Decentralized Optimal Control for Online Coordination of Connected and Automated Vehicles

Andreas A. Malikopoulos, PhD
Deputy Director, Urban Dynamics Institute
Oak Ridge National Laboratory

IPAM: New Directions in Mathematical Approaches for Traffic Flow Management
Workshop II: Traffic Estimation

October 15, 2015

Acknowledgments

- Laboratory Directed Research and Development (LDRD) Program at Oak Ridge National Laboratory (ORNL)
  - Weinberg Fellowship
  - Seed Award
  - LDRD Award

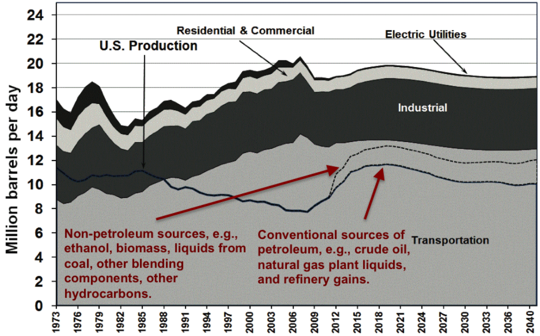
Why we should be concerned about transportation?

- Two-thirds of the oil used around the world currently goes to power transportation vehicles, of which half goes to passenger cars and light trucks.
- In emerging Asia alone, the total number of vehicles is expected to rise from 55 million in 2003 to 420 million in 2030.

Traffic jam in China

What is the impact on petroleum displacement?

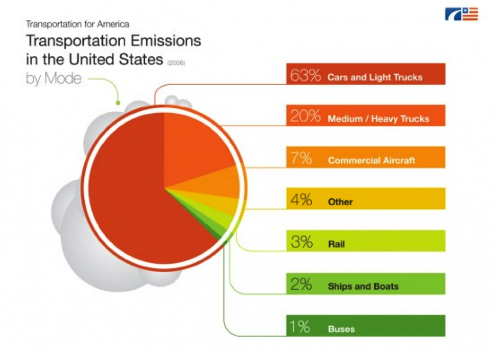
- Until 1989 the US produced enough petroleum to meet the needs of the transportation sector although petroleum needs of all other sectors were not met.



Source: Oak Ridge National Laboratory, "Transportation Energy Data Book," **32**, July 31, 2013.

What is the impact on GHG emissions?

- The transportation sector accounted for about 28% of total US greenhouse gas (GHG) emissions in 2006 making it the second behind electricity generation (34%).



Source: Transportation for America

Connected and automated vehicles

- Many stakeholders see the benefits of connected and automated vehicles (CAVs) and have started to develop business cases for their respective domains, including the automotive and insurance industries, government, and service providers.

The focus has been on safety[1]

[1] Video: courtesy of UMTRI.

Can we use CAVs to improve efficiency?

Example of an "emergent pattern" through coordination

Can we coordinate vehicles...?

Can we then scale up...?

1939 New York Word's Fair

- Norman Bel Geddes designed an exhibit at the 1939 New York World's Fair, Futurama. The idea was to present a possible model of the world 20 years into the future (1959-1960).

1939 New York Word's Fair

General Motors future car

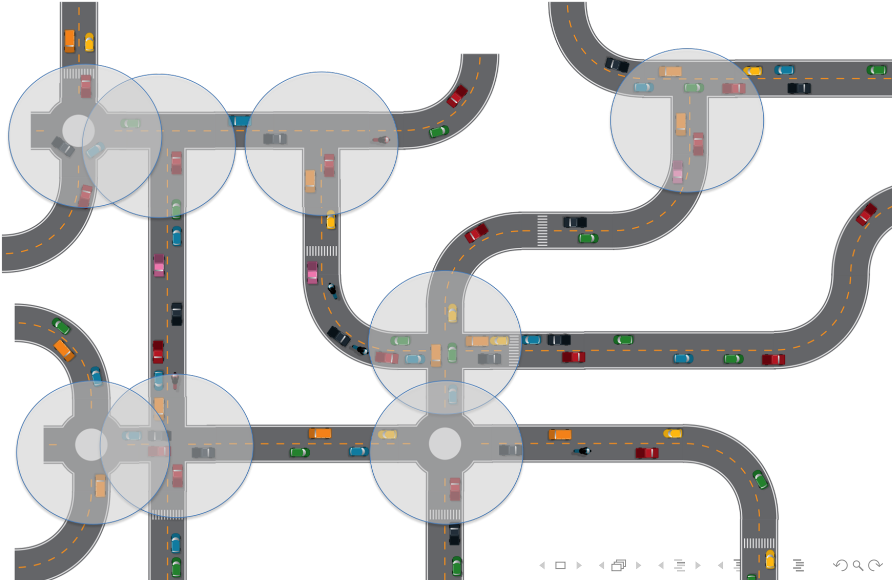
- In 1956, General Motors featured a film that looked into the far 20 year distant future 1976.

Outline

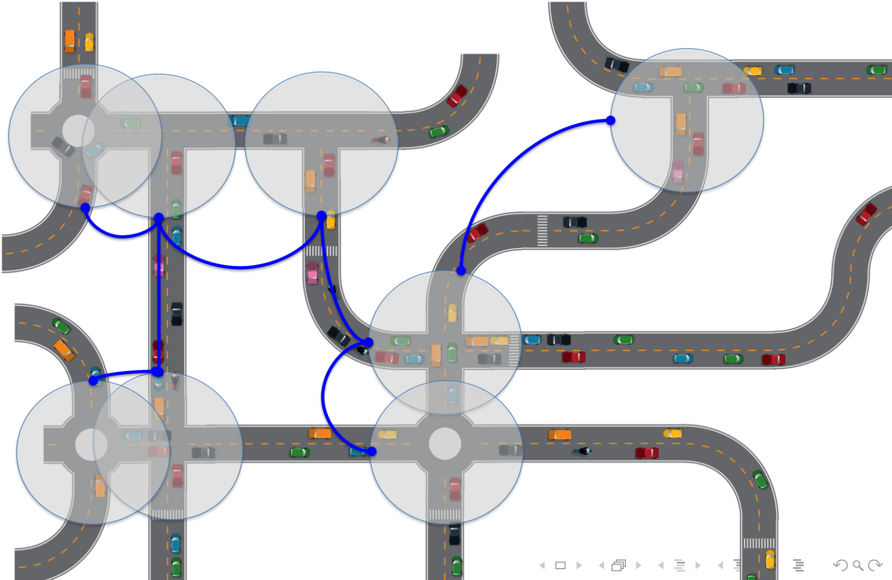
- Coordination of CAVs
  - Intersections
  - Merging roadways
- Centralized problem formulation
- Decentralized problem formulation and solution approach
- Illustrative examples
- Concluding remarks

