Applications of Hamilton Jacobi equations to network state traffic estimation and control

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Outline

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Estimation problems involving Hamilton-Jacobi equations

- Background
- Problem definition
- Optimization formulations

Application to traffic flow problems

- Estimation
- Network control
- Privacy analysis
- Cybersecurity and sensor fault detection

Conclusion

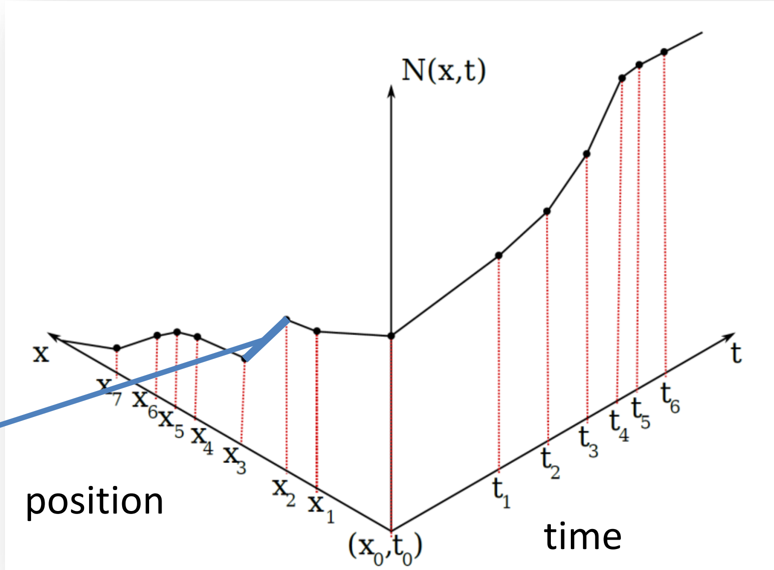
Recap: semi-analytic computational methods

We use a piecewise linear decomposition of the boundary data (which amounts to taking piecewise constant initial densities and boundary flows)

Example:

single piece of linear initial condition

$$\mathbf{M_0}_i(0,x) = \begin{cases} a_i x + b_i & \textit{if } x \in [\overline{\alpha}_i, \overline{\alpha}_{i+1}] \\ +\infty & \textit{otherwise} \end{cases}$$



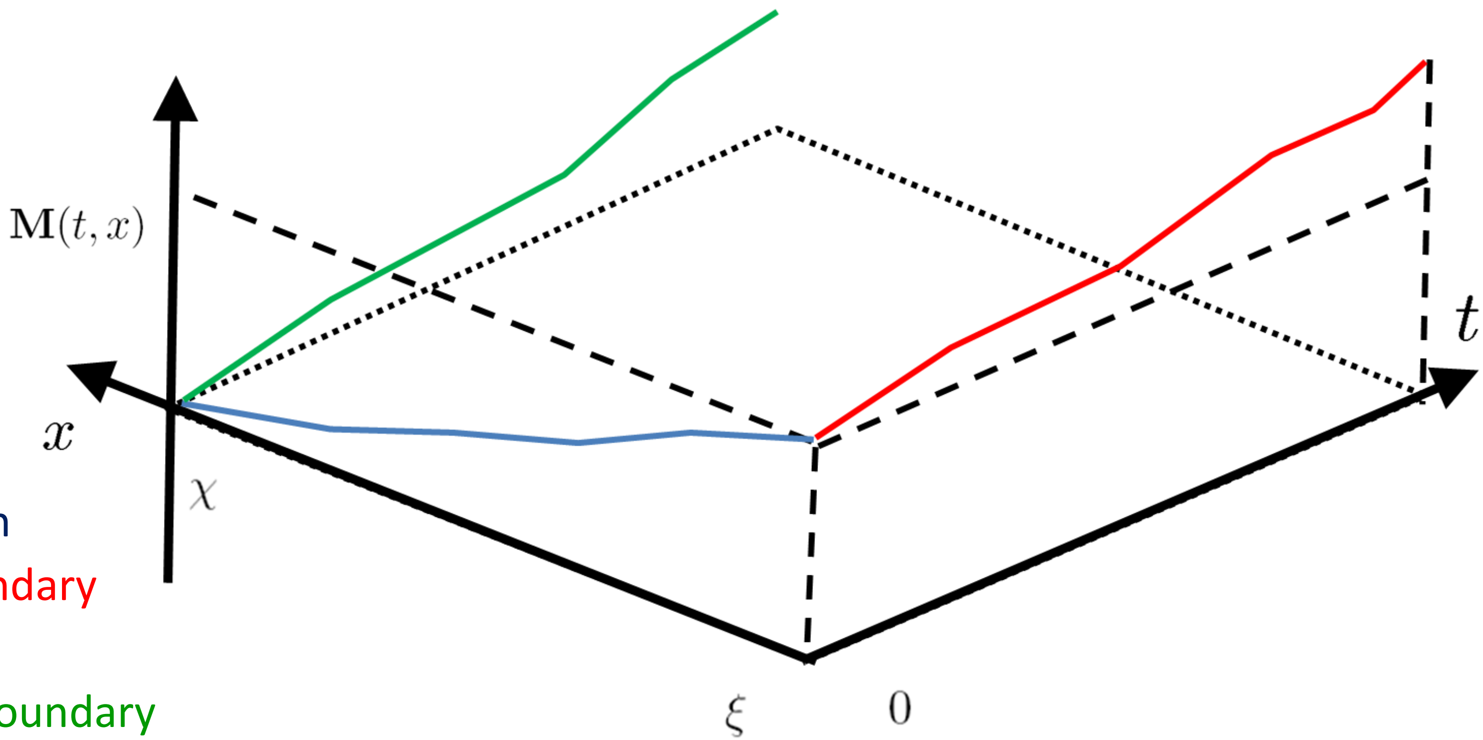
<u>Physically:</u> constant initial density in a spatial interval

no information elsewhere

[Claudel Bayen IEEE TAC part II 2010] [Mazare Dehwah Claudel Bayen, TR-B 2012]

Data fusion problems

- Solving the Hamilton Jacobi PDE requires the definition of initial and boundary conditions (Dirichlet problem)



- Initial condition
- Upstream boundary condition
- Downstream boundary condition

Data fusion problems

- Solving the Hamilton Jacobi PDE requires the definition of initial and boundary conditions (Dirichlet problem)
- **<u>In reality: this never happens</u>**

- Density sensor (ex: loop detector, camera, radar)
- Flow sensor (ex: loop detector, camera, radar)
- Probe vehicle (ex: GPS equipped vehicle)
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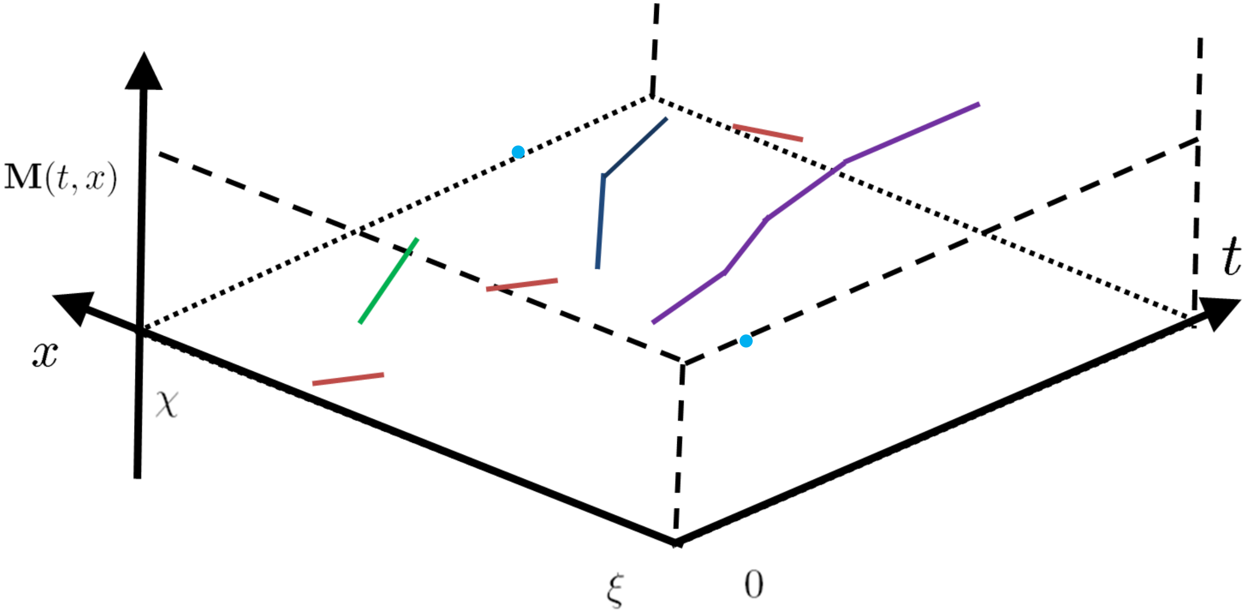
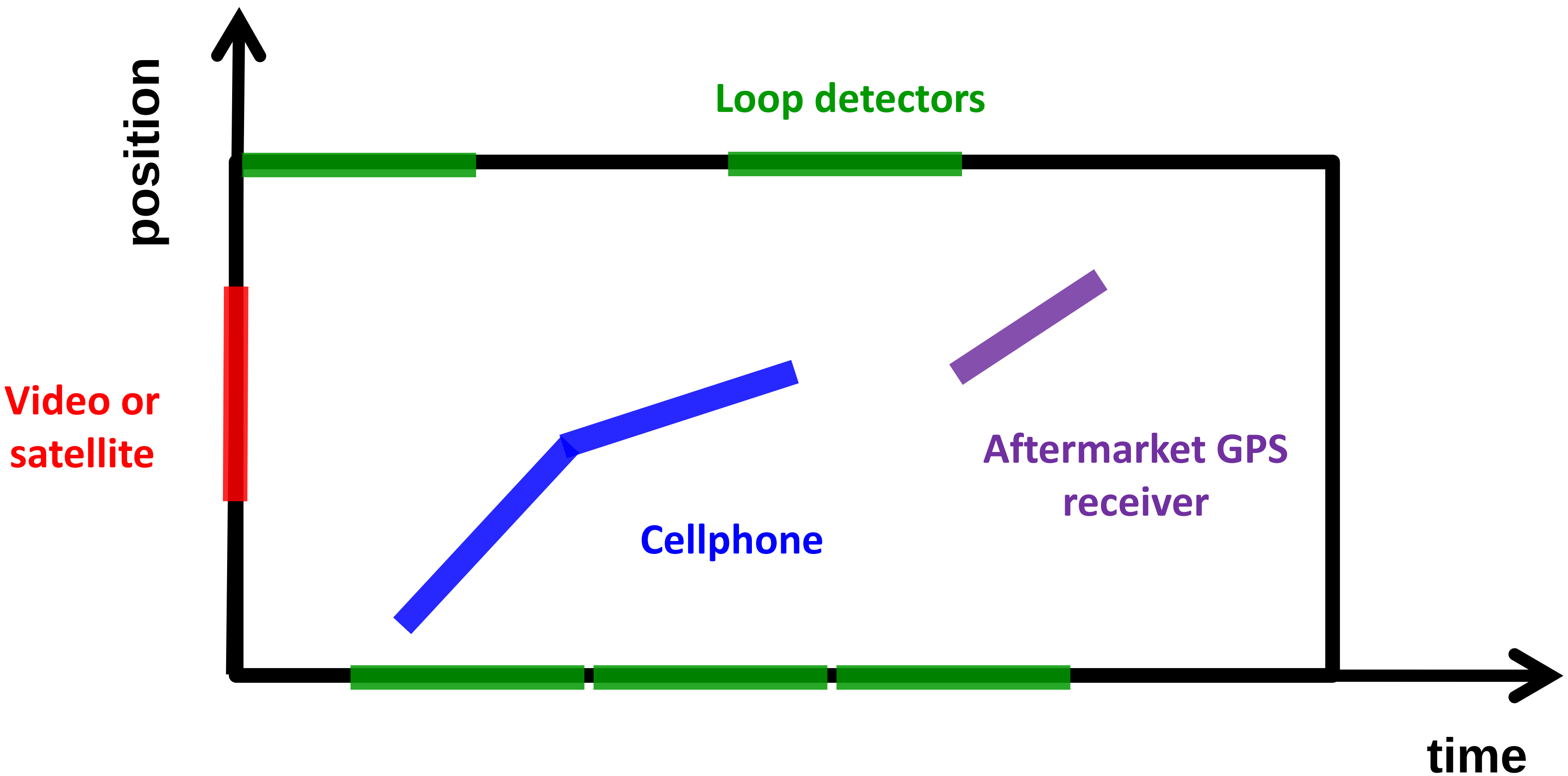
- Travel time data (ex: toll transponder)



Problem definition

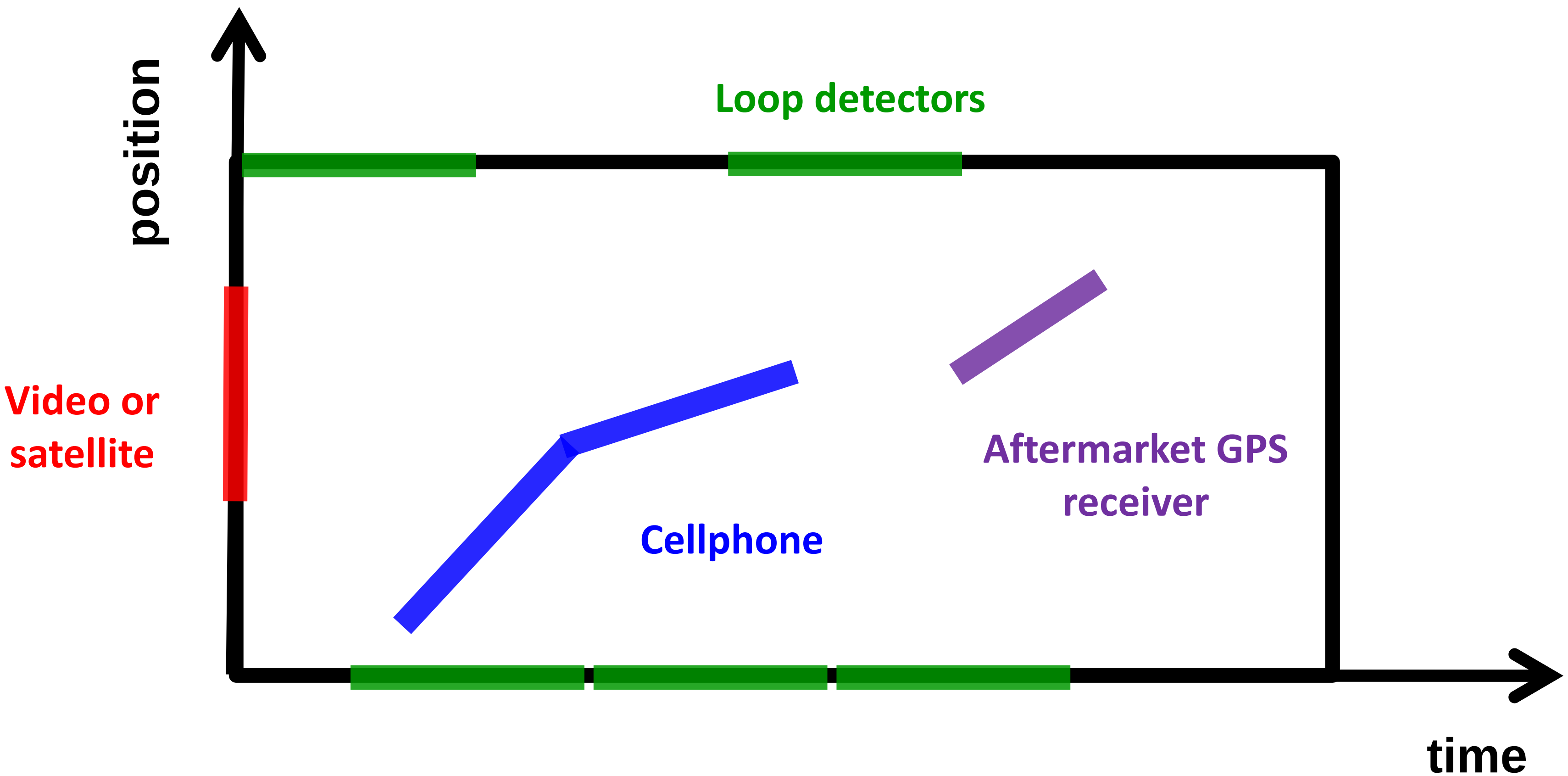
The data generated by mobile phones and fixed detectors corresponds to a (partial) set of piecewise affine initial, boundary or internal conditions



[Claudel, Bayen, ACC 10], [Claudel, Bayen, SIAM SICON 10 (in review)]

