3D rotating collapse to black holes

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Plan of the Talk

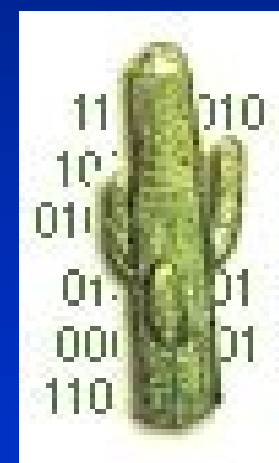
- Three years on Whisky…
- The good things about Whisky…
- The many things you can do with Whisky…
- 3D collapse to Kerr black holes
  - dynamics of matter, horizons, gravitational waves
- Conclusions and future work

- SISSA: L. Baiotti, P. Montero, LR, (B. Giacomazzo)
- AEI/LSU: I. Hawke, F. Loeffler, E. Seidel, (C. Ott)
- Valencia: T. Font; Thessaloniki: N. Stergioulas;
- Portsmouth: (A. Nerozzi); Munich: (B. Zink); Tuebingen: (D. Kobras)

The EU-network and the need of a Whisky...

Over the last three years, and funded through the European Network on Sources of GWS, we have developed **Whisky**: a 3D, parallel code in Cartesian coordinates, solving the relativistic hydrodynamics equations on a generic curved spacetime.

Whisky is *"coupled"* to **Cactus** from which it inherits parallelization, input/output infrastructures, portability on different platforms.



i.e., on each spacelike hypersurface:

Cactus → $G_{\mu\nu} = 8\pi T_{\mu\nu}$ ← **Whisky**

In practice...

The set of equations to solved besides the field equations is:

$$\nabla_\mu T^{\mu\nu} = 0 \quad ,$$

i.e. 4 conservation equations for the energy and momentum

$$\nabla_\mu (\rho u^\mu) = 0, \qquad\qquad p = p(\rho, \varepsilon)$$

i.e. conservation of baryon number and equation of state (microphysics input)

However, standard hydrodynamical quantities **(p, ε, v^i)** do not lead to a set of first-order PDEs in a flux-conservative form.

To hard-wire the conservative nature of the Euler equations, the *"primitive variables"* are replaced by the "*conserved variables*":

$$\begin{Bmatrix} \rho \\ \varepsilon \\ v^i \end{Bmatrix} \rightarrow \begin{Bmatrix} D \equiv \rho W \\ S_j \equiv \rho h W^2 v_j \\ E \equiv \rho h W^2 - p \end{Bmatrix} \quad where \quad h \equiv 1 + \varepsilon + \frac{p}{\rho}; W \equiv 1 - v^i v_i$$

As a result, a first-order flux-conservative form can be obtained

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Leftrightarrow \quad \frac{1}{\sqrt{-g}}\left(\frac{\partial \sqrt{\gamma}\,\overline{U}(\overline{w})}{\partial t} + \frac{\partial \sqrt{-g}\,\overline{F}^{i}(\overline{w})}{\partial x^{i}} \right) = \overline{S}(\overline{w})$$

Where $\overline{U}(\overline{w}) = (D, S_j, E - D)$ is the vector of conserved variables

$$\overline{F}^{i}(\overline{w}) = \left(D\left(v^{i} - \frac{\beta^{i}}{\alpha}\right), S_j\left(v^{i} - \frac{\beta^{i}}{\alpha}\right) + p\delta^{i}_{j}, E - D\left(v^{i} - \frac{\beta^{i}}{\alpha}\right) + pv^{i} \right) \qquad \text{"fluxes"}$$

$$\overline{S}(\overline{w}) = \left(0, T^{\mu\nu}\left(\frac{\partial g_{\nu j}}{\partial x^{\mu}} - \Gamma^{\delta}_{\mu\nu} g_{\delta j} \right), \alpha\left(T^{\mu 0}\frac{\partial \ln \alpha}{\partial x^{\mu}} - T^{\mu\nu}\Gamma^{0}_{\mu\nu} \right) \right) \qquad \text{"sources"}$$

Despite the added complications of a larger set of equations, progress of relativistic hydrodynamics in full GR has been good and steady!

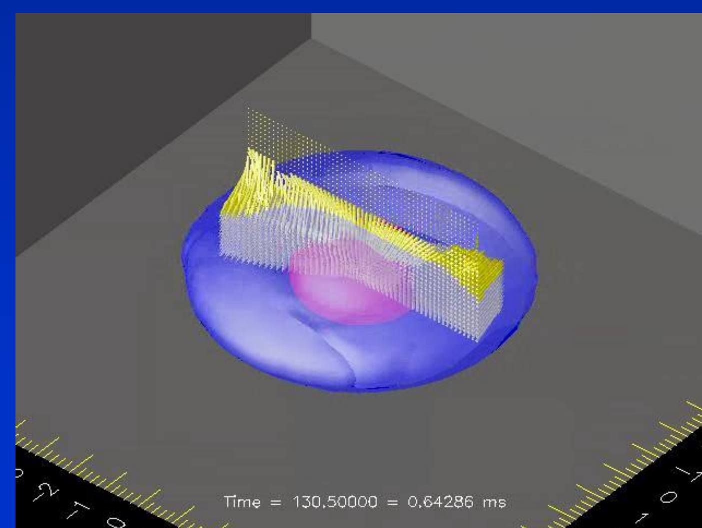
The good things about Whisky:

- <u>High-Resolution Shock-Capturing techniques</u>
  - this guarantees very high accuracy, correct treatment of shock; very small numerical dissipation
- <u>Method of Lines (MoL)</u>
  - any stable ODE integrator can be used and the control of the truncation error is *transparent;* the coupling between different treatments in the hydro or spacetime is minimized;
- <u>Excision of matter and fields</u>
- Fixed/Progressive Mesh Refinement

The things you can do with Whisky:

- Neutron star oscillations (linear and nonlinear)
- Dynamical (barmode) instability
- Differentially rotating collapse
- Binary neutron stars
- Uniformly rotating collapse

(Baiotti et al PRD 2005, Baiotti et al. PRL 2005)



Whisky's first application: 3D collapse to a bh

Gravitational collapse is among the most common phenomena in astrophysics and still remains among the most challenging processes to be investigated in full GR.

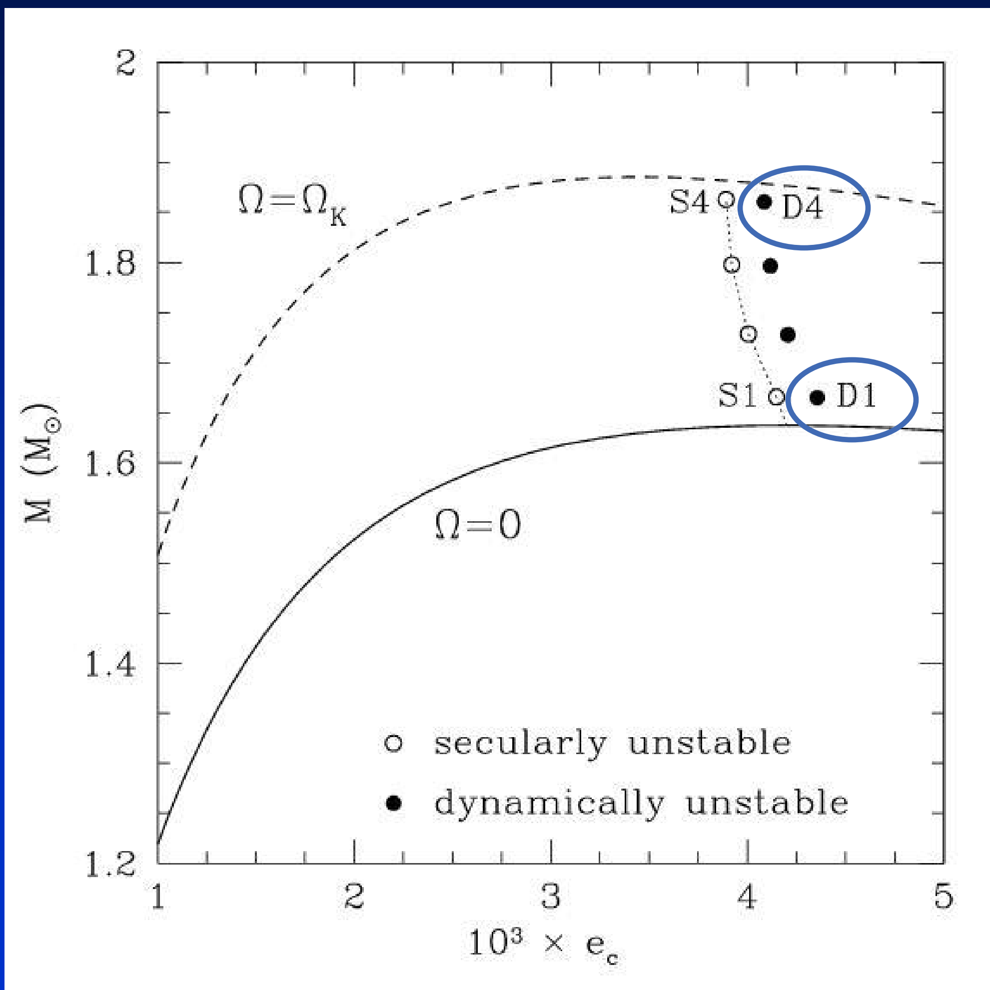
Important milestone calculations:

- May and White ('67): 1D, Lagrangian hydrodynamics, artificial viscosity
- Stark and Piran ('85): 2D, Eulerian hydrodynamics, artificial viscosity
- Shibata; Shibata, Shapiro & Baumgarte ('00-'03), 2D or 3D, Eulerian hydrodynamics; artificial viscosity (or HRSC in 2D)

However, important questions remain with only approximate answers:

- How massive is the black hole formed?
- How rapidly is it spinning?
- How much gravitational radiation is produced?
- What waveforms are expected?

