IPAM Long Program
Computational Issues in Oil Field Applications
Workshop III: Data Assimilation, Uncertainty Reduction, and Optimization for Subsurface Flow
21-24 May 2017

Model-based production optimization and history matching – some (not so) recent developments

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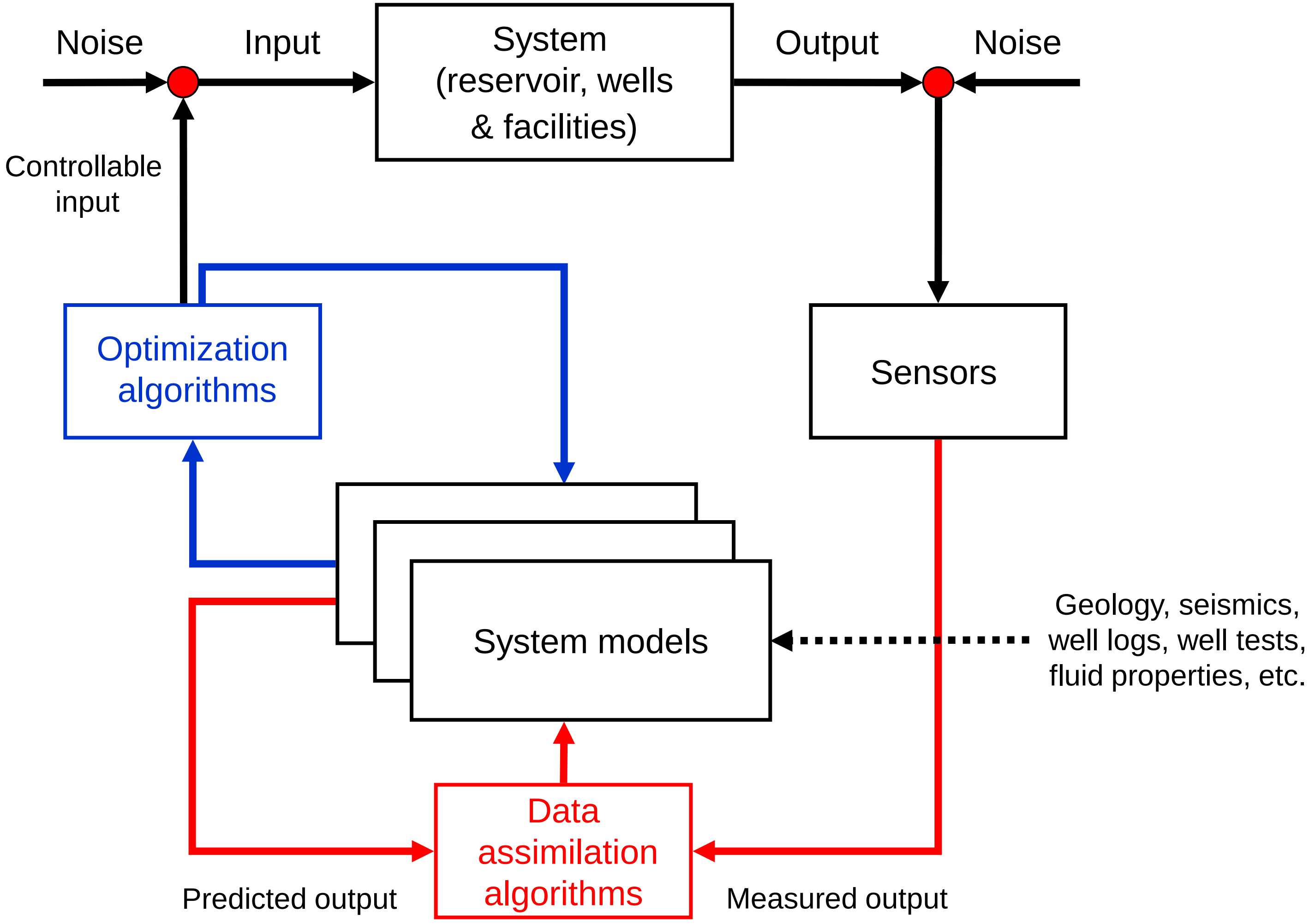
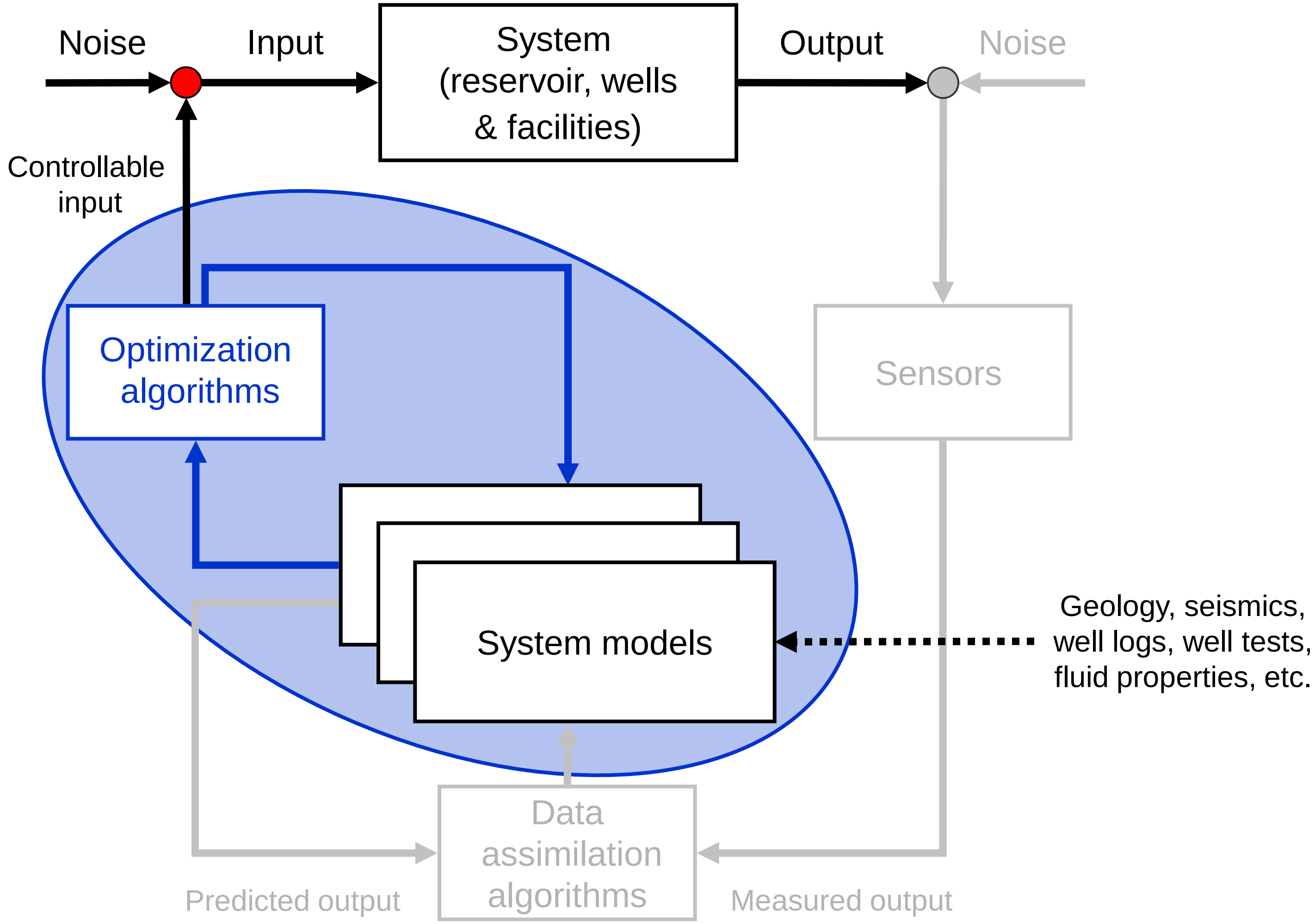
Closed-loop reservoir management



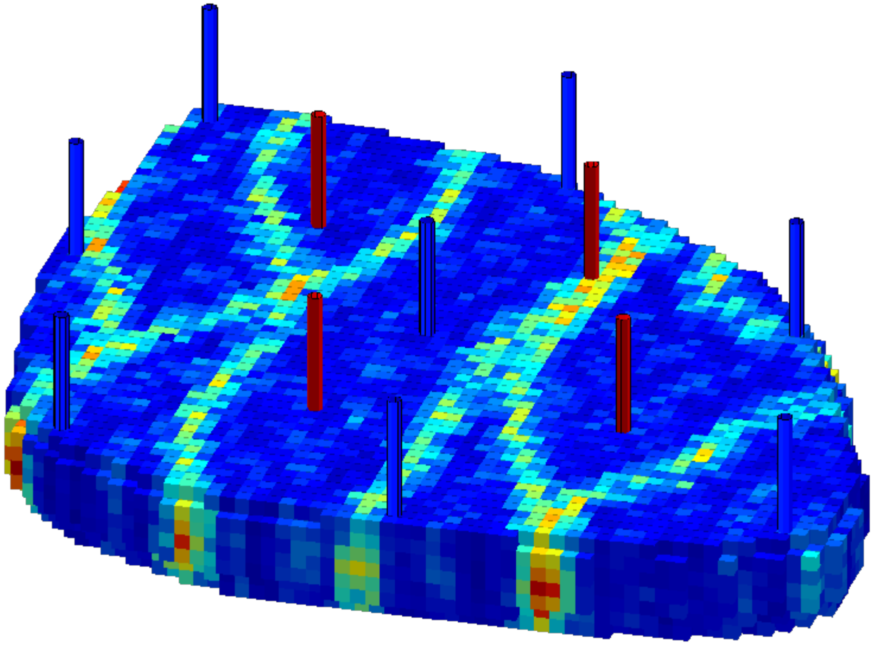
1) "Robust" open-loop production optimization



12-well example (the “egg model”)

- 8 injectors, rate-controlled
- 4 producers, BHP-controlled
- Production period of 10 years
- 12 wells x 10 x 12 time steps

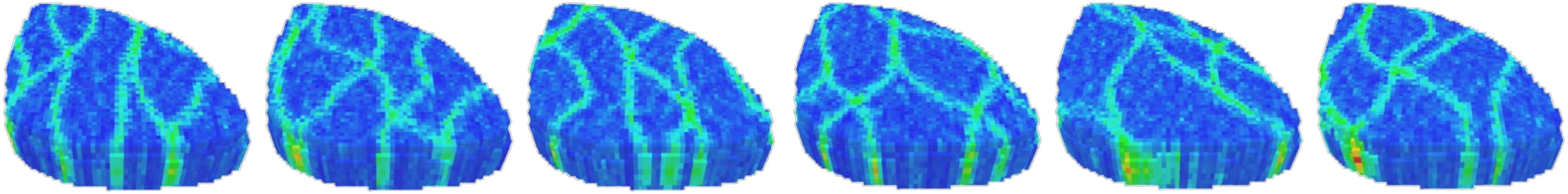
  => 1440 optimization parameters
- Bound constraints on controls



Van Essen et al., 2009

- Objective J: oil revenues minus water costs (‘NPV’)
- Forward model: fully implicit FV simulator (Dynamo MoReS, MRST)
- Optimizer: gradient- based (steepest ascent; line search with simple back tracking, gradients with adjoint formulation; projected constraints)

‘Robust’ optimization example (‘mean’ optimization)



Van Essen et al., 2009

- Number of realizations N_r = 100
- Optimize expectation of objective function J

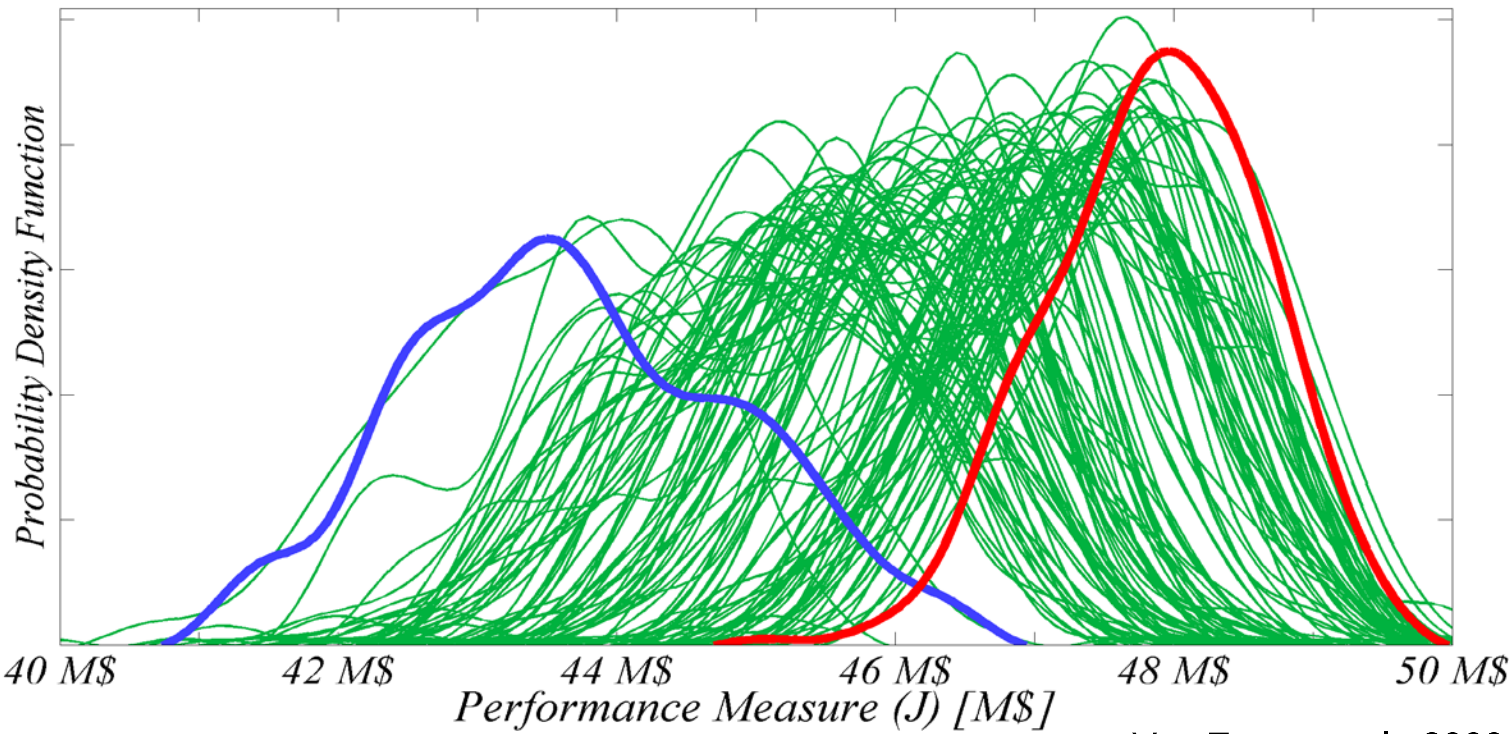
$$\max_{\mathbf{u}} \frac{1}{N_r} \sum_{i=1}^{N_r} J^i\left(\mathbf{u}, \mathbf{m}_i\right)$$

