Computational Issues in Oil Field Applications:
Fracture Propagation in Porous Media using Phase Field Approach

Workshop I: Multiphysics, Multiscale, and Coupled Problems in Subsurface Physics

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Outline

1. Introduction
2. Governing System
3. Coupling Algorithms
4. Advantages of Phase Field Approach
5. Optimization: Coupling Phase Field
6. Conclusions & Future Work

Introduction

Introduction

Phase Field

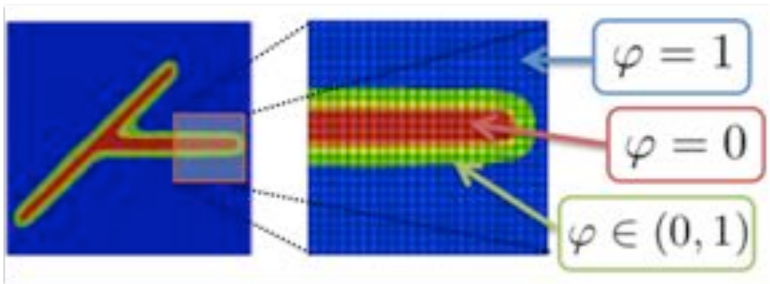
Pressurized and Fluid Filled Fracture Propagation with Transport

- Production benefits: hydraulic fracturing; shale gas, tight gas.
- Environmental issues: sinkholes, CO_2 sequestration.
- Health problems: blood and fractures.
- Coupled problem: flow and geomechanics with transport.

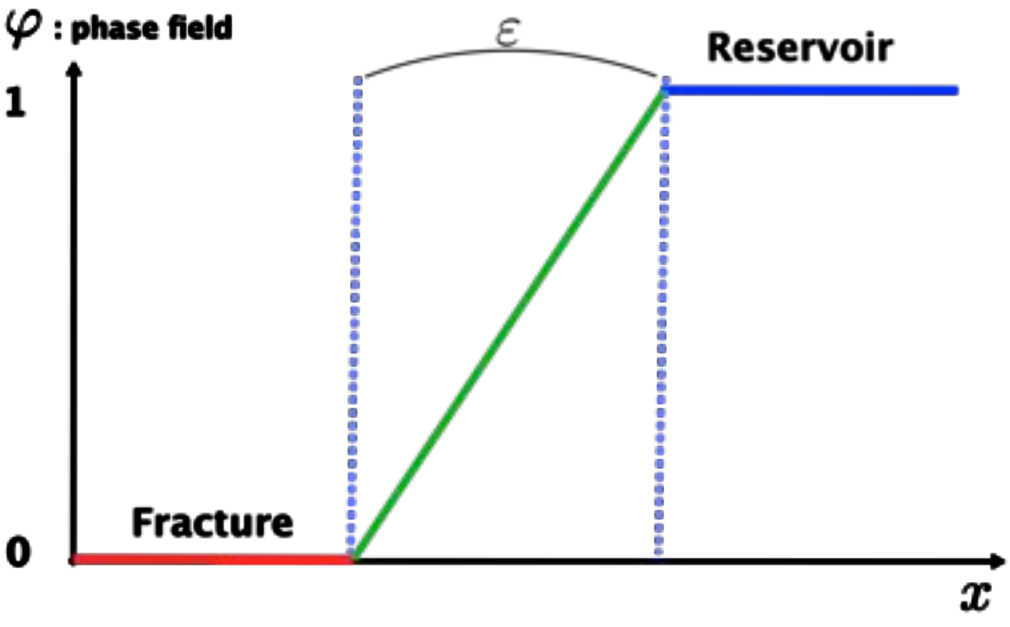
Phase Field Approach

What is a phase field function?

