dytso, D. Tuninetti and N. Devroye. "*On Gaussian Interference Channels with Mixed Gaussian and Discrete Inputs*," ISIT 2014

A. Dytso, D. Tuninetti and N. Devroye "*Interference as Noise: Friend of Foe?*" IEEE Trans. on Info Theory, June 2016.

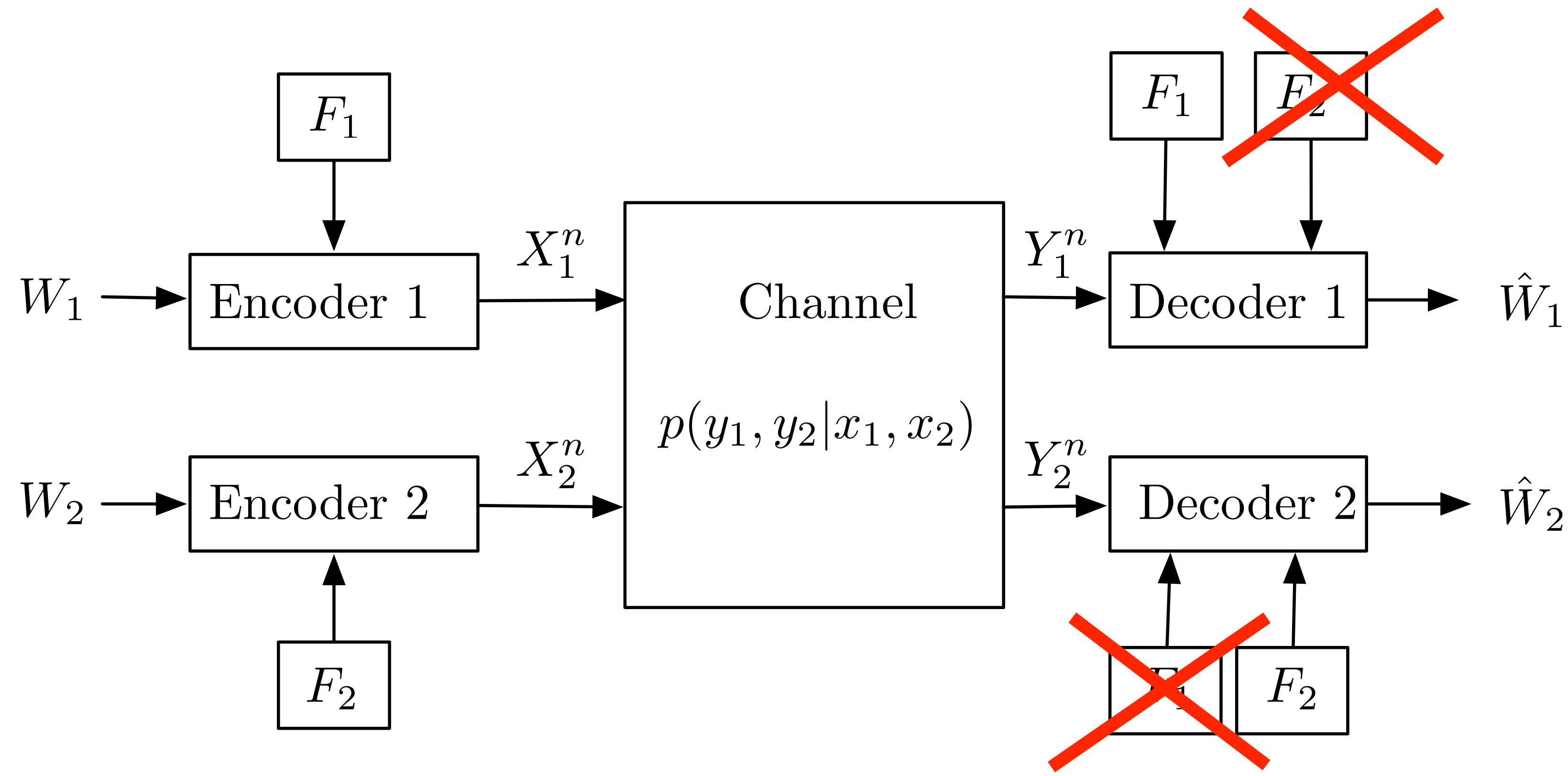
# Networks with lack of codebook knowledge

- in networks, often assume nodes know all codebooks of ALL other nodes




- this may be unrealistic sometimes….

# The interference channel




How do we use the codebook knowledge?

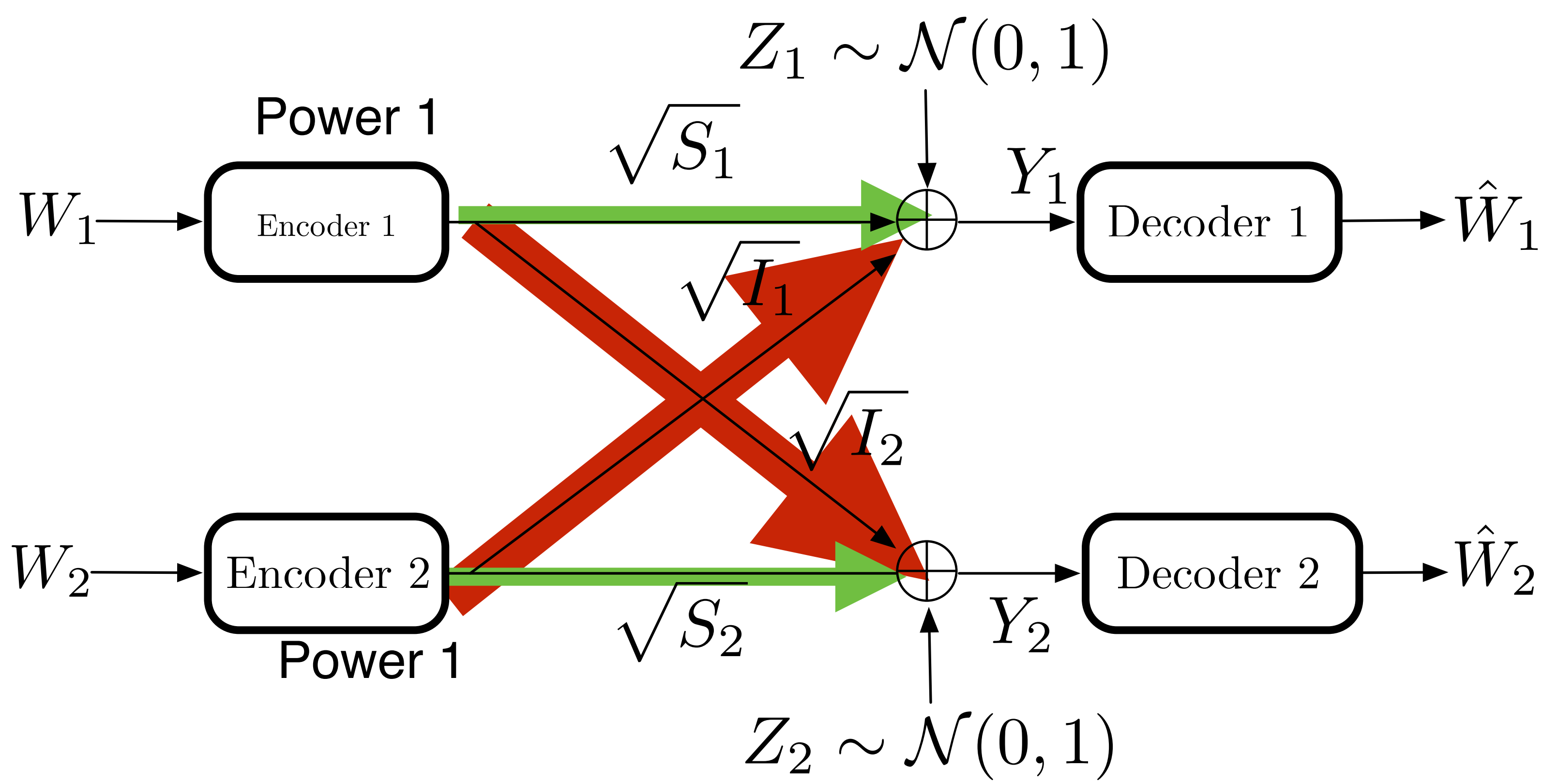
# Interfering codebook knowledge in AWGN IC

- Use it to decode interference in very strong and strong interference regimes

*joint decoding, successive interference cancellation*

A DM-IC is said to have *very strong interference* if

$$I(X_1;Y_1|X_2) \le I(X_1;Y_2)$$
$$I(X_2;Y_2|X_1) \le I(X_2;Y_1) \qquad \forall p(x_1)p(x_2)$$



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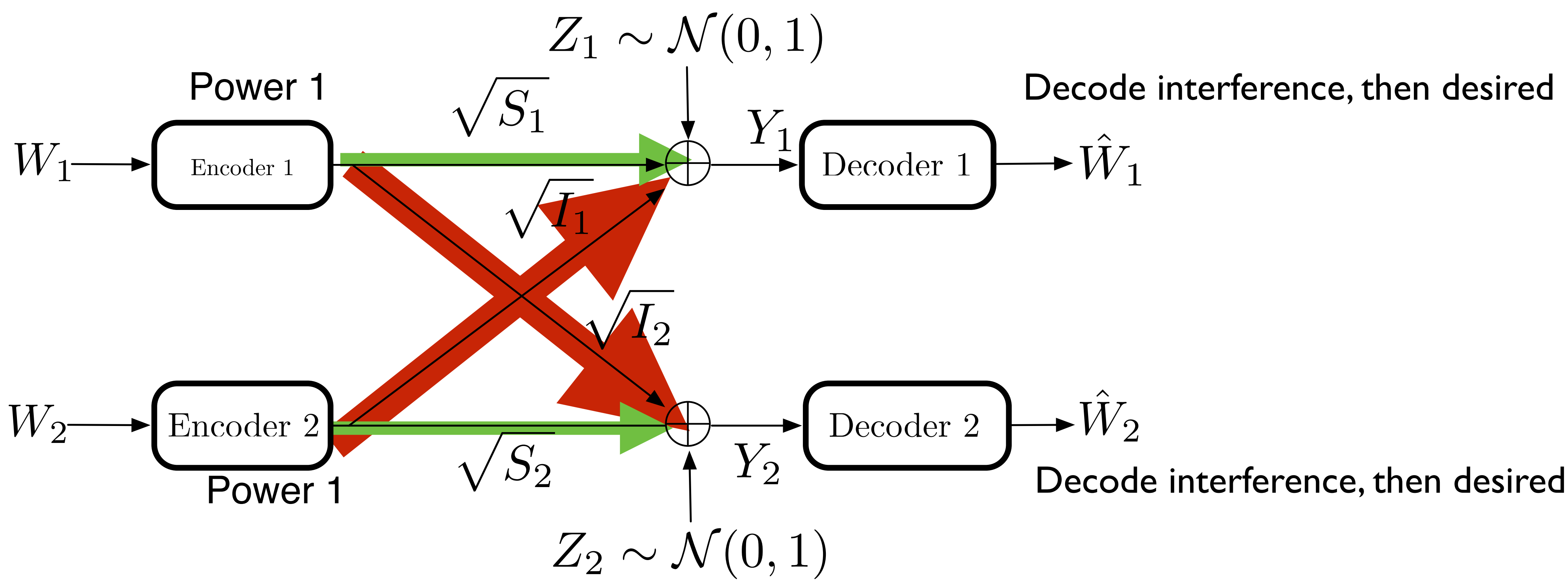


**Successive interference cancellation achieves capacity!**

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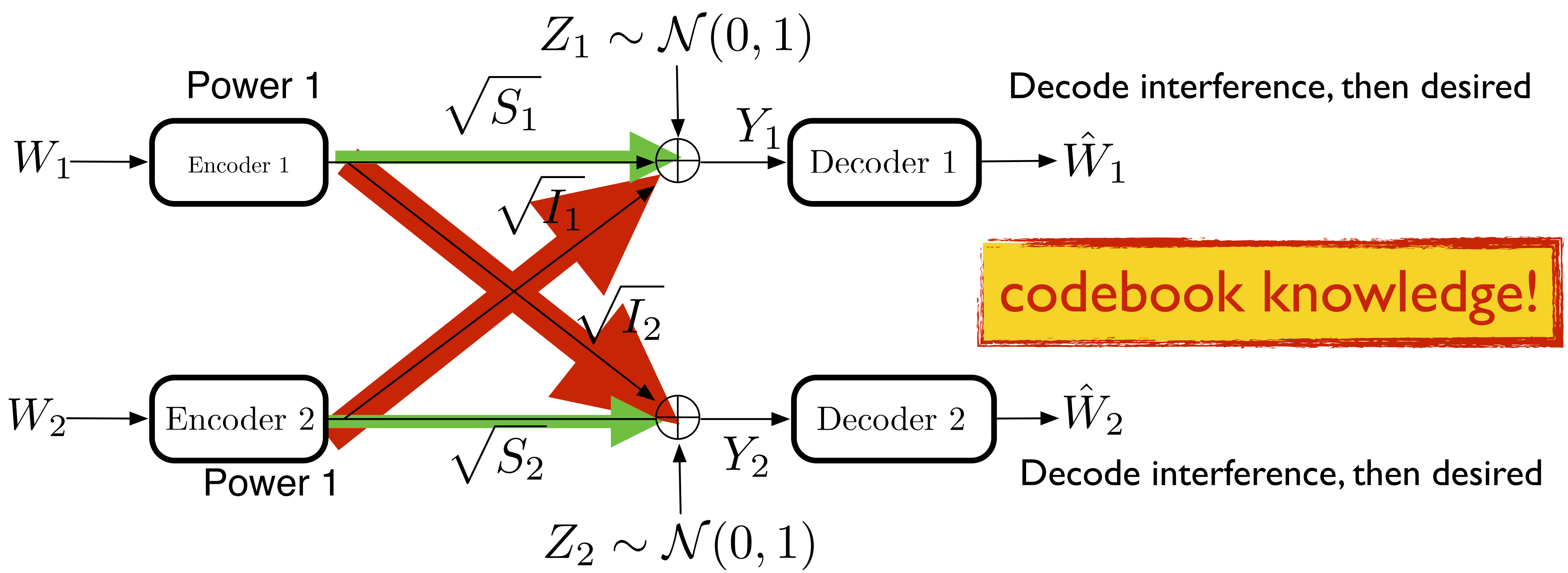


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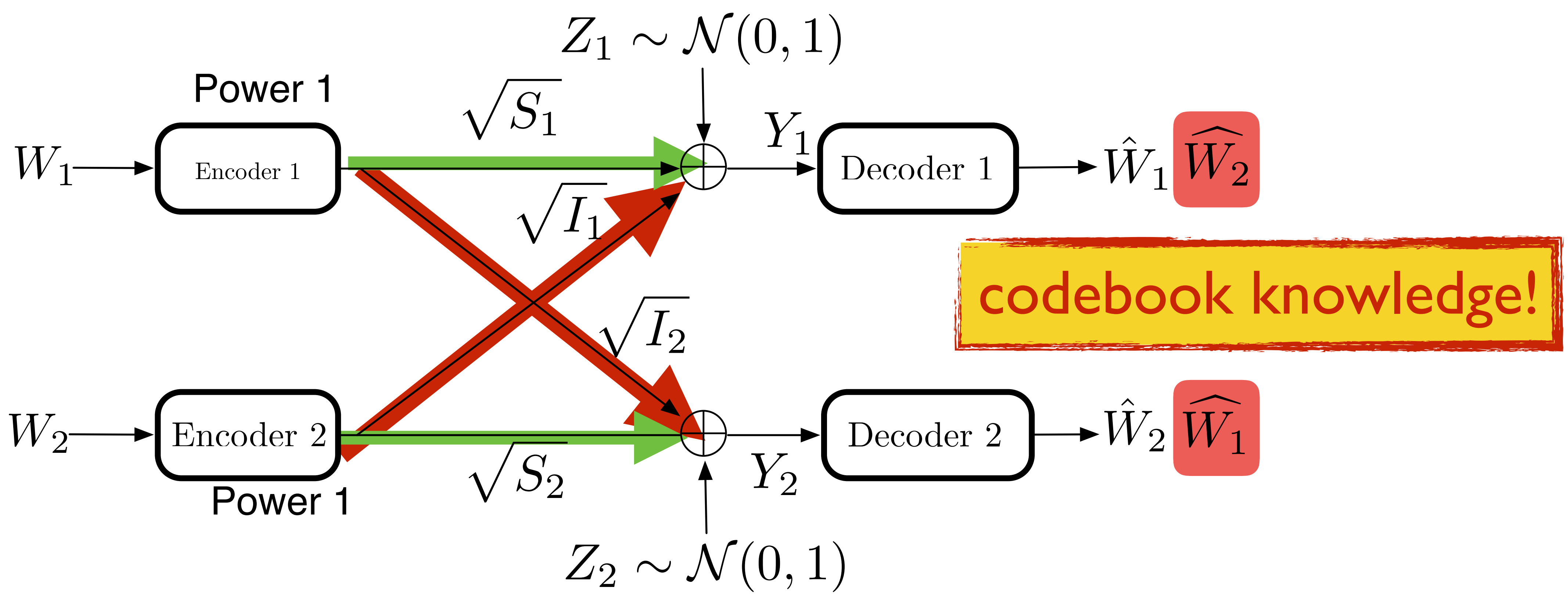


**Simultaneous decoding achieves capacity!**

# Interfering codebook knowledge in AWGN IC

- Use it to decode interference in very strong and strong interference regimes

Best general rate region

- Use it to decode public messages in Han + Kobayashi achievable rate region

# Han+Kobayashi inner bound

+

[T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," IEEE Trans. Inf. Theory, vol. 27, no. 1, pp. 49–60, 1981.]

**Theorem (Han+Kobayashi inner bound).** A rate pair $(R_1, R_2)$ is achievable for a DM-IC $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ if it satisfies

$$
\begin{aligned}
R_1 &\leq I(X_1; Y_1|U_2, Q) && (1)\\
R_2 &\leq I(X_2; Y_2|U_1, Q) && (2)\\
R_1 + R_2 &\leq I(X_1, U_2; Y_1|Q) + I(X_2; Y_2|U_1, U_2, Q) && (3)\\
R_1 + R_2 &\leq I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|Q) && (4)\\
R_1 + R_2 &\leq I(X_1, U_2; Y_1|U_1, Q) + I(X_2, U_1; Y_2|U_2, Q) && (5)\\
2R_1 + R_2 &\leq I(X_1, U_2; Y_1|Q) + I(X_1; Y_1|U_1, U_2, Q) + I(X_2, U_1; Y_2|U_2, Q) && (6)\\
R_1 + 2R_2 &\leq I(X_2, U_1; Y_2|Q) + I(X_2; Y_2|U_1, U_2, Q) + I(X_1, U_2; Y_1|U_1, Q) && (7)
\end{aligned}
$$

for some $p(q, u_1, u_2, x_1, x_2) = p(q)p(u_1, x_1|q)p(u_2, x_2|q)$ where $|\mathcal{U}_1| \leq |\mathcal{X}_1| + 4$, $|\mathcal{U}_2| \leq |\mathcal{X}_2| + 4$, and $|\mathcal{Q}| \leq 7$.

# Han+Kobayashi inner bound

 + 

## Largest single-letter achievable rate region for IC

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## Achieves capacity when we know it

[H.-F. Chong, M. Motani, H. K. Garg, and H. El Gamal, "On the Han–Kobayashi region for the interference channel," IEEE Trans. Inf. Theory, vol. 54, no. 7, pp. 3188–3195, July 2008. ]

(class of deterministic channels, approximately for class of semi-deterministic channels)


[A. El Gamal and M. H. M. Costa, "The capacity region of a class of deterministic interference channels," IEEE Trans. Inf. Theory, 1982.]

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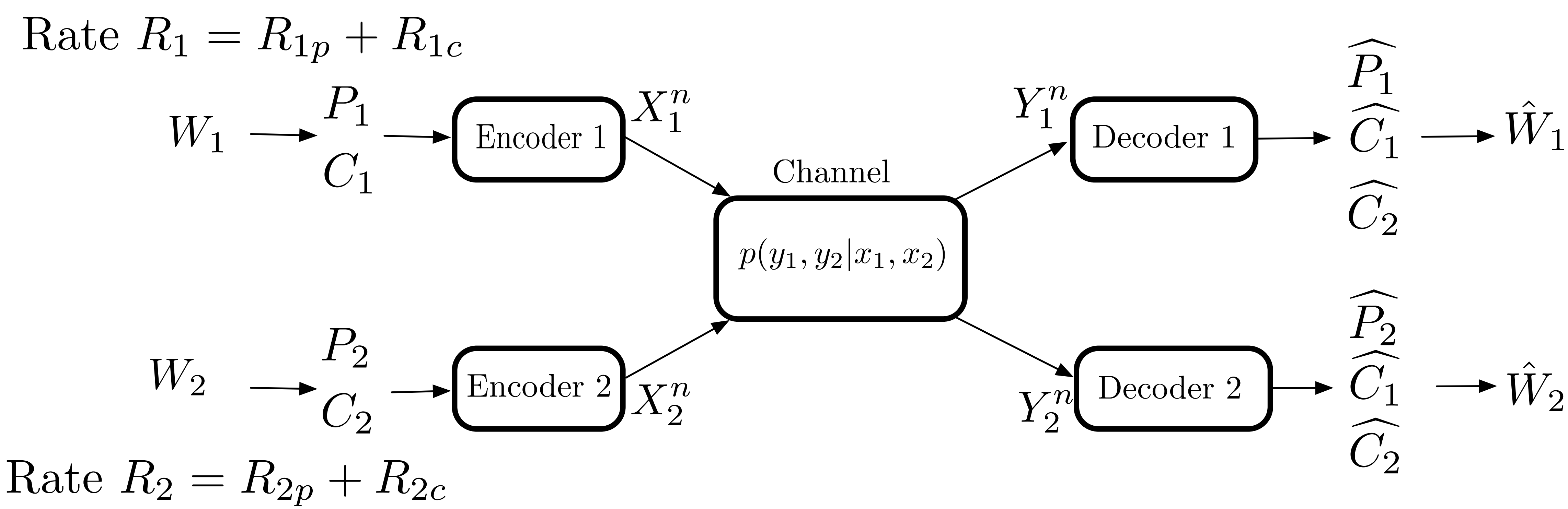
**Requires knowledge of codebooks at both receivers - WHY?**

# Split message into "public" and "private" parts




Idea: carefully split so can decode part of the interference

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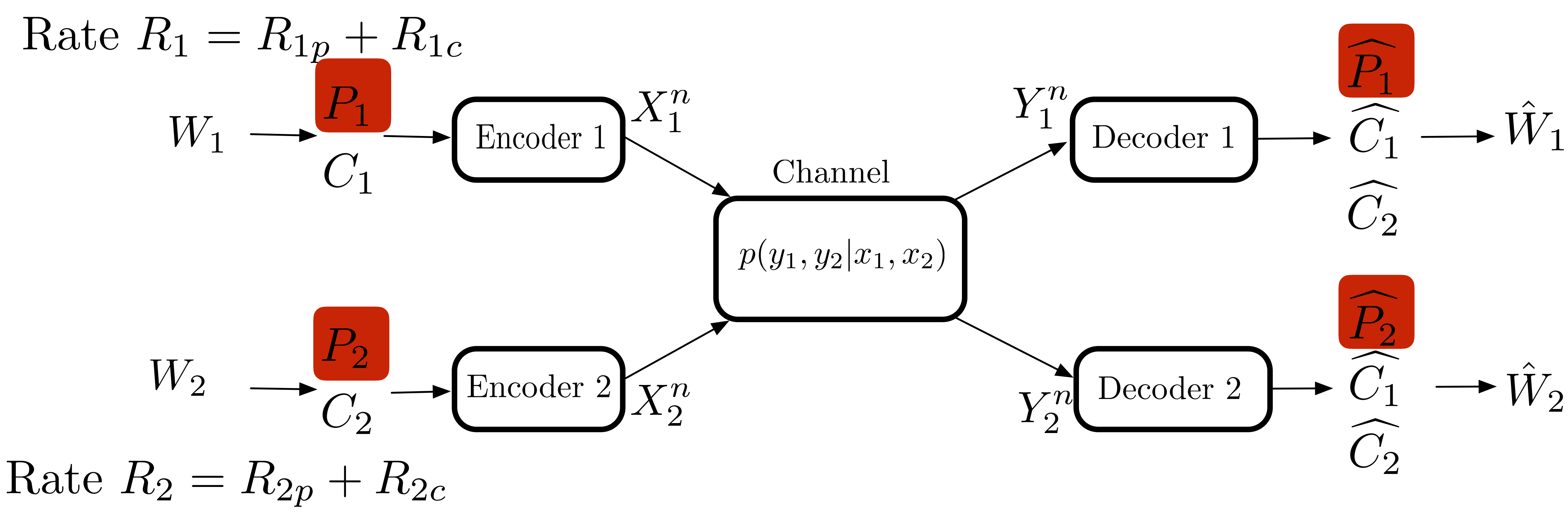


"Private" = decoded only by intended

Idea: carefully split so can decode part of the interference

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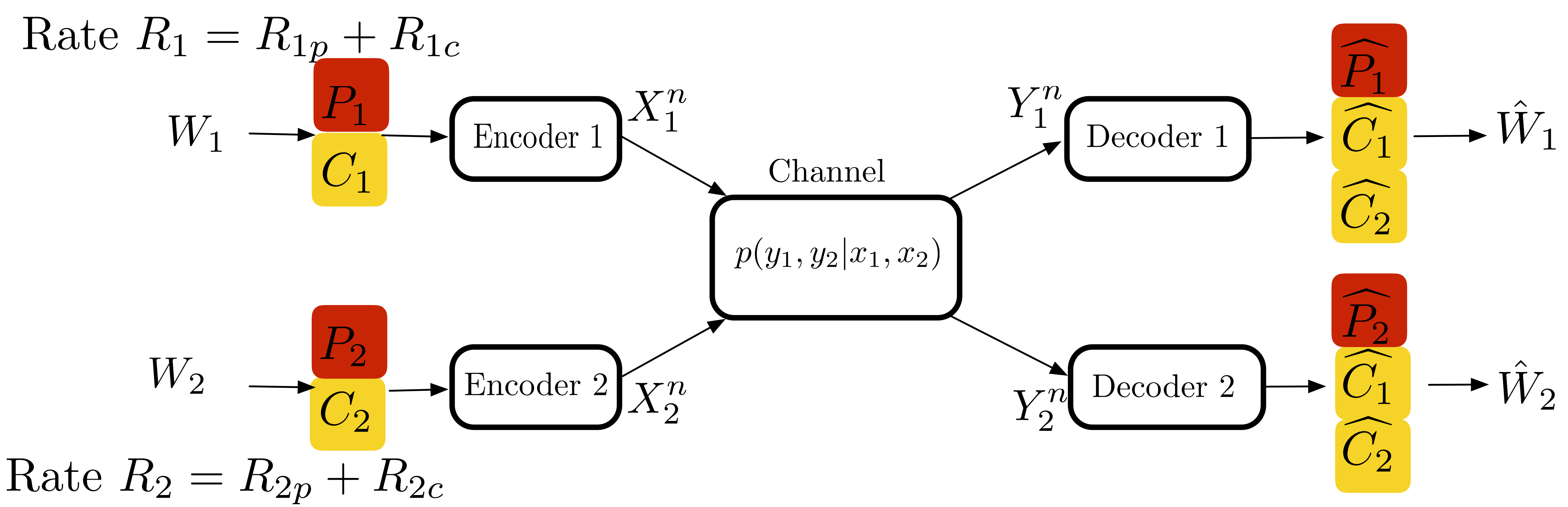