Problem definition

The measurement data alone is not sufficient to express the PWA boundary conditions uniquely (because of sensor error, constants of integration, etc)

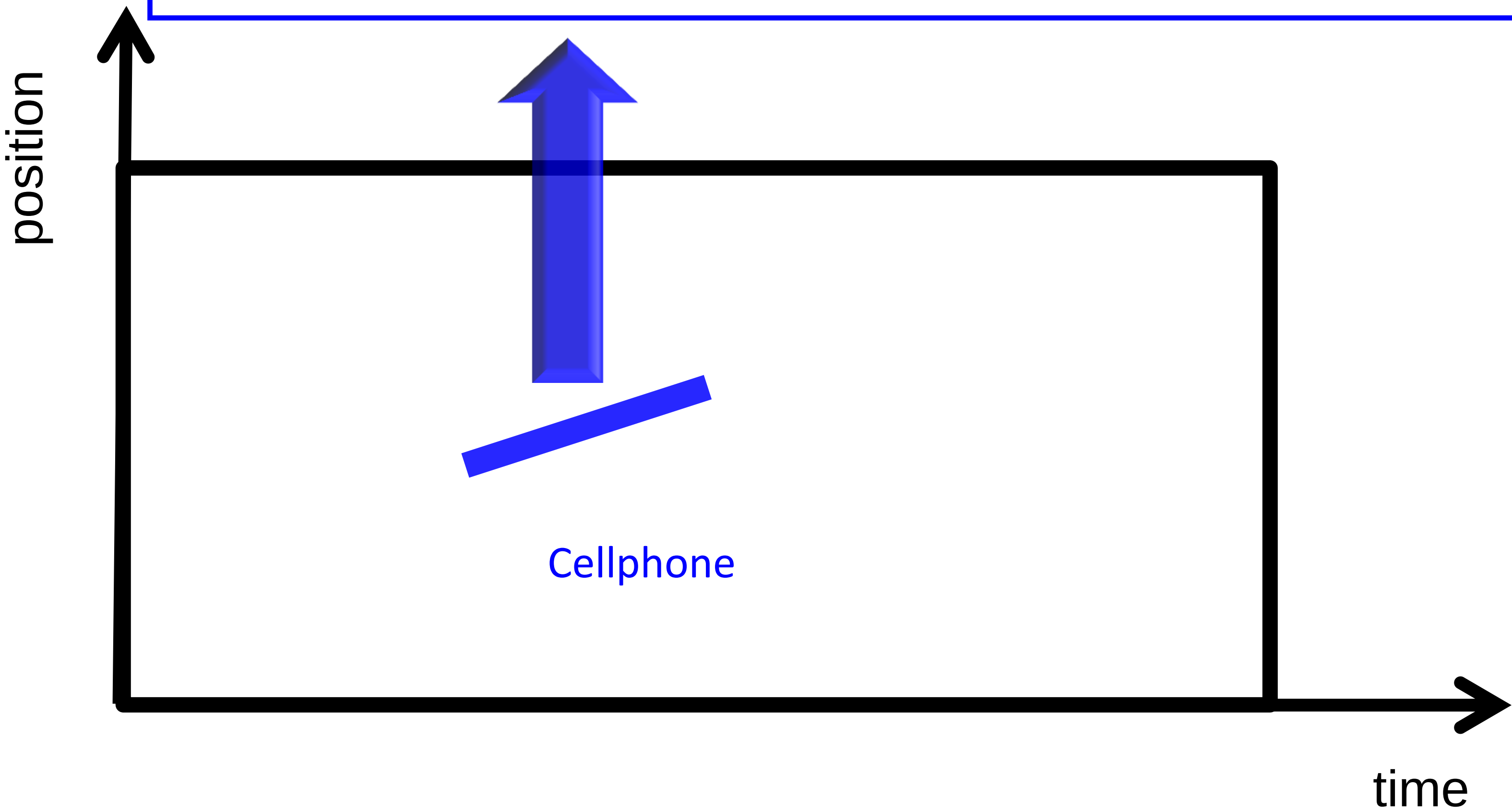


[Claudel, Bayen, ACC 10], [Claudel, Bayen, SIAM SICON 10 (in review)]

Problem definition

Example:

$$\mu_l(t,x) = \begin{cases} L_l + r_l(t - t_{\min}(l)) & \text{if } \; x = x_{\min}(l) + v^{\text{meas}}(l)(t - t_{\min}(l)) \\ & \text{and } t \in [t_{\min}(l), t_{\max}(l)] \\ +\infty & \text{otherwise} \end{cases}$$



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Coordinates

Unknown coefficients (constants of integration, or unmeasured quantities)

Approximately known from measurement data

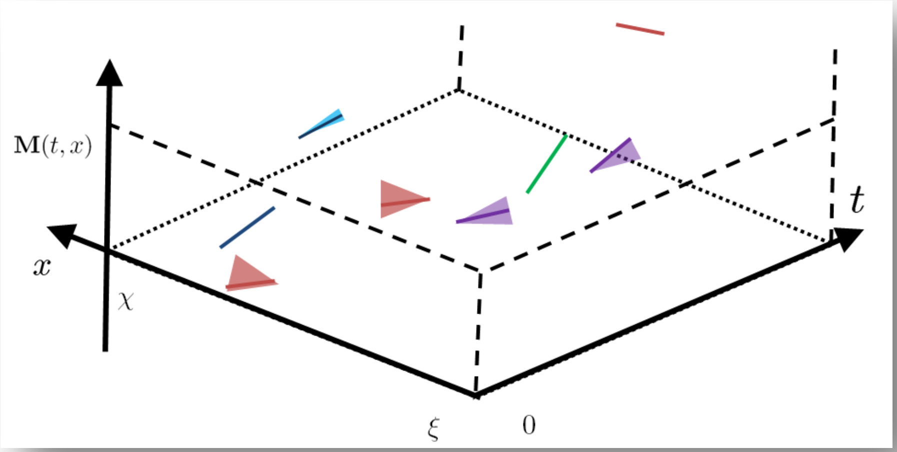
[Claudel, Bayen, ACC 10], [Claudel, Bayen, SIAM SICON 10 (in review)]

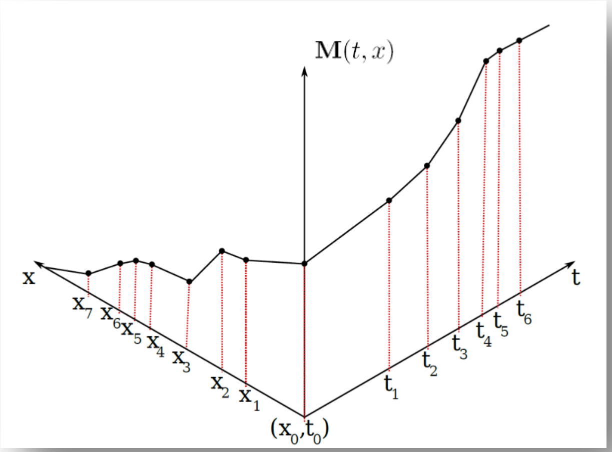
Data fusion problems

- Sensors yield constraints on the solution:

- Density sensors constrain the local density values
- Flow sensors constrain the local flow values
- Probe data constrain the local traffic velocity values
- Travel time data constrain the cumulated flow curves (or vehicle labels)

- Given an arbitrary dataset, how can one reconstruct the set of possible boundary data of the problem that are compatible with both the LWR model and the noisy measurement data?



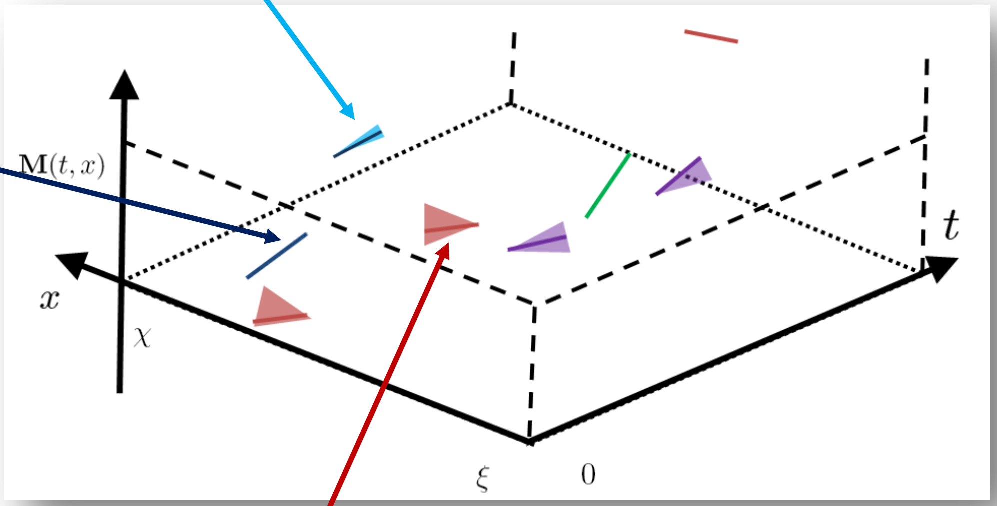


Optimization formulation

- Let us first describe all the unknowns of the initial, boundary and internal conditions generated by the sensors, and write these unknowns as a decision variable y

Boundary condition (flow sensor): unknown offset, unknown flow

Internal condition (probe vehicle): unknown offset, unknown passing rate



Density condition (density sensor): unknown offset, unknown density

- The decision variable y is constrained both by the measurement data and by the LWR model

[Claudel Bayen SIAM 2012] [Canepa, Claudel IEEE ITSC 2012]

Constraints of measurement data

- Data constraints: easy to encode, usually convex in y (in practice, linear or quadratic in y)
  - Example: suppose that we have upstream boundary flow sensors that measure the flow with 5% relative error, and that the density of the 3rd initial condition block is measured by a sensor with 10% absolute error:

$$\begin{cases} 0.95 q_{\text{in}}^{\text{measured}}(n) \leq q_{\text{in}}(n) \leq 1.05 q_{\text{in}}^{\text{measured}}(n) \quad \forall n \in [0, n_{\max}] \\ \rho(3)^{\text{measured}} - 0.1\rho_m \leq \rho(3) \leq \rho(3)^{\text{measured}} + 0.1\rho_m \end{cases}$$

[Canepa Claudel IEEE ITSC 2012]

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[Canepa Claudel IEEE ITSC 2012]

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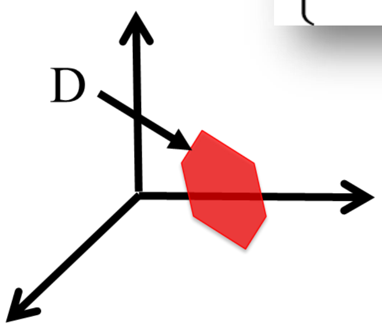


Coefficients of the PWA boundary data

Linear constraints

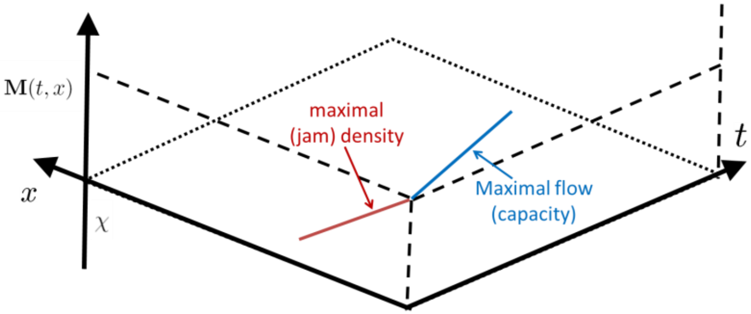
- However, PDE model constraints are much harder to encode, can even be not explicit

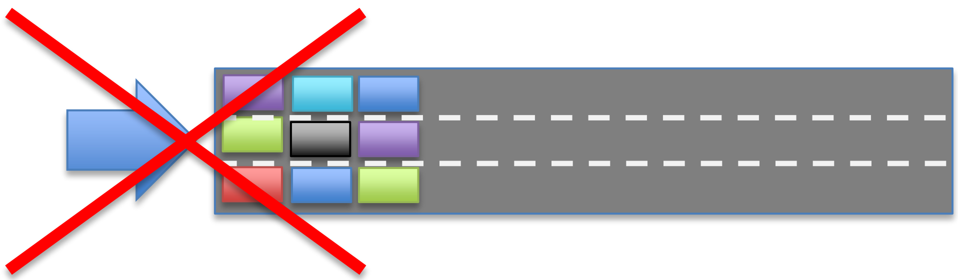
[Canepa Claudel IEEE ITSC 2012]

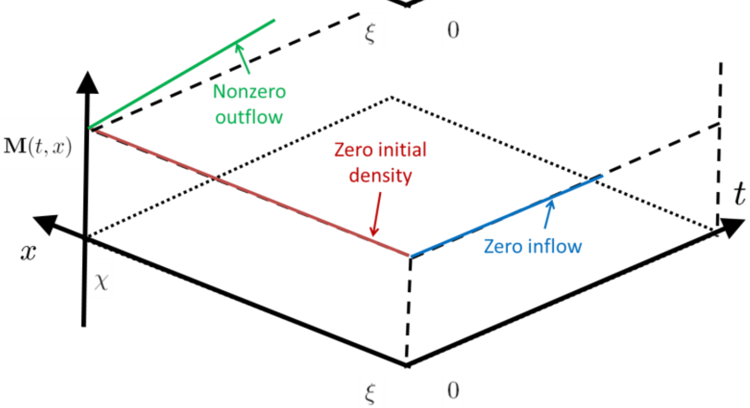
PDE model constraints

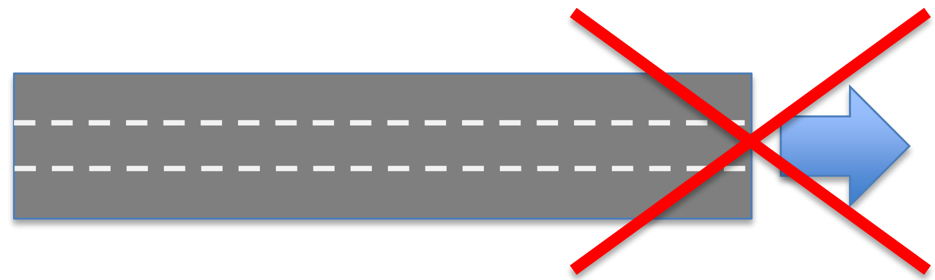
- PDE model constraints are related to the concept of “weak boundary conditions”: the initial, boundary and internal conditions cannot be set arbitrarily

- Examples:







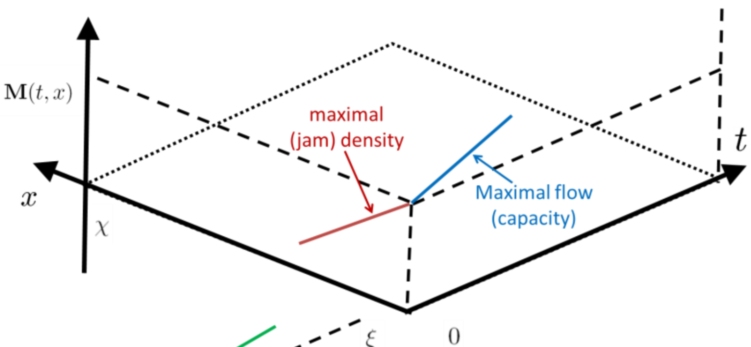


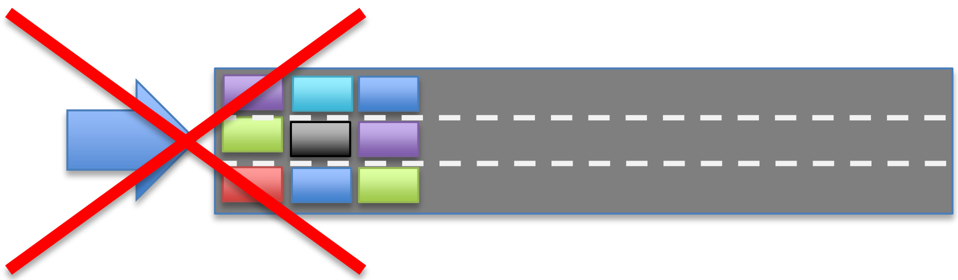
[Claudel Bayen SIAM 2012] [Canepa Claudel IEEE ITSC 2012]

