Harmonic measure and the passivation of 2D and 3D fractals

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Outline

- Geometry-adapted fast random walks

- Multifractal properties of the harmonic measure on Von Koch boundaries

  Grebenkov, Lebedev, Filoche, Sapoval, Phys. Rev. E 71, 056121 (2005)

  Grebenkov, Phys. Rev. Lett. 95, 200602 (2005).

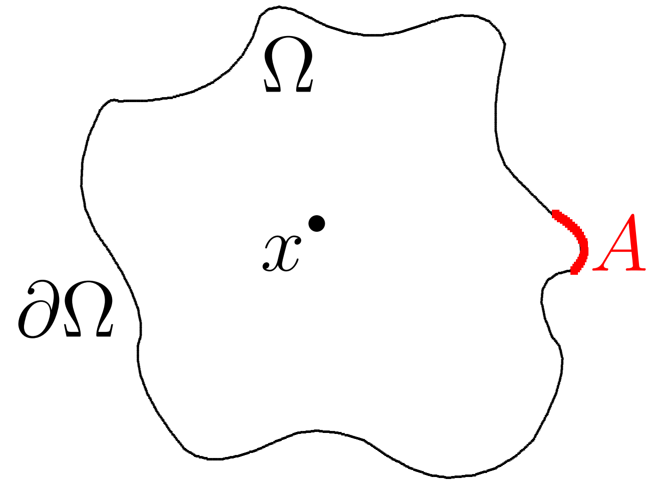
- Passivation of 2D and 3D fractals

  Filoche, Grebenkov, Andrade, Sapoval (submitted)

  Sapoval, Costa, Andrade, Filoche, Fractals 12, 381 (2004).

Harmonic measure

$\omega_x\{A\}$ is the probability for Brownian motion started from point x to reach the surface $\partial\Omega$ on the subset A



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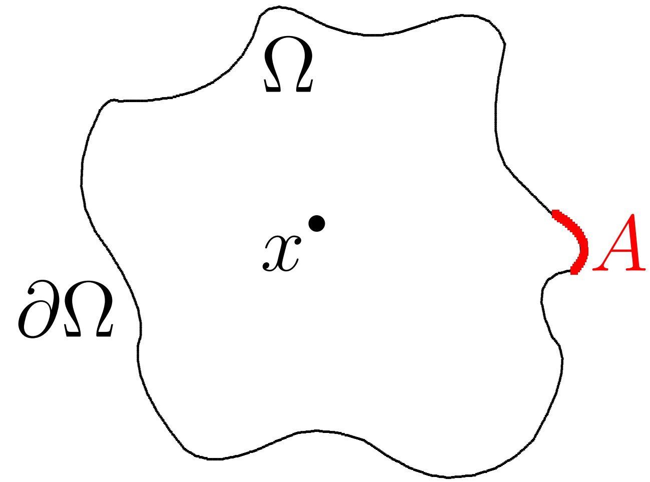

$$\Delta\omega_x\{A\} = 0 \quad (x \in \Omega)$$

$$\omega_x\{A\} = 1 \quad (x \in A)$$
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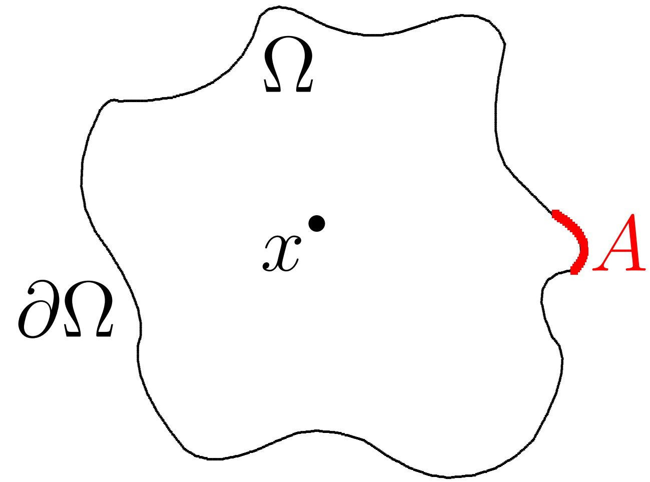

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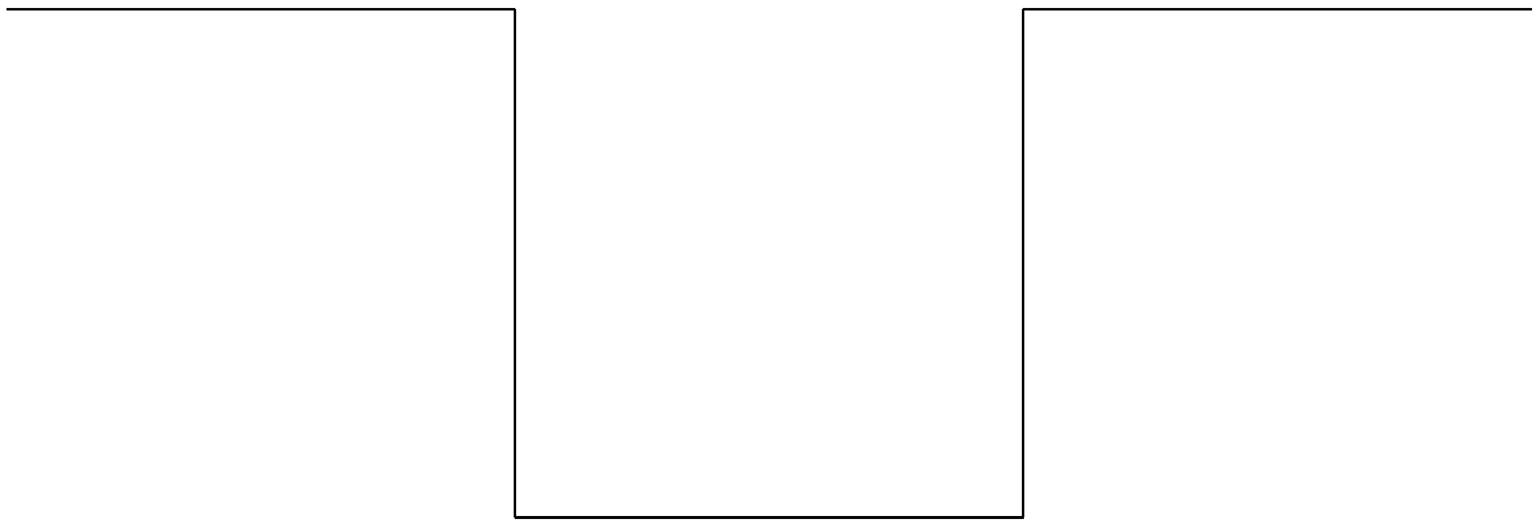
For a smooth surface, there exists a harmonic measure density $\omega_x(s)$: $\quad \omega_x\{A\} = \int\limits_A \omega_x(s) ds$

Von Koch boundaries



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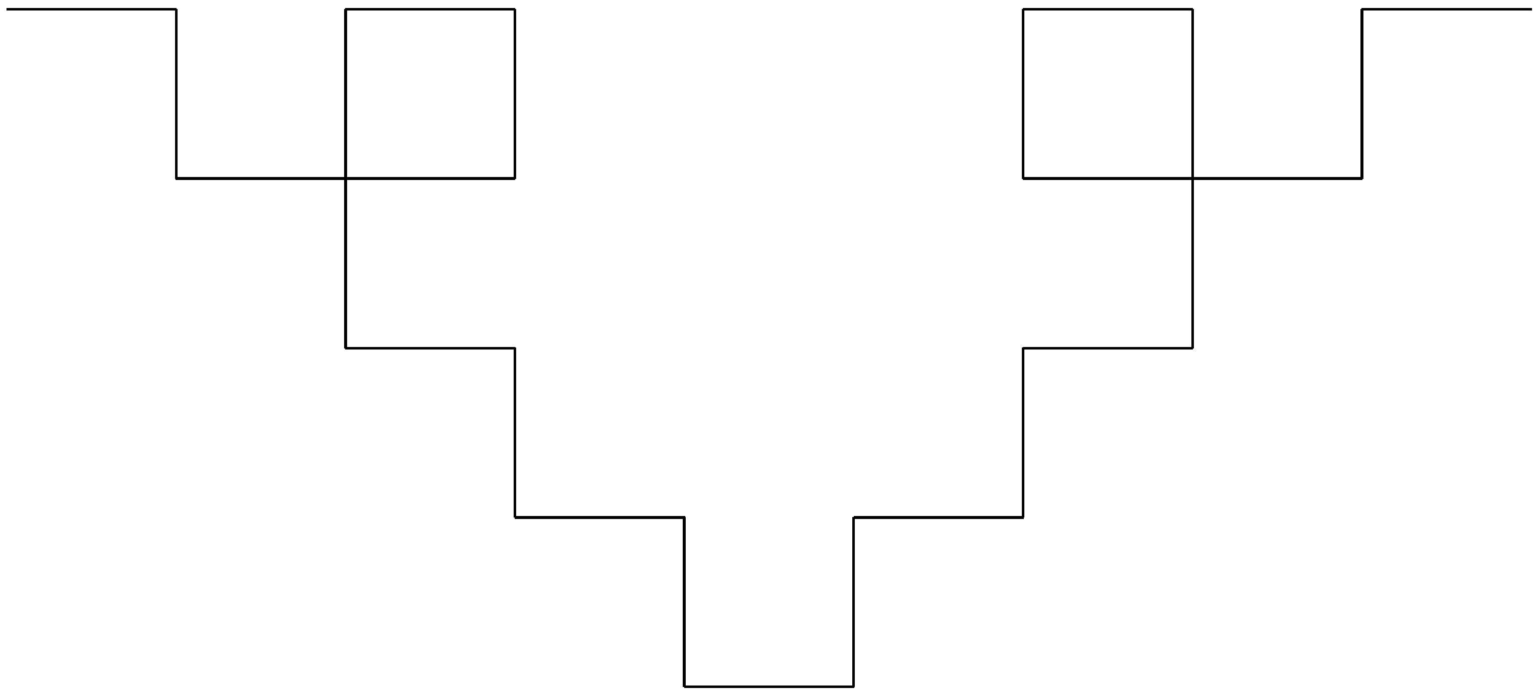

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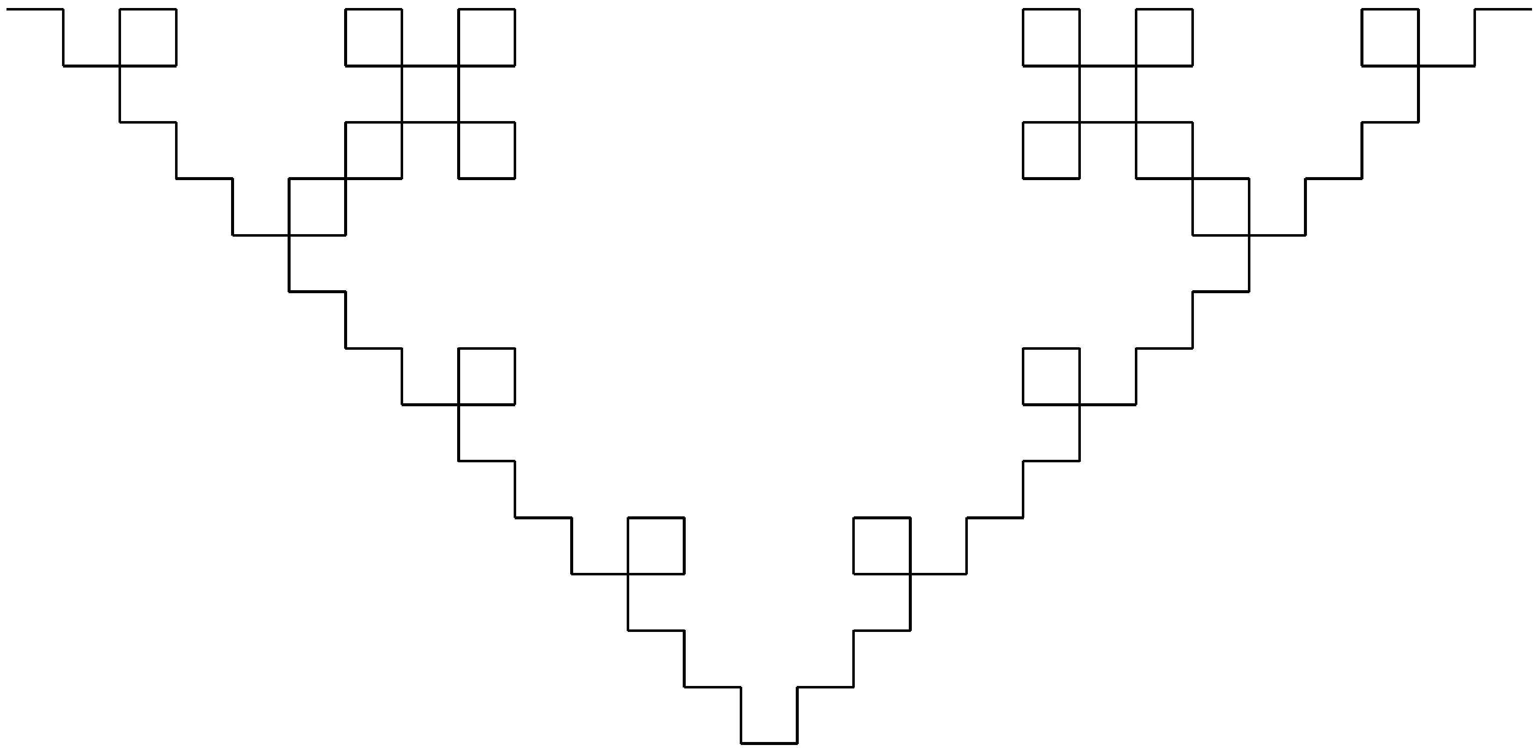

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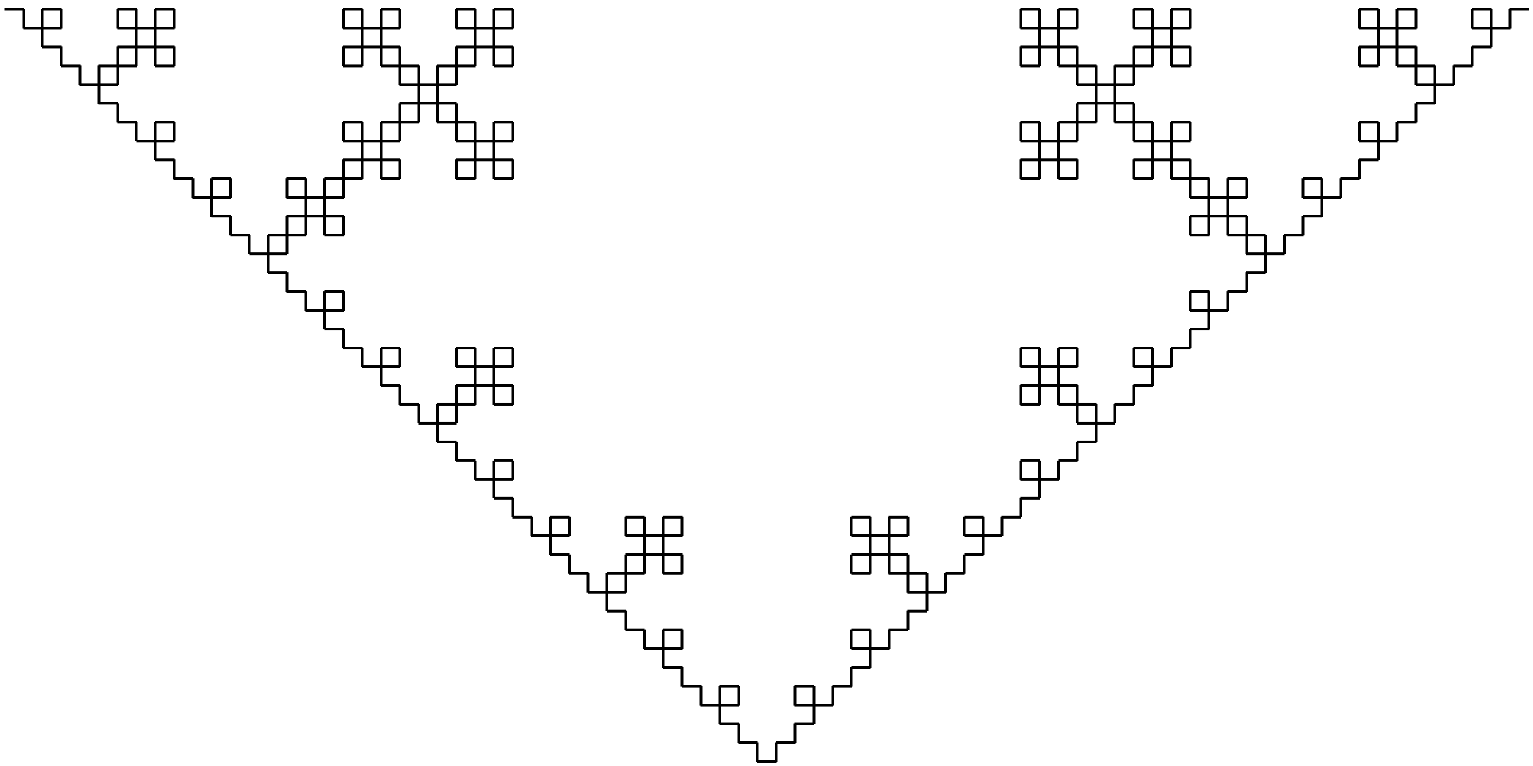
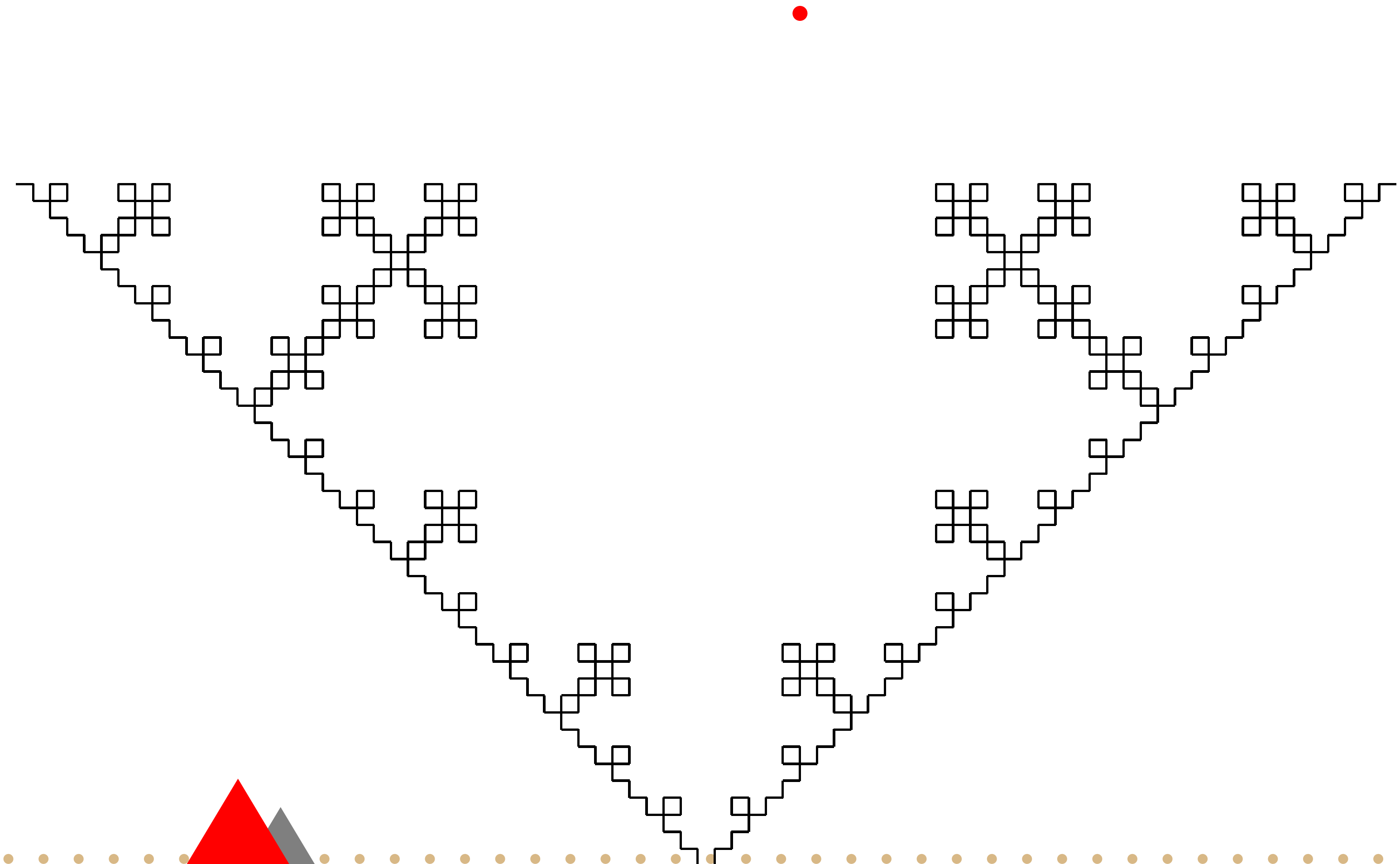


Fast random walks



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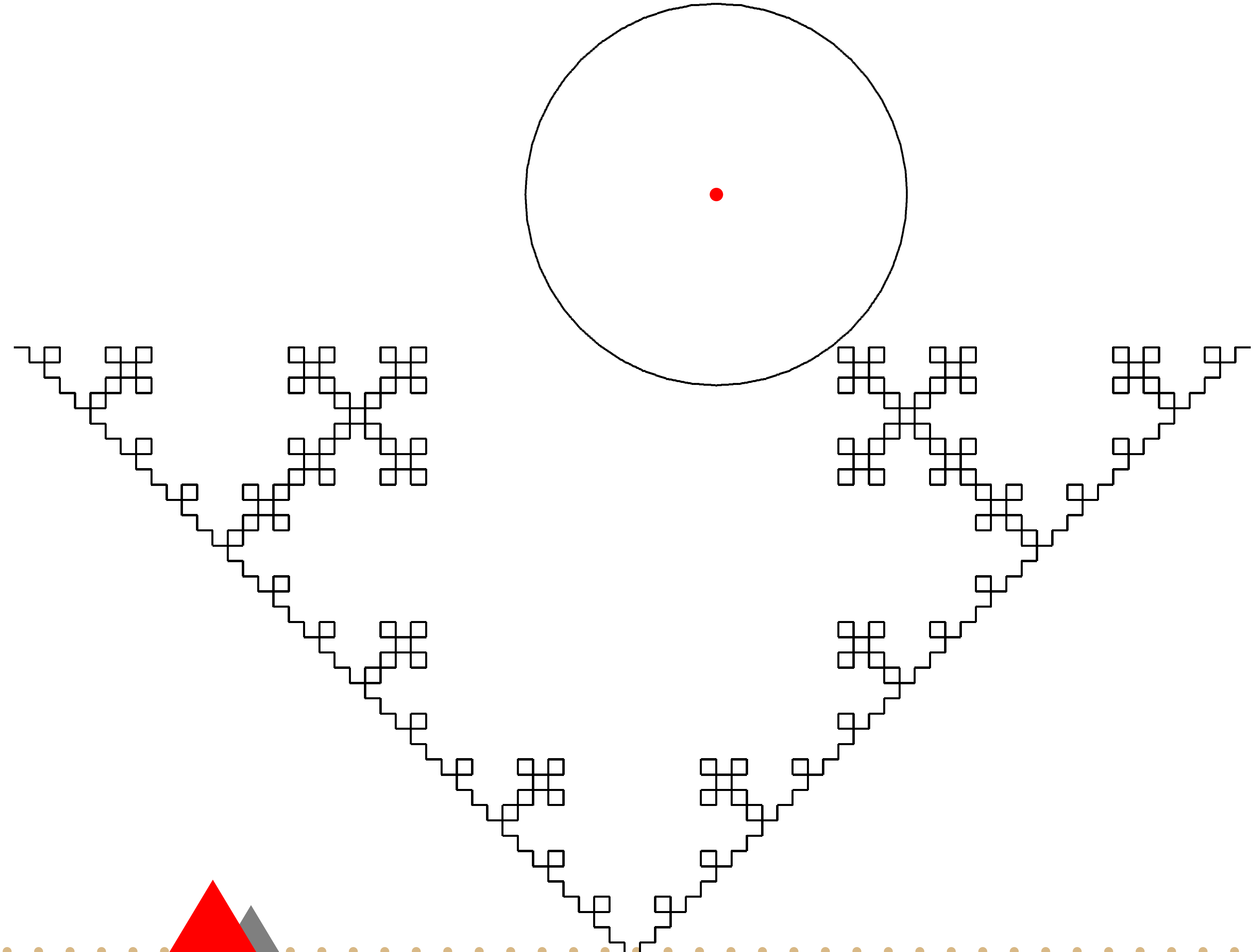

