Distributed Consensus and Cooperative Estimation

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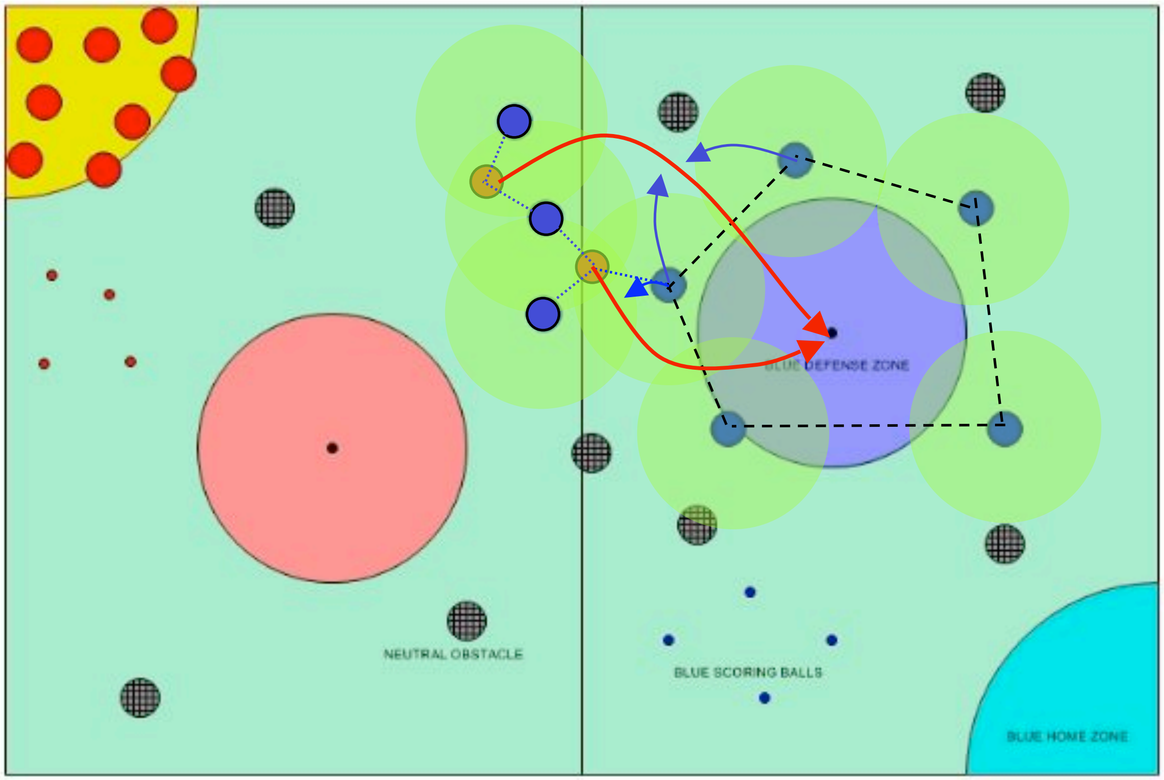
RoboFlag Subproblems



1. Formation control
- Maintain positions to guard defense zone

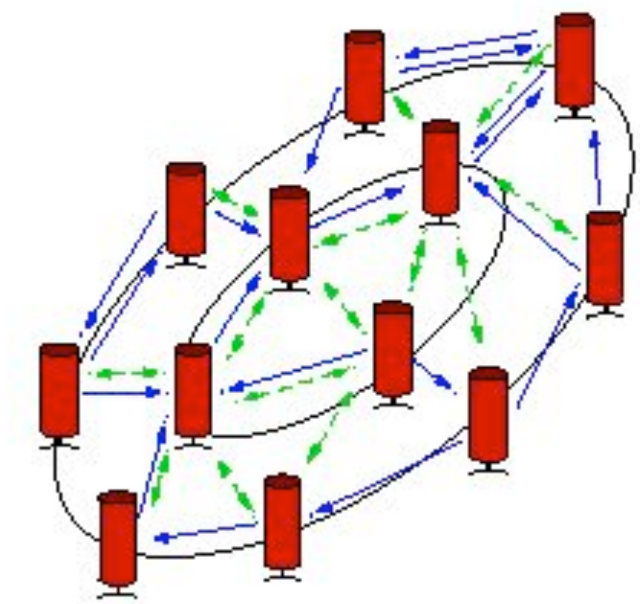
2. Distributed estimation
- Fuse sensor data to determine opponent location

3. Distributed consensus
- Assign individuals to tag incoming vehicles

Goal: develop systematic techniques for solving subproblems
- Cooperative control and graph Laplacians
- Distributed receding horizon control
- Verifiable protocols for consensus and control

→ Implement and test as part of annual RoboFlag competition

Information Flow in Vehicle Formations



Example: satellite formation

- Blue links represent *sensed* information
- Green links represent *communicated* information

Sensed information

- Local sensors can see some subset of nearby vehicles
- Assume small time delays, pos'n/vel info only

Communicated information

- Point to point communications (routing OK)
- Assume limited bandwidth, some time delay
- Advantage: can send more complex information

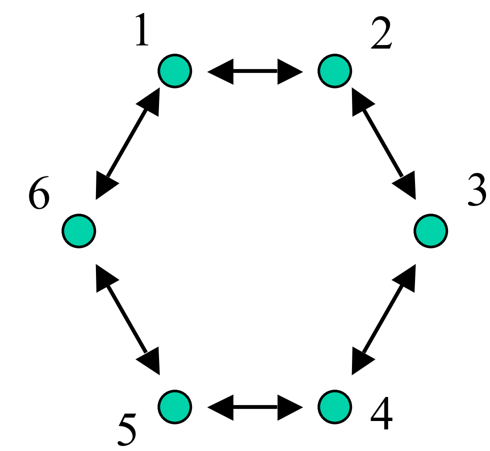
Topological features

- Information flow (sensed or communicated) represents a directed graph
- Cycles in graph ⇒ information feedback loops

Question: How does topological structure of information flow affect stability of the overall formation?

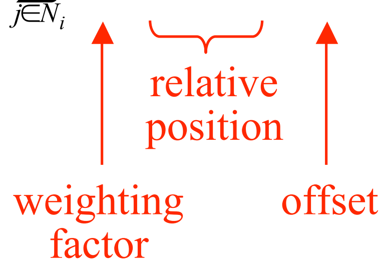
Sample Problem: Formation Stabilization

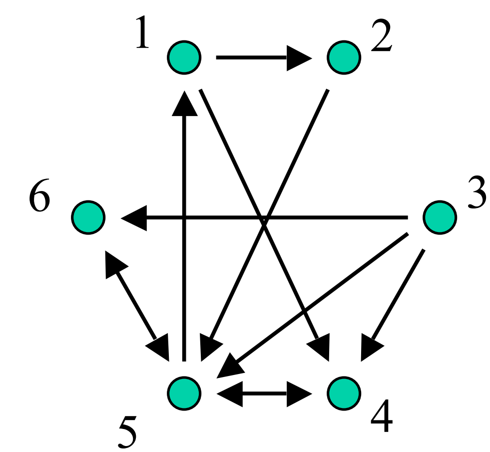
Goal: maintain position relative to neighbors
- “Neighbors” defined by graph
- Assume only sensed data for now
- Assume identical vehicle dynamics, identical controllers?



Example: hexagon formation
- Maintain fixed relative spacing between left and right neighbors

$$e_i = \sum_{j \in N_i} w_j (y_i - y_j - h_{ij})$$





Can extend to more sophisticated “formations”
- Include more complex spatia-temporal constraints

