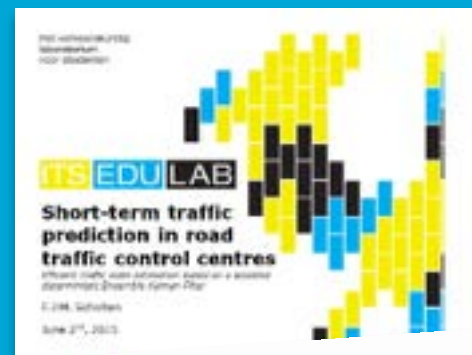


Hans van Lint, Yufei Yuan & Friso Scholten



A localized deterministic Ensemble Kalman Filter

LARGE-SCALE TRAFFIC STATE ESTIMATION

CONTENTS

- Intro: need for large-scale traffic state estimation
- Some Kalman Filter basics
- The Ensemble Kalman Filter
 - ➔ getting rid of matrix inversions
 - ➔ deterministic formulation
 - ➔ localisation
- Simulations, Results and Discussions
- Conclusions & outlook (big) plans

INTRODUCTION

Dutch National Data Warehouse Traffic Information

- Network
 - ✓ 2500 km freeways
 - ✓ 3500 km prov/urban
 - ✓ 24,000 measurement sites
- Real-time Data
 - ✓ Dynamic: Flow, Speed, Occupancy, Travel time (lane / usr class specific), FCD
 - ✓ Status: roadworks, incidents, events
 - ✓ 150,000 measurements/min
- Historical database
 - ✓ 200 TB
 - ✓ Raw data



INTRODUCTION

Dutch National Data Warehouse Traffic Information

- Ambition
 - ✓ Build national traffic observatory (for RT and archived traffic info)
- From raw data to real information:
 - ✓ Intelligent database (search terms related to traffic patterns and explanatory factors)
 - ✓ Data-based routable network graphs
 - ✓ Densities, space mean speeds
 - ✓ Capacities, other FD params
 - ✓ MFDs and other agg state vars
 - ✓ Inflows, turns, OD flows
 - ✓ Opensource tools to dive into all this

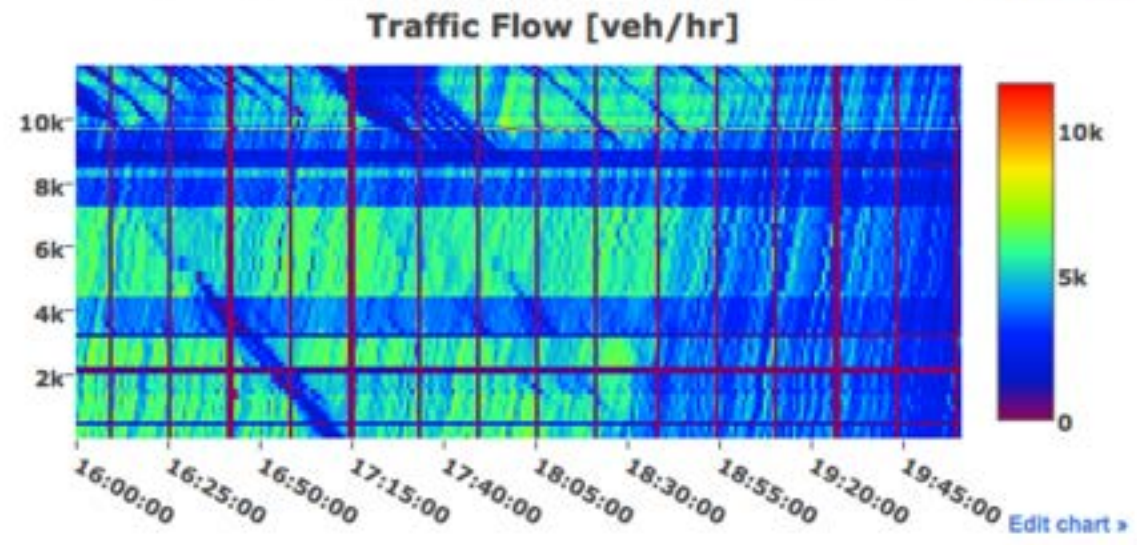
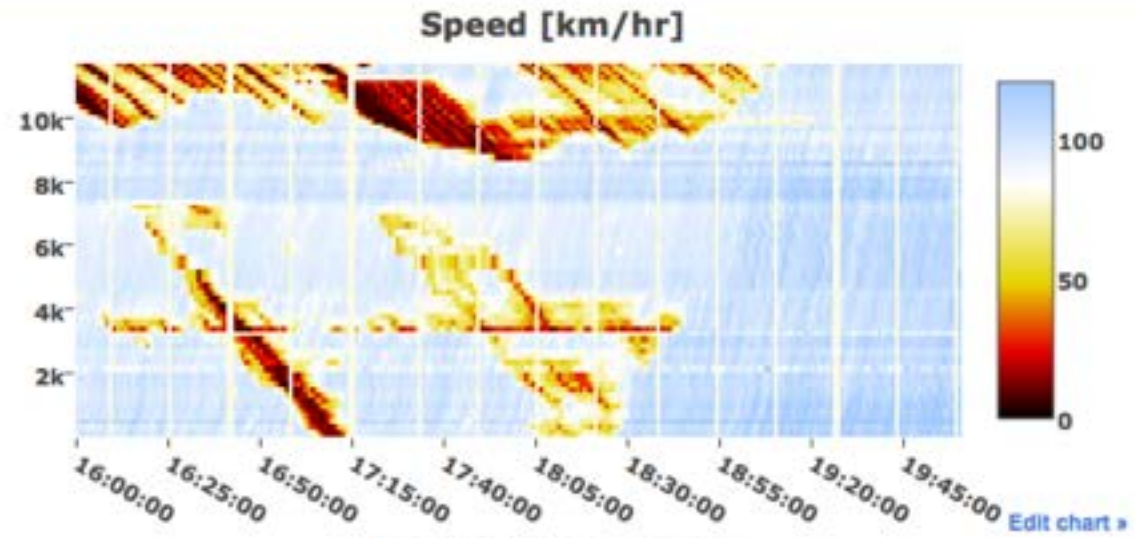
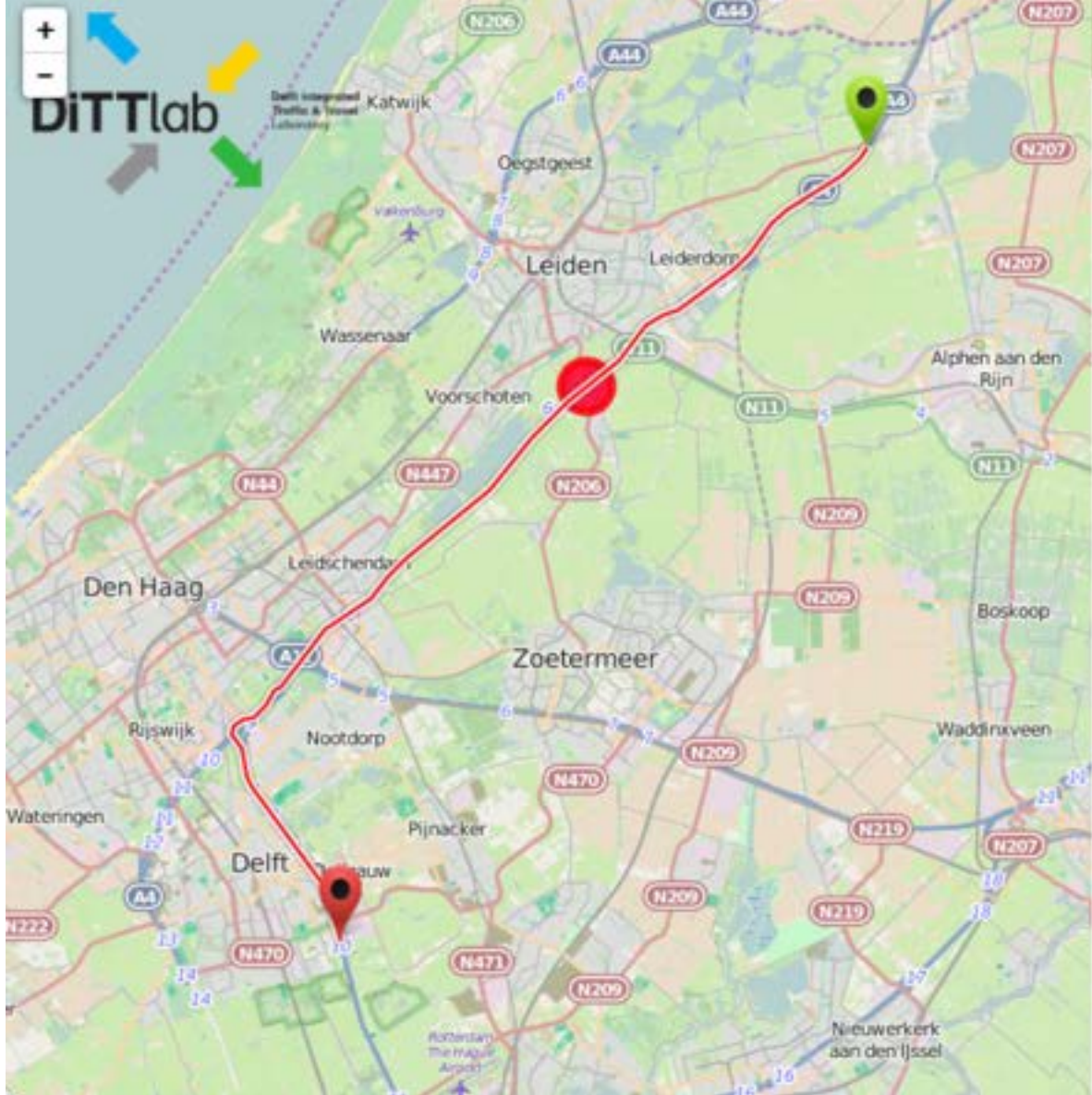


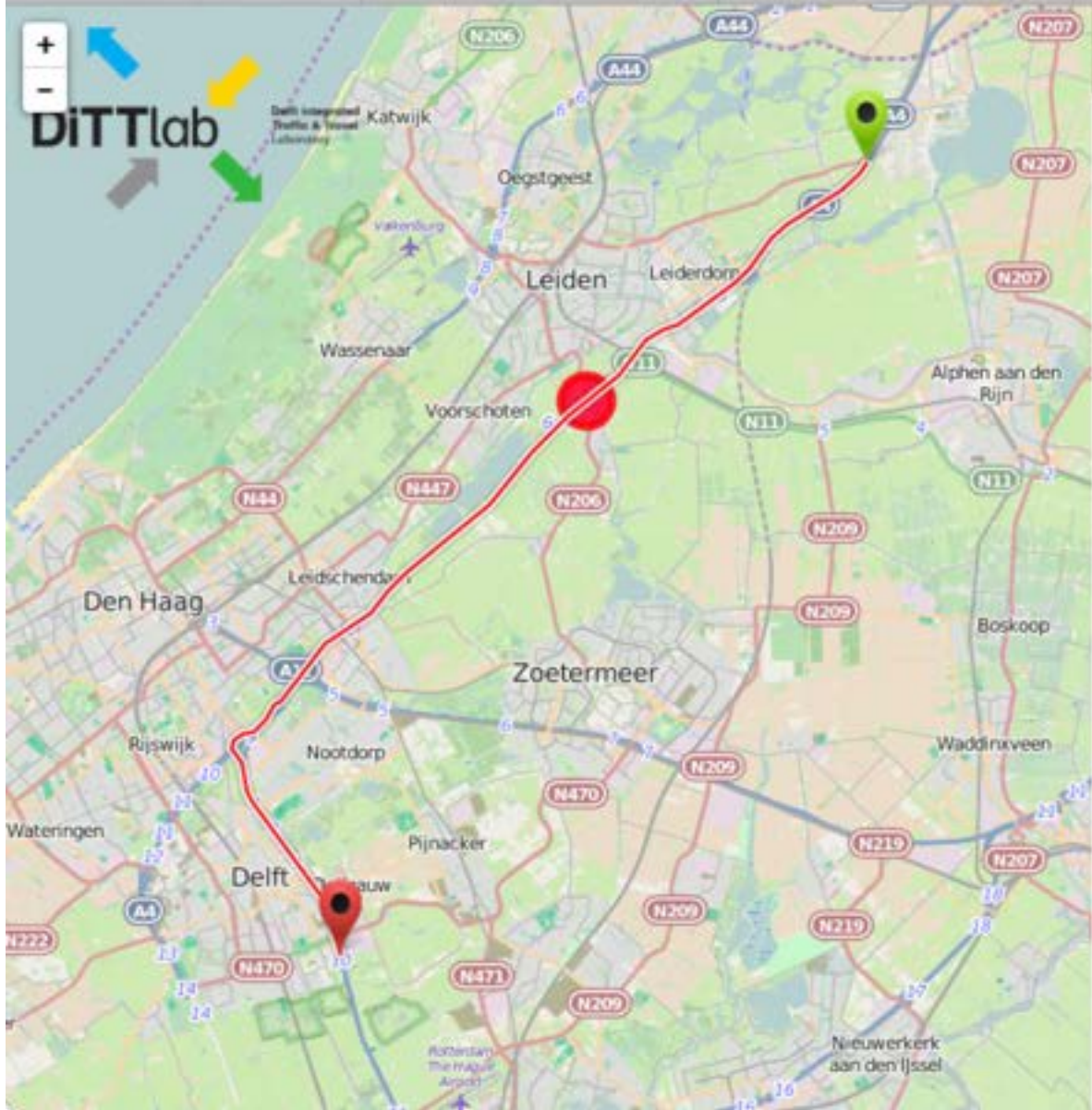
DiTTlab
Delft Integrated Traffic & Travel Laboratory

