Order and Scale Selection in Columnar Joints

or
how to attack
a 300 year
old geology
problem
using kitchen
materials

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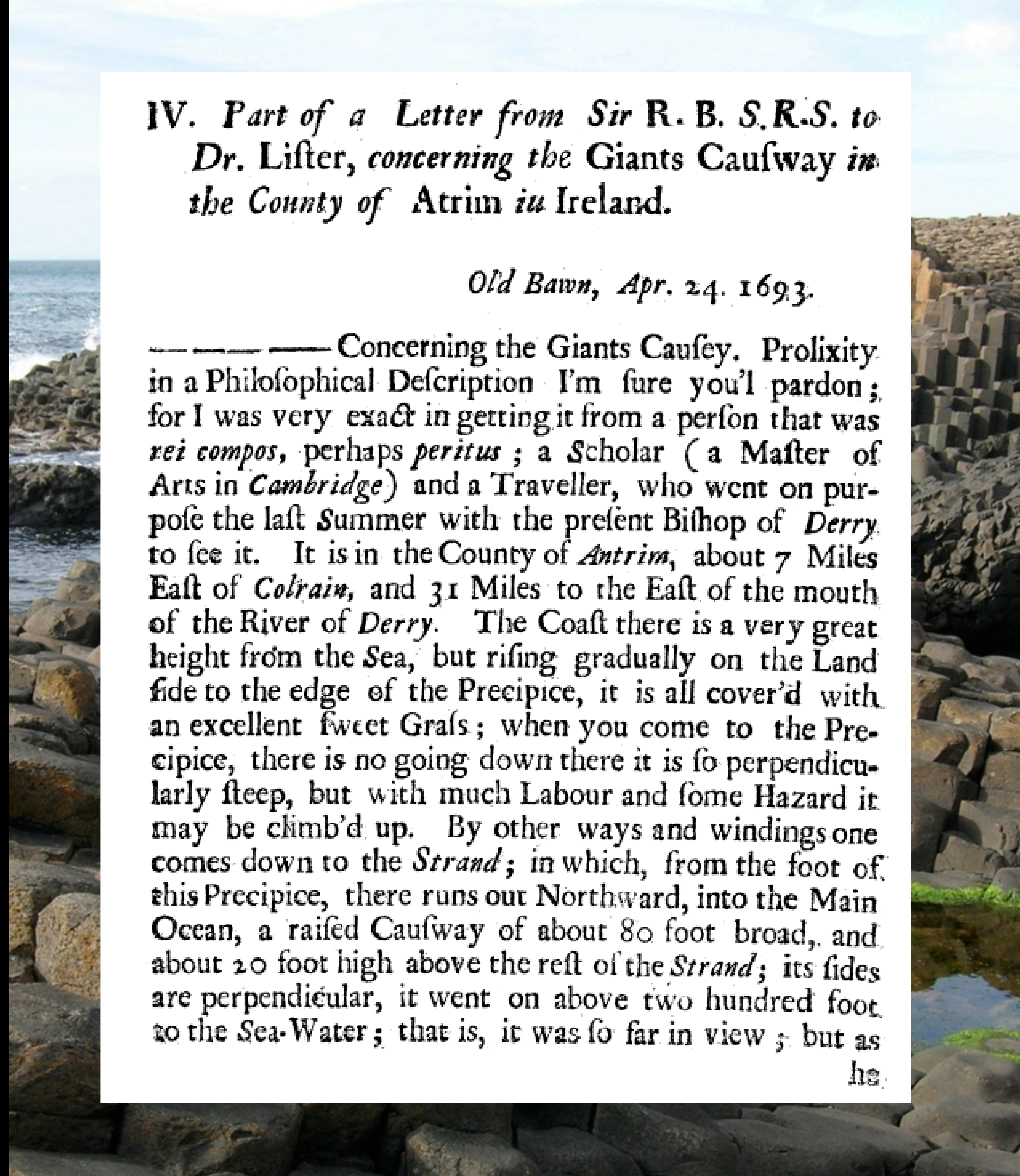
What are columnar joints?

Giant's Causeway in Ireland, first reported to the Royal Society in 1693

Sir R. B. S. R. S., *Phil. Trans. R. Soc. Lon.*, 17, 708 (1693).



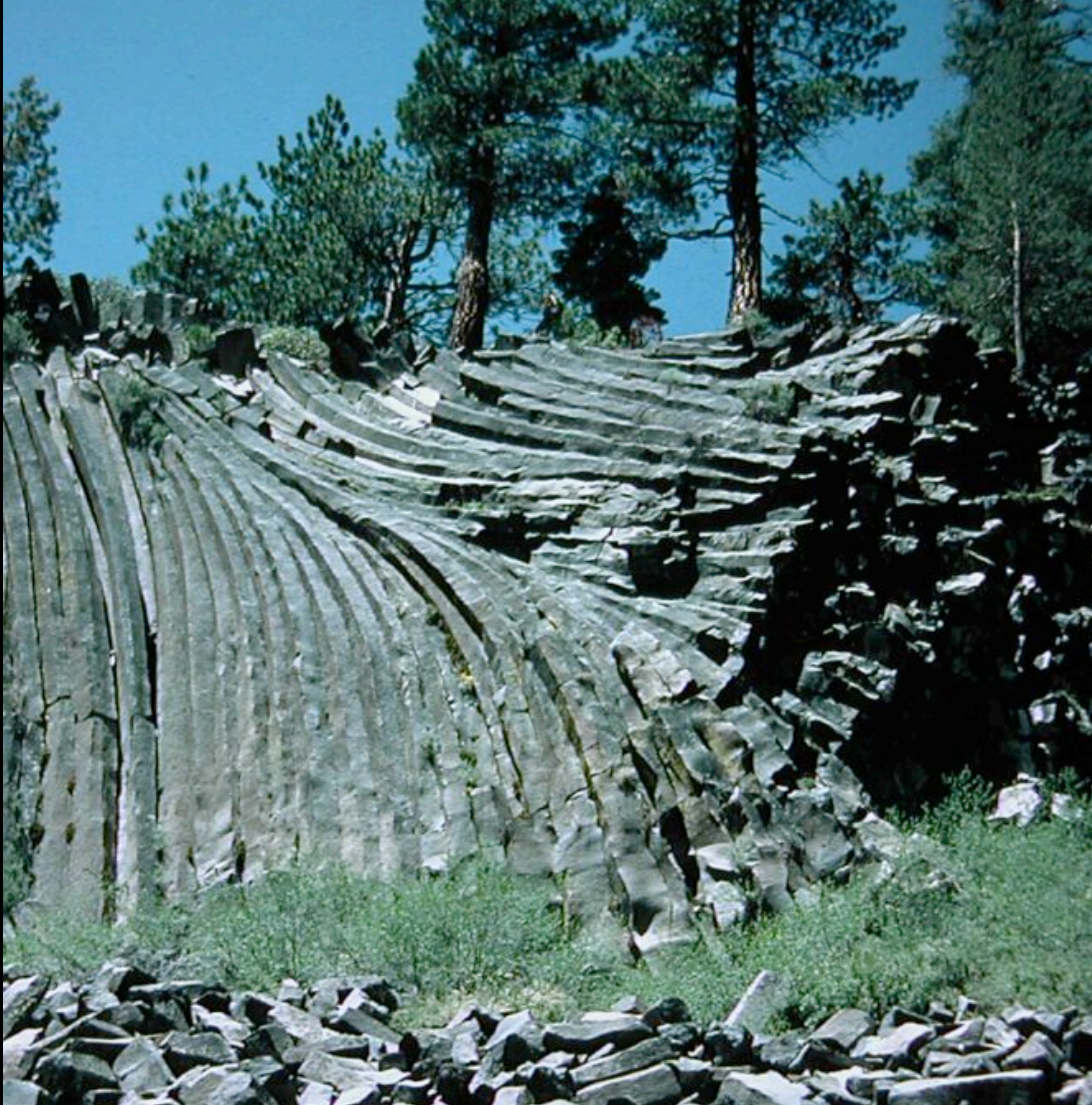
IV. *Part of a Letter from Sir* R. B. *S.R.S. to* Dr. Liſter, *concerning the* Giants Cauſway *in the County of* Atrim *iu* Ireland.

Old Bawn, Apr. 24. 1693.

——— Concerning the Giants Cauſey. Prolixity in a Philoſophical Deſcription I'm ſure you'l pardon; for I was very exact in getting it from a perſon that was *rei compos*, perhaps *peritus*; a Scholar (a Maſter of Arts in *Cambridge*) and a Traveller, who went on purpoſe the laſt Summer with the preſent Biſhop of *Derry* to ſee it. It is in the County of *Antrim*, about 7 Miles Eaſt of *Colrain*, and 31 Miles to the Eaſt of the mouth of the River of *Derry*. The Coaſt there is a very great height from the Sea, but riſing gradually on the Land ſide to the edge of the Precipice, it is all cover'd with an excellent ſweet Graſs; when you come to the Precipice, there is no going down there it is ſo perpendicularly ſteep, but with much Labour and ſome Hazard it may be climb'd up. By other ways and windings one comes down to the *Strand*; in which, from the foot of this Precipice, there runs out Northward, into the Main Ocean, a raiſed Cauſway of about 80 foot broad, and about 20 foot high above the reſt of the *Strand*; its ſides are perpendicular, it went on above two hundred foot to the Sea-Water; that is, it was ſo far in view; but as he

Devil's Postpile,
California

Devil's Postpile, California



Columns do not follow gravity


Staffa, Scotland


photo by
Robert Mehew
Fingal’s Cave, Staffa


Fingal's Cave,
Staffa
photo by
Robert Mehew


The Herdsman, Staffa

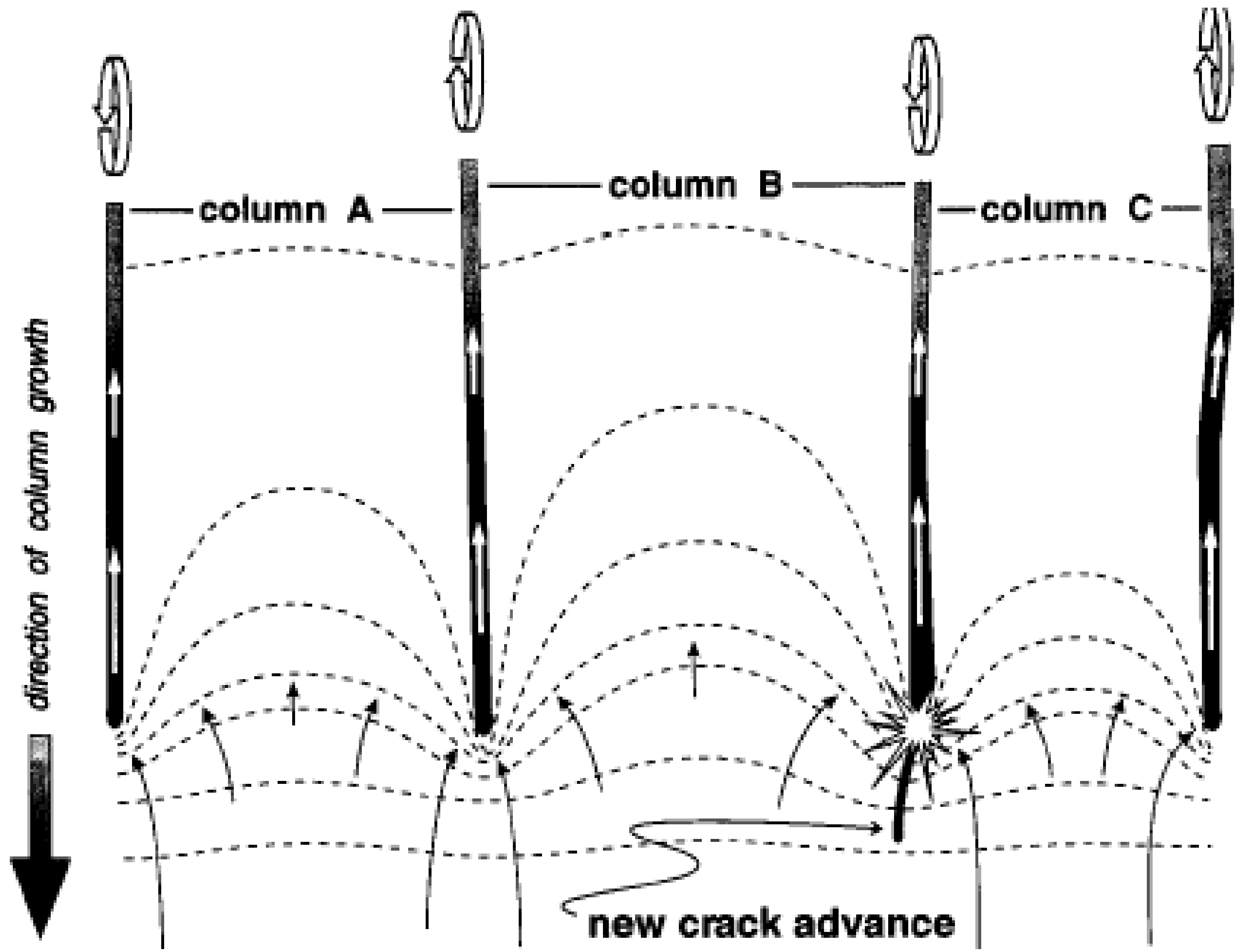

Many columns are
near-perfect hexagons
but not all
Devil's postpile,
California

Obvious Questions

- How to they form?
- What sets the length scale?
- What does this have to do with the kitchen?

What the geologists tell us:

An initially disordered, 2D crack network orders into hexagons *etc.*, as heat is extracted and as it propagates into the bulk as a *front*.



convection zone

conduction zone

Budkewitsch and Robin, *J. Volcanol. Geotherm. Res.*, 59, 213 (1994)

Each column is “chiseled out” by successive fracture advances called *striae*.

Each column is “chiseled out” by successive fracture advances called *striae*.

Rhyolite column
Steve Sparks, Bristol

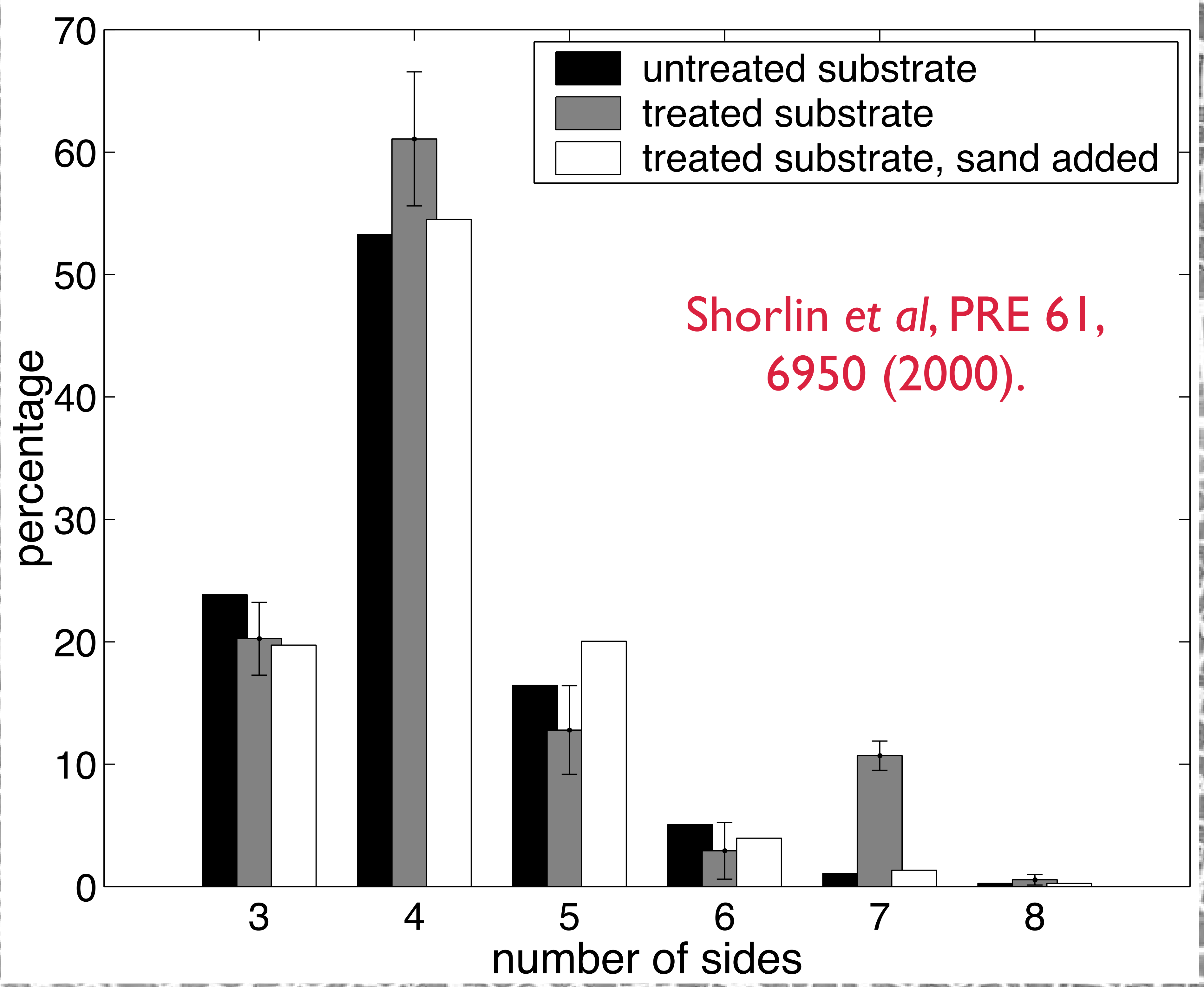
4 sides or 6?
T junctions or **Y**?

Sequential vs. iterated
crack networks

In sequential fracture of thin layers, stress steers subsequent cracks to meet existing cracks at 90 degrees.