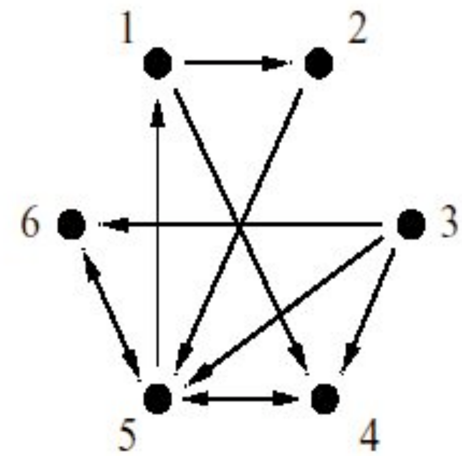
Graph Laplacian

Construction of (weighted) Laplacian

$$L = I - D^{-1}A$$

A = adjacency matrix

D = diagonal matrix, weighted by outdegree

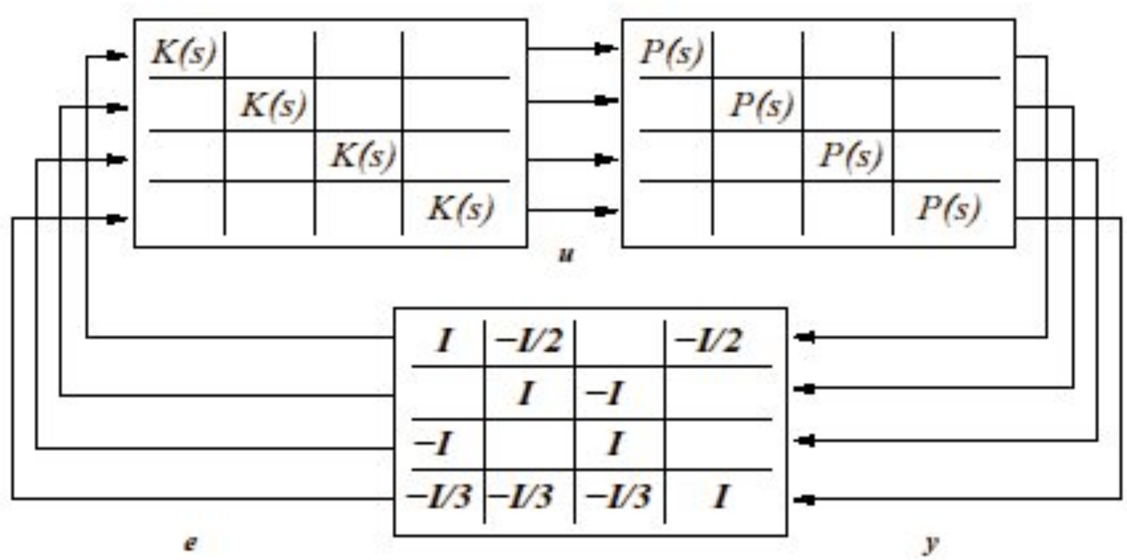


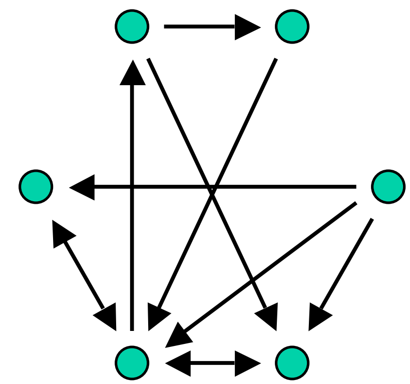
Properties of Laplacian

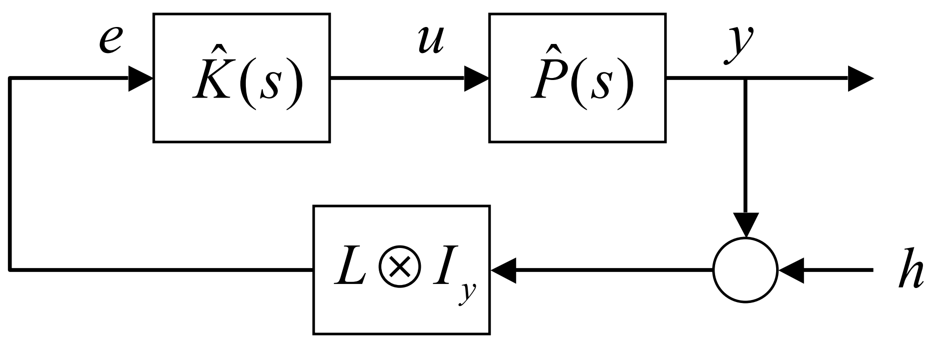
- Row sum equal 0 (stochastic matrix)
- All eigenvalues are non-negative, with at least one zero eigenvalue (from row sum)
- Multiplicity of 0 as an eigenvalue is equal to the number of strongly connected compo-nents of the graph
- All eigenvalues lie in a circle of radius one centered at 1 + 0i (Perron Frobenius)
- For bidirectional (eg, undirected) graphs, eigenvalues are all real, in [0,2]

$$L = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & 1 & -\frac{1}{3} \\ 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

Mathematical Framework





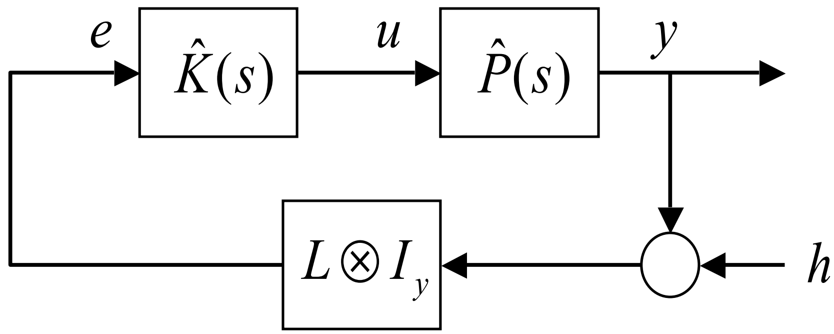


Analyze stability of closed loop

- Interconnection matrix, *L*, is the *Laplacian* of the graph
- Stability of closed loop related to eigenstructure of the Laplacian

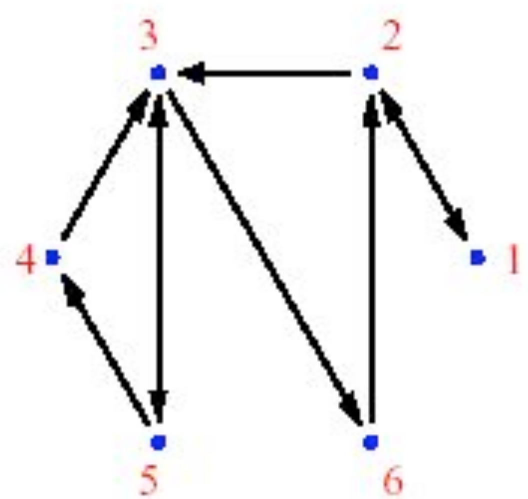
Fax and Murray
IFAC 02, TAC 04

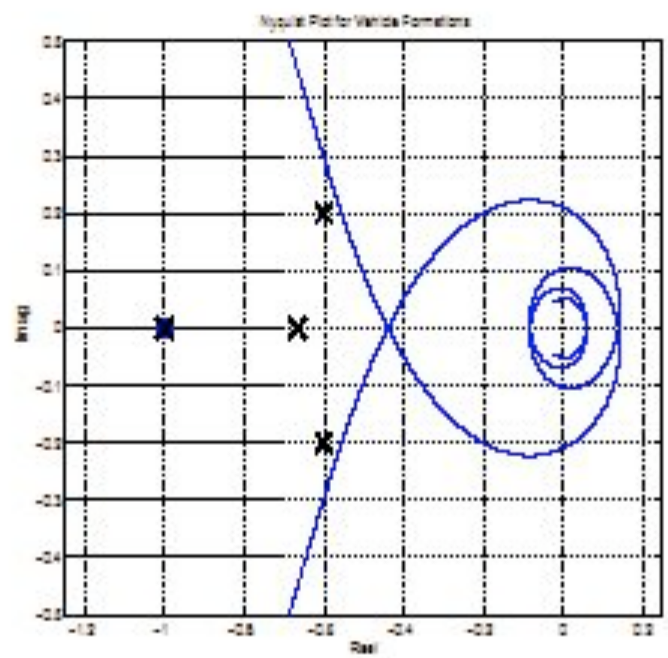
Stability Condition



Theorem *The closed loop system is (neutrally) stable iff the Nyquist plot of the open loop system does not encircle* -1/λ_i(*L*), *where* λ_i(*L*) *are the nonzero eigenvalues of L.*

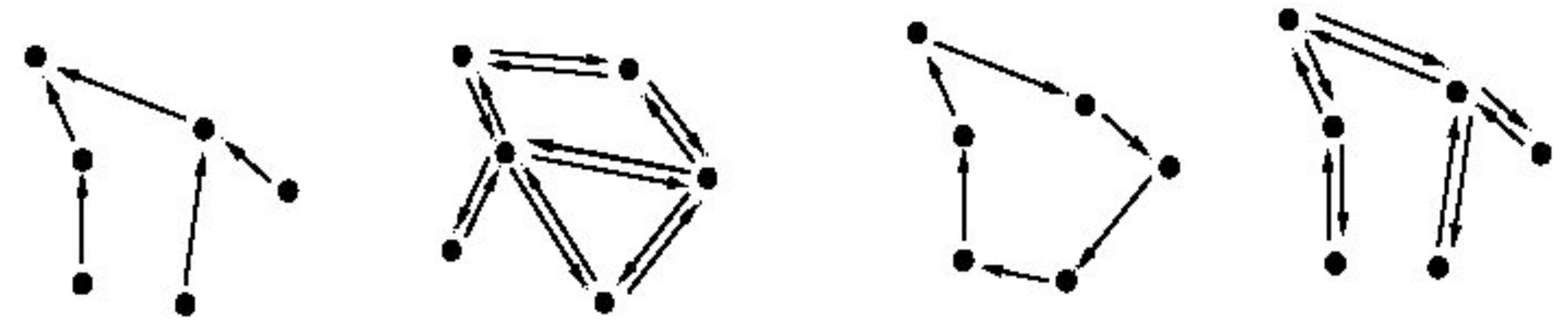
Example $P(s) = \frac{e^{-s\tau}}{s^2}$ $K(s) = K_d s + K_p$



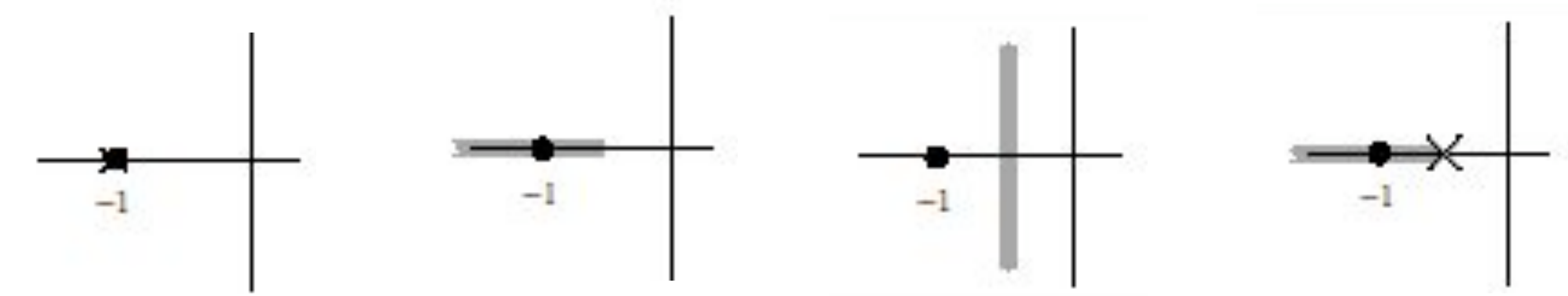


Spectra of Laplacians

Unidirectional tree	Undirected graph	Cycle	Periodic graph



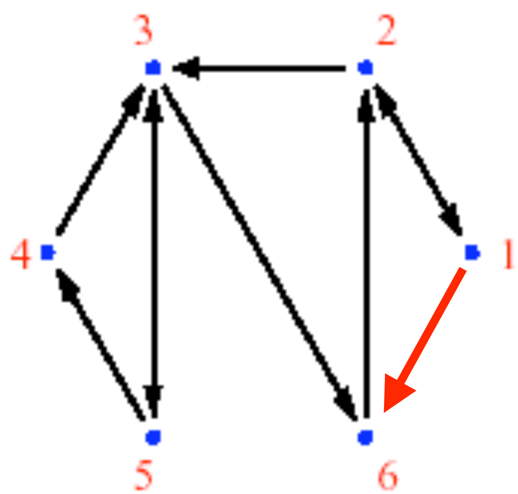
$\lambda = 0,1$	$\lambda \in [0,2]$	$\lambda_i = 1 - e^{2\pi(i-1)j/N}$	$\lambda_1 = 0, \lambda_N = 2$

