IPAM Workshop

Mathematical Approaches To Traffic Flow Management

I. Traffic Signal Control

Alex Skabardonis

UC Berkeley

Los Angeles, CA

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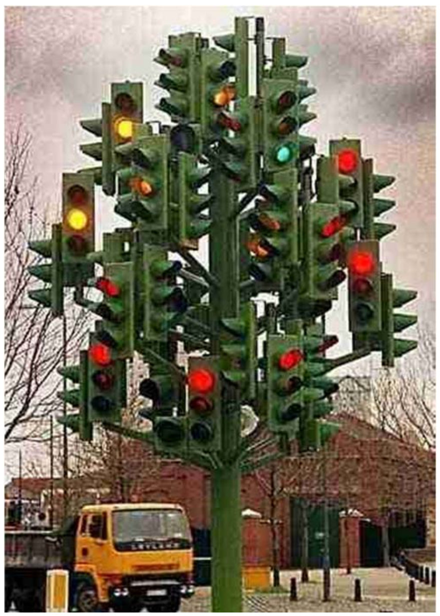
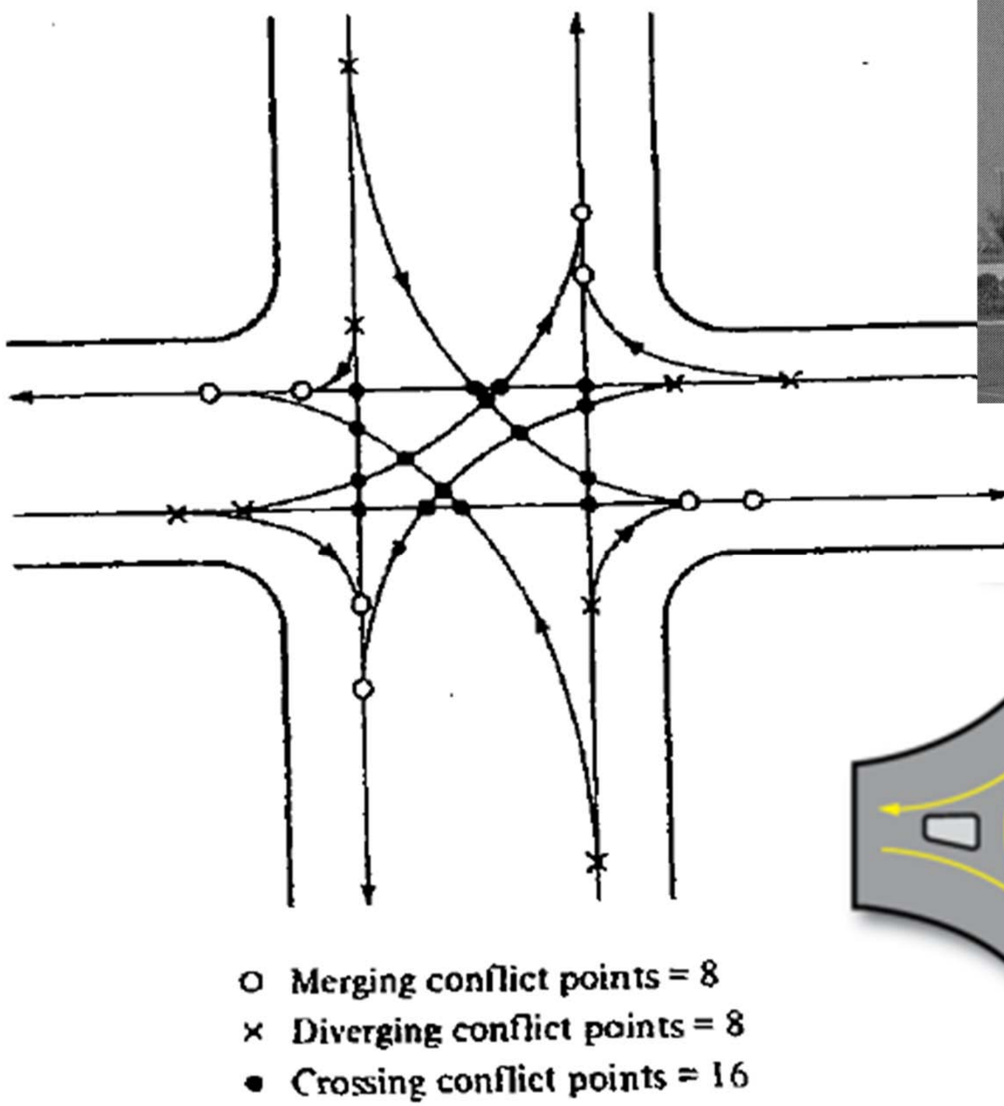
Outline

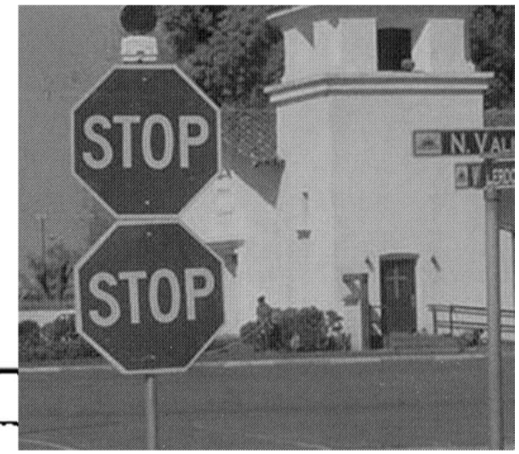
- Background: Intersection control
- Traffic Flow at Signalized Intersections
- Performance Measures
- Signal Timing Isolated Traffic Signals
- Signal Coordination
- Actuated Traffic Signals
- Traffic Responsive and Adaptive Control

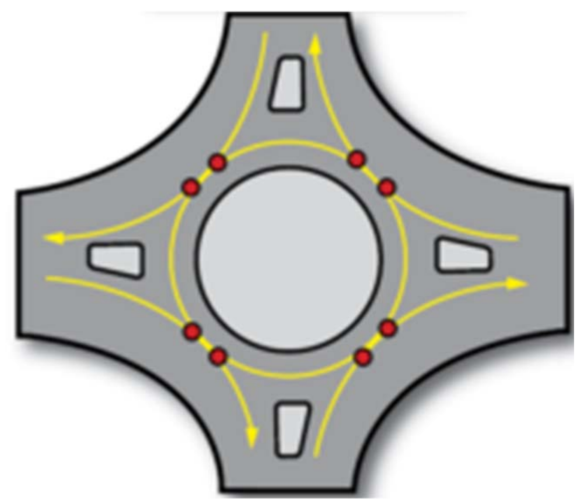
Background

Intersections: Conflict Points /Control Type

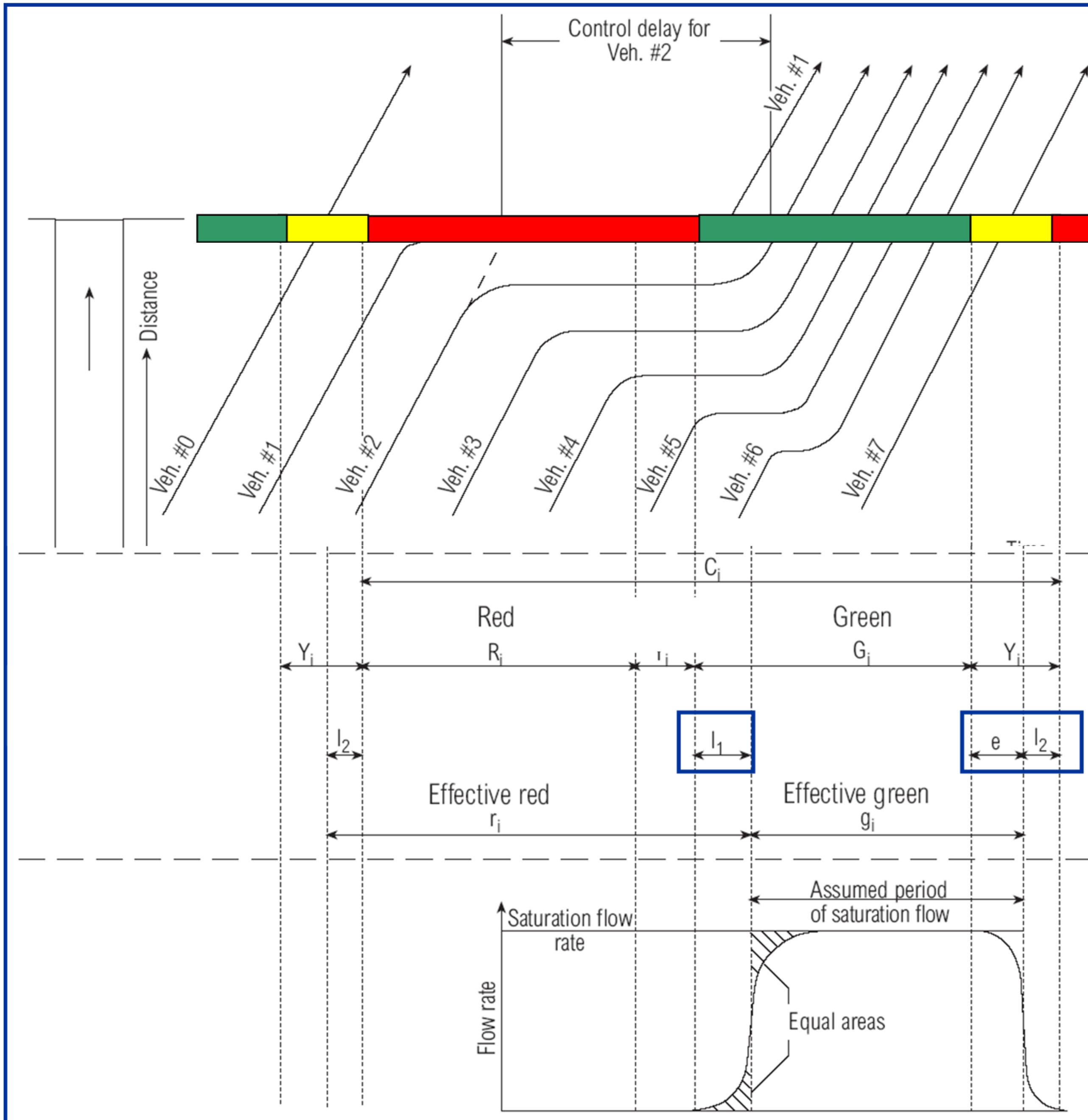








Traffic Flow at Signalized Intersections



l_1: Start-up lost time

l_2: Clearance lost time

Lost Time & Effective Green Time (1)

- **Start-up lost time (l_1):** The time lost due to acceleration of the first four vehicles at the beginning of the green
- **Clearance lost time (l_2):** Portion of the yellow (and all-red) clearance interval that is not used by vehicles to cross the intersection stopline

- **Total Lost Time, t_L**

  start-up lost time + clearance lost time

  $$t_L = l_1 + l_2$$

- **Effective Green/Red Times for a Movement**

  actual green plus yellow time - lost time

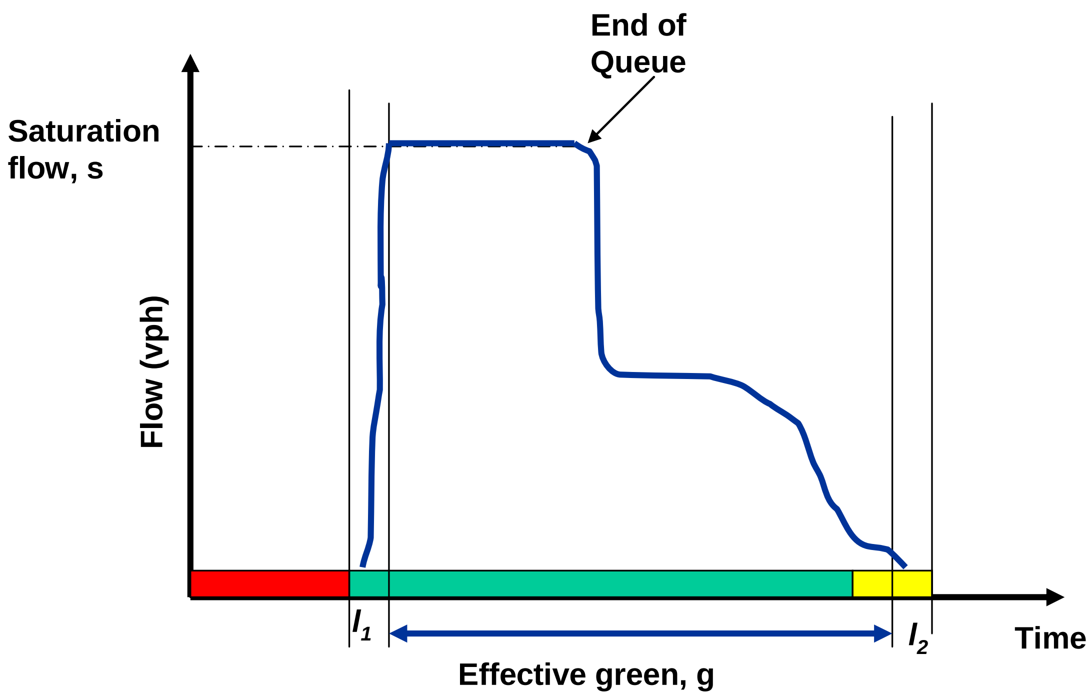
  actual red time plus lost time

  $$g_i = G_i + Y_i - t_L$$

  $$r_i = R_i + t_L$$

Saturation Flow and Capacity

<u>Saturation flow</u>: the maximum flow rate per lane at which vehicles can pass through a signalized intersection assuming that the signal was green for a full hour and a queue is present

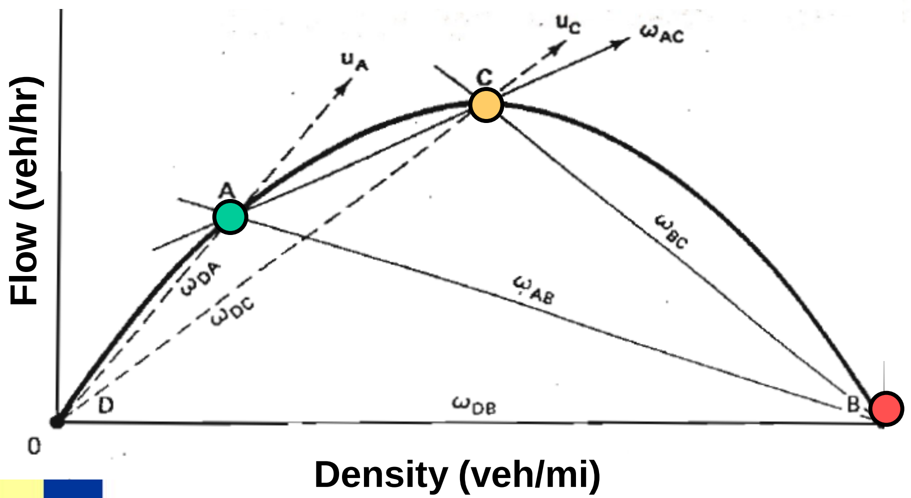


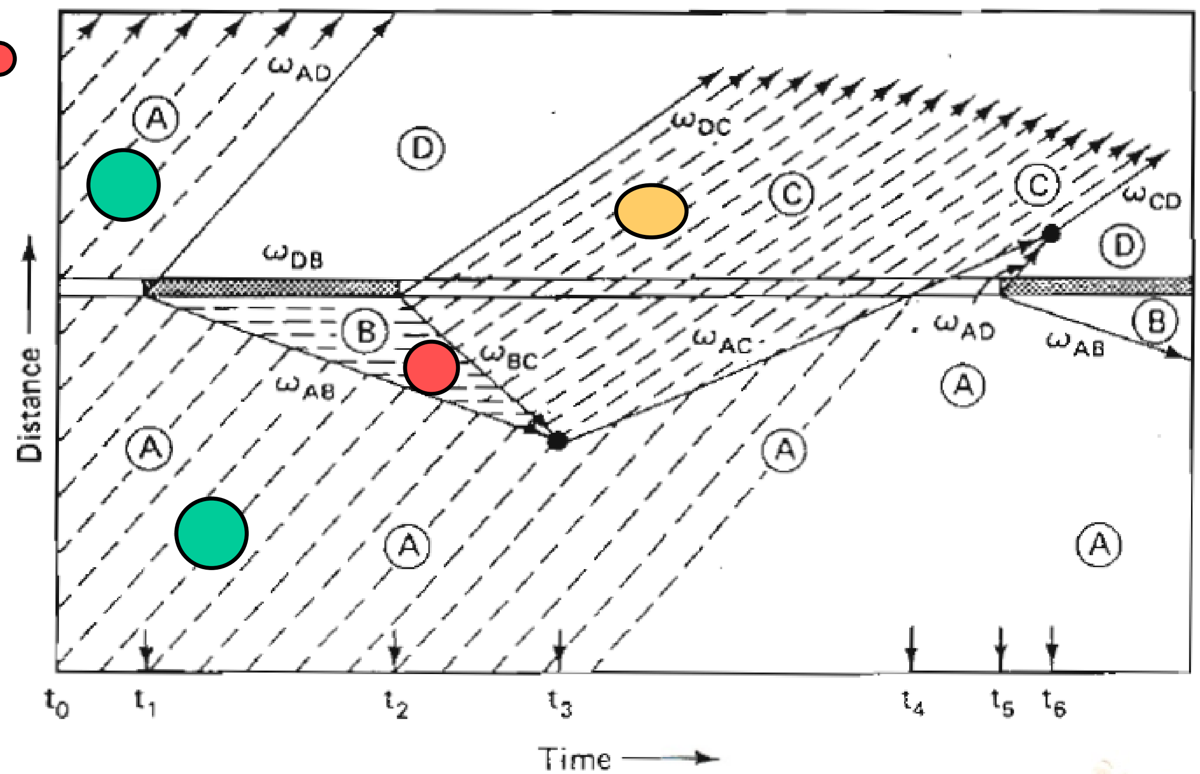
Ideal Saturation flow: 1,900 pce/l/hr/g

Capacity = (Saturation flow) * (proportion of time the signal is effectively green)

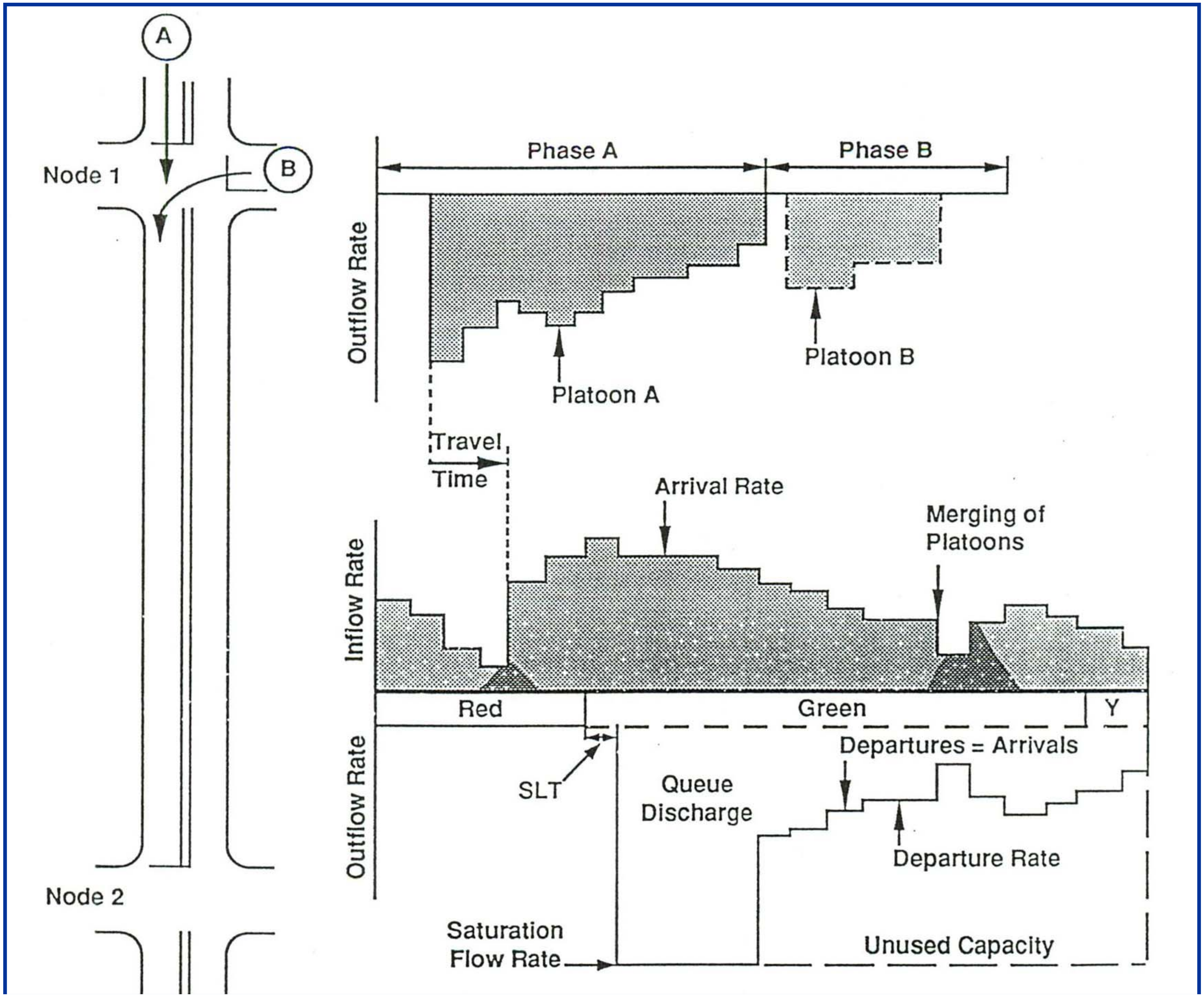
$$c = s\,(g/C) \quad \text{(vphl)}$$

Traffic Dynamics





Urban Streets: Vehicle Platoons



Performance Measures – Traffic Signals (1)

I. MOBILITY

Operating Environment	Operating Conditions	Performance Measure
Intersection	Undersaturated	Average control delay
		Max back of queue
		Cycle failure
		Green time utilization
	Oversaturated	Throughput
Arterial/ Grid Network	Undersaturated	Average travel time
		Average travel speed
		Travel time variability
		# of stops/stop rate
		Total delay
		% vehicles in the green
		Bandwidth efficiency
		Attainability
		Transit delay[1]
		Acceleration noise
	Oversaturated	Throughput
		Extend of queue
		Congestion duration

II. SAFETY

Performance Measures – Traffic Signals (2)

II, SAFETY		
Intersection/ Arterial/ Grid Network	**Undersaturated/ Oversaturated**	**# accidents per type**
		Encroachment time (ET)
		# RLR
		# vehicles in yellow
III. ENVIRONMENTAL		
Intersection/ Arterial/ Grid Network	**Undersaturated/ Oversaturated**	**Fuel Consumption**
		HC/CO/NOx/CO2/PM
		Noise

Volume/Capacity Ratio (v/c)

The volume to capacity ratio (v/c) for an intersection lane group is also called degree of saturation, X:

$$X_i = \left(\frac{v}{c}\right)_i = \frac{v_i}{s_i\left(\frac{g_i}{C}\right)} = \frac{v_i C}{s_i g_i} \qquad (16\text{-}7)$$

where

X_i = $(v/c)_i$ = ratio for lane group i,

v_i = actual or projected demand flow rate for lane group i (veh/h),

s_i = saturation flow rate for lane group i (veh/h),

g_i = effective green time for lane group i (s), and

C = cycle length (s).