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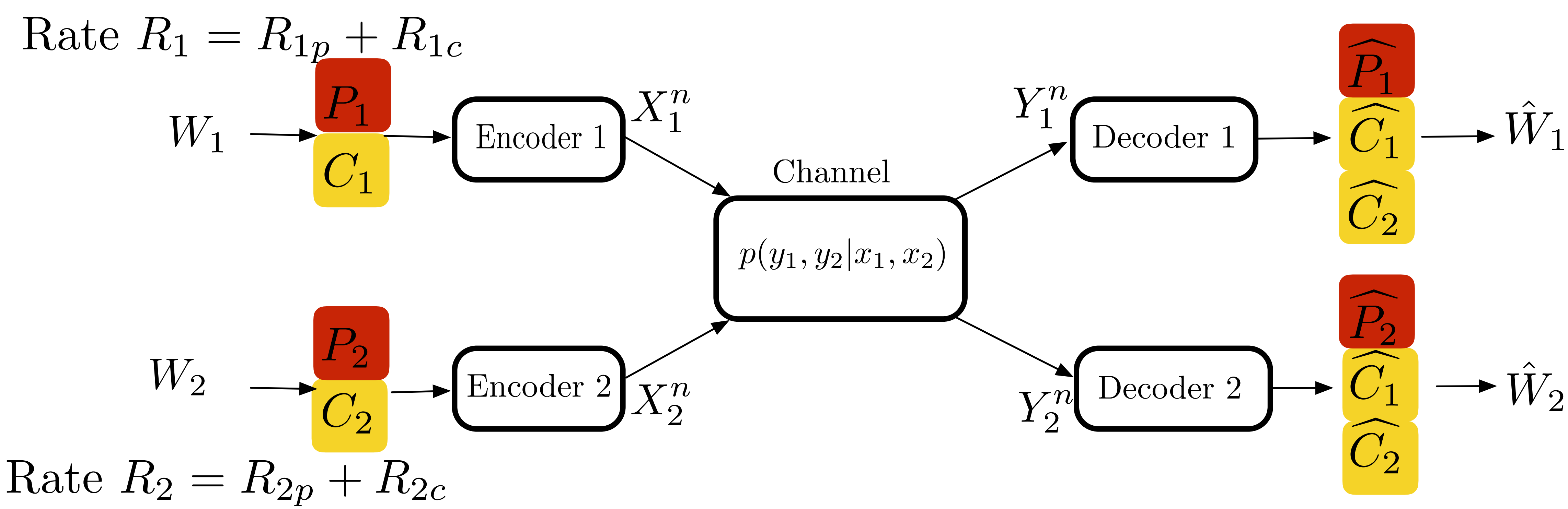
"Public" = common = decoded by everyone

Idea: carefully split so can decode part of the interference

# Split message into "public" and "private" parts

requires interfering codebooks!

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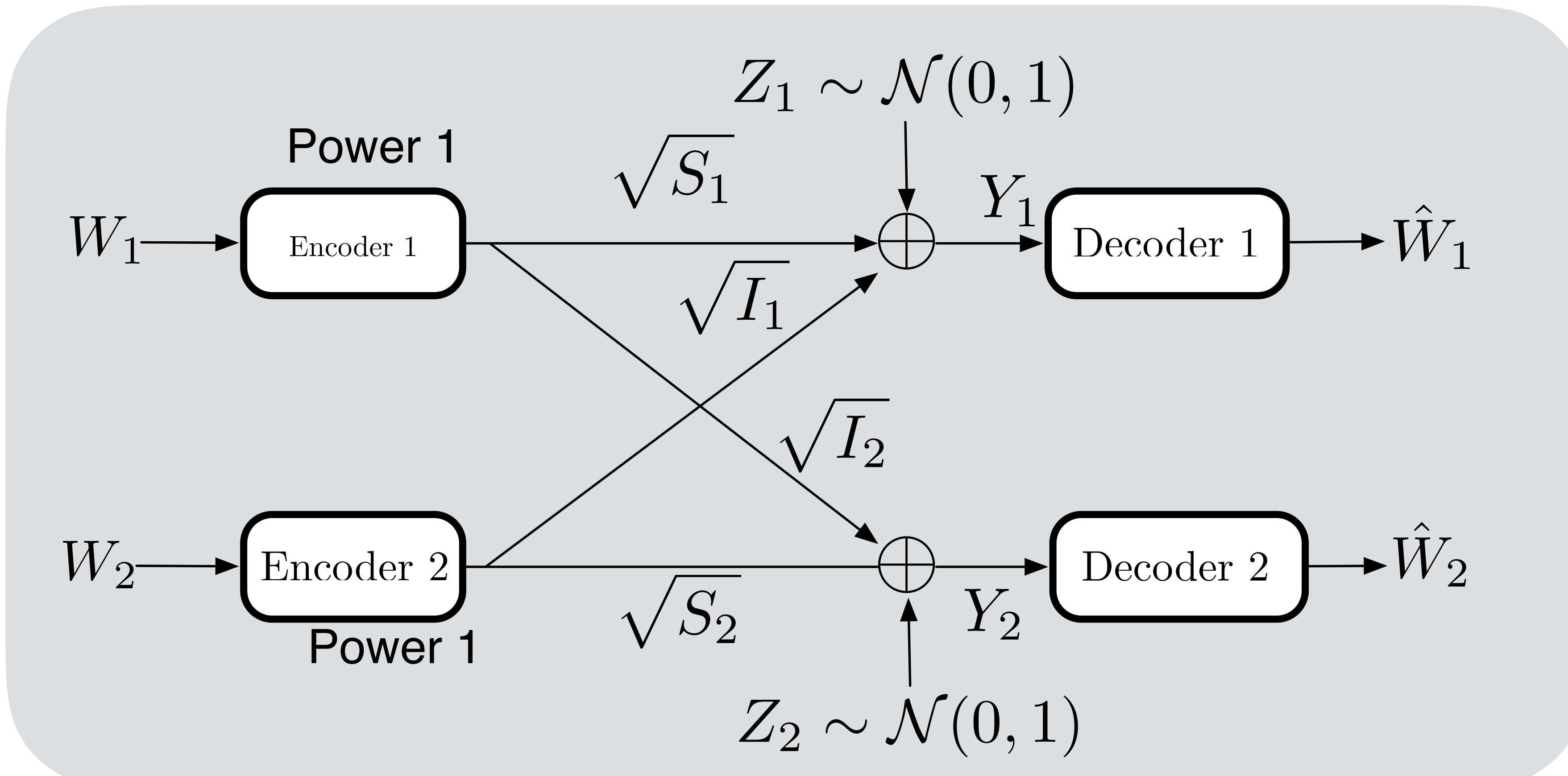
# The AWGN-IC




of practical relevance in wireless systems: cellular, wireless local area networks (WiFi), ad hoc networks (wireless sensors or nodes)



Excellent survey of results on Gaussian IC in Introduction of [R. Bustin, H.V. Poor, and S. Shamai "The Effect of Maximal Rate Codes on the Interfering Message Rate," http://arxiv.org/abs/1404.6690]

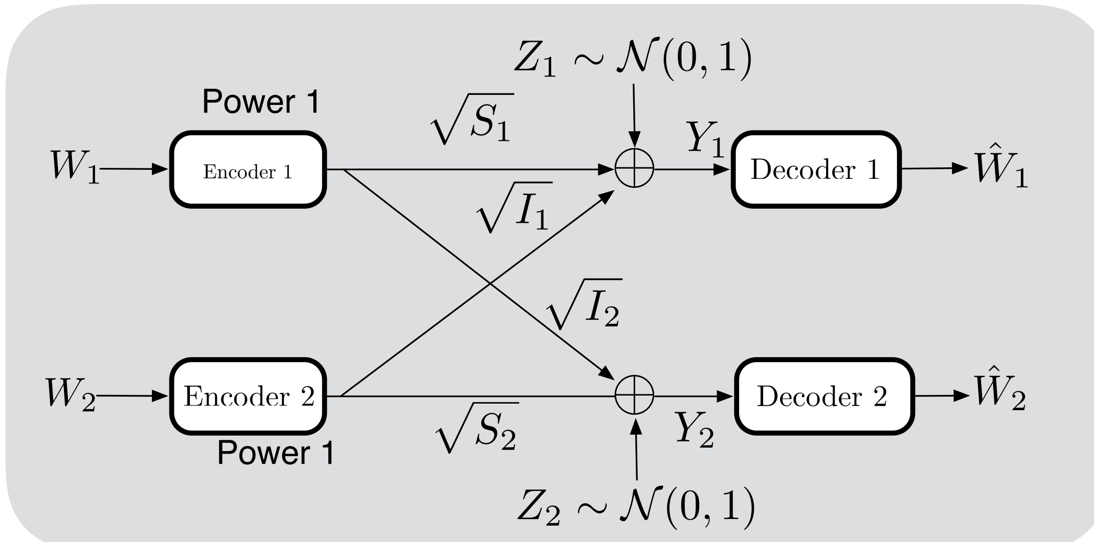
# Interfering codebook knowledge in AWGN IC

- Use it to decode interference in very strong and strong interference regimes

- Use it to decode public messages in Han + Kobayashi achievable rate region

- Use it to achieve capacity to within 1/2 bit for Gaussian noise channels

# AWGN: H+K achieves capacity to within 1/2 bit




**Theorem (gap for Gaussian IC)** If $(R_1, R_2)$ is in the outer bound $\mathcal{R}_O^{\mathrm{AWGN}}$ then $(R_1 - 1/2, R_2 - 1/2)$ is achievable.

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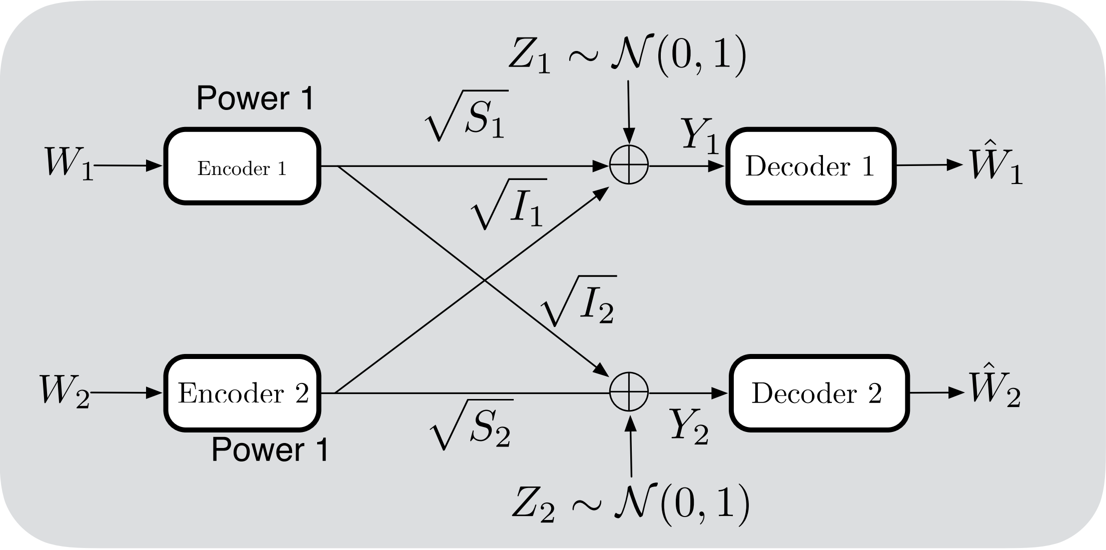


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## Etkin, Tse, Wang show how to pick **Gaussian inputs** in H+K scheme

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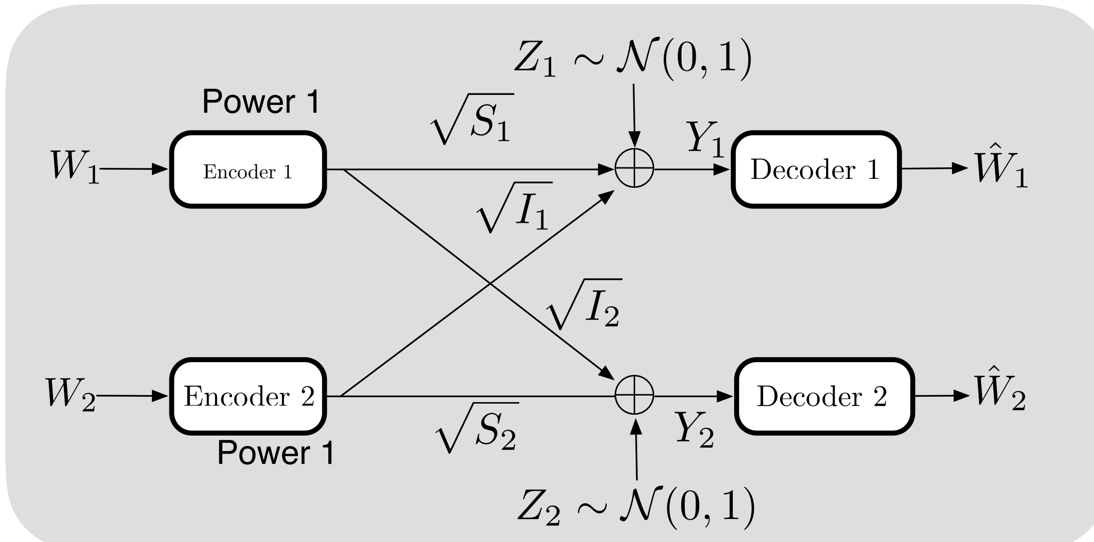


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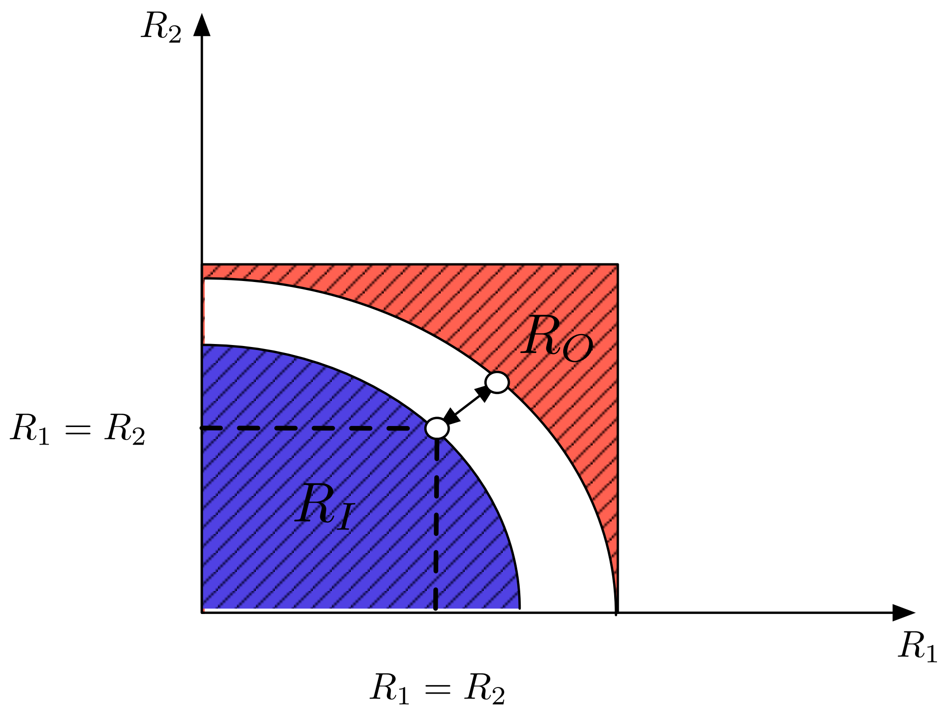
depends on the regime of operation

# AWGN: the "W" curve for the GDoF

highlights effect of interference rather than noise

$$\mathcal{D}(\alpha) := \left\{ (d_1, d_2) \in \mathbb{R}_+^2 : d_i := \lim_{\substack{\text{inr} = \text{snr}^\alpha, \\ \text{snr} \to \infty}} \frac{R_i}{\frac{1}{2}\log(1+\text{snr})}, i \in [1:2], \ (R_1, R_2) \text{ is achievable} \right\}.$$



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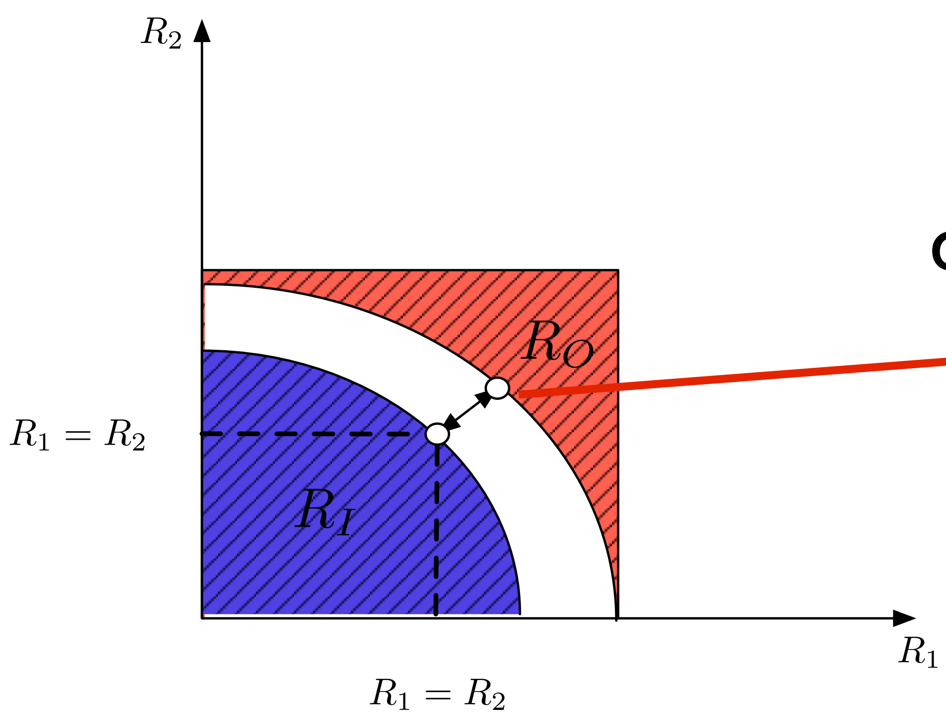
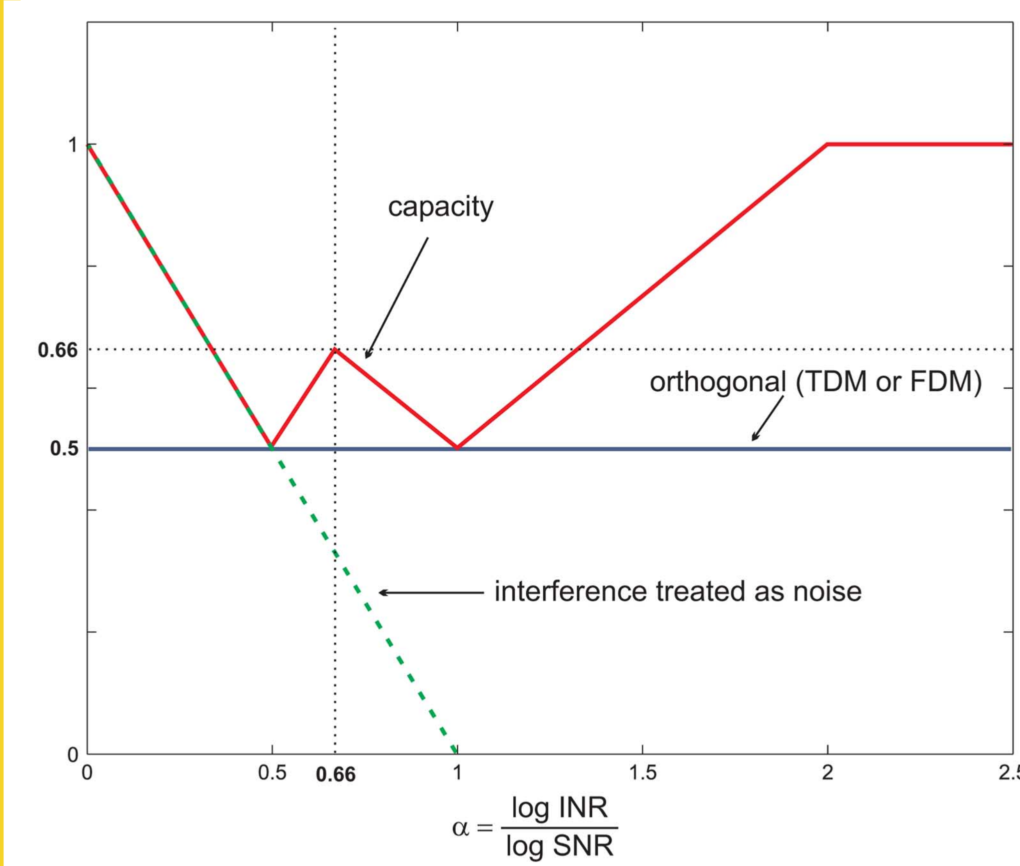
## "W" curve for $R_1=R_2$




$d_1=d_2$

generalized DoF




increasing interference

image taken from

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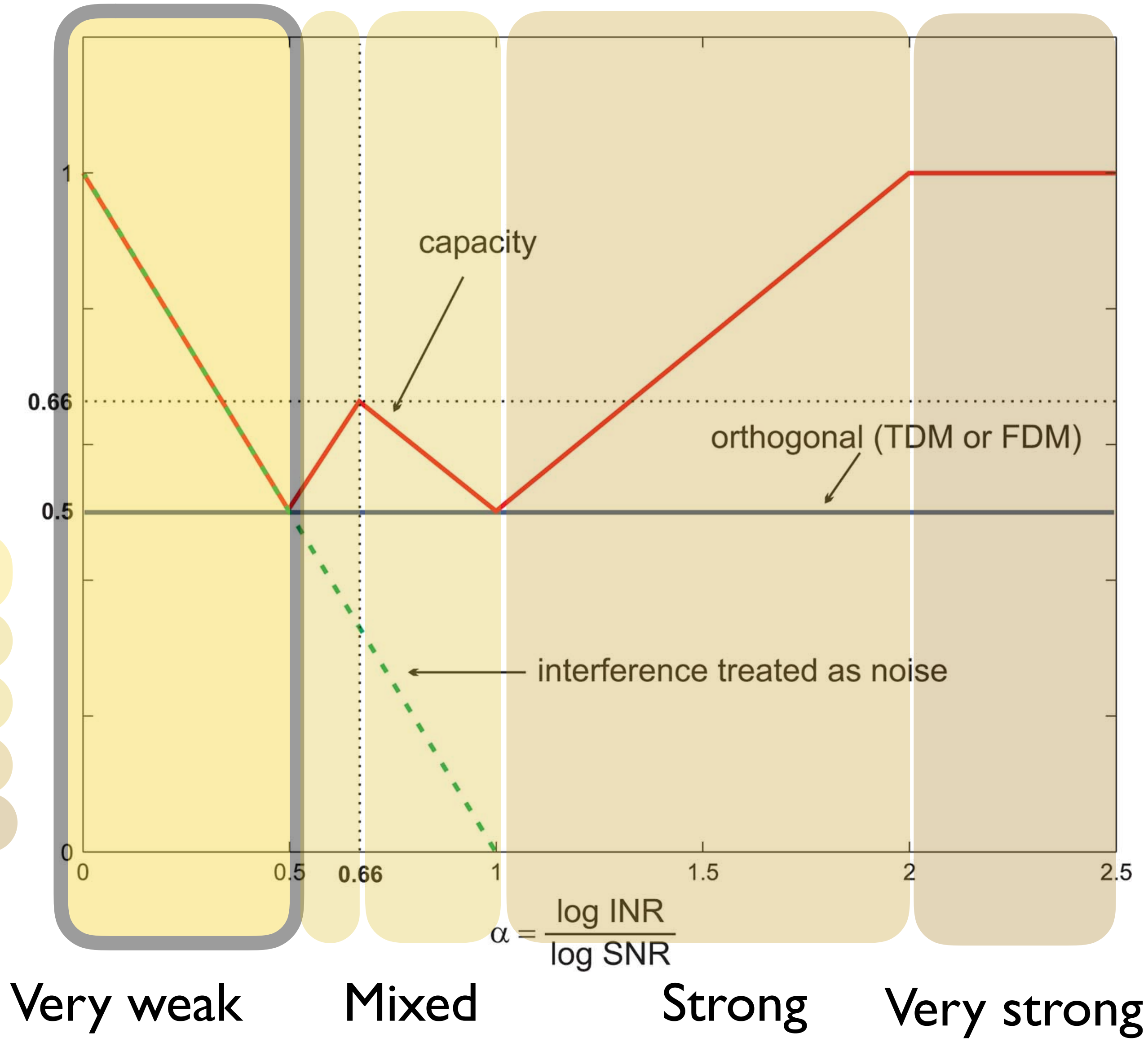


$$C_{\text{sym}} \approx \begin{cases} \log\left(\frac{\mathsf{SNR}}{\mathsf{INR}}\right) & \log \mathsf{INR} < \frac{1}{2}\log \mathsf{SNR} \\ \log \mathsf{INR} & \frac{1}{2}\log \mathsf{SNR} < \log \mathsf{INR} < \frac{2}{3}\log \mathsf{SNR} \\ \log \frac{\mathsf{SNR}}{\sqrt{\mathsf{INR}}} & \frac{2}{3}\log \mathsf{SNR} < \log \mathsf{INR} < \log \mathsf{SNR} \\ \log \sqrt{\mathsf{INR}} & \log \mathsf{SNR} < \log \mathsf{INR} < 2\log \mathsf{SNR} \\ \log \mathsf{SNR} & \log \mathsf{INR} > 2\log \mathsf{SNR} \end{cases}$$

## Regimes

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

[X. Shang, G. Kramer, and B. Chen, “A new outer bound and the noisy-interference sum-rate capacity for Gaussian interference channels,” IEEE Trans. Inf. Theory, vol. 55, no. 2, pp. 689–699, Feb. 2009.]