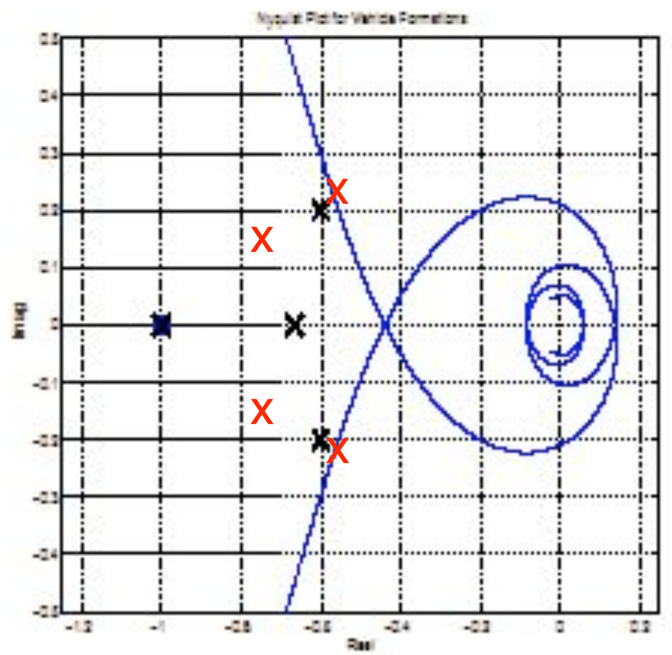


Example Revisited

Example

$$P(s) = \frac{e^{-s\tau}}{s^2} \qquad K(s) = K_d s + K$$





- Adding link increases the number of three cycles (leads to “resonances”)
- Change in control law required to avoid instability
- Q: Increasing amount of information available *decreases* stability (??)
- A: Control law cannot ignore the information ⇒ add’l feedback inserted

Fax and Murray
IFAC 02

Improving Performance through Communication

Baseline: stability only

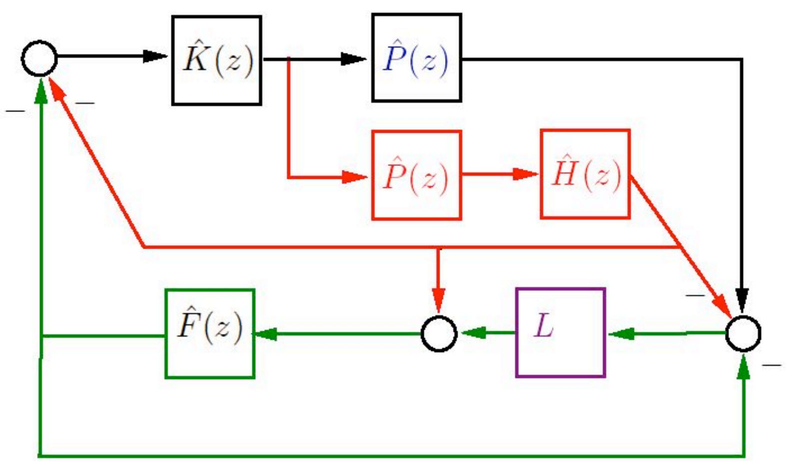
- Poor performance due to interconnection

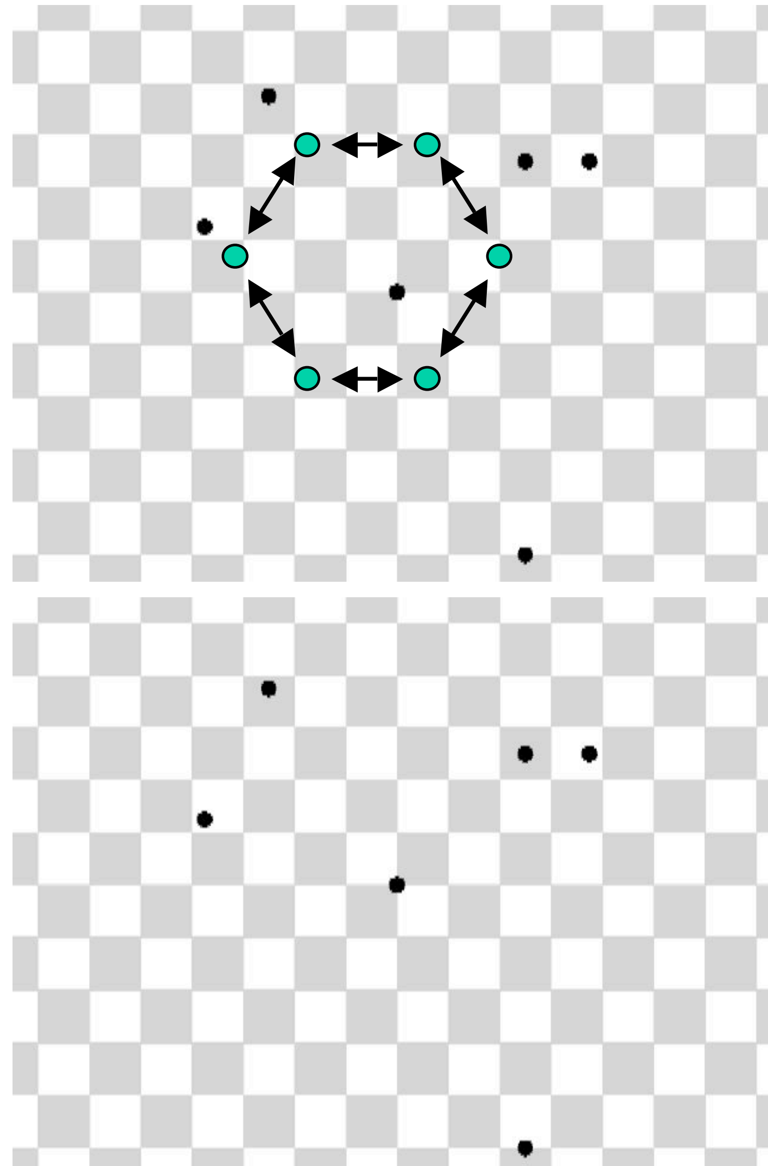
Method #1: tune information flow filter

- Low pass filter to damp response
- Improves performance somewhat

Method #2: consensus + feedforward

- Agree on center of formation, then move
- Compensate for motion of vehicles by adjusting information flow

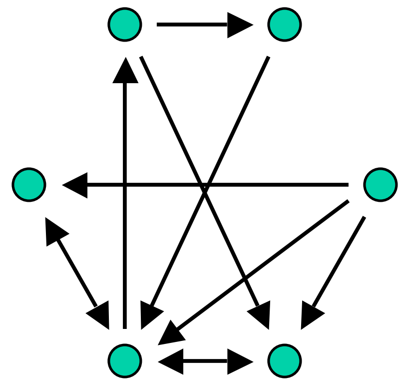


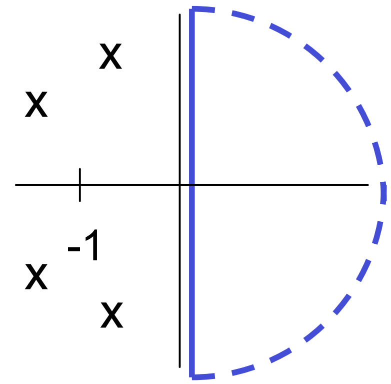


Special Case: Consensus

$$\dot{x}_i = \sum w_{ij}(x_j - x_i)$$

$$\dot{x} = -Lx$$





Consensus: agreement between agents using information flow graph

- Can prove asymptotic convergence to single value if graph is connected
- If w_{ij} = 1/(in-degree) + graph is *balanced* (same in-degree for all nodes) $\Rightarrow$ all agents converge to average of initial condition

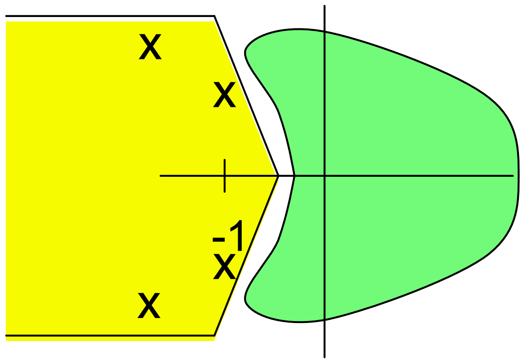
Variations and extensions (Jadbabaie, Leonard, Moreau, Morse, Olfati-Saber, Xiao, …)

- Switching (packet loss, dropped links, etc),time delays, plant uncertainty
- Nearest neighbor graphs, small world networks, optimal weights
- Nonlinear: potential fields, passive systems, gradient systems
- Distributed Kalman filtering, distributed optimization

Open Problems: Design of Information Flow (graph)

How does graph topology affect location of eigenvalues of L?

- Would like to separate effects of topology from agent dynamics



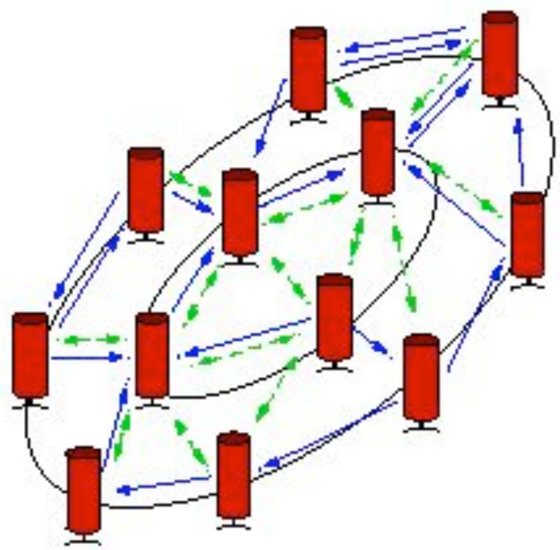
- Possible approach: exploit for of characteristic polynomial

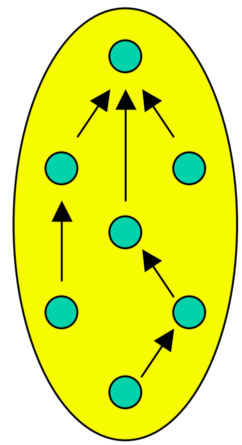
$$\lambda(s) = \text{``}s^n + \left(\sum w_i\right) + \Big(\sum_{2\ \text{cycles}} w_i w_j\Big) s^{n-2} + \Big(\sum_{3\ \text{cycles}} w_i w_j w_k\Big) s^{n-3} + \cdots + \Big(\sum_{N\ \text{cycles}} w_1 \cdots w_n\Big)\text{''}$$

Jin and Murray
CDC 04

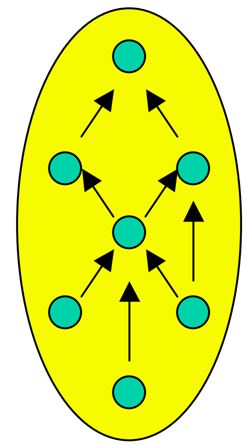
Performance

Look at motion between selected vehicles





G_1 - Control



G_2 - Performance

Theorem

For a leader-follower formation with double-graph strategy, If $\|(1-\alpha)\cdot H(s)\|_\infty = M < 1$*, then*

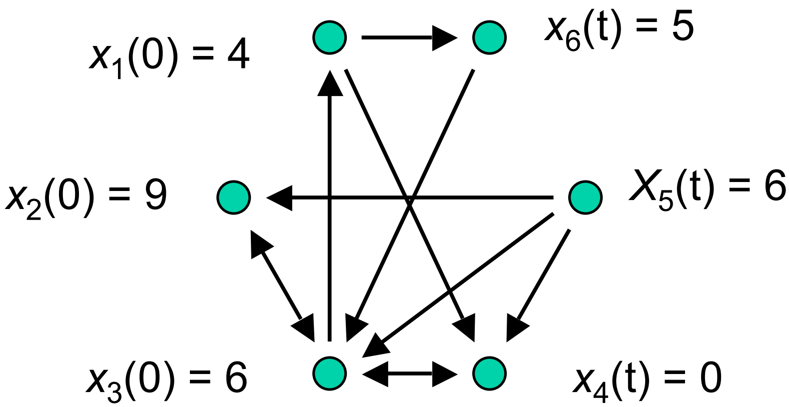
$$\frac{\|E\|_\infty}{\|V\|_\infty} \leq \left(\frac{1-M^n}{1-M} + \frac{\rho(\Theta_2)M^n}{1-\rho(\Theta_2)M}\right)\cdot \|N(s)\|_\infty$$

where $\rho(\Theta_2)$ *is the spectral radius of* Θ_2 *and* $N(s) = \frac{e_1(s)}{v_1(s)}$.