1. Pick two points on the map
SELECT ROUTE

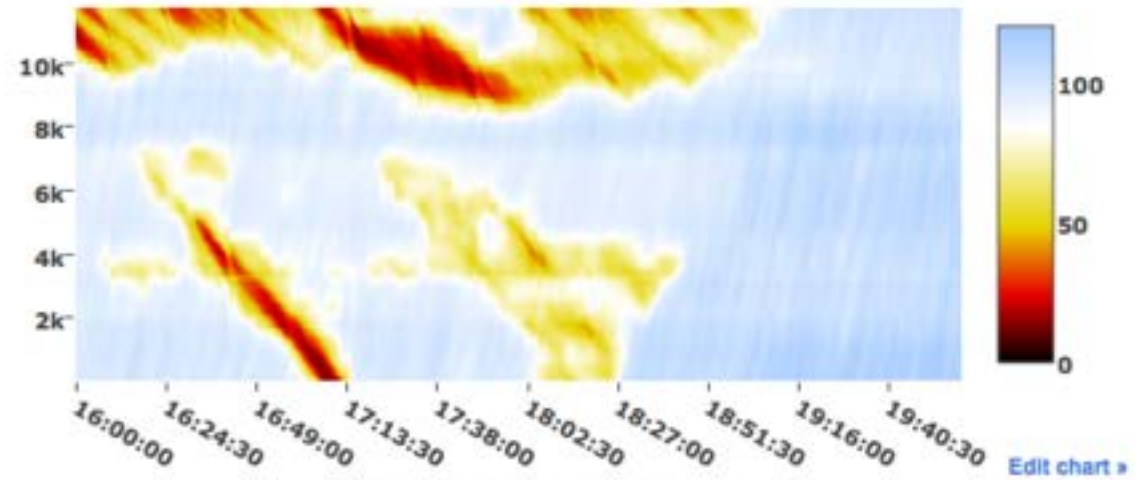
2. Pick a Date

DAILY SPEED
DAILY WEATHER
DAILY EVENTS

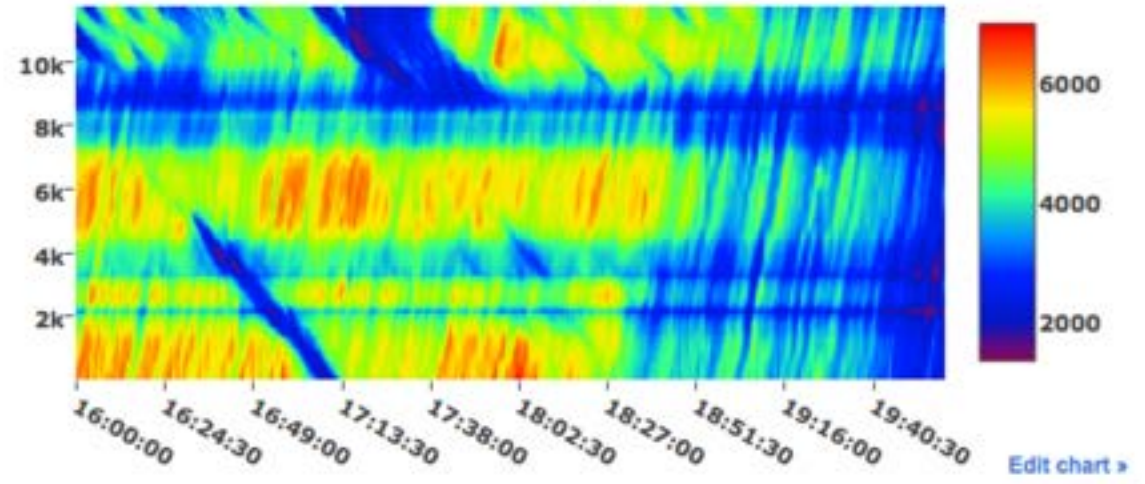




Speed [km/hr]

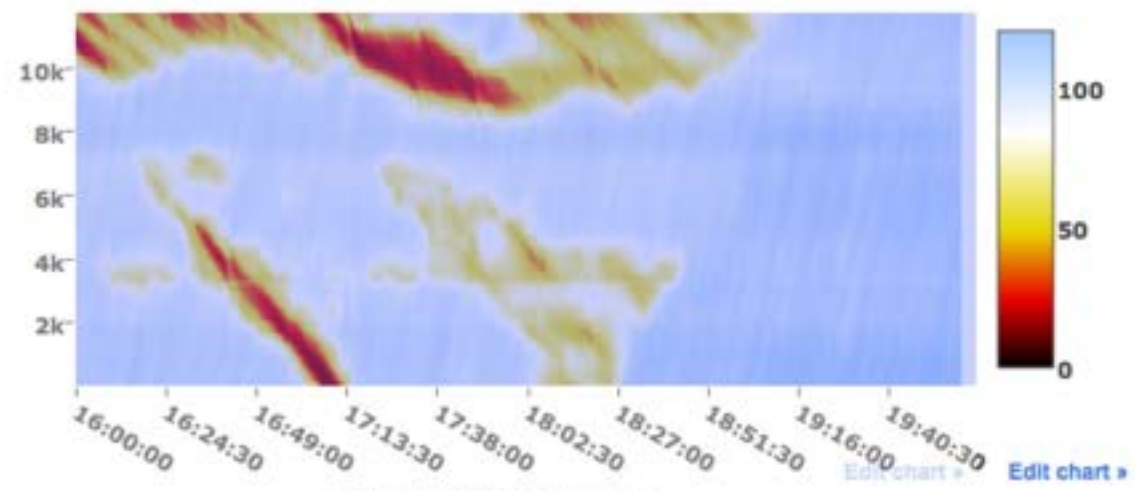


Traffic Flow [veh/hr]

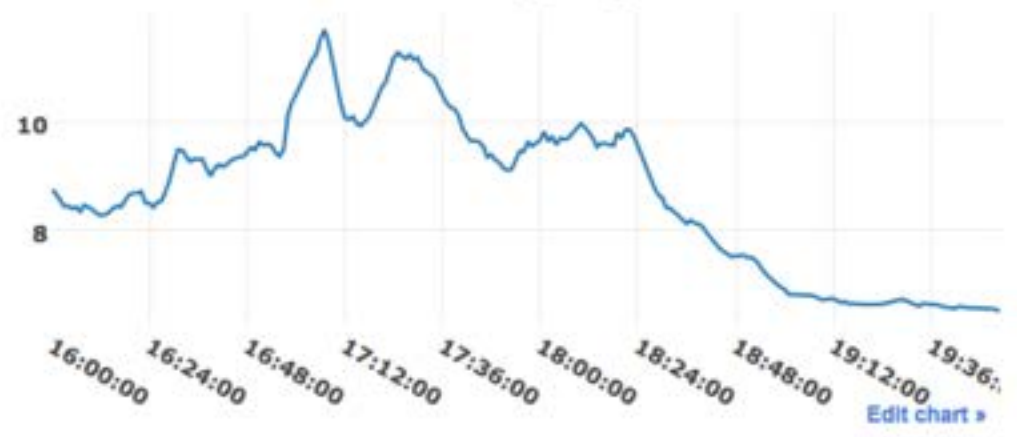




Speed [km/hr]

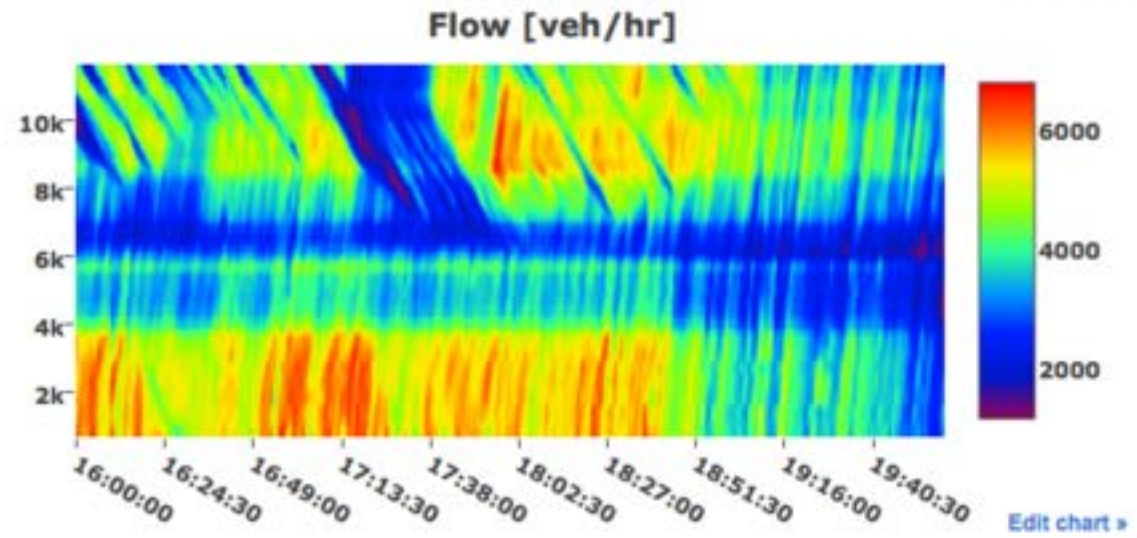
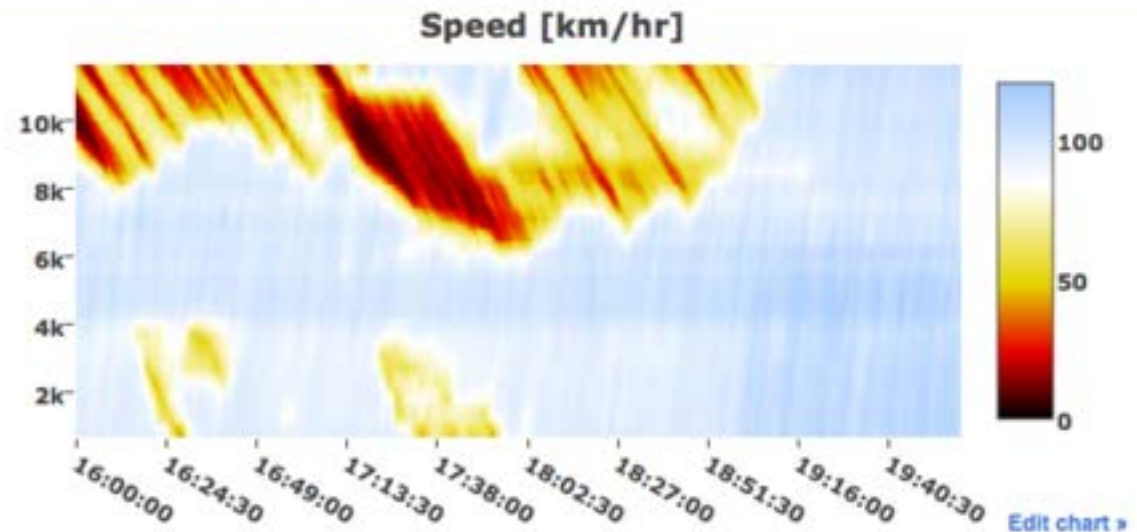
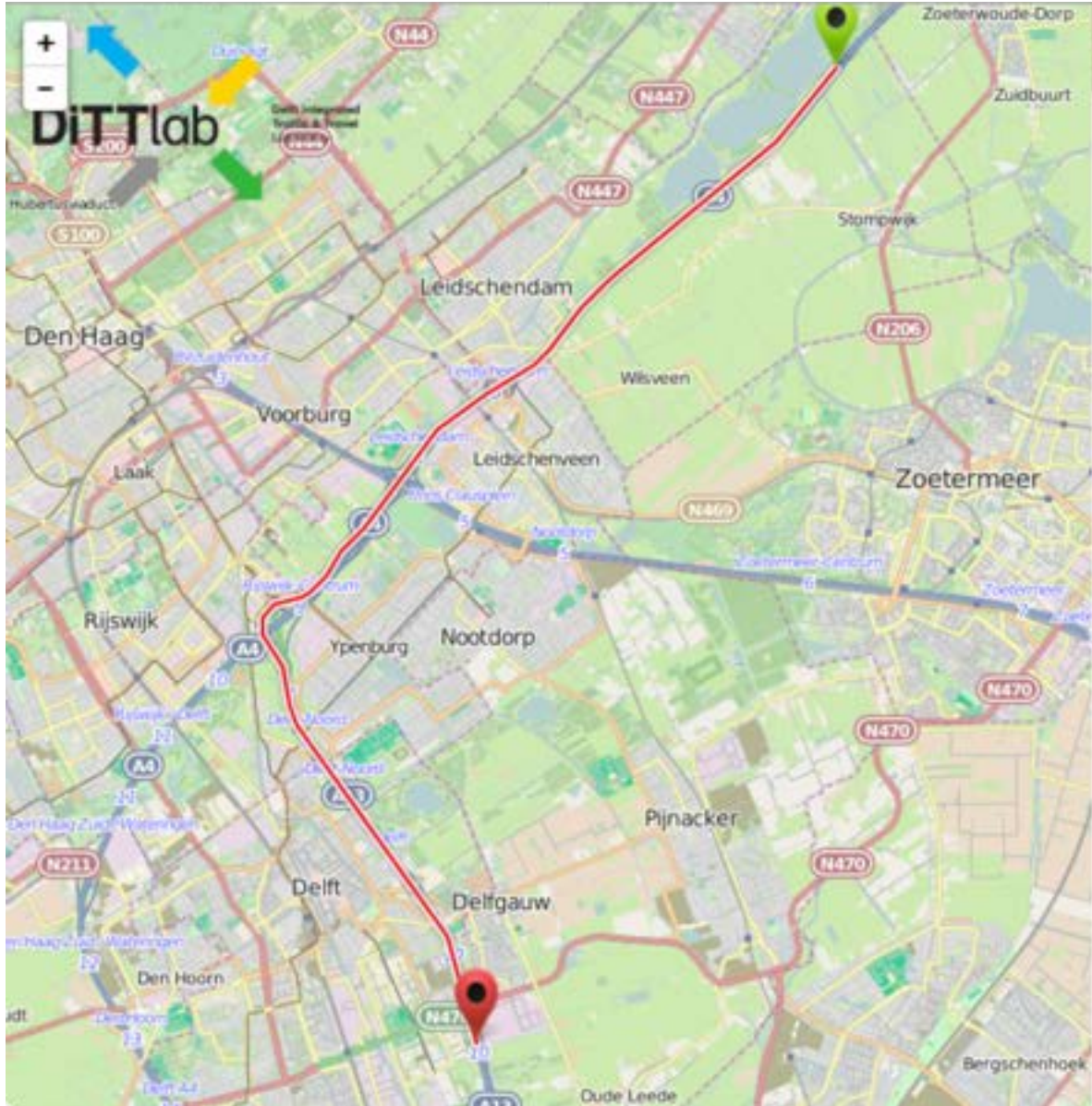


