

Electron hydrodynamics with a polygon Fermi surface

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Caleb Cook

Stanford

arXiv: 1903.05652

Strong Interactions

- ▶ goal: transport/dynamics of electrons in solids

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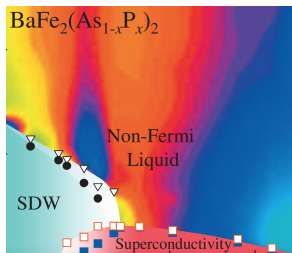
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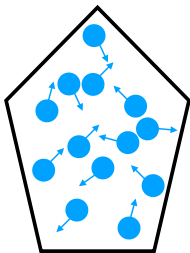
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- ▶ in general, not computationally tractable:
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 - ▶ no real time physics with quantum Monte Carlo
- ▶ real systems can be hard:



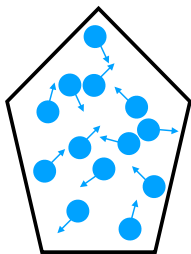
A “Solved” Problem

- ▶ analogy to a “solved” problem (hydrodynamics)?



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- ▶ hydrodynamics is valid on long time and length scales:

$$\omega\tau_{ee} \ll 1, \quad kl_{ee} \ll 1$$

where $l_{ee} = v_F\tau_{ee} =$ e-e collision mean free path

How to See the Hydrodynamic Limit

- ▶ slow modes are locally conserved quantities: in typical solid

$$\frac{\partial}{\partial t} \begin{pmatrix} \epsilon \\ \rho \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{pmatrix} \nabla^2 \begin{pmatrix} \epsilon \\ \rho \end{pmatrix}$$

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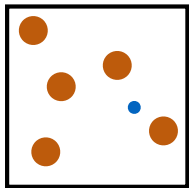
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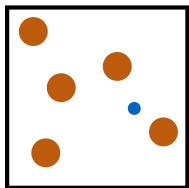
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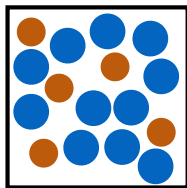
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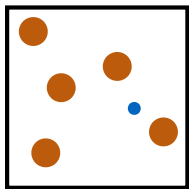
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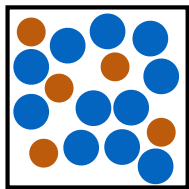
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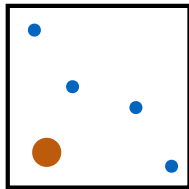
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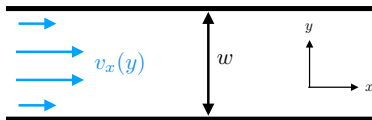


clean Fermi liquid

today's talk

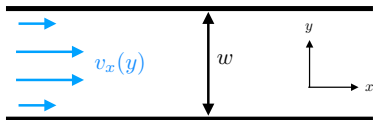
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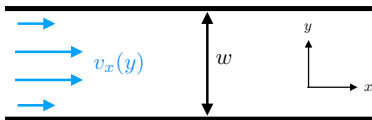


- ▶ estimate \mathcal{R} (resistance per unit length):

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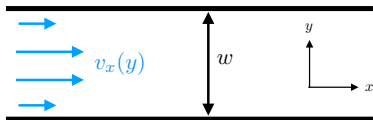
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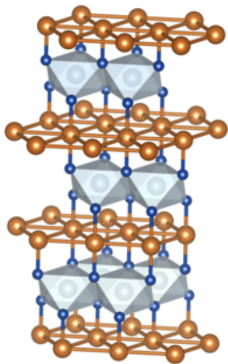
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- ▶ ballistic limit ($\ell_{ee} \gg w$): $\mathcal{R}_{\text{ballistic}} \propto \frac{1}{w} \times \frac{v_F}{w}$

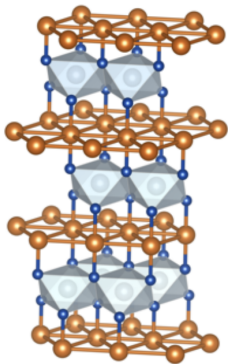


delafossite PdCoO_2 :

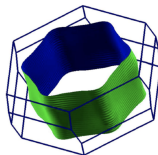




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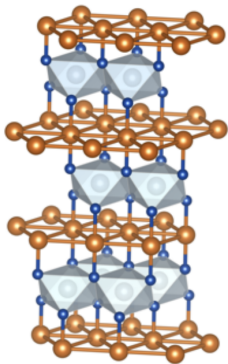