Gupta, Langbort and Murray
CDC 06

Robustness

What happens if a single node "locks up"



- Single node can change entire value of the consenus
- Desired effect for "robust" behavior: $\Delta x_i = \delta/N$

Different types of robustness (Gupta, Langbort & M)

- Type I - node stops communicating (stopping failure)
- Type II - node communicates constant value
- Type III - node computes incorrect function (Byzantine failure)

Related ideas: delay margin for multi-hop models (Jin and M)

- Improve consensus rate through multi-hop, but create sensitivity to communcations delay

Alice Overview

Team Caltech

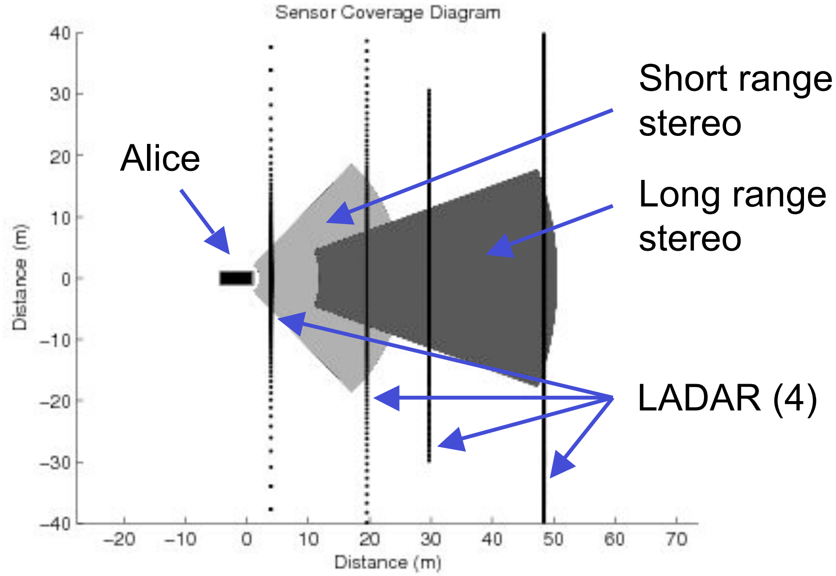
- 50 students worked on Alice over 1 year
- Course credit through CS/EE/ME 75
- Summer team: 20 SURF students + 6 graduated seniors + 4 work study + 4 grads + 2 faculty + 6 volunteers (= ~40)

Computing

- 6 Dell 750 PowerEdge Servers (P4, 3GHz)
- 1 IBM Quad Core AMD64 (fast!)
- 1 Gb/s switched ethernet

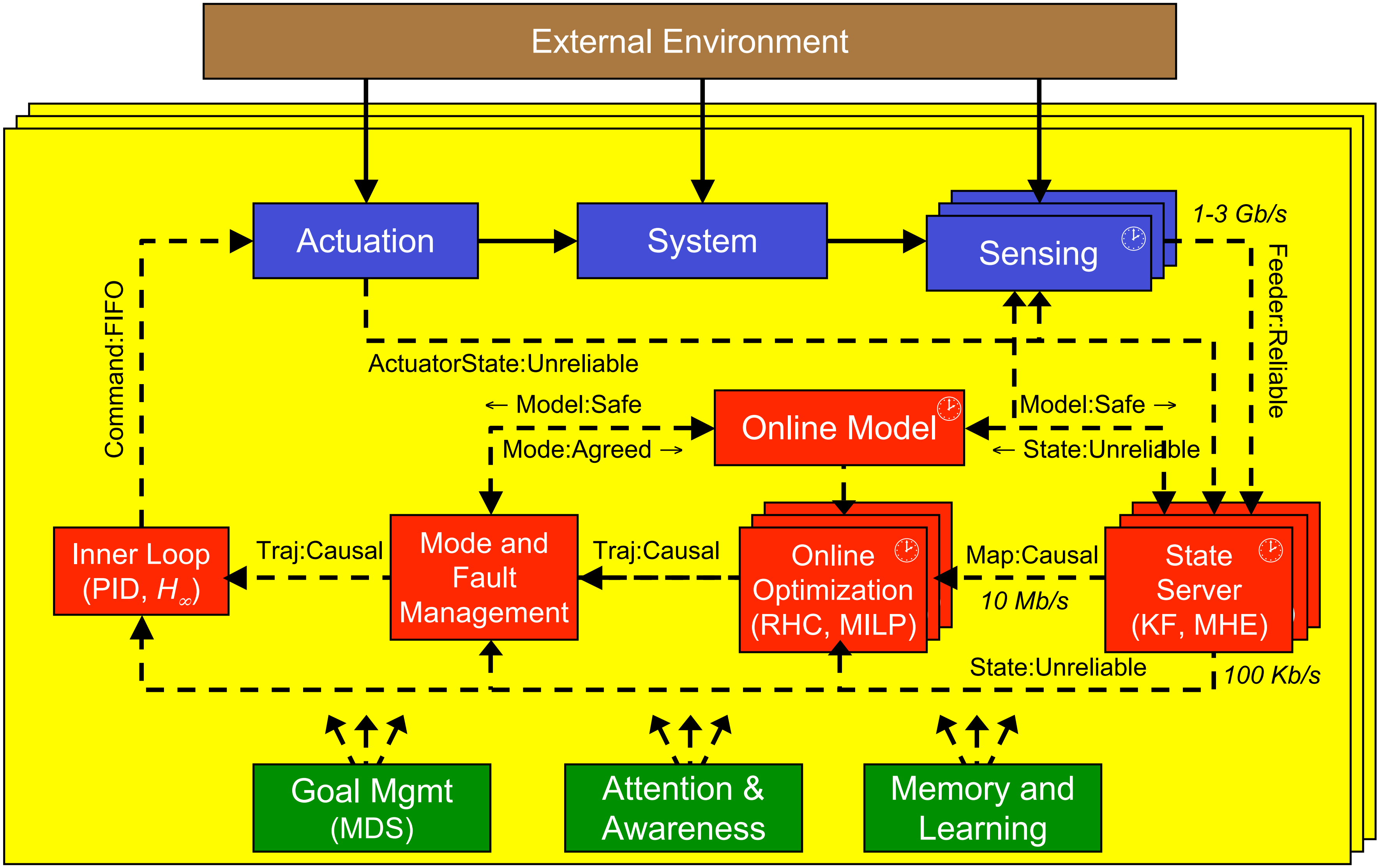
Software

- 15 individual programs with ~50 threads of execution
- Sensor fusion: separate digital elevation maps for each sensor; fuse @ 10 Hz
- Path planning: optimization-based planning over a 10-20 second horizon



An Architecture for Networked Control Systems

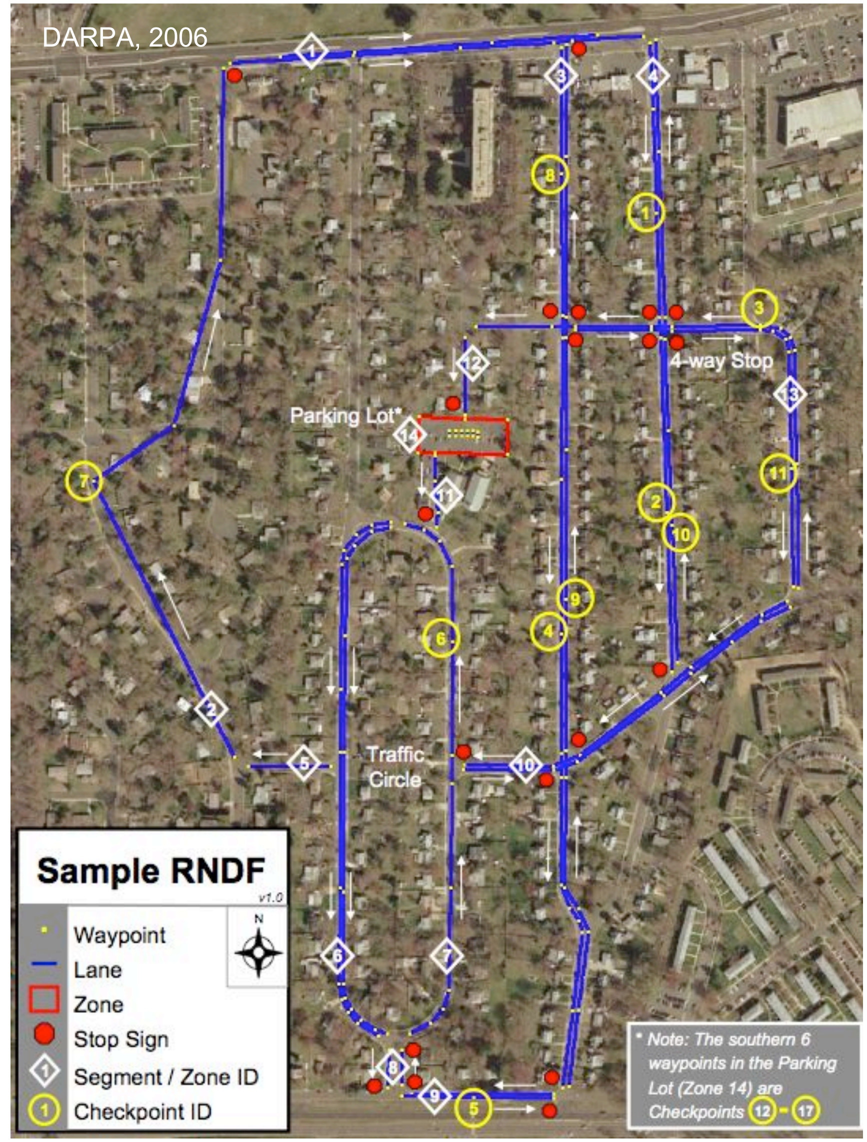
(following P. R. Kumar)