a) Real fracture b) Interface approach



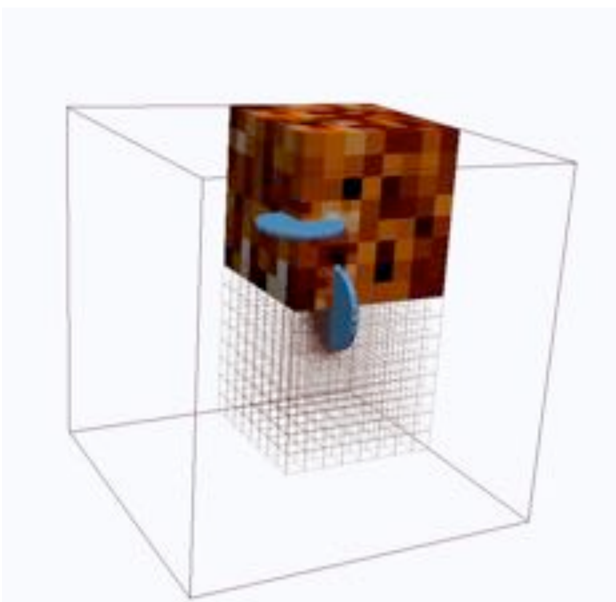
c) Diffusive approach using phase field



Phase Field Approach

What is a phase field function?

Objectives	Develop a multi-physics and fluid-flow driven **fracture propagation** method employing the phase field approach in a **poroelastic medium**



Variational methods based on energy minimization

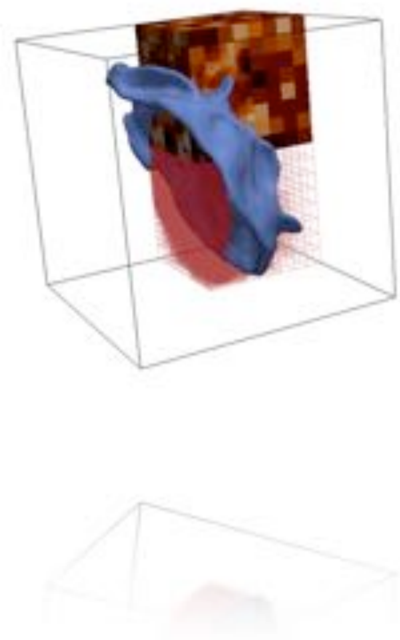
- Classical Theory of Fracture Propagation
  - Griffith 1921, Irwin 1958, Barenblatt 1962.
- Diffusive Crack Zones for Discontinuity Problems
  - Γ- convergent approximations
  - Mumford-Shah 1985, Ambrosio-Tortorelli 1992, Giacomini 2003
- Geomechanics Modeling
  - Francfort-Marigo 1998, Bourdin 2007
- Phase Field for Fractures
  - Hakim-Karma 2007, Miehe et al. 2010

Demonstrate the potential for treating practical reservoir engineering applications

Governing System

Governing System

with Phase Field



Governing System

Energy Functional

[Francfort-Marigo 1998] [Bourdin-Francfort-Marigo 1998]

$$E(\mathbf{u}, \mathcal{C}) = \int_{\Omega} \frac{1}{2} \mathcal{G}e(\mathbf{u}) : e(\mathbf{u}) \, dx - \int_{\partial\Omega} \tau \cdot \mathbf{u} \, dS + G_c \mathcal{H}^{d-1}(\mathcal{C})$$

Variables

- $\Omega = \Lambda \backslash \bar{\mathcal{C}}$, $\mathbf{u}$: displacement
- Gassman's tensor, $\mathcal{G}e(\mathbf{u}) := 2Ge(\mathbf{u}) + \lambda \text{tr}(e(\mathbf{u}))I$
- $e(\mathbf{u}) := \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$, G, λ: Lamé coefficients
- τ : tractional force
- G_c : Griffith's Criterion (fracture toughness)
- $\mathcal{H}^{d-1}(\mathcal{C})$: Hausdorff measure of the crack