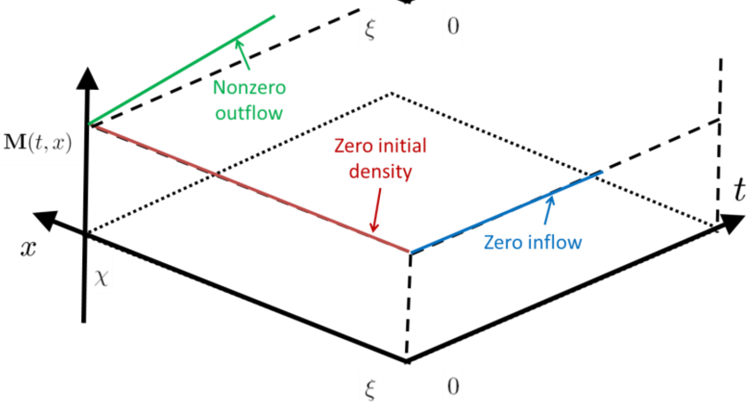
PDE model constraints

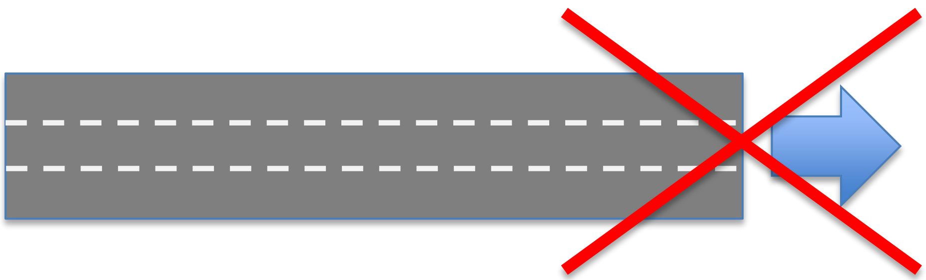
- These constraints apply to any arbitrary number of initial/boundary/internal (and the like) conditions

- <u>Examples:</u>









[Claudel Bayen SIAM 2012] [Canepa Claudel IEEE ITSC 2012]

PDE model constraints

- PDE model constraints (which ensure that all initial/boundary/internal conditions are strongly defined) are defined as follows:

Explicit solution components

Initial, boundary and internal condition data blocks

$$
\begin{cases}
\mathbf{M}_{M_k}(0,x) \geq M_p(0,x) & \forall x \in [pX,(p+1)X], \forall (k,p) \in \mathbb{K}^2 \\
\mathbf{M}_{M_k}(t,\chi) \geq \beta_p(t,x_p) & \forall t \in [pT,(p+1)T], \forall (k,p) \in \mathbb{K}^2 \\
\mathbf{M}_{M_k}(t,\xi) \geq \gamma_p(t,\xi) & \forall t \in [pT,(p+1)T], \forall (k,p) \in \mathbb{K}^2 \\
\mathbf{M}_{M_k}(t,x) \geq \mu_m(t,x) & \forall t \in [t_{\min(m)},t_{\max(m)}], x = x_{\min(m)} + \\
 & v^{\text{meas}}(m)(t - t_{\min(m)}) \forall (k,m) \in \mathbb{K} \times \mathbb{M} \\
\mathbf{M}_{\gamma_n}(t,\xi) \geq \gamma_p(t,\xi) & \forall t \in [pT,(p+1)T], \forall (n,p) \in \mathbb{N}^2 \\
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\mathbf{M}_{\mu_k}(t,x) \geq \mu_m(t,x) & \forall t \in [t_{\min(m)},t_{\max(m)}], x = x_{\min(m)} + \\
 & v^{\text{meas}}(m)(t - t_{\min(m)}) \forall (k,m) \in \mathbb{M} \times \mathbb{M}
\end{cases}
$$

[Claudel Bayen SIAM 2012] [Canepa Claudel IEEE ITSC 2012]

PDE model constraints

- <u>Problem:</u> constraints apply at each point of a continuous interval

Explicit solution components

Initial, boundary and internal condition data blocks

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\end{cases}
$$

[Claudel Bayen SIAM 2012] [Canepa Claudel IEEE ITSC 2012]

PDE model constraints

- Problem: constraints apply at each point of a continuous interval
- Solution: use piecewise linear diagrams, that have finite numbers of inflexion points, which are independent of the PWA block coefficients

Explicit solution components

$$
\begin{cases}
\mathbf{M}_{M_k}(0,x) \geq M_p(0,x) & \forall x \in [pX,(p+1)X], \forall (k,p) \in \mathbb{K}^2 \\
\mathbf{M}_{M_k}(t,\chi) \geq \beta_p(t,x_p) & \forall t \in [\ldots 1)T], \forall \ldots \mathbb{K}^2 \\
\mathbf{M}_{M_k}(t,\xi) \geq \gamma_p(t,\xi) & \forall t \in [p \ldots)T], \ldots \mathbb{K}^2 \\
\mathbf{M}_{M_k}(t,x) \geq \mu_m(t,x) & \forall t \in [t_{m} \ldots \mathrm{ax} \ldots = x_{\min(m)} + \\
 & v^{\mathrm{meas}}(m \ldots k,m) \in \mathbb{K} \times \mathbb{M} \\
\mathbf{M}_{\gamma_n}(t,\xi) \geq \gamma_p(t,\xi) & \forall t \in [pT,(\ldots p) \in \mathbb{N}^2 \\
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 & v^{\mathrm{meas}}(\ldots \min(m \ldots m) \in \mathbb{N} \times \mathbb{M} \\
\mathbf{M}_{\mu_k}(t,x) \geq \mu_m(t,x) & \forall t \in [t_{m} \ldots , t_{\max(m} \ldots x_{\min(m)} + \\
 & v^{\mathrm{meas}}(m)(t - t_{\min(m)}) \forall (k,m) \in \mathbb{M} \times \mathbb{M}
\end{cases}
$$

Initial, boundary and internal condition data blocks

[Claudel Bayen SIAM 2012] [Canepa Claudel IEEE ITSC 2012]

Convexity property

- The initial, boundary and internal condition data blocks are all linear in y
- The explicit solution components are concave in (some of) their coefficients.
  Example:

$$\mathrm{M}_{\mathcal{M}_0,i}(t,x) = \inf_{u \in \mathrm{Dom}(\varphi^*) \cap \left[\frac{\overline{\alpha}_i - x}{t}, \frac{\overline{\alpha}_{i+1} - x}{t}\right]} \left(a_i(x+tu) + b_i + t\varphi^*(u)\right), \quad \forall (t,x) \in \mathbb{R}_+^* \times X$$

is concave in both a_i and b_i

In some cases (triangular diagram) the concave function is piecewise linear

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PDE model constraints

- PDE model constraints in the present case are explicit and mixed integer linear

Explicit solution components (piecewise linear concave function of y)

$$
\begin{cases}
\mathbf{M}_{M_k}(0,x) \geq M_p(0,x) & \forall x \in [pX,(p+1)X], \forall (k,p) \in \mathbb{K}^2 \\
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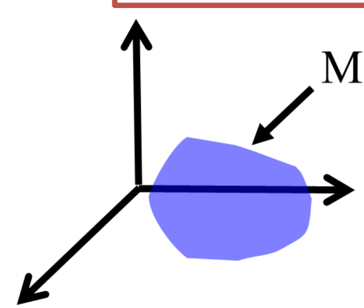
\end{cases}
$$

Initial, boundary and internal condition data blocks (linear function of y)

Linear constraints



Coefficients of the PWA boundary data

[Claudel Bayen SIAM 2012] [Canepa Claudel IEEE ITSC 2012]

Continuity constraints

- Additional constraints on y to ensure that the solution is continuous (remember, the solution to the HJ PDE is only lower semicontinuous in general)

Initial, boundary and internal condition data blocks (linear function of y)

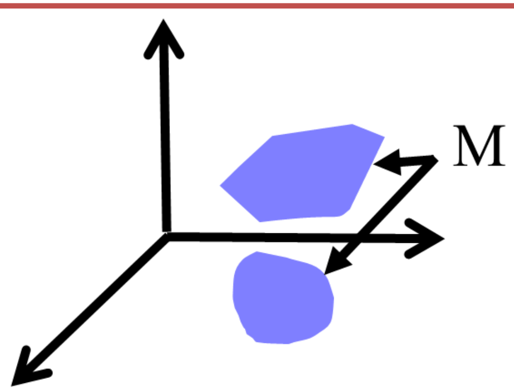
Explicit solution components (piecewise linear concave function of y)

Integer linear constraints for continuity

Mixed integer-linear constraints

$$
\begin{cases}
\mu_m(t_{\min}(m), x_{\min}(m)) \le \mathbf{M}_{M_k}(t_{\min}(m), x_{\min}(m)), \forall m \in \mathbb{M}, \forall k \in \mathbb{K} \\
\mu_m(t_{\min}(m), x_{\min}(m)) \le \mathbf{M}_{\gamma_n}(t_{\min}(m), x_{\min}(m)), \ \forall m \in \mathbb{M}, \forall n \in \mathbb{N} \\
\mu_m(t_{\min}(m), x_{\min}(m)) \le \mathbf{M}_{\beta_n}(t_{\min}(m), x_{\min}(m)), \ \forall m \in \mathbb{M}, \forall n \in \mathbb{N} \\
\mu_m(t_{\min}(m), x_{\min}(m)) \le \mathbf{M}_{\mu_p}(t_{\min}(m), x_{\min}(m)), \quad \forall (m,p) \in \mathbb{M}^2 \\
R\left(1 - b_i^m\right) + \mu_m(t_{\min}(m), x_{\min}(m)) \ge \mathbf{M}_{M_k}(t_{\min}(m), x_{\min}(m)), \\
\qquad \forall i \in [1 \ldots s_1^m], \forall m \in \mathbb{M}, \forall k \in \mathbb{K} \\
R\left(1 - b_i^m\right) + \mu_m(t_{\min}(m), x_{\min}(m)) \ge \mathbf{M}_{\gamma_n}(t_{\min}(m), x_{\min}(m)), \\
\qquad \forall i \in [s_1^m \ldots s_2^m], \forall m \in \mathbb{M}, \forall n \in \mathbb{N} \\
R\left(1 - b_i^m\right) + \mu_m(t_{\min}(m), x_{\min}(m)) \ge \mathbf{M}_{\beta_n}(t_{\min}(m), x_{\min}(m)), \\
\qquad \forall i \in [s_2^m \ldots s_3^m], \forall m \in \mathbb{M}, \forall n \in \mathbb{N} \\
R\left(1 - b_i^m\right) + \mu_m(t_{\min}(m), x_{\min}(m)) \ge \mathbf{M}_{\mu_p}(t_{\min}(m), x_{\min}(m)), \\
\qquad \forall i \in [s_3^m \ldots s_{max}^m], \forall m \in \mathbb{M}, \forall p \in \mathbb{M}
\end{cases}
\tag{31}
$$

$$
\sum_{i=1}^{s_{max}^m} b_i = 1 \tag{32}
$$

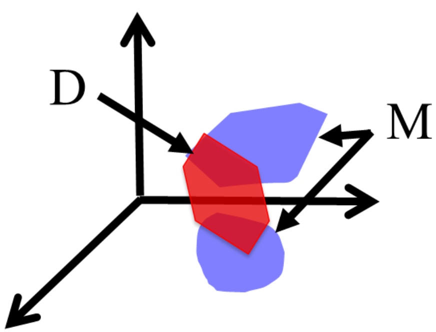


Coefficients of the PWA boundary data

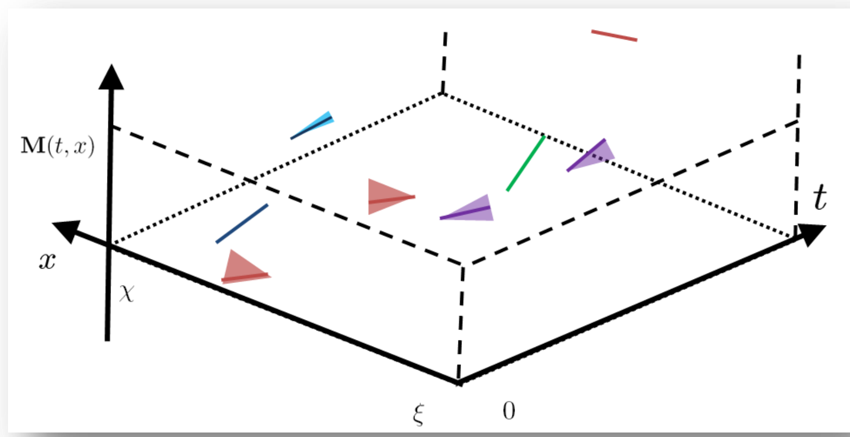
[Claudel Bayen SIAM 2012] [Canepa Claudel IEEE ITSC 2012]

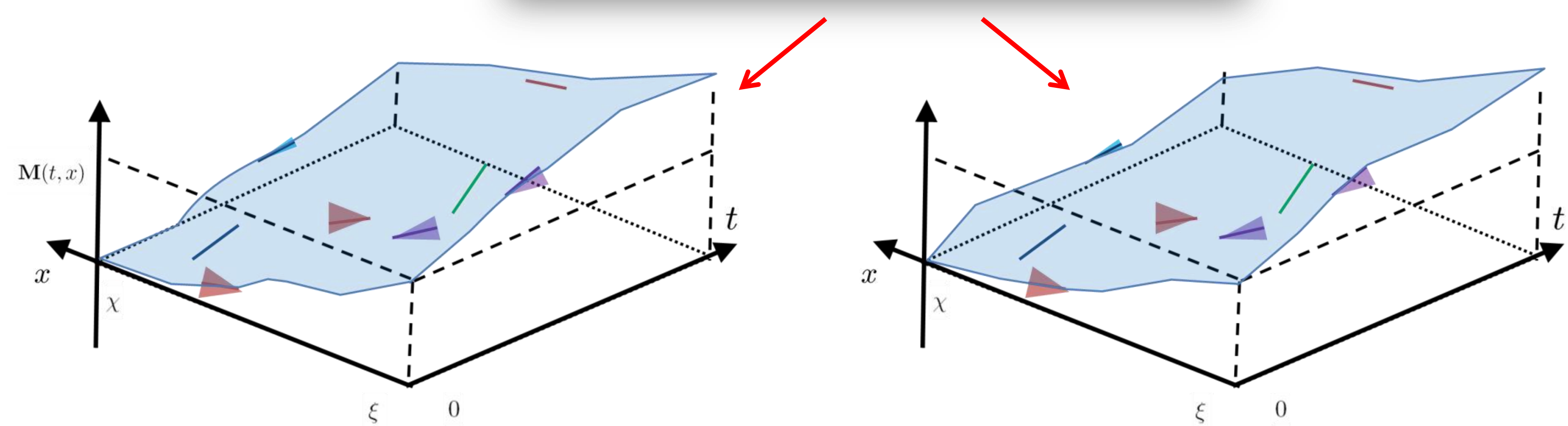
MILP formulation

- Since the sensor data constraints are linear and the model data constraints are mixed integer linear, the feasible set of the problem is a union of polyhedra



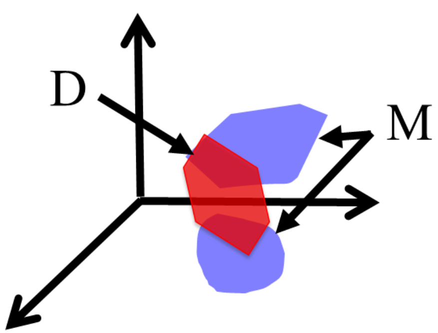
Coefficients of the PWA boundary conditions





Model parameter uncertainty

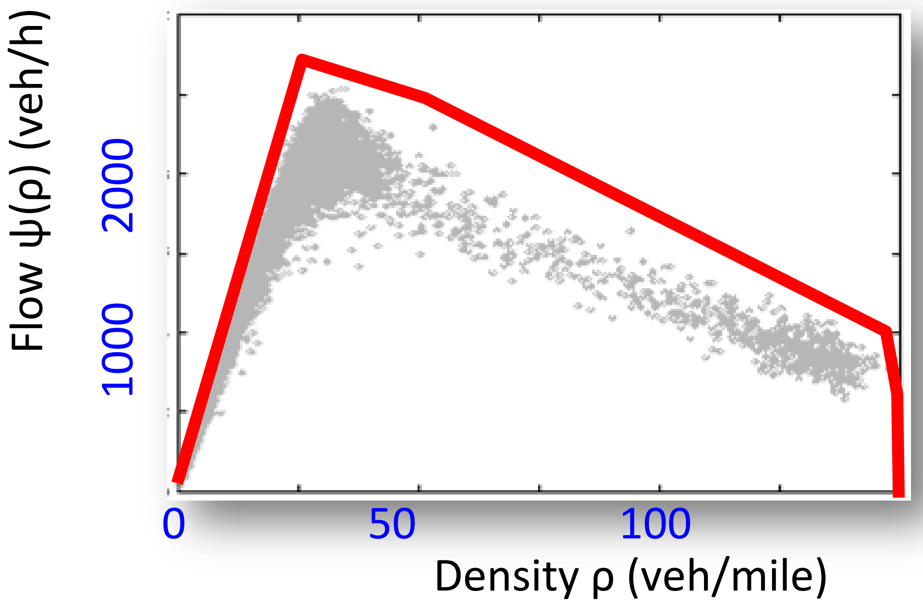
- Unlike Kalman Filtering, this approach does not consider model noise (model is assumed to be perfect)
- The model constraints (M) depend on the fundamental diagram ψ: M(ψ)
- Model noise can be added in two ways:
  - If the constraints are infeasible, one can compute the union of M(ψ) for all (possible) ψ, and compute $D \cap (\cup_{\psi \in S} M(\psi))$
  - If the constraints are feasible, we can instead compute $D \cap (\cap_{\psi \in S} M(\psi))$ if we believe in the model



