Upscaling of Compositional Models for Single-Porosity and Discrete-Fracture-Matrix Systems

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Collaborators

- Fractured systems:
  - Mohammad Karimi-Fard (Stanford)
  - Robin Hui (Chevron)
  - Brad Mallison (Chevron)
- Single-porosity systems:
  - Hangyu Li (now at Shell)

General Motivation

- Current reservoir engineering applications often require very large numbers of (automated) runs

- Key examples include field development / well control optimization and uncertainty quantification

- Compositional flow simulation, which is expensive, required for EOR and shale oil operations

- Performing $O(10^3\text{-}10^5)$ detailed compositional runs is computationally prohibitive (and not necessary)

Outline

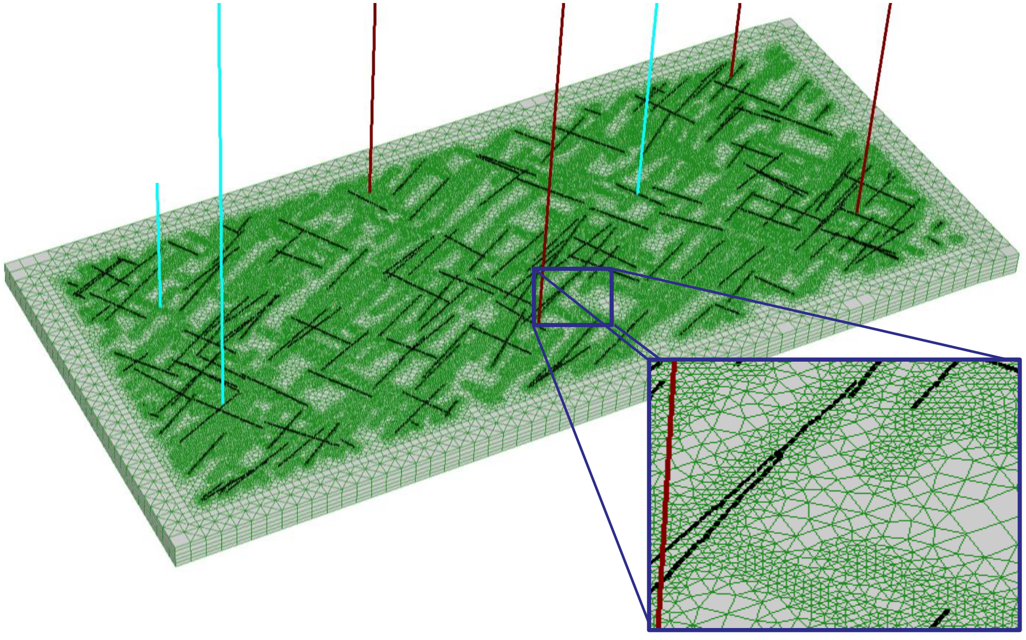
- Upscaling discrete-fracture-matrix (DFM) systems
- Global compositional upscaling for single-porosity models
- Ensemble level upscaling to generate many (approximate) upscaled models for use in UQ

Classification of Numerical Upscaling Methods

- Methods based on coarse gridding and single-phase flow computations
  - Local, global, border-region, local-global procedures
  - Provide **k***, *T**, WI*, ϕ* (not k_{rj}* etc)
- Methods based on compositional flow computations
  - Flow domains as above
  - Provide **k***, *T**, WI*, ϕ* and k_{rj}* etc
  - Expensive, so reuse is important
- Ensemble level procedures use statistical assignment for majority of upscaled models

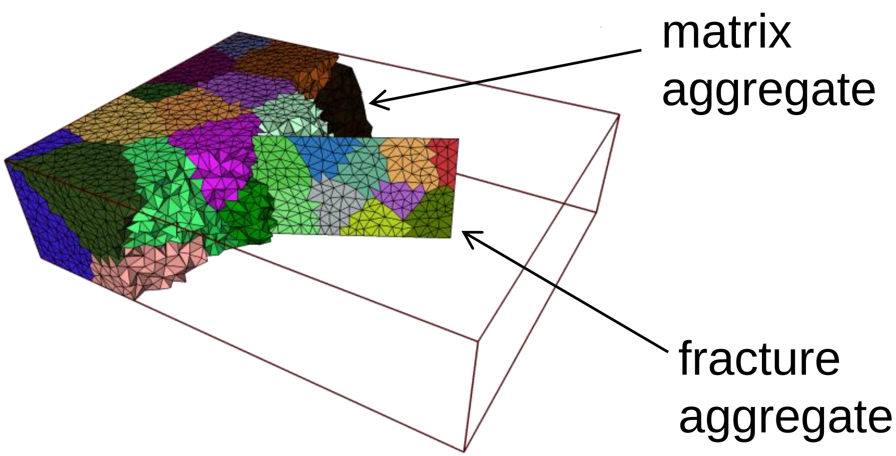
DFM Grid Generation

- Grid generation using method of Mallison et al. (2010) or Karimi-Fard (2008, 2016)
- Provides discrete-fracture-matrix model (discretization as per Brad's description)
- Could also use EDFM (Li & Lee, 2008) or a combination for fine-scale simulations

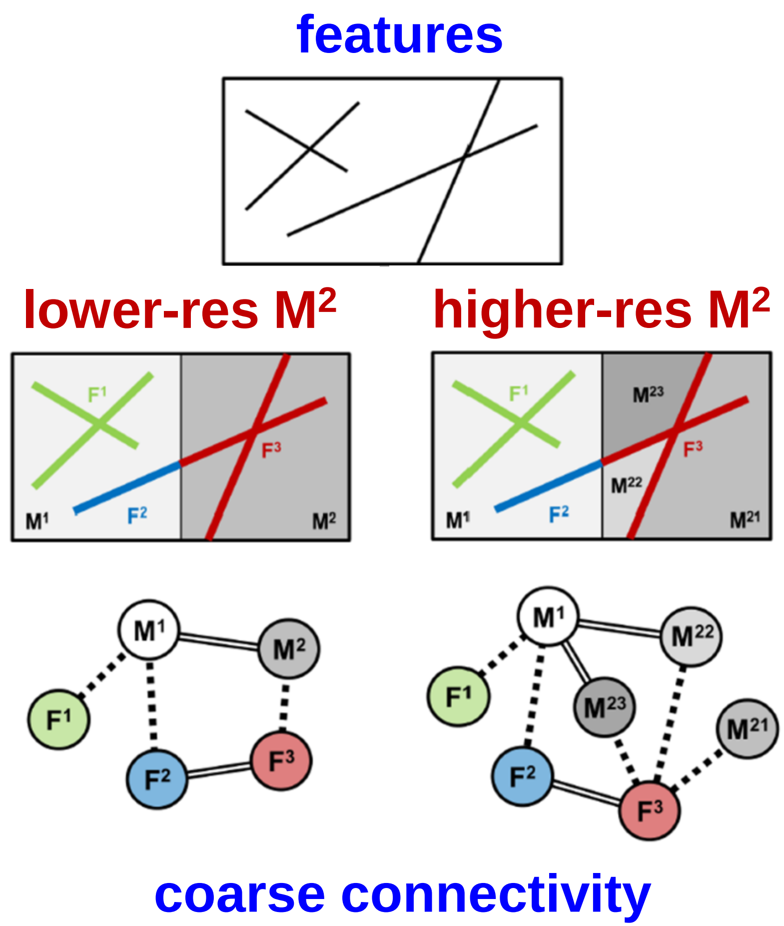


Aggregation of Fine Cells

- Coarse “control volumes” constructed via aggregation of fine cells; based on geometry, geology & flow information

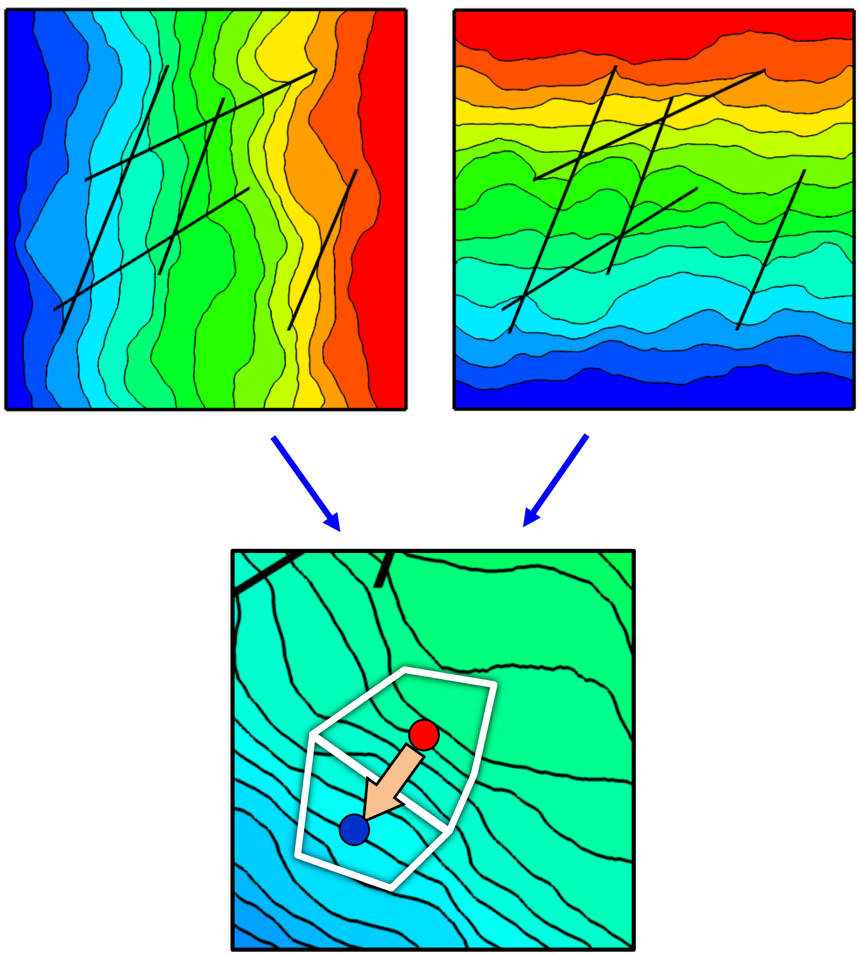


Model has a dual-continuum character; similarities with GMsFEM models (Efendiev, Chung, …)



Coarse-scale Properties (M-M & F-F)

global flow in *x* **global flow in *y***



local linear combination aligned with interface normal

$$\nabla \cdot (\mathbf{k}\nabla p)=0$$

$$p_L = 1,\ p_R = 0 \quad \text{(flow in } x\text{)}$$

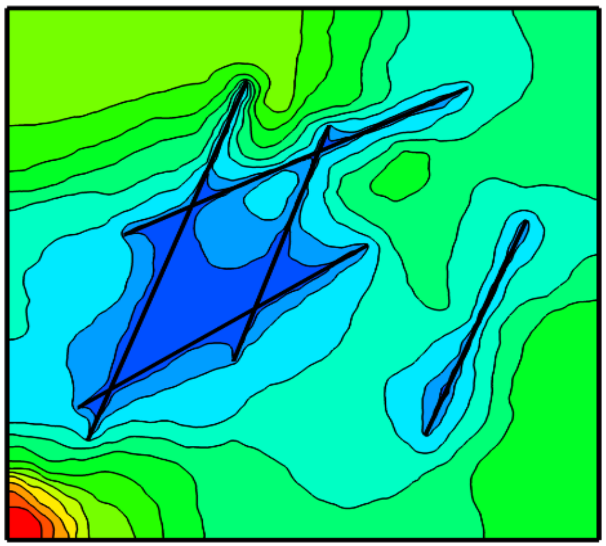
$$T^*_{IJ} = \frac{\Sigma q}{\langle p \rangle_I - \langle p \rangle_J}$$

● **coarse cell *I***

● **coarse cell *J***

Coarse-scale Properties (F-M)

fracture-matrix flow



pseudo-steady-state solution

$$\nabla \cdot (\mathbf{k} \nabla p) = \phi F$$

$$p = 0 \text{ in } \Gamma_{\mathrm{f}}$$

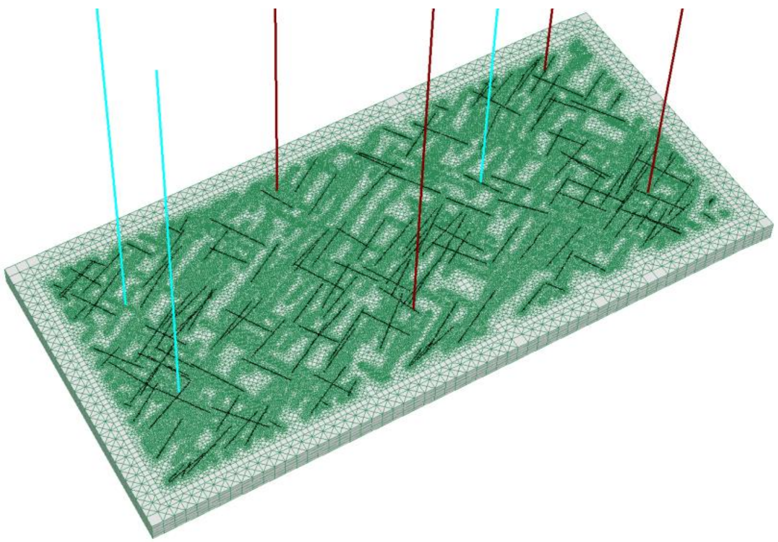
$$\partial p / \partial n = 0 \text{ on } \Gamma_{\mathrm{b}}$$

$$T_{IJ}^{*} = \frac{\Sigma q}{\langle p \rangle_I - \langle p \rangle_J}$$

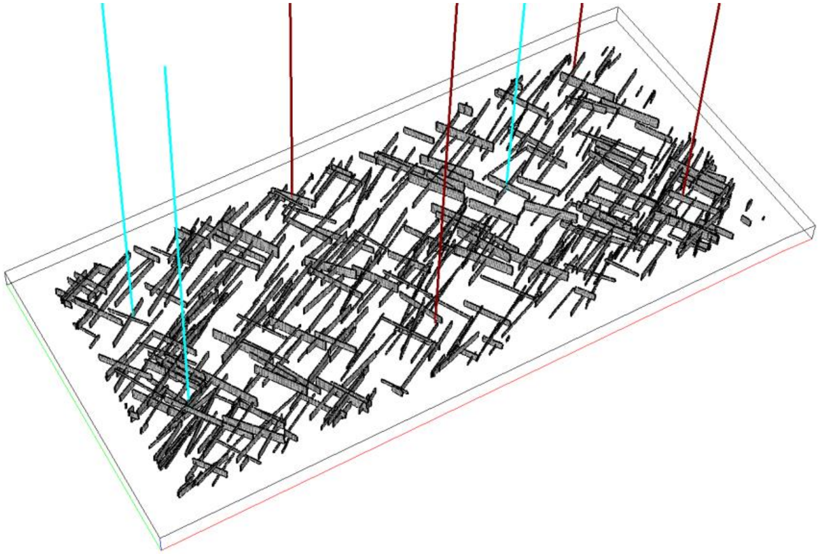
(slightly different from Brad Mallison’s treatment)

DFM Model – Problem Setup

- 663 discrete fractures in two orthogonal sets, 5° dip, k_f = 100,000 mD, k_m = 0.1 mD
- ~650,000 matrix and ~24,000 fracture control volumes
- Multiple coarse models from one set of global 1-p runs
- 6 components, 3 phases (IX runs for all models)



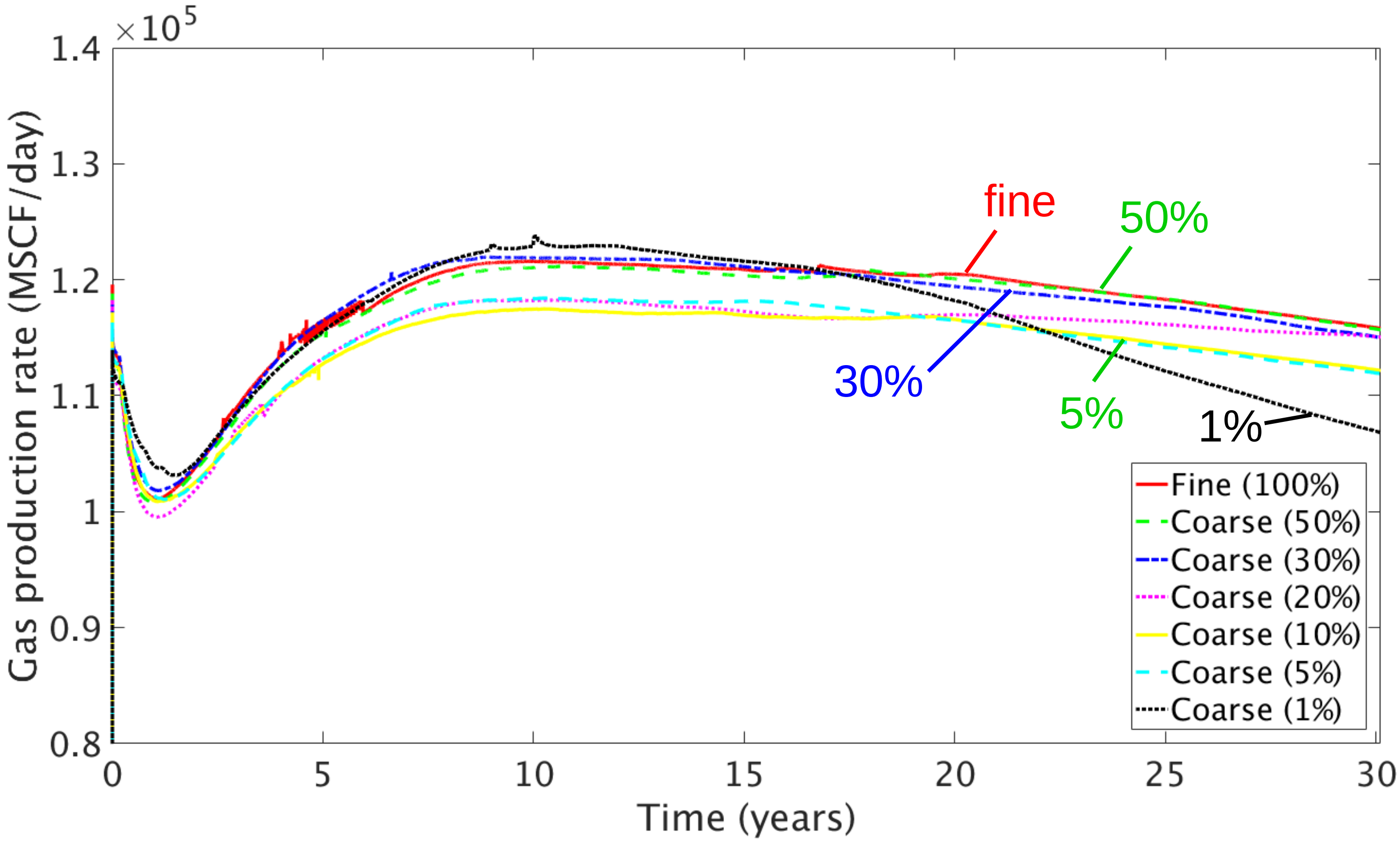
DFM grid & wells



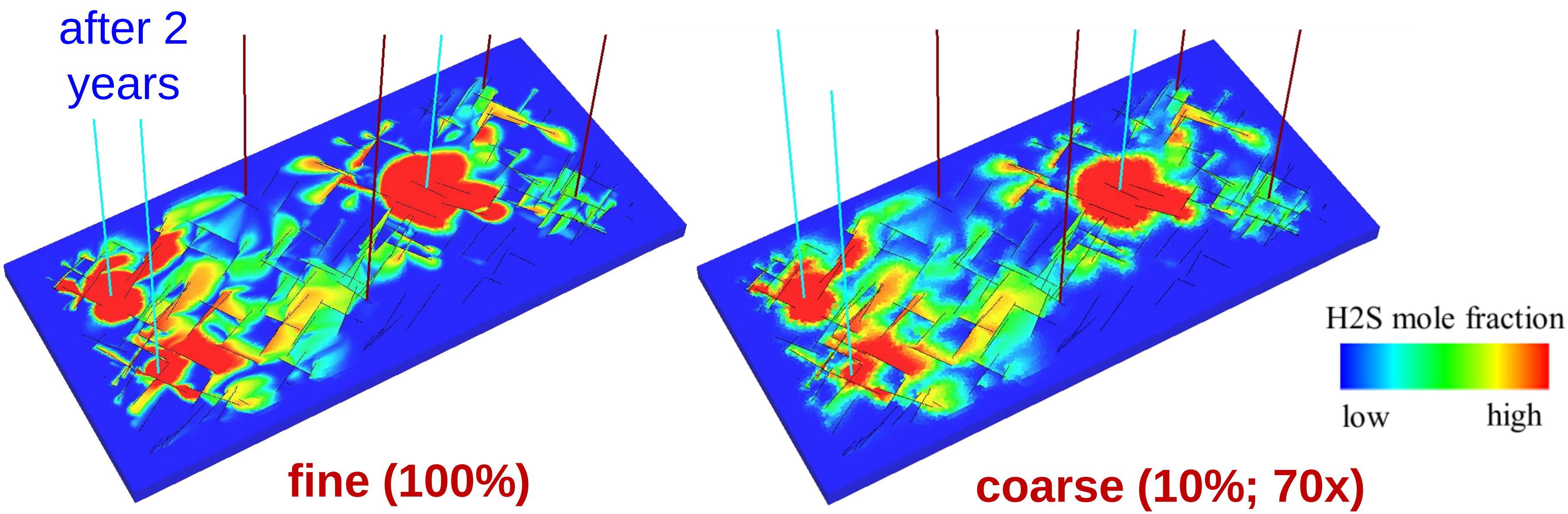
fracture cells

Gas Production Results

- Injectors: q = 30,000 bbl/day with maximum BHP; injected gas (0.23 H_2S, 0.47 CH_4, …) is miscible with reservoir oil
- Producers: q = 30,000 bbl/day with minimum BHP