Intersection Critical v/c Ratio, X_c

- **The critical v/c ratio, X_c, is the v/c ratio for the whole intersection, considering only the lane groups that have the highest flow ratio (v/s) per signal phase:**

$$X_c = \sum \left(\frac{v}{s} \right)_{ci} \left(\frac{C}{C - L} \right)$$

where

X_c = critical v/c ratio for intersection;

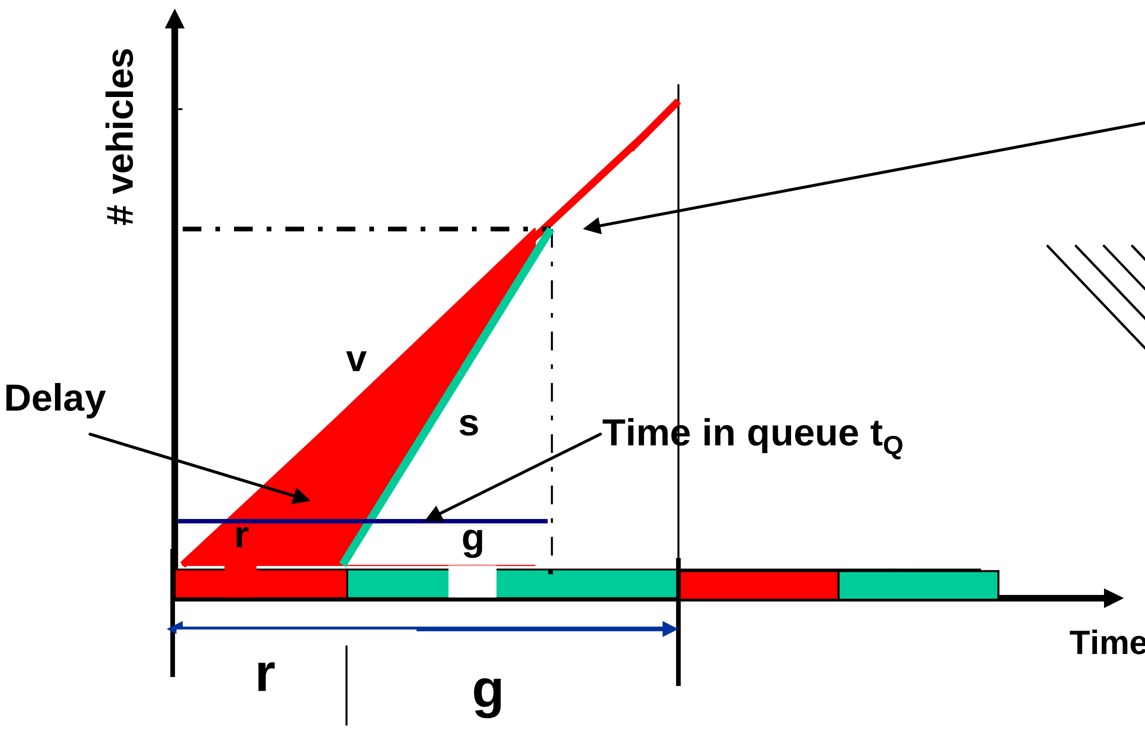
$\sum \left(\frac{v}{s} \right)_{ci}$ = summation of flow ratios for all critical lane groups i;

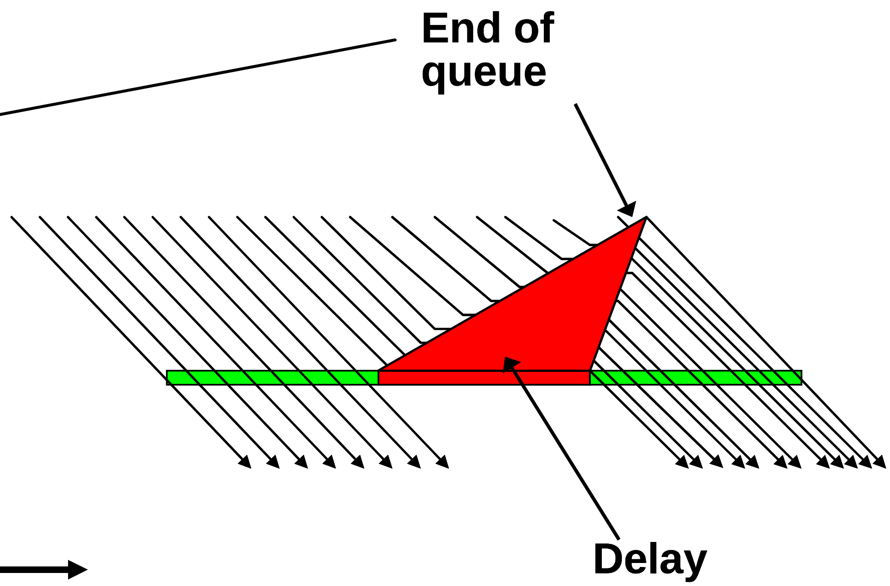
C = cycle length (s); and

L = total lost time per cycle, computed as lost time, t_L, for critical path of movements (s).

Intersection Spare Capacity = 1-X_c

Delay and Queuing





Time in the queue (sec) $$t_Q = \frac{sr}{(s-v)}$$

Queue in the beginning of green (veh) $$Q_M = \frac{vr}{3600}$$

Delay – Steady State Model--Webster

$$d = \frac{c(1-g/c)^2}{2[1-(g/c)x]} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{c}{q^2}\right)^{\frac{1}{3}} x^{2+5(g/c)}$$

where,

d = average delay per vehicle (sec),

c = cycle length (sec),

g = effective green time (sec),

x = degree of saturation (flow to capacity ratio)

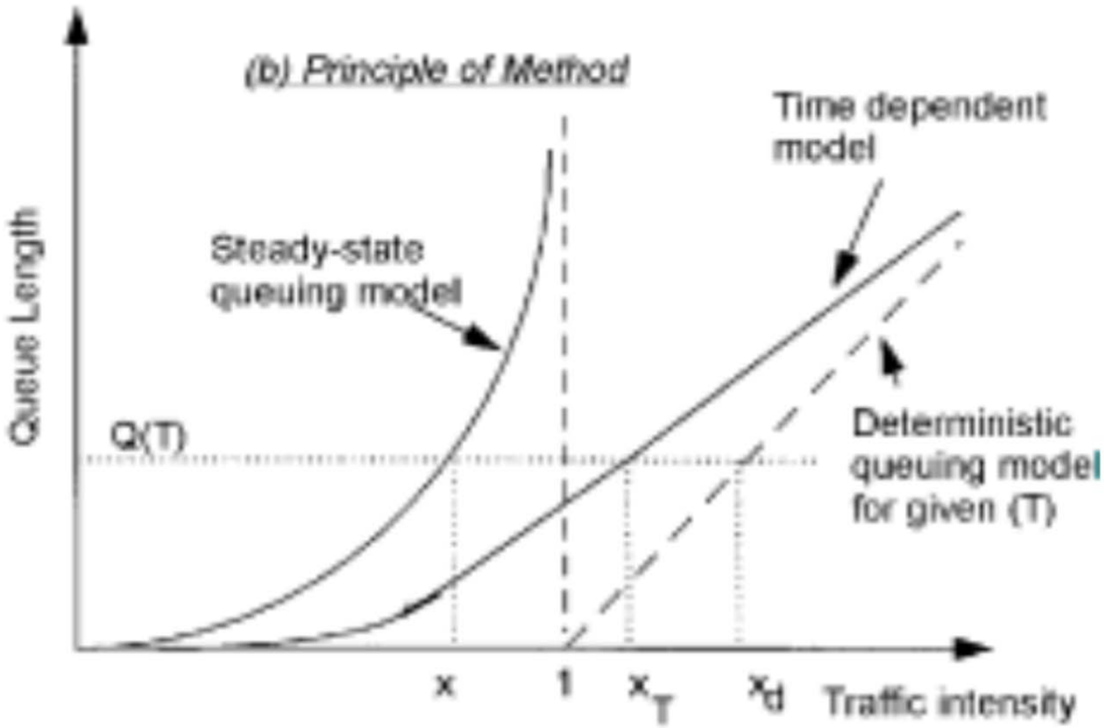
q = arrival rate (veh/sec).

Delay – Time Dependent Models

Applies to random + saturation delay

Example HCM2000

$$d_2 = 900T\left[(X-1)+\sqrt{(X-1)^2+\frac{8kIX}{cT}}\right]$$



T = analysis period

X = degree of saturation

c = capacity

K, I : adjustment factors

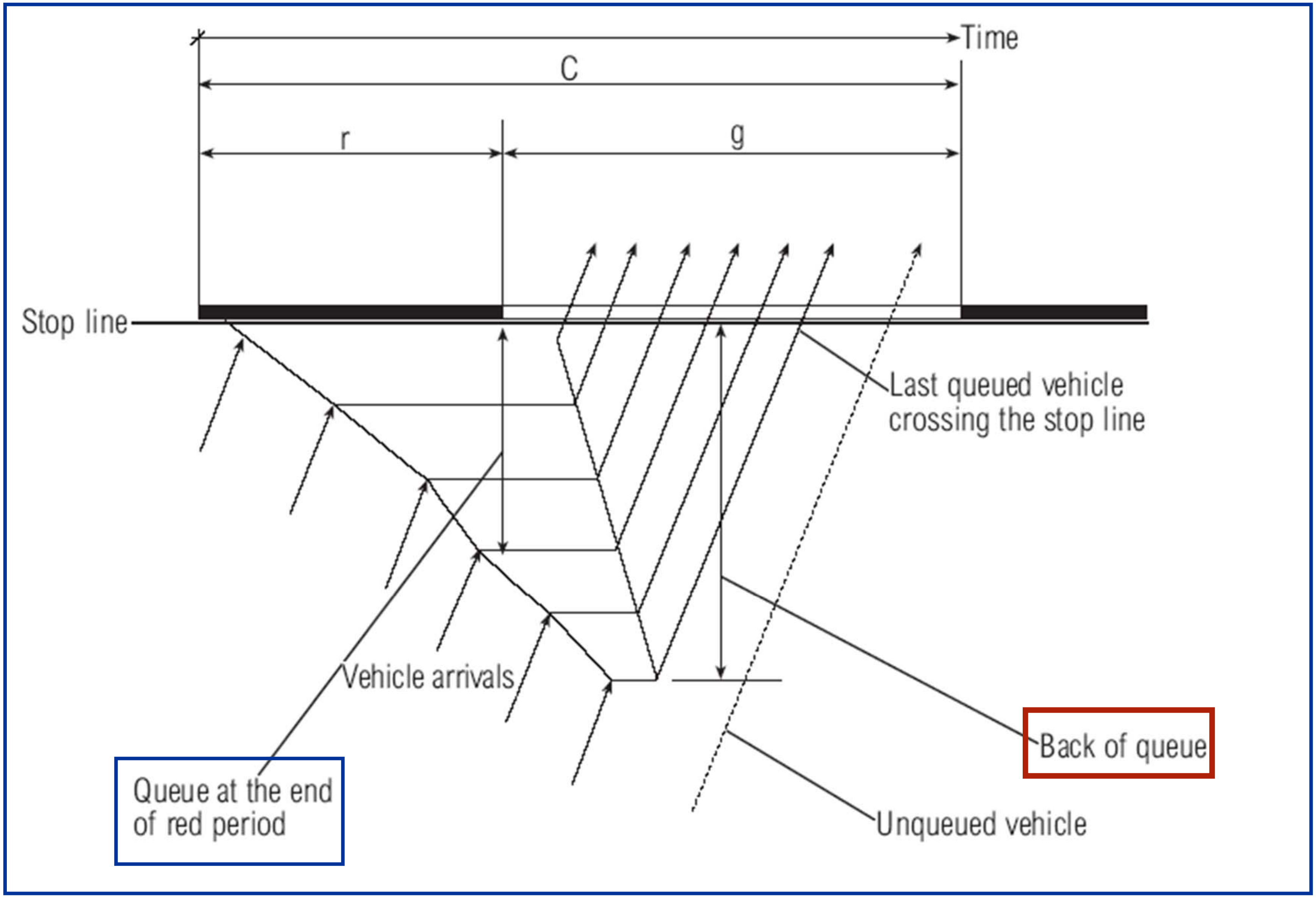
K: for coordination

I: for controller type

For Isolated fixed time signals

K = 0.5, I = 1

Back of Queue: Undersaturated Signal



Traffic Signal Operations

Isolated Signals

- **Pretimed (Fixed Time):** duration of cycle length and green times\ remain fixed throughout the analysis period
- **Actuated:** green times (and cycle length) vary based on the vehicle arrivals subject to a min and max value

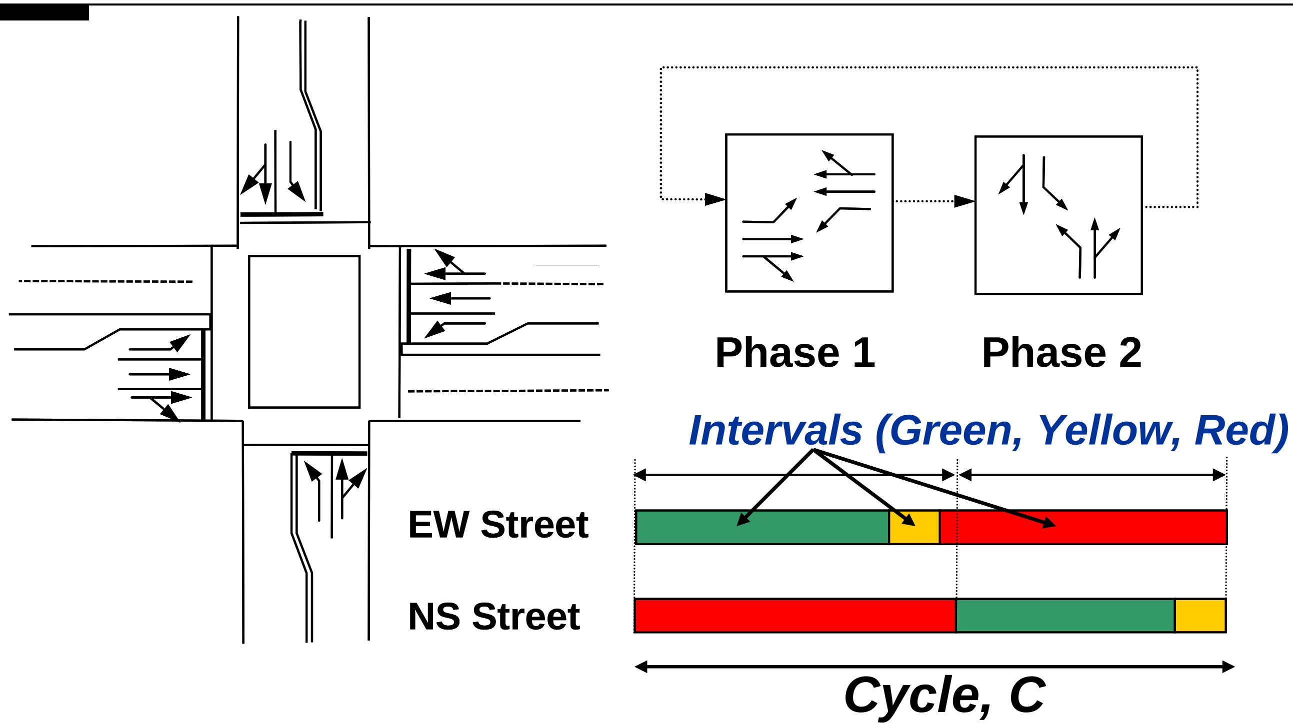
    Semi-actuated
    Fully-actuated

- **Adaptive:** advanced control based on detection and prediction of traffic arrivals

Arterials and Networks

- Coordinated fixed-time
- Coordinated actuated (semi-actuated)
- Traffic responsive & adaptive Control

Signal Timing: Definitions



- **Cycle length**: a complete sequence of signal indications
- **Phase:** portion of time that movements having the right-of-way do not change
- **Interval**: portion of time that signal indication remains unchanged

Clearance Intervals (1)

- **Yellow Time (Yellow Change)**

  Warn drivers of end of phase

$$Y = T + \frac{S}{2a + 64.4G}$$

Y = yellow change interval (sec)

T = driver reaction time (sec) – 1 sec.

S = vehicle speed (ft/s) - 85% speed or posted speed limit

a = vehicle deceleration rate (f/s^2) - 10 ft/s^2

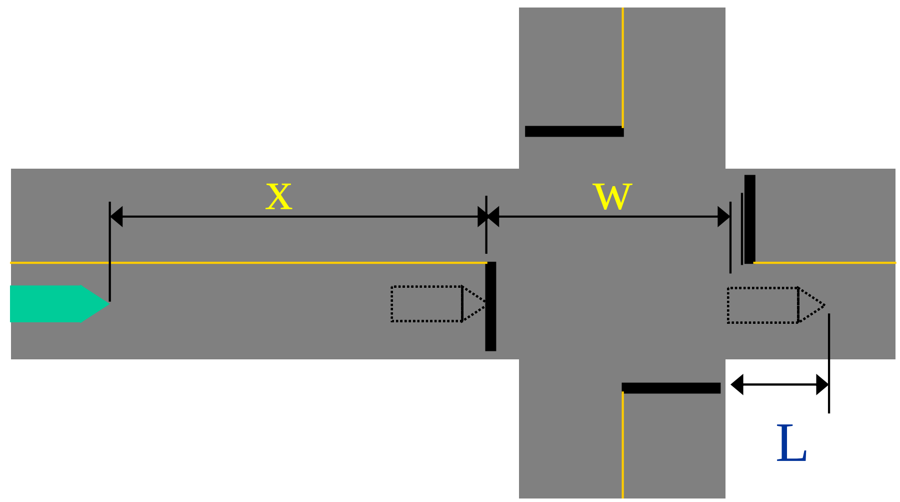
G = approach grade (%)

Clearance Intervals (2)

- **All Red (Red Clearance)**

  Use to clear vehicles in the intersection prior to changing right-of-way assignment

$$R = \frac{W + L}{S}$$



R = all-red clearance interval (sec)

S = vehicle speed (fps)

W = distance from stop line to intersection far side (ft)

L = vehicle length (ft)--- 20 ft for passenger cars

Minimum Pedestrian Green Time

- **The minimum ped green time for a phase is determined based on typical values of pedestrian walking speed (4 ft/sec or 3.5/ft/sec)**

- **Example HCM method**

$$G_p = 3.2 + \frac{L}{S_p} + \left(2.7 \frac{N_{ped}}{W_E}\right) \quad \text{for } W_E > 10 \text{ ft}$$

$$G_p = 3.2 + \frac{L}{S_p} + \left(0.27 N_{ped}\right) \quad \text{for } W_E \leq 10 \text{ ft}$$

where

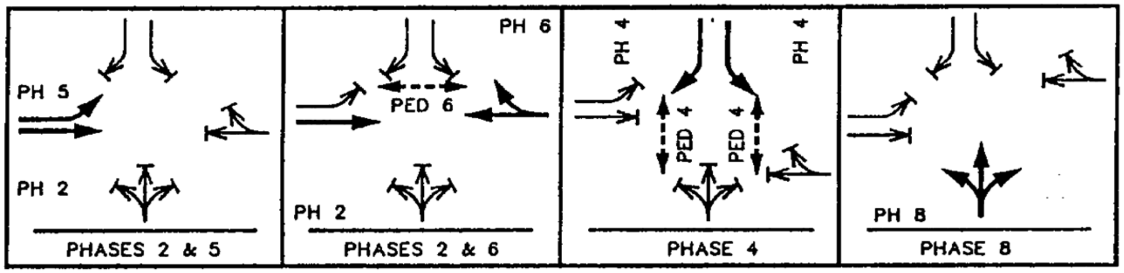
G_p = minimum green time (s),
L = crosswalk length (ft),
S_p = average speed of pedestrians (ft/s),
W_E = effective crosswalk width (ft),
3.2 = pedestrian start-up time (s), and
N_{ped} = number of pedestrians crossing during an interval (p).