Coefficients of the PWA boundary conditions

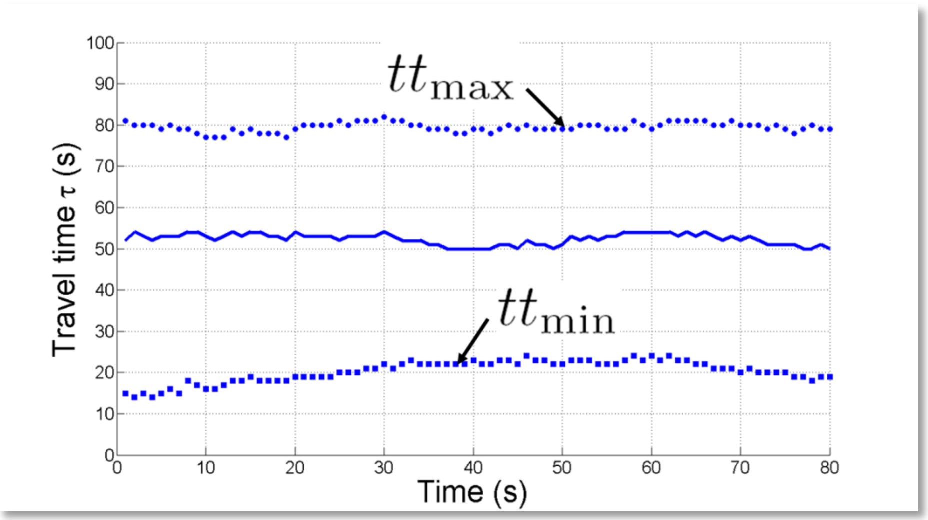
Model parameter uncertainty

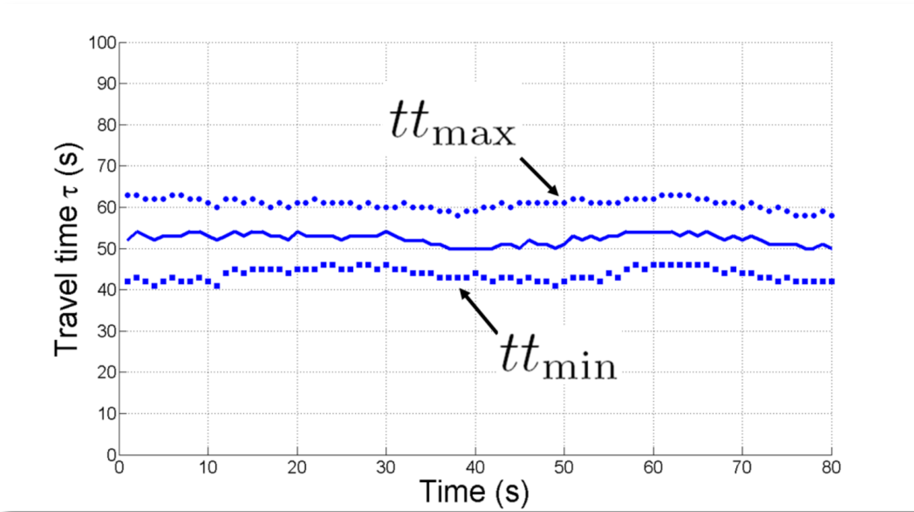
- Noteworthy: if the epigraph of the fundamental diagram contains the cloud of points, the true state of the system is guaranteed to belong to the feasible set (somewhere…)
- This assumes that the flow-density relationship applies on the complete domain



Model parameter uncertainty

- Noteworthy: if the epigraph of the fundamental diagram contains the cloud of points, the true state of the system belongs to the feasible set (somewhere…)

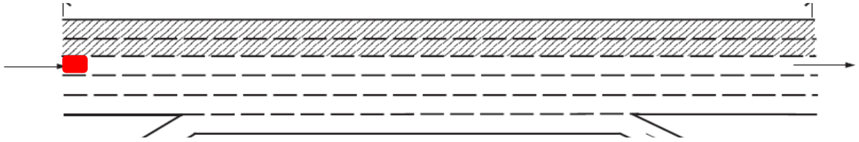




Eulerian data only

Eulerian data + 1 Lagrangian trajectory

Effect of Lagrangian data on the travel time estimation



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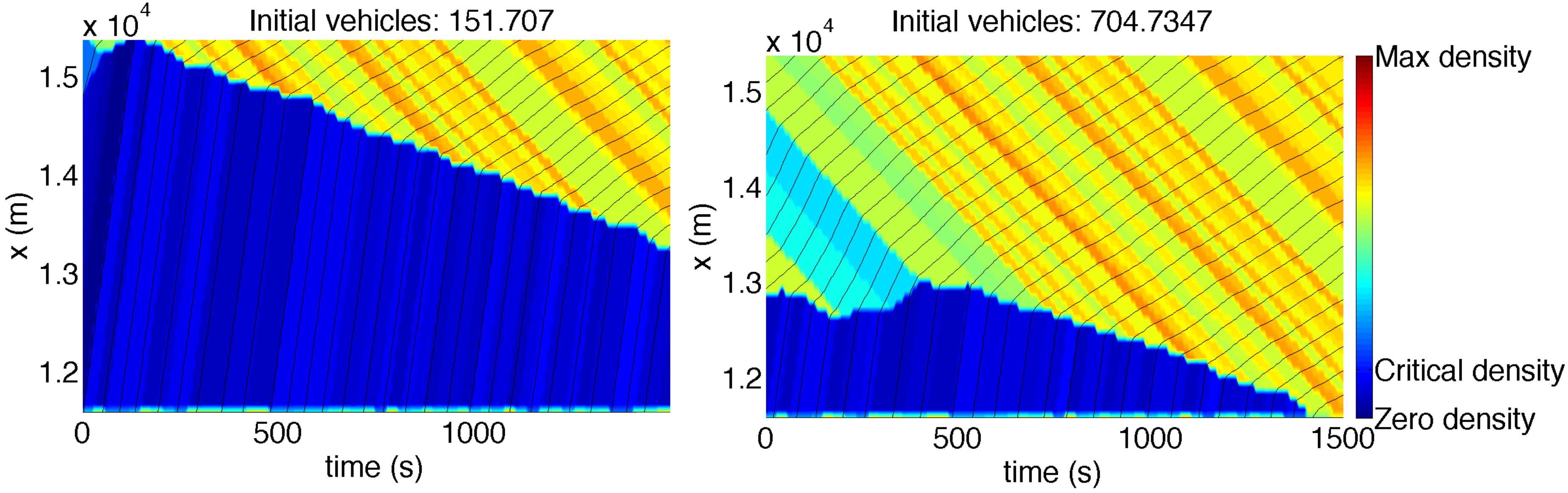
Estimation

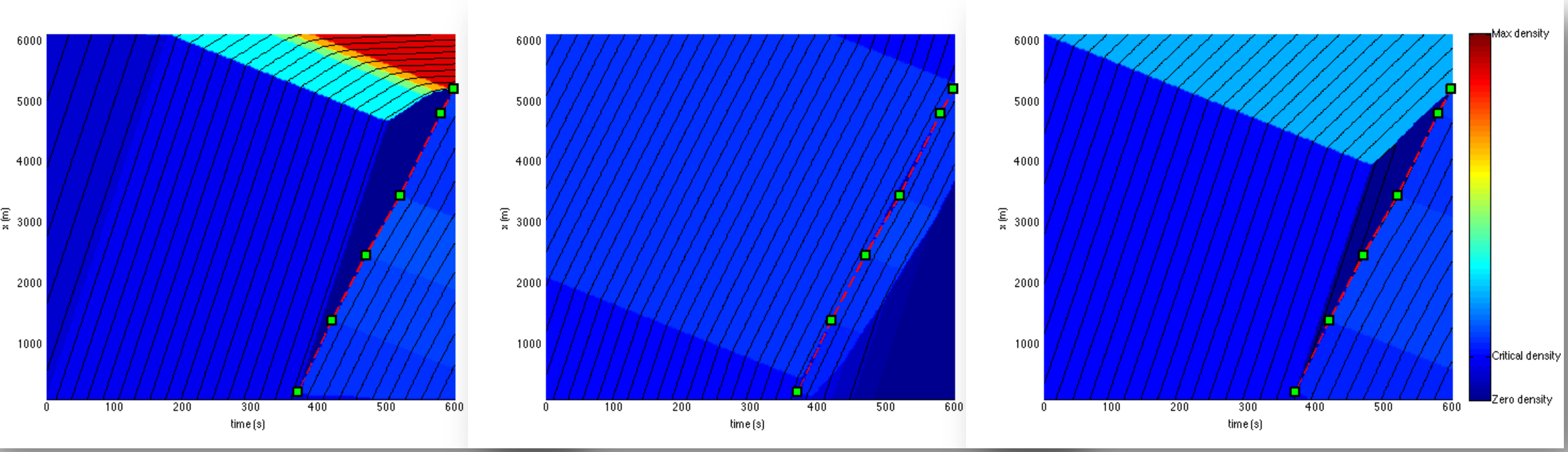
- Since we have non uniqueness in general, we select a given solution that minimizes an objective function

<u>Example:</u> minimization or maximization of the total number of vehicles at the initial time, using PeMS boundary flow measurements



Estimation

- If we take the L_1 norm of the decision variable (modulo some weights), we can do compressed sensing:



Max average density

Min average density

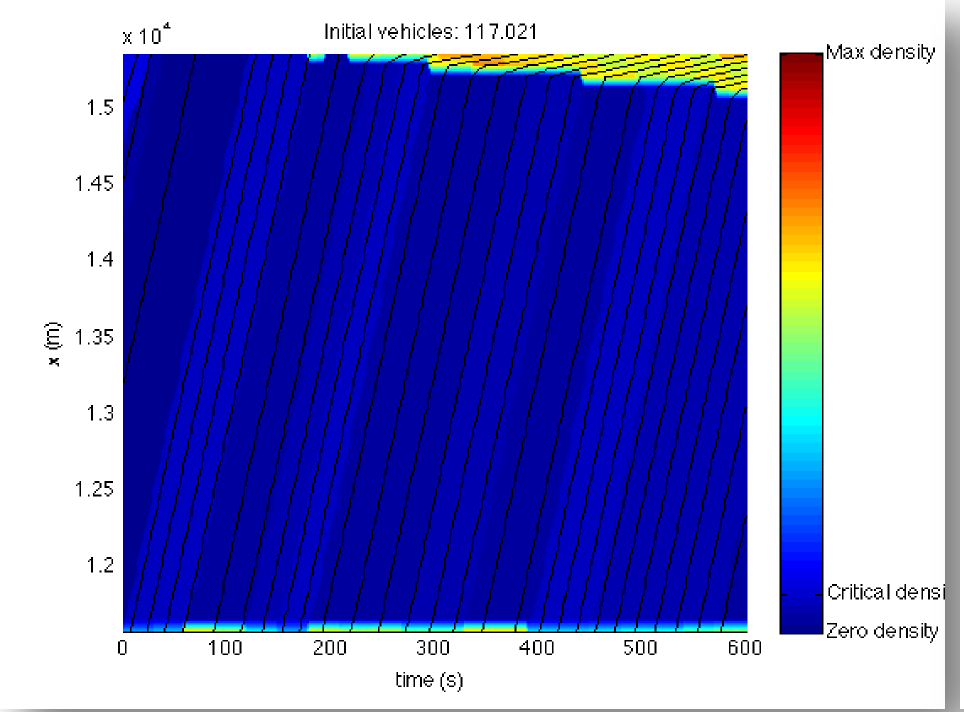
Least L_1 norm

[Canepa, Claudel (in preparation)]

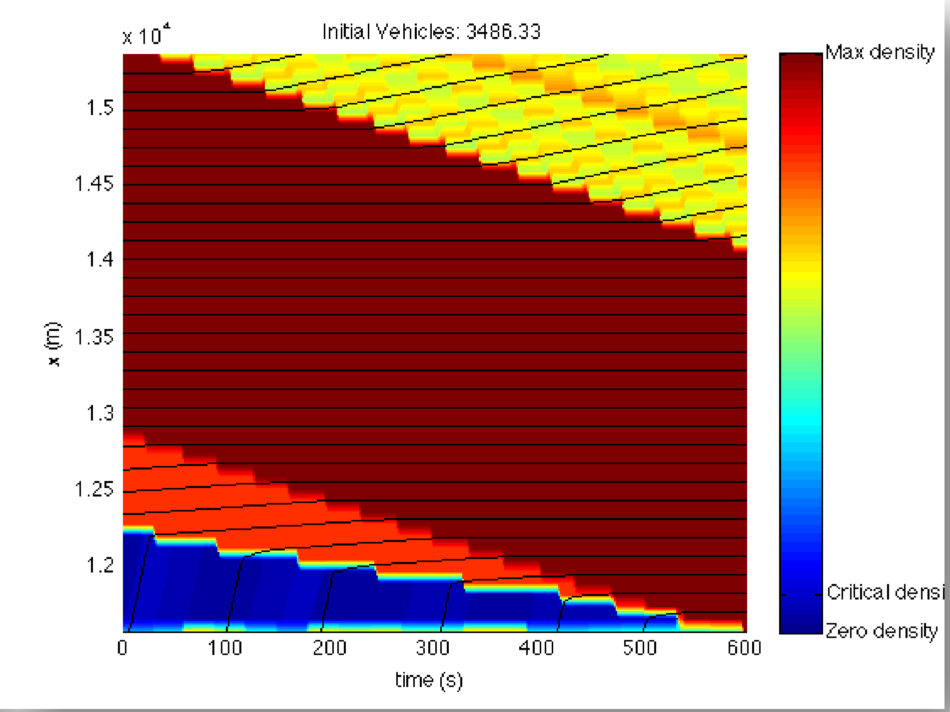
Convergence / monotonicity

- Estimation of initial number of vehicles with multiple internal conditions
  - No probe data

Min



Max

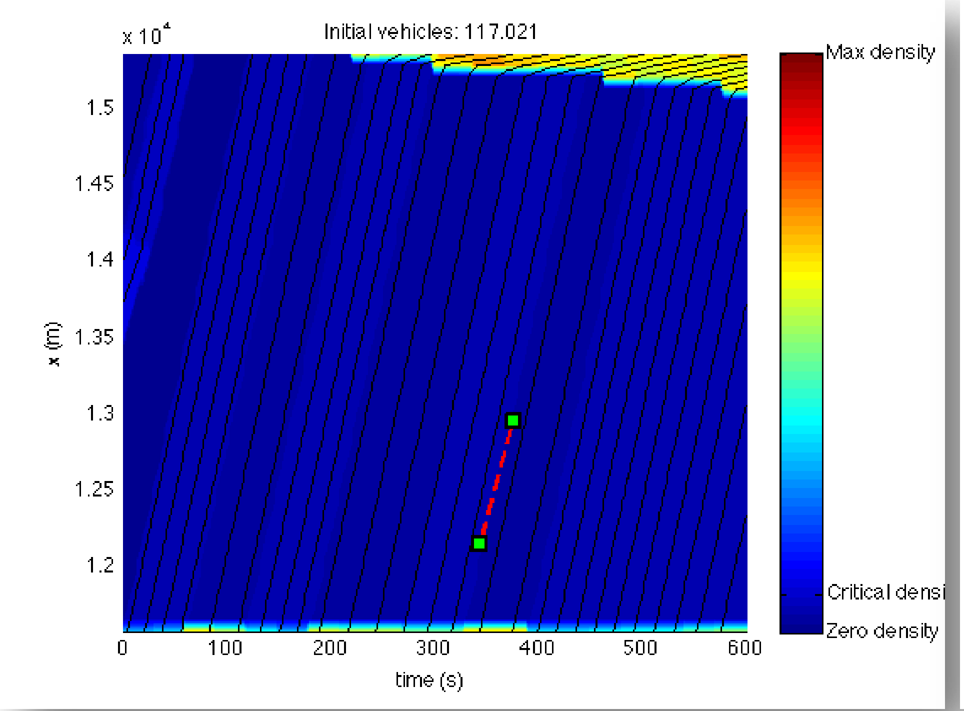


[Canepa, Claudel, ITSC 2012][Canepa, Claudel, NHM 2013][Canepa, Claudel, IEEE TAC (submitted)]