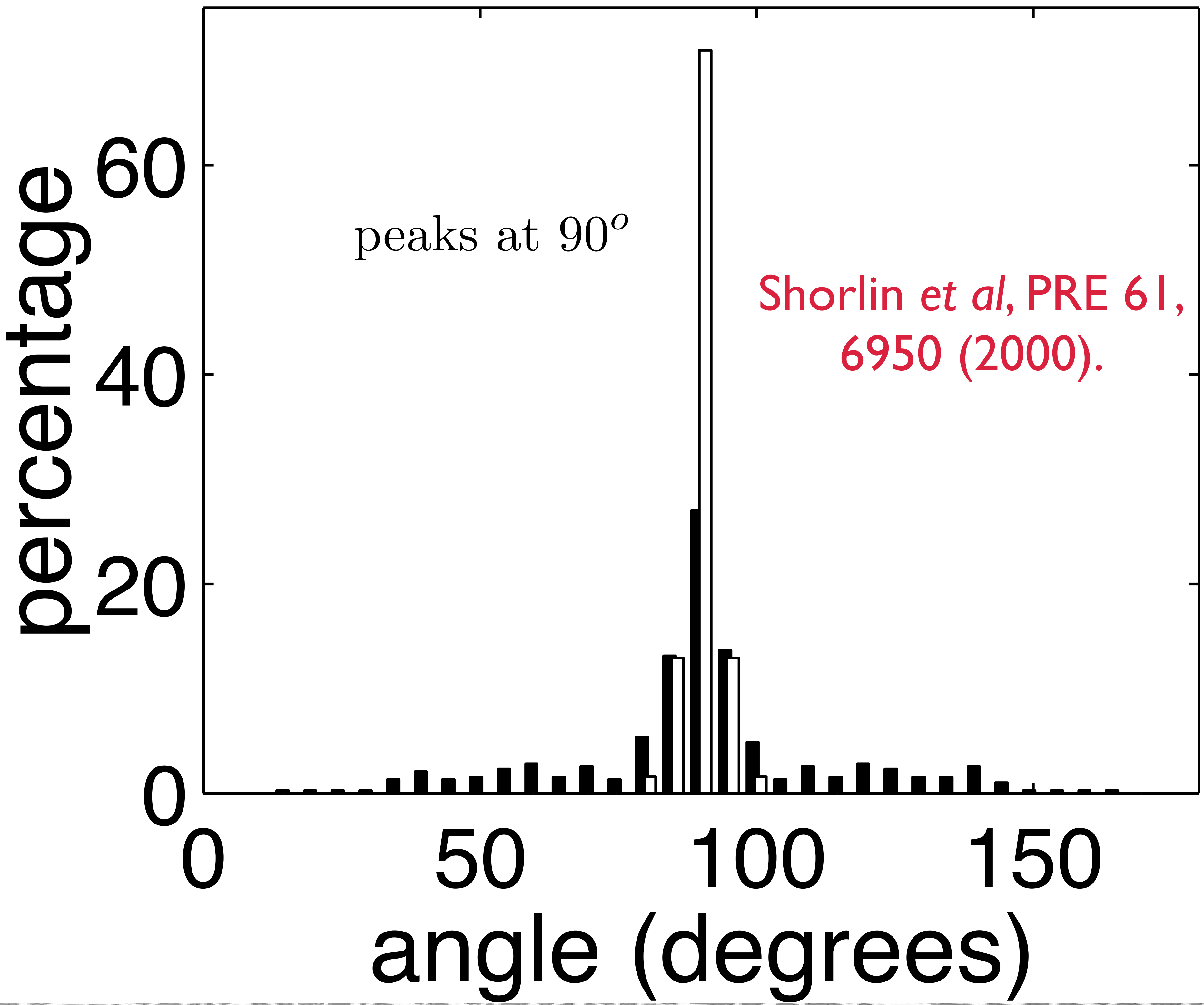
photo: mud.
Bernard Hallet

Lab experiment



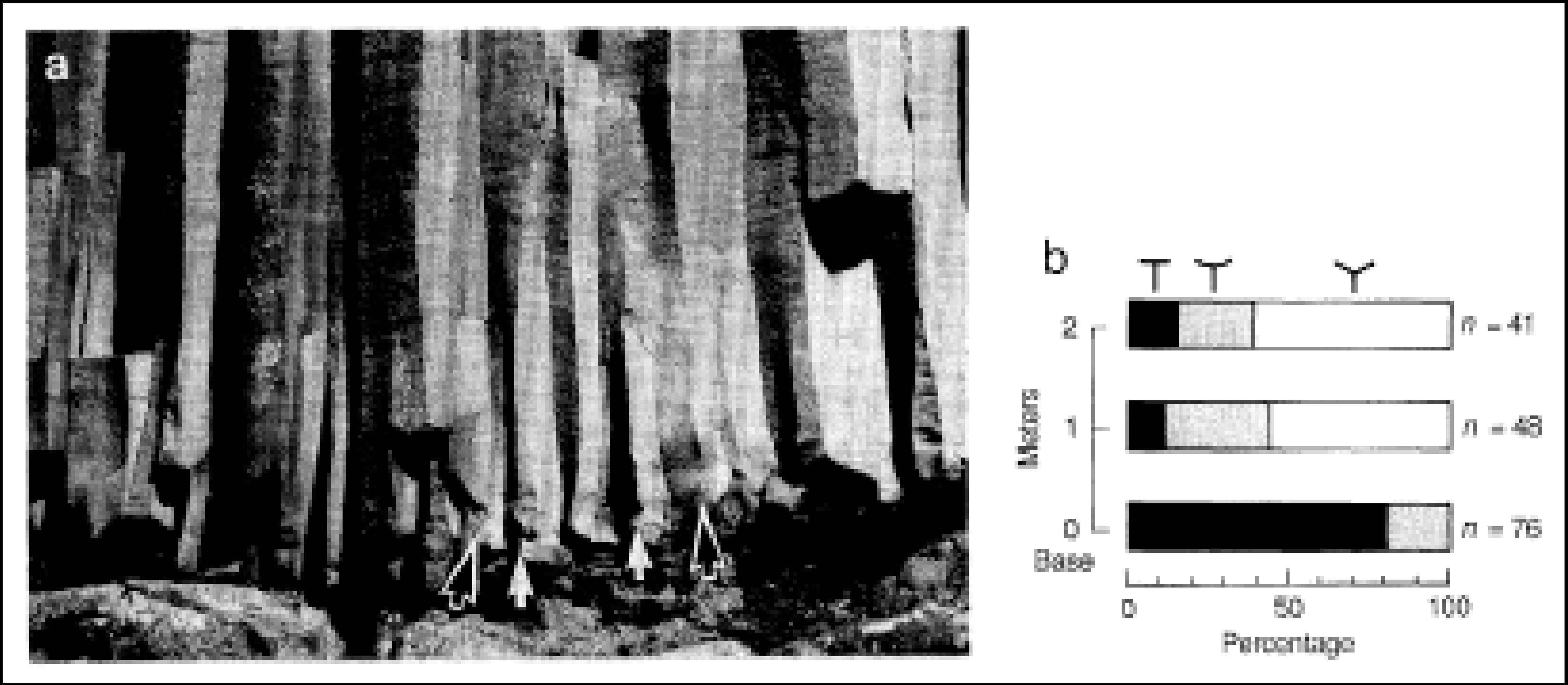
Shorlin *et al*, PRE 61, 6950 (2000).

Lab experiment



Shorlin *et al*, PRE 61, 6950 (2000).

In columnar joints, **T** junctions evolve into **Y** junctions as the cracking front advances.



(Here the cooling front moved upward)

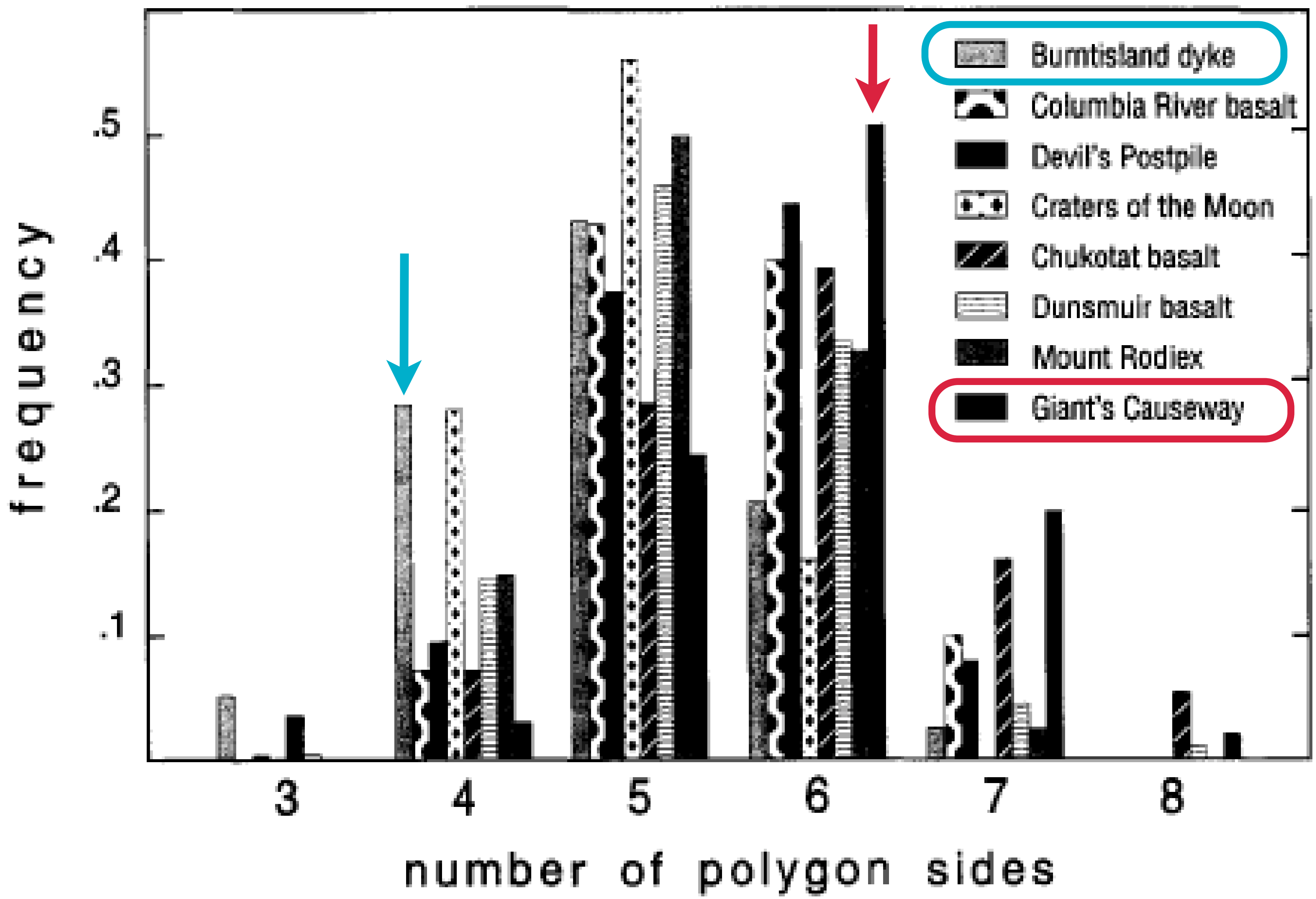
Aydin and DeGraff, *Science*, 239, 471 (1988)

The Boat Cave, Staffa

growth direction

thin ordering region

Budkewitsch and Robin, *J. Volcanol. Geotherm. Res.*, 59, 213 (1994)



immature favours 4,5

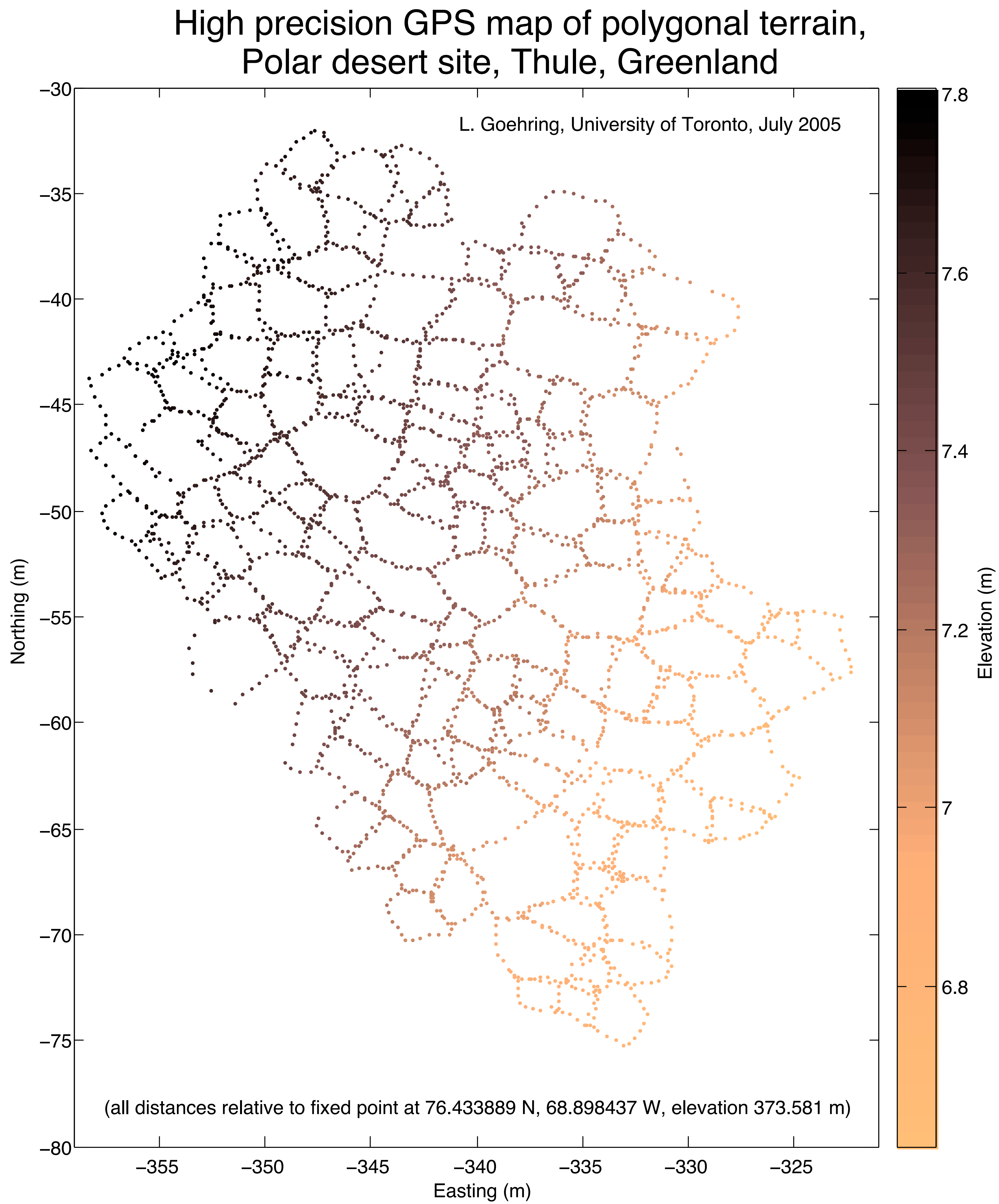
mature favours 6

Data from many different geological locales shows that 5 and 6 neighbors are most common


Freeze - thaw
patterns in
permafrost
photo by Lucas Goehring

Freeze - thaw patterns in permafrost

photo by Lucas Goehring

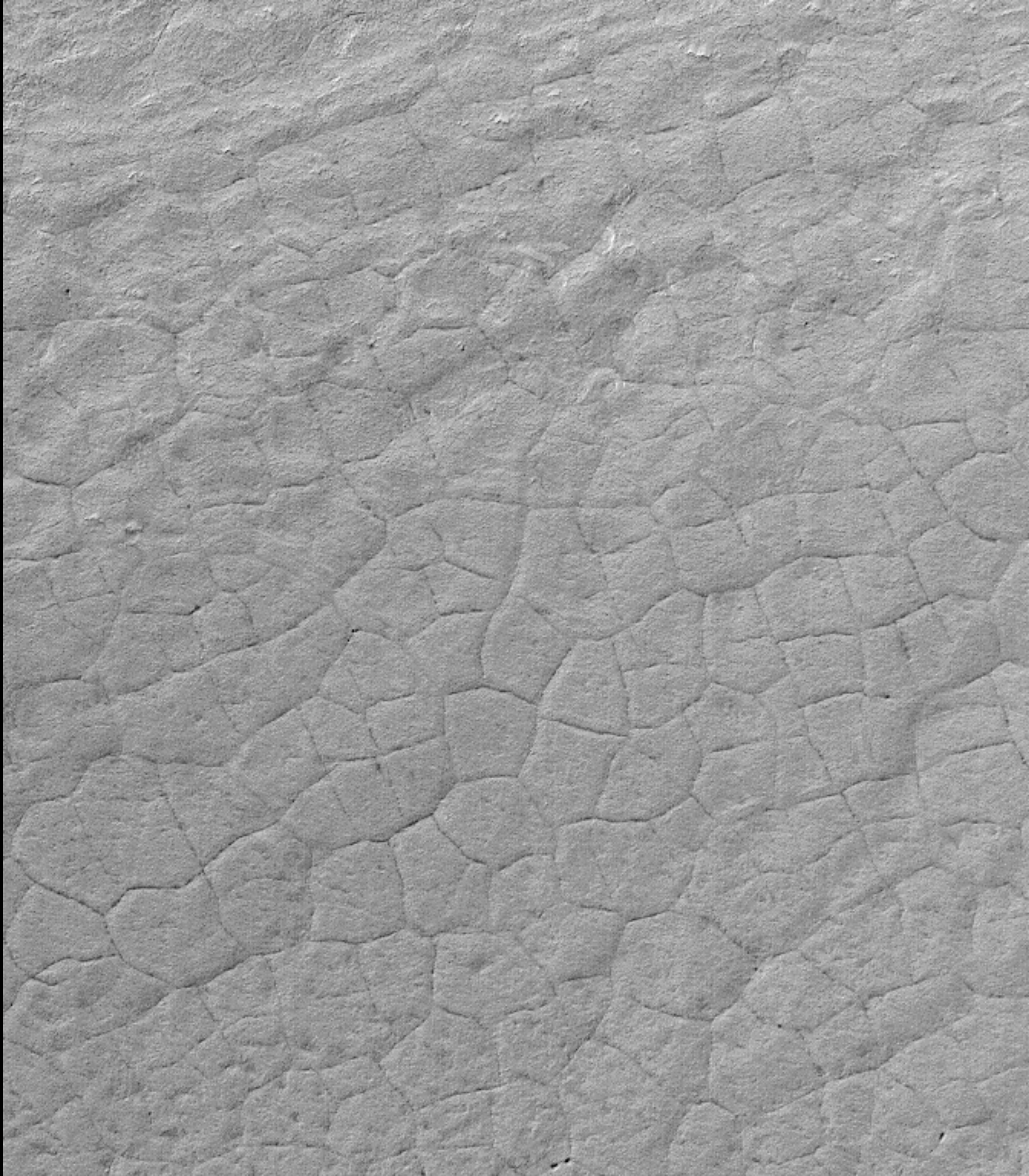


“Giant Martian Polygons”