Travel Time [min]





TotFlowRoute	=	26426.00	[veh]
AvgSpeedRoute	=	69.05	[km/h]
PercMissingSpeedDetectors	=	7.47	%
PercMissingFlowDetectors	=	7.47	%
PercMissingSpeedCrossSections	=	7.49	%
PercMissingFlowCrossSections	=	7.47	%
TotalVehicleLossHours	=	473.10	[veh-h]
TravelTimePercentile10	=	6.64	[min]
TravelTimePercentile25	=	7.24	[min]
TravelTimePercentile50	=	8.97	[min]
TravelTimePercentile75	=	9.70	[min]
TravelTimePercentile90	=	10.65	[min]
AvgTravelTime	=	8.71	[min]
StdTravelTime	=	1.45	[min]



INTRODUCTION

Dutch National Data Warehouse Traffic Information

- Ambition
 - ✓ Build national traffic observatory (for RT and archived traffic info)
- Next step:
 - ✓ Scalable efficient model-based traffic state estimation: **densities** (inflows, turns)
- TRADE OFF
 - ✓ Estimation accuracy vs computational efficiency



THE KALMAN FILTER

For any system cast in discrete state space form

- the KF is an efficient recursive solution for estimating state vector \mathbf{x} ;

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{G}_k \mathbf{u}_k + \mathbf{w}_k$$

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

- that is optimal in three ways
 - Minimum error (co)variance
 - Maximal posterior probability
 - Conditional mean estimation
- given a some (fairly strict) assumptions
 - the validity of which strongly affect the quality of the result

$$E(\mathbf{w}_k) = 0, \quad E(\mathbf{w}_k \mathbf{w}_l') = \begin{cases} \mathbf{Q}_k & k = l \\ 0 & k \neq l \end{cases}$$

$$E(\mathbf{v}_k) = 0, \quad E(\mathbf{v}_k \mathbf{v}_l') = \begin{cases} \mathbf{R}_k & k = l \\ 0 & k \neq l \end{cases}$$

THE KALMAN FILTER

- Initial conditions (reasonable choices for \mathbf{Q} and \mathbf{R} must also be made)

$$\hat{\mathbf{x}}_{0|0} = \hat{\mathbf{x}}_0, \mathbf{P}_{0|0} = \mathbf{P}_0$$

- Time-propagation (prediction):

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{G}_{k-1} \mathbf{u}_{k-1}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}$$

- Measurement adaptation (correction):

$$\mathbf{e}_k = \mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k / (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k + \mathbf{R}_k)$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{e}_k$$

$$\mathbf{P}_{k|k} = [\mathbf{I} - \mathbf{K}_k \mathbf{H}_k] \mathbf{P}_{k|k-1}$$

A CLOSER LOOK AT THE FILTER GAIN

sensitivity observation model
to changes in the state

$$\mathbf{K}_k = \frac{\mathbf{P}_{k|k-1} \mathbf{H}_k}{\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k + \mathbf{R}_k}$$

error covariance of the innovations (i.e. uncertainty in
the new information we get from observations).

A CLOSER LOOK AT THE FILTER GAIN

$$\text{Kalman Gain} = \frac{\text{uncertainty in (process) model} \times \text{Sensitivity observation model}}{\text{uncertainty in observations}}$$

So balance between uncertainty in process model and uncertainty in measurements / observation model!

TRAFFIC FLOW SIMULATION MODELS

Are naturally cast in discrete state-space form

- Conservation of vehicles
(+ possible additional
pde for speed)

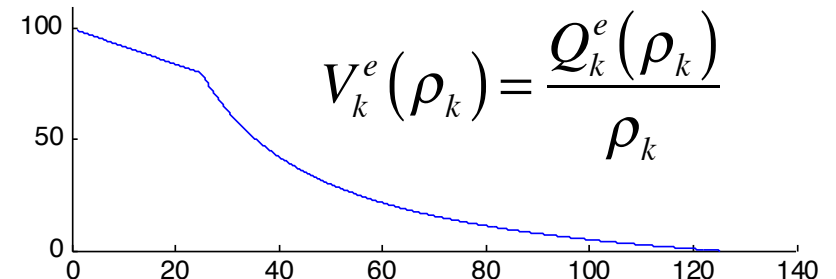
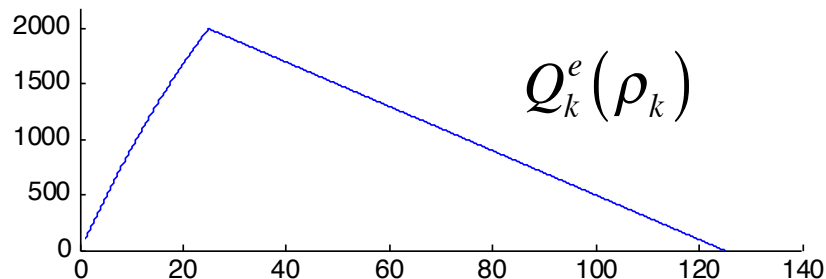
$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k$$

$$\text{typically } f : \rho_{k+1} = \rho_k + \frac{\Delta L}{\Delta t} (\mathbf{q}_k^{in} - \mathbf{q}_k^{out} + \mathbf{r} - \mathbf{s})$$

- Observation model
typically fundamental
diagram

$$\mathbf{y}_k = h_k(\mathbf{x}_k) + \mathbf{v}_k$$

$$\text{typically } h : \mathbf{q}_k = Q_k^e(\rho_k)$$



TRAFFIC STATE ESTIMATION

Extended Kalman filtering

- Nonlinear extension KF
- Assumptions
 - Process model can be approximated by 1st order Taylor expansion
 - Obs model can be approximated with 1st order Taylor expansion
 - All other KF assumptions (iid Gaussian WN, etc)

$$\mathbf{x}_{k+1} = f_k(\mathbf{x}_k) + \mathbf{w}_k$$

$$\mathbf{y}_k = h_k(\mathbf{x}_k) + \mathbf{v}_k$$

$$\mathbf{F}_k = \left. \frac{df_k(\mathbf{x})}{d\mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}_k}$$

$$\mathbf{H}_k = \left. \frac{dh_k(\mathbf{x})}{d\mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}_k}$$

EXTENDED KALMAN FILTER

- Many challenges
 - linearisation may cause adaptation in the wrong direction
 - Computing derivatives = pain in the @!\$\$...
 - Scalability: matrix manipulations & inversions expensive
 - (E)KF assumptions on violated all the time

$$\hat{\mathbf{x}}_{0|0} = \hat{\mathbf{x}}_0, \mathbf{P}_{0|0} = \mathbf{P}_0$$

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k)$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}$$

$$\mathbf{e}_k = \mathbf{y}_k - h(\hat{\mathbf{x}}_{k|k-1})$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k / (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k + \mathbf{R}_k)$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{e}_k$$

$$\mathbf{P}_{k|k} = [\mathbf{I} - \mathbf{K}_k \mathbf{H}_k] \mathbf{P}_{k|k-1}$$

ALTERNATIVE: ENSEMBLE KALMAN FILTER

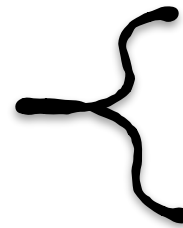


- Use ensemble of N state vectors and noisy observation replications

$$\mathbf{X}_{0|0} = [\mathbf{x}_0^1, \dots, \mathbf{x}_0^N],$$

$$\mathbf{X}_{k|k-1} = f(\mathbf{X}_{k-1|k-1}, \mathbf{U}_k)$$

- Predicting ensemble mean and variance is simple ...



$$\mathbf{x}_{k|k-1} = E(\mathbf{X}_{k|k-1}) = \frac{1}{N} \sum_{i=1:N} \mathbf{x}_{k|k-1}^i$$

$$\mathbf{P}_{k|k-1} = \mathbf{A}_{k|k-1} \mathbf{A}_{k|k-1}^T / (N-1), \text{ with } \mathbf{A}_x = \mathbf{X}_x - E(\mathbf{X}_x)$$

- Update step similar to EKF

→ Still requires linearisation ?