Governing System

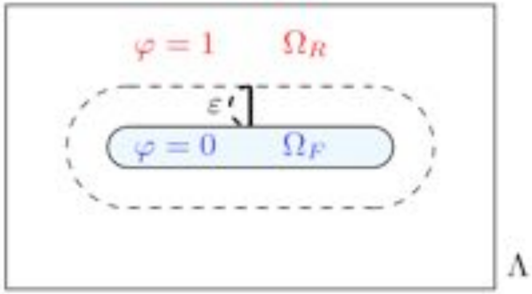
Energy Functional [Francfort-Marigo 1998] [Bourdin-Francfort-Marigo 1998]

$$E(\mathbf{u},\mathcal{C}) = \int_{\Omega} \frac{1}{2}\mathcal{G}e(\mathbf{u}) : e(\mathbf{u})\, dx - \int_{\partial\Omega} \tau \cdot \mathbf{u}\, dS + \boxed{G_c \mathcal{H}^{d-1}(\mathcal{C})}$$

Global Constitutive Dissipation Functional [Ambrosio-Tortorelli 1992] [Hofacker-Miehe-Welschinger 2013]

$$E_\varepsilon(\mathbf{u},\varphi) = \int_{\Lambda} \frac{1}{2}((1-k)\varphi^2 + k)\mathcal{G}e(\mathbf{u}) : e(\mathbf{u})\, dx - \int_{\partial\Lambda} \tau \cdot \mathbf{u}\, dS$$
$$+ \boxed{G_c \int_{\Lambda} \left(\frac{1}{2\varepsilon}(1-\varphi^2) + \frac{\varepsilon}{2}|\nabla\varphi^2| \right) dx}$$

- $\Omega = \Lambda \backslash \bar{\mathcal{C}}$
- $\varphi \in [0,1]$: phase field.
- ε: length of the phase field zone,
- k : positive regularization parameter $k \ll \varepsilon$.



Governing System

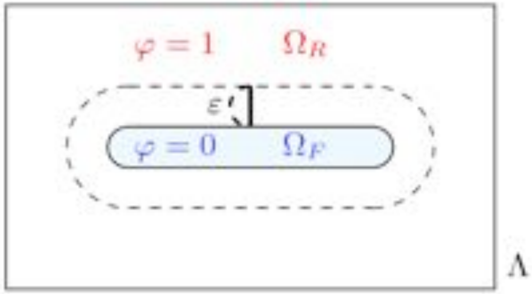
Energy Functional [Francfort-Marigo 1998] [Bourdin-Francfort-Marigo 1998]

$$E(\mathbf{u}, \mathcal{C}) = \int_\Omega \frac{1}{2}\mathcal{G}e(\mathbf{u}) : e(\mathbf{u})\, dx - \int_{\partial\Omega} \tau \cdot \mathbf{u}\, dS + G_c \mathcal{H}^{d-1}(\mathcal{C})$$

Global Constitutive Dissipation Functional [Bourdin et al. 2013] [Mikelic-Wheeler-Wick 2014]

$$E_\varepsilon(\mathbf{u}, p, \varphi) = \int_\Lambda \frac{1}{2}((1-k)\varphi^2 + k)\mathcal{G}e(\mathbf{u}) : e(\mathbf{u})\, dx - \int_{\partial\Lambda} \tau \cdot \mathbf{u}\, dS$$
$$- \int_\Lambda (\alpha - 1)\varphi^2 p \operatorname{div} \mathbf{u}\, dx + \int_\Lambda (\varphi^2 \nabla p)\mathbf{u}\, dx + G_c \int_\Lambda \left(\frac{1}{2\varepsilon}(1-\varphi^2) + \frac{\varepsilon}{2}|\nabla \varphi^2| \right) dx$$

- $\alpha \in (0,1)$: Biot Coefficient, p : pressure,
- $\alpha = 0$: pressurized, $\alpha = 1$: fluid filled.



