Neutrino Transport Near Compact Objects

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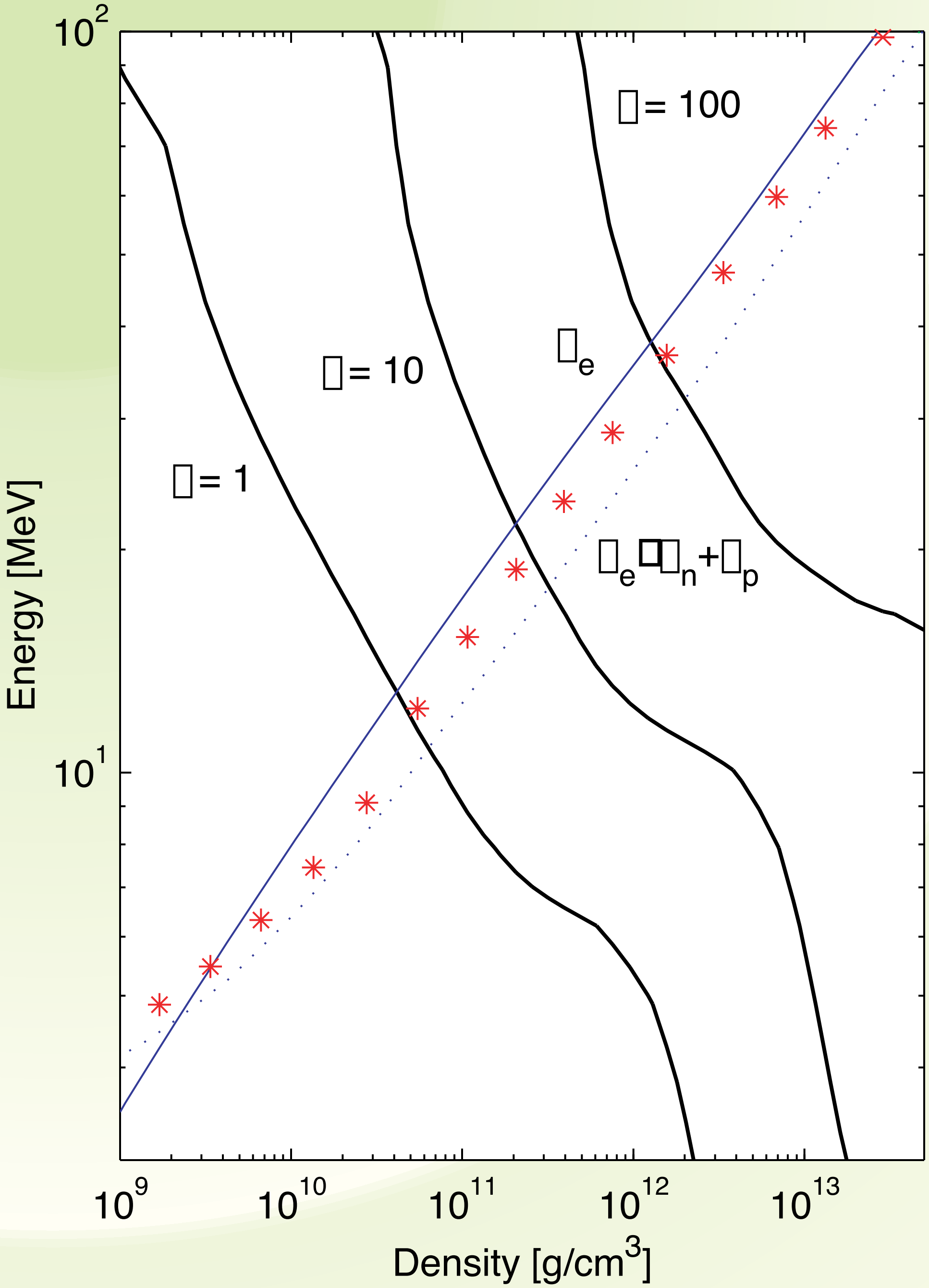
- Environments of neutrino transport & neutrino-matter interactions
  (introductory...)

- Boltzmann neutrino transport in spherical symmetry
  (technical...)

- Neutrino transport in supernova simulations
  (visual...)

IPAM 2005

Regions of semi-transparent neutrino transport



Bethe 1990:

$$\lambda_\nu = 1.0 \times 10^8 \rho_{12}^{-1} [(N^2/6A)X_h + X_n]^{-1} \varepsilon_\nu^{-2} \text{ cm} .$$

$$\tau(R) = \frac{N^2}{6A} \frac{\varepsilon_\nu^2}{60} \rho_{12}^{2/3} .$$

interesting range: 10^11 - 10^12 g/cm^3

size:

(1 Msol) / (10^11 g/cm^3)
~ 4pi/3 (170 km)^3

compact
& instable!

time scale:

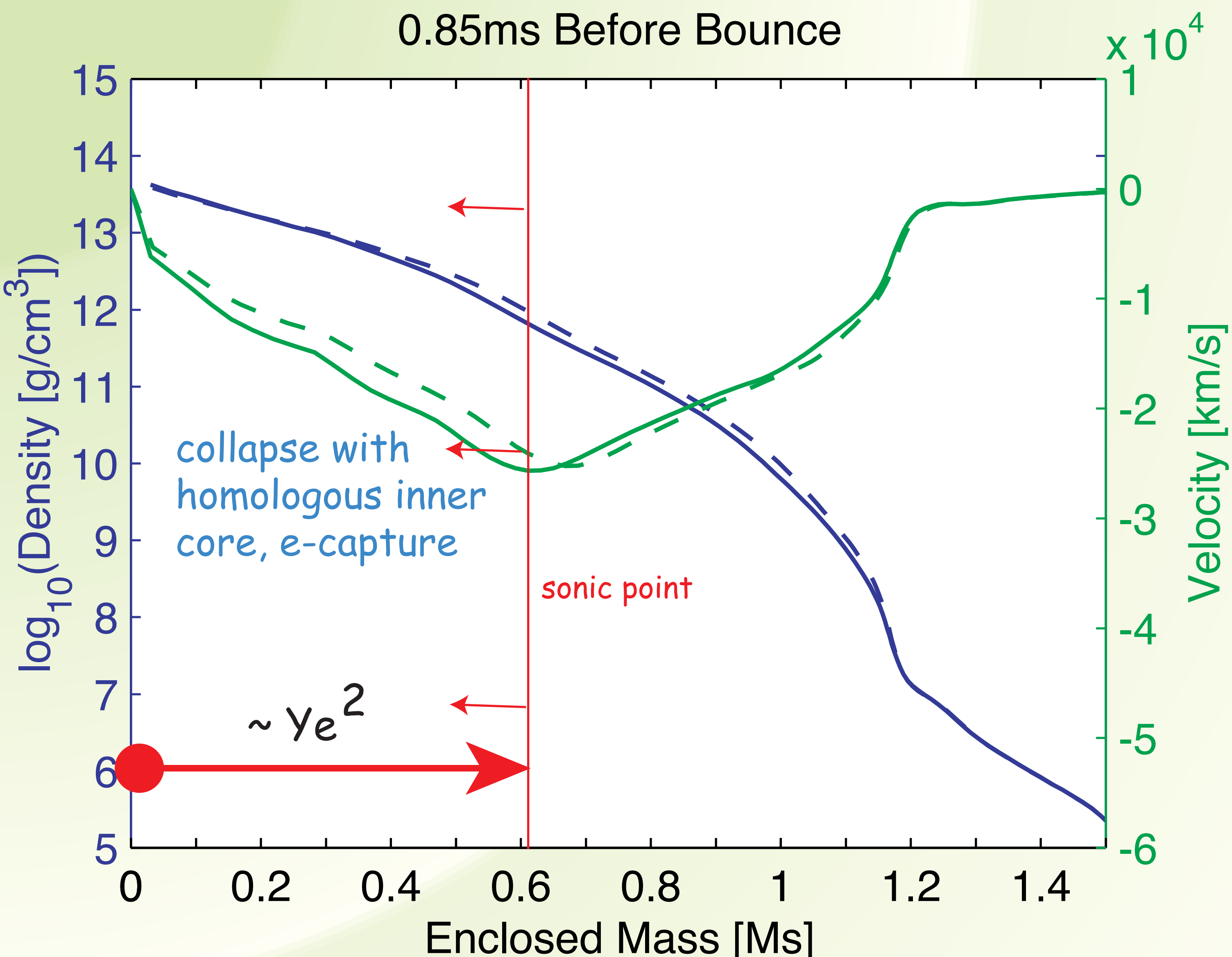
1/sqrt(G*10^11 g/cm^3)
~ 12 ms

fast!

--> relativistic effects

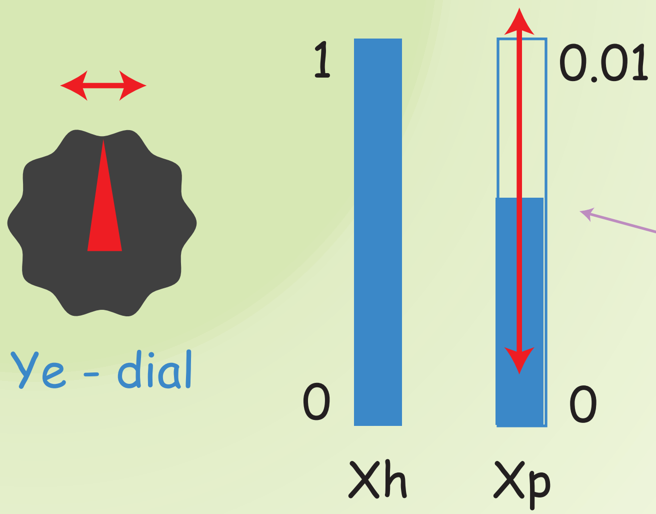
Stellar core collapse

(1)



dashed = Newtonian
solid = general relativistic

Electron Capture in Core Collapse



Convergence of Ye and sonic point if:

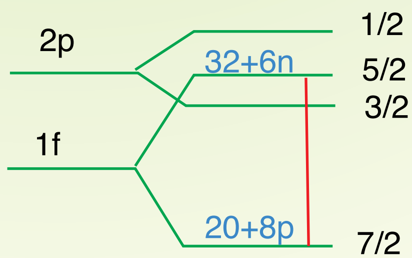
(1) target has low abundance

(2) abundance is sensitive to Ye

(3) the reaction dominates

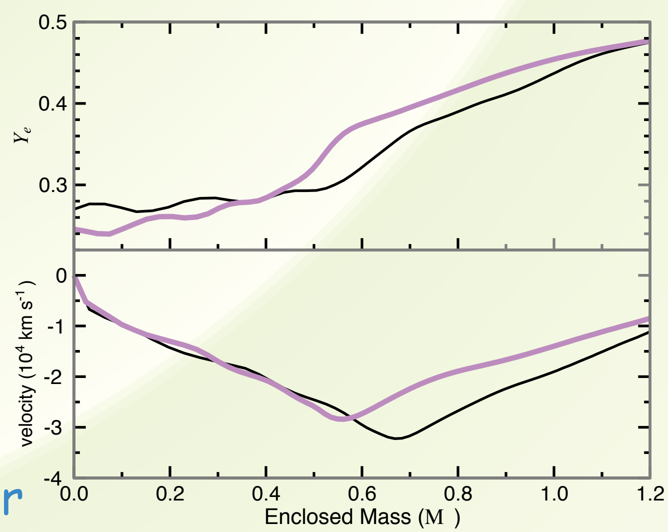
Messer et al. 200X

Old standard e capture rates:
(independent particle model)

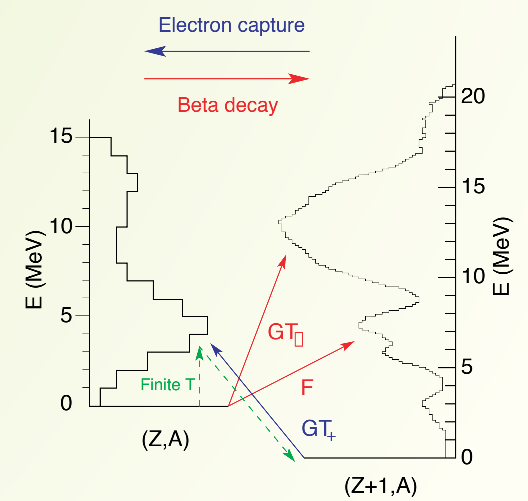


Initial inner iron cores have similar density profiles.
Bounce occurs at same nuclear matter density.