Transportation network

Step 1: Address specific transportation segments



Step 2: Scale up to the whole network

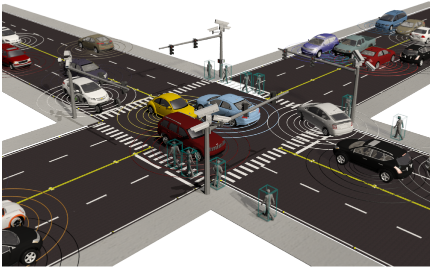


Vehicle coordination: state of the art

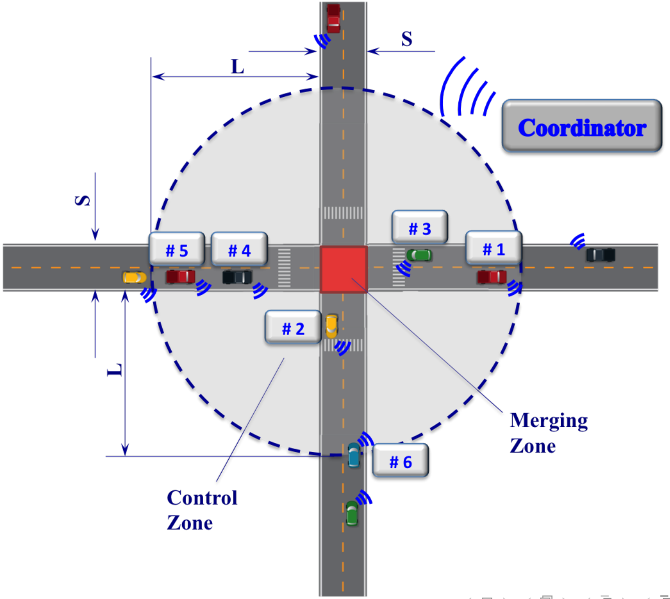
- Various approaches have been proposed in the literature[2].
- Centralized:
  - There is at least one task in the system that is globally decided for all vehicles by a single central controller.
- Decentralized:
  - Each vehicle obtains information from other vehicles and roadside infrastructure to optimize specific performance criteria (e.g., efficiency, travel time).

[2] Rios-Torres, J., Malikopoulos, A.A., Pisu, P., "A Survey on Driver Feedback Systems and Coordination of Connected and Automated Vehicles," *IEEE Transactions on Intelligent Transportation Systems. (in review)*

Vehicle coordination in intersections

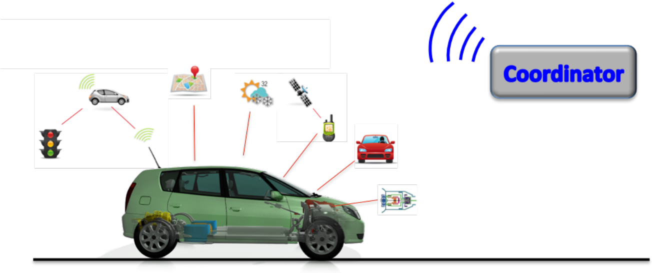


Optimal control problem



Local information and observation

- Assumption: Each vehicle i has proximity sensors and can observe and/or estimate local information that can be shared with other vehicles.



The model

- $\mathcal{N}(t) = \{1, \ldots, N(t)\}$

$$\begin{aligned} \dot{p}_i &= v_i(t) \\ \dot{v}_i &= u_i(t) \end{aligned} \tag{1}$$

  where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$, and $t \in \mathbb{R}^+$.
- $\mathcal{P}_i$, $\mathcal{V}_i$ and $\mathcal{U}_i$, $i \in \mathcal{N}(t)$, are complete and totally bounded subsets of $\mathbb{R}$.

The model

- $x_i(t) = \begin{bmatrix} p_i(t) & v_i(t) \end{bmatrix}^T$, with $x_i^0 = \begin{bmatrix} 0 & v_i^0 \end{bmatrix}^T$

$$\dot{x}_i = f(t, x_i, u_i), \qquad x_i(t_i^0) = x_i^0, \tag{2}$$

- $\forall (t_i^0, x_i^0)$ and $\forall\ u(t) \in \mathcal{U}_i$, (2) has a unique solution $x(t)$ on some $[t_i^0, t_i^f]$.
  - f is continuous in u and continuously differentiable in x.
  - f_x, is continuous in u.
  - $u(t)$ is continuous in t.

The model

- Control and state constraints

$$u_{min} \leq u_i(t) \leq u_{max}, \quad \text{and}$$
$$0 \leq v_{min} \leq v_i(t) \leq v_{max}, \quad \forall t \in [t_i^0, t_i^f], \tag{3}$$

- Assumption: The vehicle speed inside the merging zone is constant.

Feasibility sets

Definition

Rear-end control interval

$$R_i \triangleq \Big\{u_i(t) \in [u_{min}, u_{max}] \mid s_i(t) = p_k(t) - p_i(t) \geq \delta, \\ v_i(t) \in [v_{min}, v_{max}], \forall i, k \in \mathcal{N}, i \neq k, |\mathcal{N}| > 1, \forall t \in [t_i^0, t_i^f]\Big\}. \quad (4)$$

Definition

Lateral zone set

$$M_i \triangleq \Big\{p_i(t) \mid p_i(t) \in [L, L+S], \forall i \in \mathcal{N}, |\mathcal{N}| > 1, \forall t \in [t_i^0, t_i^f]\Big\}. \quad (5)$$

Feasibility sets

Definition

Critical control interval

$$\Gamma_i \triangleq \Big\{ u_i(t) \in R_i \mid p_i(t) \in M_i, \forall i \in \mathcal{N}, \\ |\mathcal{N}| > 1, \forall t \in [t_i^0, t_i^f] \Big\}. \quad (6)$$

- Lateral collision constraint

$$\Gamma_i \bigcap \Gamma_j = \varnothing, \forall t \in [t_i^0, t_i^f]. \quad (7)$$

