gemalto

Can RSA Keys be (Non-)Malleable?

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Gemplus/Gemalto, Cryptography & Innovation

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1. Preliminaries
   - Impossibility Results for RSA-based Cryptography
   - RSA and Related Computational Problems
   - The Example of RSA Encryption
2. Instance-Malleability of RSA Key Generation
   - A Definition
   - The Fundamental Property
3. Impossibility Results for RSA Encryption
   - One-Wayness kills Chosen-Ciphertext Security
   - Indistinguishability kills Plaintext-Checking Security
   - IND-CCA $[\mathcal{E}] \not\equiv$ INV [Gen]
4. Non-Malleability in Other Settings
5. Computational Number Theoretic Challenges

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RSA Encryption

the CCA security of RSA-based encryption *e.g.* RSA-OAEP and similar schemes cannot be equivalent to inverting RSA in the standard model.

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These results contradict standard proofs

standing in the RO model

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Non-Malleable Key Generation

Non-malleability: Given $n \leftarrow \mathrm{Gen}(1^k)$, a factoring oracle $\mathrm{FACT}(n' \neq n)$ does not help to factor n

Instance Non-malleability: Given $(n, e) \leftarrow \mathrm{Gen}(1^k)$, a root extraction oracle $\mathrm{RSA}((n', e') \neq (n, e))$ does not help to extract an e-th root modulo n.

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consists in computing $x = y^{1/e} \bmod n$ given random integers $n, e \leftarrow \mathbb{N}$ such that $\gcd(e, \phi(n)) = 1$ where $\phi(n) = |\mathbb{Z}_n^*|$ and $y \leftarrow \mathbb{Z}_n$.

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Hard Instances

We define an instance generator Gen. Given $k > 0$, $\mathrm{Gen}(1^k)$ generates a hard instance (n, e), as well as the side information $d = e^{-1} \bmod \phi(n)$.

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A probabilistic algorithm $\mathcal{A}$ is said to (ε, τ)-invert Gen when

$$\Pr\left[(n, e, d) \leftarrow \mathrm{Gen}(1^k), y \leftarrow \mathbb{Z}_n \ : \ \mathcal{A}(n, e, y) = y^d \bmod n\right] \geq \varepsilon \ ,$$

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Breaking INV [Gen] means (ε, τ)-inverting Gen for $\tau = \mathrm{poly}(\mathrm{k})$ and $\varepsilon > 1/\mathrm{poly}(\mathrm{k})$

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(ε, τ)-breaking ℓ-OM [Gen] for $\ell, \tau = \mathrm{poly}(\mathsf{k})$ and $\varepsilon > 1/\mathrm{poly}(\mathsf{k})$.

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Padding functions

$$\begin{array}{rcl} \mu : \{0,1\}^{|\mathsf{m}|} \times \{0,1\}^{|\mathsf{r}|} & \rightarrow & \{0,1\}^{\mathsf{k}} \\ (m, r) & \rightarrow & \mu(m, r) \end{array}$$

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μ is invertible

there is a function μ^{-1} such that $\forall\, m \in \{0,1\}^{|\mathsf{m}|}, \forall\, y \in \{0,1\}^{\mathsf{k}}$,

$$\begin{array}{rcl} \mu^{-1}(y) & = & m \quad \text{iff} \quad \exists\, r \in \{0,1\}^{|\mathsf{r}|} \quad \text{s.t.} \quad y = \mu(m, r) \\ & = & \perp \quad \text{otherwise} \end{array}$$

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- *chosen-ciphertext attack* (CCA), a decryption oracle

Relations among Security Notions

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solving RSA for $(n, e) \leftarrow \mathrm{Gen}(1^k)$ is easier when given unlimited access to $\mathrm{RSA}(n', e', \cdot)$ for $(n', e') \neq (n, e)$ and $(n', e') \in \mathrm{Range}\left(\mathrm{Gen}(1^k)\right)$

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$$\mathrm{RSA}(n, e_1 e_2) \Leftarrow \mathrm{RSA}(n, e_1) + \mathrm{RSA}(n, e_2)$$

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Can you see non-trivial examples?

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Do these RSA generators feature instance-non-malleability in a strong sense?

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Proof (Sketchy and Visual)

Skip proof

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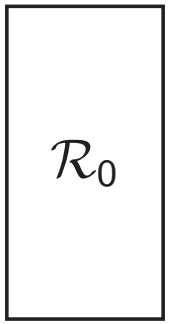

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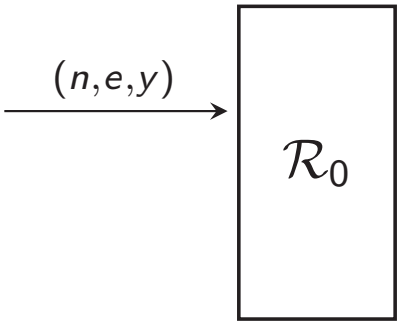

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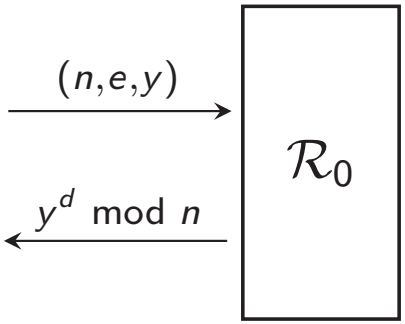

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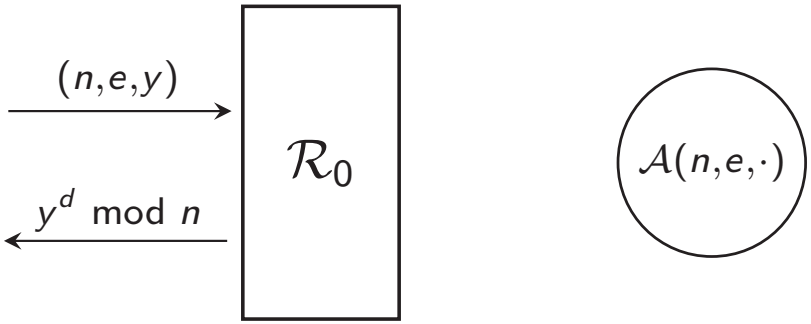

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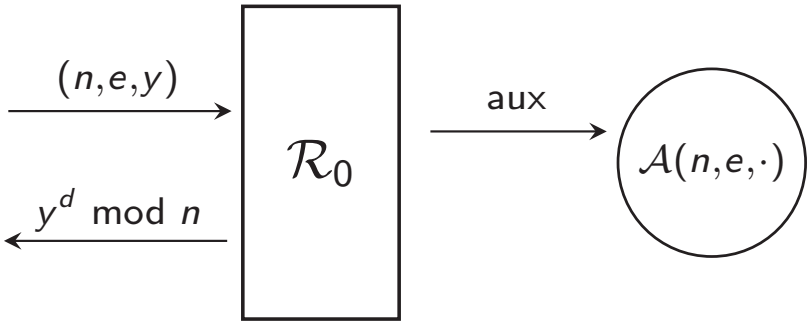

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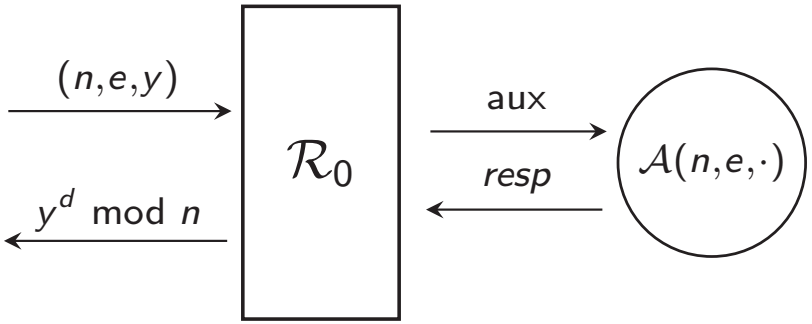

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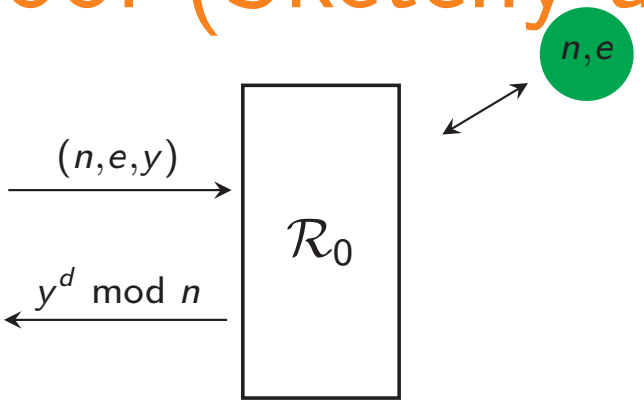

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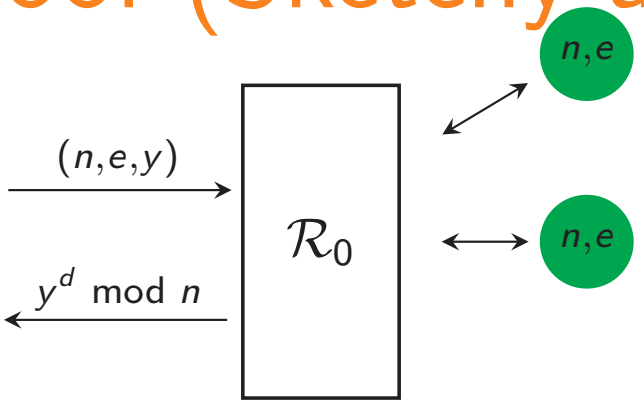

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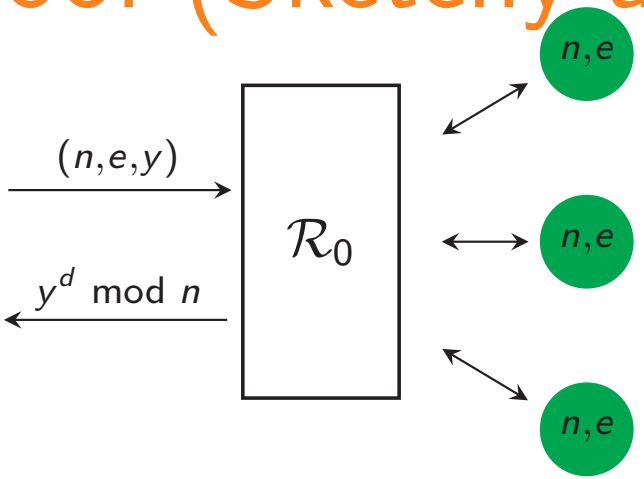