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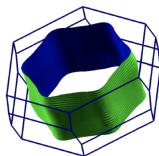




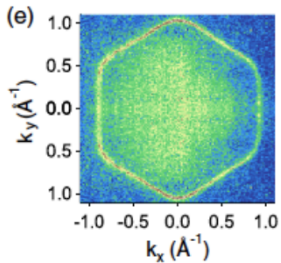
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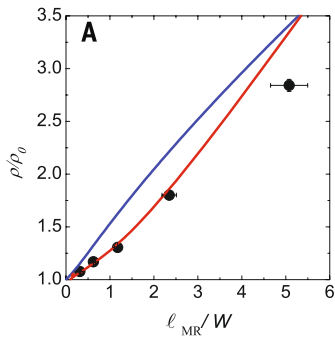


► big hexagonal FS:



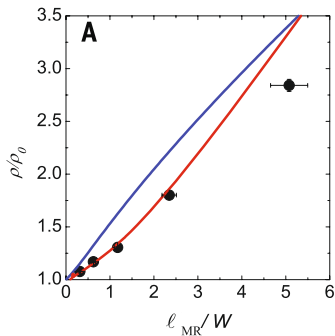
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experiment:

[Moll *et al*; 1509.05691]

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- ▶ simulations: w -dependence compatible with onset of hydrodynamics?

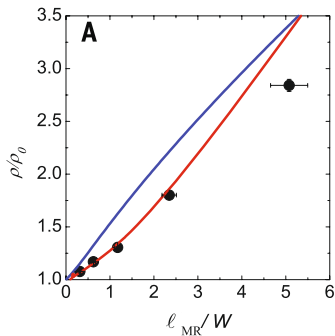
$$\rho \sim w^{-1} \quad (\text{ballistic})$$

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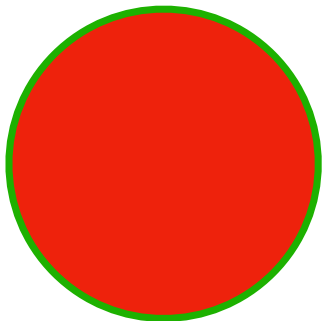
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- ▶ T -dependence?

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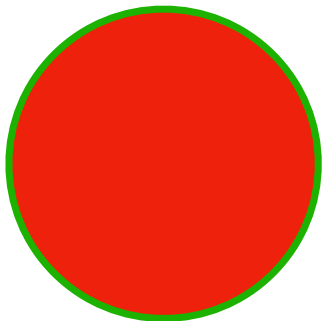
Circle vs. Hexagon?

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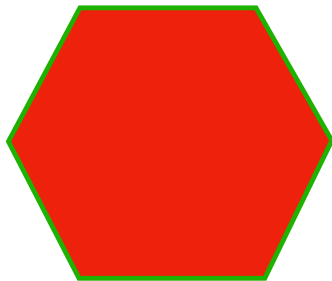


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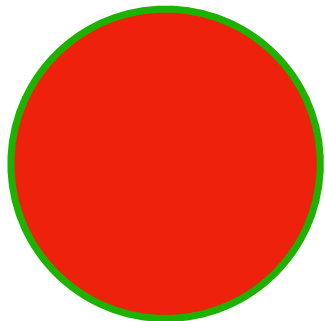


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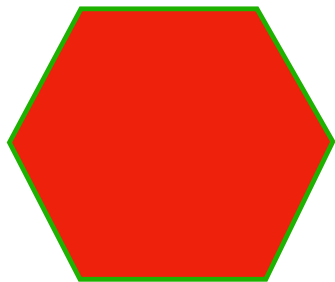


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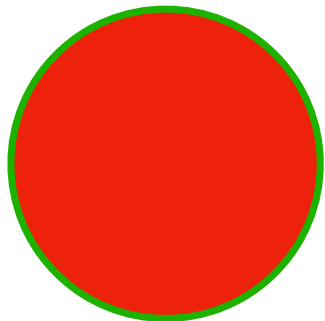
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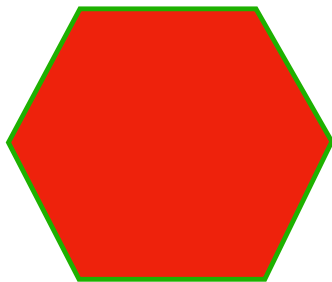
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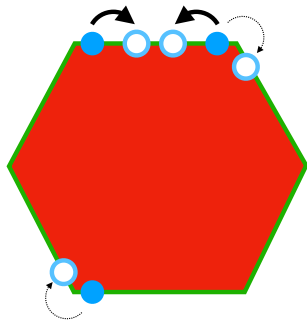