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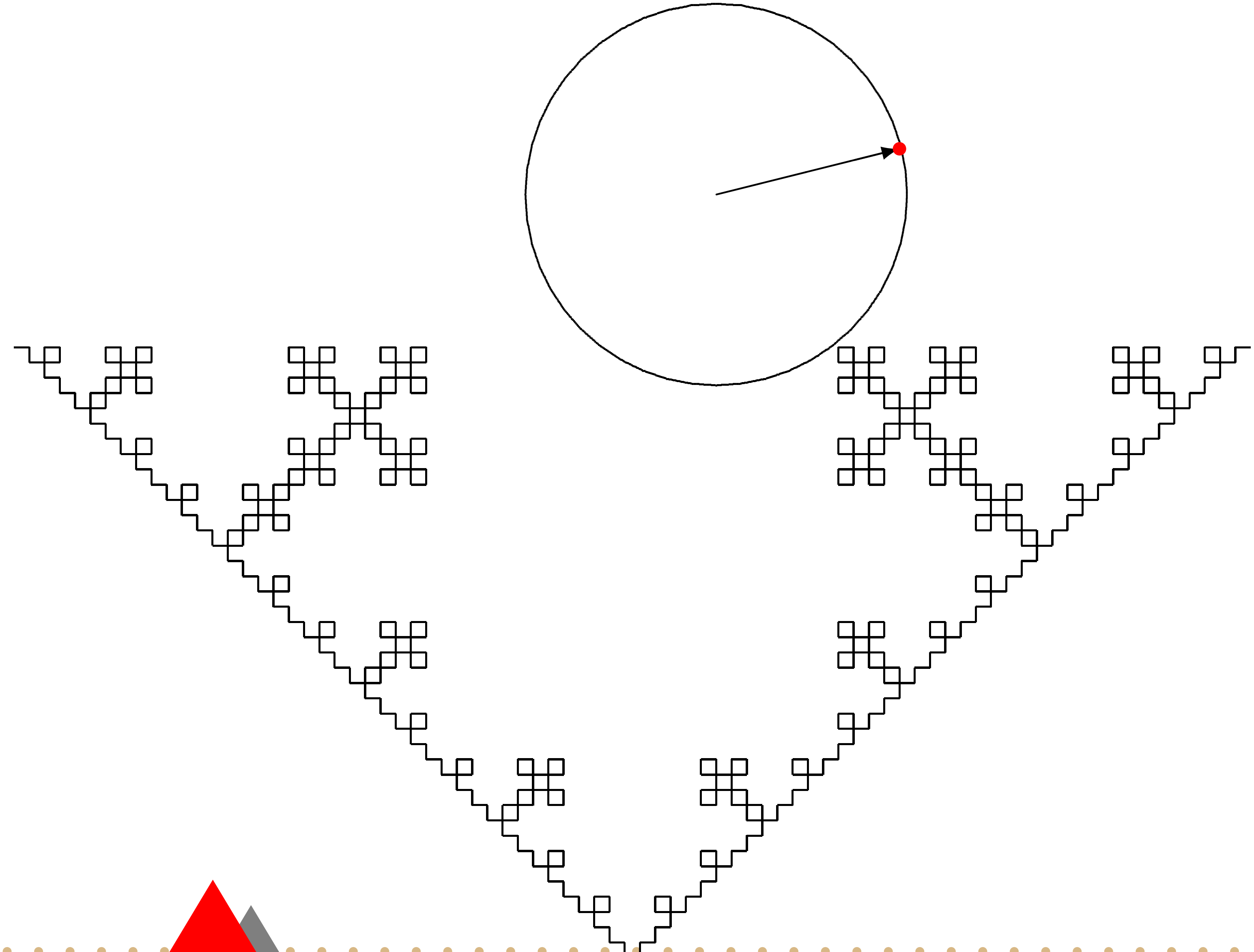

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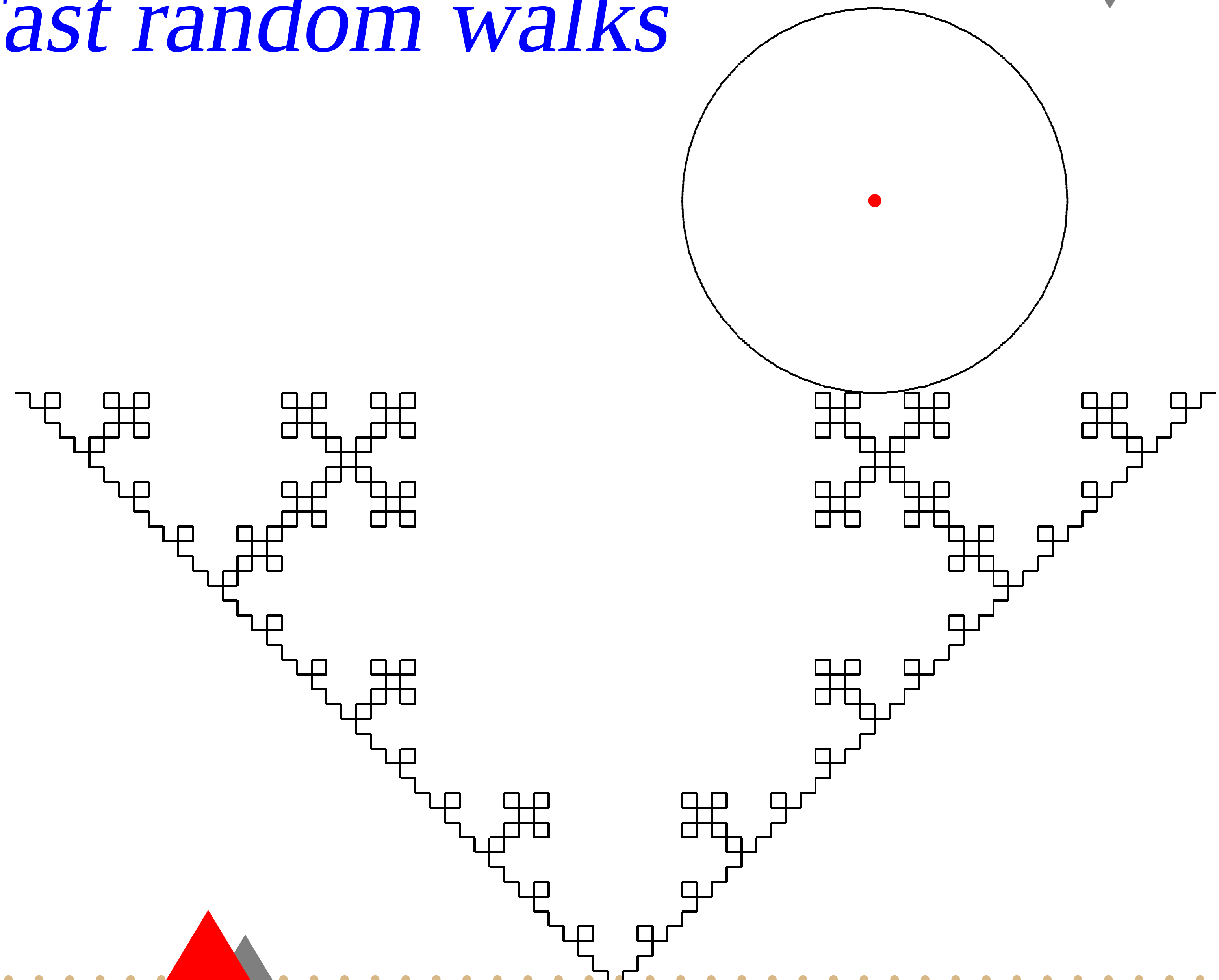

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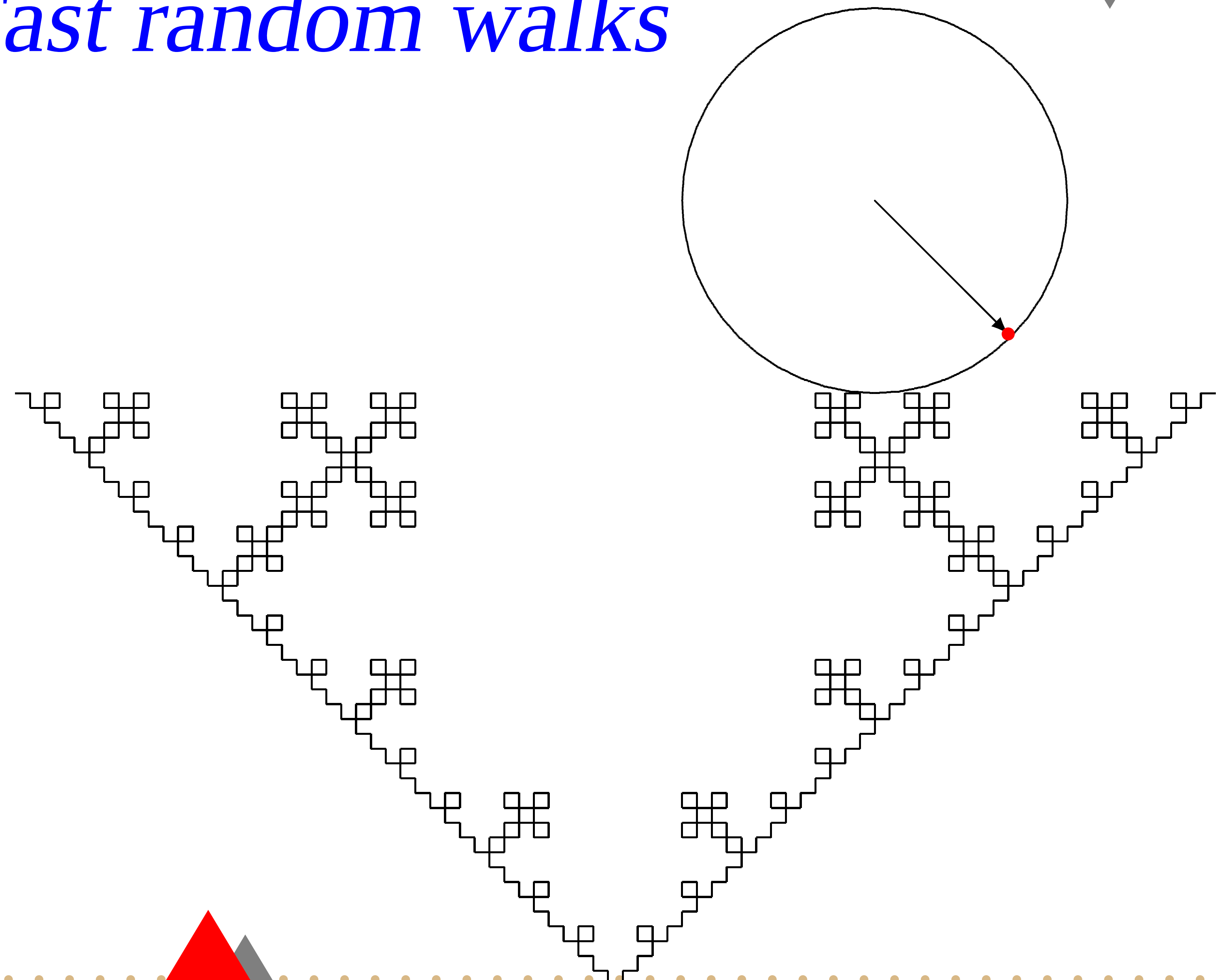

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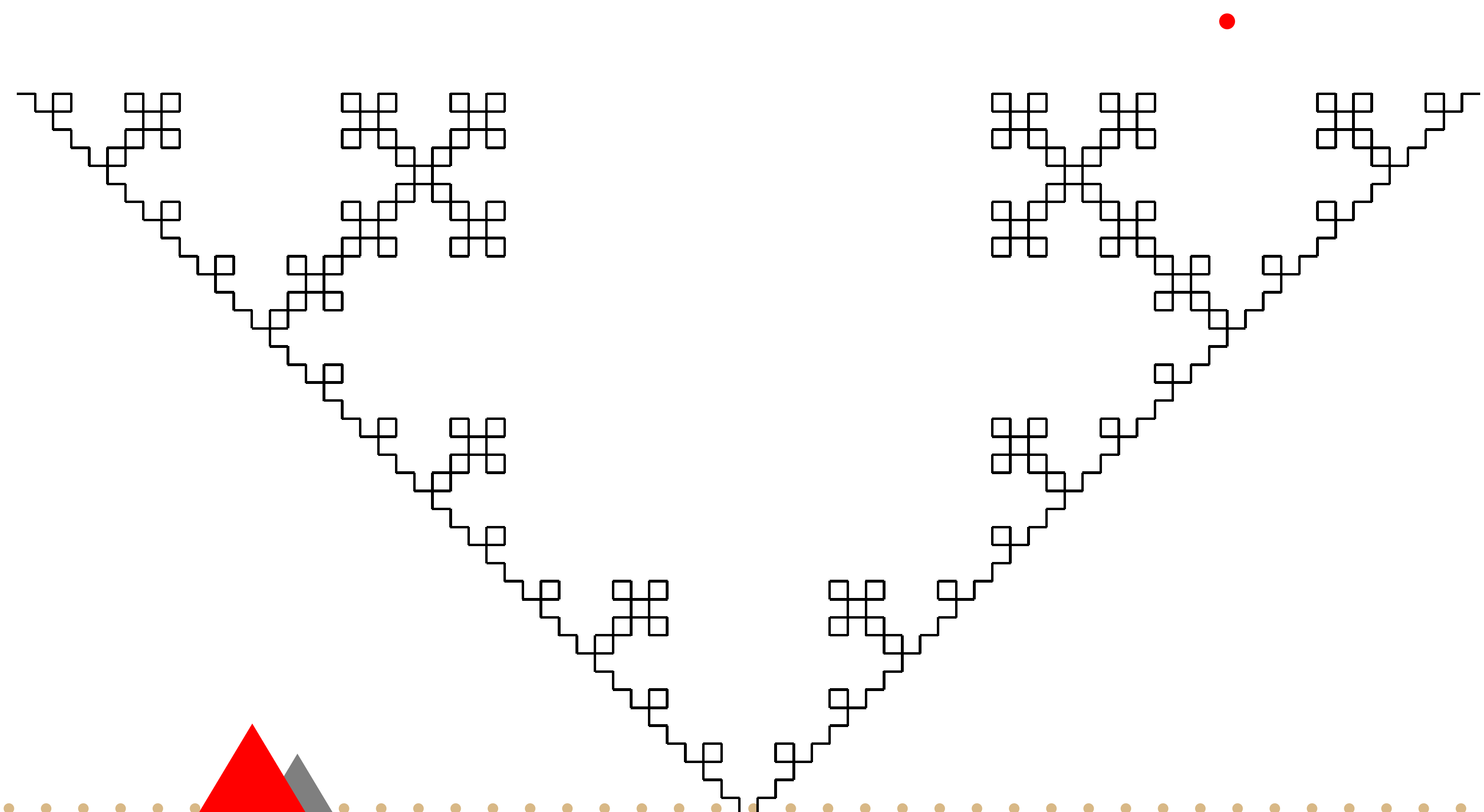

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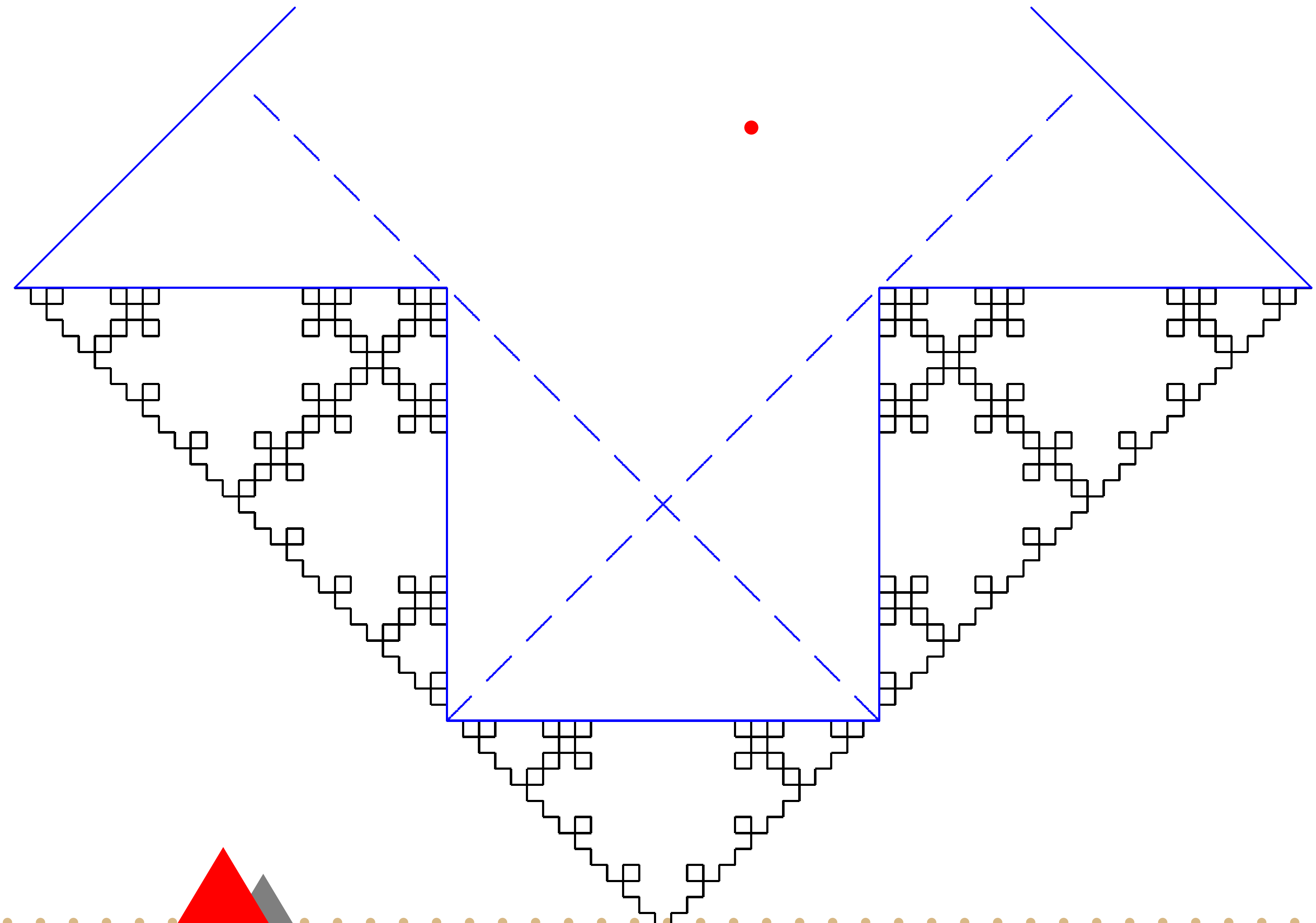
Calculation of distance



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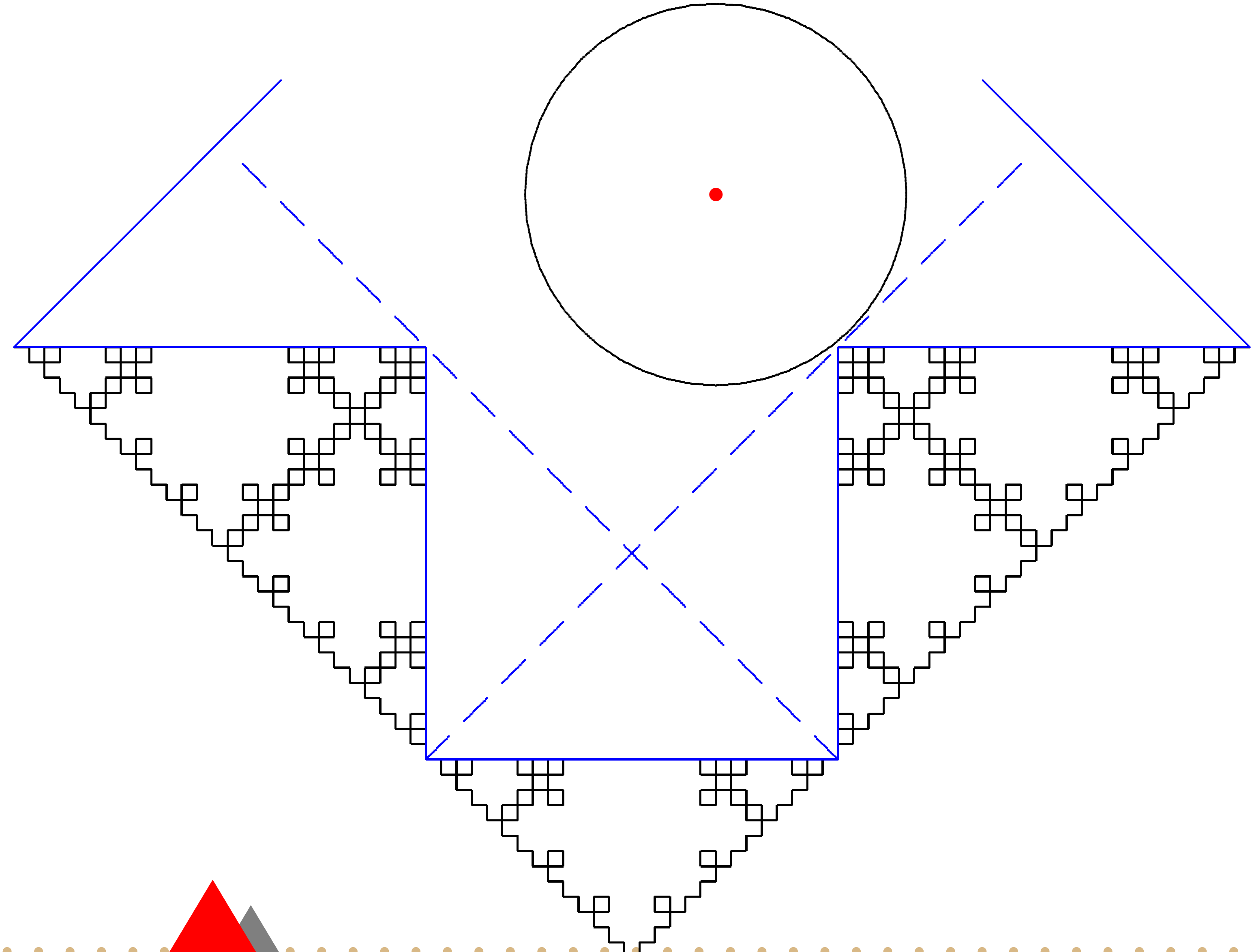

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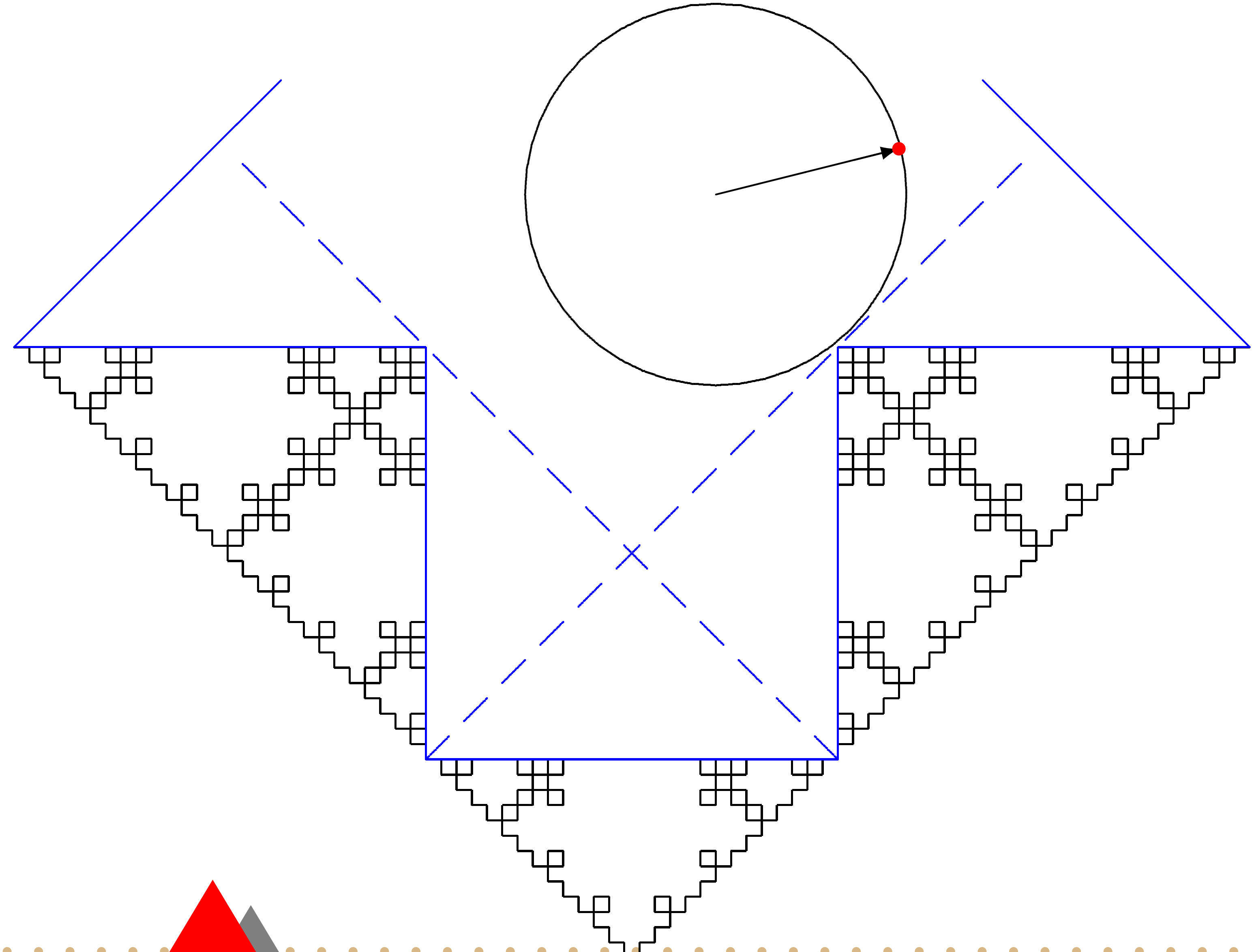