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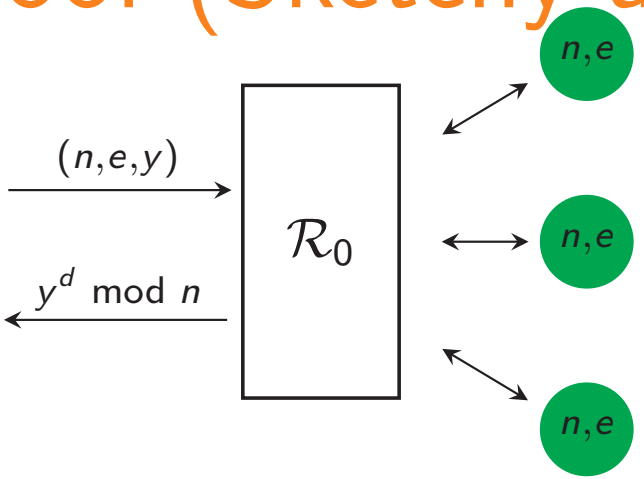
GAME 0



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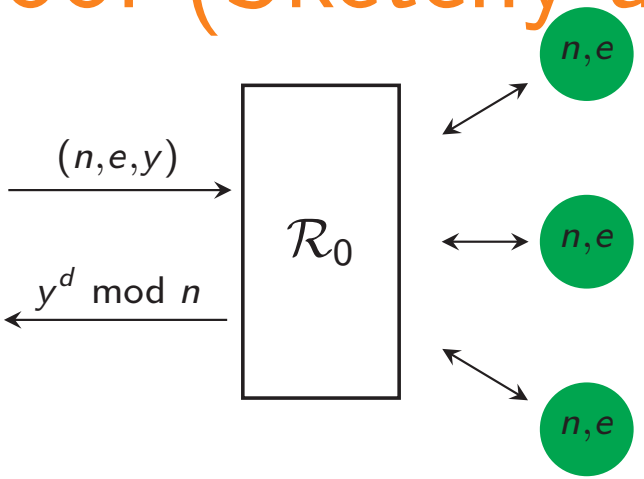

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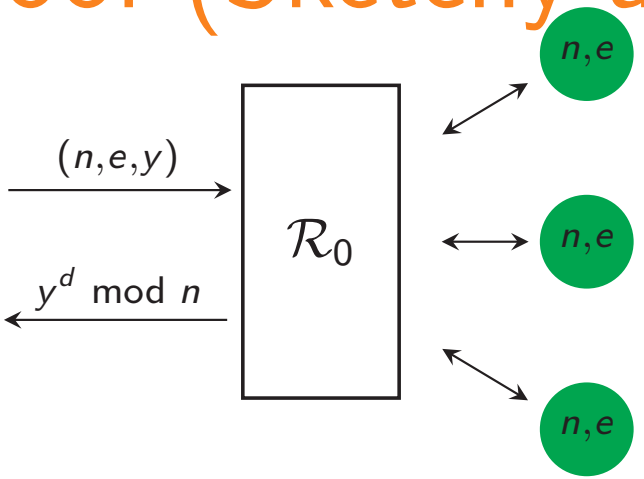

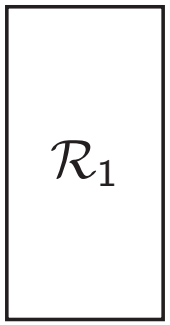

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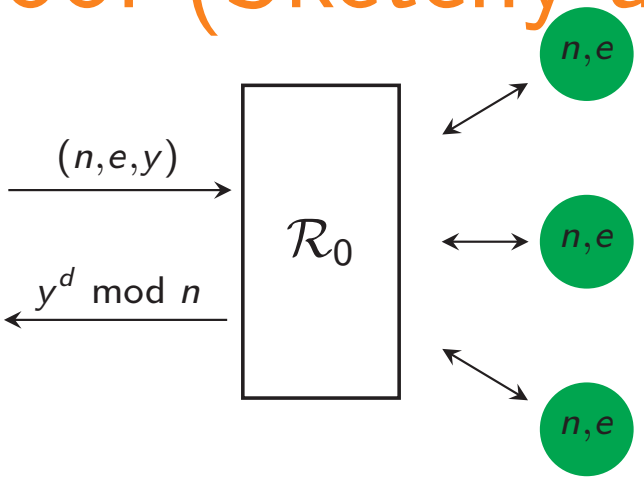
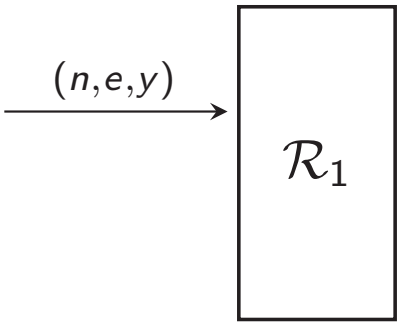
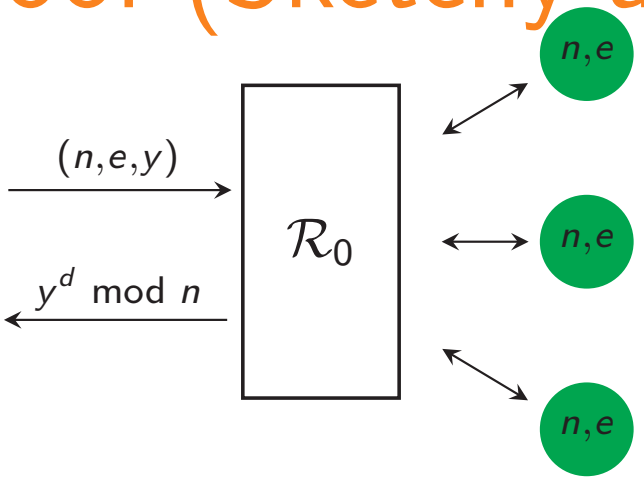


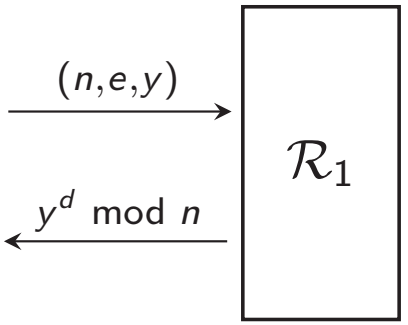


Proof (Sketchy and Visual)

Skip proof

GAME 0





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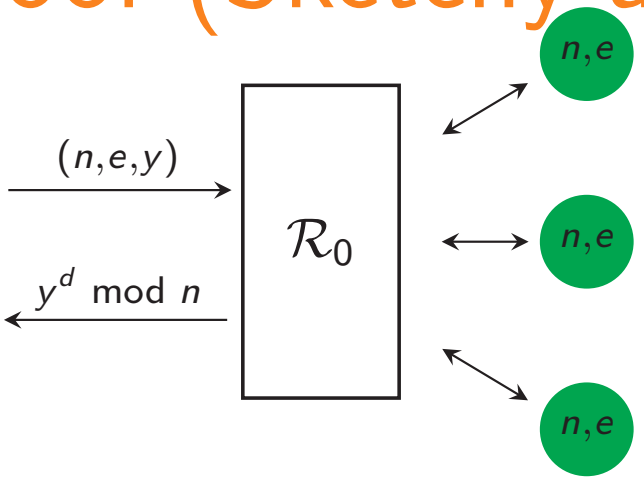

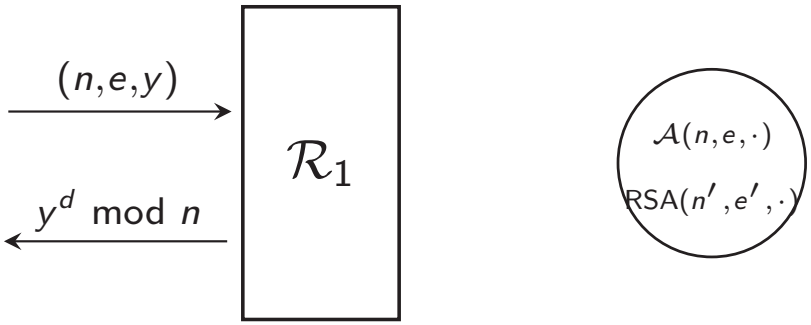

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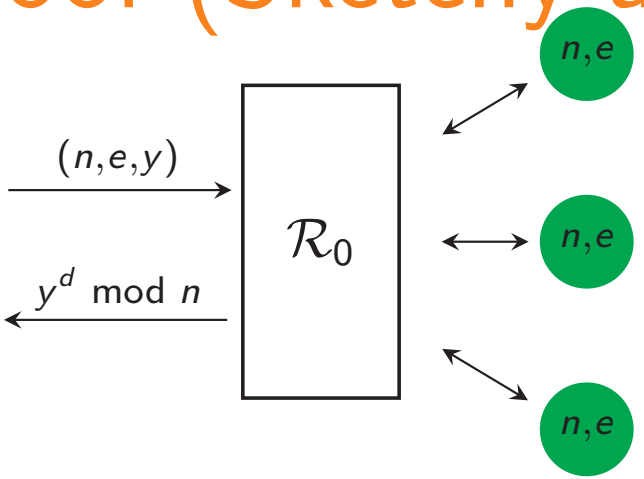

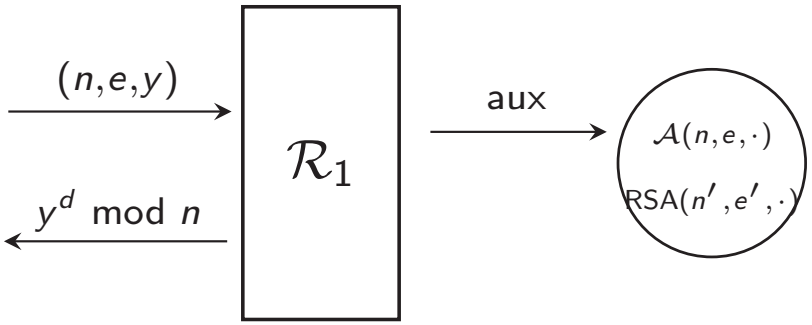

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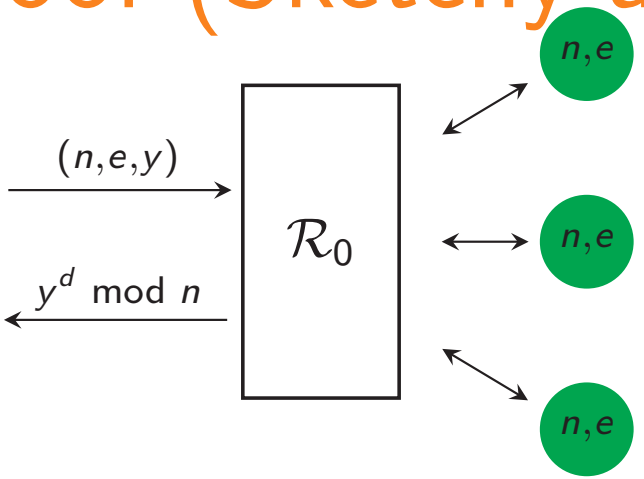

