The University Of Sheffield.

Sequential Monte Carlo Methods for Tracking and Inference with Applications to Intelligent Transportation Systems

Dr Lyudmila Mihaylova

Department of Automatic Control and Systems Engineering

University of Sheffield, United Kingdom

Email: L.S.Mihaylova@sheffield.ac.uk

Outline

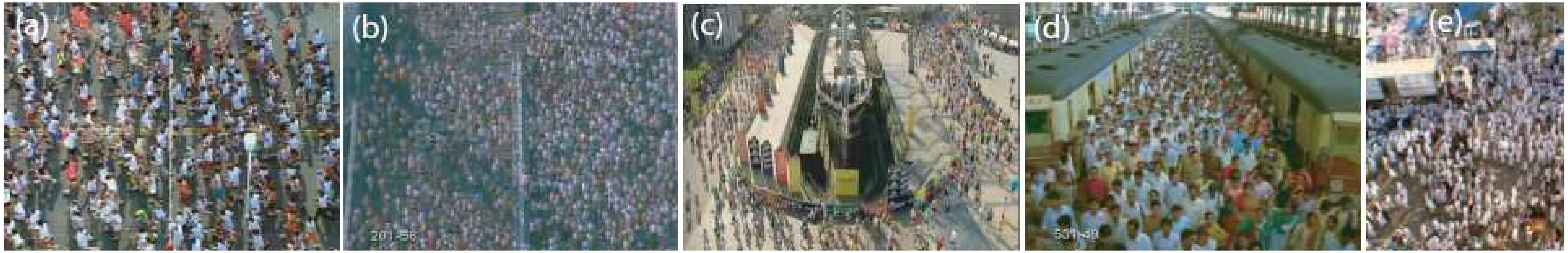
- Problems of Interest and Bayesian Formulation
- Key Related Works in the Area
- Modelling Interactions Between Pedestrians with the Social Force Model
- The Convolution Particle Filter and the Box Particle Filter for Group Tracking
- Dealing with Big Volumes of Data
  - Subsampling in Sequential Markov Chain Monte Carlo Methods
- Conclusions and Future Work

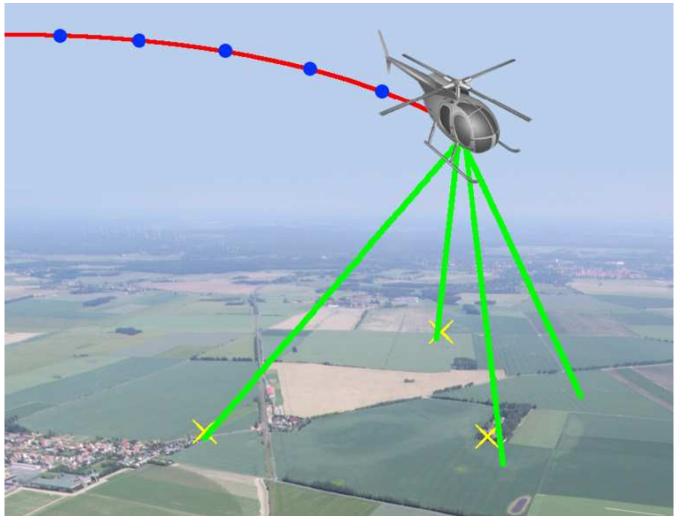
Background & Motivation

Groups and Extended Objects

- **Groups**: are structured objects, formations of entities moving in a coordinated fashion, whose number vary in time
- Groups can split, merge, can be near to each other, can spawn new groups.
- Group formations maintain a certain pattern of motion

Small versus Large Groups



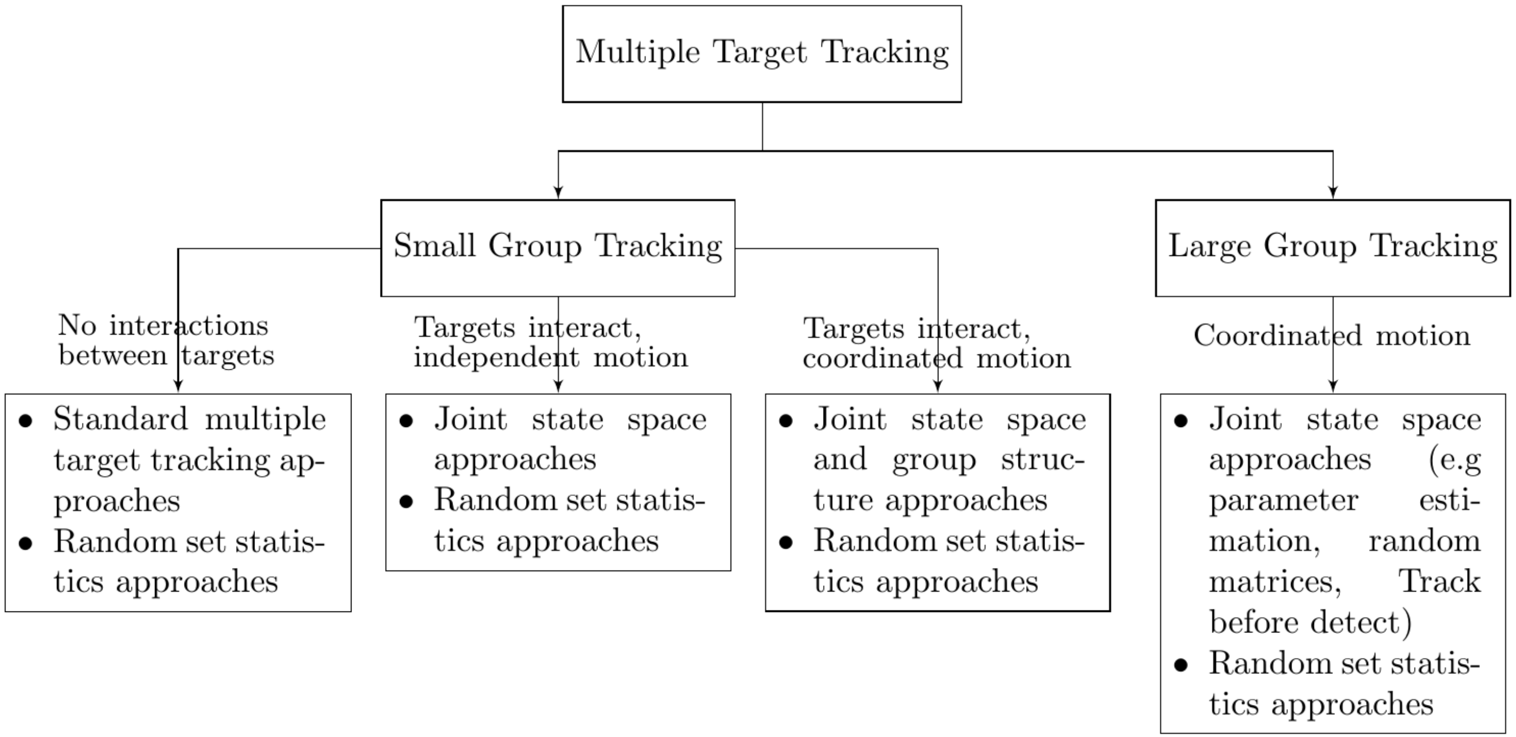


- Small groups: typically with up to 20 components
- Big groups: with 100s, 1000s of elements

- Examples of high density crowd scenes. (a)-(c) Hundreds of people participating in marathons. (d) A scene from a densely packed railway station in India. (e) A group of people moving in opposite directions.
- S. Ali, K. Nishino, D. Manocha, W. Shah (Eds.), Modelling, Simulation and Visual Analysis of Crowds, Springer, 2013.

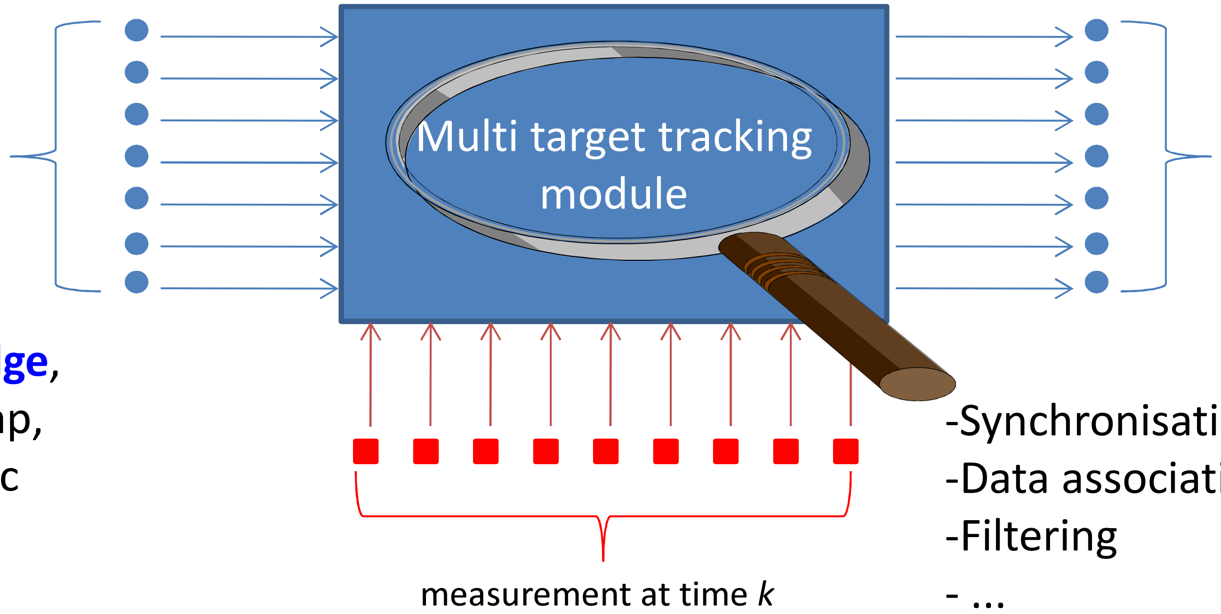
- With aggregated traffic models, e.g. :
- Wang et al. 2006,
- Work et al. 2008, 2010,
- C. De Wit, 2007

Literature Overview & Objectives



- Estimation methods robust to:
  - A large number of objects
  - A large number of measurements
  - High-dimensional systems

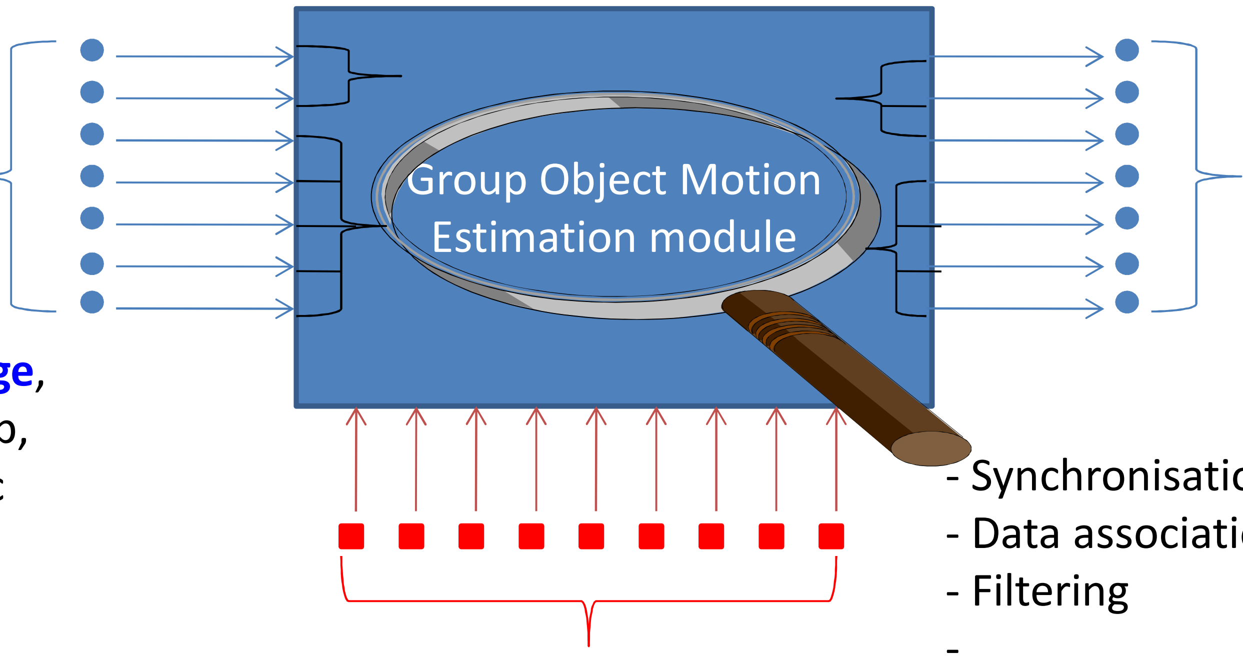
Multi Object Tracking



Prior knowledge, including a map, constraints, etc

-Synchronisation
-Data association
-Filtering
- ...

Group Object Motion Estimation



Prior knowledge, including a map, constraints, etc

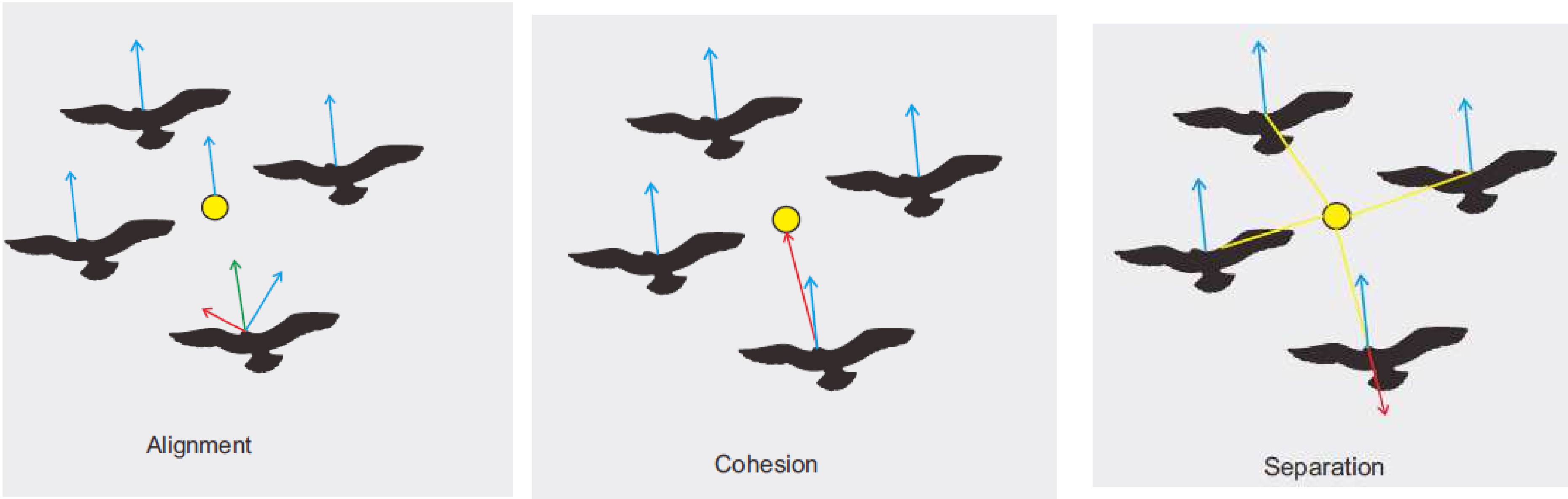
- Synchronisation
- Data association
- Filtering
- ...