H_2S Mole Fraction & Errors in Cumulatives



Resolution	Error oil (%)	Error gas (%)	Error inj (%)	Speedup factor
50%	0.2	0.3	0.3	5
5%	0.2	2.7	3.2	187
1%	3.4	1.5	2.8	634

- Fine IX simulation required 62 hours on 64 CPUs

Global Compositional Upscaling

- Mass conservation on fine scale:

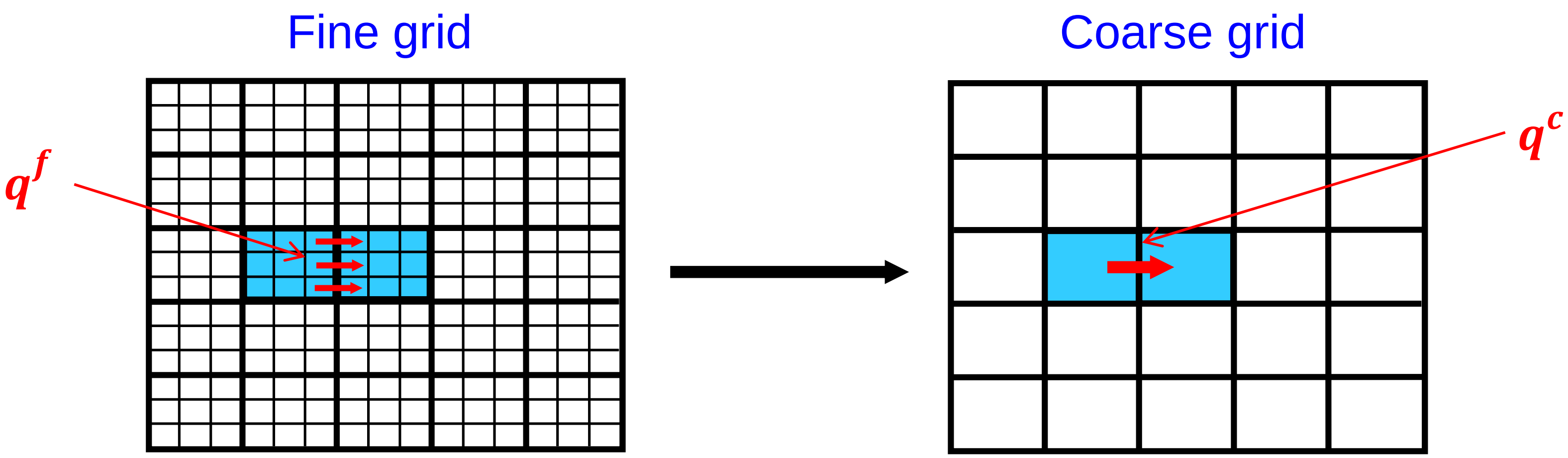
$$\nabla \cdot \left(\sum_{j=1}^{n_p} x_{ij} \rho_j \, \lambda_j \mathbf{k} \cdot \nabla p \right) - \sum_{j=1}^{n_p} x_{ij} \rho_j \, q_j^w = \frac{\partial}{\partial t} \left(\phi \sum_{j=1}^{n_p} x_{ij} \rho_j S_j \right), \quad i = 1, n_c$$

- Mass conservation on coarse scale:

$$\nabla \cdot \left(\sum_{j=1}^{n_p} x_{ij}^c \alpha_{ij} \rho_j^c \lambda_j^* \, \mathbf{k}^* \cdot \nabla p^c \right) - \sum_{j=1}^{n_p} x_{ij}^c \alpha_{ij} \rho_j^c \, q_j^{w,c} = \frac{\partial}{\partial t} \left(\phi^* \sum_{j=1}^{n_p} x_{ij}^c \rho_j^c S_j^c \right)$$

(α-factor model after Barker and Fayers, 1994;
also Iranshahr, Chen & Voskov, 2014; Salehi, Voskov & Tchelepi, 2017)

Global Upscaling Procedure (1)

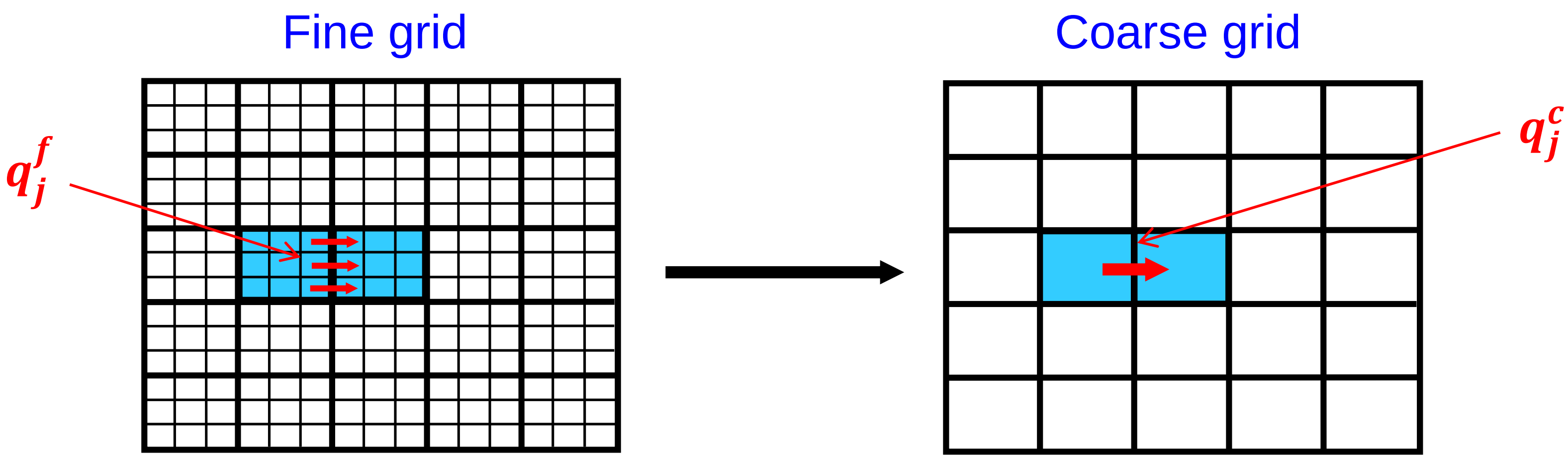


- Single-phase parameters T^* (solve $\nabla \cdot (\mathbf{k} \cdot \nabla p) = q^w$)

$$\sum q^f = \sum T^f \Delta p^f \quad \Longleftrightarrow \quad q^c = T^* \Delta p^c$$

- Transmissibility: $T^* = \dfrac{\sum q^f}{\langle p^f \rangle_n - \langle p^f \rangle_{n+1}}$

Global Upscaling Procedure (2)

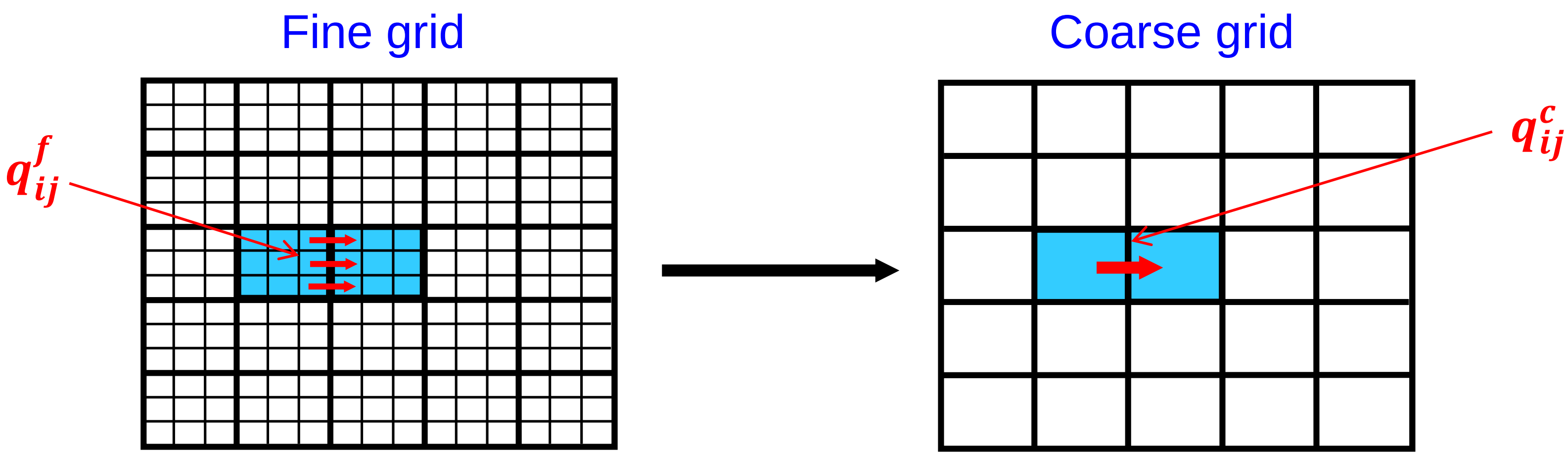


- Two-phase functions k_{rj}^* (solve full compositional model)

$$\sum q_j^f = \sum T^f \Delta p^f \frac{k_{rj}^f}{\mu_j^f} \rho_j^f \qquad \Longleftrightarrow \qquad q_j^c = T^* \Delta p^c \frac{k_{rj}^*}{\mu_j^c} \rho_j^c, \quad j = o, g$$

- Relative permeability: $k_{rj}^*(S_g^c) = \dfrac{\langle \mu_j^f \rangle \sum q_j^f}{T^* \langle \rho_j^f \rangle \{\langle p^f \rangle_n - \langle p^f \rangle_{n+1}\}}$

Global Upscaling Procedure (3)

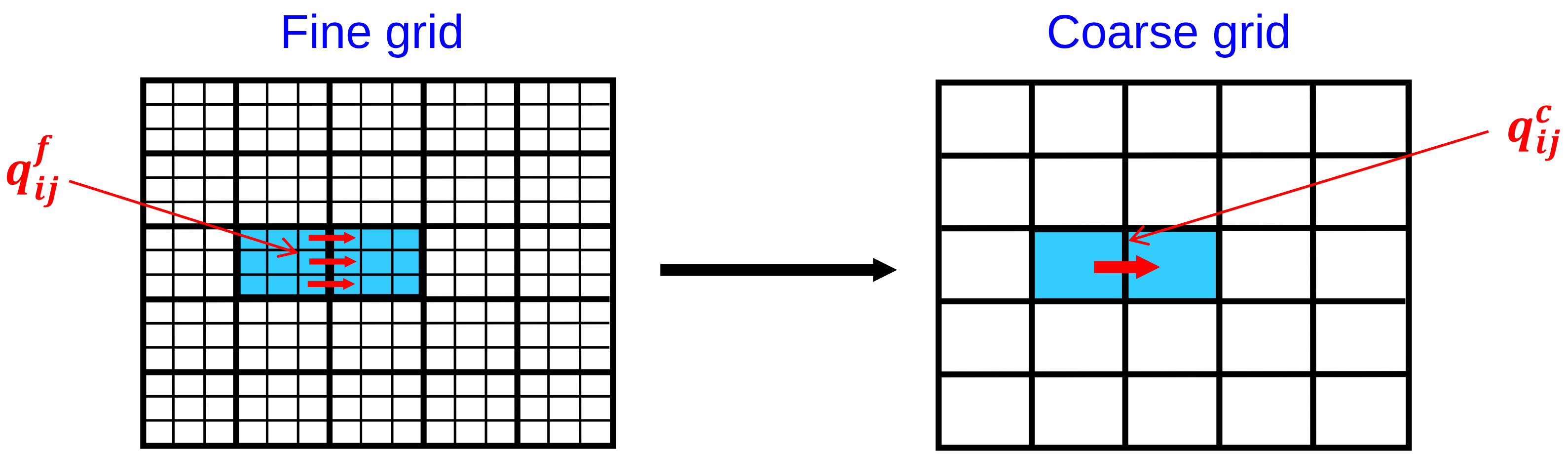


- Compositional functions α_{ij} (same global solution)

$$\sum q_{ij}^f = \sum T^f \Delta p^f \frac{k_{rj}^f}{\mu_j^f} \rho_j^f x_{ij}^f \Longleftrightarrow q_{ij}^c = T^* \Delta p^c \frac{k_{rj}^*}{\mu_j^c} \rho_j^c \alpha_{ij} x_{ij}^c$$

- α-factor: $\alpha_{ij}(S_g^c) = \dfrac{\langle \mu_j^f \rangle \sum q_j^f}{T^* k_{rj}^* \langle x_{ij}^f \rho_j^f \rangle \{\langle p^f \rangle_n - \langle p^f \rangle_{n+1}\}}$

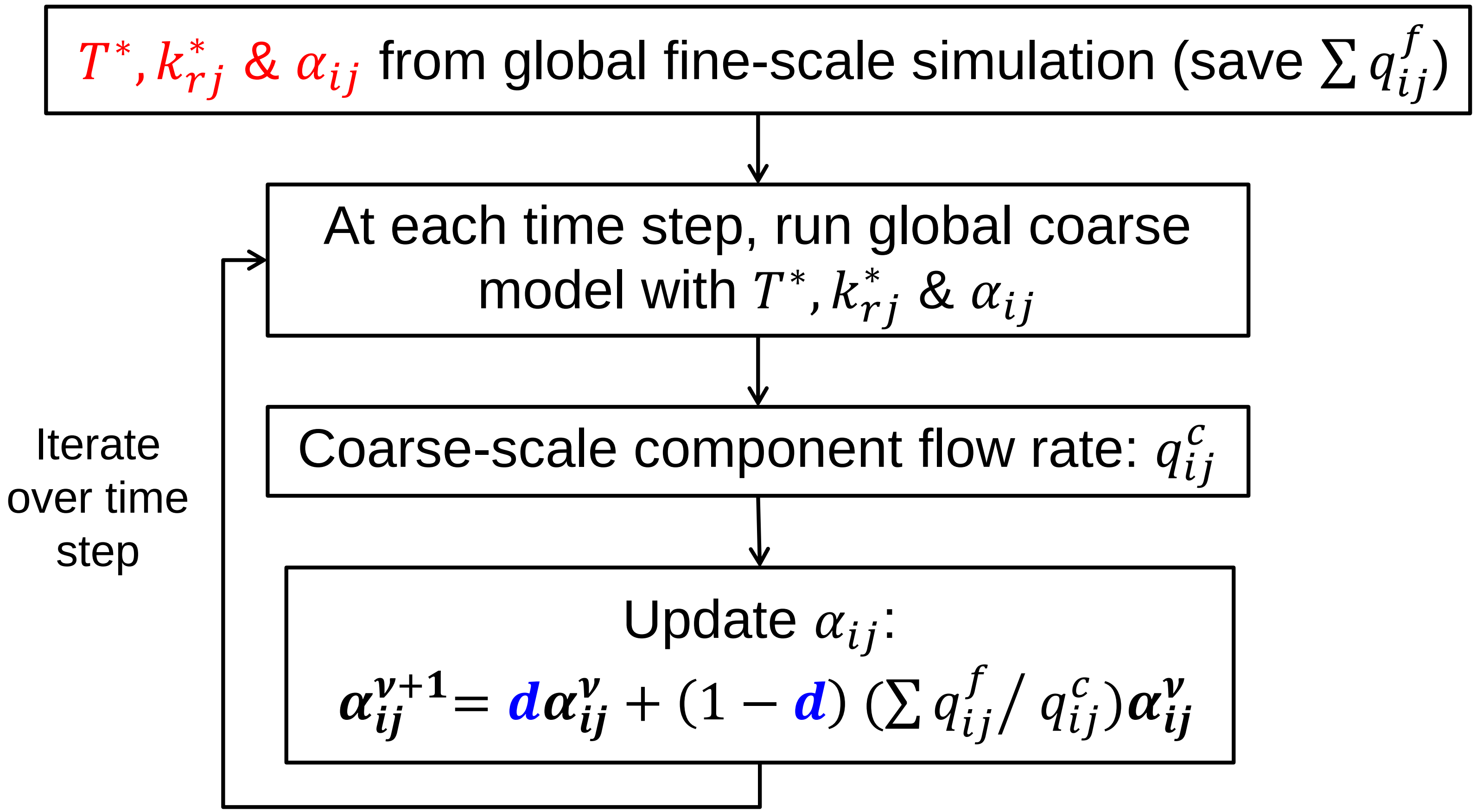
Global Upscaling Procedure (4)



- Upscaled functions for well terms also computed
- Use of T^*, k_{rj}^* & α_{ij} does not assure that $q_{ij}^c = \sum q_{ij}^f$
- Iterate on α_{ij} to minimize $\left| q_{ij}^c - \sum q_{ij}^f \right|$

(after Y. Chen, Mallison and Durlofsky, 2008)