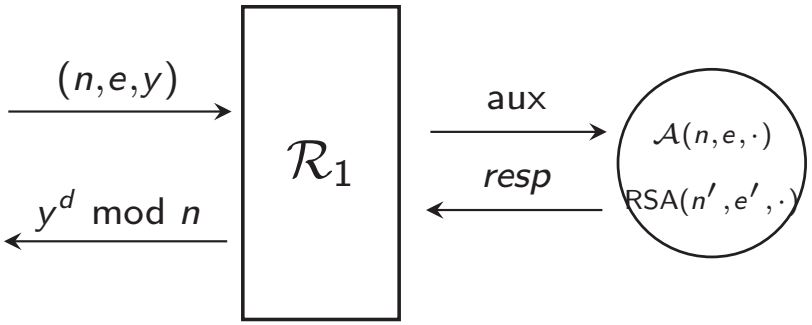

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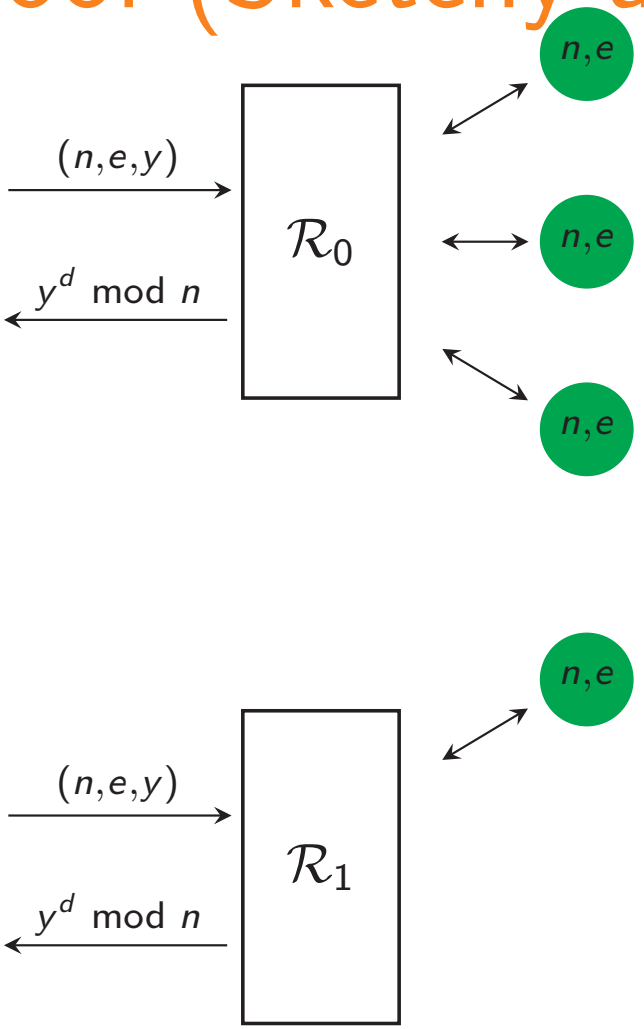

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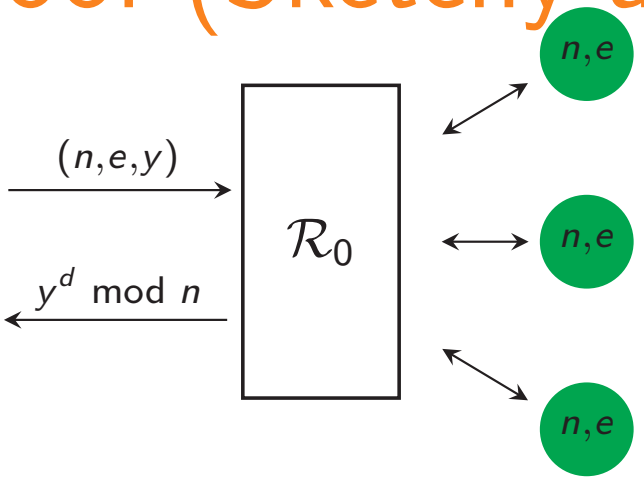

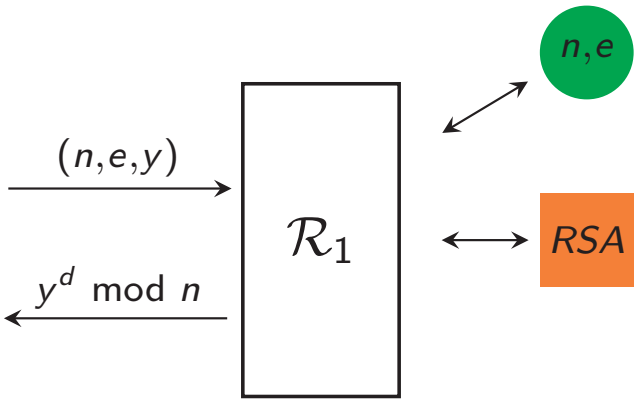

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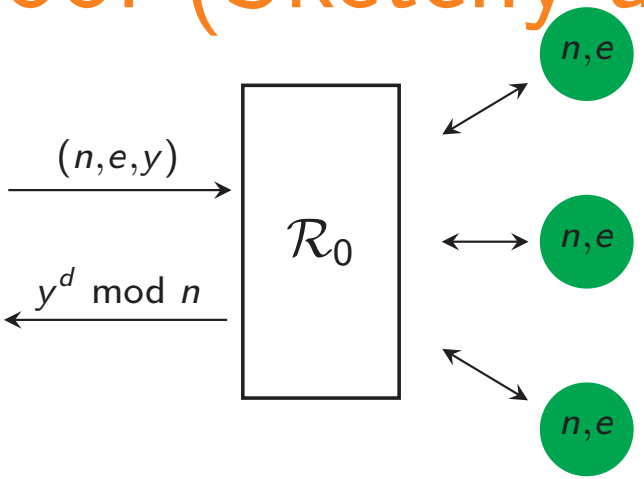

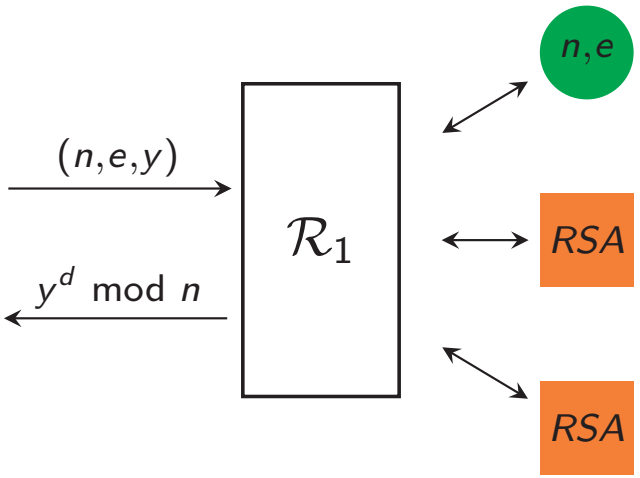

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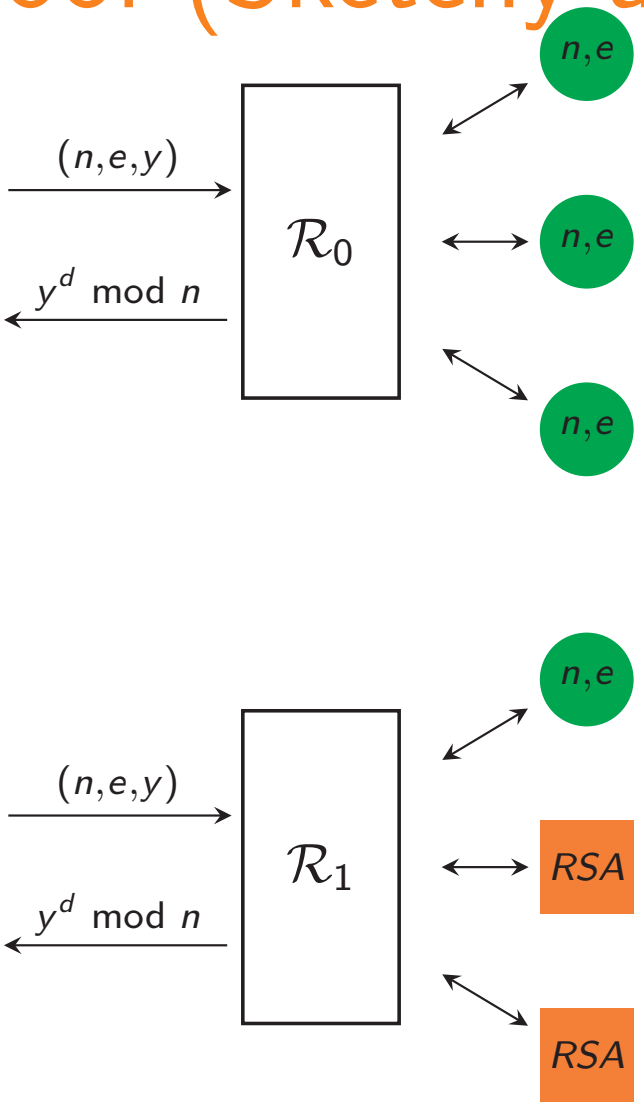

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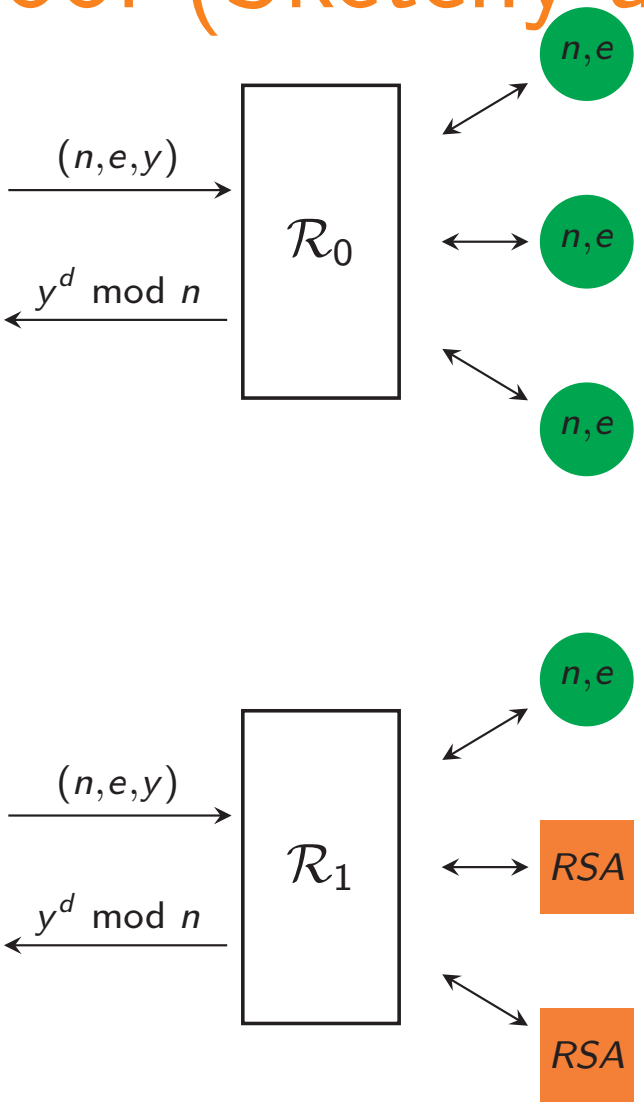