Initial Data: uniformly rotating polytropes



We have built sequences of uniformly rotating polytropes with constant angular momentum.

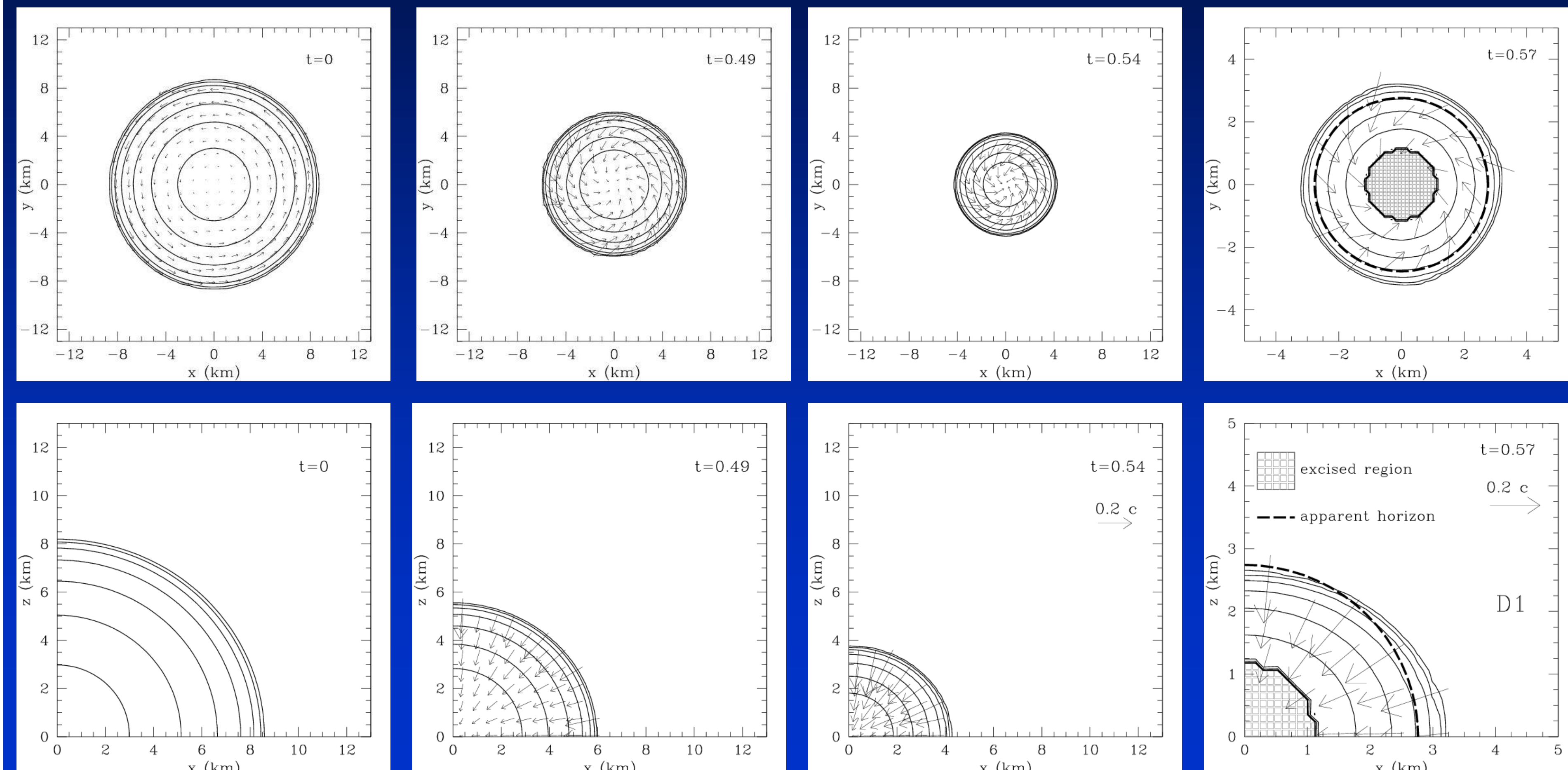
To compare with previous studies, here the initial stellar models are modeled as polytropes with EOS

$$p = K\rho^{\Gamma};\ \textit{or}$$

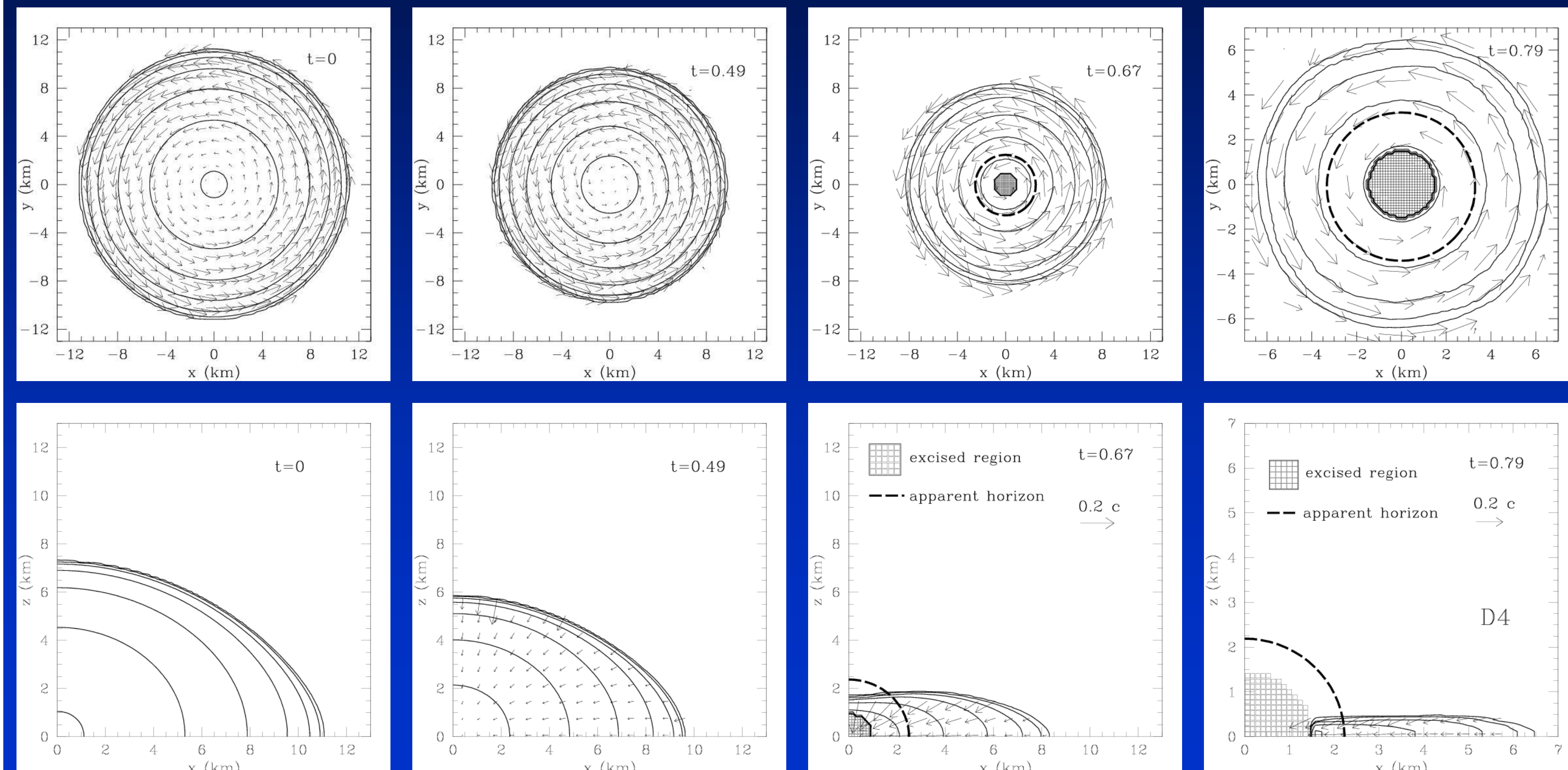
$$p = \rho\varepsilon(\Gamma - 1);$$

$$K = 100, \qquad \Gamma = 2$$

Collapse of D1: slowly rotating model



Collapse of D4: rapidly rotating model



Collapse of D4

Initial Data:

Rapidly rotating neutron star near mass-shedding:

$M = 1.66\, M_{\odot}$

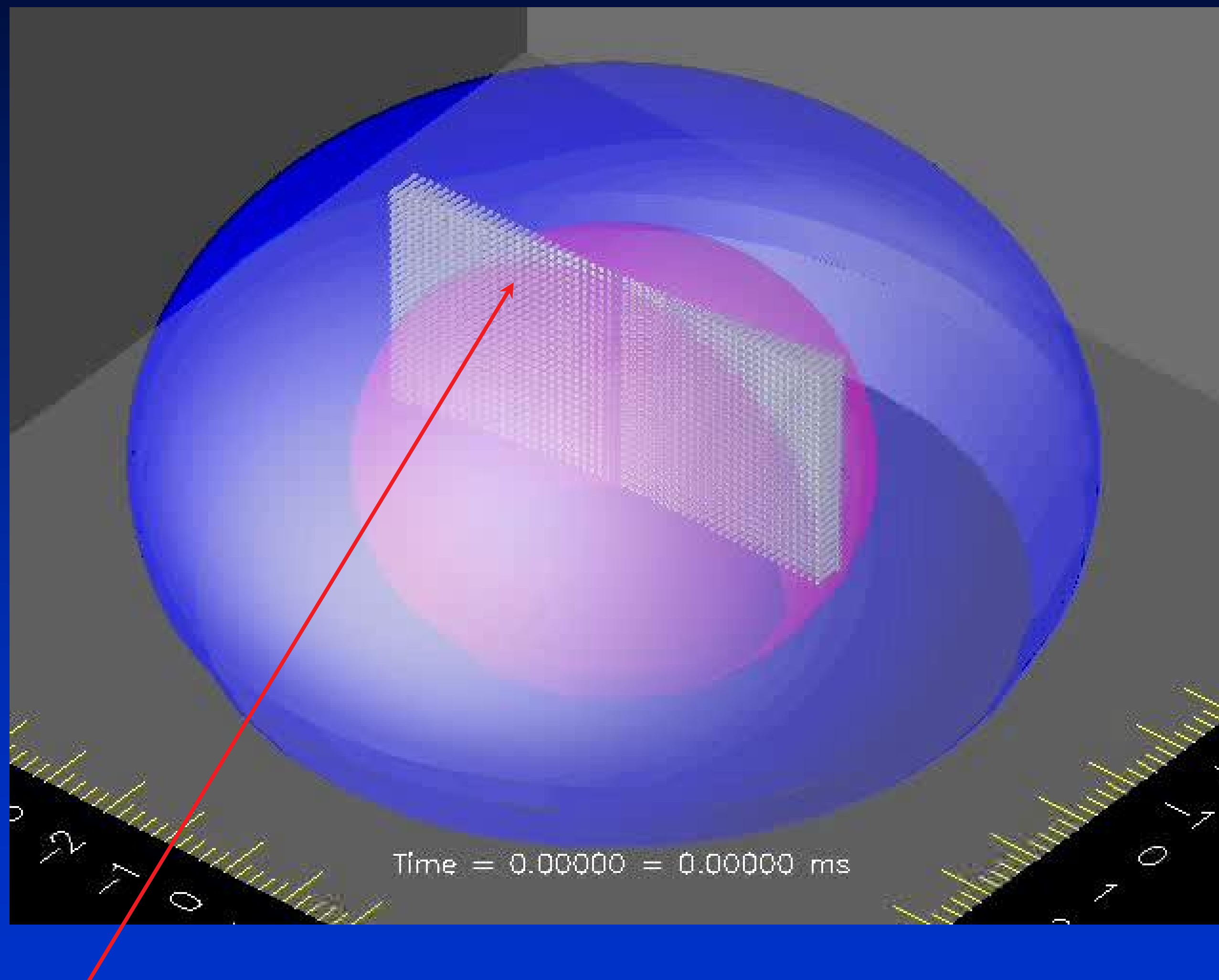
$r_p/r_e = 0.65$

$J/M^2 = 0.543 < 1$

(i.e. no maximal Kerr can be produced)

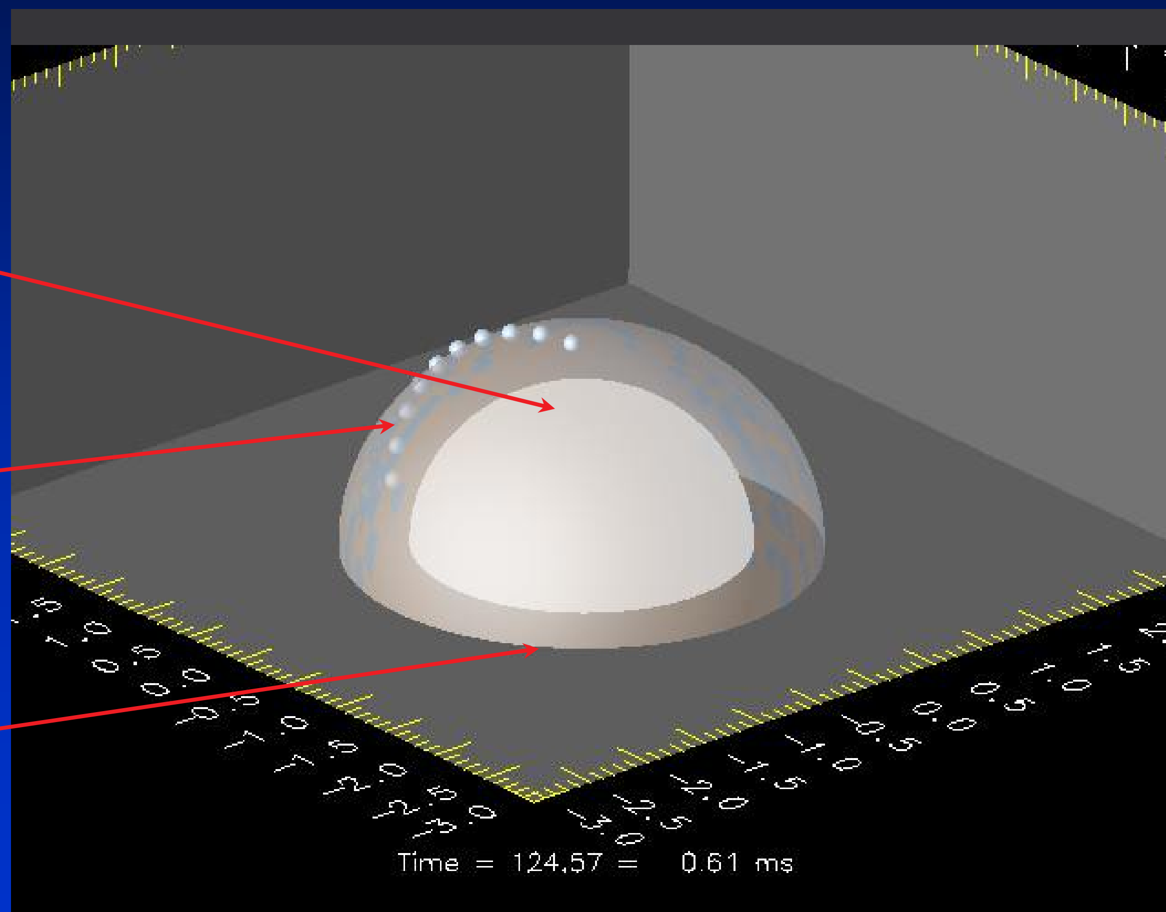
$T/|W| = 0.0767 < 0.27$

(i.e. no dynamical instability is expeected)