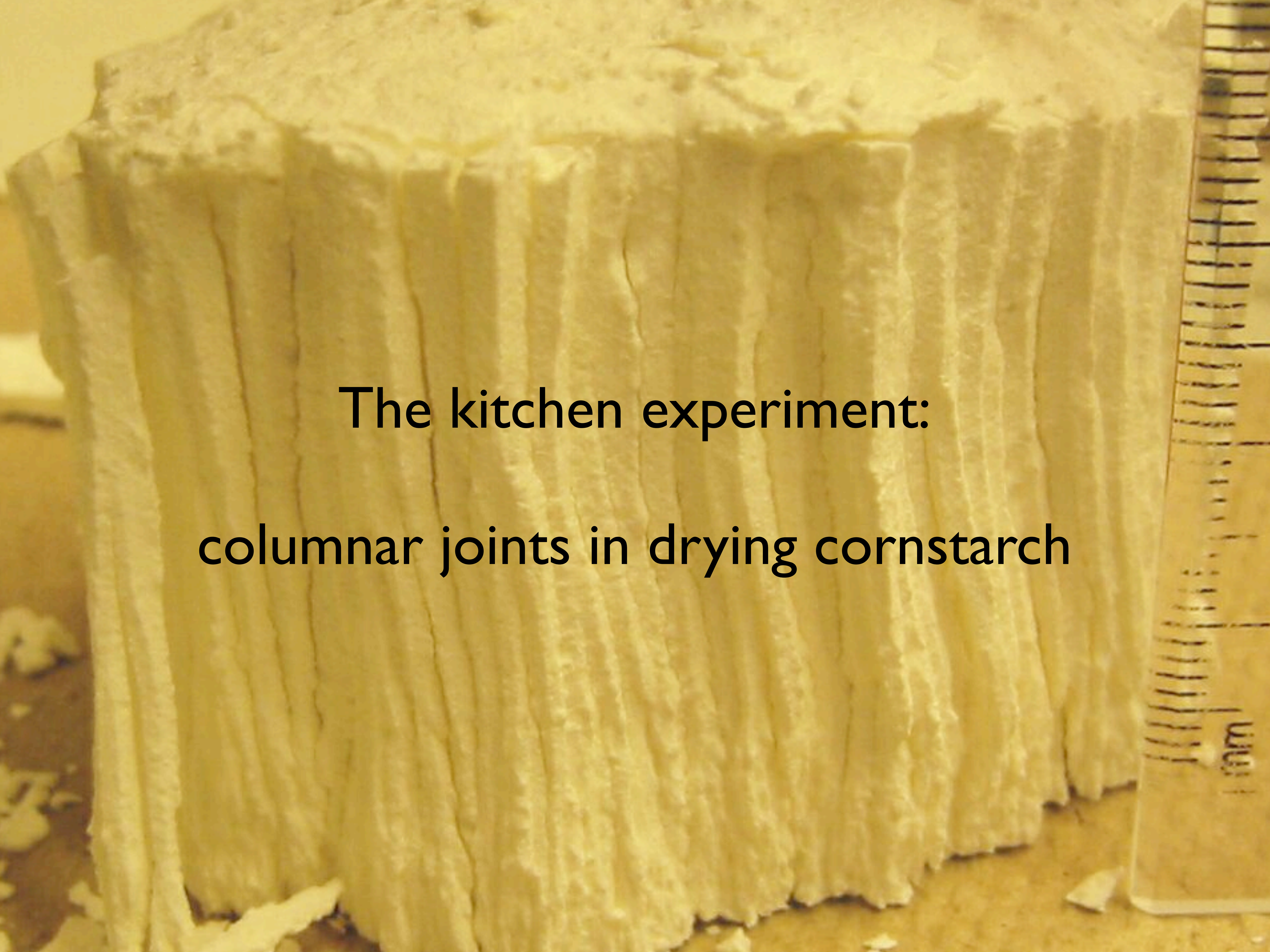
Permafrost patterns on Mars?

This Mars Global Surveyor (MGS) Mars Orbiter Camera (MOC) picture displays polygons outlined by cracks in the martian south polar region.

image 3 km wide

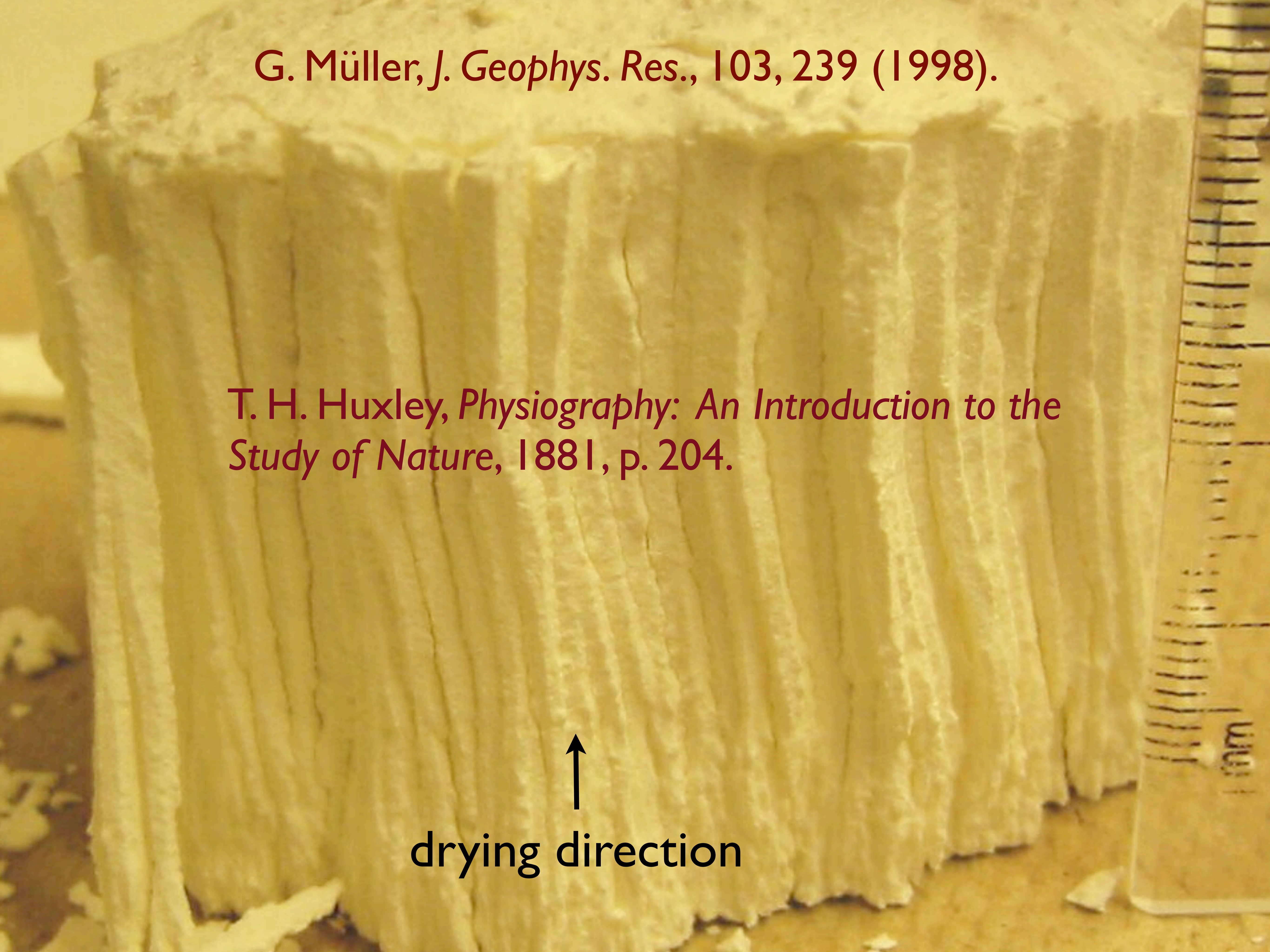


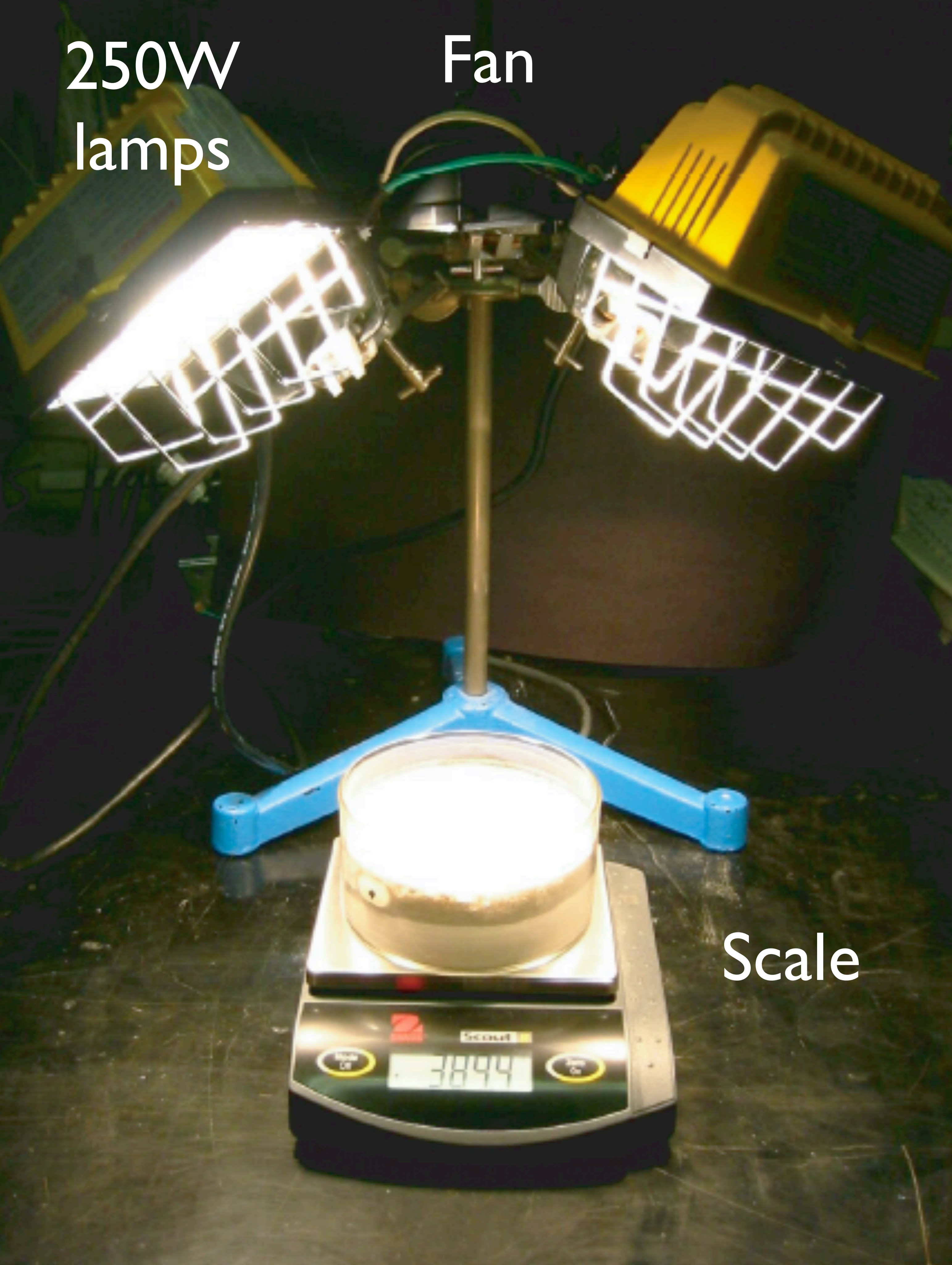
Some real mud cracks *may* be iterated by successive rains



The kitchen experiment:

columnar joints in drying cornstarch

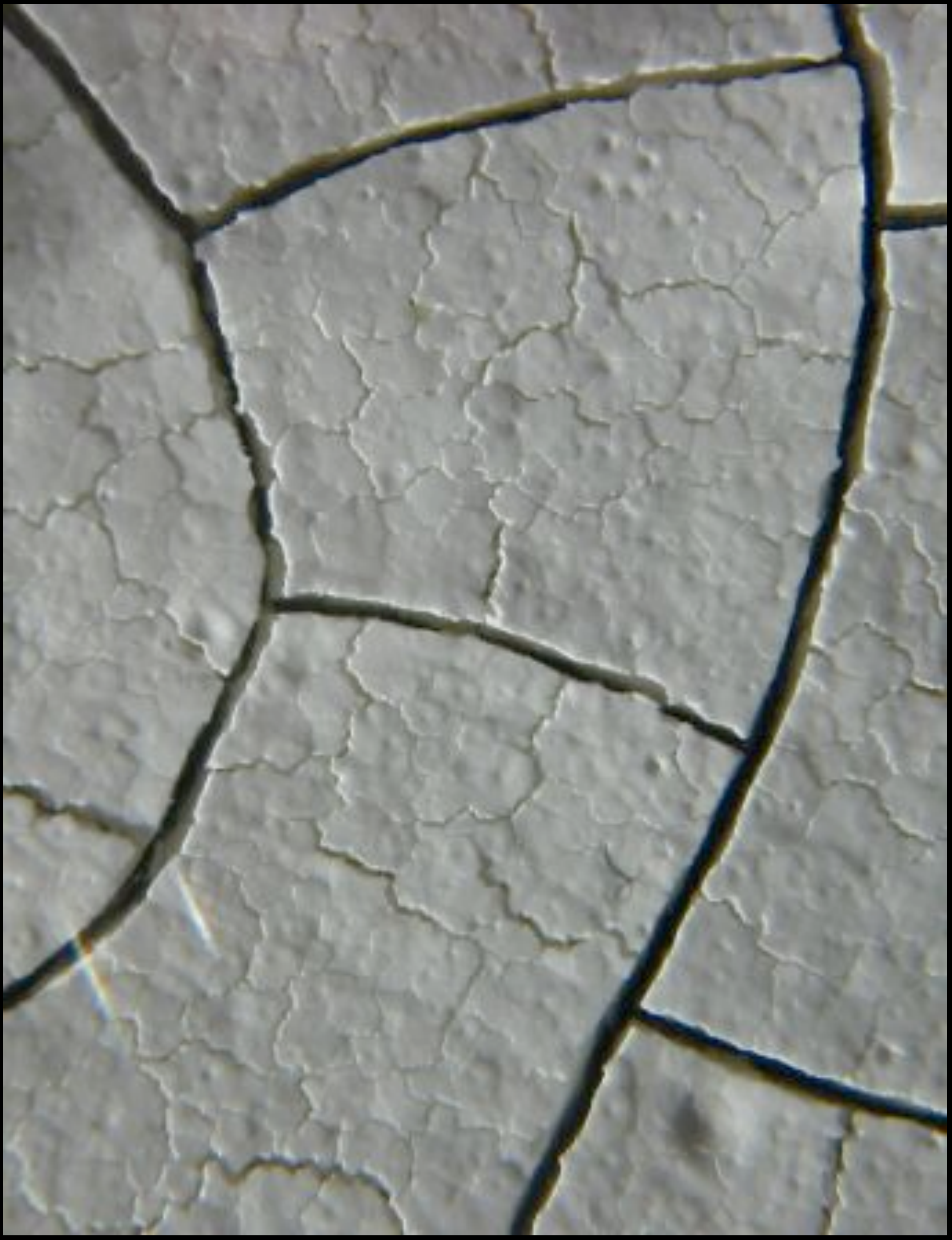

G. Müller, *J. Geophys. Res.*, 103, 239 (1998).
T. H. Huxley, *Physiography: An Introduction to the Study of Nature*, 1881, p. 204.
drying direction



The Apparatus

Dry corn starch slurries under heat lamps, weighing continuously.

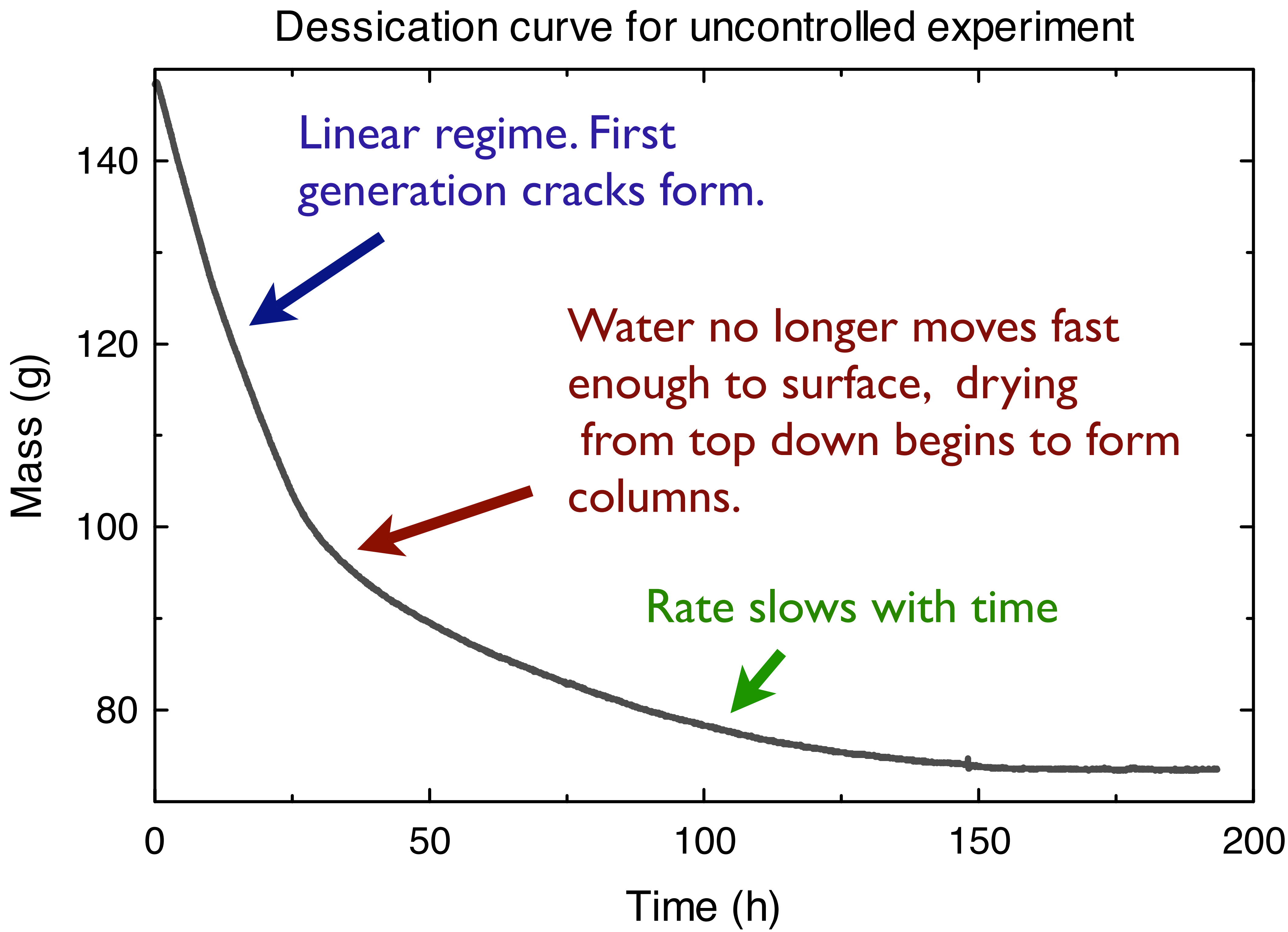
Use a computer to feedback control drying rate by modulating the lights, or leave uncontrolled.