- $\mathbf{u}$: inputs (well rates, pressures) for all optimization time steps
- $\mathbf{m}$: parameters (permeabilities)

Robust optimization results

3 control strategies applied to set of 100 realizations:

reactive control, nominal optimization, robust optimization



Van Essen et al., 2009

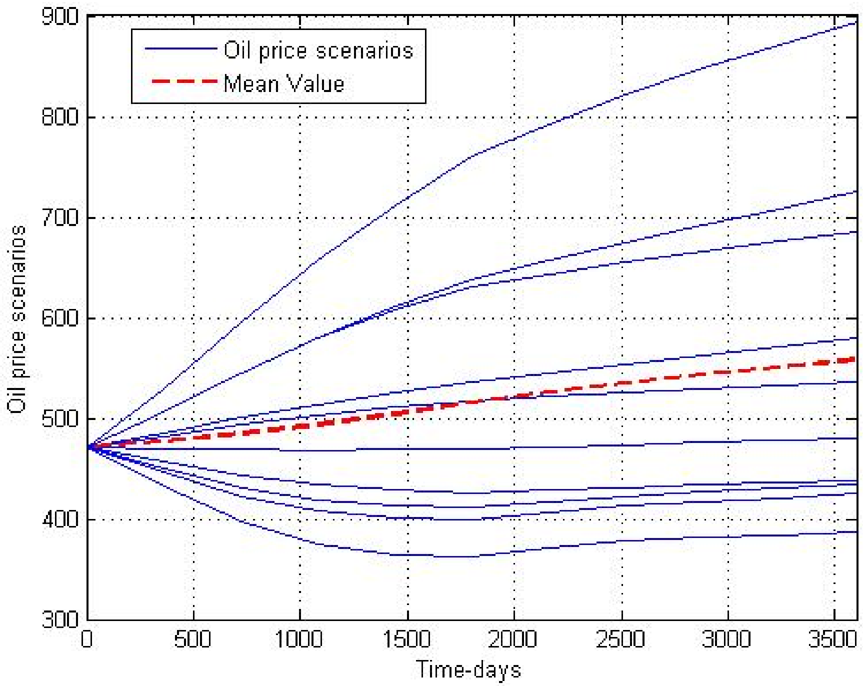
Oil price uncertainty – time series

- Various complex models:
  - Prospective Outlook on Long-term Energy Systems (POLES) (EU and French Government)
  - National Energy Modeling System (NEMS) (US DoE)
- We use: Auto-Regressive-Moving-Average model (ARMA) (Ljung, 1999)

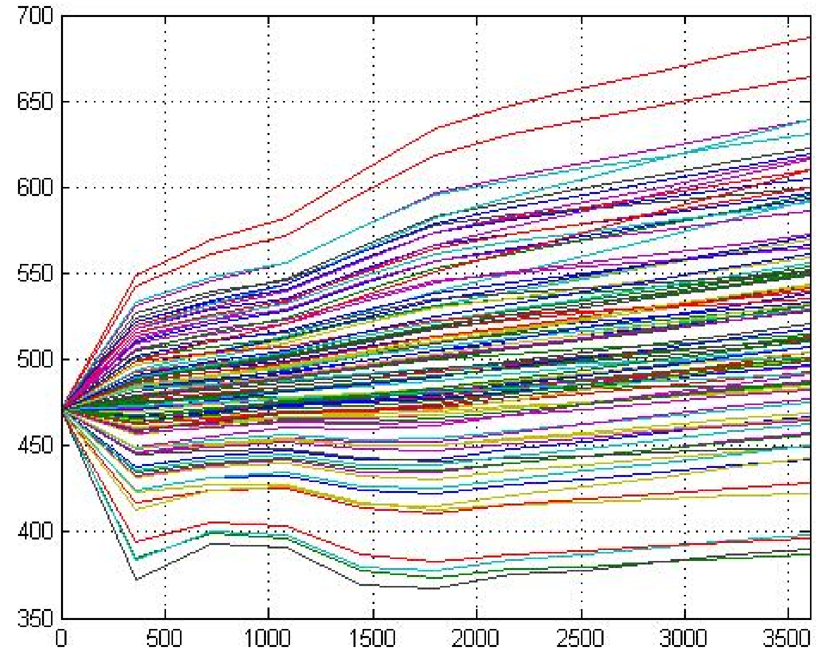
$$r_k = a_0 + \sum_{i=1}^{6} a_i r_{k-i} + \sum_{i=1}^{6} b_i e_{k-i}$$

- r_k = oil price
- e_k = white noise sequence
- a_0, a_i, b_i are constants

Oil price uncertainty – ensemble



$n = 10$



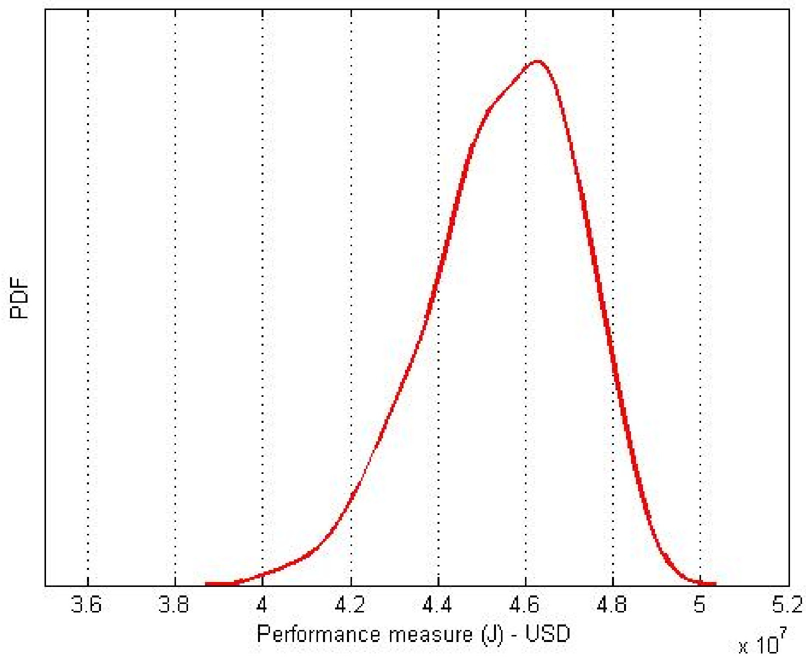
Siraj et al. 2015

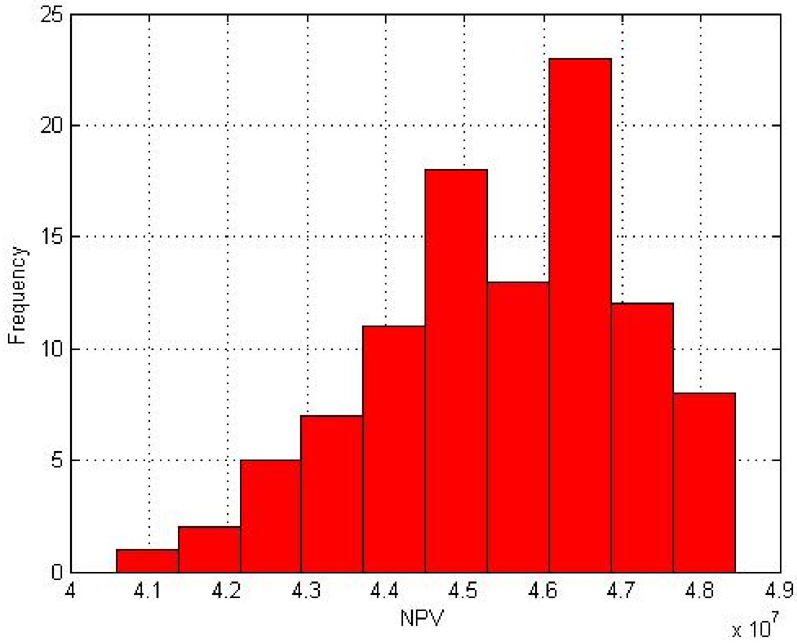
$n = 100$

- Base oil price 471 $/m^3 = 75 $/bbl

Mean optimization (MO)

$$J_{\mathrm{MO}} = \frac{1}{N_r} \sum_{i=1}^{N_r} J^i\left(\mathbf{u}, \mathbf{m}_i\right)$$

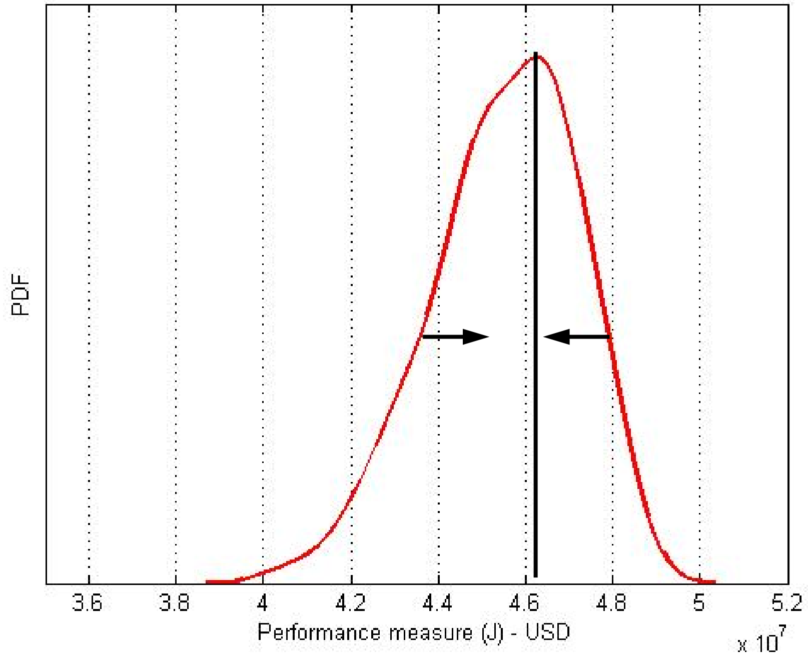




Mean-variance optimization (MVO)

$$J_{\mathrm{MVO}} = J_{\mathrm{MO}} - \gamma J_{\mathrm{V}} = \frac{1}{N_r}\sum_{i=1}^{N_r} J^i - \gamma \frac{1}{N_r - 1}\sum_{i=1}^{N_r}\left(J_{\mathrm{MO}} - J^i\right)^2$$

H. Markowitz (1952), Yeten et al. (2003), Bailey et al. (2005), Yasari et al. (2013), Capolei et al. (2015), Siraj et al. (2015), Liu and Reynolds (2016)

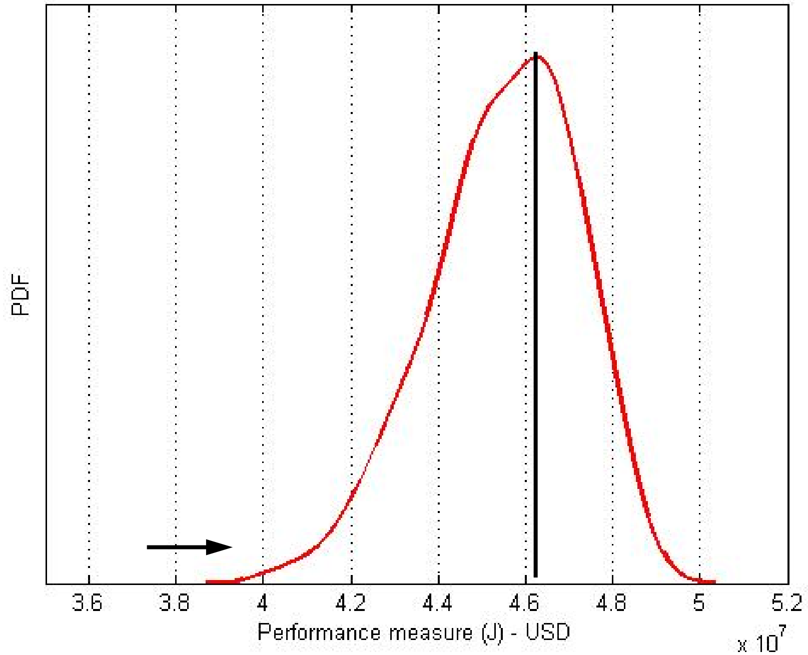


- Symmetric ‘risk measure’
- Penalizes the best cases
- Decision makers are mainly concerned with worst cases

Worst-case optimization (WCO)

$$\max_{\mathbf{u}} \min_{m_i} J\left(\mathbf{u}, m_i\right) \quad \forall i$$

- Min operator on discrete set is non-differentiable
- Reformulate with slack variable z



$$\max_{\mathbf{u}, z} z \quad \text{s.t.} \quad z \leq J\left(\mathbf{u}, m_i\right) \quad \forall i$$

- N_r inequality constraints
- Asymmetric 'risk measure'
- Sensitive to outliers
- Usually very conservative

Optimizer KNITRO

- Large-scale non-linear constrained optimization
- Both interior-point (barrier) and active-set methods;
- Programmatic interfaces: C/C++, Fortran, Java, Python;
- Modeling language interfaces: AMPL ©, AIMMS ©, GAMS ©, MATLAB ©, MPL ©, Microsoft Excel Premium Solver ©;


ziena® OPTIMIZATION LLC Optimization Software, Modeling, and Consulting

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Discussion Forum is at a Google group called "KNITRO". Anyone can view the discussions. To post a message you need to sign into Google, but Google accounts are free.