Signal Phasing

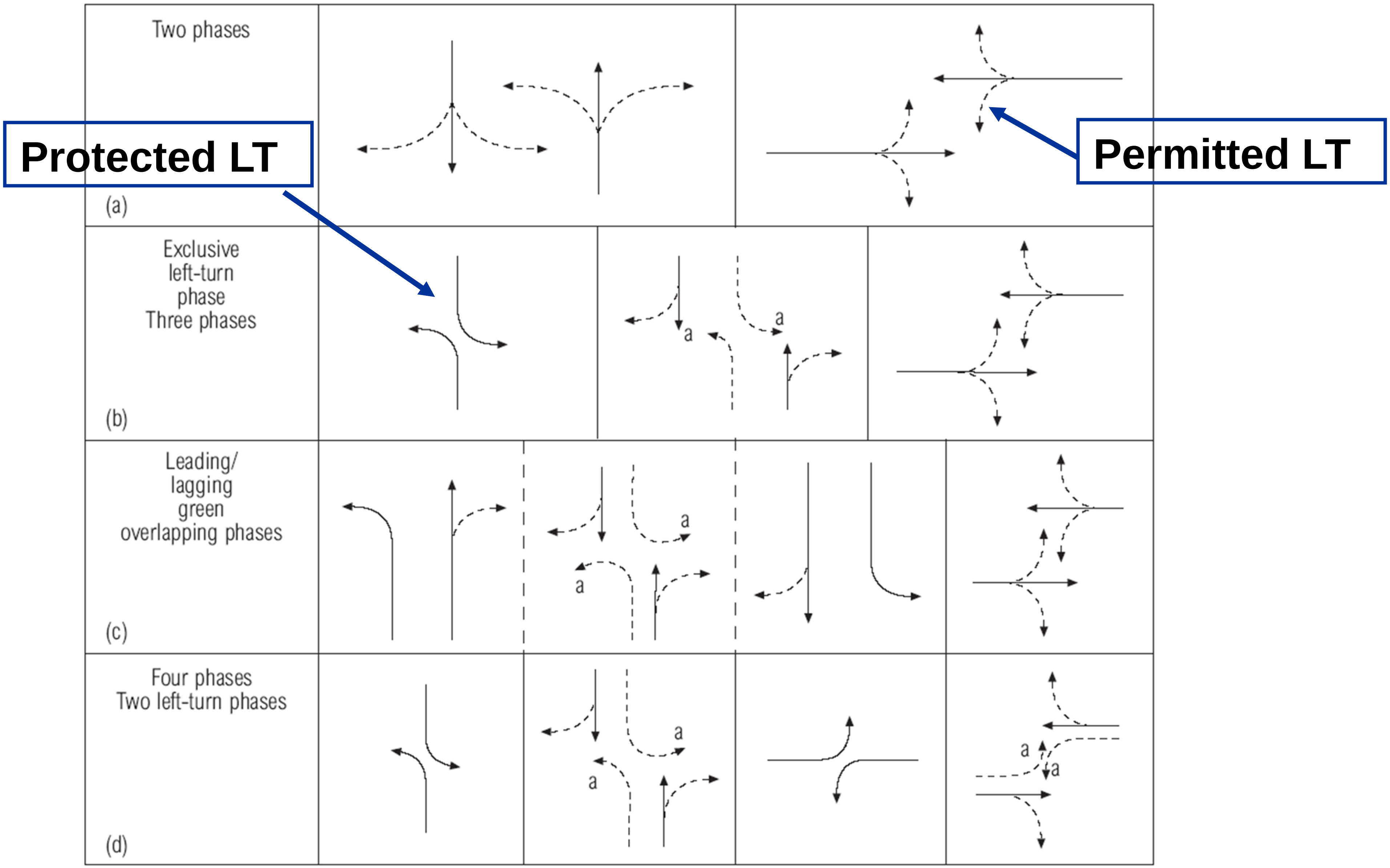
- Permitted phasing: turning movement is made through the opposing vehicle flow.

  Example: Left turns concurrent with the opposing through movement, and right-turns concurrent with pedestrian crossings in a conflicting crosswalk.

- Protected phasing: turns are made during an exclusive left-turn phase
- Protected-Permissive Left Turns
- Prohibited Left Turns
- Split Phasing



Typical Phasing Schemes



Criteria for Left Turn Phasing (1)

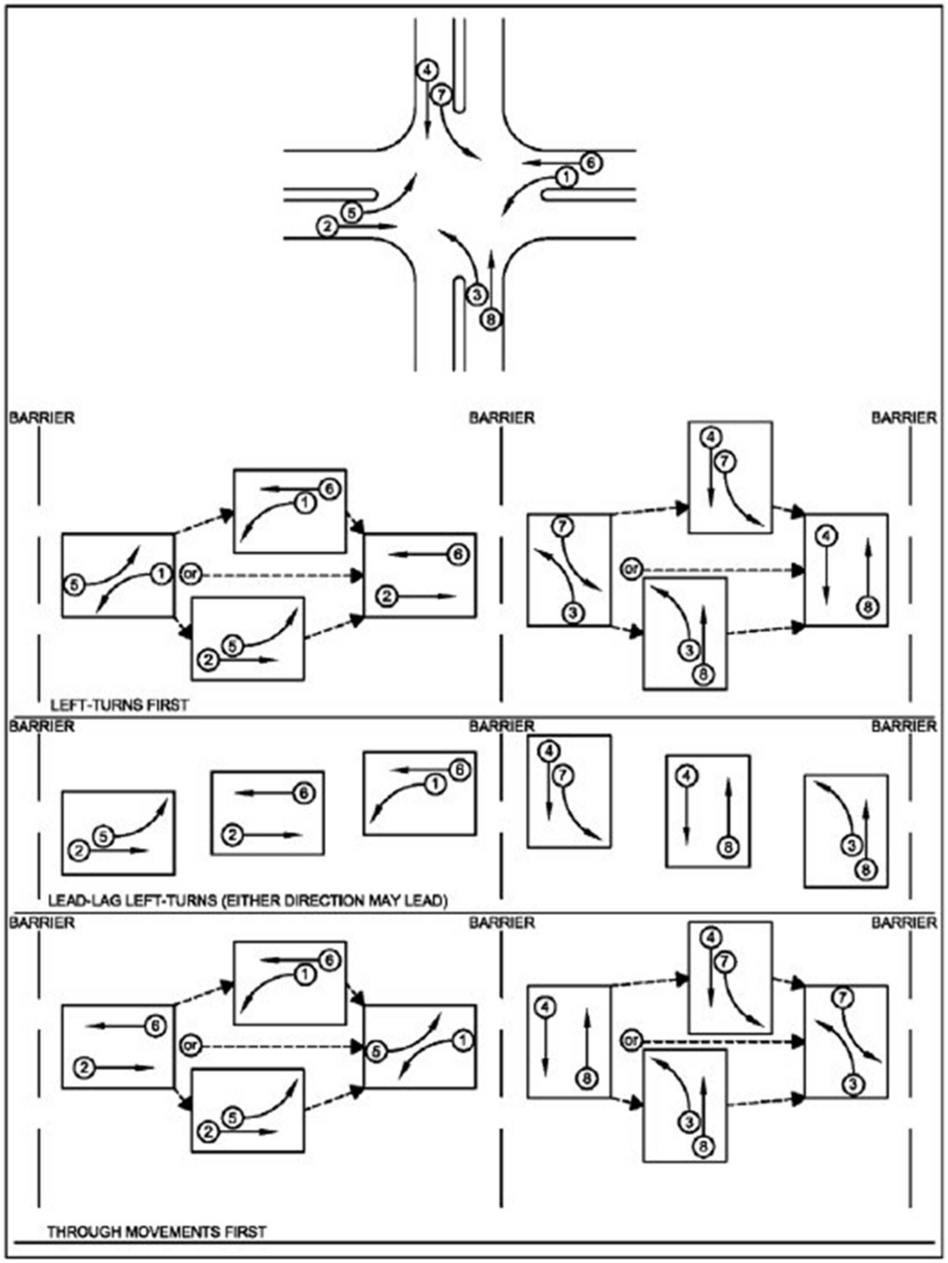
- **Intersection Design Characteristics**
  - # of left-turn lanes
  - # of opposing through lanes
  - Speed of opposing traffic
  - Sight distance
- **Traffic Volumes**
  - Left-turn volume (veh/cycle) V_{lt}
  - Opposing through + right-turn volume V_o
- **Delay to Left-Turning Vehicles**
- **Accident History**

Example:

$V_{lt} * V_o > 50{,}000$

Protected LT

Phase Sequence



Pretimed Controllers: Signal Timing

- **Cycle length:**

  Minimum green time to serve critical traffic demand/phase:

$$g = (v/s)(C-L) = y(C-L)$$

Minimum Cycle Time: $C = L/(1-Y)$

s.t. sum of min phase lengths

Optimum cycle length (Webster):

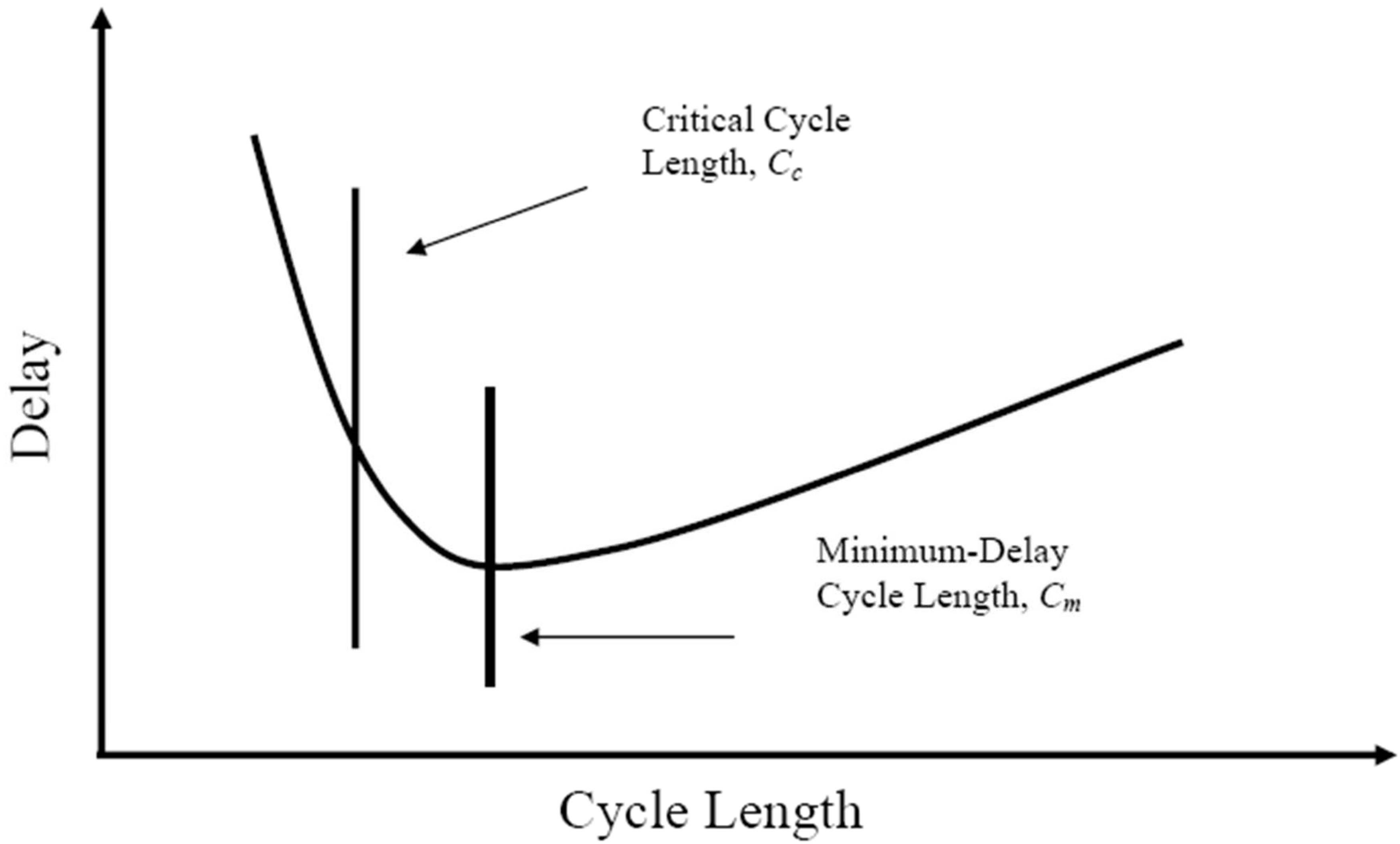
$$C_o = \frac{1.5L + 5}{1 - \sum_i \left(v_j / s_j\right)_i^c}$$

C_o = Optimum delay cycle length (sec)

L = Total lost time per cycle (sec)

$y = (v/s)_i^c$ = Critical (flow/saturation flow) ratio for phase i

Signal Delay vs. Cycle Length



Signal Timing: Green Splits

- **Phase Green Times**

  Effective green time per phase g_i

  Equal Degree Of Saturation (EQUISAT)

$$g_i = \frac{(v/s)_i^c}{\sum_i (v/s)_i^c}(C-L)$$

$$g == y(C-L)/Y$$

s.t. min phase lengths for peds/veh

$g_i = G_i + Y - t_L$

G_i = actual green time for phase i (sec)

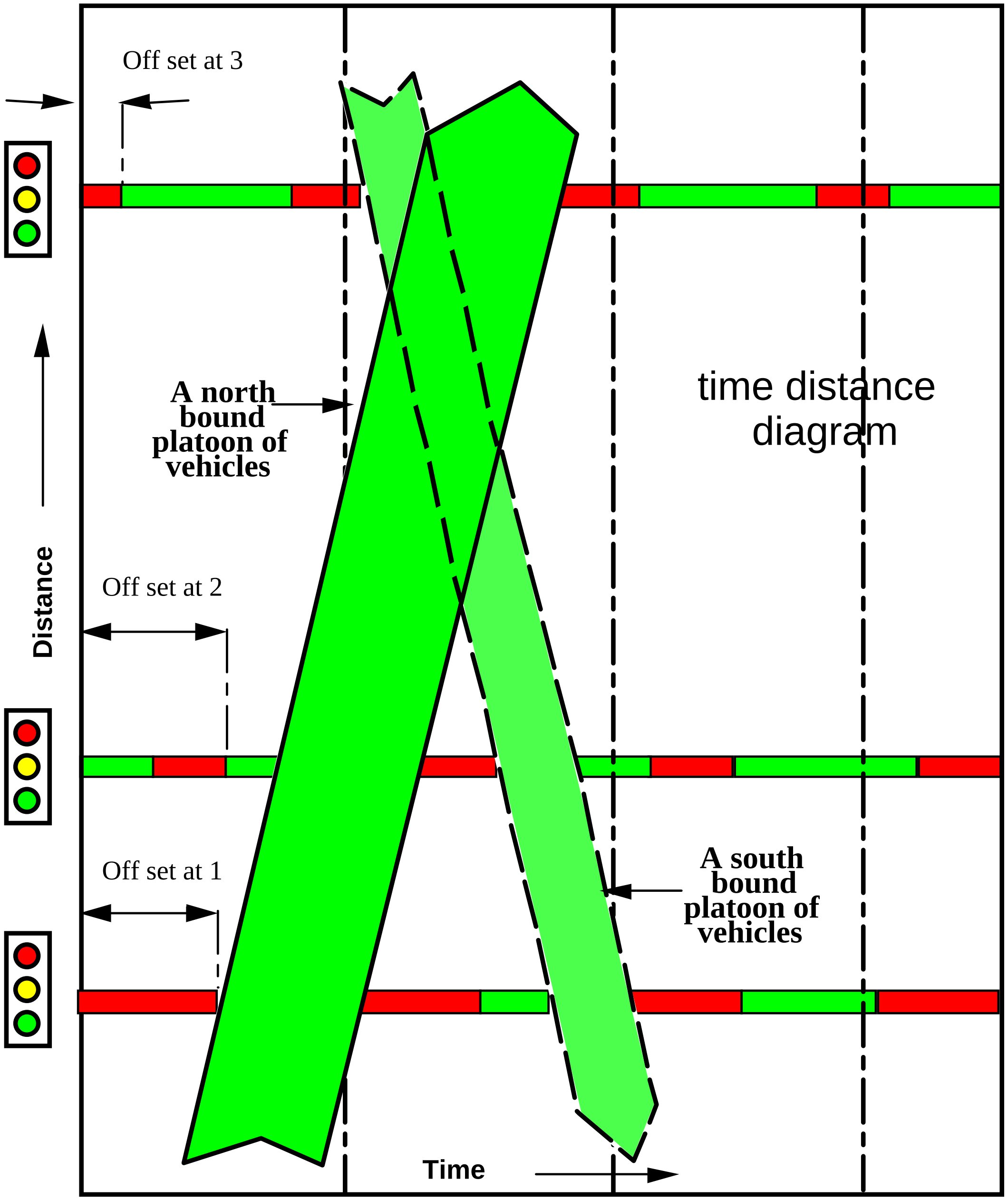
Y_i = clearance = yellow + all-red time (sec)

t_L = total lost time for phase (sec)

C = cycle length (sec)

L = lost time per cycle (sec)

Coordinated Signals (Arterials/Networks): Signal Control Parameters



System Cycle length C

Phase sequence ps_i

Green Times g_i

Offsets O_i

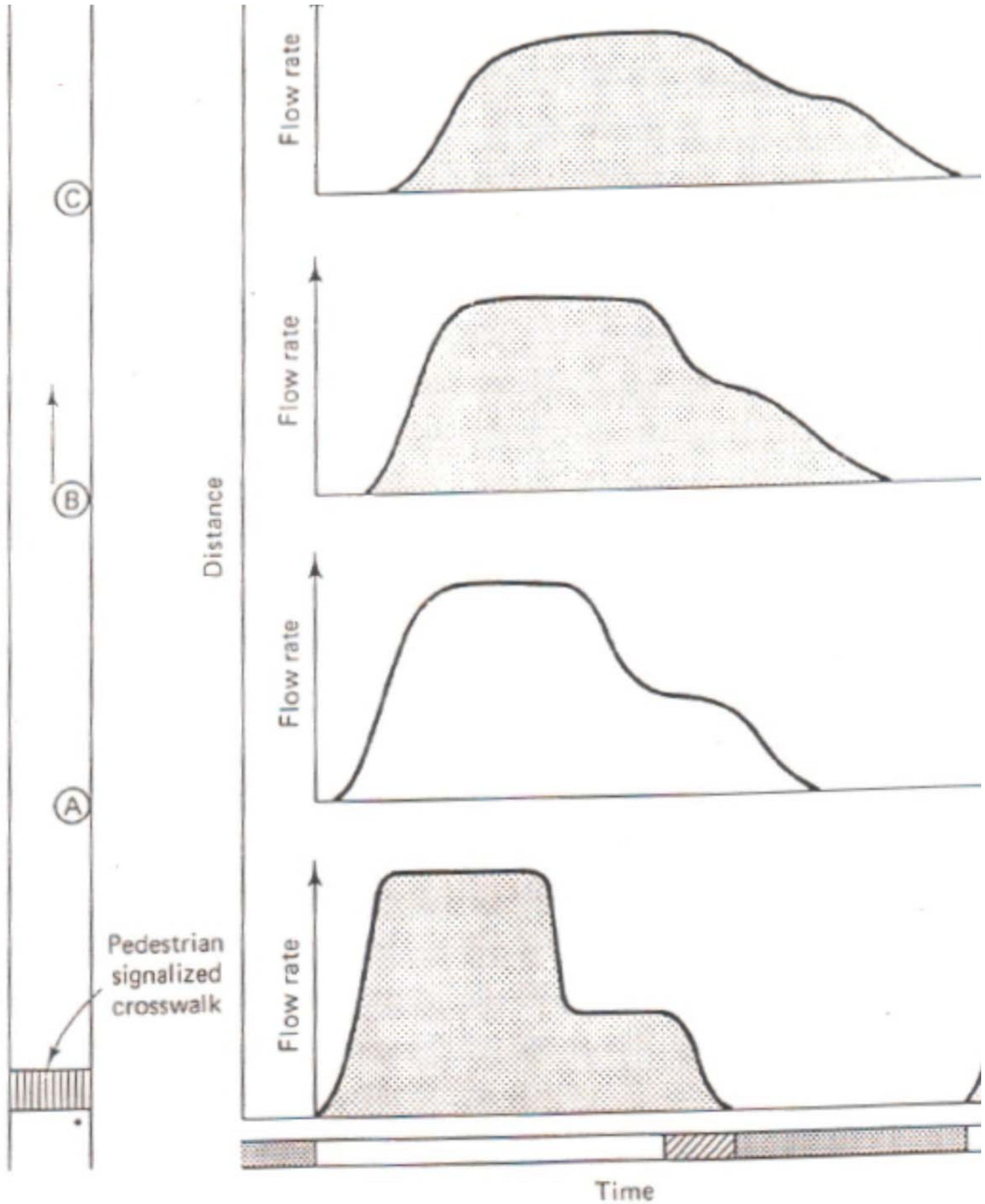
Fixed:

 # Phases

Yellow (all red) intervals

Minimum green times

Platoon Dispersion



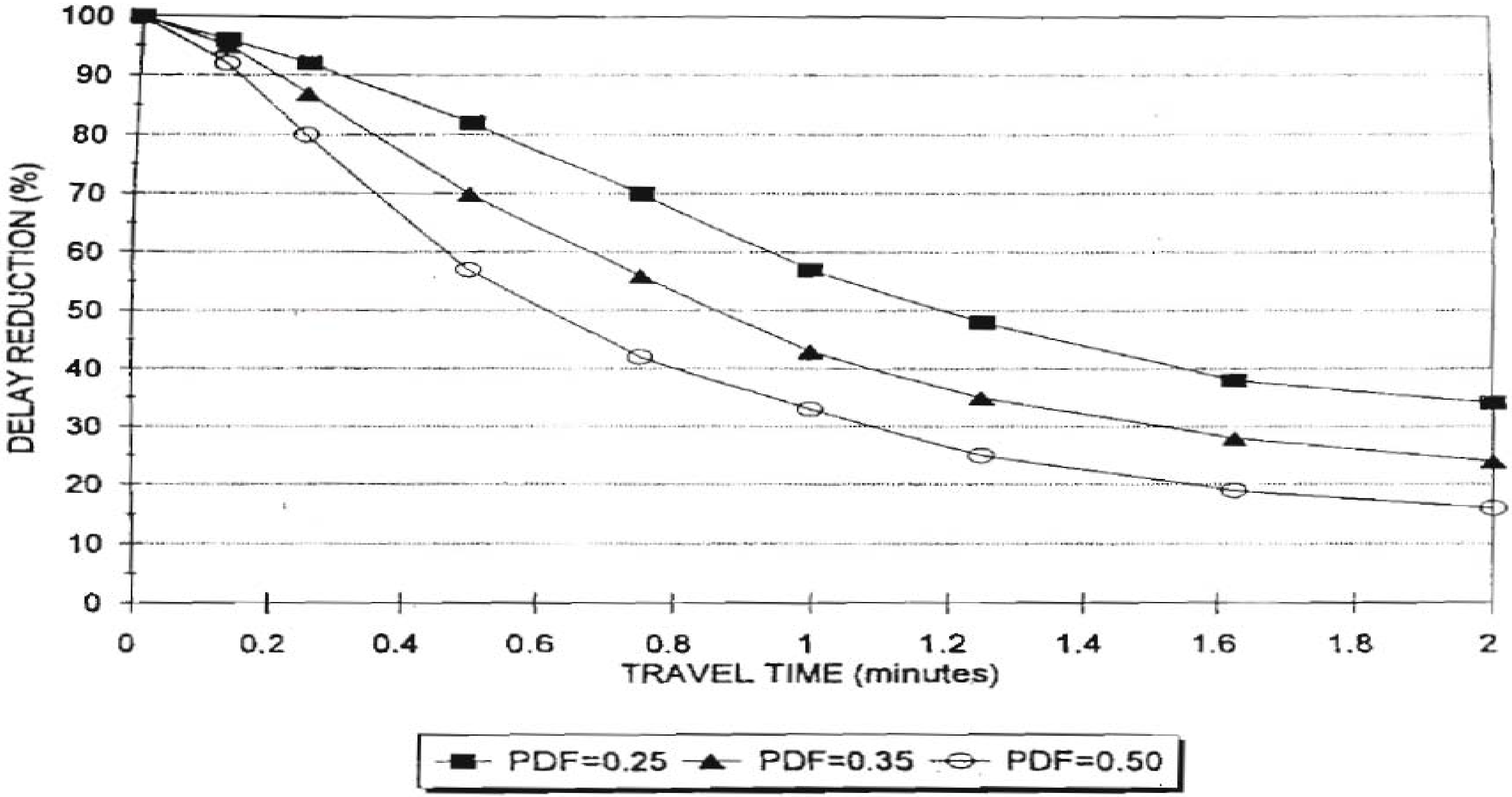
Robertson's platoon dispersion formula:

$q_2(i+t) = F\, q_1(i) + (1-F)\, q_2(i+1+t)$

where:
$F = 1/(1+\alpha t)$
$t = 0.8TT$, $\alpha = 0.50$

α platoon dispersion factor (PDF)

Benefits of Signal Coordination



Concepts: Cycle length

- All signals operate on a <u>common</u> cycle length
- Typically the critical intersection dictates the system cycle length
- Signals may operate on multiples of cycle length (half-cycle)

Concepts: Offsets

- Time difference between two reference points
- Defined by phase and interval
- Value: 0 ~ cycle length (seconds)

The sum of offsets in two directions must be equal to an integer number of cycles

$Off_{ij} + Off_{ji} = n\,C$

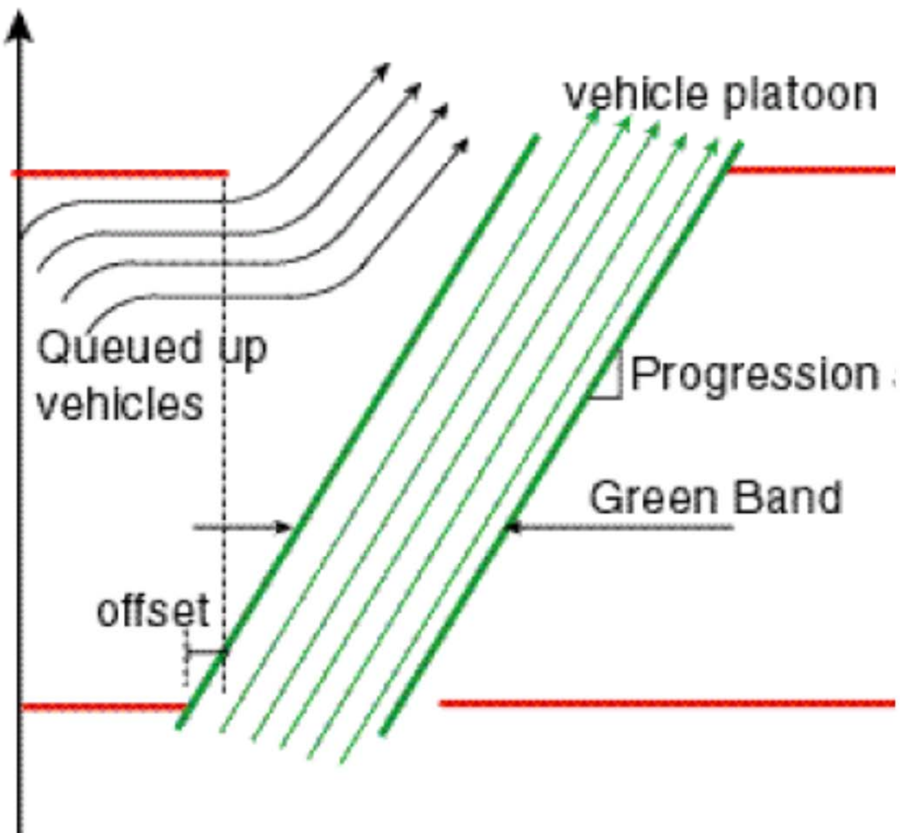
Ideal Offsets:

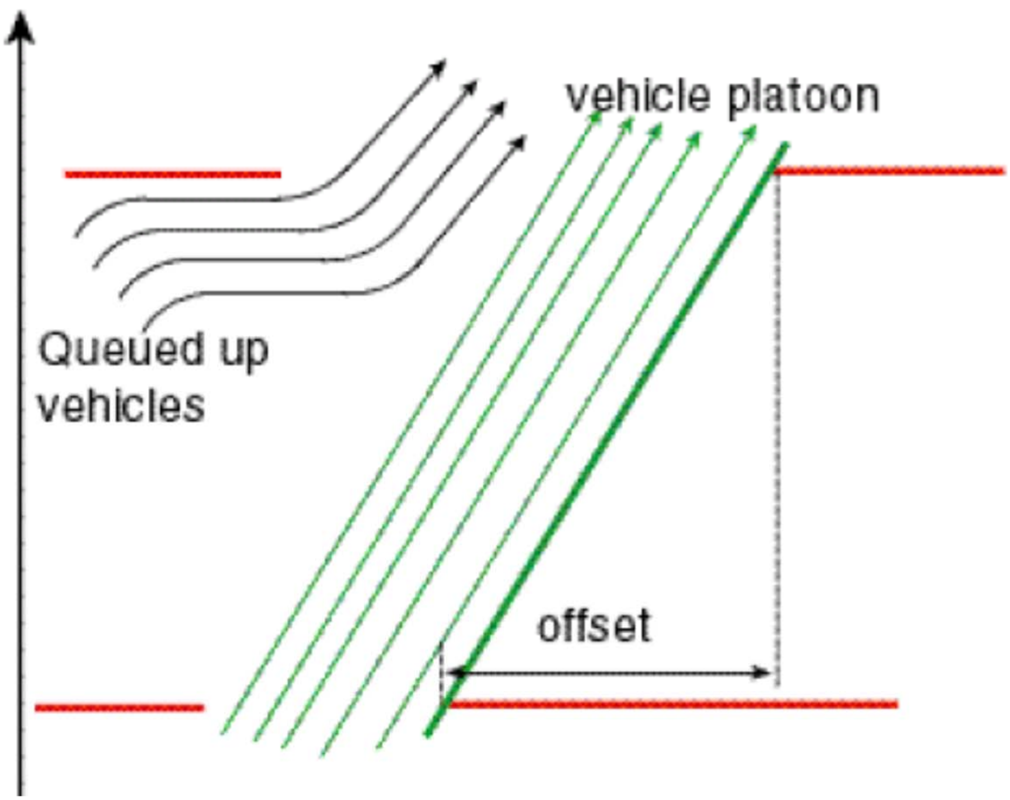
Offset = L/v

L: Signal spacing (ft)

V: Speed (ft/sec)

Concepts: Progression

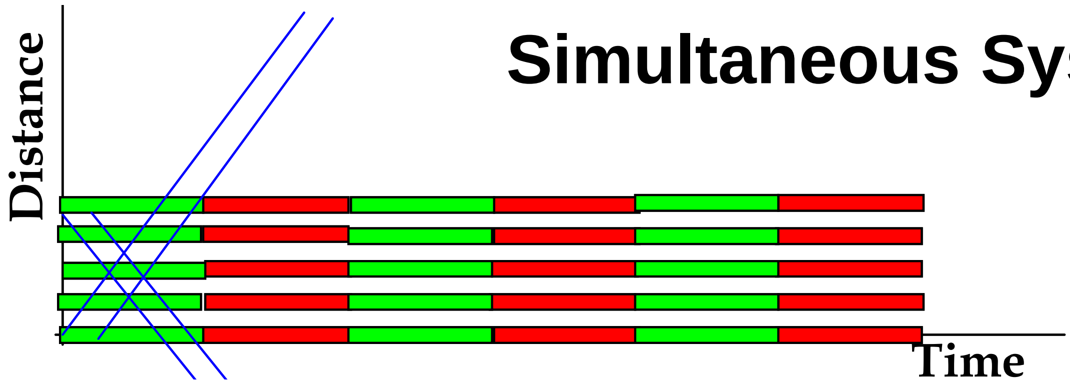


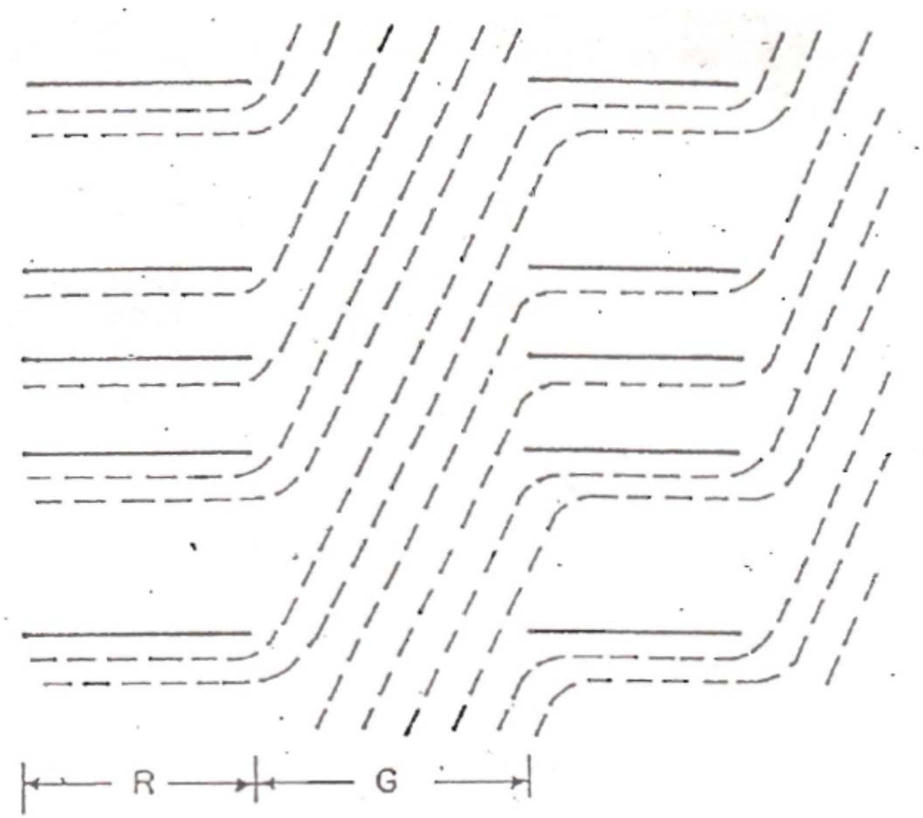


Offset measured at the start of green

Offset measured at the end of green

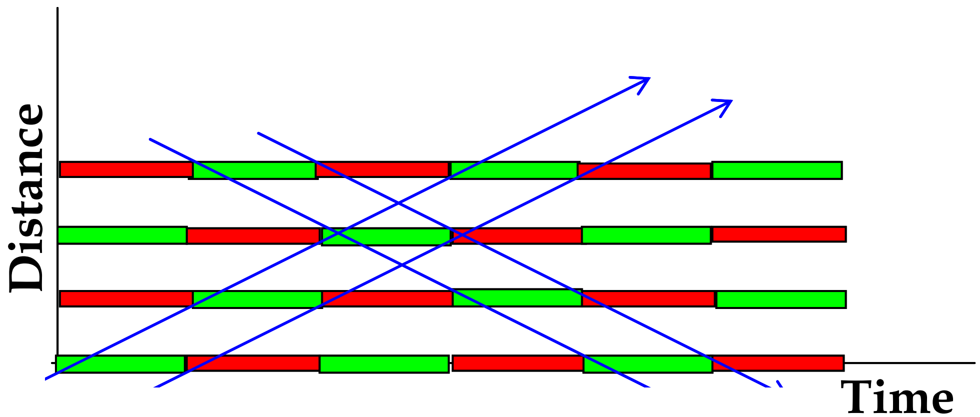
Common Synchronization Schemes (1)





Common Synchronization Schemes (2)

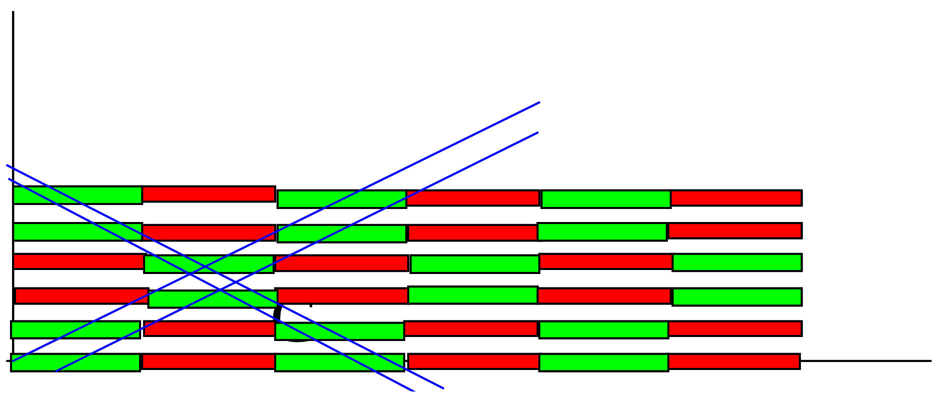
- Traffic demands per travel direction are balanced
- The signal spacing is approximately equal



Alternate System

C = 2L/v

Bandwidth = green time



Double Alternate System

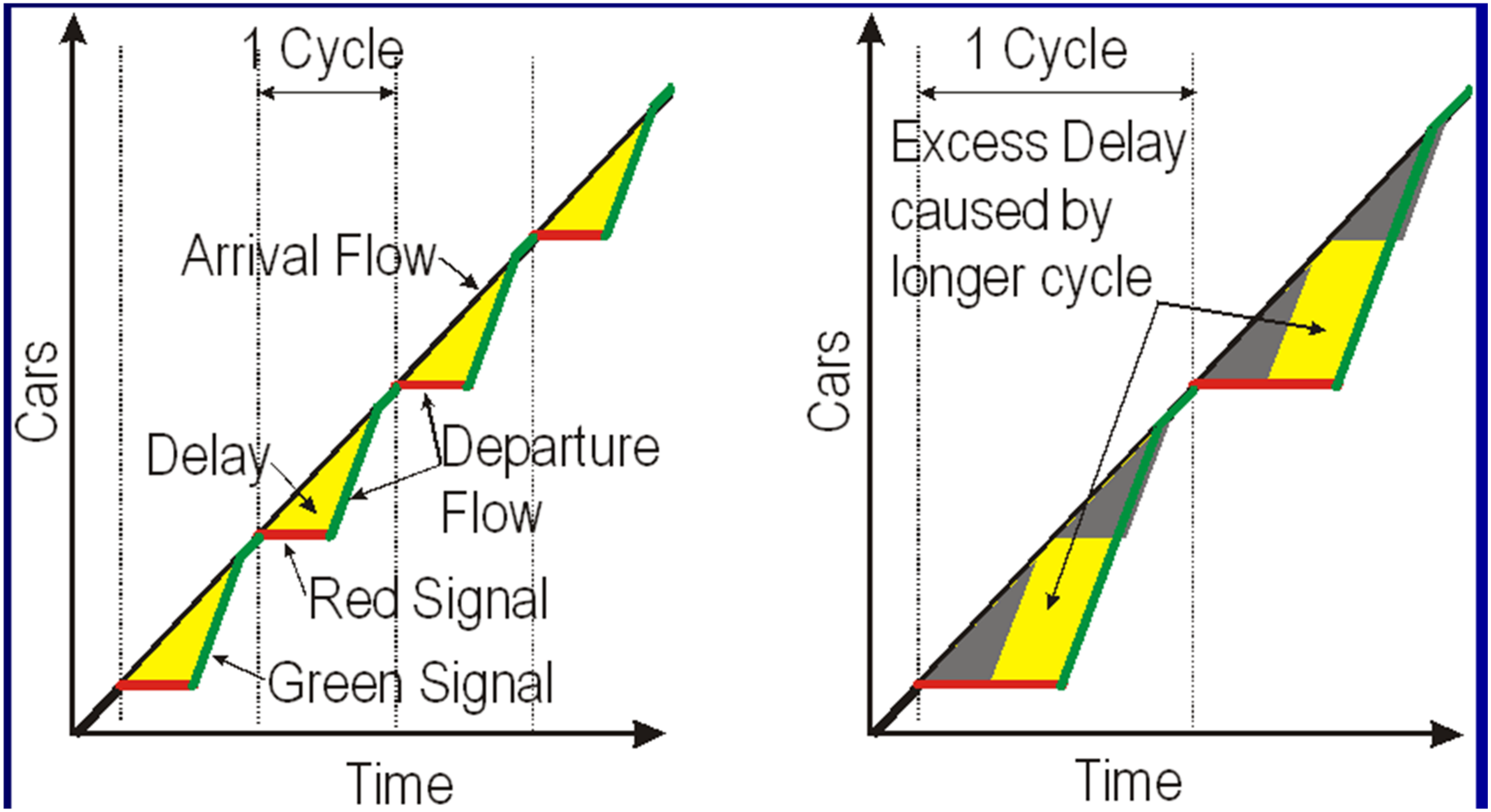
C = 4L/v

Bandwidth = (green time)/2

C = system cycle length (sec)
L = signal spacing (ft)
V = average link speed (ft/sec)