1^{st} Generation Cracks

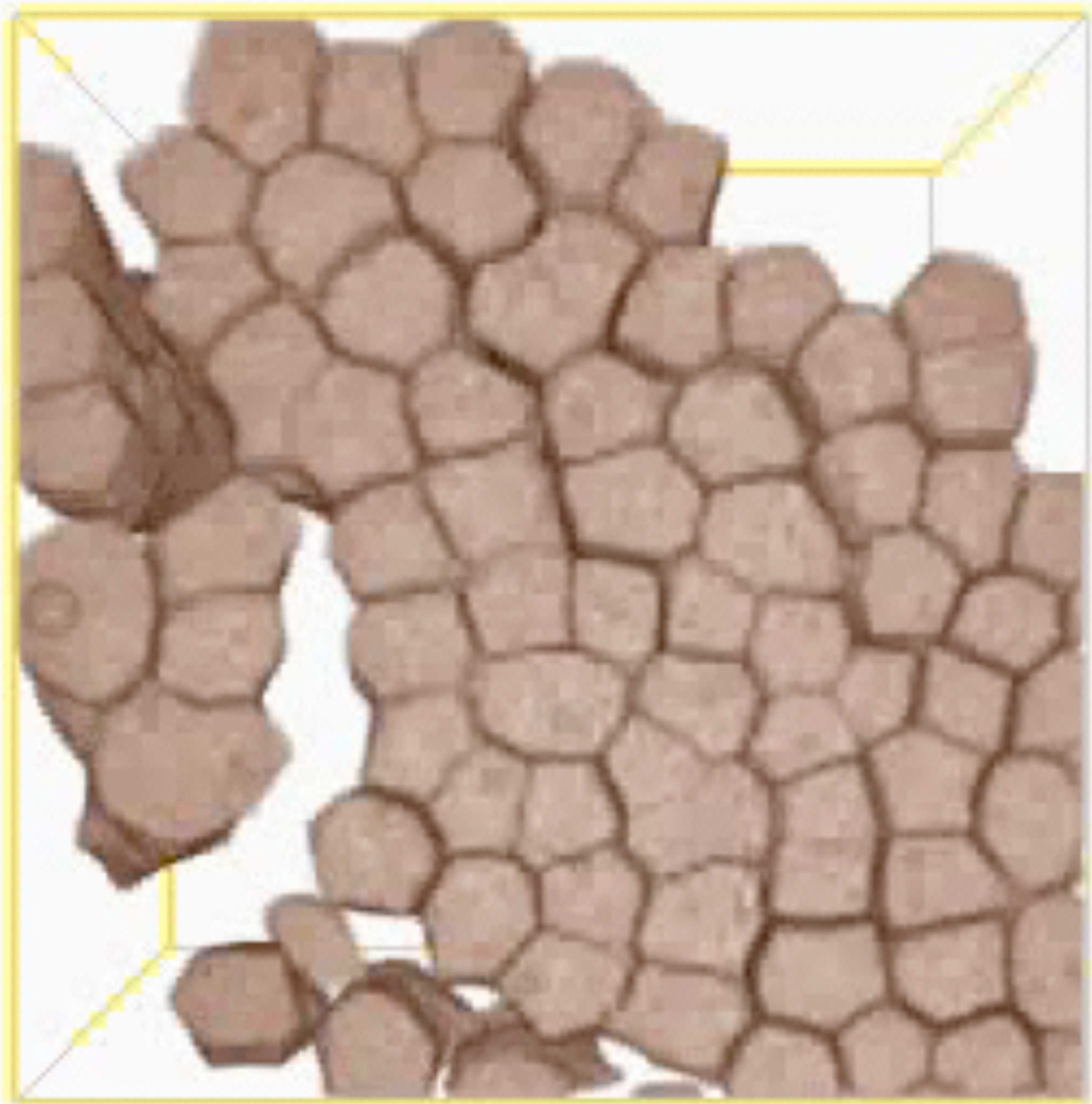
These form before columnar jointing occurs. Meet at 90 degrees.

They crack the sample from top to bottom, breaking the sample into independent regions with free boundary conditions.



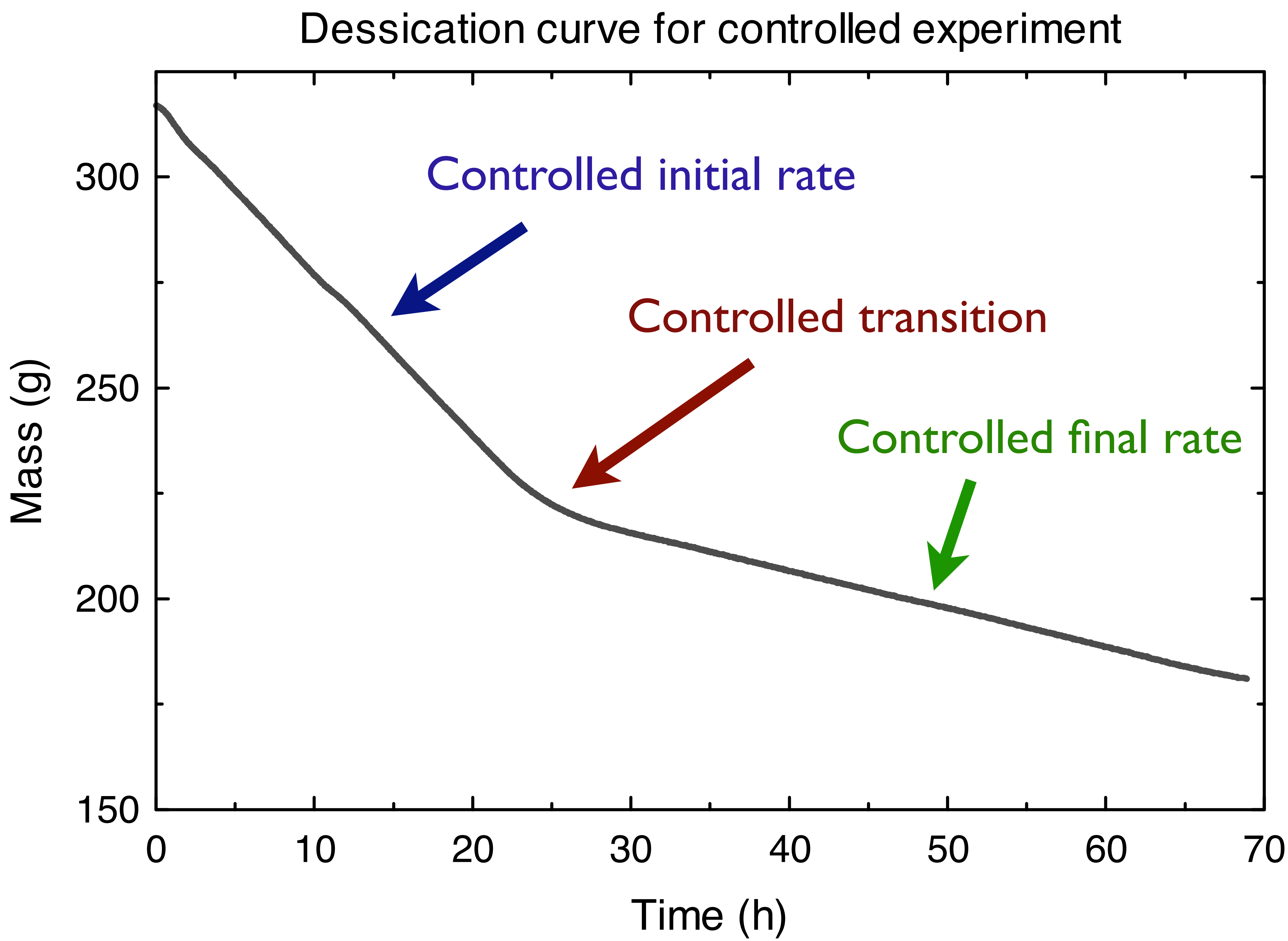
Uncontrolled drying gives columns that *coarsen* with time/depth.

Micro X-ray tomography of starch

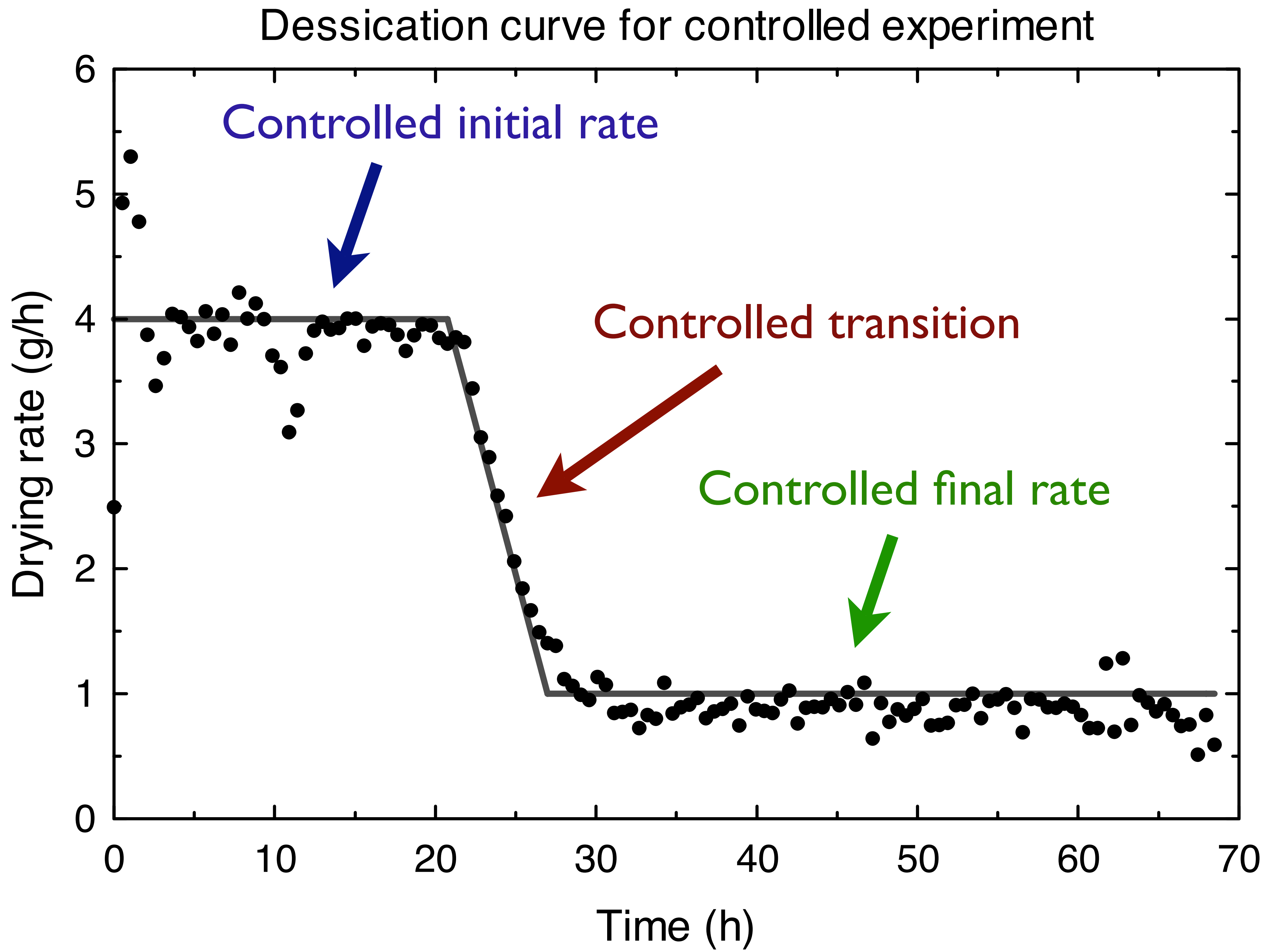


Thanks to Mark Henkelman, MICe, Hospital for Sick Children

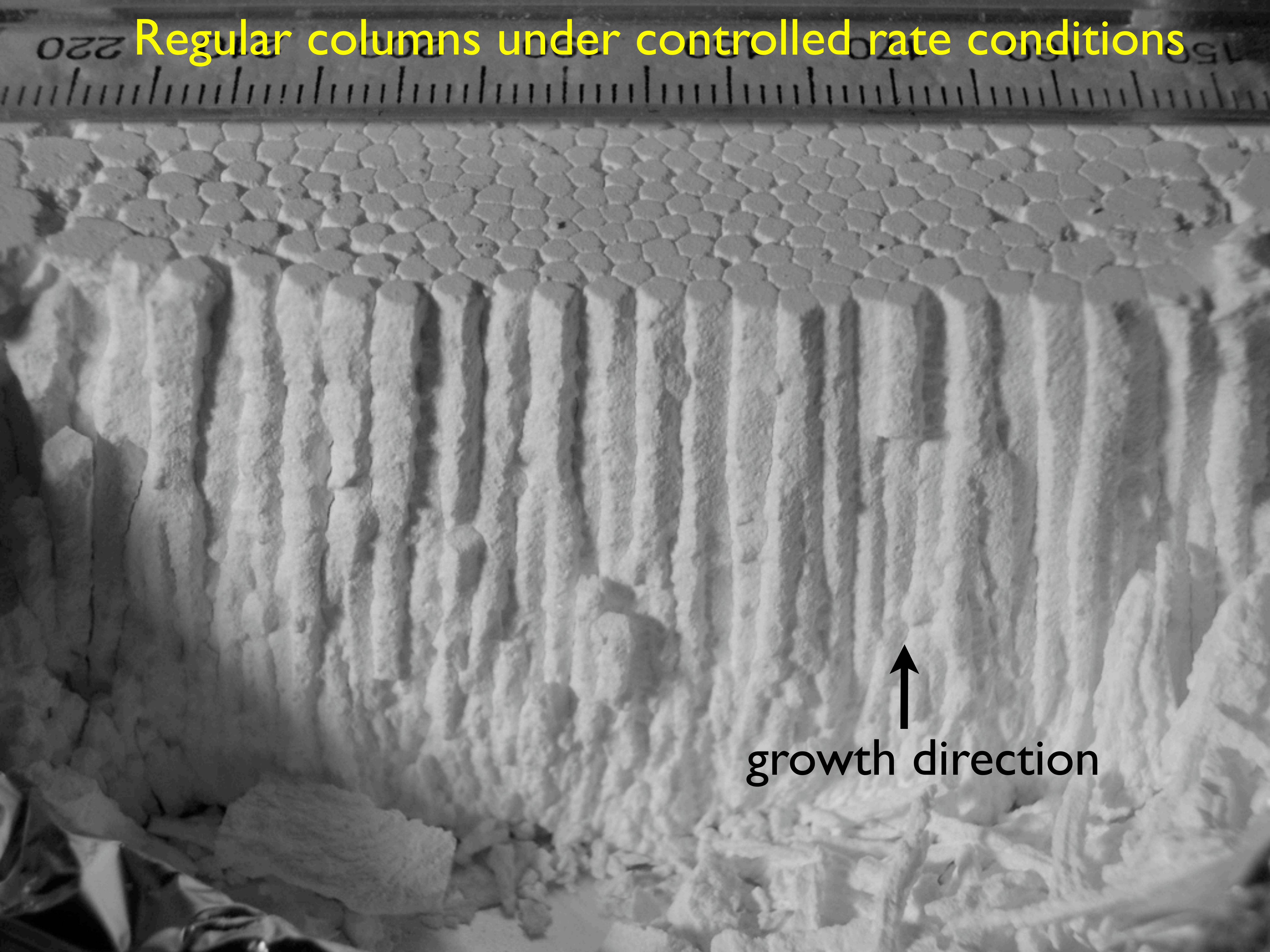
X-ray micro CT movie



Dessication curve for controlled experiment
Controlled initial rate
Controlled transition
Controlled final rate
Mass (g)
300
250
200
150
0
10
20
30
40
50
60
70
Time (h)



Controlled drying gives regular columns.


Regular columns under controlled rate conditions
growth direction

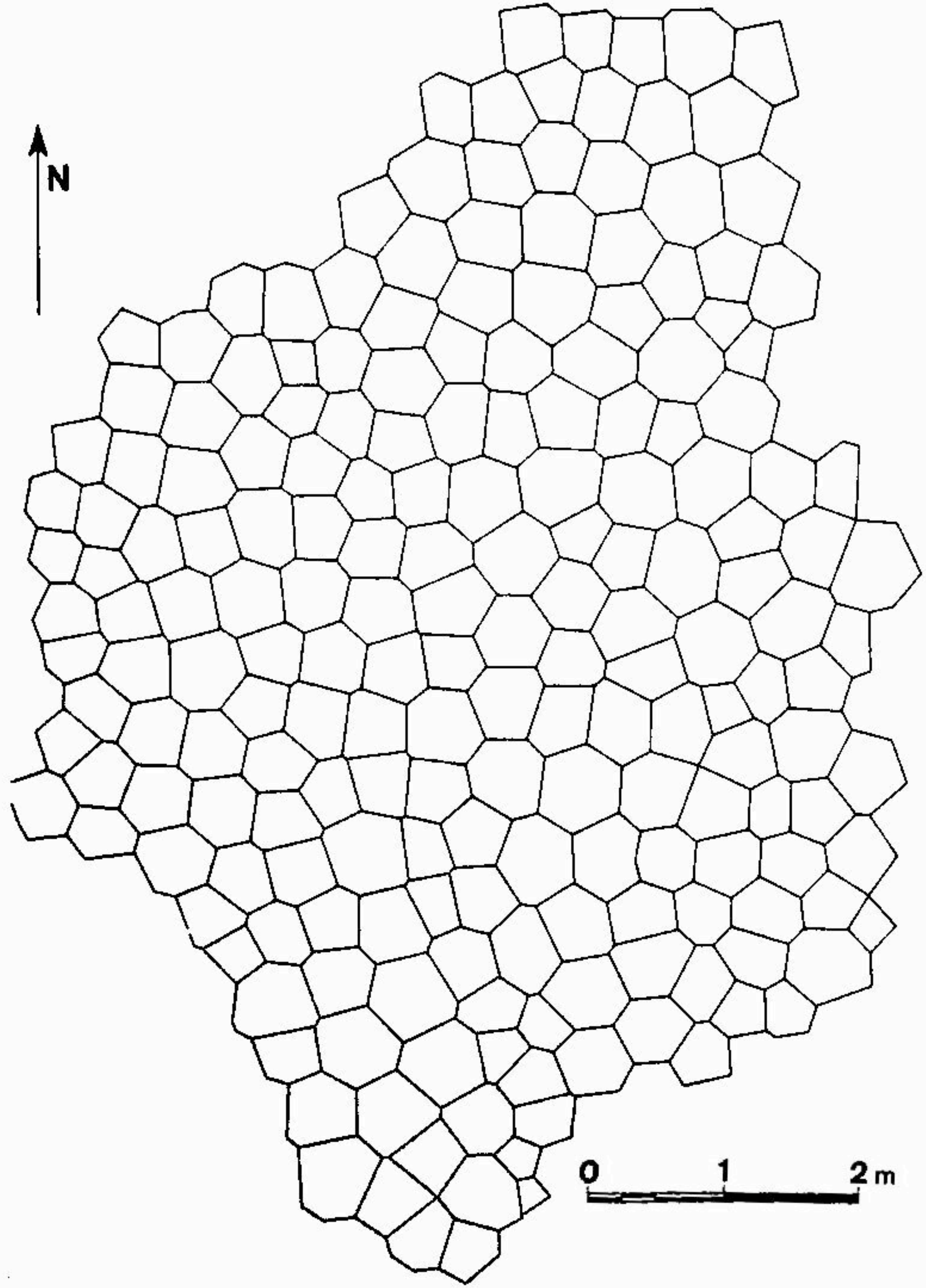
X-ray micro CT movie

Statistical measures of disorder in the Giant's Causeway

Is the disorder intrinsic?