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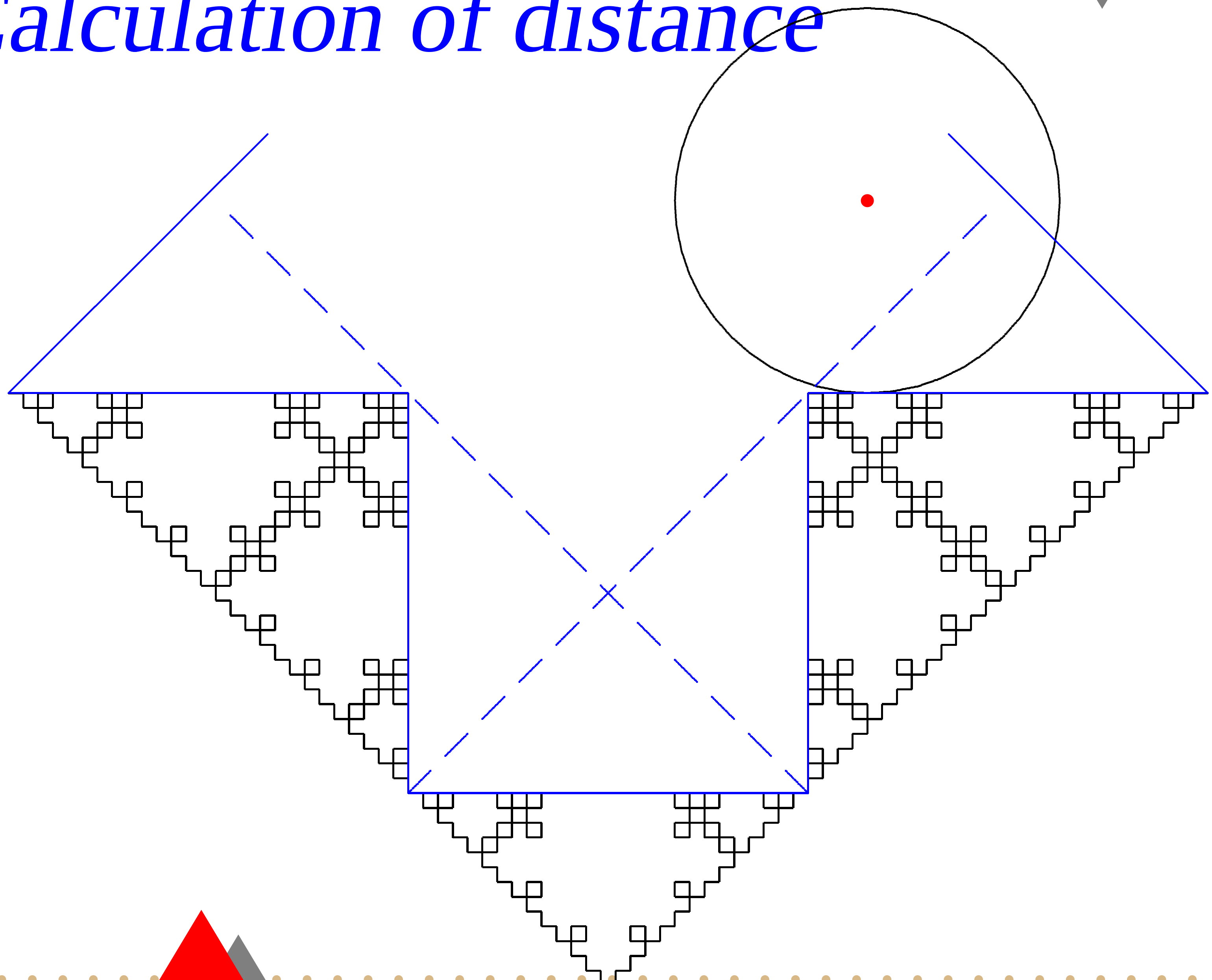

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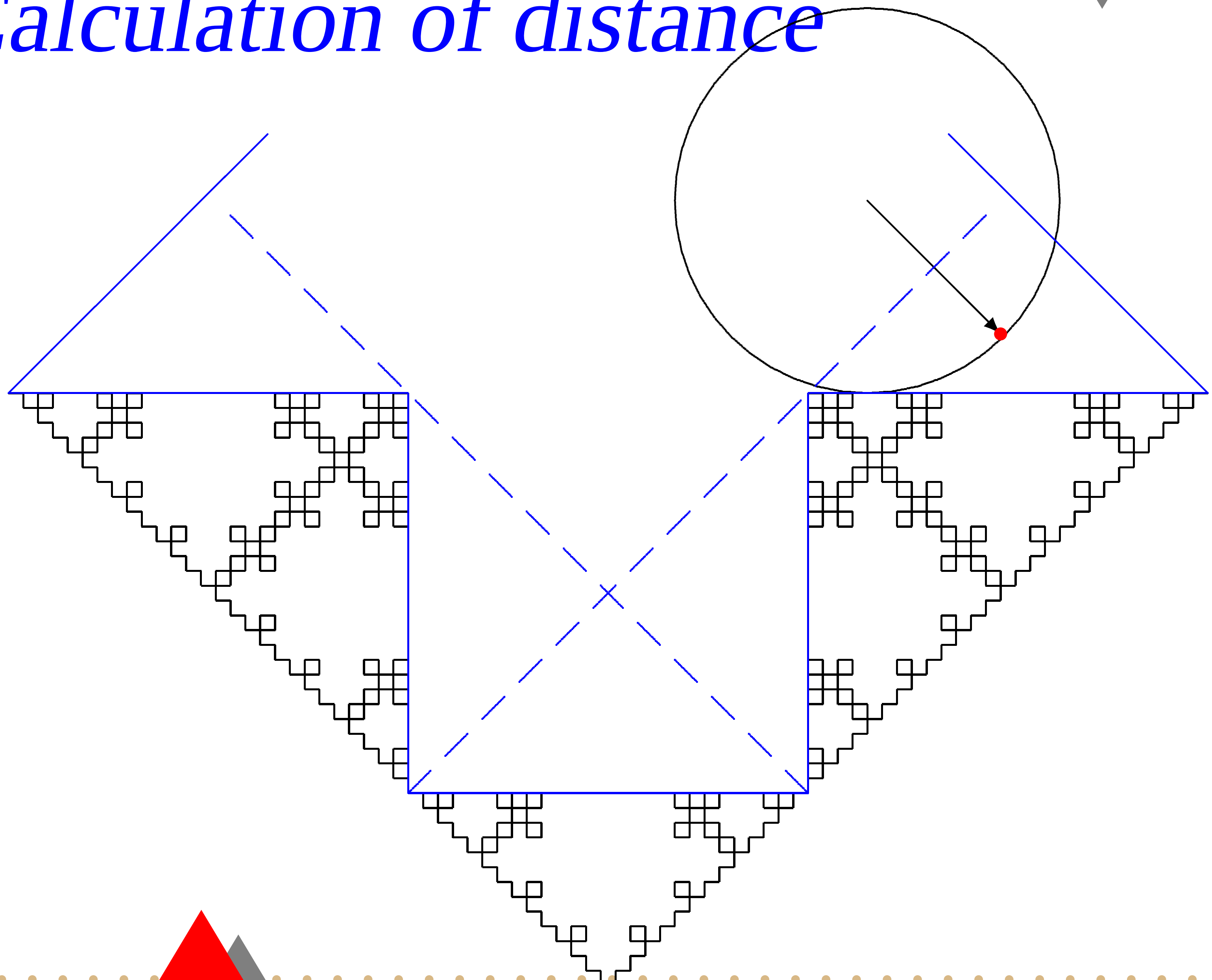

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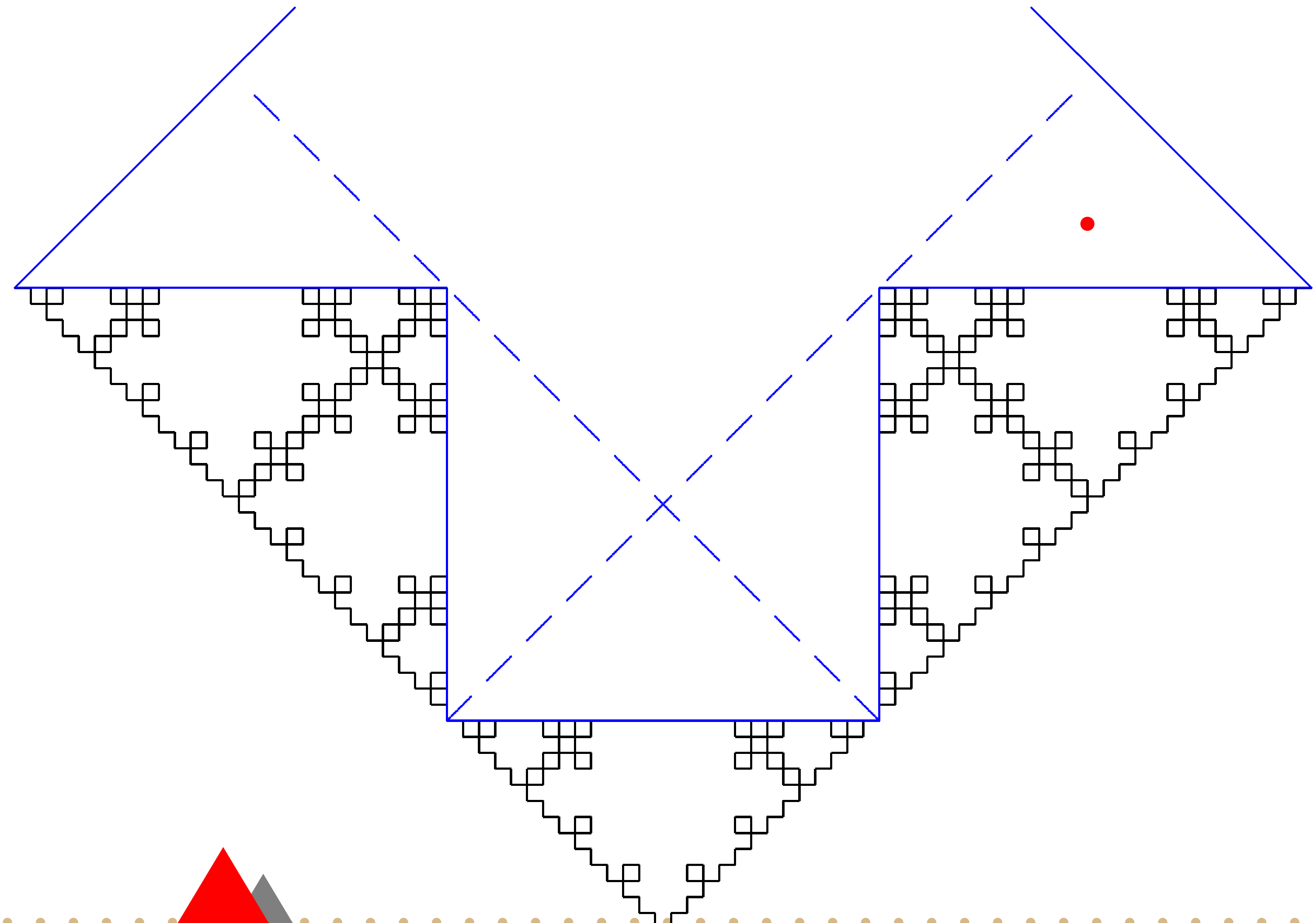

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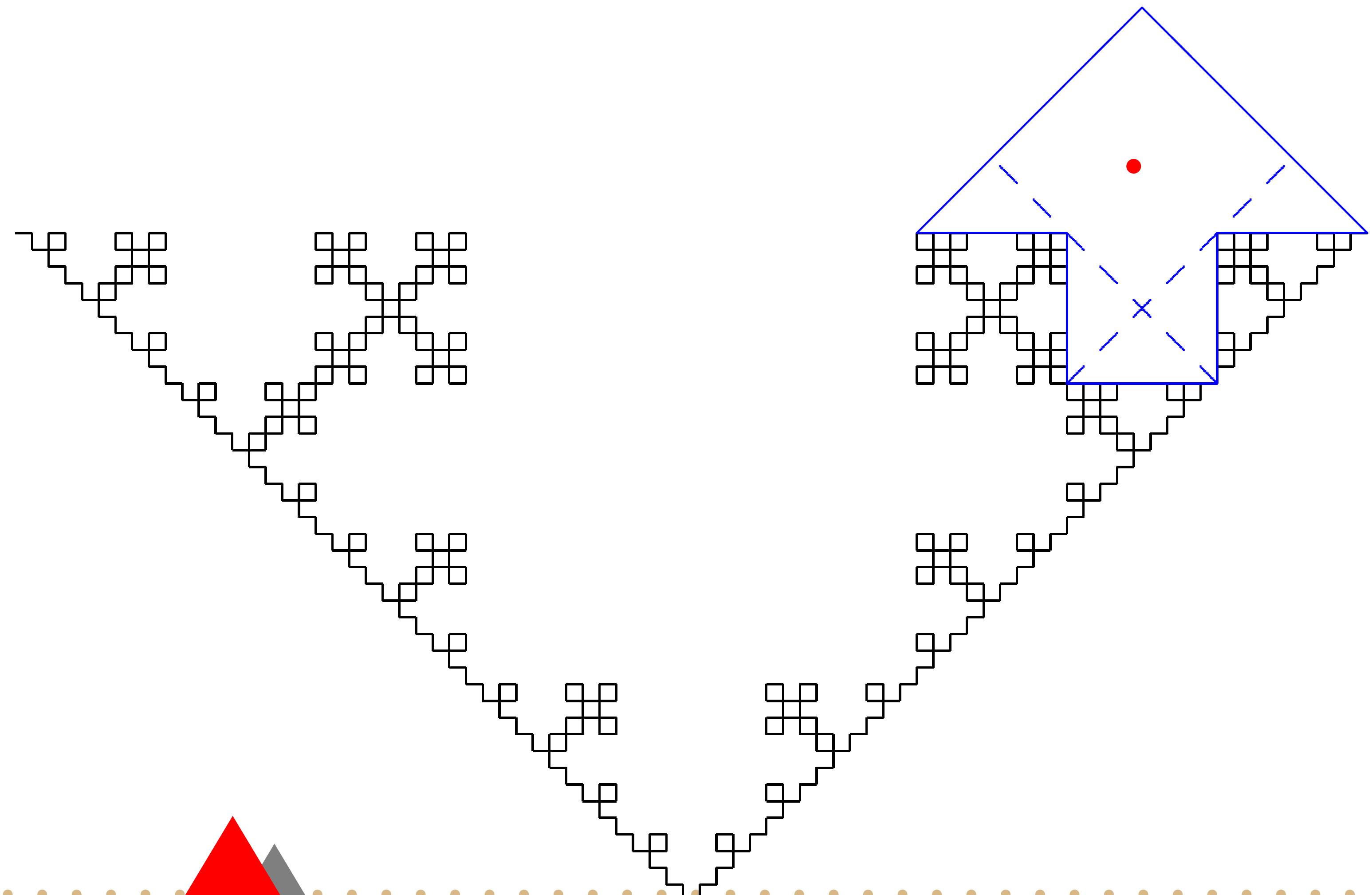

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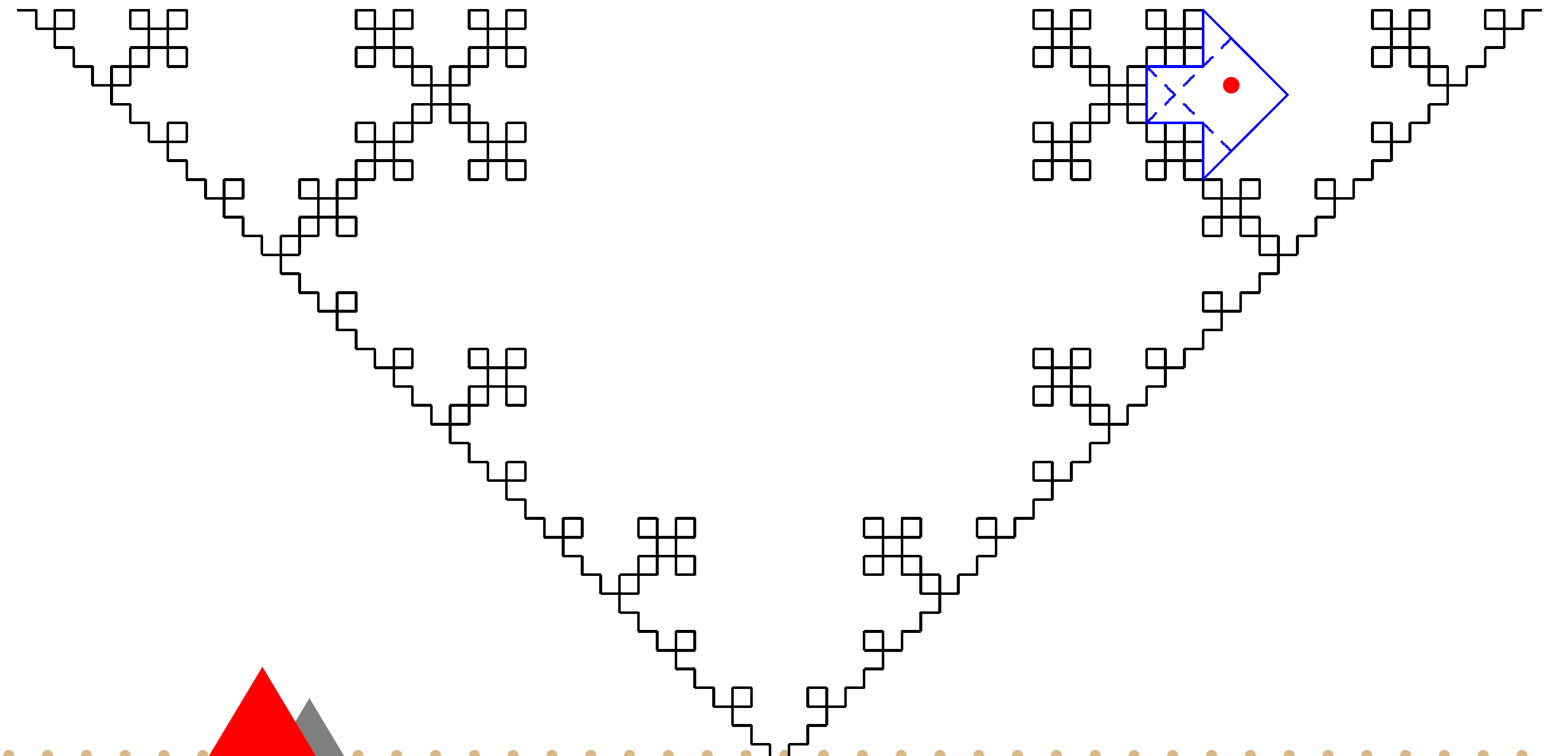

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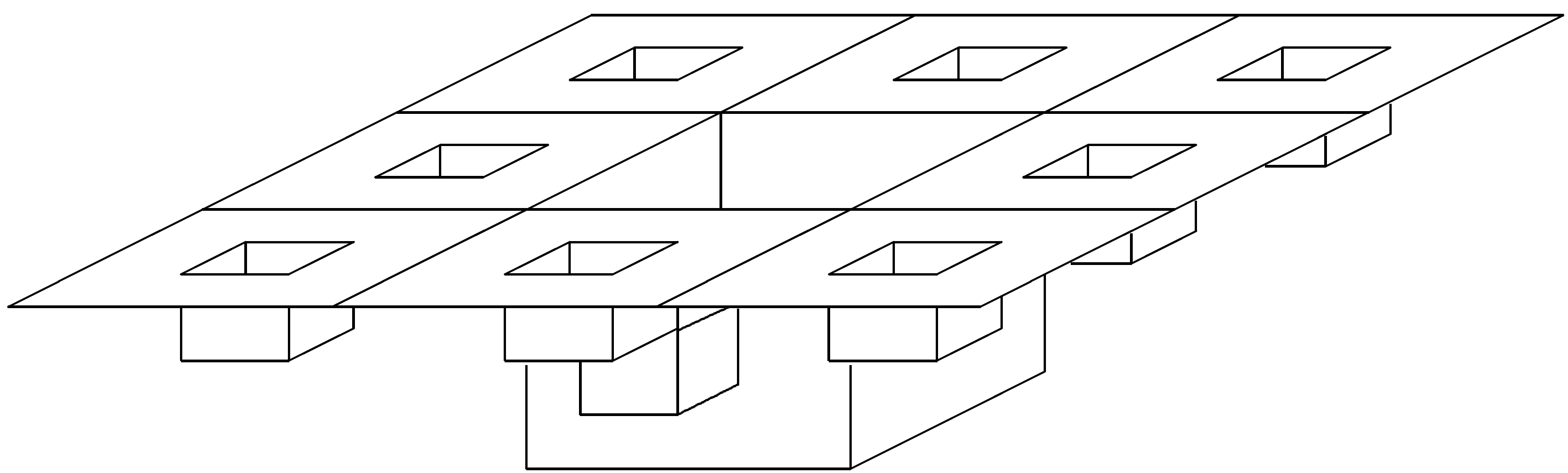
Realization in 3D



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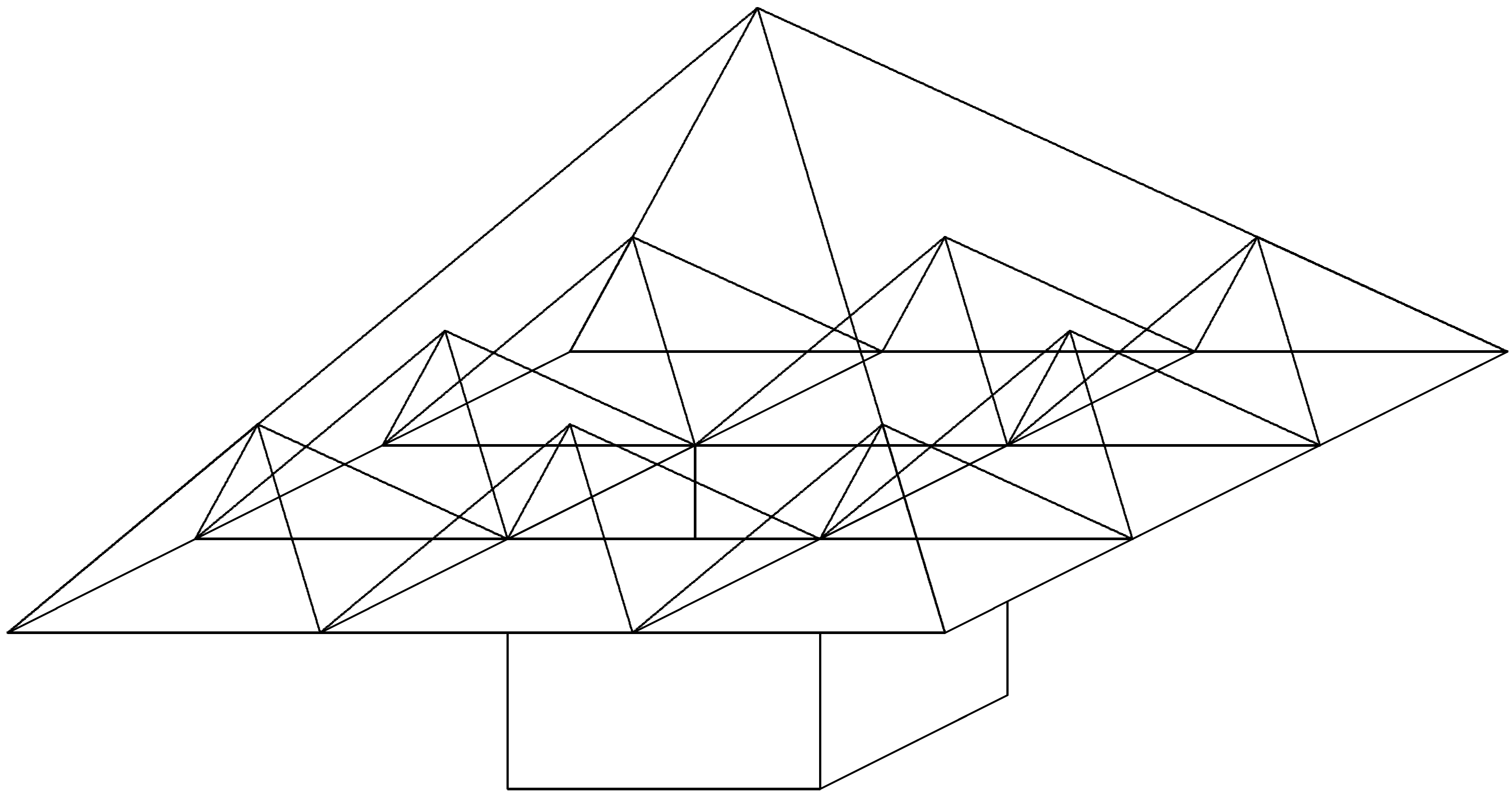

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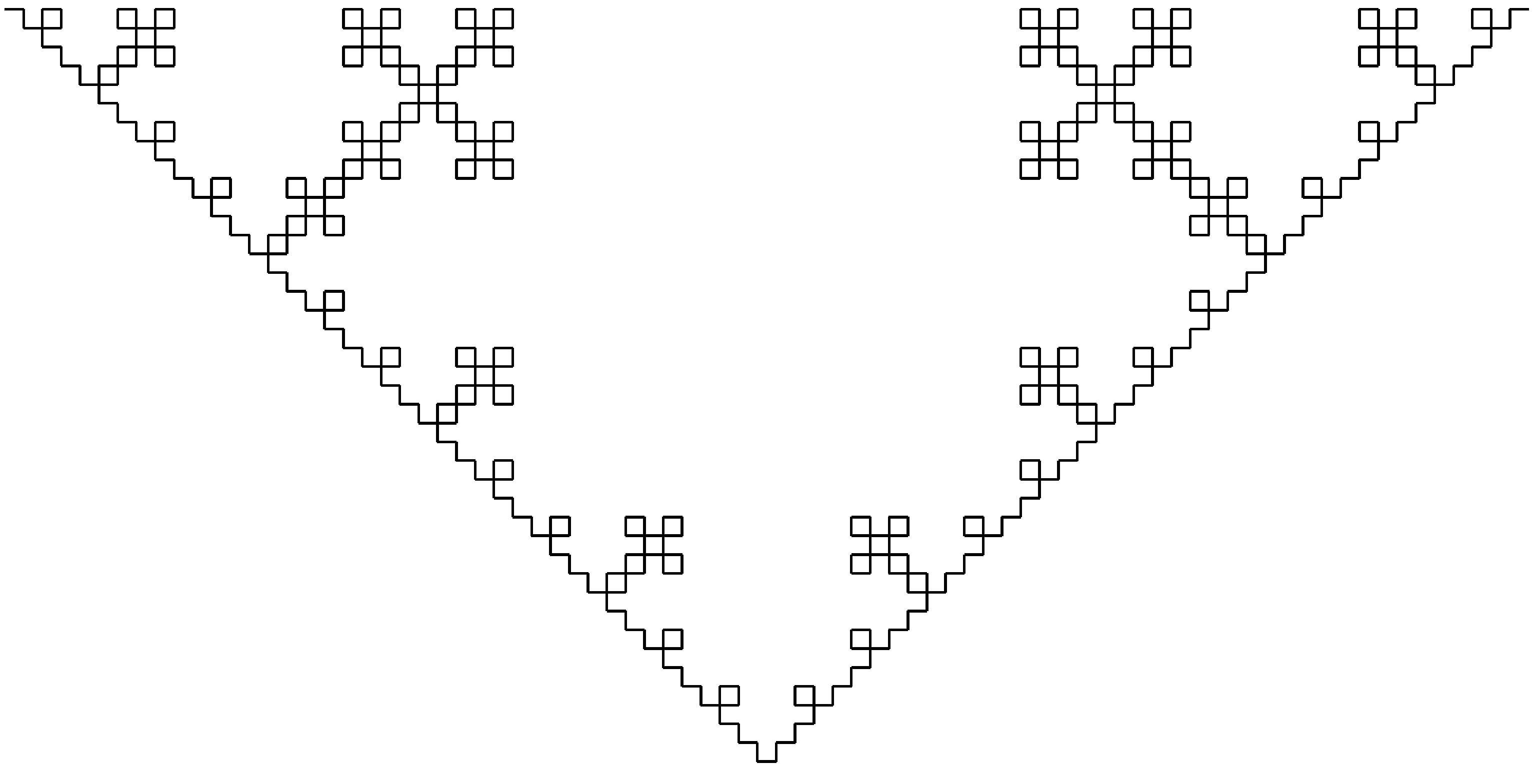
Multifractal analysis: Triangular cover