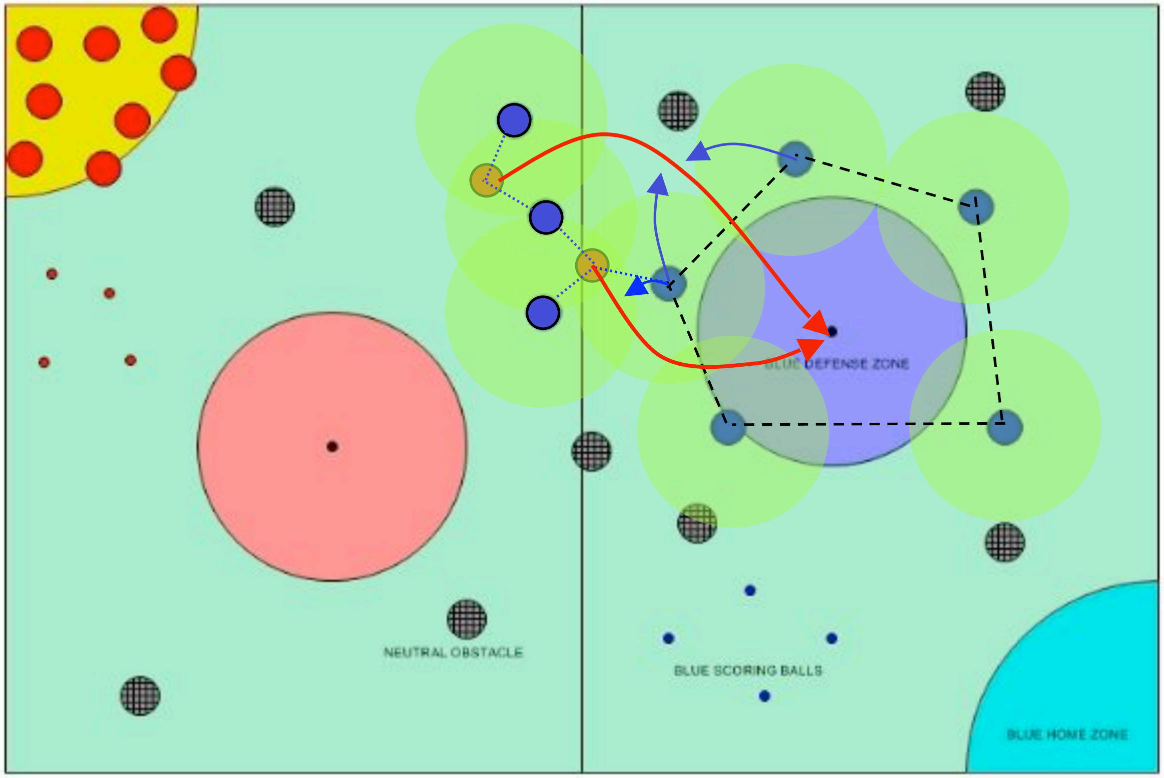
2007 Urban Challenge - 3 November 2007

Autonomous Urban Driving

- 60 mile course, less than 6 hours
- City streets, obeying traffic rules
- Follow cars, maintain safe distance
- Pull around stopped, moving vehicles
- Stop and go through intersections
- Navigate in parking lots (w/ other cars)
- U turns, traffic merges, replanning
- Prizes: $2M, $500K, $250K



Summary: Networked Control Systems



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   - Maintain positions to guard defense zone
2. **Distributed estimation**
   - Fuse sensor data to determine opponent location
3. **Distributed consensus**
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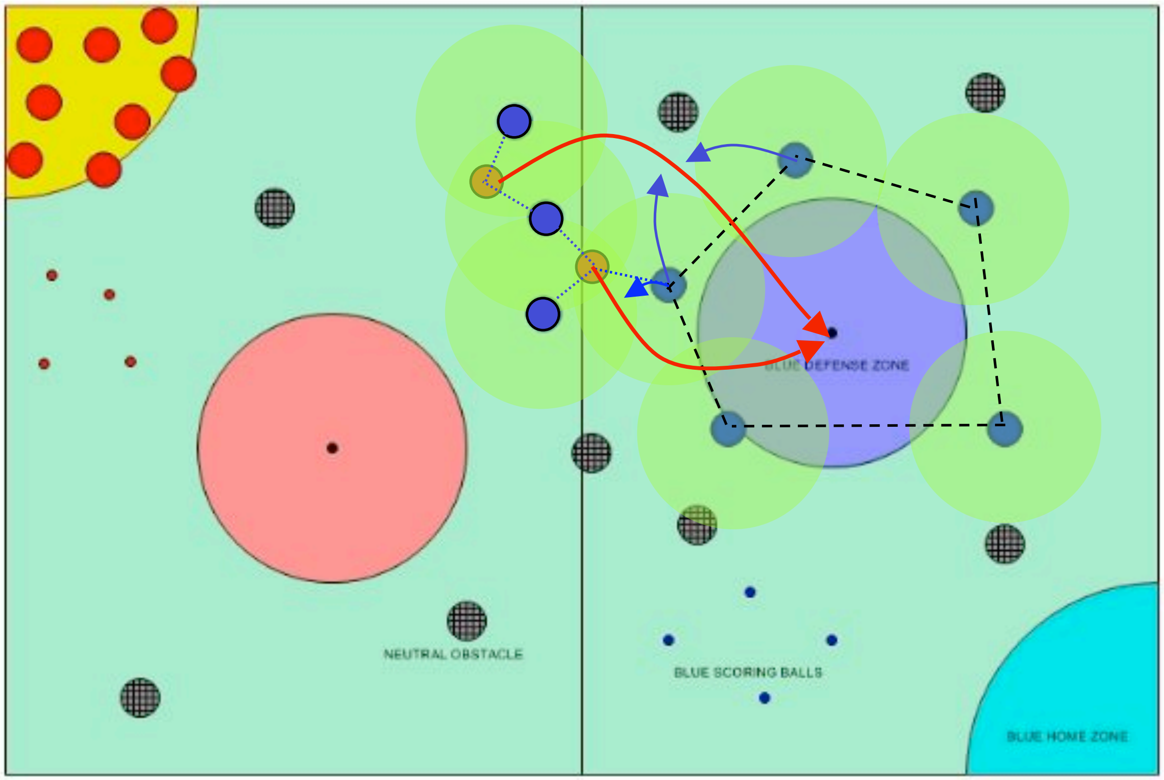
Integration of computer science, communications, and control

- Mixture of techniques from computer science, communications, control
- Increased need for reasoning at higher levels of abstraction (strategy)

RoboFlag Subproblems



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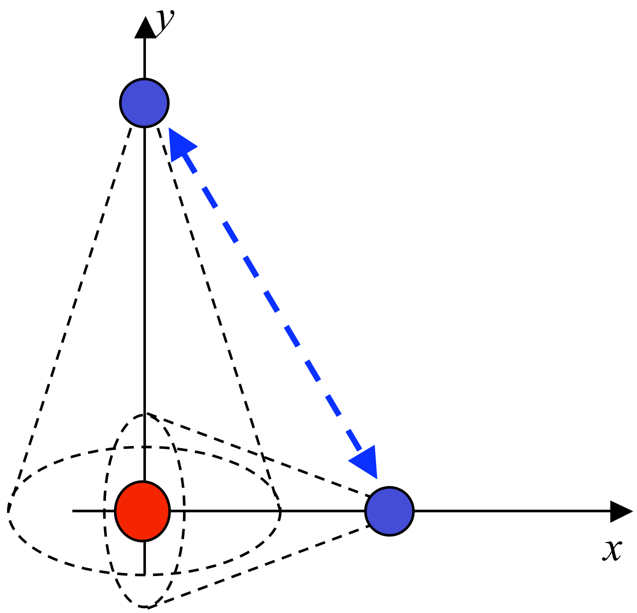
Distributed Sensor Fusion



Two agents viewing single object

$$\dot{x} = Ax + Bu$$

$$\dot{y}_1 = C_1 x \qquad \dot{y}_2 = C_2 x$$

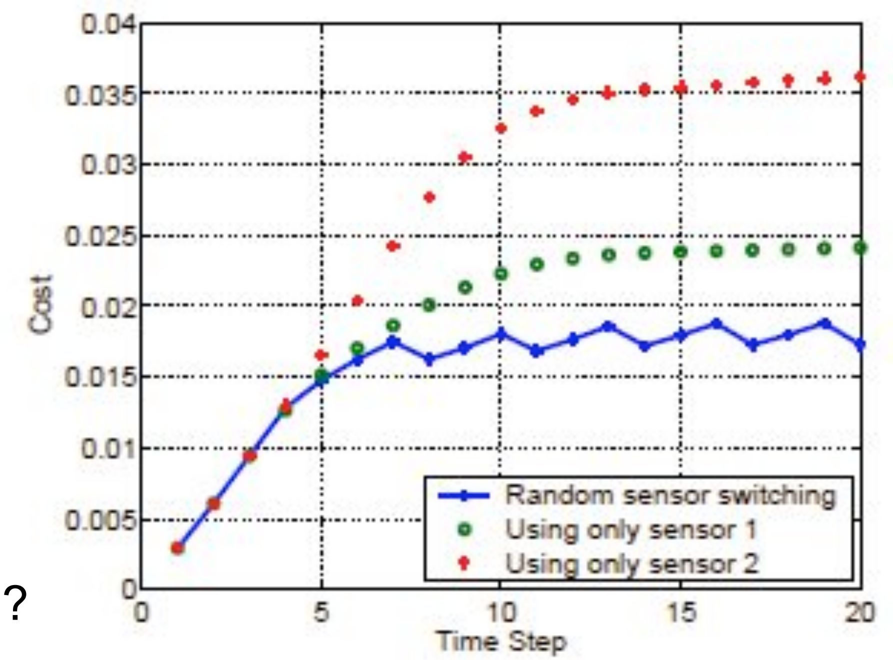
- Each sensor maintains its own estimate
- Sensors can communicate w/ packet loss