TECHNICAL REFERENCES

The primary technical reference is:
R. H. Byrd, J. Nocedal, and R. A. Waltz, "KNITRO: An Integrated Package for Nonlinear Optimization" in *Large-Scale Nonlinear Optimization*, G. di Pillo and M. Roma, eds, pp. 35-59 (2006), Springer-Verlag. The *KNITRO User Manual* also contains a comprehensive list of technical references.

The Interior Point algorithms are described in:
R. Byrd, M. E. Hribar, and J. Nocedal, "An Interior Point Method for Large Scale Nonlinear Programming" *SIAM J. Optimization*, 9,4, pp. 877-900 (1999).

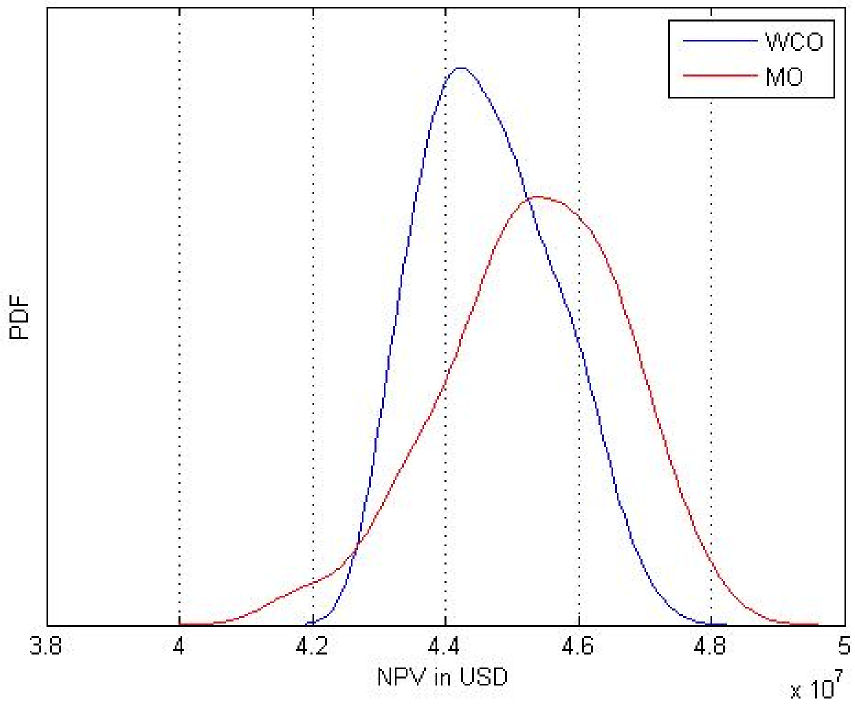
Supporting theory is given in:
R. Byrd, J.C. Gilbert, and J. Nocedal, "A Trust Region Method Based on Interior Point Techniques for Nonlinear Programming", *Mathematical Programming A*, 89: 149-185 (2000).

R. Byrd, Guanghui Liu, and J. Nocedal, "On the Local Behavior of an Interior Point Method for Nonlinear Programming" in *Numerical Analysis*, D. F. Griffiths and D. J. Higham, eds, pp. 37-56 (1997), Addison Wesley Longman.

KNITRO User Manual
Complete details for installing, using, and understanding Knitro.

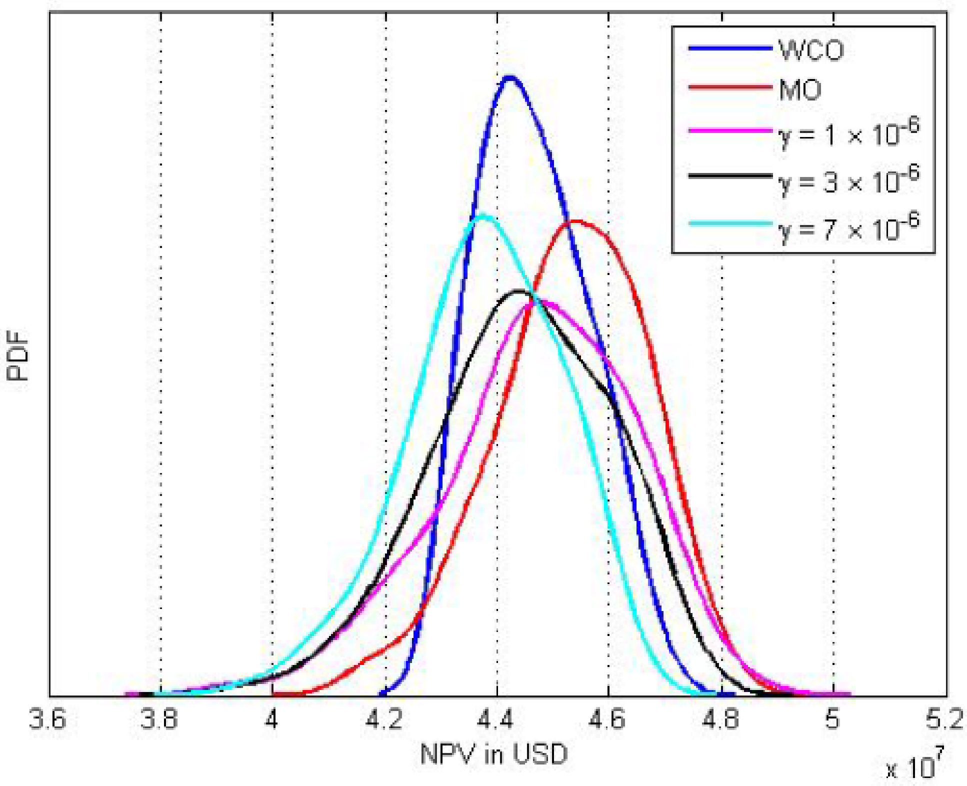
Ziena License Manager User's Manual
The manual for configuring and using Ziena licenses with KNITRO and AMPL.

Worst-case optimization (WCO) (geology)



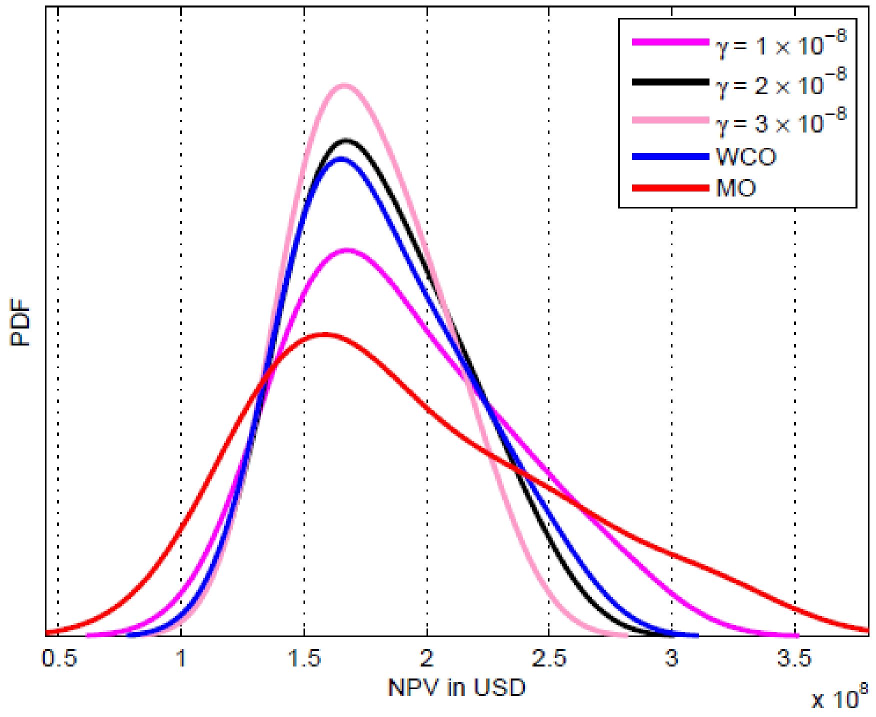
- Worst-case increase: 3.60 %
- Average decrease: 1.54 %

MO, MVO and WCO (geology)



- MVO and WCO all reduce upside

MO, MVO and WCO (oil price)



- Note: WCO = single optimization with lowest oil price
- Same story: MVO and WCO all reduce upside

Mean worst-case optimization (MWCO)

$$J_{\text{WCO}} = \max_{\mathbf{u}} \min_{m_i} J\left(\mathbf{u}, m_i\right)$$

- J_{WCO} is usually very conservative
- Can be controlled ad-hoc with weighted formulation:

$$J_{\text{MWCO}} = J_{\text{MO}} - \lambda J_{\text{WCO}}$$

- Will not be pursued any further

Conditional value at risk (CVaR)

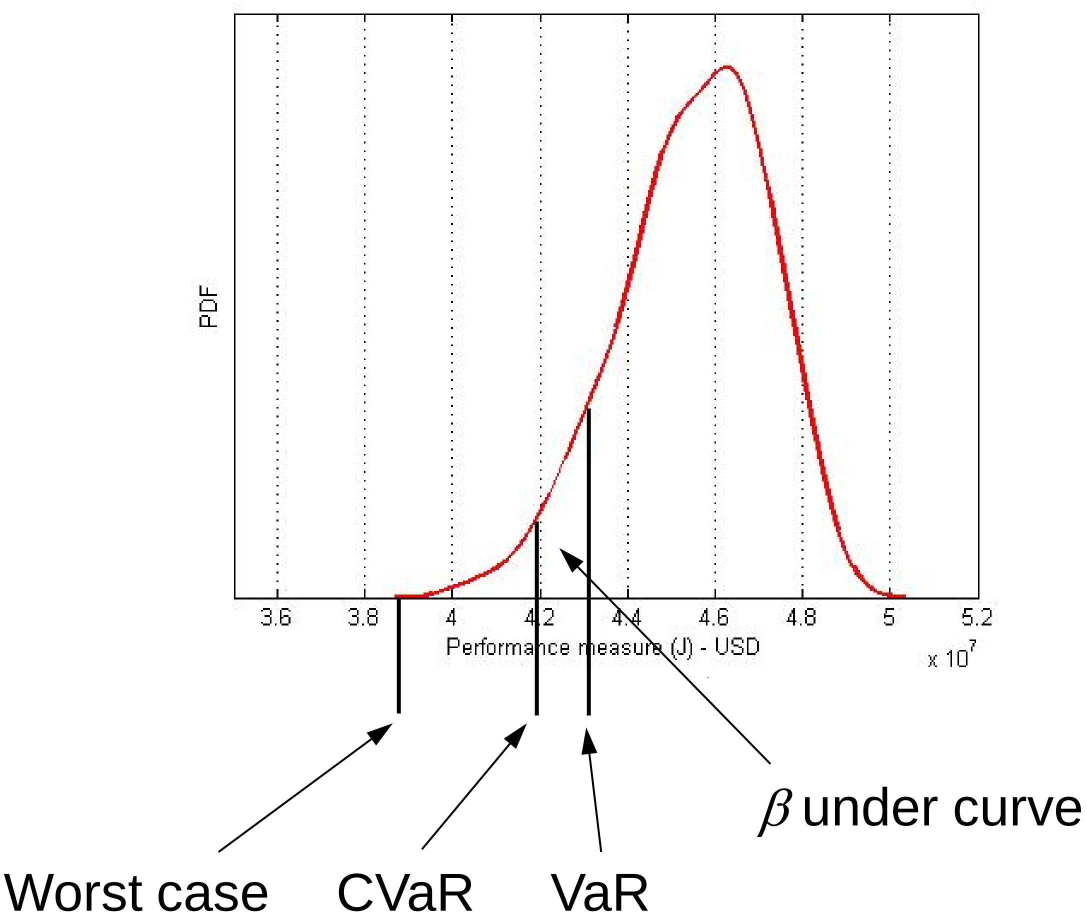
- Value at risk (VaR):

$$\alpha_\beta(x) = \min\left\{z \middle| F_x(z) \le \beta\right\}$$

- x is a random variable
- $F_x(z)$ is the cdf $P(x \leqslant z)$
- $\beta \in]0,1[$ is the confidence level
- In words: β fraction of objective function distribution
- Conditional Value at Risk (CVaR):

$$\varphi_\beta(x) = E\left\{x \middle| x \le \alpha_\beta\right\}$$

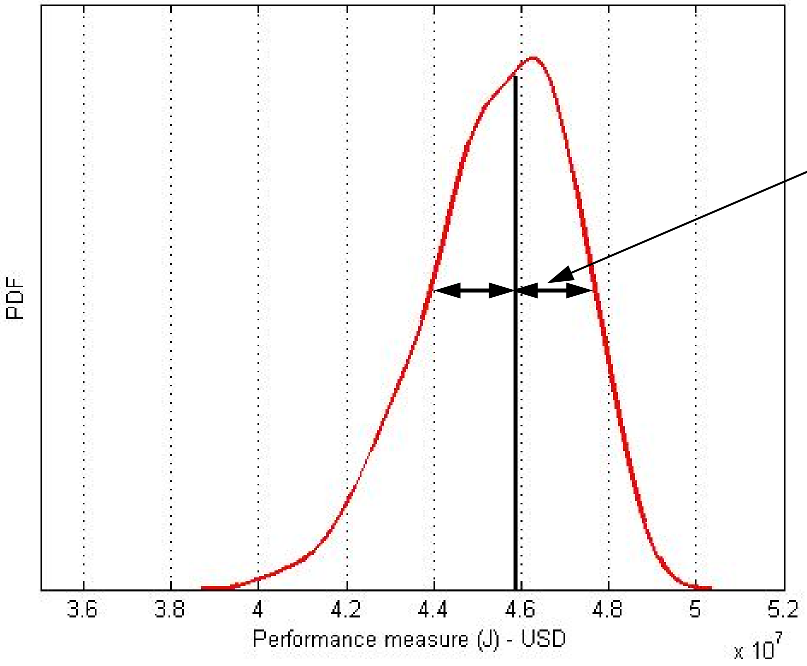
Worst case, VaR, and CVaR



Semi variance

$$Var_{+}\left(x\right)=E\left\{\max\left[x-E\left(x\right),0\right]\right\}^{2}$$