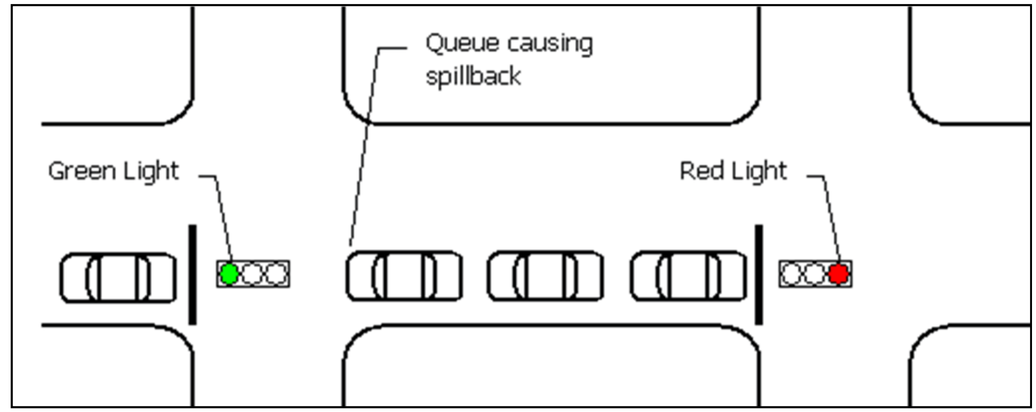
System Cycle Length: Trade-offs

Select Cycle length for Critical intersection

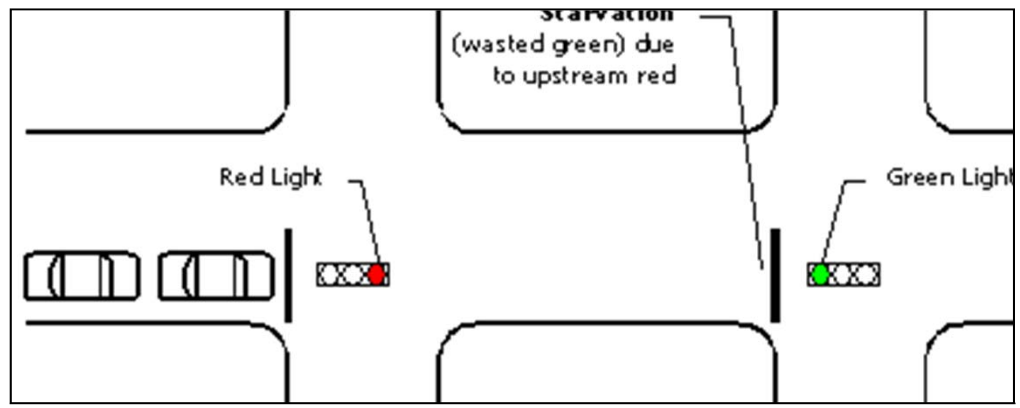


Offsets: Issues

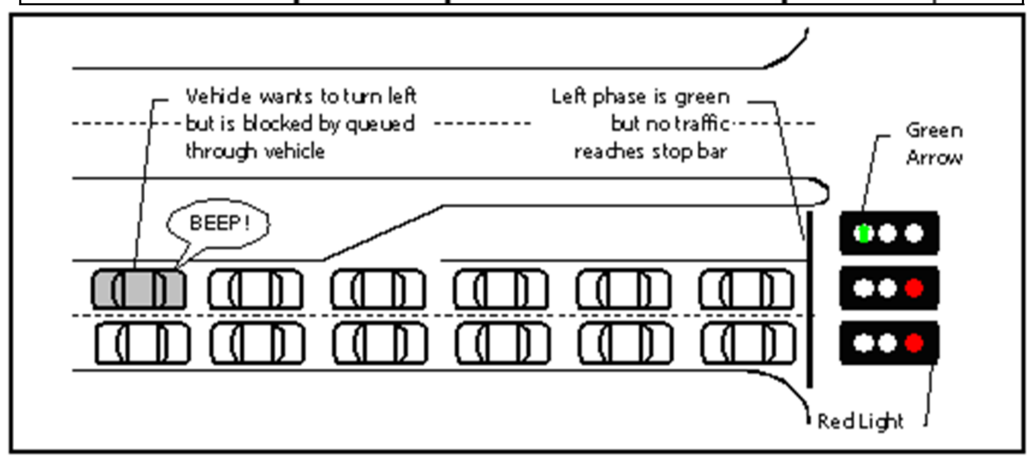
Incorporate Queue Clearance time



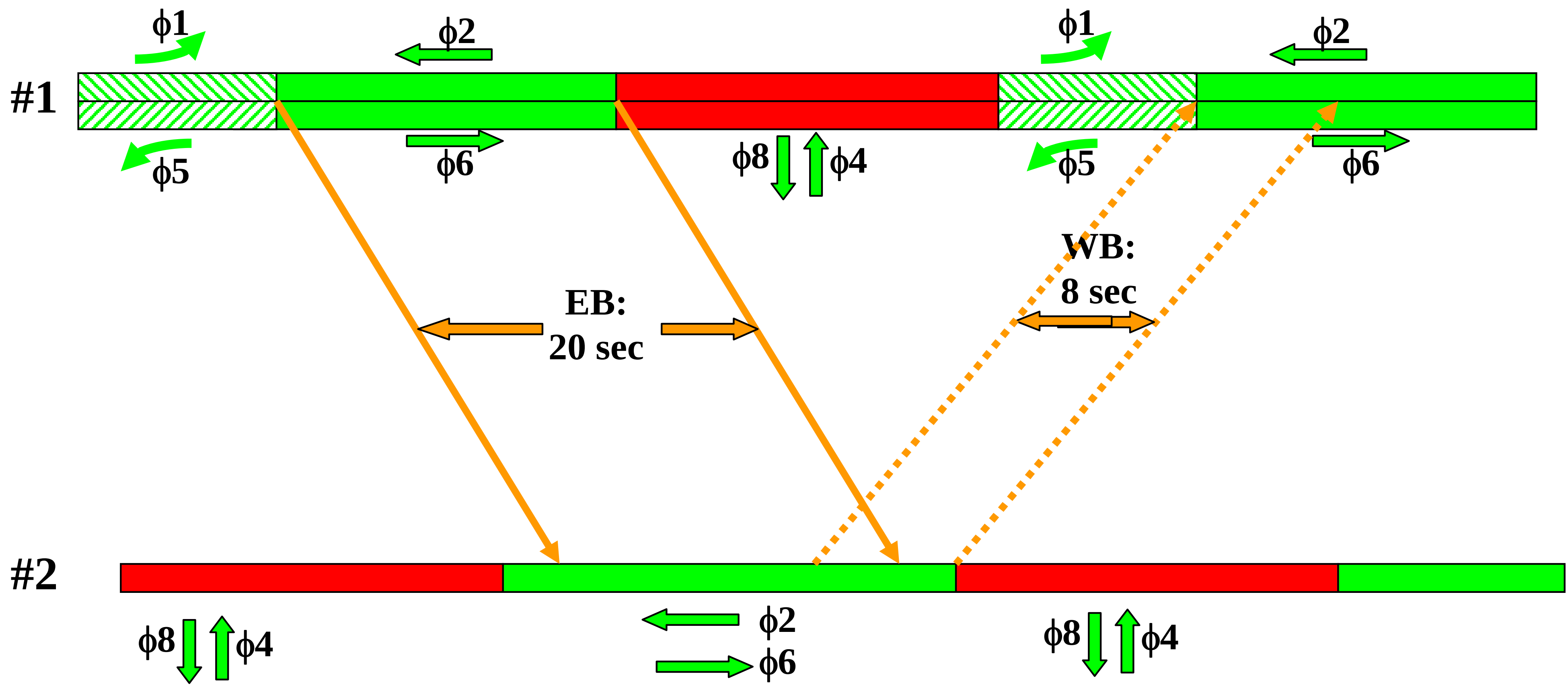
Reduce wasted Green time



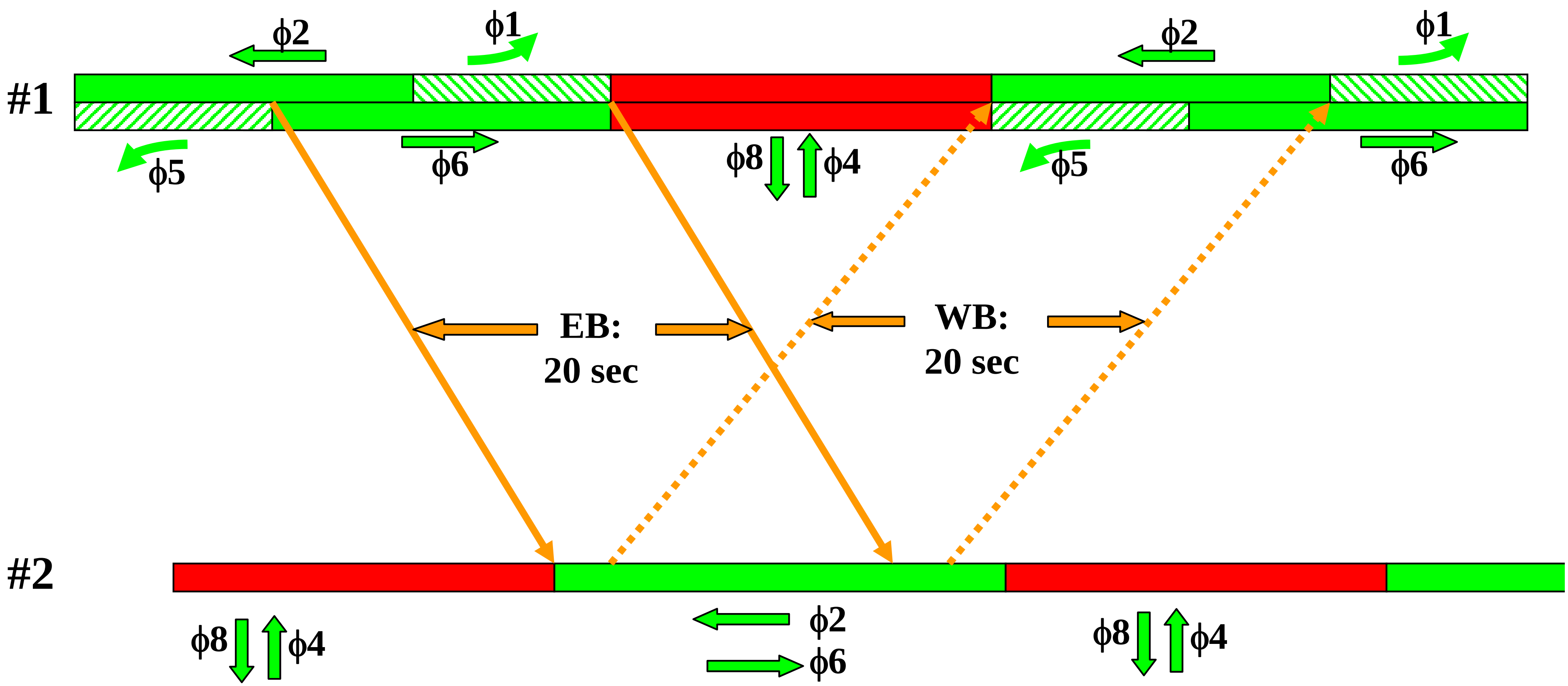
Reduce upstream blocking & consider Lag Left phasing sequence



Phase Sequence & Arterial Progression (1) Leading Lefts



Phase Sequence & Arterial Progression (2) Lead-Lag



Optimization Objective Functions

Maximum Bandwidth

- **Efficiency:** Proportion of the cycle length used by the bandwidth

    Efficiency (%) = (Bandwidth /2*Cycle Length)

- **Attainability:** Proportion of the through green utilized by the bandwidth (%)

    Attainability (%)= (Bandwidth/Min. Artery Through Green)

Minimum Delay and Stops

Signal Timing Optimization Models

Model	Objective Function	Control Parameters
TRANSYT-7F	Min Delay & Stops	C, g, O
PASSER (MAXBAND)	Max Bandwidth	C, ps, O
TSD4	Max Bandwidth	ps, O
SYNCHRO	Min Delay	C, ps, g,O

Example: PASSER IV-96 Model

- **Optimization Objective Function:**

Maximize $B = W_1 b_o + W_2 b_i$

b_o = outbound green band

b_i = inbound green band

W = weighting factors

subject to:

Minimum green times

- **Assumptions**

No Platoon Dispersion

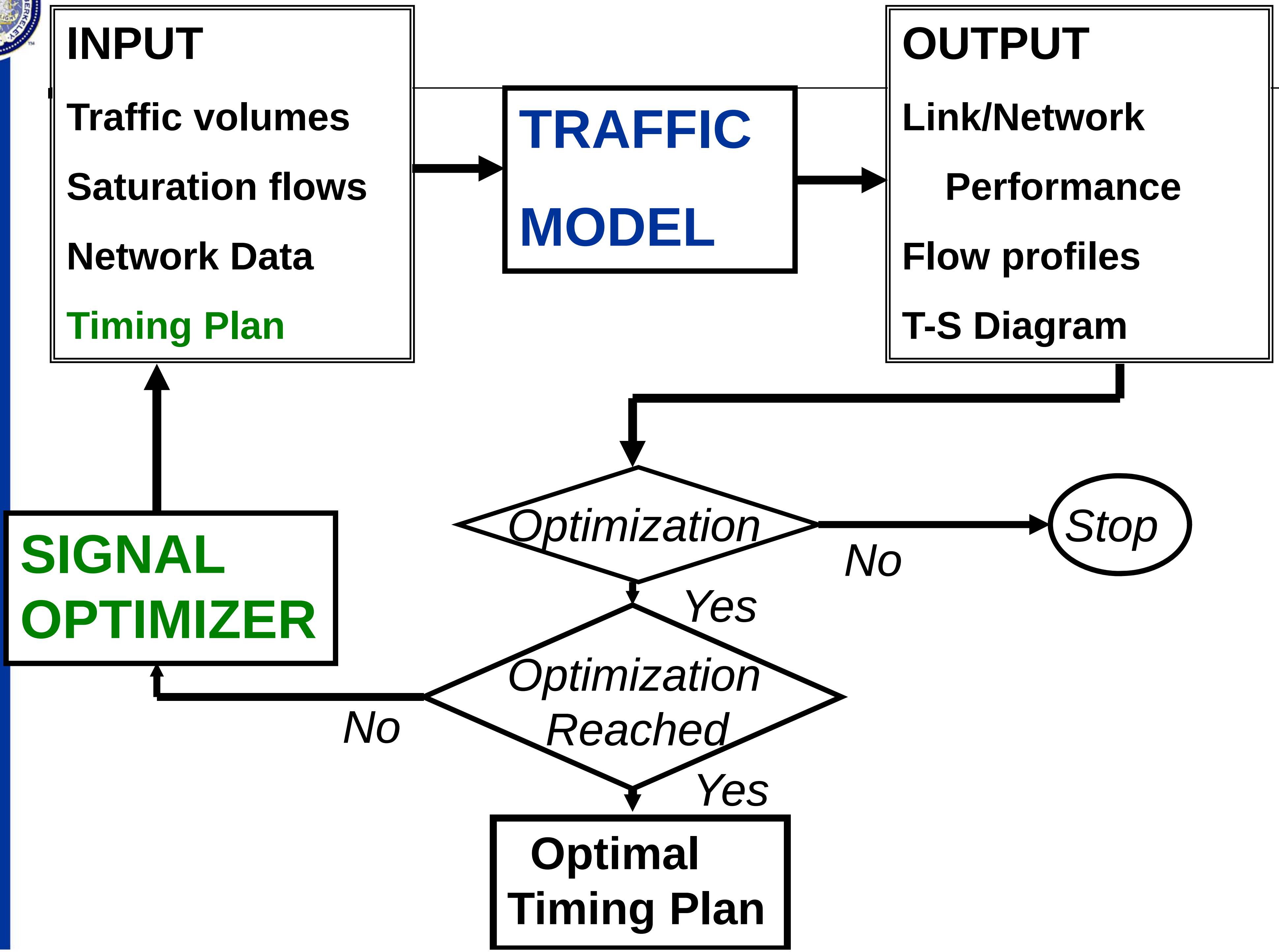
Turning Traffic

Green Times calculated (Webster formula)

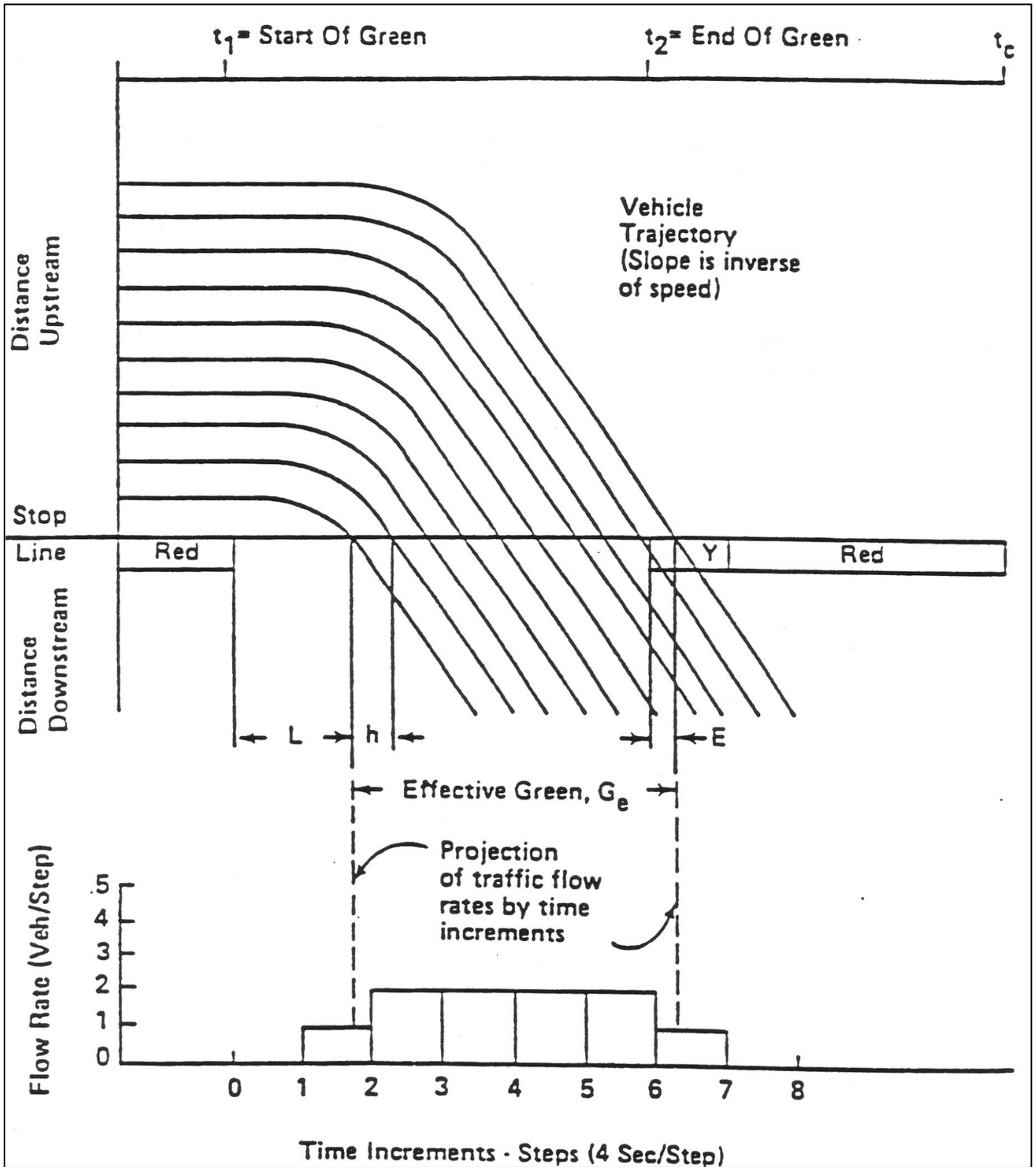
TRANSYT-7F Model

TRAffic Network StudY Tool

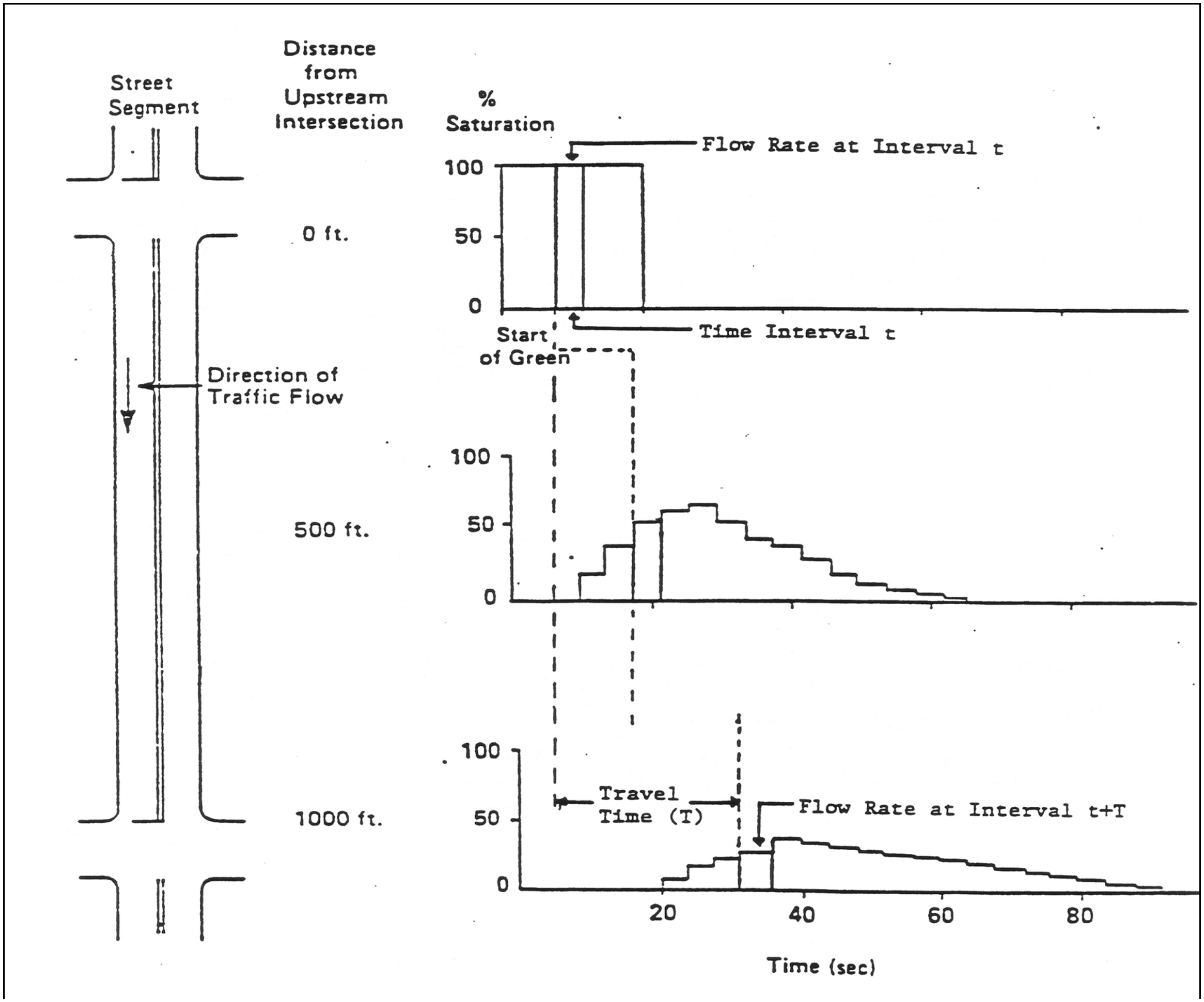
- **Macroscopic Deterministic Simulation**
- **Optimization of Cycle/Splits/Offsets**
- **Arterials/Grid Networks**
- **Multiple Vehicle Classes**


INPUT
Traffic volumes
Saturation flows
Network Data
Timing Plan
TRAFFIC
MODEL
OUTPUT
Link/Network
Performance
Flow profiles
T-S Diagram
Optimization
No
Stop
Yes
SIGNAL
OPTIMIZER
Optimization
Reached
No
Yes
Optimal
Timing Plan