Simulation results

- Exchanging information, even intermittently, decreases error

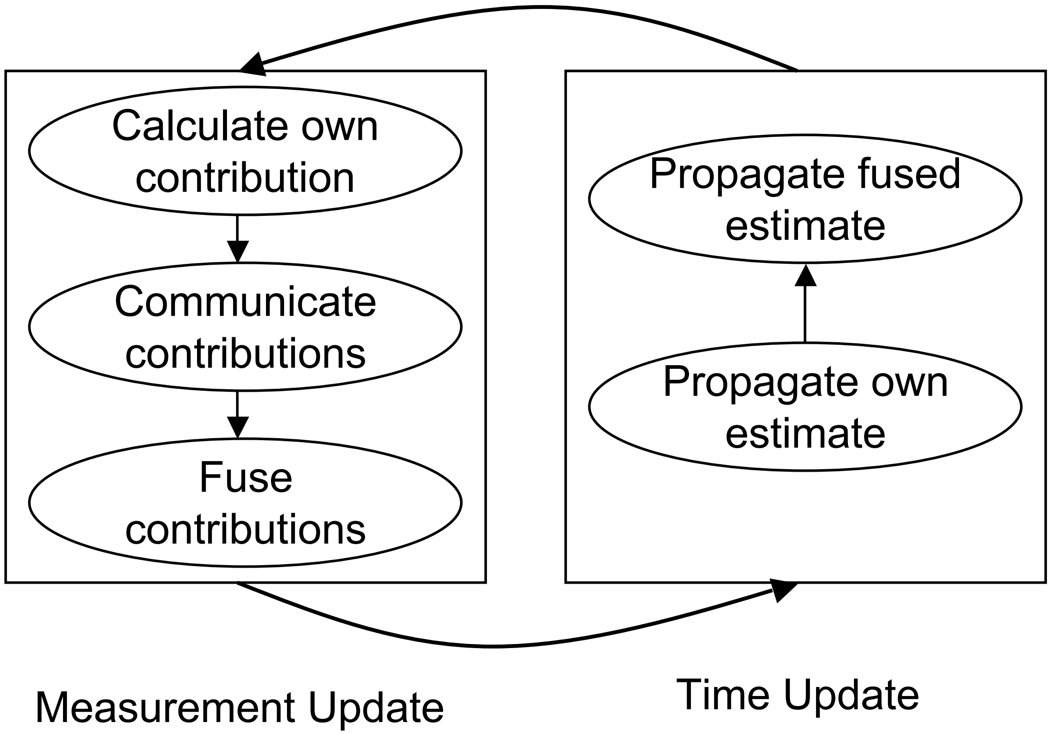
Optimal estimation

- Q: what should sensors communicate?
- How should packet loss be handled?



Gupta, M & Hassibi
CDC 2004 (s)

Decentralized Estimation Algorithm



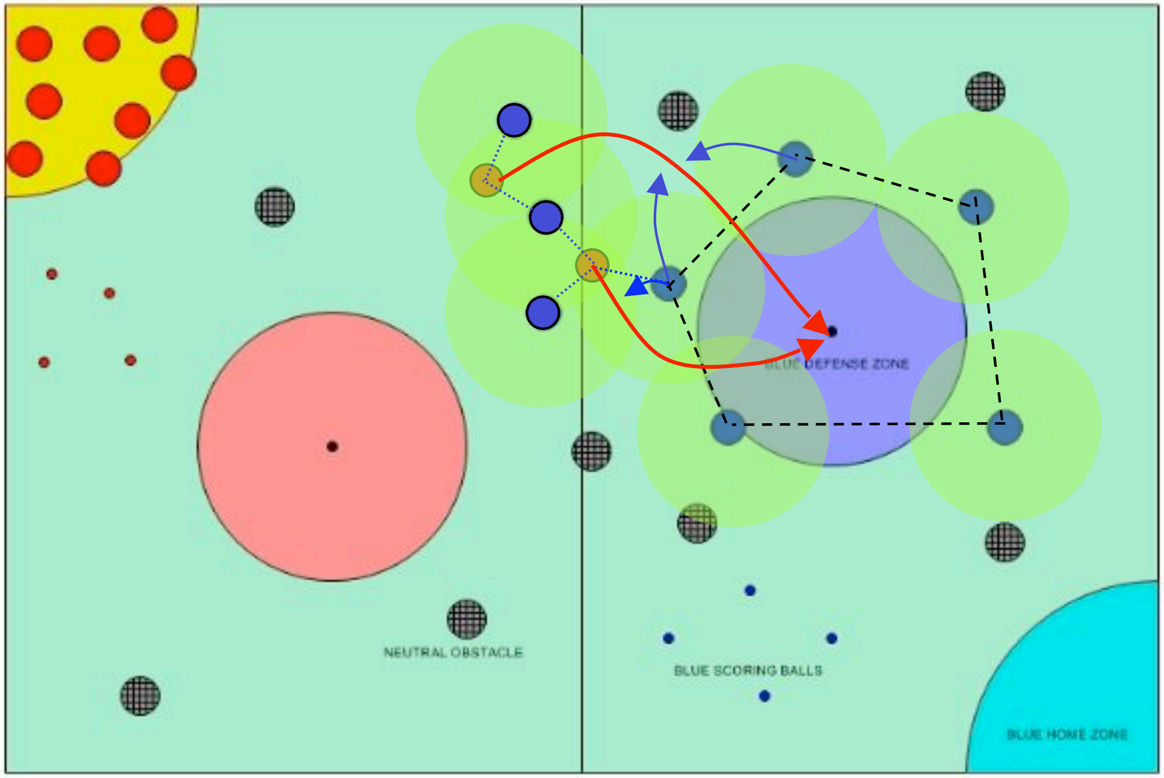
Two sensor case

- Optimal estimator can be decoupled into two contributions
- Sensor *i* can compute contribution to estimate *j* and transmit
- If information not received, use local info to propagate estimate
- In *n* sensor case, decom-position not as straight-forward; suboptimal

$$P_{k|k}^{-1}\hat{x}_{k|k} = \Gamma_k\Gamma_{k-1}\cdots\Gamma_1 P_{0|-1}^{-1}\hat{x}_{0|-1}^{-1} + \sum_i \left[A_k^i + \Gamma_k\Lambda_{k-1}^i + \cdots + (\Gamma_k\Gamma_{k-1}\cdots\Gamma_1)\Lambda_0^i\right]$$

$$\Lambda_k^i = \left(P_{k|k}^i\right)^{-1}\hat{x}_{k|k}^i - \left(P_{k|k-1}^i\right)^{-1}\hat{x}_{k|k-1}^i \qquad \Gamma_k = \left(P_{k|k-1}^i\right)^{-1} A P_{k-1|k-1}$$

RoboFlag Subproblems



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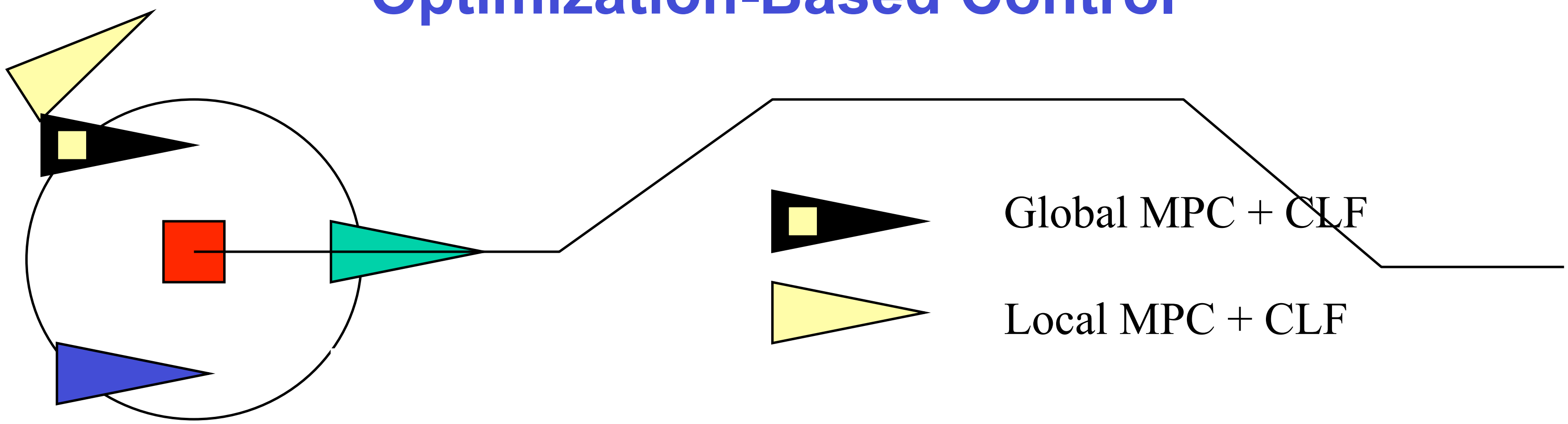
Dunbar and M
IFAC 02

Optimization-Based Control

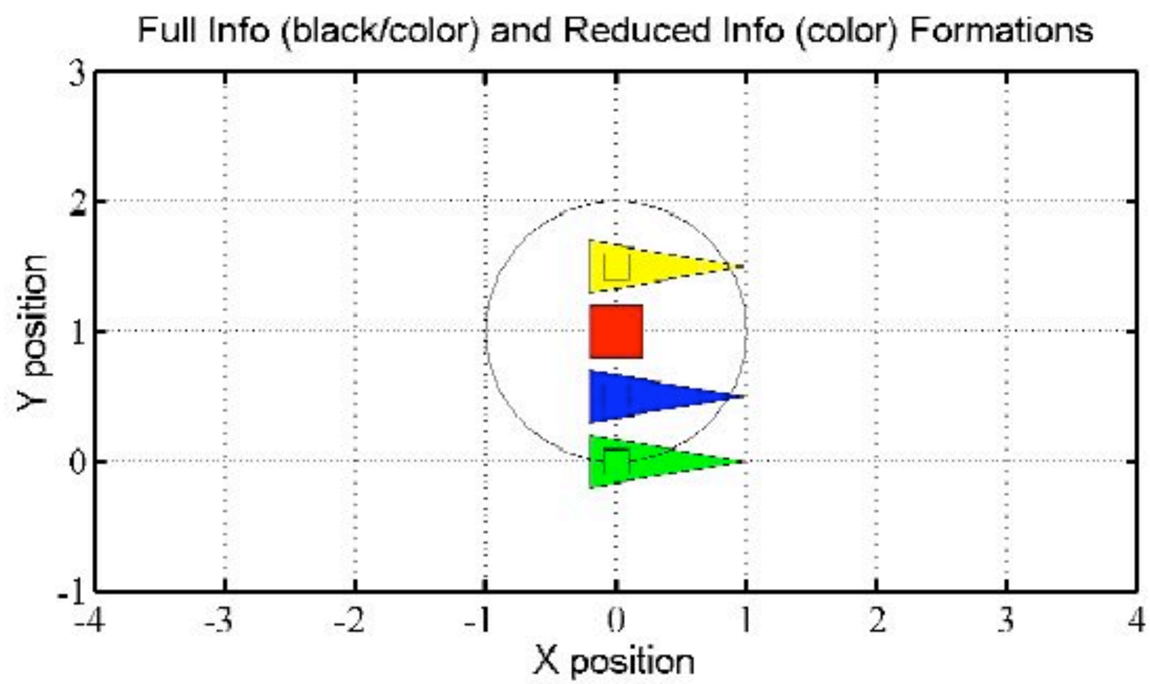


Task:

- Maintain equal spacing of vehicles around circle
- Follow desired trajectory for center of mass

Parameters:

- Horizon: 2 sec
- Update: 0.5 sec



Main Idea: Assume Plan for Neighbors

Individual optimization:

$$\min_{u_3(\cdot)} \left\{ \int_{t_k}^{t_k+T} L_3(z_3(\tau), \hat{z}_2(\tau), u_3(\tau)) \, d\tau + G_3(z_3(t_k+T)) \right\}$$

$$\text{s.t.} \quad \dot{z}_3(t) = f_3(z_3(t), u_3(t))$$

$$u_3(t) \in U_3, \quad z_3(t_k+T) \in Z_{f3}$$

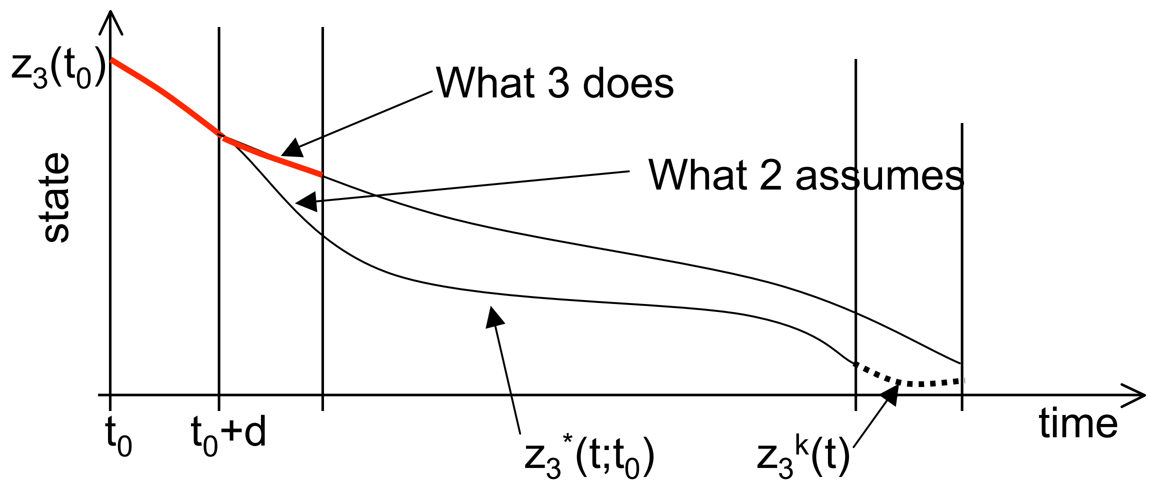
$$||z_3(t) - \hat{z}_3(t)|| \le \delta^2 \kappa$$

Compatibility constraint:

- each vehicle transmits plan to neighbors
- stay w/in bounded path of what was transmitted

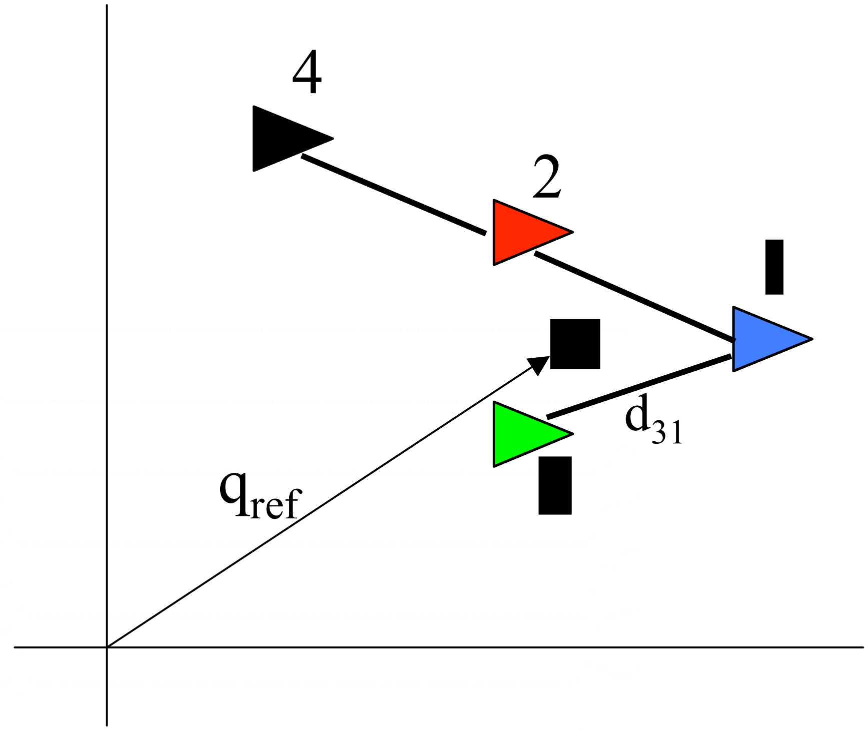
Theorem. Under suitable assumptions, vehicles are stable and converge to global optimal solution.

Pf Detailed Lyapunov calculation (Dunbar thesis)



Example: Multi-Vehicle Fingertip Formation

Four vehicles with $\ddot{q}_i = u_i$.



Constraints:
$U_i = \{(v_1, v_2) \in \mathbb{R}^2 : -1 \leq v_{1,2} \leq 1\}$.

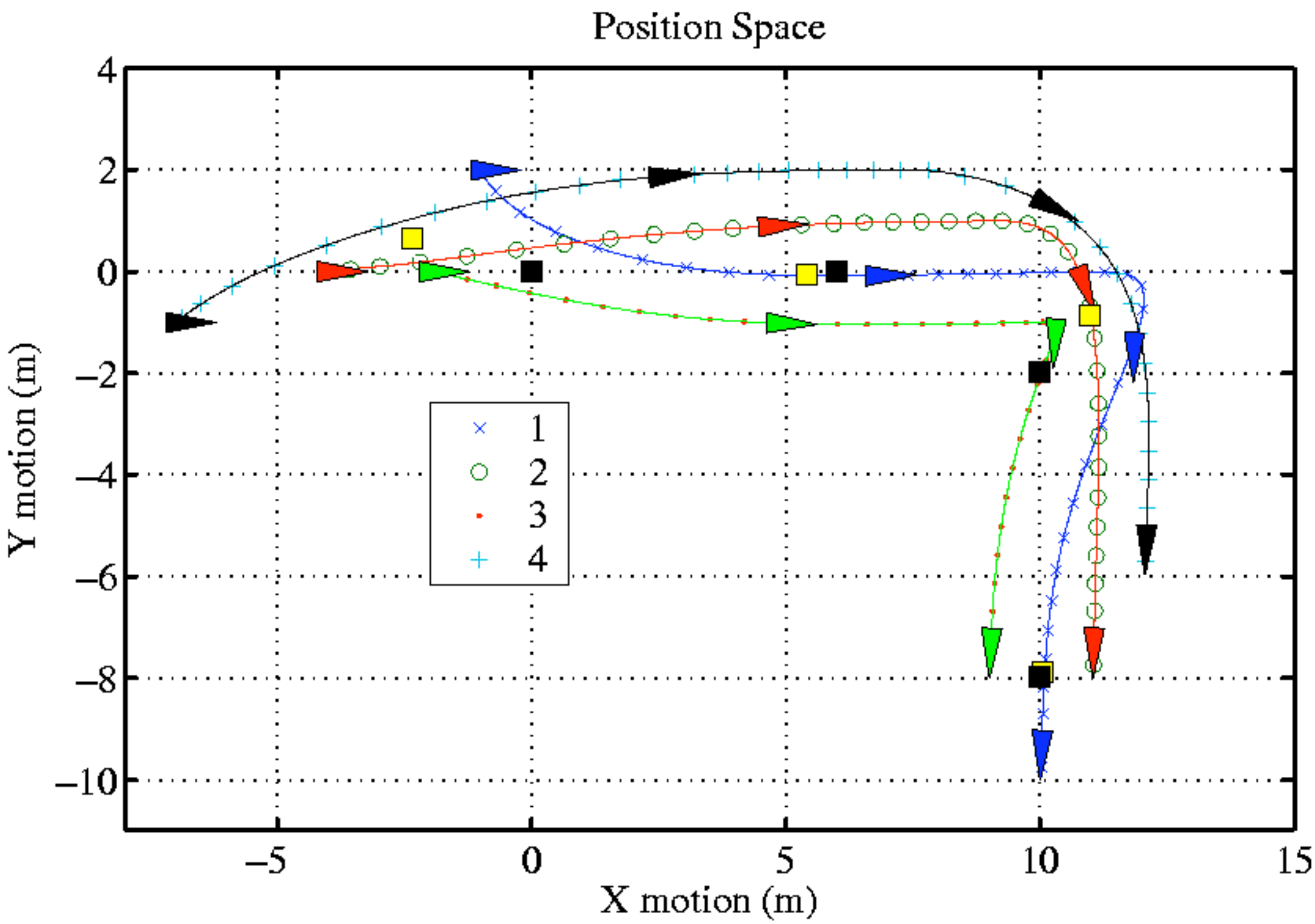
Formation defined by:

- Relative vectors d_{ij}
- COM of $\{1, 2, 3\}$ tracking signal $(q_{\text{ref}}, \dot{q}_{\text{ref}})$.