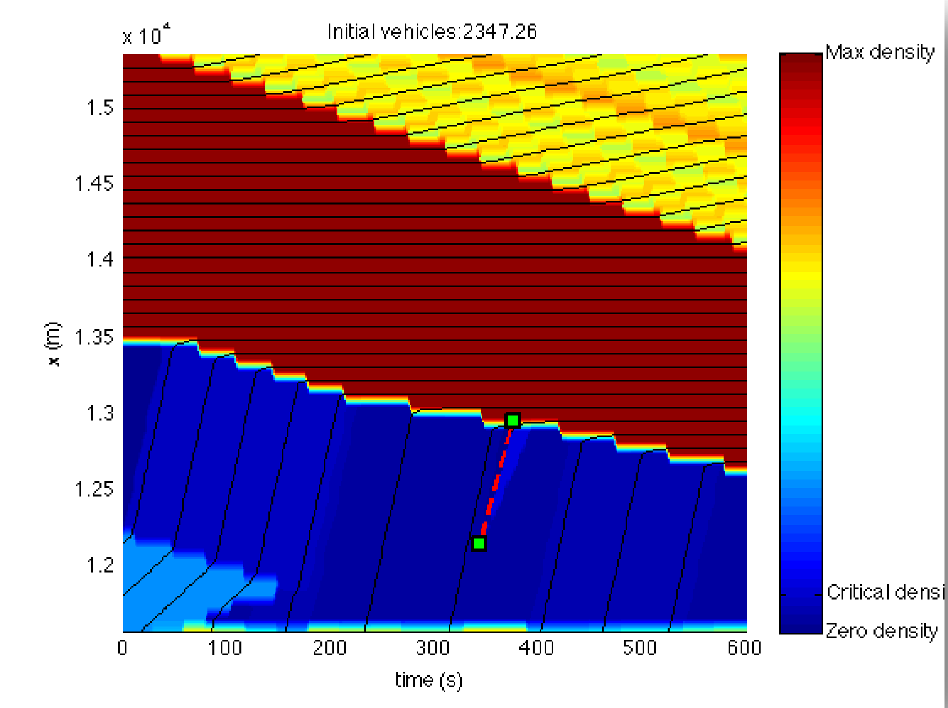
Convergence / monotonicity

- Estimation of initial number of vehicles with multiple internal conditions - 1 probe data

Min



Max

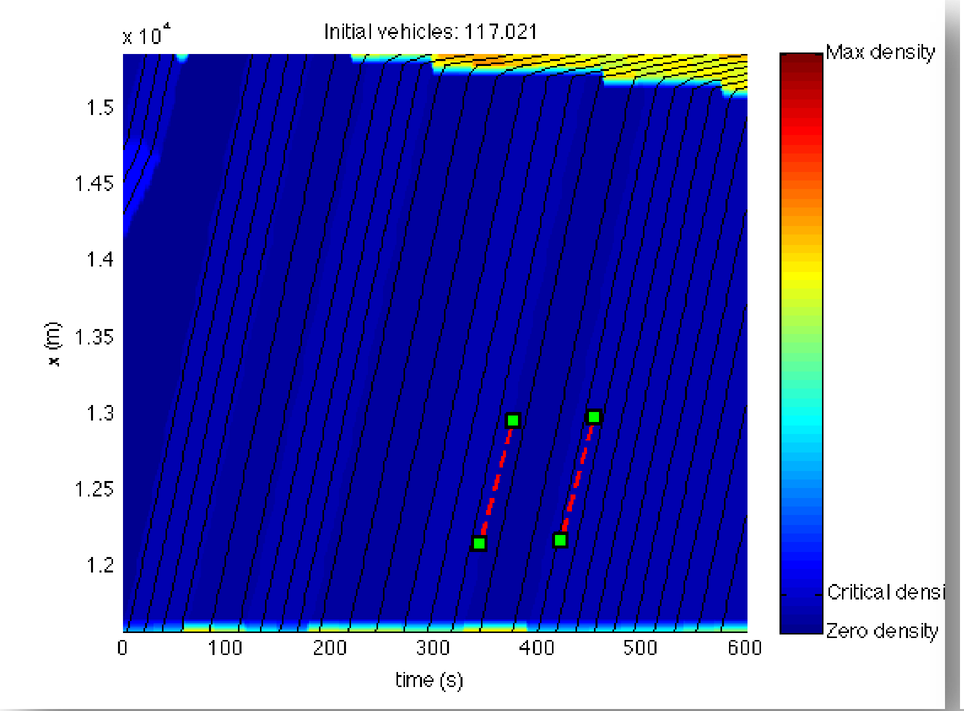


[Canepa, Claudel, ITSC 2012][Canepa, Claudel, NHM 2013][Canepa, Claudel, IEEE TAC (submitted)]

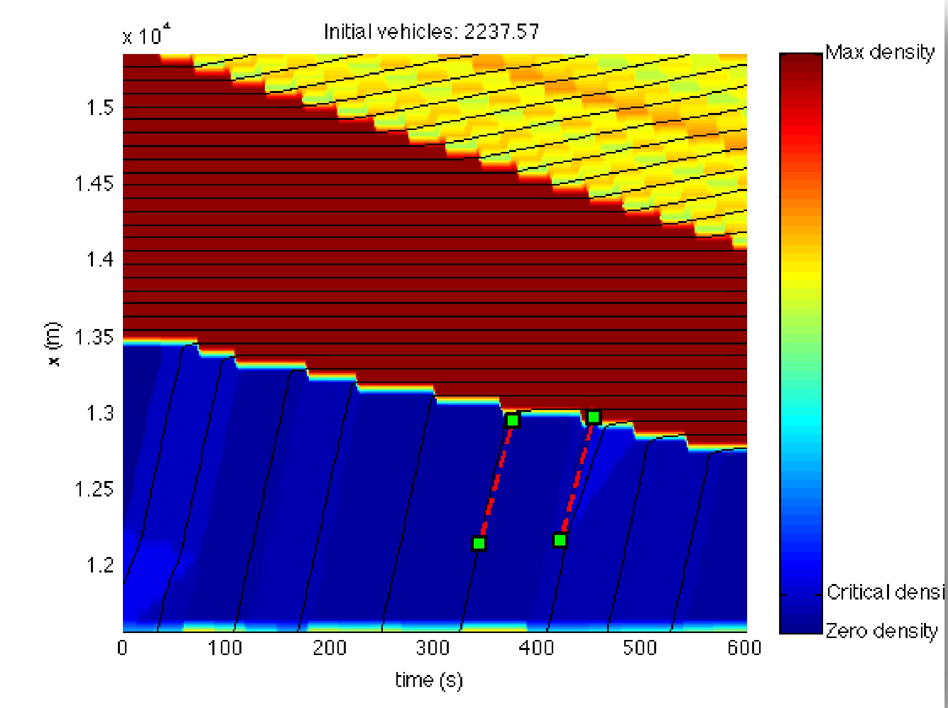
Convergence / monotonicity

- Estimation of initial number of vehicles with multiple internal conditions
  - 2 probe data

Min



Max

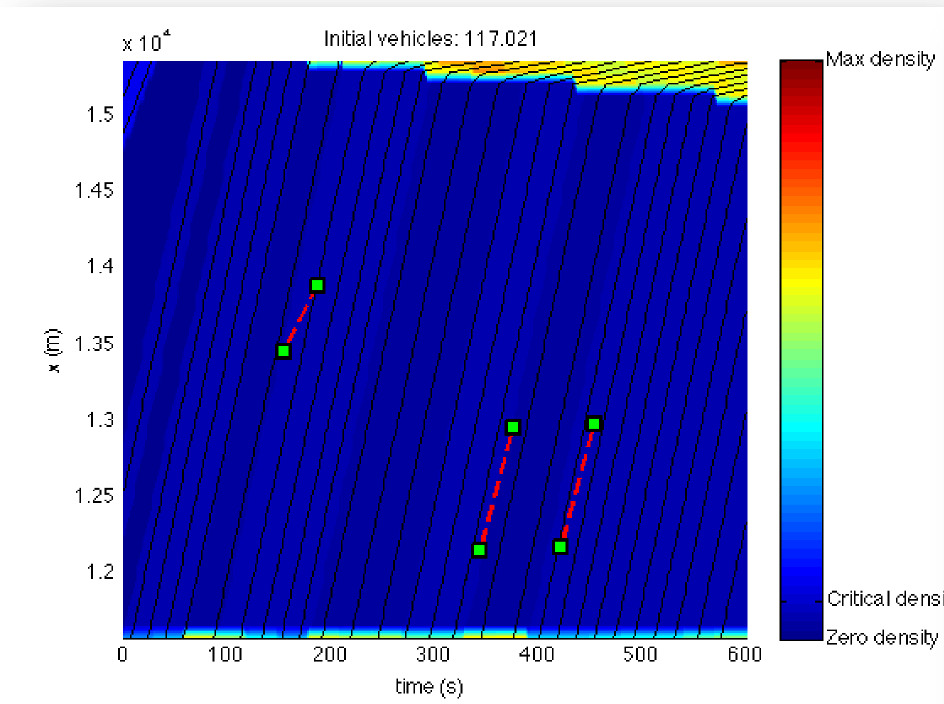


[Canepa, Claudel, ITSC 2012][Canepa, Claudel, NHM 2013][Canepa, Claudel, IEEE TAC (submitted)]

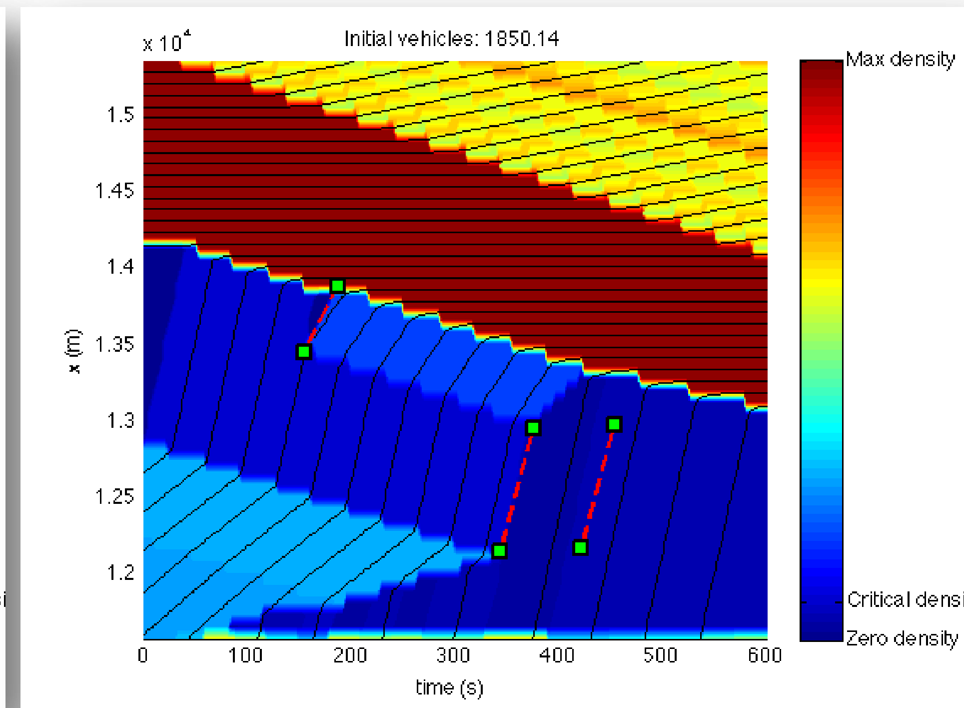
Convergence / monotonicity

- Estimation of initial number of vehicles with multiple internal conditions
  - 3 probe data

Min



Max

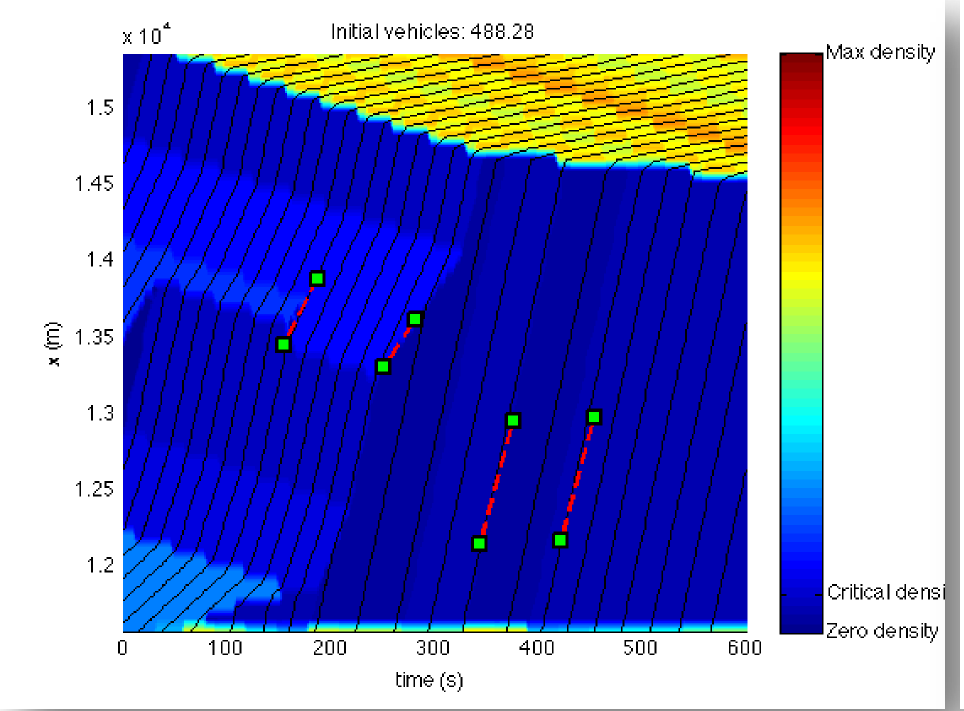


[Canepa, Claudel, ITSC 2012][Canepa, Claudel, NHM 2013][Canepa, Claudel, IEEE TAC (submitted)]

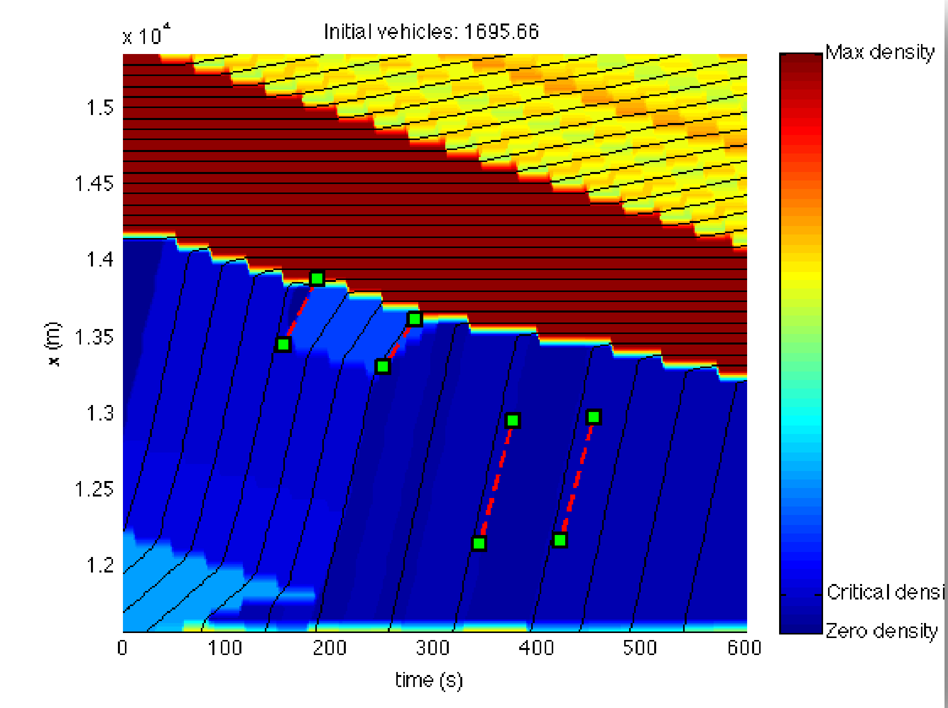
Convergence / monotonicity

- Estimation of initial number of vehicles with multiple internal conditions - 4 probe data