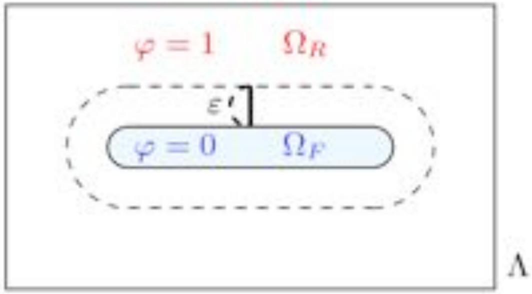
Governing System

Energy Functional (Biot) [Mikelic-Wheeler-Wick 2014] – continued from A. Mikelic's talk

$$E_\varepsilon(\mathbf{u},p,\varphi) = \int_\Lambda \frac{1}{2}((1-k)\varphi^2+k)\mathcal{G}e(\mathbf{u}) : e(\mathbf{u})\,dx - \int_{\partial\Lambda} \tau\cdot\mathbf{u}\,dS$$

$$-\int_\Lambda (\alpha-1)\varphi^2 p \,\mathrm{div}\,\mathbf{u}\,dx + \int_\Lambda (\varphi^2\nabla p)\mathbf{u}\,dx + G_c\int_\Lambda \left(\frac{1}{2\varepsilon}(1-\varphi^2)+\frac{\varepsilon}{2}|\nabla\varphi^2|\right)dx$$

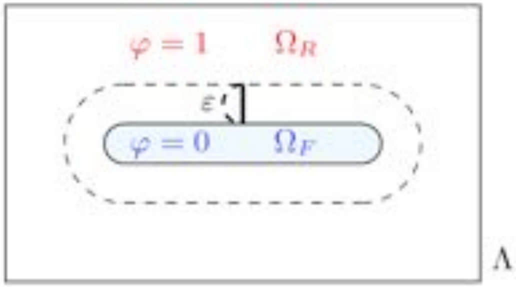
- $\partial_t\varphi \le 0$ (Irreversibility condition)
  - Simple Penalization
  - Augmented Lagrangian
  - Prime-dual Active Set Method [Hintermller et al. 2014]



Governing System

Pressure Diffraction System [Mikelic-Wheeler-Wick 2015]

- Lubrication approximation
- Flow in the non-fracture zone is also important.
- Slightly compressible fluid ($c_F \ll 1$)

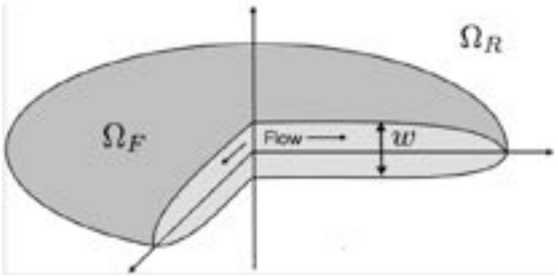


$$\varphi \left(\rho_R \partial_t \left(\frac{1}{M} p_R + \alpha \nabla \cdot \mathbf{u}\right) - \nabla \cdot \frac{K_R \rho_R}{\eta_R} (\nabla p_R - \rho_R \mathbf{g}) = q_R \right) \quad \text{in} \quad \Omega_R(t) \times (0, T]$$

$$(1 - \varphi) \left(\rho_F \partial_t (c_F p_F) - \nabla \cdot \frac{K_F \rho_F}{\eta_F} (\nabla p_F - \rho_F \mathbf{g}) = q_F - q_L \right) \quad \text{in} \quad \Omega_F(t) \times (0, T]$$

Variables

- Ω_R: Reservoir domain, Ω_F: Fracture domain
- M: Biot modulus, $\alpha \in (0, 1)$: Biot Coefficient,
- $\rho_{F,R}$: fluid density, $K_{F,R}$: permeability, $\eta_{F,R}$: fluid viscosity
- $q_{F,R}$: source/sink term [Peaceman 1978],
- q_L : leakage term
- Fluid compressibility coefficient $c_F \ll 1$, slightly compressible



Governing System

Pressure Diffraction System
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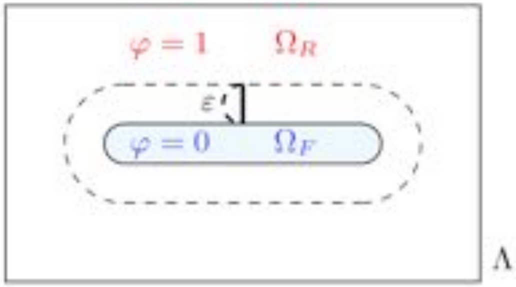

$$\varphi \left(\rho_R \partial_t \left(\frac{1}{M} p_R + \alpha \nabla \cdot \mathbf{u}\right) - \nabla \cdot \frac{K_R \rho_R}{\eta_R} (\nabla p_R - \rho_R \mathbf{g}) = q_R \right) \quad \text{in} \quad \Omega_R(t) \times (0, T]$$