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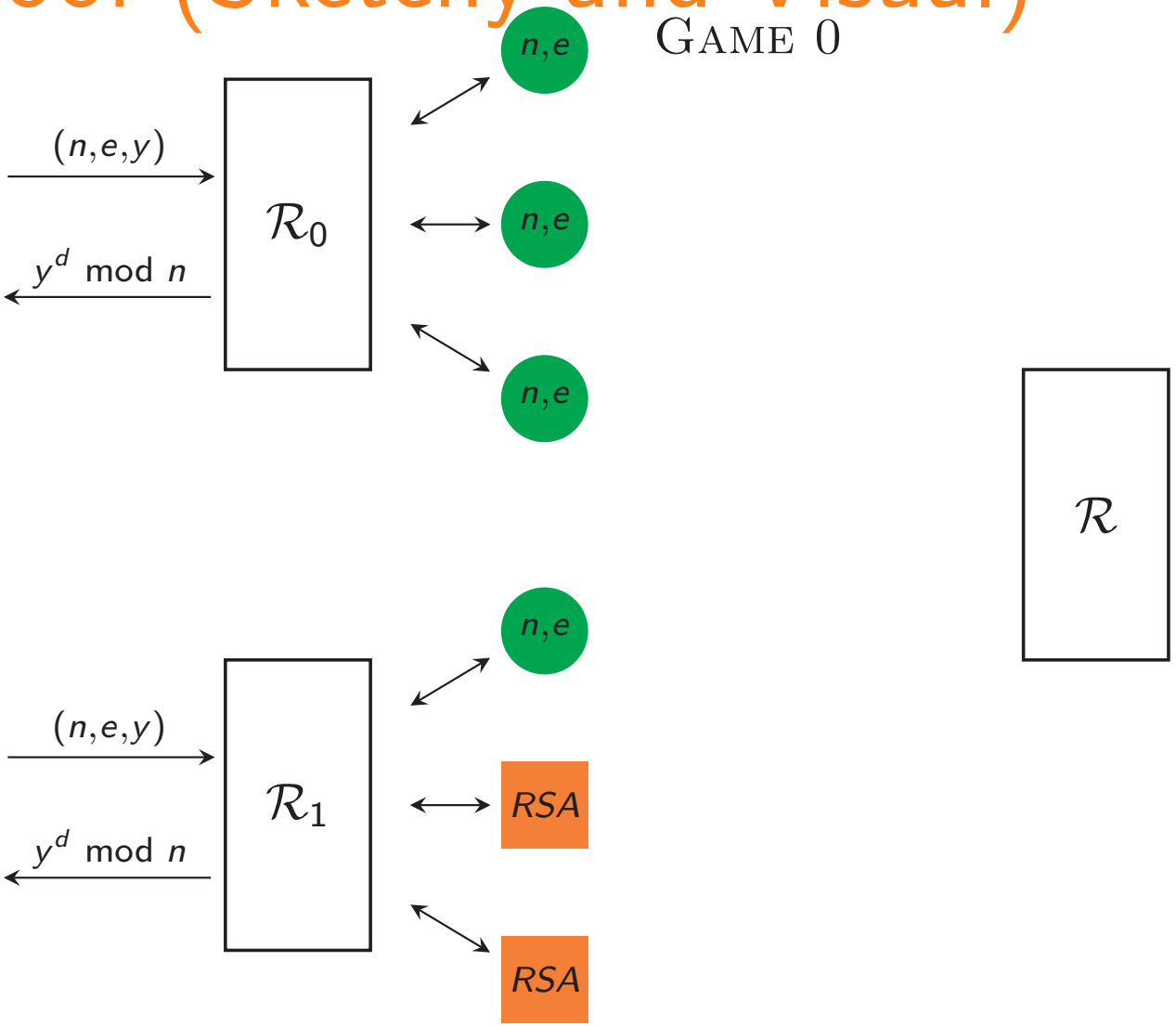

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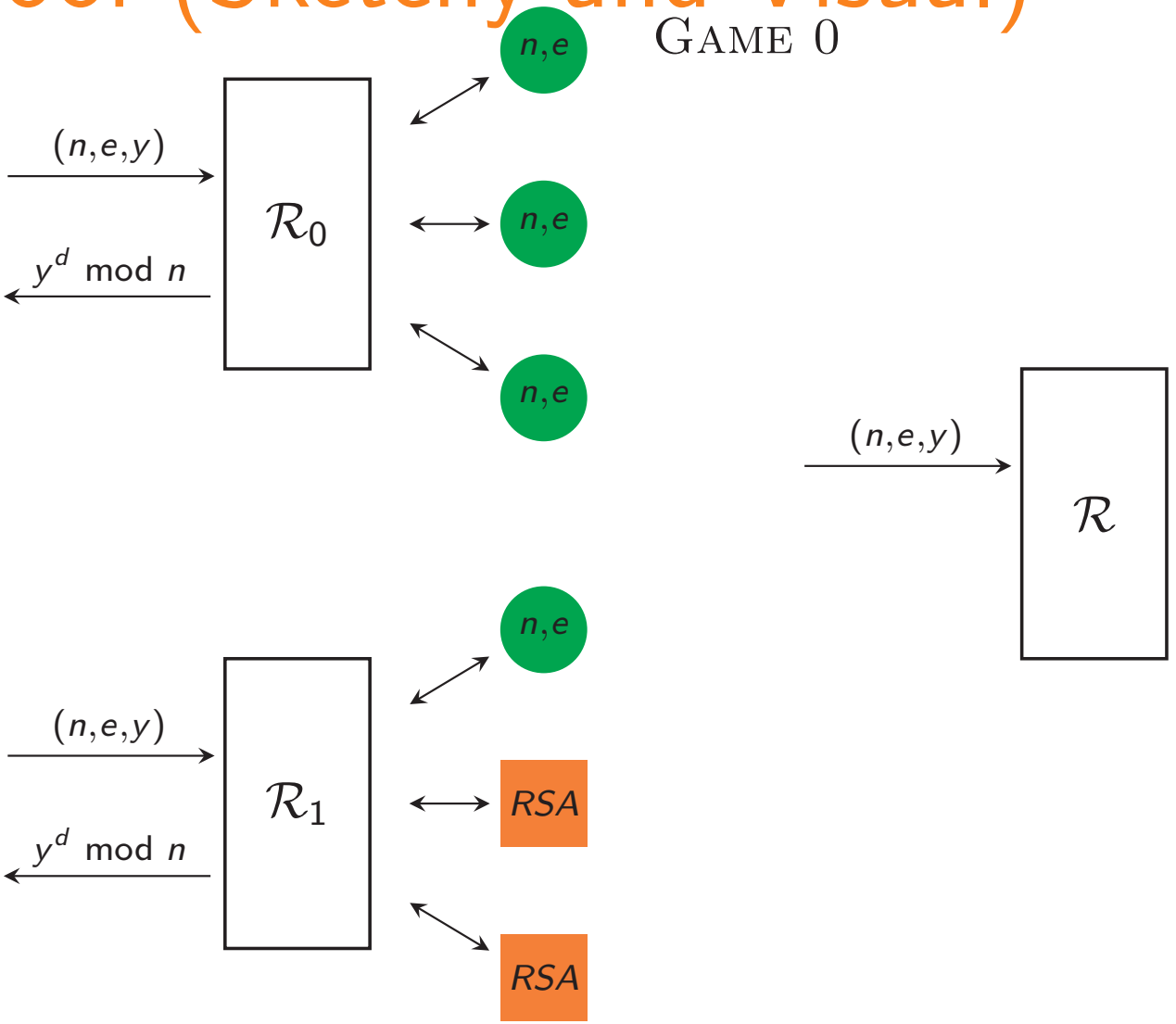

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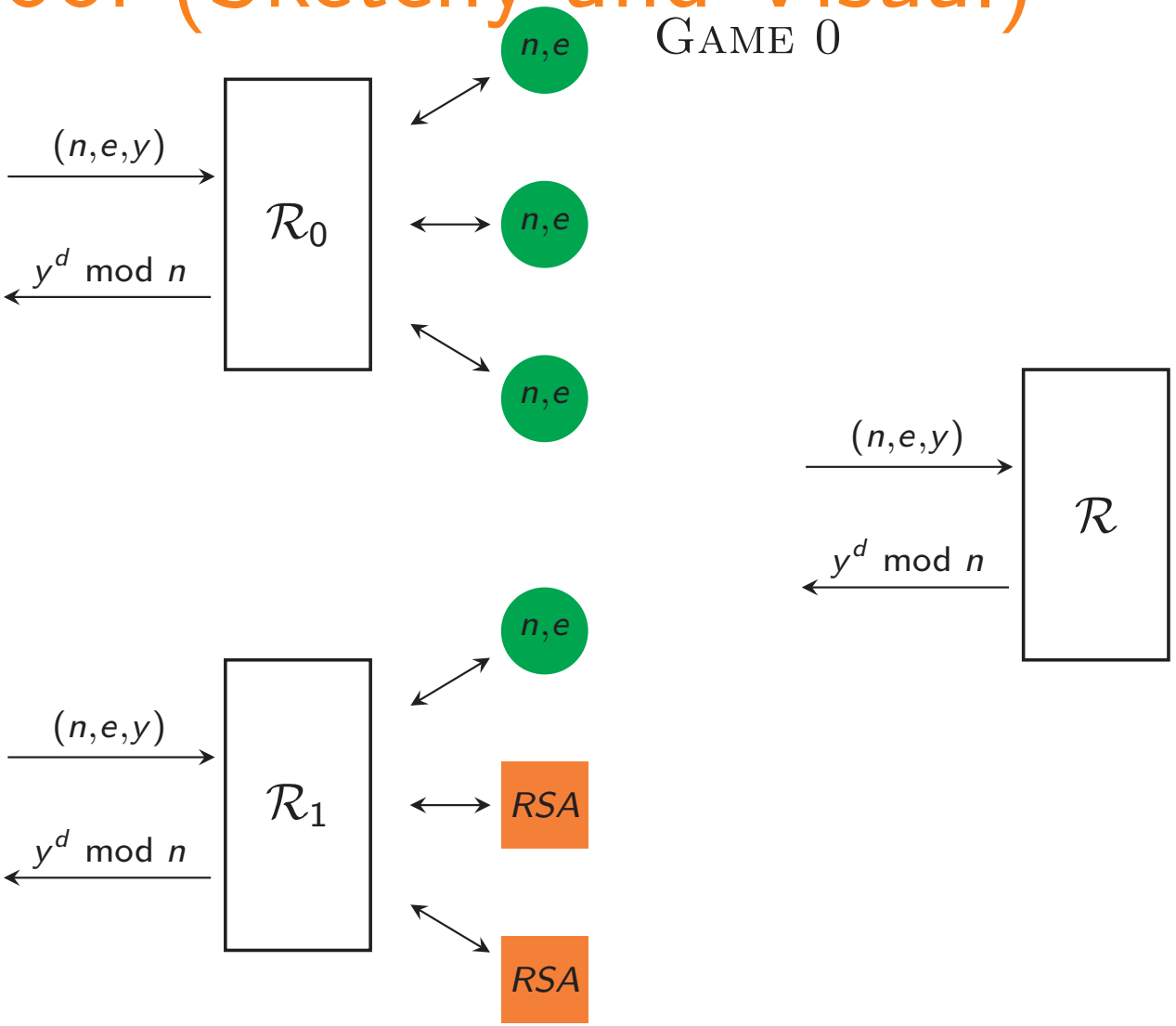