Min



Max

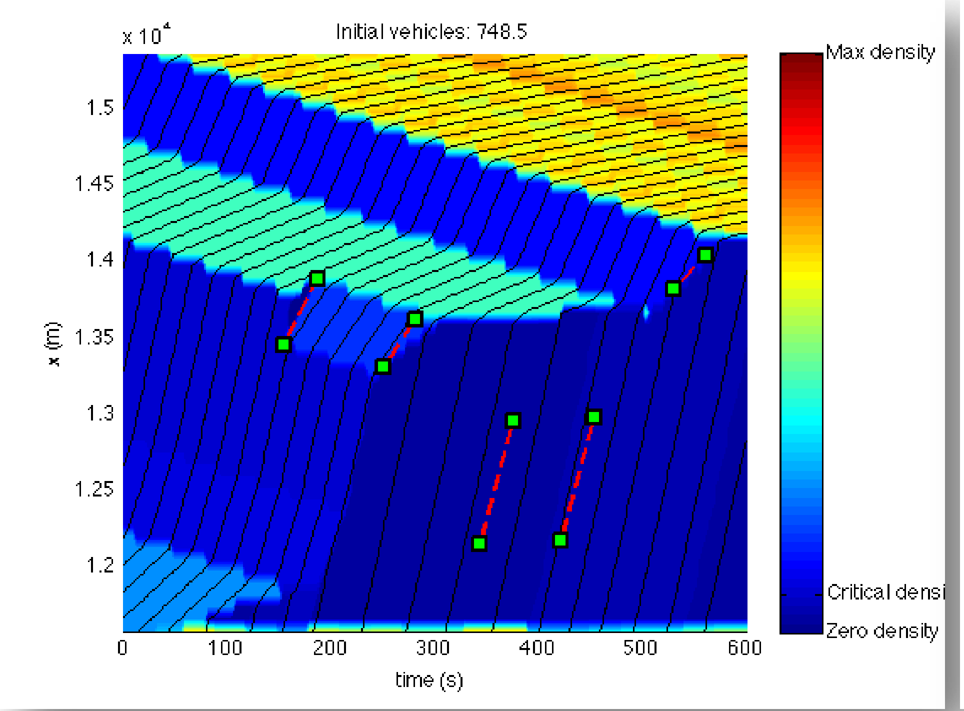


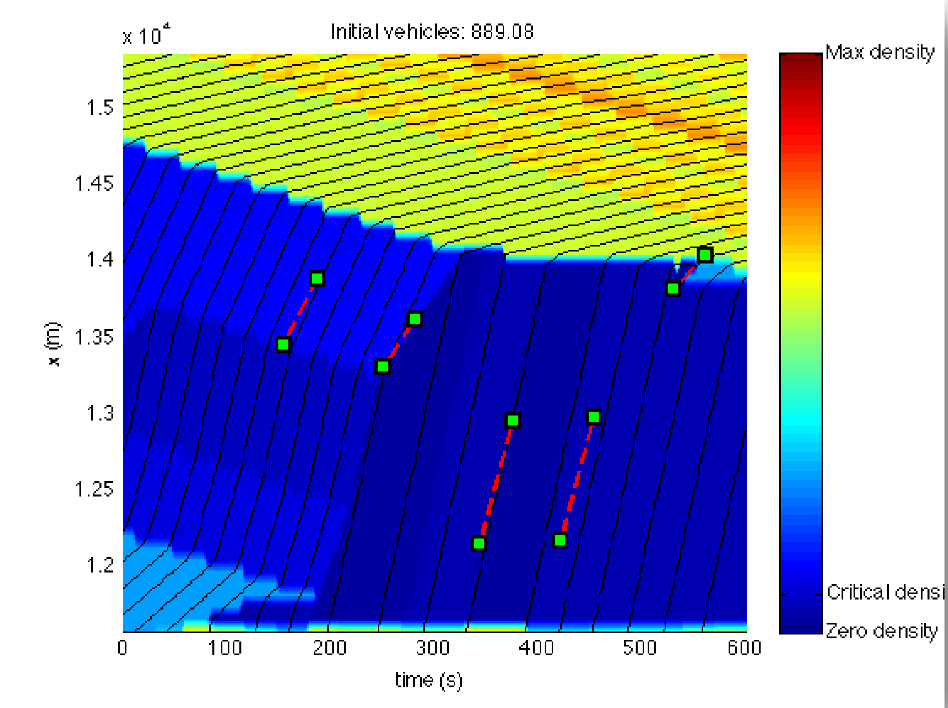
[Canepa, Claudel, ITSC 2012][Canepa, Claudel, NHM 2013][Canepa, Claudel, IEEE TAC (submitted)]

Convergence / monotonicity

- Estimation of initial number of vehicles with multiple internal conditions
  - 5 probe data

Min **Max**



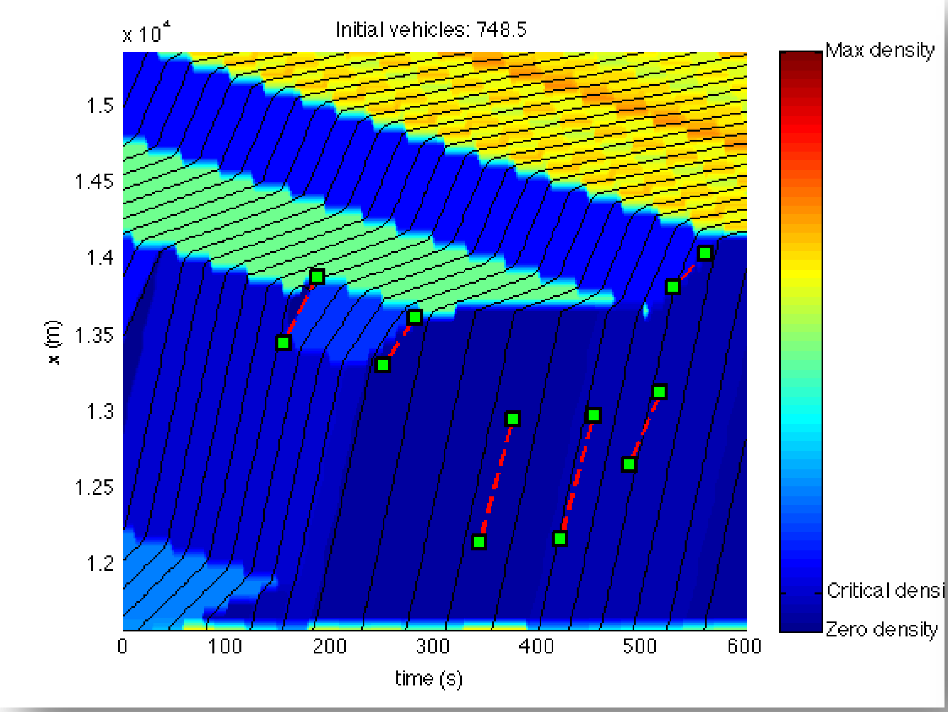


[Canepa, Claudel, ITSC 2012][Canepa, Claudel, NHM 2013][Canepa, Claudel, IEEE TAC (submitted)]

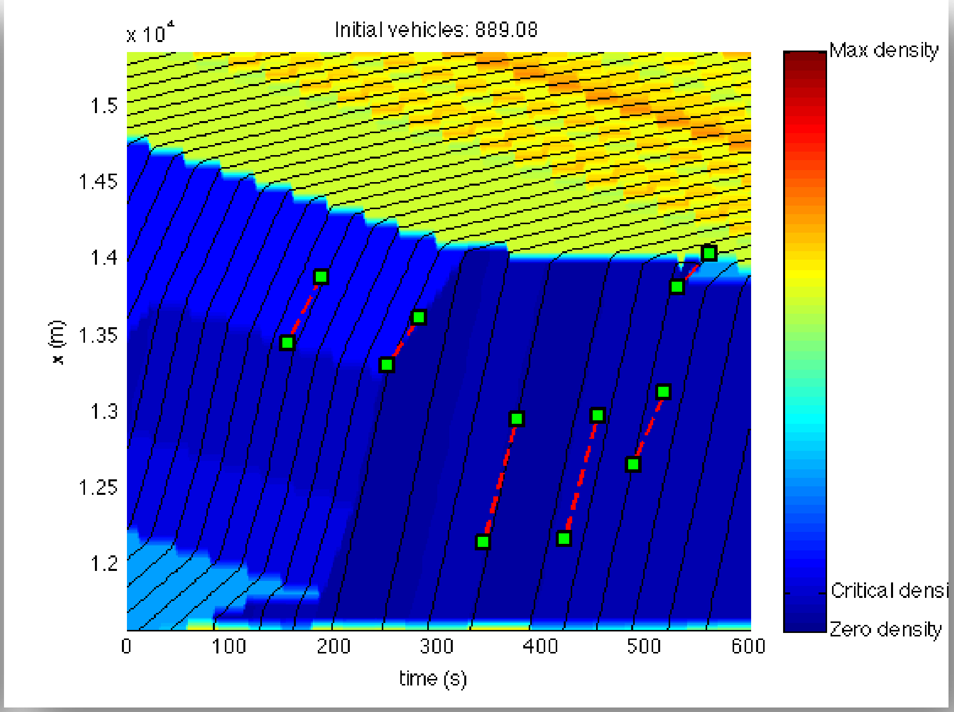
Convergence / monotonicity

- Estimation of initial number of vehicles with multiple internal conditions
  - 6 probe data

Min



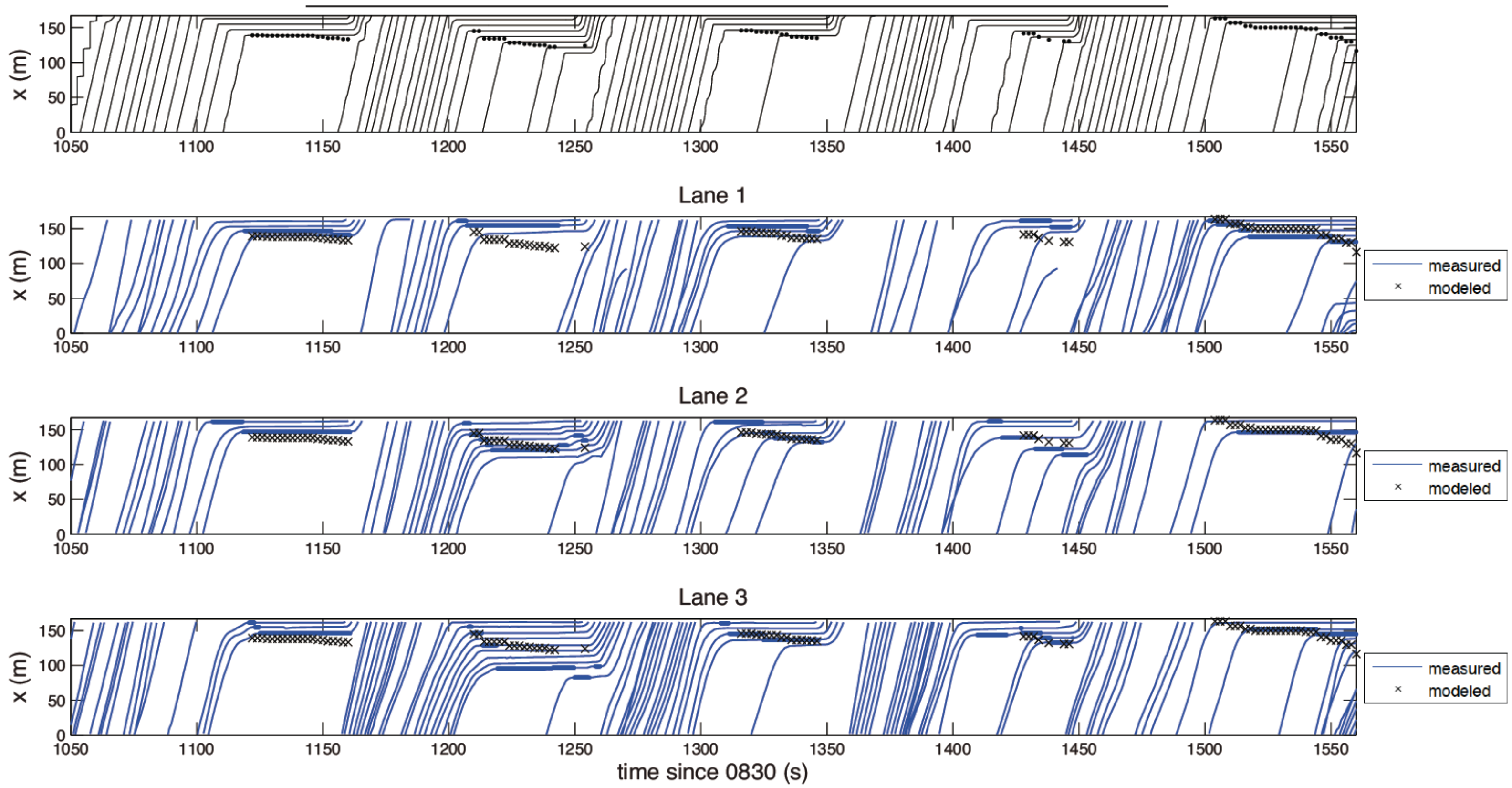
Max



[Canepa, Claudel, ITSC 2012][Canepa, Claudel, NHM 2013][Canepa, Claudel, IEEE TAC (submitted)]

Traffic state estimation

- Traffic state estimation using flow/travel time data



[Anderson Canepa Horowitz Claudel Bayen 2014]

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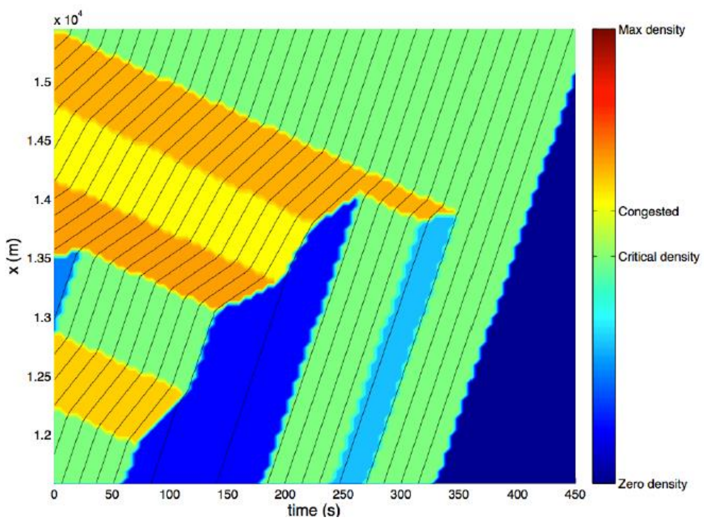
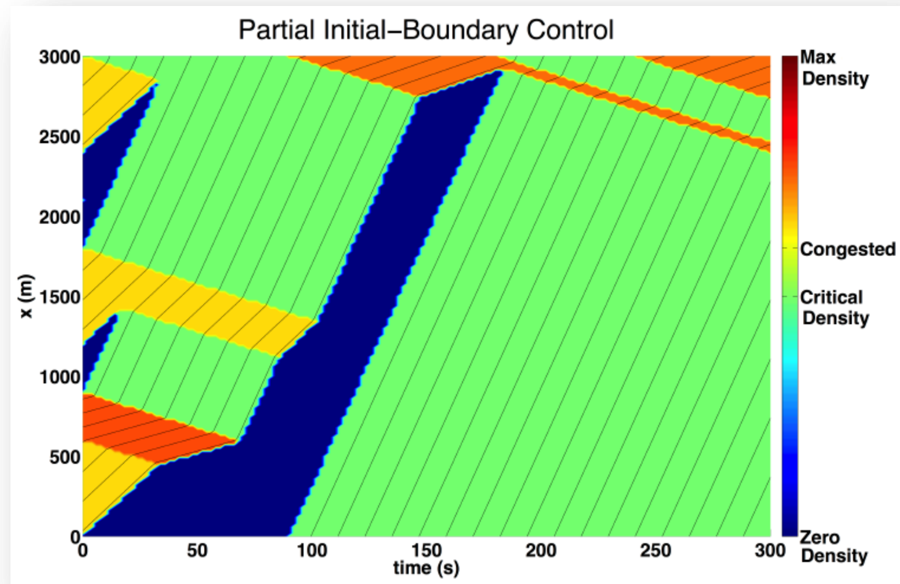
Conclusion