Dynamics of trapped surfaces

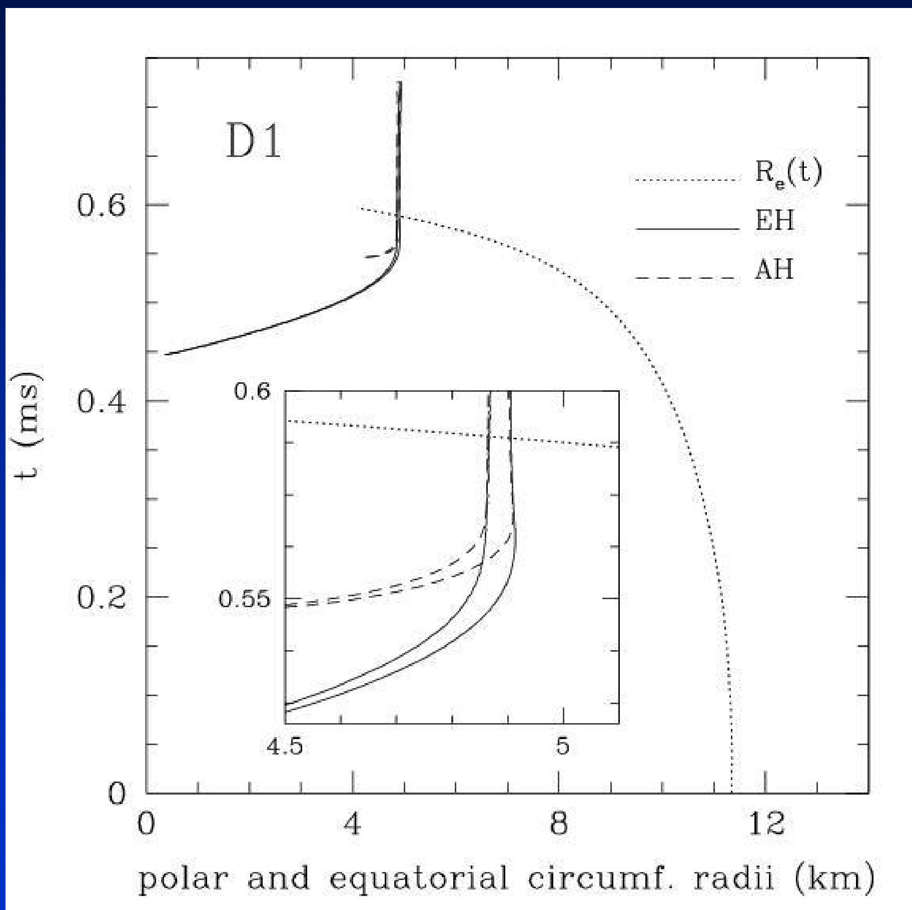
- White surface: apparent horizon
- Filled circles: event horizon generators
- Grey surface: event horizon

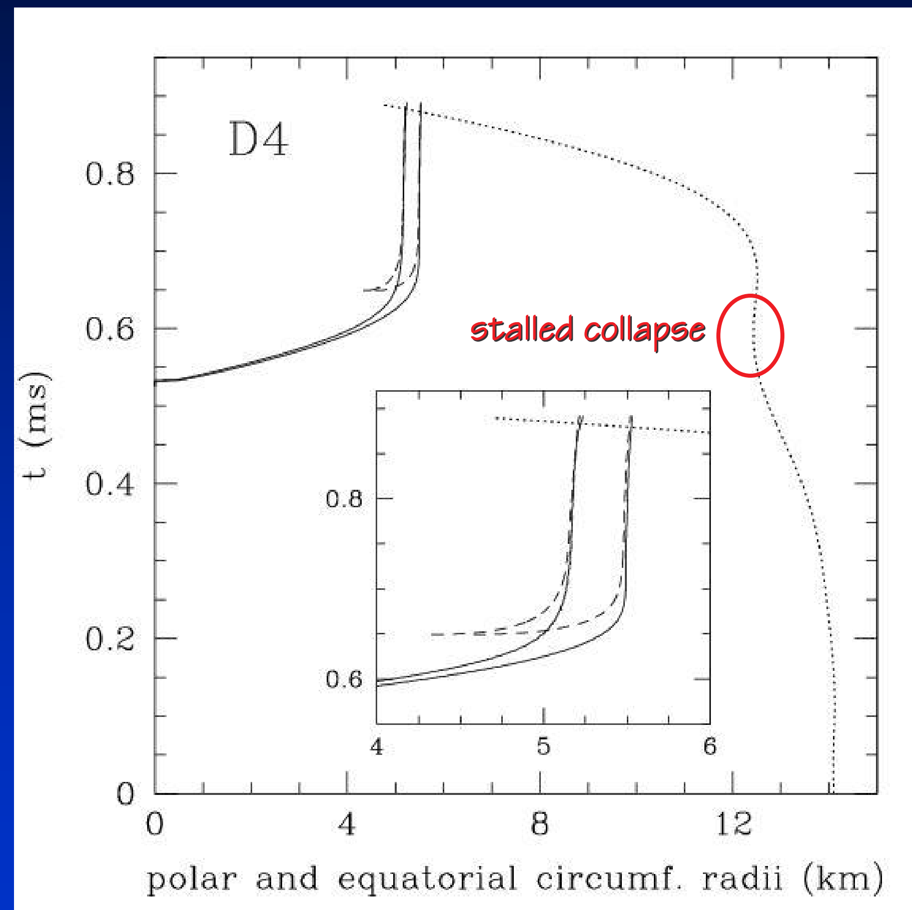


Visualization by P. Diener (LSU)

Reconstruncting the global spacetime

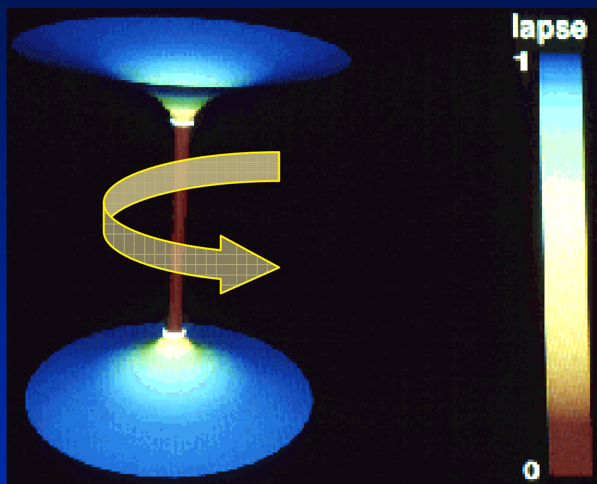
Slowly rotating star | *Rapidly rotating star*





Each point on these diagrams summarizes ~10^{10} operations!

Measuring the black hole...



+

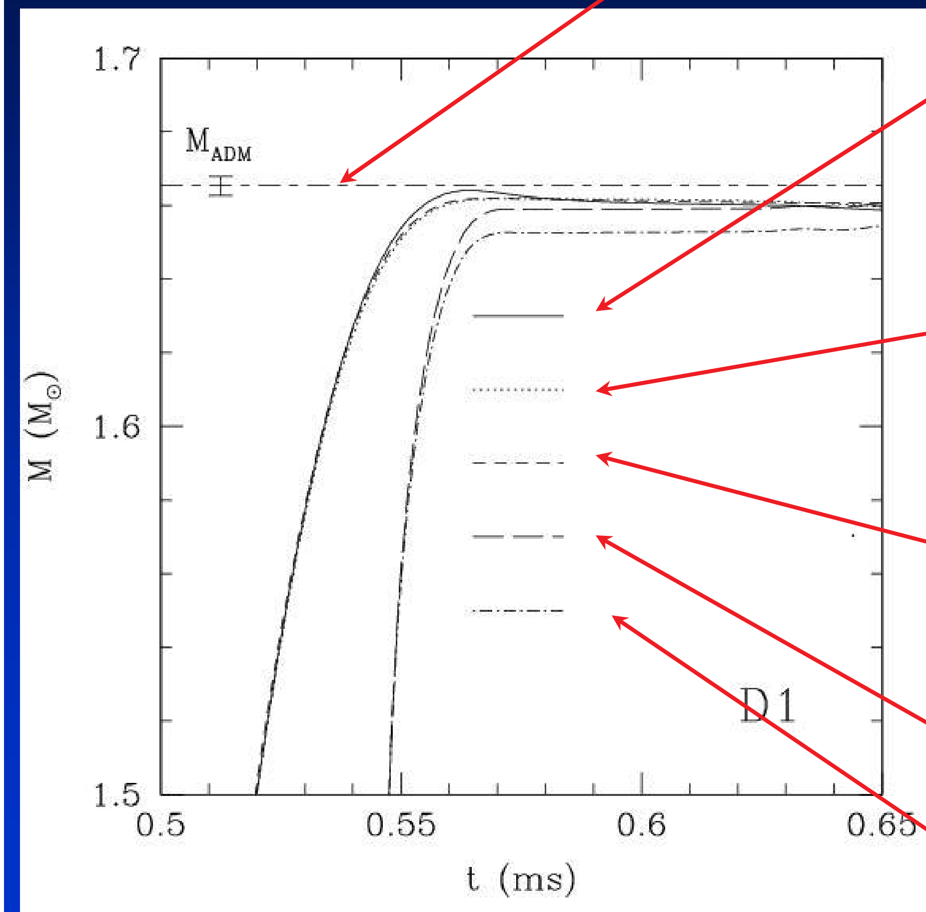
= ?...

Not a trivial task at all !...