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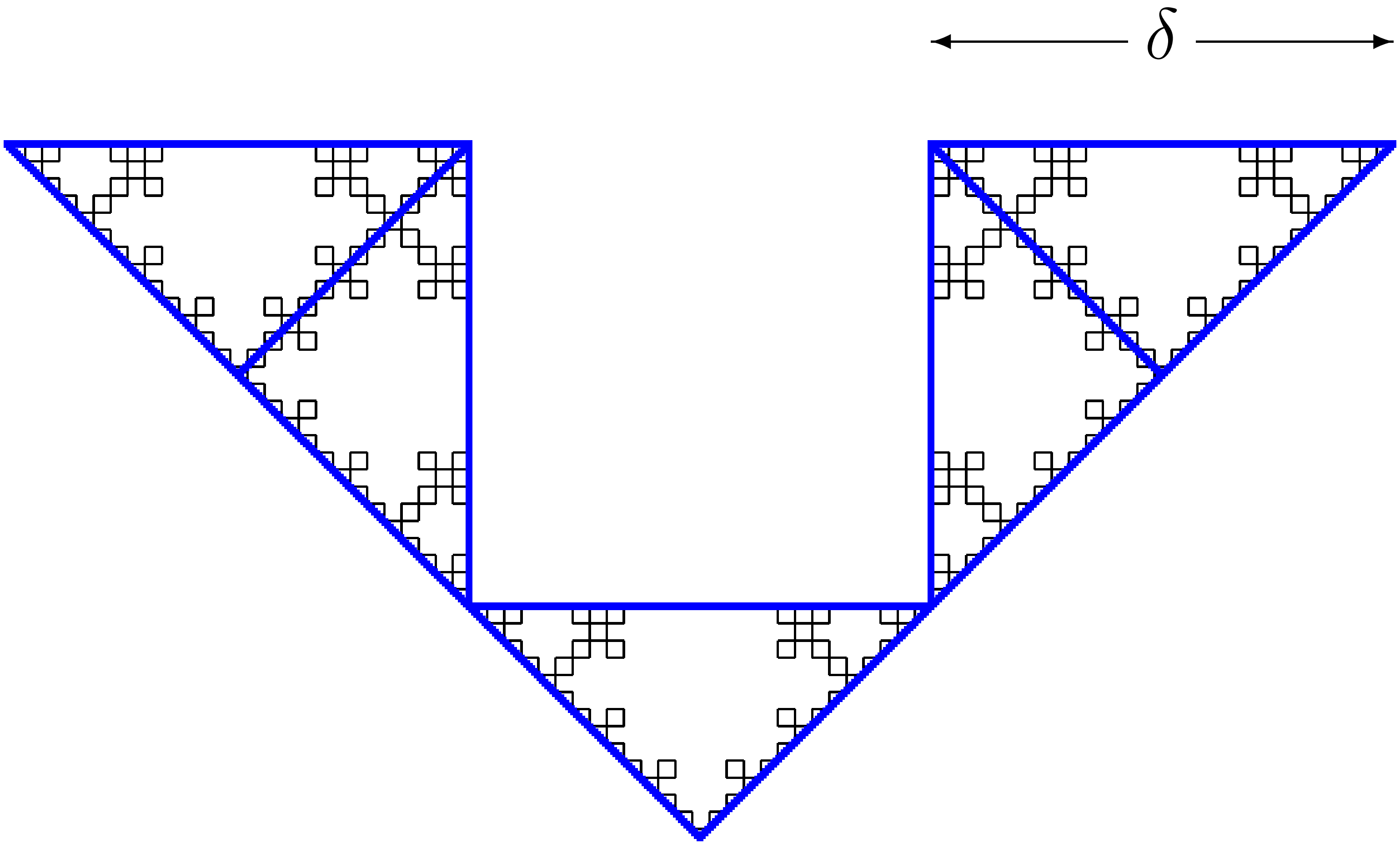

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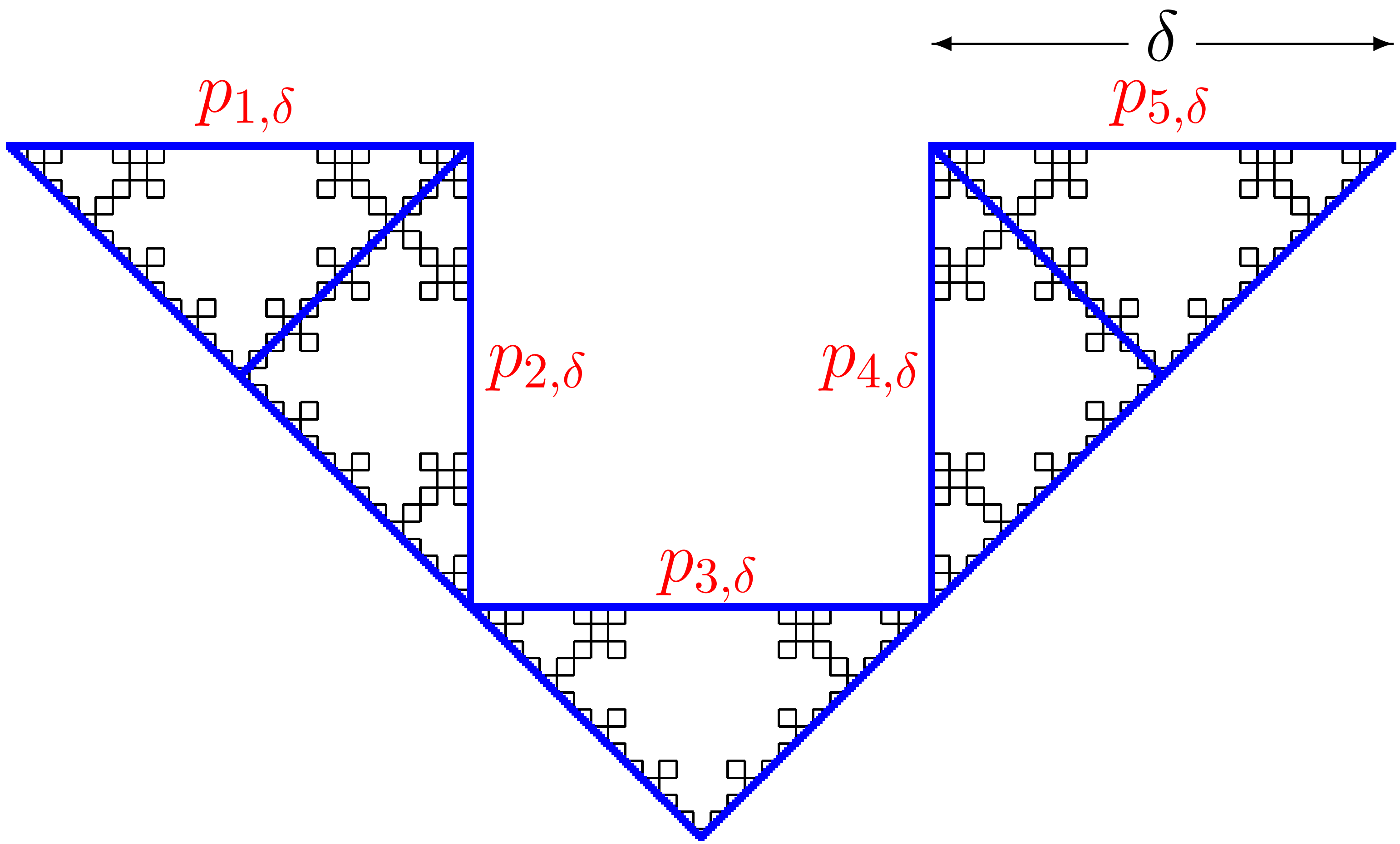

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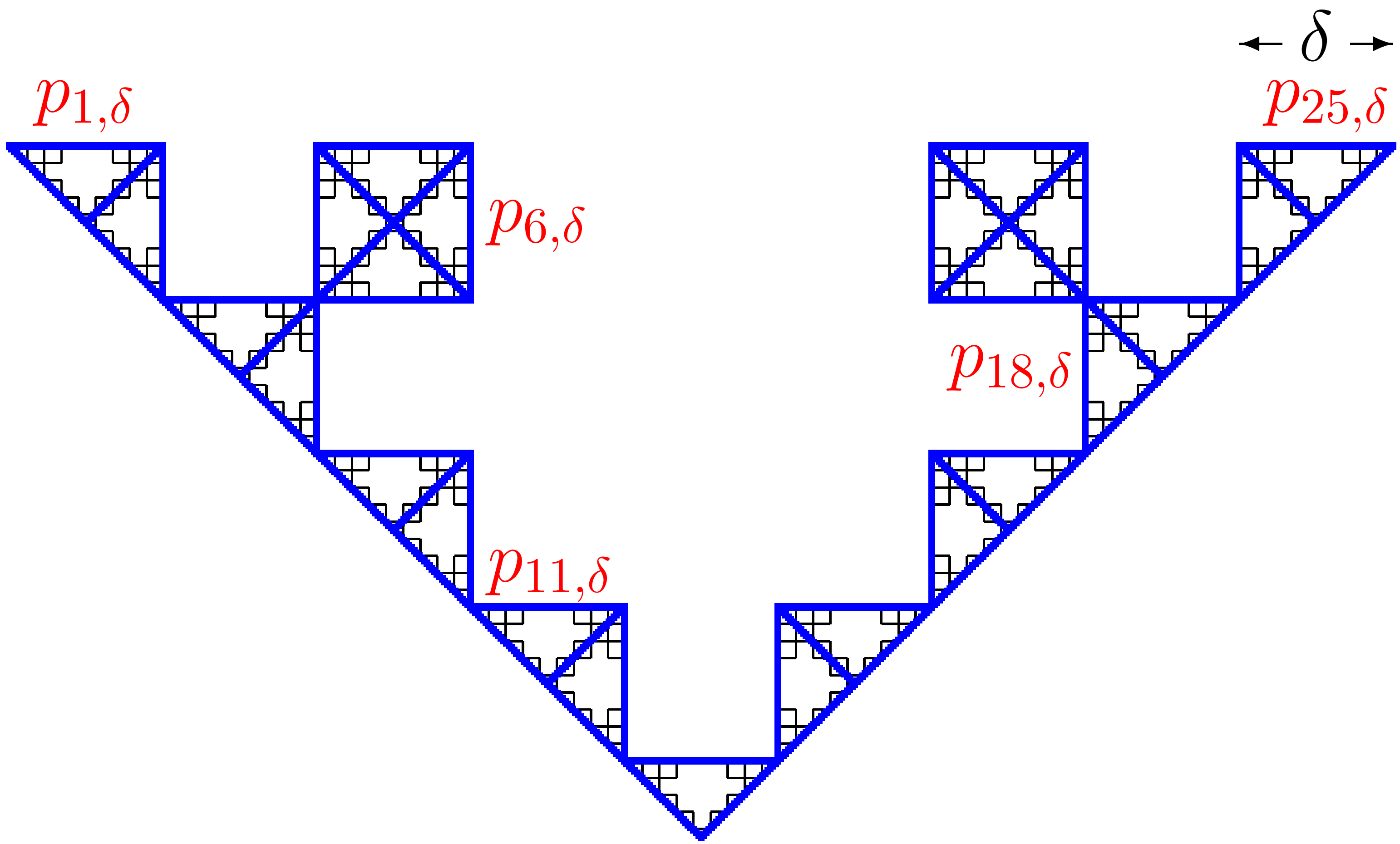

Multifractal dimensions

$$\zeta(q,\delta) = \sum_k (p_{k,\delta})^q$$

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$$D_{q,\delta} = \frac{\ln[\zeta(q,\delta)]^{\frac{1}{q-1}}}{\ln \delta}$$

$$D_q = \lim_{\delta \to 0} D_{q,\delta}$$

Logarithmic development: a smooth boundary

$$p_{k,\delta} \simeq \delta^{d-1}\,\omega(s_k)$$

$\omega(s)$ is the harmonic measure density

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Logarithmic development: an irregular boundary ?

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Logarithmic development: application to fractals

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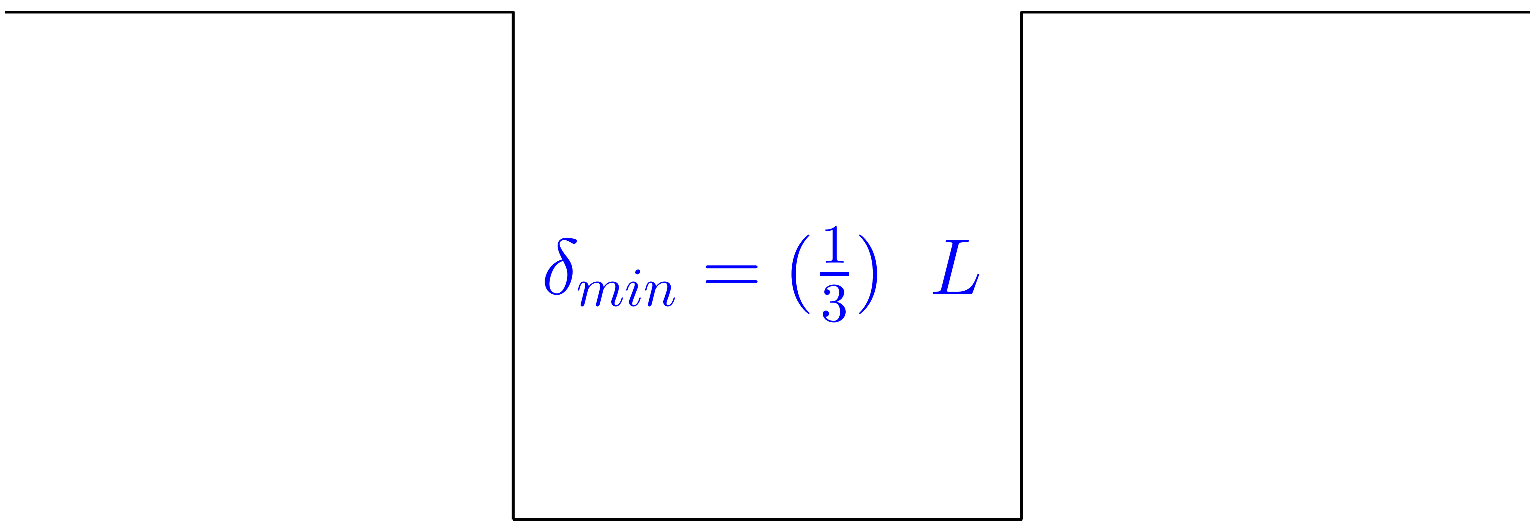

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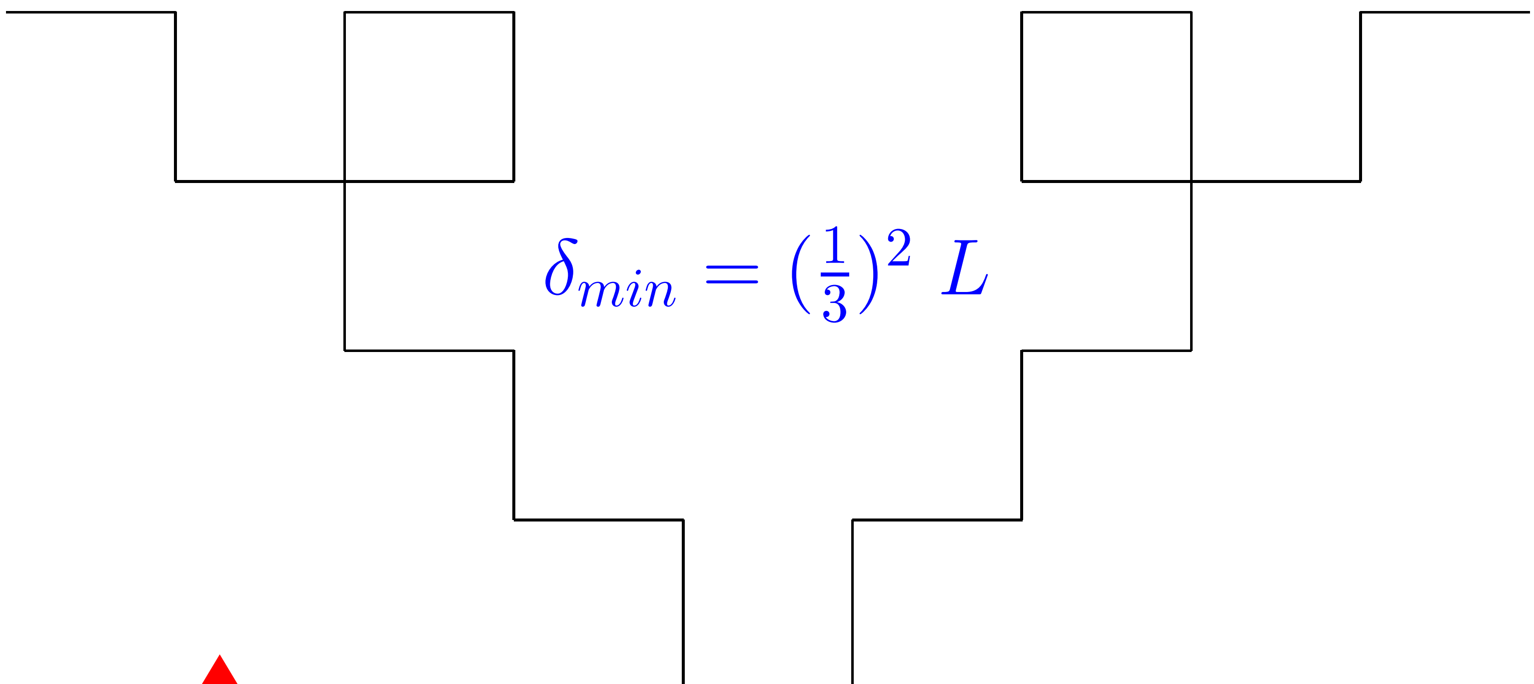

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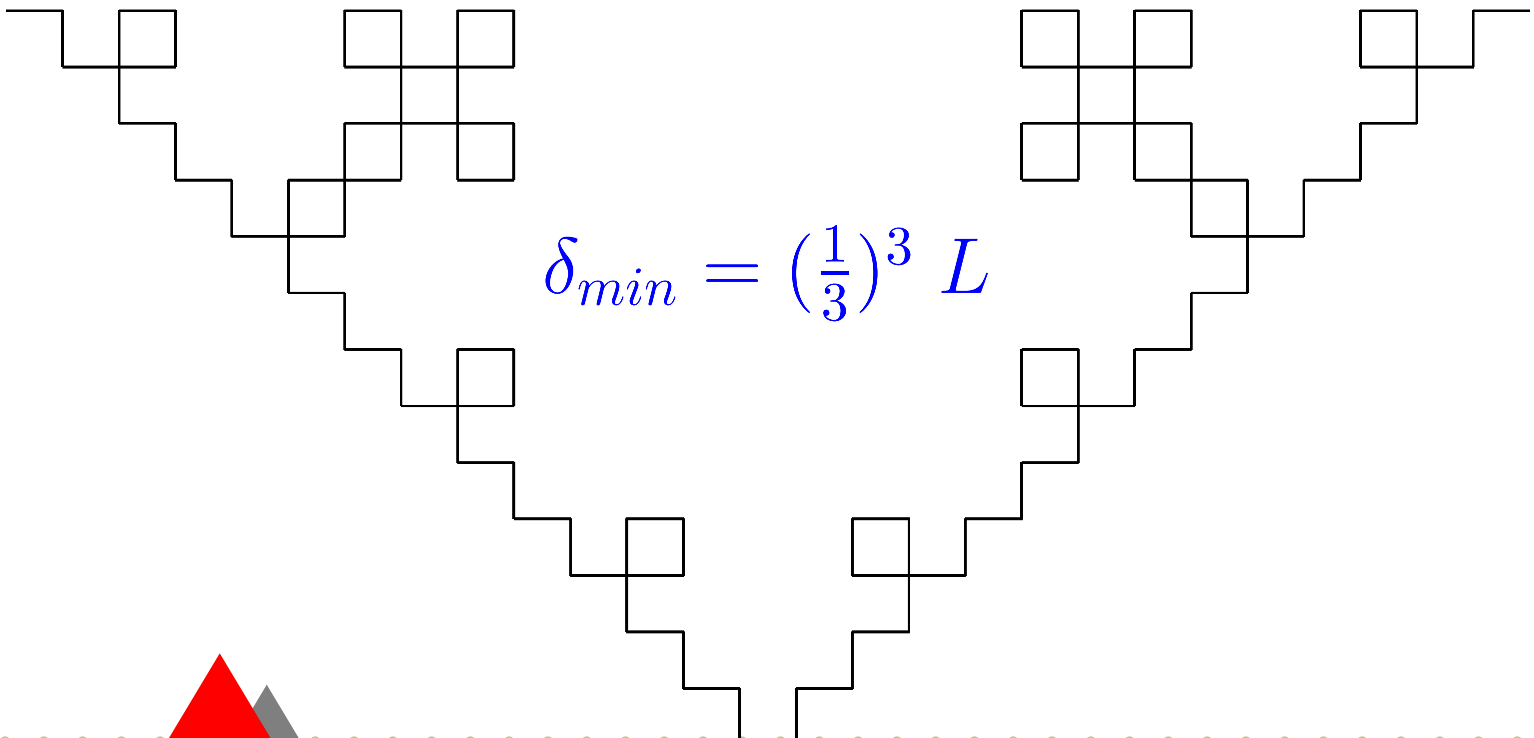

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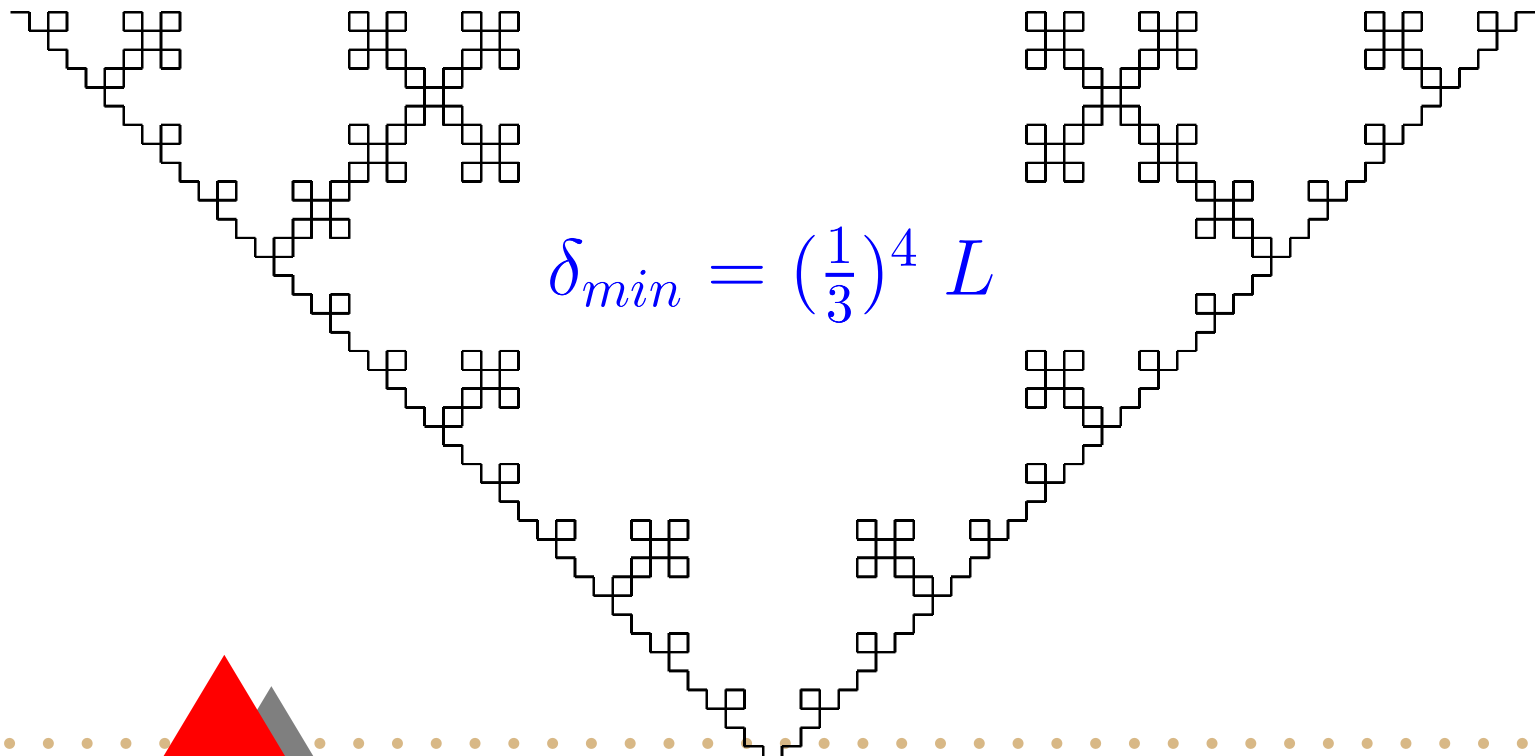