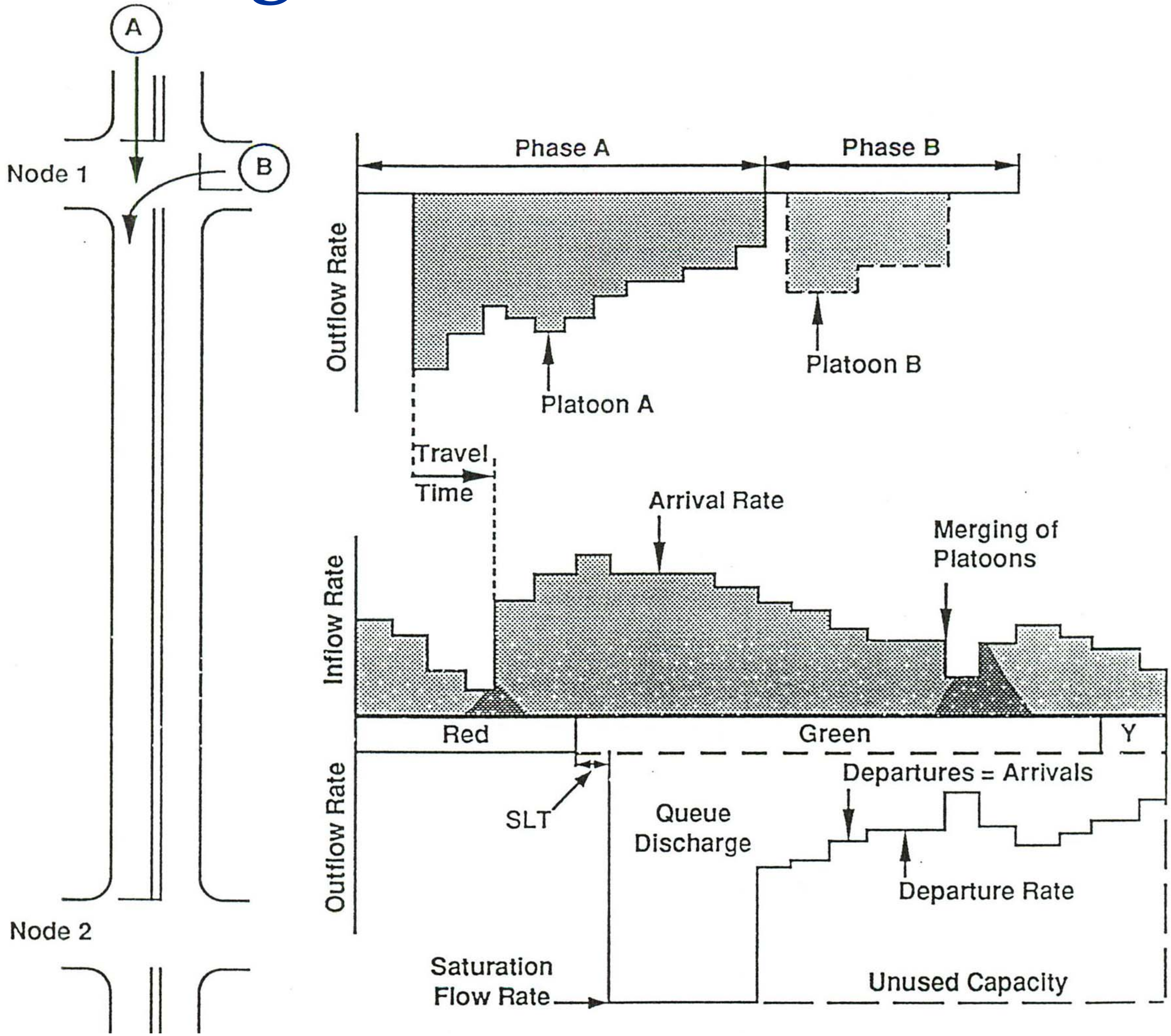
Modeling Traffic Flow--Stopline



Modeling Traffic Flow — Platoons (1)



Modeling Traffic Flow — Platoons (2)



TRANSYT-7F: Signal Optimization

$$PI = \sum_{i=1}^{N} W_{Di} \; D_i + K W_{si} \; S_i$$

Di: delay on link i (veh-hr)

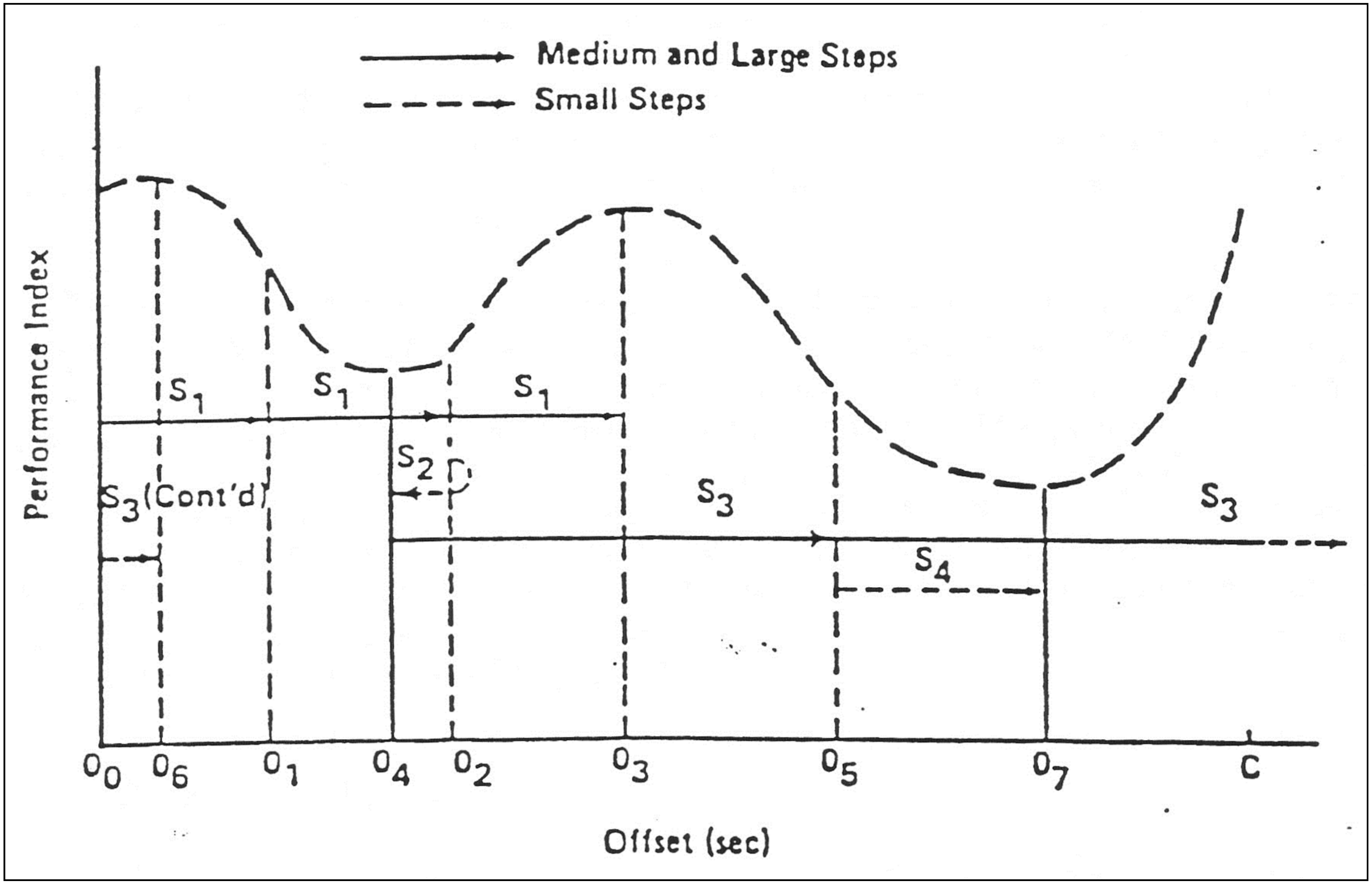
Si: number of stops on link i (veh/sec)

K: system-wide "stop penalty": the weight given to number of stops relative to delay.

W: weighting factors (default =1)

TRANSYT-7F Optimization Process

- **Hill Climbing algorithm**
- **Global Optimum not guaranteed**



SYNCHRO Model

- **Capacity Analysis**

  HCM 2000, ICU, Signal/Stops

- **Signal Timing**

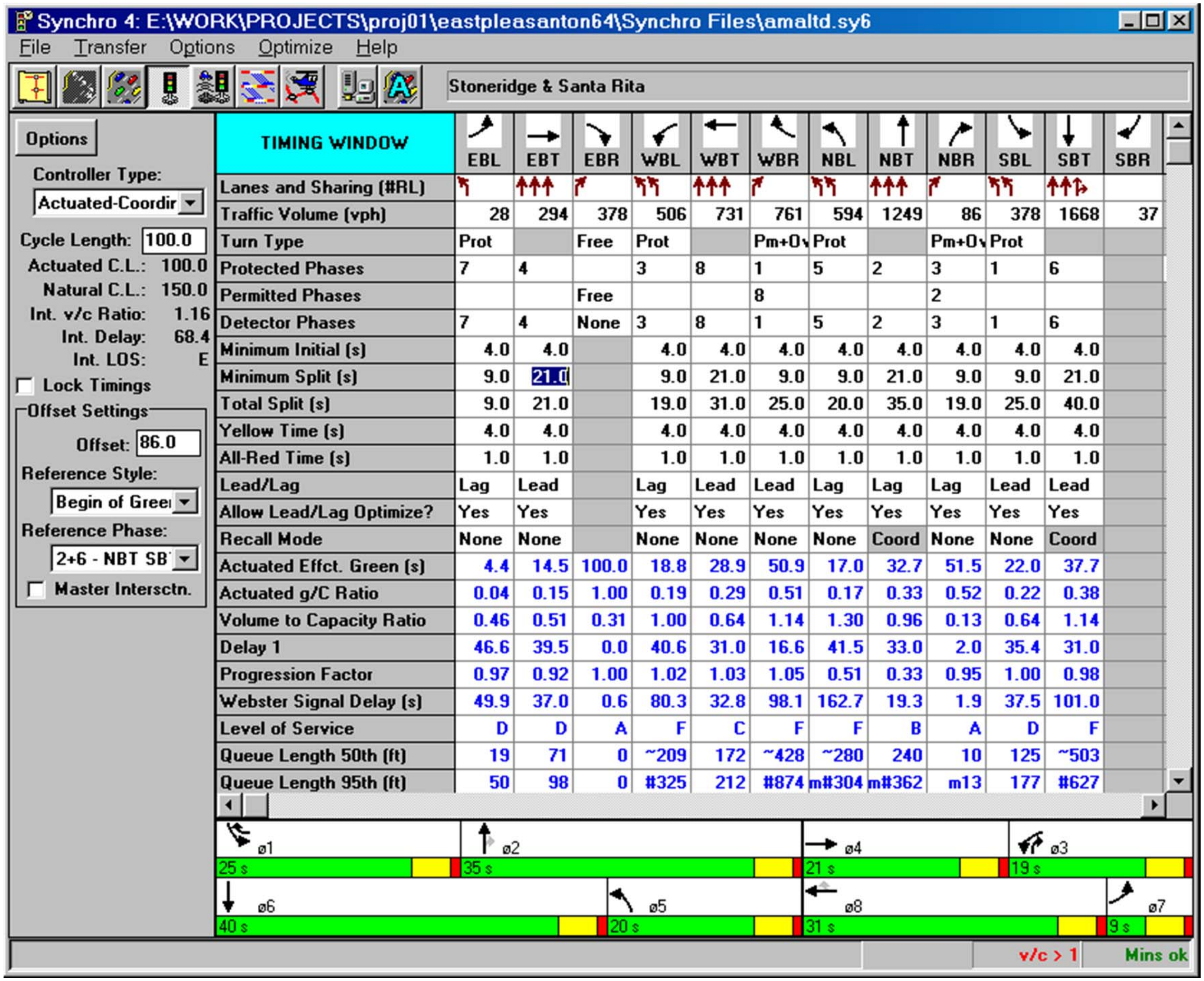
  Actuated Signals

  Optimization – ***Minimum Delay***

  Coordination

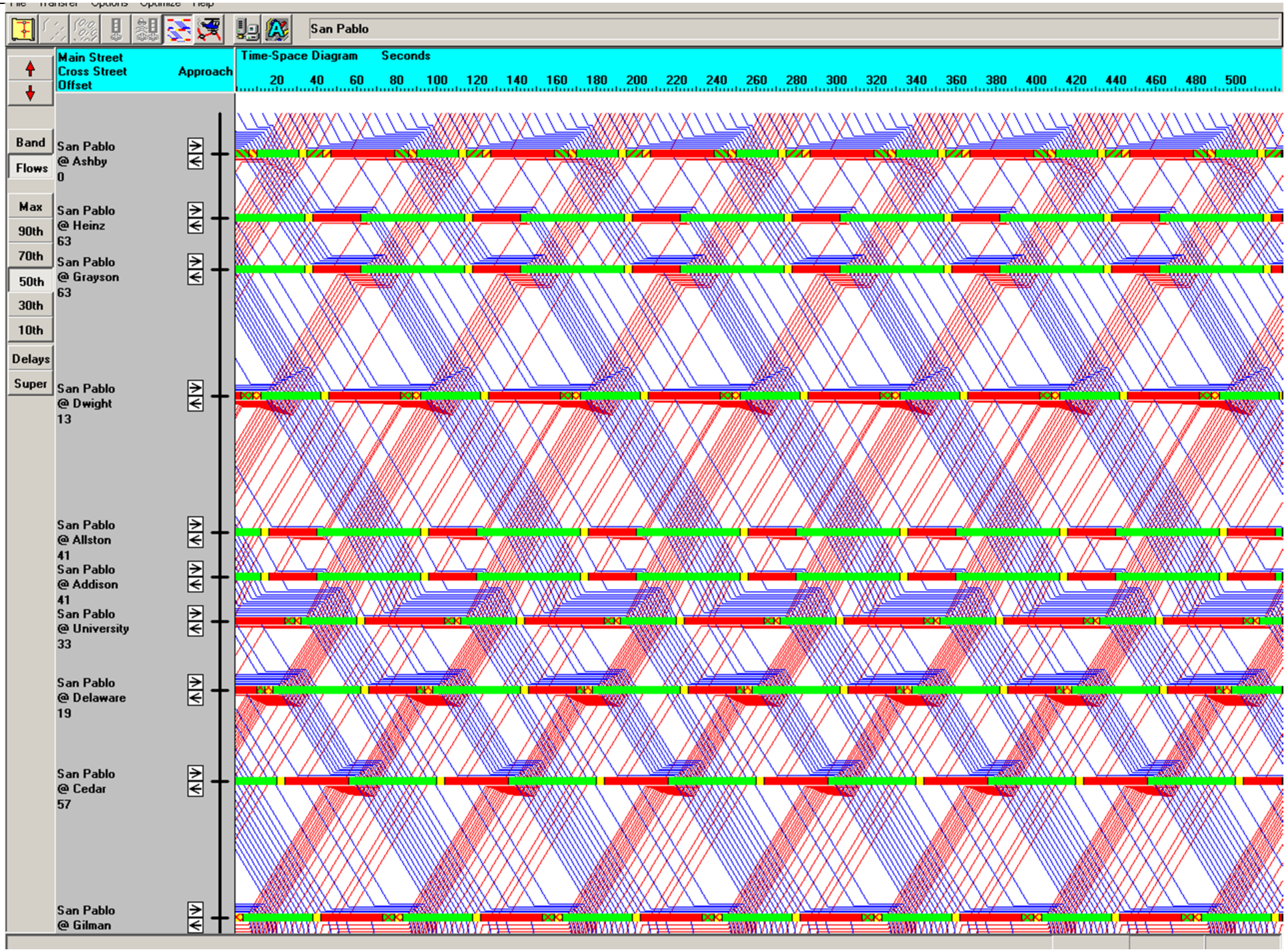
- **www.trafficware.com**

SYNCHRO Signal Timing Window

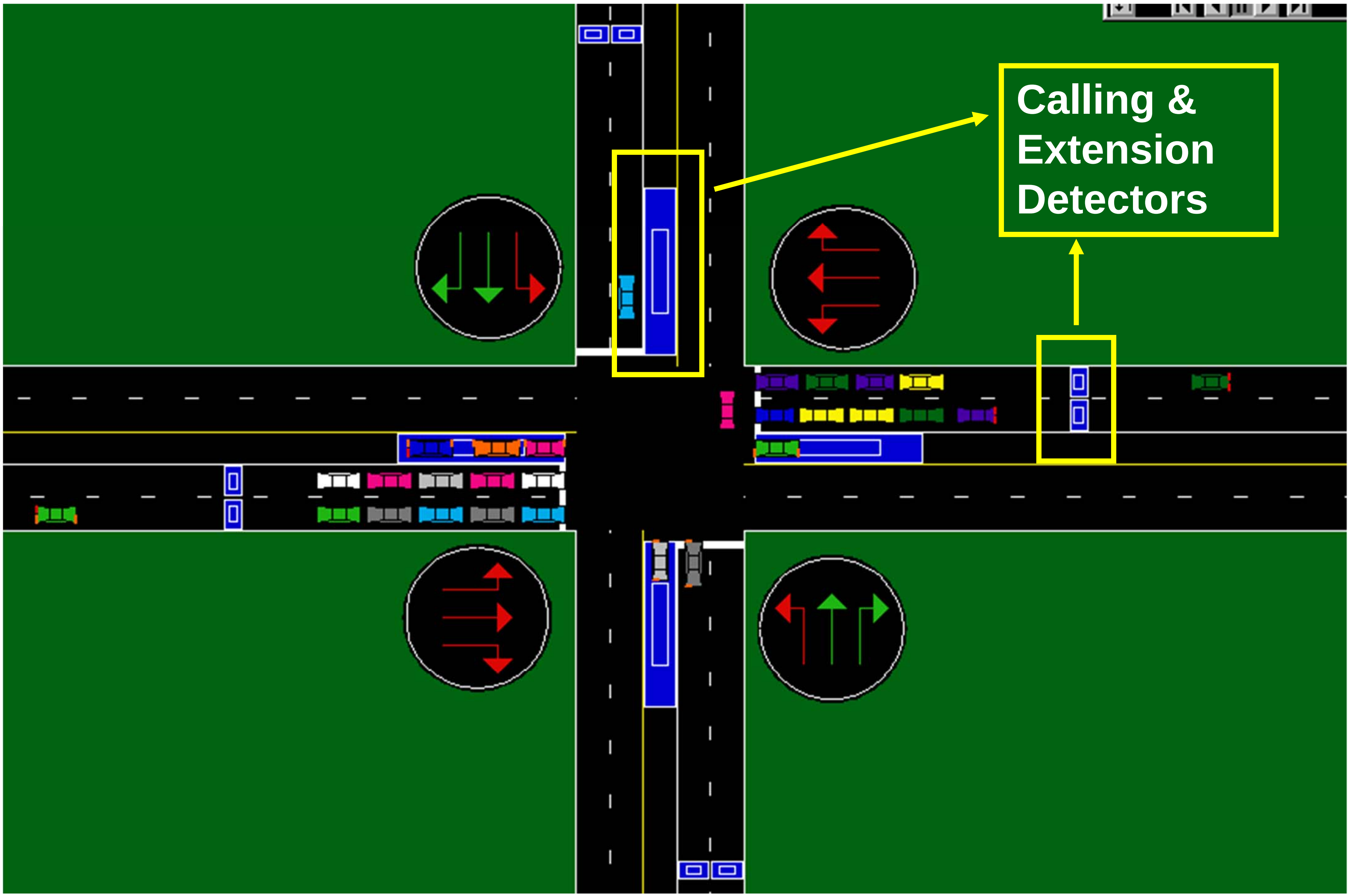


TIMING WINDOW	EBL	EBT	EBR	WBL	WBT	WBR	NBL	NBT	NBR	SBL	SBT	SBR
Lanes and Sharing (#RL)												
Traffic Volume (vph)	28	294	378	506	731	761	594	1249	86	378	1668	37
Turn Type	Prot		Free	Prot		Pm+Ov	Prot		Pm+Ov	Prot		
Protected Phases	7	4		3	8	1	5	2	3	1	6	
Permitted Phases			Free			8			2			
Detector Phases	7	4	None	3	8	1	5	2	3	1	6	
Minimum Initial (s)	4.0	4.0		4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	
Minimum Split (s)	9.0	21.0		9.0	21.0	9.0	9.0	21.0	9.0	9.0	21.0	
Total Split (s)	9.0	21.0		19.0	31.0	25.0	20.0	35.0	19.0	25.0	40.0	
Yellow Time (s)	4.0	4.0		4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	
All-Red Time (s)	1.0	1.0		1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
Lead/Lag	Lag	Lead		Lag	Lead	Lead	Lag	Lag	Lag	Lead	Lead	
Allow Lead/Lag Optimize?	Yes	Yes		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
Recall Mode	None	None		None	None	None	None	Coord	None	None	Coord	
Actuated Effct. Green (s)	4.4	14.5	100.0	18.8	28.9	50.9	17.0	32.7	51.5	22.0	37.7	
Actuated g/C Ratio	0.04	0.15	1.00	0.19	0.29	0.51	0.17	0.33	0.52	0.22	0.38	
Volume to Capacity Ratio	0.46	0.51	0.31	1.00	0.64	1.14	1.30	0.96	0.13	0.64	1.14	
Delay 1	46.6	39.5	0.0	40.6	31.0	16.6	41.5	33.0	2.0	35.4	31.0	
Progression Factor	0.97	0.92	1.00	1.02	1.03	1.05	0.51	0.33	0.95	1.00	0.98	
Webster Signal Delay (s)	49.9	37.0	0.6	80.3	32.8	98.1	162.7	19.3	1.9	37.5	101.0	
Level of Service	D	D	A	F	C	F	F	B	A	D	F	
Queue Length 50th (ft)	19	71	0	~209	172	~428	~280	240	10	125	~503	
Queue Length 95th (ft)	50	98	0	#325	212	#874	m#304	m#362	m13	177	#627	

SYNCHRO: Time-Space Diagram



Traffic Actuated Signal Control



Actuated Signals – Basic Signal Settings

- **Minimum Green**

  Time to serve vehicles in the queue (stopline to detector location)

  - 10 s main street through movements

- **Vehicle Extension (Gap/Passage Time)**

  - Advanced Loop Detectors:

    Vehicle extension (s) = DS/(1.47·V)