- All the measures are not asymptotic (gauge dependence)
- The bh produced is not (yet) stationary and thus not (yet) Kerr
- During the simulation matter is still falling and gws are emitted

Measuring the bh mass: slowly rotating star

M_{ADM} from initial data using a compactified space



$$M = \frac{(C_{eq})_{eh}}{4\pi}$$ (true for Kerr if eh is found)

$$M^2 = \frac{A}{8\pi}\left(\frac{M_{hor}}{a}\right)^2\left[1-\sqrt{1-\left(\frac{a}{M_{hor}}\right)^2}\right]$$

(using fit to the bh quasi-normal ringing)

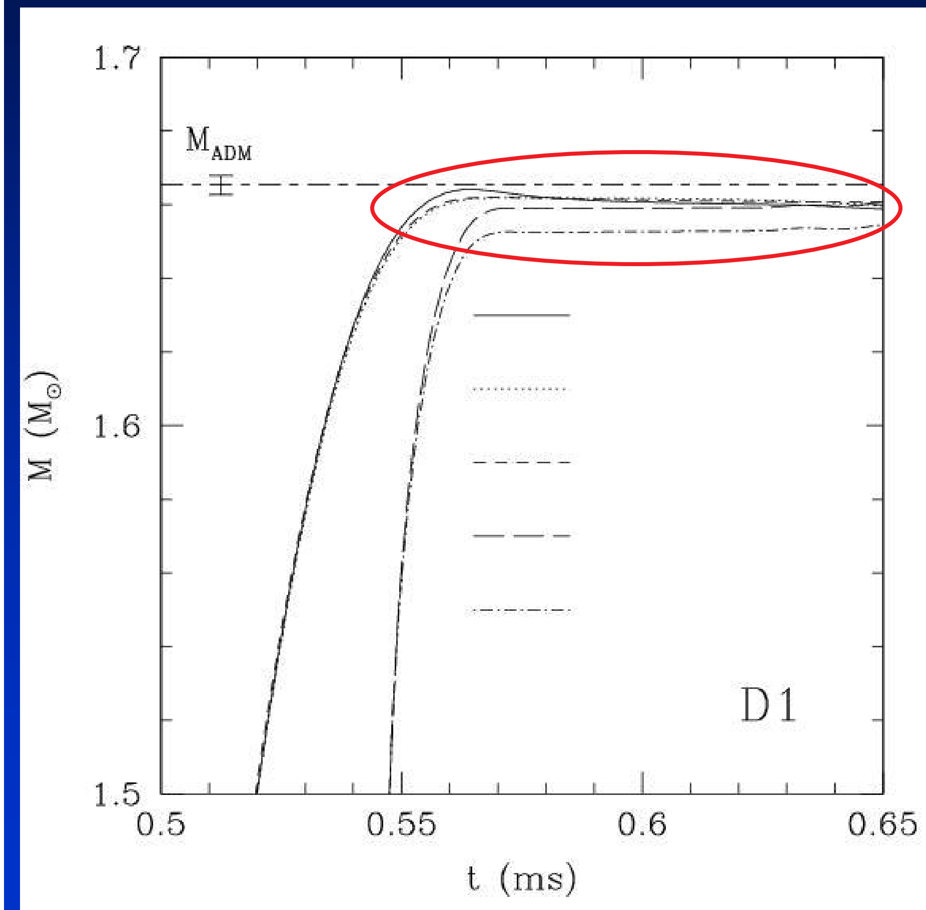
$$M^2 = \frac{A_{eh}}{16\pi - 4A_{eh}\omega_{eh}}; \quad \omega_{eh} \equiv -\frac{g_{t\phi}}{g_{\phi\phi}}$$

(horizon generators if the eh is found)

Isolated/dynamical horizon framework

$$M^2 = \frac{A_{eh}}{16\pi}$$ irreducible mass from eh

Measuring the bh <u>mass</u>: **slowly** rotating star



SUMMARY:

Different approaches provide comparable mass estimates.

Even the irreducible mass is only few percent off the expected value. We can set a <u>measured</u> upper limit of 0.5% M_{ADM} to the energy lost to GWs.

The error is larger for the rapidly rotating case, upper limit of 0.5% M_{ADM}.

The true value is going to be much smaller (more later).

What about waveforms?...

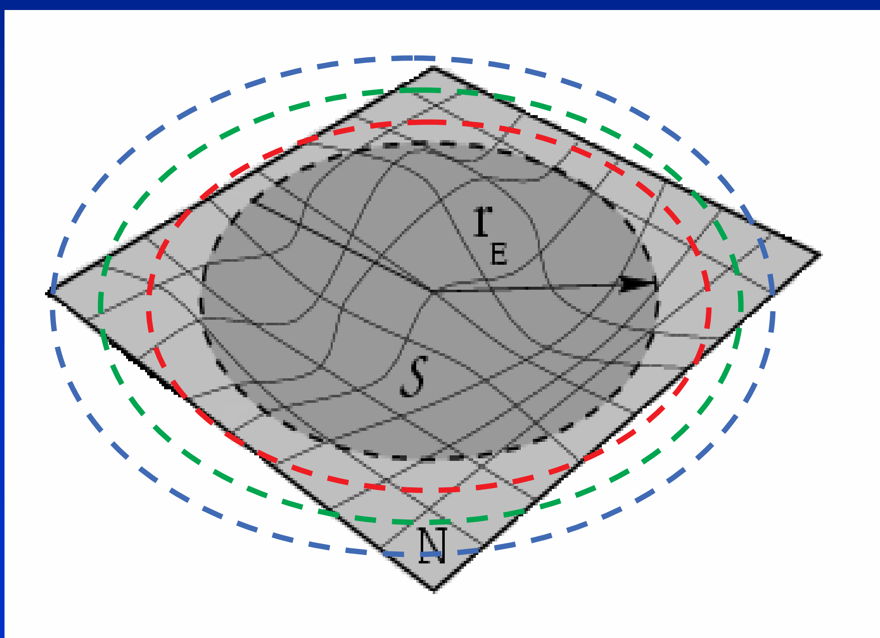
This is the ultimate goal of all the collapse simulations. Progress in 3D simulations so far has been hindered by:

- use of Cartesian grids (Stark & Piran in '85 were using non-uniform spherical polar coordinates in 2D)
- use of uniform grids and present computer resources impose to have outer boundaries very close to the source (not enough memory)
- signal intrinsically very small ($\Delta M/M \sim 10^{-6}$-10^{-7})

Wave-extraction techniques

Several ways of extracting gws from relativity codes (perturbative matching, characteristic matching, conformal compactification, Weyl scalars, etc.) all with different degrees of success.

We use a *gauge-invariant extraction* of perturbations of a *Schwarzschild* black hole.



"*Observers*" placed on 2-spheres at r_E decompose the metric into tensor spherical-harmonics to calculate the odd and even-parity perturbations Q_{lm} of a Schwarschild black hole (LR et al., PRD 1998)

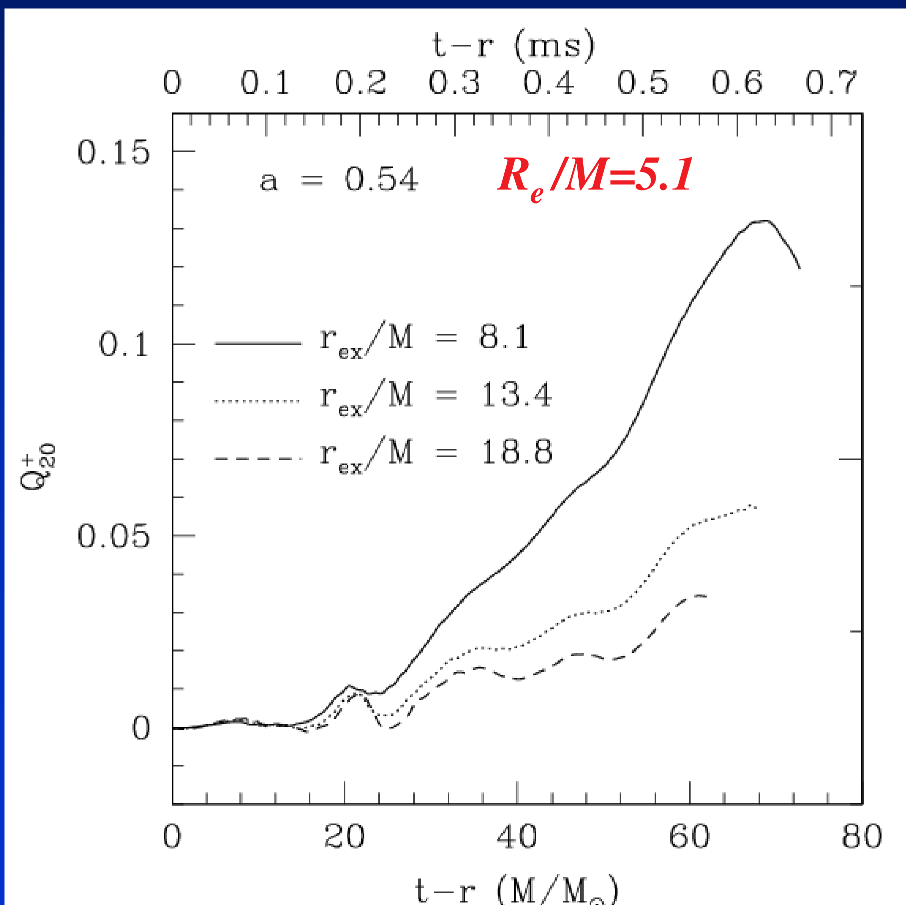
All of this can be repeated on several nested 2-spheres.