O'Reilly's 1879 survey of the Giant's Causeway is the basis for many later geometrical studies

J. P. O'Reilly, *Trans. R. Irish Acad.*, 26, 641 (1879)



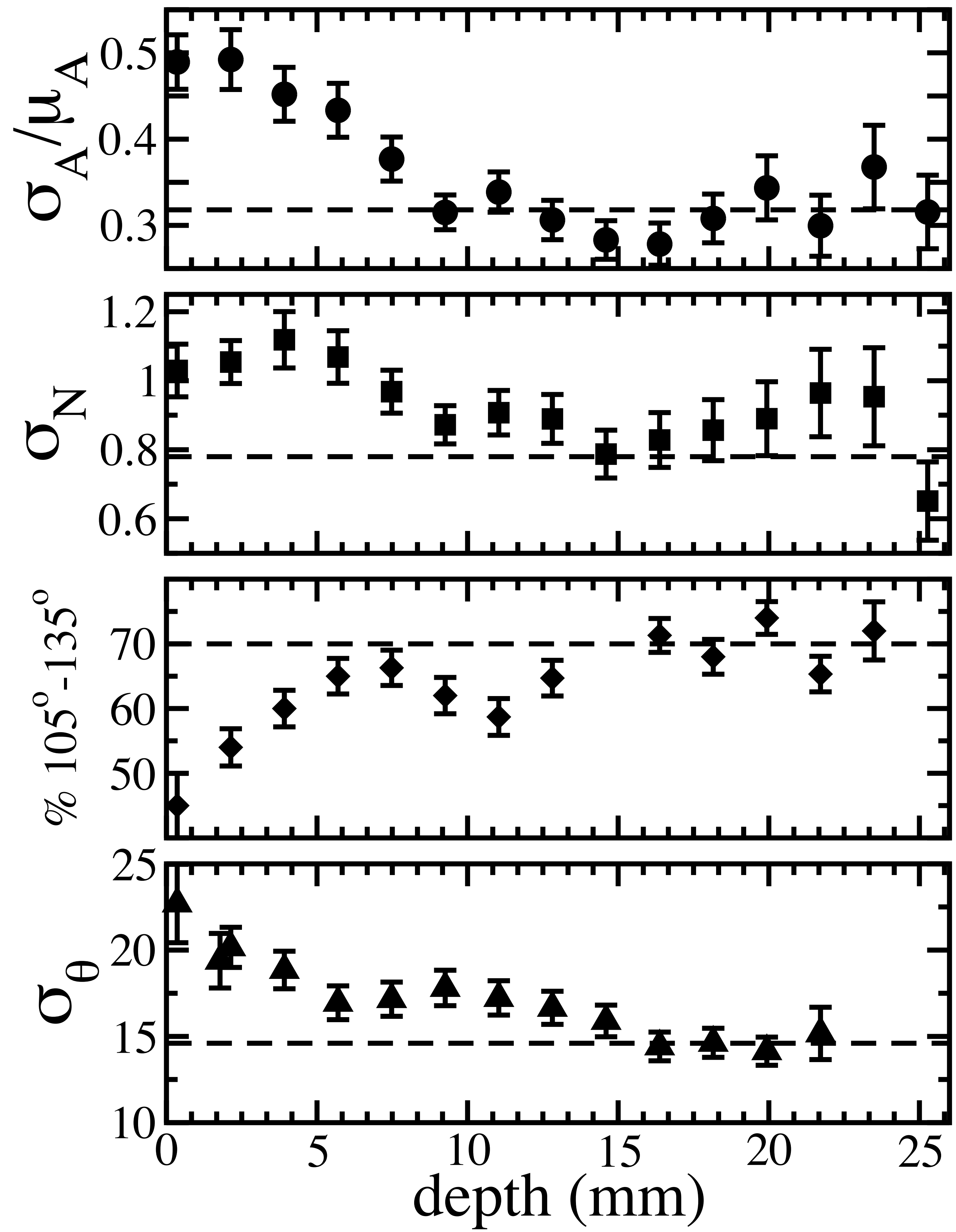
Statistical measures of disorder in the drying starch

(both controlled and uncontrolled)

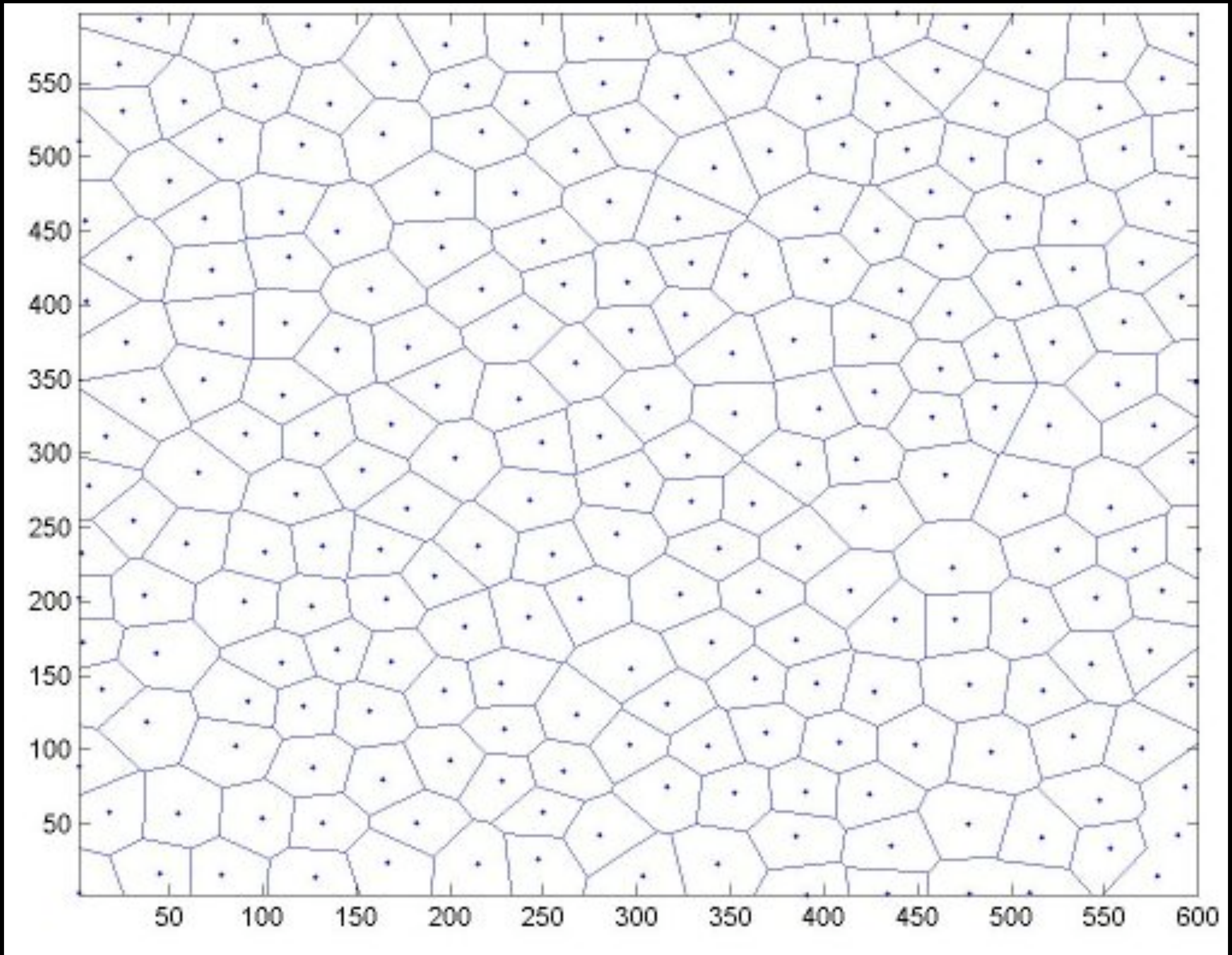
All measures converge to Giant's Causeway values.

Residual disorder could be intrinsic to the dynamics of the process.

L. Goehring, S. Morris, *Europhys Lett.*, 69, 739 (2005).



Geometrical models

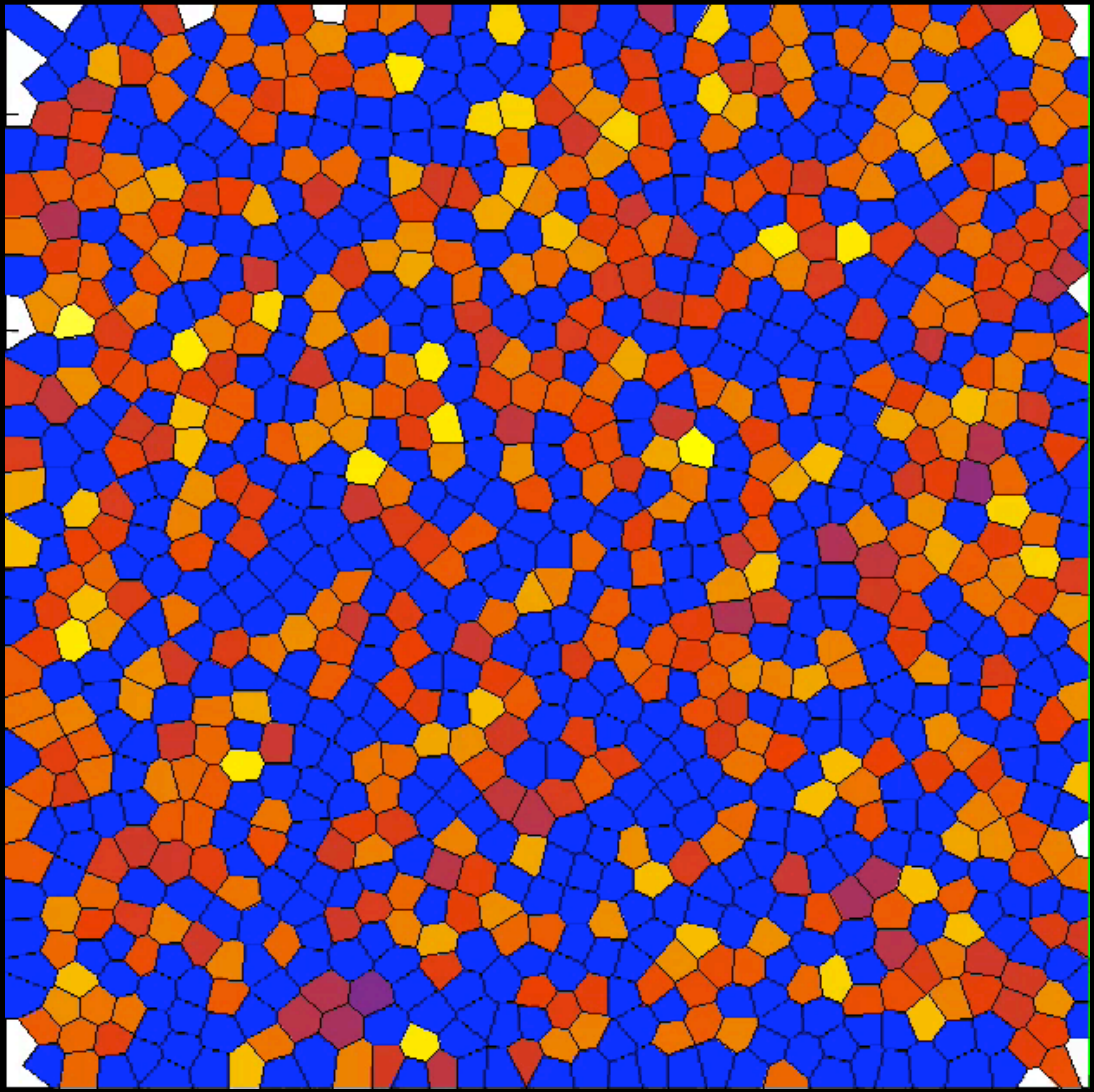


Smally's model is a Voronoi tessellation of random, anticorrelated points.

Budkewitsch & Robin add an iterative update rule that moves the Voronoi points toward the centroid of each polygon.

Smally, *Geol. Mag.*, 103, 110 (1966)

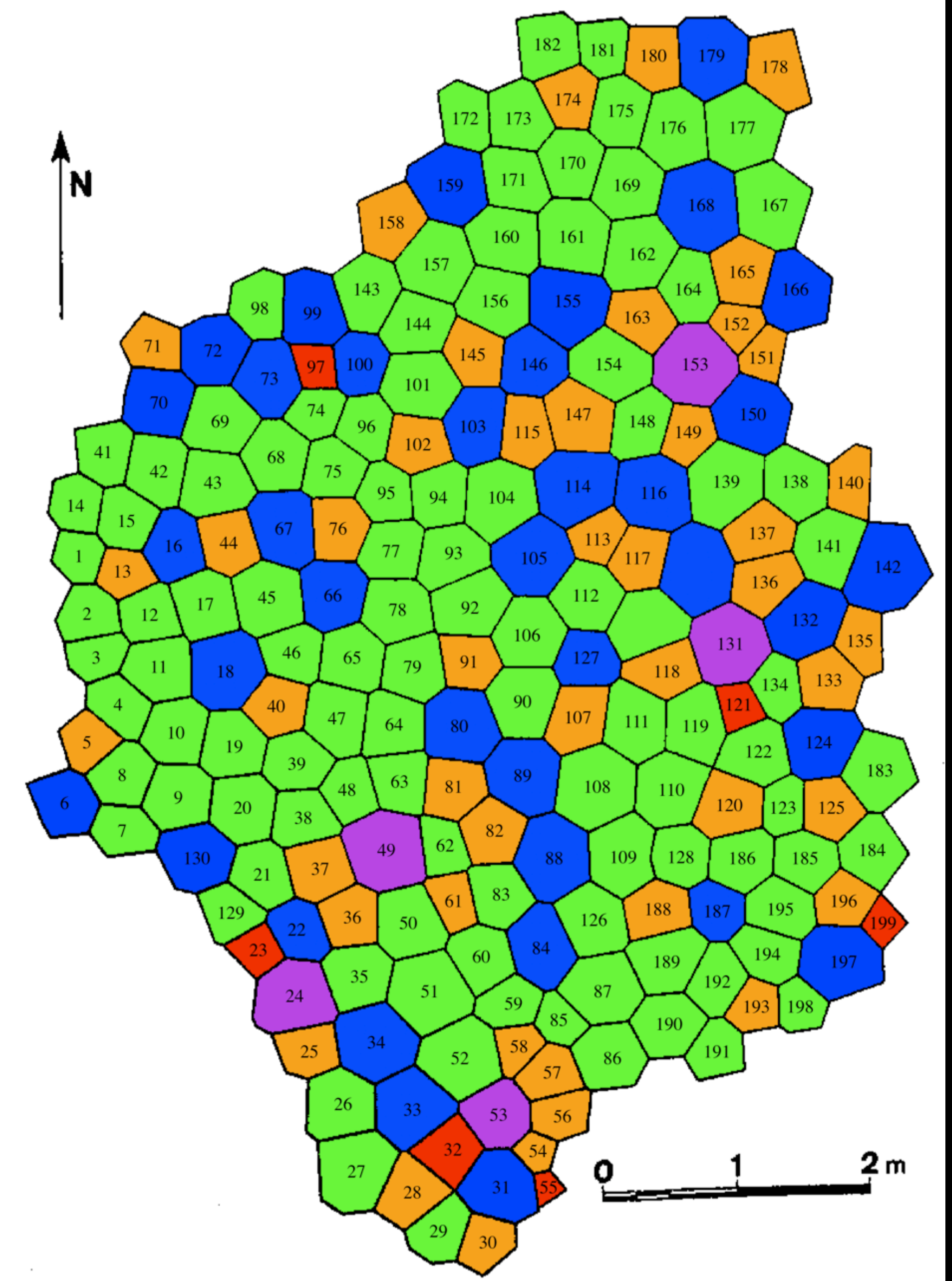
Budkewitsch and Robin, *J. Volcanol. Geotherm. Res.*, 59, 213 (1994)



Budkewitsch and Robin, *J. Volcanol. Geotherm. Res.*, 59, 213 (1994)

O'Reilly's 1879 survey of the Giant's Causeway, *colorized.*

See some chains of defects?



J. P. O'Reilly, *Trans. R. Irish Acad.*, 26, 641 (1879)

Comparative statistics for "mature" patterns

System	σ_A/μ_A	σ_N	σ_θ	% 105°-135°
Dessicated starch (Goehring, Morris, 2005)	0.32 ± 0.01	0.86 ± 0.05	$14.6 \pm 0.3°$	$70.1 \pm 1.7\%$
Giant's Causeway (O'Reilly1879)	0.32 ± 0.02	0.75 ± 0.06	$14.3 \pm 0.6°$	$69.7 \pm 1.4\%$
Random close packed polygons	0.142 ± 0.001	0.669 ± 0.005	$16.4 \pm 0.05°$	$61.1 \pm 0.2\%$
Iterated Voronoi model (Budkewitsch and Robin)	0 - 0.05	0 - 0.5	0 - 8°	0 - 6%

Budkewitsch and Robin model produces too much order!