+

- Group object structure
- Group motion modelling,
- Modelling the interactions within groups and between groups

How to model the group structure?
How to update the group structure?
How to take into account the uncertainty on the group structure?

Characteristics of Group Behaviour



- **Alignment**: align towards the **average heading** of local flockmates
- **Cohesion**: steer to move toward the **average position** of local flockmates
- **Separation**: steer to avoid crowding local flockmates

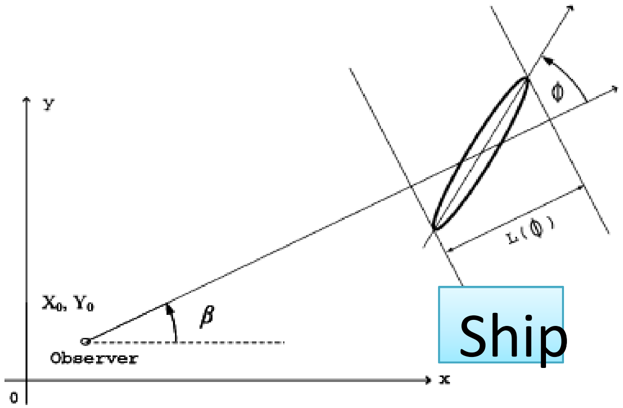
http://www.red3d.com/cwr/boids/

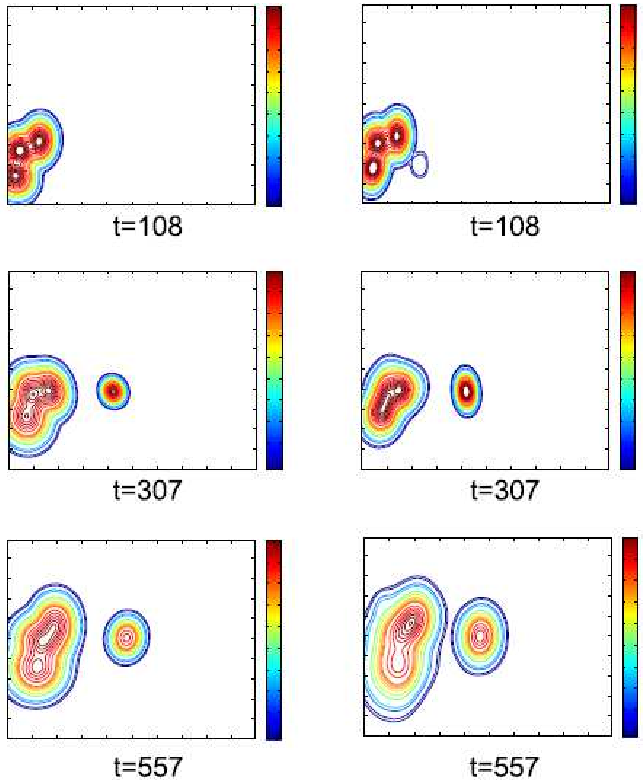
Models of Groups

- Leader – follower
- Models inspired from attraction and repulsion forces (Pang, Li, Godsill et al, 2011)
- Mechanical spring
- Social force models (based on ideas from Newtonian laws)

- D. Helbing and P. Molnar, Social Force Model for Pedestrian Dynamics, *Physical Review E*, vol. 51, pp. 4282–4286, 1995.
- R. Mazzon and A. Cavallaro, “Multi-camera tracking using a multi-goal social force model,” *Neurocomputing*, vol. 100, pp. 41–50, 2013.

Extended Objects





True average concentration of the radio active cloud is shown in left panel, estimated (right), Septier, Godsill, Carmi, Fusion, 2009, LIDAR data

- **Extended objects:** objects that are not considered as points, but instead have a spatial extent characterising their size and volume
- Modelled usually with geometric shapes such as circles, ellipses, rectangles or in 3D volumes, e.g. spheres, ellipsoides
- Similarly to groups, extended objects give rise to **multiple measurements**

Methods for Groups Tracking

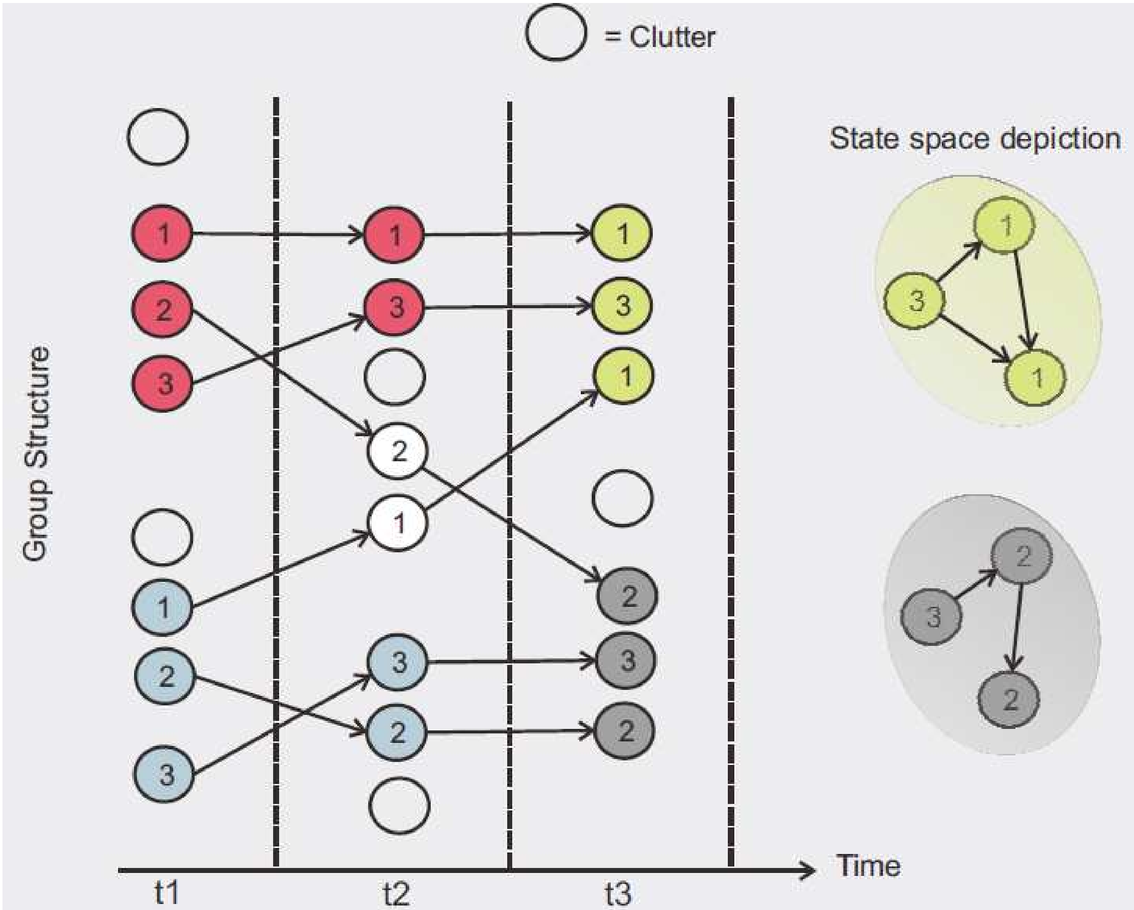
* The **standard methods** do not consider the interactions between the objects and track each object individually. One algorithm per object.

- **Small groups of objects**
  - Estimate the state vector of each object individually
  - Take into account the communications between objects within the group
  - Estimate the structure of the group and parameters (e.g. size) of the group
  - Many measurements, splitting, merging, new groups birth

- **Large groups of objects**
  - Track the centre of the extended object/ cluster and its extent
  - Individual objects difficult to identify/estimate
  - The sensor data typically do not allow to track each object individually
  - Many measurements, splitting, merging of groups, new groups birth

- L. Mihaylova, A. Carmi, F. Septier, A. Gning, S.K. Pang, S. Godsill, Overview of Sequential Bayesian Monte Carlo Methods for Group and Extended Object Tracking, *Digital Signal Processing*, February, 2014, Vol. 25, pp. 1-16.

More on the Challenges



Dynamics of a small group structure over time

- Splitting takes place from time t_1 to t_2, resulting with a new group (white).
- At time t_2, the white group splits again and merges with the red and blue groups, giving rise to entirely new (green and grey) configurations. The arrows represent the entities movement direction.

Bayesian Methodology

$$\Pr(X \mid D) = \frac{\Pr(D \mid X)\Pr(X)}{\Pr(D)}$$

Bayes Rule

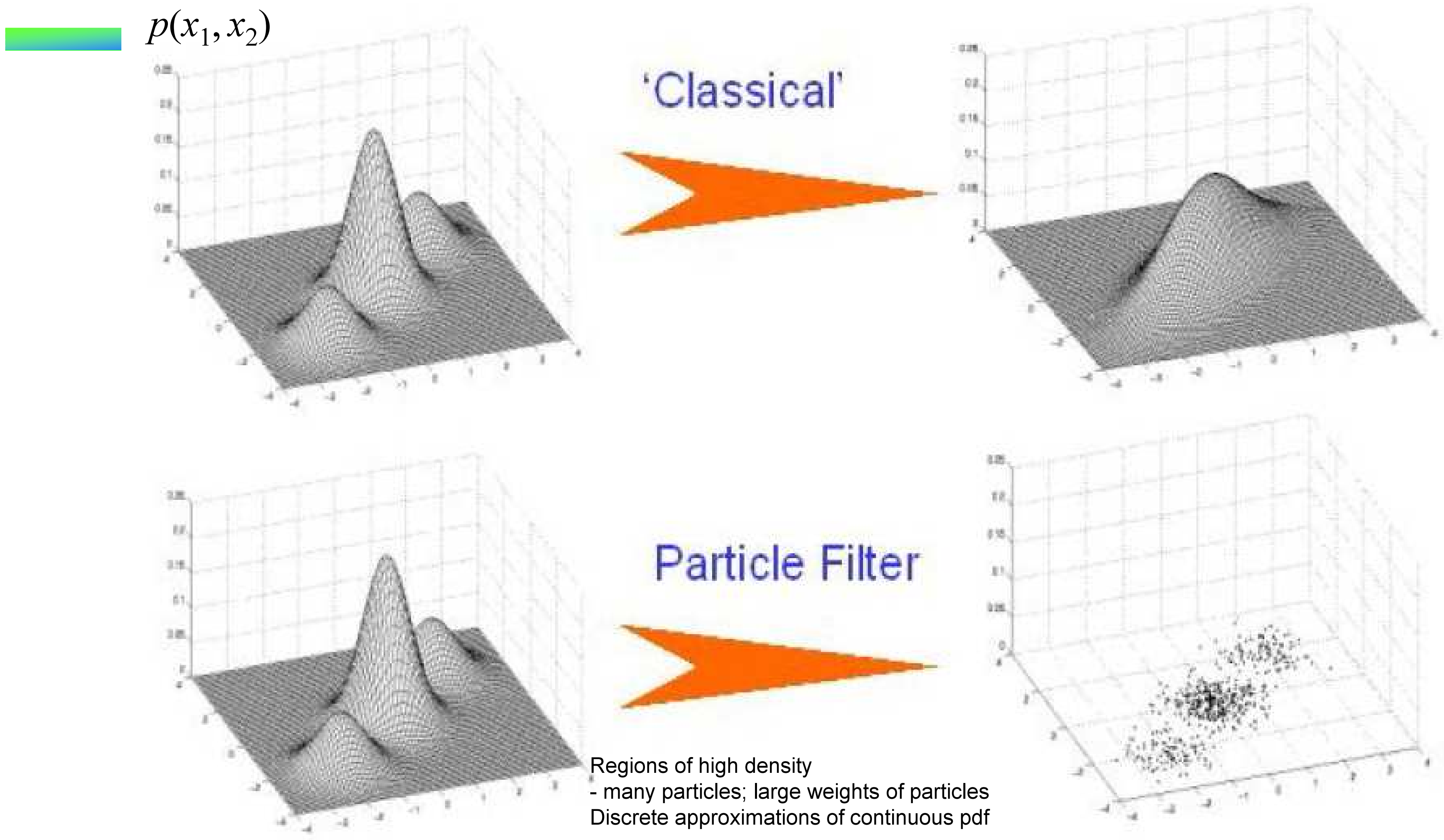
- X: state, D: data
- Prediction
- Correction step

Posterior PDF = likelihood * prior / evidence

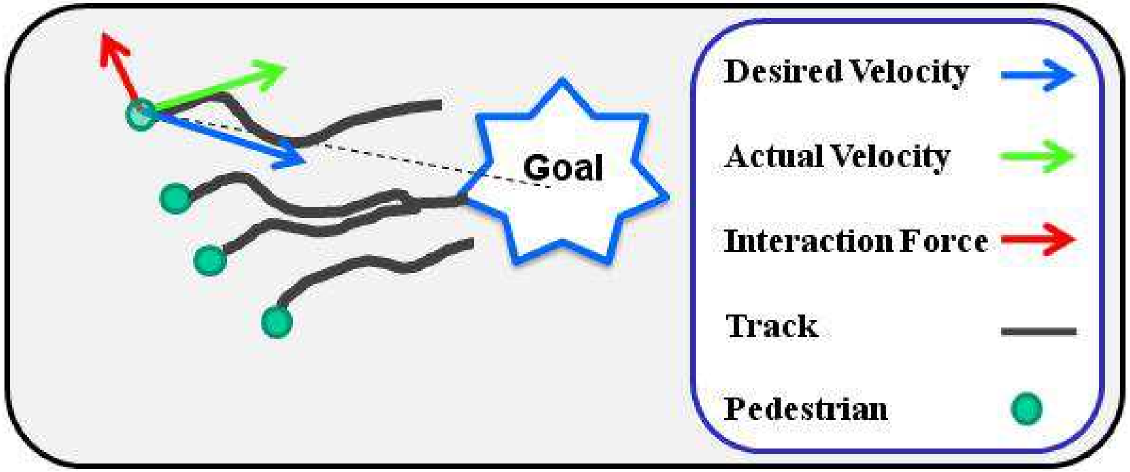
Bayesian Framework for Nonlinear Estimation Problems

- System model
$$x_k = f(x_{k-1}, v_{k-1}),$$
$$z_k = h(x_k, n_k),$$
- The posterior state probability density function (PDF) is estimated given the data
$$z_{1:k} \triangleq \{z_1, \ldots, z_k\}$$
- The sensor information updates recursively the state distribution.
- **Prediction**:
$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1})dx_{k-1}$$
- **Update**:
$$p(x_k|z_k) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})}$$

Sequential Monte Carlo Methods



Multiple People Tracking with Social Force Model and a Particle Filter



- Model originated from the Newton's laws
- Repulsive and attraction forces
- Given a pedestrian i with position coordinates ${p_k}^i$, the set of all pedestrians that influence pedestrian i at time k are defined as N_i.
- The set of static obstacles for i is defined as W_i.