Centralized problem formulation

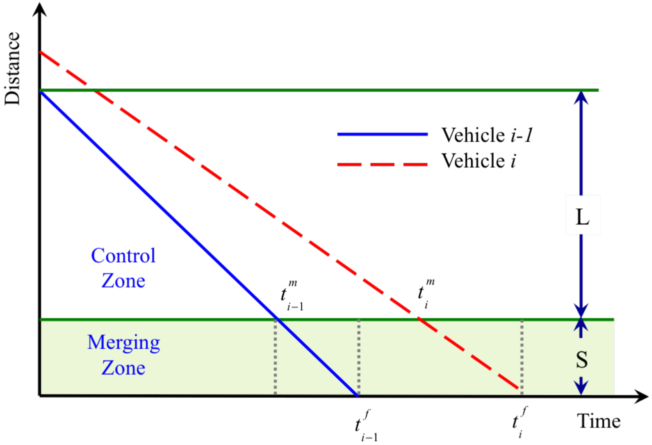
- Centralized control problem

$$\min_{u_i} \left(\frac{1}{2} \sum_{i=1}^{N(t)} \int_{t_i^0}^{t_i^f} u_i^2 \, dt + \sum_{i=2}^{N(t)} \left(t_{i-1}^m - t_i^m \right) \right) \tag{8}$$

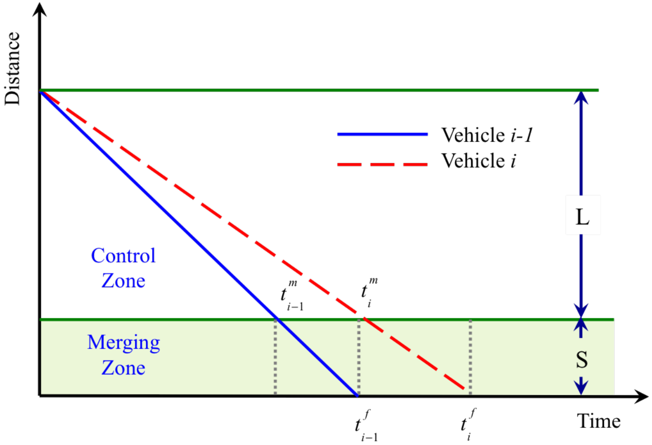
$$\begin{aligned} \text{Subject to :} \quad & \text{vehicle dynamics} \\ & \text{state and control bounds} \\ & \text{rear-end collision} \\ & \text{lateral collision,} \end{aligned}$$

where t_i^m is the time for each vehicle i that enters the merging zone.

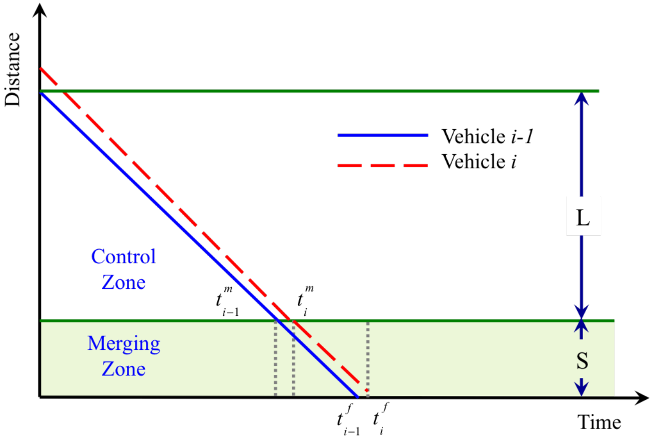
Centralized problem formulation



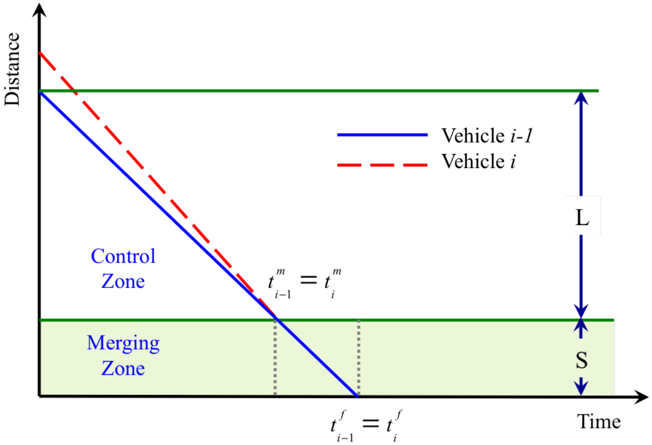
Centralized problem formulation



Centralized problem formulation



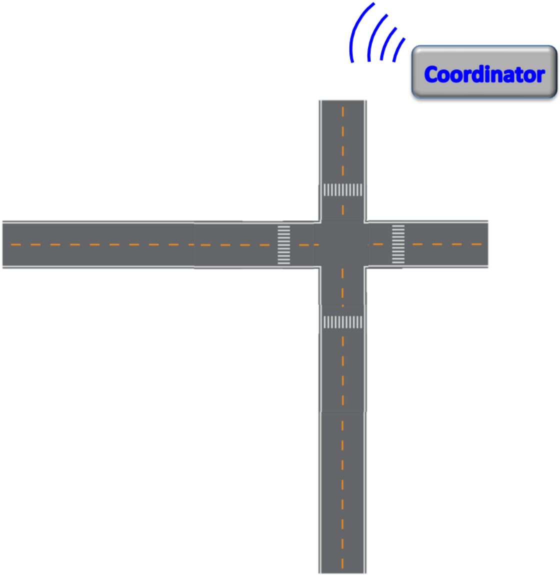
Centralized problem formulation



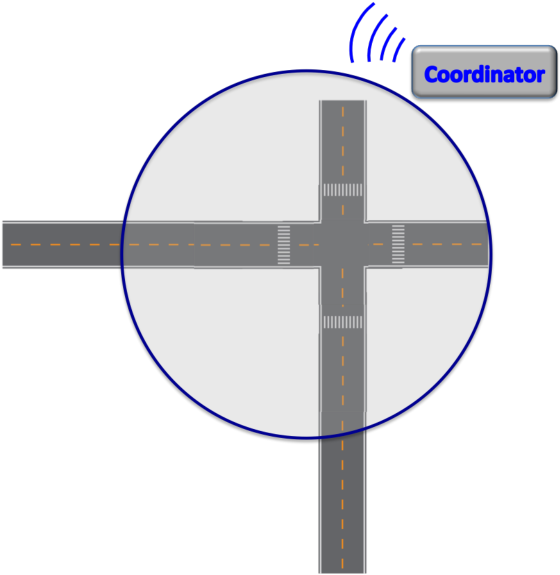
Challenges

- The computational burden associated with deriving the optimal solution in (8).
- Online implementation requirement.
- Problem dimensionality as the number of vehicles inside the control zone increases.