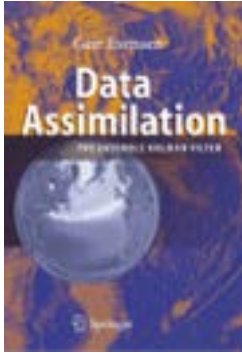
$$\mathbf{D}_k = [\mathbf{d}_k^1, \dots, \mathbf{d}_k^N], \text{ with } \mathbf{d}_k^i = \mathbf{y}_k + \mathbf{e}_k^i$$

$$\mathbf{E}_k = \mathbf{D}_k - h(\mathbf{X}_{k|k-1})$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} / (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k + \mathbf{R}_k)$$

$$\mathbf{X}_{k|k} = \mathbf{X}_{k|k-1} + \mathbf{K}_k \mathbf{E}_k$$

ALTERNATIVE: ENSEMBLE KALMAN FILTER

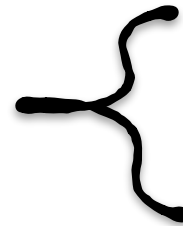


- Use ensemble of N state vectors and noisy observation replications

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$$\mathbf{X}_{k|k-1} = f(\mathbf{X}_{k-1|k-1}, \mathbf{U}_k)$$

- Predicting ensemble mean and variance is simple ...



$$\mathbf{x}_{k|k-1} = E(\mathbf{X}_{k|k-1}) = \frac{1}{N} \sum_{i=1:N} \mathbf{x}_{k|k-1}^i$$

$$\mathbf{P}_{k|k-1} = \mathbf{A}_{k|k-1} \mathbf{A}_{k|k-1}^T / (N-1), \text{ with } \mathbf{A}_x = \mathbf{X}_x - E(\mathbf{X}_x)$$

$$\mathbf{D}_k = [\mathbf{d}_k^1, \dots, \mathbf{d}_k^N], \text{ with } \mathbf{d}_k^i = \mathbf{y}_k + \mathbf{e}_k^i$$

$$\mathbf{E}_k = \mathbf{D}_k - h(\mathbf{X}_{k|k-1})$$

~~$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$~~

$$\mathbf{X}_{k|k} = \mathbf{X}_{k|k-1} + \mathbf{K}_k \mathbf{E}_k$$

$$\mathbf{K}_k = \frac{1}{N-1} \mathbf{A}_{k|k-1} (\mathbf{H}\mathbf{A})^T / \mathbf{P}_{k|k}^*$$

with $\mathbf{P}_{k|k}^* = \frac{1}{N-1} \mathbf{H}\mathbf{A} (\mathbf{H}\mathbf{A})^T + \mathbf{R}_k$

and $[\mathbf{H}\mathbf{A}^i] = h(\mathbf{x}_{k|k-1}^i) - \frac{1}{N} \sum_{j=1:N} h(\mathbf{x}_{k|k-1}^j)$

A CLOSER LOOK AT THIS FILTER GAIN

distance prior state
to mean ensemble
state

distance predicted
observations to mean
ensemble prediction

$$\mathbf{K}_k = \frac{\mathbf{A}_{k|k-1} (\mathbf{H}\mathbf{A})^T}{\mathbf{H}\mathbf{A} (\mathbf{H}\mathbf{A})^T + \mathbf{R}_k}$$

total error variance (uncertainty)
measurement equation

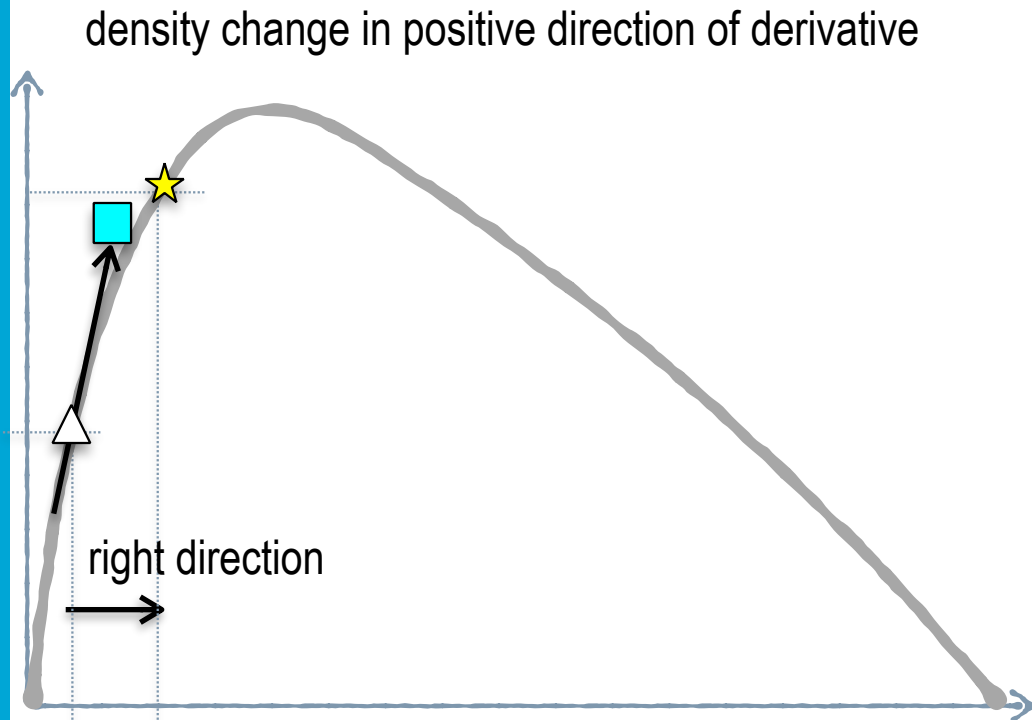
A CLOSER LOOK AT THIS FILTER GAIN

$$\text{Kalman Gain} = \frac{\text{uncertainty in model predictions (sign?)}}{\text{uncertainty in observations}}$$

So again balance between uncertainty in model and uncertainty in measurements!

EKF VS ENKF

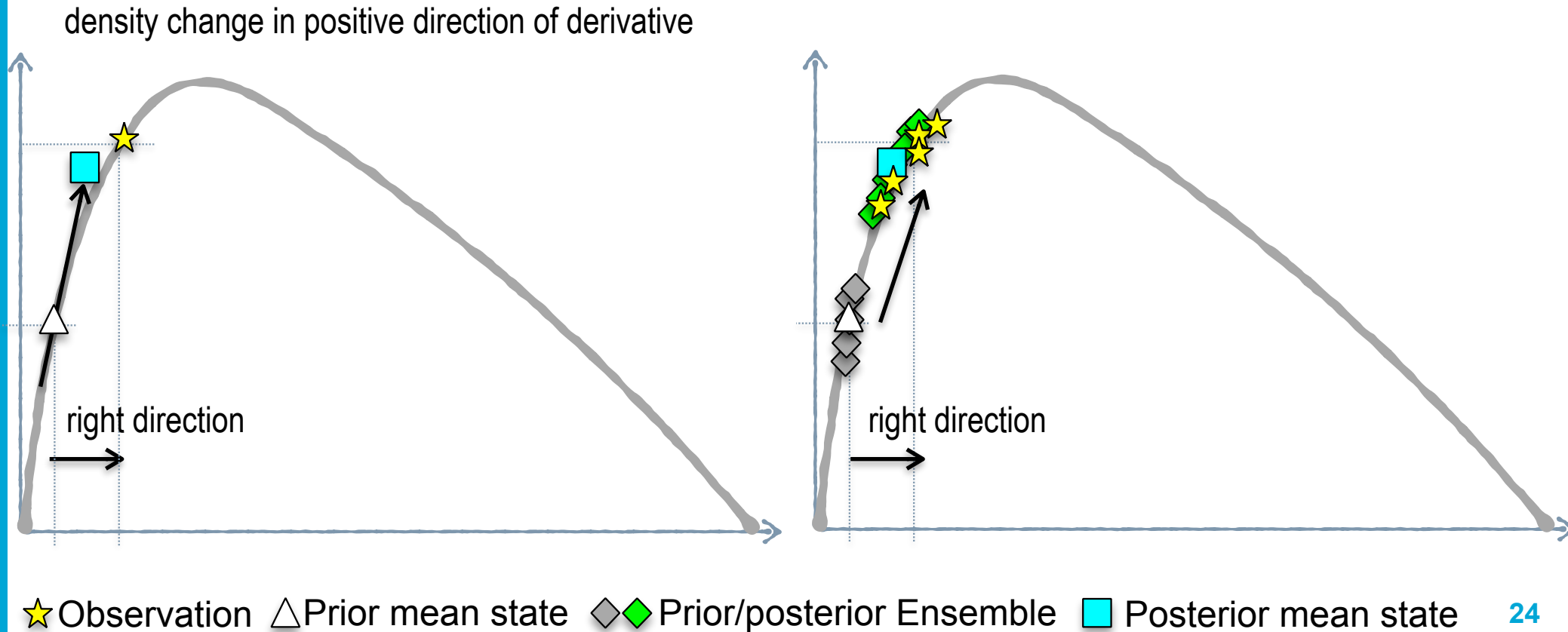
EnKF Improves upon the 'wrong sign' adaptation problem



★ Observation △ Prior mean state ◆ Prior/posterior Ensemble ■ Posterior mean state