[V. S. Annapureddy and V. V. Veeravalli, “Gaussian interference networks: Sum capacity in the low interference regime and new outer bounds on the capacity region,” IEEE Trans. Inf. Theory, vol. 55, no. 7, pp. 3032–3050, July 2009. ]

[A. S. Motahari and A. K. Khandani, “Capacity bounds for the Gaussian interference channel,” IEEE Trans. Inf. Theory, vol. 55, no. 2, pp. 620–643, Feb. 2009. ]

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image + formula taken from

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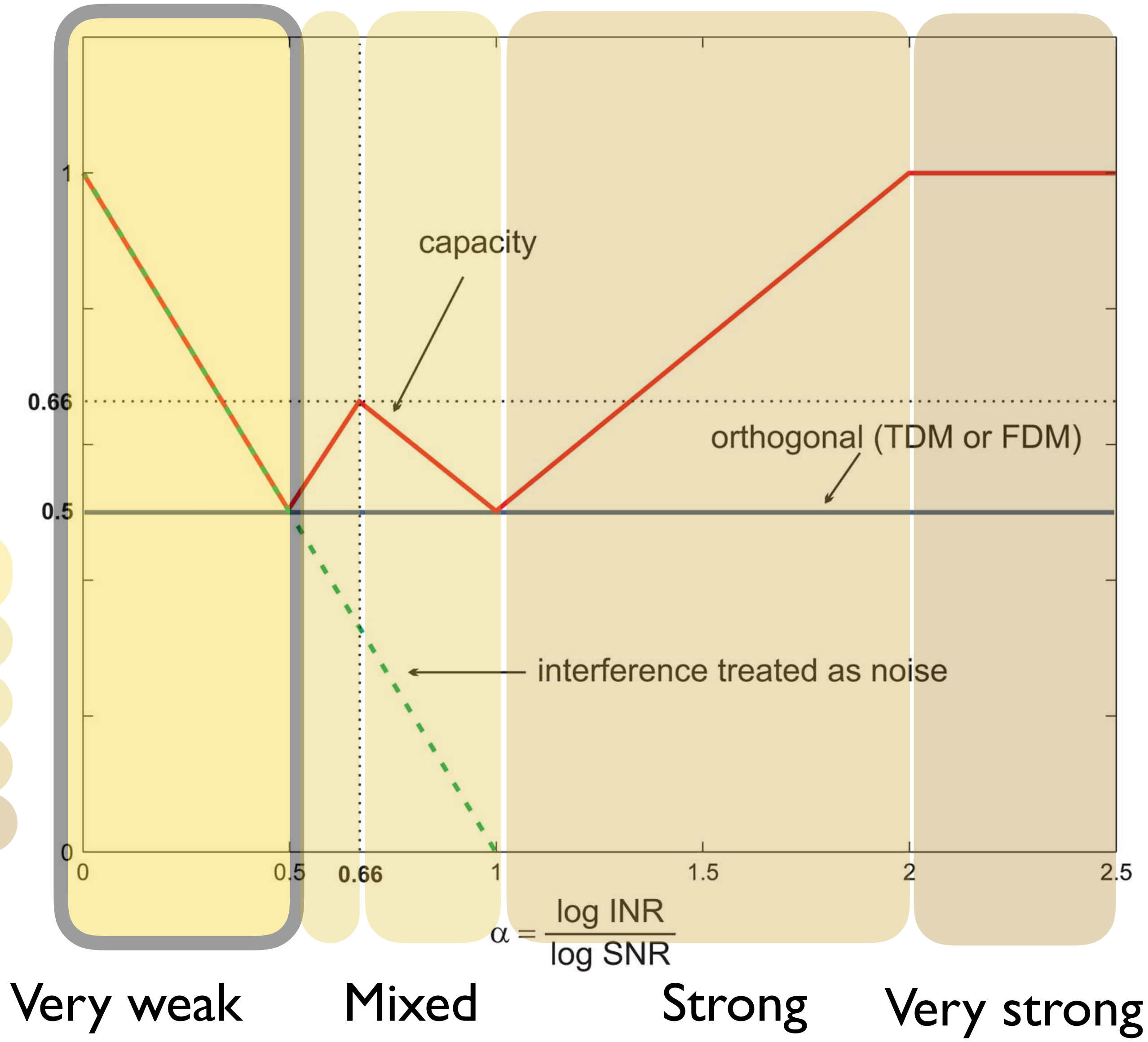


## Regimes

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

## Treating interference as noise inner bound (with Gaussian inputs):

$$R_1 \le \frac{1}{2}\log\left(1 + \frac{S_1}{1+I_1}\right), \quad R_2 \le \frac{1}{2}\log\left(1 + \frac{S_2}{1+I_2}\right)$$

# AWGN: the "W" curve for the GDoF

image + formula taken from

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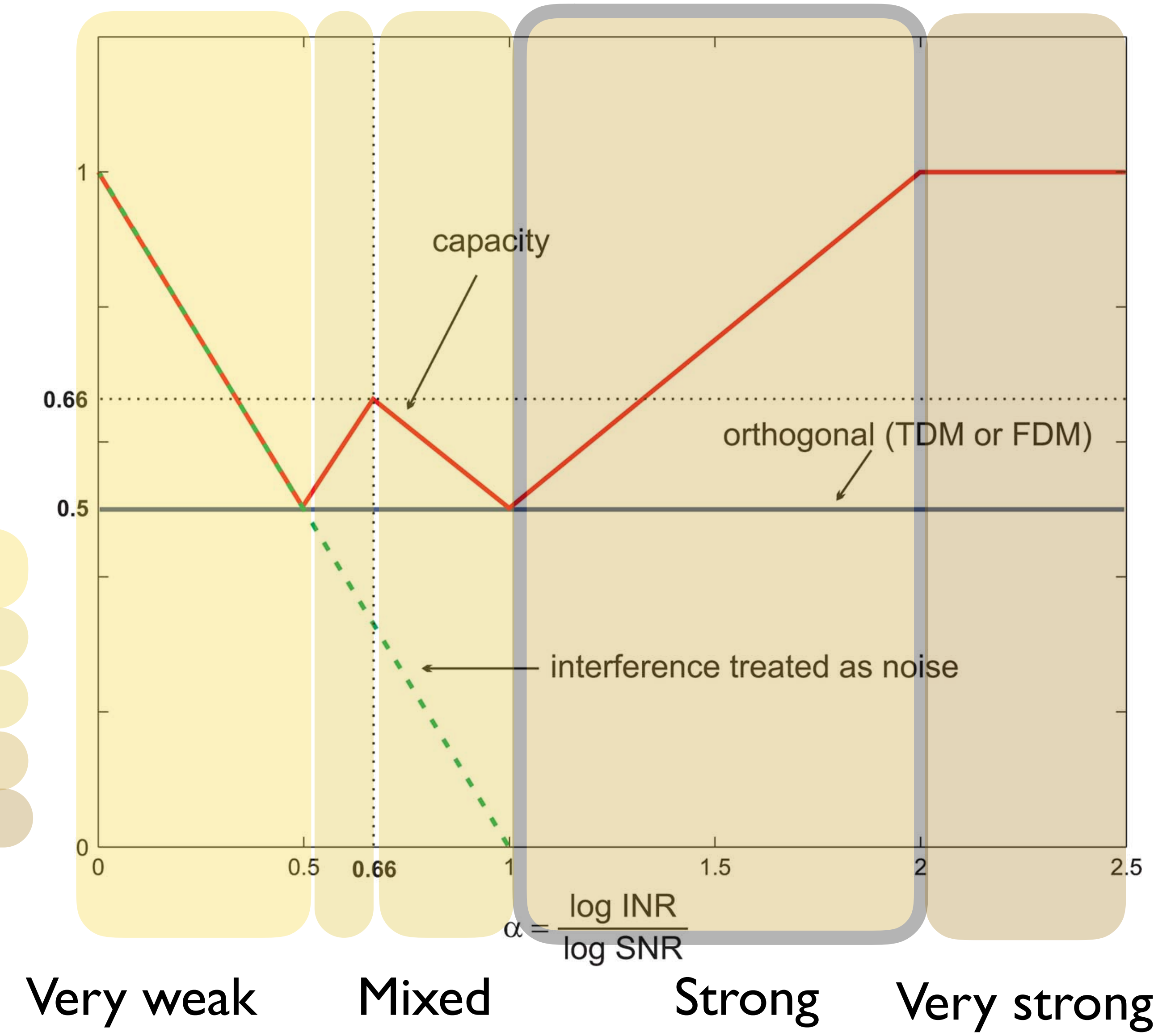


## Regimes

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

Strong: jointly decoding both messages at both receivers is capacity optimal, capacity known

# AWGN: the "W" curve for the GDoF

image + formula taken from

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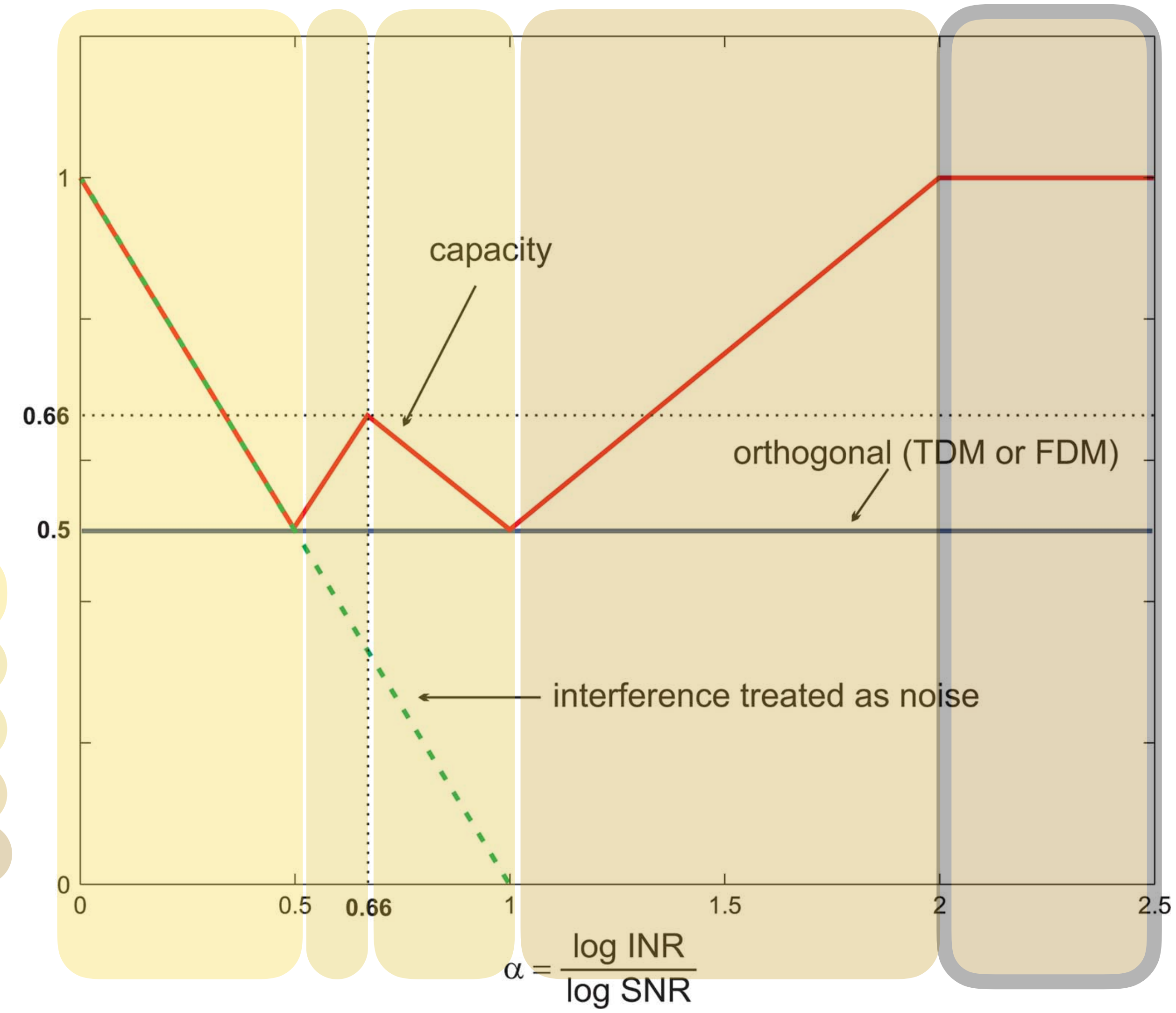


## Regimes

Very weak  Mixed  Strong  Very strong

Very weak: treating interference as noise is sometimes sum-capacity optimal, capacity partially known

Strong: jointly decode both messages at both receivers is capacity optimal, capacity known

Very strong: first decode interference then desired is capacity optimal, capacity known

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image + formula taken from

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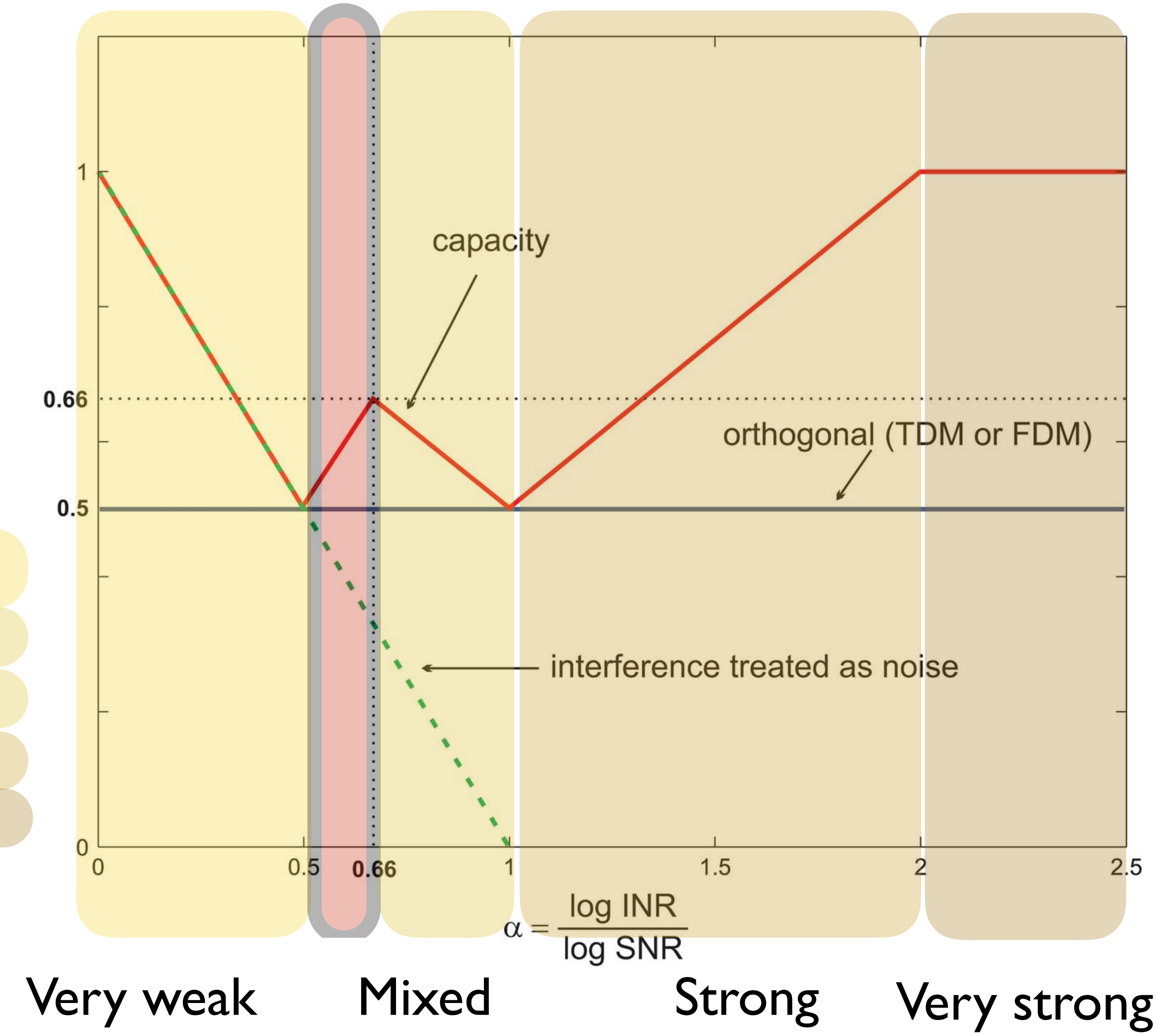


## Regimes

Very weak: treating interference as noise is sometimes capacity optimal, capacity partially known

Mixed 1: partially decode interference H+K is gDoF optimal — larger INR, cancel more, capacity unknown

Strong: jointly decode both messages at both receivers is capacity optimal, capacity known

Very strong: first decode interference then desired is capacity optimal, capacity known

# AWGN: the “W” curve for the GDoF

image + formula taken from

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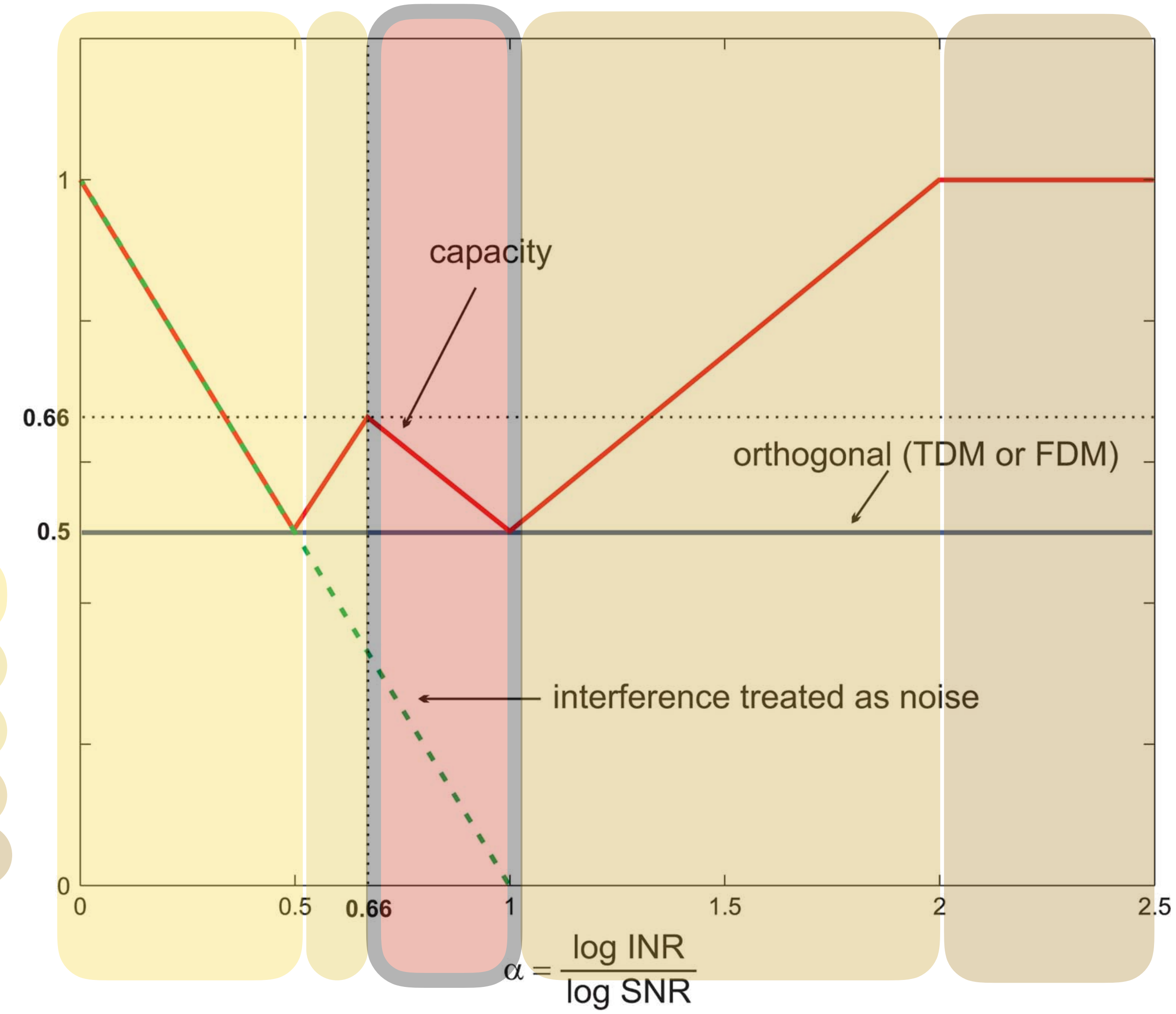



## Regimes

Very weak | Mixed | Strong | Very strong

Very weak: treating interference as noise is sometimes capacity optimal, capacity partially known

Mixed 1: partially decode interference H+K is gDoF optimal — larger INR, cancel more, capacity unknown

Mixed 2: partially decode interference H+K is gDoF optimal — larger INR hurts, capacity unknown

Strong: jointly decode both messages at both receivers is capacity optimal, capacity known

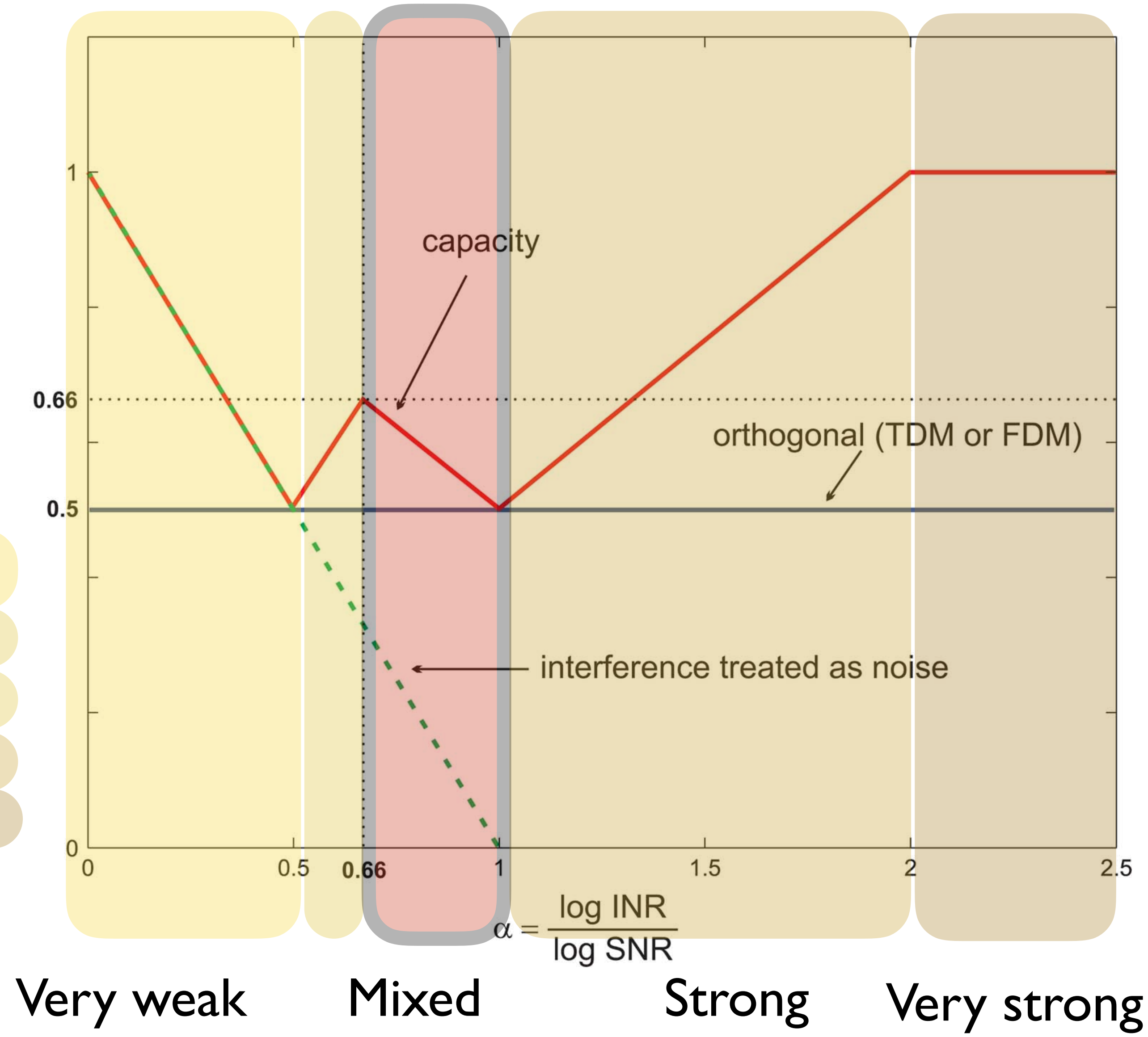
Very strong: first decode interference then desired is capacity optimal, capacity known

# AWGN: the "W" curve for the GDoF

image + formula taken from

[R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008. ]

$$C_{\mathrm{sym}} \approx \begin{cases} \log\left(\frac{\mathsf{SNR}}{\mathsf{INR}}\right) & \log \mathsf{INR} < \frac{1}{2}\log \mathsf{SNR} \\ \log \mathsf{INR} & \frac{1}{2}\log \mathsf{SNR} < \log \mathsf{INR} < \frac{2}{3}\log \mathsf{SNR} \\ \log \frac{\mathsf{SNR}}{\sqrt{\mathsf{INR}}} & \frac{2}{3}\log \mathsf{SNR} < \log \mathsf{INR} < \log \mathsf{SNR} \\ \log \sqrt{\mathsf{INR}} & \log \mathsf{SNR} < \log \mathsf{INR} < 2\log \mathsf{SNR} \\ \log \mathsf{SNR} & \log \mathsf{INR} > 2\log \mathsf{SNR} \end{cases}$$