Boundary control

Similar to estimation modulo a change in objective functions

- Example: single link control





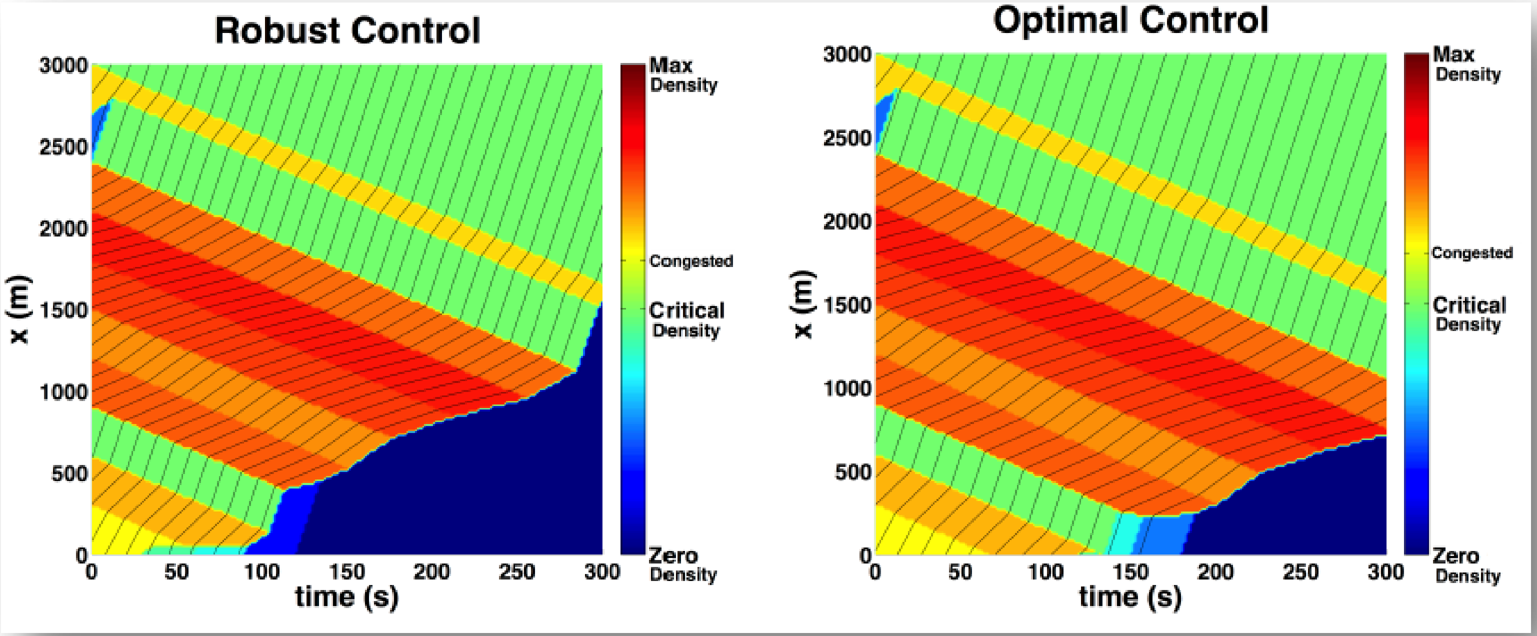
- Can be extended to other control problems
  - Robust control
  - Network control

[Li Canepa Claudel 2014]

Boundary control

Similar to estimation modulo a change in objective functions

- Robust (with respect to initial conditions uncertainty) control:

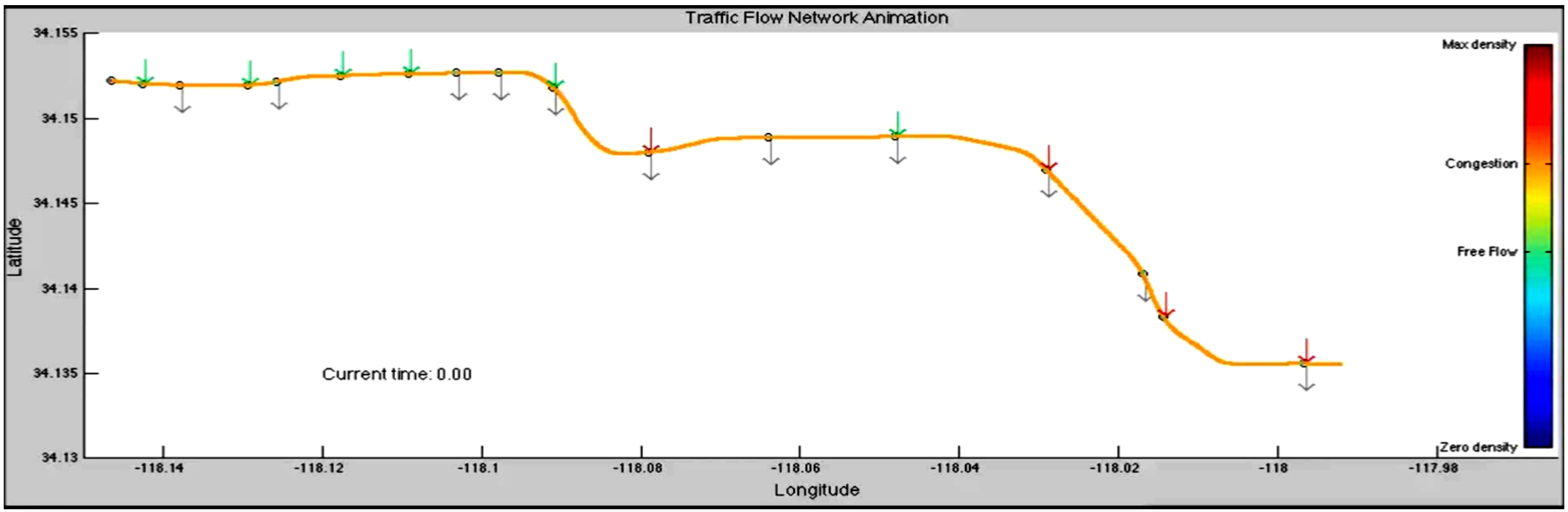


[Li Canepa Claudel 2014]

Network control

In network control, junctions cause the problem to become non-convex, **unless** we assume all junctions to be controllable

On ramp switched traffic flow control example on I210 near Pasadena, CA (ramp metering). Optimal control input is computed in 0.19 s on an iMac



[Li Canepa Claudel 2014]

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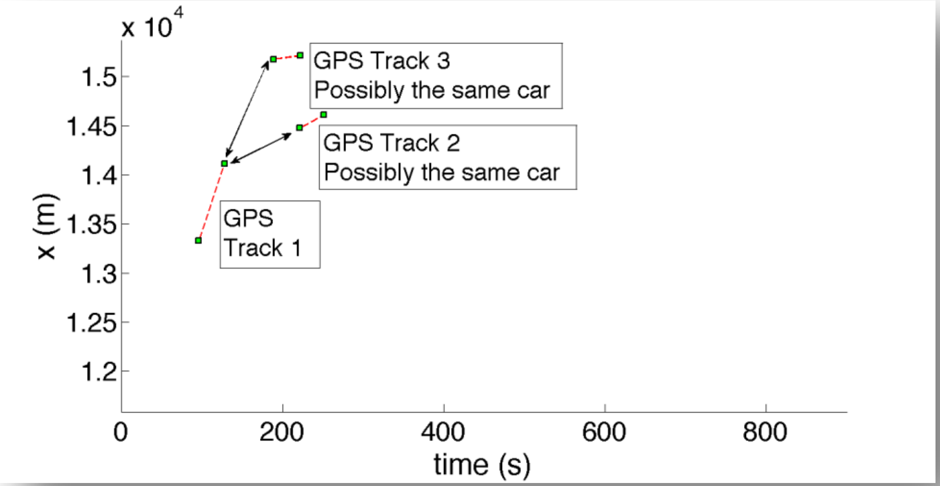
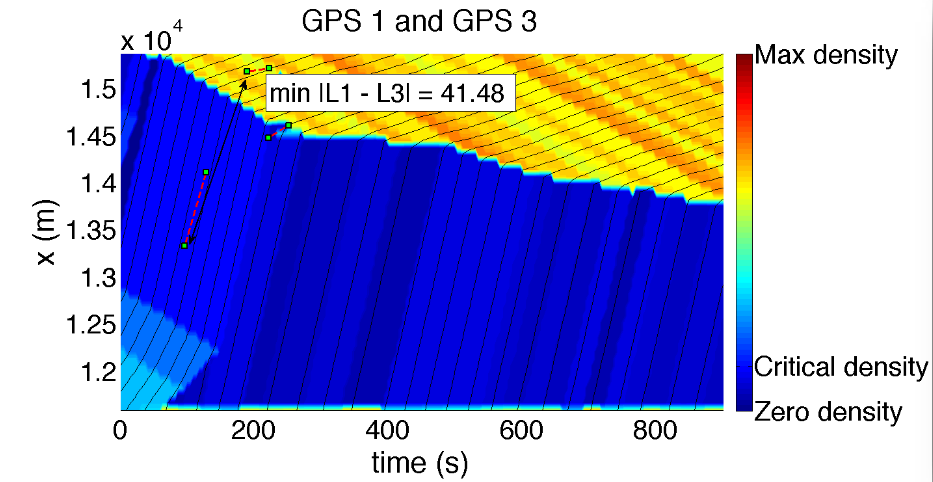
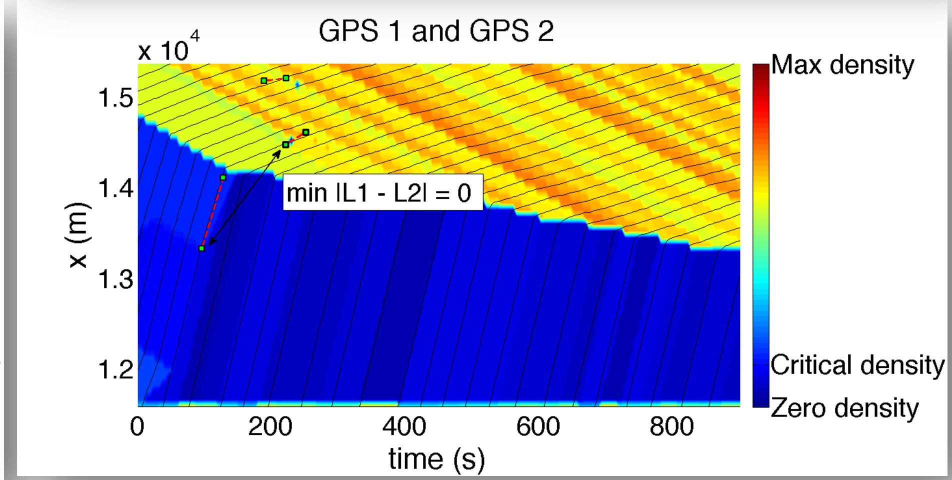
Privacy analysis

- Setup: identify which GPS track (among two candidates) was generated by a given test vehicle
- Objective function: minimize label difference
- Minimum label of zero means that the track is a possible candidate

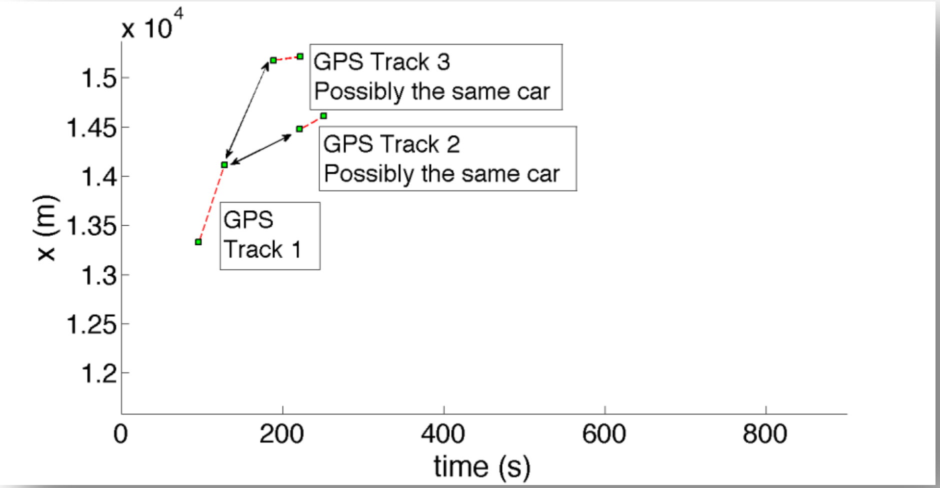


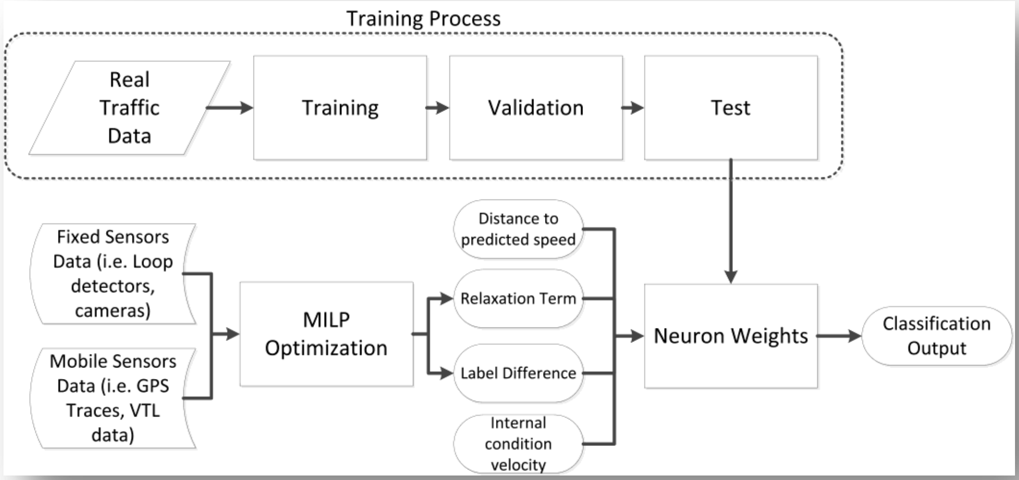




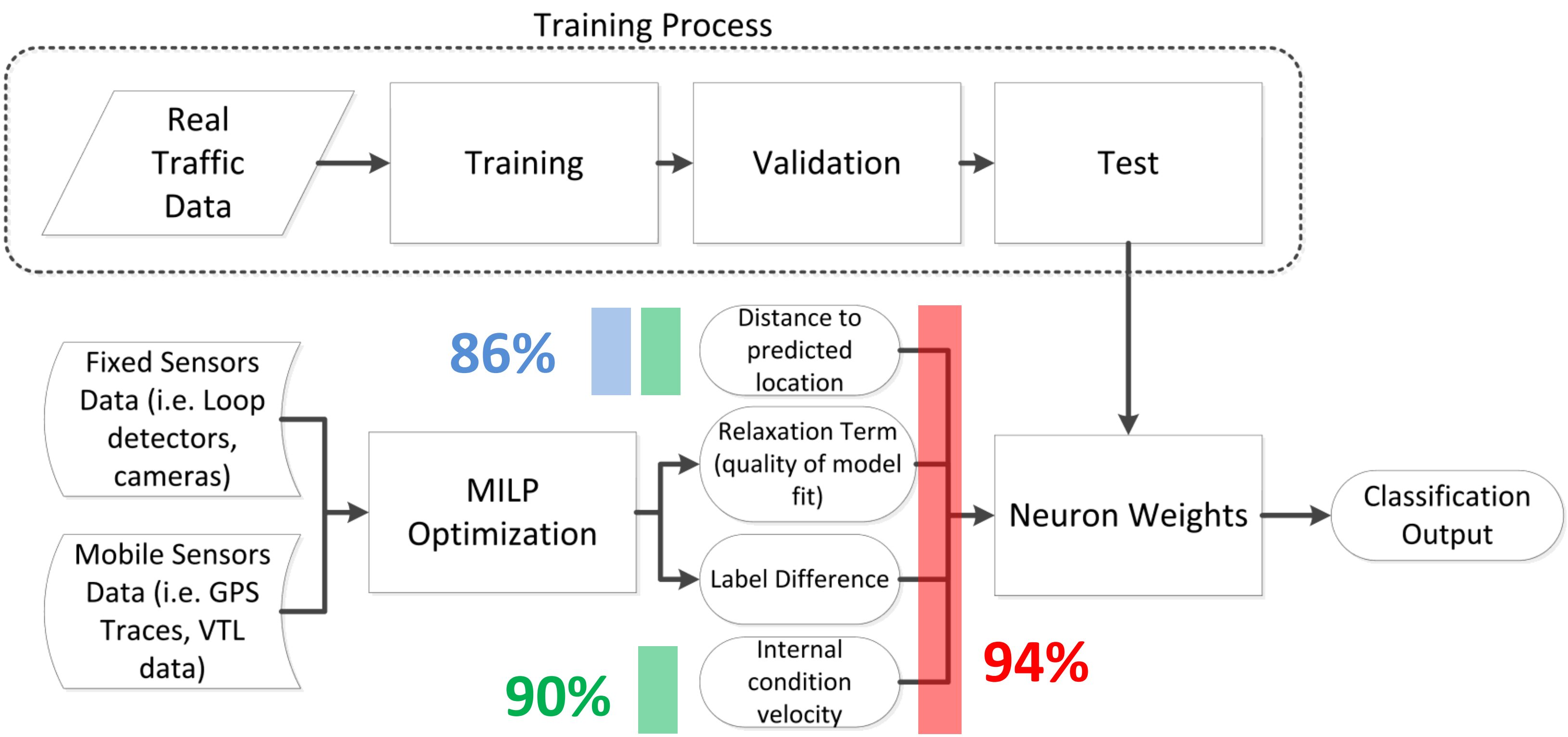
Privacy analysis

- <u>Problem</u>: performs poorly in average (not significantly better than naïve reidentification)
- Sensitive to model parameters
- LWR model is too simplistic in free flow
- Solution: compute metrics with the model, use ML to identify where the model is useful





Reidentification performance



Validation over 5000 test reidentification problems (Mobile Century data)
http://traffic.berkeley.edu

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Sensor fault detection

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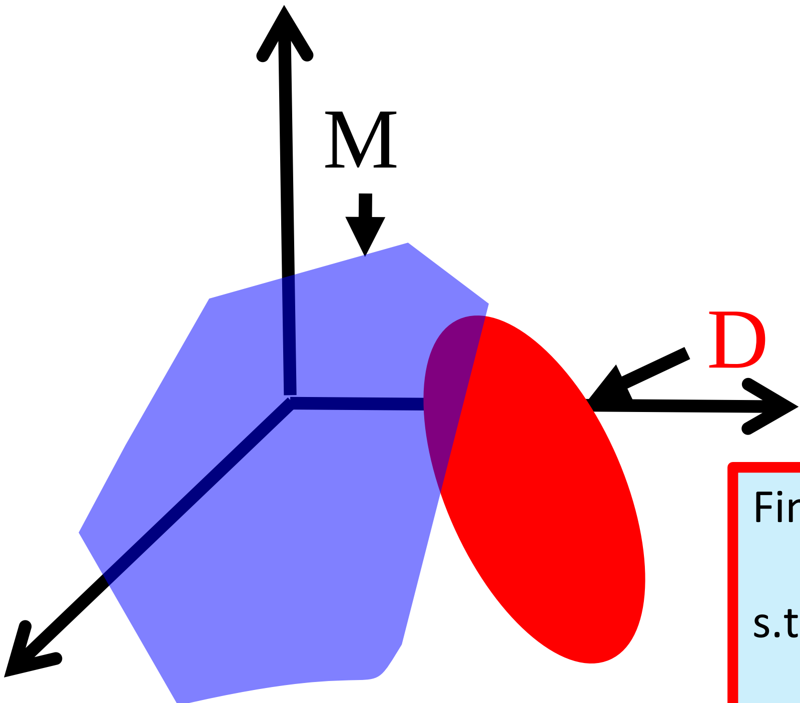
Data consistency check:

Is the data generated by a set of sensors compatible with the model constraints ?