A Social Force Model

- The overall force applied to a pedestrian is represented as a sum of driving and repulsive forces

$$F_i = F_i^{drv} + \sum_{j \in N_i} F_i^{rep} + \sum_{w \in W_i} F_{iw}^{rep}$$

- The state evolution of the pedestrian at time k can be described as

$$\begin{bmatrix} \mathbf{p}_k(\varphi_i) \\ \mathbf{v}_k(\varphi_i) \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{k-1}(\varphi_i) + \mathbf{v}_{k-1}\Delta t + \frac{1}{2}\frac{\mathcal{F}(\varphi_i)}{m}\Delta t^2 \\ \mathbf{v}_{k-1}(\varphi_i) + \frac{\mathcal{F}(\varphi_i)}{m}\Delta t \end{bmatrix} + \xi_k$$

- **p**: position vector, **v**: velocity vector, φ_i – interaction mode, m – average mass of the pedestrian, $\mathcal{F}(\varphi_i)$ – social force

A Particle Filter with a Social Force Model

- Estimate links between pedestrians
- Consider three possible modes on a link: attraction, repulsion and no interaction.
- Create all possible combinations (3^L) of modes
- Predict $N_s \times 3^L$ particles at every time step according to the equation of motion
- Use a particle filter to estimate posterior distribution of the state

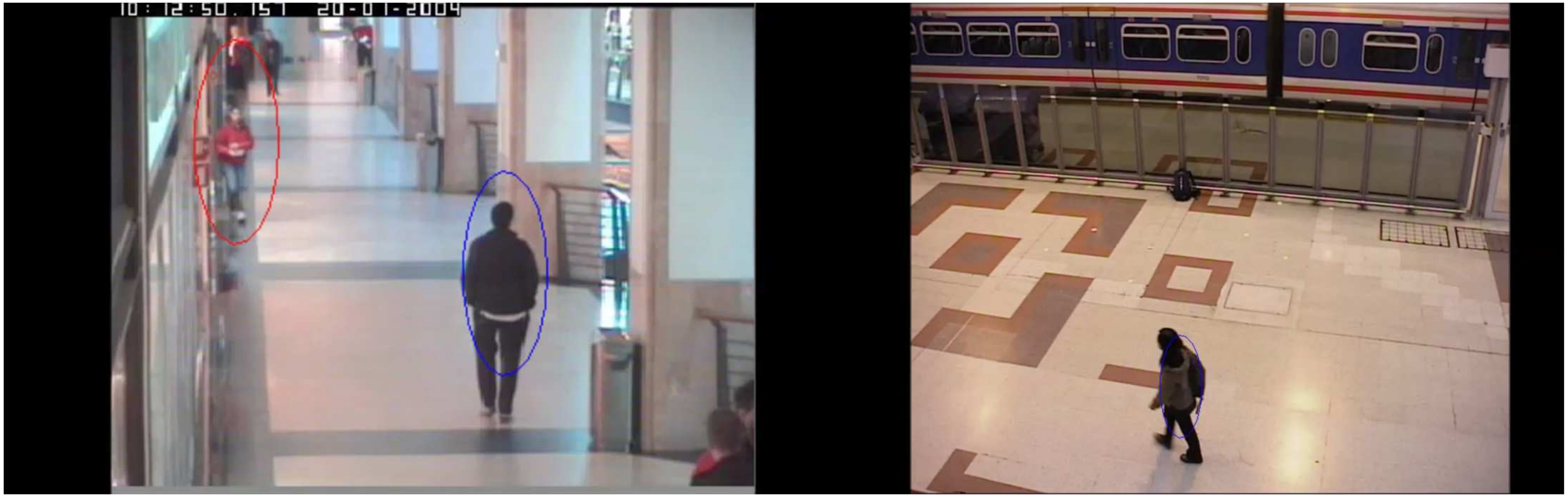
Link 1

- Attraction
- Repulsion
- No interaction

Link 2

- Attraction
- Repulsion
- No interaction

Multiple Pedestrian Tracking with a Social Force Model



A.-ur Rehman, S.M. Naqvi, L. Mihaylova, J. Chambers, Multi-target Tracking and Occlusion Handling with Learned Variational Bayesian Clusters and a Social Force Model, *IEEE Transactions on Signal Processing*, 2015, under review.

The University Of Sheffield.

THE 1ST STUDENT EXPERIENCE SURVEY 2014-15

Conjugacy

- x - state, z – data
- $p(x)$: prior distribution, $p(x|z)$: posterior distribution
- If the posterior distributions $p(x|z)$ are in the same family as the prior probability distribution, $p(x)$, the prior and posterior are called **conjugate distributions.** The prior is called a **conjugate prior** for the likelihood function.
- Examples:
  – The Gaussian family - conjugate with respect to a Gaussian likelihood function; Others: Poisson - Gamma, Binomial - Beta
- Examples of where to use them:
  – For kinematic states of extended objects and groups – modelled by Gaussian distributions (Koch, Feldbaum et al., 2008, 2009, 2011)
  – Extent of the objects – random matrix, by Inverted Wishart distribution (Koch et al.)
  – Clutter rates distribution – Gamma
  – Used in a number of filters

Overview of Related Works

- Approach with **random matrices,** Gaussian and Inverted Wishart (GIW) distributions
  - M. Feldmann, W. Koch, Road-map Assisted Convoy Track Maintenance Using Random Matrices, *Proc. of the 11th International Conf. on Information Fusion*, Germany, 2008
  - J. W. Koch, Bayesian Approach to Extended Object and Cluster Tracking Using Random Matrices, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, Issue 3, pp. 1042–1059, July 2008.
  - W. Koch and M. Feldmann, Cluster Tracking Under Kinematical Constraints Using Random Matrices, *Robotics and Autonomous Systems*, vol. 57, no. 3, pp. 296–309, 2009.
  - M. Feldmann, D. Fränken, W. Koch, Tracking of Extended Objects and Group Targets Using Random Matrices, *IEEE Trans. on Signal Processing*, Vol. 59, No. 4, pp. 1409-1420, 2011.
- Random finite sets, e.g. PHD filters, no conjugate priors: Mahler, Vo & Vo, 2007, 2010, Grandstroem, Orguner, 2012, 2014
- Cardinality PHD (CPHD) filters, no conjugacy
- PHD, CPHD filters with conjugate prior distributions
  - K. Granström and U. Orguner, A PHD filter for tracking multiple extended targets using random matrices, *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 5657–5671, 2012.Gamma Gaussian Inverted Wishart (GGIW) distribution
  - B.-T. Vo, V.-Ng Vo, Labeled Random Finite Sets and Multi-Object Conjugate Priors, *IEEE Transactions on Signal Processing*, Vol. 61, No. 13, pp. 3460–3475, 2013
- Labelled Multi-Bernoulli PHD filters, Mahler, Vo and Vo 2014, 2015
- Grandström et al., 2010, 2014, S. Reuter, Fusion 2015: Gamma, Gaussian, Inverted Wishart distributions
- Hanebeck et al, 2010, 2013: exact solutions to these problems
- X.Rong Li et al: solutions for linear systems and with multiple model approaches
- Convolution Particle Filters, Box Particle Filters and Sequential Monte Carlo filters combined with random graphs: Mihaylova and team

Extended/ Group Object Tracking within the SMC Framework

- The system dynamics is given by:

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{\eta}_{k-1})$$

where $\boldsymbol{x}_k = \left(\boldsymbol{X}_k^T, \boldsymbol{\Theta}_k^T\right)^T \in \mathbb{R}^{n_x}$ is consisting of the kinematic state vector $\boldsymbol{X}_k$ and extent parameters $\boldsymbol{\Theta}_k \in \mathbb{R}^{n_\Theta}$ and $\boldsymbol{\eta}_k$ is the system noise.

- The sensors measurement model is

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k, \boldsymbol{w}_k)$$

  - where $\boldsymbol{w}_k$ is the measurement noise.
  - in the state vector other parameters such as clutter rate, probability of detection, number of targets can be included

Convolution Particle Filters

- Convolution particle filters (CPFs): sub-family of particle filters with improved efficiency in state estimation of nonlinear dynamic systems.

- Rely on convolution kernel density estimation and regularisation of both state and observation distributions.
- Useful, when the likelihood computation is analytically intractable or it is not available in an analytical form

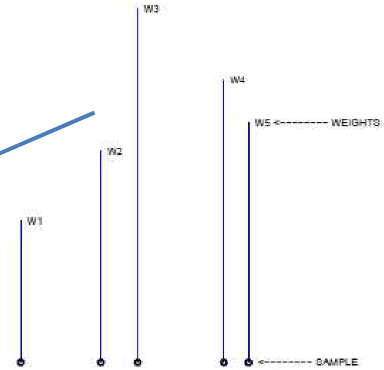
Advantages of CPFs:

- Likelihood-free types of filters

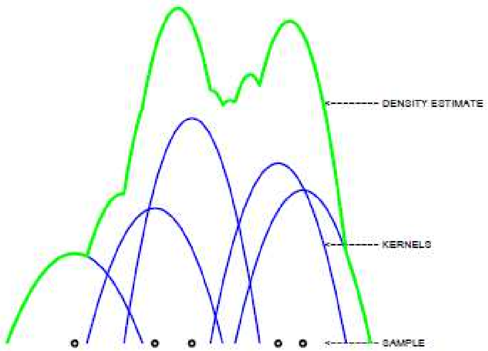
- C. Musso, N. Oudjane and F. Le Gland, Improving Regularized Particle Filters, Ch. 12 in A. Doucet, N. de Freitas and N. Gordon (Eds.), Sequential Monte Carlo Methods in Practice, New York, Springer, 2001, pp. 247-273.

Convolution PFs for Groups and Extended Objects Tracking

$$p(\mathbf{X}_k \mid \mathbf{Z}_{1:k}) \approx \sum_{i=1}^{N} w_k^{(i)} \delta(\mathbf{X}_k - \mathbf{X}_k^{(i)})$$



Weighted empirical measure



Regularized measure

$$p(X_k \mid Z_{1:k}) \approx \sum_{i=1}^{N} w_k^{(i)} K_h^X (X_k - X_k^{(i)})$$

$$K_h^X(X) = \frac{1}{h^{n_x}} K(X/h)$$

Kernel density, h > 0 the bandwidth, nx – state dimension

CPF advantages

1) Ability to deal with multiple measurements, including high level of clutter,
2) Ability to resolve data association problems, without the need to estimate clutter parameters,
3) Estimation of dynamically changing extent parameters of group/ extended objects jointly with the dynamic kinematic states.

Regularised Particle Filters

Sample from the joint state and measurement distribution: $\left\{\mathbf{X}_k^{(i)}, \mathbf{Z}_k^{(i)}, i=1,\ldots,N\right\}$

$$p(\mathbf{X}_k,\mathbf{Z}_{1:k}) \approx \frac{1}{N}\sum_{i=1}^{N}\delta(\mathbf{X}_k-\mathbf{X}_k^{(i)},\mathbf{Z}_{1:k}-\mathbf{Z}_{1:k}^{(i)})$$

Next, the kernel approximation of the joint density is obtained by a convolution of the empirical estimate with an appropriate kernels

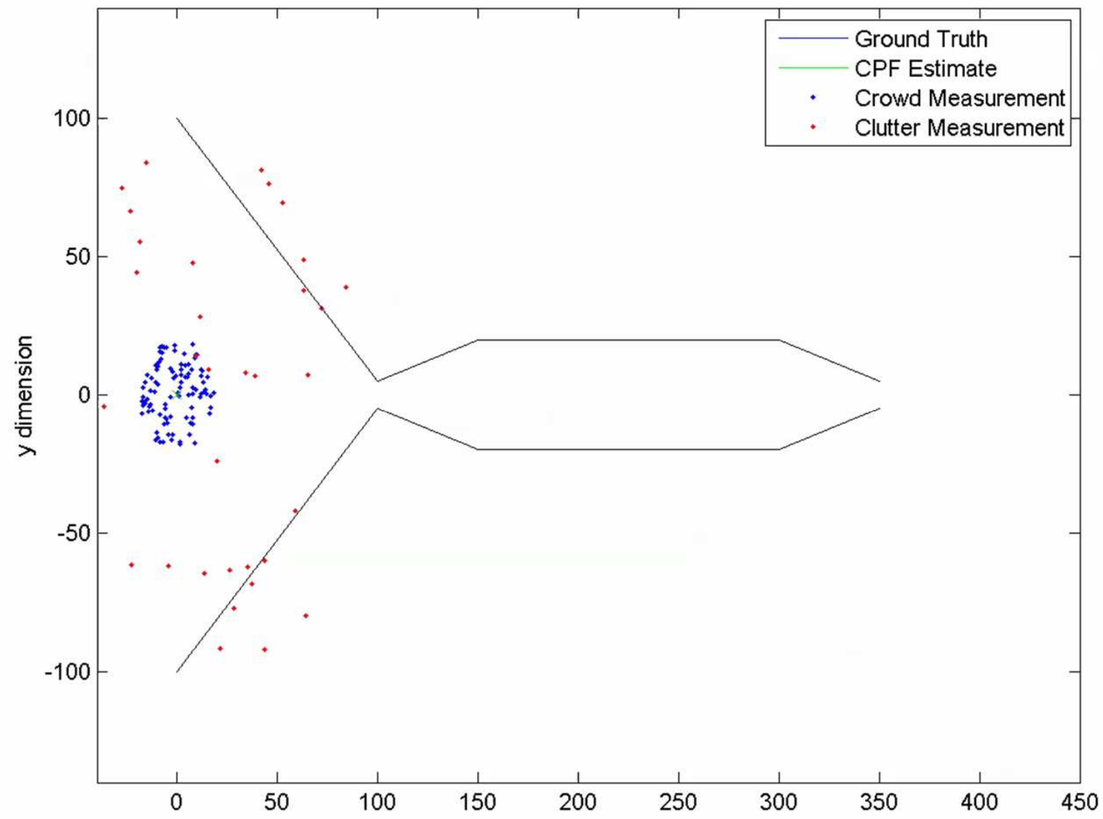
$$p_k^N(\mathbf{X}_k,\mathbf{Z}_{1:k}) = \frac{1}{N}\sum_{i=1}^{N}K_h^X(\mathbf{X}_k-\mathbf{X}_k^{(i)})K_h^{\bar{Z}}(\mathbf{Z}_{1:k}-\mathbf{Z}_{1:k}^{(i)})$$

$$K_h^{\bar{Z}}(\mathbf{Z}_{1:k}-\mathbf{Z}_{1:k}^{(i)}) = \prod_{j=1}^{k}K_h^Z(\mathbf{Z}_j-\mathbf{Z}_j^{(i)})$$

$$p_k^N(\boldsymbol{X}_k|\boldsymbol{Z}_{1:k}) = \frac{\sum_{i=1}^{N}K_h^X(\boldsymbol{X}_k-\boldsymbol{X}_k^{(i)})K_h^{\bar{Z}}(\boldsymbol{Z}_{1:k}-\boldsymbol{Z}_{1:k}^{(i)})}{\sum_{i=1}^{N}K_h^{\bar{Z}}(\boldsymbol{Z}_{1:k}-\boldsymbol{Z}_{1:k}^{(i)})}$$

- D. Angelova, L. Mihaylova, N. Petrov, A. Gning, A convolution particle filtering approach for tracking elliptical extended objects, *Proc. of the International Conf. Information Fusion*, 2013.
- A. de Freitas, N. Petrov, L. Mihaylova, A. Gning, D. Angelova, V. Kadirkamanathan, Autonomous Crowds Tracking with Box Particle Filtering and Convolution Particle Filtering, *Automatica*, 2015.