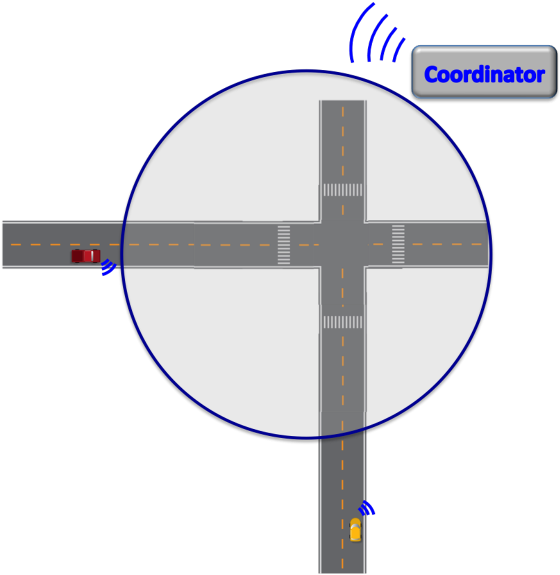
CAV coordination: decentralized control



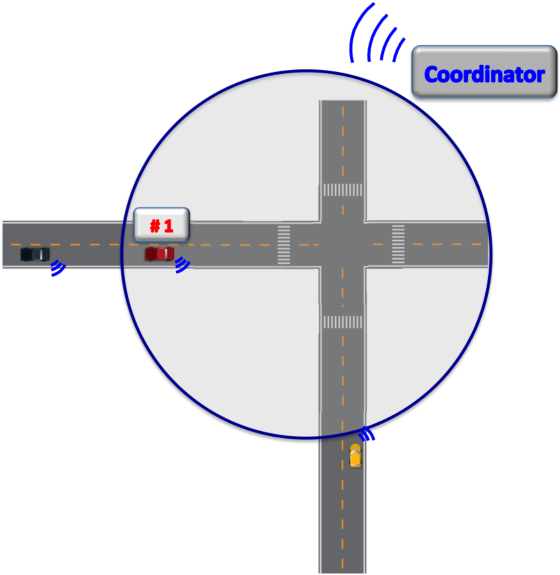
CAV coordination: decentralized control



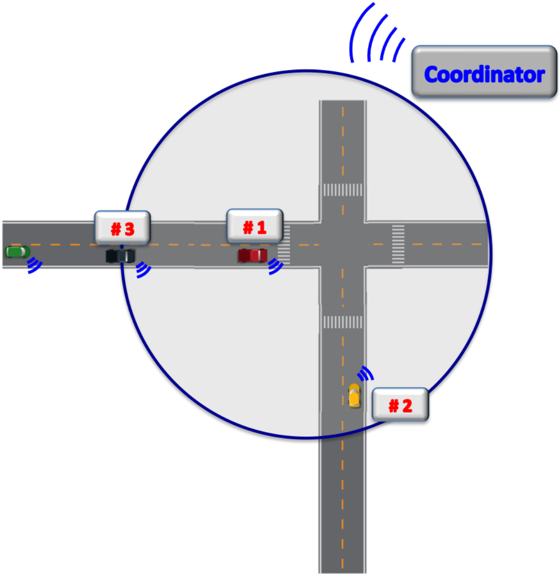
CAV coordination: decentralized control



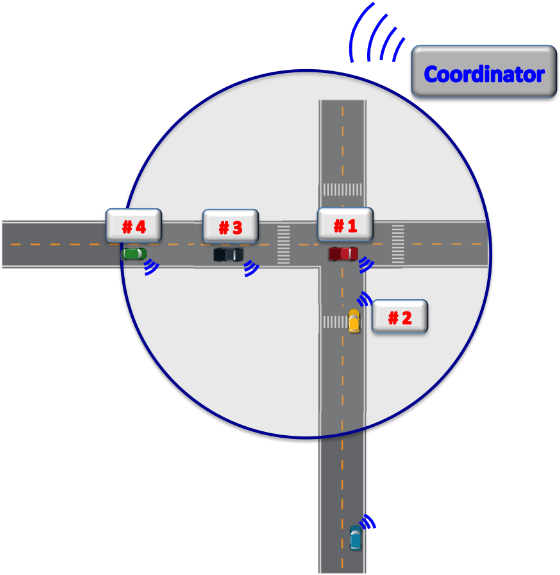
CAV coordination: decentralized control



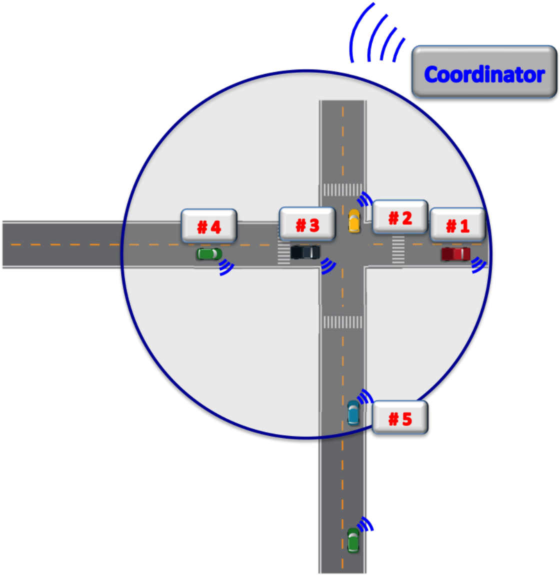
CAV coordination: decentralized control



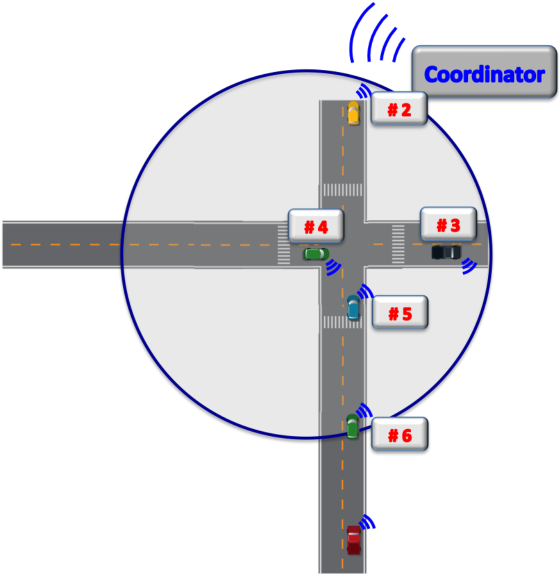
CAV coordination: decentralized control



CAV coordination: decentralized control



CAV coordination: decentralized control



Coordinator's identity

Definition

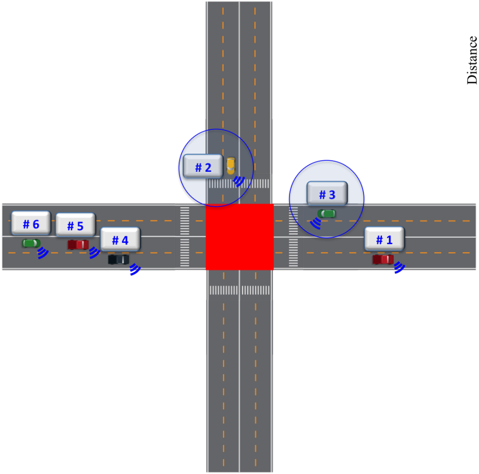
The unique identity that the coordinator assigns to each vehicle $i \in \mathcal{N}$, at the time t when the vehicle arrives at control zone, is a pair (i, j),

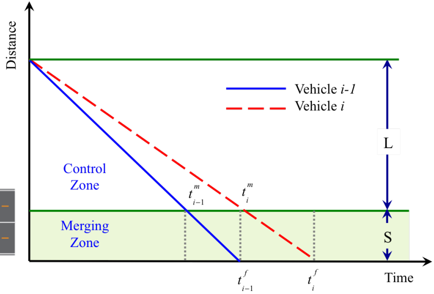
where

- $i = N_z(t) + 1$ is an integer representing the location of the vehicle in the FIFO queue $\mathcal{N}(t)$ and