## Regimes

Very weak: treating interference as noise is sometimes capacity optimal, capacity partially known

Mixed 1: partially decode interference H+K is gDoF optimal — larger INR, cancel more, capacity unknown

Mixed 2: partially ~~decode interference H+K is gDoF optimal — larger INR hurts, capacity unknown~~

Strong: jointly de

**requires interfering codebooks to (partially) decode interference!**

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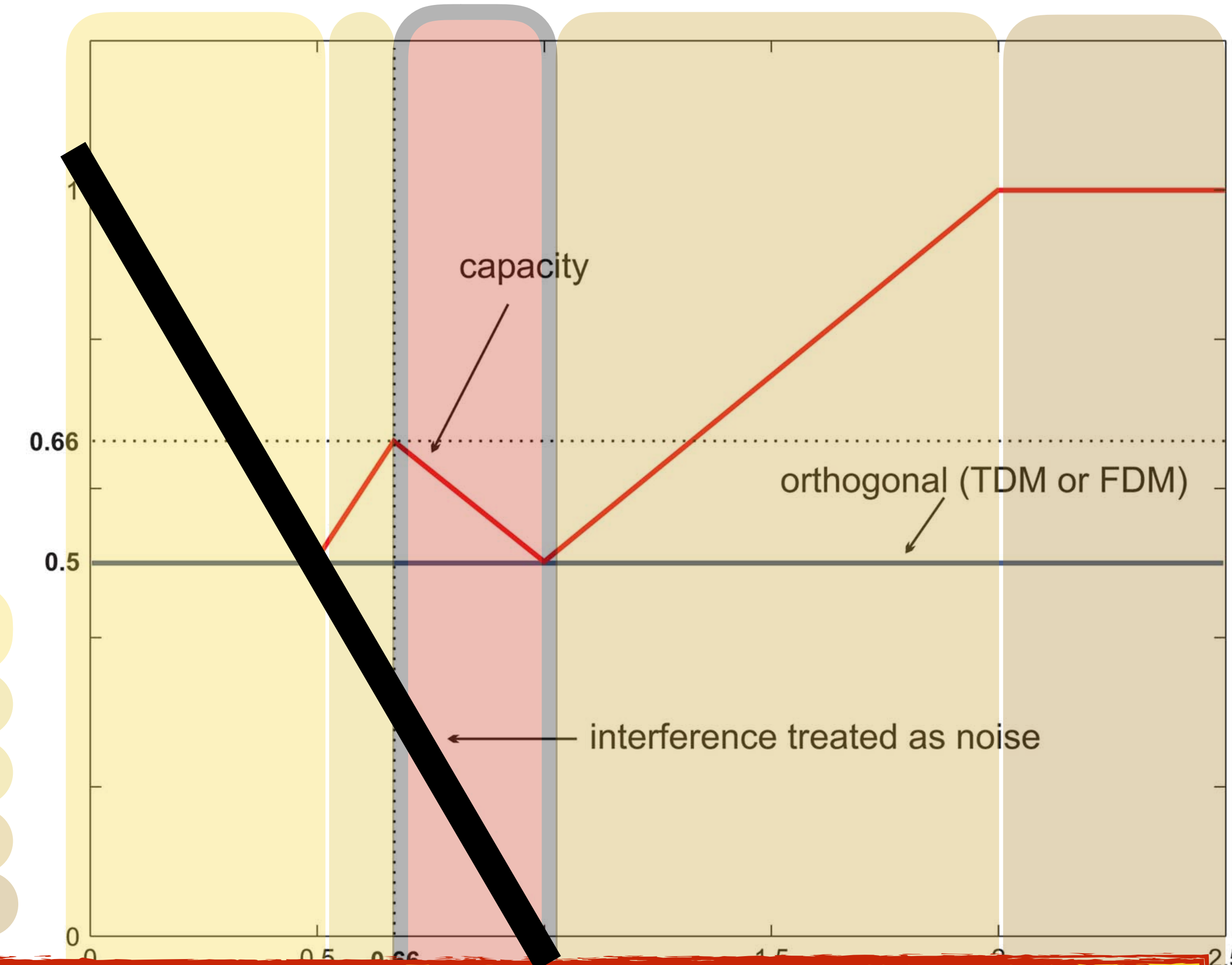


is this treating interference as noise all we can do!?

## Regimes

Very weak: treating interference as noise is sometimes capacity optimal, capacity partially known

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Alex Dysto

# Back to the question

Daniela Tuninetti

Natasha Devroye

## How does lack of codebook knowledge affect capacity of the Gaussian IC?

some slides taken from Alex Dytso's Ph.D. defense, May 2016

## Idea: use **non-Gaussian inputs** in a Gaussian interference channel!

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## How does lack of codebook knowledge affect capacity of the Gaussian IC?

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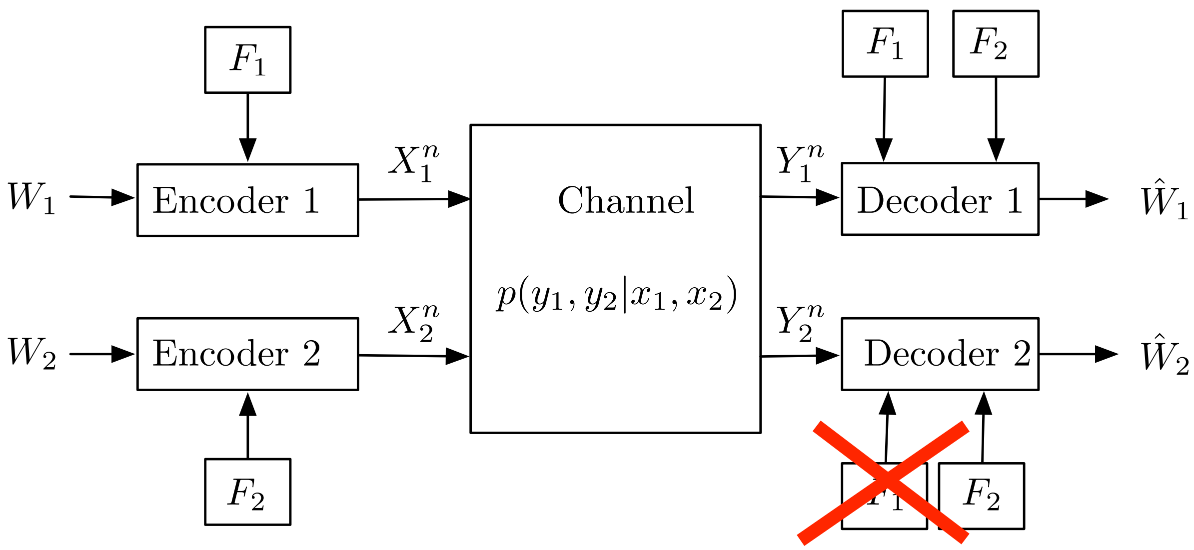
## Idea: use **non-Gaussian inputs** in a Gaussian interference channel!

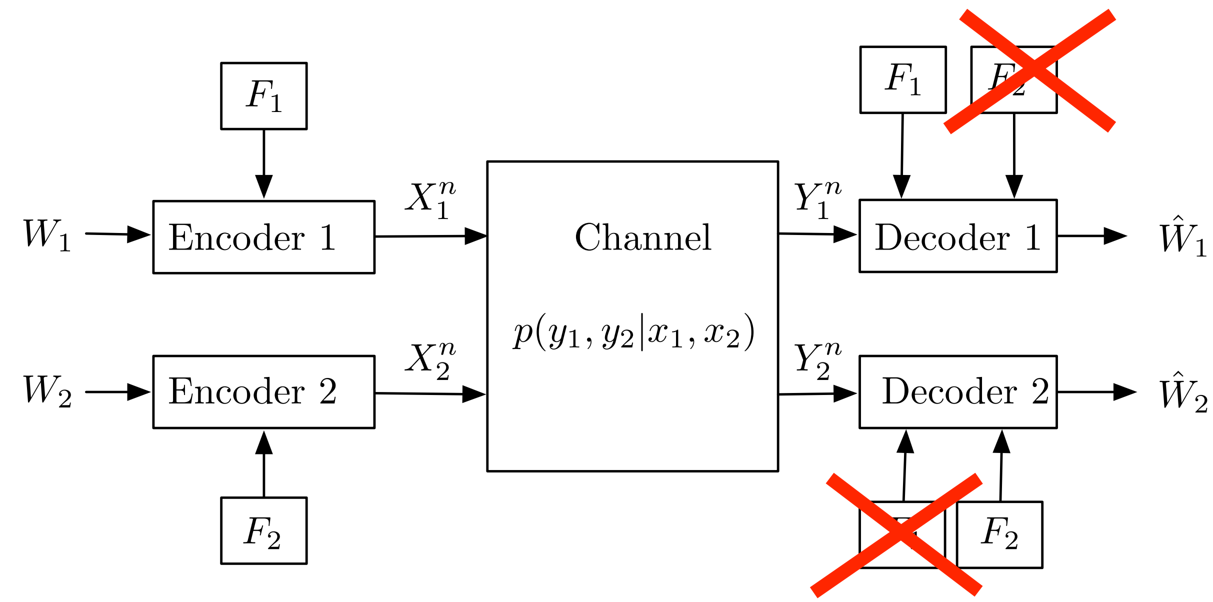
**Decent performance, and can``estimate" and strip off!**

# ICs with lack of codebook knowledge

*Our motivation:*




**IC with one oblivious Rx**




**IC with two oblivious Rx**

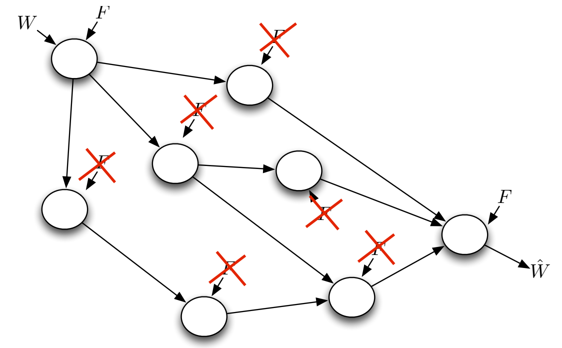
A. Dytso, N. Devroye, and D. Tuninetti, "*On the capacity of interference channels with partial codebook knowledge*," ISIT 2013

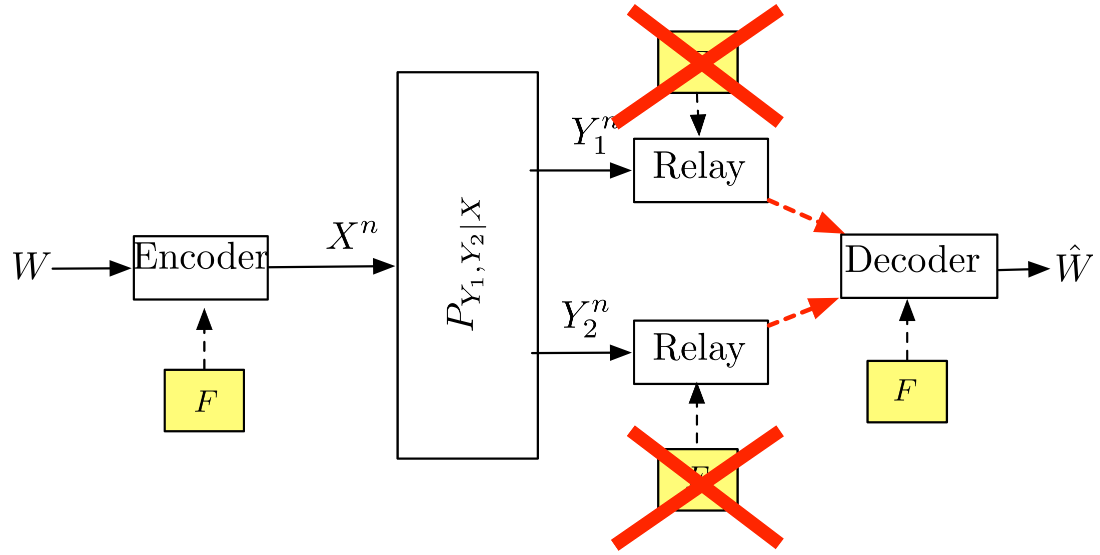
A. Dytso, D. Tuninetti and N. Devroye, ``On the Two-User Interference Channel With Lack of Knowledge of the Interference Codebook at One Receiver,'' IEEE Transactions on Information Theory, Vol. 61, No. 3, pp. 1256-1276, March 2015.

A. Dytso, D. Tuninetti and N. Devroye. "*On Gaussian Interference Channels with Mixed Gaussian and Discrete Inputs*," ISIT 2014

A. Dytso, D. Tuninetti and N. Devroye "*Interference as Noise: Friend of Foe?*" IEEE Trans. on Info Theory, June 2016.

# Past work: lack of codebooks leads to non-Gaussians outperforming Gaussians






A. Sanderovich, S. Shamai, Y. Steinberg, and G. Kramer, "*Communication via decentralized processing*," IT July 2008.

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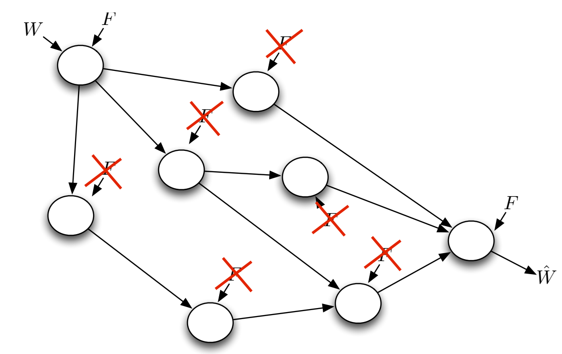


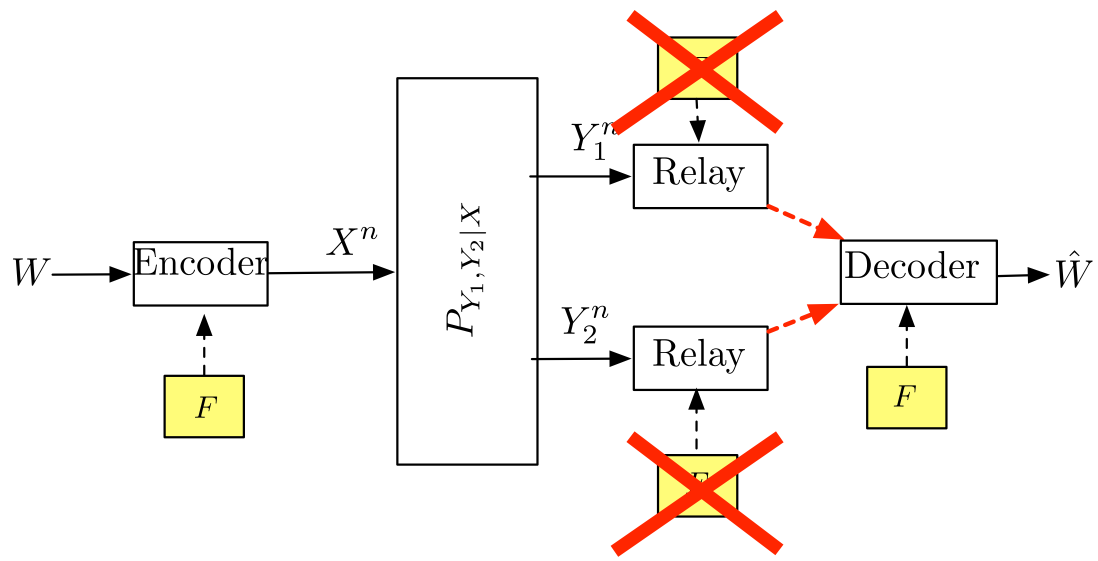


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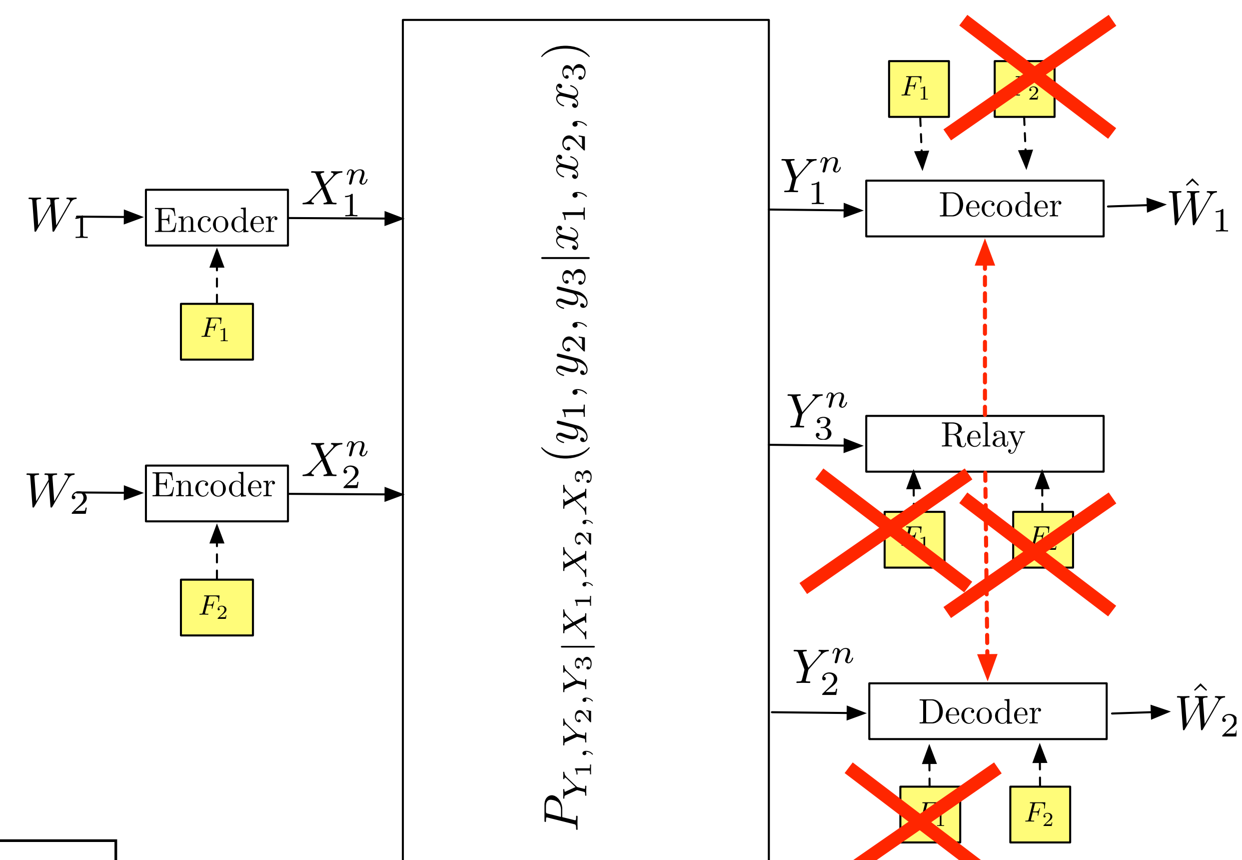


O. Simeone, E. Erkip, and S. Shamai, "*On codebook information for interference relay channels with out-of-band relaying*," IT May 2011.

1. Primitive relay channel: capacity with compress forward
2. IC+R+Oblivious receivers: capacity with compress forward and TIN
3. Gaussian noise: optimizing input unknown

$$\mathcal{C}^{\text{IC-OR}} = \bigcup_{P_Q P_{X_1|Q} P_{X_2|Q}} \left\{ \begin{array}{l} R_1 \leq I(X_1;Y_1|Q) \\ R_2 \leq I(X_2;Y_2|Q) \end{array} \right\}$$

[Ye Tian and Aylin Yener, Relaying for Multiuser Networks in the Absence of Codebook Information, IEEE Transactions on Information Theory, 61(3), pp. 1247-1256, Mar. 2015.]

# Discrete inputs in Gaussian channels — deeper?

## Other supporting arguments

- E. Abbe and L. Zheng, *"A coordinate system for Gaussian networks,"* IT 2012.
- E. Calvo, J. Fonollosa, and J. Vidal, *"On the totally asynchronous interference channel with single-user receivers,"* ISIT 2009

- No gDoF Gain
- Discrete input conclusions are simulation based

# Questions

- loss in performance in IC due to lack of interfering codebook knowledge?
- are there inputs that outperform Gaussians in the AWGN IC under these conditions?
- can we show analytical gains?

# How we tackle discrete inputs for G-IC

- best inner bound for Gaussian IC is the complex H+K scheme

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Similar results as

S. Li, Y.-C. Huang, T. Liu, and H.D. Pfister, "On the limits of treating interference as noise in the two-user Gaussian symmetric interference channel," ISIT 2015.

# IC channel capacity is known.... sort of

**Capacity:** $\mathcal{C} \quad = \lim_{n\to\infty} \mathrm{co}\left(\bigcup_{P_{X_1^n X_2^n} = P_{X_1^n} P_{X_2^n}} \left\{ \begin{array}{c} 0 \le R_1 \le \frac{1}{n} I(X_1^n; Y_1^n) \\ 0 \le R_2 \le \frac{1}{n} I(X_2^n; Y_2^n) \end{array} \right\}\right)$

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**Treat Interference as Noise Inner Bound:**

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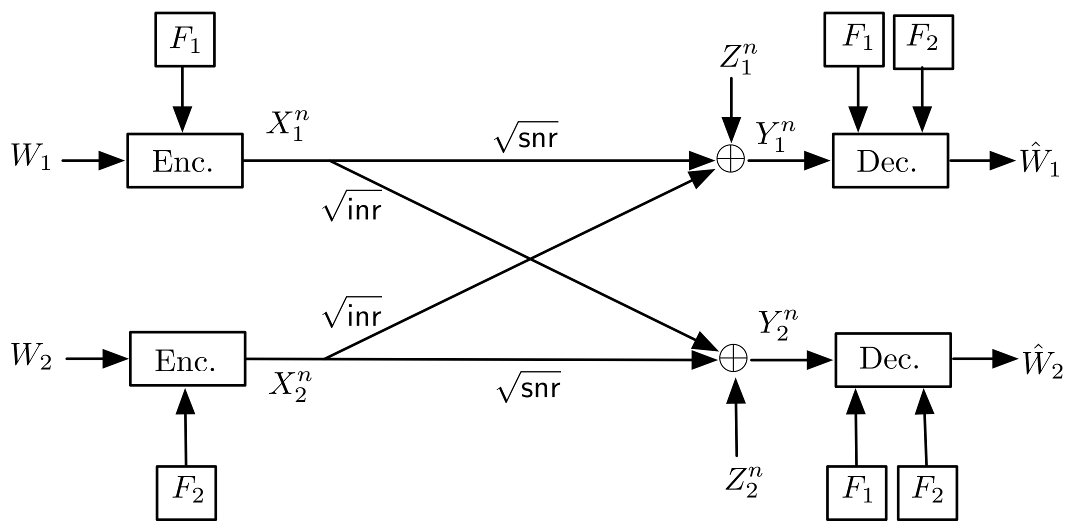
**How far away is TINnoTS from capacity?**

**Is it really "treating interference as noise"?**

# Gaussian channels with discrete inputs




$$Z_1, Z_2 \sim \mathcal{N}(0,1)$$

$$Y_1 = \sqrt{\mathsf{snr}}X_1 + \sqrt{\mathsf{inr}}X_2 + Z_1$$
$$Y_2 = \sqrt{\mathsf{inr}}X_1 + \sqrt{\mathsf{snr}}X_2 + Z_2$$

- instead of taking $X_1$ and $X_2$ to be Gaussian, take them to be discrete
- difficulty: how to evaluate mutual information expressions with discrete and Gaussian mixtures

$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{P_{X_1X_2}=P_{X_1}P_{X_2}} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1;Y_1) \\ 0 \le R_2 \le I(X_2;Y_2) \end{array} \right\}$$



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# Tools for Discrete Inputs

# Discrete+mixed inputs

- Discrete input $$X_D \sim P(X_D) = \sum_{i=1}^{|X|} p_i \delta(x_i)$$

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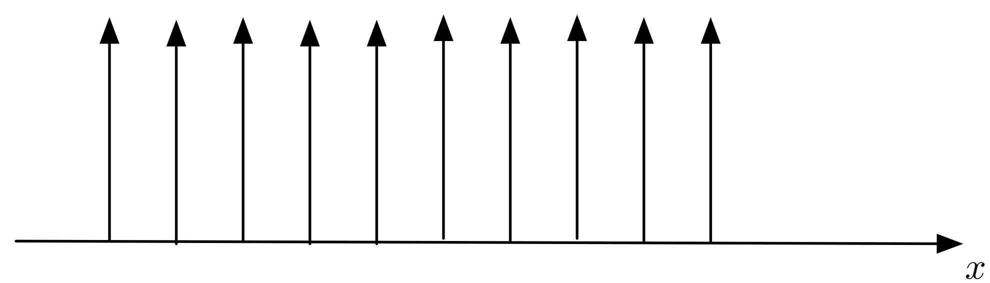
$$X_D \sim \mathrm{PAM(N), |X| = N, p_i = \frac{1}{N} \text{ for all } i \in [1, ..., N]}$$

- PAM input



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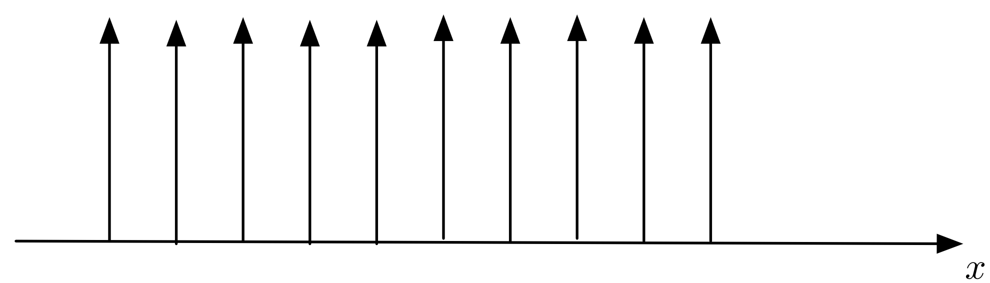


- Minimum distance

$$d_{\min}(X_D) = \min_{x_i, x_j : i \neq j} \|x_i - x_i\|$$

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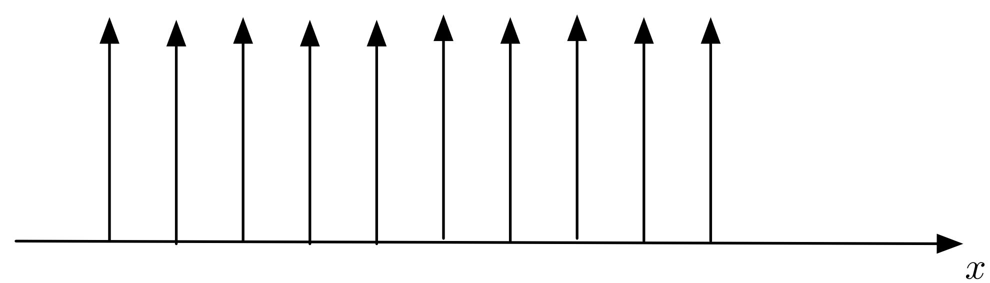


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- Mixed inputs

$$X_{\mathrm{mix}} = \sqrt{1-\delta} X_D + \sqrt{\delta} X_G,$$
$$\delta \in [0,1],$$
$$X_G \sim \mathcal{N}(0,1)$$
$$\mathbb{E}[X_D^2] \leq 1$$

# Discrete+mixed inputs

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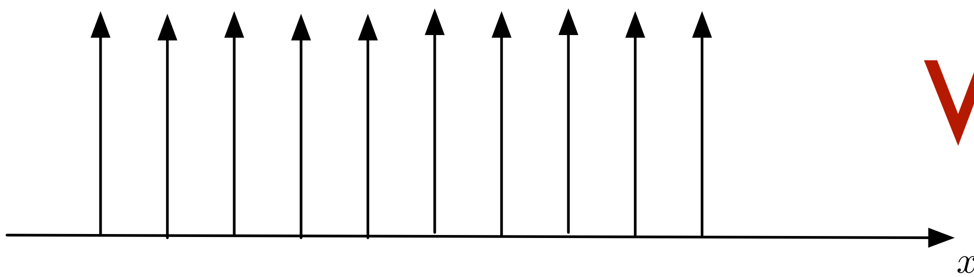


Why PAM?