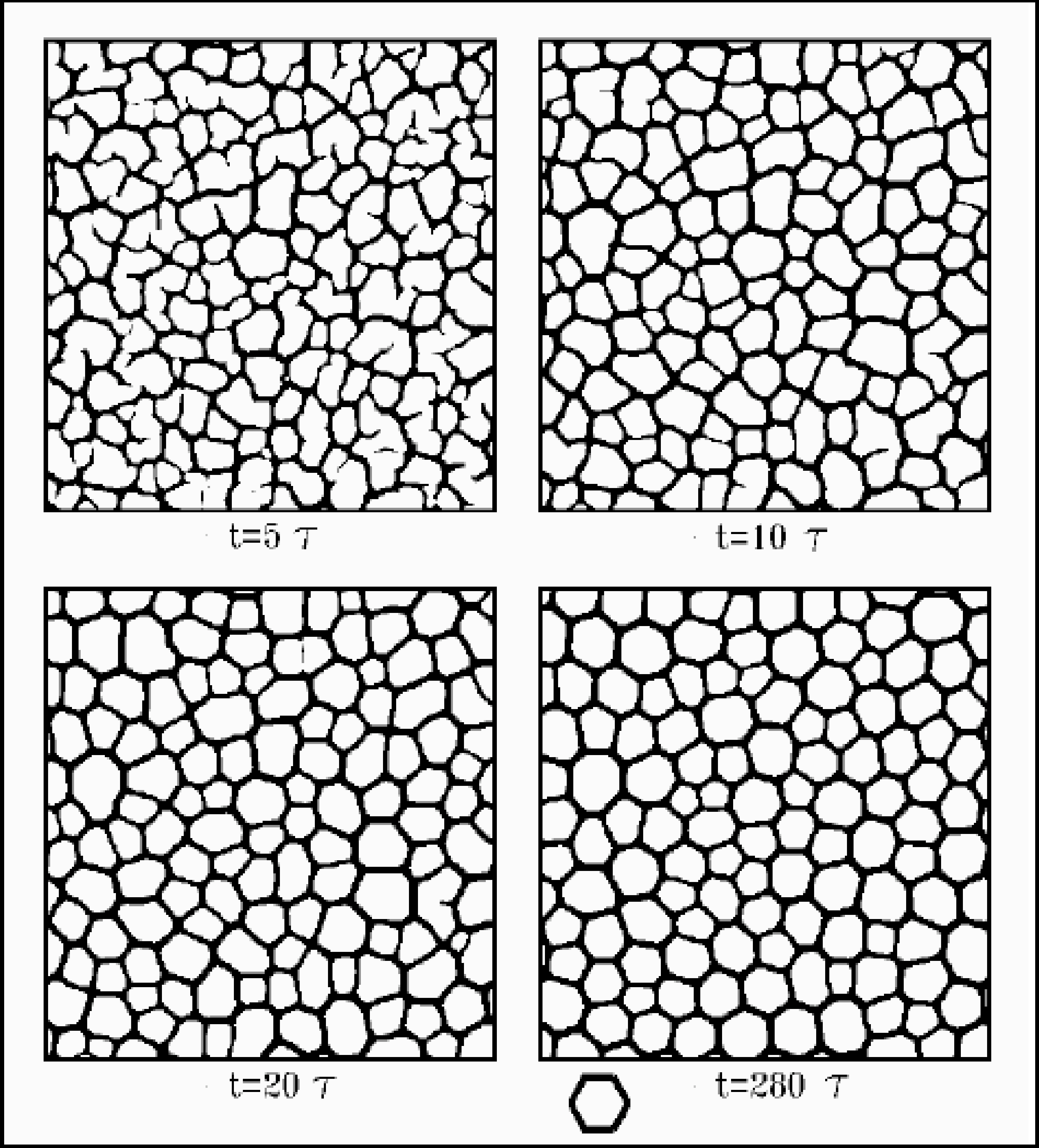
A 2D phase field model

Dynamically minimize a model free energy with a "soft" phase which represents the cracks.

The global free energy minimum is a perfect hexagonal lattice of a certain size.

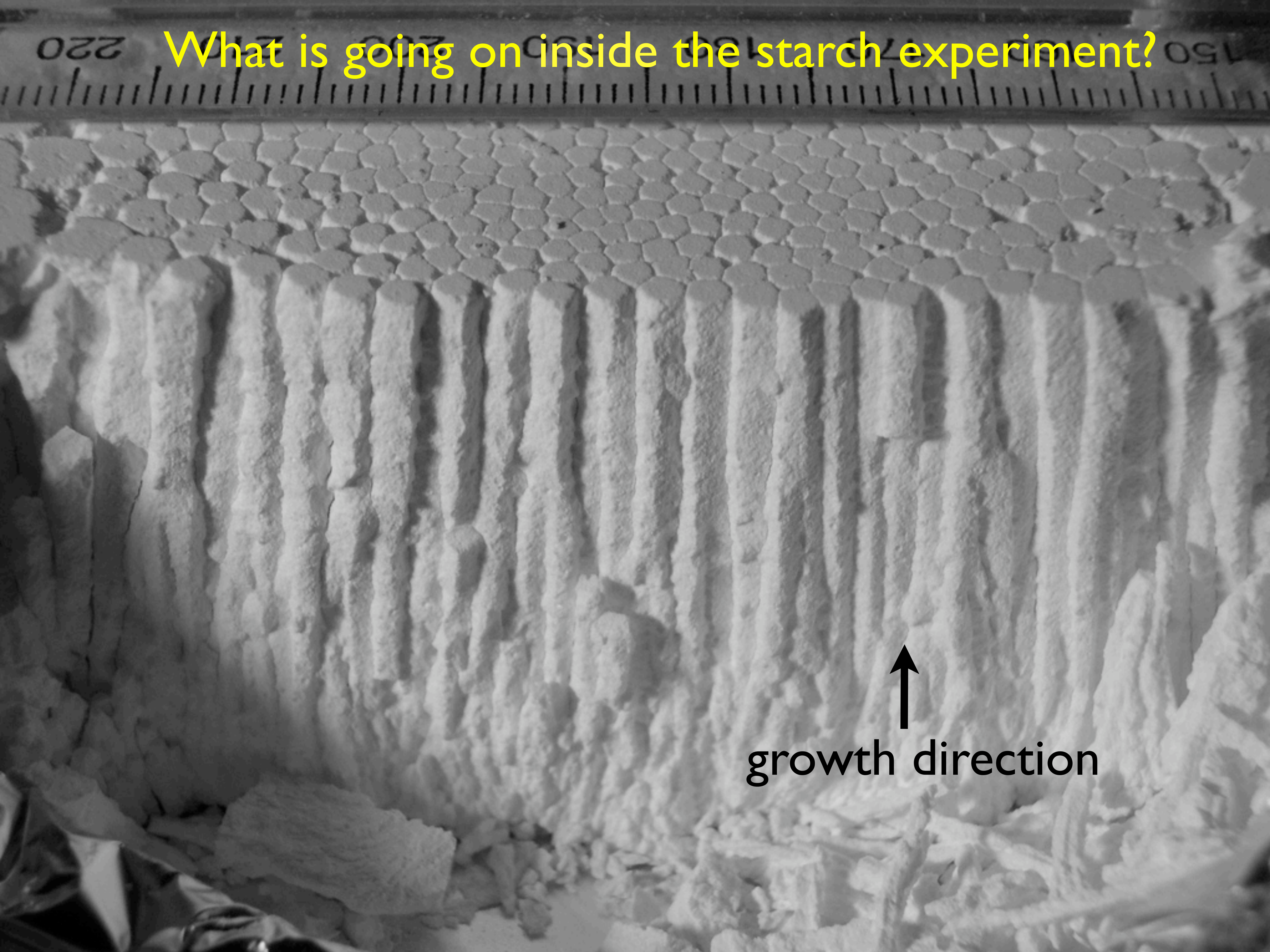
System gets stuck in a local free energy minimum.

Is this "optimization" paradigm correct?



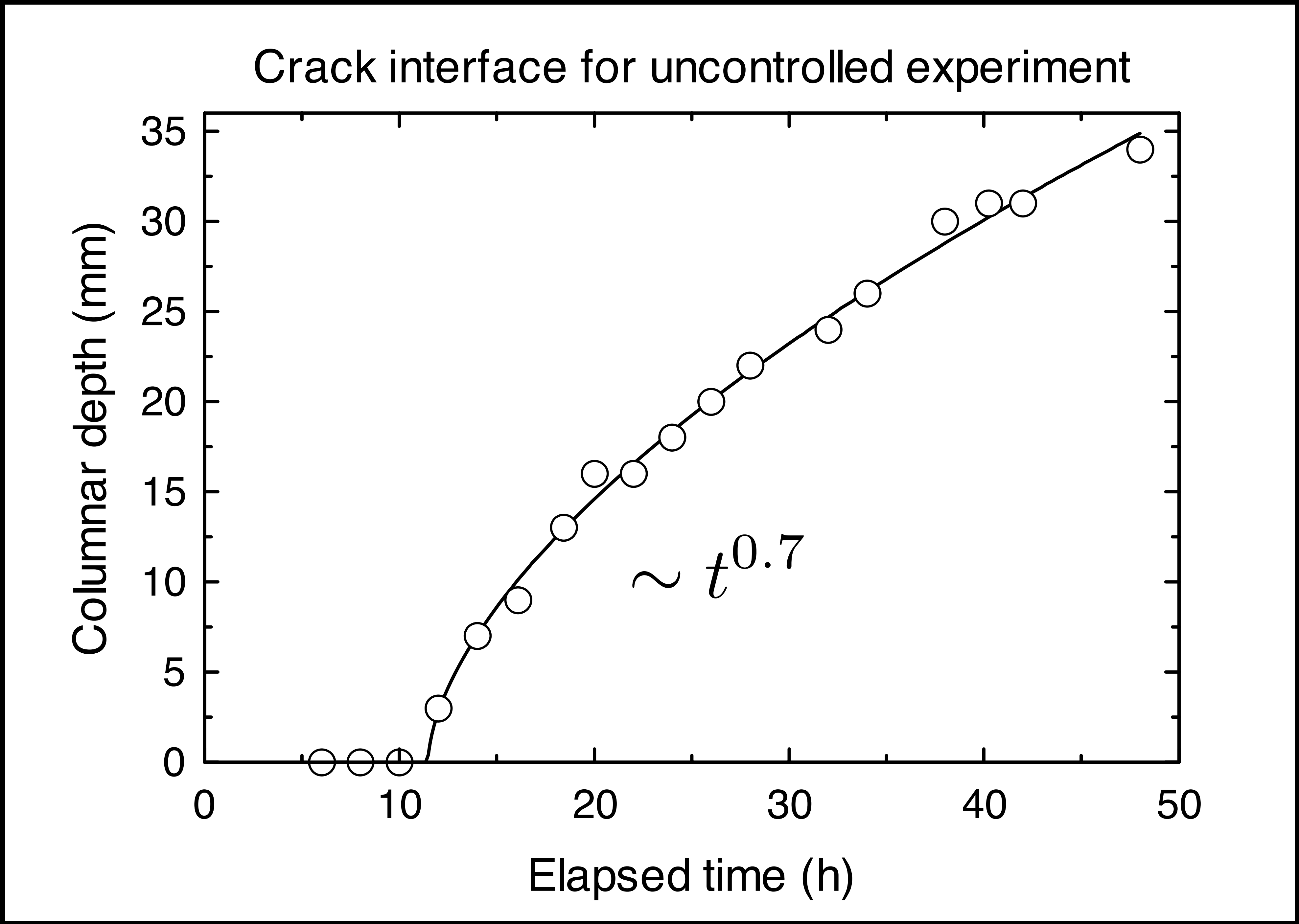
E. A. Jagla, Phys. Rev. E 69, 056212 (2004)

What is going on inside the starch experiment?



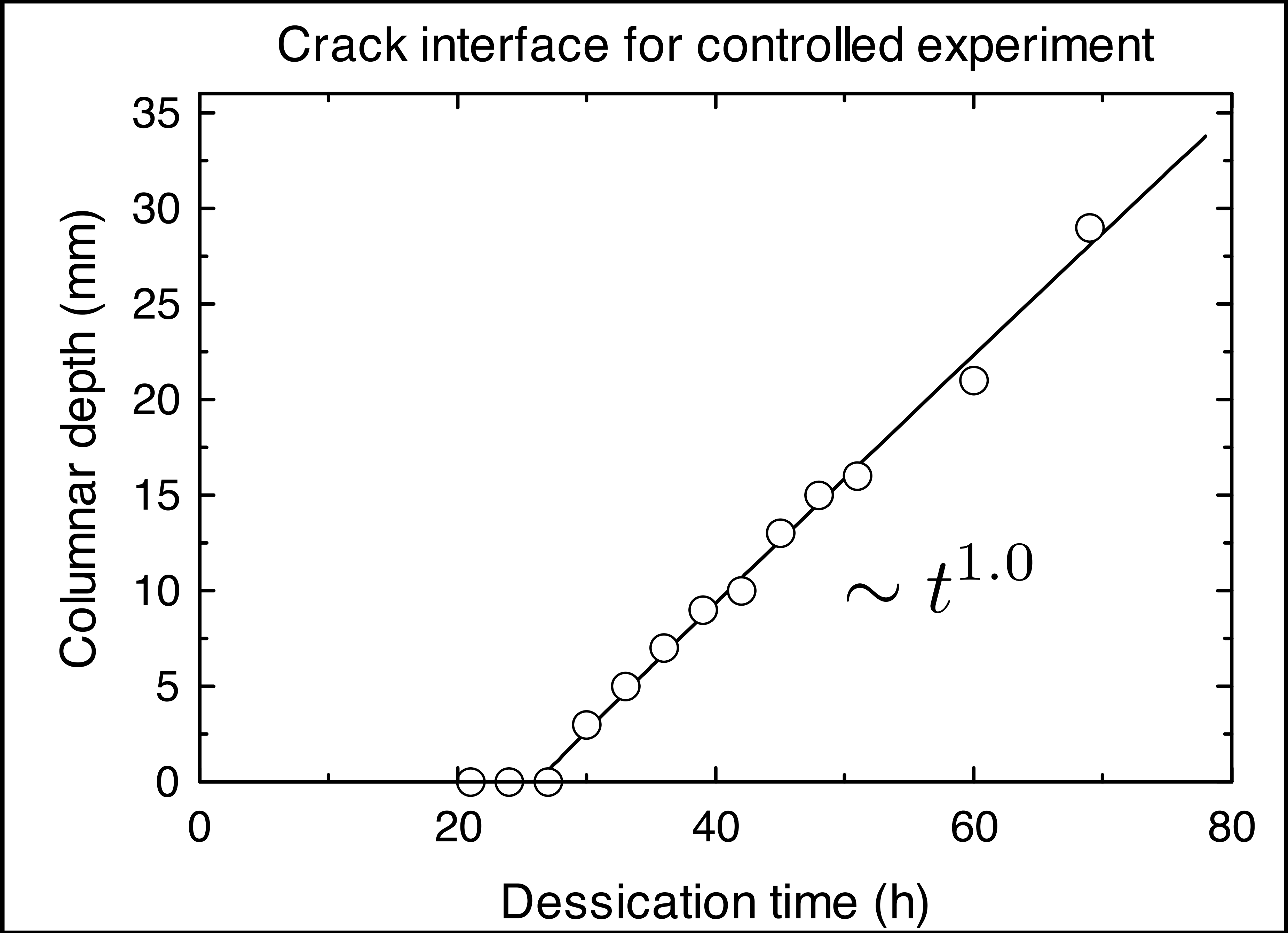
Progress of the cracks into the sample

(measured by destroying many samples)



Uncontrolled columns coarsen and slow down

Progress of the cracks into the sample
(measured by destroying many samples)



Controlled columns advance at a constant rate

Soil mechanics model of water transport in starch

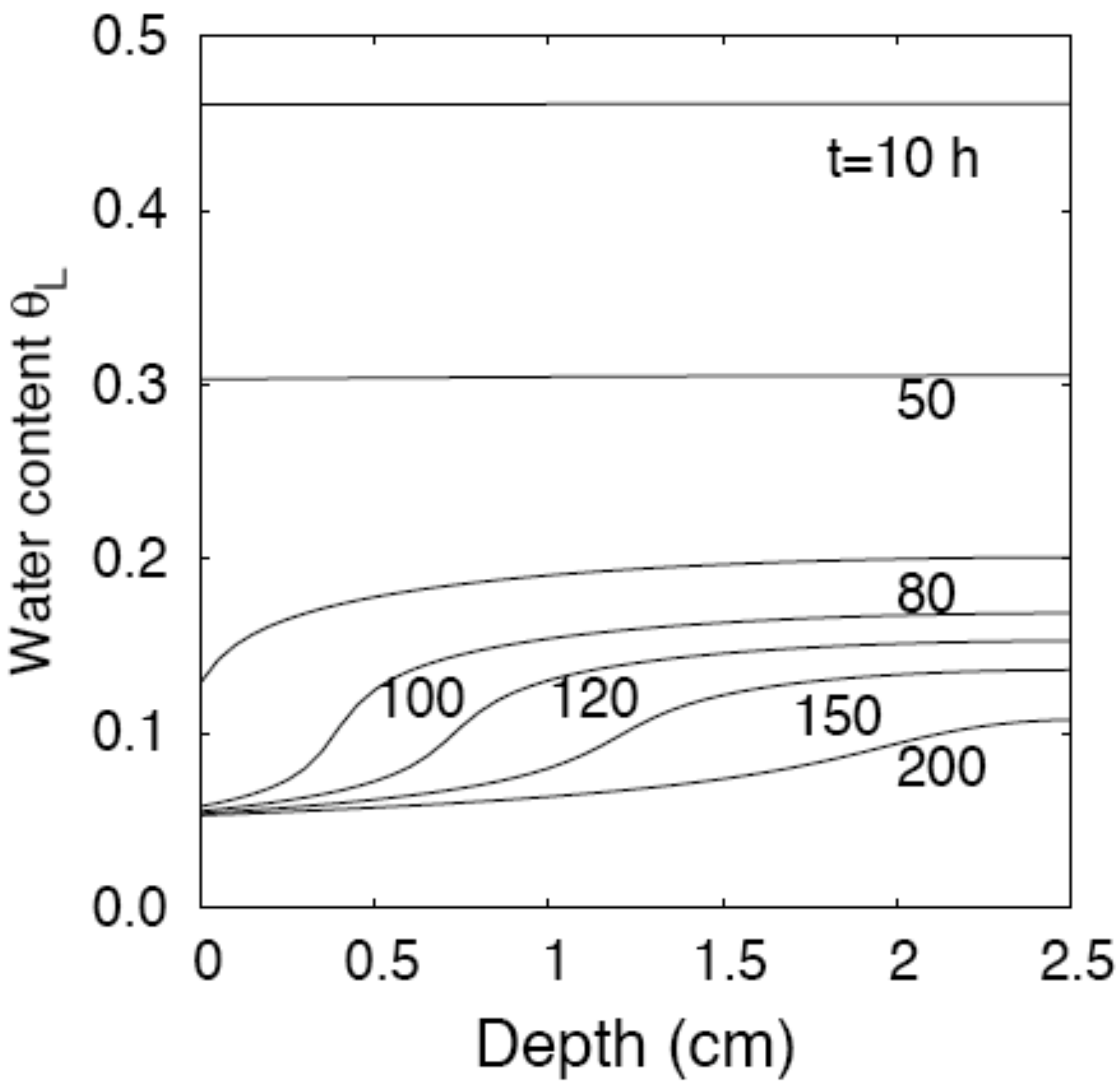


FIG. 5: Volumetric water content θ_L vs depth z.