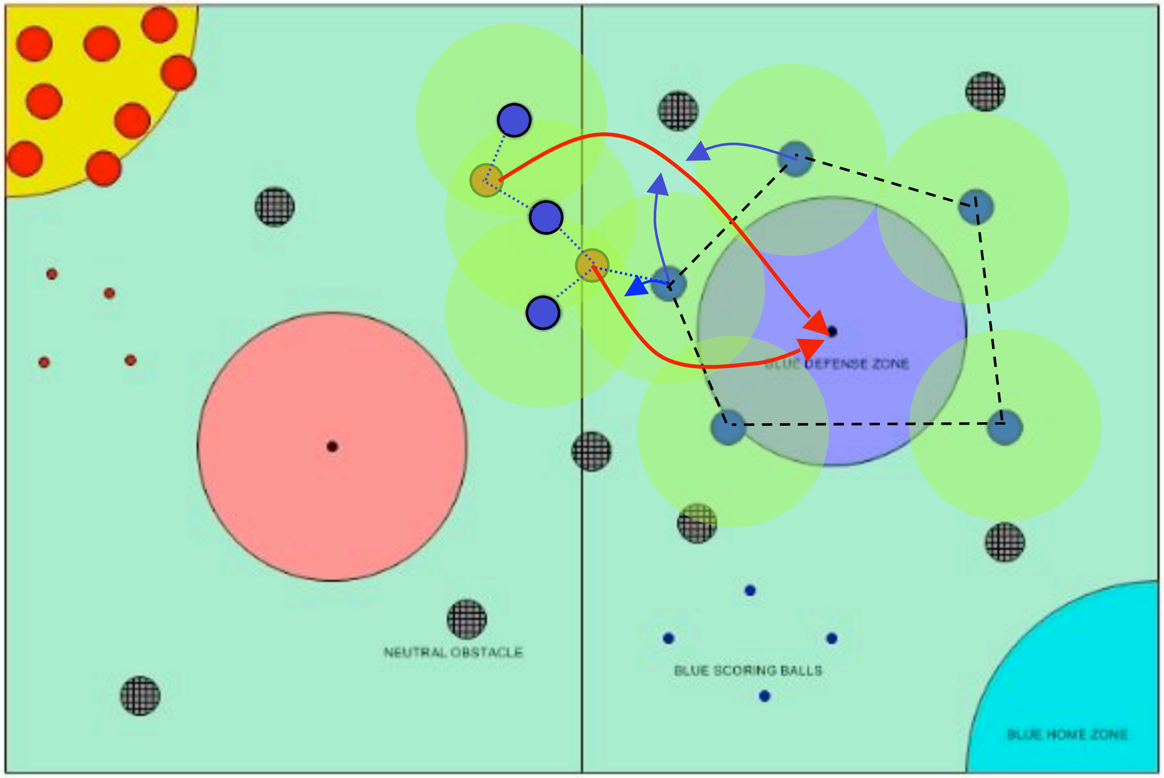
Coupled objective function

$$\begin{aligned} L(z,u) \; &= ||q_3 - q_1 + d_{31}||^2 + ||q_2 - q_1 + d_{21}||^2 + ||q_4 - q_2 + d_{42}||^2 \\ &\quad + ||q_{\text{ref}} - (q_1 + q_2 + q_3)/3||^2 + \sum_{i=1}^{4} ||\dot{q}_{\text{ref}} - \dot{q}_i||^2 + ||u_i||^2. \end{aligned}$$

Simulation Results



RoboFlag Subproblems



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CCL: Computation and Control Language

Formal Language for Provably Correct Control Protocols

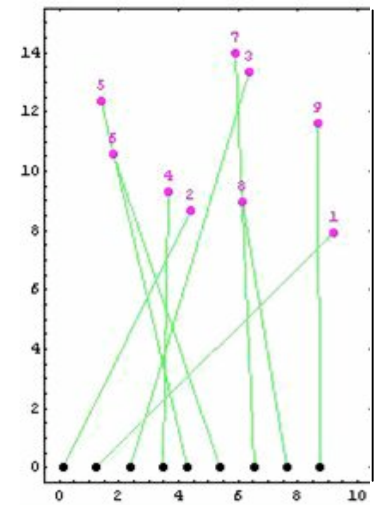
```
P(k1,k2) := {
  initializers
  guard1:rule1
  guard2:rule2
  ...
}
```

"soup" of guarded commands

composition = union

non-shared variables remain local to component programs

```
S(k1,k2):=P(k1,k2)+C(k1+1) sharing y,u
```



CCL Protocol for Decentralized Target Allocation

<u>CCL Interpreter</u>

Formal programming language for control and computation. Interfaces with libraries in other languages.

<u>Formal Results</u>

Formal semantics in transition systems and temporal logic. *RoboFlag* drill formalized and basic algorithms verified.

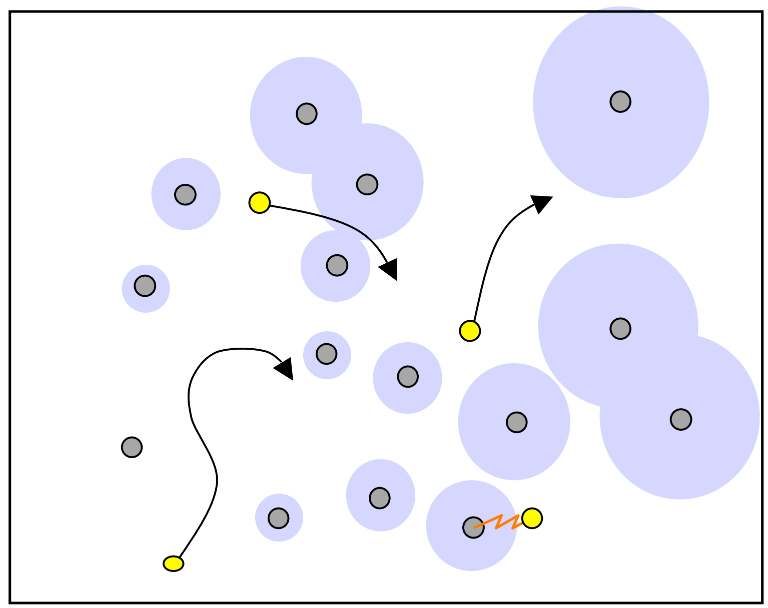
<u>Automated Verification</u>

CCL encoded in the *Isabelle* theorem prover; basic specs verified semi-automatically. Investigating various model checking tools.

Example #1: Situational Awareness

Communications complexity

- Maintain “situational awareness”
- Assume point-to-point communications and that each robot knows its own position
- Q: how many messages are required for each robot to keep track of all other robots w/in ε?
- A: $O(n^2)$ messages (worst case)



Method #1: Distance Modulated Communication - *O*(*n* log *n*)

- Maintain position estimates to within $k \Pi x_i - x_j \Pi$
- Communicate more often with robots that are closer

Method #2: Wandering Communication Scheme - *O*(*n*)

- Only moving robots need to keep track of position
- Robots transfer knowledge when they stop/start

Proof of correctness using CCL

Klavins WAFR 02

Klavins
CDC, 03

Example #2: RoboFlag Drill

Specification 1 Red Robot Dynamics: $\Pi_{red}(i)$

Initial:

$x_i \in [min, max] \wedge y_i > \max$

Clauses:

$y_i - \delta > 0 \; : \; y_i' = y_i - \delta$

Specification 2 Blue Robot Control: $\Pi_{blue}(i)$

Initial:

$z_i \in [min, max] \; \wedge \; z_i < z_{i+1}$

Clauses:

$z_i < x_{\alpha(i)} \wedge z_i < z_{i+1} - 2\delta \; : \; z_i' = z_i + \delta$

$z_i > x_{\alpha(i)} \wedge z_i > z_{i-1} + 2\delta \; : \; z_i' = z_i - \delta$

Specification 3 Assignment Protocol: $\Pi_{proto}(n)$

Initial:

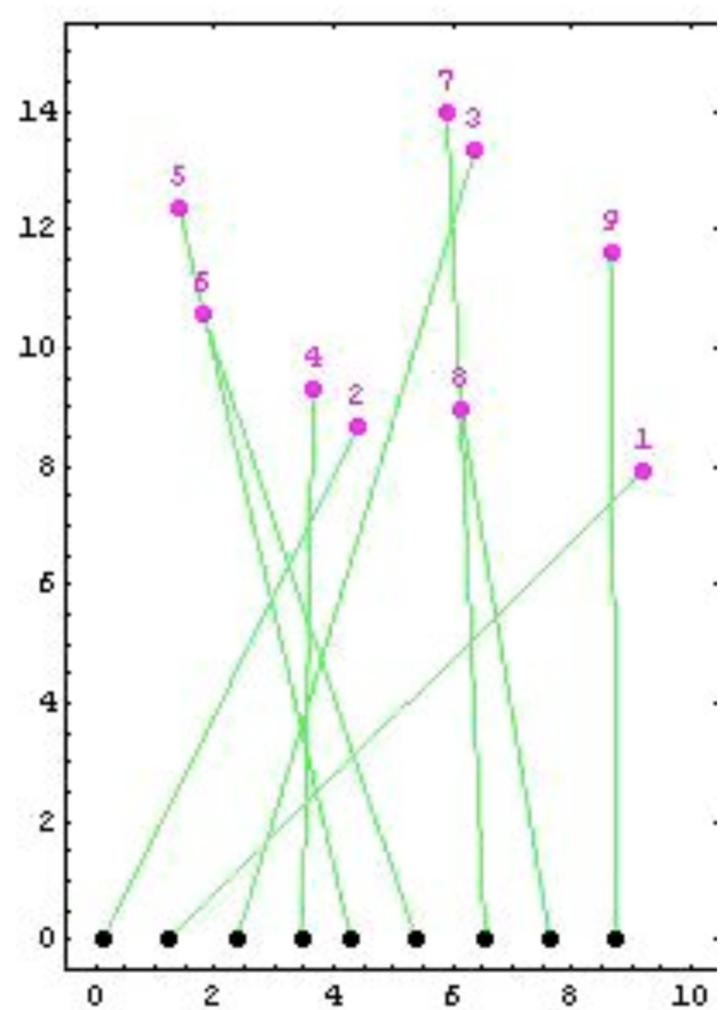
α is a bijection from $\{1, ..., n\}$ to $\{1, ...n\}$.

Clauses:

$switch_{1,2} \; : \; (\alpha(1)', \alpha(2)') = (\alpha(2), \alpha(1))$

...

$switch_{n-1,n} \; : \; (\alpha(n-1)', \alpha(n)') = (\alpha(n), \alpha(n-1))$



Formal Specifications

Safety (Defenders do not collide)

$$z_i < z_{i+1} \ \mathbf{co} \ z_i < z_{i+1}$$

Stability (switch stays false)

Robots are "far enough" apart.

$$\forall i \ . \ \underbrace{y_i > 2\delta \wedge z_i + 2\delta < z_{i+1}} \wedge \neg switch_{i,i+1} \ \mathbf{co} \ \neg switch_{i,i+1}$$

Lyapunov Stability:

$$\forall i \ . \ z_i + 2\delta m < z_{i+1} \wedge \exists j \ . \ switch_{j,j+1} \wedge V = m \ \mathbf{co} \ V < m$$

Example #3: Observation of CCL Programs

Del Vecchio & Klavins, CDC'03

Problem: Determine state of communications protocol used by a group of robots given their physical movements.

Assumptions: Protocol and motion control are described in CCL like language.

Results:
- Defns of observability, etc. for CCL programs
- Construction and analysis of observer that converges when the system is "weakly" observable
- Construction of an efficient observer for Roboflag drill in particular
- Everything specified in CCL