Stadium Testing Example



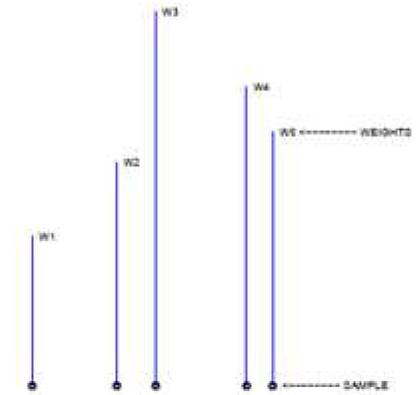
Convolution PF, N = 1000

Sequential Monte Carlo Framework with Boxes

Traditional Particles vs. Boxes

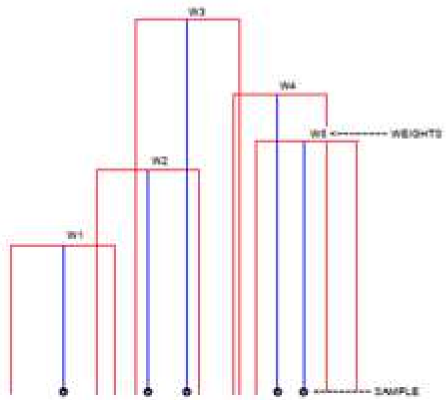
- Prediction: $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}) = \int_{\mathbb{R}^{n_x}} p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})d\boldsymbol{x}_{k-1}$
- Update: $p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k}) = \dfrac{p(\boldsymbol{z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1})}{p(\boldsymbol{z}_k|\boldsymbol{z}_{1:k-1})}$

$\mathrm{p}(x_k|z_{1:k}) \approx$



Weighted empirical measure

$$\sum_{p=1}^{N} w^{(p)} \delta^{(p)}(x_k)$$



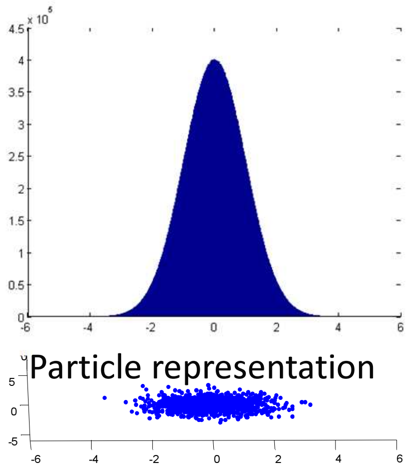
Box measure

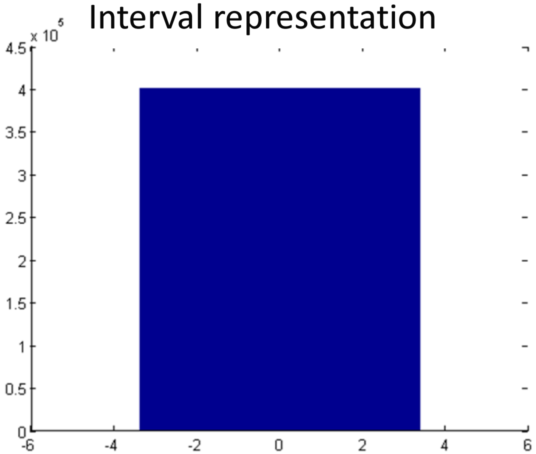
$$\sum_{p=1}^{N} w^{(p)} U_{[x^{(p)}]}(x_k)$$

Recursive Bayesian Estimation

Drawbacks of the generic Particle Filter

- A significant number of particles are required for exploring the space of interest.
- Sensitivity to different types of errors in the measurements.





System Model for Group Object Tracking

- The system state model

$$[\boldsymbol{x}_k] = [f]([\boldsymbol{x}_{k-1}], [\boldsymbol{\eta}_k])$$
$$= [f]([\underline{\boldsymbol{x}}_{k-1}, \overline{\boldsymbol{x}}_{k-1}], [\underline{\boldsymbol{\eta}}_k, \overline{\boldsymbol{\eta}}_k]),$$
$$\boldsymbol{z}_k = h(\boldsymbol{x}_k, \boldsymbol{w}_k)$$

- The system state model: with kinematic state and extent parameters

$$[\boldsymbol{x}_k] = \left([\boldsymbol{X}_k]^T, [\boldsymbol{\Theta}_k]^T\right)^T \subset \mathbb{R}^{n_x}$$
$$[\boldsymbol{X}_k] \subset \mathbb{R}^{n_X}, \quad [\boldsymbol{\Theta}_k] \subset \mathbb{R}^{n_\Theta}$$
$$[\boldsymbol{\eta}_k] = ([\boldsymbol{\eta}_{s,k}]^T, [\boldsymbol{\eta}_{p,k}]^T)^T$$

- Interval measurements $[\boldsymbol{z}_k] = [\underline{\boldsymbol{z}}_k, \overline{\boldsymbol{z}}_k] \subset \mathbb{R}^{n_z}$

Box Particle Filter: Prediction

- Posterior Distribution at the previous time step:

$$p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1}) \approx \sum_{p=1}^{N} w_{k-1}^{p} U_{[\boldsymbol{x}_{k-1}^{(p)}]}(\boldsymbol{x}_{k-1})$$

- Predicted Posterior Distribution:

$$\begin{aligned} p(\boldsymbol{x}_k|\boldsymbol{z}_{1:k-1}) &\approx \sum_{p=1}^{N} w_{k-1}^{(p)} U_{[f]([\boldsymbol{x}_{k-1}^{(p)}])+[\boldsymbol{\eta}_k]}(\boldsymbol{x}_k) \\ &= \sum_{p=1}^{N} w_{k-1}^{(p)} U_{[\boldsymbol{x}_{k|k-1}^{(p)}]}(\boldsymbol{x}_k). \end{aligned}$$

- A. Gning, L. Mihaylova, F. Abdallah, B. Ristic, Particle Filtering Combined with Interval Methods for Tracking Applications, *Integrated Tracking, Classification, and Sensor Management: Theory and Applications*, Eds. M. Mallick, V. Krishnamurthy, B.-N. Vo, John Wiley & Sons, 2011.
- A. Gning, B. Ristic, L. Mihaylova, A. Fahed, Introduction to the Box Particle Filtering, *IEEE Signal Processing Magazine*, Vol. 30, No. 4, pp. 166 - 171, July, 2013.
- Matlab Central code for the Box Particle Filter and Bernoulli Box Particle Filter, http://www.mathworks.com/matlabcentral/fileexchange/43012-box-particlefilter-and-bernoulli-box-particle-filter, 2013.

Box Particle Filter: Update

- Generalised Likelihood of the SIR PF: uniform pdf (clutter)*likelihood of the objects' originated measurements

$$p(\boldsymbol{Z}_k|\boldsymbol{x}_k) = \prod_{m=1}^{M_k}\left(1 + \frac{\lambda_T}{\rho}p(\boldsymbol{z}_k^m|\boldsymbol{x}_k)\right)$$

$$\approx \prod_{m=1}^{M_k}\left(1 + \frac{\lambda_T}{\rho}U_{r(\boldsymbol{x}_k)}(\boldsymbol{z}_k^m)\right)$$

K. Gilholm, D. Salmond, Spatial distribution model for tracking extended objects, *IEE Proc. Radar, Sonar Navig.*, Vol. 152, No. 5, pp. 364 - 371, 2005.

- Updated Posterior Distribution in the Box PF:

$$p(\boldsymbol{x}_k|\boldsymbol{Z}_{1:k}) = \frac{1}{\alpha_k}p(\boldsymbol{Z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{Z}_{1:k-1})$$

$$= \frac{1}{\alpha_k}\sum_{p=1}^{N} w_{k-1}^{(p)} \prod_{m=1}^{M_k}\left(U_{[\boldsymbol{x}_{k|k-1}^{(p)}]}(\boldsymbol{x}_k) + \frac{\lambda_T}{\rho}\boxed{U_{[\boldsymbol{x}_{k|k-1}^{(p)}]}(\boldsymbol{x}_k)U_{r(\boldsymbol{x}_k)}(\boldsymbol{z}_k^m)}\right)$$

↓ Constraint Satisfaction Problem

Box Particle Filter: Constraints Propagation

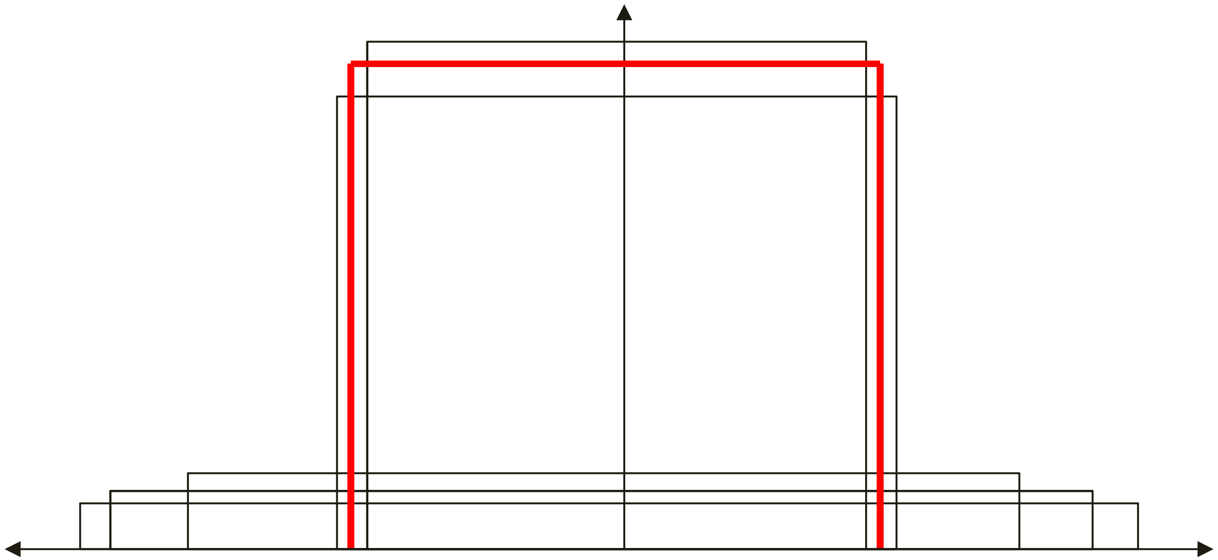
$$
\begin{aligned}
[\tilde{x}_m^{(p)}] &= [x^{(p)}] \cap \left([z_1^m] \mp \frac{[a^{(p)}]}{2} \cdot [0,1] \right), \\
[\tilde{\dot{x}}_m^{(p)}] &= [\dot{x}^{(p)}] \cap \left(\frac{[\tilde{x}_m^{(p)}(k)] - [\tilde{x}_m^{(p)}(k-1)]}{\frac{1}{\alpha_x}(1 - e^{-\alpha_x T_s})} \right), \\
[\tilde{y}_m^{(p)}] &= [y^{(p)}] \cap \left([z_2^m] \mp \frac{[b^{(p)}]}{2} \cdot [0,1] \right), \\
[\tilde{\dot{y}}_m^{(p)}] &= [\dot{y}^{(p)}] \cap \left(\frac{[\tilde{y}_m^{(p)}(k)] - [\tilde{y}_m^{(p)}(k-1)]}{\frac{1}{\alpha_y}(1 - e^{-\alpha_y T_s})} \right), \\
[\tilde{a}_m^{(p)}] &= [a^{(p)}] \cap \pm 2 \left(\frac{[z_1^m] - [\tilde{x}_{m,s}^{(p)}]}{[0,1]} \right), \\
[\tilde{b}_m^{(p)}] &= [b^{(p)}] \cap \pm 2 \left(\frac{[z_2^m] - [\tilde{y}_{m,s}^{(p)}]}{[0,1]} \right), \\
[\tilde{z}_1^{m,(p)}] &= [z_1^m] \cap \left([\tilde{x}_m^{(p)}] \pm \frac{[\tilde{a}_m^{(p)}]}{2} \cdot [0,1] \right), \\
[\tilde{z}_2^{m,(p)}] &= [z_2^m] \cap \left([\tilde{y}_m^{(p)}] \pm \frac{[\tilde{b}_m^{(p)}]}{2} \cdot [0,1] \right).
\end{aligned}
$$

Box Particle Filter: Update II

- Updated Posterior Distribution:

$$p(\boldsymbol{x}_k|\boldsymbol{Z}_{1:k}) = \frac{1}{\alpha_k}\sum_{p=1}^{N} w_{k-1}^{(p)}\left(\left(U_{[\boldsymbol{x}_{k|k-1}^{(p)}]}(\boldsymbol{x}_k)\right)^{M_k} + \sum_{m=1}^{M_k}\sum_{j=1}^{\binom{M_k}{m}}\left(U_{[\boldsymbol{x}_{k|k-1}^{(p)}]}(\boldsymbol{x}_k)\right)^{M_k-m}\prod_{i\in\mathcal{A}_j^m}\frac{\lambda_T}{\rho}\frac{1}{|r(\boldsymbol{x}_k)|}\frac{|[\tilde{\boldsymbol{x}}_{k,i}^{(p)}]|}{|[\boldsymbol{x}_{k|k-1}^{(p)}]|}U_{[\tilde{\boldsymbol{x}}_{k,i}^{(p)}]}(\boldsymbol{x}_k)\right)$$

Relaxed Intersection



L. Jaulin, Robust Set-membership State Estimation: Application to Underwater Robotics, *Automatica,* Vol. 45, No. 1, pp. 202-206, 2009.