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Quadratic Von Koch curve

	$g = 5$
$D_{1,\delta}$	1.0345
$D_{2,\delta}$	0.9263

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Quadratic Von Koch curve

	$g = 5$	$g = 6$	$g = 7$	$g = 8$	$g = 9$	$g = 10$
$D_{1,\delta}$	1.0345	1.0287	1.0246	1.0215	1.0191	1.0172
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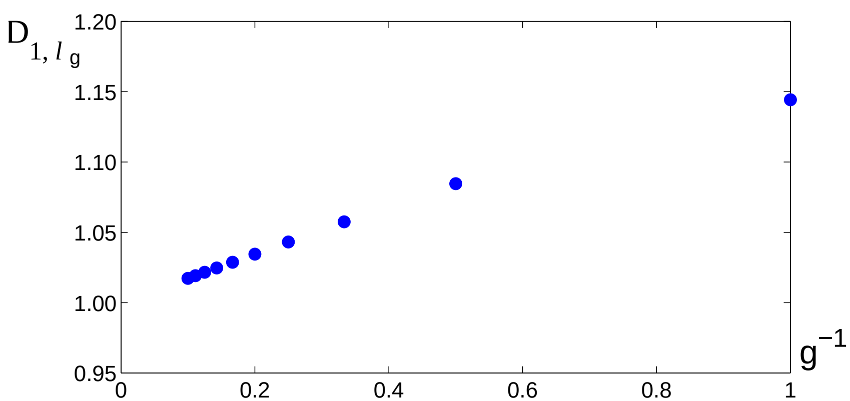

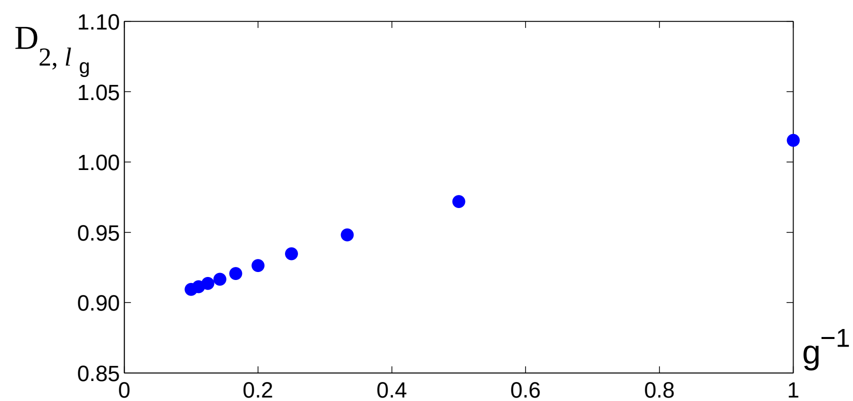

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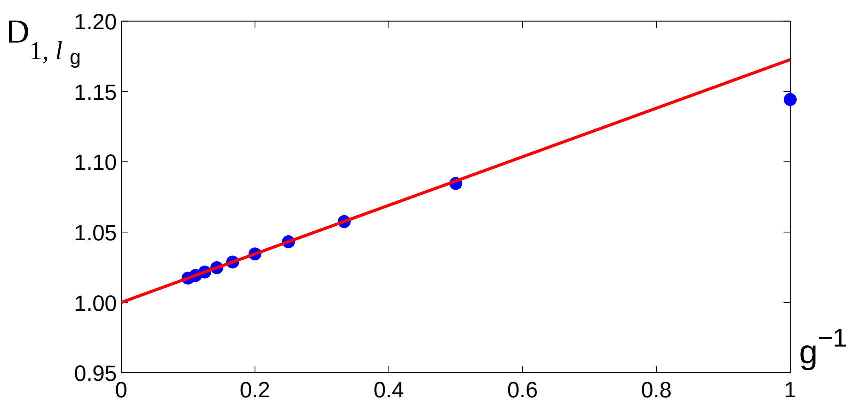

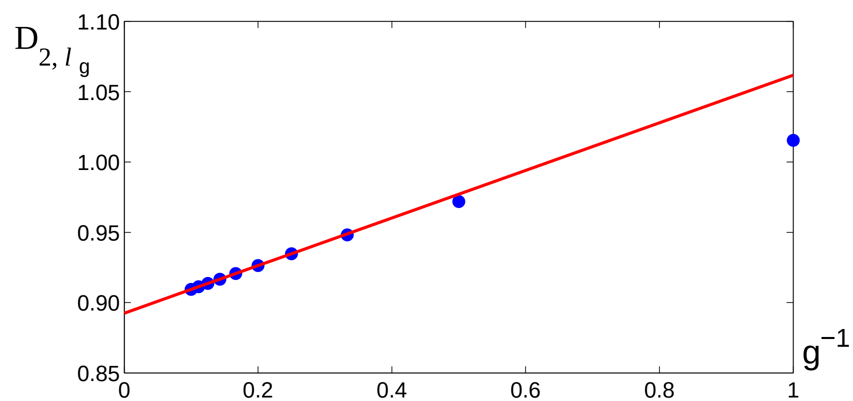

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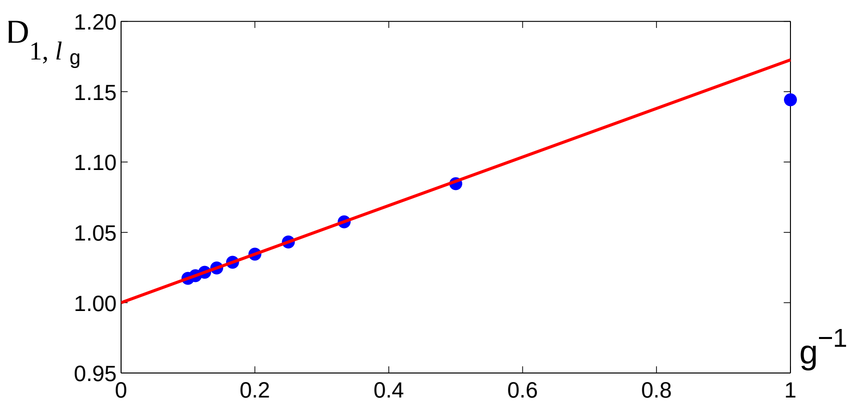

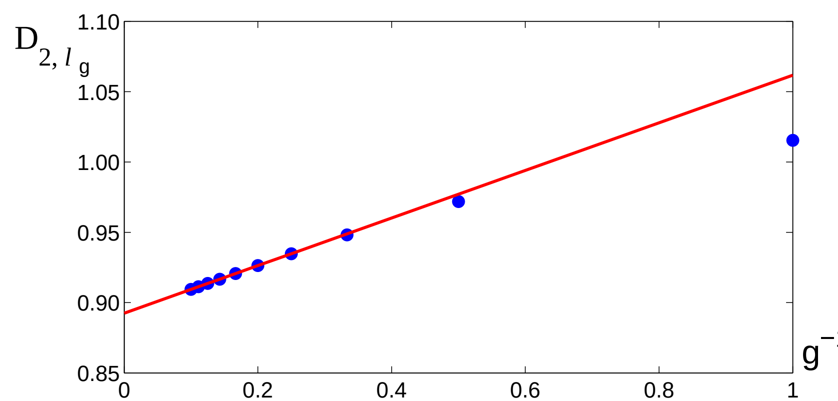

$D_1 = 1.0000 \pm 0.0003$ $\qquad$ $D_2 = 0.8925 \pm 0.0007$

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Cubic Von Koch surface

	$g=1$	$g=2$	$g=3$	$g=4$	$g=5$	$g=6$
$D_{1,\delta}$	2.0815	2.0569	2.0426	2.0341	2.0286	2.0249
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Grebenkov, Lebedev, Filoche, Sapoval, Phys. Rev. E 71, 056121 (2005)

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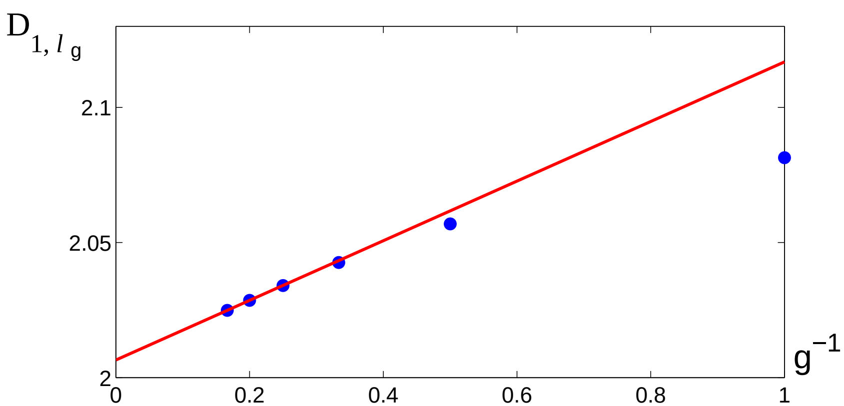

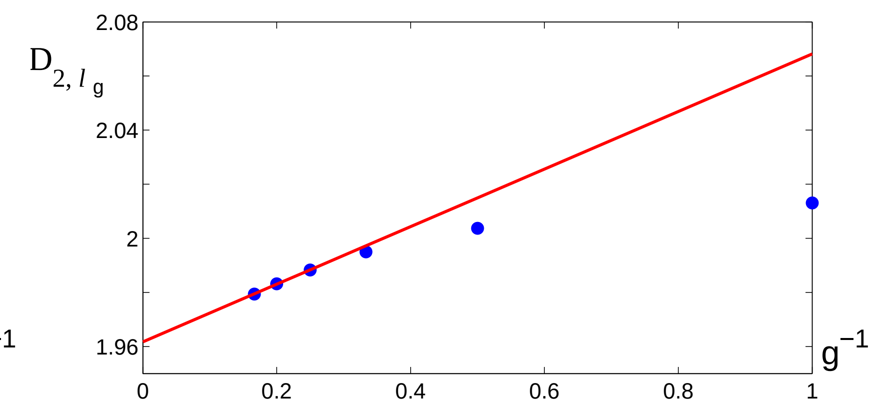

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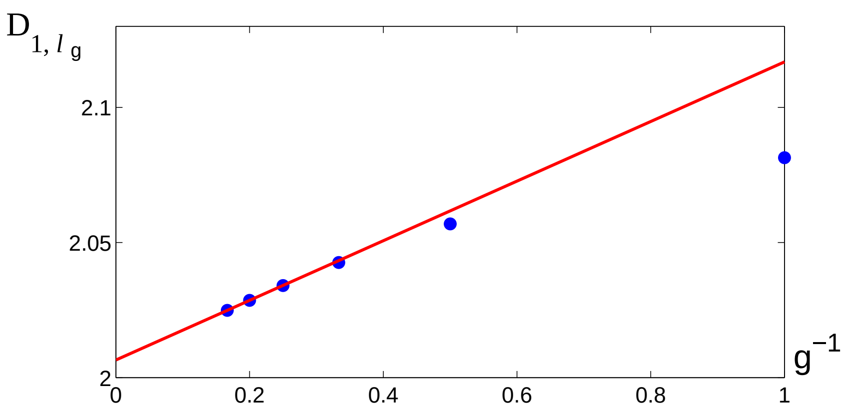
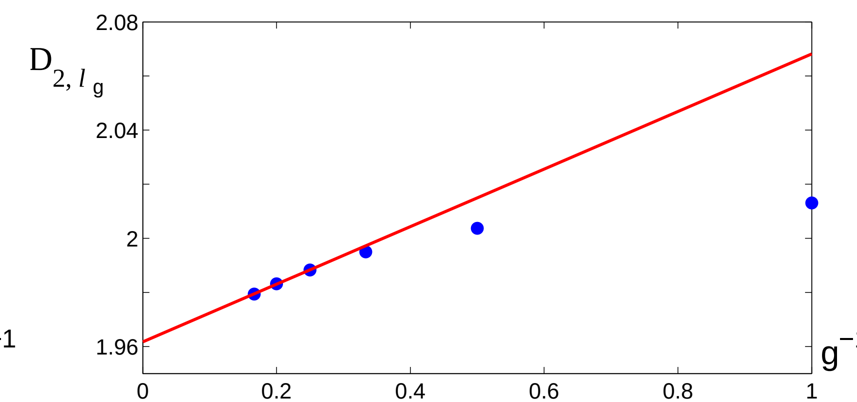


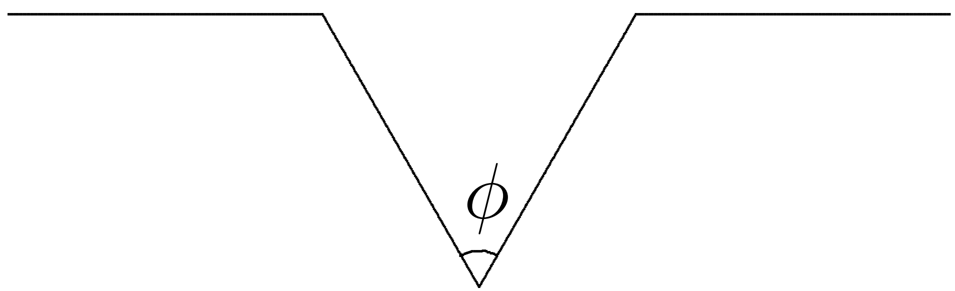


$D_1 = 2.007 \pm 0.002$ $D_2 = 1.963 \pm 0.006$

Grebenkov, Lebedev, Filoche, Sapoval, Phys. Rev. E 71, 056121 (2005)

Von Koch curves of variable dimension

$$D_0 = \frac{\ln 4}{\ln 2(1 + \sin \phi/2)}$$



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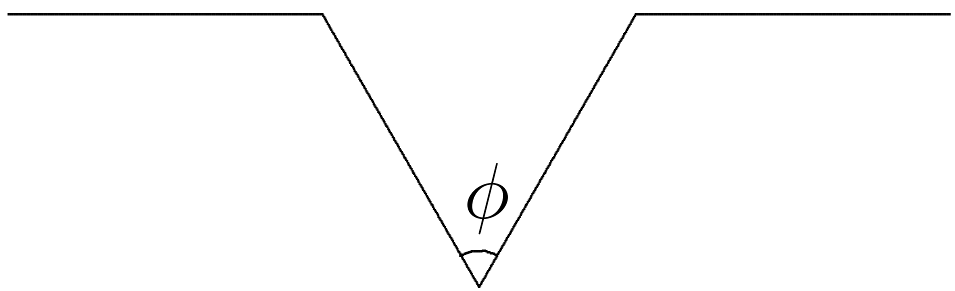
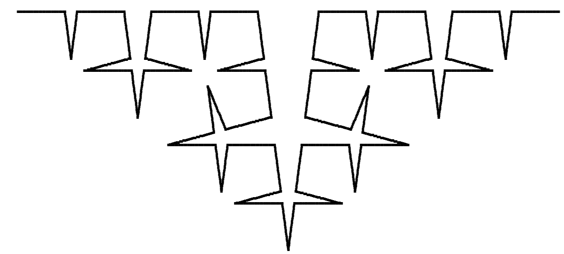
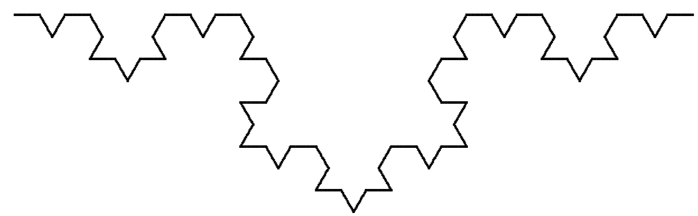


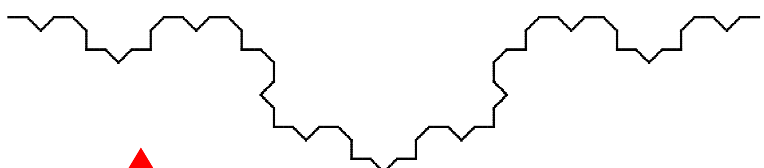
$D_0 = 1.699$



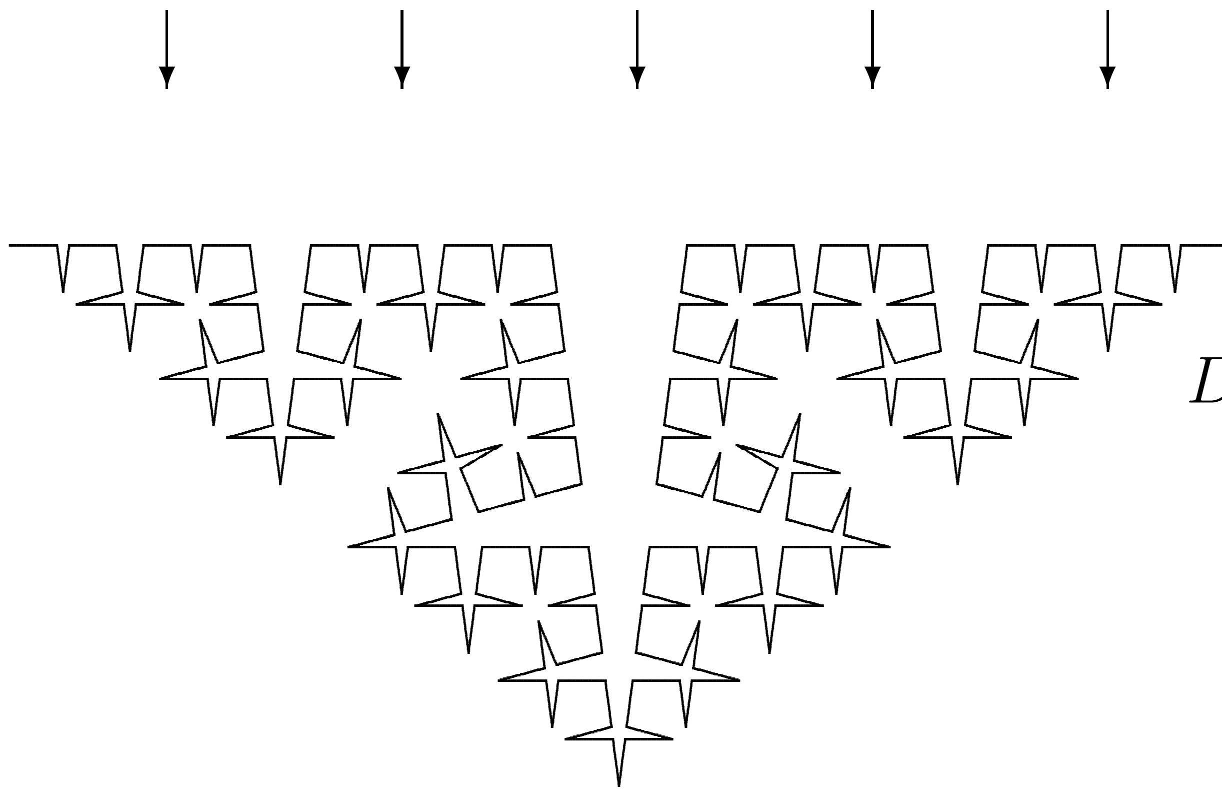
$D_0 = 1.262$



$D_0 = 1.129$



Two families of curves



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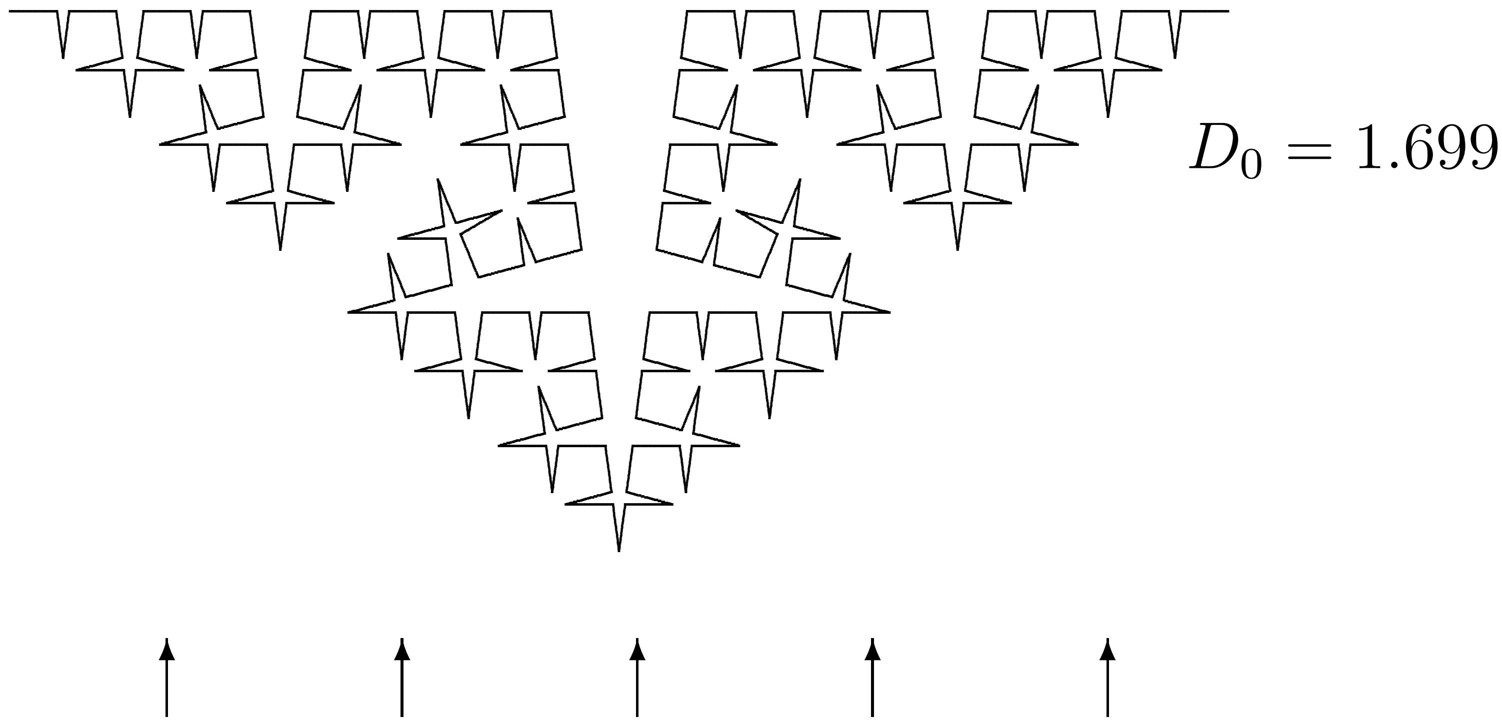
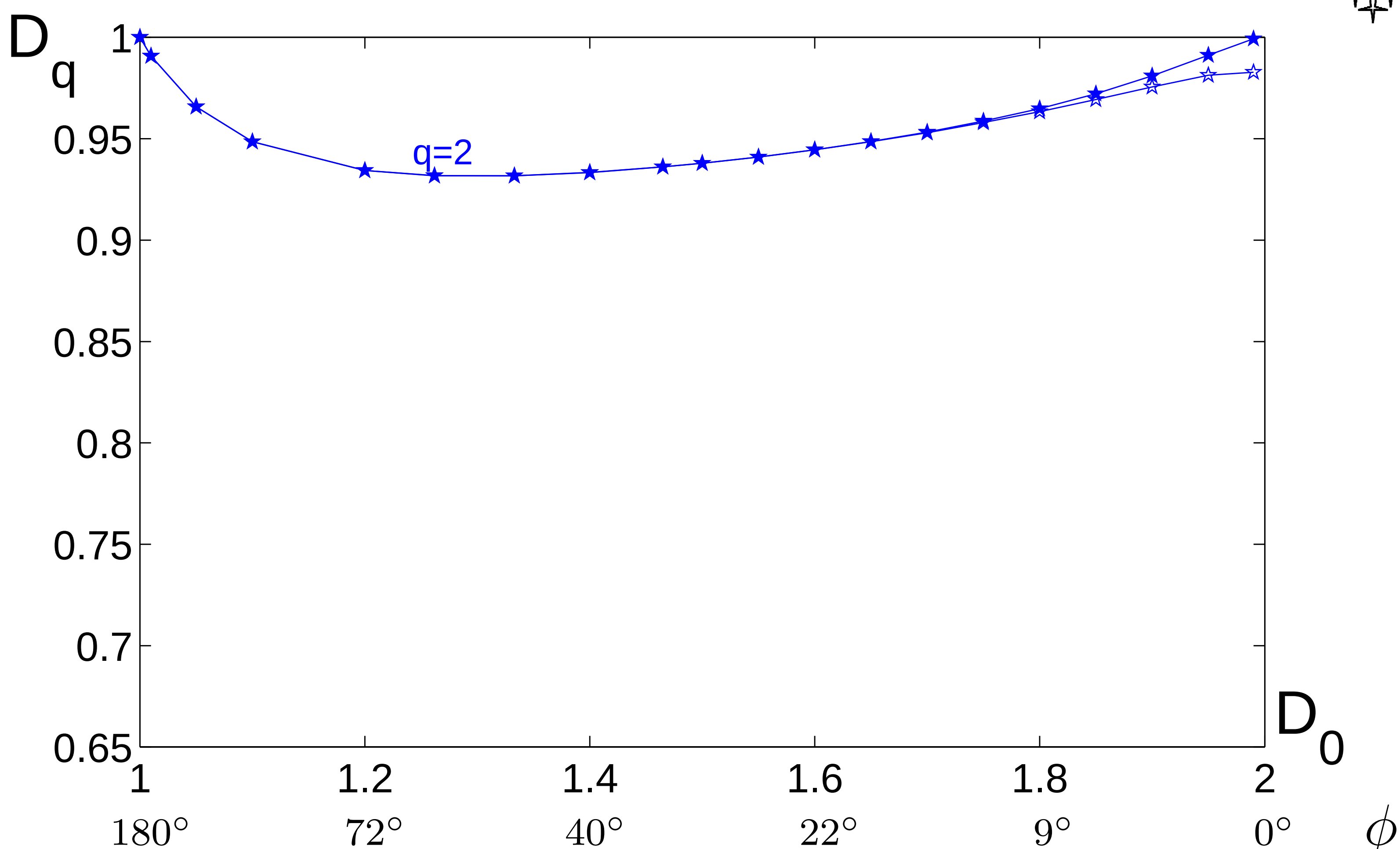
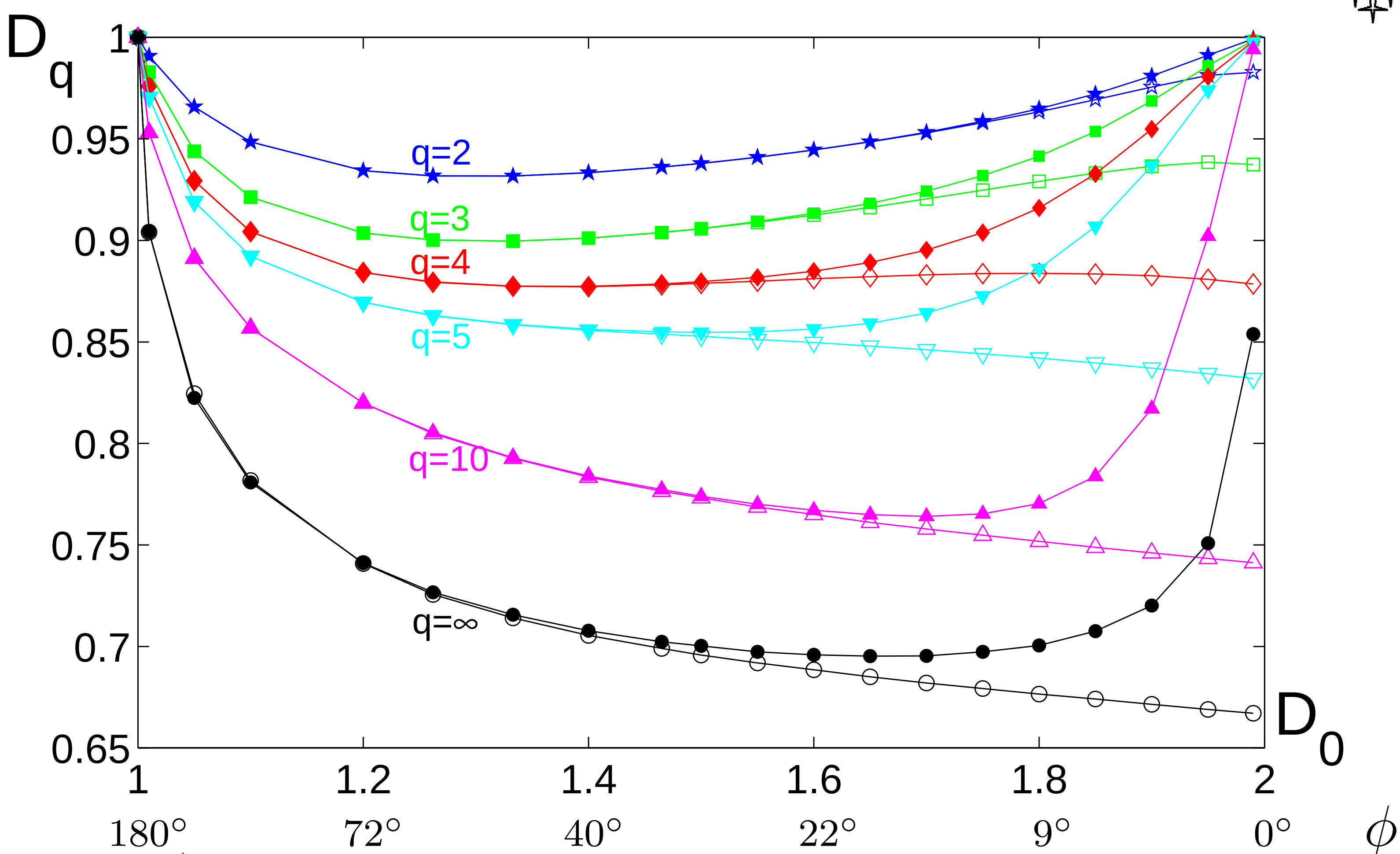