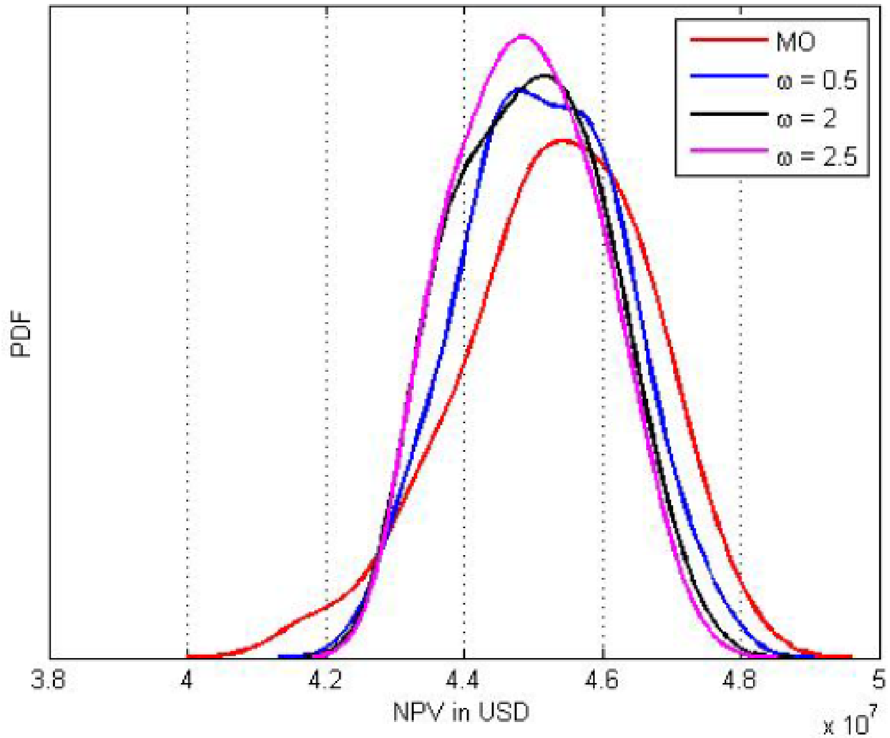
$$Var_{-}\left(x\right)=E\left\{\max\left[E\left(x\right)-x,0\right]\right\}^{2}$$



Semi standard deviation

MCVaR (geology)

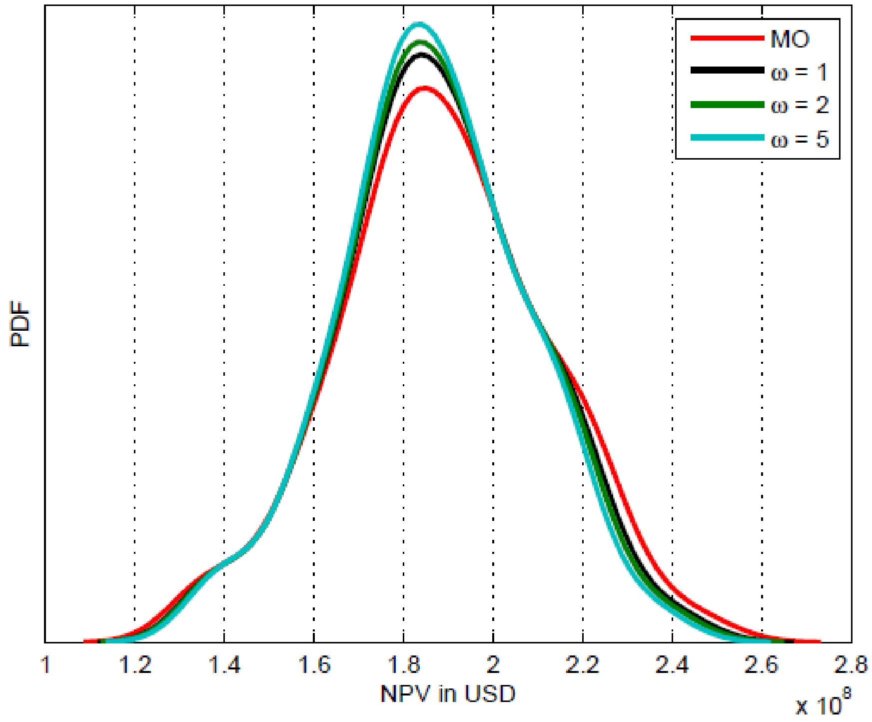
$$J_{\text{MCVaR}} = J_{\text{MO}} - \omega J_{\text{VaR}}$$



- Computationally tedious (integration)

MCVaR (oil price)

$$J_{\mathrm{MCVaR}} = J_{\mathrm{MO}} - \omega J_{\mathrm{VaR}}$$

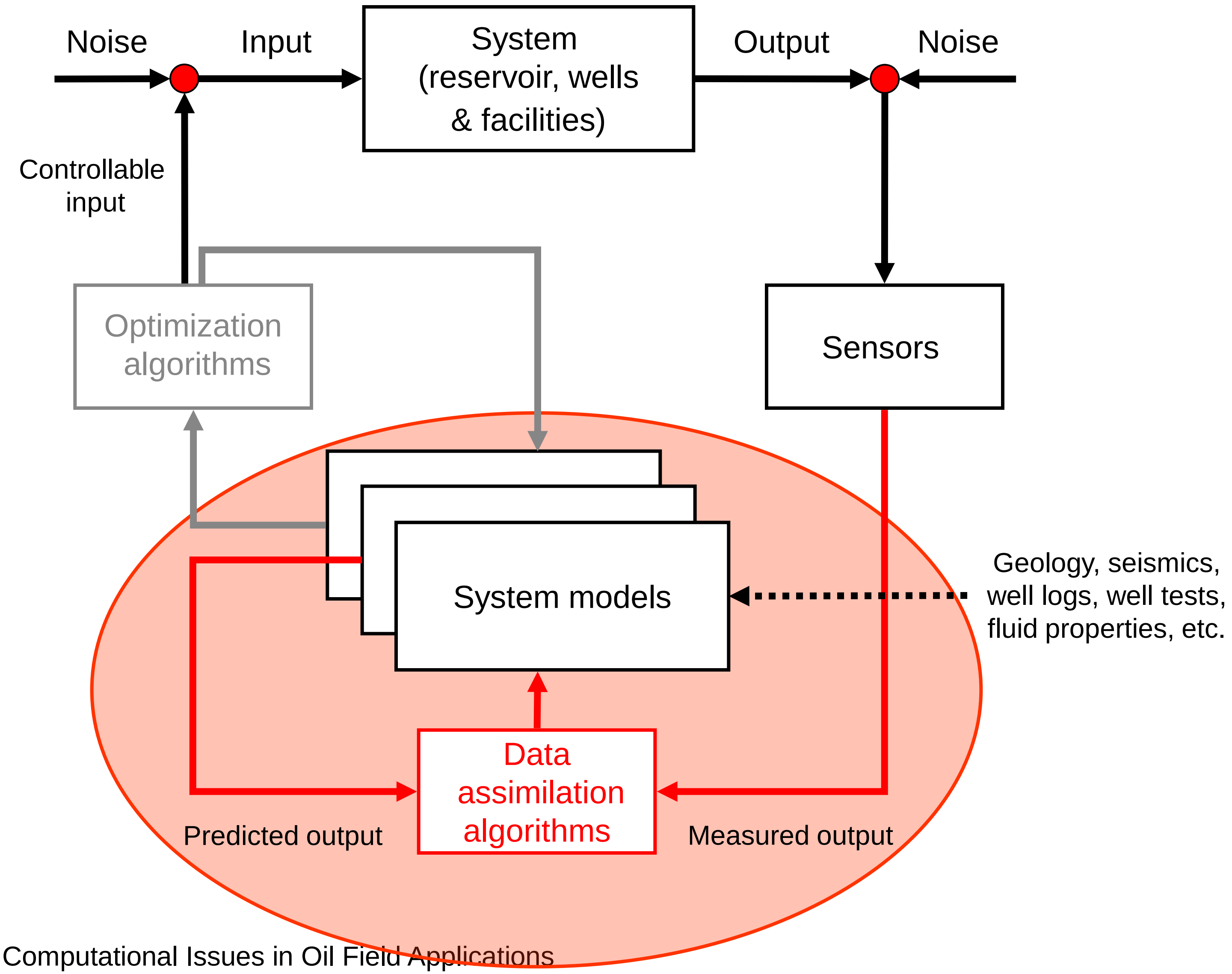


- Not convincingly successful

Conclusions ‘risk measures’

- MVO (symmetric) leads to strong reduction in upside
- Asymmetric risk measures (WCO, CVaR, SV and their ‘mean’ varieties) improve the situation somewhat
- MCVaR seems to perform best, but is computationally demanding and requires choice of weighting parameter
- Improvements under oil price uncertainty lower than expected
- Joint geological - oil price scenarios not yet tested

2) Computer-assisted history matching



Upper/lower economic bounds

Idea:

- Explicitly search for HM-models that provide upper and lower bounds of economic forecasts (for a given production strategy)
- Proposed solution: hierarchical optimization
- Motivation: after obtaining a history match there is still a lot of room in the parameter space to optimize a second objective

- Van Essen et al., *Computational Geosciences* (2016)*;* ECMOR (2010)

Hierarchical optimization

- Order objectives according to importance
    1. Good history-match (V)
    2. Maximize/minimize (economic) forecasts (J)
- Optimize objectives sequentially
- Optimality of upper objective constrains optimization of lower one
- Use *redundant* degrees of freedom (DOF) in decision variables, after meeting primary objective (take a walk in the null space)

Null space wandering in 3D

Hierarchical optimization

$$V_{\min} := \min_{\mathbf{m}} V\left(\bar{\mathbf{u}}, \mathbf{m}\right)$$

$$s.t. \quad \mathbf{g}_k\left(\bar{\mathbf{u}}_k, \mathbf{x}_{k-1}, \mathbf{x}_k, \mathbf{m}\right) = \mathbf{0}, \;\; k = 1, \ldots, K, \;\; \mathbf{x}_0 = \bar{\mathbf{x}}_0$$

primary optimization problem

$$\max_{\mathbf{m}} J\left(\bar{\mathbf{u}}, \mathbf{m}\right) \quad / \quad \min_{\mathbf{m}} J\left(\bar{\mathbf{u}}, \mathbf{m}\right)$$

$$s.t. \quad \mathbf{g}_k\left(\bar{\mathbf{u}}_k, \mathbf{x}_{k-1}, \mathbf{x}_k, \mathbf{m}\right) = \mathbf{0}, \;\; k = 1, \ldots, K, \;\; \mathbf{x}_0 = \bar{\mathbf{x}}_0$$

$$V\left(\mathbf{m}\right) - V_{\min} \leq \varepsilon$$

relaxation of constraint

secondary optimization problem

Formal method: Null-space approach

Idea: find 'free' directions and use these to optimize second objective function

1. Find optimal match $\mathbf{m}$ for primary objective V
2. Determine null-space N of input parameter space $S_{\mathbf{m}}$ around $\mathbf{m}$. (N relates to those directions in $S_{\mathbf{m}}$ to which V is insensitive)
3. Find improving direction $\mathbf{d}$ for secondary objective J
4. Project $\mathbf{d}$ onto basis of N to get projected direction $\mathbf{d}^*$ ($\mathbf{d}^*$ is improving direction for J but does not affect V)
5. Update $\mathbf{m}$ using projected direction $\mathbf{d}^*$
6. Perform steps 2 – 5 until convergence

Alternative: switching method

Idea: alternate unconstrained step to optimize J
with correction step to return to V_{min}

- New objective function $W = \Omega_1(V)\cdot V + \Omega_2(V)\cdot J,$

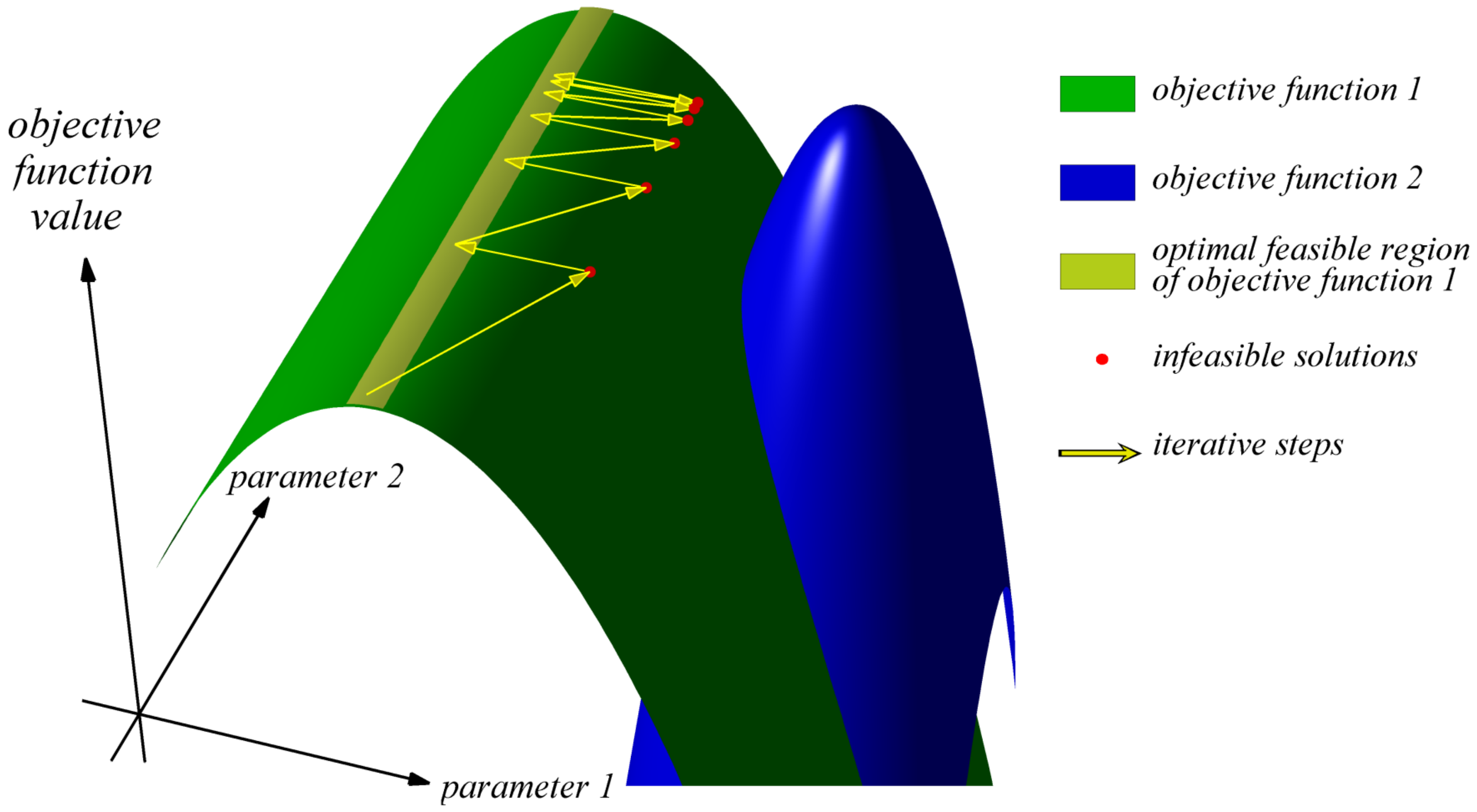
- $\Omega_1(V) = \begin{cases} 1 & \text{if } V - V_{\min} > \varepsilon \\ 0 & \text{if } V - V_{\min} \le \varepsilon \end{cases}, \qquad \Omega_2(V) = \begin{cases} 0 & \text{if } V - V_{\min} > \varepsilon \\ 1 & \text{if } V - V_{\min} \le \varepsilon \end{cases}$