    DS = detection setback (ft)

    V = speed (mph)

  - **Stopbar Detection: *Vehicle extension close to zero***

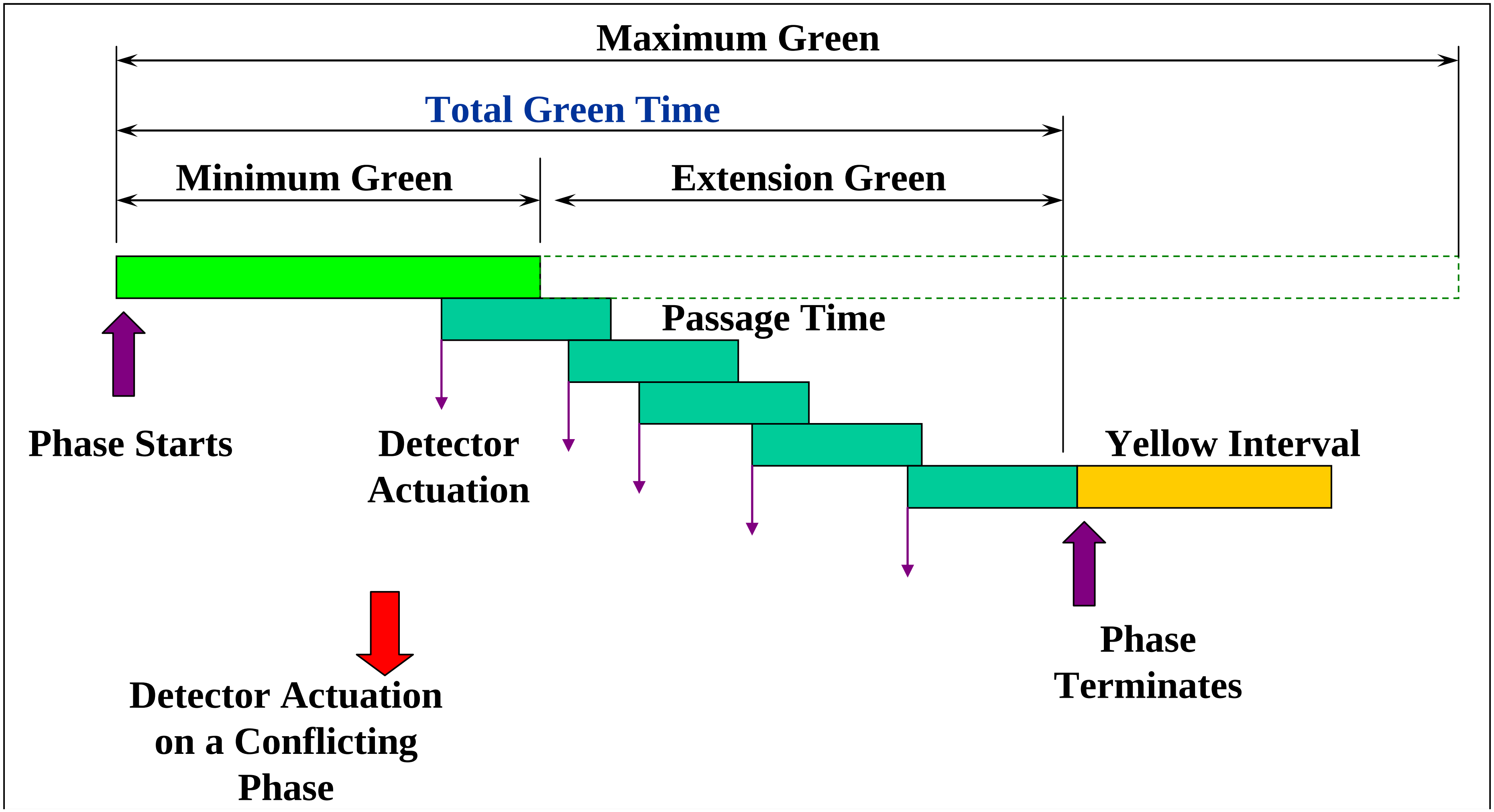
Maximum Green

Max green time allowed for the green phase

Maximum green time = 1.5 x Green time (X = 0.95)

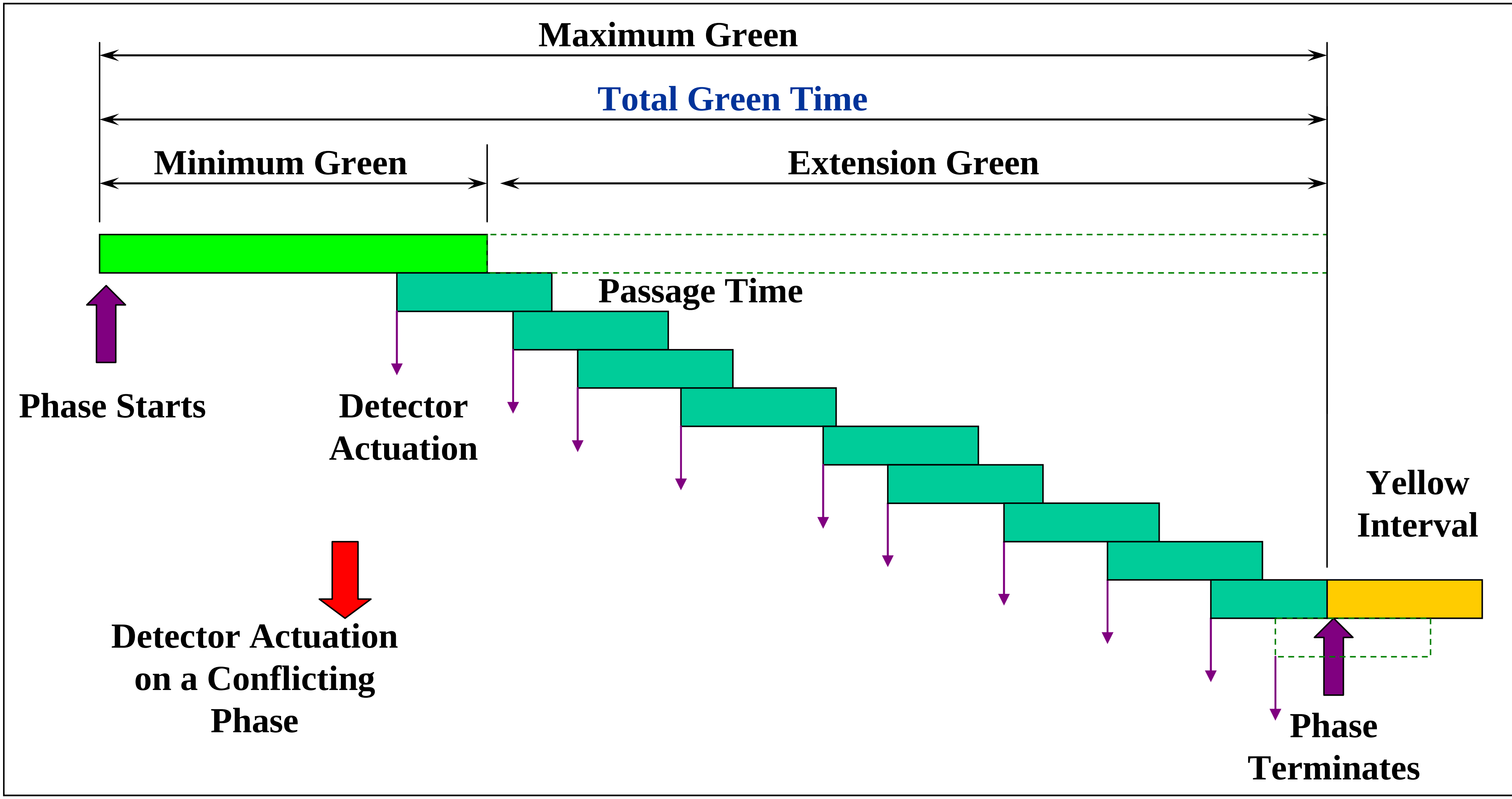
Operation of an Actuated Phase (1)

Case 1: Max Green Not Reached (Gap out)



Operation of an Actuated Phase (2)

Case 2: Max Green Reached (Max out)



NEMA/170/2070 (Volume/Density) Controllers

- **Variable Initial**

  Function of vehicle arrivals during yellow & all-red

- **Vehicle Extension (Gap) Reduction**

  Start with high gap time and reduce once traffic is moving

- **Max I/Max II:**

  Different Settings of Max Green

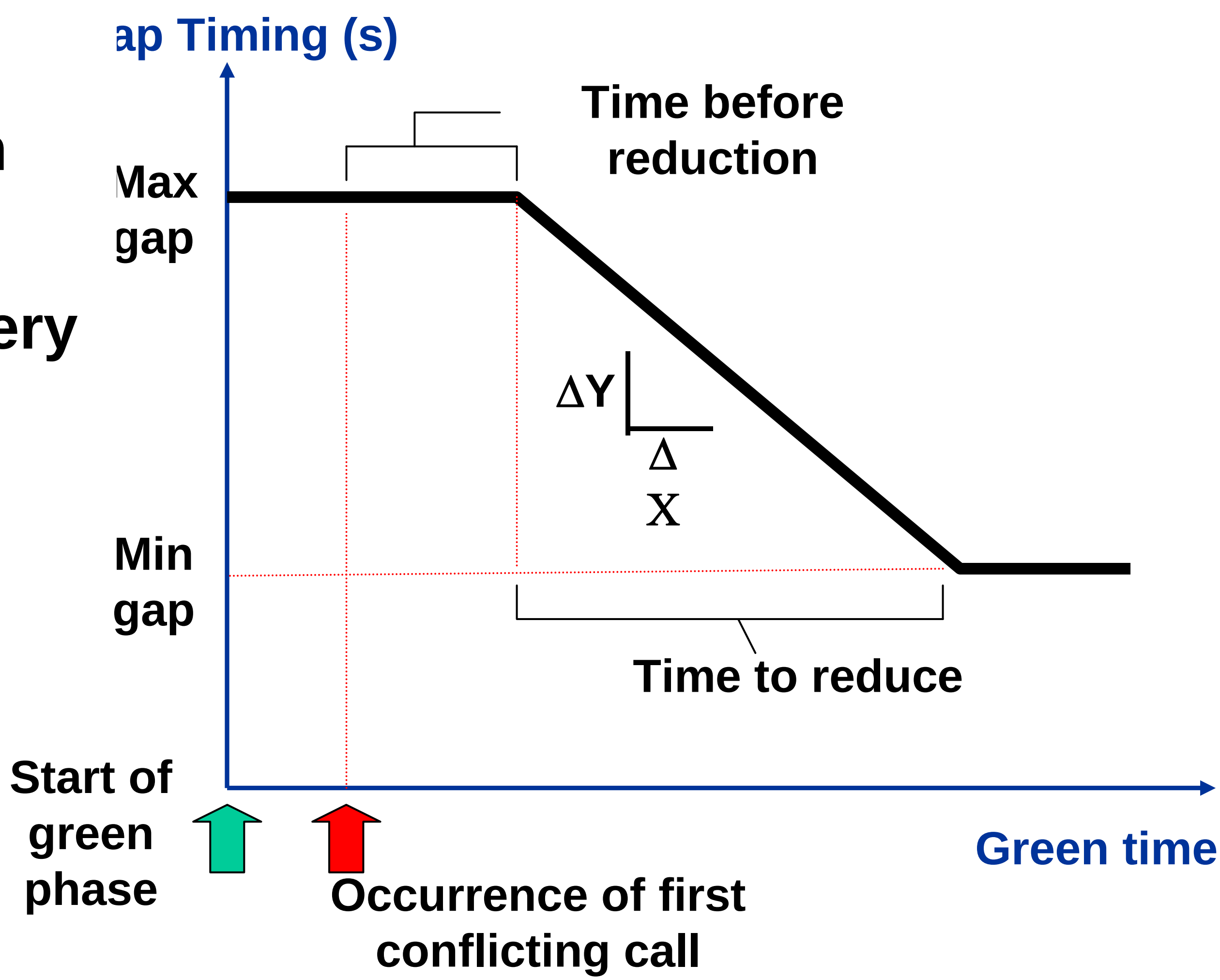
  Time-of-day or Coordinated/Free operation

Other Controller Functions

Phase skip
Recall: min, max, ped
Dual/single entry
Conditional service

Volume/Density Settings: Vehicle Extension

- Min and Max Gap
- Time Before Reduction
- Time to Reduce
- Reduce By/Reduce Every



Min gap = 2-3 sec

Max gap =
Min (Veh extension, 5 sec)

Time before reduction =
33 % of max green

Time to reduce =
80 % of max green

Coordination of Actuated Signals

Cycle Length (sec): All signals operate on a common fixed cycle. Typically the cycle length for the critical intersection. Usually determined for X = 0.95

Split (sec): The phase green + yellow + all red intervals

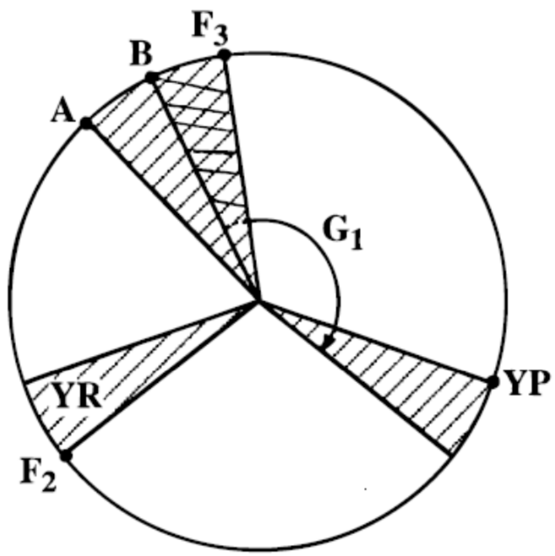
Offset or Yield Point (sec): the difference between a reference point (system zero) and the end (beginning) of the through (sync) phase green interval at the intersection.

Force Off (sec): fixed point in the signal cycle where actuated phases terminate.

Permissive Period (sec): amount of time (in seconds) within the cycle, during which non-coordinated, phases are allowed to be serviced

Coordinated Actuated Signals: Operation (1)

a. Phase 2 Is "Forced-Off," Phase 3 "Gaps-Out"

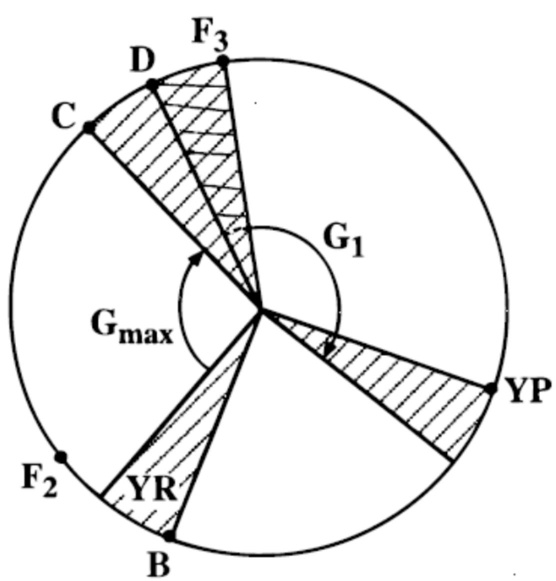


F_2 = Phase 2 Ends At The Force-Off Point

A = Phase 3 Ends Before Force-Off, F_3. Position Of Point A Depends On Vehicle Headways And Extension Interval (Gap).

BF_3 = Spare Green Time Available To The Sync Phase.

b. Phase 2 "Gaps-Out," Phase 3 "Maxes-Out"

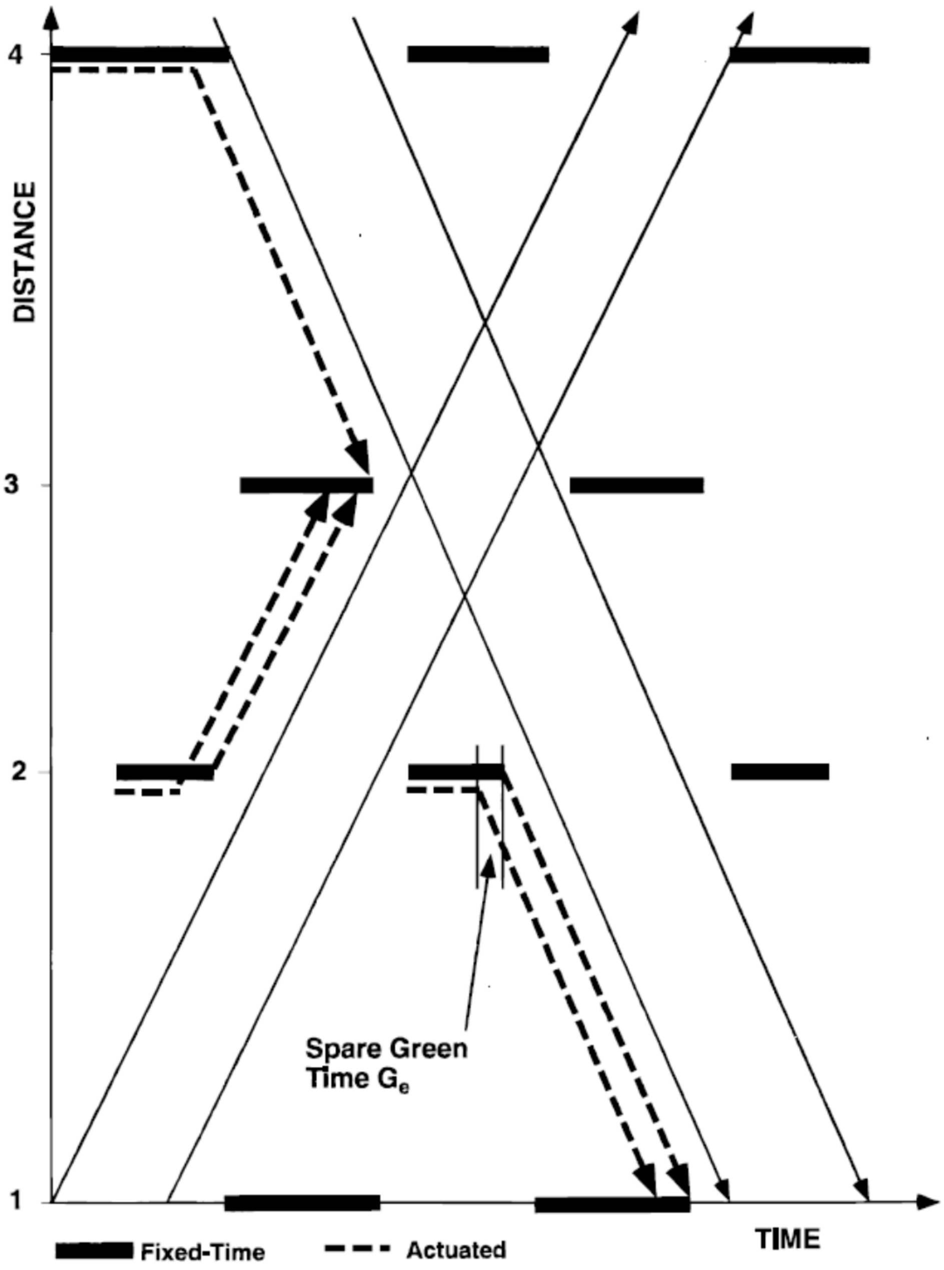


B = Phase 2 Ends Before Force-off, F_2

C = Phase 3 ends, sync phase starts. Position of point C depends on the max green **G_{max}** for Phase 3.