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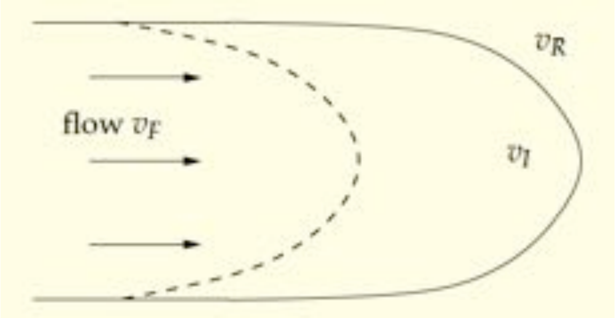
Interface conditions:

$$p_R = p_F$$

$$\frac{K_R \rho_R}{\eta_R} (\nabla p_R - \rho_R \mathbf{g}) \cdot \mathbf{n} = \frac{K_F \rho_F}{\eta_F} (\nabla p_F - \rho_F \mathbf{g}) \cdot \mathbf{n}$$



Leak off

$$v_R \cdot \mathbf{n} = v_I \cdot \mathbf{n} = v_F \cdot \mathbf{n}$$

Governing System

Width Computation using a Level Set

[Nguyen et al. 2016] [L.-Wheeler-Wick 2016 JCAM]

$$K_F = \frac{w(\mathbf{u})^2}{12}$$, $w(\mathbf{u}) \approx 2\mathbf{u} \cdot \mathbf{n}_F$: crack openning displacement (width).

Definition of a level set function

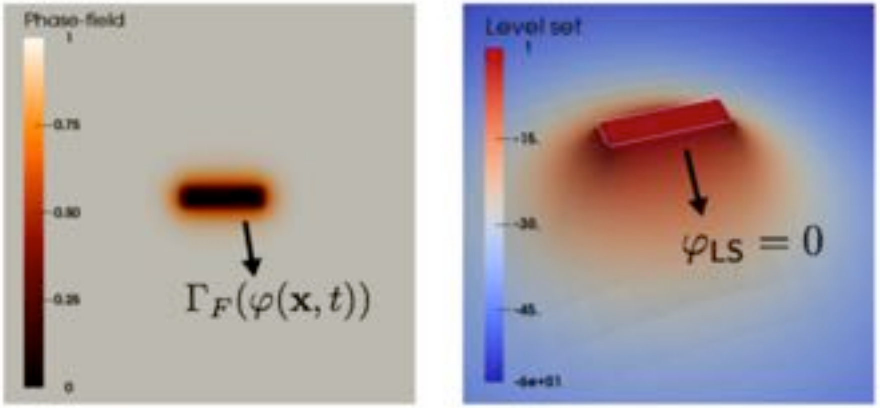
$$\varphi_{\mathrm{LS}} > 0, \quad \mathbf{x} \in \Omega_R, \; \Omega_R := \{\mathbf{x} \in \Lambda \mid \varphi(\mathbf{x}, t) > C_{LS}\}$$

$$\varphi_{\mathrm{LS}} < 0, \quad \mathbf{x} \in \Omega_F, \; \Omega_F := \{\mathbf{x} \in \Lambda \mid \varphi(\mathbf{x}, t) < C_{LS}\}.$$

$$\varphi_{\mathrm{LS}} = 0, \quad \mathbf{x} \in \Gamma_F, \; \Gamma_F := \{\mathbf{x} \in \Lambda \mid \varphi(\mathbf{x}, t) = C_{LS}\}, \quad C_{LS} \in (0, 1) \quad C_{LS} = 0.1$$

$\varphi_{\mathrm{LS}} = \varphi - C_{LS}$. ▸ $\Gamma_F := \{\mathbf{x} \in \Lambda \mid \varphi_{\mathrm{LS}}(\mathbf{x}, t) = 0\}$

$$w(\mathbf{u}) \approx -2\mathbf{u} \cdot \frac{\nabla \varphi_{\mathrm{LS}}}{\|\nabla \varphi_{\mathrm{LS}}\|} \text{ on } \Gamma_F,$$



(a) Phase field φ.