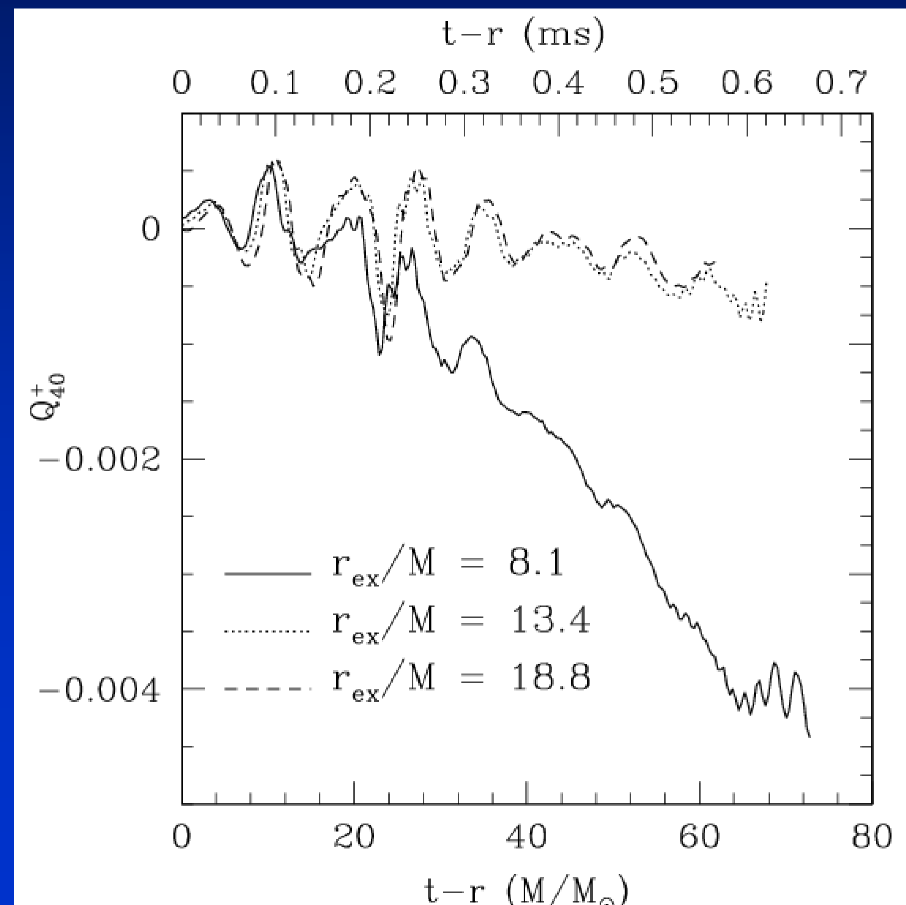
The functions Q_{lm} are then used to reconstruct the asymptotic gw quantities: h_+, $h_\times$, E, J, etc.

Results with uniform grids

Asymptotic form for Q_{20} from the simulation discussed before (typical unigrid extraction)

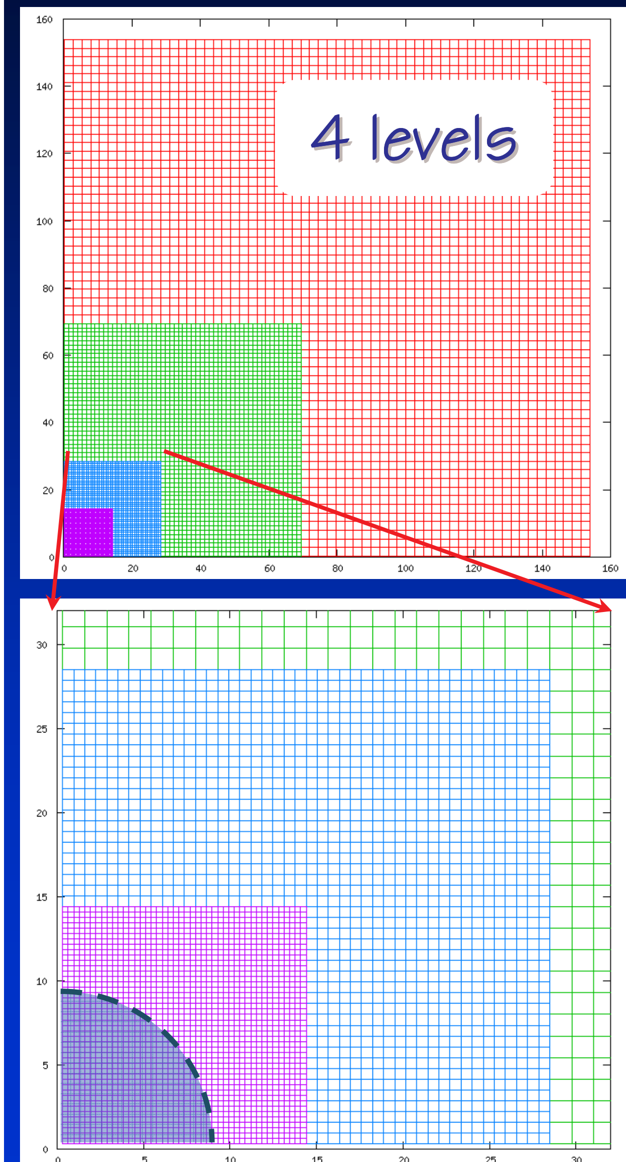
The same but for Q_{40}. Note that the high-order modes converge more rapidly