- Minimum distance

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# Bounds on mutual information

**Setup:**

$$Y = \sqrt{\mathrm{snr}} X + Z,$$
$$Z \sim \mathcal{N}(0,1)$$

**We define:**

$$I(X; Y_{\mathrm{snr}}) = I(X, \mathrm{snr})$$
$$\mathbb{E}\left[(X - \mathbb{E}[X|Y_{\mathrm{snr}}])^2\right] = \mathrm{mmse}(X, \mathrm{snr})$$

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**Interested in:**

$$[H(X_D) - \mathrm{gap}]^+ \leq I(X_D, \mathrm{snr}) \leq H(X_D)$$

**Want the tightest version of the "gap" term for a given PMF**

# Bounds on mutual information

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## Ozarow-Wyner-A

$$\text{gap}_{\text{OW-A}} \le \xi \log \frac{1}{\xi} + (1-\xi)\log\frac{1}{1-\xi} + \xi \log(N-1)\ ,\ \ \xi := 2Q\left(\frac{\sqrt{\text{snr}}\, d_{\min(X_D)}}{2}\right)$$

## Ozarow-Wyner-B

$$\text{gap} \le \frac{1}{2}\log\left(\frac{\pi \mathrm{e}}{6}\right) + \frac{1}{2}\log\left(1 + \frac{\text{lmmse}(X, \text{snr})}{d_{\min}(X_D)^2}\right)$$

L. Ozarow and A. Wyner, "On the capacity of the Gaussian channel with a finite number of input levels," IEEE Trans. Inf. Theory, vol. 36, no. 6, pp. 1426–1428, Nov 1990.

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## DTD-ITA` 14-B

$$\left[ -\log\left( \sum_{(i,j)\in[1:N]^2} \frac{p_i p_j}{\sqrt{4\pi}} \mathrm{e}^{-\frac{\mathrm{snr}(x_i - x_j)^2}{4}} \right) - \frac{1}{2} \log(2\pi \mathrm{e}) \right]^+ \leq I(X_D, \mathrm{snr}) \leq H(X_D)$$

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$$\left[-\log\left(\sum_{(i,j)\in[1:N]^2} \frac{p_i p_j}{\sqrt{4\pi}} e^{-\frac{\text{snr}(x_i - x_j)^2}{4}}\right) - \frac{1}{2}\log(2\pi e)\right]^+ \le I(X_D, \text{snr}) \le H(X_D)$$

$$\text{gap}_{\text{ITA}} \le \frac{1}{2}\log\left(\frac{e}{2}\right) + \log\left(1 + (N-1)e^{-\frac{\text{snr}d^2_{\min(X_D)}}{4}}\right)$$

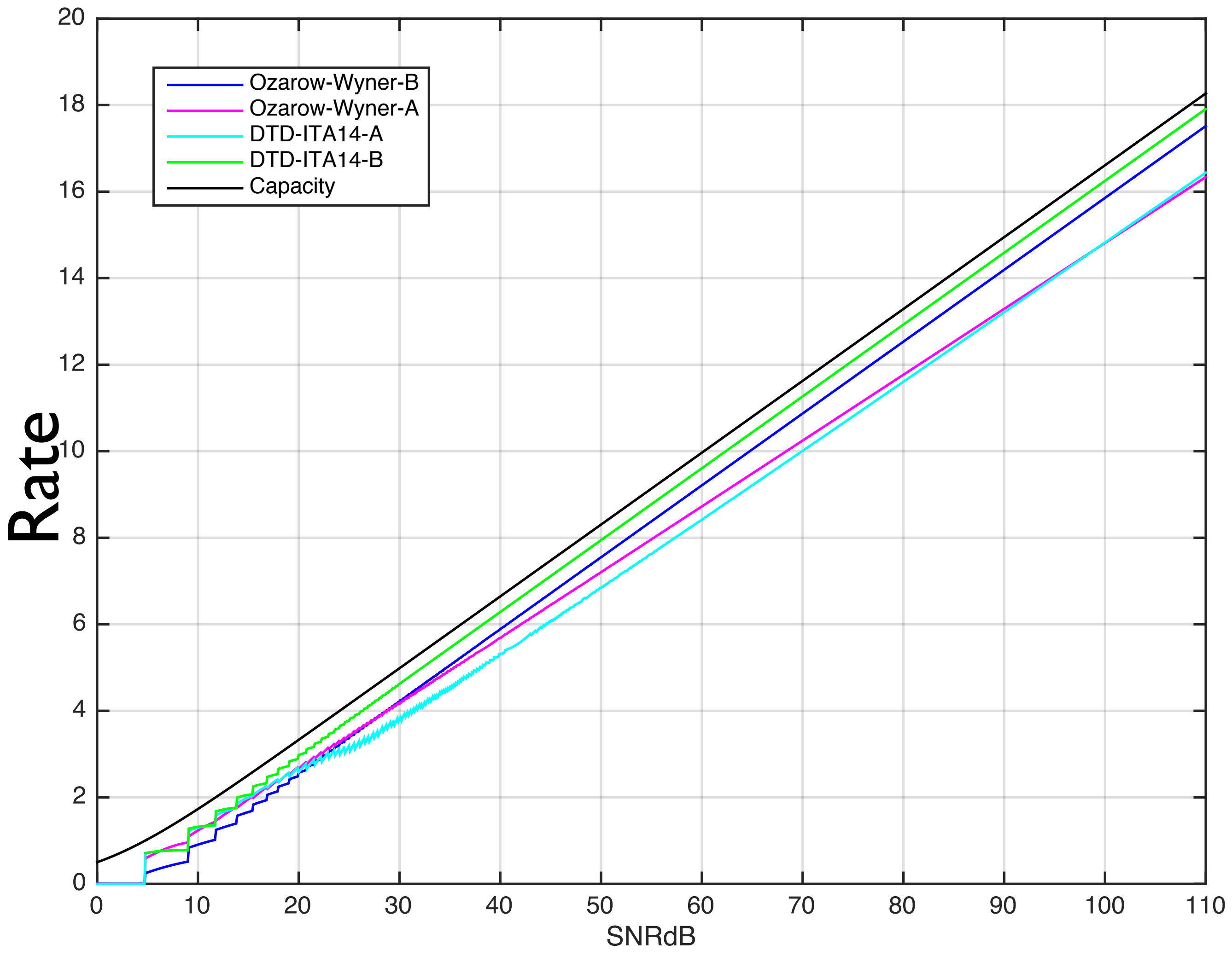
## DTD-ITA` 14-A

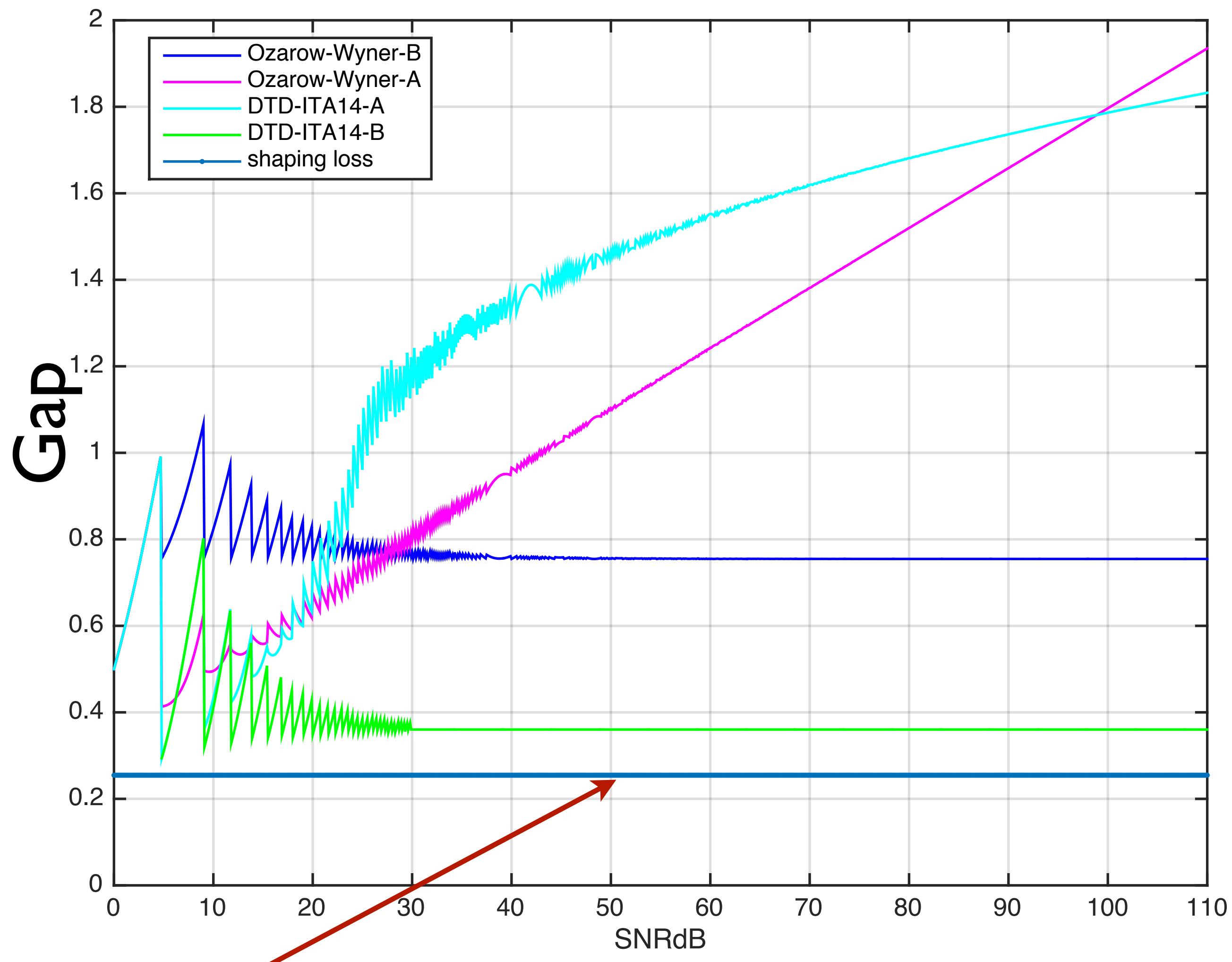
Dytso, A.; Tuninetti, D.; Devroye, N., "On discrete alphabets for the two-user Gaussian interference channel with one receiver lacking knowledge of the interfering codebook," ITA, 2014 , vol., no., pp.1,8, 9-14 Feb. 2014

# Comparison of bounds

## Input: PAM with number of points

$$N = \lfloor \sqrt{1+\mathrm{snr}} \rfloor \Rightarrow H(X) = \log(N) \approx \frac{1}{2}\log(1+\mathrm{snr})$$




shaping loss of uniform lattice

# More recent tighter bounds

## Tighter Ozarow-Wyner bounds on gaps available, based on the MMPE:

A. Dytso, R. Bustin, D. Tuninetti, N. Devroye, H.V. Poor and S. Shamai "On the Minimum Mean p-th Error in Gaussian channels and its Application," under submission to TransIT, 2016.

$$\text{gap} \le \frac{1}{2}\log\left(\frac{\pi e}{6}\right) + \frac{1}{2}\log\left(1 + \frac{\text{lmmse}(X,\text{snr})}{d_{\min}(X_D)^2}\right),$$

↓ tighter

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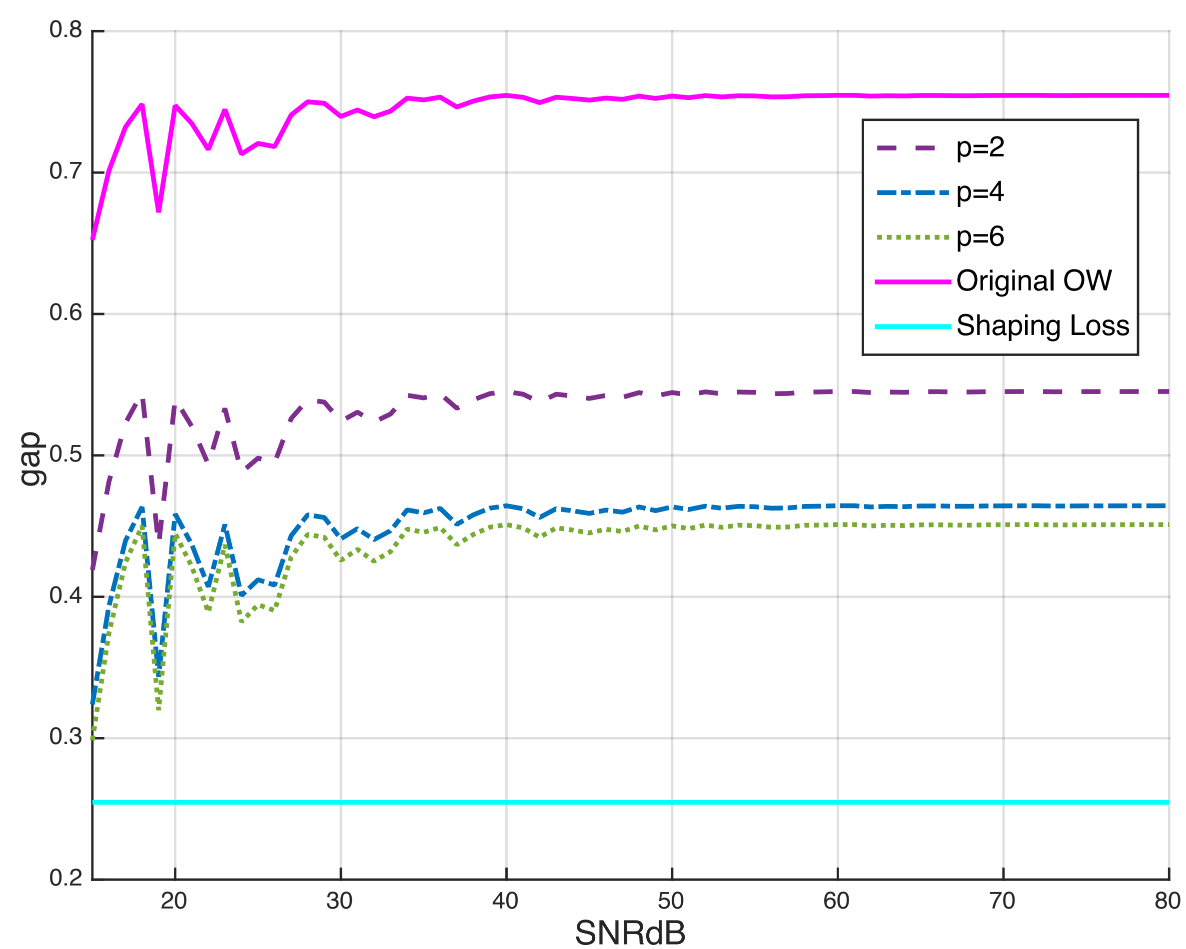
$$n^{-1}\text{gap}_{\mathsf{p}} \le \inf_{\mathbf{U}\in\mathcal{K}_{\mathsf{p}}} \left(G_{1,\mathsf{p}}(\mathbf{U},\mathbf{X}_D) + G_{2,\mathsf{p}}(\mathbf{U})\right),$$

$$G_{1,\mathsf{p}}(\mathbf{U},\mathbf{X}_D) = \log\left(\frac{\|\mathbf{U}+\mathbf{X}_D - f_{\mathsf{p}}(\mathbf{X}_D|\mathbf{Y})\|_{\mathsf{p}}}{\|\mathbf{U}\|_{\mathsf{p}}}\right)$$

$$\overset{\textit{for } \mathsf{p}\ge 1}{\le} \log\left(1 + \frac{\text{mmpe}^{\frac{1}{\mathsf{p}}}(\mathbf{X}_D,\text{snr},\mathsf{p})}{\|\mathbf{U}\|_{\mathsf{p}}}\right),$$

$$G_{2,\mathsf{p}}(\mathbf{U}) = \log\left(\frac{k_{n,\mathsf{p}}\cdot n^{\frac{1}{\mathsf{p}}}\cdot\|\mathbf{U}\|_{\mathsf{p}}}{e^{\frac{1}{n}h_e(\mathbf{U})}}\right).$$



# Why is discrete good?

**good input**

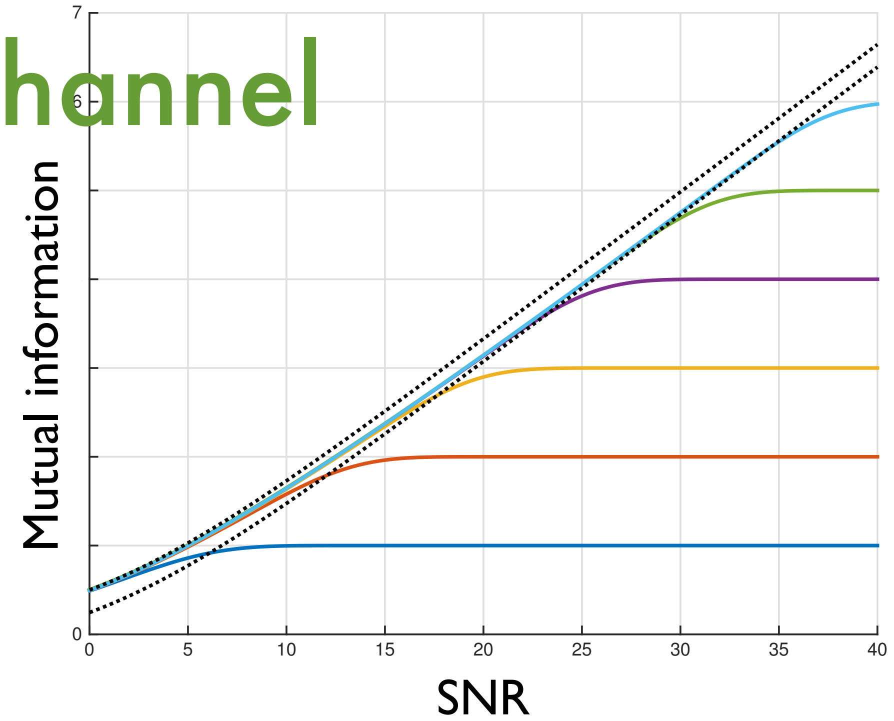
**good interferer**

# Discrete is a good input.

## 1. Point-to-point Gaussian noise Channel

$$Y = \sqrt{\text{snr}}X + Z_G :$$

$$E[X^2] \leq 1, Z_G \sim \mathcal{N}(0,1)$$




## Capacity

$$C = \frac{1}{2}\log(1+\text{snr})$$

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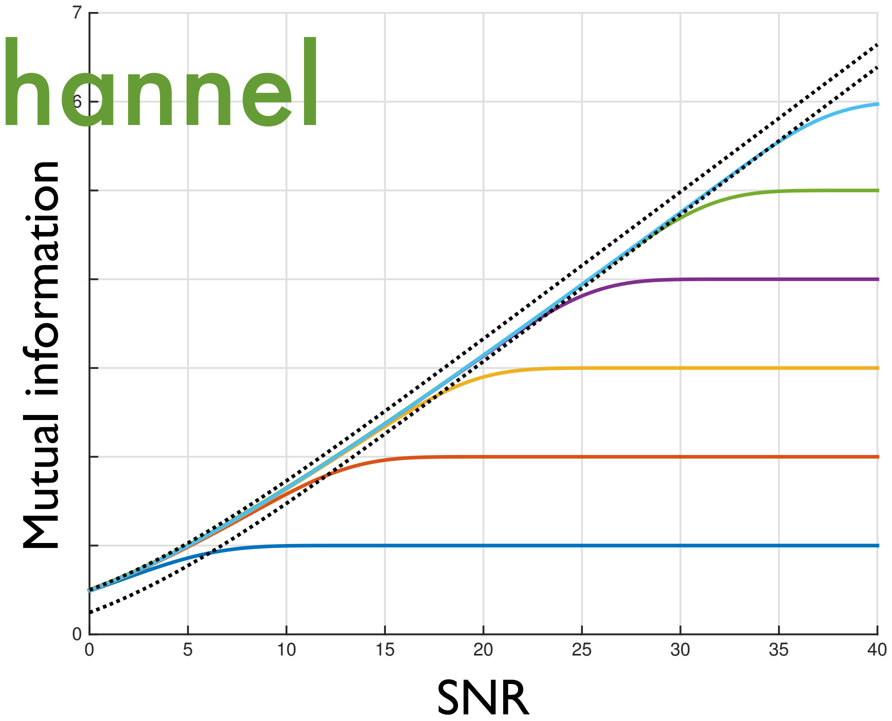


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**with PAM:**

$$N = \lfloor \sqrt{1+\text{snr}} \rfloor$$

$$C \geq \frac{1}{2}\log(1+\text{snr}) - \text{gap}$$

$$\text{gap} = \frac{1}{2}\log\left(\frac{4\pi e}{3}\right)$$

# Discrete is a good interferer.

## 2. Point-to-point Gaussian noise Channel with State

$Y = \sqrt{\mathrm{snr}}X + hT + Z_G :$

$E[X^2] \leq 1, Z_G \sim \mathcal{N}(0,1),$

$T \sim \text{discrete: } |T| = N \text{ and } d^2_{\min(T)} > 0$

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## Discrete Interference

$$C \geq I(X_G; \sqrt{\text{snr}}X_G + hT + Z_G)$$
$$\geq \frac{1}{2}\log(1+\text{snr}) - \text{gap}$$

$$\text{gap} = \frac{1}{2}\log\left[\frac{2\pi e}{12}\left(1 + \frac{12}{d^2_{\min}(T)}\frac{|h|^2\mathcal{E}_T}{|h|^2\mathcal{E}_T + 1 + \text{snr}}\right)\right]$$

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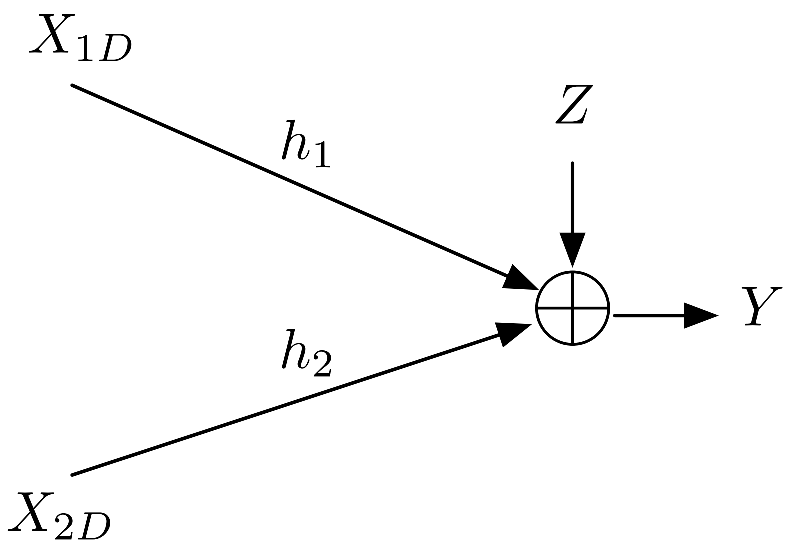
**Gaussian Interference**

$$C \geq I(X_G; \sqrt{\text{snr}}X_G + hT_G + Z_G)$$

$$= \frac{1}{2}\log\left(1 + \frac{\text{snr}}{1 + |h|^2\mathcal{E}_T}\right)$$

# Discrete inputs in multi-user channels

## More complex in multi-user scenarios




"sum-set"

$$h_1 X_{1D} + h_2 X_{2D} = \{h_1 x_{1D} + h_2 x_{2D} | x_1 \in X_{1D}, x_2 \in X_{2D}\}$$

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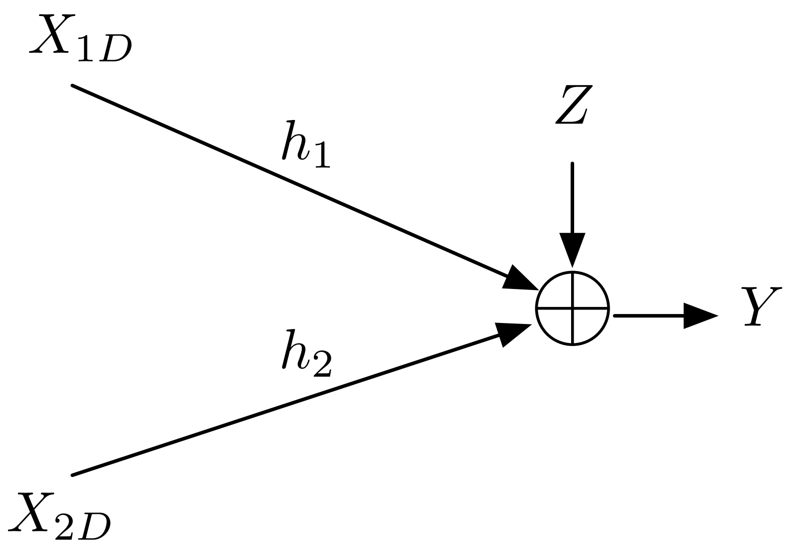


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$$|h_1 X_{1D} + h_2 X_{2D}| = |\{h_1 x_{1D} + h_2 x_{2D} | x_1 \in X_{1D}, x_2 \in X_{2D}\}|$$ ???

$$d_{\min}(h_1 X_{1D} + h_2 X_{2D}) = \min\{|s_i - s_j| : s_i, s_j \in h_1 X_{1D} + h_2 X_{2D}, i \neq j\}$$ ???