New improved e capture rates
(thermal excitations and correlations)



Langanke et al., Hix et al. 2003

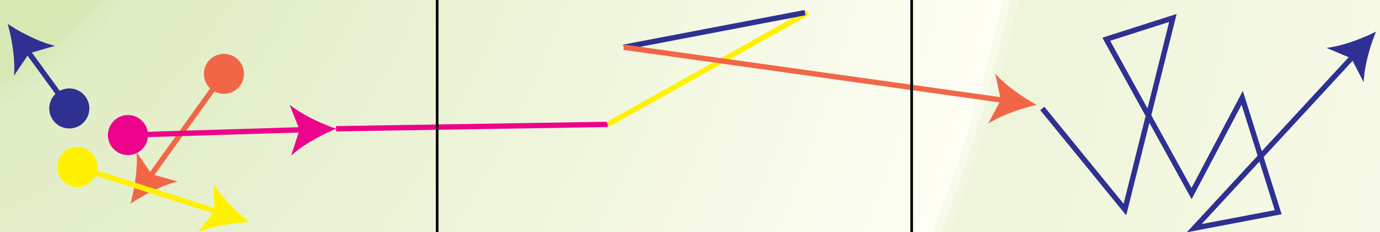


Langanke & Martinez-Pinedo, 2002

Neutrino Escape

High matter density

Emission & Absorption	Inelastic Scattering	Isoenergetic Scattering
Can be Pauli-blocked in diffusive regime	Probability enhanced by isoenergetic scattering	Strong energy dependence, ~E^2
Type Energy Direction	Energy Direction	Direction



Low matter density

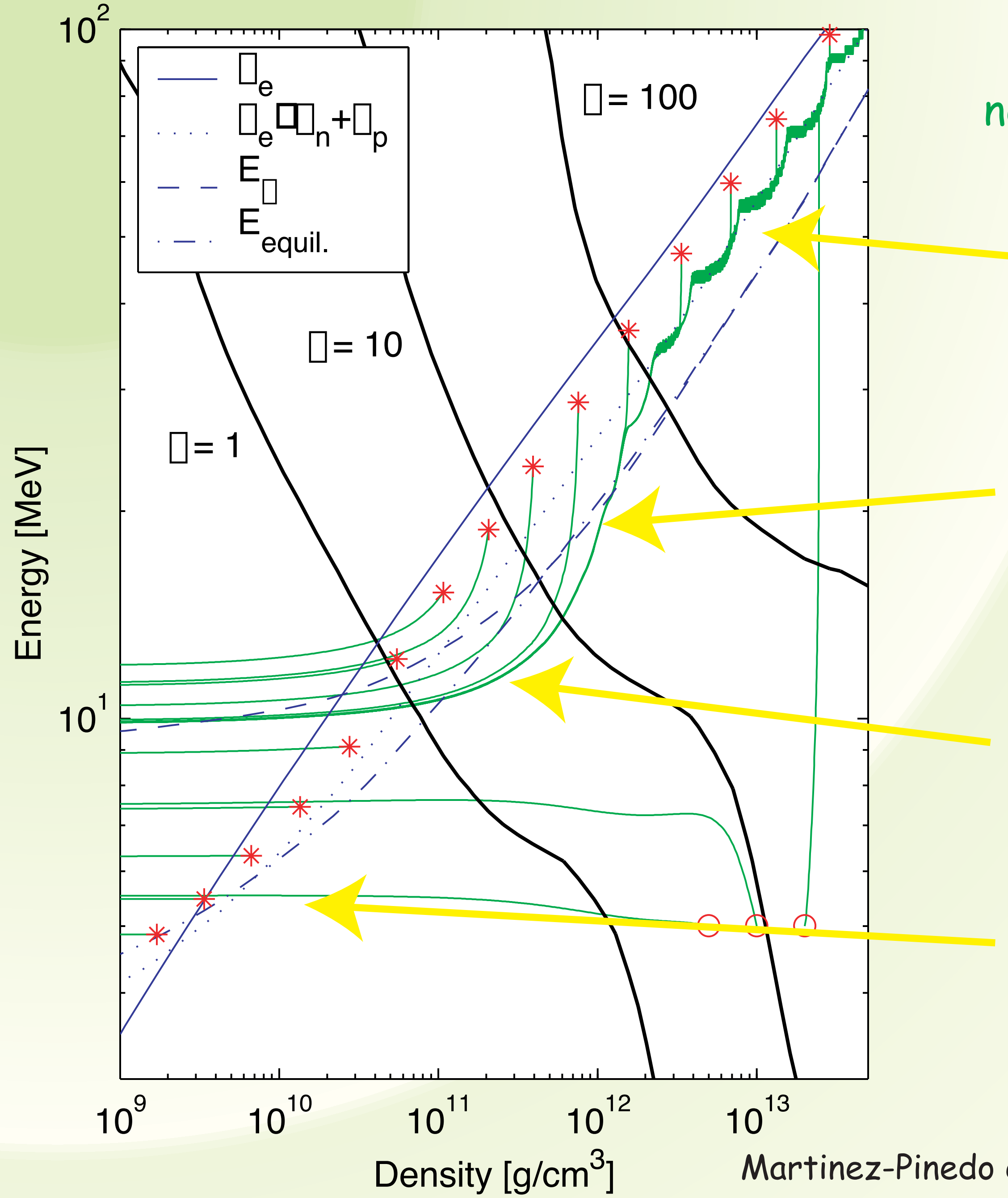
Number Sphere | Energy Sphere | Transport Sphere

Description:

Rates & Equilibrium	Mean free path & Fluid state	Mean free path & Diffusion time scale

Raffelt (2001)

Neutrino escape from core collapse



neutrinos are:

trapped and degenerate
Fermi surface is relevant

thermalizing
thermalization time scale
is relevant

diffusing
opacities and neutrino energies
are relevant

free streaming
productions rate is relevant

Martinez-Pinedo et al., astro-ph/0412091

Weak interactions

(Bruenn, ApJS 58, 1985)

In supernova example:

Collapse

source
opacity
thermalization

Breakout

source
absorption
opacity

Accretion

source
absorption
opacity

mu/tau source
absorption
opacity
thermalization

Process

Coherent scattering of neutrinos on nuclei

$$\nu + (A,Z) \rightleftharpoons \nu + (A,Z)$$

Neutrino absorption on nuclei

$$\nu_e + (A,Z) \rightleftharpoons e^- + (A,Z+1)$$

Neutrino absorption on nucleons

$$\nu_e + n \rightleftharpoons e^- + p$$

$$\bar{\nu}_e + p \rightleftharpoons e^+ + n$$

Neutrino-nucleon scattering

$$\nu + N \rightleftharpoons \nu + N$$

Neutrino-electron scattering

$$\nu + e \rightleftharpoons \nu + e$$

Neutrino production from pair annihilation

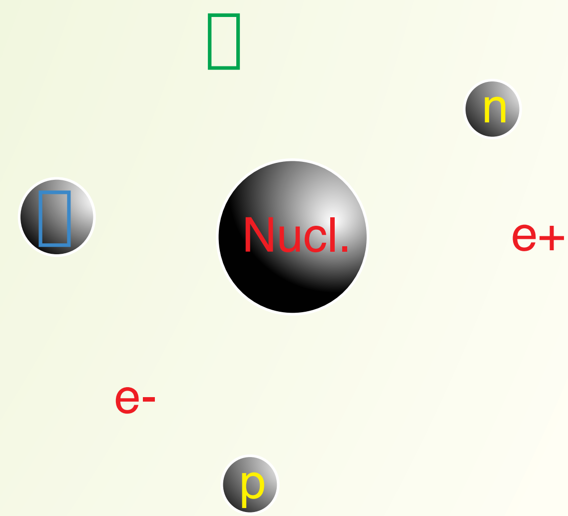
$$e^- + e^+ \rightleftharpoons \nu + \bar{\nu}$$

Nucleon-Nucleon bremsstrahlung (Thompson et al. 2002)

Electron-□ pair annihilation --> muon-□ pair creation (Buras et al. 2003)

Liquid drop equation of state

(Lattimer-Swesty 1991)



Why are Neutrinos so Important for SNe?

(a) The observation of 24 neutrinos from SN1987A in Kamiokande, IMB, and Baksan confirmed the general idea of stellar core collapse. The neutrinos transport information about buried layers to the outside.

(b) Neutrino physics determines the condition of matter in the supernova: Most regions are in nuclear statistical equilibrium. Free parameters to specify the fluid state are only the

1 density   2 temperature or entropy   3 electron fraction Ye=np/(np+nn)

For example

(e-) + p <==> n + ve	reduces Ye	reduces entropy
(e+) + n <==> p + vebar	increases Ye	reduces entropy

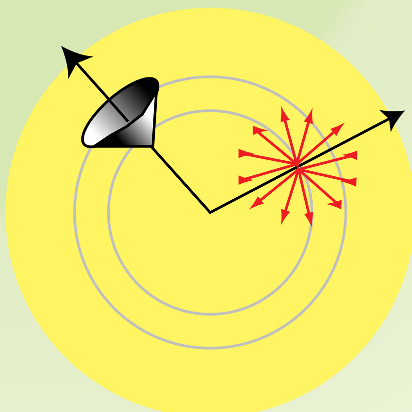
2 degrees of freedom: weak equilibrium & thermal balance

(c) They are responsible for the explosion in the v-driven mechanism (distorted view! - actually they are responsible for the failure:
- without neutrinos, Chandrasekhar mass ~ Ye^2 ~ 1.2 solar Masses
- homologous collapse, bounce, dissociation & explosion!)