  where Ω_1 and Ω_2 are ‘switching’ functions

$$\frac{\partial W}{\partial \mathbf{m}} = \Omega_1(V)\cdot\frac{\partial V}{\partial \mathbf{m}} + \Omega_2(V)\cdot\frac{\partial J}{\partial \mathbf{m}}$$

- Gradients of W with respect to the model parameters

Switching method



Modified switching method

- Goal is to keep V close to V_{min} with update in J -direction
- Projection of the gradients J onto the first-order approximation of the null-space of V:

$$\frac{\partial \tilde{J}}{\partial \mathbf{m}} := \frac{\partial J}{\partial \mathbf{m}} \cdot \left[\mathbf{I} - \frac{\partial V}{\partial \mathbf{m}}^T \cdot \frac{\partial V}{\partial \mathbf{m}} \right],$$

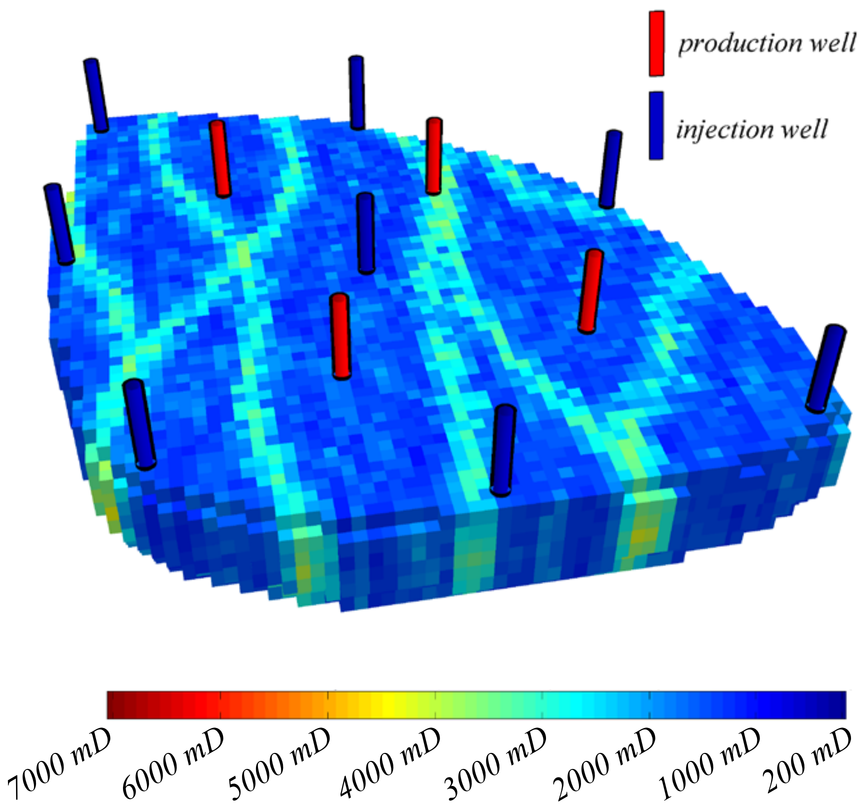
gives an alternative switching search direction $\mathbf{d}$

$$\mathbf{d} = \Omega_1(V) \cdot \frac{\partial V}{\partial \mathbf{m}} + \Omega_2(V) \cdot \frac{\partial J}{\partial \mathbf{m}} \cdot \left[I - \left.\frac{\partial V}{\partial \mathbf{m}}\right|^T \cdot \frac{\partial V}{\partial \mathbf{m}} \right]$$

Example 1: egg model

As before, except:

- Production history of 1.5 years (monthly measurements)
- Forecasts for next 4.5 years



Example 1: optimization method

- In-house reservoir simulator (fully-implicit black oil)
- Minimization with adjoint-based gradients, steepest-descent and line search
- Twin approach: ‘truth’ to generate synthetic; uniform model (correct mean) as prior for history match
- History match objective (first optimization):

$$V = \sum_{k=1}^{K} \left(\mathbf{d}_k - \mathbf{y}_k \right)^T \mathbf{P}_{d_k}^{-1} \left(\mathbf{d}_k - \mathbf{y}_k \right)$$

  where $\mathbf{d}$ are measured data and $\mathbf{y}$ predicted data

- Economic objective (second optimization):

$$J = \sum_{k=1}^{K} \left\{ \sum_{i=1}^{N_{inj}} r_{wi} \cdot \left(u_{wi,i} \right)_k + \sum_{j=1}^{N_{prod}} \left[r_{wp} \cdot \left(y_{wp,j} \right)_k + r_o \cdot \left(y_{o,j} \right)_k \right] \cdot \Delta t_k \right\}$$

Example 1: hierarchical optimization

Primary optimization problem

History-matching

0 – 1.5 years

- Simulation run by prescribing:
  - injection rates (from history)
  - BPHs producers (from history)
- Minimize V (mismatch between measured & simulated data)
- Data (288 points):
  - BHPs of injectors
  - Oil/water flow rates producers
- Controls: grid block perms

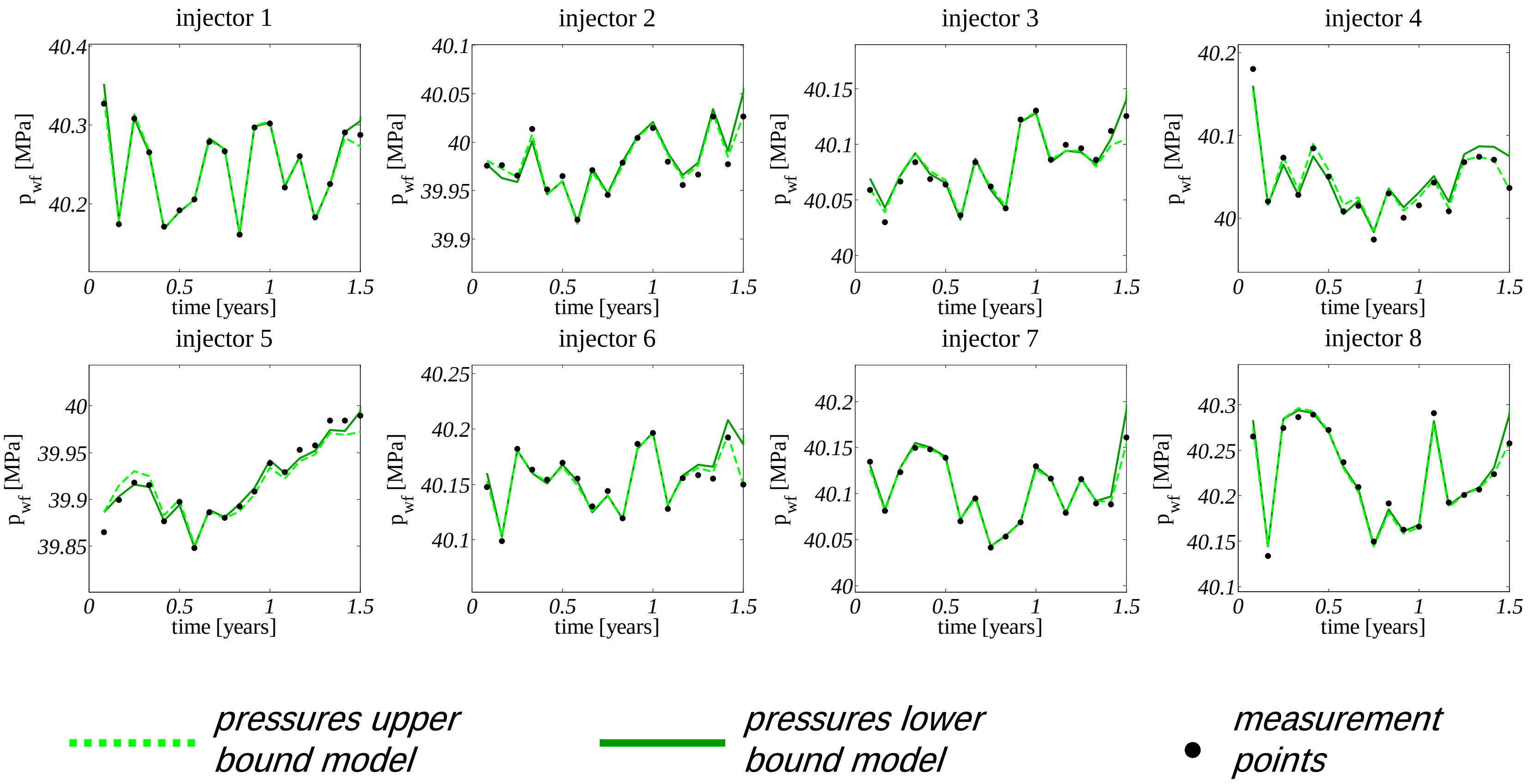
Secondary optimization problem

Bounds on economic forecast

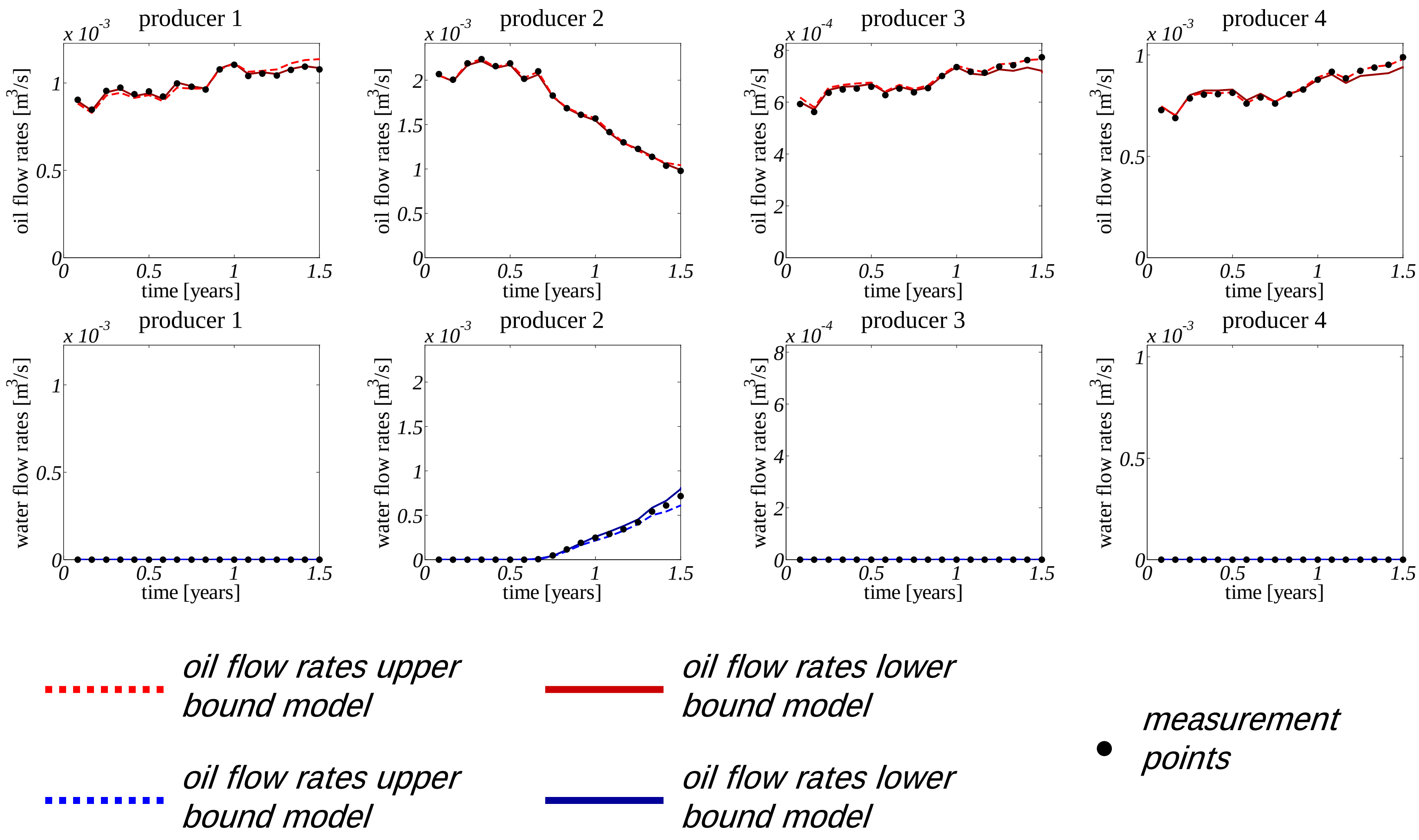
1.5 – 6 years

- Simulation run by prescribing:
  - injection rates (constant)
  - BHPs producers (constant)
- Maximize/minimize J (NPV over 4.5 years)
- r_o =9 \$/bbl, r_w = -1 \$/bbl, 0 disc.
- Weakly constrained by minimum primary objective V_{min}
- Controls: grid block perms