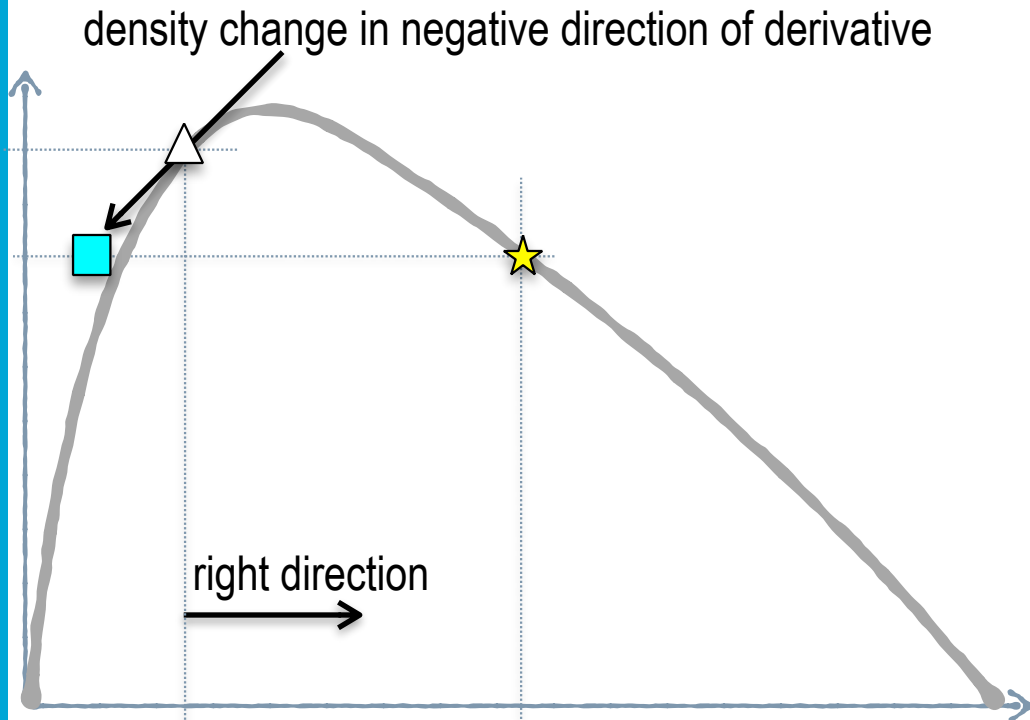
EKF VS ENKF

EnKF Improves upon the 'wrong sign' adaptation problem



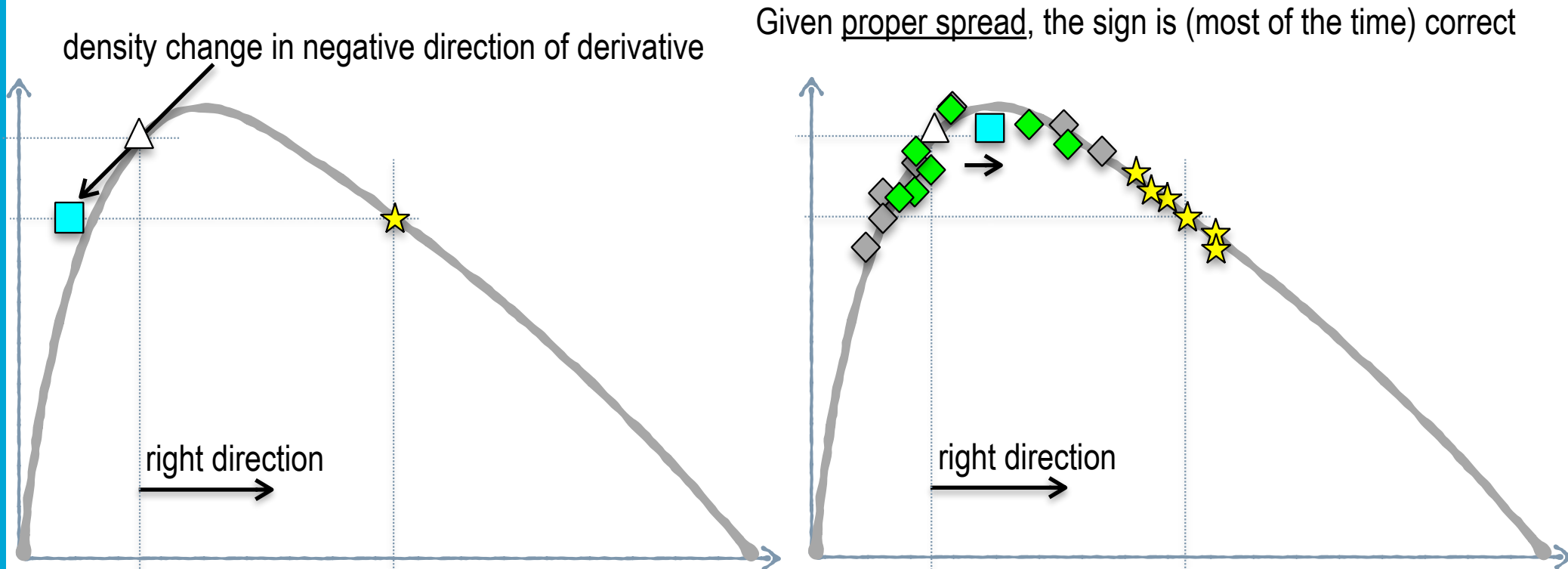
EKF VS ENKF

EnKF Improves upon the 'wrong sign' adaptation problem



EKF VS ENKF

EnKF Improves upon the 'wrong sign' adaptation problem

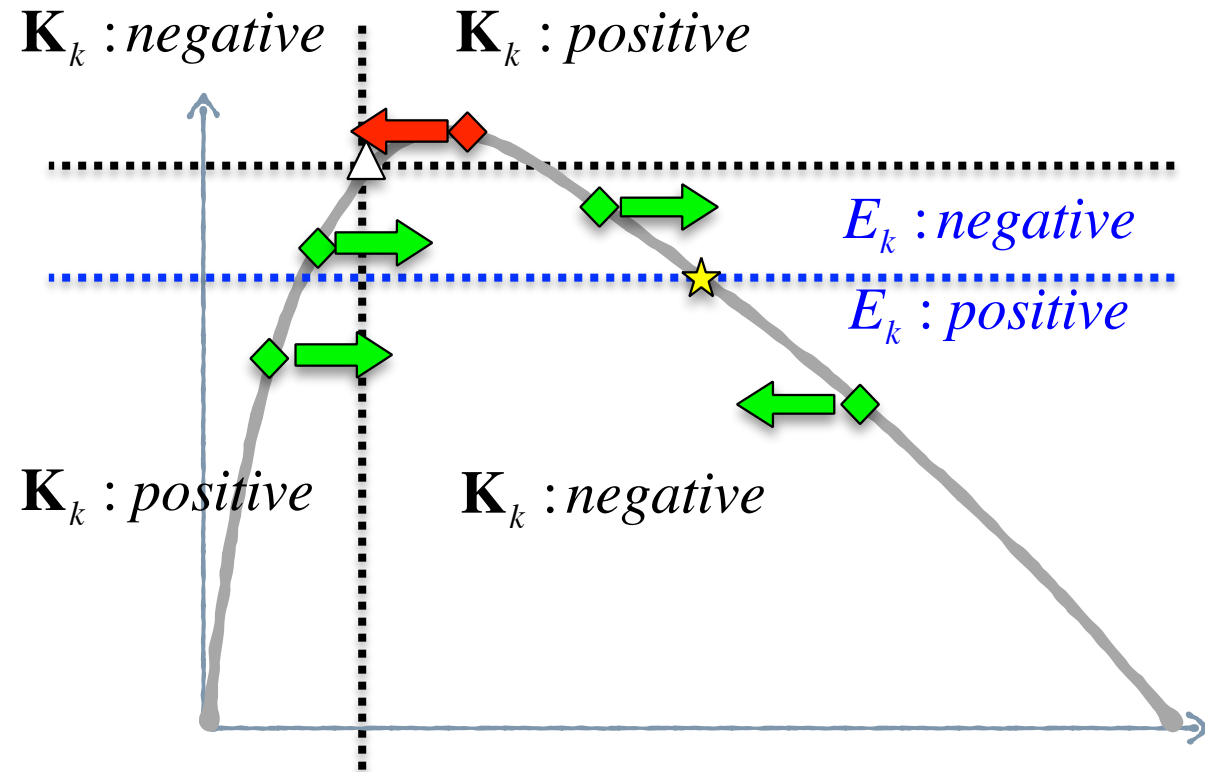


EKF VS ENKF

EnKF Improves upon the 'wrong sign' adaptation problem

$$\mathbf{X}_{klk} = \mathbf{X}_{klk-1} + \mathbf{K}_k \mathbf{E}_k$$

In this case most ensemble members move in the correct direction: so does the ensemble mean



$$\mathbf{K}_k = \frac{1}{N-1} \mathbf{A}_{klk-1} (\mathbf{H}\mathbf{A})^T / \mathbf{P}_{klk}^*$$

$$\text{with } \mathbf{P}_{klk}^* = \frac{1}{N-1} \mathbf{H}\mathbf{A} (\mathbf{H}\mathbf{A})^T + \mathbf{R}_k$$

$$\text{and } [\mathbf{H}\mathbf{A}^i] = h(\mathbf{x}_{klk-1}^i) - \frac{1}{N} \sum_{j=1:N} h(\mathbf{x}_{klk-1}^j)$$

ENKF FOR LARGE-SCALE ESTIMATION

Three further adaptations

- Getting rid of expensive matrix inversion
- Deterministic instead of stochastic resampling of **D**
- Localisation of the filter

ENKF FOR LARGE-SCALE ESTIMATION

Three further adaptations

- Getting rid of expensive matrix inversion
 - ➔ Computational cost traditional EnKF: $\mathbf{O}(m^3 + m^2N + mN^2 + nN^2)$
 - cubic in nr measurements: (2 as much \Rightarrow 8 times more cost)
 - quadratic in state variables and ensemble members
 - ➔ Computational cost SMW alternative: $\mathbf{O}(N^3 + mN^2 + nN^2)$
 - linear in measurements and state size
 - quadratic in ensemble members: 8

Sherman-Morrison-Woodbury (SMW) formula, e.g. W.Hager, SIAM Rev., 31(2), 221–239

$$\mathbf{K}_k = \frac{1}{N-1} \mathbf{A}_{k|k-1} (\mathbf{HA})^T \underbrace{\left(\frac{1}{N-1} \mathbf{HA} (\mathbf{HA})^T + \mathbf{R}_k \right)^{-1}}_{\Downarrow}$$
$$\mathbf{R}^{-1} \left[\mathbf{I} - \frac{1}{N-1} (\mathbf{HA}) \left(\mathbf{I} + (\mathbf{HA})^T \mathbf{R}^{-1} \frac{1}{N-1} (\mathbf{HA}) \right)^{-1} (\mathbf{HA})^T \mathbf{R}^{-1} \right]$$

ENKF FOR LARGE-SCALE ESTIMATION

Three further adaptations

- Getting rid of expensive matrix inversion
- Deterministic instead of stochastic resampling of \mathbf{D}
 - ➔ Correct: ensemble spans about the same (model) subspace as the forecasting errors (else: filter divergence looms ...)
 - ➔ Stochastic resampling may lead to clustering ensemble

$$\mathbf{D}_k = [\mathbf{d}_k^1, \dots, \mathbf{d}_k^N], \text{ with } \mathbf{d}_k^i = \mathbf{y}_k + \mathbf{e}_k^i$$
$$\mathbf{X}_{k|k} = \mathbf{X}_{k|k-1} + \mathbf{K}_k (\mathbf{D}_k - h(\mathbf{X}_{k|k-1})) \quad \begin{matrix} \nearrow \\ \searrow \end{matrix} \quad \begin{matrix} \mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{K}_k (\mathbf{d}_k - h(\mathbf{x}_{k|k-1})) \\ \mathbf{A}_{k|k} = \mathbf{A}_{k|k-1} + \mathbf{K}_k^A (\mathbf{D}_k - \mathbf{A}\mathbf{H}_k) \end{matrix}$$

- ➔ Deterministic approach manipulates \mathbf{K}_k^A so that innovation / residuals $\mathbf{A}\mathbf{H}$ ALWAYS spread entire measurement space (with cov \mathbf{R})

Sakov, P., & Oke, P. R. (2008). A deterministic formulation of the ensemble kalman filter: an alternative to ensemble square root filters. Tellus A, 60 (2), 361-371.

ENKF FOR LARGE-SCALE ESTIMATION

Three further adaptations

- Getting rid of expensive matrix inversion
- Deterministic instead of stochastic resampling of **D**
- Localisation of the filter
 - ➔ Further reduces computational costs (chops up Cov matrix => update equations in small bits)
 - ➔ Avoids spurious correlations
 - ➔ Increases effective ensemble size (and spread!)
 - ➔ *Note: localisation does minimise effect SMW formulation*

Different techniques:

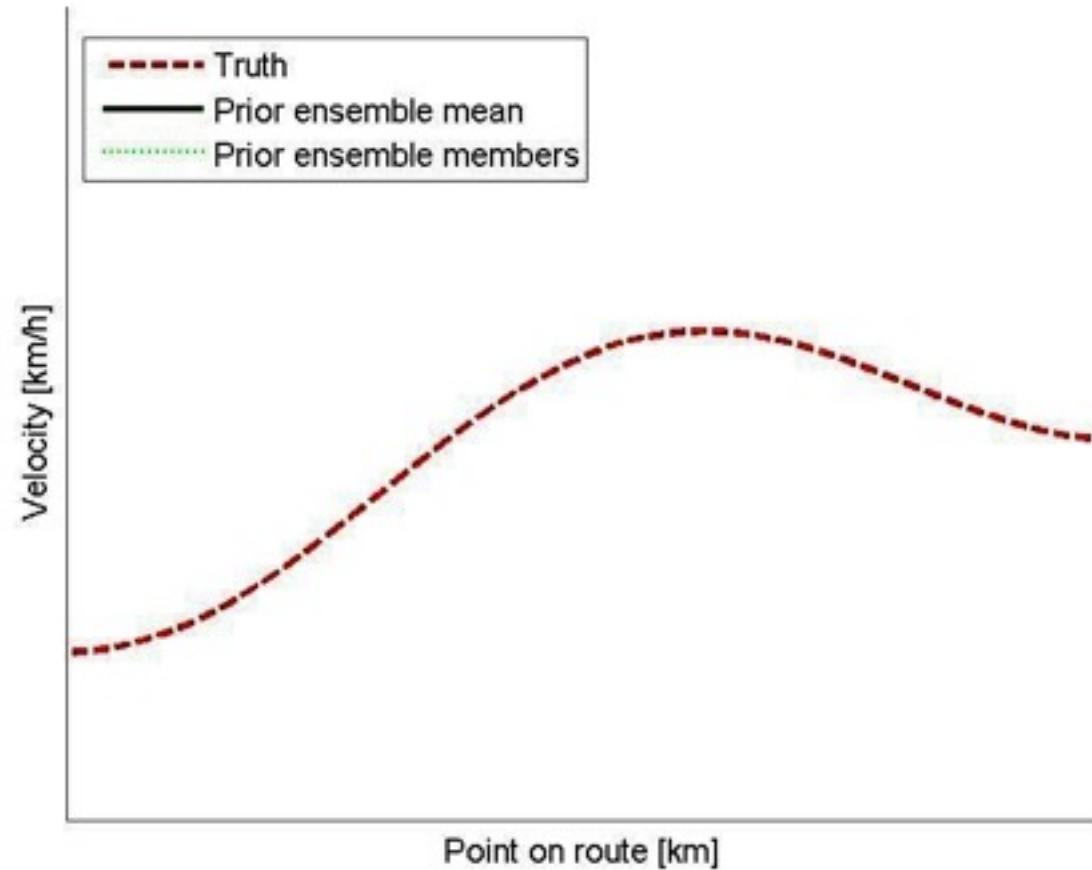
least efficient (state based)