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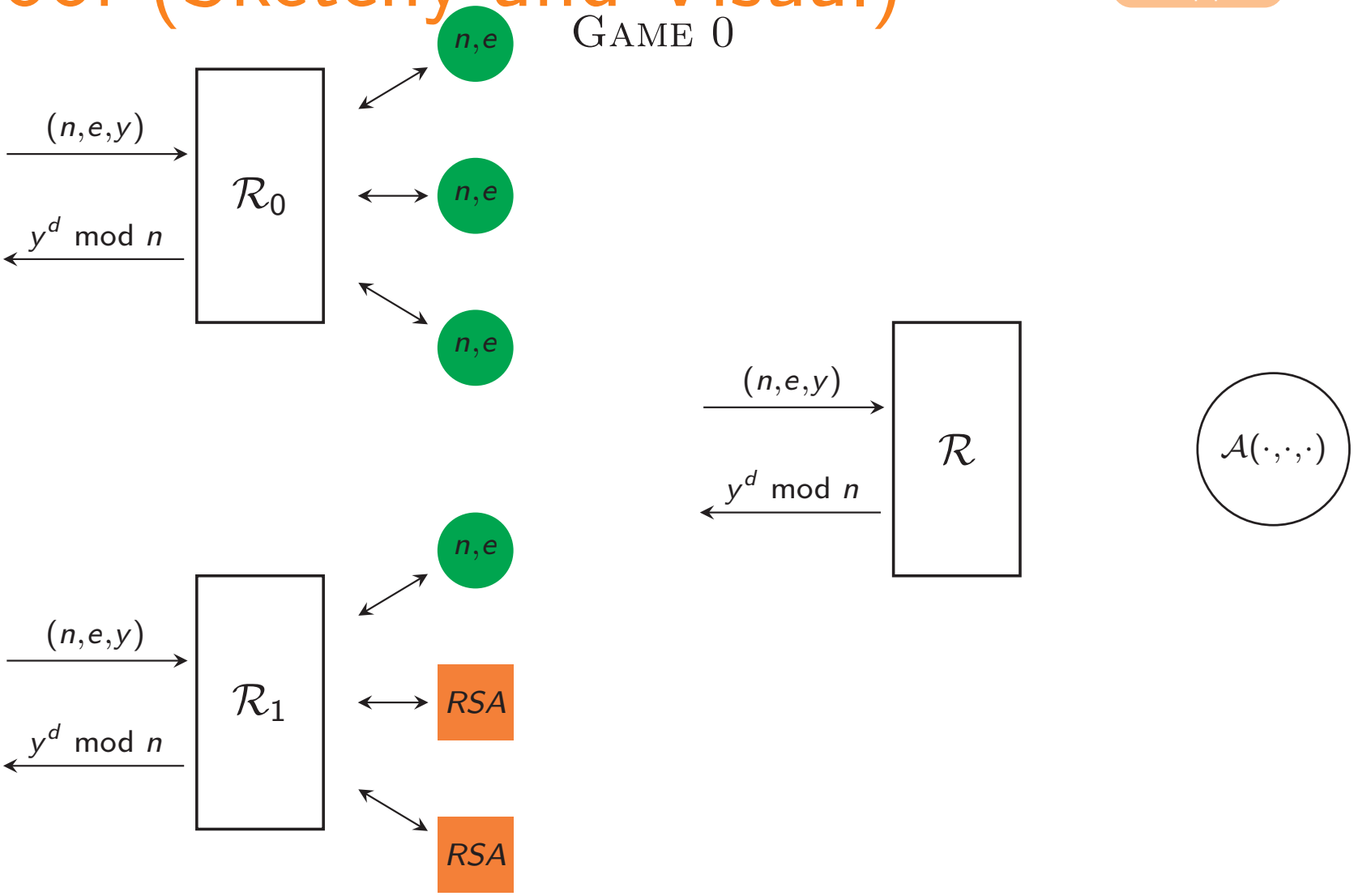

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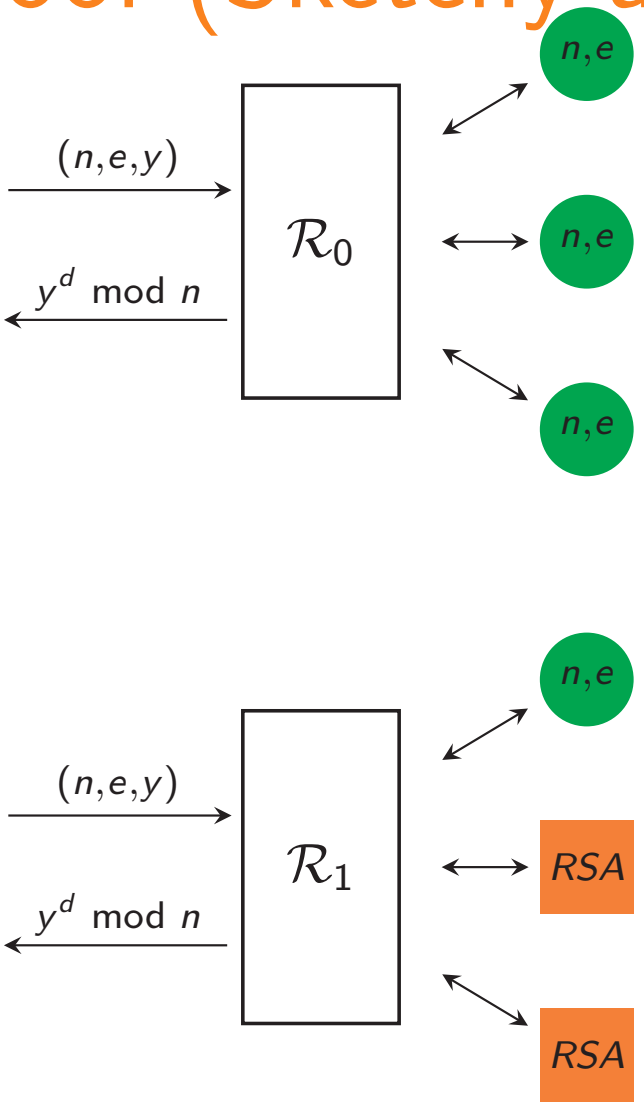
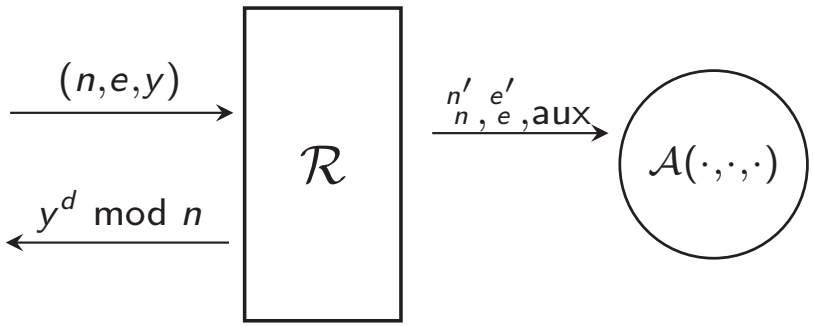
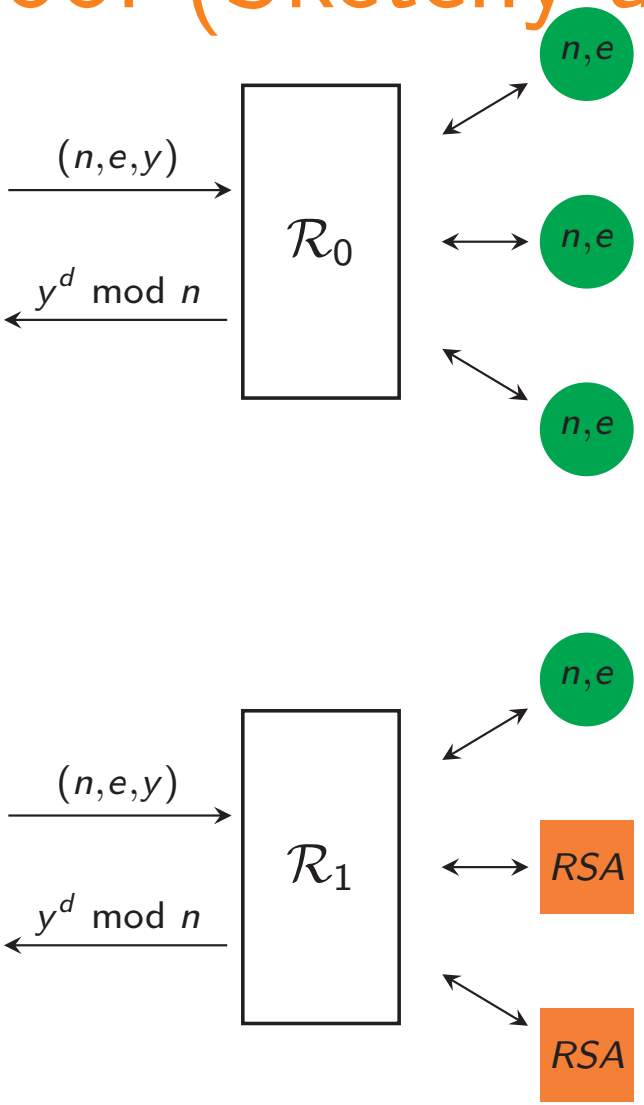


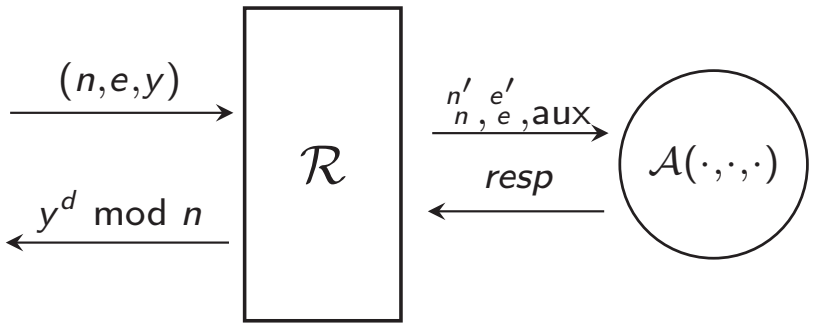


GAME 1

Proof (Sketchy and Visual)