Boltzmann Neutrino Transport in Spherical Symmetry

AGILE-BOLTZTRAN
Liebendörfer et al. 2004

= time-implicit in three phase-space dimensions



Direct calculation of the distribution function f(time,radius,angle,energy).
Dimensionality:

- ~30'000 time steps
  102 adaptive mass zones
  6 propagation angles
  12 energy groups
- GR dynamics and energy conserv.
- GR redshift
- GR light bending

	Newtonian & O(v/c)	general relativistic
quasi-stationary	Mezzacappa & Bruenn 1993 Burrows et al. 2000	Yamada, Janka & Suzuki 1999
dynamic	Rampp & Janka 2000 Mezzacappa et al. 2001 Thompson, Burrows & Pinto 2003	Wilson 1971 Liebendörfer et al. 2001 Liebendörfer et al. 2004
dynamic (spanning both columns)	Rampp & Janka 2002	

Boltzmann Neutrino Transport in Spherical Symmetry

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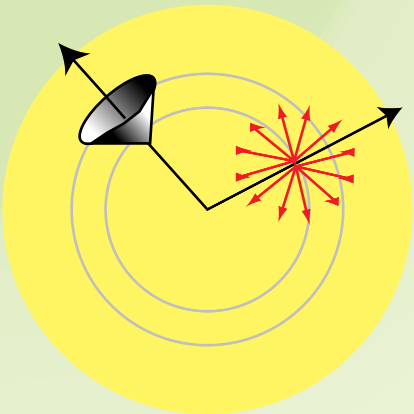
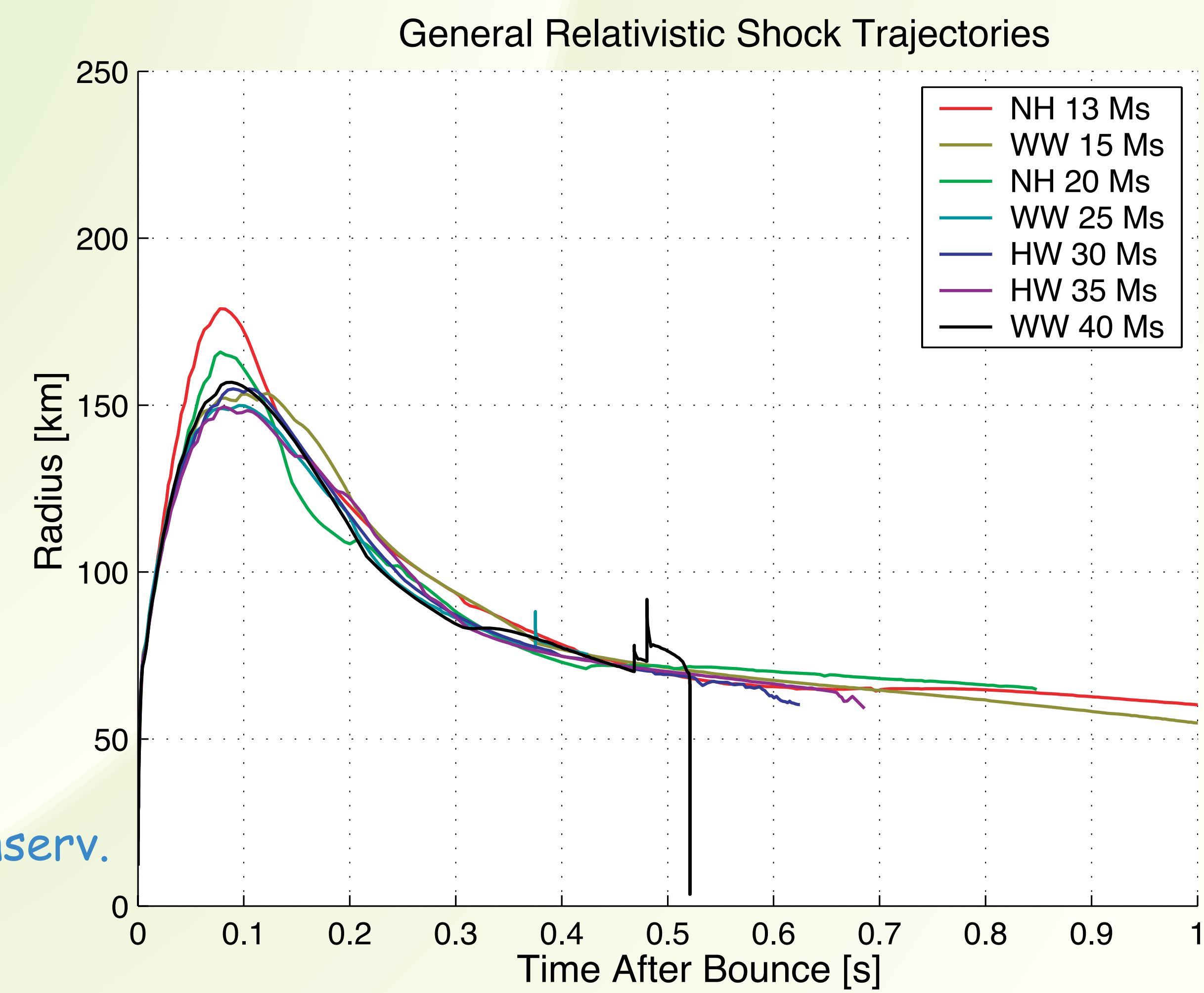




Shock recedes - no explosion found!

- ~30'000 time steps
  102 adaptive mass zones
  6 propagation angles
  12 energy groups
- GR dynamics and energy conserv.
- GR redshift
- GR light bending

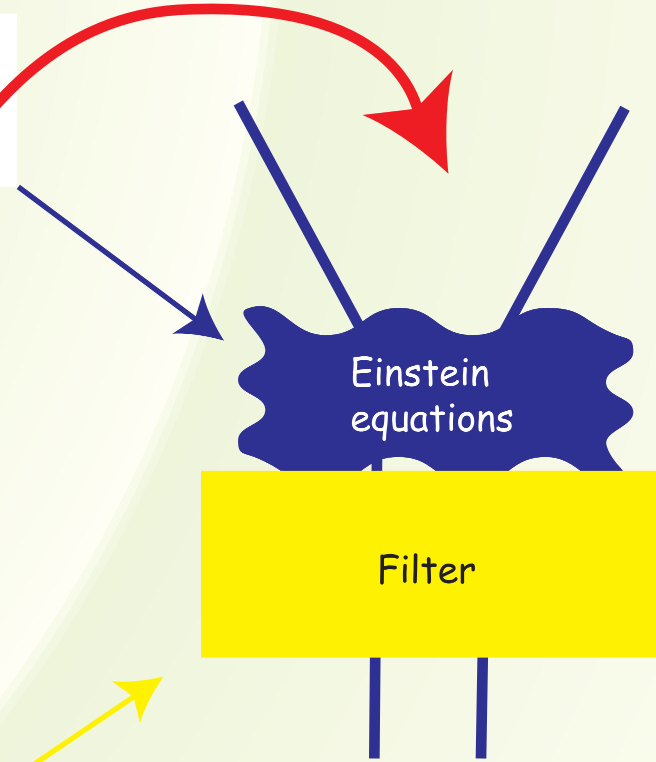
GR Hydrodynamics Equations

Metric in spherical symmetry:

$$ds^2 = -\alpha^2 dt^2 + \left(\frac{r'}{\Gamma}\right)^2 da^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right), \quad (1)$$

Stress-energy tensor:

$$T^{tt} = \rho(1 + e + J),$$
$$T^{ta} = T^{at} = H$$
$$T^{aa} = p + \rho K,$$
$$T^{\vartheta\vartheta} = T^{\varphi\varphi} = p + \frac{1}{2}\rho(J - K). \quad (2)$$



Conservation quantities:

$$\frac{1}{D} = \frac{\Gamma}{\rho}, \quad (3)$$

$$\tau = \Gamma(e + J) + \frac{2}{\Gamma + 1}\left(\frac{1}{2}u^2 - \frac{m}{r}\right) + uH, \quad (4)$$

$$S = u(1 + e + J) + \Gamma H. \quad (5)$$

"continuity equation"

$$\frac{\partial}{\partial t}\left(\frac{1}{D}\right) = \frac{\partial}{\partial a}(4\pi r^2 \alpha u), \quad (6)$$

$$\frac{\partial \tau}{\partial t} = -\frac{\partial}{\partial a}\left[4\pi r^2 \alpha(up + u\rho K + \Gamma\rho H)\right], \quad (7)$$

"energy equation"

AGILE

Implicit general relativistic hydrodynamics on dynamically adaptive grid

$$\frac{\partial}{\partial t}\left[\frac{1}{D}\right] = \frac{\partial}{\partial a}\left[4\pi r^2 \alpha u\right] \quad (1)$$

specific volume

$$\frac{\partial \tau}{\partial t} = -\frac{\partial}{\partial a}\left[4\pi r^2 \alpha \left(up + u\rho K + \Gamma \rho H\right)\right] \quad (2)$$

energy

$$\begin{aligned}\frac{\partial S}{\partial t} &= -\frac{\partial}{\partial a}\left[4\pi r^2 \alpha \left(\Gamma p + \Gamma \rho K + u\rho H\right)\right] \\ &- \frac{\alpha}{r}\left[\left(1 + e + \frac{3p}{\rho} + J + 3K\right)\frac{m}{r} - \left(1 - \frac{2m}{r}\right)\left(J - 3K\right)\right. \\ &+ \left. 8\pi r^2 \left(\left(1 + e + J\right)\left(p + \rho K\right) - \rho H^2\right) - 2\left(\frac{p}{\rho} + K\right)\right] \quad (3)\end{aligned}$$

radial momentum

$$\frac{\partial V}{\partial a} = \frac{1}{D} \quad (4)$$

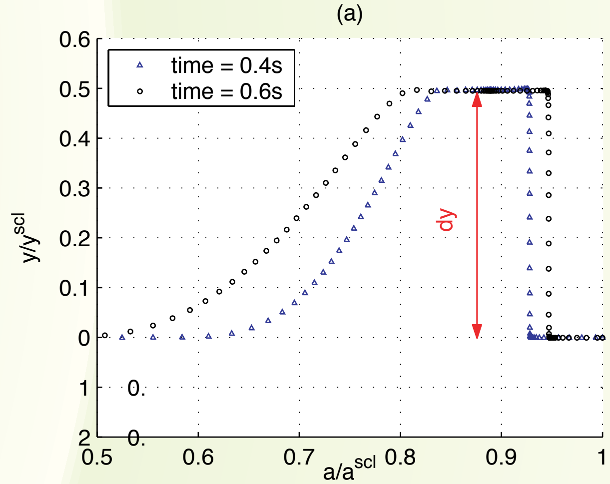
baryon density

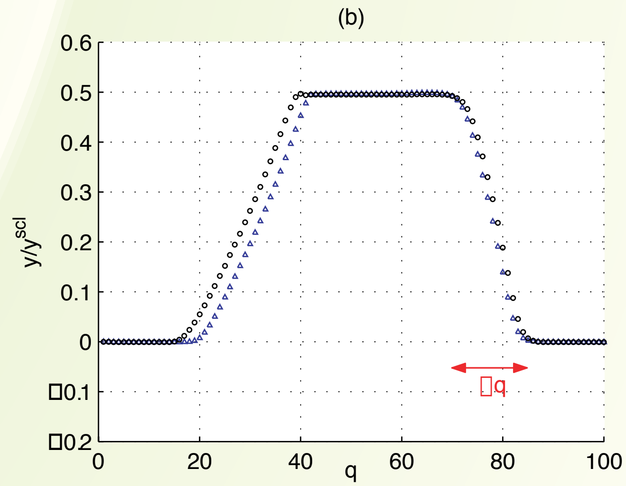
$$\frac{\partial m}{\partial a} = 1 + \tau \quad (5)$$

Poisson Eq.

$$\frac{\partial}{\partial t}\left[\frac{1}{4\pi r^2 \rho} H\right] = -\left(1 + e + J\right)\frac{\partial \alpha}{\partial a} - \frac{1}{\rho}\frac{\partial}{\partial a}\left[\alpha\left(p + \rho K\right)\right] + \frac{\alpha}{3VD}\left(J - 3K\right). \quad (6)$$

lapse function





(Liebendörfer et al., ApJS, 2002)

GR Boltzmann Equation

Neutrino distribution function:

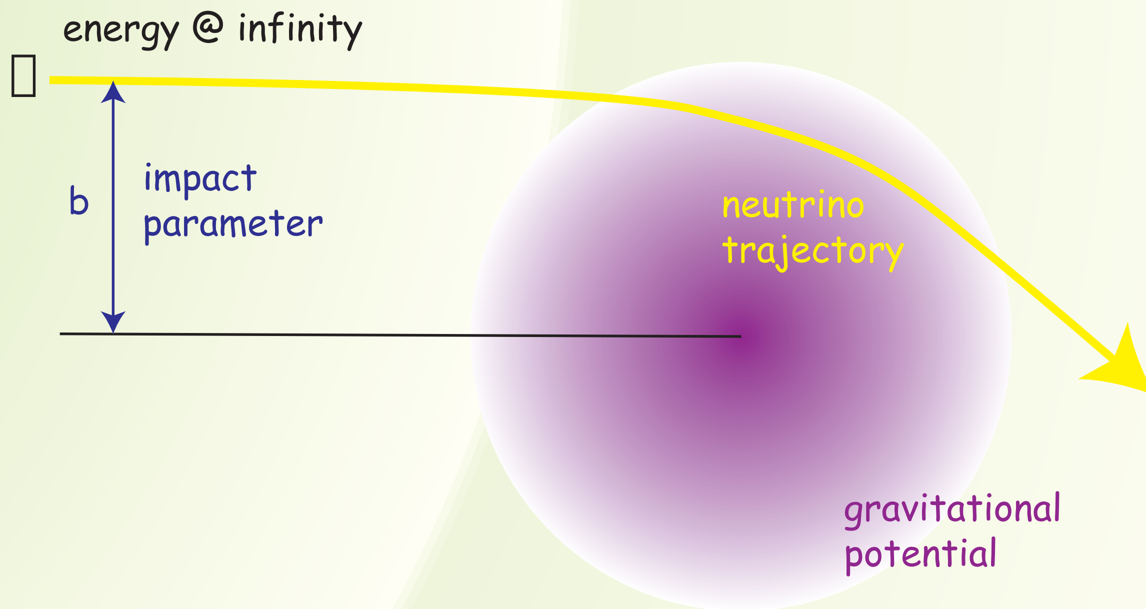
$$\bar{f}(t,a,b,\varepsilon)$$

time

enclosed mass (location)

impact parameter

energy @ infinity



Derivatives with respect to b and ε vanish along phase flow
--> Boltzmann transport equation is very simple ???

$$\frac{1}{\alpha}\frac{\partial \bar{f}}{\partial t}+\mu\frac{\Gamma}{r'}\frac{\partial \bar{f}}{\partial a}+\frac{4\pi r}{\Gamma+u\mu}$$
$$\times\left[\mu\rho\left(1+e+\frac{p}{\rho}\right)-(1+\mu^2)q\right]$$
$$\times\left(-\varepsilon\frac{\partial \bar{f}}{\partial \varepsilon}+b\frac{\partial \bar{f}}{\partial b}\right)=j+\chi\bar{f}. \qquad (43)$$

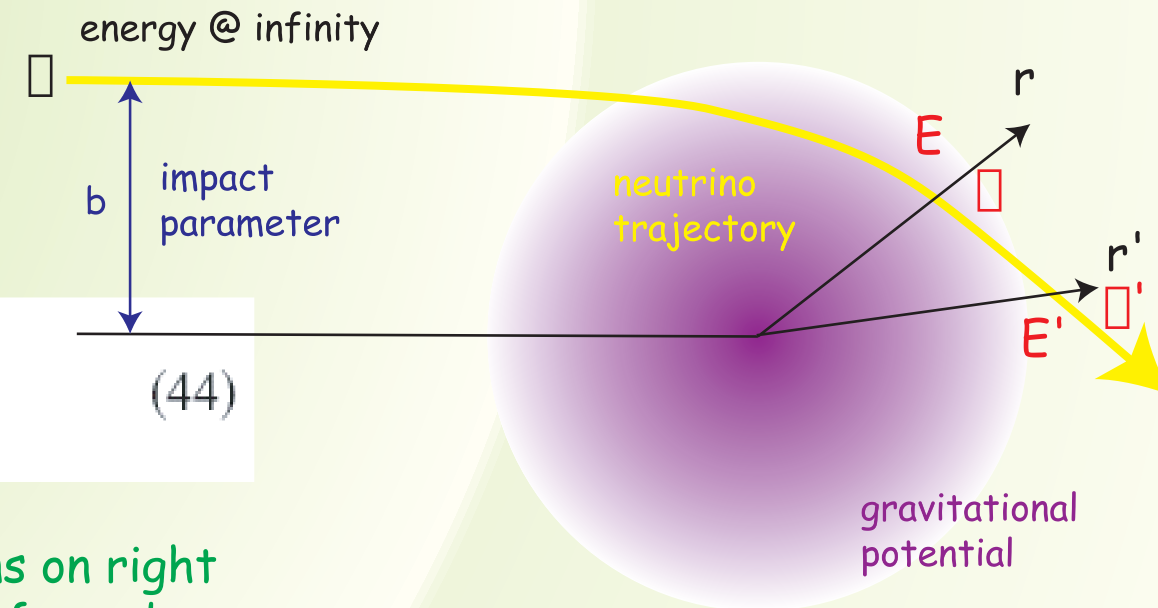
small terms coming from extended mass-energy in --> neglect! comparison to point mass

Liebendörfer, Mezzacappa, Thielemann 2001

GR Boltzmann Equation

Boltzmann equation for decoupled momentum phase space:

$$\frac{1}{\alpha}\frac{\partial \bar{f}}{\partial t} + \mu \frac{\Gamma}{r'}\frac{\partial \bar{f}}{\partial a} = j + \chi \bar{f}. \qquad (44)$$



For neutrino-matter interactions on right hand side comoving frame is preferred:

$$dN = f(t,a,\mu,E)E^2 dE\, d\mu \frac{dV}{\Gamma} = F(t,a,\mu,E)E^2 dE\, d\mu\, da.$$

The transformation to the comoving frame is given by:

$$b = r\frac{\sqrt{1-\mu^2}}{\Gamma + u\mu}$$

$$\varepsilon = (\Gamma + u\mu)E.$$

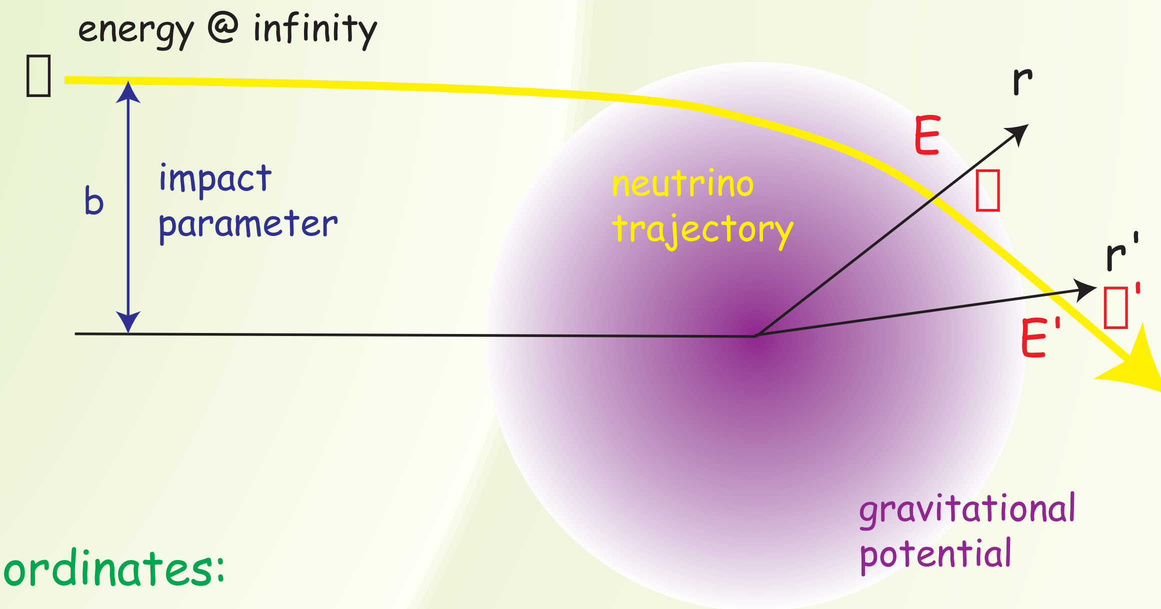
→ calculate all partial derivatives for Boltzmann equation in comoving frame...

GR Boltzmann Equation

The transformation to the comoving frame is given by:

$$b = r\frac{\sqrt{1-\mu^2}}{\Gamma + u\mu}$$

$$\varepsilon = (\Gamma + u\mu)E.$$



Transport equation in comoving coordinates:
[Lindquist, Ann. Phys., 1966; etc.]

$$C_t + D_a + D_\mu + D_E + O_\mu + O_E = C_c, \qquad (15)$$

with

Lagrangian time derivative

$$C_t = \frac{\partial F}{\alpha \partial t}, \qquad (16)$$

spatial advection

$$D_a = \frac{\mu}{\alpha}\frac{\partial}{\partial a}(4\pi r^2 \alpha \rho F), \qquad (17)$$

angular advection

$$D_\mu = \Gamma\left(\frac{1}{r} - \frac{1}{\alpha}\frac{\partial \alpha}{\partial r}\right)\frac{\partial}{\partial \mu}\left[(1-\mu^2)F\right], \qquad (18)$$

gravitational frequency shift

$$D_E = -\mu\Gamma\frac{1}{\alpha}\frac{\partial \alpha}{\partial r}\frac{1}{E^2}\frac{\partial}{\partial E}(E^3 F), \qquad (19)$$

Doppler frequency shift

$$O_E = \left[\mu^2\left(\frac{\partial \ln \rho}{\alpha \partial t} + \frac{3u}{r}\right) - \frac{u}{r}\right]\frac{1}{E^2}\frac{\partial}{\partial E}(E^3 F), \qquad (20)$$

angular aberration

$$O_\mu = \left(\frac{\partial \ln \rho}{\alpha \partial t} + \frac{3u}{r}\right)\frac{\partial}{\partial \mu}\left[\mu(1-\mu^2)F\right], \qquad (21)$$

collision term

$$C_c = \frac{j}{\rho} - \chi F. \qquad (22)$$

Microscopic Transport --- Macroscopic Properties

Interest in time evolution of macroscopic quantities,
e.g. the evolution of neutrino energy:

$$b = r\frac{\sqrt{1-\mu^2}}{\Gamma + u\mu}$$

$$\varepsilon = (\Gamma + u\mu)E.$$

$$dN = F(t, a, \mu, E)E^2 dE\, d\mu\, da.$$

operator

occupation probability

Laboratory frame:

$$\frac{\partial}{\partial t}\int (\Gamma + u\mu)FE^3 dE\, d\mu.$$

time derivative

expectation value

Chain rule for time derivative

$$C_t = \frac{\partial F}{\alpha \partial t},$$

$$C_t + D_a + D_\mu + D_E + O_\mu + O_E = C_c,$$

GR Boltzmann equation

Microscopic Transport --- Macroscopic Properties

Interest in time evolution of macroscopic quantities,
e.g. the evolution of neutrino energy:

Numerically, these are trouble, not beautiful!

Laboratory frame:

$$\frac{\partial}{\partial t}\int (\Gamma + u\mu) F E^3 dE\, d\mu.$$

time derivative

expectation value

Many cancellations among partial derivatives because □ is conserved along phase flow!