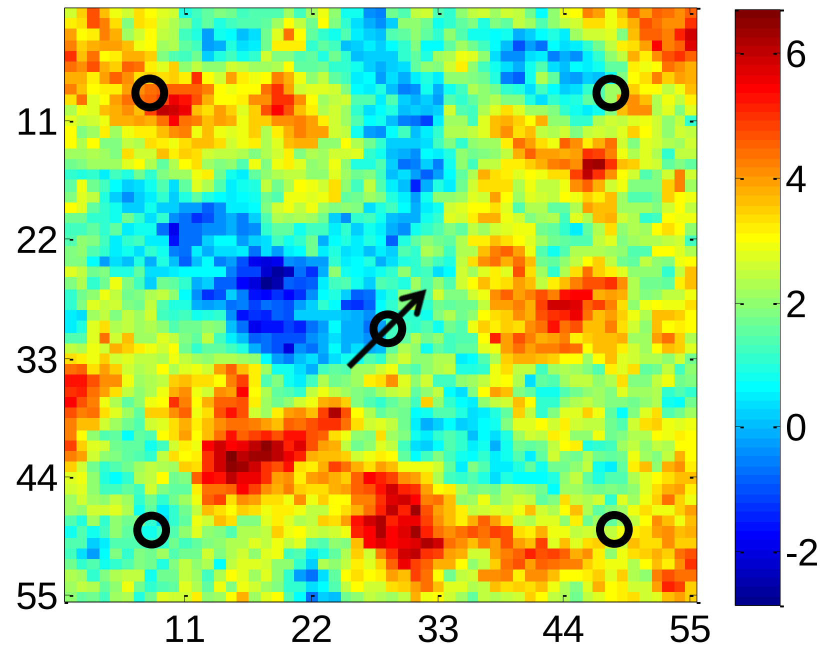
Flowchart of Global Upscaling



$\boldsymbol{d}$: damping factor

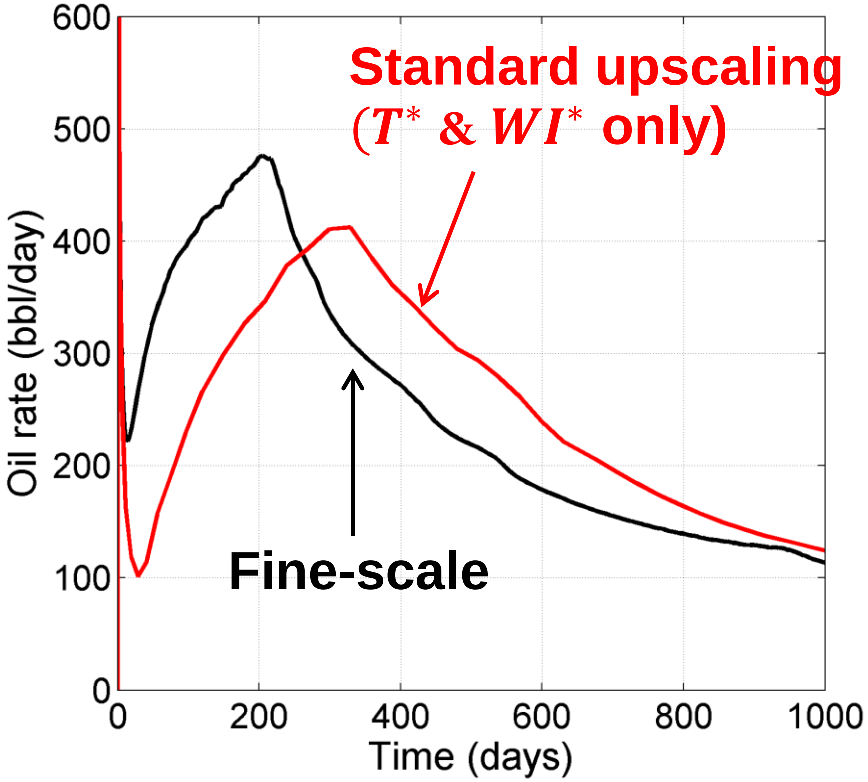
Eight-Component Near-Miscible System

- Initial conditions:
  - 1150 psi, 373 K
  - {C_1(10%), CO_2(1%), C_2(1%), C_3(1%), C_4(10%), C_6(10%), C_8(20%), C_{15}(47%)}
- Permeability using sGsim
  - $\sigma_{\ln k}$ =1.6, l_x = 0.3, l_y = 0.3
  - 55 x 55 → 11 x 11 (uniform grid)
- Inject {C_1(20%), CO_2(80%)} at 1700 psi
- Produce at 750 psi



Oil and Gas Field Production Rates

Oil rate



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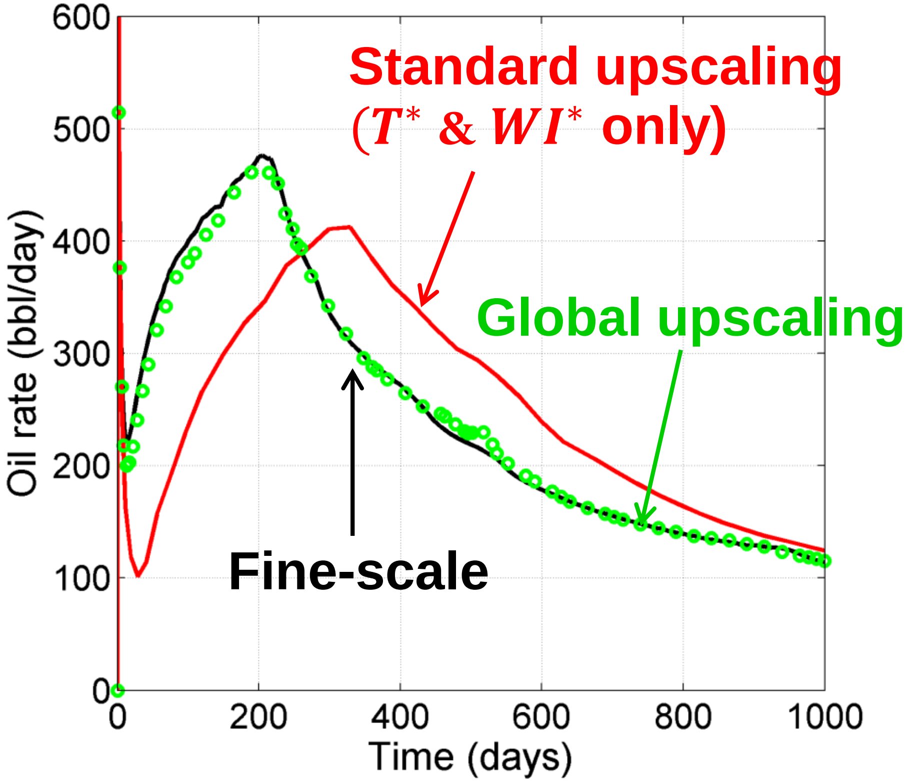

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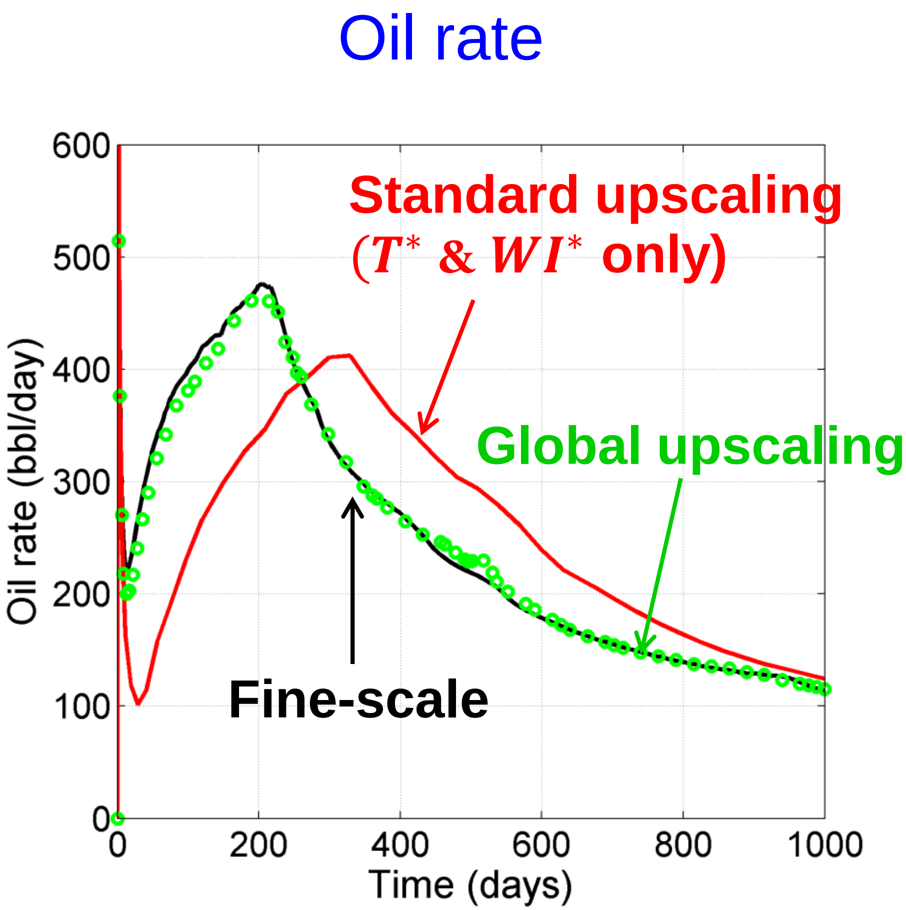
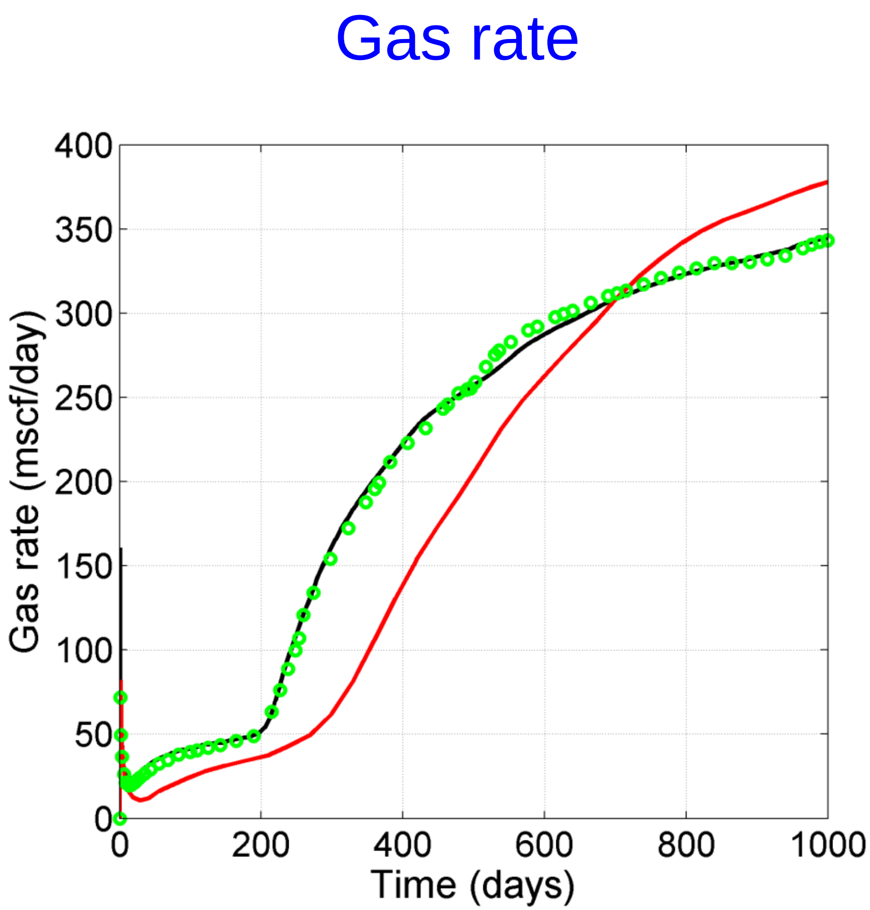
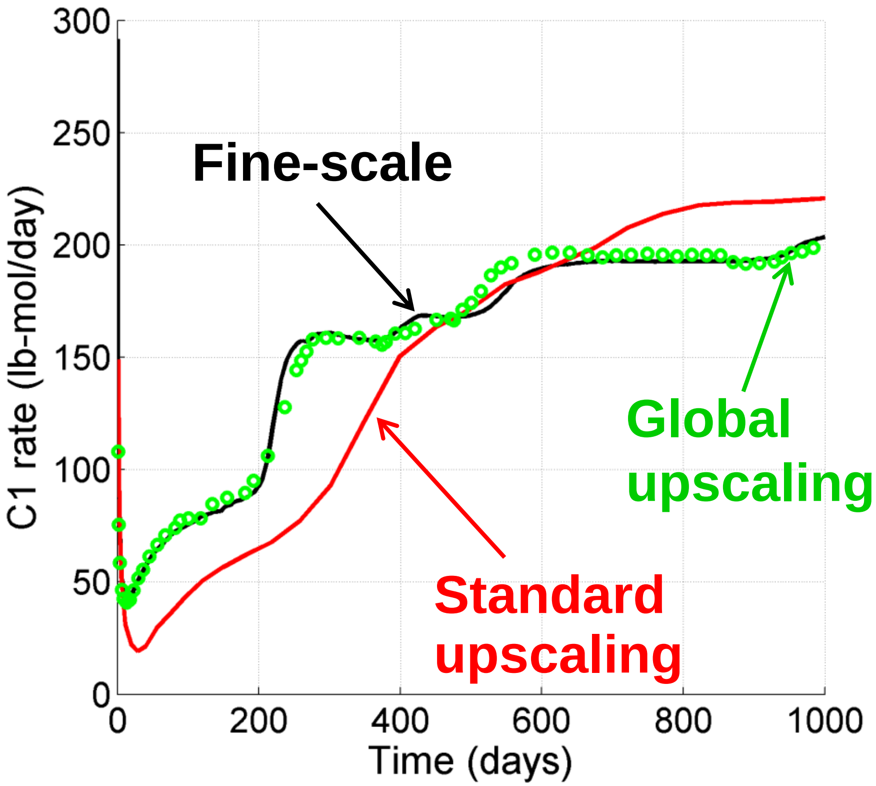


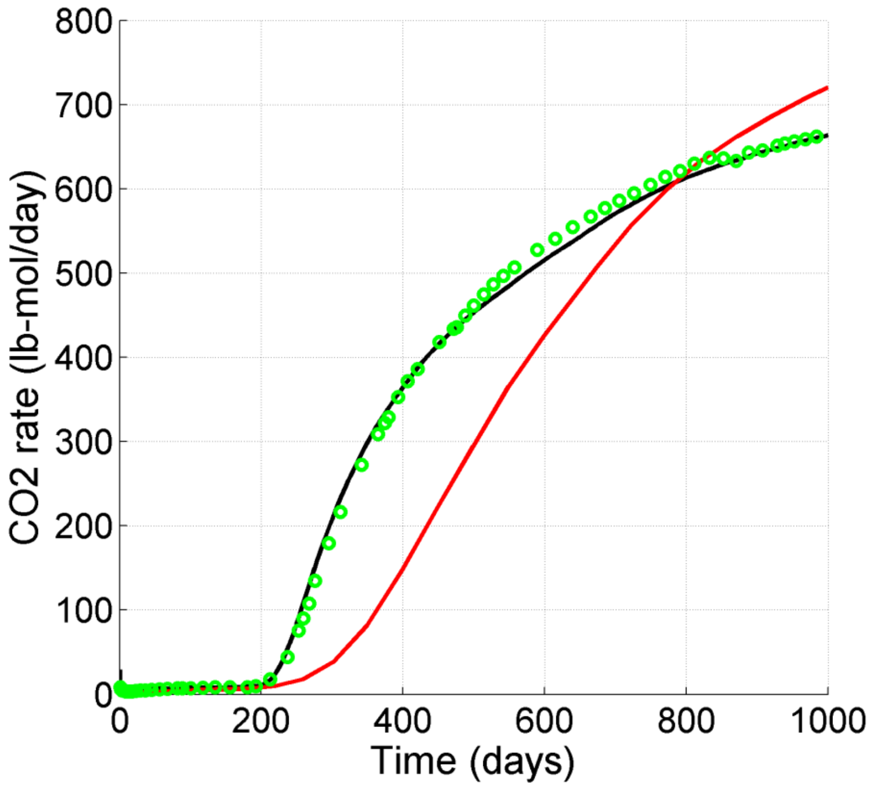


C_1 and CO_2 Field Production Rates

C_1 rate

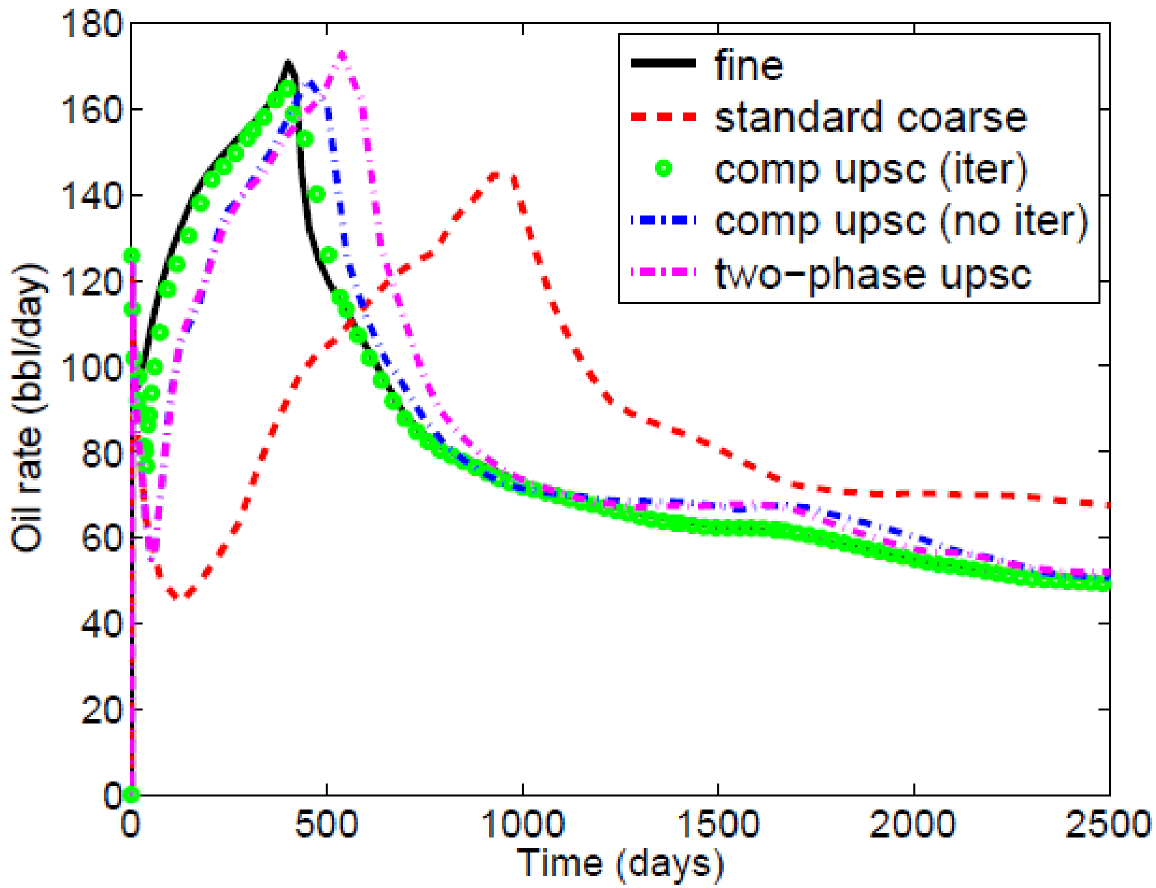


CO_2 rate



Impact of α_{ij} and Iteration

- Different (4-component) system, C_1 injected
- Five-spot, 99 x 99 → 11 x 11, $\sigma_{lnk} = 1.8$