Skip proof





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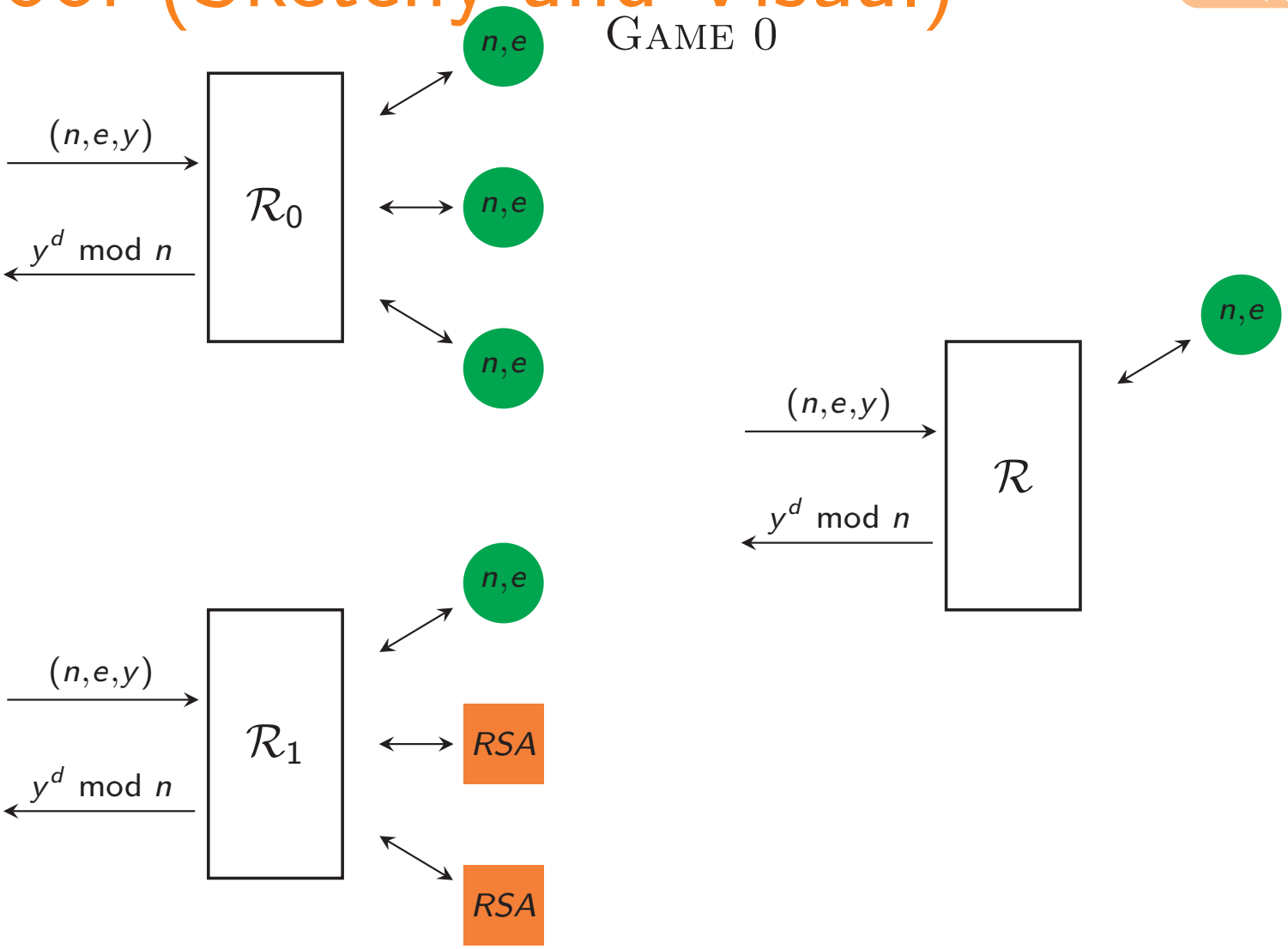

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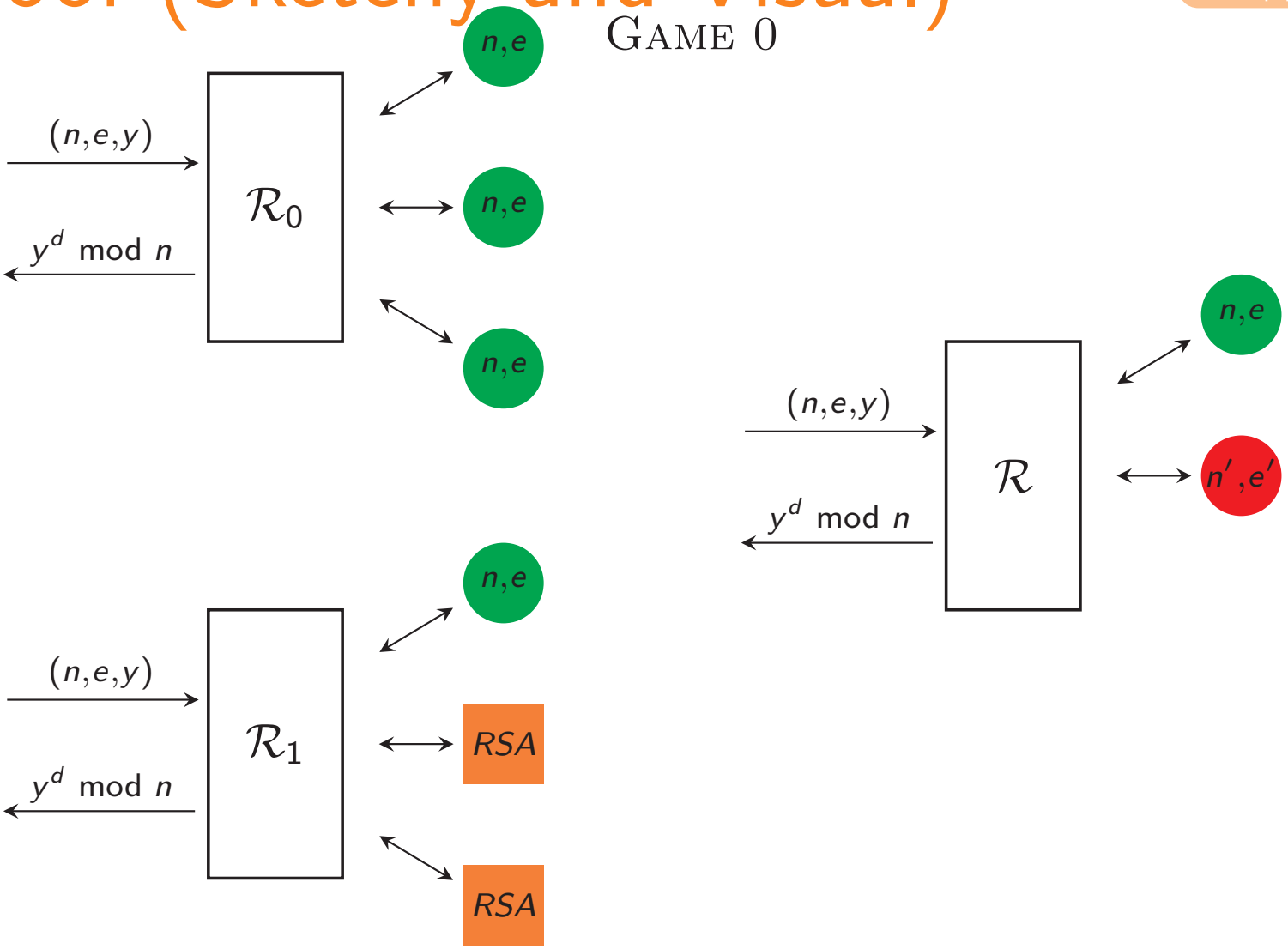

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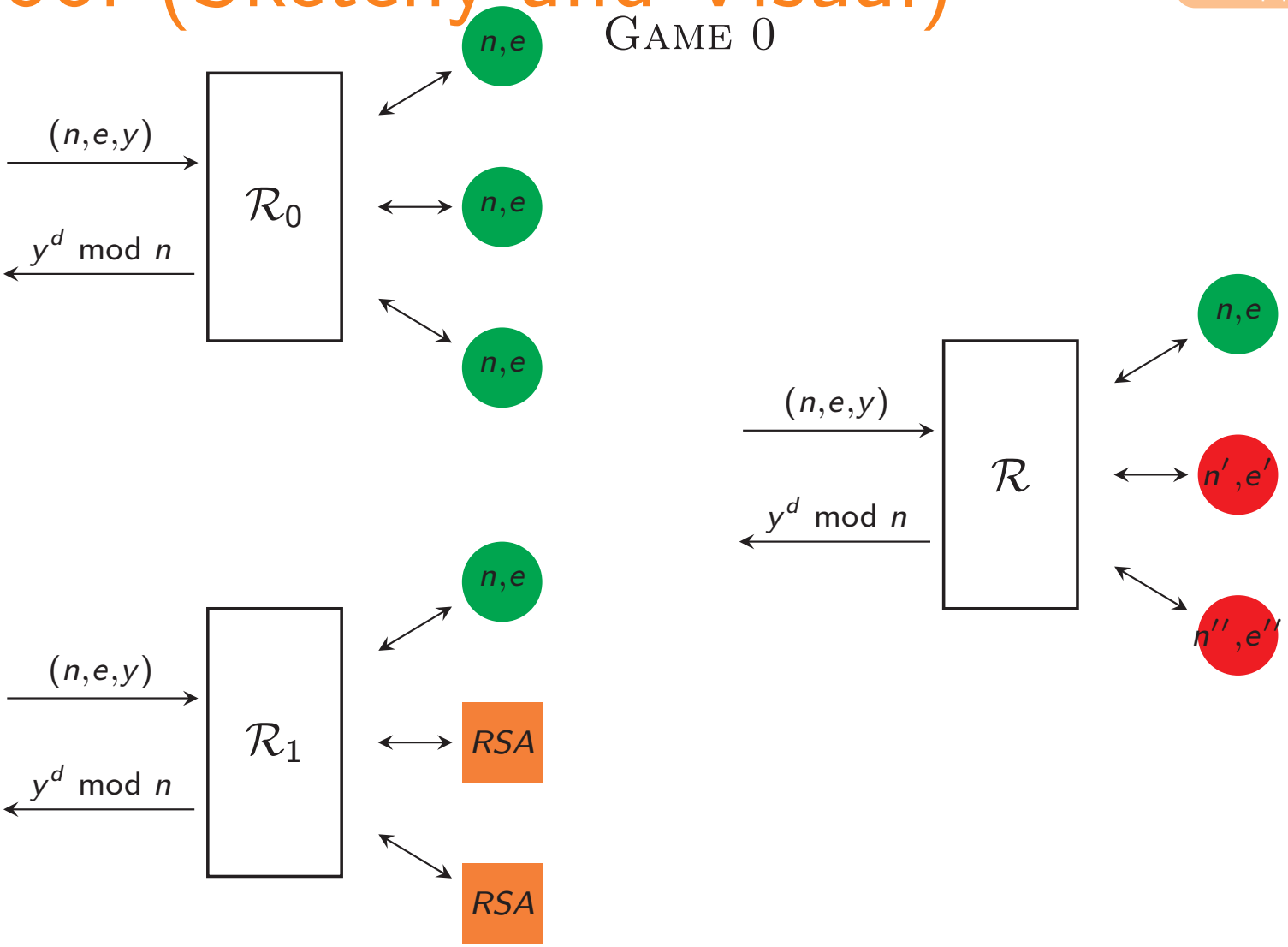