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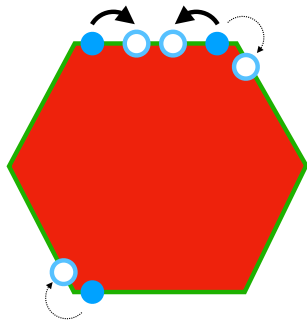


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 - ▶ less obvious: ≥ 2 scattering times

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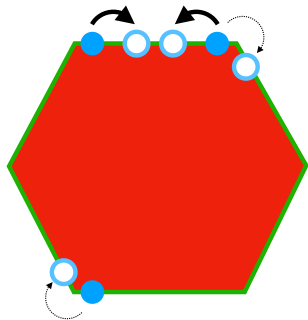


- ▶ (at least) two scattering rates (up to logs):

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- ▶ similar, but not equivalent, to discrepancy in parity-even/odd scattering rates [Ledwith *et al*; 1708.01815]

The True Hydrodynamic Limit for the Hexagon

- on time scales $\omega \ll \gamma_s$...**hydrodynamics:**

$$\begin{aligned}\partial_t n + \partial_i(n_0 v_i - D_0 \partial_i n) &= 0, \\ mn_0 \partial_t v_i + mv_s^2 \partial_i n - \eta_{jikl} \partial_j \partial_k v_l &= 0\end{aligned}$$

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$$\eta_{ijkl} = \eta (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \delta_{ij} \delta_{kl}) + \tilde{\eta} \epsilon_{ij} \epsilon_{kl}$$

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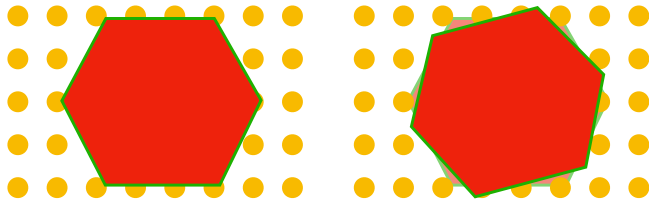
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- ▶ $\tilde{\eta} > 0$ is **dissipative** – not Hall viscosity!

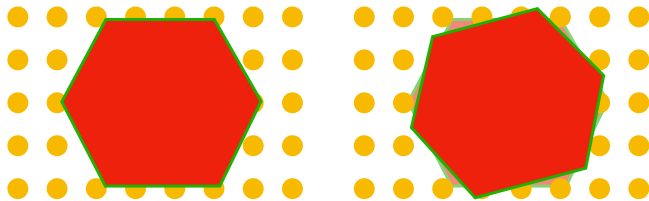
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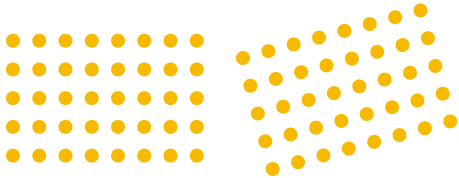


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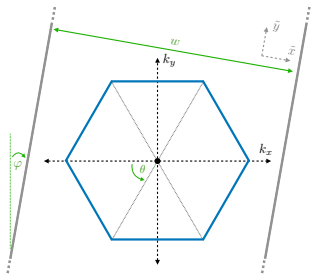


- ▶ contrast: the ionic lattice **spontaneously breaks isotropy**: it (w/ electrons) must have local angular momentum conservation ($\tau_{ij}^{\text{universe}} = \tau_{ji}^{\text{universe}}$)



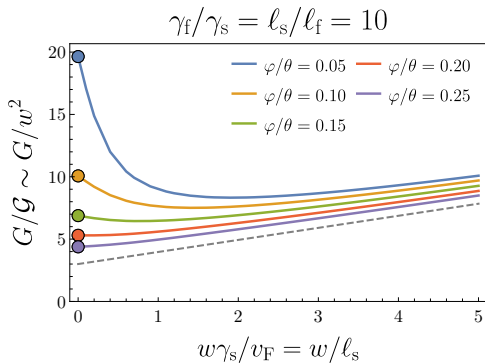
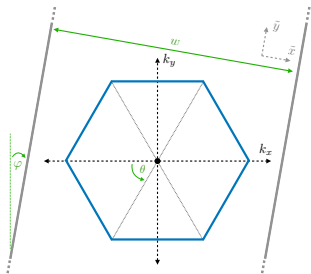
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- ▶ Gurzhi effect with hexagonal FS?