- $j \in \{1, \dots, 4\}$ is an integer based on a one-to-one mapping from $\{\mathcal{C}(t), \mathcal{R}(t), \mathcal{O}(t), \mathcal{L}(t)\}$ onto $\{1, \dots, 4\}$.

Subsets of $\mathcal{N}(t)$

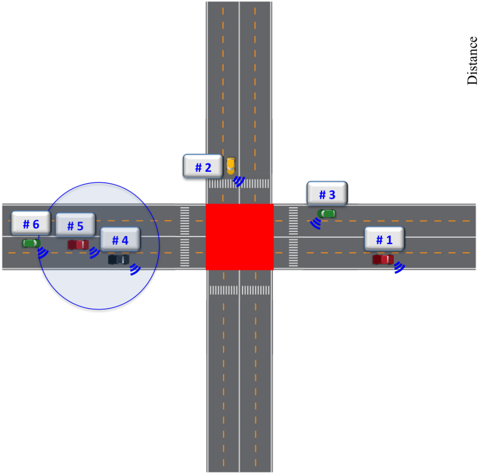
- $\mathcal{C}(t)$ contains i-1 and i when traveling on different roads with destinations that can cause collision, e.g., # 2 and #3.

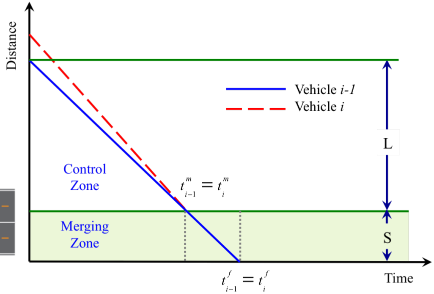




Subsets of $\mathcal{N}(t)$

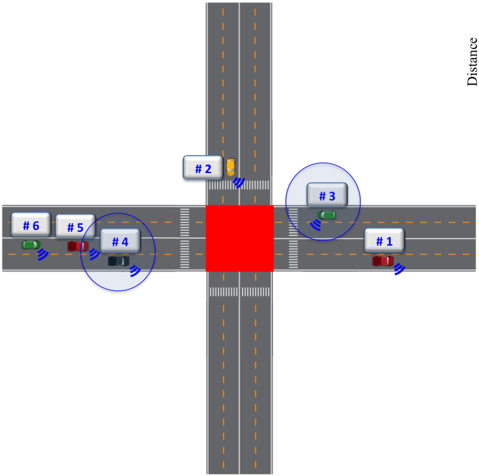
- $\mathcal{R}(t)$ contains i-1 and i when traveling on the same road, towards the same direction but on different lanes, e.g., # 4 and #5.

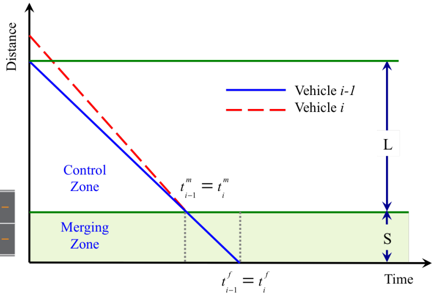




Subsets of $\mathcal{N}(t)$

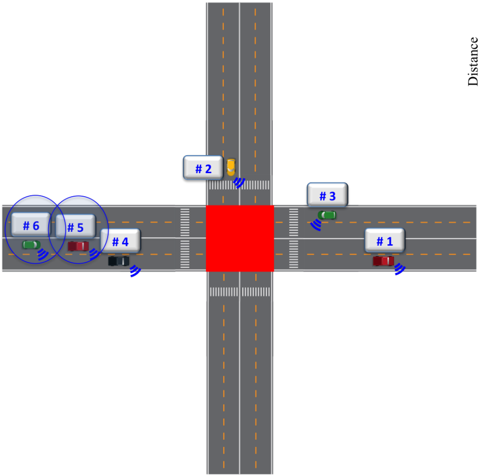
- $\mathcal{O}(t)$ contains i-1 and i when traveling on the same road and opposite destinations, e.g., # 3 and #4.

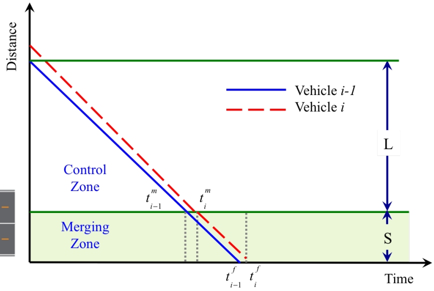




Subsets of $\mathcal{N}(t)$

- $\mathcal{L}(t)$ contains i-1 and i when traveling on the same road and lane e.g., # 5 and #6.