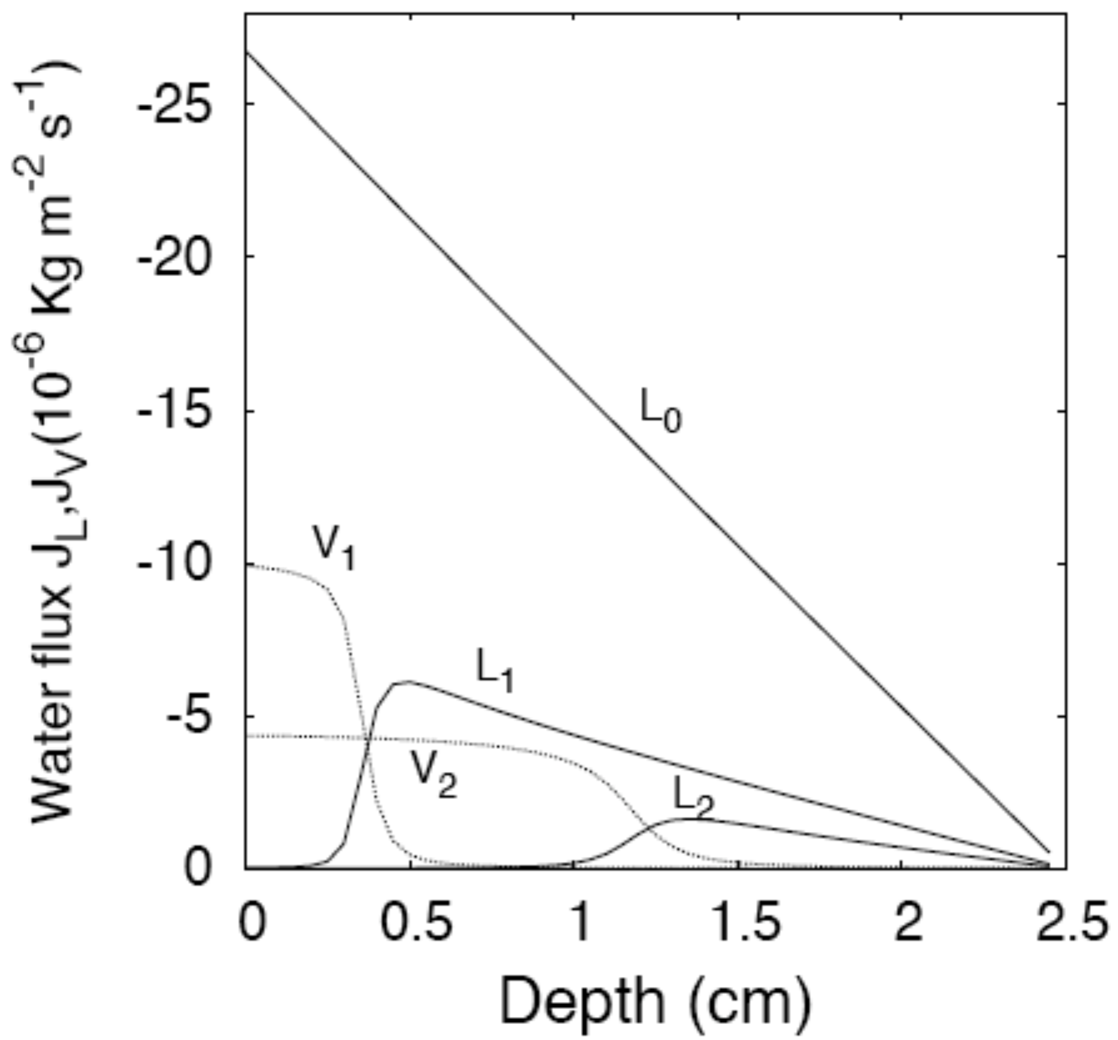


FIG. 6: Flux of liquid water J_L and flux of vapor J_V vs depth z. L_0: $J_L(t = 50\text{h})$, L_1: $J_L(t = 100\text{h})$, V_1: $J_V(t = 100\text{h})$, L_2: $J_L(t = 150\text{h})$ and V_2: $J_V(t = 150\text{h})$. $J_V(t = 50\text{h})$ is on the order of 10^{-11} Kg m^{-2} s^{-1}.

Akihiro Nishimoto, Tsuyoshi Mizuguchi, So Kitsunezaki
http://arxiv.org/abs/cond-mat/0702083

Soil mechanics model of water transport in starch

There is a sharp minimum in the effective diffusion constant D at the position of the front.

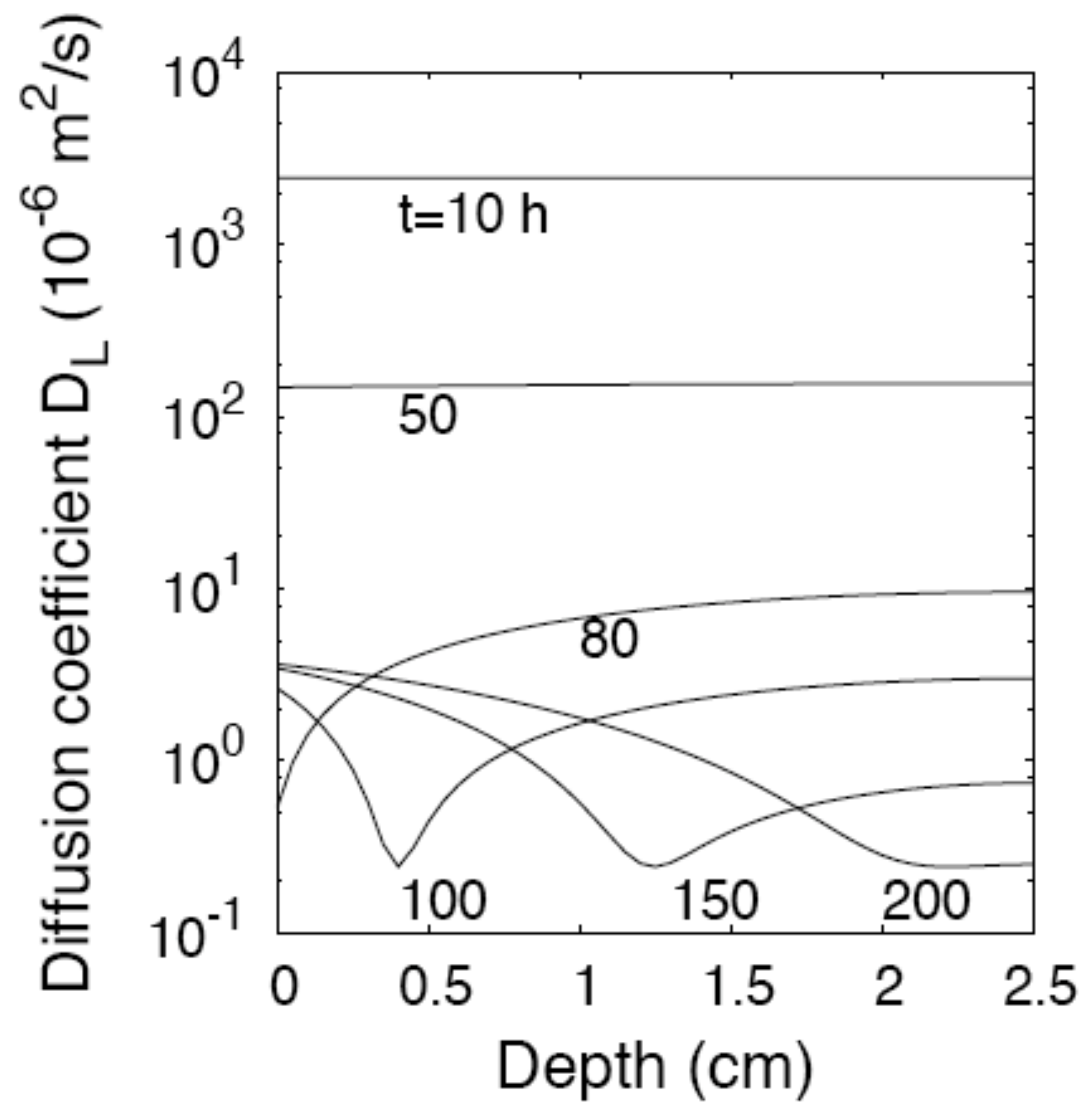


FIG. 14: Diffusion coefficient D_L defined by Eq. (33).

Akihiro Nishimoto, Tsuyoshi Mizuguchi, So Kitsunezaki

http://arxiv.org/abs/cond-mat/0702083

Dynamical similarity between starch and basalt columns

Both starch and basalt form under constant flux conditions.

The only length is

$$L \sim D/J$$

L. Mahadevan

The flux is proportional to the crack advance rate

$$J \sim v$$

Dynamical similarity between starch and basalt columns

Both starch and basalt form at a similar Péclet number:

$$Pe = \frac{vL}{D} \sim 0.2$$

L. Goehring, Z Lin and S. Morris, *Phys. Rev.* E 74, 036115 (2006), *J. Geophysical Res.*, submitted.

Dynamical similarity between starch and basalt columns

Exploit the striae as proxies for the stress field, use some thermal modeling to deduce the heat flux *Q* and hence the mean fracture speed v.

It comes down to a constant heat flux per unit cross sectional length of crack of

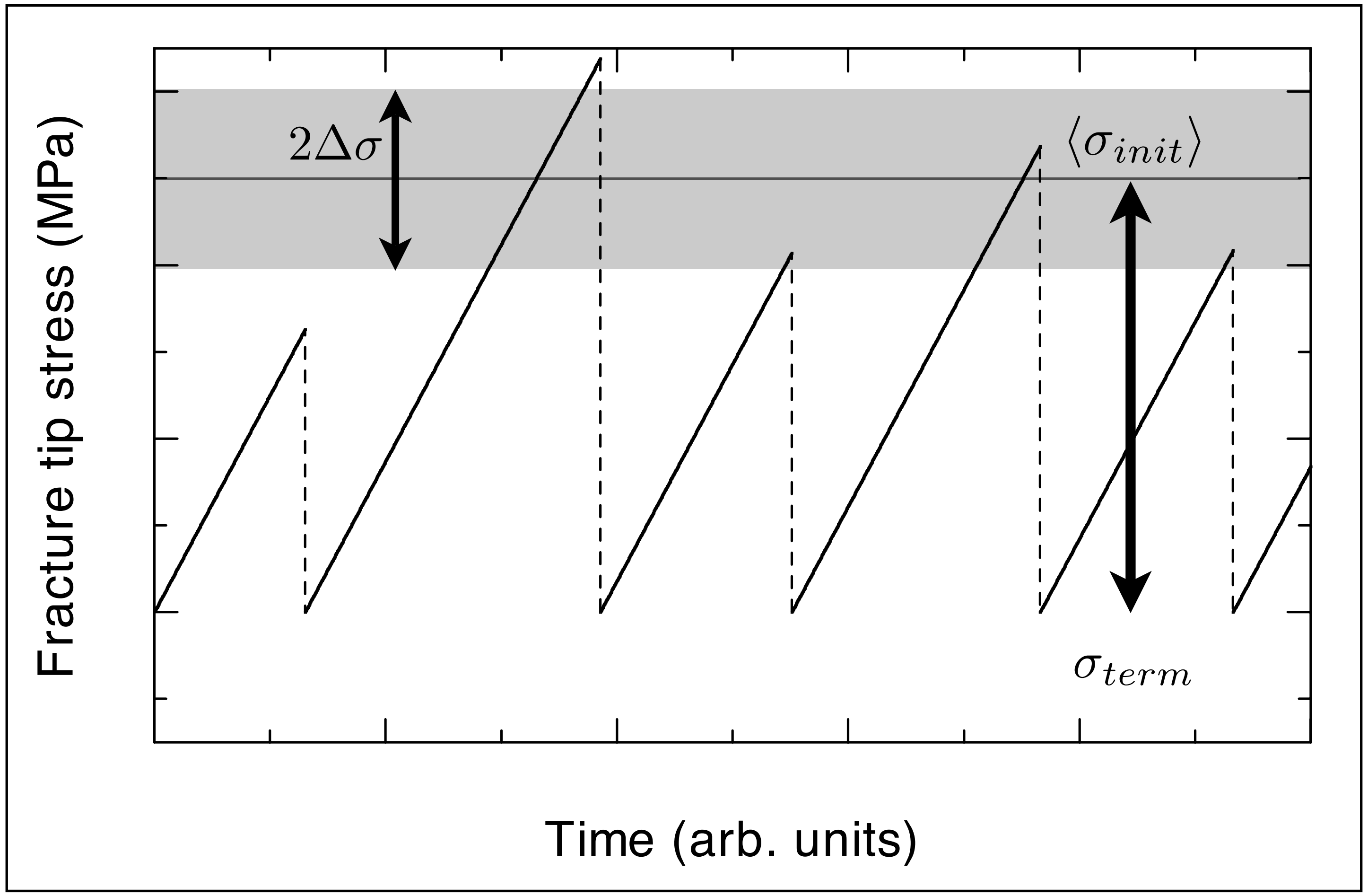
$$q_c = 500 \pm 200 \quad \mathrm{W/m}$$

which corresponds to 0.2 ml/s of water boiled off per meter of crack.

L. Goehring and S. Morris, *Journal of Geophysical Research*, submitted.

Thermal model

Assume stria height $\langle s \rangle$ spans starting and stopping stresses which map to certain isotherms.



$$\langle \sigma_{init} \rangle,\ \sigma_{term} \to T_{init},\ T_{term}$$

Near the flow margin:

$\partial_t T = \kappa \nabla^2 T$ and $\langle s \rangle \propto z$ Striae grow linearly with height

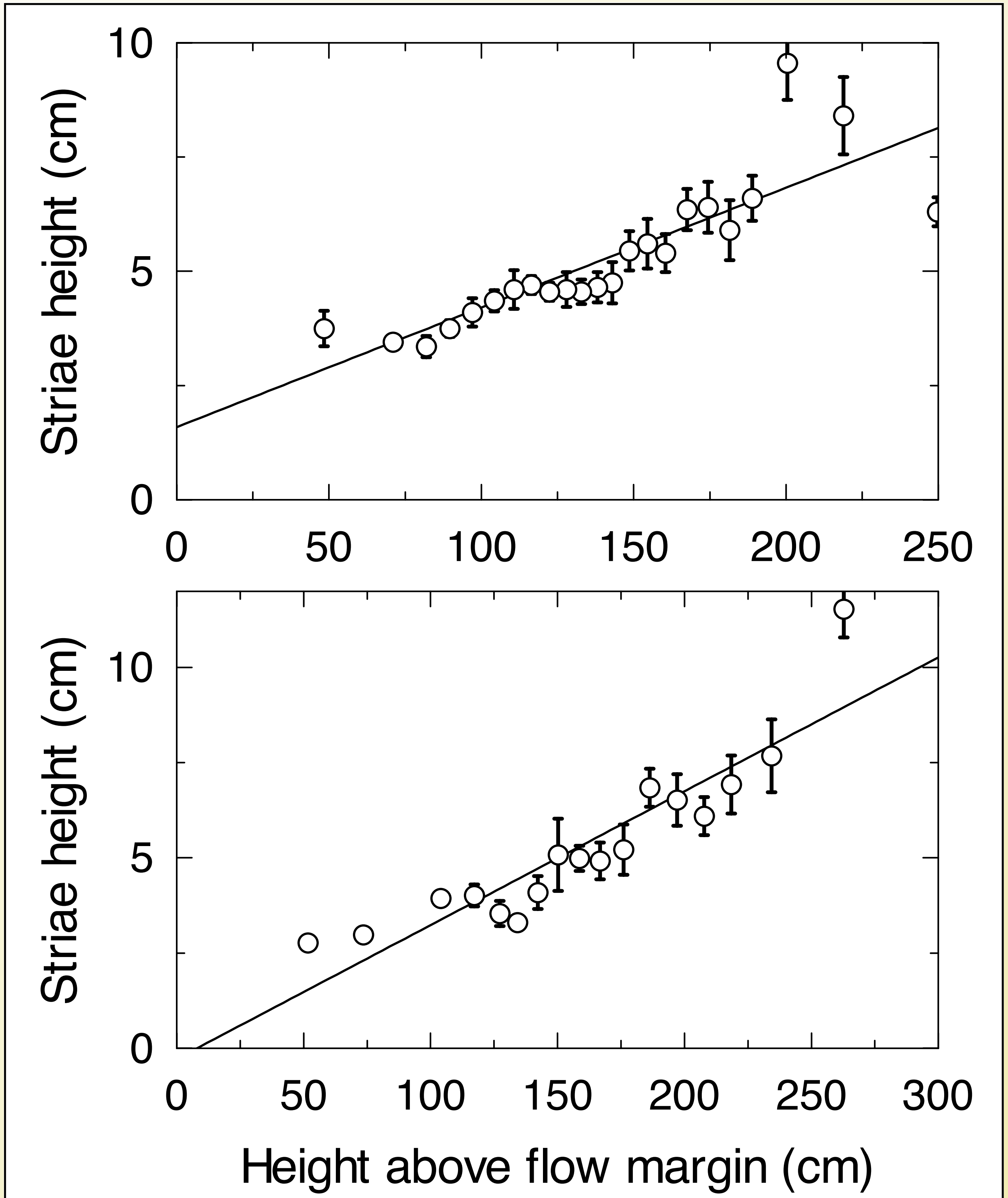
Far from a flow margin: in the nonequilibrium steady state:

$\partial_t T = \kappa \nabla^2 T + v \partial_z T = 0$ and the heat flux

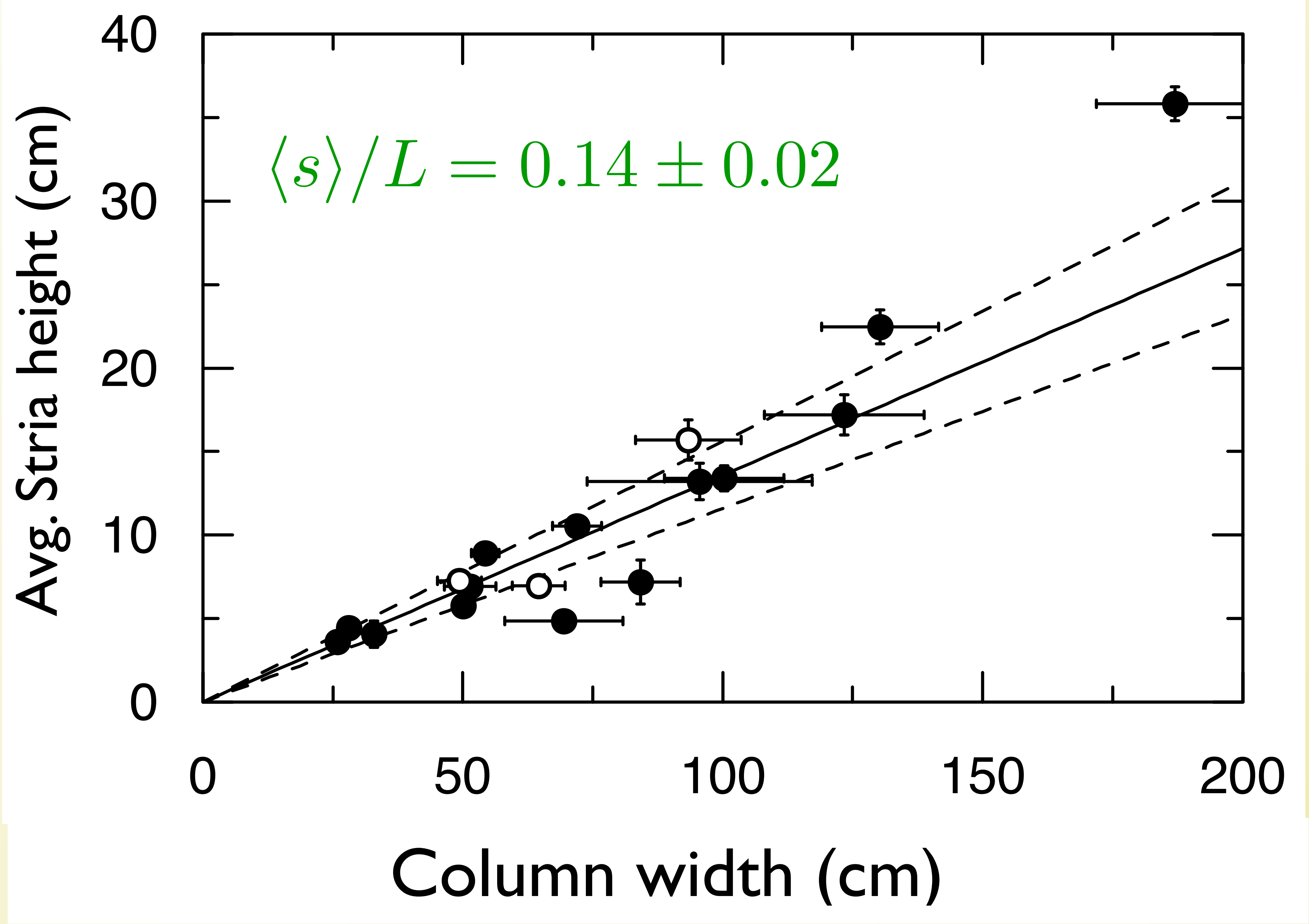
$$Q = \rho v (c_p (T_1 - T_0) + \mathcal{L}) = \frac{\rho \kappa}{\langle s \rangle} (c_p (T_1 - T_0) + \mathcal{L}) \log \left[\frac{1 - U_{init}}{1 - U_{term}} \right].$$

$U(T) = (T - T_0)/(T_1 - T_0)$ $\qquad v \propto Q \propto 1/\langle s \rangle$

Near a flow margin, striae increase in size linearly with position, but become constant in the interior of the flow.