From microscopic GR Boltzmann equation:

$$\begin{aligned} 0 = &\frac{\partial}{\partial t}(\Gamma J + uH) + \frac{\partial}{\partial a}\left[4\pi r^2 \alpha \rho (uK + \Gamma H)\right] \\ &+ 4\pi r \alpha \rho \left(1 + e + \frac{p}{\rho}\right) H \\ &- \alpha \Gamma \int \left(\frac{j}{\rho} - \chi\right) E^3 dE\, d\mu + \alpha u \int \chi F E^3 dE \mu d\mu. \qquad (27) \end{aligned}$$

From macroscopic Einstein equations:

$$\frac{\partial \tau}{\partial t} = -\frac{\partial}{\partial a}\left[4\pi r^2 \alpha (up + u\rho K + \Gamma \rho H)\right], \qquad (7)$$

$$\tau = \Gamma(e + J) + \frac{2}{\Gamma + 1}\left(\frac{1}{2}u^2 - \frac{m}{r}\right) + uH, \qquad (4)$$

Of course, conservation equations exist because of conservation of energies in the laboratory frame! They are only expressed in terms of comoving frame quantities.

Microscopic Transport --- Macroscopic Properties

Interest in time evolution of macroscopic quantities,
e.g. the evolution of neutrino energy:

Laboratory frame:

$$\frac{\partial}{\partial t}\int(\Gamma+u\mu)FE^3\,dE\,d\mu.$$

time derivative

expectation value

Many cancellations among partial derivatives because □ is conserved along phase flow!

Cancellations don't happen in discretized world!

Discretized transport equation:

$$\frac{\partial f_{i'}}{\partial t}+c\,\frac{f_{i'}-f_{i'-1}}{dx_{i'}}=0.$$

$$\frac{\partial}{c\partial t}\sum_{i=1}^{n} g_{i'}f_{i'}\,dx_{i'}=\sum_{i=1}^{n} g_{i'}\frac{\partial f_{i'}}{c\partial t}dx_{i'}=-\sum_{i=1}^{n} g_{i'}\left(f_{i'}-f_{i'-1}\right)$$

$$=-\sum_{i=1}^{n} g_{i'}f_{i'}+\sum_{i=0}^{n-1} g_{i'+1}f_{i'}$$

$$=\sum_{i=1}^{n}\frac{g_{i'+1}-g_{i'}}{dx_{i'}}f_{i'}\,dx_{i'}-\left(g_{n'+1}f_{n'}-g_{0'+1}f_{0'}\right).$$

evolution of macroscopic property...

macroscopic operator inherits finite differencing from advection term

... as function of expectation value

Chain rule and integration by parts in discrete space!

Microscopic Transport --- Macroscopic Properties

Reason	Suggesting	Problem
Discrete Ordinates - adaptive mesh refinement - parallelization	Discretization of all dimensions	Low resolution
Implicit time integration - reactive equilibria - stationary state	First order advection	Inaccurate evolution of expectation values e.g. diffusion limit
Comoving coordinates - evaluation of collistion term - general relativistic curvature	Large cancellations	Inaccurate evolution in laboratory frame e.g. conservation laws

These problems have been solved in spherical symmetry by tuning of the coefficients in front of partial derivatives to reproduce the diffusion limit and conservation laws.

e.g. Liebendörfer et al., ApJS, 2004 and references therein

? And for more-dimensional neutrino transport simulations ?

e.g. Cardall & Mezzacappa, PRD, 2003

Microscopic Transport --- Macroscopic Properties

Reason	Suggesting	Problem
Discrete Ordinates - adaptive mesh refinement - parallelization	Discretization of all dimensions	Low resolution
Implicit time integration - reactive equilibria - stationary state	~~First order advection~~	Inaccurate evolution of expectation values e.g. diffusion limit
~~**Comoving coordinates** - evaluation of collistion term - general relativistic curvature~~		Inaccurate evolution in laboratory frame e.g. conservation laws

Solution 1: Solve macroscopic equations and add microscopic information

Variable Eddington Tensor Method, e.g. Burrows et al., ApJ, 2000
Rampp & Janka, A&A, 2002

MGFLD with geometric flux limiter, e.g. Bruenn et al., ApJ, 2001

- Solution 2 -

Adaptive mesh refinement → **Adaptive algorithm refinement** →

- hydrodynamics
- diffusion limit
- transport
- free streaming

Stellar evolution

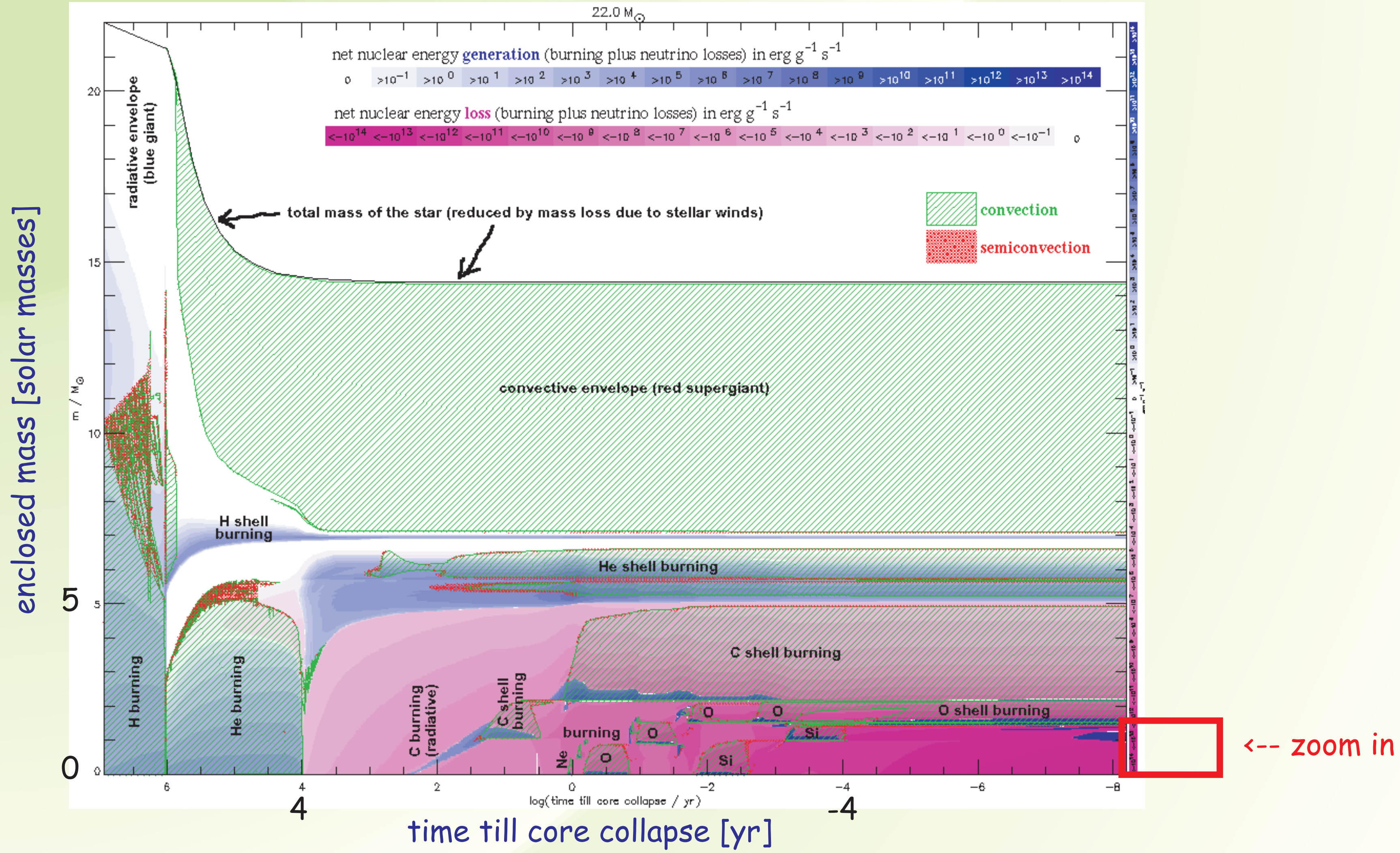
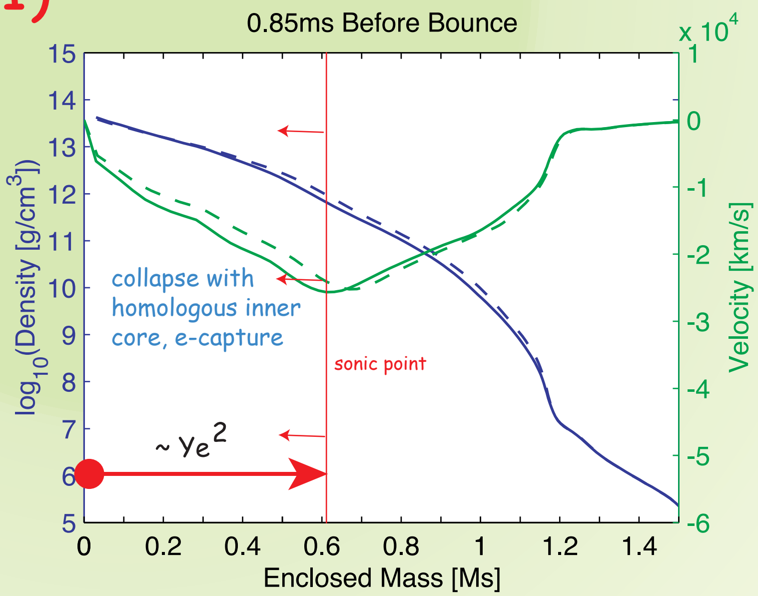


FIG. 6 Energy history of a 22 M star as a function of time till core collapse. The y-axis defines the included mass from the center. Hydrogen and helium core and shell burning are major energy sources. In the later burning stages, following oxygen core burning, neutrino losses related to weak processes in the stellar interior become increasingly important and can dominate over the nuclear energy production. Convection plays an important role in the envelope outside the helium burning shell, but also in shells during oxygen and silicon burning (from Heger and Woosley, 2001).

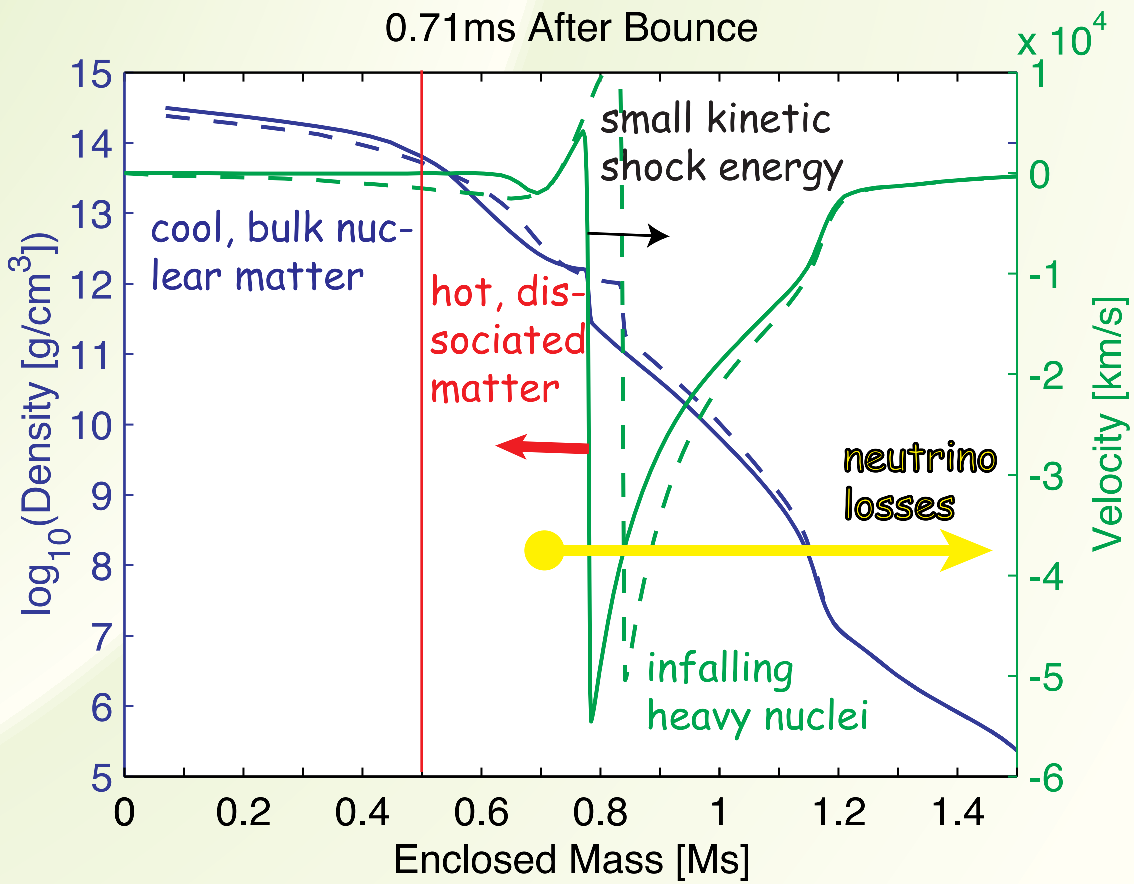
Neutrino-Driven Supernovae?