Local observation set

Definition

Local observation set

$$Y_i(t) \triangleq \left\{p_i(t), v_i(t), \mathcal{Q}_j, j = 1, \ldots, 4, s_i(t), t_i^f\right\}, \quad \forall t \in [t_i^0, t_i^f], \tag{9}$$

where $\mathcal{Q}_j \in \{\mathcal{R}(t),\ \mathcal{L}(t),\ \mathcal{C}(t),\ \mathcal{O}(t)\}$, is the subset assigned to vehicle i by the coordinator.

Time to exit the merging zone

- t_i^f is computed as follows:

$$t_i^f = \begin{cases} t_1^f, & \text{if } i = 1, \\ t_{i-1}^f, & \text{if } i-1 \in \mathcal{R}(t) \text{ (or } \mathcal{O}(t)), \\ t_{i-1}^f + \frac{\delta}{v_i(t_{i-1}^f)}, & \text{if } i-1 \in \mathcal{L}(t), \\ t_{i-1}^f + \frac{S}{v_i(t_{i-1}^f)}, & \text{if } i-1 \in \mathcal{C}(t). \end{cases} \tag{10}$$

Decentralized problem formulation

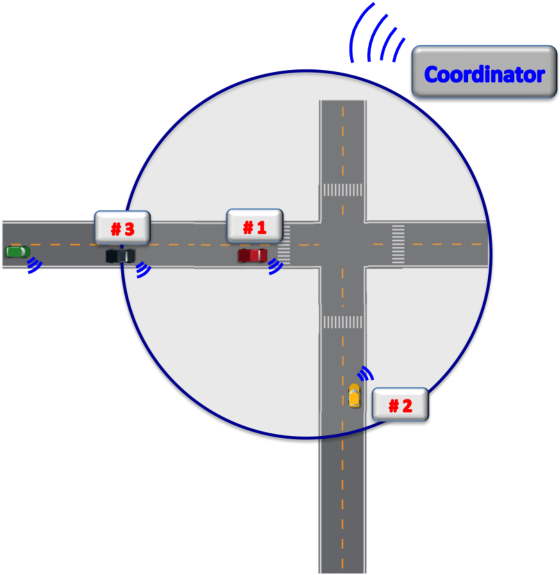
Lemma

The decentralized communication structure aims each vehicle i to solve an optimal problem for $t \in [t_i^0, t_i^f]$ the solution of which depends only on the solution of the vehicle i-1.

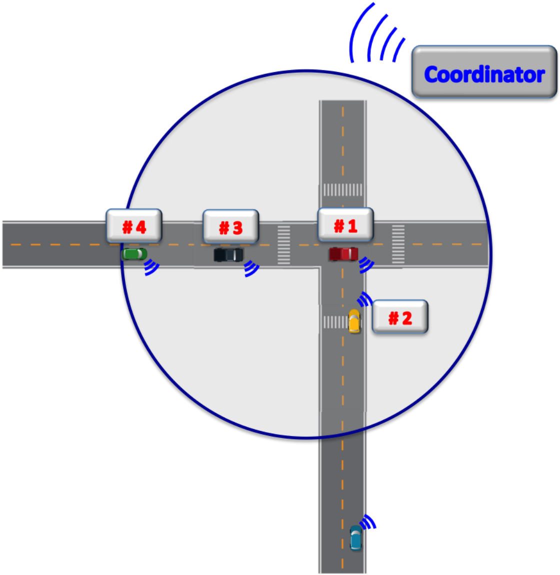
It is shown[3] by induction that for each vehicle i, t_i^f depends only on t_{i-1}^f.

[3] Zhang, Y.Z, Malikopoulos, A.A., Cassandras, C.G., "Optimal Control and Coordination of Connected and Automated Vehicles at Urban Traffic Intersections," *Proceedings of the 2016 American Control Conference, 2016. (in review)*

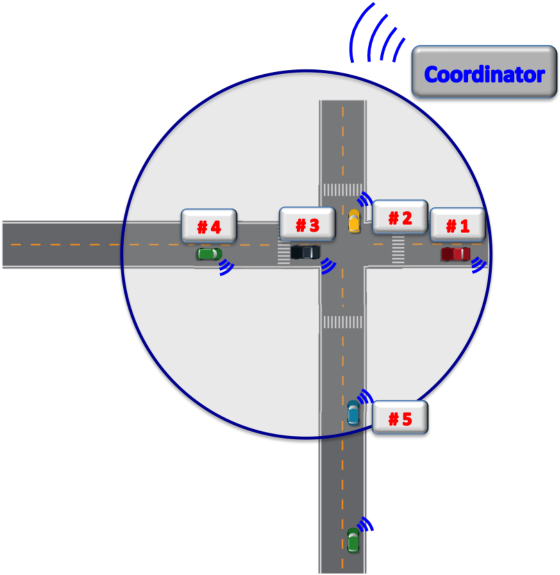
Sketch of proof with intuition



Sketch of proof with intuition



Sketch of proof with intuition



Decentralized problem formulation

- We formulate $N(t)$ decentralized tractable problems that can be solved online

$$\min_{u_i} \frac{1}{2} \int_{t_i^0}^{t_i^f} u_i^2 \, dt \tag{11}$$

$$\begin{aligned} \text{Subject to :} \quad & \text{vehicle dynamics} \\ & \text{state and control bounds,} \end{aligned}$$

where t_i^f is given by (10), $\forall i \in \mathcal{N}$.

Analytical solution

$$H_i\big(t, x(t), u(t)\big) = L_i\big(t, x(t), u(t)\big) + \lambda^T \cdot f_i\big(t, x(t), u(t)\big) \qquad (12)$$
$$+\mu^T \cdot g_i\big(t, x(t), u(t)\big),$$

where

$$g_i\big(t, x(t), u(t)\big) = \begin{cases} u_i(t) - u_{max} \leq 0, \\ u_{min} - u_i(t) \leq 0, \\ v_i(t) - v_{max} \leq 0, \\ v_{min} - v_i(t) \leq 0, \\ p_i(t) - p_{i-1}(t) + \delta \leq 0, i-1, i \in \mathcal{N}. \end{cases}$$

- Optimal control input

$$u_i^*(t, p_i(t), v_i(t)) = a_i(t, p_i(t), v_i(t)) \cdot t + b_i(t, p_i(t), v_i(t)). \qquad (13)$$

Comparison between centralized and decentralized solution

Theorem

The optimal solution of the centralized problem (8) preserves the queue $\mathcal{N}$.