# New phenomenon

## Example, BPSK:

$$X_{1D} = X_{2D} = \{-1, +1\}$$

$$h_1 X_{1D} + h_2 X_{2D} \overset{(h_1=1, h_2=2)}{=} \{3, -1, 1, 3\}$$

$$\overset{(h_1=1, h_2=1)}{=} \{1, 0, -1\}$$

“Cardinality is Sensitive to Channel Gain Values.”

# Overall proposition / tool

- cardinality of the sum-set $\{h_x X + h_y Y\}$

**Proposition:** Let $X \sim \mathsf{PAM}(|X|, d_{\min(X)})$ and $Y \sim \mathsf{PAM}(|Y|, d_{\min(Y)})$. Then for $(h_x, h_y) \in \mathbb{R}^2$

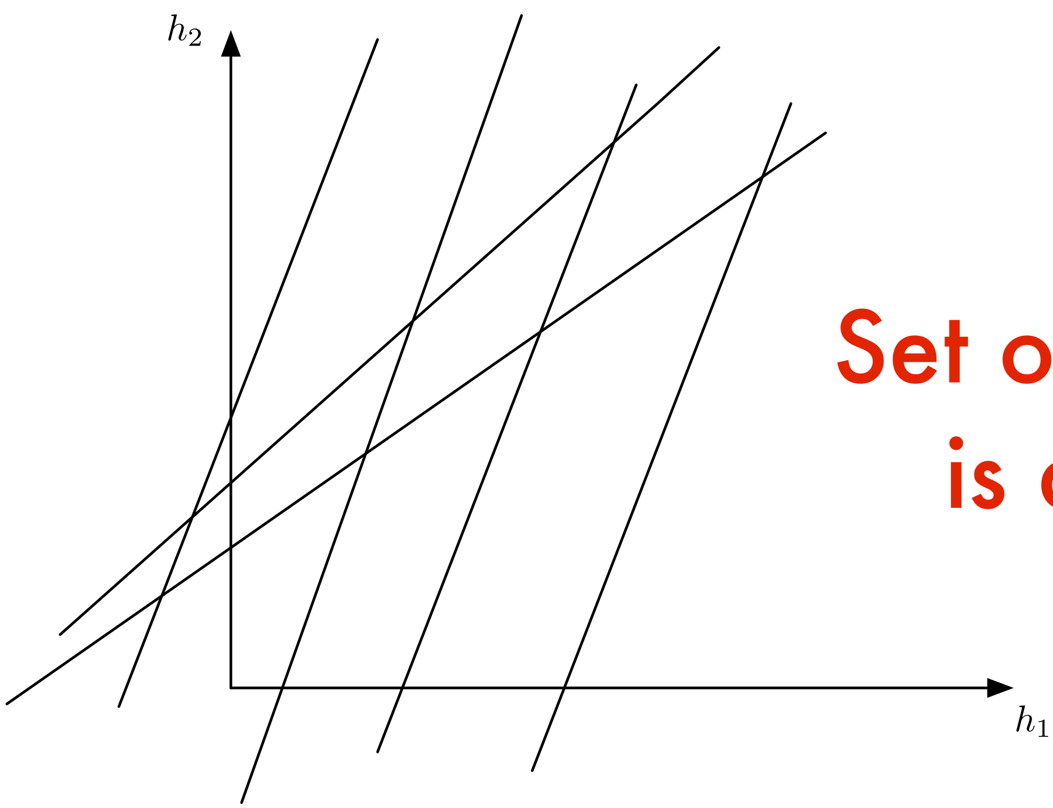
$$|h_x X + h_y Y| = |X||Y| \text{ almost everywhere (a.e.)}, \tag{1}$$

- minimum distance of the sum-set

and $d_{\min(h_x X + h_y Y)} \geq \text{......?}$

# Cardinality

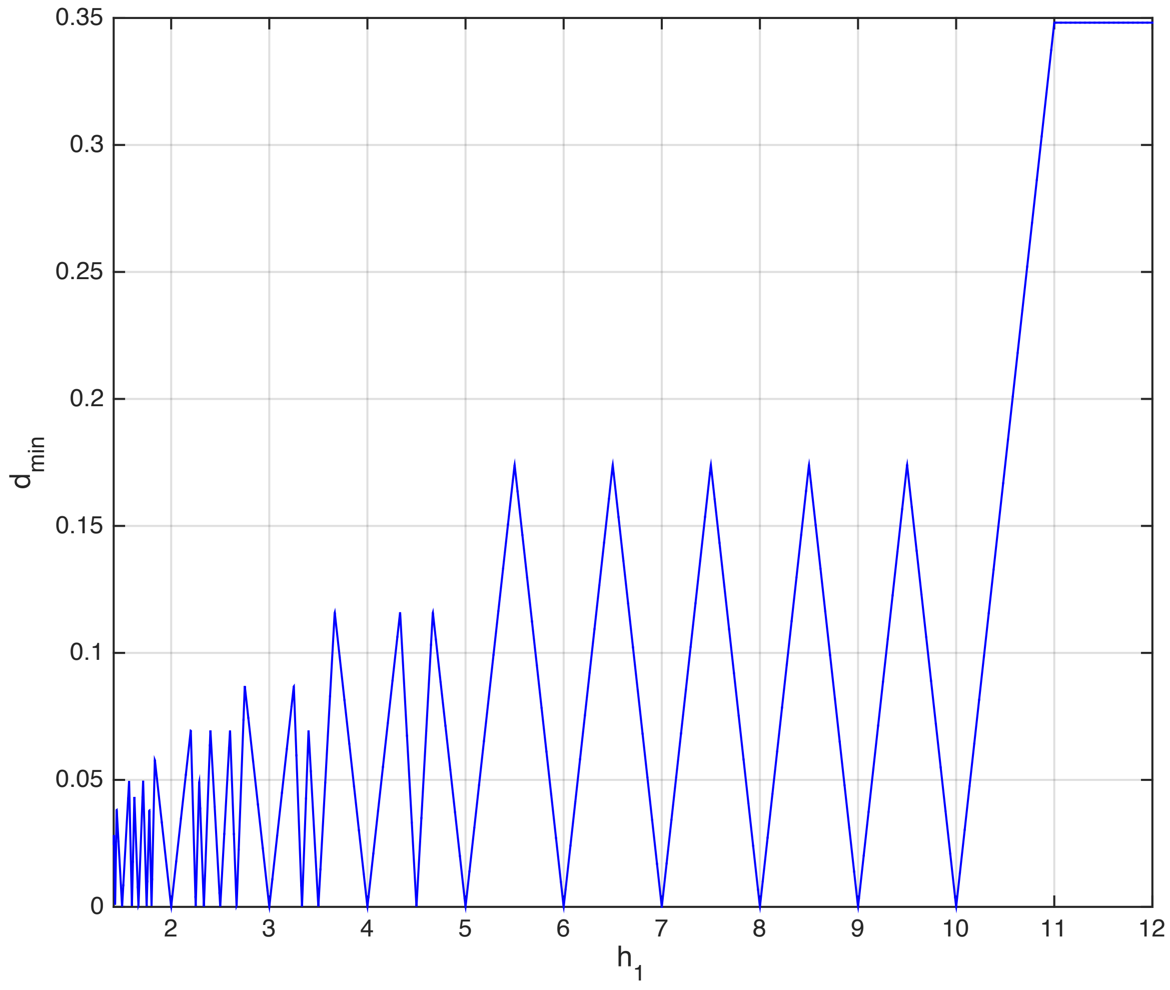
$$|h_x X + h_y Y| = |X||Y| \text{ almost everywhere (a.e.)}$$




Set of values where cardinality is less is a union of lines, of measure 0

# Minimum distance

## Example:h2=1,  N1=N2=10




Very Irregular

Can we even have a lower bound?

$$\mathrm{gap}_{\text{OW-B}} \leq \frac{1}{2}\log\left(\frac{\pi \mathrm{e}}{6}\right) + \frac{1}{2}\log\left(1 + \frac{12}{\mathrm{snr}\ d^2_{\min(X_D)}}\right)$$

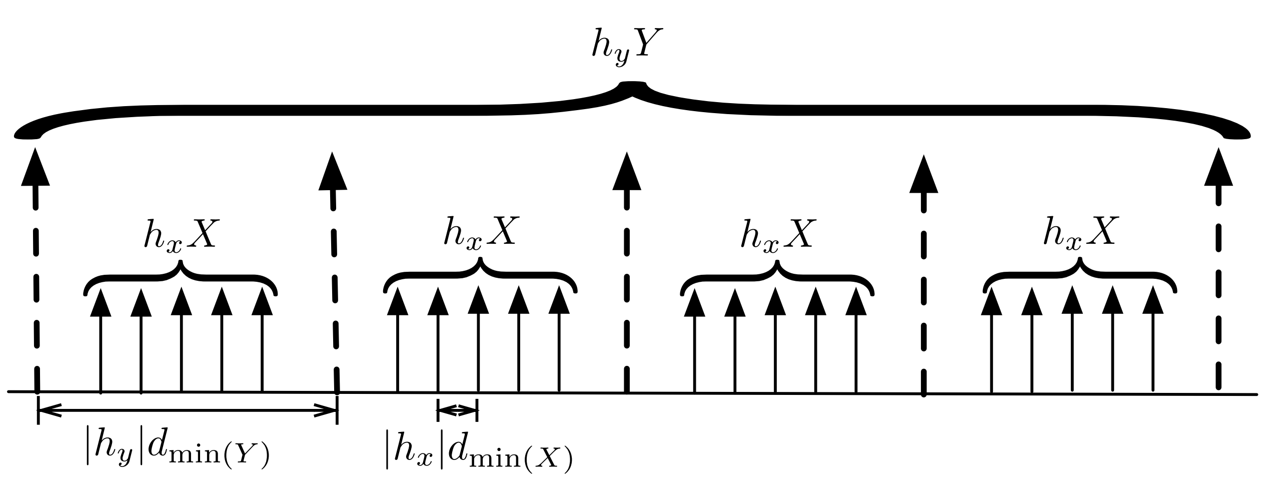
# Minimum distance, case 1: no overlap

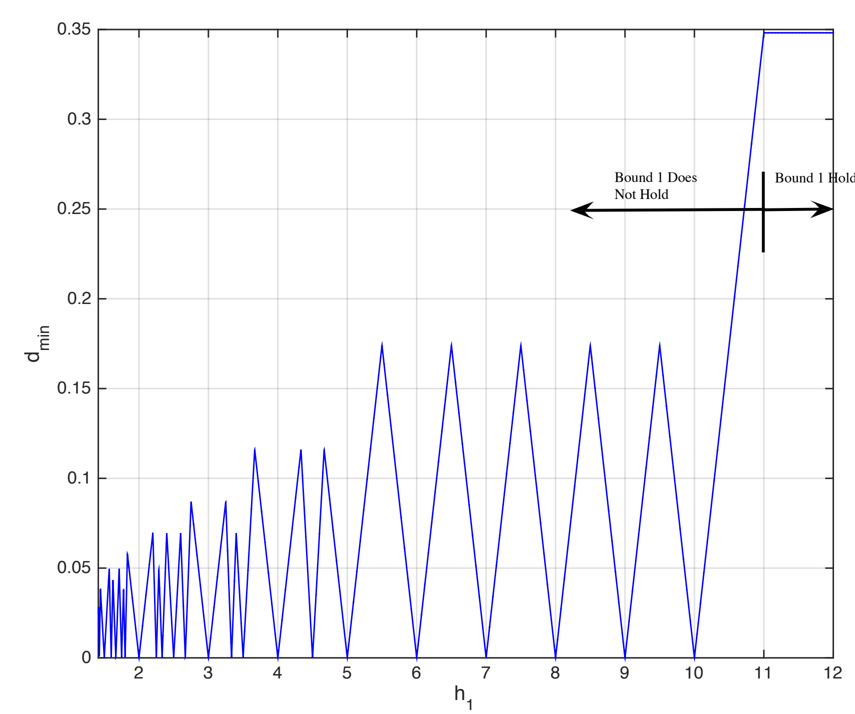
We have

$$d_{\min(h_x X + h_y Y)} = \min\left(|h_x| d_{\min(X)}, |h_y| d_{\min(Y)}\right)$$

under the following conditions

$$\begin{aligned} \text{either} \quad & |Y||h_y| d_{\min(Y)} \leq |h_x| d_{\min(X)}, \\ \text{or} \quad & |X||h_x| d_{\min(X)} \leq |h_y| d_{\min(Y)} \text{ (shown below).} \end{aligned}$$



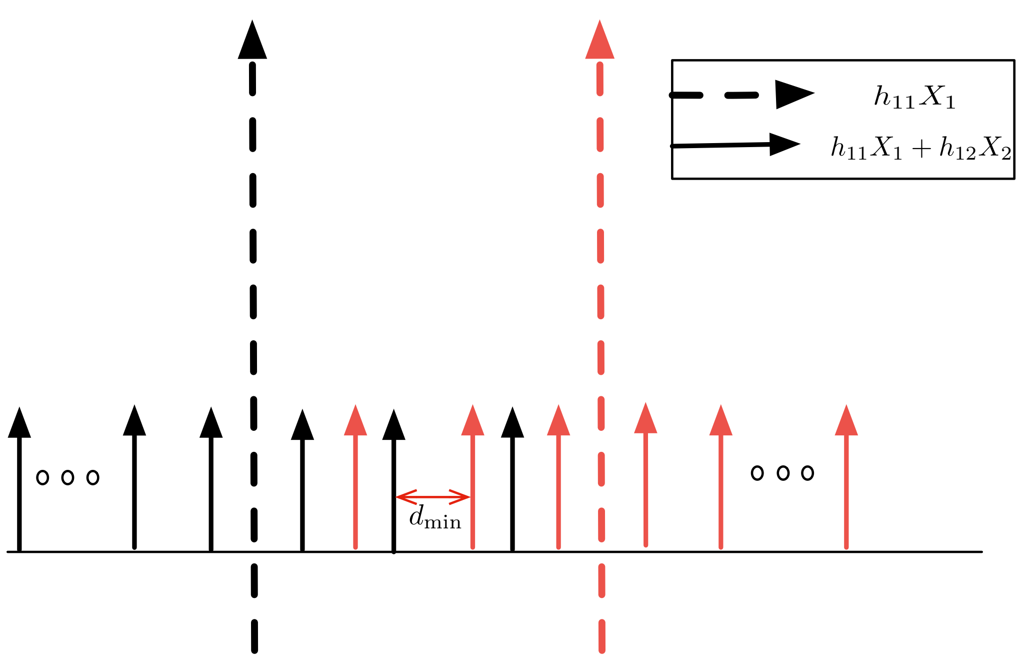
# Minimum distance, case 2: with overlap

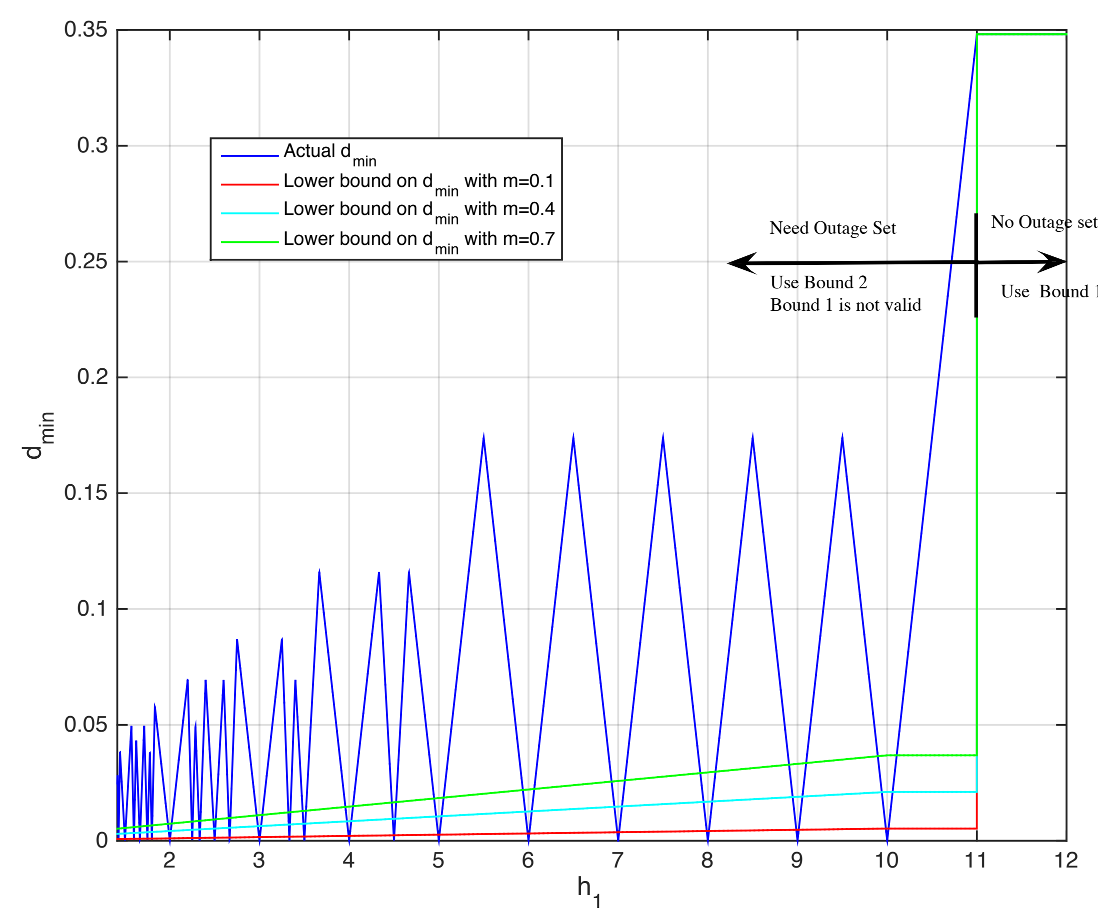
Then, up to a set of $(h_x, h_y)$ of measure no more than $\gamma$, we have

$$d_{\min(h_x X + h_y Y)} \geq \kappa_{\gamma,|X|,|Y|} \cdot \min\left(|h_x| d_{\min(X)}, |h_y| d_{\min(Y)}, \xi_{|h_x|,|h_y|,|X|,|Y|}\right),$$

$$\kappa_{\gamma,|X|,|Y|} := \frac{\gamma/2}{1 + \ln(\max(|X|, |Y|))},$$

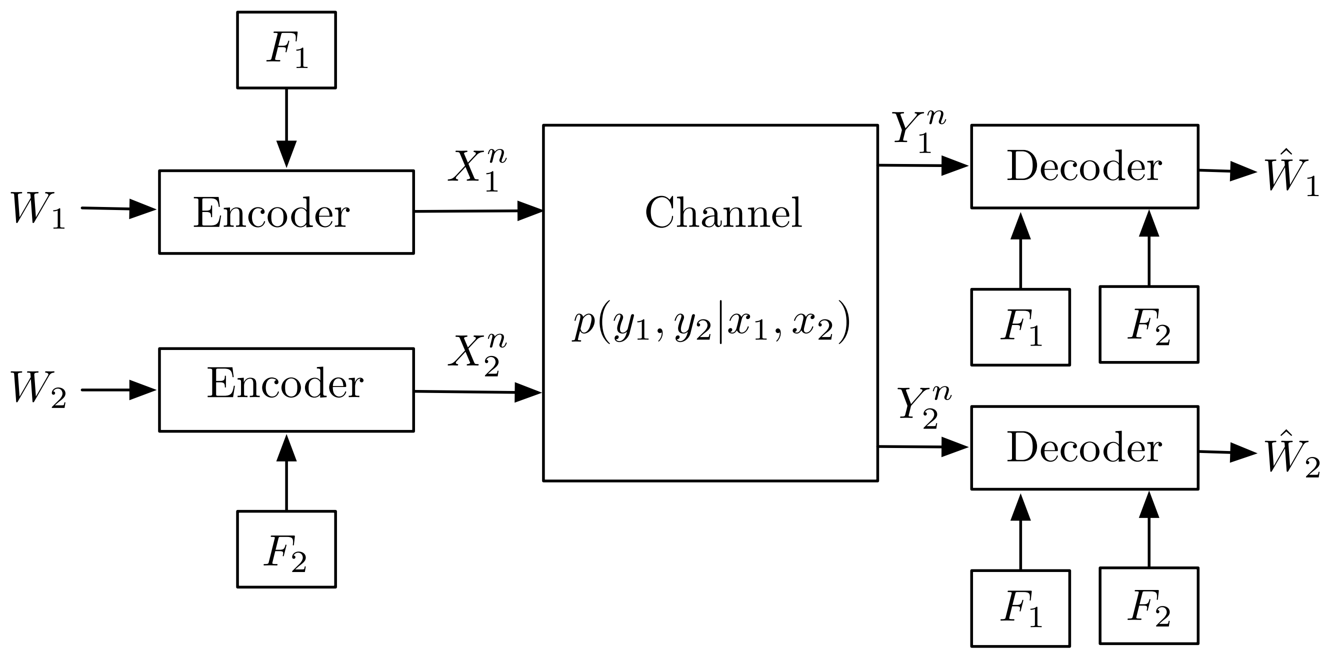
$$\xi_{|h_x|,|h_y|,|X|,|Y|} := \max\left(\frac{|h_x| d_{\min(X)}}{|Y|}, \frac{|h_y| d_{\min(Y)}}{|X|}\right),$$



# Applications of discrete inputs

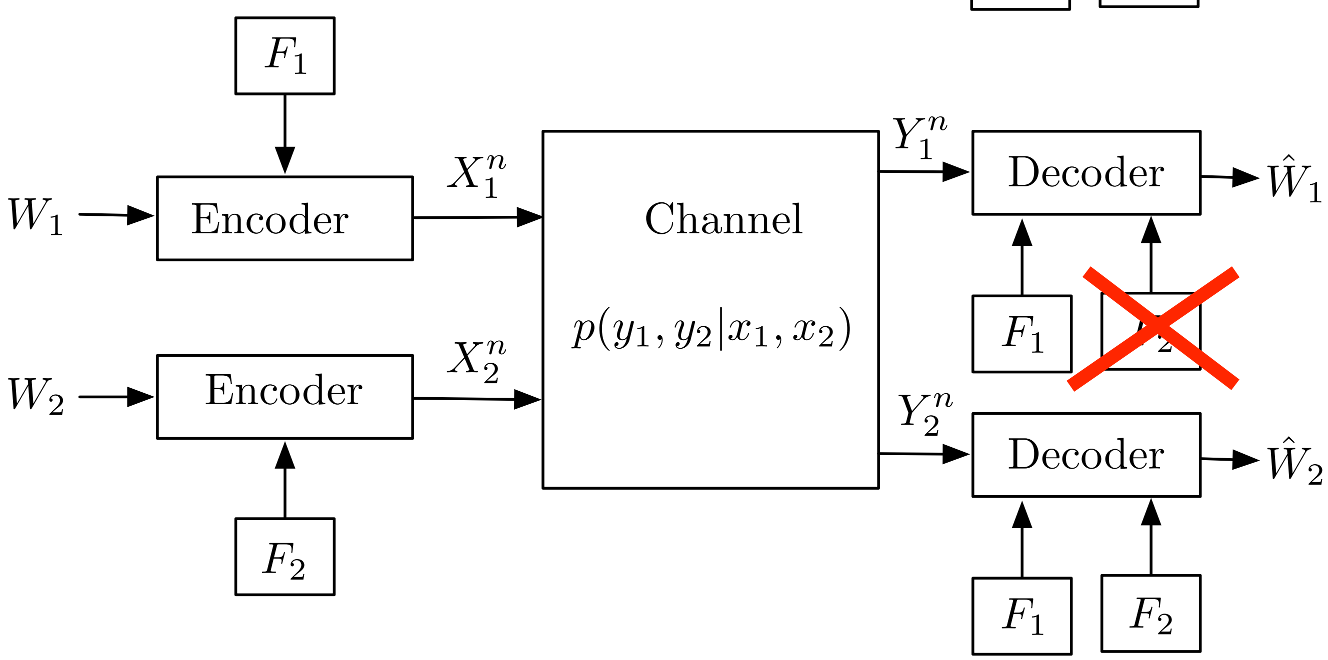
# Approximate capacity without codebooks




HK+Gaussian Inputs

1/2 bit

R. Etkin, D. Tse, and H. Wang, “Gaussian interference channel capacity to within one bit,” IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.




“One-sided” HK+

Mixed Inputs

3.34 bits

A. Dytso, D. Tuninetti, and N. Devroye, “On the two-user interference channel with lack of knowledge of the interference codebook at one receiver,” IEEE Trans. Inf. Theory, vol. 61, no. 3, pp. 1257–1276, March 2015.