Results: Box PF

Simulation Parameters:

$$\lambda_T = 100$$

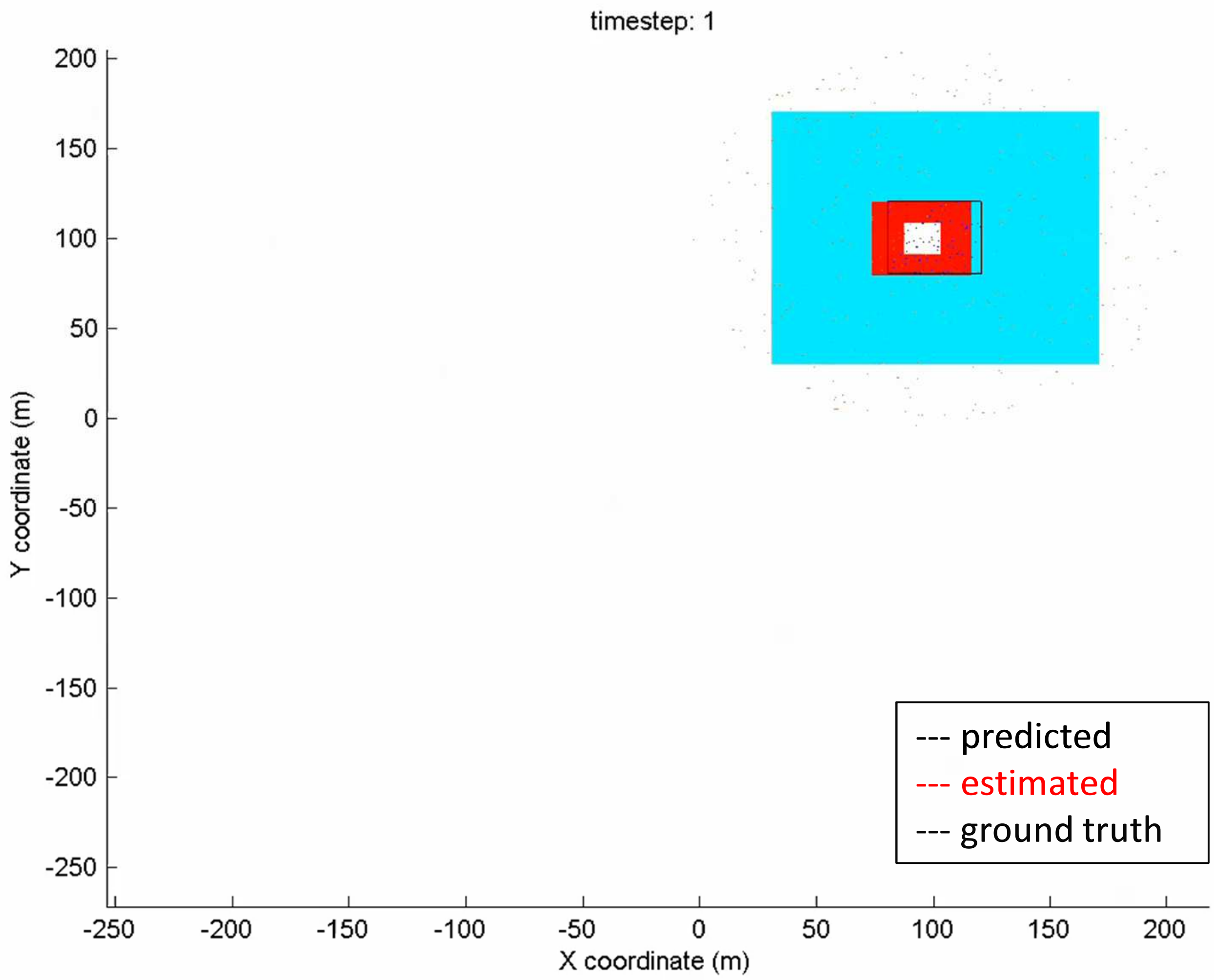
$$\rho = 1 \times 10^{-2}$$

$$A_0 = B_0 = 40\text{m}$$

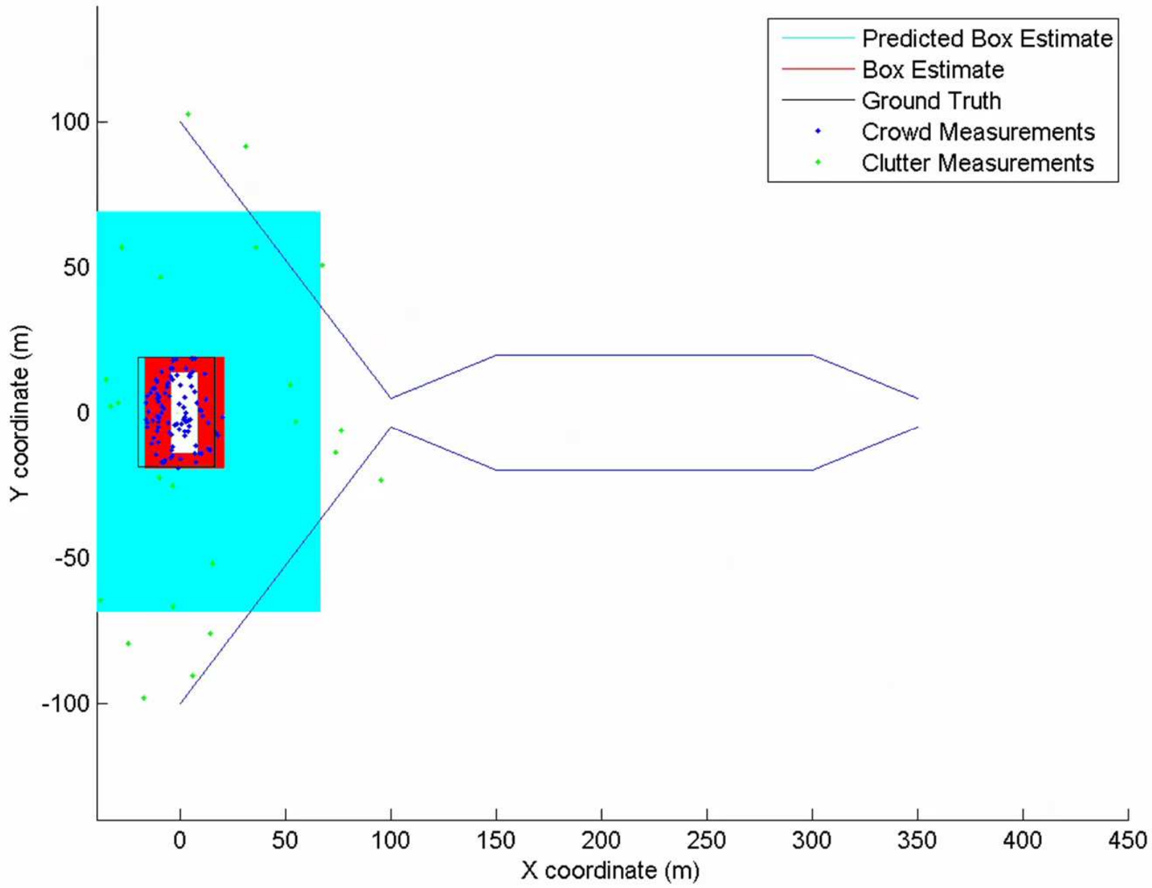
$$\sigma_{AB} = 1\text{m}$$

Filter Parameters:

$$N = 4$$



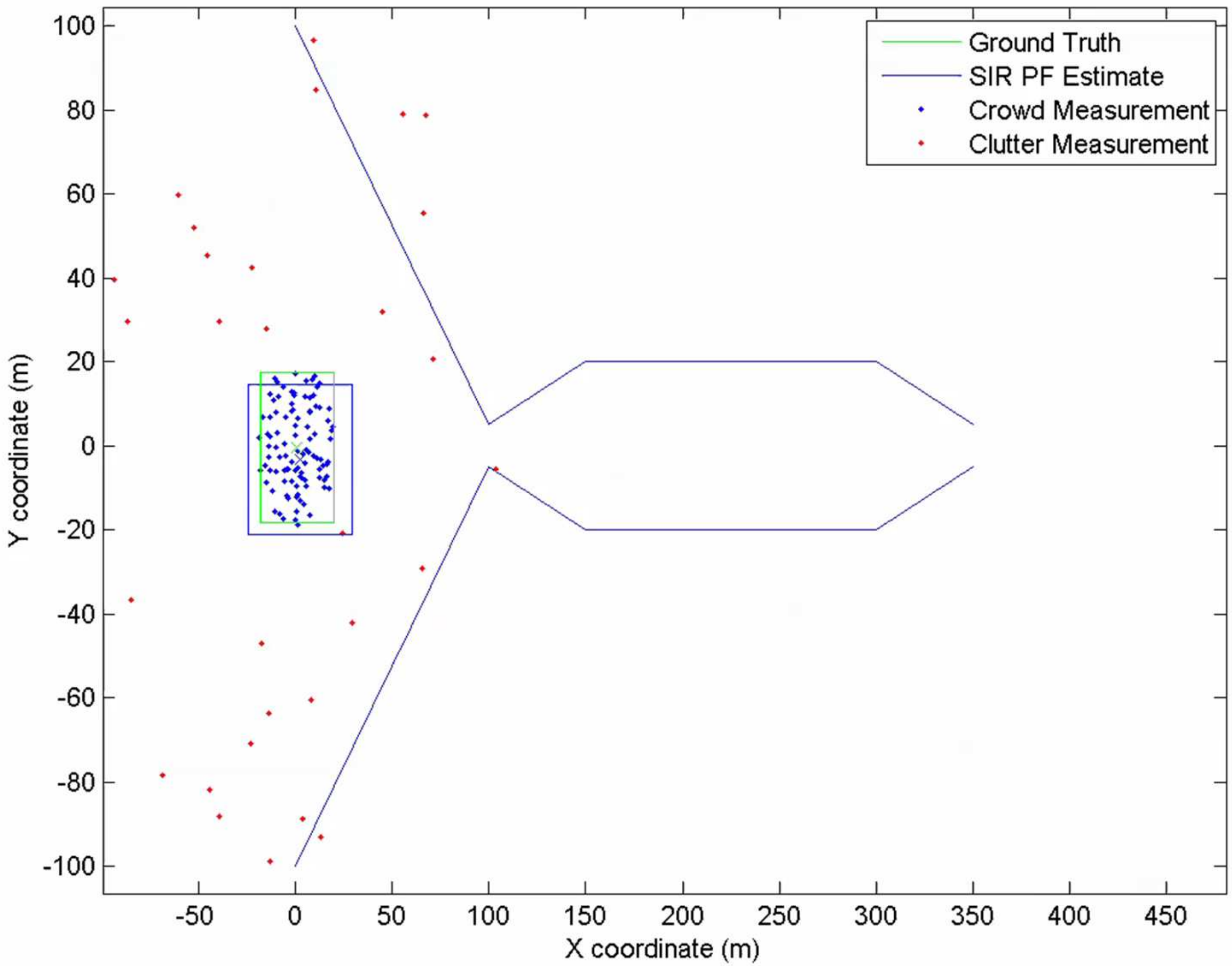
Stadium Testing Example



Box PF, N = 4

A. de Freitas, L. Mihaylova, A. Gning, D. Angelova, V. Kadirkamanathan, Autonomous Crowds Tracking with Box Particle Filtering and Convolution Particle Filtering, *Automatica*, 2015.

SIR Particle Filter



Results: SIR PF

Simulation Parameters:

$\lambda_T = 100$

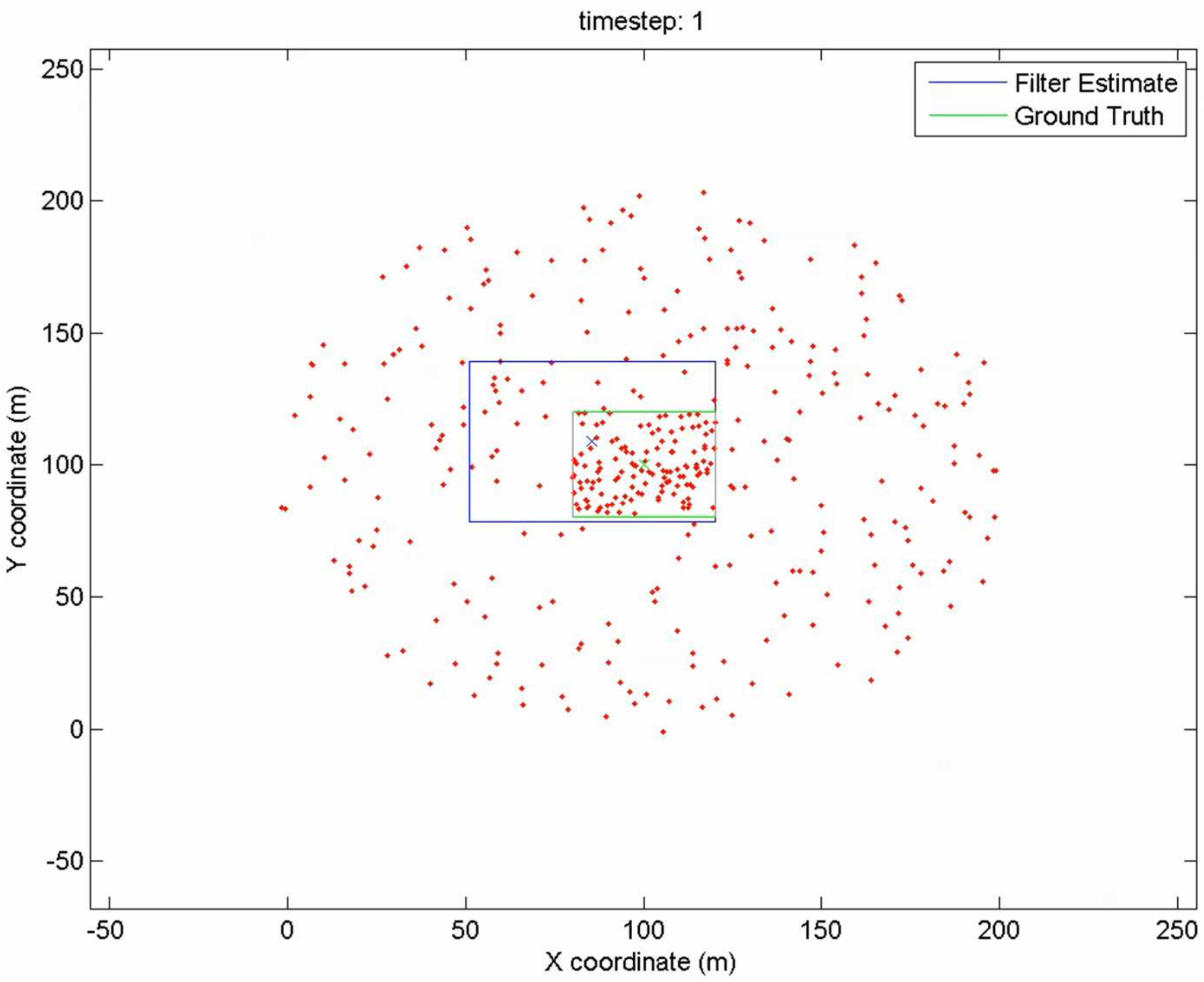
$\rho = 1 \times 10^{-2}$

$A_0 = B_0 = 40\text{m}$

$\sigma_{AB} = 1\text{m}$

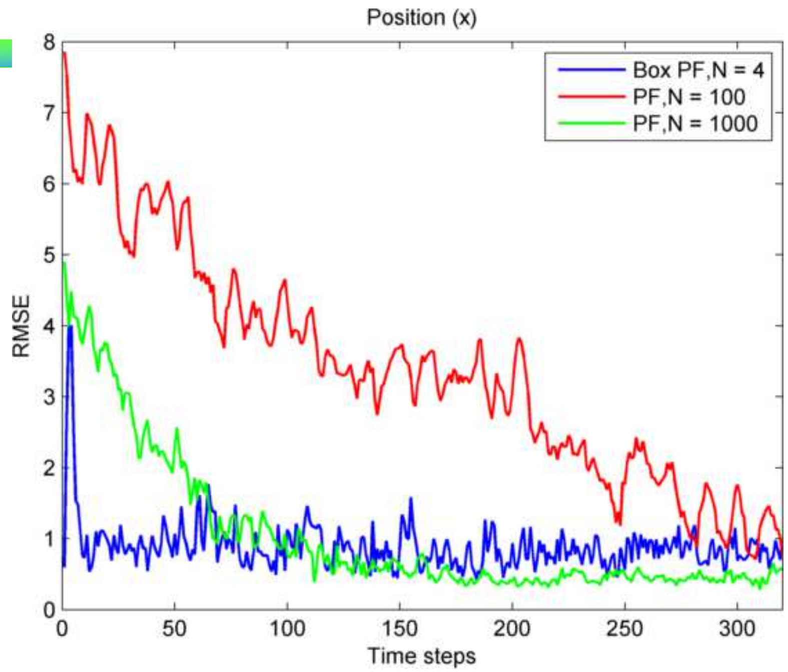
Filter Parameters:

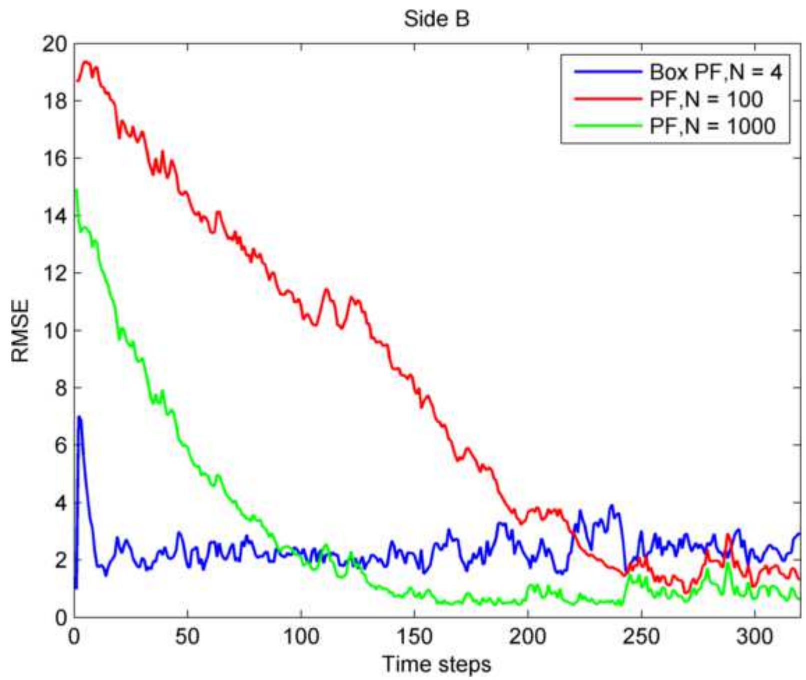
$N = 100$



Likelihood function calculated as in Gilholm and Salmond, 2005

Results

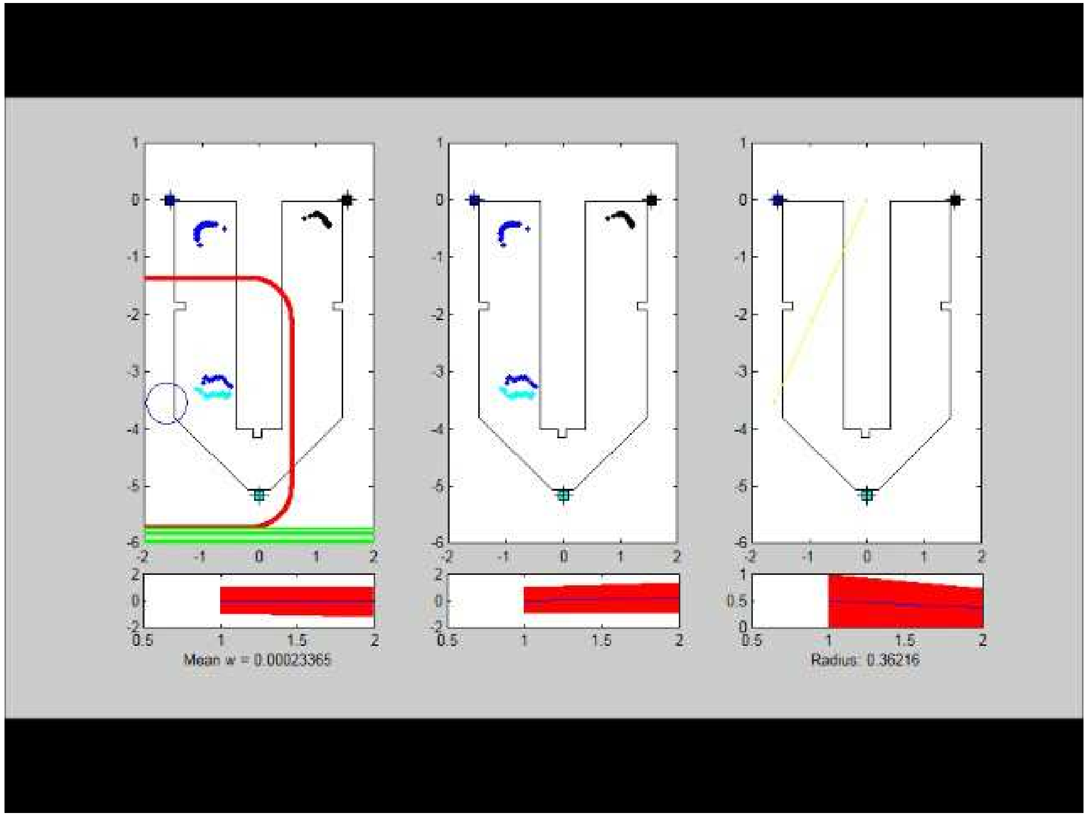




Filter	Computational Time (s)
Box PF, N = 4	12
SIR PF, N = 100	8
SIR PF, N = 1000	50

- SIR PF from: K. Gilholm, D. Salmond, Spatial distribution model for tracking extended objects, *IEE Proc. Radar, Sonar Navig.*, Vol. 152, No. 5, pp. 364 - 371, 2005.

Results



With laser range finder data, provided by FKIE Fraunhofer

Markov Chain Monte Carlo Methods for Group Object Tracking

- In high dimensional spaces: depletion of particles is possible while calculating posterior probability density functions
- Remedies to this:
  - Sequential Markov Chain Monte Carlo (MCMC)
  - Metropolis within Gibbs
- The MCMC step:
  - moves particles into more likely regions and hence improves the performance
  - allows to simulate complex systems
  - allows to sample only a part of the state, thus reducing the search space of the state

- A. Carmi, L. Mihaylova, A. Gning, P. Gurfil, S. Godsill, Inferring Leadership from Group Dynamics Using Markov Chain Monte Carlo Methods, Chapter in Modeling, Simulation, and Visual Analysis of Crowds, Eds. S. Ali, K. Nishino, D. Monacha, M. Sha, Springer, pp. 335-357, 2014.

Volume, Variety, Velocity and Veracity in Big Data

The University Of Sheffield.

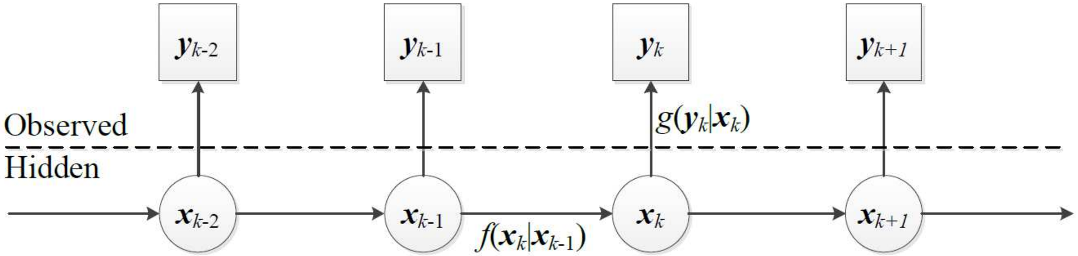
The University Of Sheffield.

How Can Subsampling Reduce Complexity in Sequential MCMC Methods and Deal with Big Data in Target Tracking?

- A. De Freitas, F. Septier, L. Mihaylova, S. Godsill, How Can Subsampling Reduce Complexity in Sequential MCMC Methods and Deal with Big Data in Target Tracking?, *Proceedings of the 2015 International Conference on Information Fusion*, 9-11 July 2015, Washington DC, USA
- A. De Freitas, F. Septier, L. Mihaylova, Sequential MCMC for Big Data, *IEEE Transactions on Signal Processing*, under review
- A. Gelman, A. Vehtari, P. Jylnki, C. Robert, N. Chopin, and J. P. Cunningham, Expectation Propagation as a Way of Life, preprint, http://arxiv.org/abs/1412.4869, 2014.

Sequential Markov Chain Monte Carlo Methods

- State space modelling



- MCMC – sample from an equilibrium distribution

$$\boldsymbol{x}^* \sim q(\cdot|\boldsymbol{x}^{m-1})$$

- Metropolis Hastings algorithm $u < \dfrac{\pi(\boldsymbol{x}^*)q(\boldsymbol{x}^{m-1}|\boldsymbol{x}^*)}{\pi(\boldsymbol{x}^{m-1})q(\boldsymbol{x}^*|\boldsymbol{x}^{m-1})}$
- Draw samples from a joint posterior distribution:

$$p(\boldsymbol{x}_k, \boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k}) \propto p(\boldsymbol{z}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1})$$

$$p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{1:k-1}) \approx \frac{1}{N_p}\sum_{j=1}^{N_p}\delta(\boldsymbol{x}_{k-1} - \boldsymbol{x}_{k-1}^{(j)})$$

Key Idea

- To reduce the computational complexity, we introduce a Monte Carlo (MC) approximation for the log likelihood ratio:

$$\Lambda_1^{S_{m,k}}(x_k^{m-1}, x_k^*) = \frac{1}{S_{m,k}} \sum_{i=1}^{S_{m,k}} \log \left[\frac{p(z_k^{i,*} | x_k^*)}{p(z_k^{i,*} | x_k^{m-1})} \right]$$

- We have a subset

$$z_k^* = \{z_k^{1,*}, \ldots, z_k^{S_{m,k},*}\}$$

from the original set of measurements

- How to select a minimum value for *Sm,k* for the set of subsampled measurements that contains enough information to make the correct decision in the MH step.

Sequential Markov Chain Monte Carlo

- Here we propose: $\{\boldsymbol{x}_k^*, \boldsymbol{x}_{k-1}^*\} \sim q_1\left(\boldsymbol{x}_k, \boldsymbol{x}_{k-1} | \boldsymbol{x}_k^{m-1}, \boldsymbol{x}_{k-1}^{m-1}\right)$
- Accept if: $u \leq \dfrac{p(\boldsymbol{x}_k^*, \boldsymbol{x}_{k-1}^* | \boldsymbol{z}_{1:k}) q_1\left(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_{k-1}^{m-1} | \boldsymbol{x}_k^*, \boldsymbol{x}_{k-1}^*\right)}{p(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_{k-1}^{m-1} | \boldsymbol{z}_{1:k}) q_1\left(\boldsymbol{x}_k^*, \boldsymbol{x}_{k-1}^* | \boldsymbol{x}_k^{m-1}, \boldsymbol{x}_{k-1}^{m-1}\right)}$
- We can further represent this expression with the likelihood isolated:

$$\frac{1}{M_k} \log\left[u \frac{p(\boldsymbol{x}_k^{m-1} | \boldsymbol{x}_{k-1}^{m-1}) p(\boldsymbol{x}_{k-1}^{m-1} | \boldsymbol{z}_{1:k-1}) q_1\left(\boldsymbol{x}_k^*, \boldsymbol{x}_{k-1}^* | \boldsymbol{x}_k^{m-1}, \boldsymbol{x}_{k-1}^{m-1}\right)}{p(\boldsymbol{x}_k^* | \boldsymbol{x}_{k-1}^*) p(\boldsymbol{x}_{k-1}^* | \boldsymbol{z}_{1:k-1}) q_1\left(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_{k-1}^{m-1} | \boldsymbol{x}_k^*, \boldsymbol{x}_{k-1}^*\right)}\right] \leq \frac{1}{M_k} \sum_{i=1}^{M_k} \log\left[\frac{p(\boldsymbol{z}_k^i | \boldsymbol{x}_k^*)}{p(\boldsymbol{z}_k^i | \boldsymbol{x}_k^{m-1})}\right]$$

$$\psi(\,\cdot\,) \leq \Lambda^{M_k}(\,\cdot\,)$$

- R. Bardenet, A. Doucet, and C. Holmes, “Towards scaling up Markov chain Monte Carlo: an adaptive subsampling approach,” in Proc. of the Int. Conf. on Machine Learning, 2014, pp. 405–413.
- R. Bardenet and O.-A. Maillard, “Concentration inequalities for sampling without replacement,” To appear in Bernoulli, 2015. [Online]. Available: arxiv.org/abs/1309.4029

Key Idea

- Calculate an approximation of the likelihood
- Standard sequential MCMC: 2*N iterations *Mk* measurements
- To reduce the computational complexity we introduce a Monte Carlo approximation of the log likelihood ratio:

$$\Lambda_1^{S_{m,k}}(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*) = \frac{1}{S_{m,k}} \sum_{i=1}^{S_{m,k}} \log \left[\frac{p(\boldsymbol{z}_k^{i,*} | \boldsymbol{x}_k^*)}{p(\boldsymbol{z}_k^{i,*} | \boldsymbol{x}_k^{m-1})} \right]$$

- Obtain a bound on the deviation of the MC approximation from the complete log-likelihood ratio

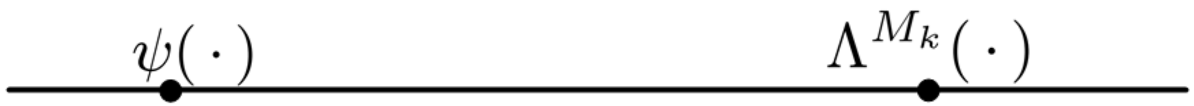
$$P(|\Lambda_1^{S_{m,k}}(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*) - \Lambda_1^{M_k}(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*)| \leq c_{S_{m,k}}) \geq 1 - \delta_{S_{m,k}}$$

$\delta_{S_{m,k}} > 0$ and $c_{S_{m,k}}$ depend on the inequality used.