$O(m^3n + m^2nN + mnN^2)$

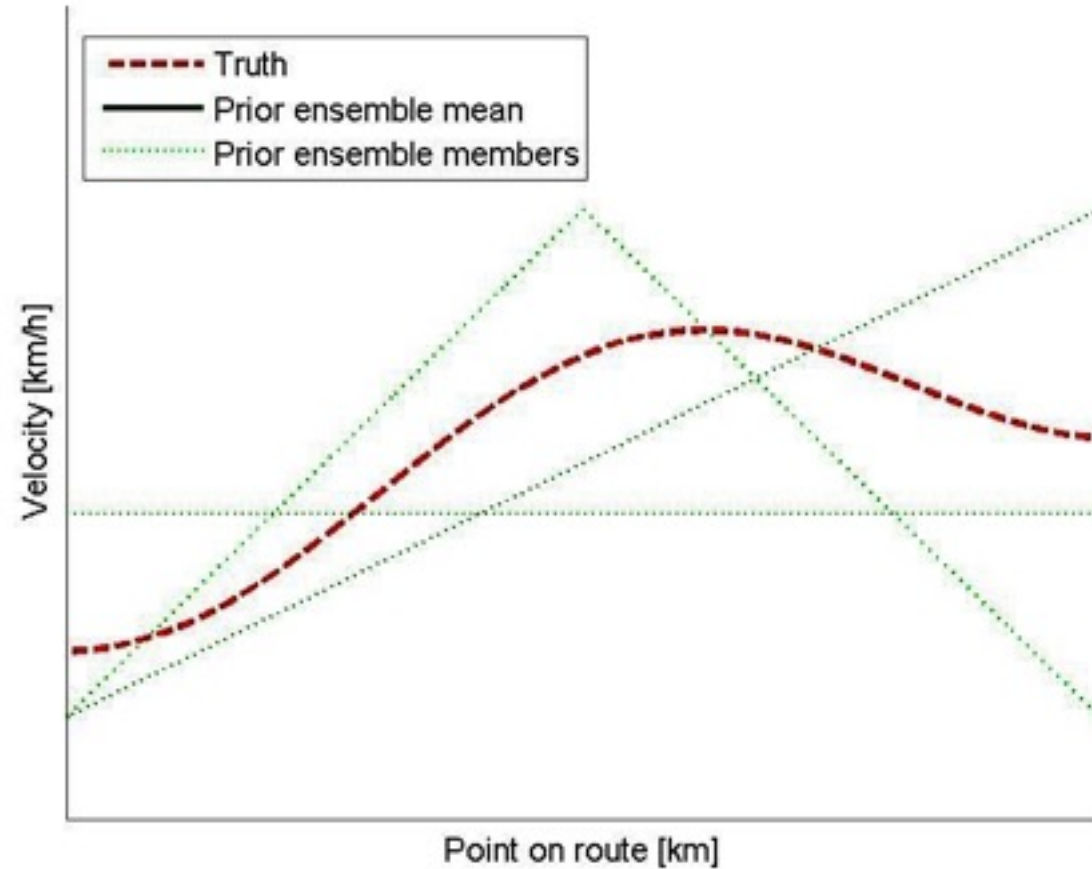
most efficient (obs based)