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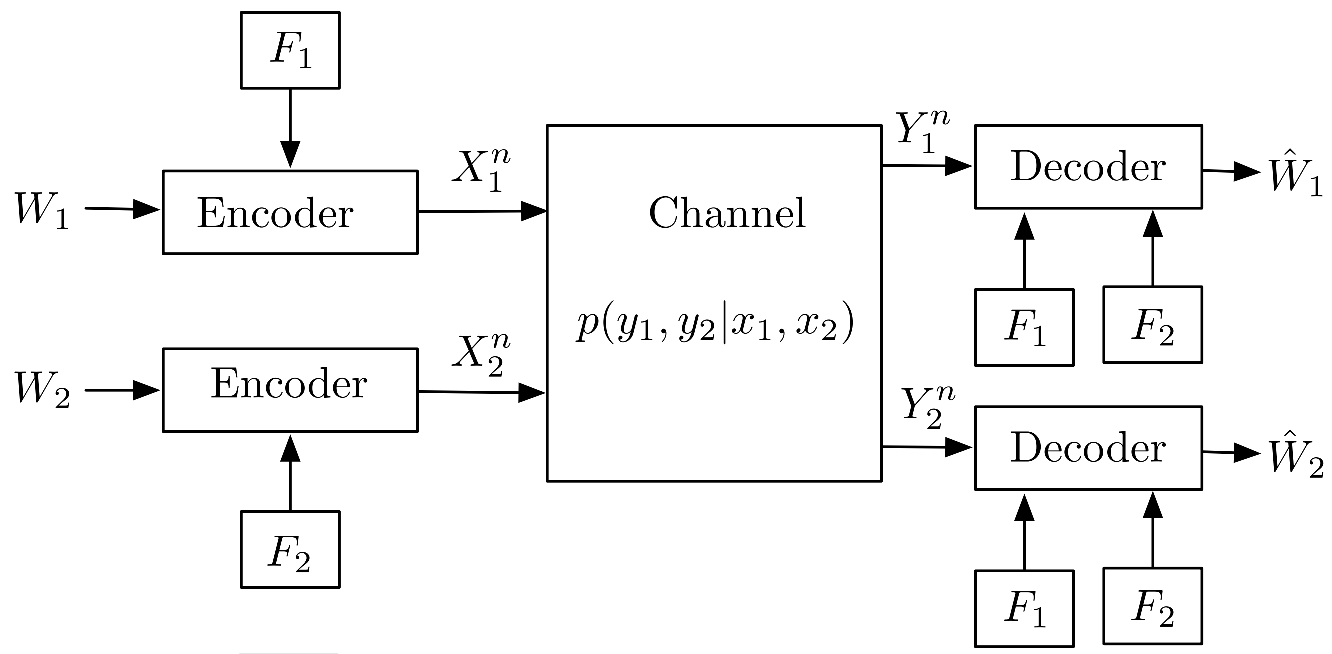


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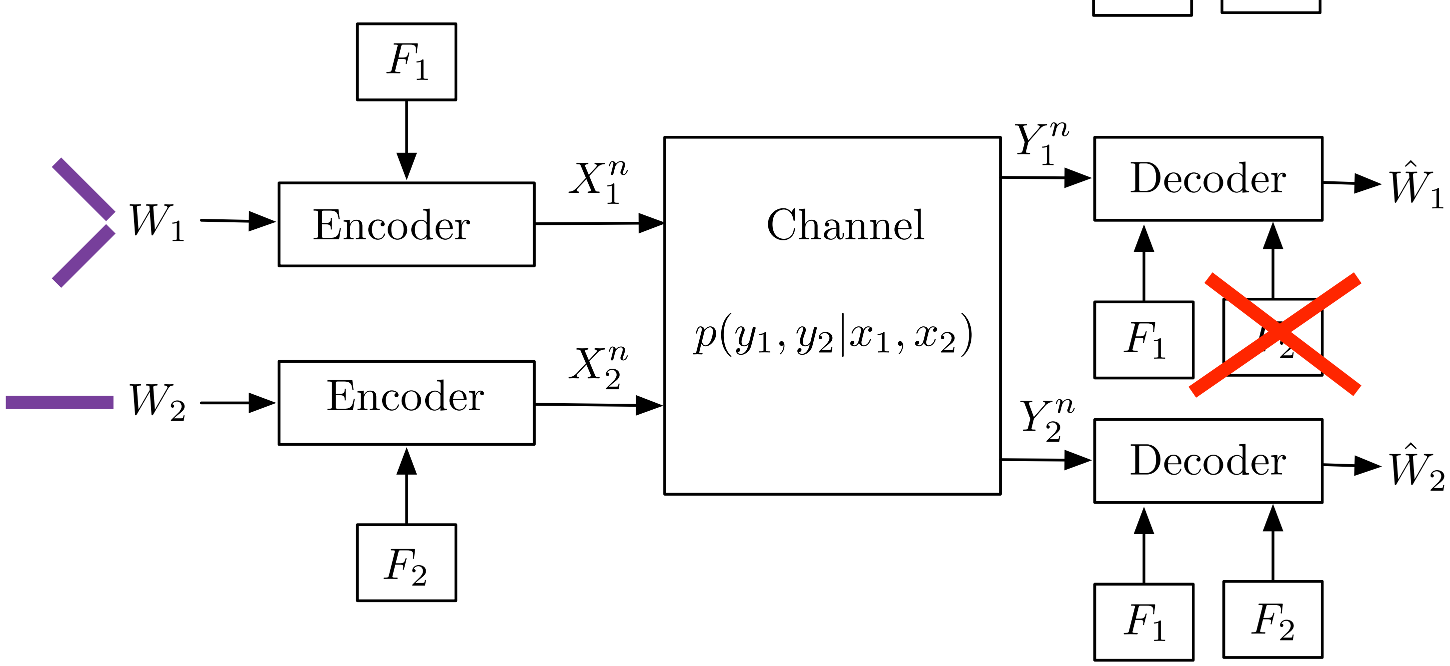


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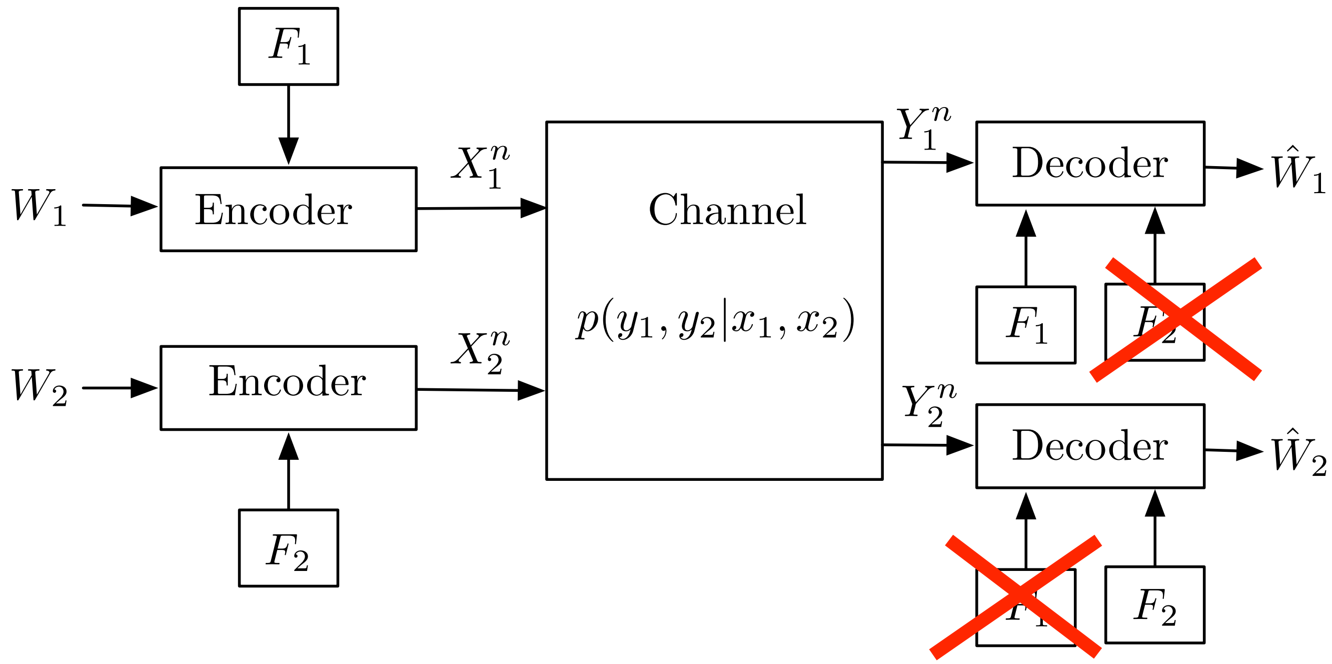


**TINnoTS + Mixed Inputs constant or log-log gaps**

A. Dytso, D. Tuninetti, and N. Devroye, "Interference as Noise: Friend of Foe?" IEEE Trans. Inf. Theory, Vol. 62, No. 6, pp. 3561 -- 3596, June, 2016.

$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{N_1, N_2, \delta_1, \delta_2} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1; Y_1) \\ 0 \le R_2 \le I(X_2; Y_2) \end{array} \right\}$$

**with**

$$\begin{aligned} X_i &= \sqrt{1-\delta_i}\ X_{iD} + \sqrt{\delta_i}\ X_{iG}, \\ &\delta_i \in [0,1], \\ &X_{iD} \sim \text{PAM}\left(N_i\right), \\ &X_{iG} \sim \mathcal{N}(0,1), \\ &i = 1,2. \end{aligned}$$

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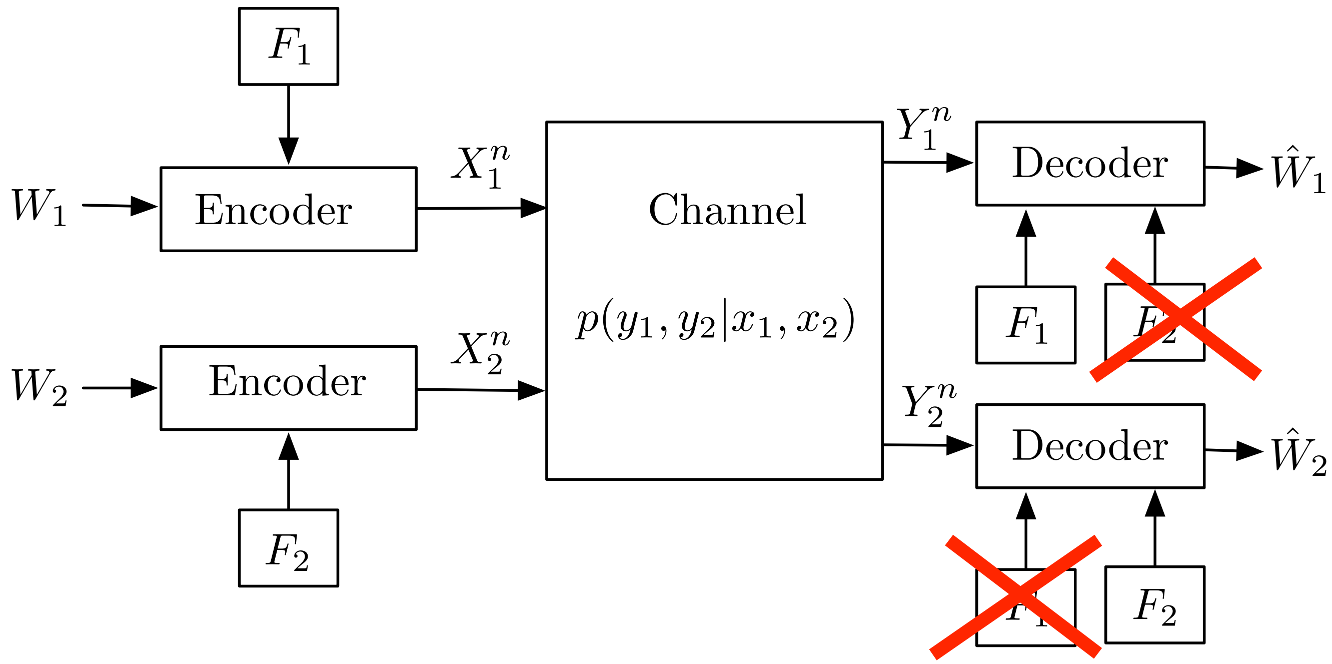


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Choice of $N_i$, $\delta_i$ looks like

discrete $\Longleftrightarrow$ public

Gaussian $\Longleftrightarrow$ private

# Approximate optimality of TINnoTS in Gaussian-IC

| Very Weak | Weak I | Weak II | Strong | VeryStrong |
| --- | --- | --- | --- | --- |
| gap $= 1/2$ | gap $= 3.79$ | gap $= O(\log\log(\min(\mathrm{S},\mathrm{I})))$ up to an outage of controllable measure | | gap $= 1.25$ |

$1/2$ $2/3$ $1$ $2$ $\alpha$

| Gaussian | Mixed | Mixed | Discrete | Discrete |
| --- | --- | --- | --- | --- |

$$X_i = \sqrt{1-\delta_i}\; X_{iD} + \sqrt{\delta_i}\; X_{iG}, \;\; i \in [1:2],$$

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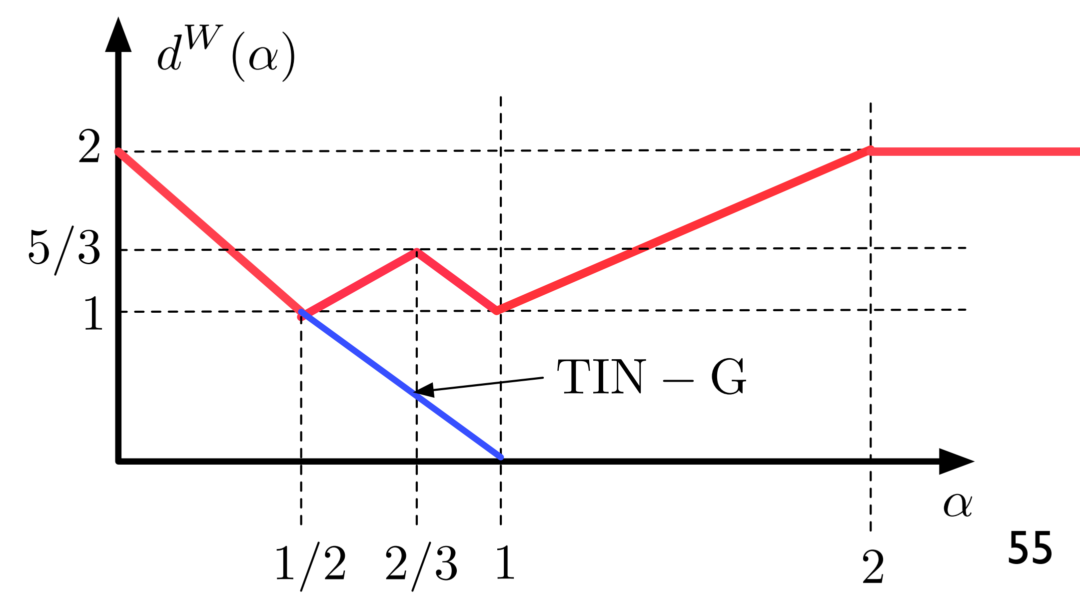
**DoF gain over Gaussians with TINnoTS!**



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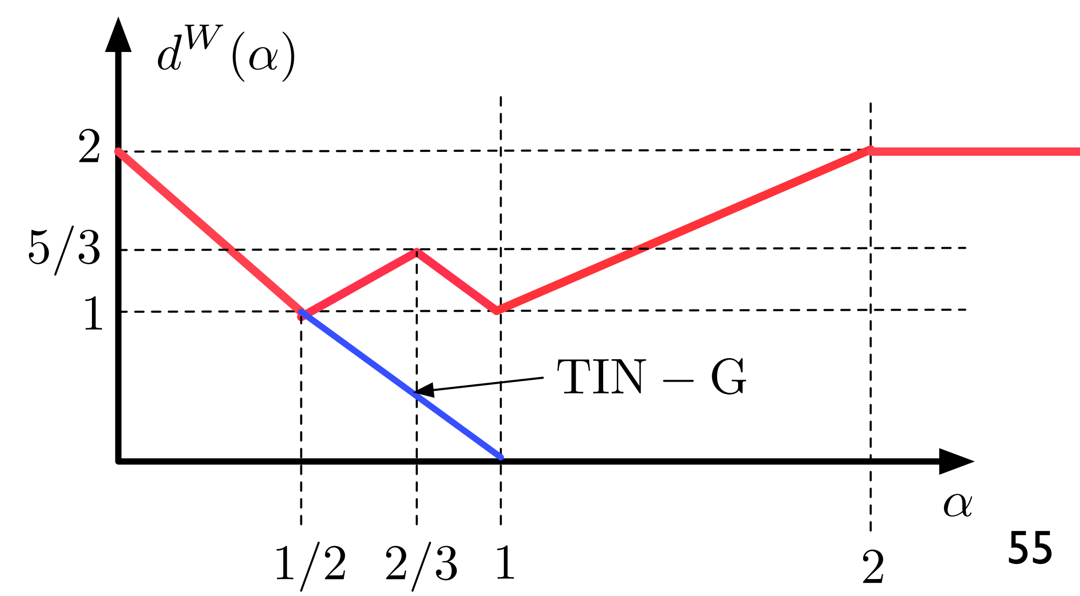

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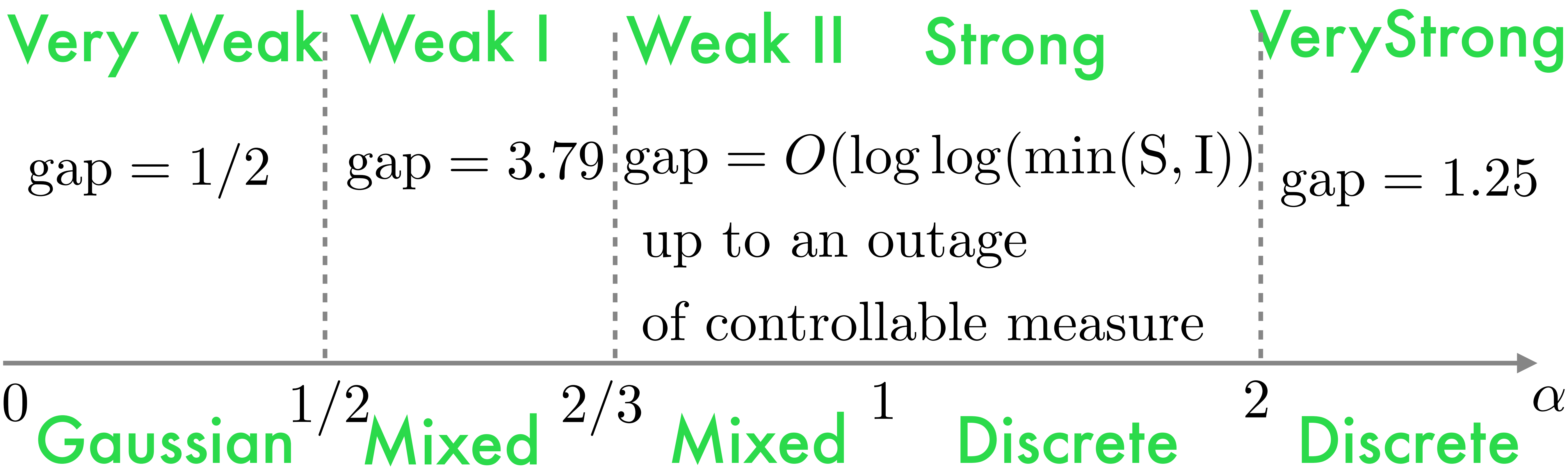


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<u>Closed-form</u> expressions for number of points, power splits and gap

# Key ideas + open problems

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- general tools on bounding dmin, mutual information applicable elsewhere?
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- **OPEN:** better constellation than PAM? What about higher dimensions?
- **OPEN:** can we develop a smart set of multi-letter discrete inputs and evaluate these in the capacity achieving expression for the G-IC?

**Capacity:** $$\mathcal{C} = \lim_{n\to\infty} \mathrm{co}\left(\bigcup_{P_{X_1^n X_2^n} = P_{X_1^n} P_{X_2^n}} \left\{ \begin{array}{l} 0 \le R_1 \le \frac{1}{n} I(X_1^n; Y_1^n) \\ 0 \le R_2 \le \frac{1}{n} I(X_2^n; Y_2^n) \end{array} \right\}\right)$$

R. Ahlswede, “Multi-way communication channels,” in Proc. IEEE Int. Symp. Inf. Theory, March 1973, pp. 23–52.

# Discussion: relation to…………

## Characterizing when (low INR) TIN is optimal in larger interference networks:

[Chunhua Geng, Syed A. Jafar, On the Optimality of Treating Interference as Noise: Compound Interference Networks , IEEE Trans. on Info. Theory, Accepted, 2016.]

[Hua Sun, Syed A. Jafar, On the Optimality of Treating Interference as Noise for K user Parallel Gaussian Interference Networks , IEEE Transactions on Information Theory, Vol. 62, No. 4, Pages: 1911-1930, April 2016.]

[Chunhua Geng, Syed A. Jafar, On the Optimality of Treating Interference as Noise: General Message Sets, IEEE Transactions on Information Theory, Vol. 61, No. 7, Pages: 3722-3736, July 2015.]

[Chunhua Geng, Navid Naderializadeh, Salman Avestimehr, Syed A. Jafar, On the Optimality of Treating Interference as Noise, IEEE Transactions on Information Theory, Vol. 61, No. 4, Pages: 1753-1767, April 2015.]

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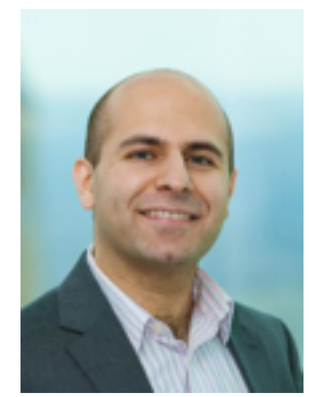

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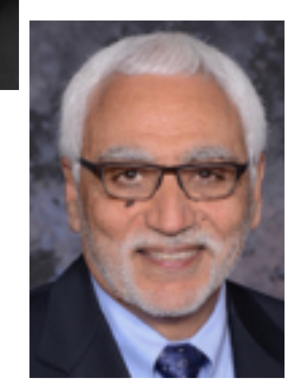
## Using point-to-point codes in interference networks:

[F. Baccelli, A. El Gamal, D. Tse, "Interference Networks with Point-to-Point Codes," IEEE Trans. on Info. Theory, Vol. 57, No. 5, May 2011.]

[J. Sebastian, C. Karakus, S. Diggavi "Approximately achieving the feedback interference channel capacity with point-to-point codes" ISIT 2016.]

[Young Han Kim et al. http://circuit.ucsd.edu/~yhk/pdfs/swcm.pdf]

[B. Bandemer, A. El Gamal, Y.-H. Kim, "Optimal achievable rates for interference networks with random codes" Trans IT 2015.]



# Discussion: relation to…………

## Characterizing when (low INR) TIN is optimal in larger interference networks:

[Chunhua Geng, Syed A. Jafar, On the Optimality of Treating Interference as Noise: Compound Interference Networks , IEEE Trans. on Info. Theory, Accepted, 2016.]

[Hua Sun, Syed A. Jafar, On the Optimality of Treating Interference as Noise for K user Parallel Gaussian Interference Networks , IEEE Transactions on Information Theory, Vol. 62, No. 4, Pages: 1911-1930, April 2016.]

[Chunhua Geng, Syed A. Jafar, On the Optimality of Treating Interference as Noise: Gener… Message Sets, IEEE Transactions on Information Theory, Vol. 61, No. 7, Pages: 3722-37…

[Chunhua Geng, Navid Naderializadeh, Salman Avestimehr, S… Optimality of Treating Interference as Noise, IEE… Theory, Vol. 61, No. 4, P…

**Our message: use discrete inputs and TIN in interference networks for simple comm. where do not have synchronism or codebook knowledge**

## U… …es in interference networks:

[F. Ba… …, D. Tse, "Interference Networks with Point-to-Poin… …odes," IEEE Trans. on Info. Theory, Vol. 57, No. 5, May 2011.]

[J. Sebastian, C. Karakus, S. Diggavi "Approximately achieving the feedback interference channel capacity with point-to-point codes" ISIT 2016.]