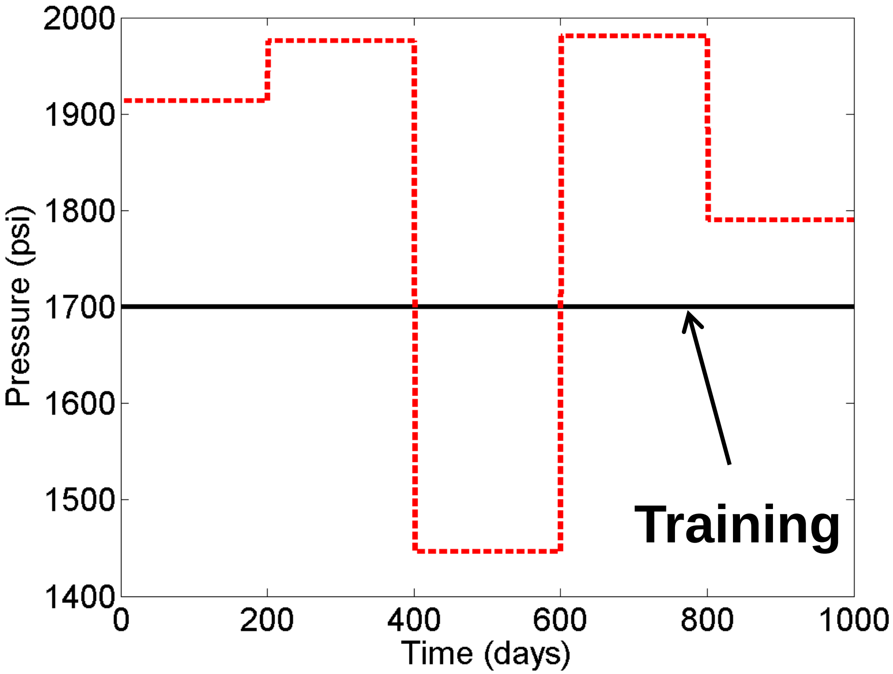
Field oil rate

(Li, 2014)

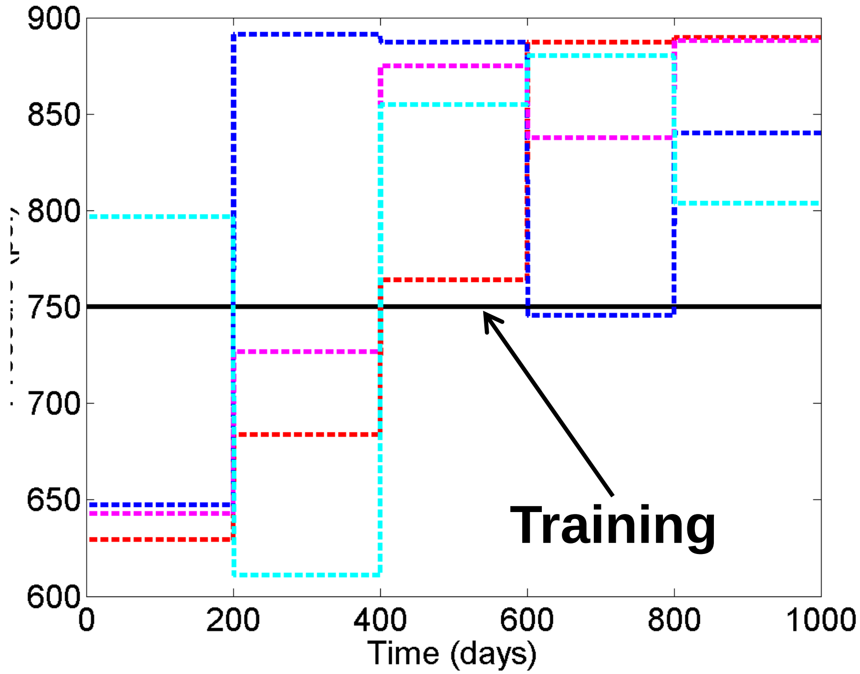
Robustness Test (8-comp system)

- "Training" case: BHP_{inj} = 1700 psi, BHP_{prod} = 750 psi
- "Test" case: vary BHPs by ±20% every 200 days (to mimic optimization run) – *use upscaled functions from training run*

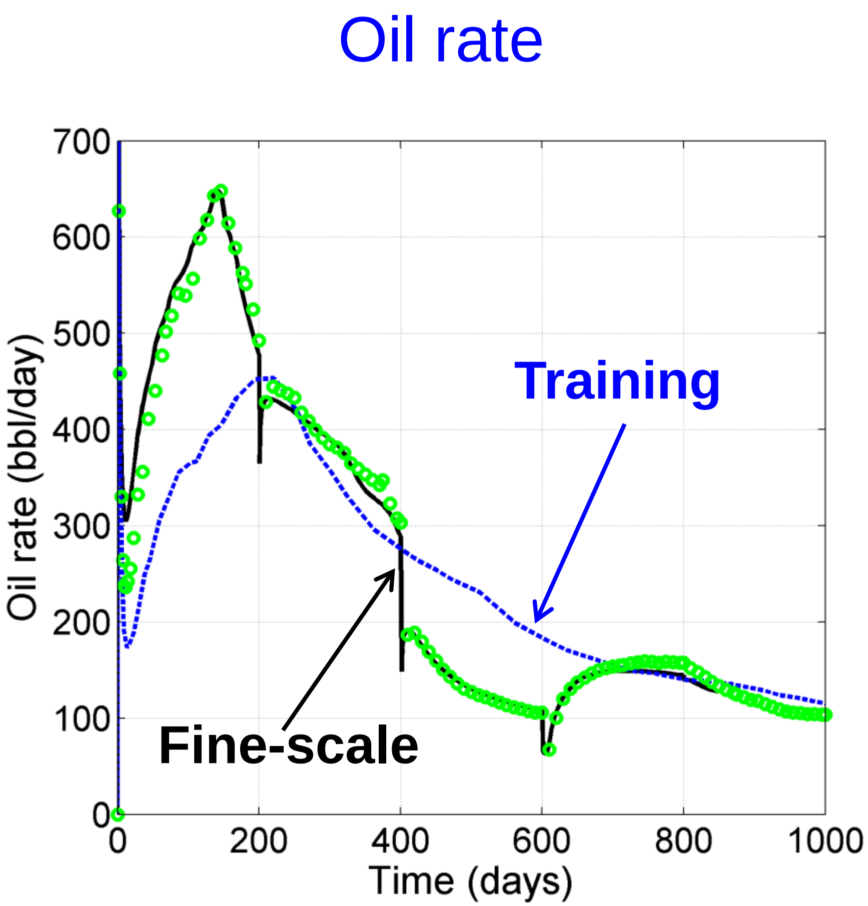
BHP_{inj}

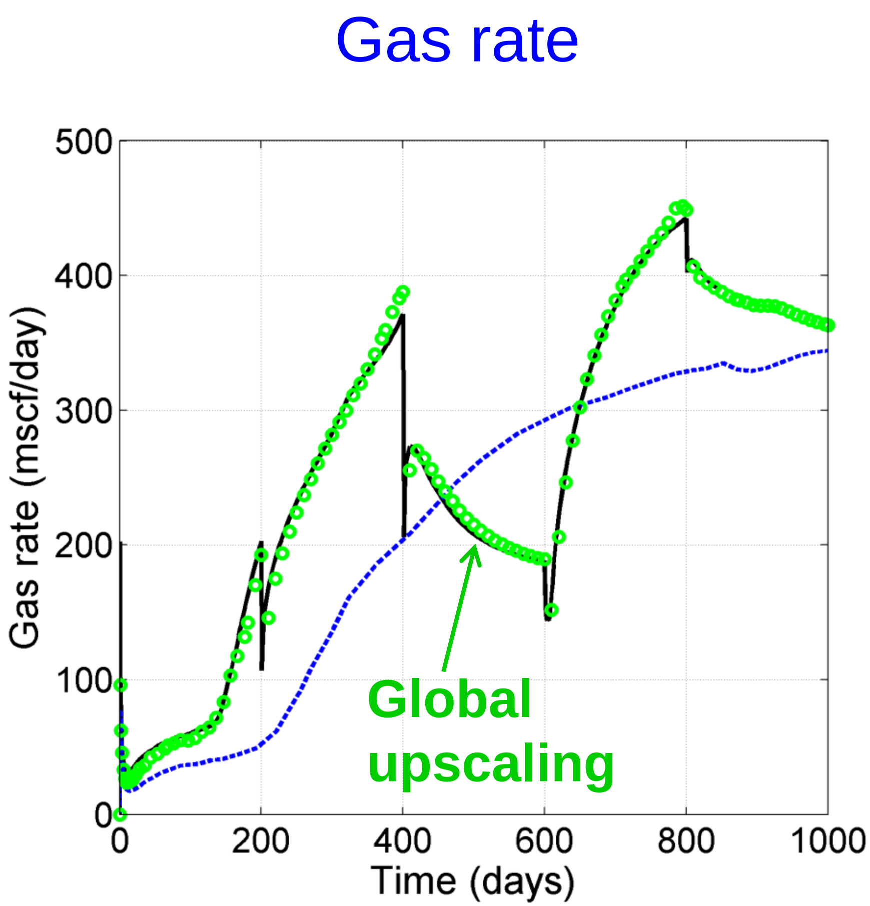


BHP_{prod}



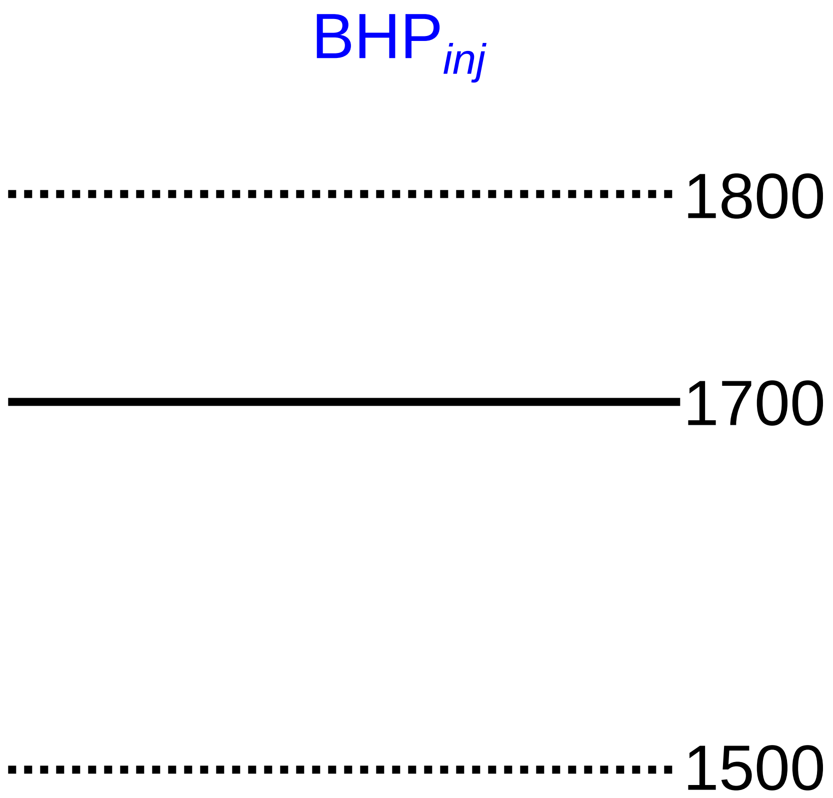
Oil and Gas Field Production Rates





Multiple Test Cases

- Training case: BHP_{inj} = 1700 psi, BHP_{prod} = 750 psi
- 100 test cases with time-varying BHPs



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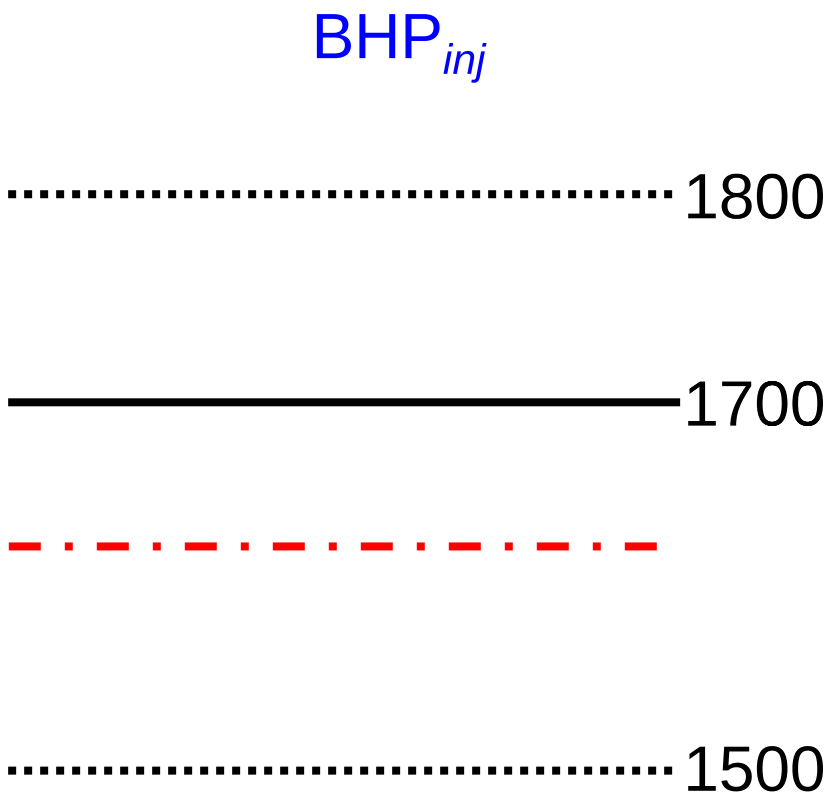

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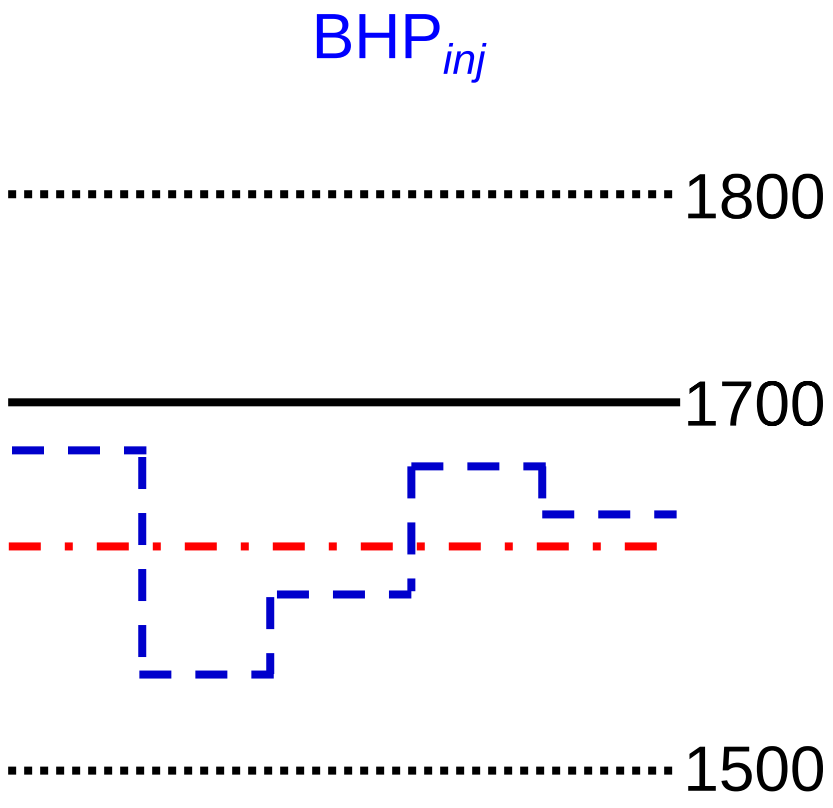