Example 1: HM results - pressures



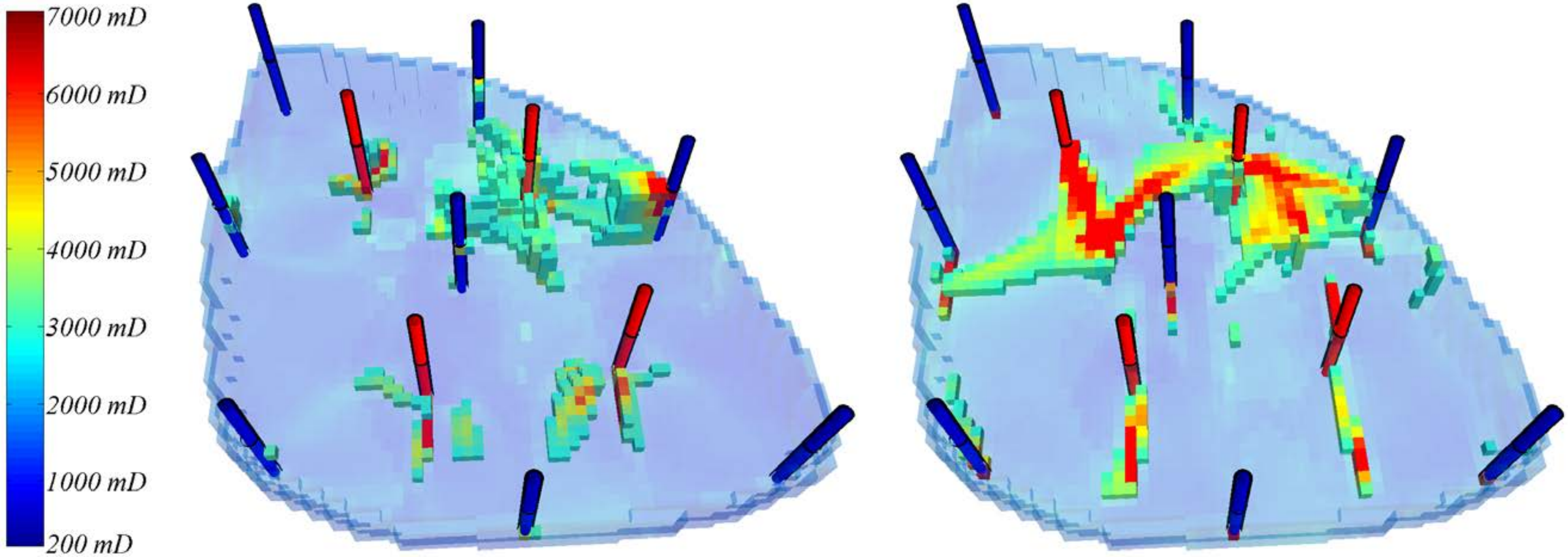
Example 1: HM results – flow rates



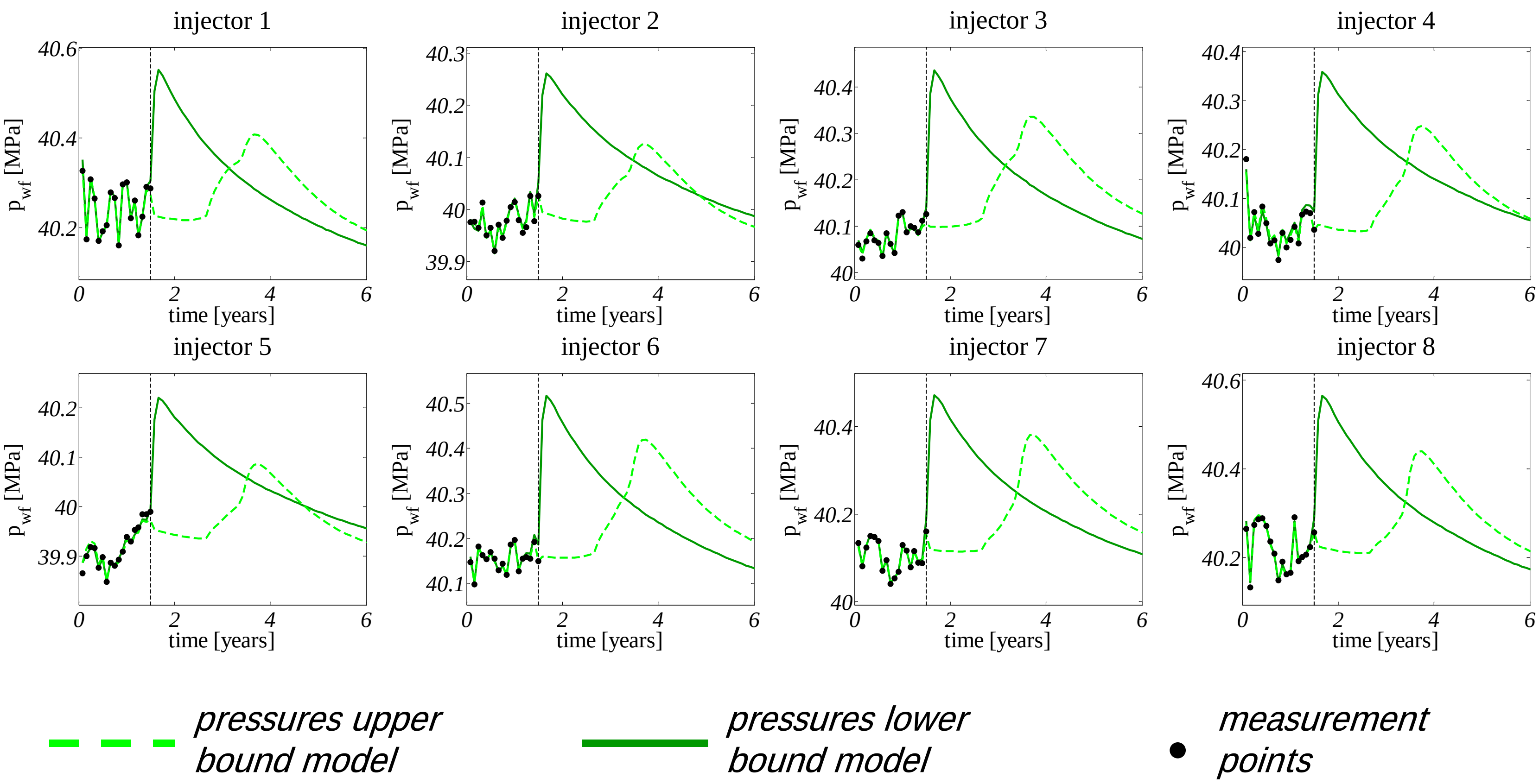
Example 1: incremental permeability fields