(b) Level set φ_{LS}

Governing System

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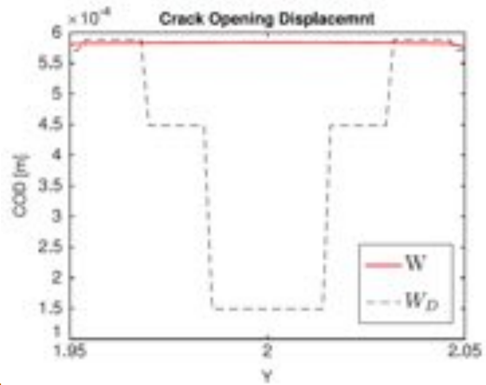

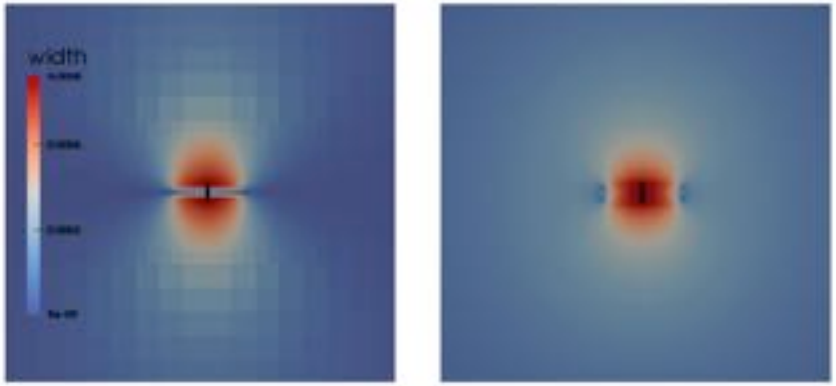

a) Width by level set (WD) b) Interpolated width (W)

Governing System

Width Computation using a Level Set [Nguyen et al. 2016] [L.-Wheeler-Wick 2016 JCAM]

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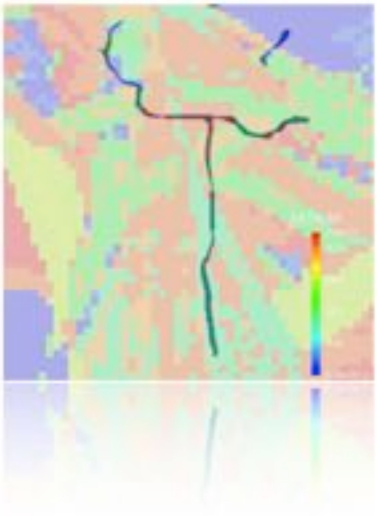
$$w(\mathbf{u}) \approx -2\mathbf{u} \cdot \frac{\nabla \varphi_{LS}}{\|\nabla \varphi_{LS}\|} \text{ on } \Gamma_F,$$

- Power-law lubrication Flow in Ω_F: [L.-Mikelic-Wheeler-Wick 2016 CMAME]

$$\rightarrow \mathbf{v}_F = \frac{K_F \rho_F}{\eta_F^{1/n}} |\mathbf{f} - \nabla p_F|^{\frac{1}{n}-1} (\mathbf{f} - \nabla p_F)$$

Coupling Algorithms

for fracture propagation



A Monolithically-Coupled Formulation : Resulting Euler-Lagrange System

- **Displacement-Phase Field : a monolithically-coupled formulation**
  - High accuracy of coupling conditions. [Gerasimov/Lorenzis; 2016, CMAME]
  - Numerical stability and implicit discretization. [Mikelic-Wheeler-Wick 2014-5]