Multiple Test Cases

- Training case: BHP_{inj} = 1700 psi, BHP_{prod} = 750 psi
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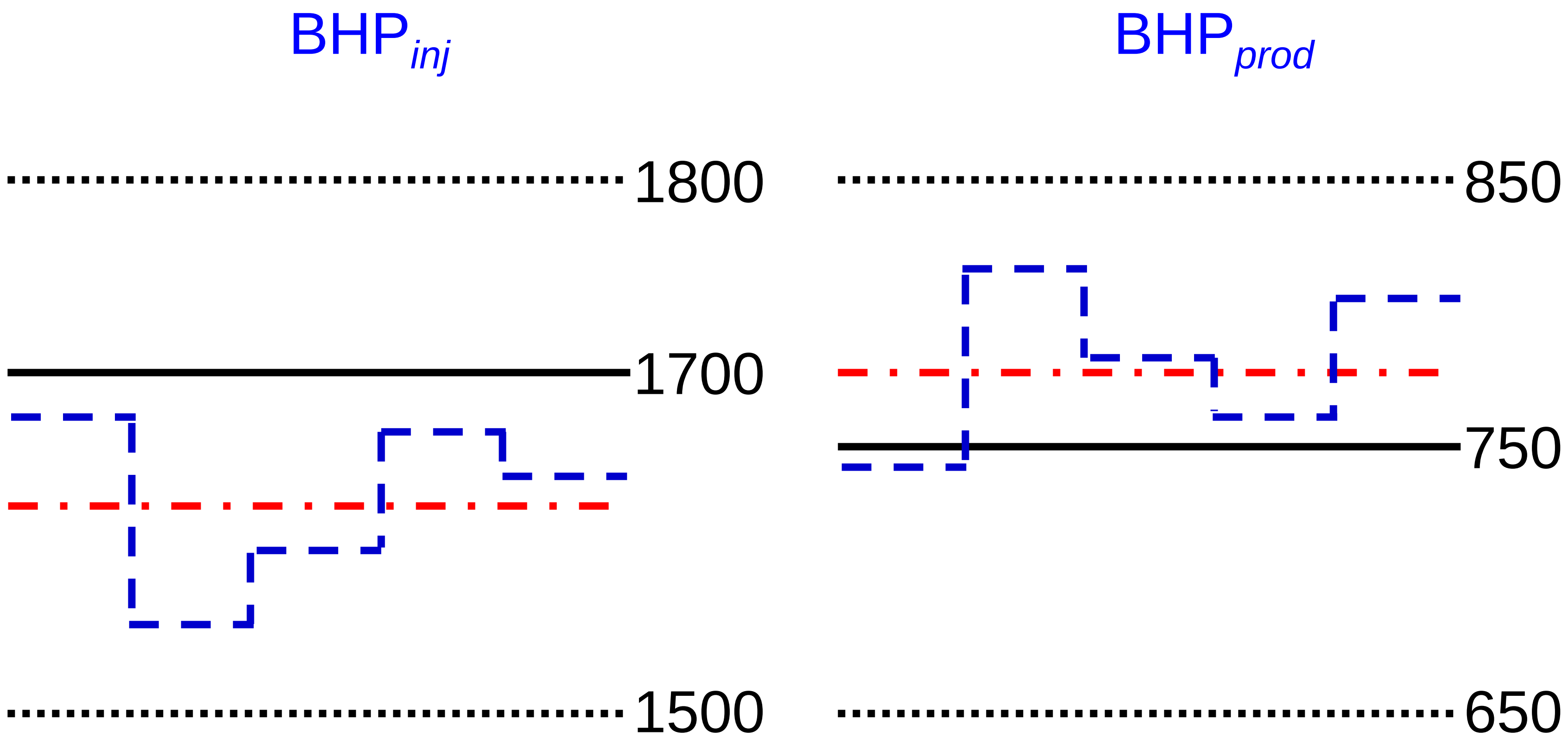
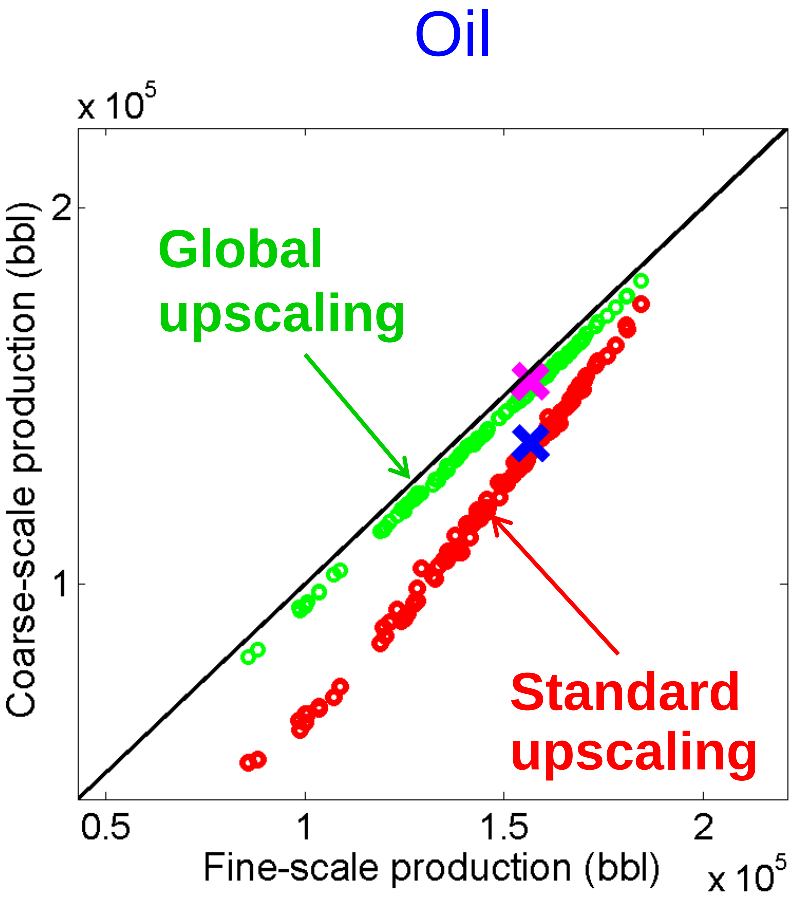
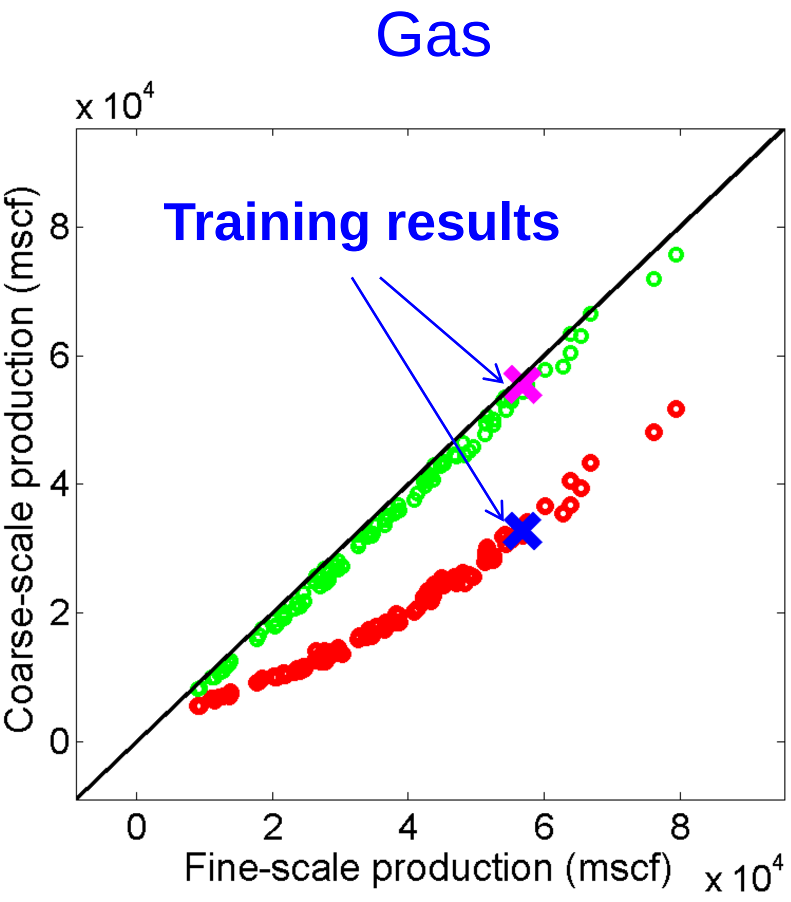


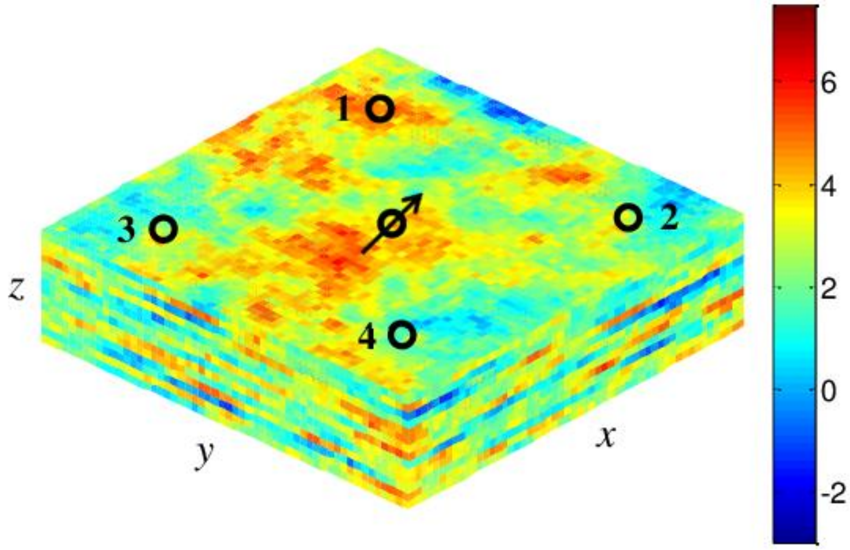
Cumulative Production at 500 days



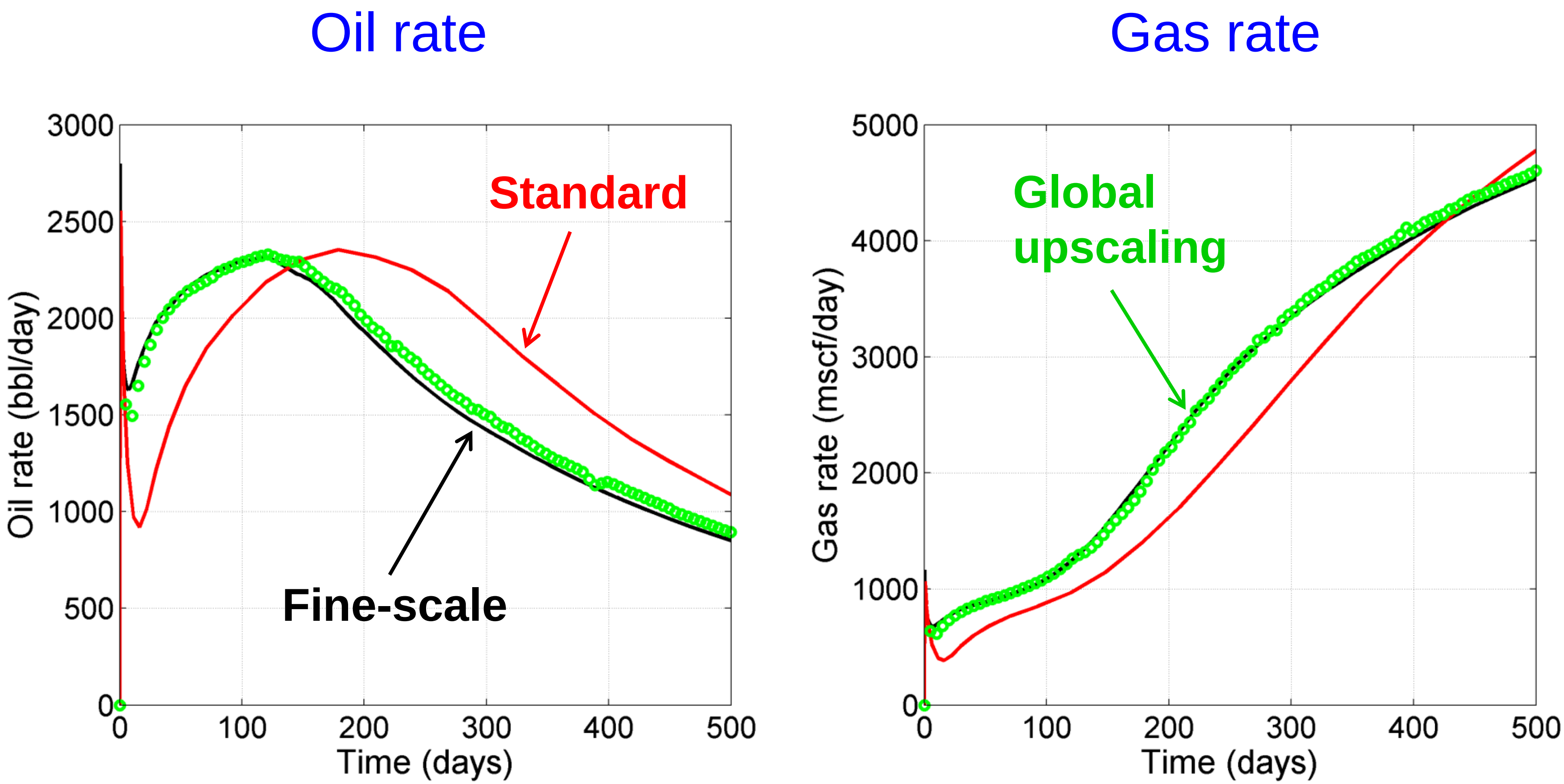


3D Model (5-spot pattern)

- Initial conditions:
  - 1150 psi, 334 K
  - {C_1(0.2), CO_2(0.01), C_4(0.29), C_{10}(0.5)}
- Permeability using sGsim
  - $l_x = 0.3$, $l_y = 0.3$, $l_z = 0.1$
  - 55 x 55 x 15 → 11 x 11 x 5
- Inject {**C_1(10%), CO_2(90%)**} at 1500 psi
- Produce at 750 psi

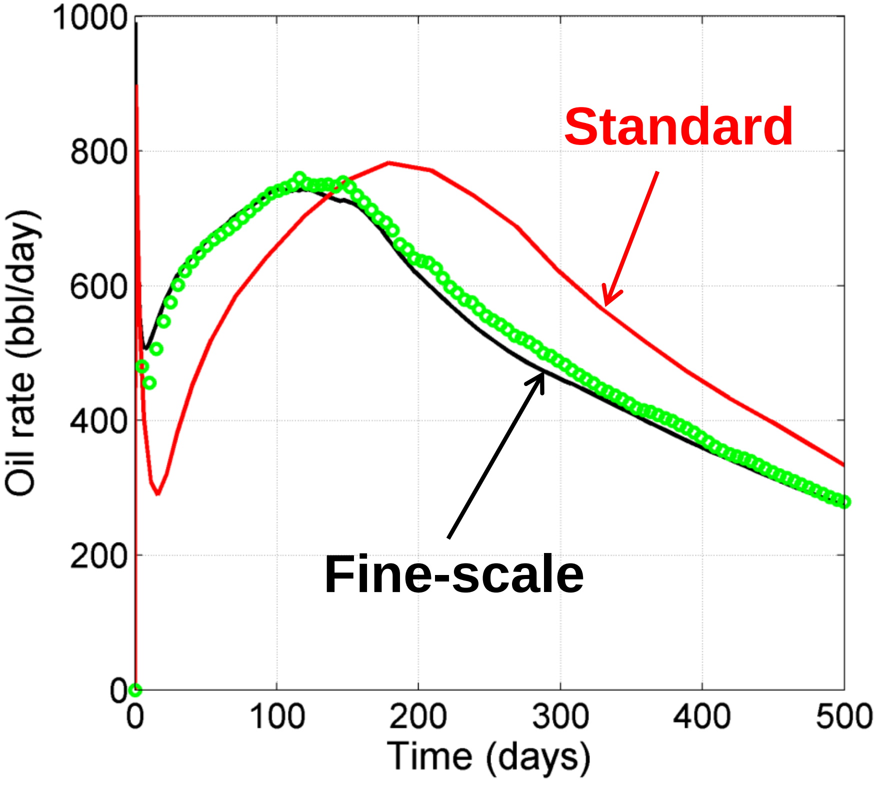


Oil and Gas Field Production Rates

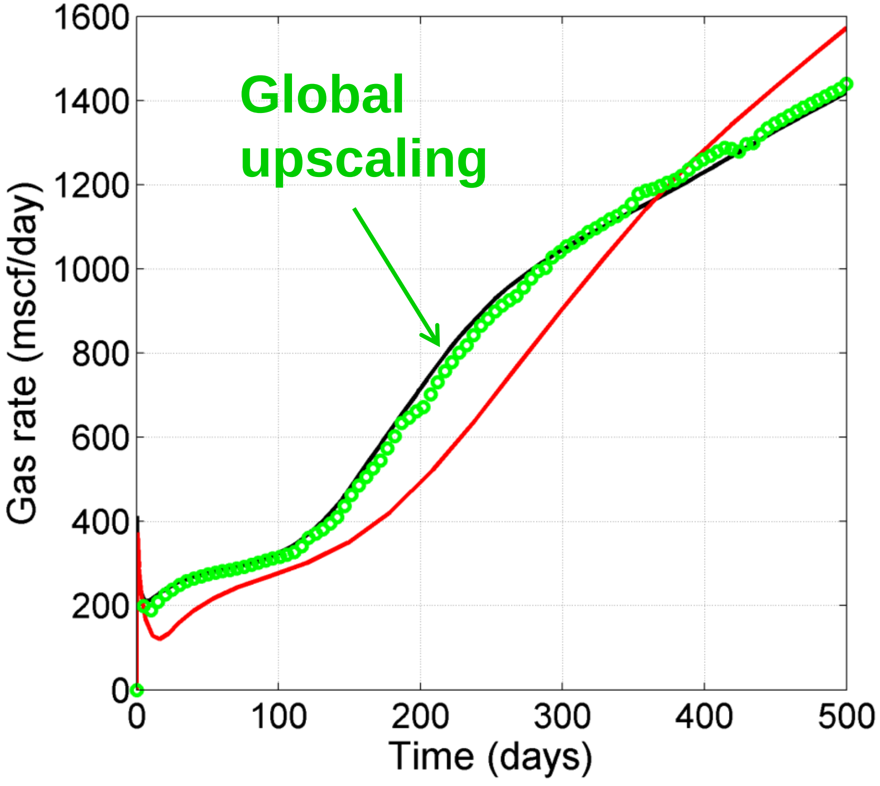


Oil and Gas Production Rates (Producer 4)