Multifractal dimensions



Grebenkov, Phys. Rev. Lett. 95, 200602 (2005)

Multifractal dimensions



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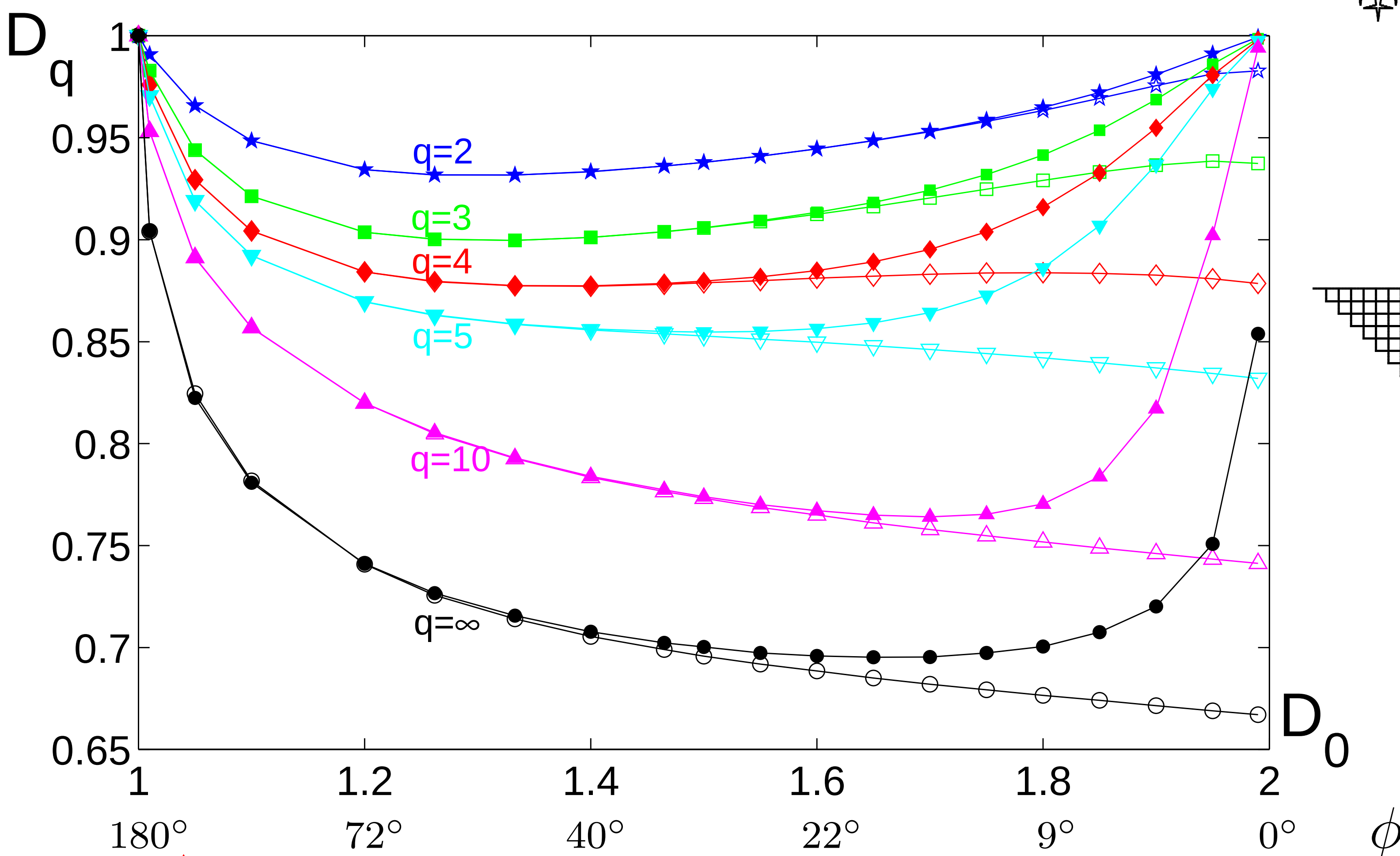

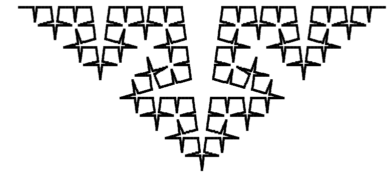

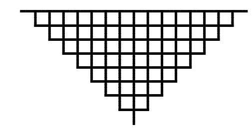

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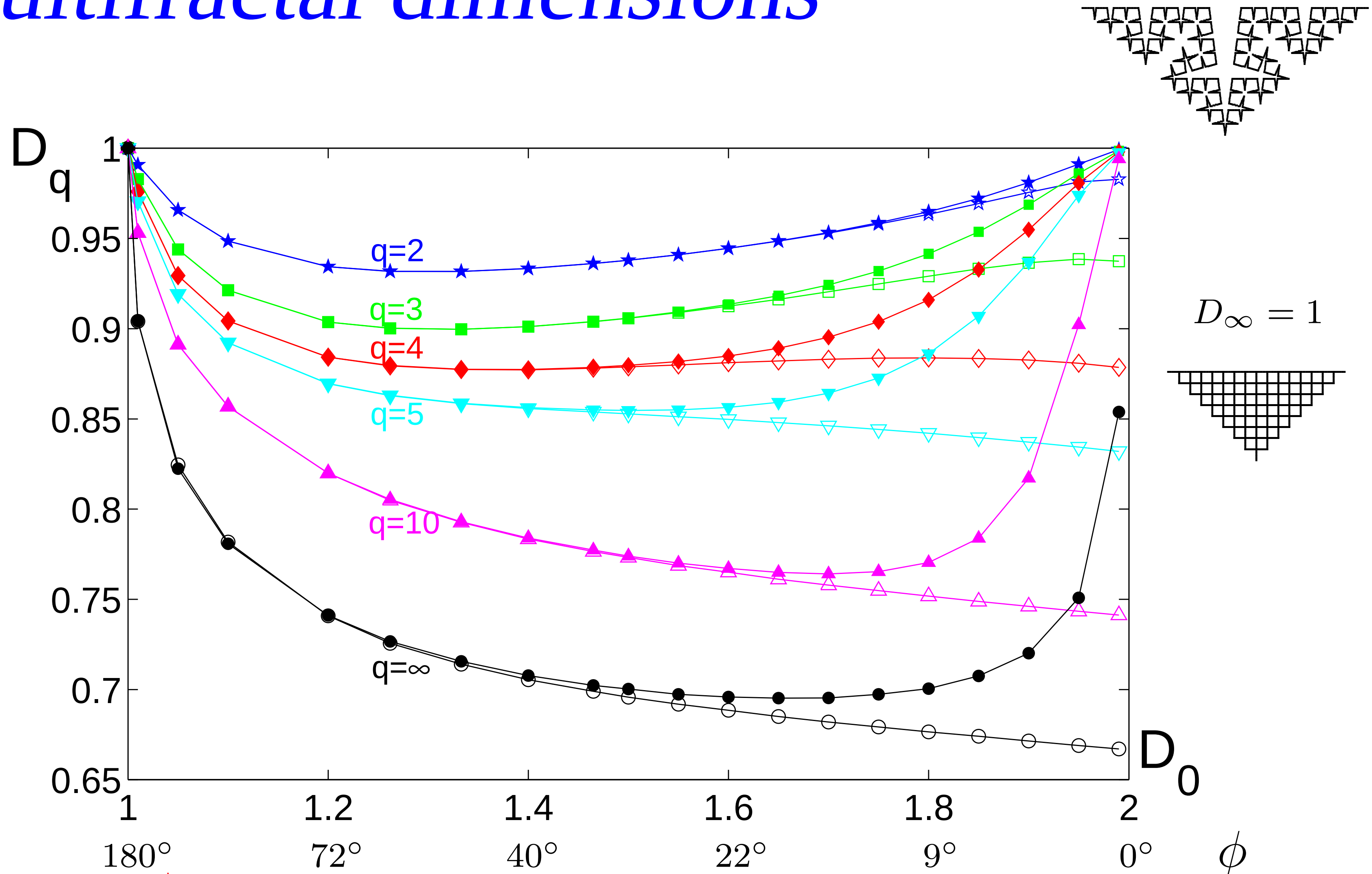

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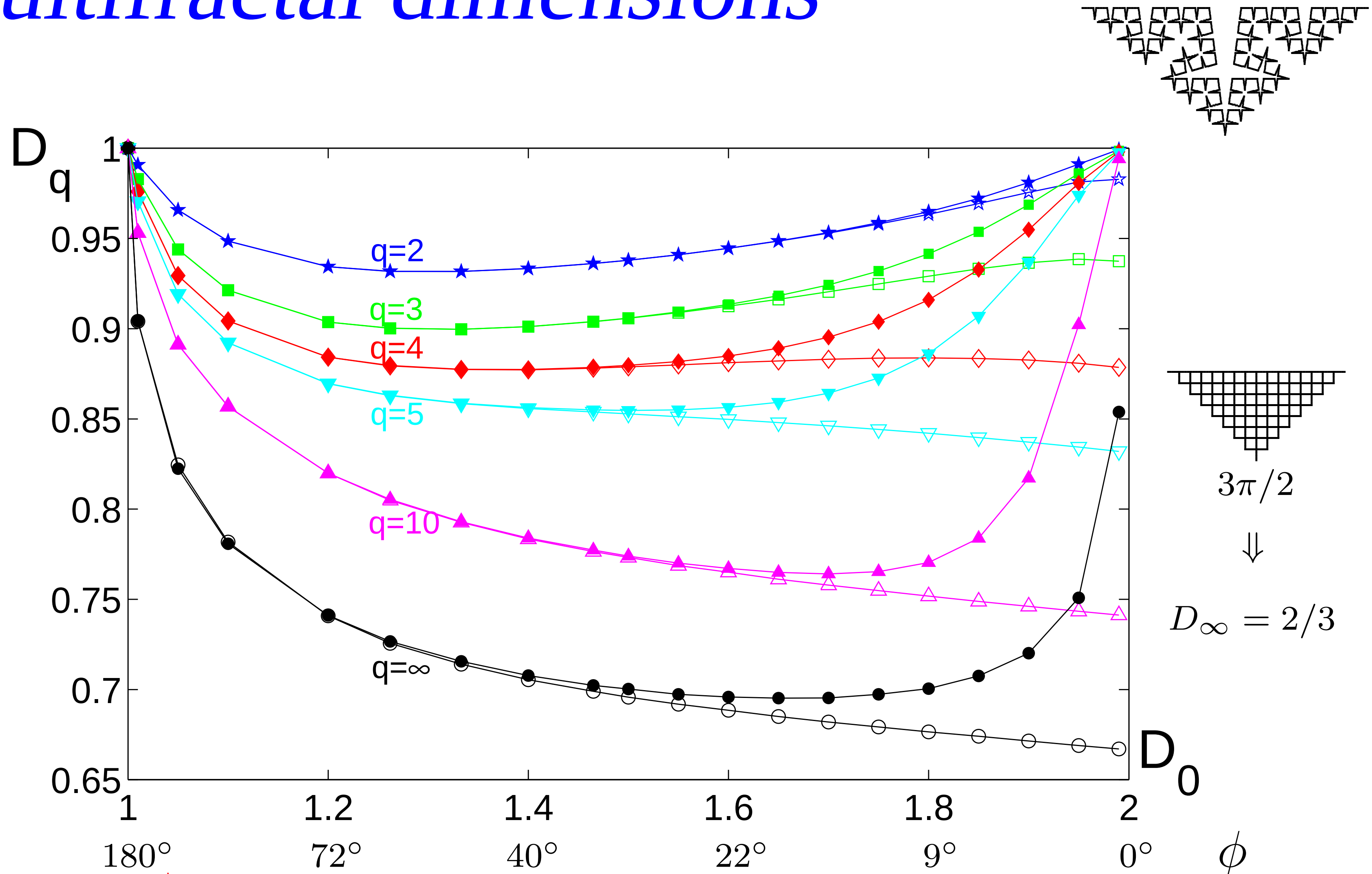

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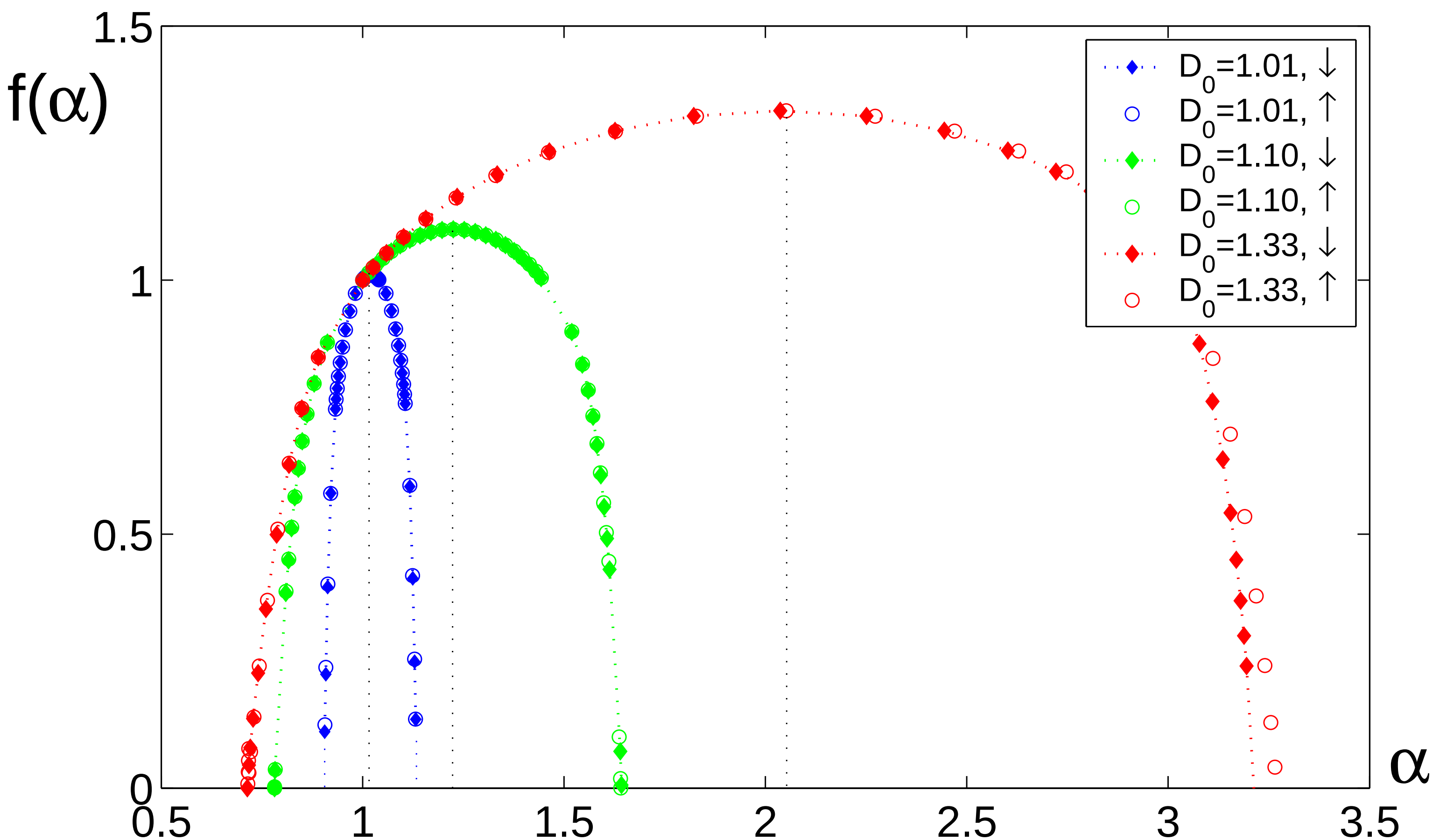
Multifractal spectrum



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Outline

- Geometry-adapted fast random walks

- Multifractal properties of the harmonic measure on Von Koch boundaries

  Grebenkov, Lebedev, Filoche, Sapoval, Phys. Rev. E 71, 056121 (2005)

  Grebenkov, Phys. Rev. Lett. 95, 200602 (2005).

- Passivation of 2D and 3D fractals

  Filoche, Grebenkov, Andrade, Sapoval (submitted)

  Sapoval, Costa, Andrade, Filoche, Fractals 12, 381 (2004).

Heterogeneous catalysis

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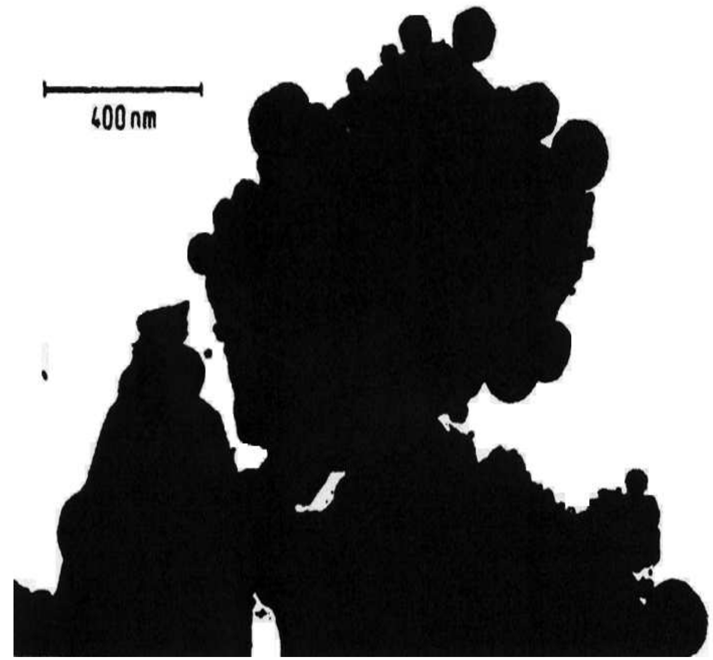
Catalytic reaction
on the boundary

$$A + C \longrightarrow A^* + C$$



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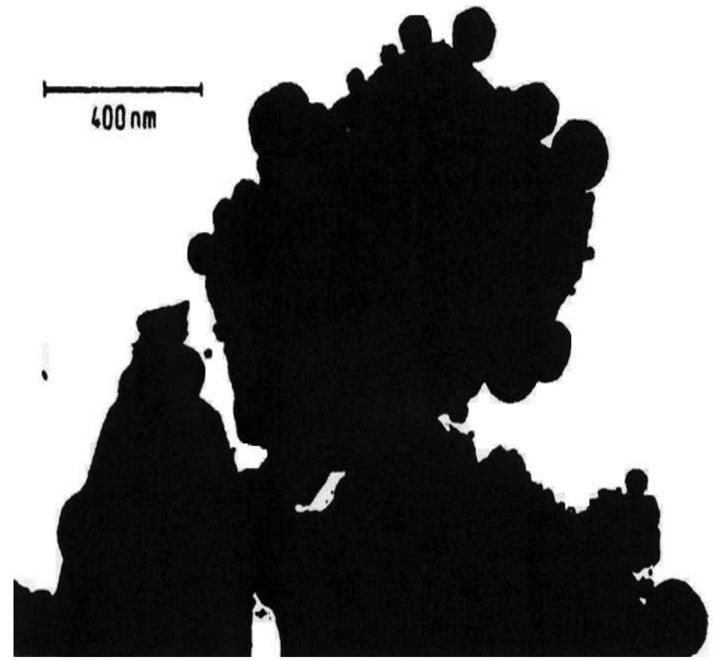
Larger surface

$\downarrow \quad \downarrow \quad \downarrow$

Higher outcome



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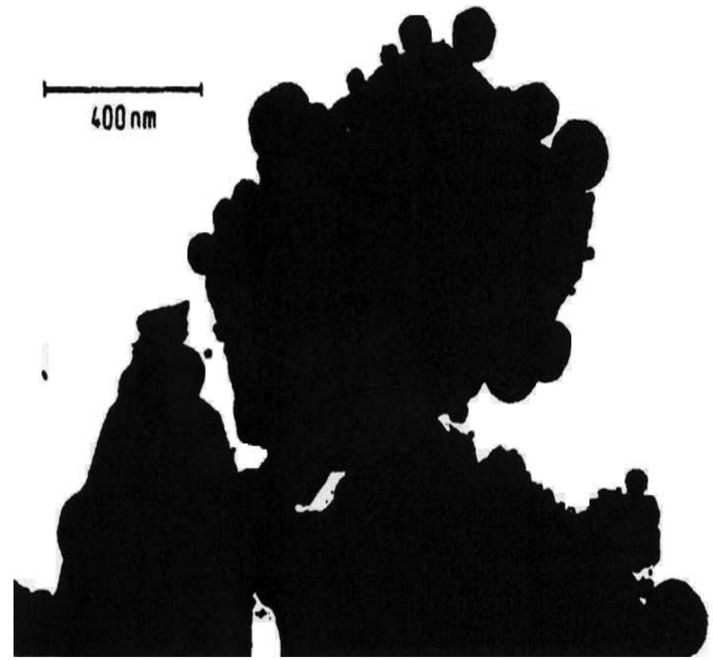