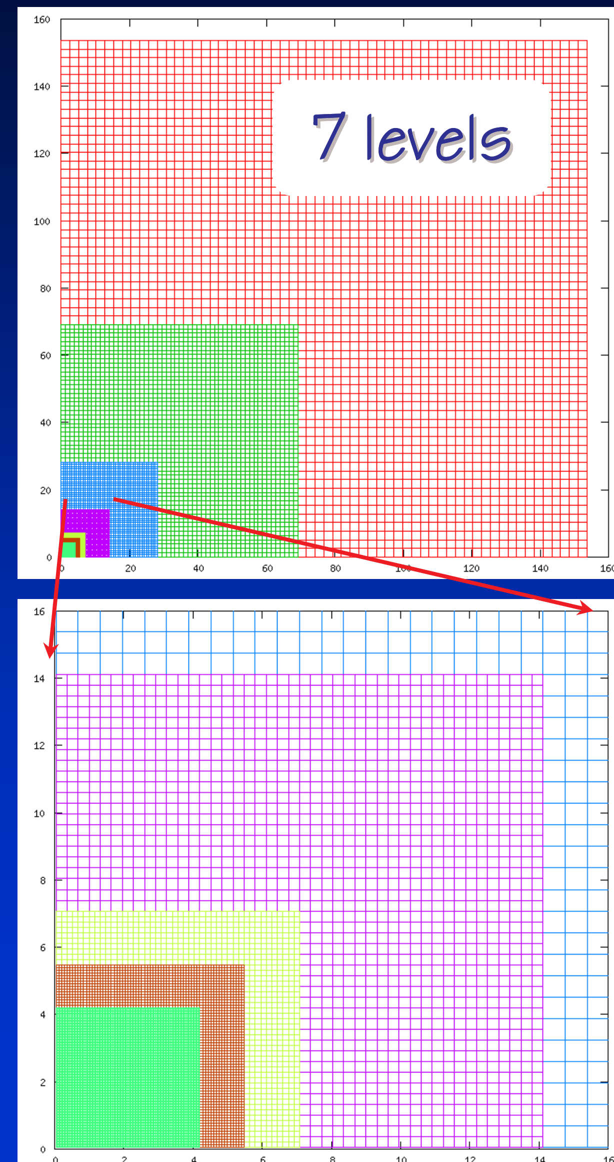


Carpet: progressive fixed mesh refinement

Initial meshes



Final meshes



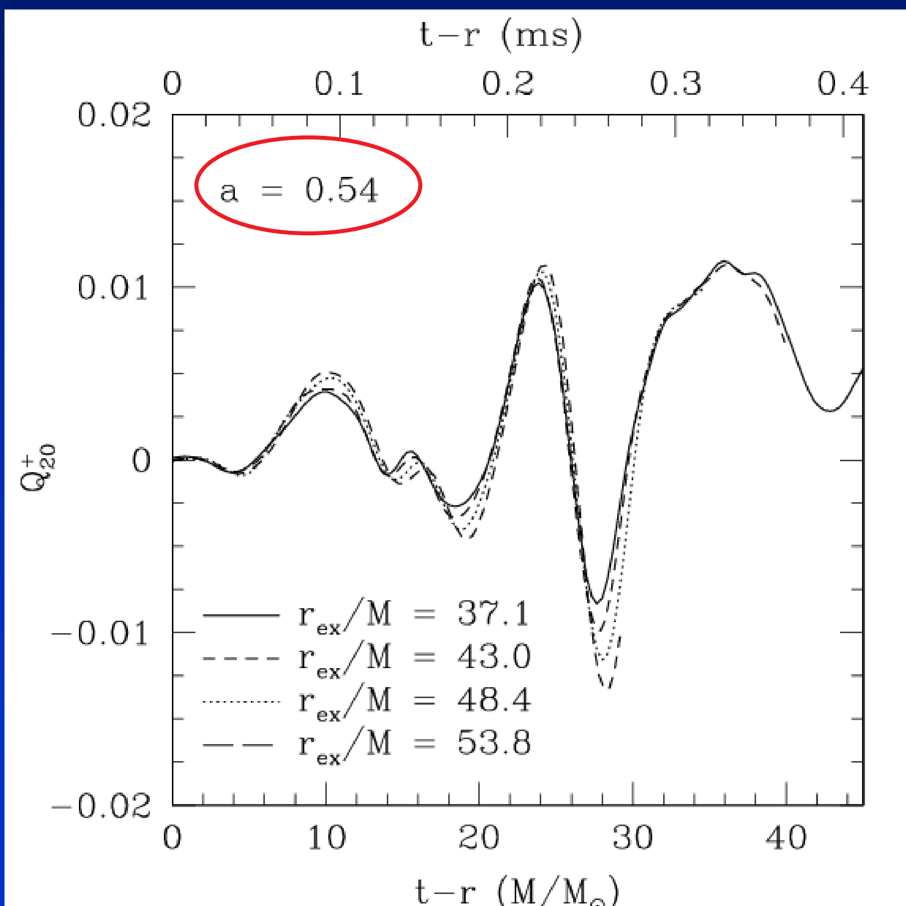
- Carpet has been recently incorporated in the Cactus-Whisky codes and has removed the constraints of unigrids (E. Schnetter, AEI).
- Carpet allows for fixed meshes to be set up. While the meshes are fixed in location, they can be "switched on" during the evolution or moved through the grid.
- Here we have used 7 levels of refinement with $\Delta x/M$=0.02-1.38.

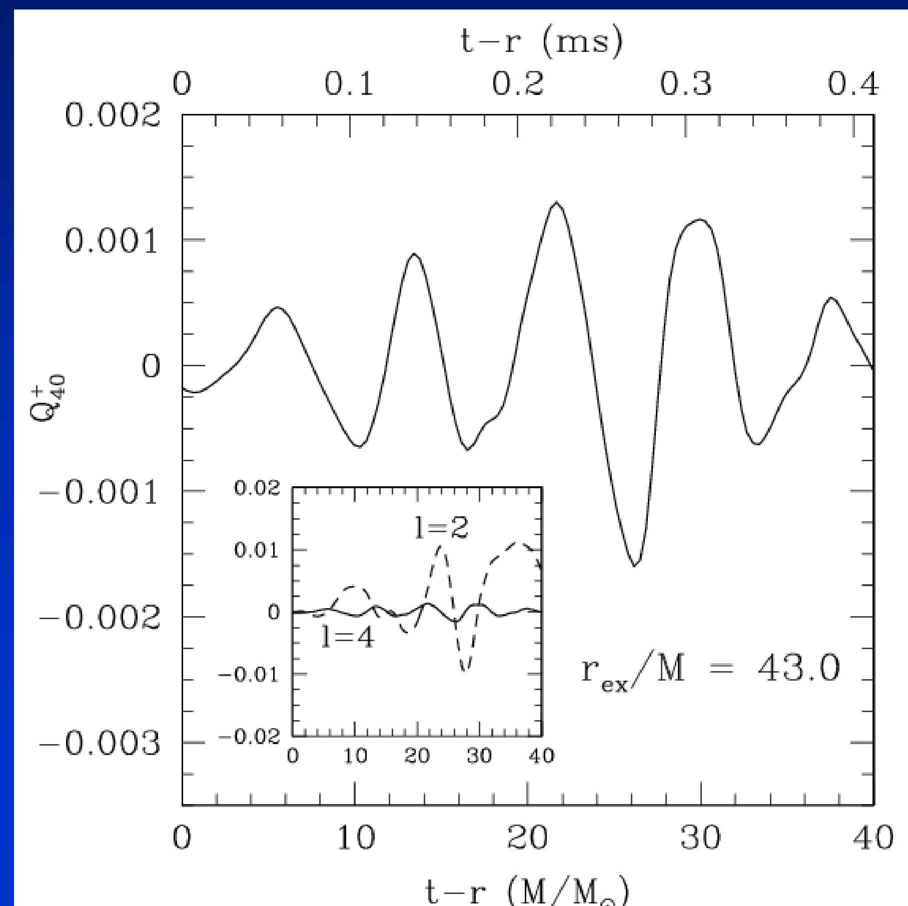
Waveforms at last!...

The overlapping of waveforms extracted by different detectors indicate the reached wave nature.

Most of the signal is in the lowest multipole: Q_{20}

Note that $h_+ >> h_\times$ (essentially axisymmetric spacetime)



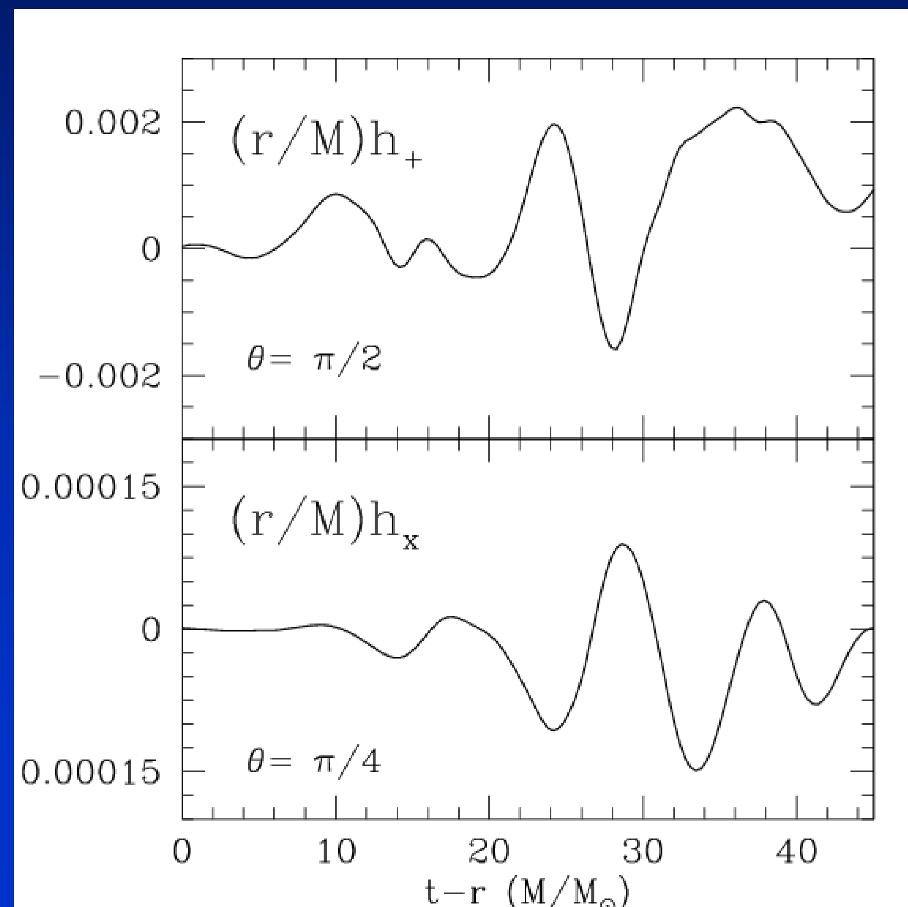


Waveforms at last!...