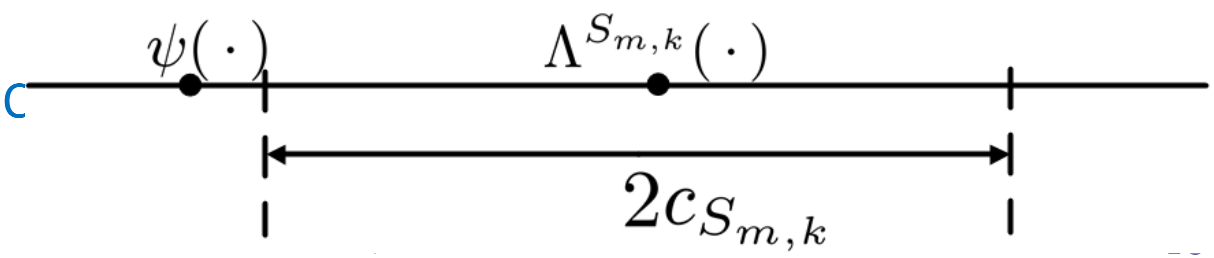
Adaptive Subsampling Approach

- Concentration inequality: $P(|\Lambda^{S_{m,k}}(\cdot) - \Lambda^{M_k}(\cdot)| \leq c_{S_{m,k}}) \geq 1 - \delta_{S_{m,k}}$
- Empirical Bernstein inequality: $c_{S_{m,k}} = \sqrt{\frac{2V_{S_{m,k}} \log(3/\delta_{S_{m,k}})}{S_{m,k}}} + \frac{3R_k \log(3/\delta_{S_{m,k}})}{S_{m,k}}$
- V: sample variance of the approximated log-likelihood ratio, R - range

- Standard SMCMC



- Adaptive Subsampling SMCMC



Adaptive Subsampling Approach

- Range: $$R_k = \max_{1\le i\le M_k}\left\{\log\left[\frac{p(\boldsymbol{z}_k^i|\boldsymbol{x}_k^*)}{p(\boldsymbol{z}_k^i|\boldsymbol{x}_k^{m-1})}\right]\right\} - \min_{1\le i\le M_k}\left\{\log\left[\frac{p(\boldsymbol{z}_k^i|\boldsymbol{x}_k^*)}{p(\boldsymbol{z}_k^i|\boldsymbol{x}_k^{m-1})}\right]\right\}$$
- Introduce the proxy function: $$\wp_i(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*) \approx \log\left[\frac{p(\boldsymbol{z}_k^i|\boldsymbol{x}_k^*)}{p(\boldsymbol{z}_k^i|\boldsymbol{x}_k^{m-1})}\right]$$

$$\Lambda_1^{S_{m,k}}(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*) = \frac{1}{S_{m,k}}\sum_{i=1}^{S_{m,k}}\log\left[\frac{p(\boldsymbol{z}_k^{i,*}|\boldsymbol{x}_k^*)}{p(\boldsymbol{z}_k^{i,*}|\boldsymbol{x}_k^{m-1})}\right] - \wp_i(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*)$$

- Taylor Approximation: $$\wp_i(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*) = (\nabla\ell_i)_{\boldsymbol{x}^+}^T \cdot (\boldsymbol{x}_k^* - \boldsymbol{x}_k^{m-1})$$

$$\ell_i(\boldsymbol{x}) = \log p(\boldsymbol{z}^i|\boldsymbol{x}) \qquad \hat{\ell}_i(\boldsymbol{x}) = \ell_i(\boldsymbol{x}^+) + (\nabla\ell_i)_{\boldsymbol{x}^+}^T \cdot (\boldsymbol{x} - \boldsymbol{x}^+)$$

- We derive the upper bound on Range: $R_k^B \ \geq \ R_k$

  and use the Taylor-Lagrange inequality

$$\text{if } |f^{(n+1)}(\boldsymbol{x})| \le Y, \text{ then } |B_k(\boldsymbol{x})| \le \tfrac{Y|\boldsymbol{x}-\boldsymbol{x}^+|^{n+1}}{(n+1)!}$$

$$R_k^B = 2\left|\left|B_k(\boldsymbol{x}_k^{m-1})\right| + \left|B_k(\boldsymbol{x}_k^*)\right|\right|$$

$B_k(\boldsymbol{x}) = \ell_i(\boldsymbol{x}) - \hat{\ell}_i(\boldsymbol{x})$ the reminder of the Taylor series approximation

Key Point

- How to approximate the proxy term

$$\wp_i(\boldsymbol{x}_k^{m-1}, \boldsymbol{x}_k^*) = (\nabla \ell_i)_{\boldsymbol{x}^+}^T \cdot (\boldsymbol{x}_k^* - \boldsymbol{x}_k^{m-1})$$

- For the considered example, a single object, moving with constant velocity, in the presence of clutter, Poisson distributed

$$\ell_i(\boldsymbol{x}_k) = \log\left(\lambda_X \mathcal{N}(\boldsymbol{z}_{\boldsymbol{k}}^{\boldsymbol{i}}; \boldsymbol{x}_k, \boldsymbol{\Sigma}) + \frac{\lambda_C}{A_C}\right)$$

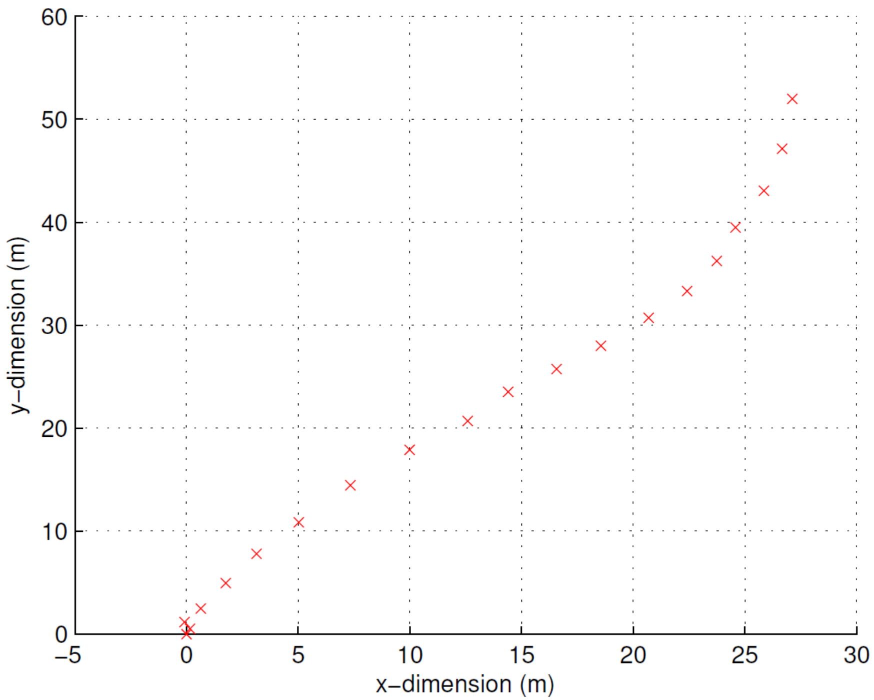
$$\nabla \ell_i = \frac{\lambda_X \boldsymbol{\Sigma}^{-1}(\boldsymbol{z}_k^i - \boldsymbol{x}_k)\mathcal{N}(\boldsymbol{z}_k^i; \boldsymbol{x}_k, \boldsymbol{\Sigma})}{\lambda_X \mathcal{N}(\boldsymbol{z}_k^i; \boldsymbol{x}_k, \boldsymbol{\Sigma}) + \frac{\lambda_C}{A_C}}$$

Hessian

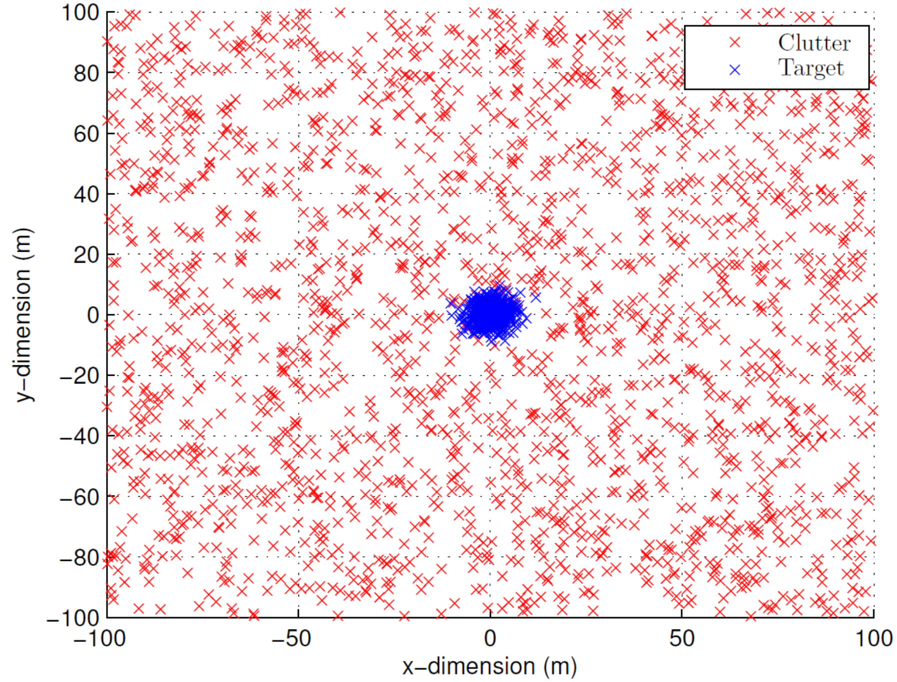
$$\boldsymbol{H} = \frac{-\lambda_X \boldsymbol{\Sigma}^{-1}\mathcal{N}(\boldsymbol{z}_k^i; \boldsymbol{x}_k, \boldsymbol{\Sigma})\left(\lambda_X \mathcal{N}(\boldsymbol{z}_k^i; \boldsymbol{x}_k, \boldsymbol{\Sigma}) + \frac{\lambda_C}{A_C}\right)}{\left(\lambda_X \mathcal{N}(\boldsymbol{z}_k^i; \boldsymbol{x}_k, \boldsymbol{\Sigma}) + \frac{\lambda_C}{A_C}\right)^2} + \frac{\frac{\lambda_C \lambda_X}{A_C}\boldsymbol{\Sigma}^{-1}(\boldsymbol{z}_k^i - \boldsymbol{x}_k)\left(\boldsymbol{\Sigma}^{-1}(\boldsymbol{z}_k^i - \boldsymbol{x}_k)\right)^T \mathcal{N}(\boldsymbol{z}_k^i; \boldsymbol{x}_k, \boldsymbol{\Sigma})}{\left(\lambda_X \mathcal{N}(\boldsymbol{z}_k^i; \boldsymbol{x}_k, \boldsymbol{\Sigma}) + \frac{\lambda_C}{A_C}\right)^2}$$

Results

Target trajectory



Measurements



$\lambda_X = 500, \lambda_C = 2000, \sigma_z^2 = 10$

Likelihood function as in (Gilholm, Salmond, 2005)

$$p(z_k|x_k) \propto \prod_{i=1}^{M_k} \lambda_X p_X(z_k^i|x_k) + \lambda_C p_C(z_k^i)$$

Results

- Metrics:
  - Estimate Accuracy:

$$RMSE = \sqrt{\frac{1}{N_I}\sum_{i=1}^{N_I}(\hat{X}_i - X_i)^2}$$

  - Relative computational efficiency: $D = \frac{1}{T}\sum_{k=1}^{T}\frac{\sum_{m=1}^{N}(S_{m,k})_{JD} + (S_{m,k})_R}{2NM_k}$
  - D : normalised number of sub-sampled measurements required for the likelihood calculation

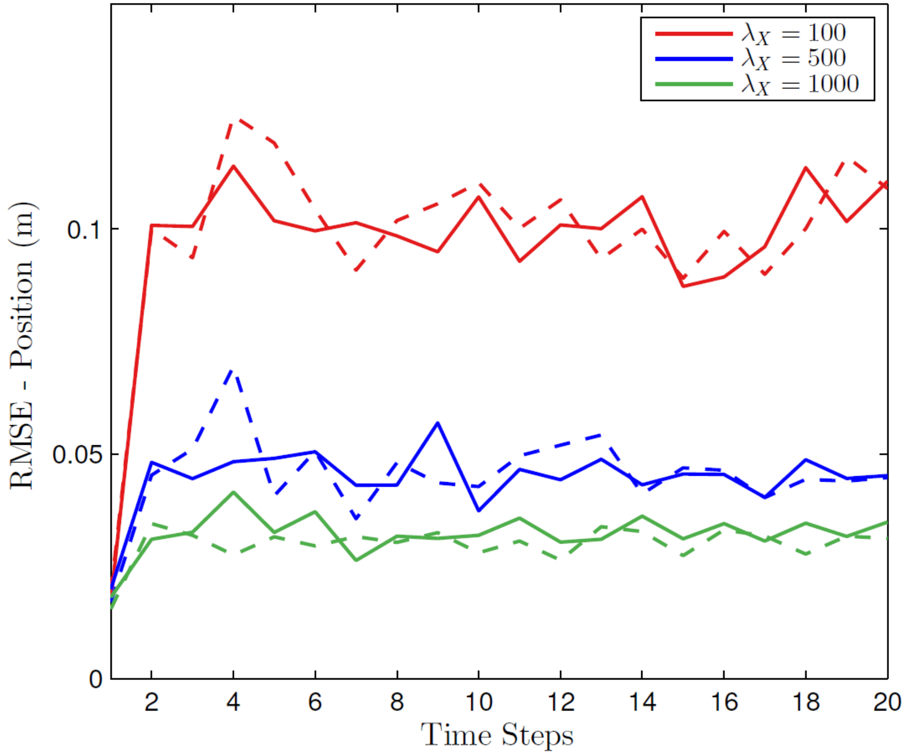
$(S_{m,k})_{JD}$ Number of subsampled measurements from the joint draw step