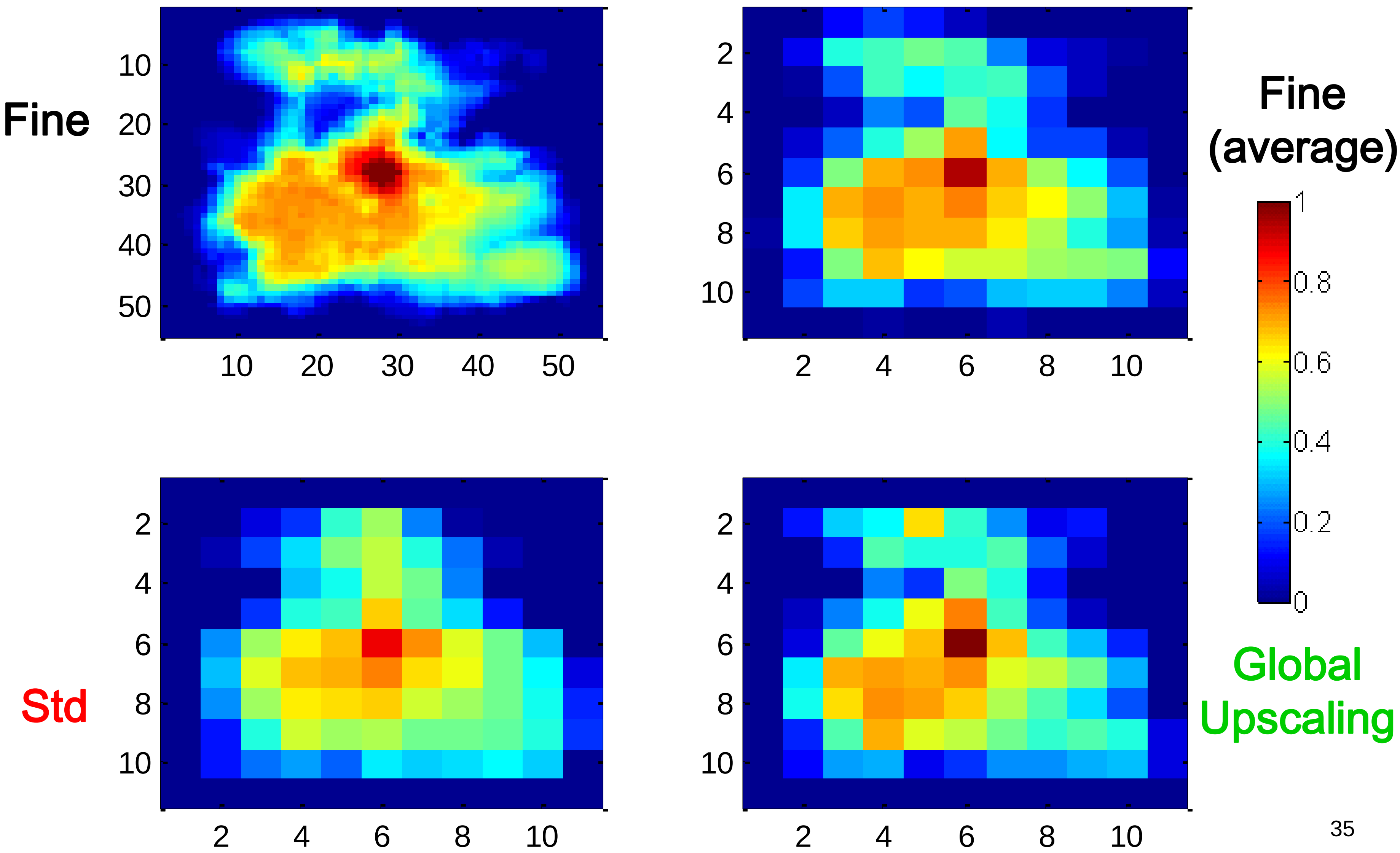
Oil rate



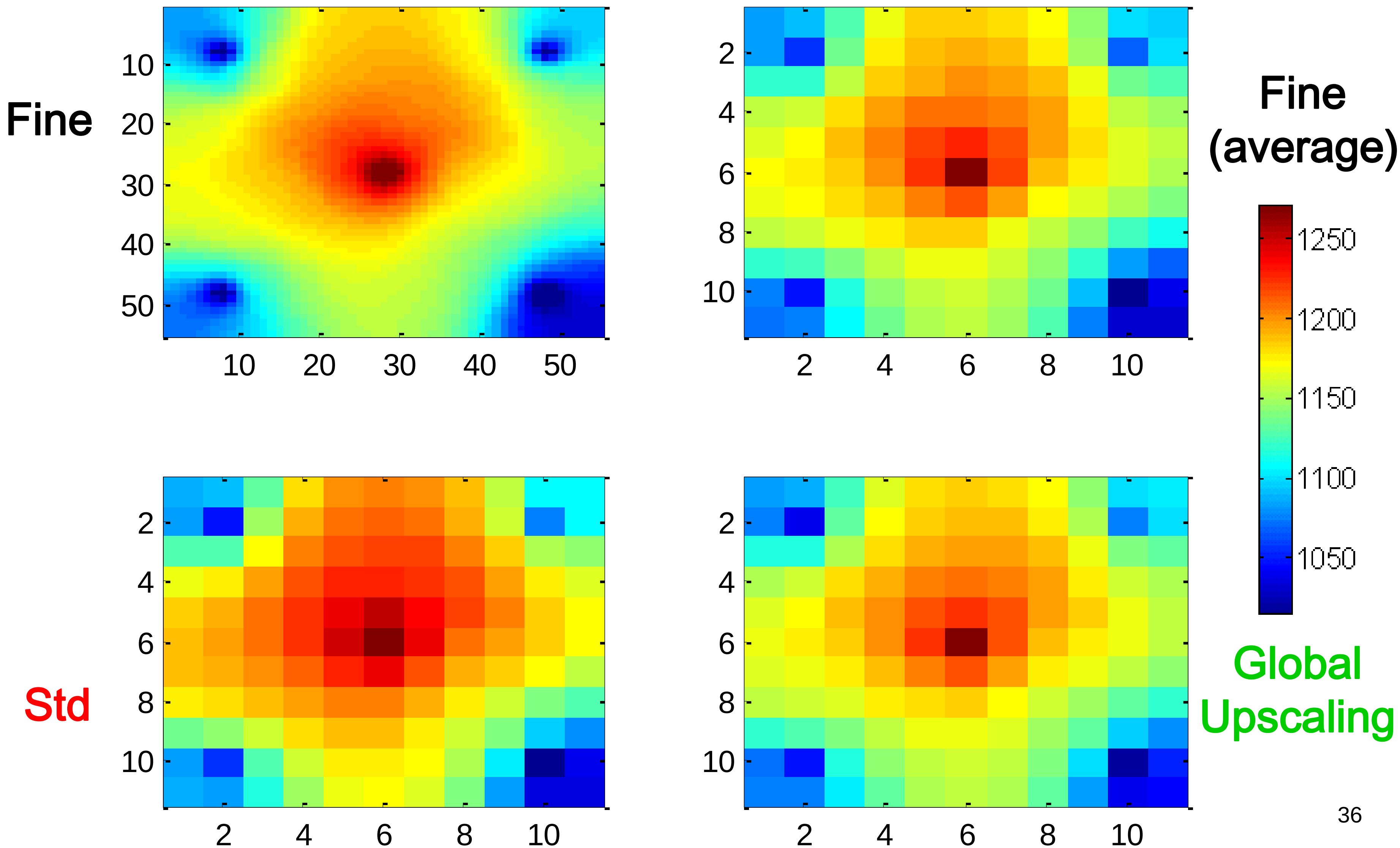
Gas rate



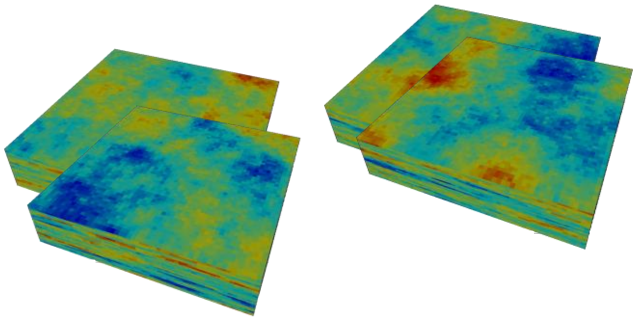
Gas Saturation Distribution
(300 days, coarse layer 4)



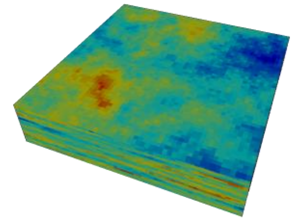
Pressure Distribution (300 days, coarse layer 4)



Ensemble Level Upscaling for UQ



1. Generate multiple fine-scale geological realizations



$k_{rg}^*, k_{ro}^*, \alpha_{ij}$

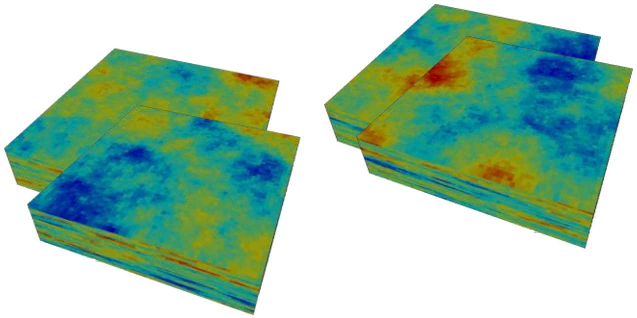
2. Apply full flow-based global upscaling to a small fraction of realizations

3. Calibrate the numerically upscaled $k_{rg}^*, k_{ro}^*, \alpha_{ij}$ to coarse-scale features

4. Statistically assign k_{rg}^*, k_{ro}^* & α_{ij} for new realizations

(figure after Y. Chen)

Ensemble Level Upscaling for UQ



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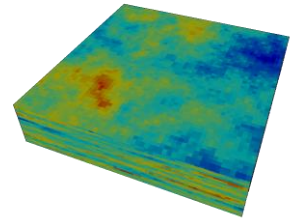

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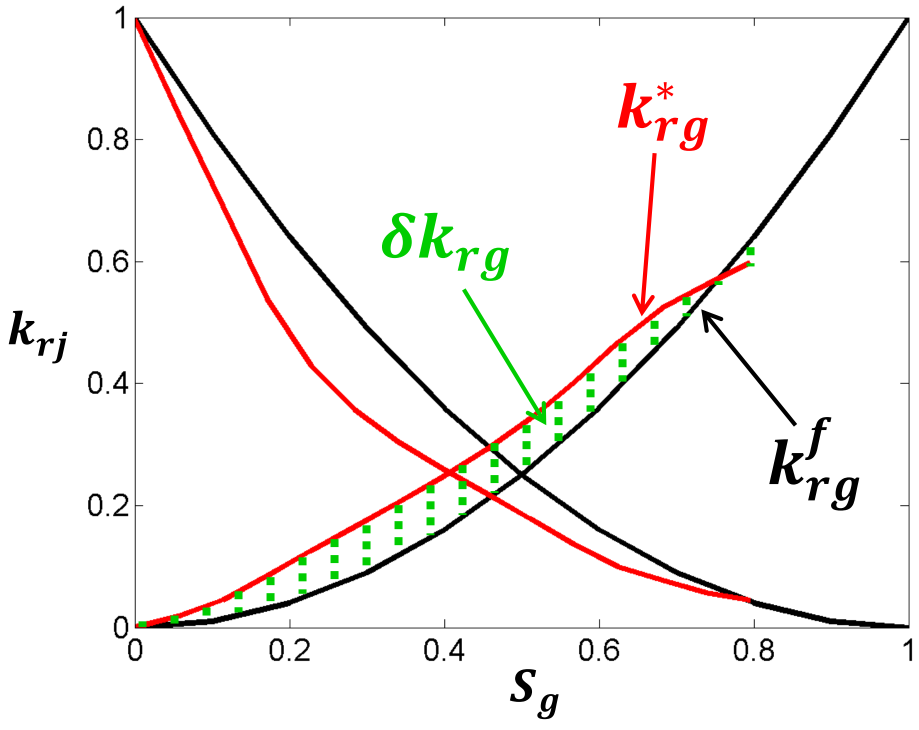
(figure after Y. Chen)

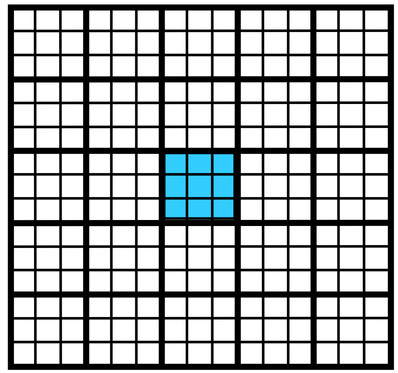
Parameterize k_{rg}^*

- Parameterize by the difference between k_{rg}^* and k_{rg}^f

$$\boldsymbol{\delta k_{rg} = \frac{1}{N}\sum(k_{rg}^* - k_{rg}^f)}$$



Calibrate (k_{rg}^{*}) for Upscaled Realizations



Features:

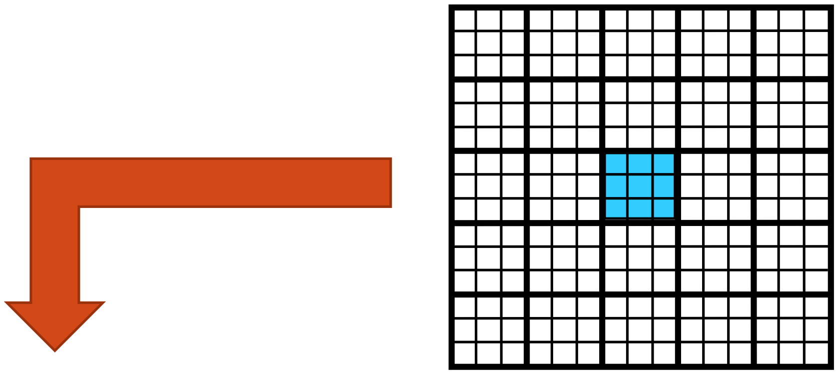
- $\boldsymbol{\sigma(\ln k)}$
- $\boldsymbol{\sigma(u_{1p})/\overline{u_{1p}}}$
- $\boldsymbol{\sum q^{f}}$

$\boldsymbol{\delta k_{rg}}$($f_1$, f_2, f_3)

Numerically upscaled function ($\boldsymbol{k_{rg}^{*}}$):

$\boldsymbol{k_{rg}^{*}} \rightarrow \boldsymbol{\delta k_{rg}}$

δk_{rg} from Features for New Realizations

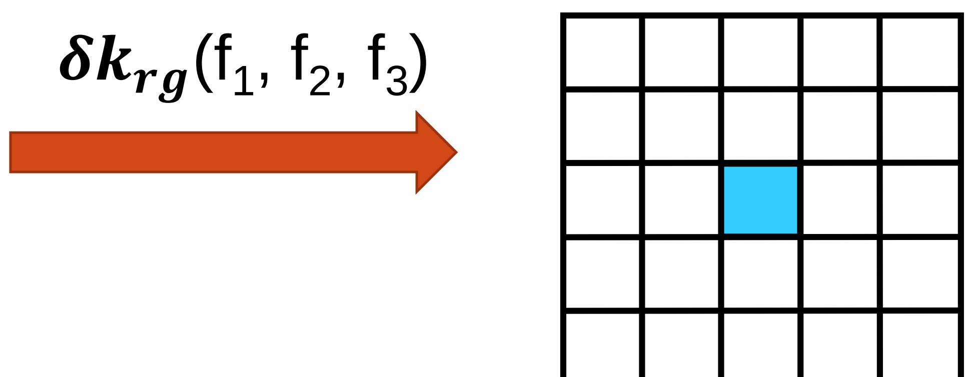


Features:

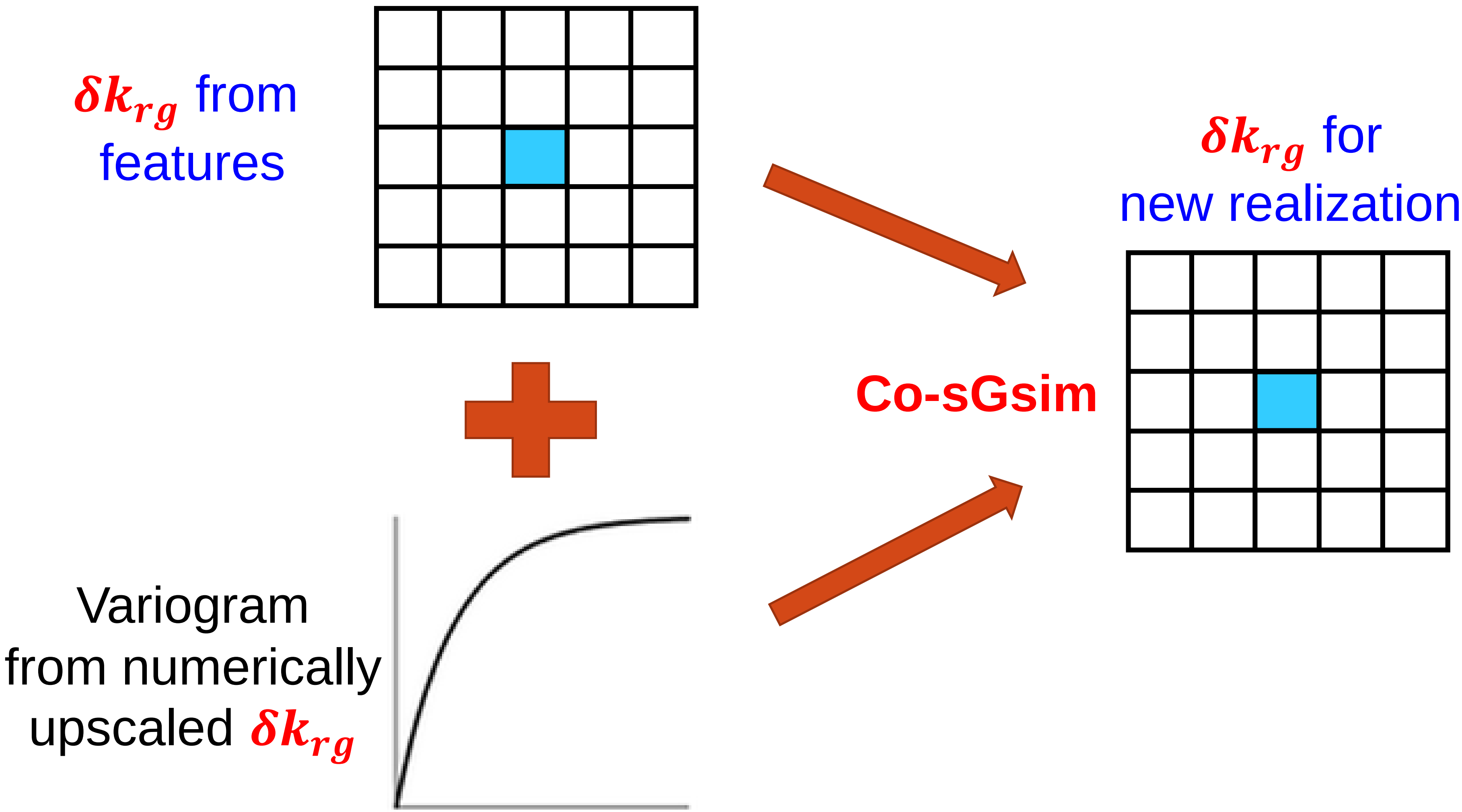
- $\boldsymbol{\sigma(\ln k)}$
- $\boldsymbol{\sigma(u_{1p})/\overline{u_{1p}}}$
- $\boldsymbol{\sum q^f}$

$\boldsymbol{\delta k_{rg}}$(f$_1$, f$_2$, f$_3$)

$\boldsymbol{\delta k_{rg}}$ from features:

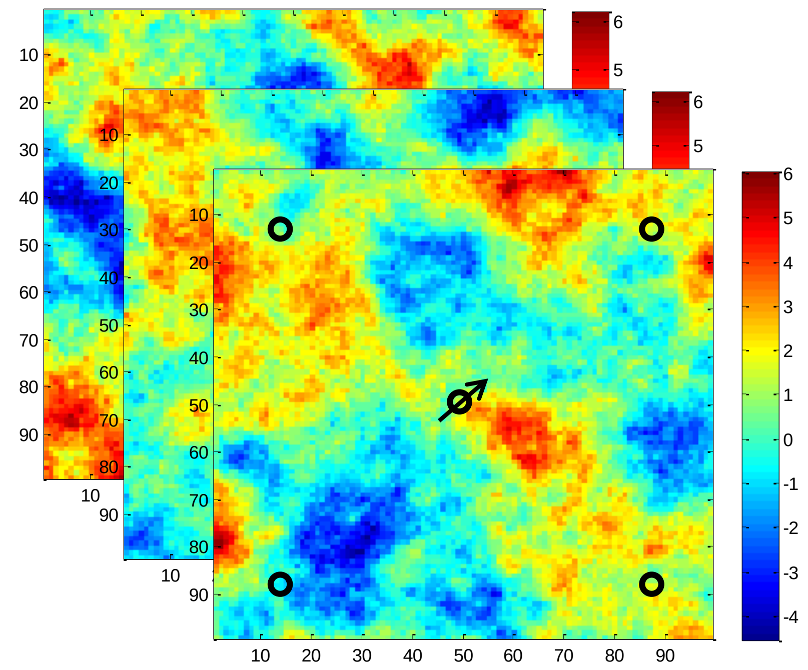


Statistically Assign δk_{rg} for New Realizations



Multiple Realizations (4-component system)

- Permeability using sGsim
  - Initial oil: {C_1(0.1), CO_2(0.18), C_4(0.38), C_{10}(0.34)}
  - **100 realizations**:
    99 x 99→11 x 11

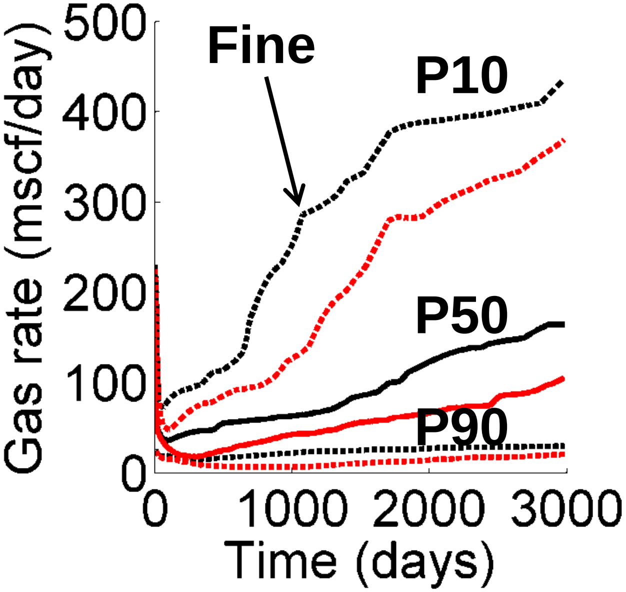


- Inject pure C_1 at 1500 psi, produce at 500 psi

- **10** realizations **upscaled globally, EnLU** applied for other **90** realizations