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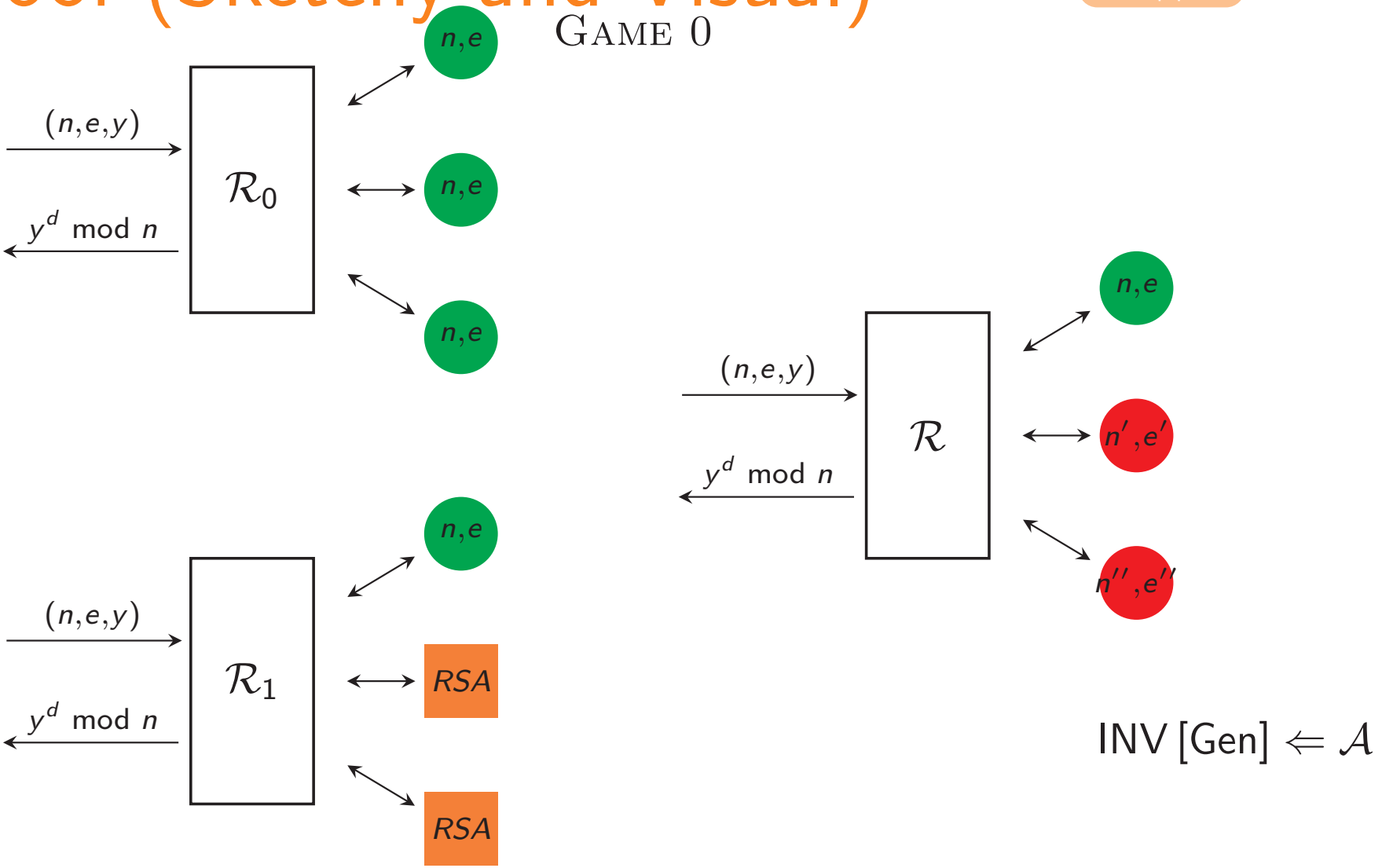

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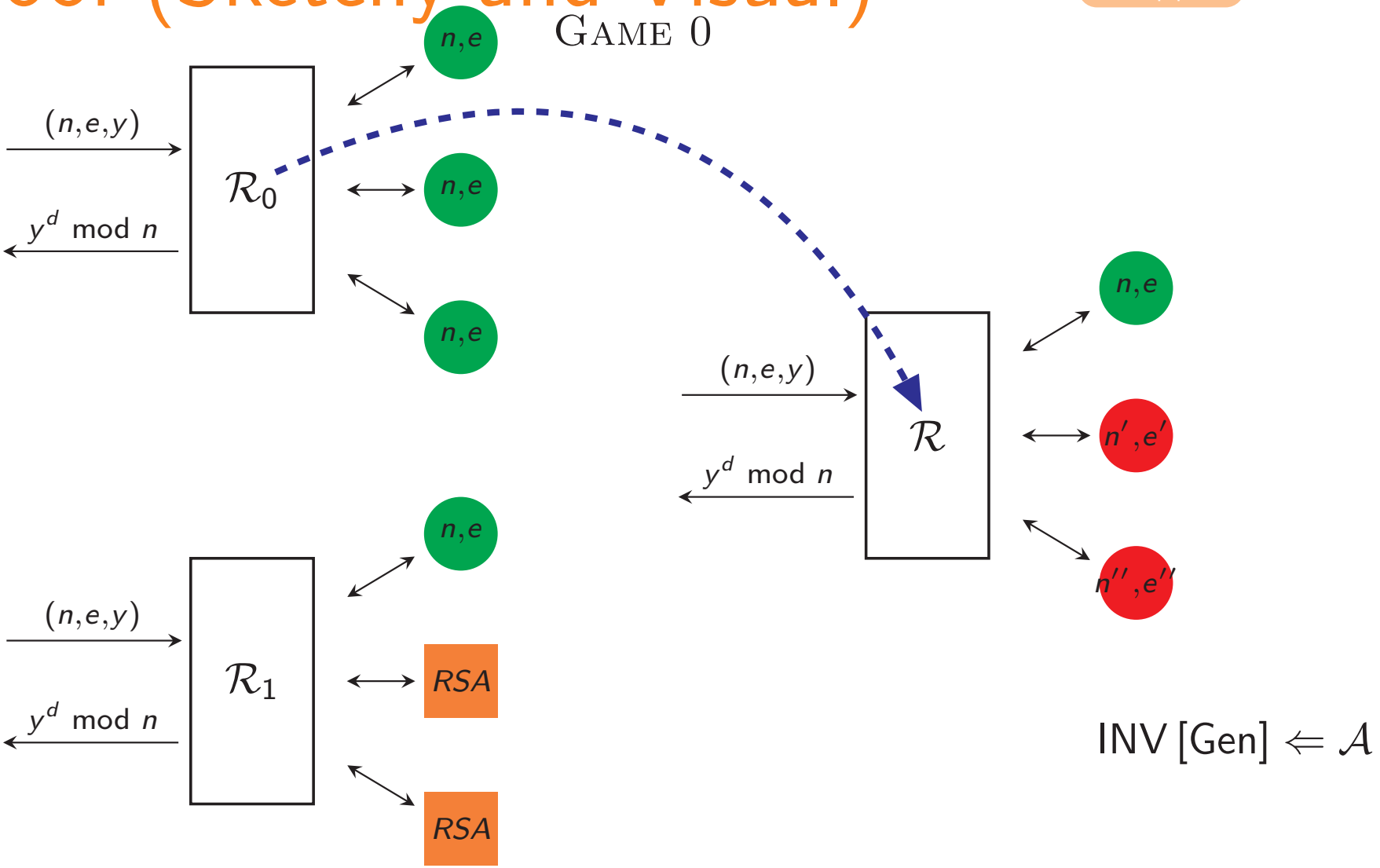
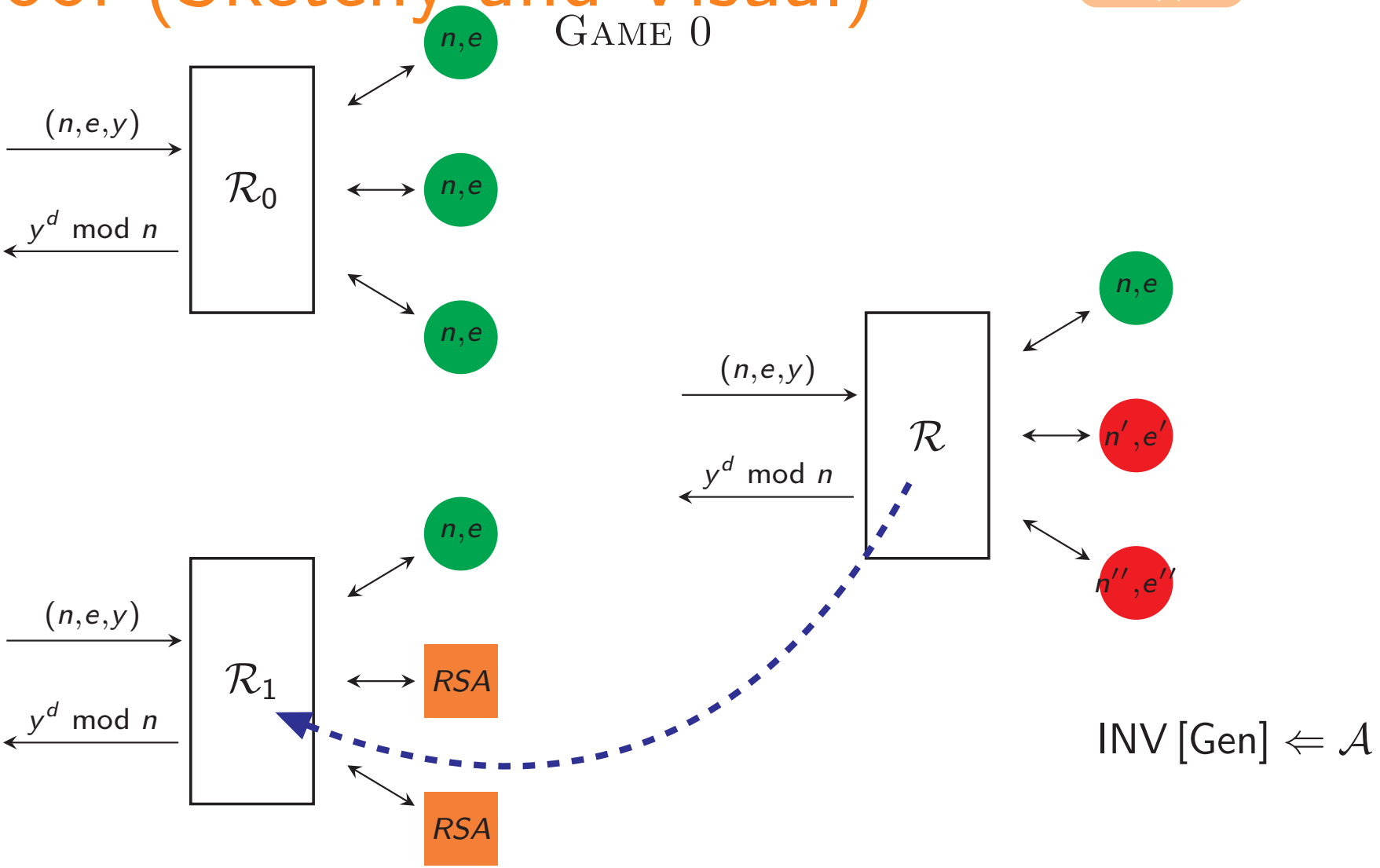