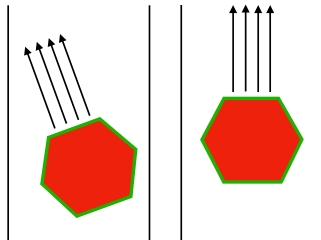
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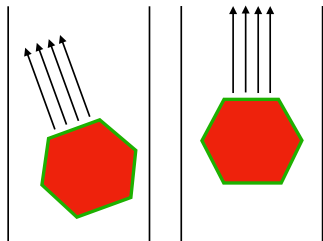
Origin of Non-Monotonicity

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- ▶ hydrodynamic limit (no slip boundary conditions):

$$G = \frac{e^2 n^2 w^3}{12 \eta_{xyxy}(\phi)}$$

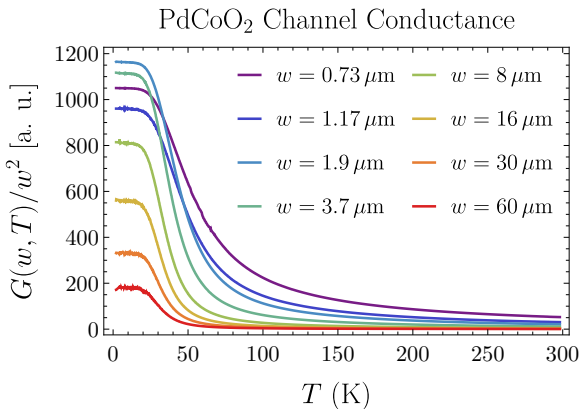
for the hexagon, $\eta_{xyxy}(\phi) = \eta + \tilde{\eta}$.

Replotting Data

- ▶ is our model relevant at all for PdCoO₂? not yet...

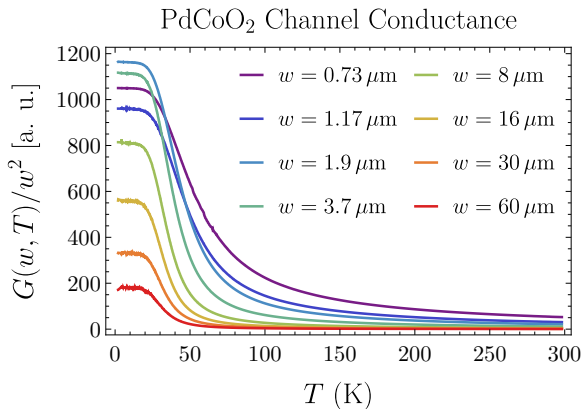
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- ▶ why small w curves don't collapse as $T \rightarrow 0$?

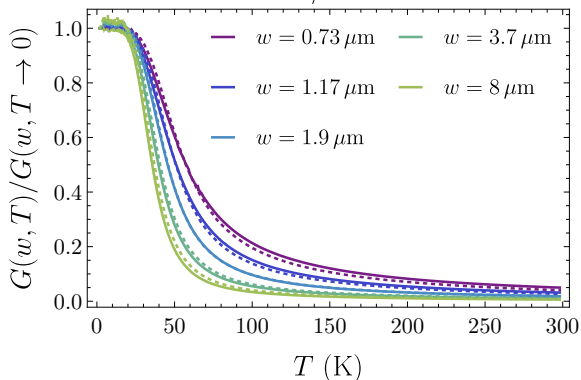
Ohmic Fit?

- ▶ we fit these curves to the model

$$\frac{G(w, T)}{G(w, 0)} = \frac{A_{\text{ballistic}} + \rho_{\text{bulk}}(0)w}{A_{\text{ballistic}} + \rho_{\text{bulk}}(T)w}$$

with $A_{\text{ballistic}}$ only fit parameter, same for all w

Fit to Ohmic/Ballistic Model



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- ▶ PdCoO₂ – still some open questions...?