Coefficients of the PWA boundary conditions

Find y

s.t.

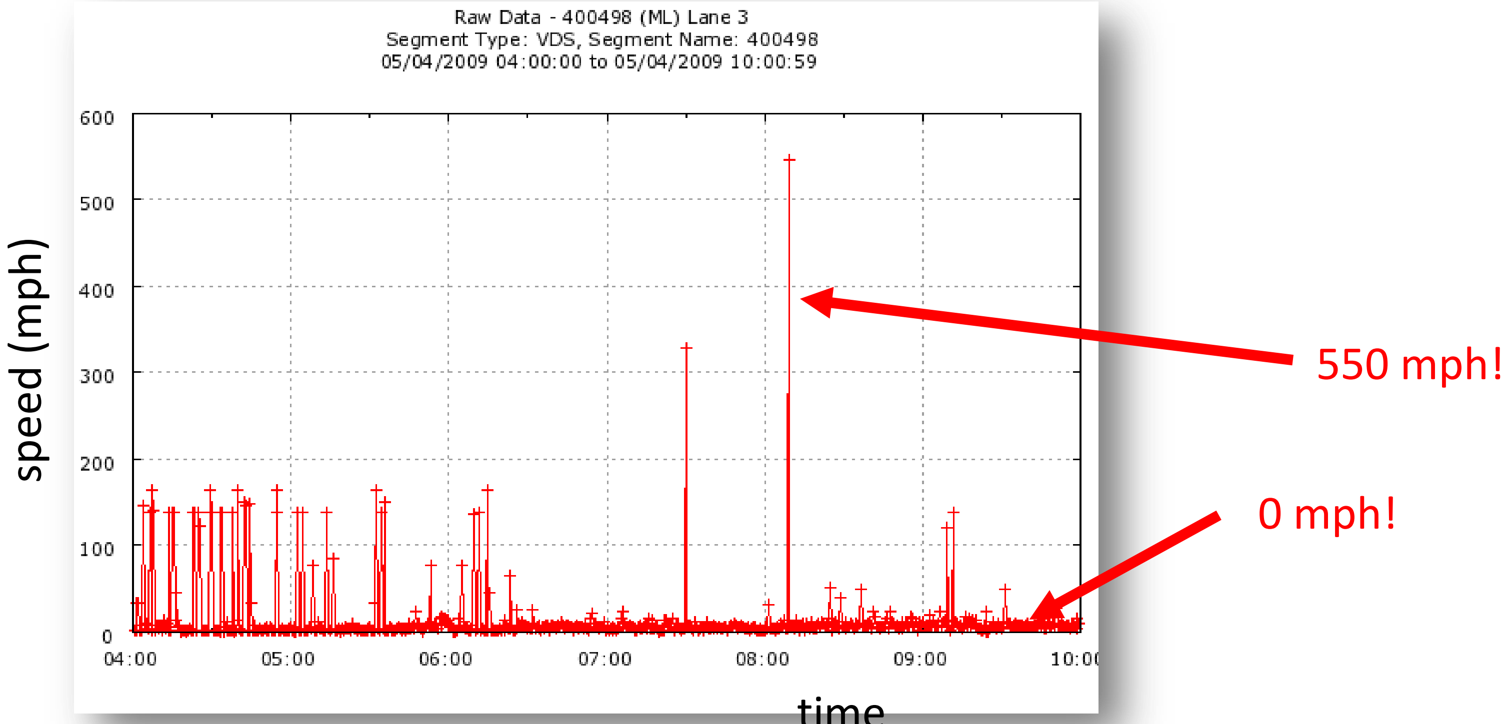
- y satisfies the convex inequality constraints(M)
- y satisfies the convex inequality constraints (D)

[Claudel, Bayen, Allerton CCC 2009]

Sensor fault detection

PeMS loop detector network:

- 1200 sensors in the San Francisco Bay Area
- poor reliability (70% availability in average)
- detecting sensor failures is a big problem



[Claudel, Bayen, Allerton CCC 2009]

Sensor fault detection

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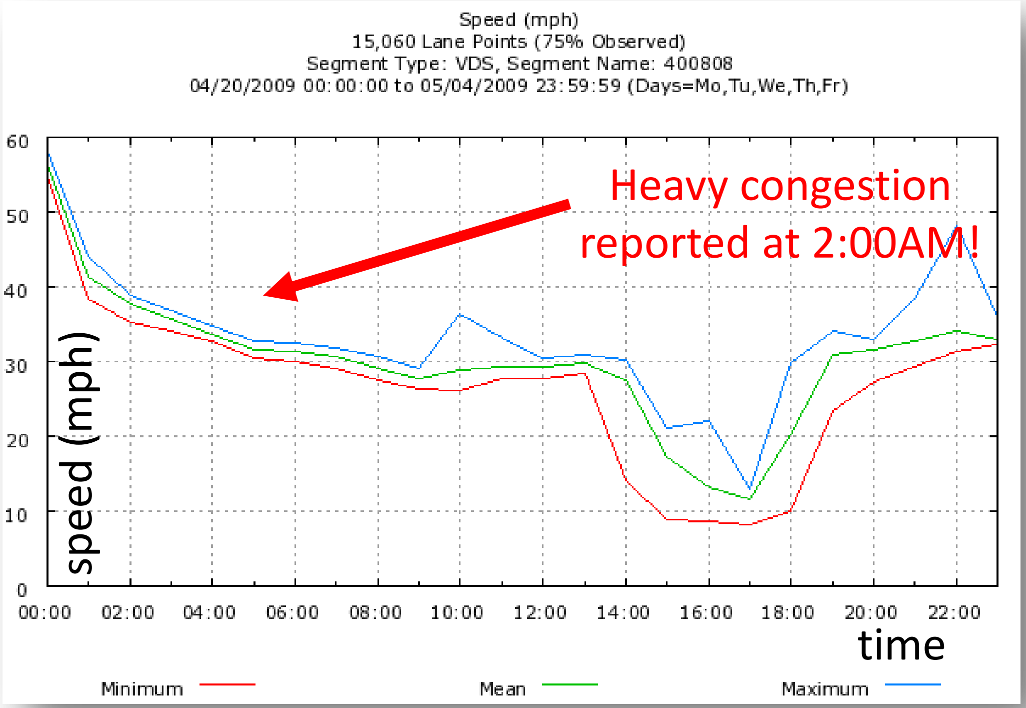
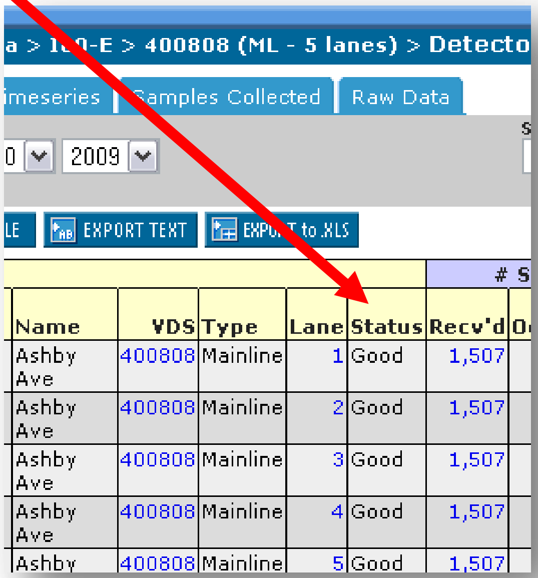


Status: good!



Name	VDS	Type	Lane	Status	Recv'd	O
Ashby Ave	400808	Mainline	1	Good	1,507	
Ashby Ave	400808	Mainline	2	Good	1,507	
Ashby Ave	400808	Mainline	3	Good	1,507	
Ashby Ave	400808	Mainline	4	Good	1,507	
Ashby	400808	Mainline	5	Good	1,507	

[Claudel, Bayen, Allerton CCC 2009]

Sensor fault detection

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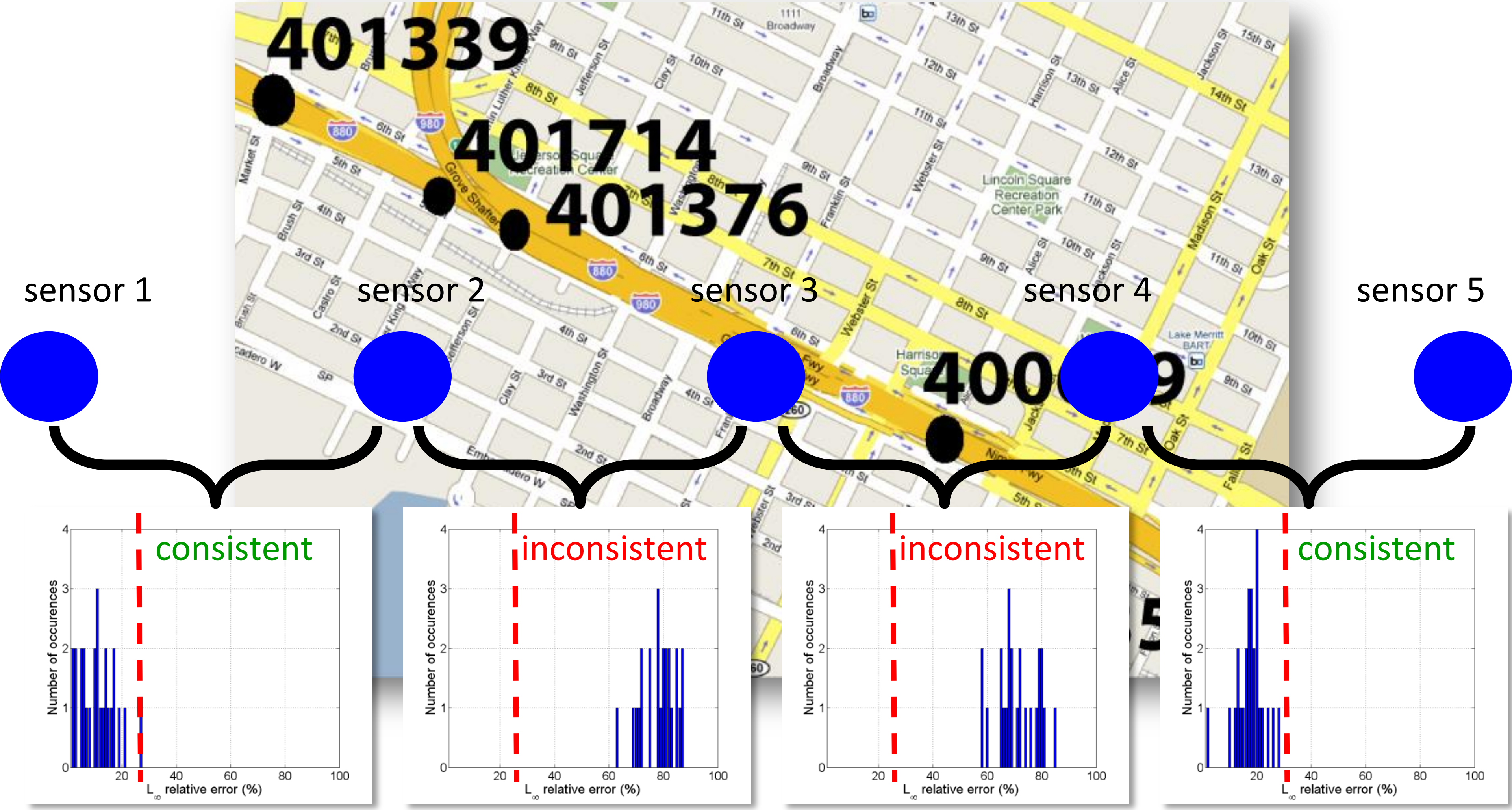
Example of sensor fault detection (actually sensor misplacement)

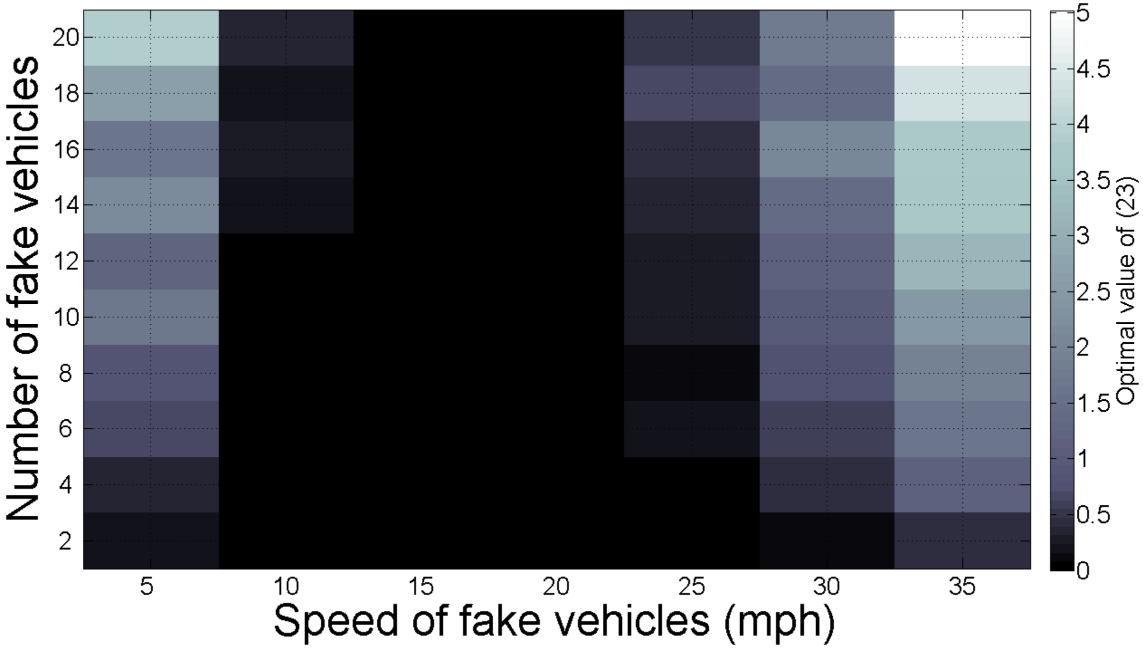


Sensor fault detection

Example of sensor fault detection (actually sensor misplacement)

Cybersecurity

- The same framework can be applied to cybersecurity problems
- Idea: measure the deviation between the model and the data (distance between D and M



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Very flexible framework for integrating the LWR model into a variety of problems (estimation, control, etc)

Less Boolean variables than frameworks based on the discretization of the LWR PDE (1 per junction, vs 1 per cell)