INV [Gen] $\Leftarrow \mathcal{A}$

Proof (Sketchy and Visual)

Skip proof



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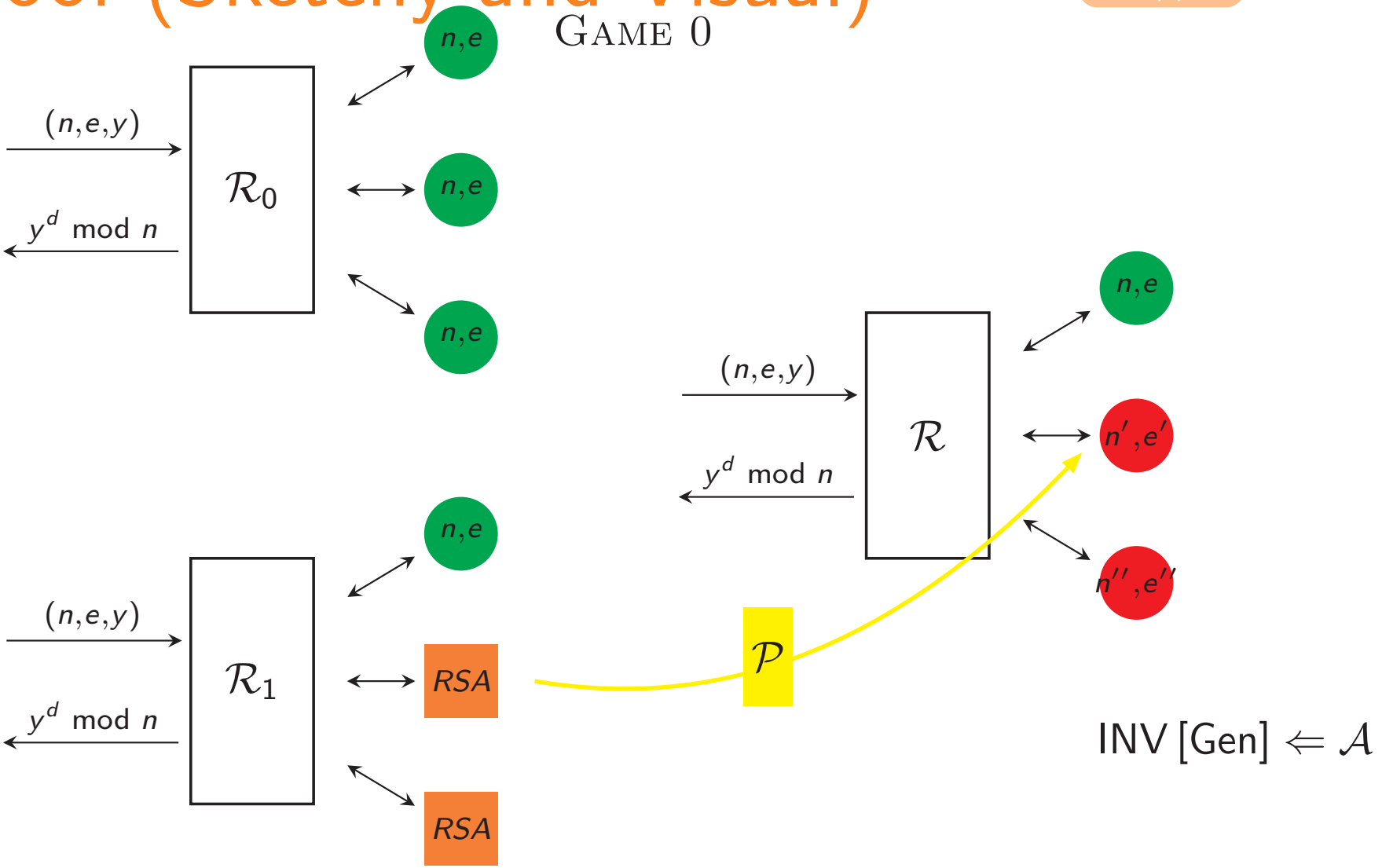

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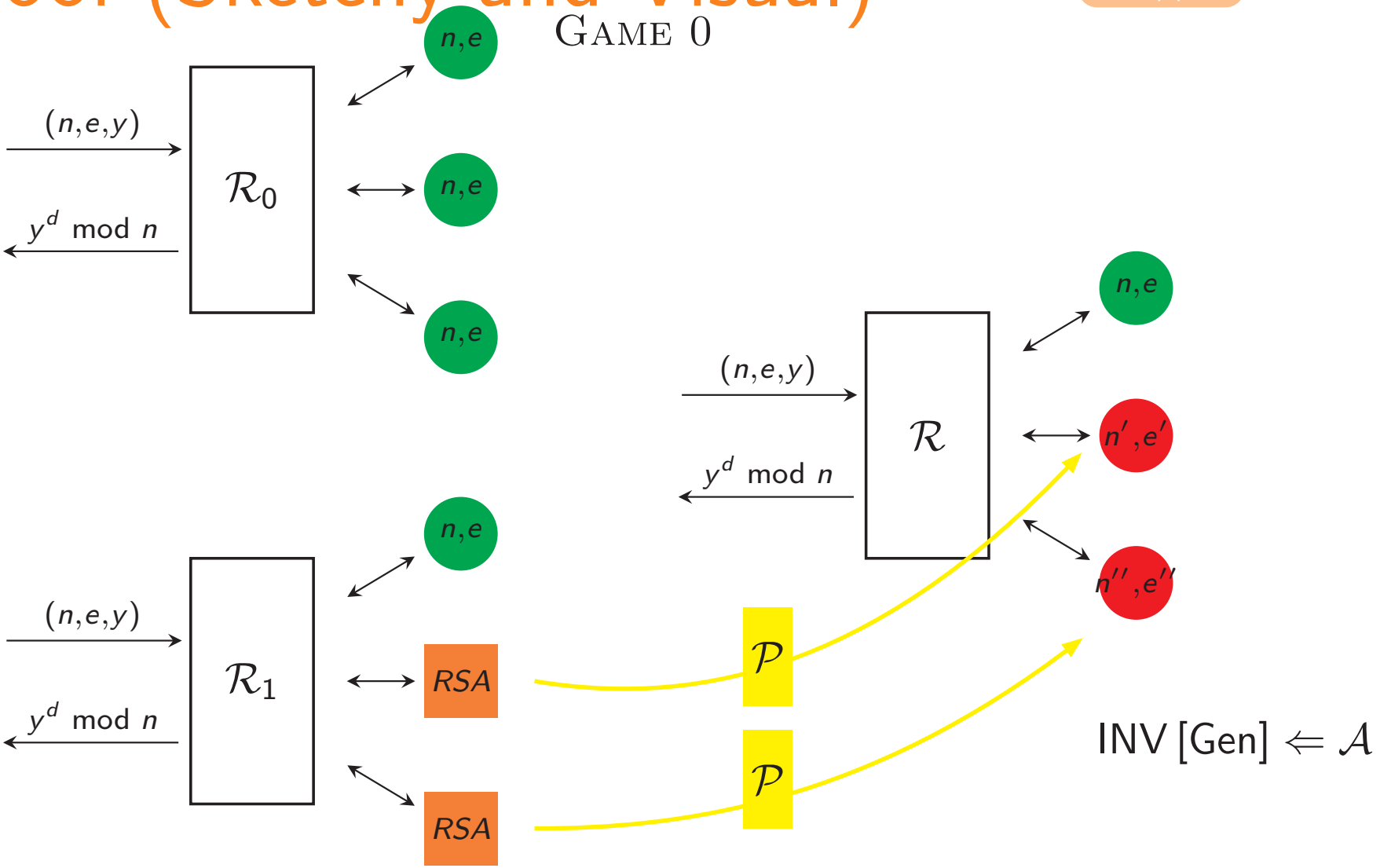

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Essentially

$$\textsc{Game}\ 0 \leq \mathsf{INV}\,[\mathsf{Gen}] \Leftarrow \mathcal{A} \leq \textsc{Game}\ 1$$

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Consequently

$$\textsc{Game}\ 0 \simeq \mathsf{INV}\,[\mathsf{Gen}] \Leftarrow \mathcal{A} \simeq \textsc{Game}\ 1$$

One-Wayness kills CCA Security

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Theorem

Assume that INV [Gen] $\Leftarrow_{\mathcal{R}}$ OW-CPA [$\mathcal{E}$]

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- hence winning $\textsc{Game}\ 0$ is easy too if Gen is instance-non-malleable
- finally note that winning $\textsc{Game}\ 0$ is exactly breaking RE-CCA [$\mathcal{E}$]

Indistinguishability kills PCA Security

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$$\boxed{\varepsilon_{\mathcal{M}} \geq \varepsilon_{\mathcal{R}} \text{ and } \tau_{\mathcal{M}} \leq \tau_{\mathcal{R}} + \text{poly}(\ell, k)}$$

Proof technique

Identical.

IND-CCA $[\mathcal{E}] \not\equiv$ INV [Gen]

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The ROM is way too optimistic

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There is no hope for having

- IND-CCA $[\mathcal{E}] \equiv$ INV [Gen]
- OW-CCA $[\mathcal{E}] \equiv$ INV [Gen]
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The ROM is way too optimistic

In the ROM, there are encryptions *e.g.* RSA-OAEP for which

$$\boxed{\text{IND-CCA}\left[\mathcal{E}^{H}\right] \equiv \text{INV [Gen]} \text{ !!}}$$

Conclusion

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The programmability of random oracles makes them too powerful and unrealistic

What is the real security level of RSA-OAEP?

We don't know!

Non-Malleability in Other Settings

Assume $\Delta(\text{poly}(k)) = \text{negl}(k)$.

RSA Signatures

the unforgeability of RSA-based signatures such as FDH, PSS, PSS-R and many others cannot be equivalent to INV [Gen]

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Transposable Signatures *e.g.* PSS

Even worse as UF-KOA $[\mathcal{S}] \not\equiv$ OM [Gen]!

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Factoring-Based Signatures

The unforgeability of Rabin signatures, etc. is $\not\equiv$ FACT [Gen]

Computational Number Theoretic Challenges

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Malleable RSA moduli

$(n, n') \leftarrow \mathrm{Gen}(1^k)$ such that

- n' can be recovered from n alone
- factoring n' allows to factor n easily