"Lower bound" model

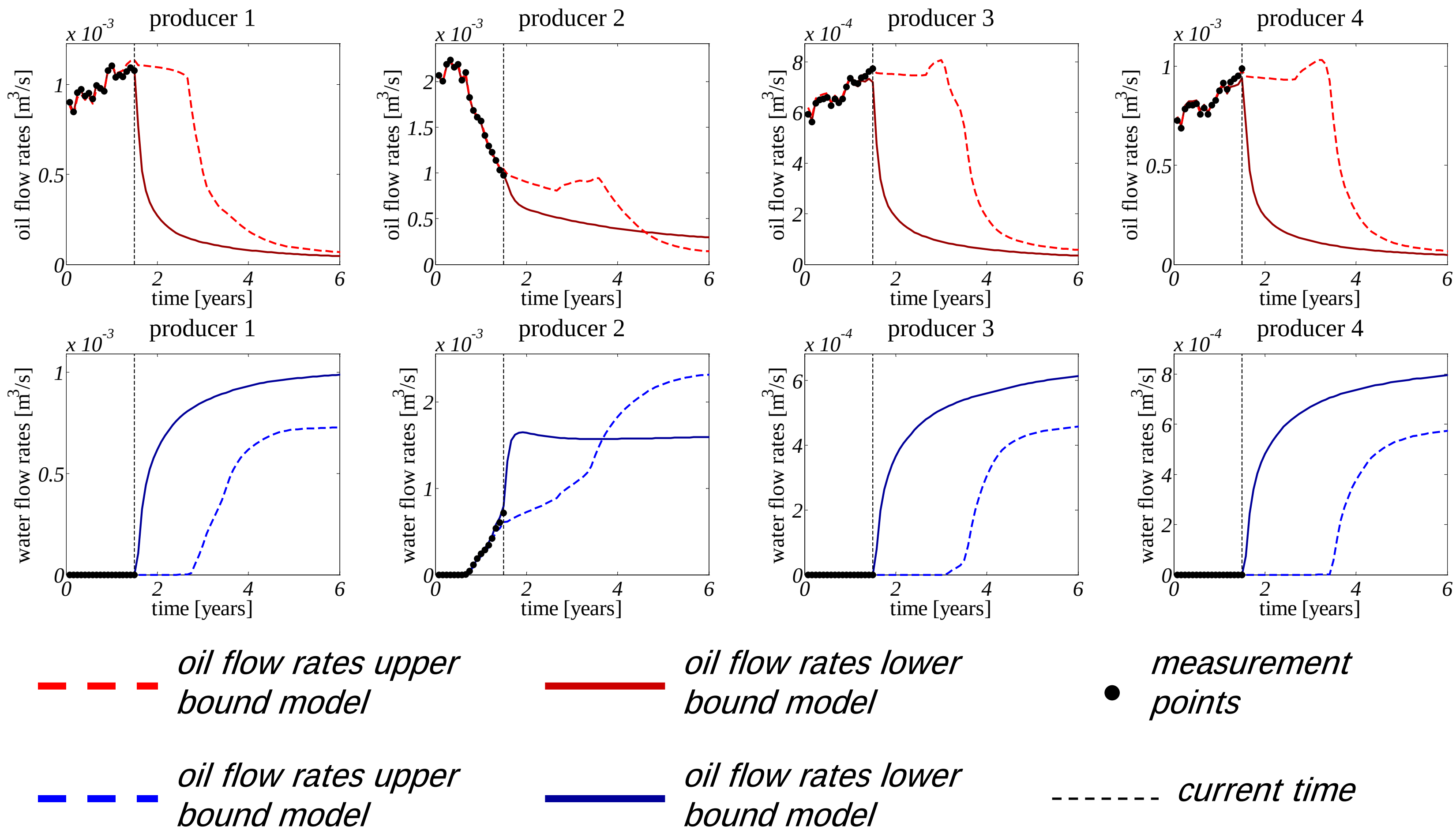
"Upper bound" model



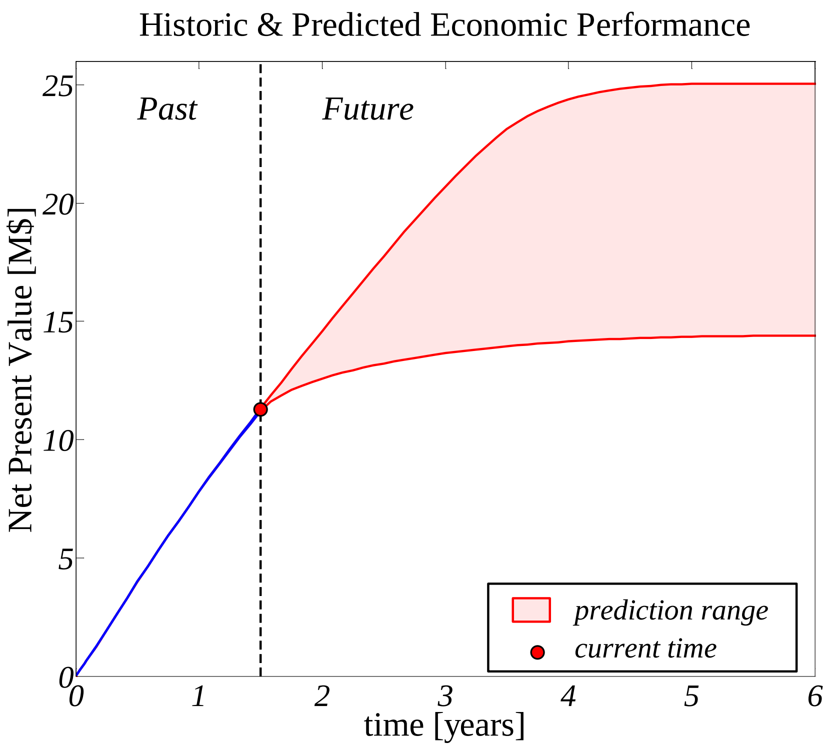
Example 1: HM & forecast – pressures

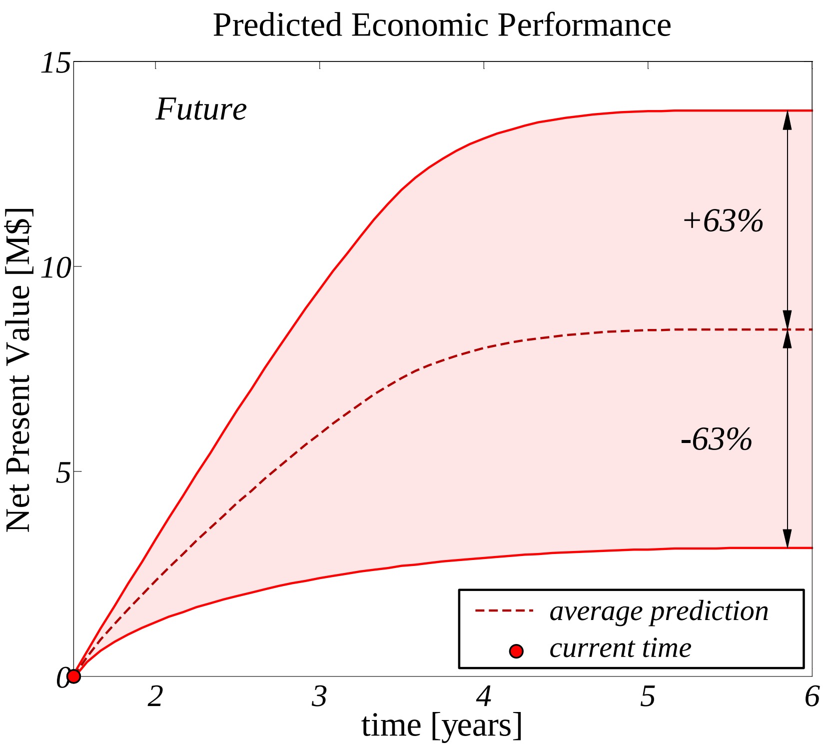


Example 1: HM & forecast – flow rates

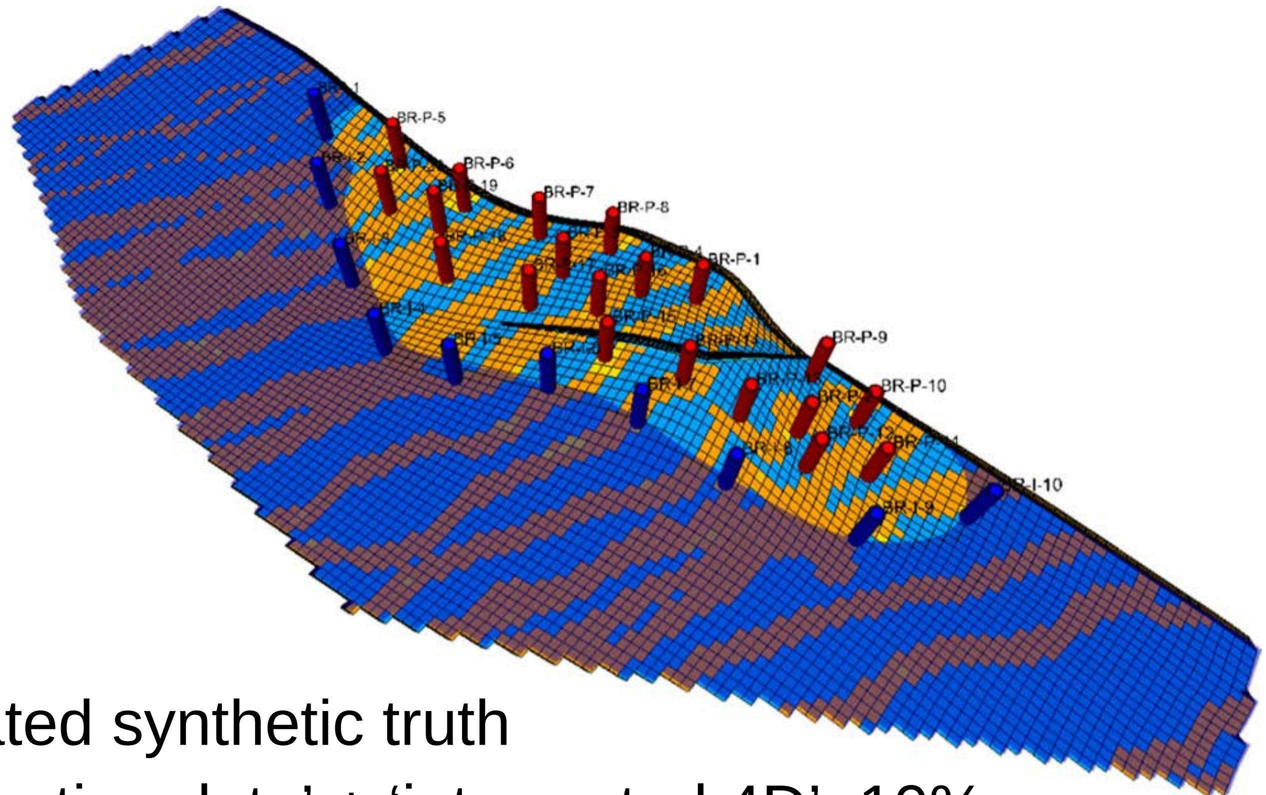


Example 1: forecast range in NPV



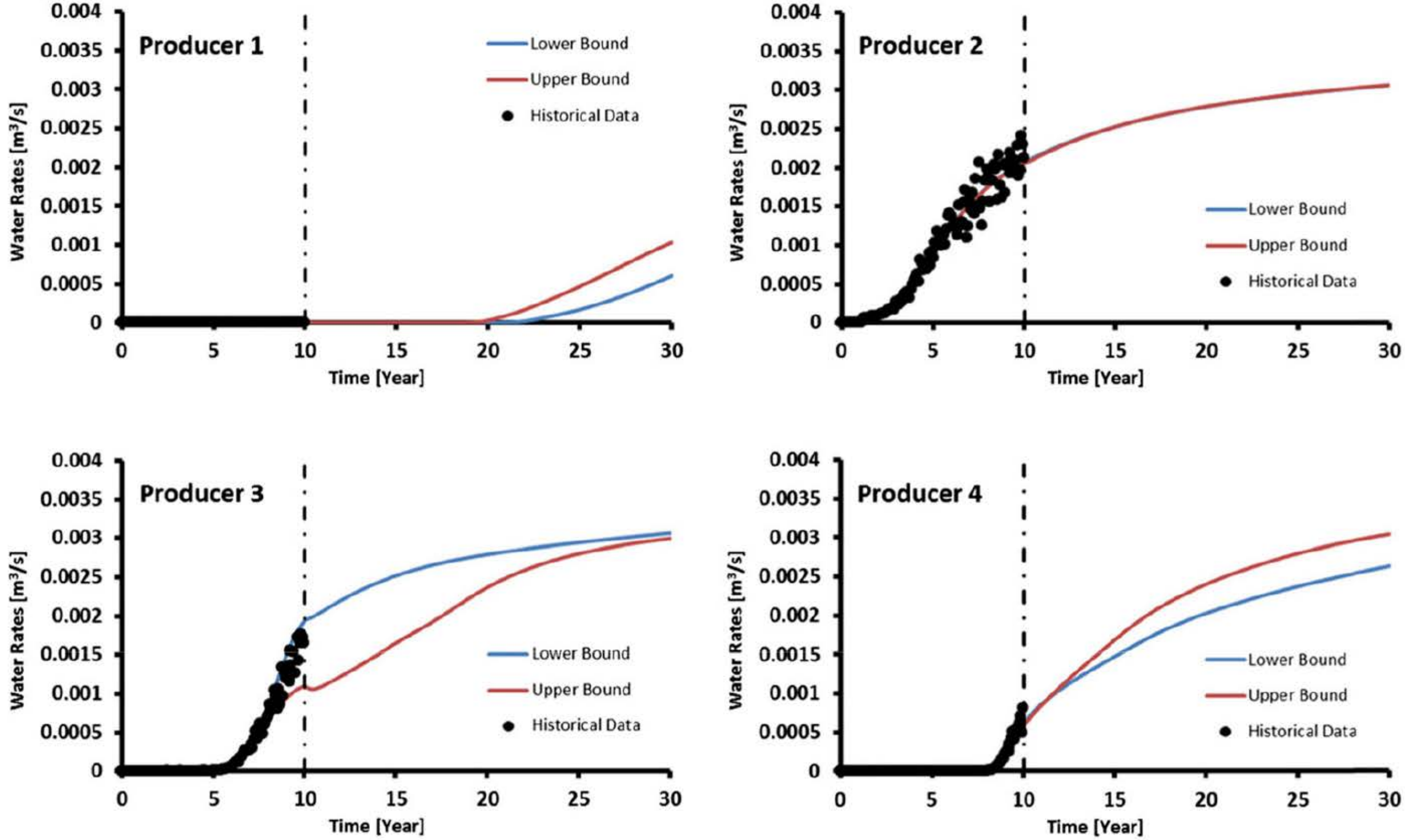


Example 2: Brugge field



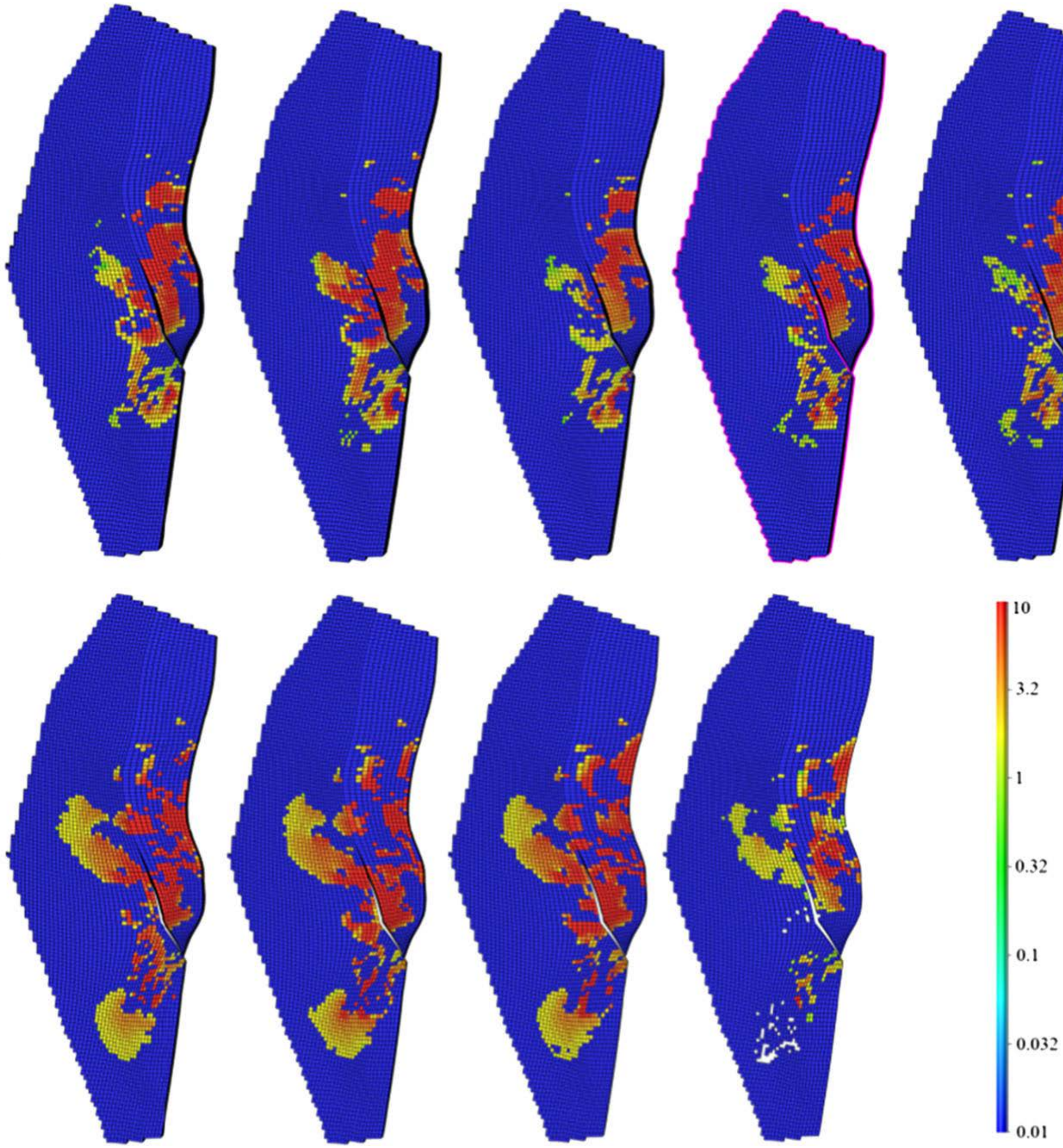
- 60,048 cells
- Own-generated synthetic truth
- 10 yrs ‘production data’ + ‘interpreted 4D’; 10% error
- Starting model for HM randomly selected out of ensemble
- 11 producers, BHP-controlled with bounds; reactive
- 20 injectors, fixed rate-controlled