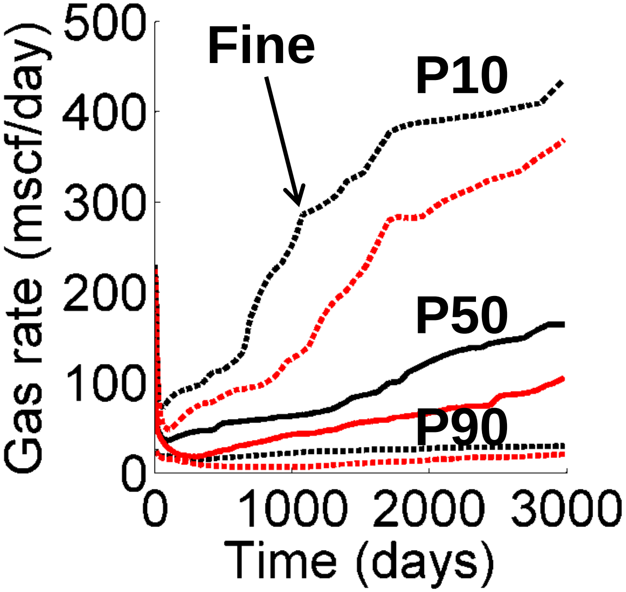
Flow Statistics of Gas Production Rates

Standard
upscaling
(T^* & WI^* only)

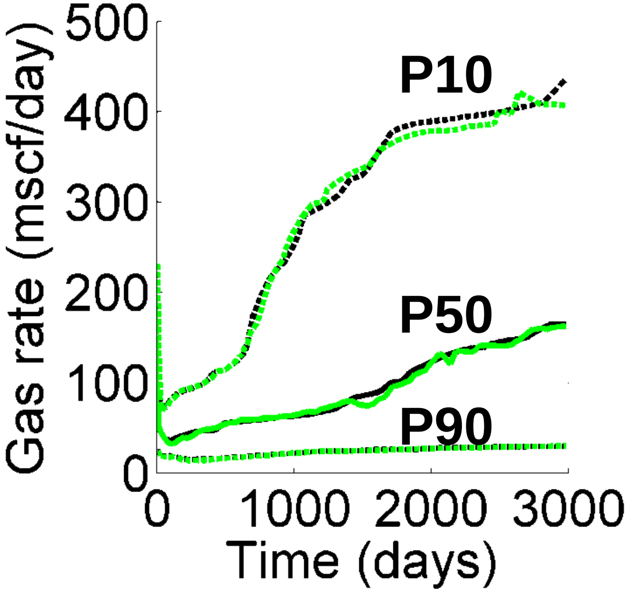


Flow Statistics of Gas Production Rates

Standard upscaling (T^* & WI^* only)

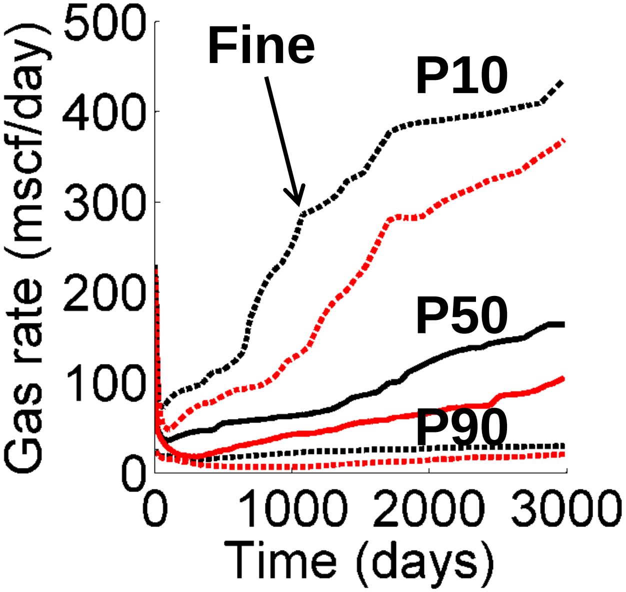


Full global compositional upscaling (ref)

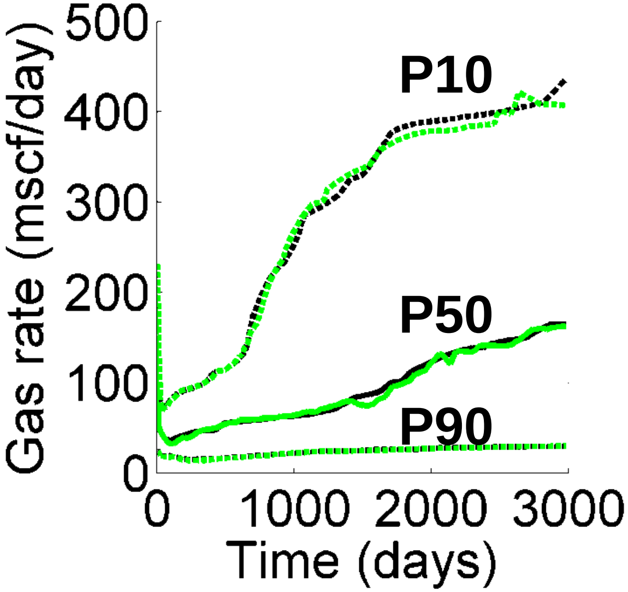


Flow Statistics of Gas Production Rates

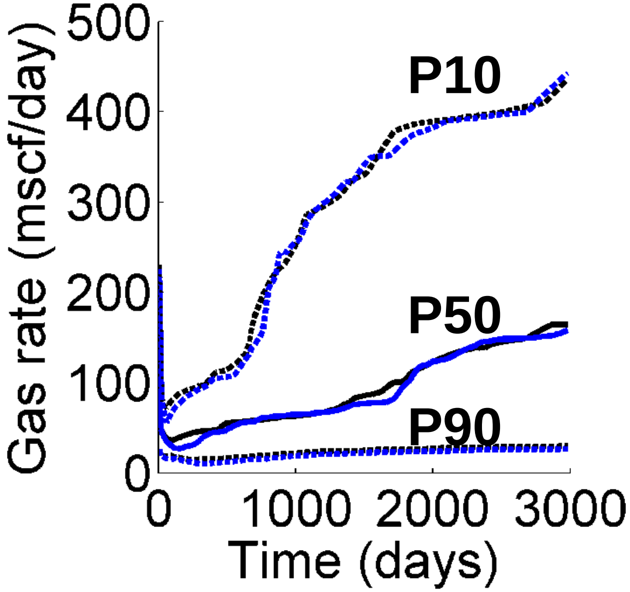
Standard upscaling (T^* & WI^* only)



Full global compositional upscaling (ref)



EnLU compositional upscaling

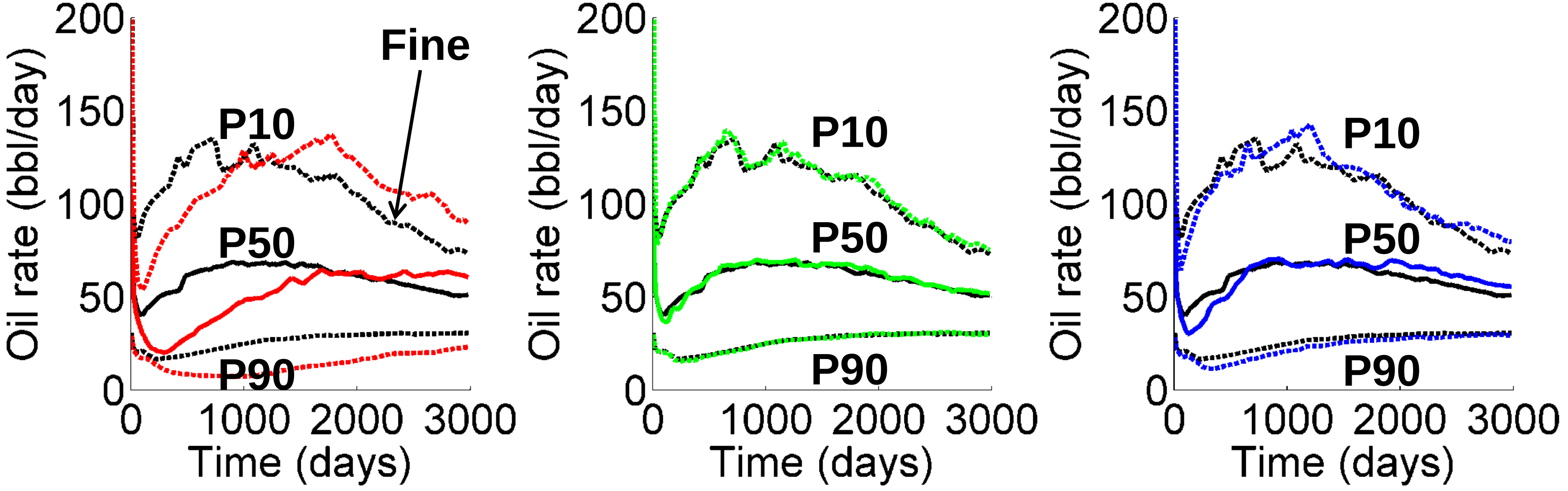


Flow Statistics of Oil Production Rates

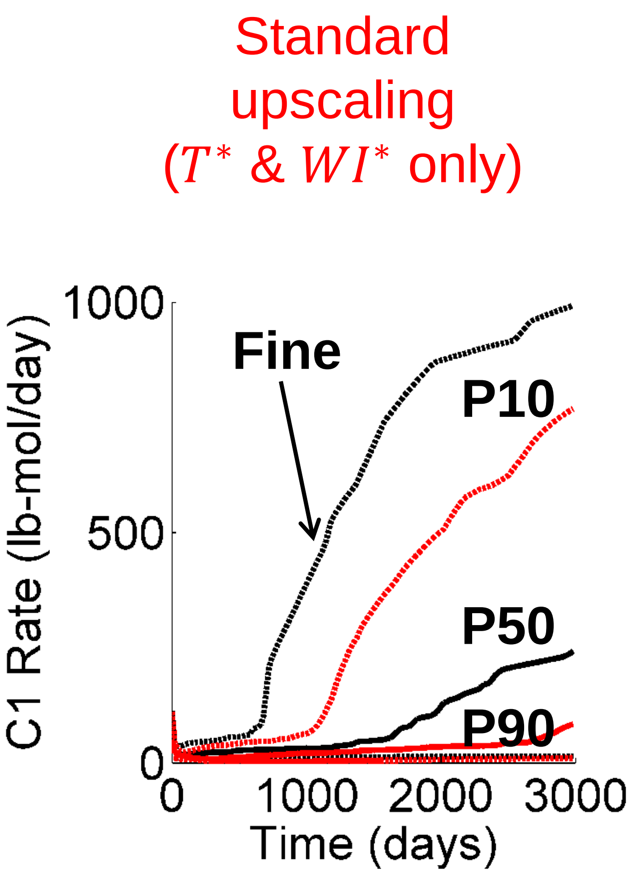
Standard upscaling (T^* & WI^* only)

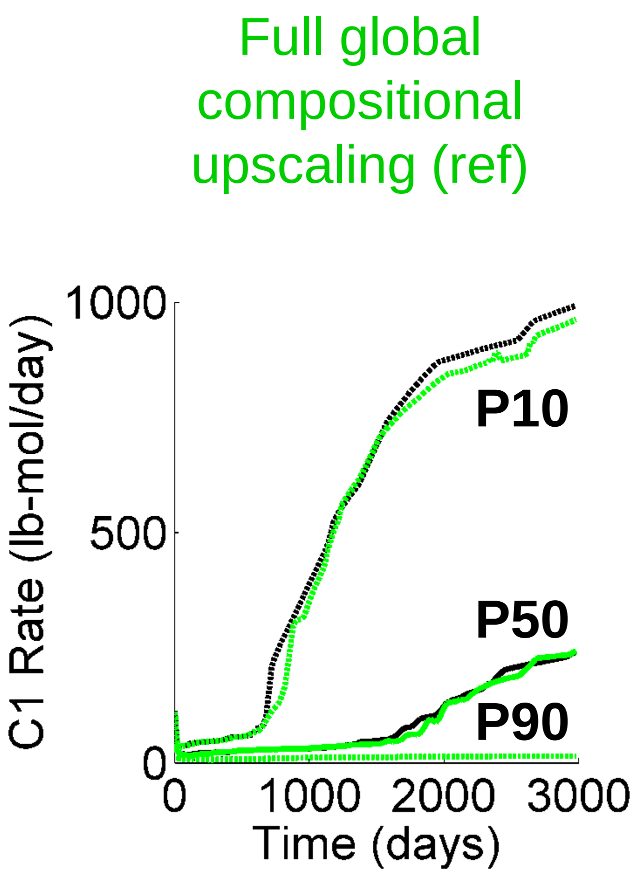
Full global compositional upscaling (ref)