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Higher outcome

BUT

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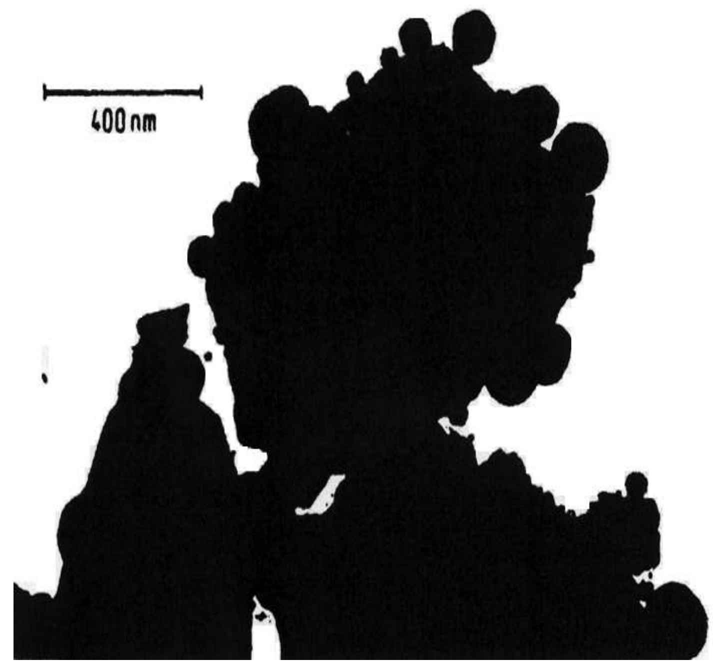


BUT
- Laplacian screening
- Passivation of the catalyst

Heterogeneous catalysis

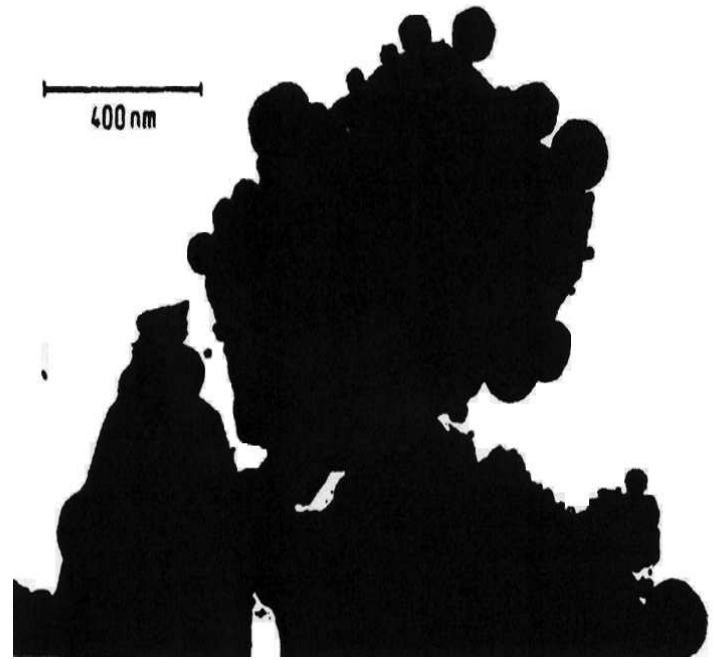
Catalytic reaction
on the boundary

$$A + C \longrightarrow A^* + C$$
$$A + C \longrightarrow B$$

Larger surface

$\downarrow \quad \downarrow \quad \downarrow$

Higher outcome



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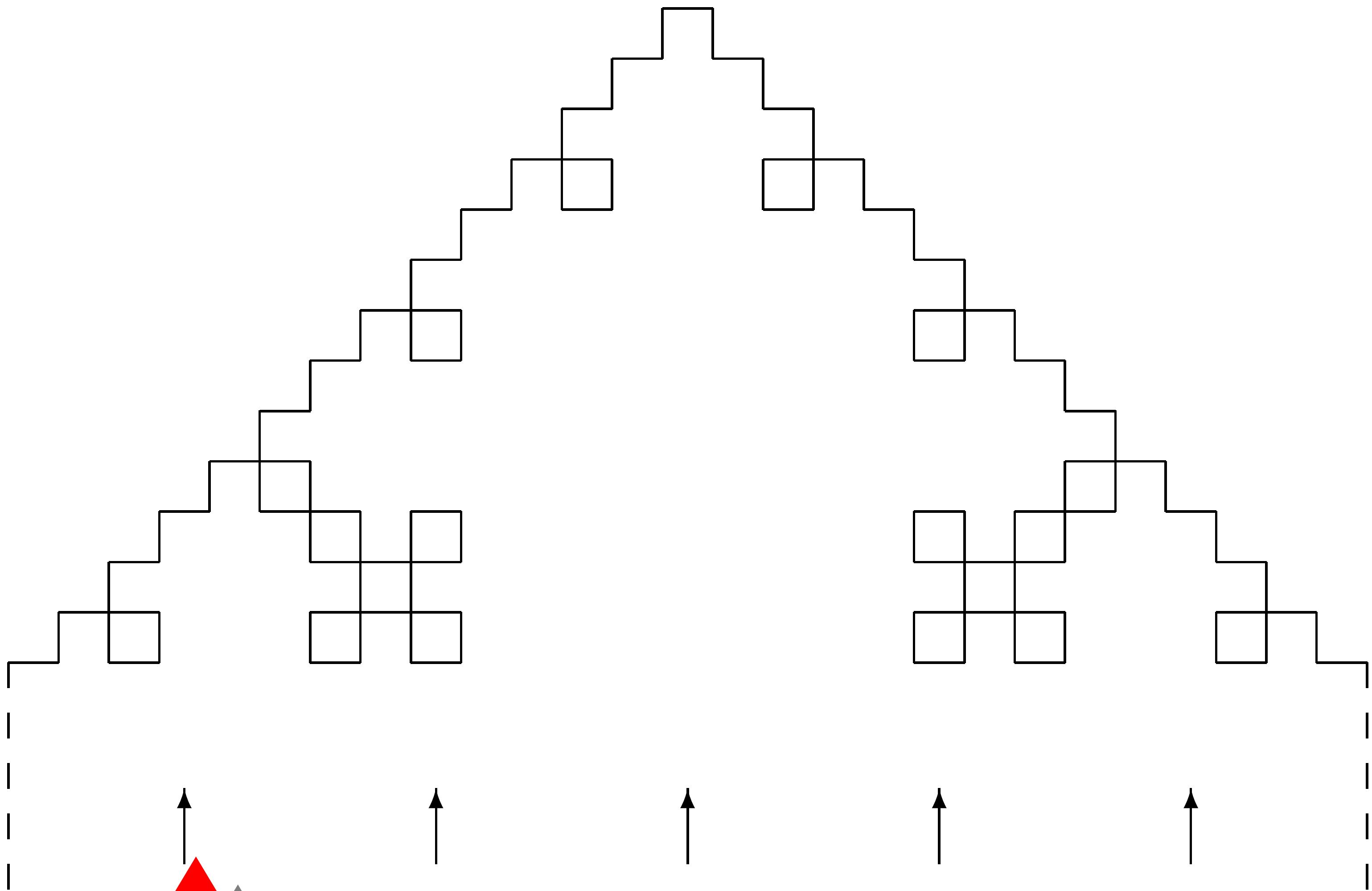
Passivation process



Sapoval, Costa, Andrade, Filoche, Fractals 12, 381 (2004).

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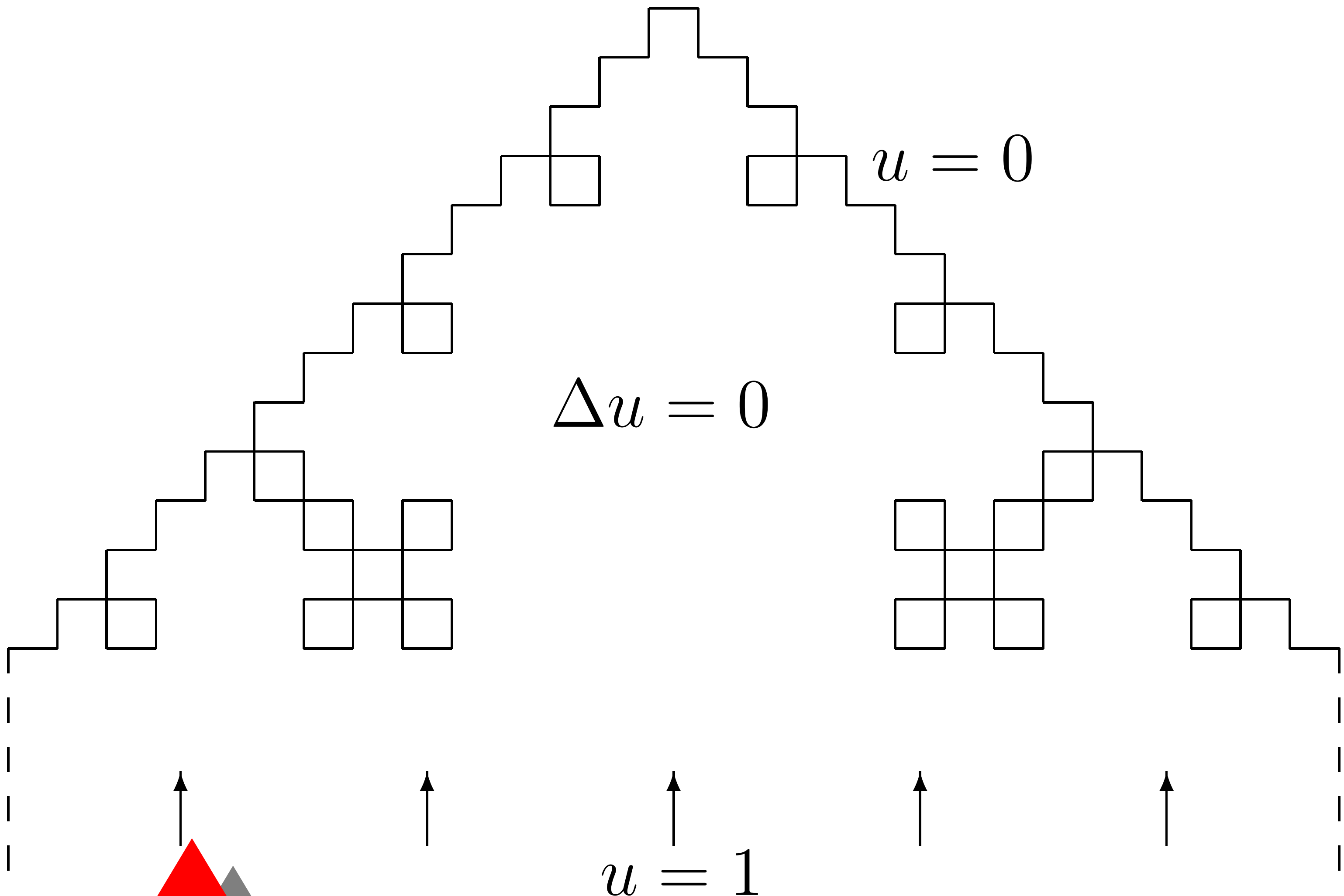

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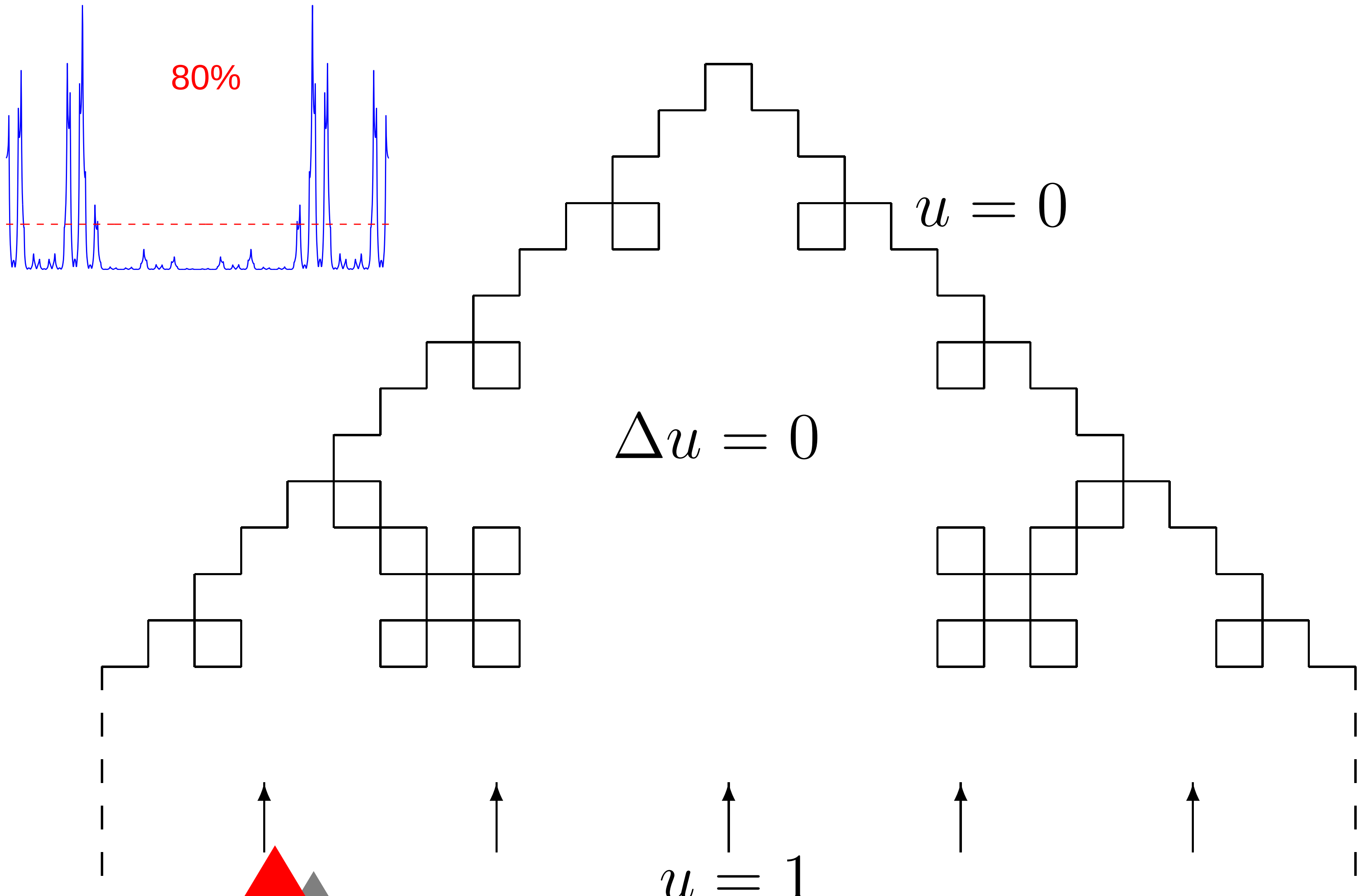

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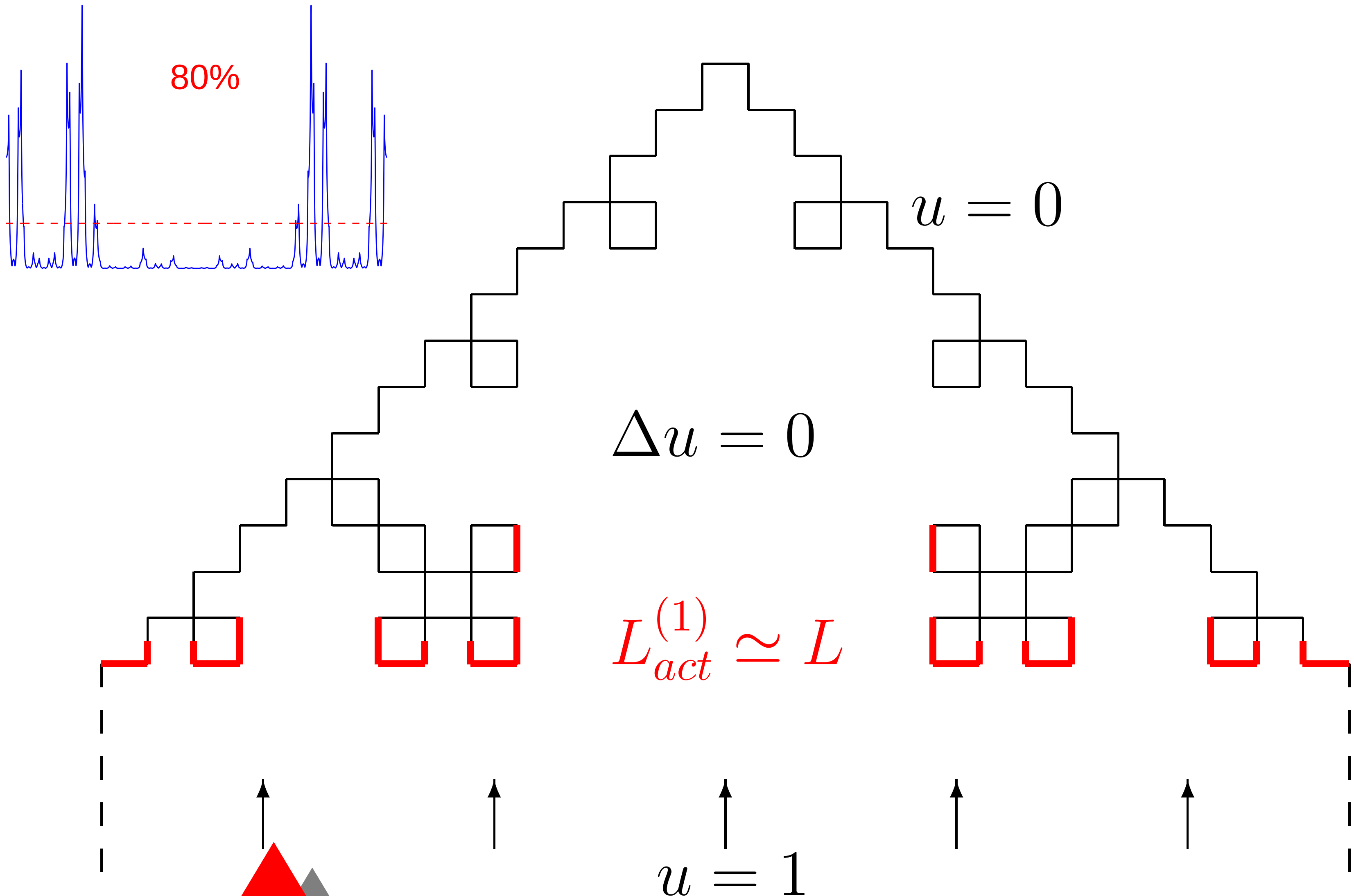

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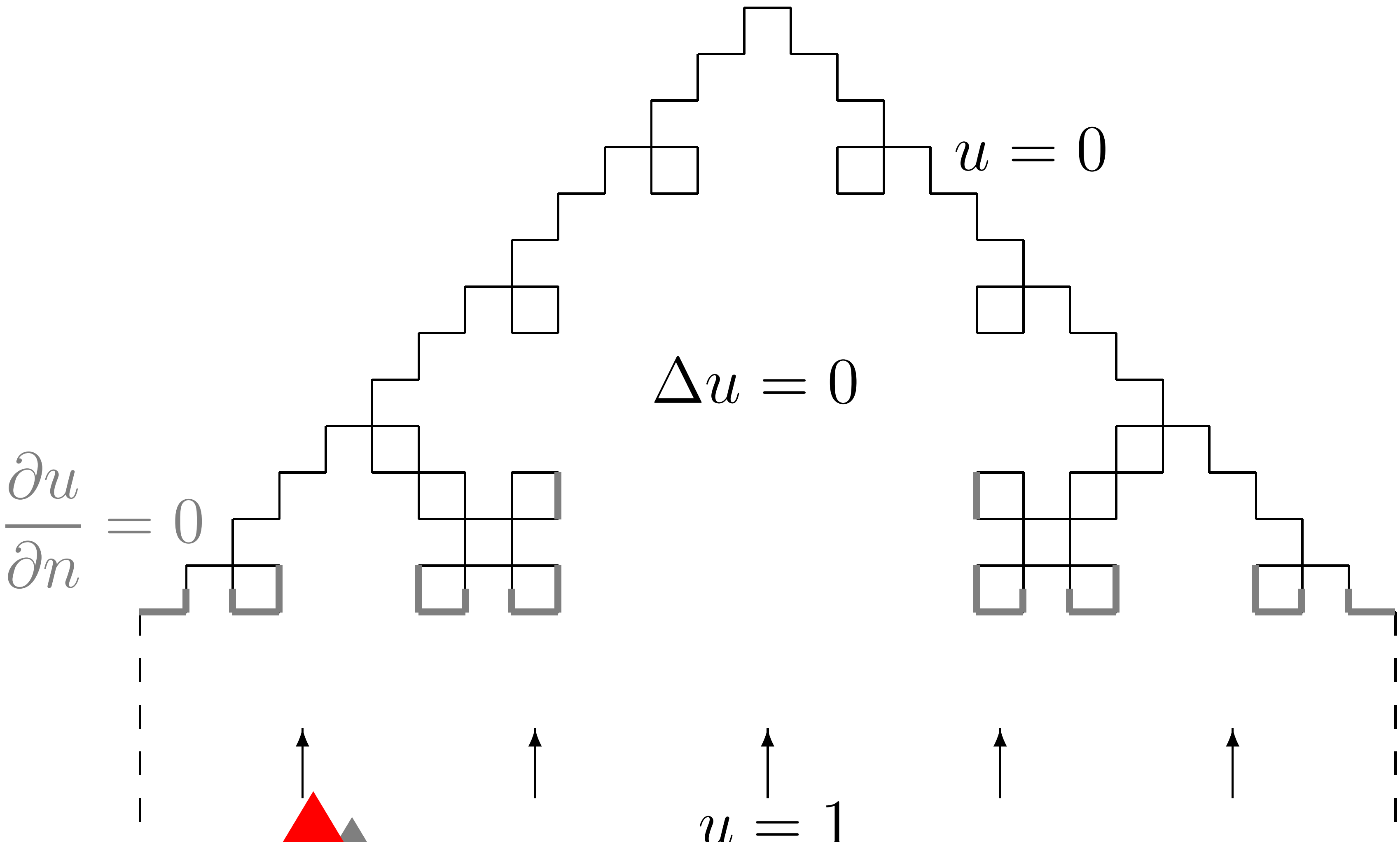

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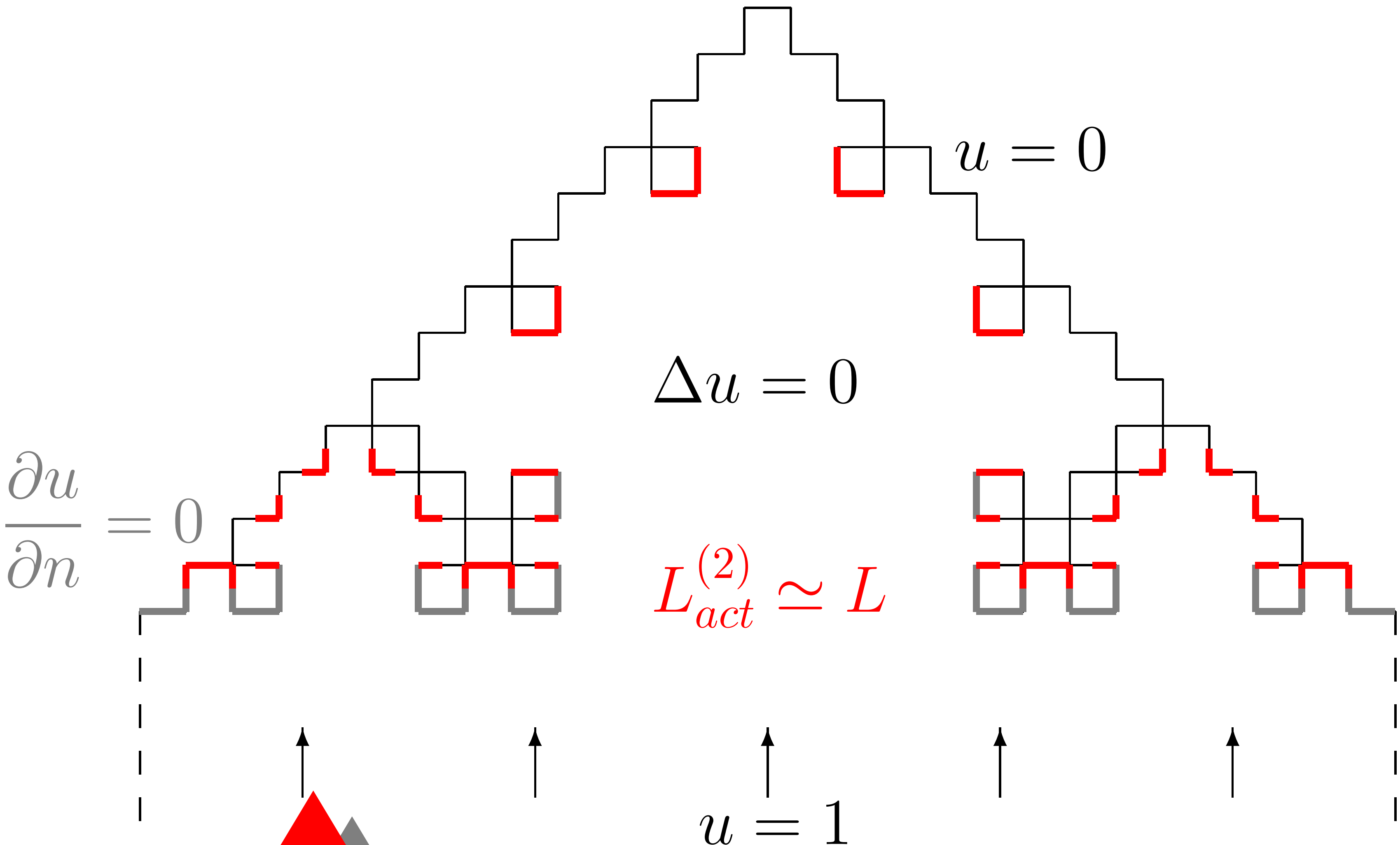