The analysis of the centralized solution shows[4] that the queue of the vehicles is preserved.

Corollary

The solution of the decentralized control problem is equivalent to the solution of the centralized problem.

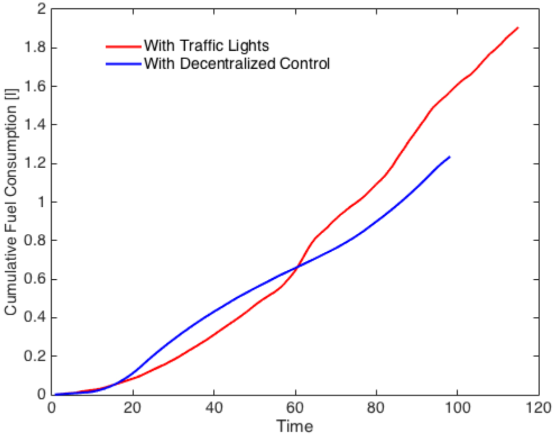
[4] Malikopoulos, A.A. and Rios-Torres, J., "Decentralized Optimal Control for Vehicle Coordination at Intersections," *Proceedings of the 2016 American Control Conference, 2016. (in review)*

Vehicle coordination with decentralized control

Vehicle coordination with decentralized control

Impact on fuel consumption and travel time

- Fuel consumption (55% improvement) and travel time (17% improvement)



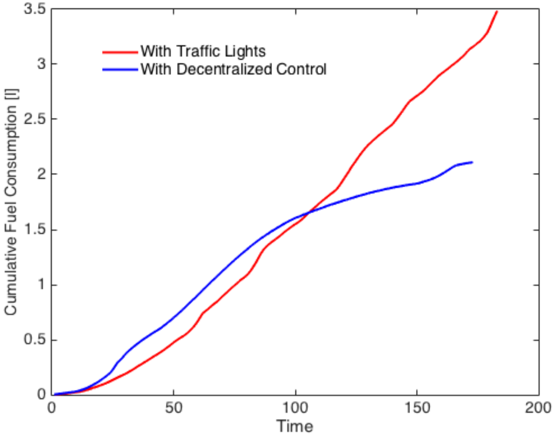
Vehicle coordination with decentralized control[5]

[5] Malikopoulos, A.A. and Rios-Torres, J., "Decentralized Optimal Control for Vehicle Coordination at Intersections," *Proceedings of the 2016 American Control Conference, 2016. (in review)*

Vehicle coordination with decentralized control

Impact on fuel consumption and travel time

- Fuel consumption (65% improvement) and travel time (6% improvement)

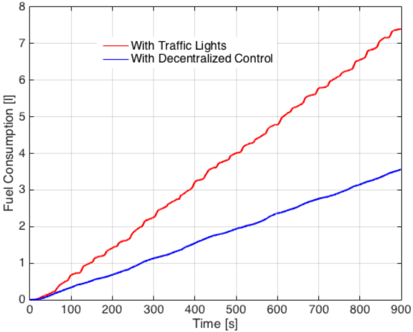


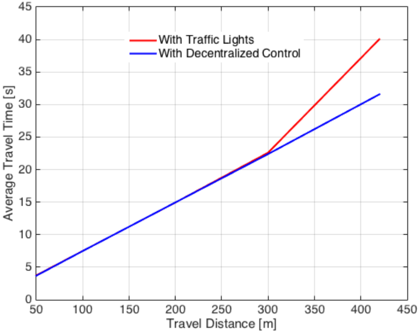
Vehicle coordination - VISSIM

- With traffic lights and with decentralized control of CAVs

Impact on fuel consumption and travel time

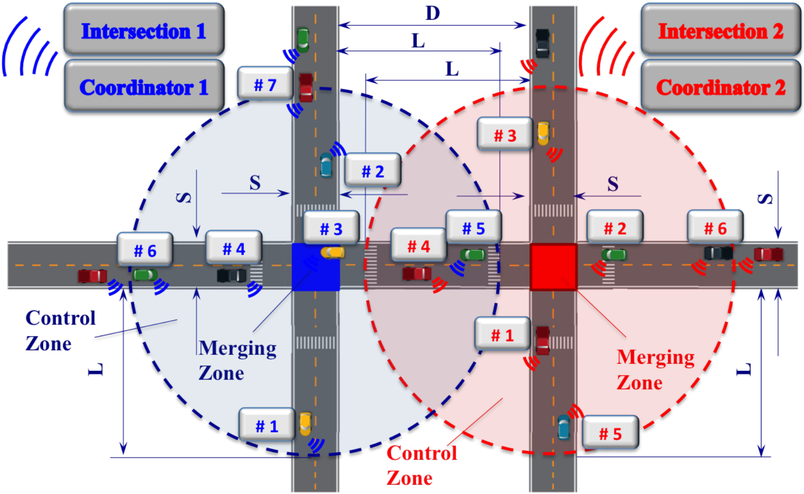
- For 470 vehicles that crossed the intersection
- Fuel consumption (52% improvement) and travel time (21% improvement)





Decentralized stochastic control: intersection

- Co-investigator: Professor Christos Cassandras (BU)



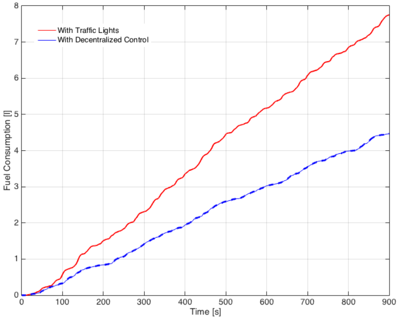
Boston University Intersection - VISSIM[6]

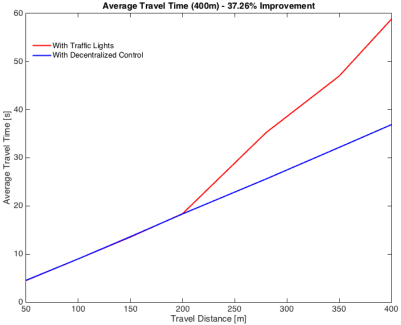
- With traffic lights and with decentralized control of CAVs

[6] Zhang, Y.Z, Malikopoulos, A.A., Cassandras, C.G., "Optimal Control and Coordination of Connected and Automated Vehicles at Urban Traffic Intersections," *Proceedings of the 2016 American Control Conference, 2016. (in review)*