Let $W_{in} := \{w \in H^1(\Lambda) | w \leq \varphi^{old} \leq 1 \text{ a.e on } \Lambda\}$ and let $p \in H^1(\Lambda)$ be given.
Find $\{\mathbf{u}, \varphi\} \in V \times W$ such that

$$\big(((1-k)\tilde{\varphi}^2 + k)\mathcal{G}^+ e(\mathbf{u}), e(\mathbf{w})\big) + (\mathcal{G}^- e(\mathbf{u}), e(\mathbf{w})) \\ - (\alpha - 1)(\varphi^2 p, \nabla \cdot \mathbf{w}) + (\varphi^2 \nabla p, \mathbf{w}) = 0, \ \forall \mathbf{w} \in V$$

$$(1-k)(\varphi \mathcal{G}^+ e(\mathbf{u}) : e(\mathbf{u}), \psi - \varphi) - 2(\alpha - 1)(\varphi p \nabla \cdot \mathbf{u}, \psi) + 2(\varphi \nabla p \mathbf{u}, \psi) \\ + G_c \left(-\frac{1}{\varepsilon}(1-\varphi, \psi - \varphi) + \varepsilon(\nabla\varphi, \nabla(\psi - \varphi)) \right) \geq 0, \forall \psi \in W_{in} \cap L^{\infty(\Lambda)}$$

- **Displacement-Phase Field–and pressure : ** Fully coupled. previous talk by A.Mikelic

[Mikelic-Wheeler-Wick 2016]

=> Sequential iterative coupling

Coupling Algorithms

Iterative Coupling : Fixed Stress Iteration

[Settari et al. 1998] [Chin et al. 2002] [Gai 2004] [Deen et el. 2006] [Kim et al. 2009, 11] [Castelletto et al. 2015] more..

- For each time t^n

Step 1. Flow Equation

Impose constant volumetric mean total stress :

$$\sigma_v = \sigma_{v,0} + K_{dr}\nabla\cdot(\mathbf{u}-\mathbf{u}^0) - \alpha(p-p_0).$$

$$K_{dr} = \frac{2E\nu}{(1+\nu)(1-2\nu)}$$

$$\left(\frac{1}{M} + \frac{\alpha^2}{K_{dr}}\right)\partial_t p_R^{l+1} - \nabla\cdot\frac{K_R}{\eta_R}(\nabla p_R^{l+1} - \rho_R \mathbf{g})$$

$$= q_R - \alpha\nabla\cdot\partial_t \mathbf{u}^l + \frac{\alpha^2}{K_{dr}}\partial_t p_R^l \quad \text{in} \quad \Omega_R(t)\times(0,T]$$

$$(c_F + \gamma_{dr})\partial_t p_F^{l+1} - \nabla\cdot\frac{K_F}{\eta_F}(\nabla p_F^{l+1} - \rho_F \mathbf{g})$$

$$= q_F - q_L + \gamma_{dr}\partial_t p_F^l \quad \text{in} \quad \Omega_F(t)\times(0,T]$$

Step 2. Mechanics

$$\min E_\varepsilon(\mathbf{u}^{l+1}, p^{l+1}, \varphi^{l+1}) \quad \text{respect to } \partial_t\varphi^{l+1} \le 0 \quad \text{in } \Lambda(t)\times(0,T]$$

Coupling Algorithms

Contraction convergence analysis:

- Biot : [Mikelic-Wheeler 2014]
- With a fracture (interface) : [Kumar-Almani-Girault-Wheeler 2015]
- With a fixed phase field fracture : [Almani-L.-Wheeler-Wick SPE RSC 182610-MS 2017]

Fixed stress algorithm with a fixed phase field

The scheme contracts on the L_2 norm of following composite quantity:

$$\delta\sigma_v^{n,k} := \delta p_h^{n,k} - \frac{\theta_\alpha}{\tilde{L}} \nabla \cdot \delta u_h^{n,k}$$

Theorem 1. (Banach fixed point iteration) Contraction Result:

$$\left\| \delta\sigma_v^{n+1,k} \right\|^2 \leq \left(\frac{C}{1+C}\right)\left(\frac{\tilde{L}}{\theta+\tilde{L}}\right)^2 \left\| \delta\sigma_v^{n,k} \right\|^2, \qquad \left(\frac{C}{1+C}\right)\left(\frac{\tilde{L}}{\theta+\tilde{L}}\right) < 1$$

=> Phase field is involved as a heterogeneous coefficient.