Once computed, the different multipoles Q^{+}_{lm} and $Q^{\times}_{lm}$ can be combined to obtain the asymptotic gravitational wave amplitudes in the TT-gauge (we use up to l=m=5).

$$h_{+} - \mathrm{i}h_{\times} = \frac{1}{2r}\sum_{\ell m}\Big(Q^{+}_{\ell m} - \mathrm{i}\int_{-\infty}^{t} Q^{\times}_{\ell m}(t')dt'\Big)\,{}_{-2}Y^{\ell m}$$

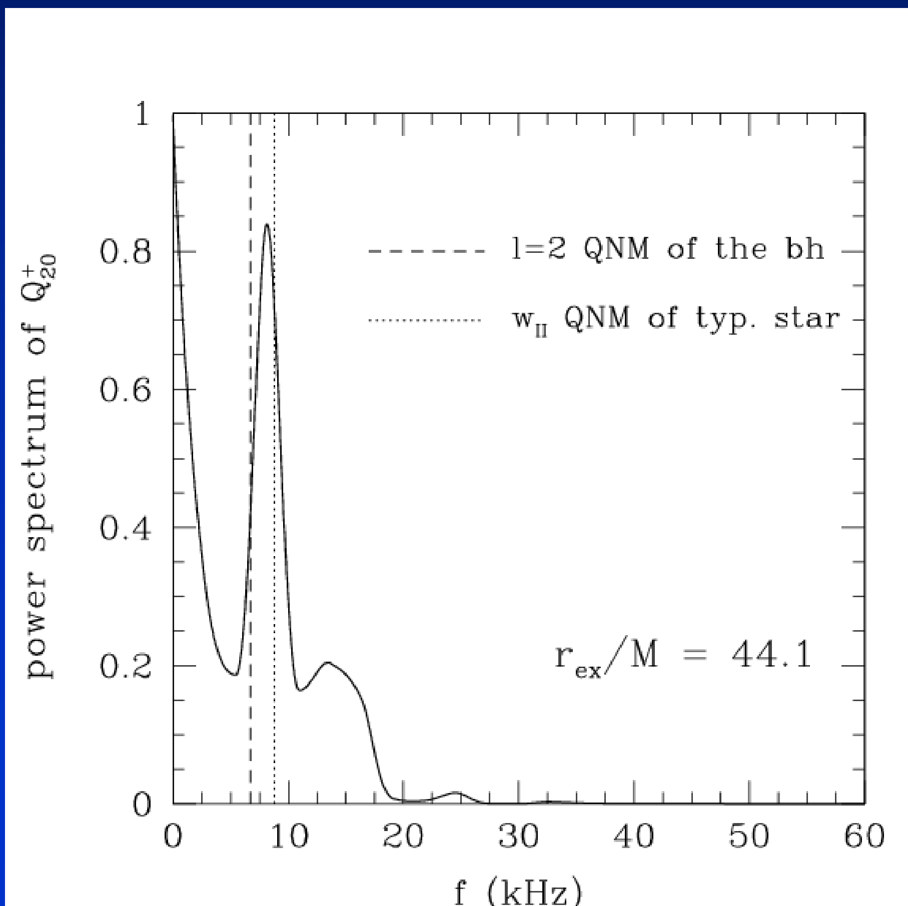
- $h_{+} >> h_{x}$ (the collapse is essentially axisymmetric)
- the computed amplitudes are one order of magnitude smaller than those of S&P (pressure is fundamental)

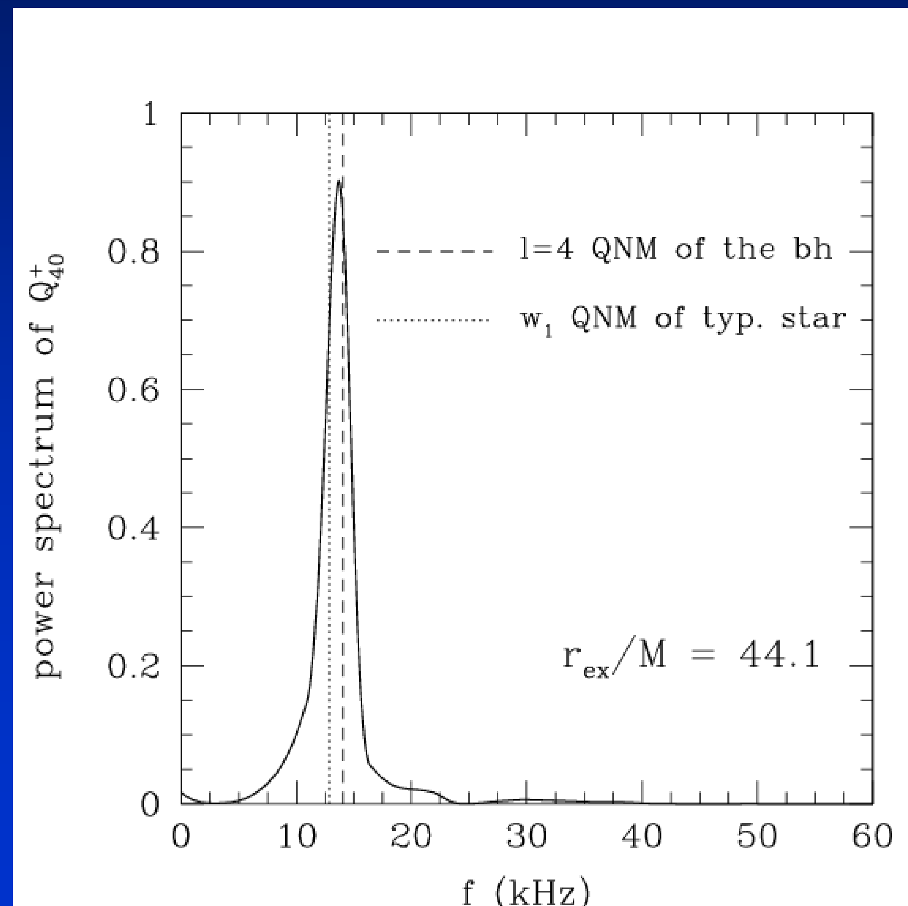


Are these waveforms realistic ?

- Convergence in the wave zone is an important requirement but it does not guarantee that the result is correct.

- Is there consistency with the expected frequencies?





GW Amplitude and Energetics

Knowledge of the waveforms is not enough. Large amplitudes are essential for a positive detection. Following Thorne (1987) we define the characteristic frequency as

$$f_c \equiv \left[\int_0^\infty \frac{|\hat{h}(f)|^2}{S_h(f)} f df \right] \Bigg/ \left[\int_0^\infty \frac{|\hat{h}(f)|^2}{S_h(f)} df \right]$$

with $\hat{h}(f)$ the FFT of the waveform and $S_h(f)$ the PSD of the instrument. The characteristic amplitude is then

$$h_c \equiv \left[\int_0^\infty \frac{S_h(f_c)}{S_h(f)} |\hat{h}(f)|^2 \, f df \right]^{1/2}$$

In this way we estimate the frequency and the amplitude at which the simulates waveform is seen by the detector

GW Amplitude and Energetics

These are the numbers we find for a star rotating at the mass-shedding limit and collapsing at 10 kpc:

$$h_c = 9.57 \times 10^{-22} \quad M/M_\odot$$

$$f_c = 931\,\mathrm{Hz}$$

VIRGO

$$h_c = 5.46 \times 10^{-22} \quad M/M_\odot$$

$$f_c = 531\,\mathrm{Hz}$$

LIGO I

NOTE: $\hat{h}(f)$ has maxima at ~ 6 kHz (l=2) and at 14 kHz (l=4)

Energy losses

In turn, the wave-amplitudes can be used to calculate the energy lost to gravitational waves as

$$\frac{dE}{dt} = \frac{1}{32\pi} \sum_{\ell m} \left(\left| \frac{dQ^{+}_{\ell m}}{dt} \right|^2 + \left| Q^{\times}_{\ell m} \right|^2 \right)$$

whose integral yields

$$\Delta E \sim 1.5 \times 10^{-6}\, M/M_{\odot}$$

Smaller efficiency than calculated by S&P (i.e. $\Delta E \sim 1.5 \times 10^{-4\text{-}5}\ M/M_{\odot}$), but *consistent* with what obtained in core-collapse to protoneutron stars (i.e. $\Delta E \sim 1.5 \times 10^{-8\text{-}9}\, M/M_{\odot}$) (Mueller et al. ApJ 2004).

Detectability

Even more significant than these amplitudes and energetics is the signal-to-noise ratio. Again, for a star uniformly rotating near mass-shedding at 10kpc

$$\frac{S}{N} \equiv \frac{h_c}{h_{rms}(f_c)} = \frac{h_c}{\sqrt{f_c S_h(f_c)}} = 0.25 - 4$$

VIRGO/LIGO I → 0.25; Advanced → 4

NOTE:

The gw emission can be increased significantly if the initial data is rotating differentially and is supra-Kerr ***(a>1)***.

Conclusions

- We have recently built and tested **Whisky** a new parallel, 3D code for hydrodynamical simulations in full GR.
- Many *"good things"* about **Whisky**: HRSC techniques, excision, method of lines, progressive mesh refinement...
- This is the most accurate description of collapse to rotating BHs and the first calculations of GW emission in 3D.
- Intense work is in progress both on source modelling (differential rotation, EOSs, dynamical instabilities, binary NSs) and on code development (MHD, radiation transport).

*We expect a lot of **Whisky** in our near future*
and this cannot be a bad thing...