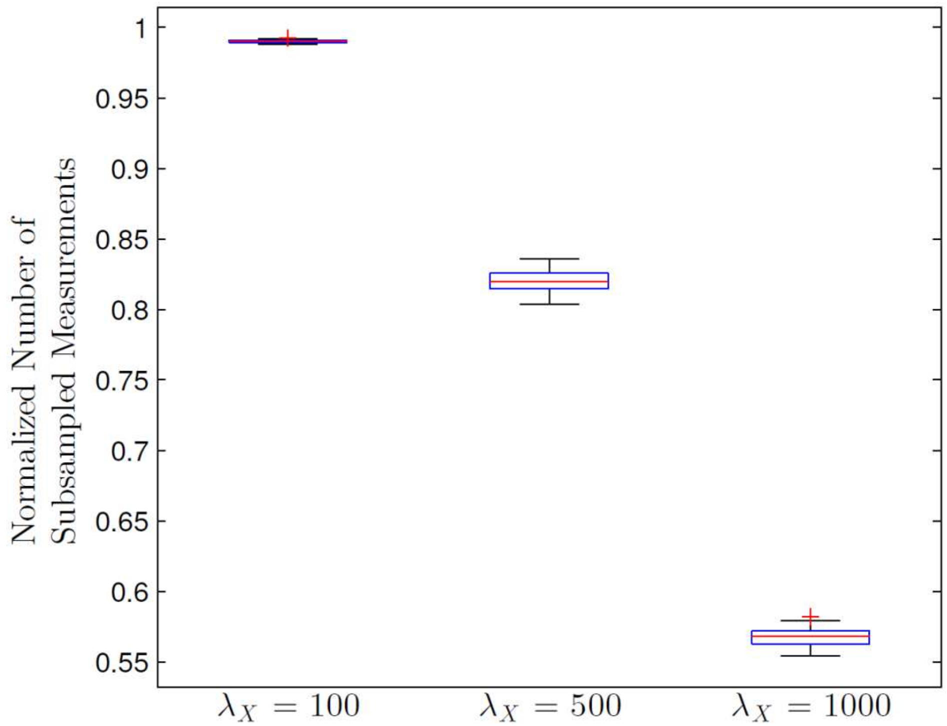
$(S_{m,k})_R$ Number of measurements at the refinement step, N Monte Carlo samples

$M_k = M_k^x + M_k^c$: Total number of measurements: from the object + clutter

Results

Dotted line – standard SMCMC, full line – adaptive SMCMC

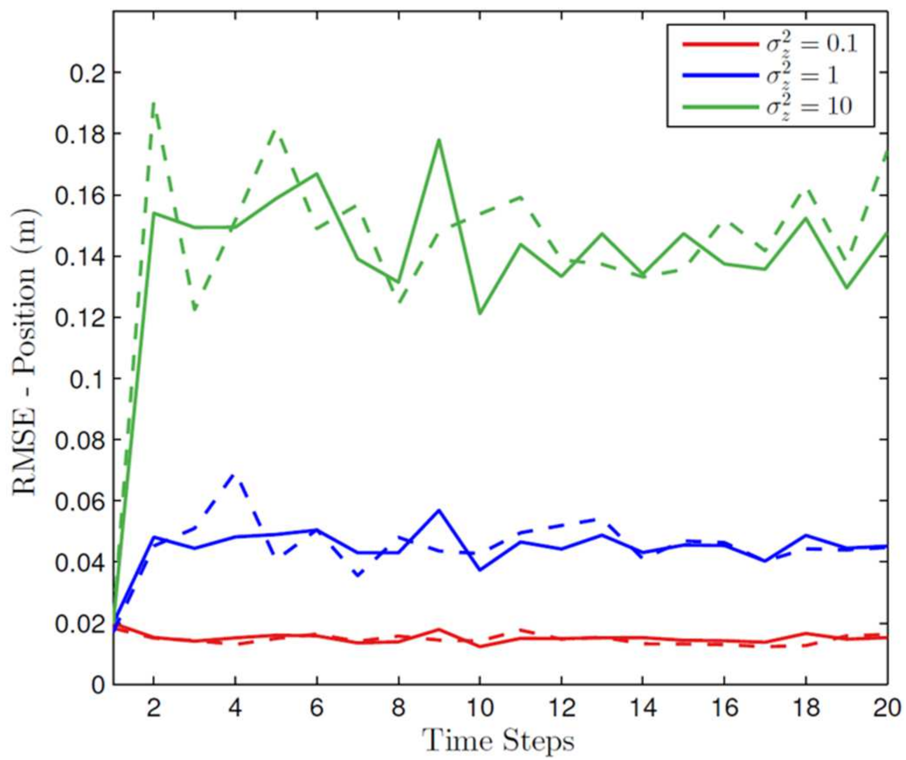


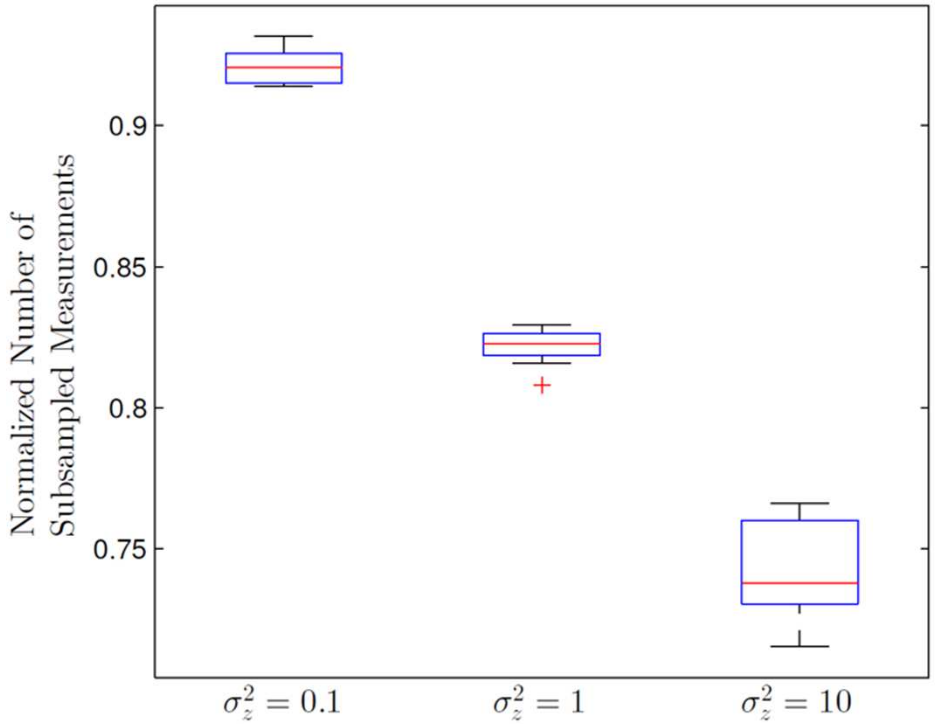


Varying number of target measurements

Results

Dotted line – standard SMCMC, full line – adaptive SMCMC

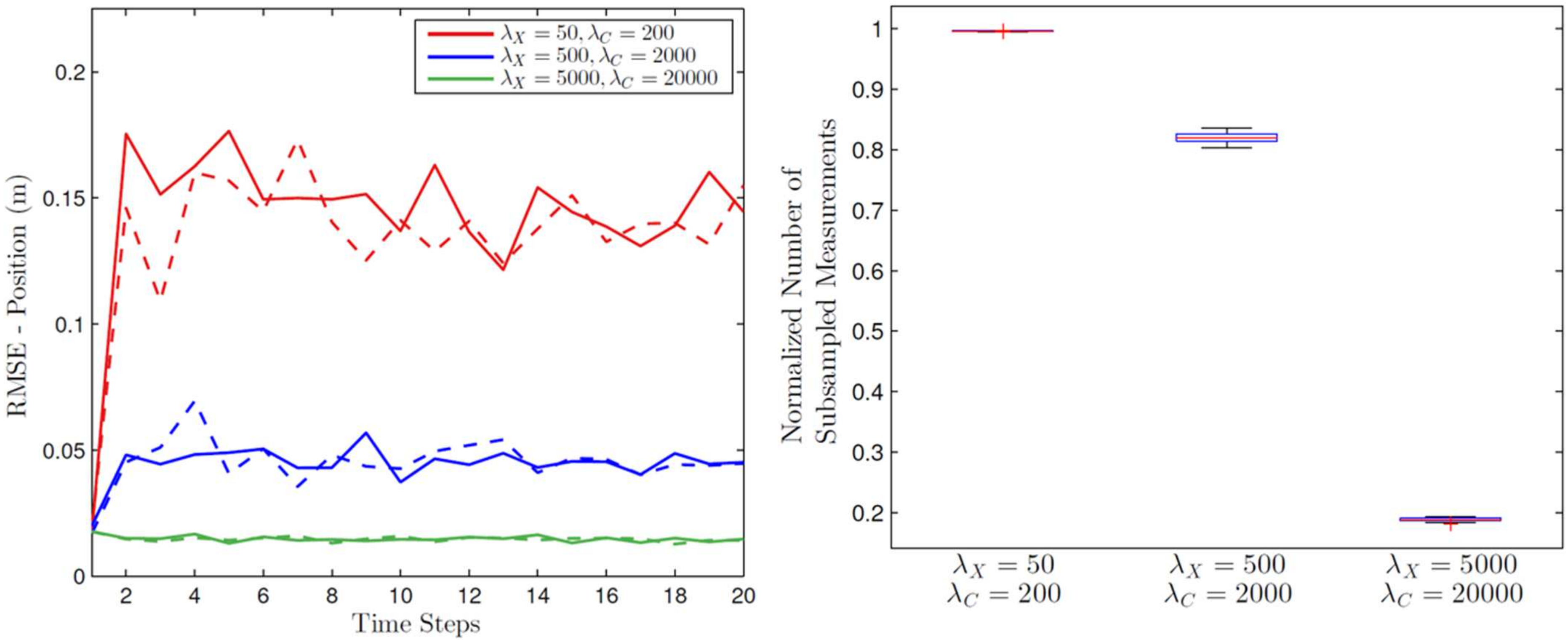




Varying the sensor covariance $\left(\boldsymbol{\Sigma} = \sigma_z^2 \boldsymbol{I}\right)$

Results

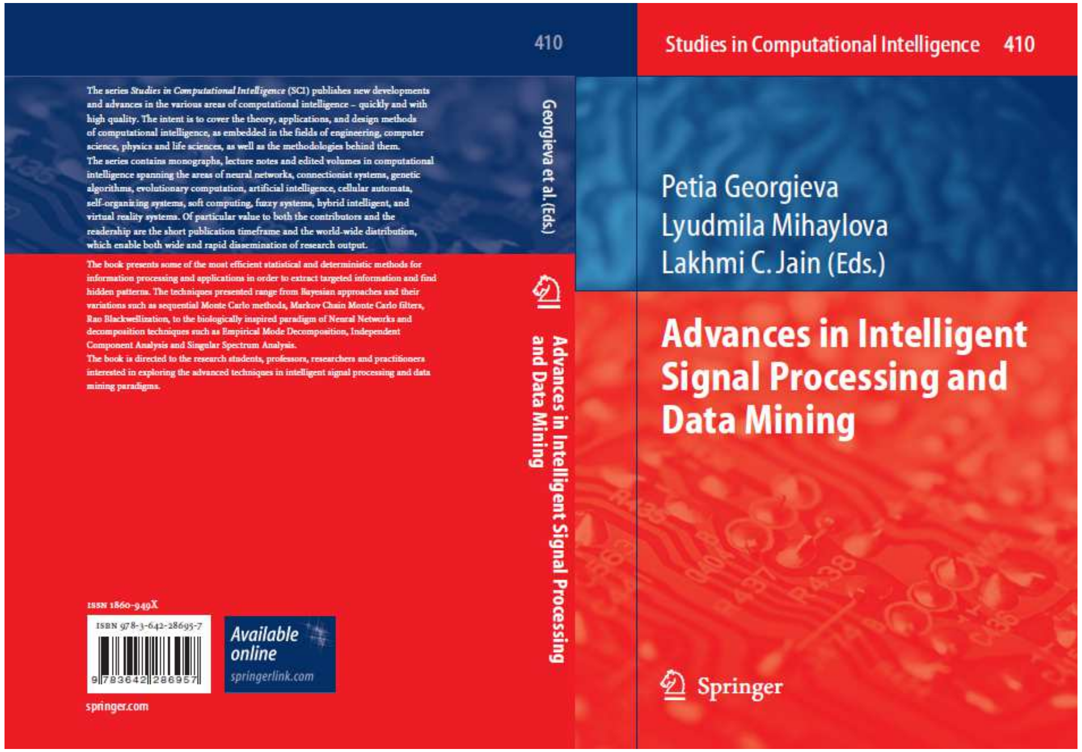
Dotted line – standard SMCMC, full line – adaptive SMCMC



Varying the total number of measurements

More than 80% reduction of the computational time thanks to subsampling, in the case with 25000 measurements.

Compressed Sensing for Dealing with Big Data



Conclusions & Future Work

The University Of Sheffield.

- Solutions to pedestrians and crowds tracking for autonomous systems and large volumes of data
- Convolution particle filters, sequential MCMC and box particle filters – can solve efficiently these problems, with uncertainties

Challenges:

- Scalable methods for groups and extended objects: deal with 100s and 1000s of objects, model the interactions
- Large volumes of measurements
  - Performance improves with an increase in data volume.
  - Showed up to 80% computational savings and accurate estimation
- **Future work:**
  - Large scale systems and Big Data, data fusion for challenges in urban areas, improved mobility of passengers, distributed localisation and network systems
  - Techniques for driver assistance systems, linked with Internet of Vehicles and networks; air pollution prediction in networks

Acknowledgements

- We Acknowledge the support from the EC Seventh Framework Programme [FP7 2013-2017] TRAcking in compleX sensor systems (TRAX) Grant no.: 607400

TRAX

- BTaRoT: Bayesian Tracking and Reasoning over Time, EPSRC project, EP/K021516/1

- Selex ES, UK

Thank you !

The University Of Sheffield.

THE 1ST STUDENT EXPERIENCE SURVEY 2014-15