$O(mnN^2 + nN)$

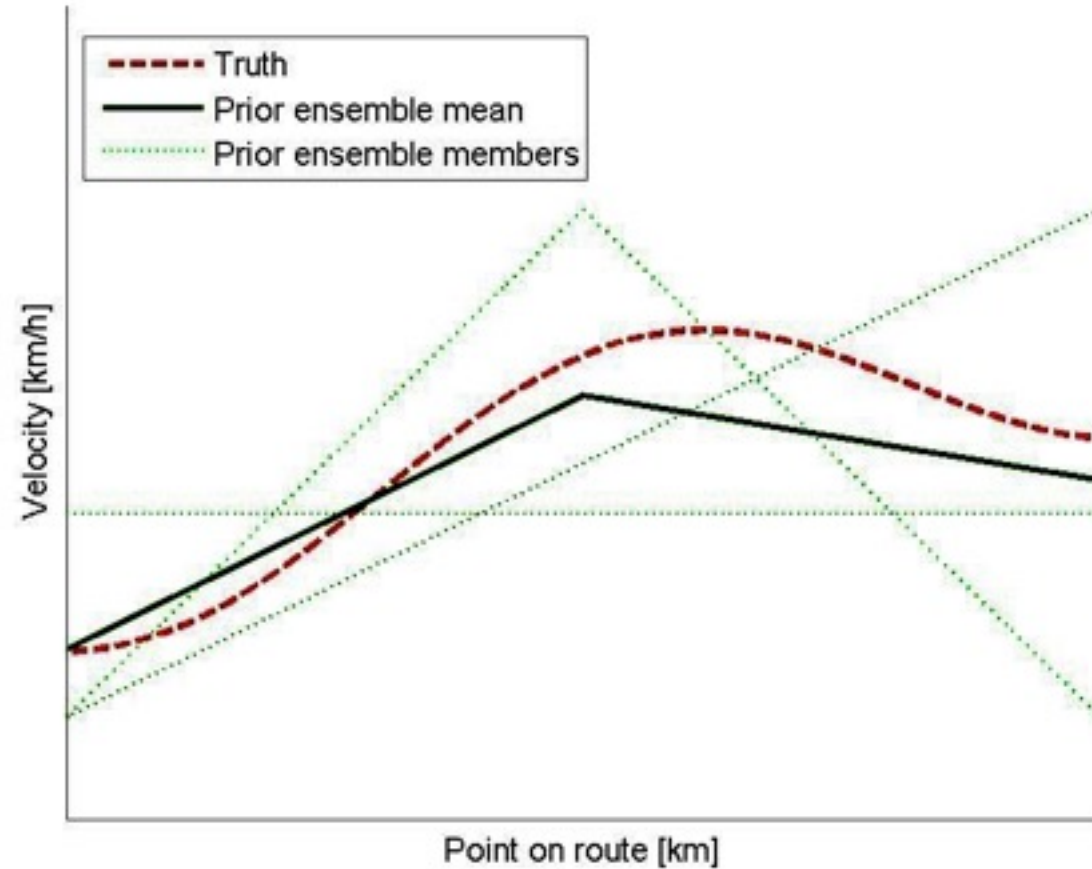
LOCALISATION INCREASES EFFECTIVE ENSEMBLE SIZE



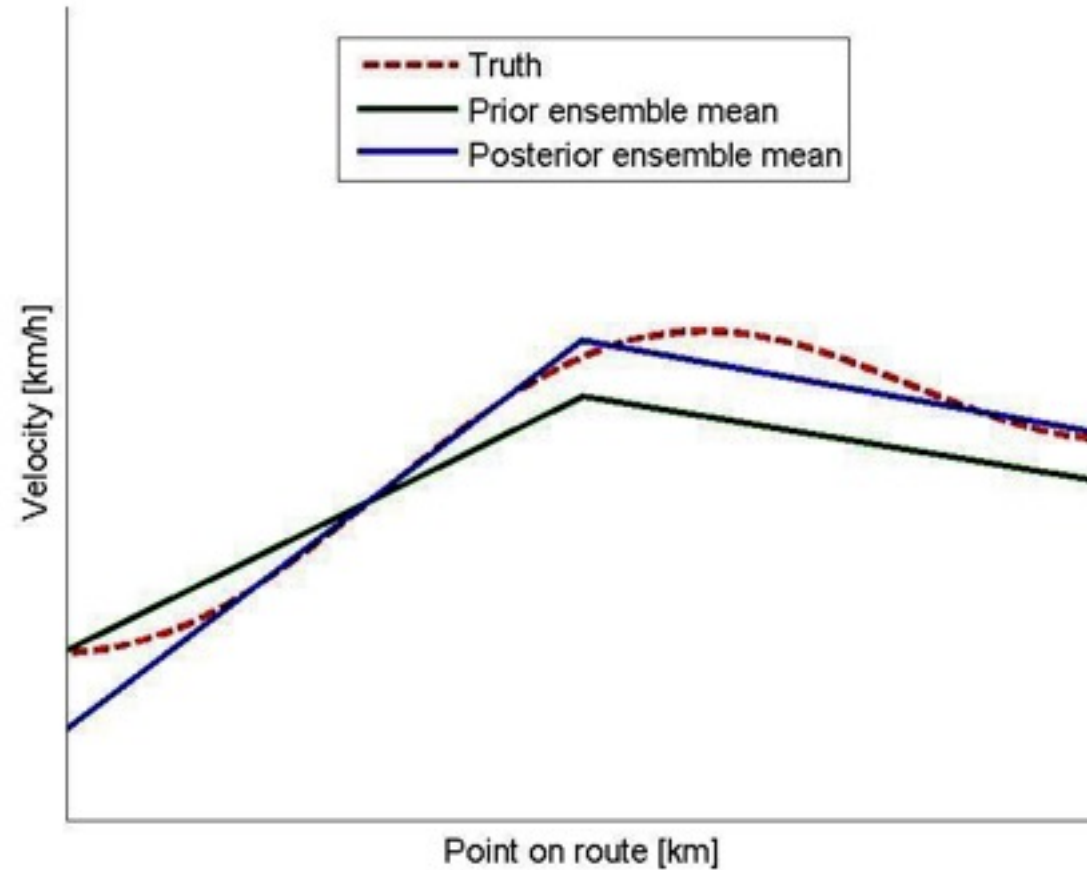
LOCALISATION INCREASES EFFECTIVE ENSEMBLE SIZE



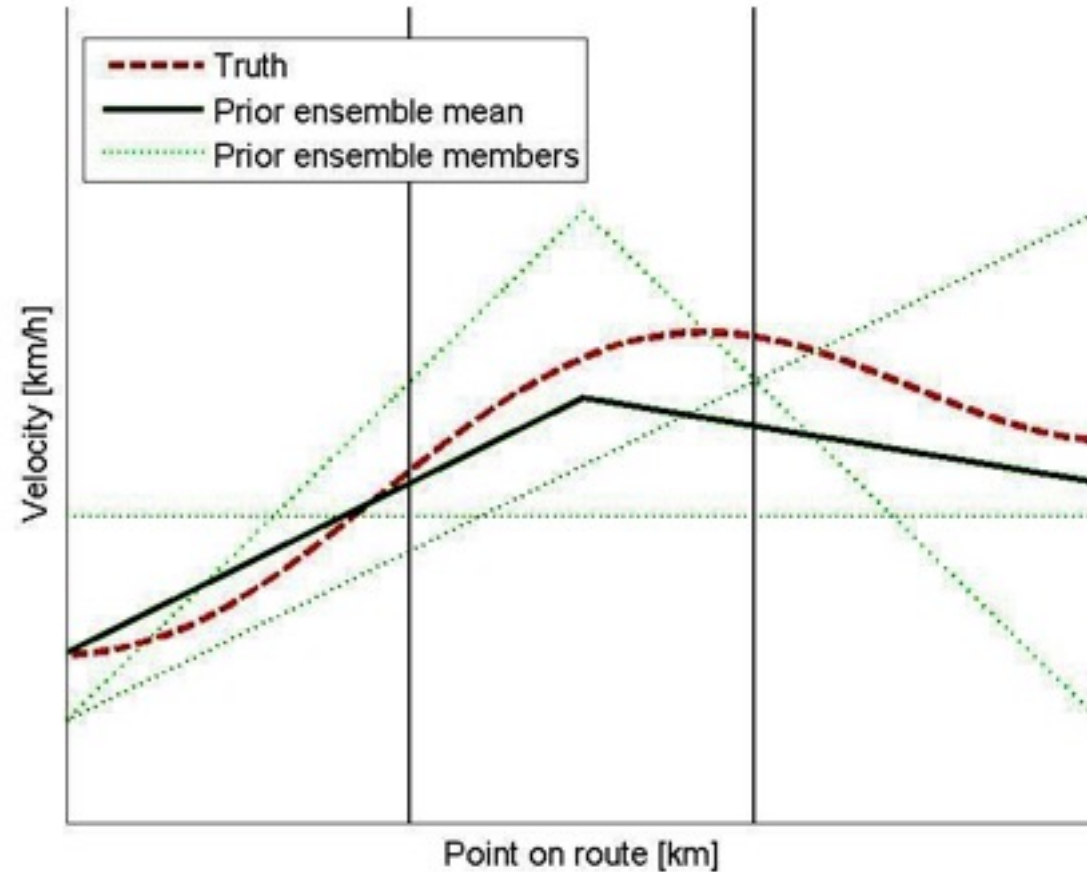
LOCALISATION INCREASES EFFECTIVE ENSEMBLE SIZE



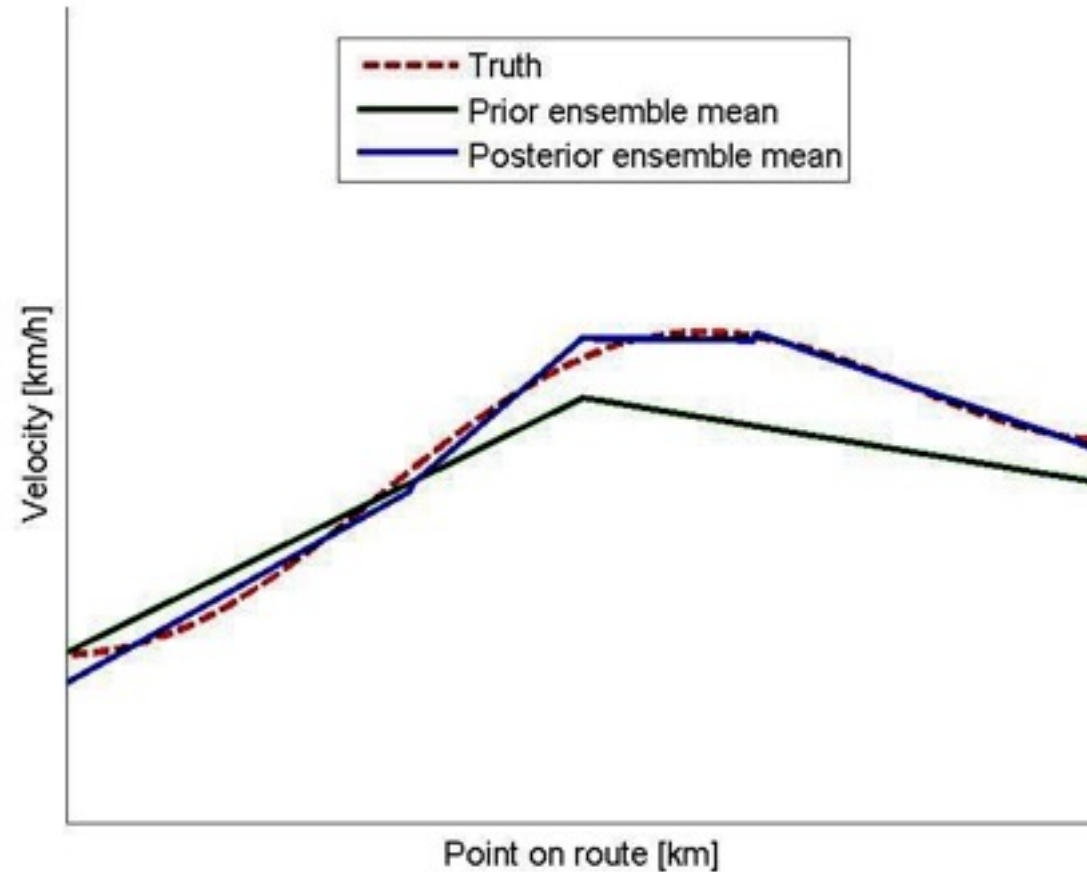
LOCALISATION INCREASES EFFECTIVE ENSEMBLE SIZE



LOCALISATION INCREASES EFFECTIVE ENSEMBLE SIZE

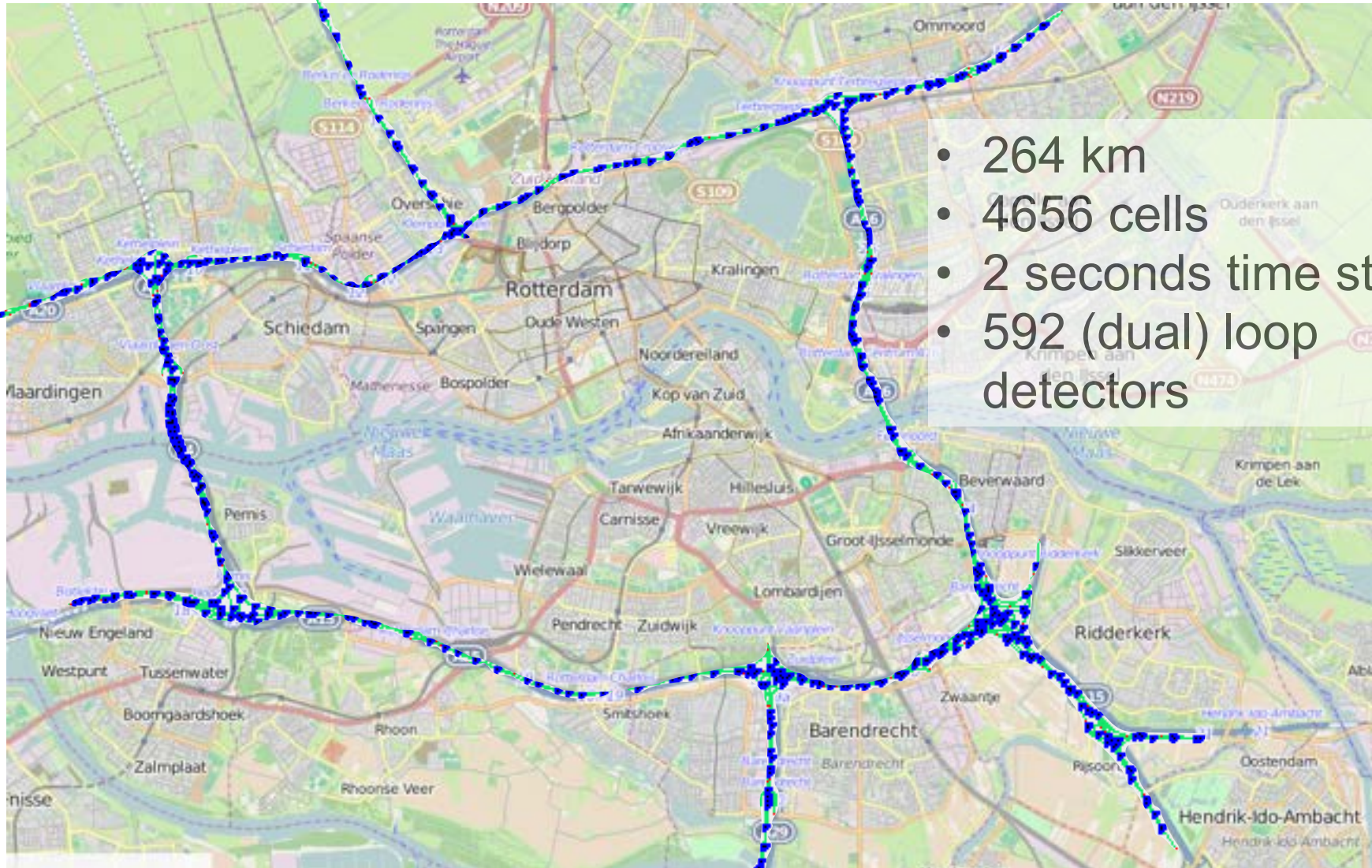


LOCALISATION INCREASES EFFECTIVE ENSEMBLE SIZE

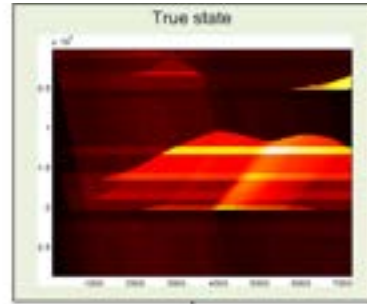


Note: Localisation may induce “jumps” in the state, which may not be realistic

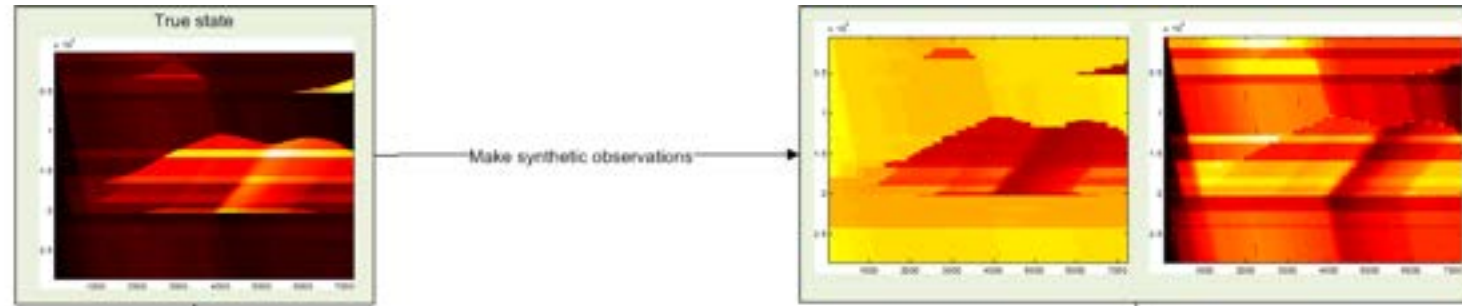
SIMULATION NETWORK



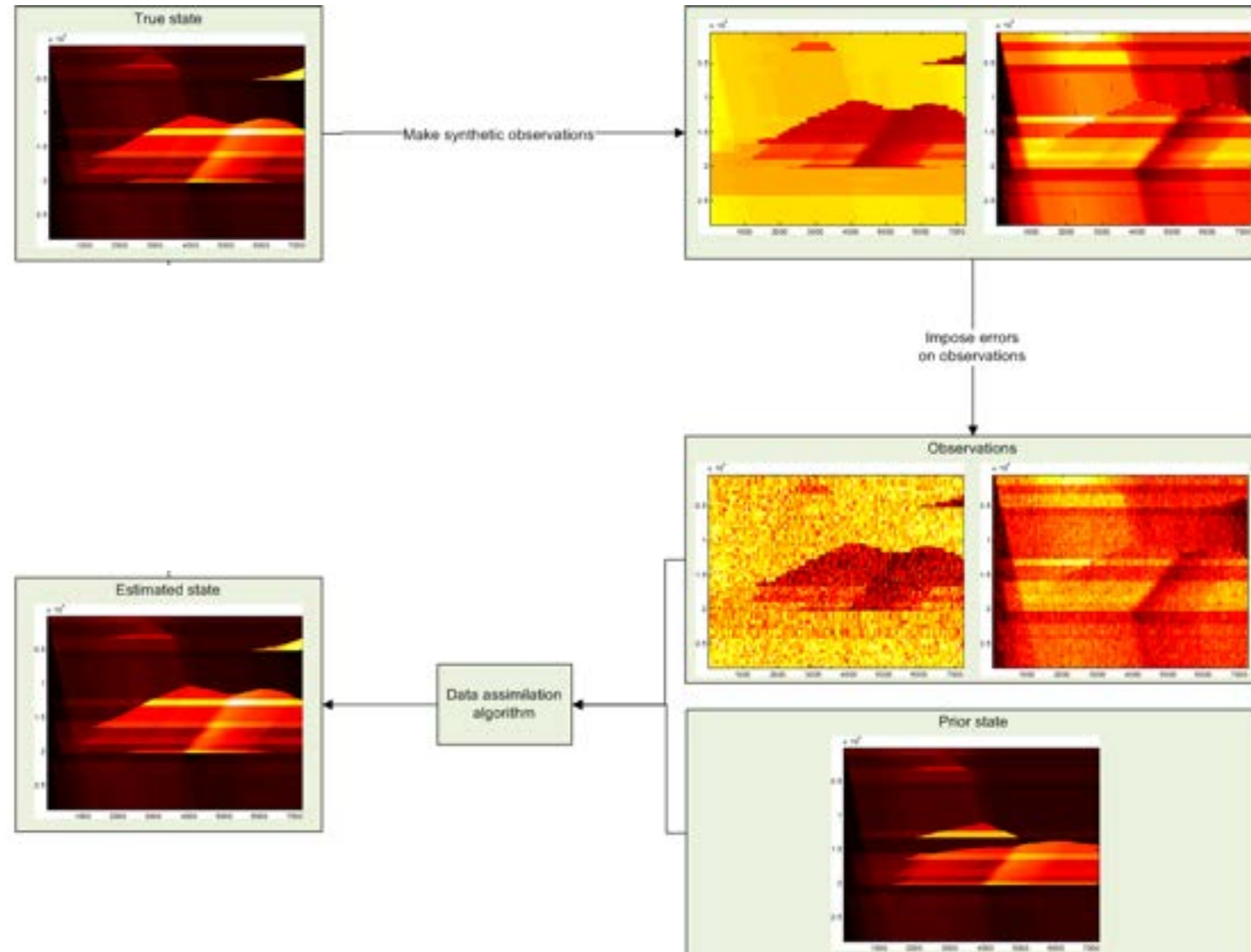
EXPERIMENTAL SETUP



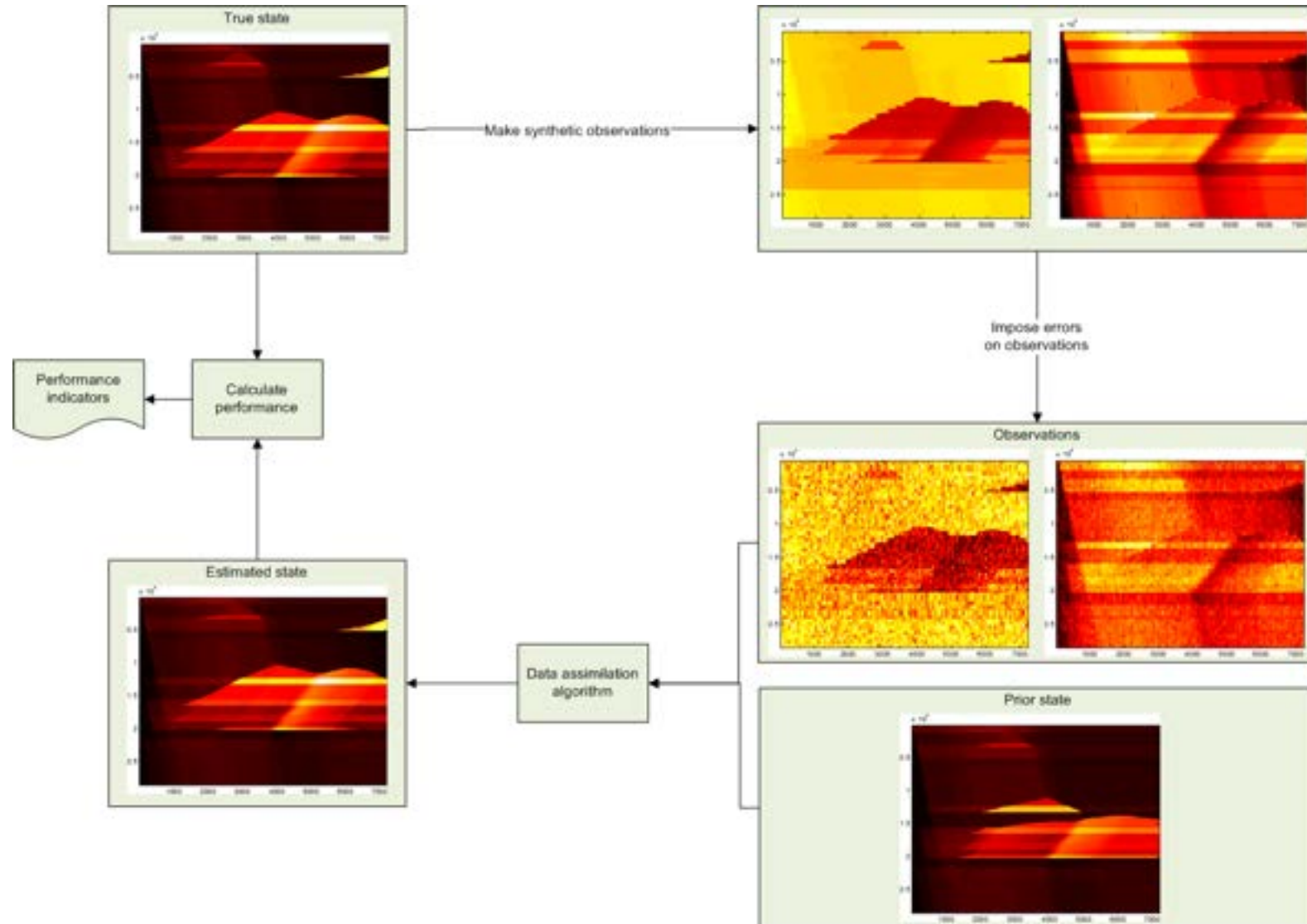
EXPERIMENTAL SETUP



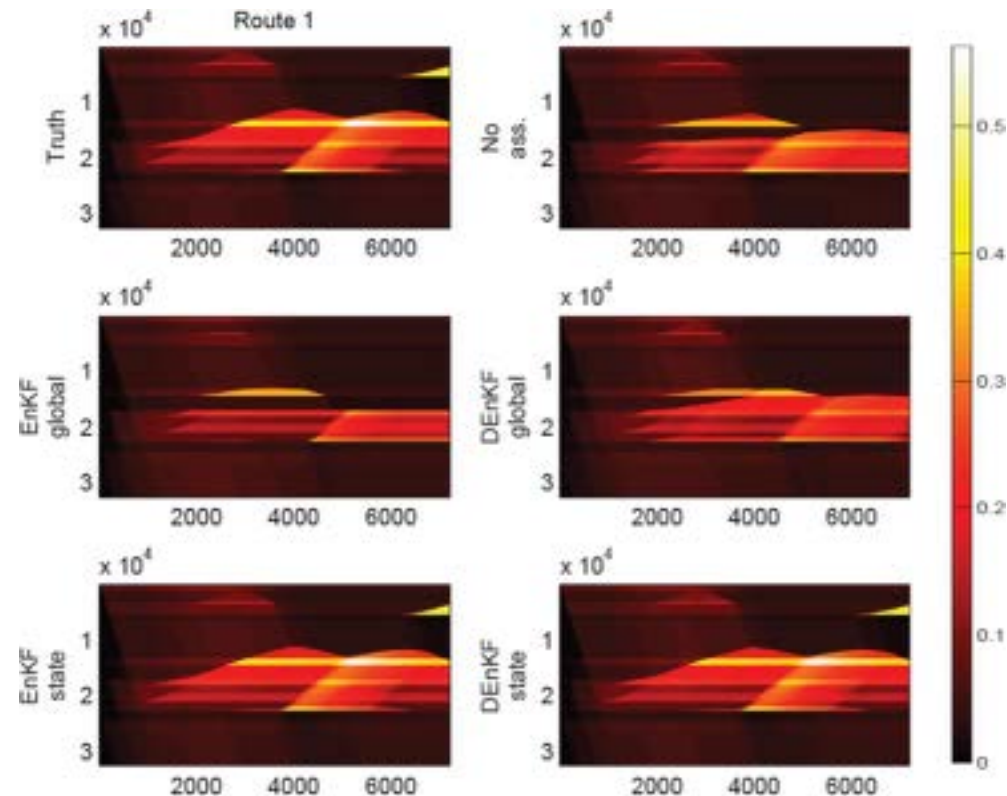
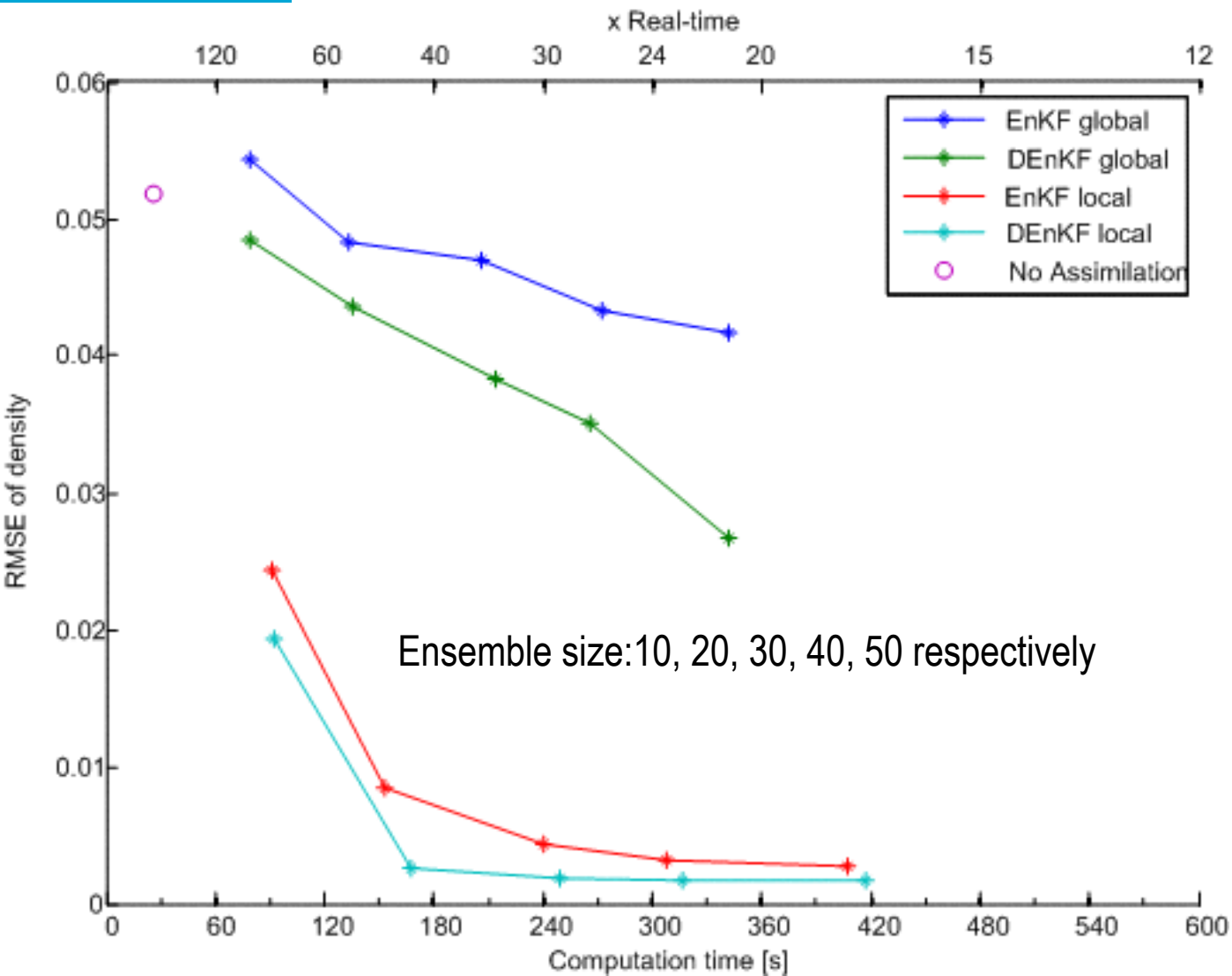
EXPERIMENTAL SETUP



EXPERIMENTAL SETUP



RESULTS

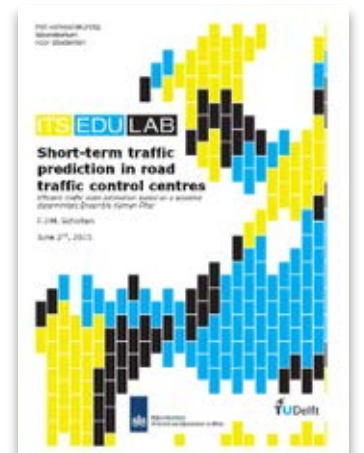


RESULTS

	Localized EKF	Localized (D)EnKF
Reference	Van Hinsbergen et al. (2012)	<i>This research</i>