(1)

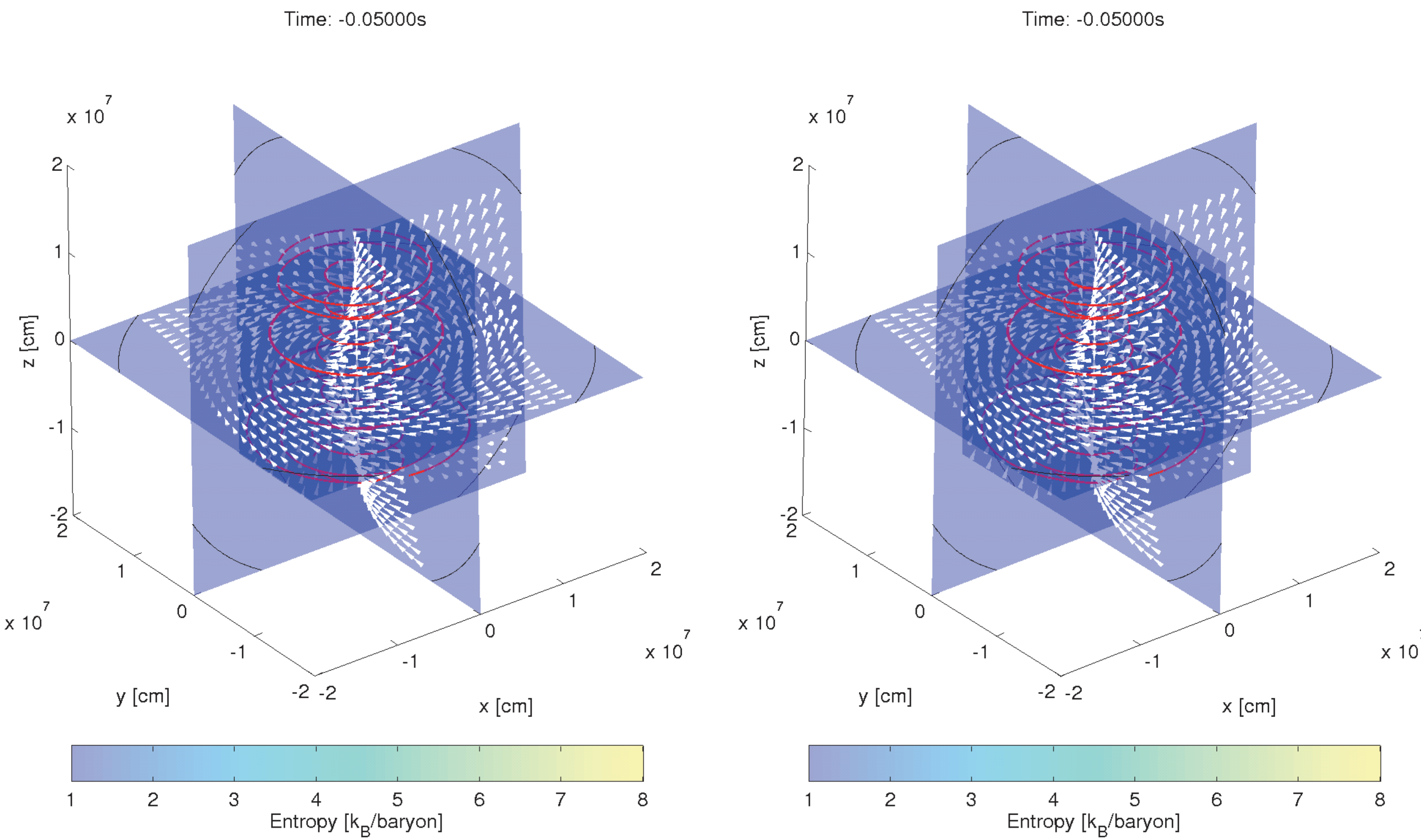


(2)

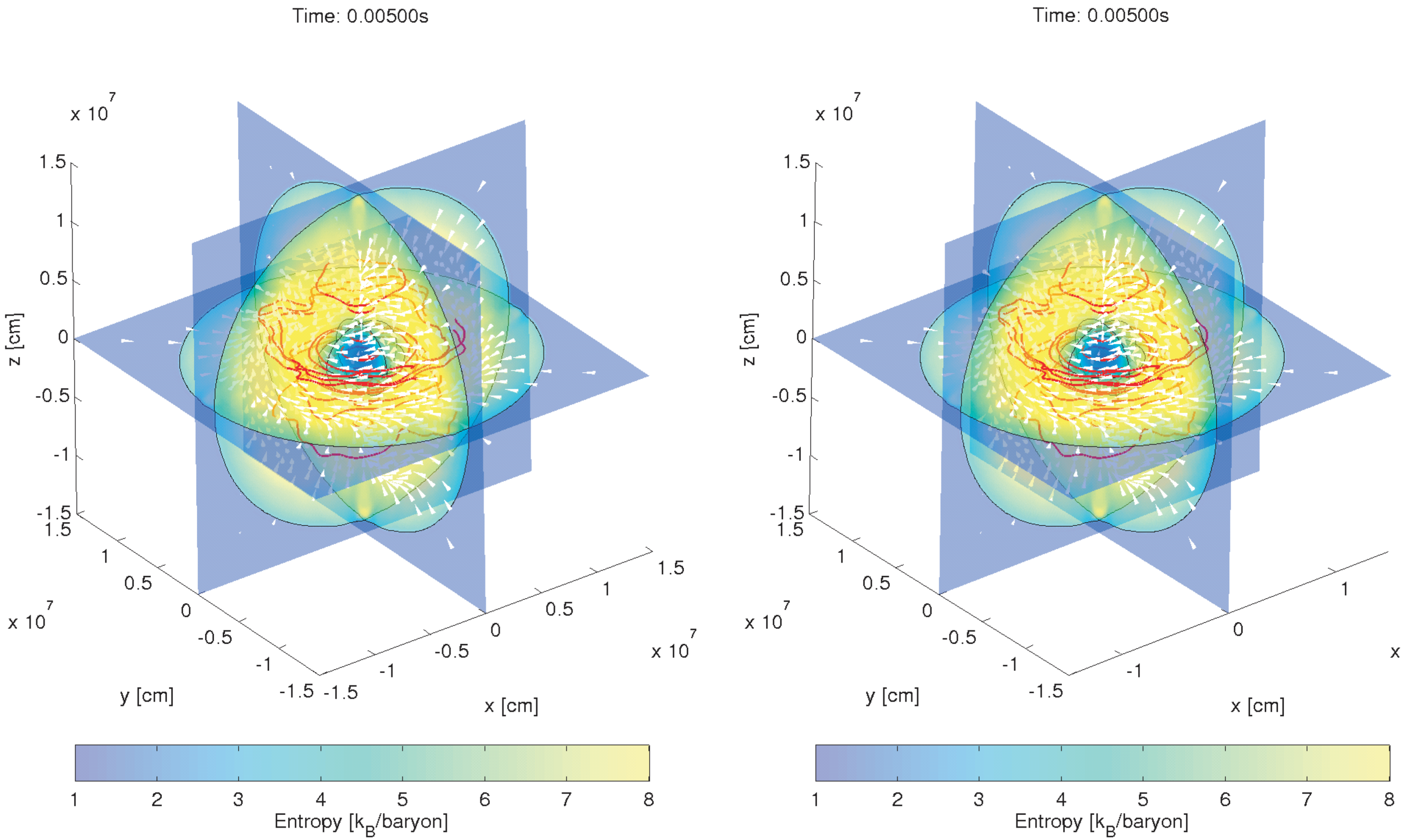
dashed = Newtonian
solid = general relativistic



- Bounce at nuclear density
- Dynamic bounce-shock
- Dissociation of infalling material costs ~10^51 erg per 0.1 Ms
- Neutrino losses from ~5ms after bounce on
- Conversion to accretion shock



This is a cross-view stereogram: Cross your view to see four blurry pictures in the background. Now relax the viewing angle until the inner two overlap. Then wait for focus without changing the viewing angle.



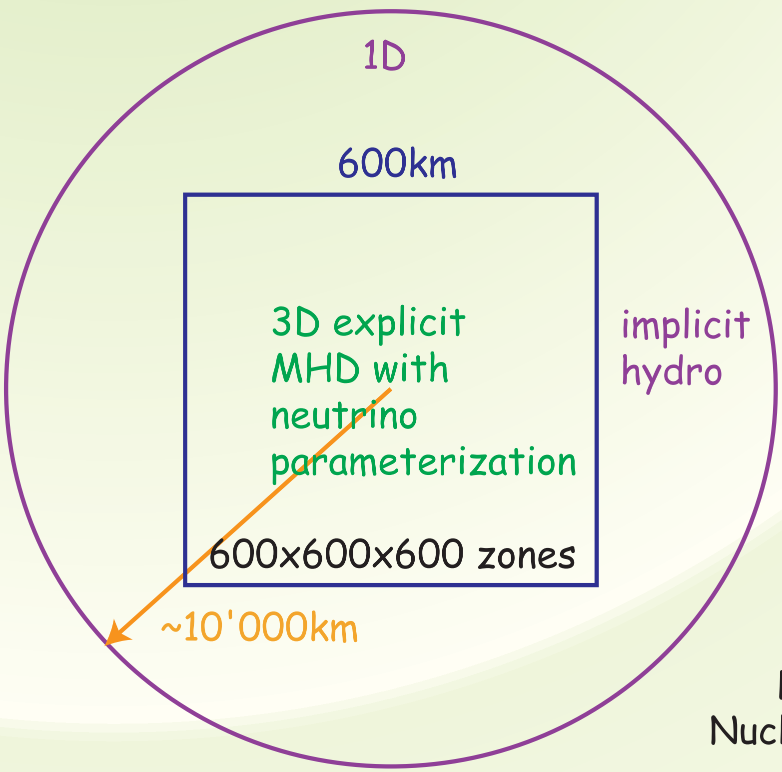
This is a cross-view stereogram: Cross your view to see four blurry pictures in the background. Now relax the viewing angle until the inner two overlap. Then wait for focus without changing the viewing angle.