DF_3 = Spare Green Time Available To The Sync Phase

Coordinated Actuated Signals: Operation (2)



"Early Return" To Green Problem

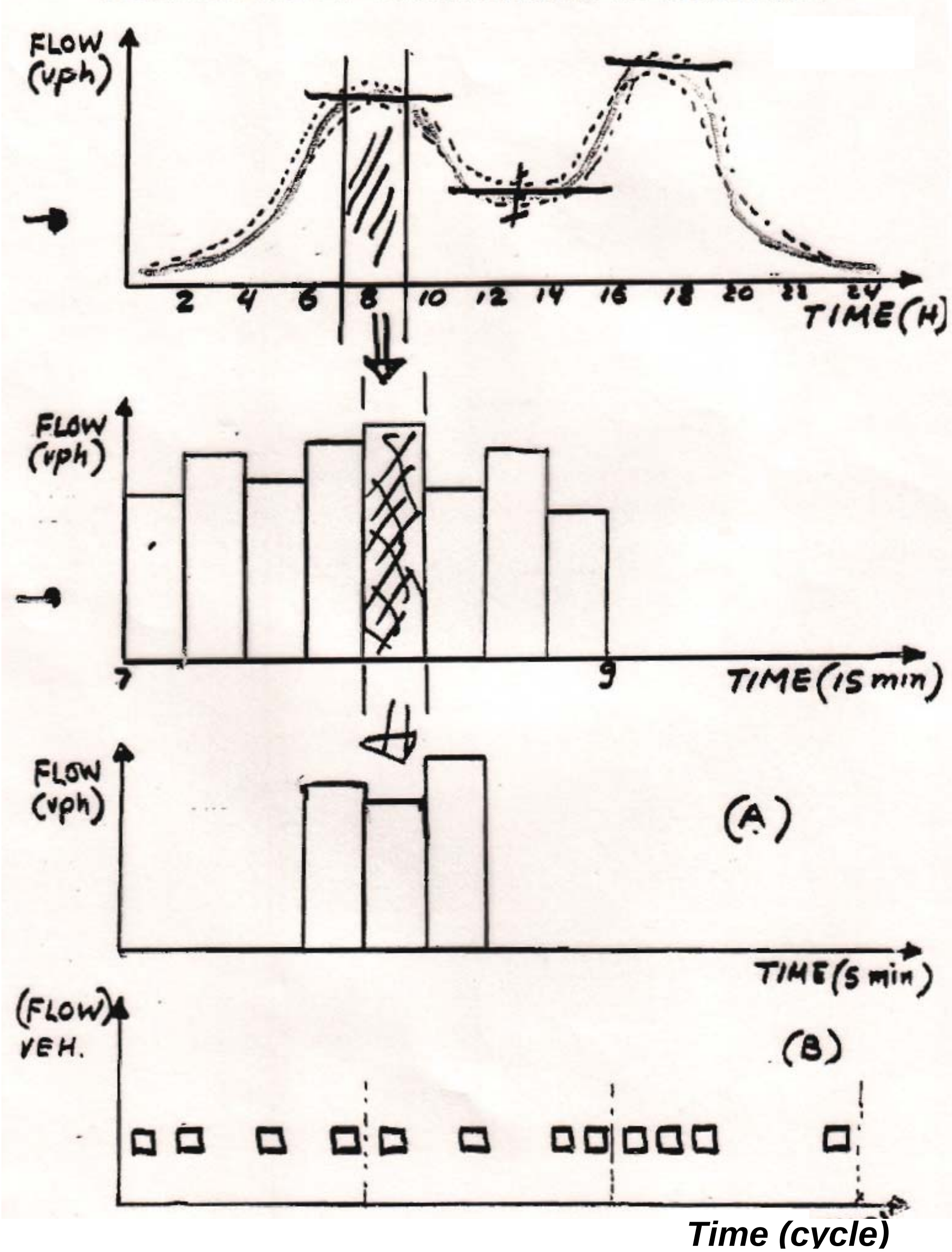
Traffic Flow Variability vs. Control

A
- Fixed-Time Plans
- Time of Day (TOD)
- No Detection
- May be actuated

B
- Fixed time plans
- Traffic responsive plan selection
- System detection

C
- Traffic responsive control
- On-line timing development
- Approach & system detection

D
- Adaptive control
- Measure & predict arrivals per cycle
- Extensive detection

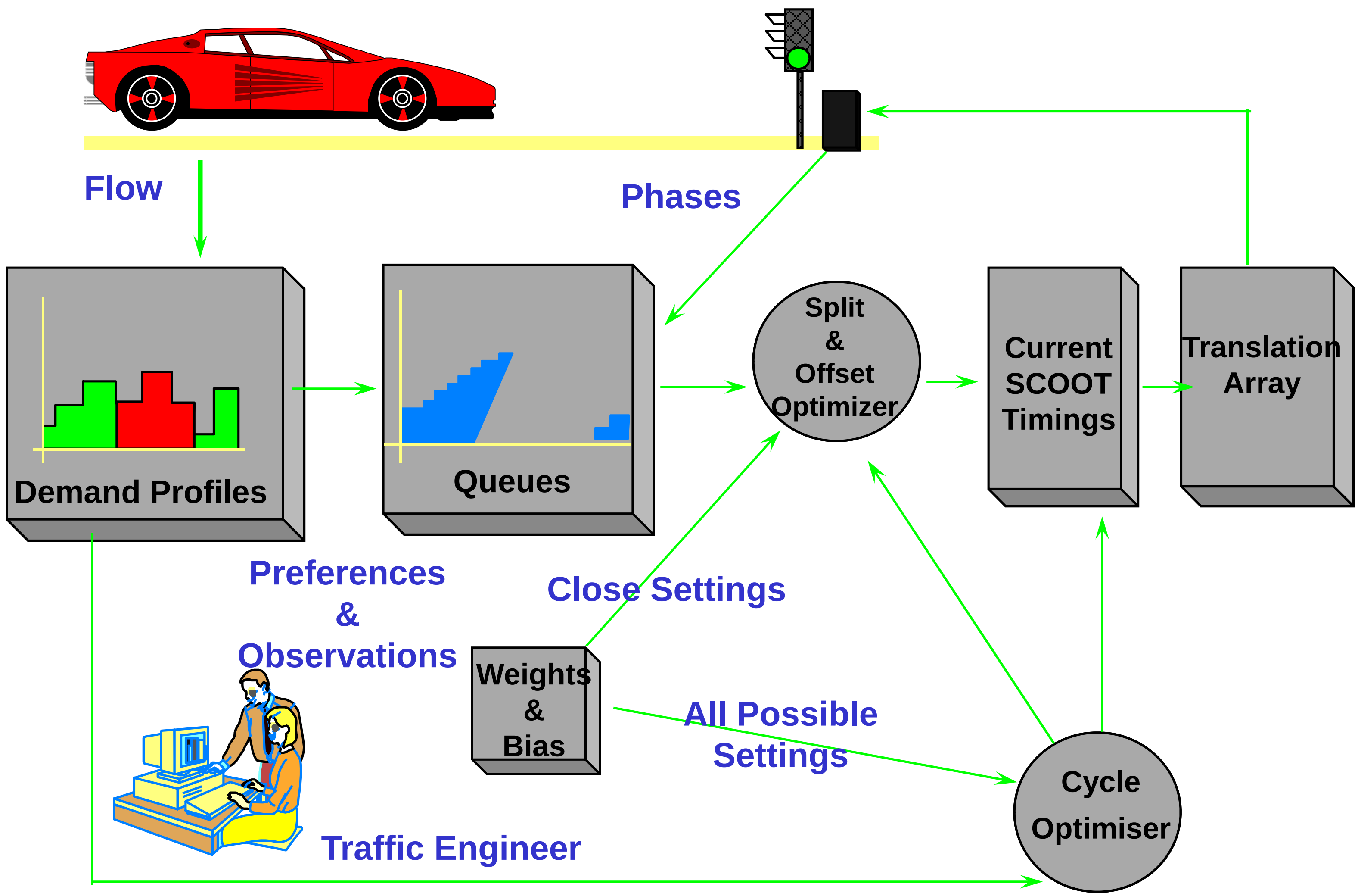


Overview: Traffic Control Strategies

CONTROL STRATEGY	TIMINGS DEVELOPED	TIMINGS IMPLEMENTED	TRAFFIC DATA	DETECTORS
1st Generation	Off-line	Time-of-Day (Traffic Responsive)	Historical	N/A System
1.5 Generation	Off-line	Operator	Estimated (Real-time)	System
2nd Generation (SCOOT, SCATS)	On-line	On-line	Predicted/ Real-Time	Approach
3rd Generation/Adaptive OPAC, RHODES	On-line	On-line	Predicted/ Real-Time	Approach

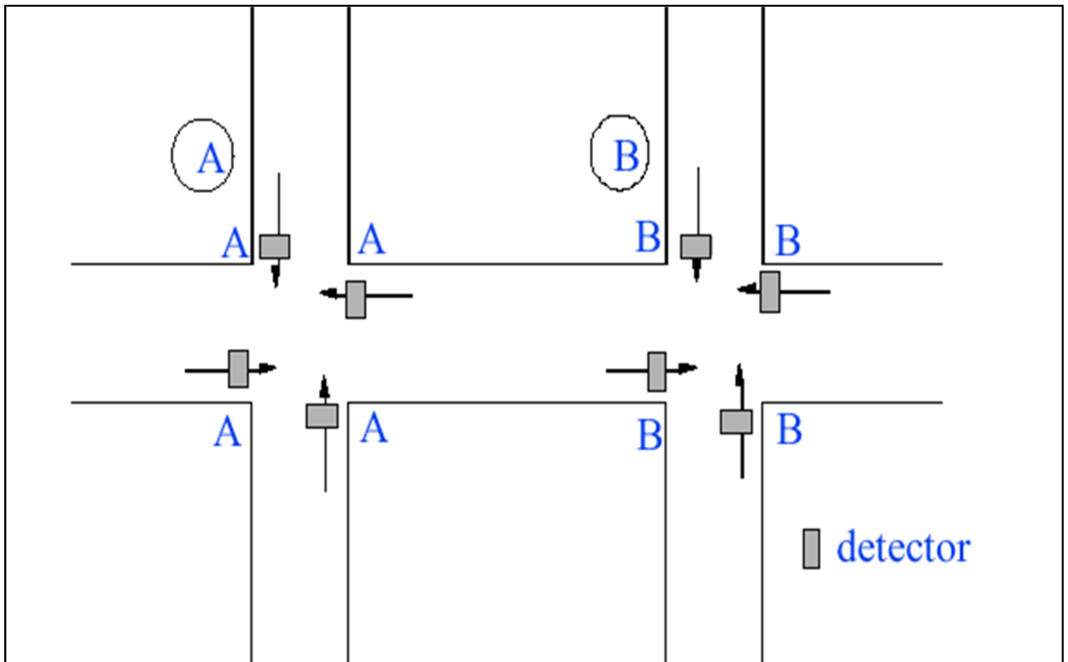
http://www.signalsystems.org.vt.edu/documents.html

SCOOT:
Split Cycle Offset Optimization Technique



SCATS
Sydney Coordinated Adaptive Traffic System

- **Stopline Detection**
- **Network subdivided into *Regions***
  - **Each region has homogenous flow characteristics**
  - **Each region subdivided into links and nodes**
  - **Calculate degree of saturation (DS) for all nodes**

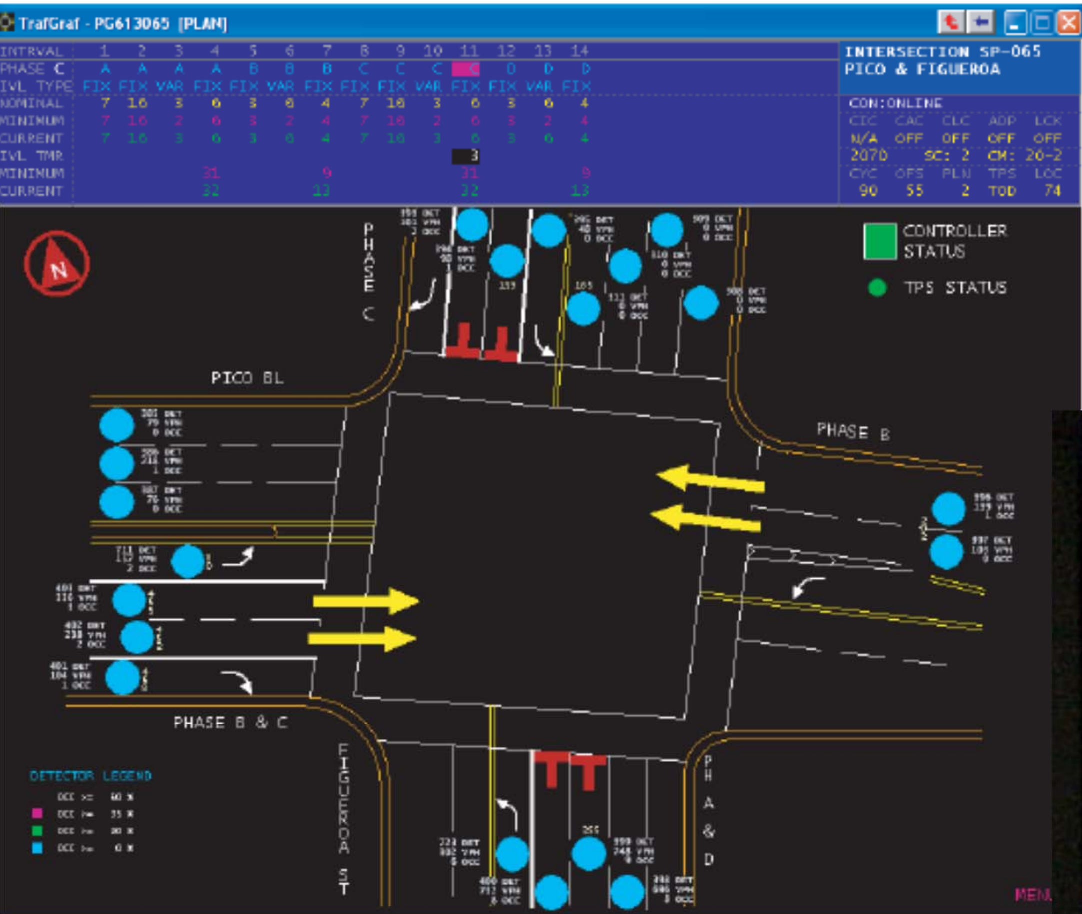


DS = [green-(unused green)]/green

LADOT'S ATCS
Adaptive Traffic Control System (1)

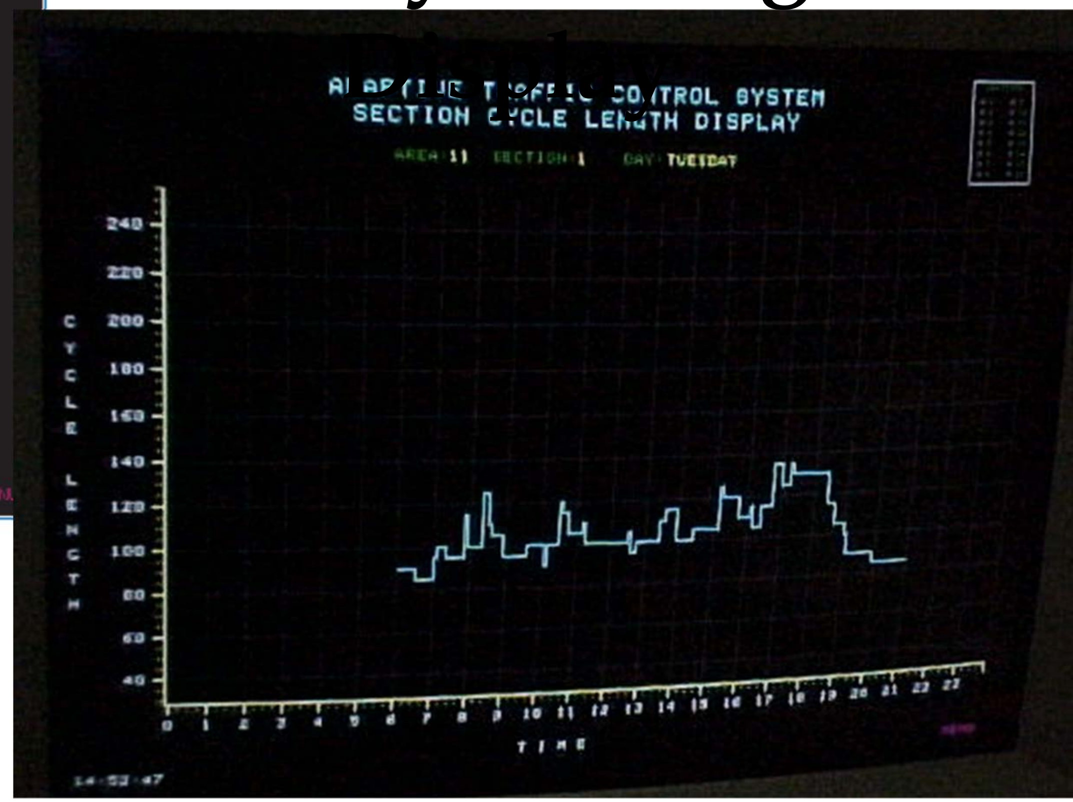
- **Adjust signal timing on a cycle-by-cycle basis:**
  - Cycle Length
  - Splits (Critical Intersection Control)
  - Offset (Critical Link Control)
- **Data:**
  - Detectors located 200-300 ft upstream of the stopline
  - Volume & occupancy data collected every second
  - Data smoothed and used every cycle
- **Constraints:**
  - Phase sequence is fixed
  - Minimum green times

LADOT ATCS (2)

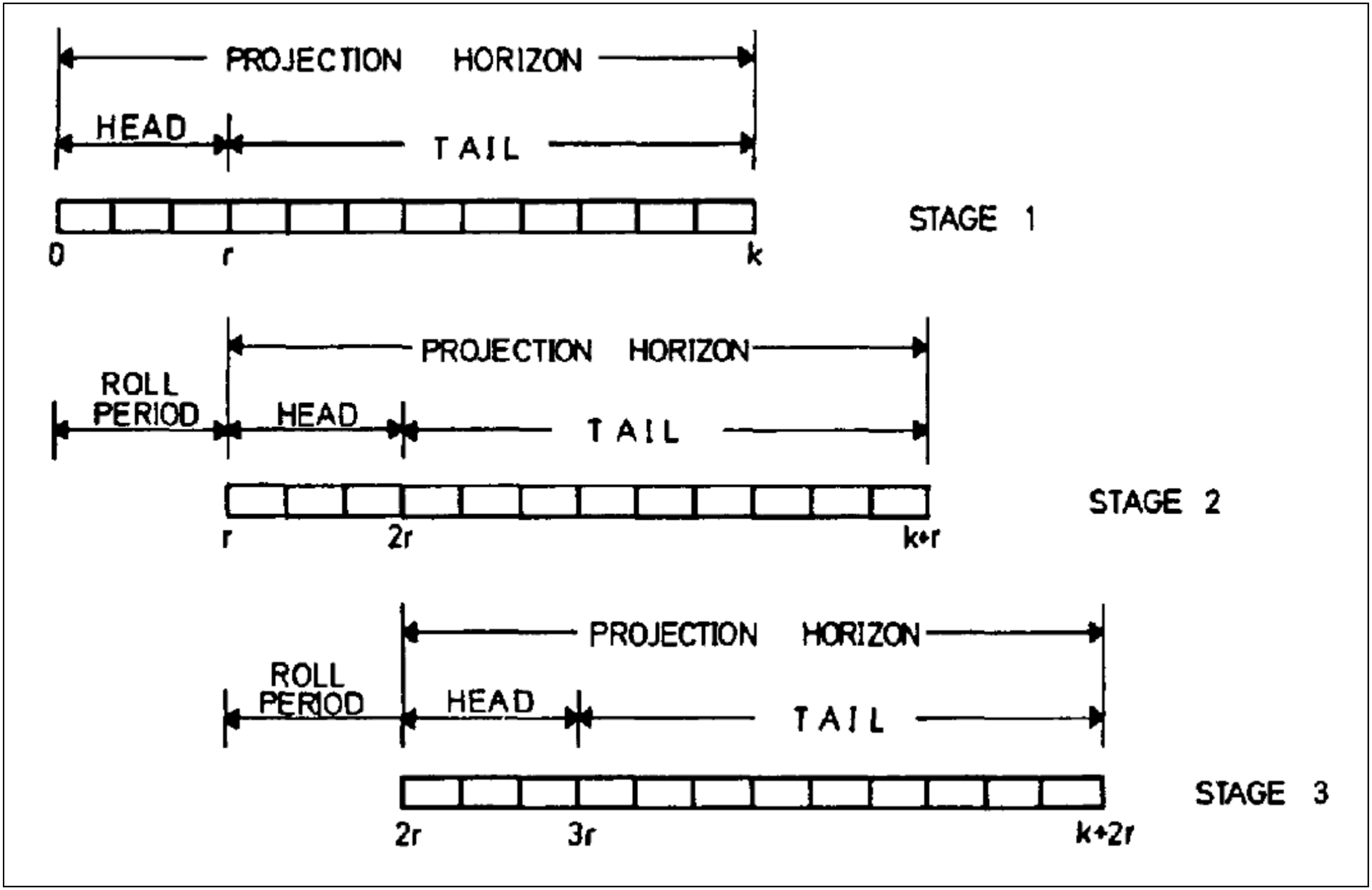


Intersection Display

Cycle Length

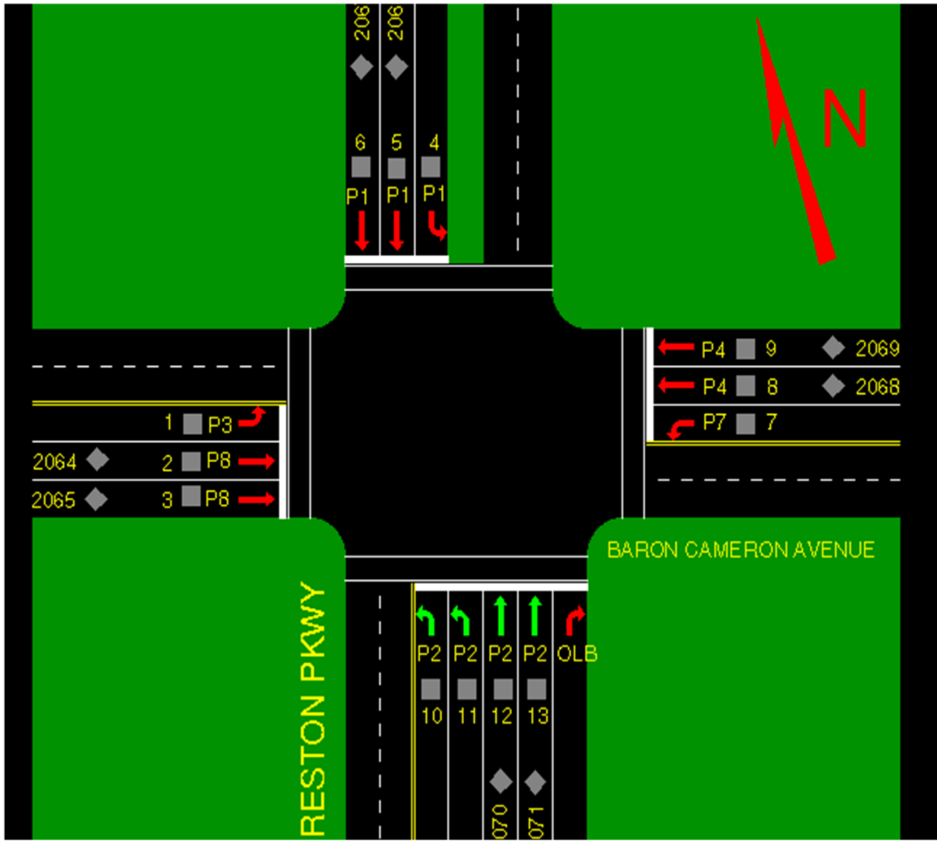


Adaptive Control: Rolling Horizon



OPAC/ RHODES
Adaptive Control: No Fixed Cycle

- Measured and Predicted Vehicle Arrivals
- Optimization: Min Queues
- Rolling Horizon



- Upstream detectors can provide an actual history for a short portion of the profile
- Smoothed volume can be used for uniform profiles
- Platoon identification and smoothing can be used for cyclic profiles