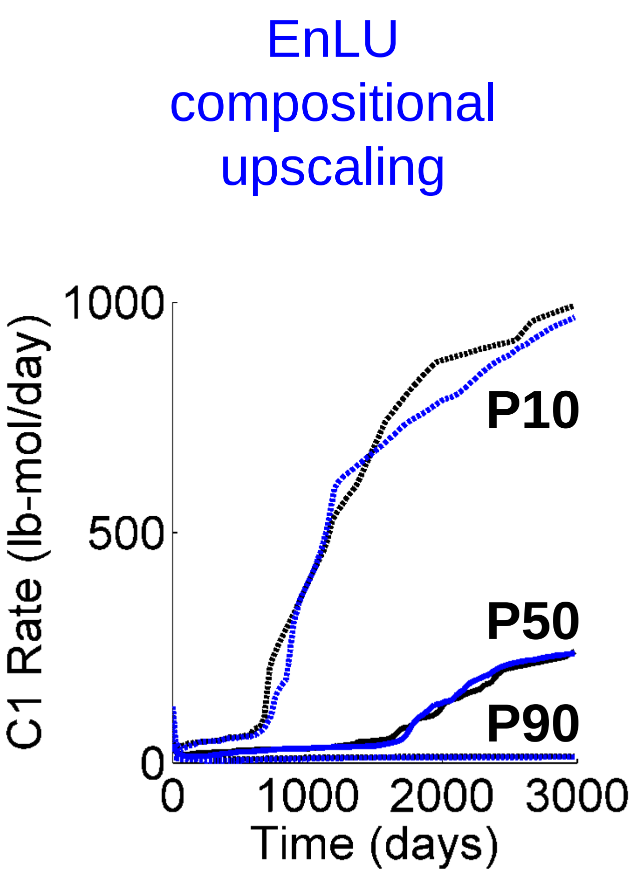
EnLU compositional upscaling



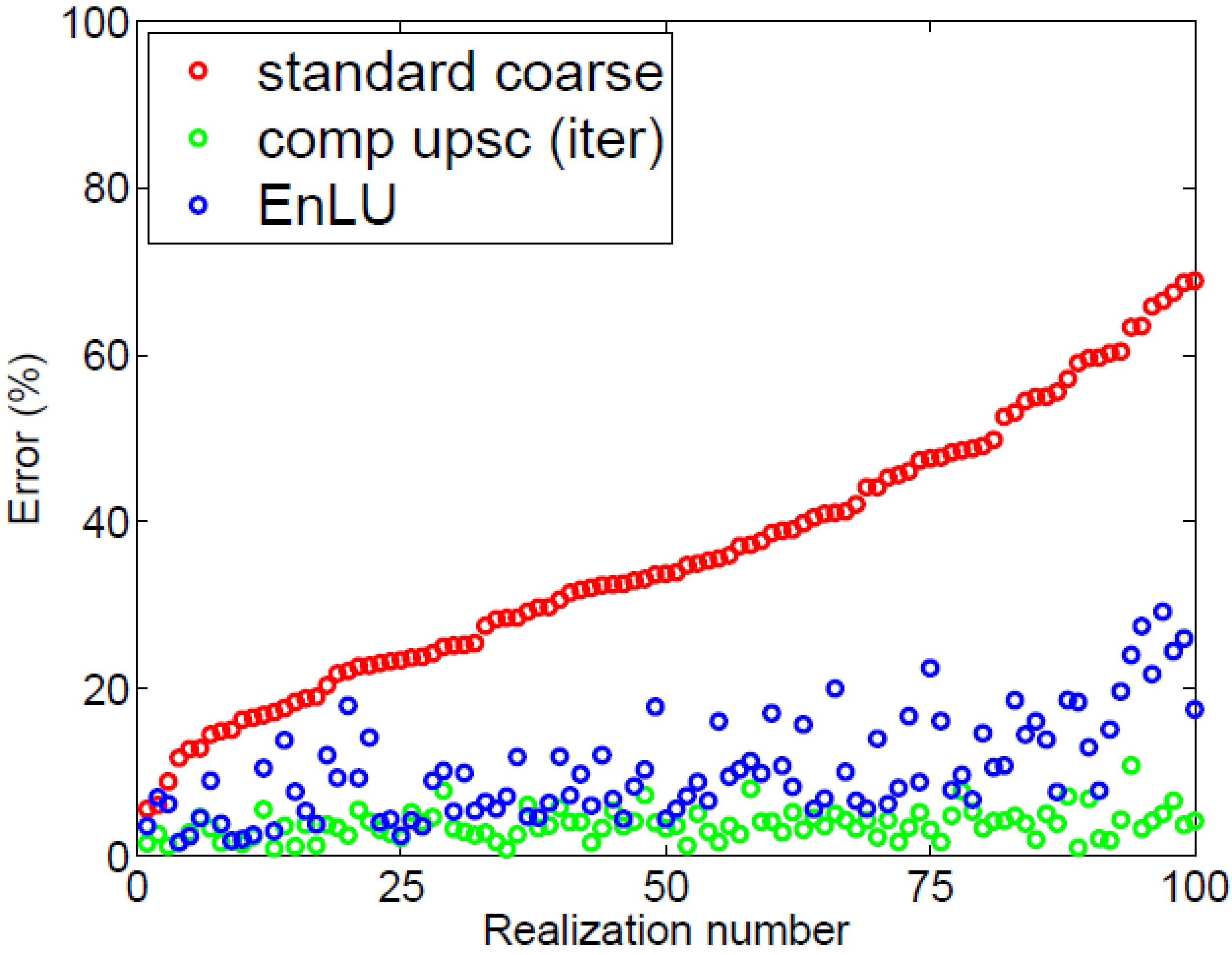
Flow Statistics of C_1 Production Rates



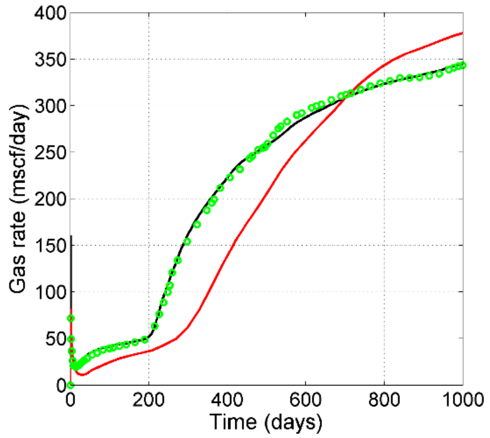




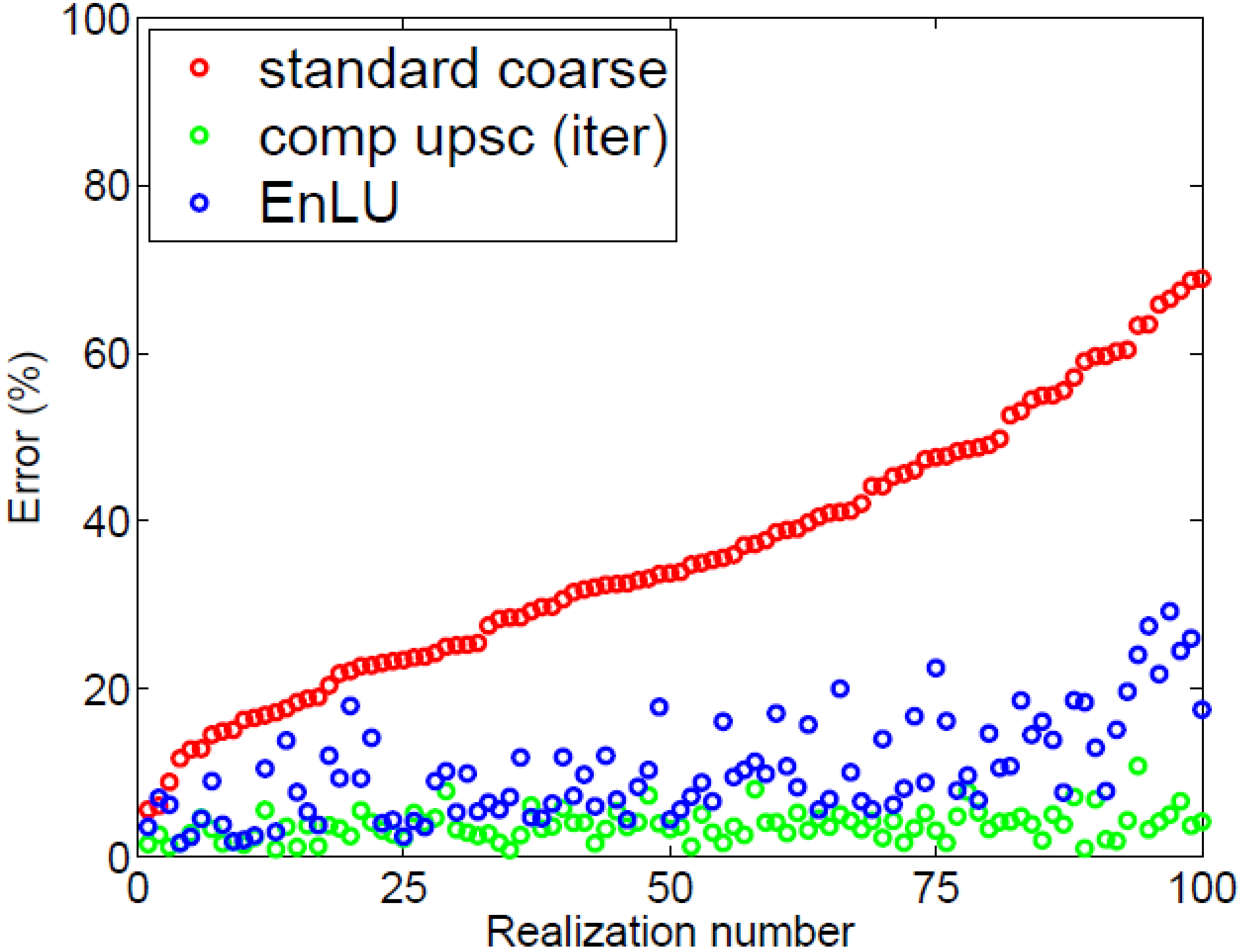
Realization-by-Realization Error (Gas Rate)



$$E_j = \frac{\int_t \left| q_j^c - q_j^f \right| dt}{\int_t q_j^f \, dt}$$



Realization-by-Realization Error (Gas Rate)



$$E_j = \frac{\int_t \left| q_j^c - q_j^f \right| dt}{\int_t q_j^f \, dt}$$

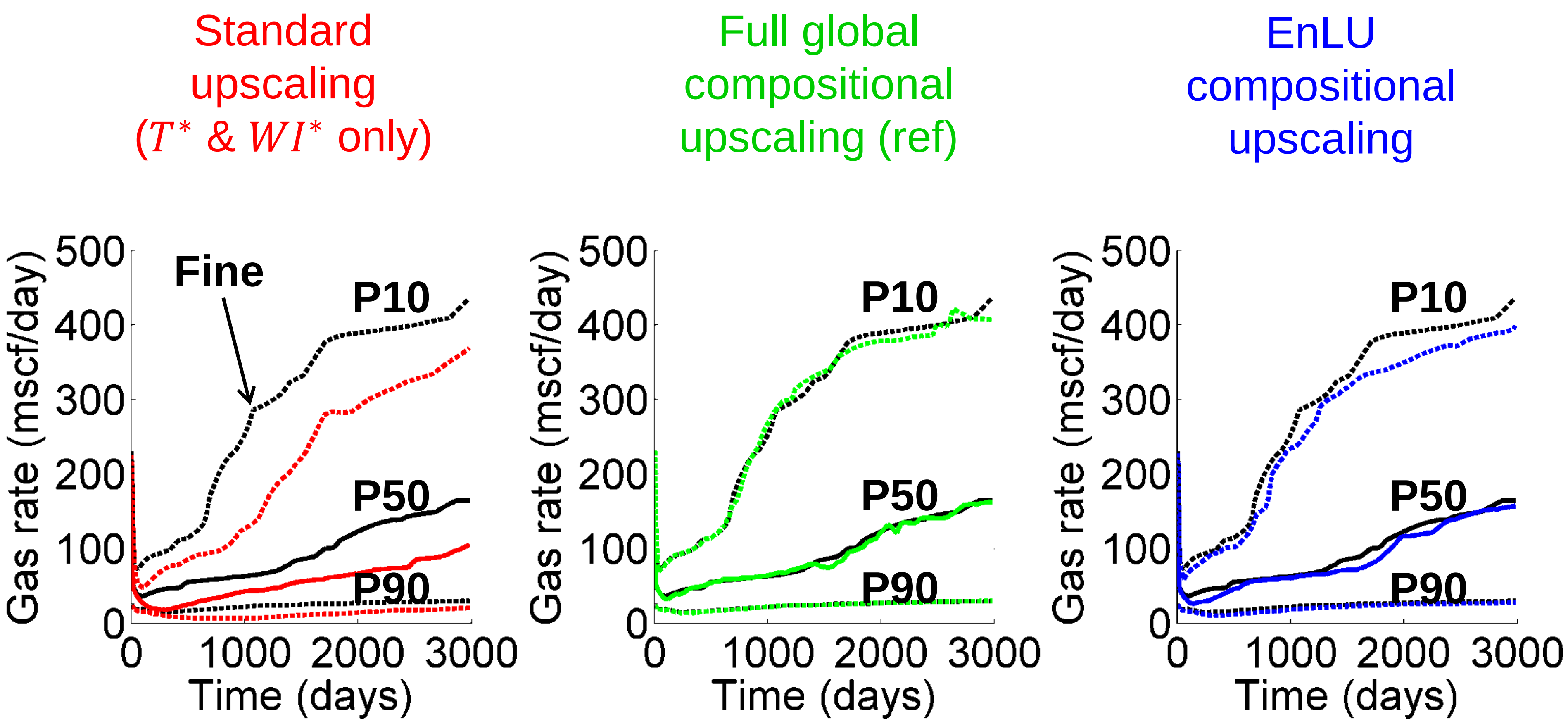
Median errors

std coarse: 34%

comp upsc: 3.3%

EnLU: 9.3%

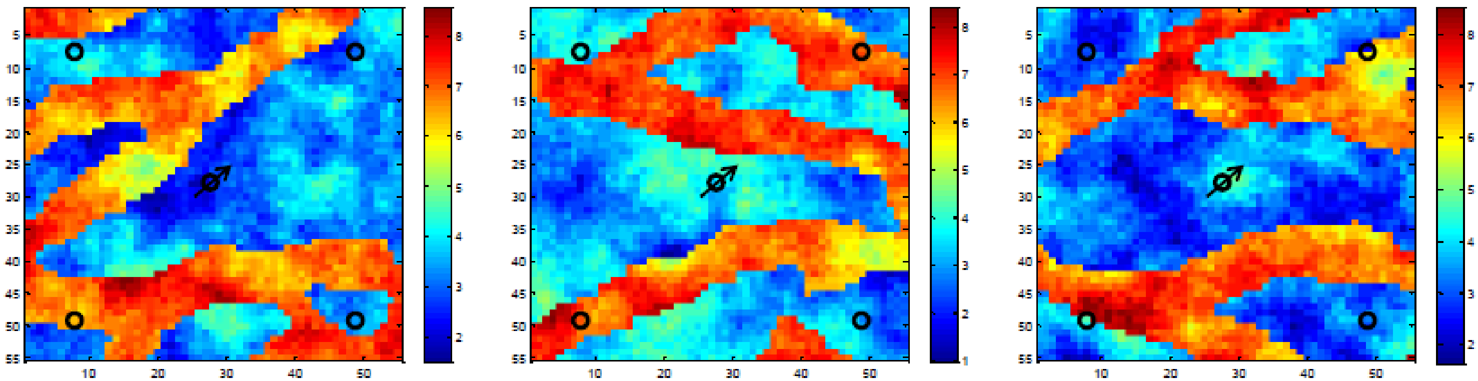
Flow Statistics of Gas Production Rates (5 realizations upscaled)



Channel Realizations (4 components)

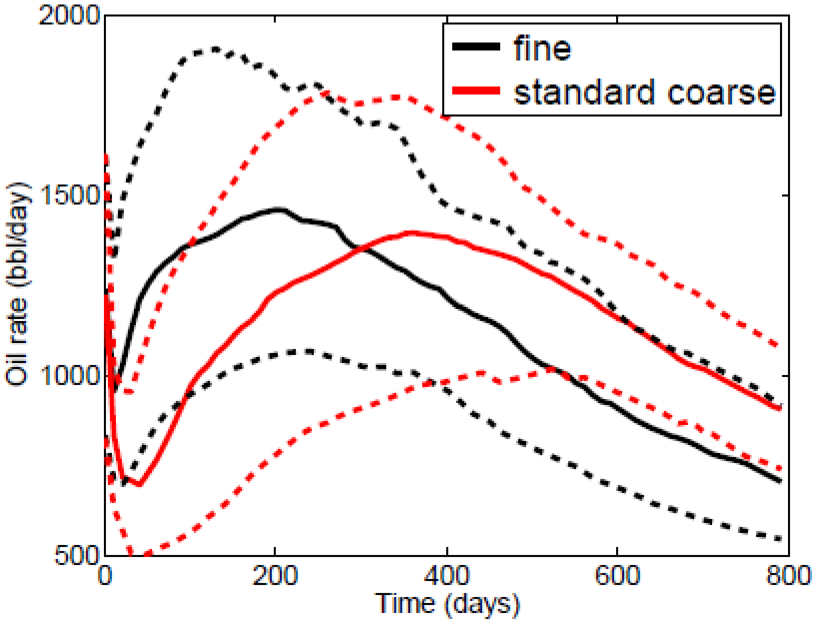
- Problem setup
  - Initial oil: {C_1(0.2), CO_2(0.01), C_4(0.29), C_{10}(0.5)}
  - 100 realizations**:** 55 x 55 →11 x 11

- 10 realizations upscaled globally, EnLU applied for other 90 realizations
- Inject CO_2 at 1500 psi, produce at 500 psi

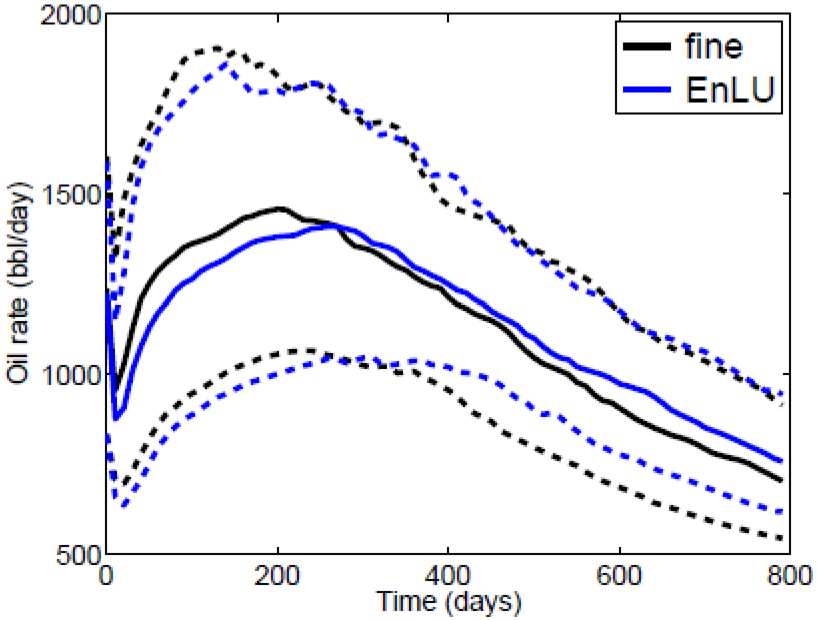


Flow Statistics – Oil Rates (channel model)

Standard upscaling



EnLU



Median (realization) rate errors

std coarse: 20.5%

comp upsc: 1.3%

EnLU: 5.7%

Summary

- Described single-phase (for DFMs) and compositional upscaling approaches
- Aggregation-based method can efficiently generate a range of coarsened models
- Global compositional upscaling is expensive but provides high accuracy and acceptable robustness
- Ensemble level upscaling provides reasonable flow statistics with 5-10% of models run on fine-scale

Current and Future Work

- Combine aggregation-based treatments with multilevel Monte Carlo for UQ
- Apply/extend compositional upscaling to shale oil simulations
- Further develop error models to correct upscaled models and guide fine-scale simulation

References for Results Presented

- Karimi-Fard, M., Durlofsky, L.J. A general gridding, discretization, and coarsening methodology for modeling flow in porous formations with discrete geological features. *Advances in Water Resources*, **96**, 354-372 (2016).

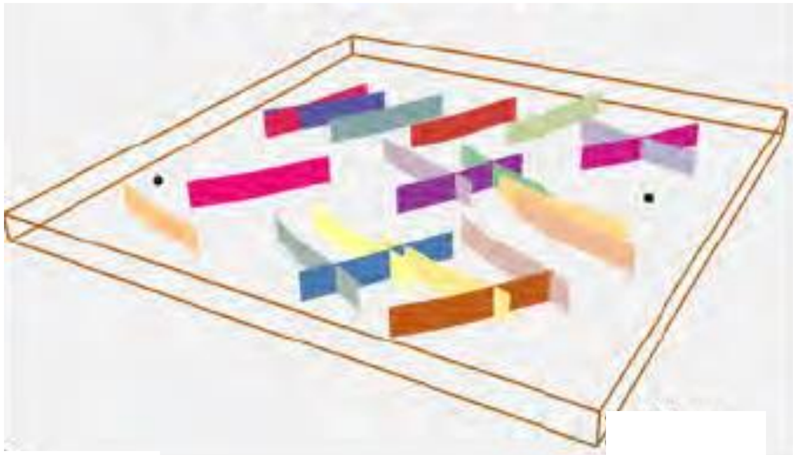
- Hui, M., Karimi-Fard, M., Mallison, B., Durlofsky, L.J. A general modeling framework for simulating complex recovery processes in fractured reservoirs at different resolutions. SPE paper 182621, presented at the SPE Reservoir Simulation Conference, Montgomery, TX, Feb. 20-22 (2017).

- Li, H., Durlofsky, L.J. Upscaling for compositional reservoir simulation. *SPE Journal*, **21**, 873-887 (2016).

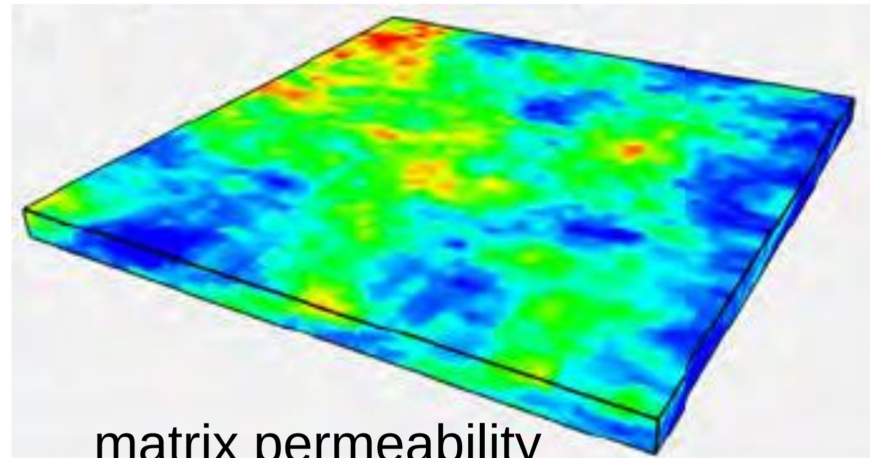
- Li, H., Durlofsky, L.J. Ensemble level upscaling for compositional flow simulation. *Computational Geosciences*, **20**, 525-540 (2016).

- Li, H., Durlofsky, L.J. Local-global upscaling for compositional subsurface flow simulation. *Transport in Porous Media*, **111**, 701-730 (2016).

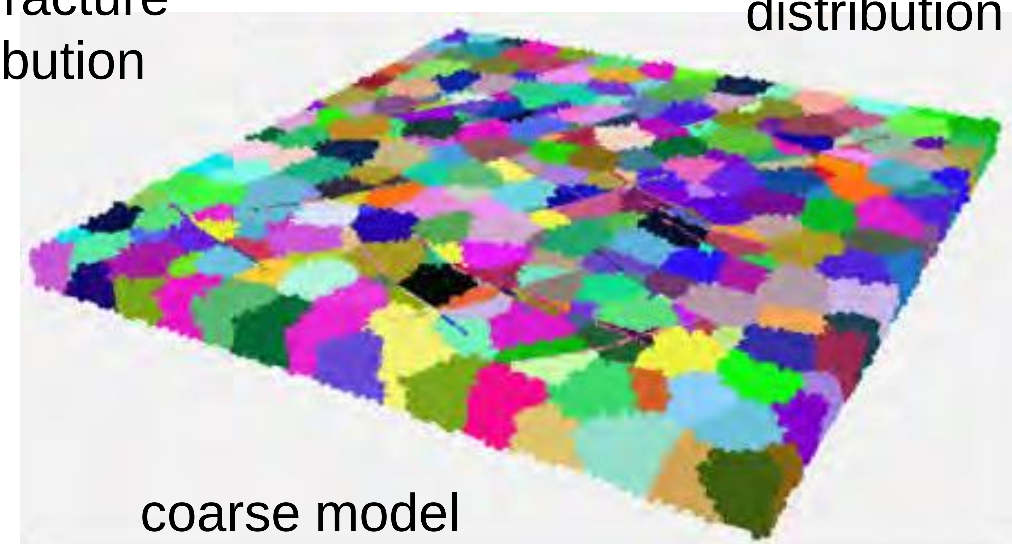
Alternative DFM Gridding Framework



local fracture
distribution



matrix permeability
distribution



coarse model