3D MHD simulations

3D MHD (Pen, Arras & Wong, ApJS 2003)
TVD second order, energy-conserving and divergence free

New MPI implementation and velocity decomposition, loosely based on (Trac & Pen, astro-ph/0309599)

smooth bulk velocity field combined with entropy equation
turbulent local velocities combined with energy equation

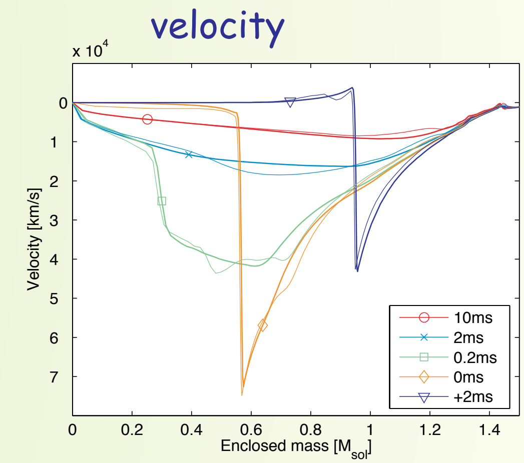


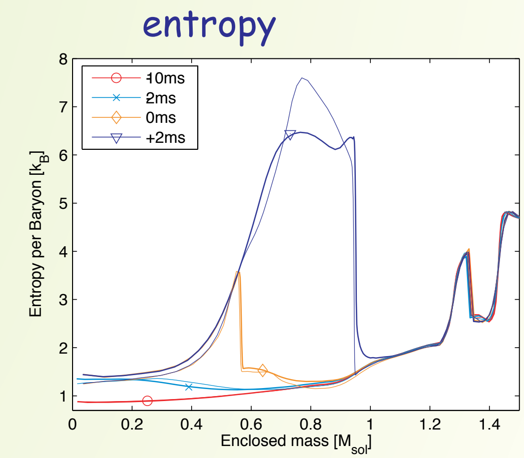
Lattimer-Swesty (1991) equation of state

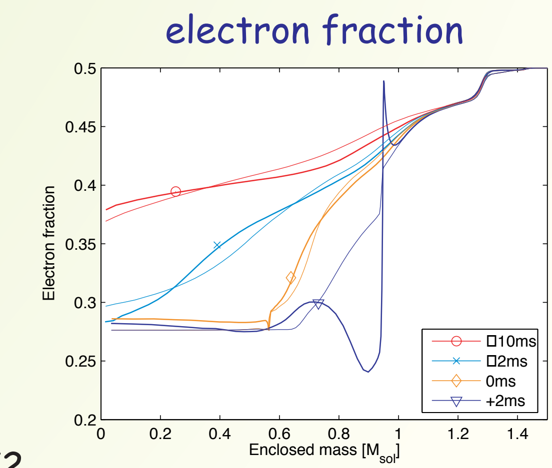
Shperical gravity, all Newtonian

Parameterization of neutrino physics for collapse phase

Liebendörfer, Pen, & Thompson
Nucl. Phys. A 2005; astro-ph/0504072



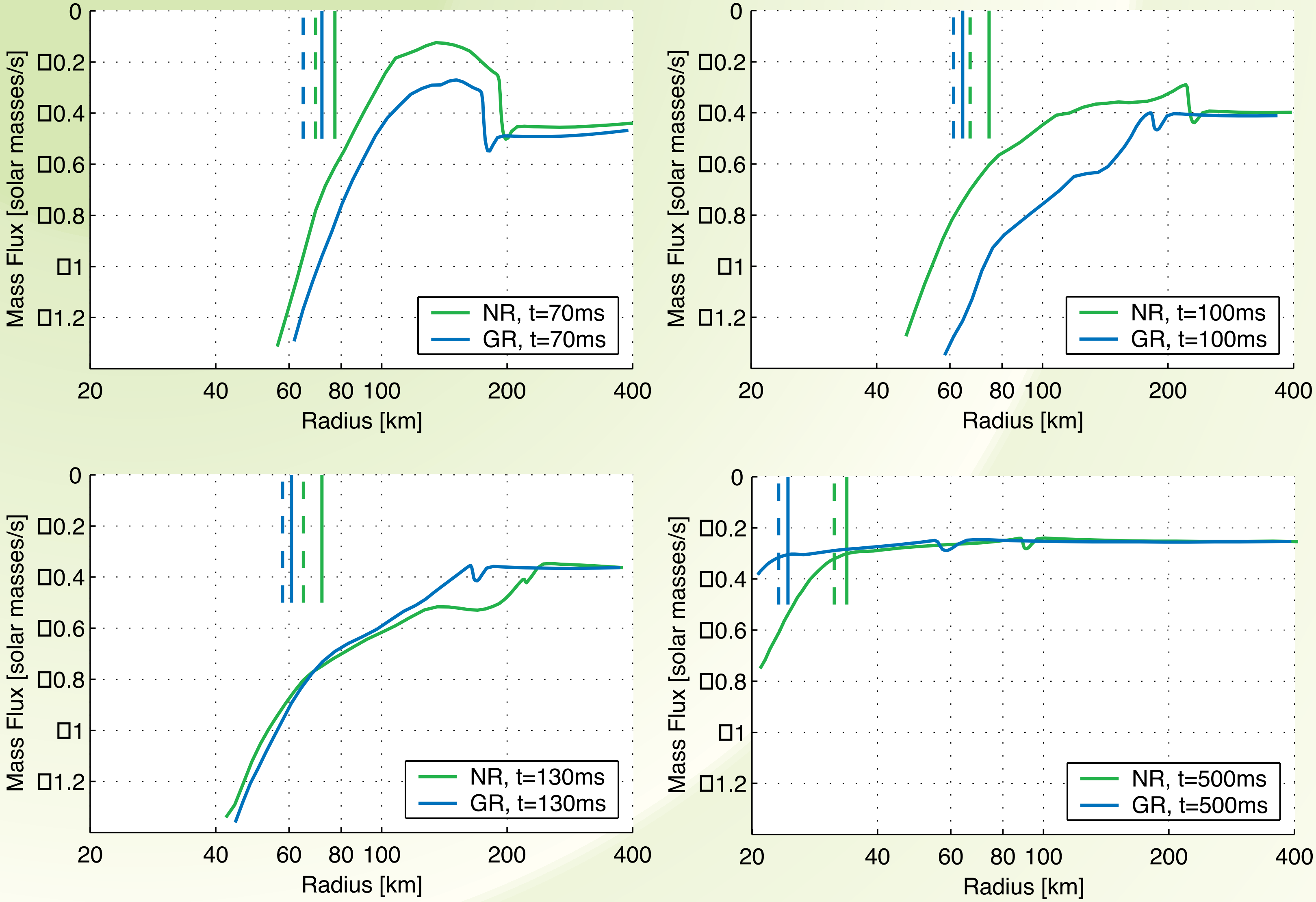




Time evolution of mass flux

rapid accretion

Liebendörfer et al. 2001



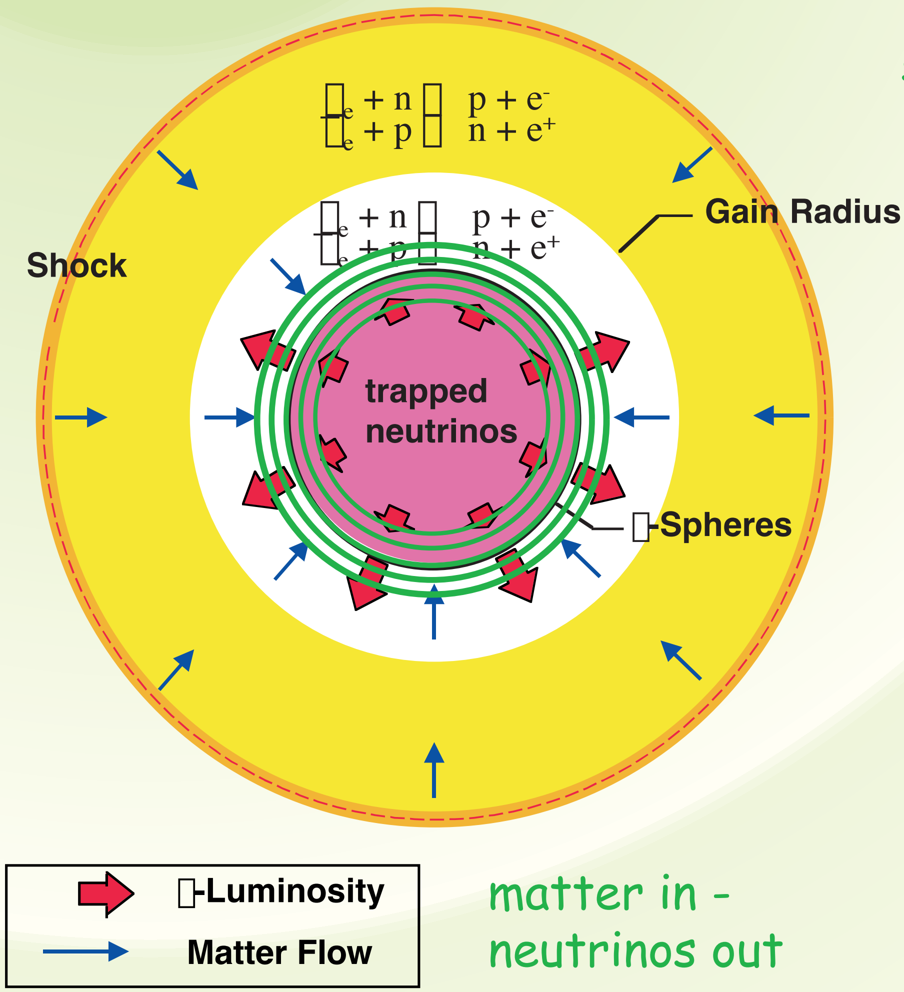
"It must be suspected that excessive neutrino emission in the cooling layer, causing mass and energy loss from the gain layer, may have been the main reason why spheri cally symmetric simulations ulti- mately failed to produce explosions"

Janka 2001

A side-effect? A surface-effect?

"It is important to note that one is not obliged to unbind the inner core ... as well; the explosion is a phenomenon of the outer mantle at ten times the radius (50-200 kilometers)."