Example 2: HM results (prod. only) – water rates



- 0.5% deviation allowed in objective function value
- 19.5 % difference in NPV

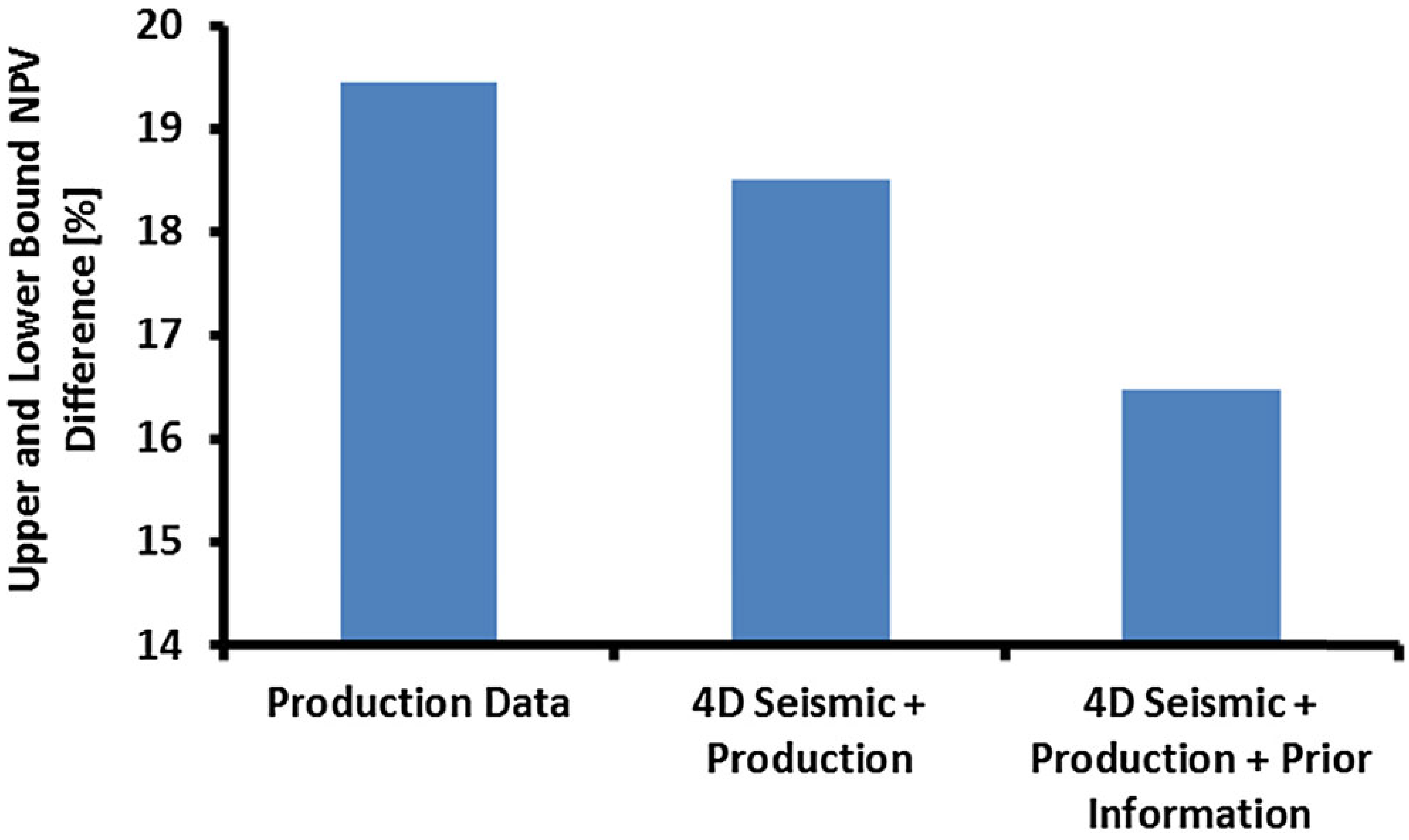
Example 2: Updated permeability fields



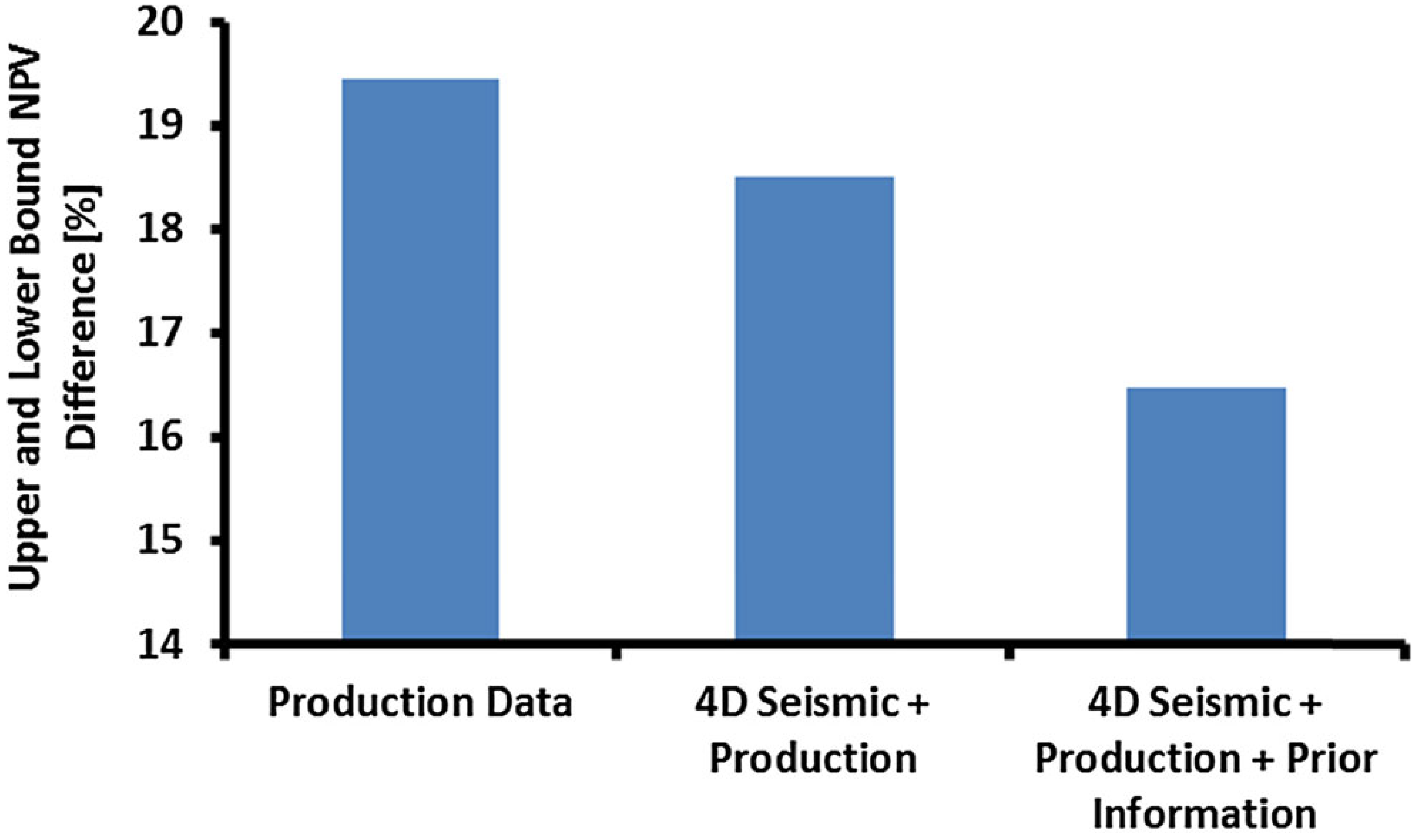
natural log mD

Differences in permeabilities in 9 layers

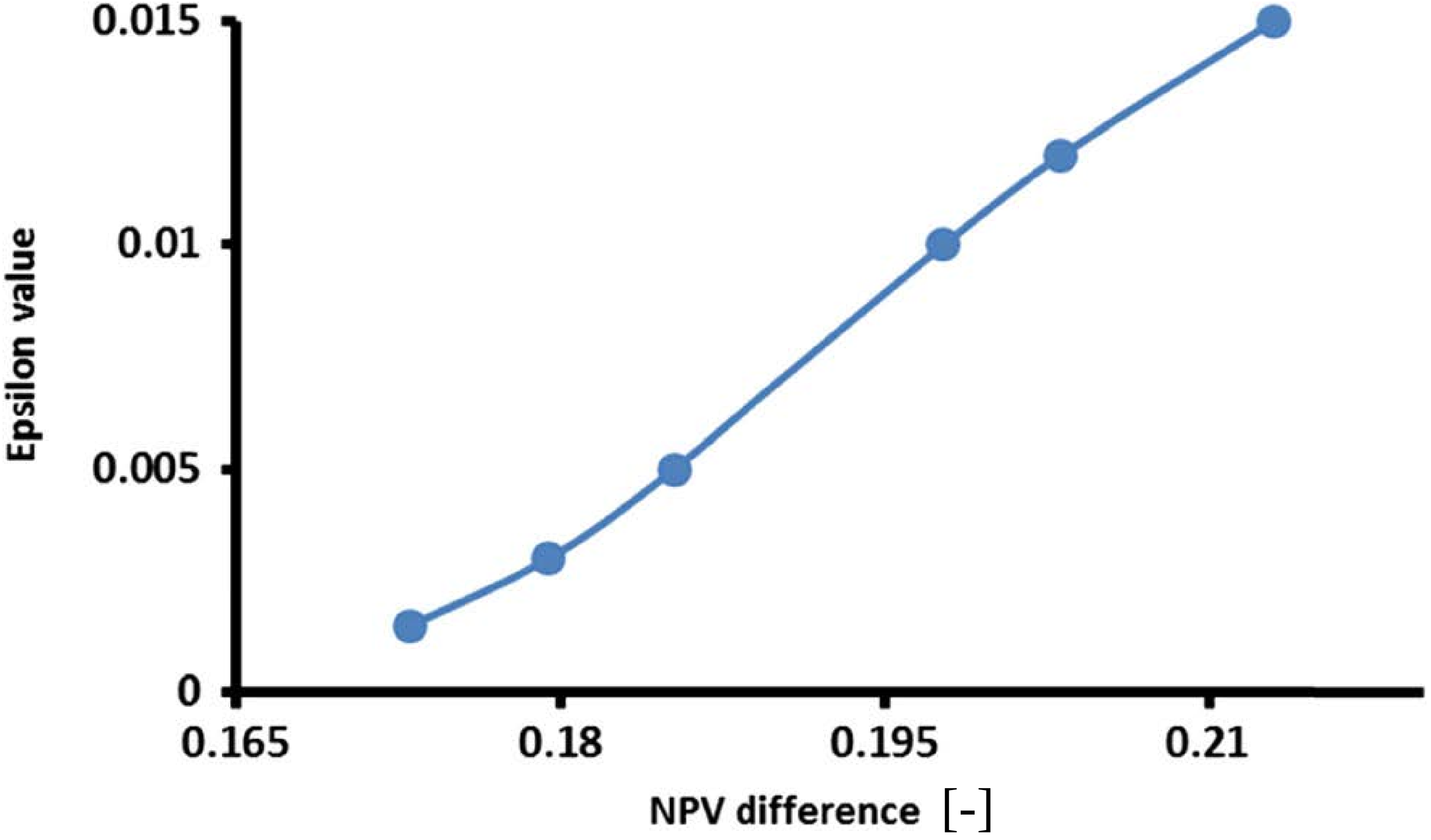
Example 2: HM results – effect of ‘data type’



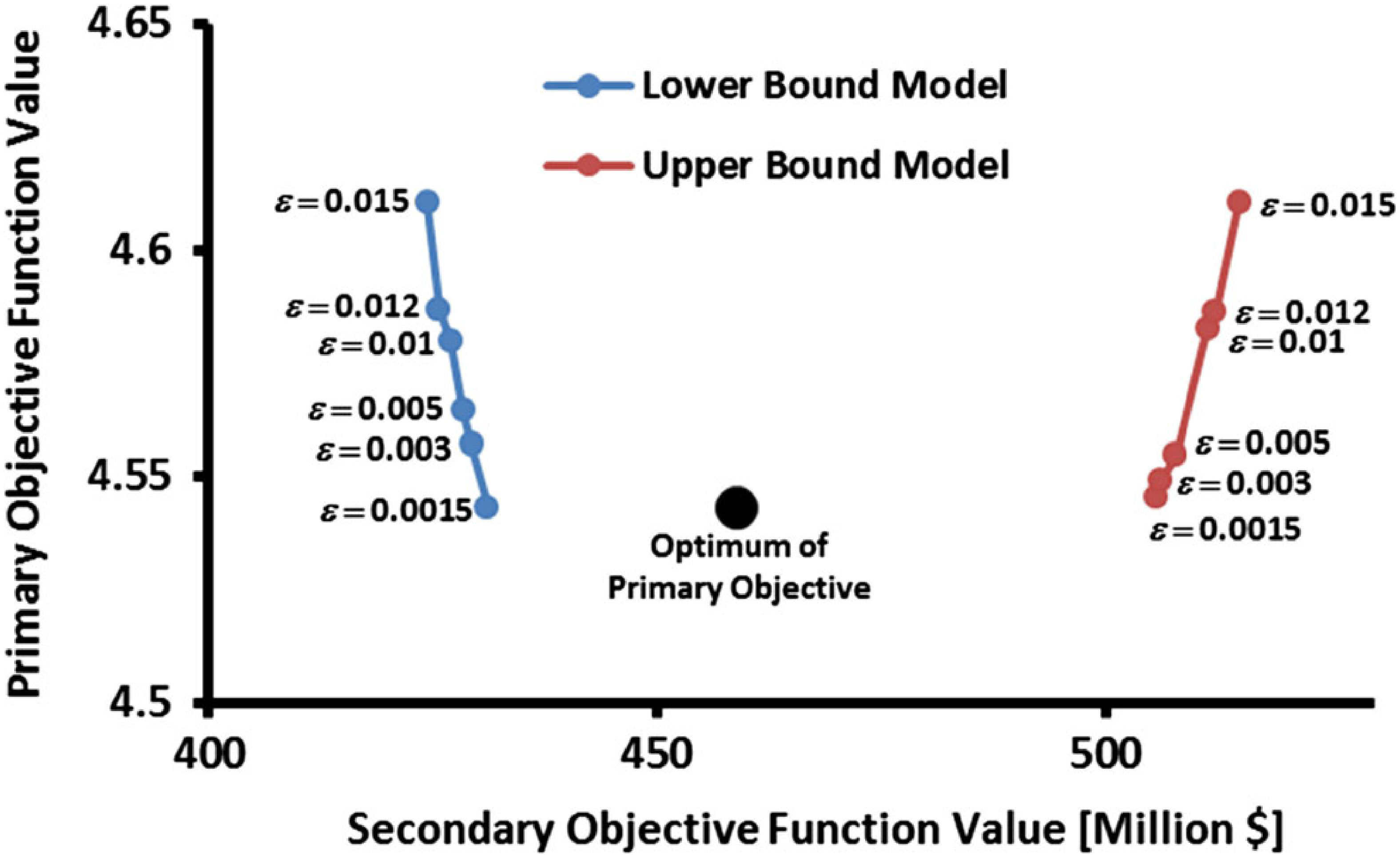
Example 2: HM results – effect of 'data type'



Example 2: HM results – effect of threshold value (1)



Example 2: HM results – effect of threshold value (2)



Conclusions ‘upper and lower bounds’

- Method can be used to gain more insight in the possible economic consequences of the lack of information in the data
  - NPV, total production, ultimate recovery, or other.
  - Economic impact alternative data sources, e.g. 4D seismic data
- No guaranteed lower/upper bounds, due to local optima
- Considerable number of iterations required until convergence
  - May be improved using more efficient optimization scheme (Quasi-Newton, conjugate gradient method, …)
- Wandering in the null space can be useful after all

References

- Robust optimization

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- Upper/lower bounds

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