Impact on fuel consumption and travel time

- For 448 vehicles that crossed the intersection
- Fuel consumption (42.4% improvement) and travel time (37.3% improvement)



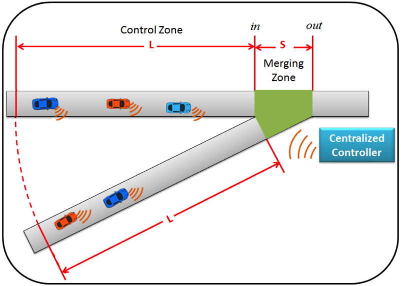


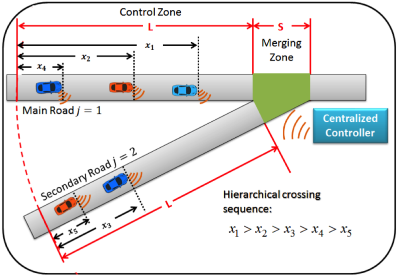
Automated intersection: futuristic movie[7]

[7] Courtesy of Fernando Livschitz

Traffic flow merging with vehicle coordination[8],[9]

- Coordination of 30 vehicles (15 vehicles on each road)





---

[8] Rios-Torres, J., Malikopoulos, A.A., Pisu, P, "Online Optimal Control of Connected Vehicles for Efficient Traffic Flow at Merging Roads," *in Proceedings of 2015 IEEE 18th International Conference on Intelligent Transportation Systems, pp. 2432-2437, 2015.*

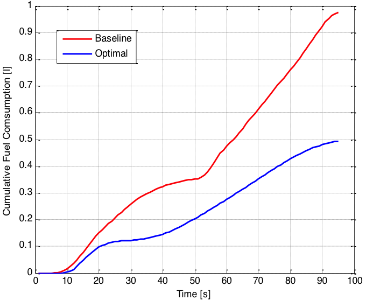
[9] Rios-Torres, J. and Malikopoulos, A.A., Pisu, P., "Coordination of Connected and Automated Vehicles for Efficient Traffic Flow," *IEEE Transactions on Intelligent Transportation Systems. (in review)*

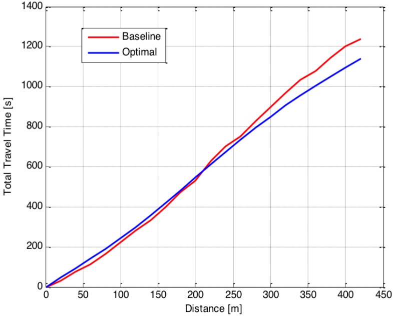
Traffic flow merging without vehicle coordination

Traffic flow merging with vehicle coordination

Traffic flow merging with vehicle coordination

- Fuel consumption (50% improvement) and travel time (7% improvement)





Traffic flow merging with vehicle coordination and different speed

Desired traffic flow "emergence" with CAVs[10]

[10] Video: The future of self-driving cars.

Penetration of CAVs in the transportation network[11]

[11] Toolbox for Urban Mobility Simulation (TUMS)

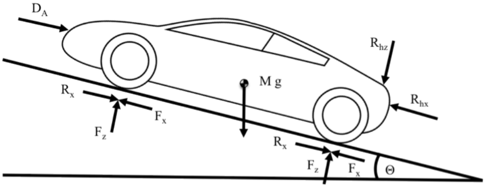
How about connected vehicles with a driver...?

- Each individual driving style is different.
- Variation in fuel economy can be as much as 30%[12].


[12] Rios-Torres, J., Malikopoulos, A.A., Pisu, P., "A Survey on Driver Feedback Systems and Coordination of Connected and Automated Vehicles," *IEEE Transactions on Intelligent Transportation Systems. (in review)*

Driver's feedback system

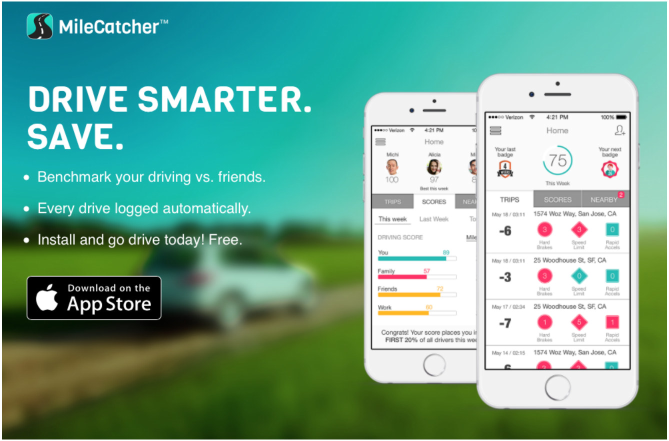
- We identified the driving-style factors that have a major impact on fuel economy and developed an optimization framework to optimize individual driving styles with respect to these factors[13],[14].



[13] Malikopoulos, A.A. and Aguilar, J.P., "An Optimization Framework for Driver Feedback Systems," *IEEE Transactions on Intelligent Transportation Systems, Vol.* ***14****, No. 2, 2013, pp.955-964.*

[14] Malikopoulos, A.A. and Aguilar, J.P., "Optimization of Driving Styles for Fuel Economy Improvement," *in Proceedings of 2012 15th International IEEE Conference on Intelligent Transportation Systems, Anchorage, AK, September 16-19, 2012.*

Technology can be integrated in an app[15]



15 Malikopoulos, A.A., Driver Feedback for Fuel Efficiency, United States Patent Application Serial No. 61/877,446, September 2013.

Concluding remarks

- The results presented here address the problem of coordinating online a continuous flow of CAVs crossing an intersection.
- We presented a decentralized control framework and a closed-form solution that yields for each vehicle the optimal acceleration/deceleration at any time aimed at minimizing fuel consumption.
- What is next? In this "new world" of massive amount of data how we can improve the efficiency in mega cities?

Future research direction

What else we should be expecting to see...?[16]

[16] Courtesy of Fernando Livschitz

Connected cities

- Connected people, vehicles and infrastructure

Complex urban systems

- Develop system-wide optimal solutions using scalable data and informatics

System-wide optimal solutions in complex urban systems[17]

[17] Synthesis of various videos: courtesy of Siemens.

Thank you for your Attention!

Andreas A. Malikopoulos, Ph.D.
Deputy Director, Urban Dynamics Institute
Oak Ridge National Laboratory

e-mail: andreas@ornl.gov
URL: http://www.andreas-malikopoulos.com