Hardware	3.0 GHz dual-core, 2 GB RAM	2.6 GHz quad-core i5-3230M, 8 GB RAM
Length network	272 km	264 km
Model time step	5 s	2 s
Number of cells	1911	4656
Number of detectors in network	531	592
Number of used measurements	398	1184
Computational speed	$\approx 20\times$ real-time	$\approx 40\times$ real-time

CONCLUSIONS

- Localized DEnKF overall winner in this study ...
 - ➔ Faster than Localised EKF on similar case
 - ➔ More accurate and faster than all other EnKF alternatives
 - ➔ No need (on the contrary) for SWM reformulation
- Localization of EnKF is absolutely crucial:
 - ➔ Higher estimation accuracy (larger effective ensemble)
 - ➔ Lower computational burden
- Deterministic sampling beneficial
 - ➔ outperforms stochastic slightly
 - ➔ more robust wrt smaller ensemble sizes



NEXT STEPS

Dutch National Data Warehouse Traffic Information

- In the coming few years we are going to work on implementing and further developing these ideas on real-data
- Many additional puzzles:
 - ➔ Network graph updating
 - ➔ Parameter estimation (dual filtering?)
 - ➔ Inflows / turns / ODs / Paths
 - ➔ Urban / provincial (different data, different state variables)