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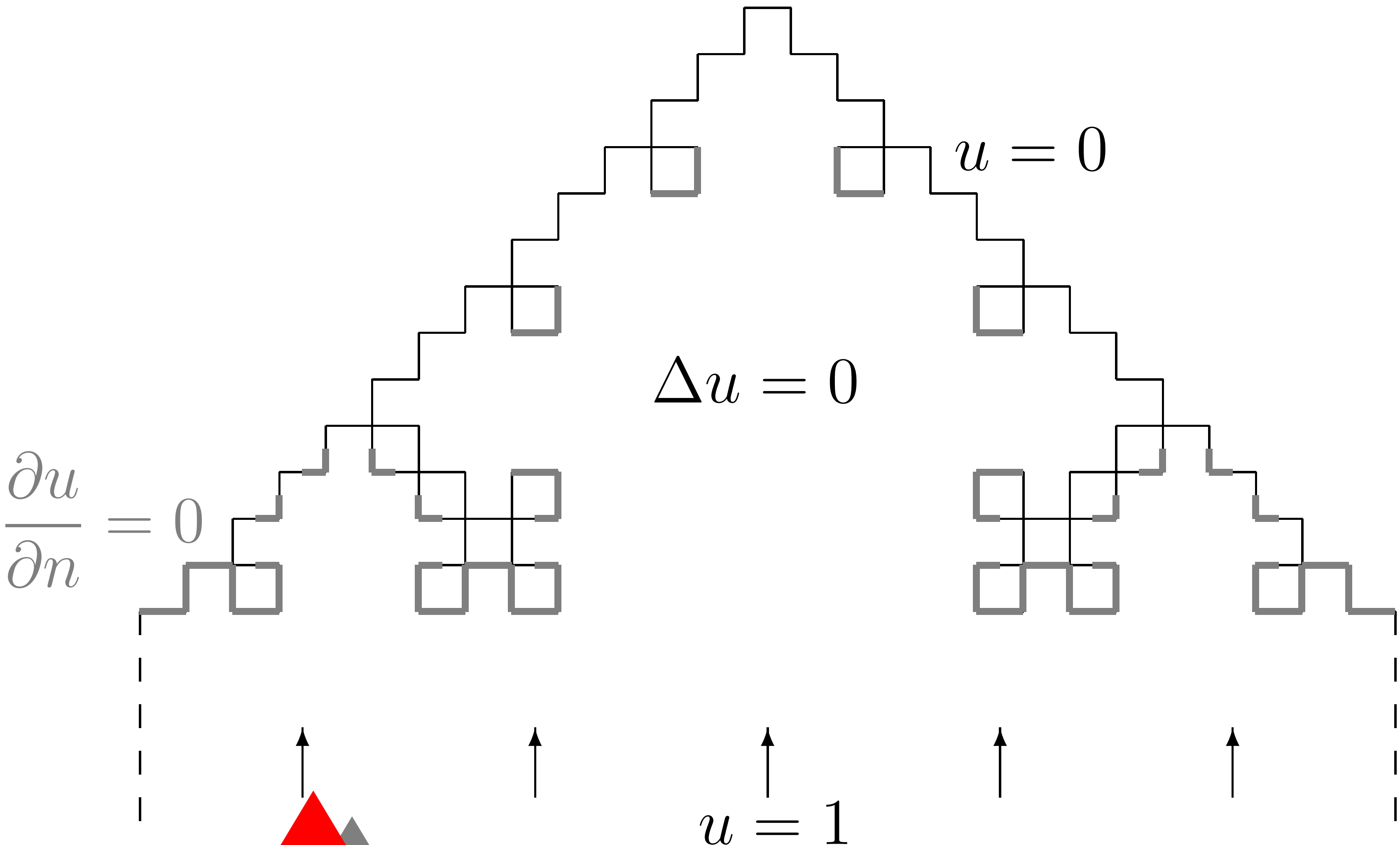

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Numerical scheme and hints

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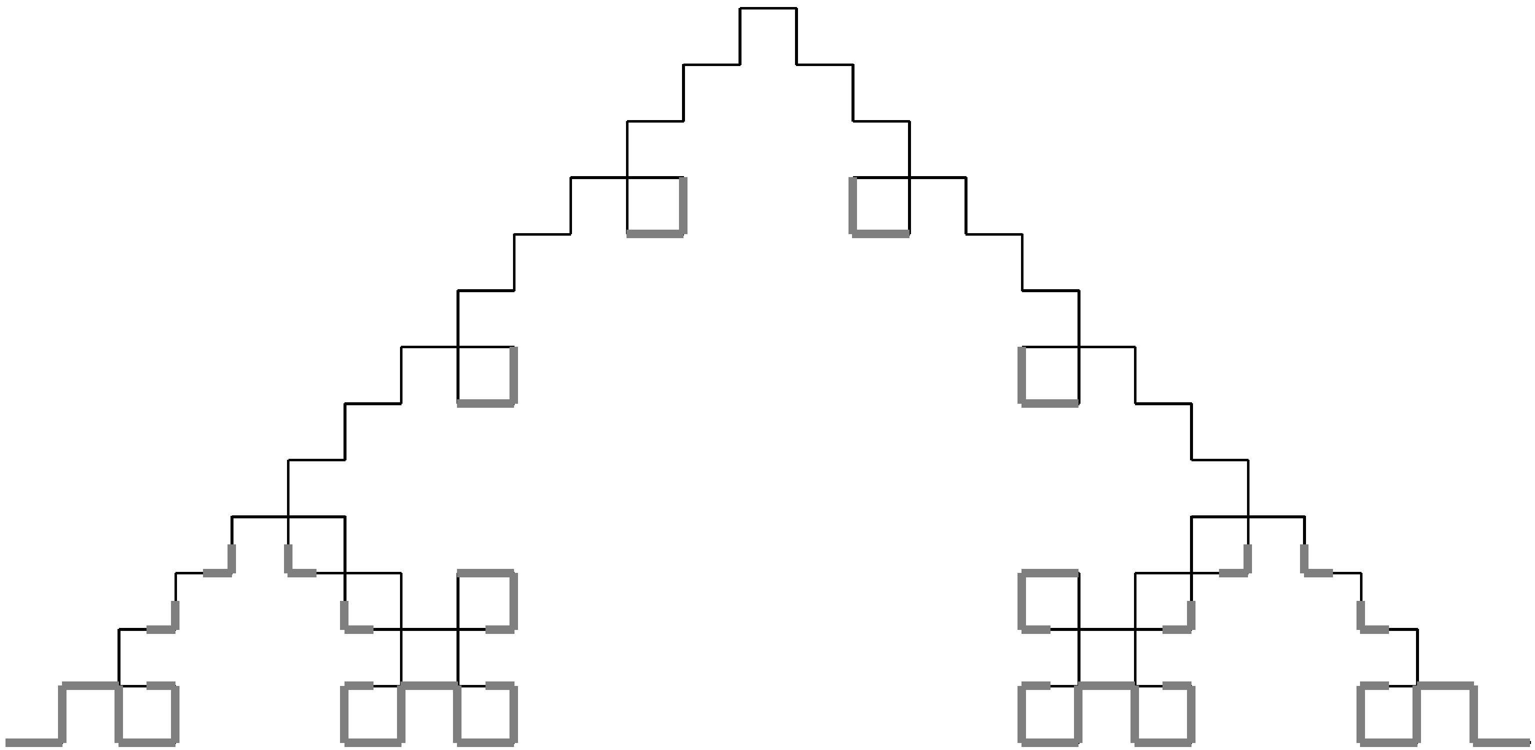
- *etc.*

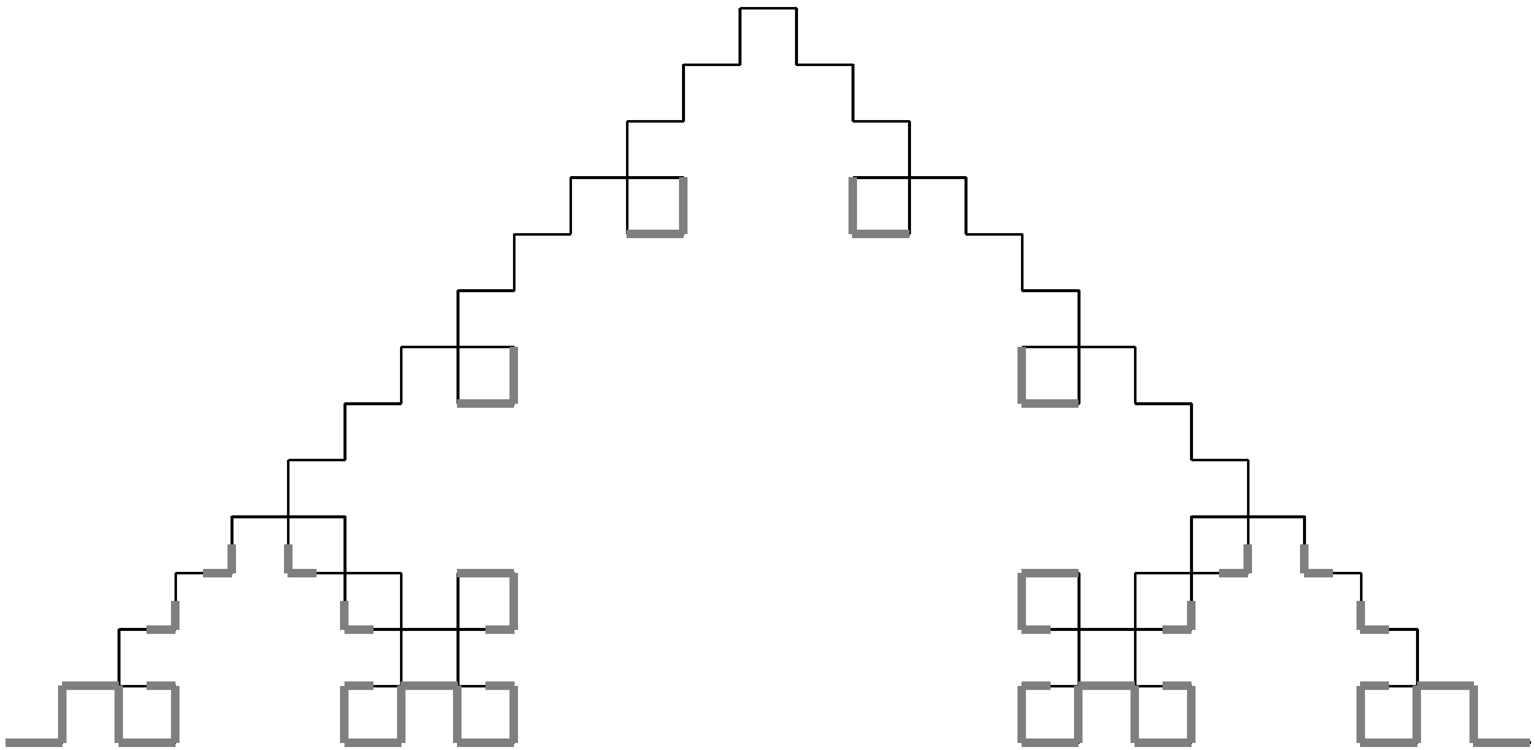
Numerical scheme and hints



The most accessible regions are passivated,

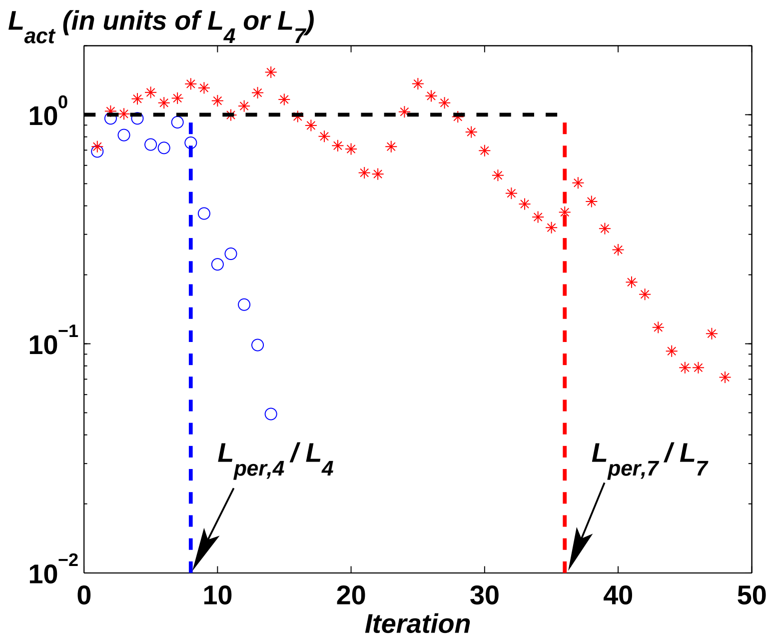
the other regions are harder and harder to access

Numerical scheme and hints



Use the arrival probability distribution at stage $n-1$ as an effective source of particles for the stage n

Passivation in 2D/3D



Filoche, Grebenkov, Andrade, Sapoval (submitted)

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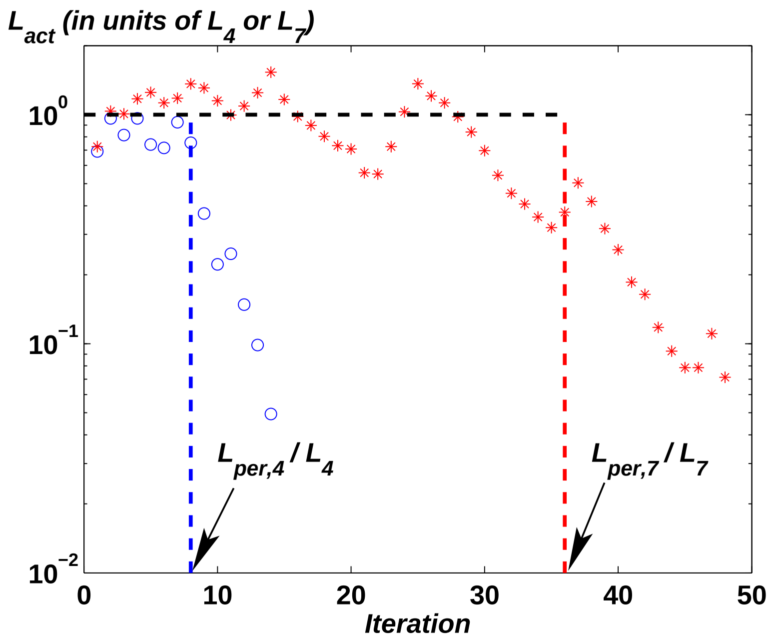

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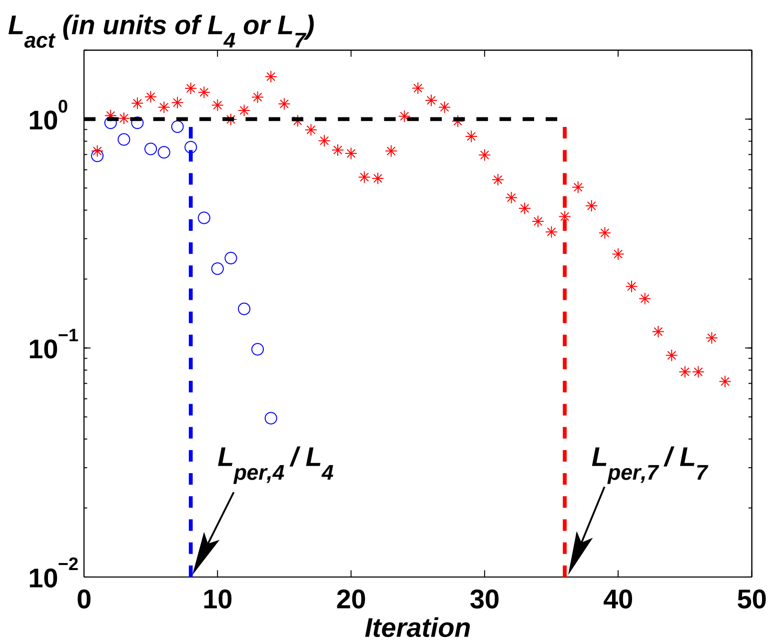

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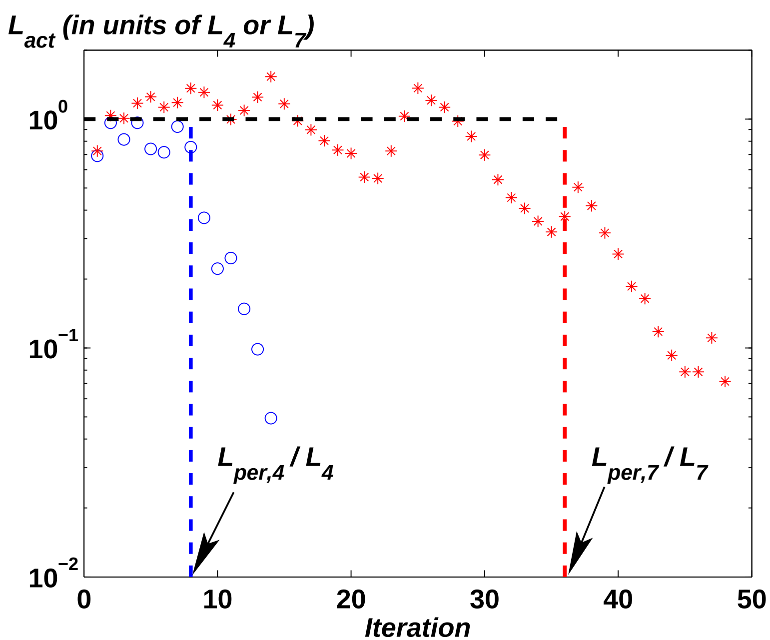

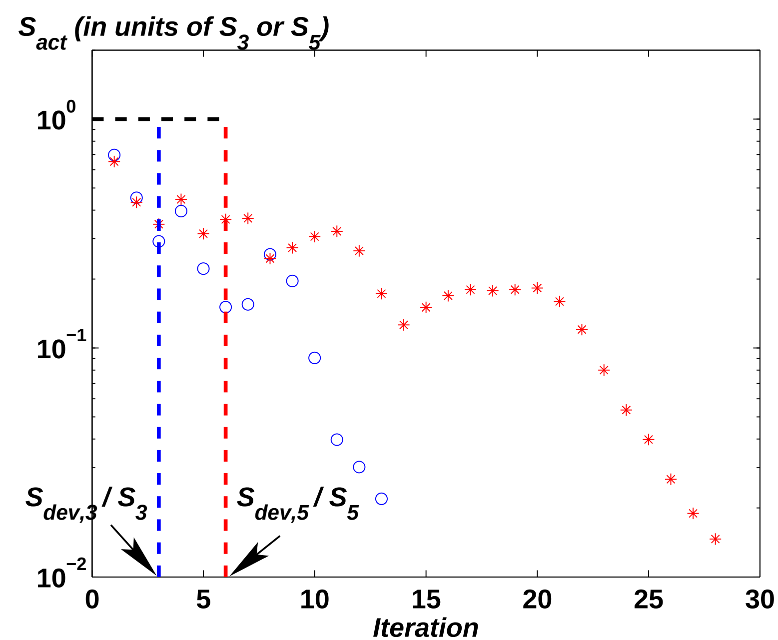

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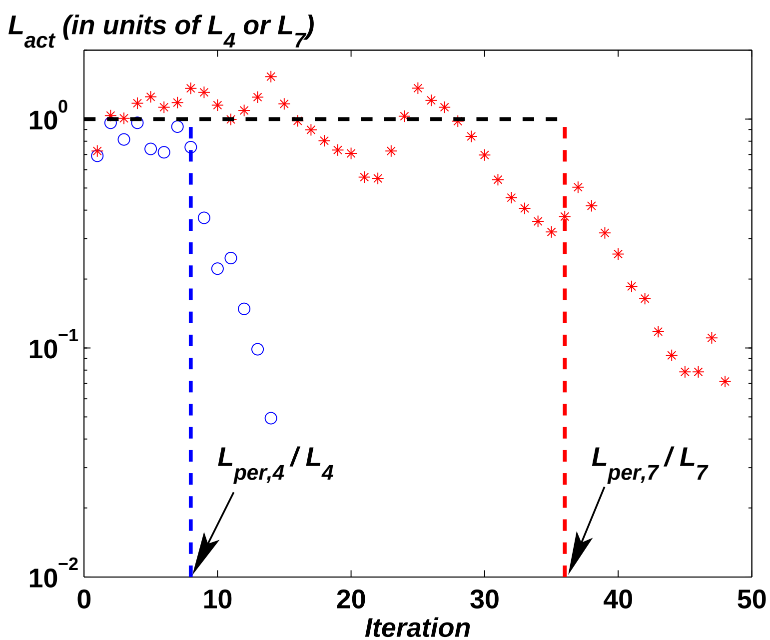

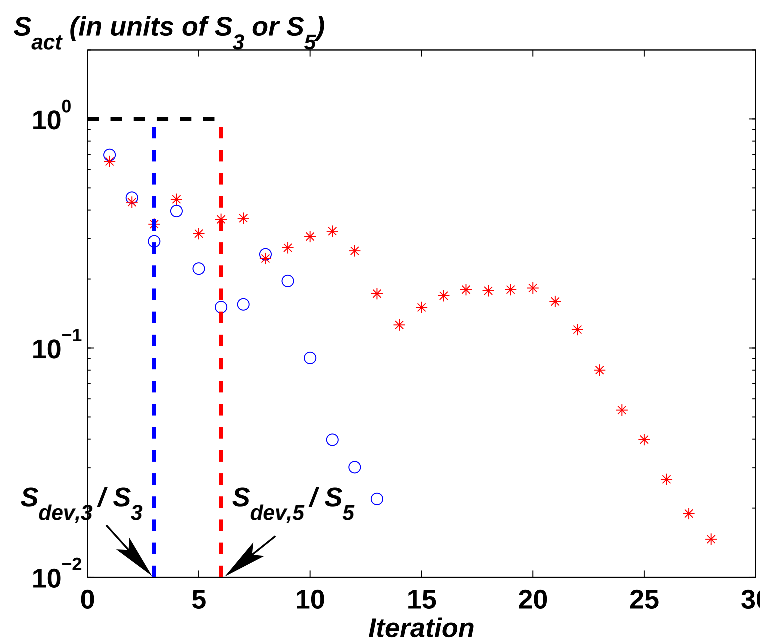

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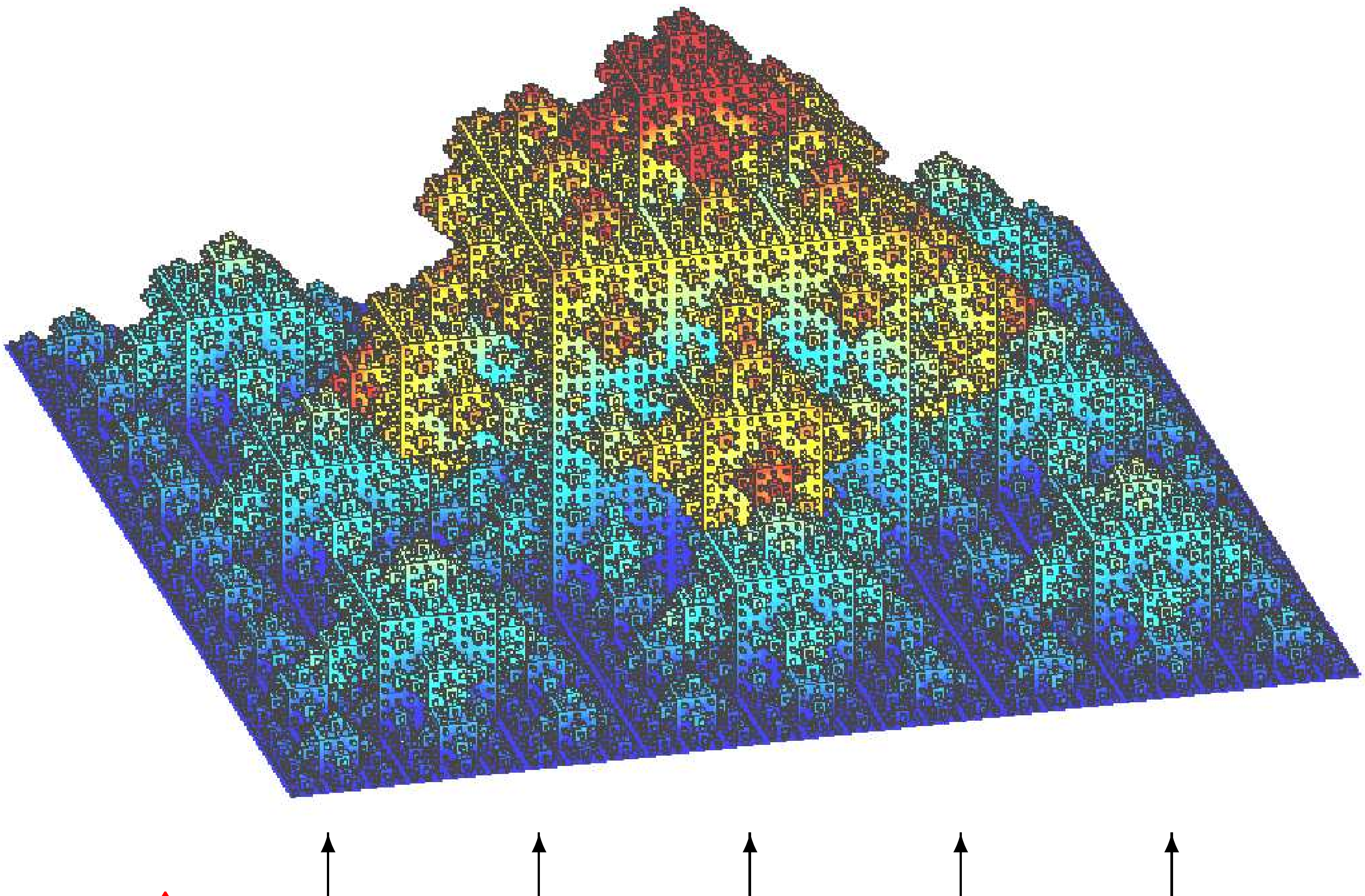
Passivation in 3D



Passivation in 3D

In sharp contrast with 2D:

- active zones progressively decrease
- passivation is much longer
- catalyst may still contain worth matter and behave as exhausted

Summary

- *Geometry-adapted fast random walk algorithm*

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- *Passivation is very different in 2D and 3D*

E N D