Far from margin, stria heights scale with column width L



Use this to deduce heat flux Q and hence fracture speed v

Will show that L and v are simply related

by a dynamic similarity argument.

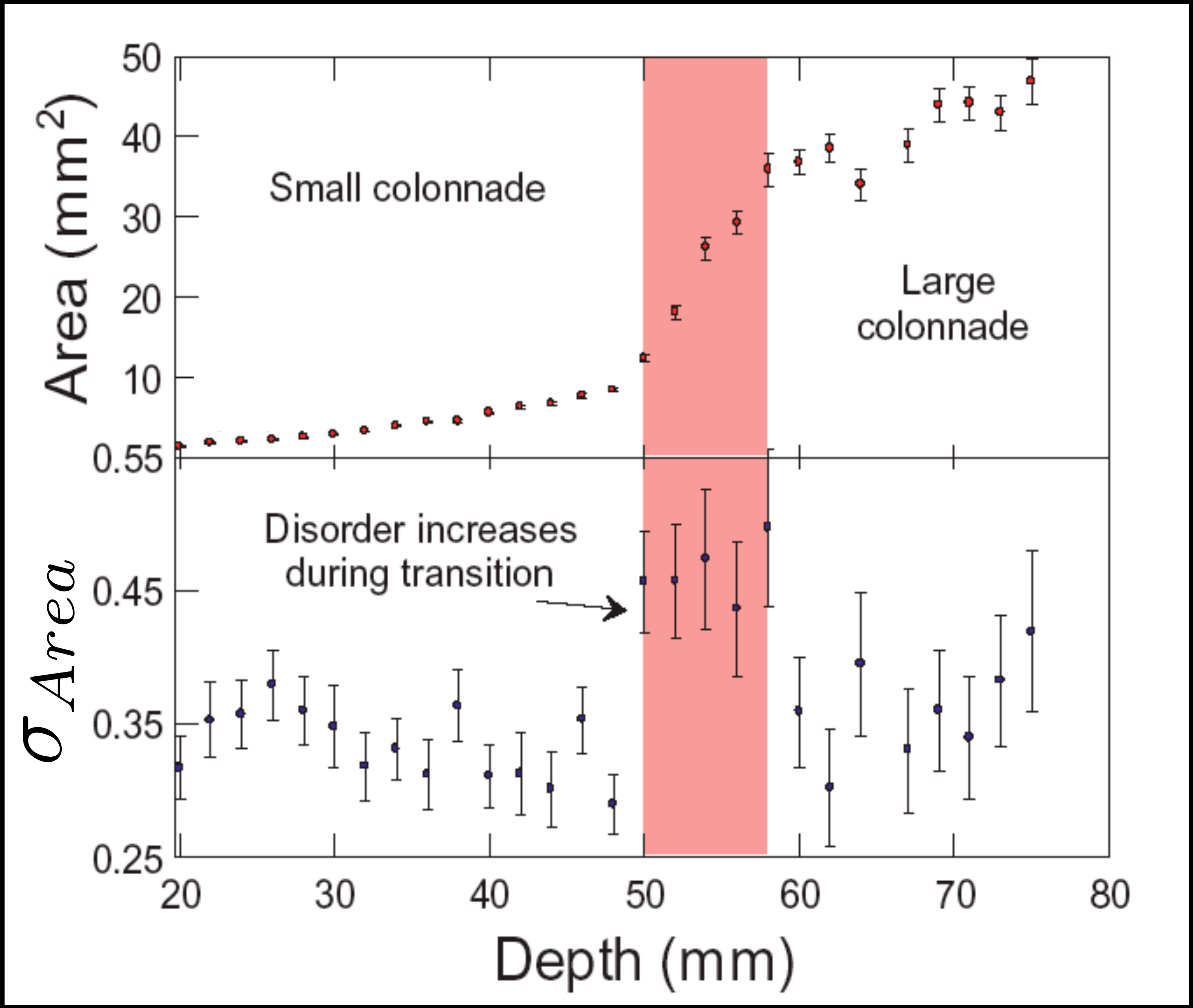
Conjecture:

Péclet number controls the stability, and hence the overall scaling of the columns.

The columns are a dynamically selected pattern, not the result of some optimization of a "free energy".

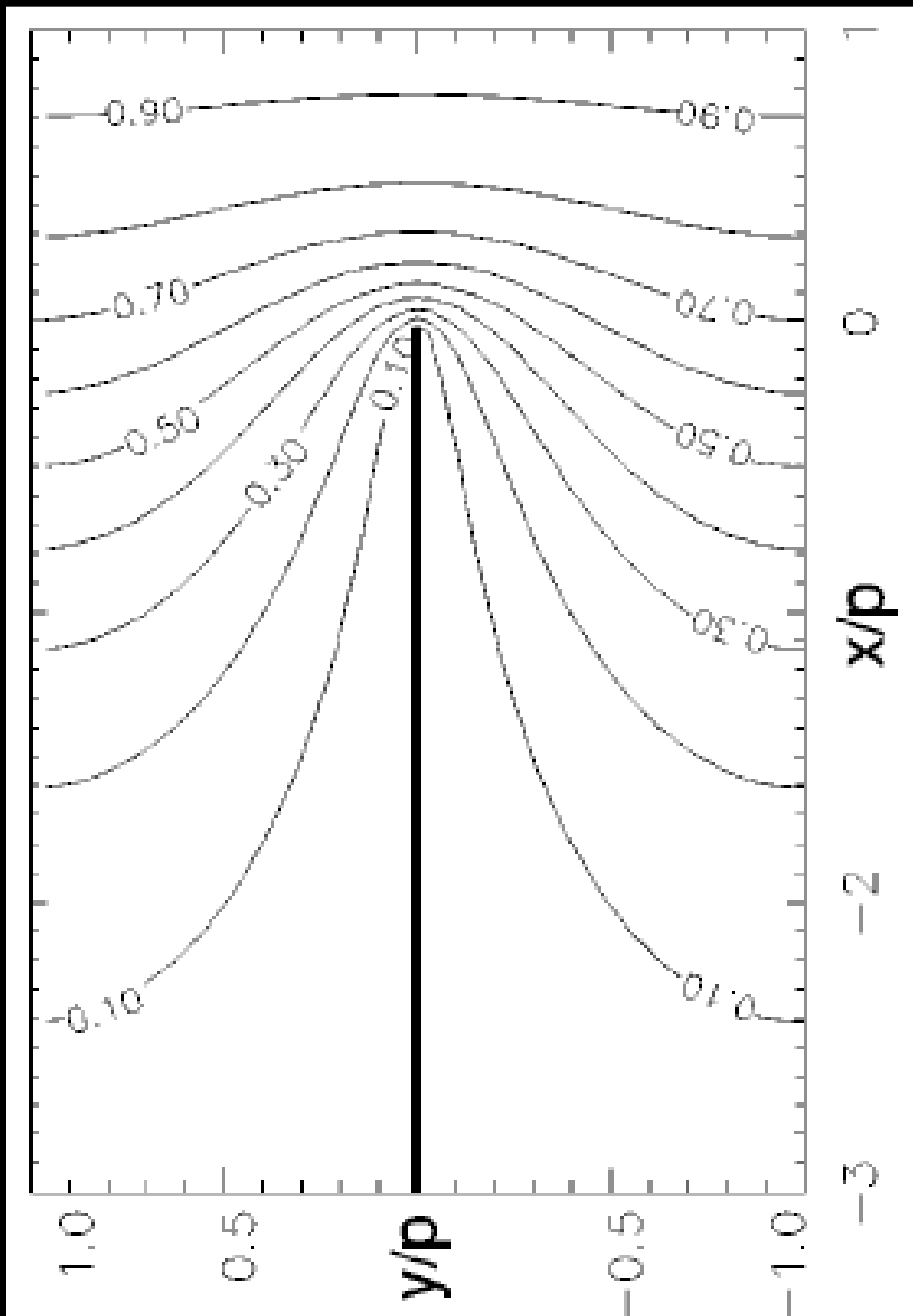
Sudden scale transitions in starch experiment

Bursts of column mergers, also seen in basalts

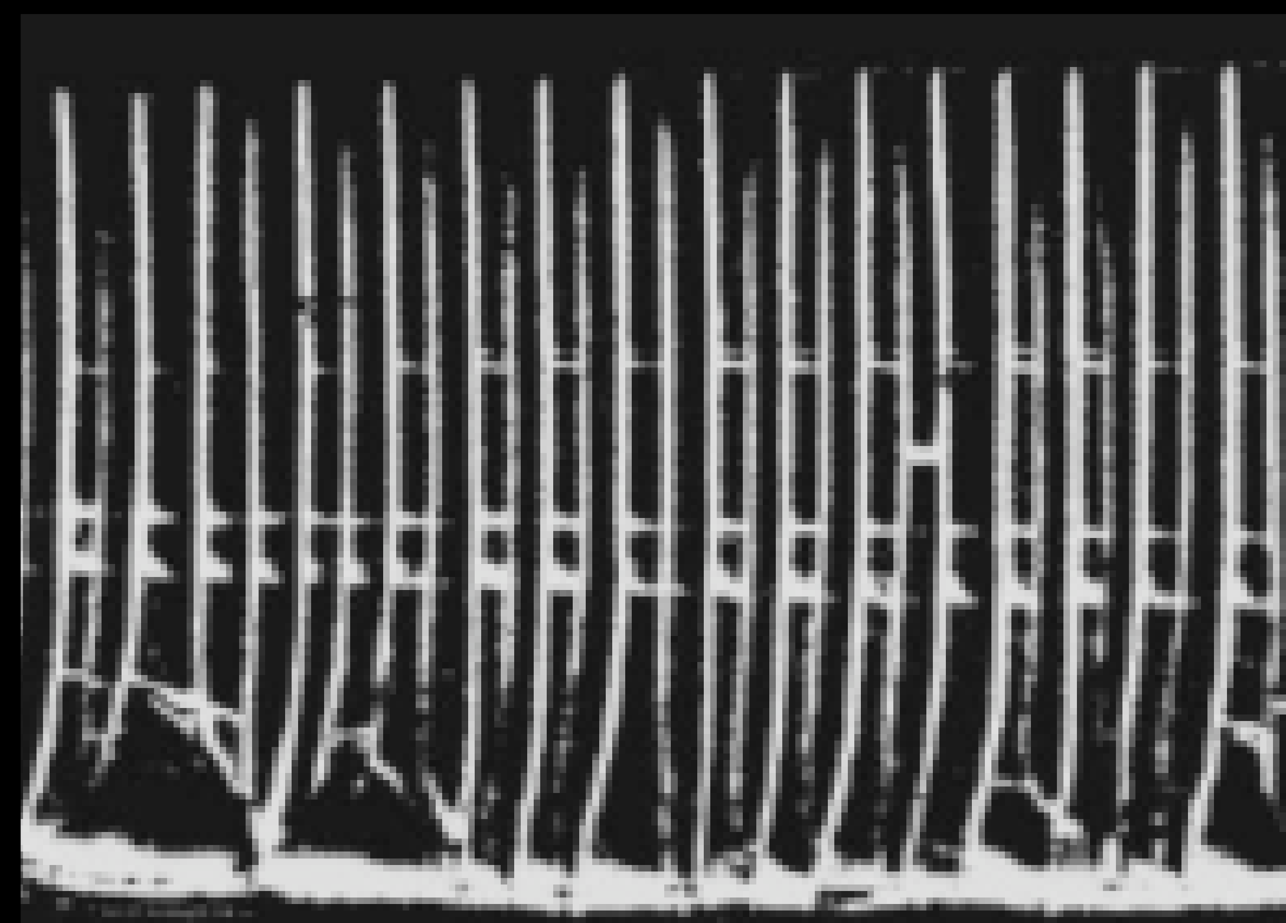


Are these jumps a dynamical effect?

2D thermal analog - fractured glass



Starting with 2D advection-diffusion equations, Boeck *et al.* have described a period-doubling instability in glass fractures.



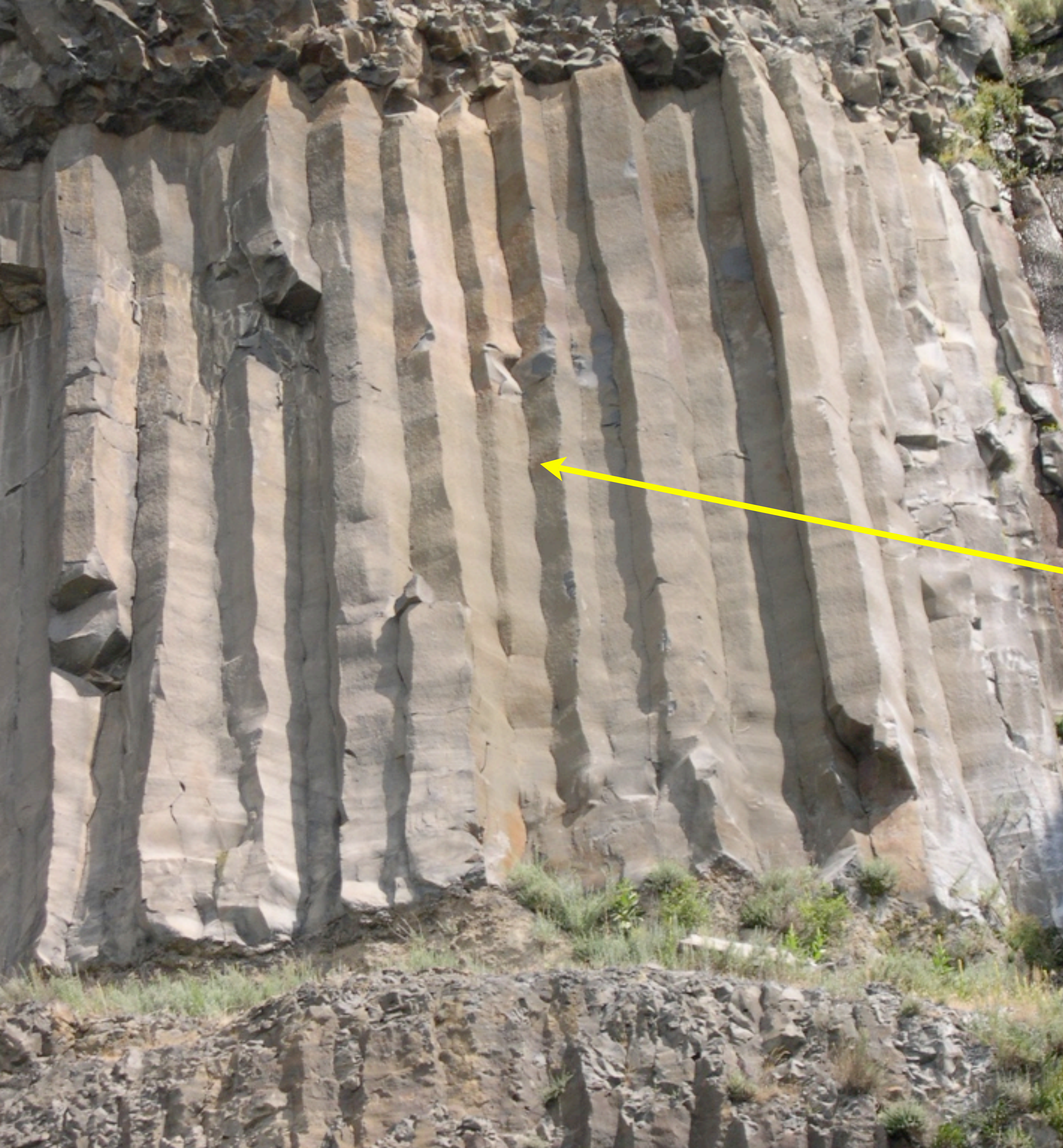
T Boeck *et al.*, *Phys. Rev. E*, **59**, 1408 (1999).

Hysteresis and multistability in the column scale near *Pe* ~ 0.2

Suggests a finite band of column scales are stable at any given fracture advance speed, depending on history of preparation.

This is to be expected if the column scale is selected dynamically.

Are there secondary instabilities?



Wavy columns?

Grand Coulee

Are there secondary instabilities?

Wavy columns?

Devil's Postpile, California

Conclusion

Still many open questions.

Dynamic stability of the advancing columns seems to be the right approach, rather than some global optimization.