Burrows & Thompson 2002



Extended layer with high neutrino opacities at subnuclear densities (roughly 10^12 g/cm^3)

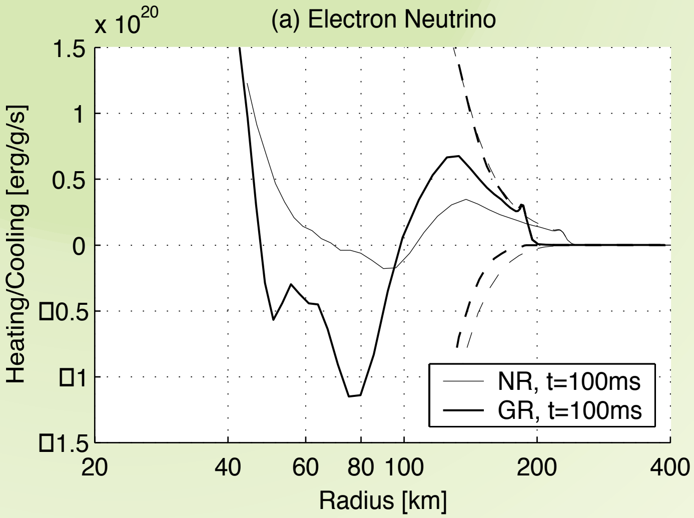
It shields the region with the history of core collapse and many uncertainties in microscopic physics input from the surface on a long neutrino diffusion time scale

This layer is convectively unstable in simulations that explode (e.g. Wilson & Mayle '93, Herant et al. '94) and convectively stable in simulations that don't (Buras et al. '03)

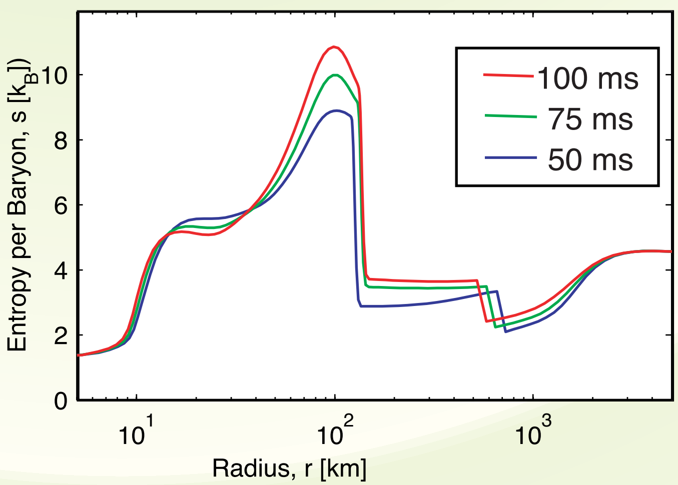
matter in -
neutrinos out

Convective Turnover in the Heating Region

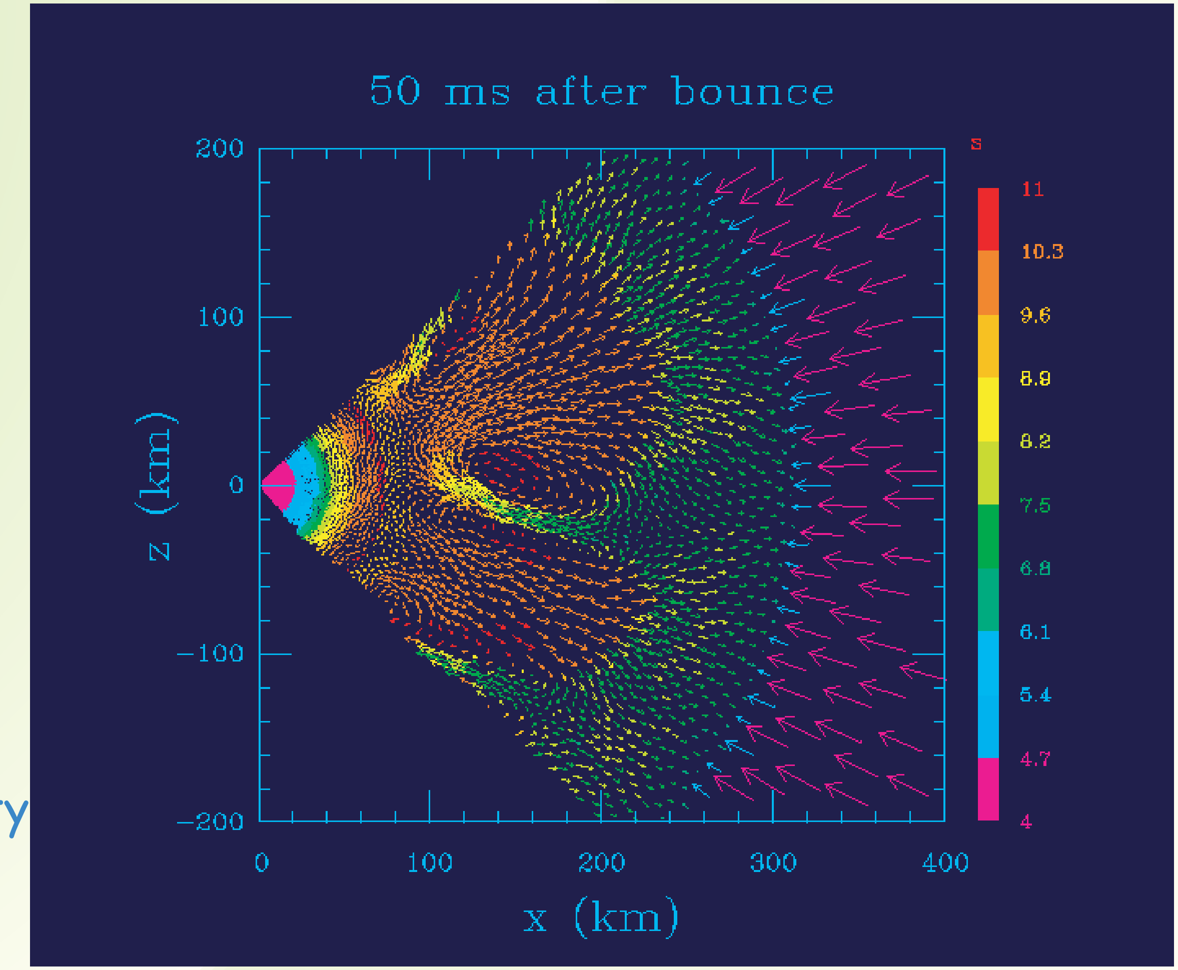
Neutrino heating is strongest at the base of the gain region:



The negative entropy gradient leads to hydrodynamical instability



1D GR simulations very pessimistic!



(Herant et al., ApJ, 1994)

- grey transport (Miller, Wilson & Mayle, 1993)
- grey transport (Burrows, Hayes & Fryxell 1995)
- parameter study (Janka & Mueller, 1996)
- energy dependent imported neutrino transport (Mezzacappa et al., 1998)

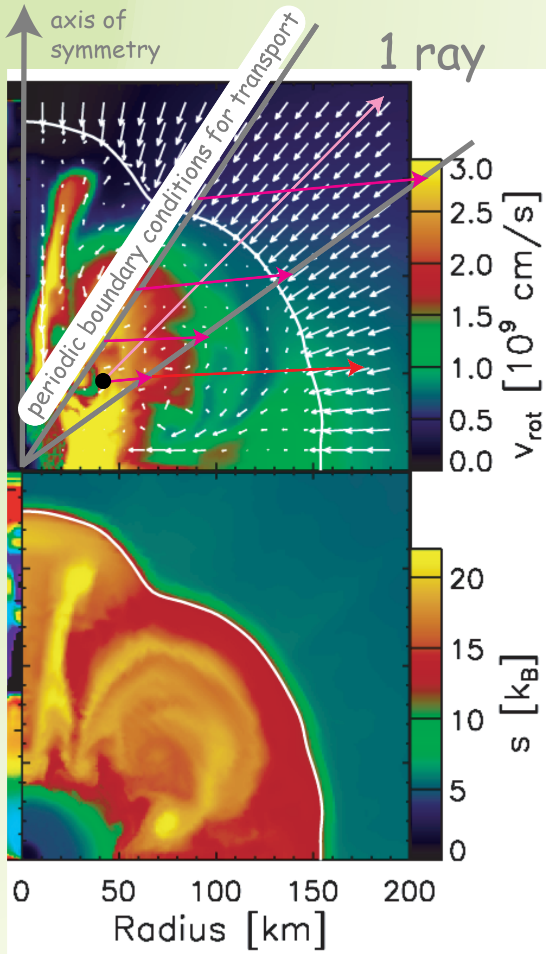
Current state-of-the-art

2D + ray-by-ray

Improved models of stellar core collapse and still no explosions: what is missing?

R. Buras, M. Rampp, H.-Th. Janka, K. Kifonidis, Phys. Rev. Lett., 2003

Snapshots of the stellar structure for the rotating model s15r at 198 ms after shock formation. The panels display the rotational velocity (top) and the entropy (in kB per nucleon) as functions of radius. The arrows indicate the velocity field, the white line marks the shock front.



2D MGFLD

R. Walder, A. Burrows, C.D. Ott, E. Livne, I. Lichtenstadt, M. Jarrah, astro-ph/0412187

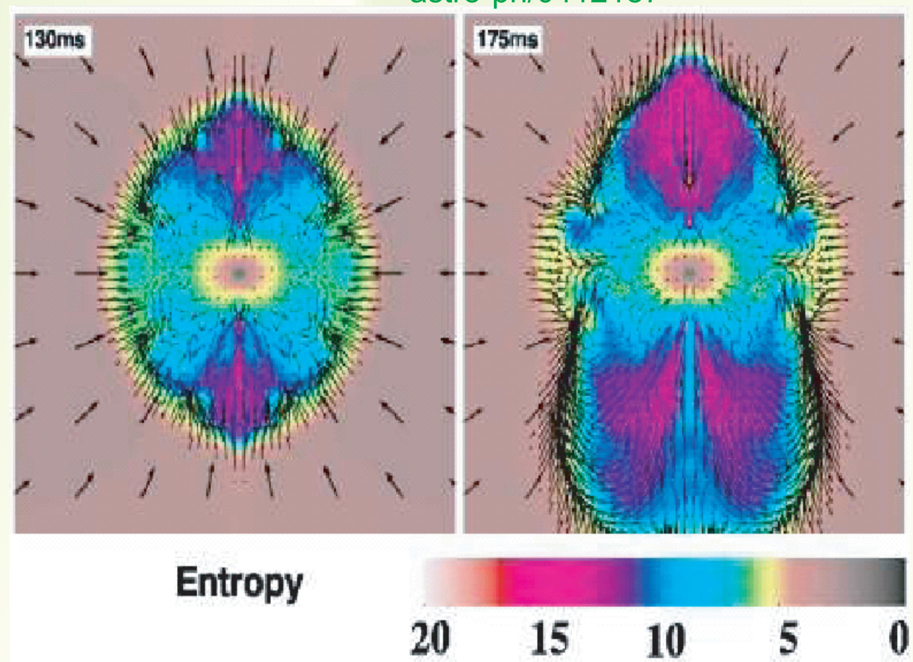


Fig. 4.— Color map of the entropy (per baryon per Boltzmann's constant), with the r-z velocities (arrows) superposed, of the fast rotating model A ($\Omega = 2.68\ s^{-1}$). Shown is the inner 600 km on a side at times 40 ms (upper left panel), 85 ms (upper right panel), 130 ms (lower left panel), and 175 ms (lower right panel) after bounce. The entropy in the polar direction is about a factor of two higher than in the equatorial regions at the same radius.

Milestones 2D Boltzmann

- Cardall & Mezzacappa, PRD, 2003
- Livne et al., ApJ, 2004

Simple neutrino physics

- Fryer & Warren, ApJ, 2002/4
- Kotake et al., PRD, 2004

Parameterized neutrino physics

- Scheck et al., PRL, 2003
- Liebendörfer et al., Nucl. Phys. A, 2005, astro-ph/0504072

Conclusions

- Semi-transparent neutrino transport in astrophysics happens in small volumes and on fast time scales!

- Discrete Ordinates techniques on large volumes suffer from the combination of low resolution, low-order advection, and artificially created cancellation of large quantities.

1D

- done!
- difficulties can be fixed, even in GR.
- different methods (MGFLD, VEF, SN) have been compared and give similar results on the scale of physical relevance.
- continue to be useful as reference and to explore new input physics.

2D

- on the path to success.
- different methods/approximations are being implemented.
- significantly more realistic than 1D, but still only weak explosions.
- How are results compared? How is stochastic information separated from deterministic information?

3D

- Explorations...
- feasible when based on approximations.
- interesting for MHD and the study of possible dynamo action.
- adaptive mesh refinement not sufficient, adaptive algorithms are needed!