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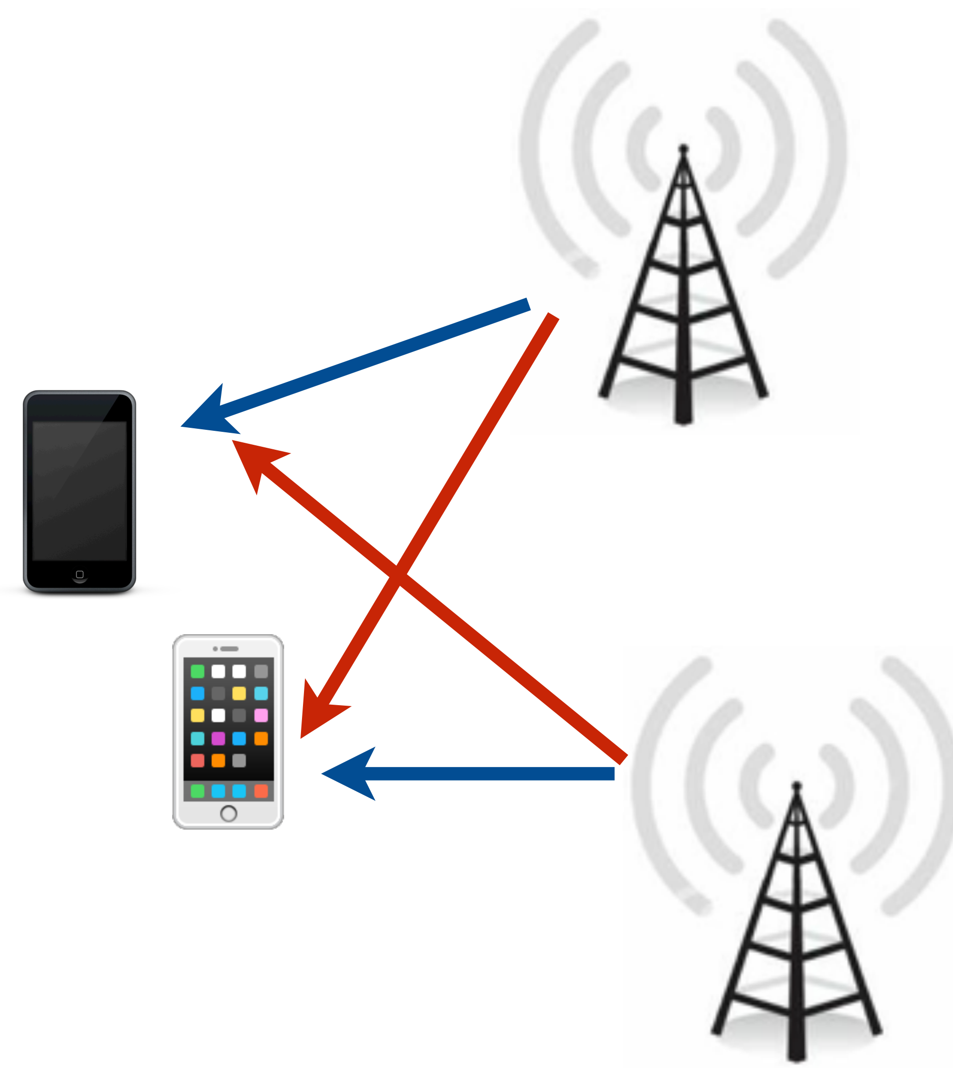
# Basic model for interfering wireless channels: the interference channel

codebook knowledge

synchronization

channel state

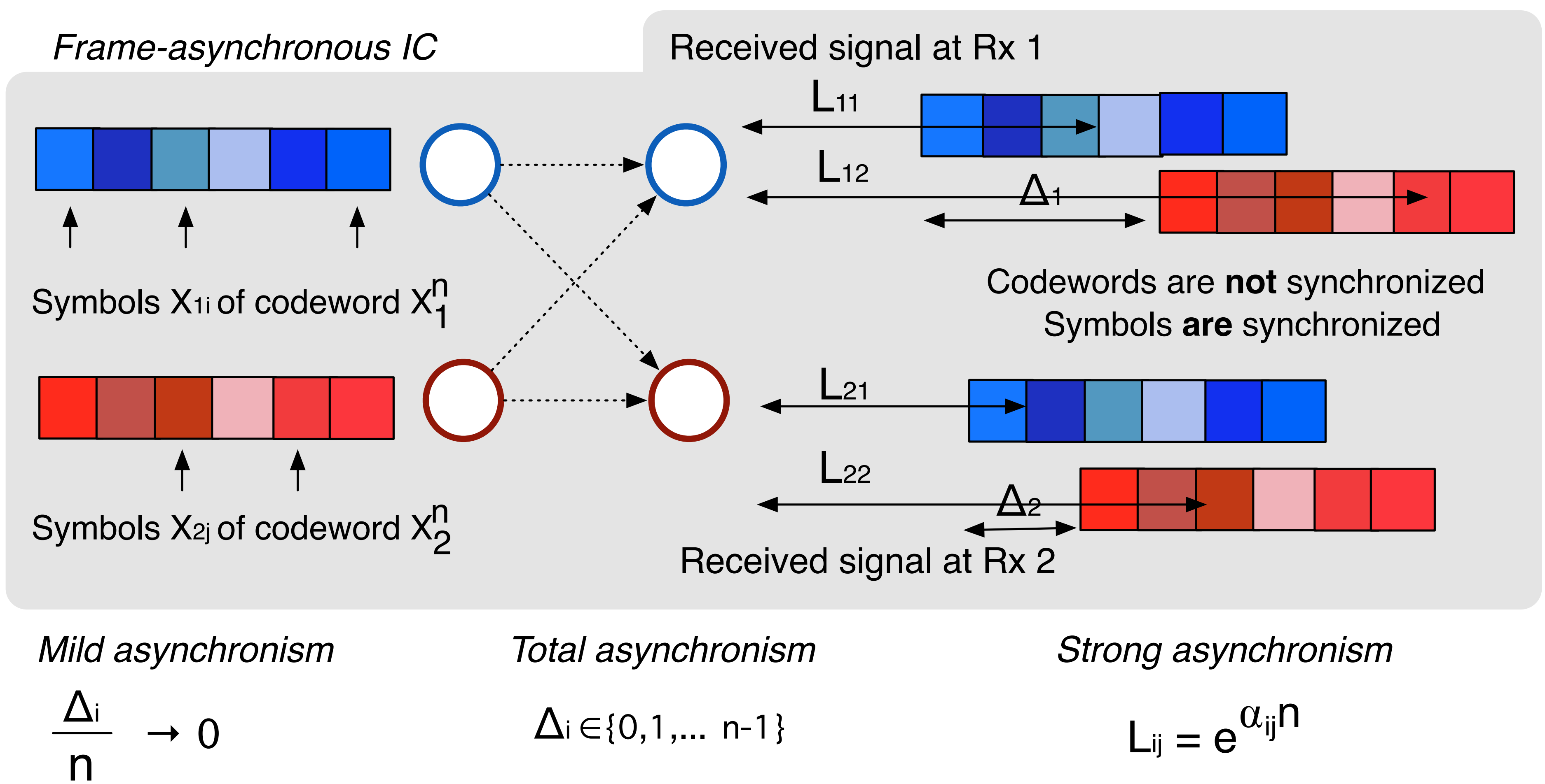
small versus zero error



use this channel model to exemplify the effect (or lack thereof) of several commonly made network information theoretic assumptions

# Approximate capacity of ICs with lack of synchronization

- in networks, often assume all nodes are synchronized




- this may be unrealistic sometimes….

# Approximate capacity of ICs with **total asynchronism**

## Treat Interference as Noise without Time Sharing Inner Bound:

$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{P_{X_1 X_2} = P_{X_1} P_{X_2}} \left\{ \begin{array}{l} 0 \leq R_1 \leq I(X_1; Y_1) \\ 0 \leq R_2 \leq I(X_2; Y_2) \end{array} \right\}$$

No Time Sharing

[E. Calvo, J.R. Fonollosa, J. Vidal, ``On the Totally Asynchronous Interference Channel with Single-User Receivers" ISIT 2009.]

- this is achievable by asynchronous G-IC, so our approximate gap to capacity results apply even without synchronization!

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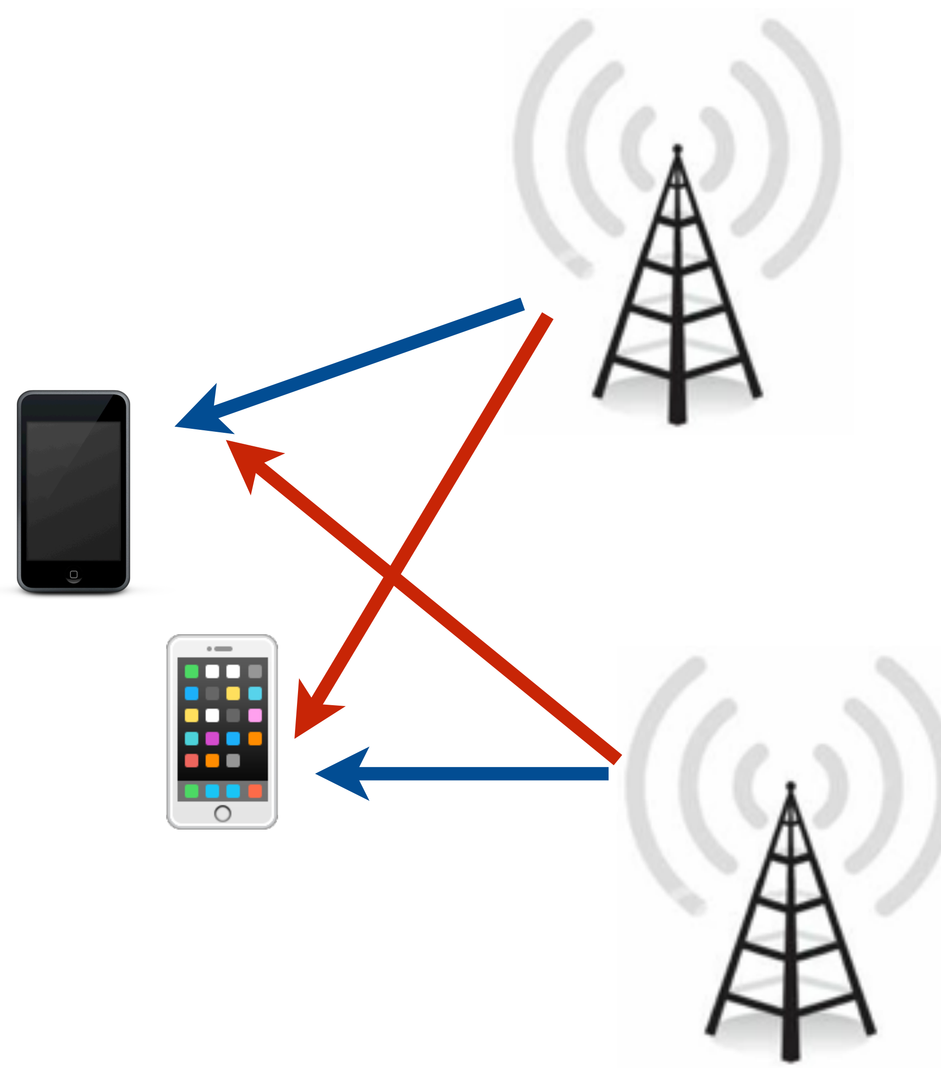

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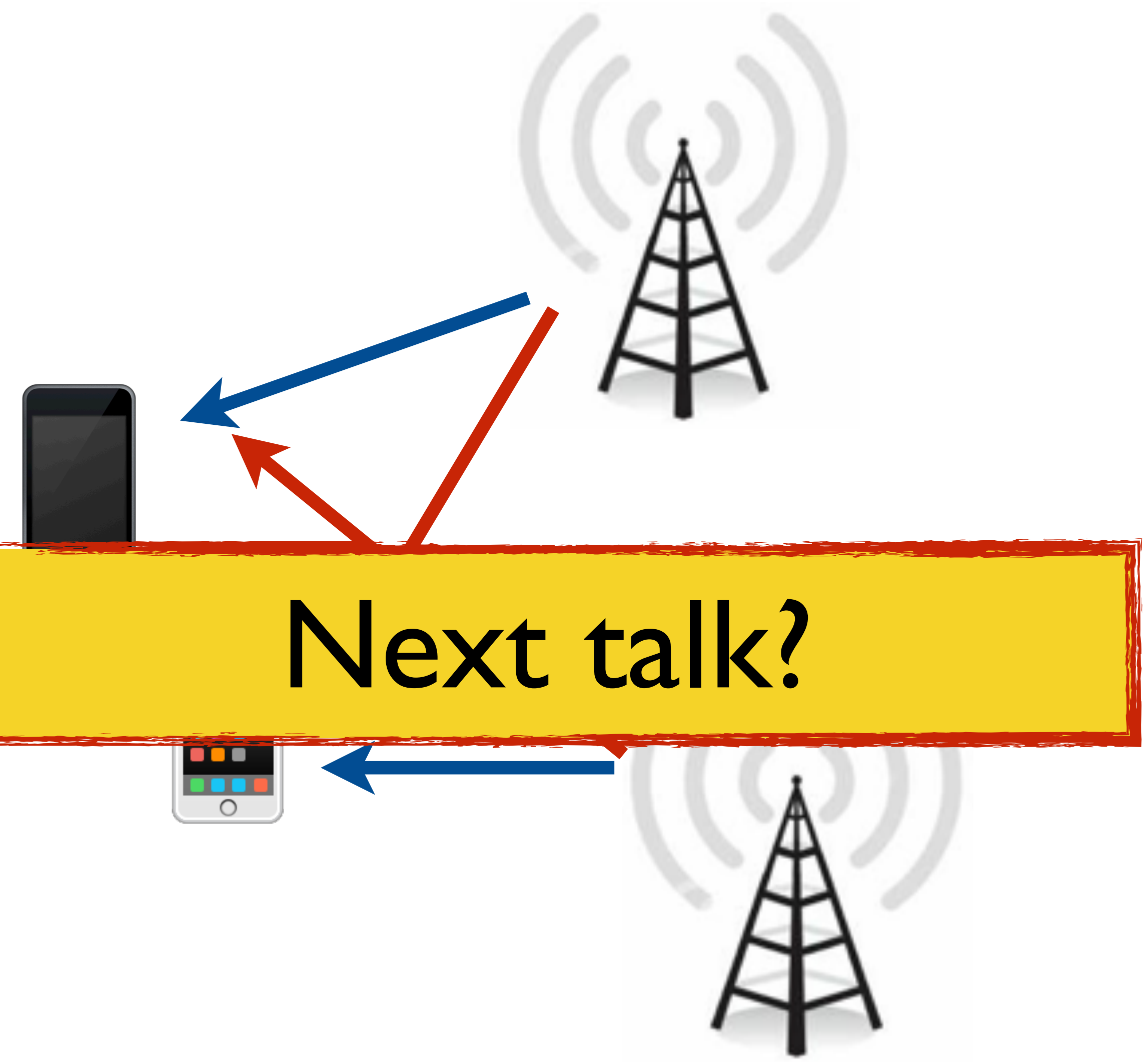


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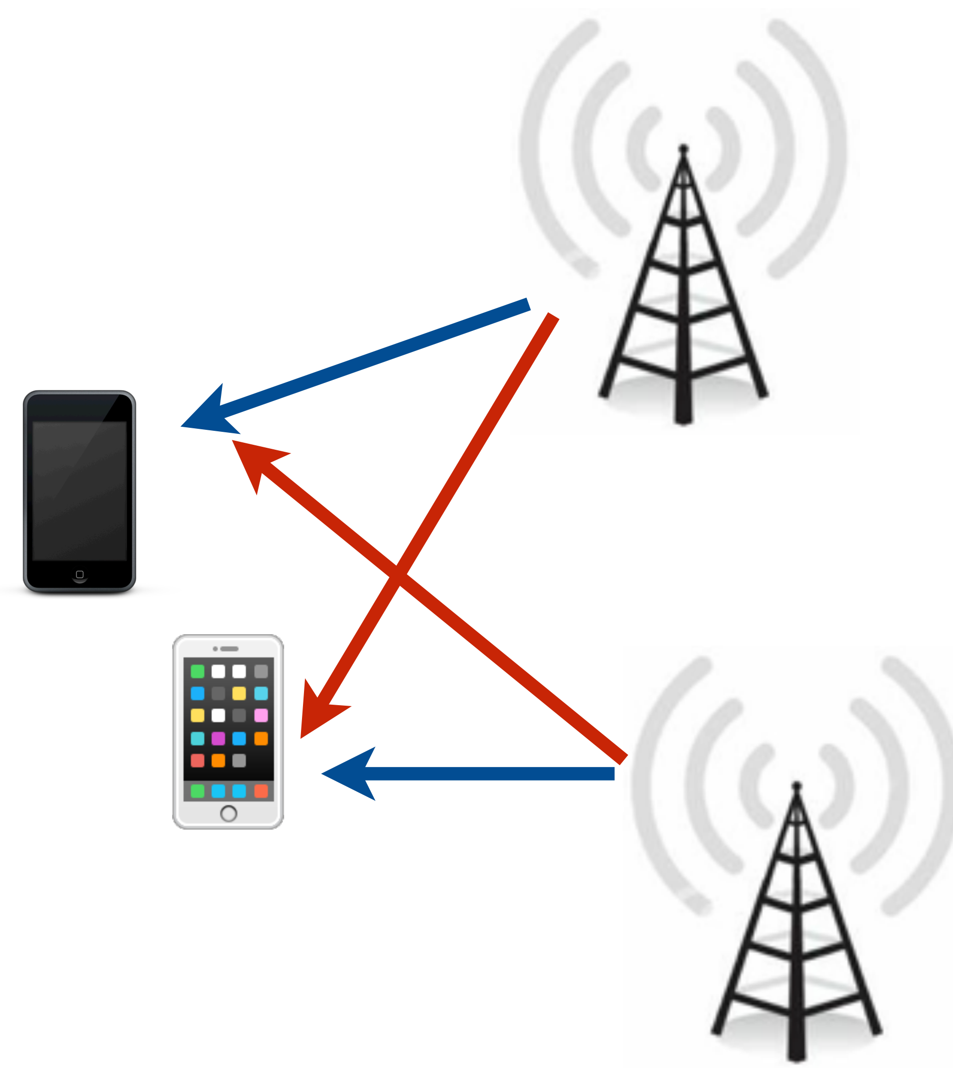

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Network coding: [T. Chan and A. Grant, ``On capacity regions of non-multicast networks" ISIT 2010]

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later talks / speakers?

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zero versus epsilon error affects things!

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# Conclusion for interference channel

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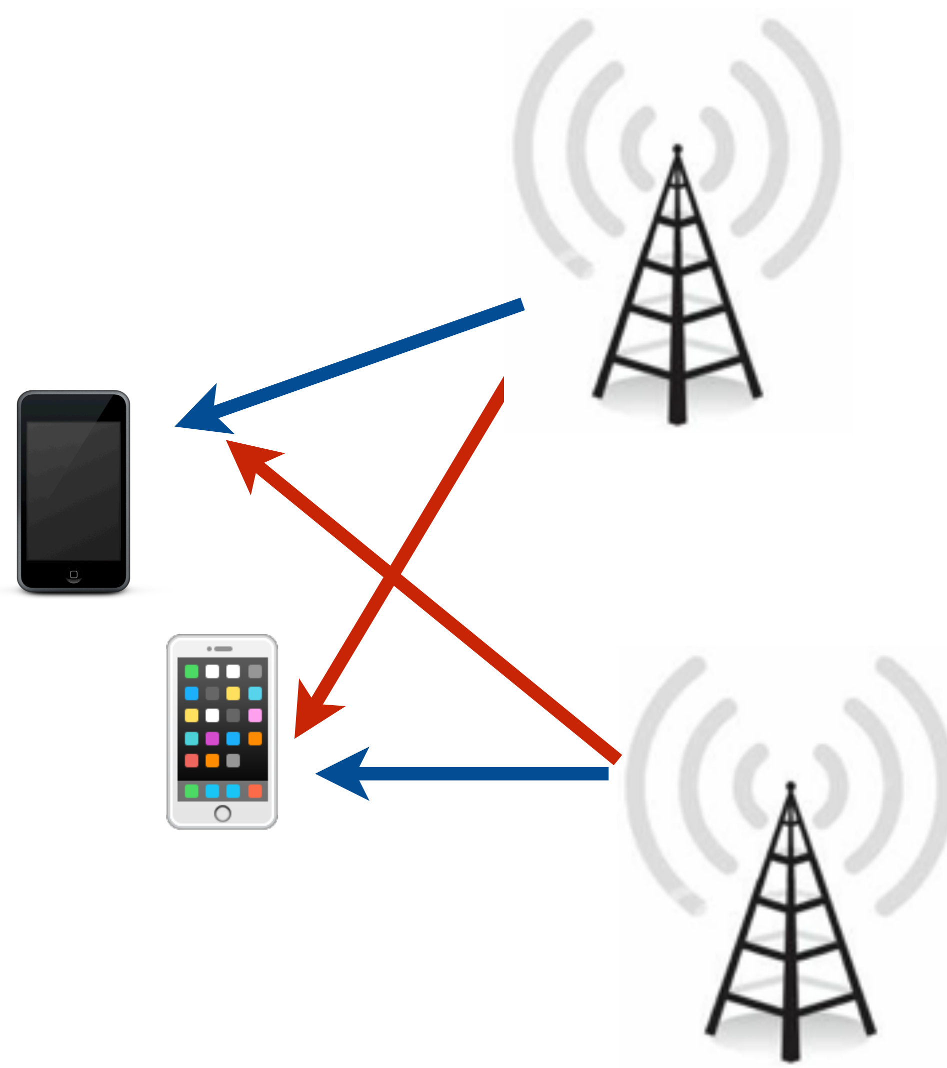

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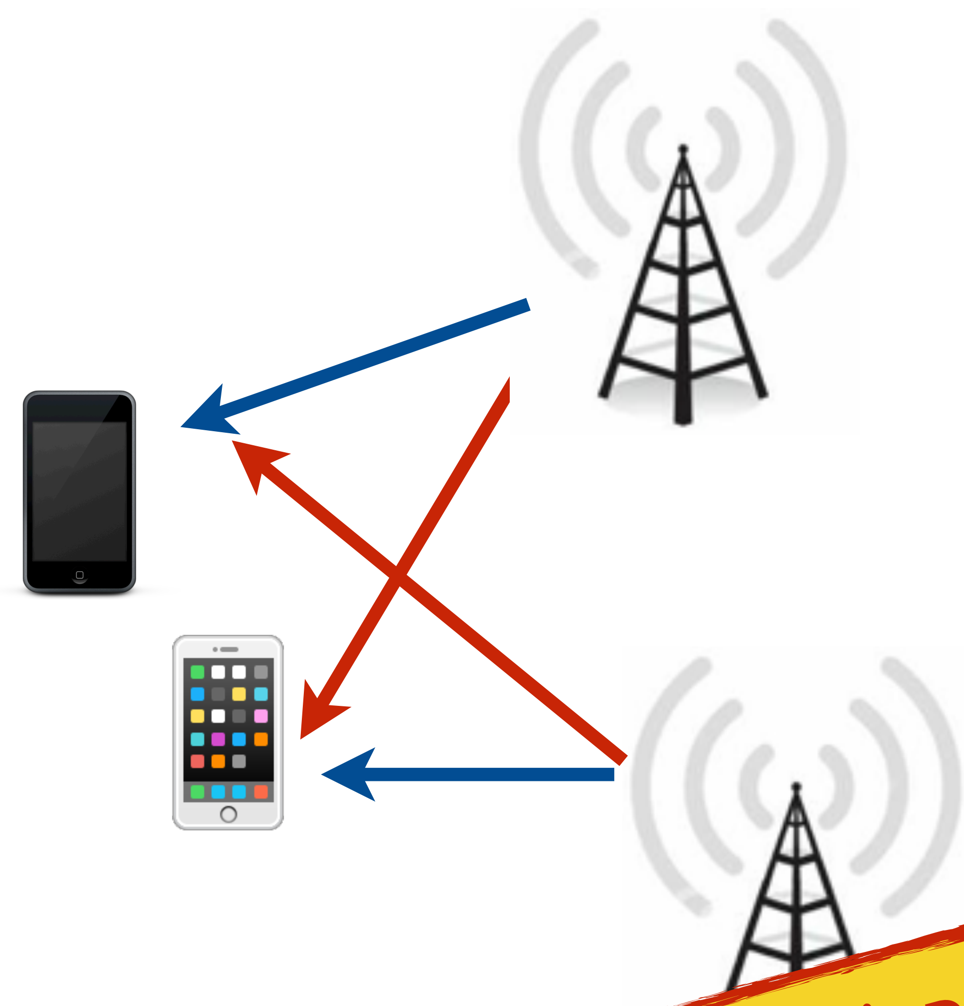

channel state

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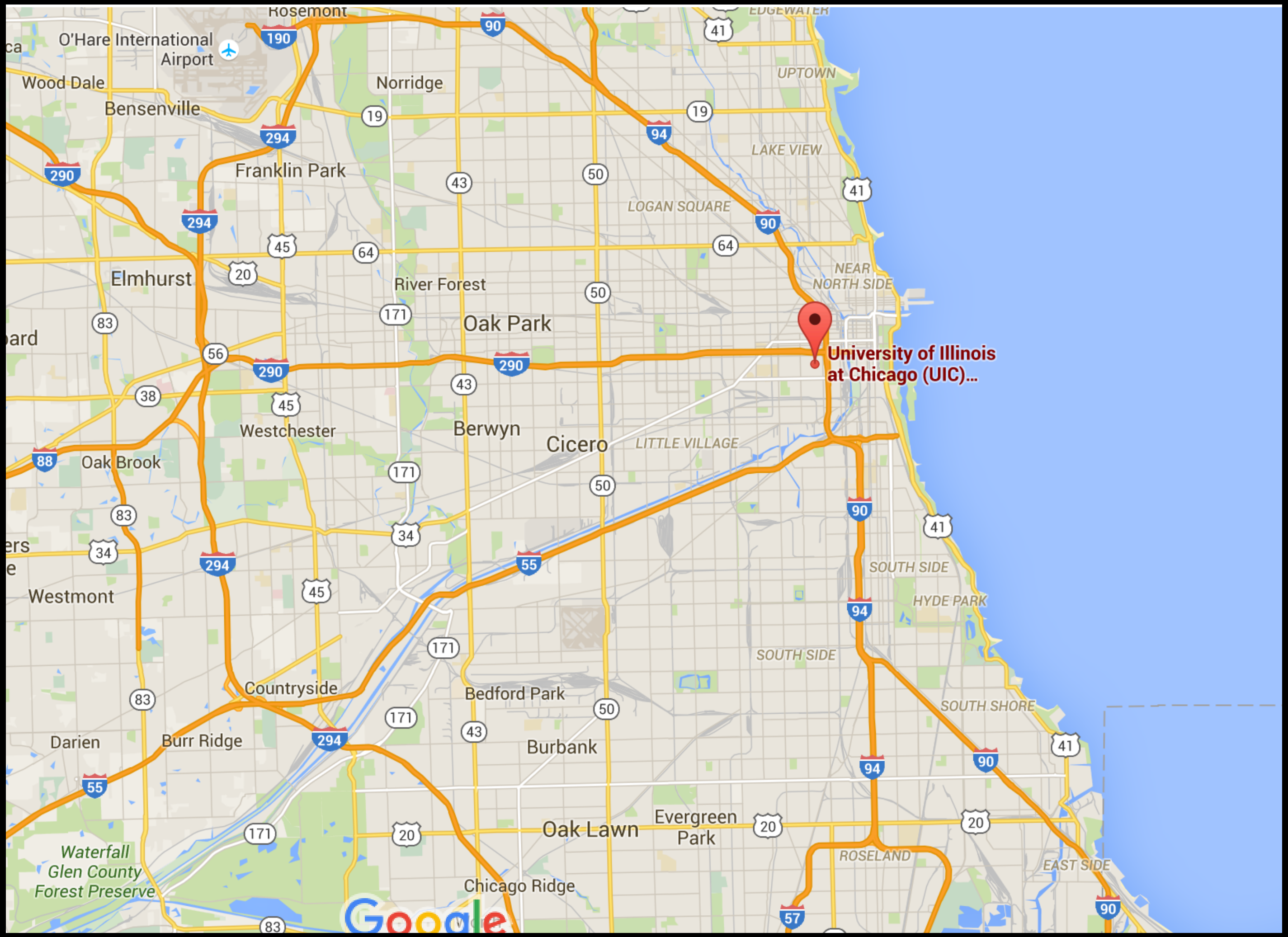
does not affect things dramatically if use discrete inputs in Gaussian channels! “estimate” interference?


# UIC is a public school in downtown Chicago




# UIC is a great place to visit

- home of the "NICEST" lab

**Networks Information Communications and Engineering Systems Laboratory**

Natasha Devroye

Hulya Seferoglu

Besma Smida

Daniela Tuninetti

- home of the best "Brutalist" architecture in the world

# Questions + discussions now, later, email are always welcome

*Natasha Devroye*
devroye@uic.edu
www.ece.uic.edu/Devroye

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