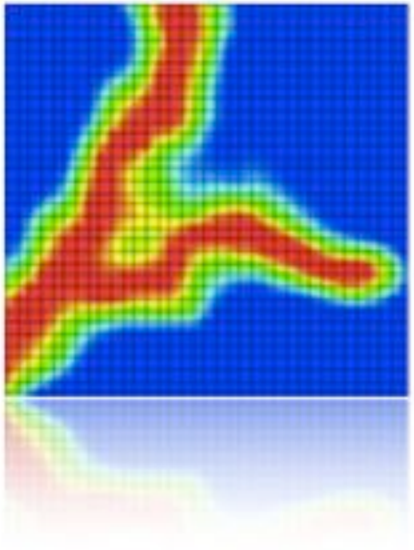
n : iteration number, k : time step number,

$$\theta_\alpha = \chi_{\Omega_R}\alpha \quad \tilde{L} = \frac{\alpha\theta_\alpha}{2\lambda} \quad \theta = \chi_{\Omega_R}\frac{1}{M} + \chi_{\Omega_F} c_F$$

Advantages

Advantages

Why Phase Field ?



Advantages of Phase Field Approach

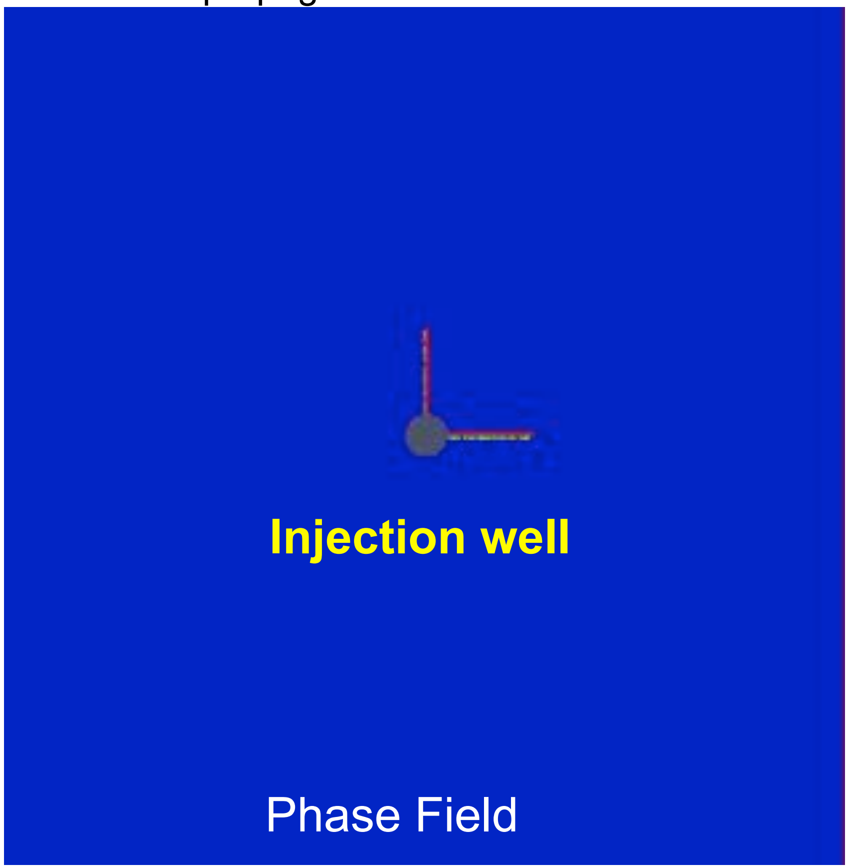
- 1. **Crack propagation paths are automatically determined by energy minimization.**
- 2. Joining and branching of non-planar cracks are simple to handle for two/three dimension.
- 3. Computationally robust: verifications and validations are available.
- 4. Multiscale and multiphysical extensions are possible.

Advantages of Phase Field Approach

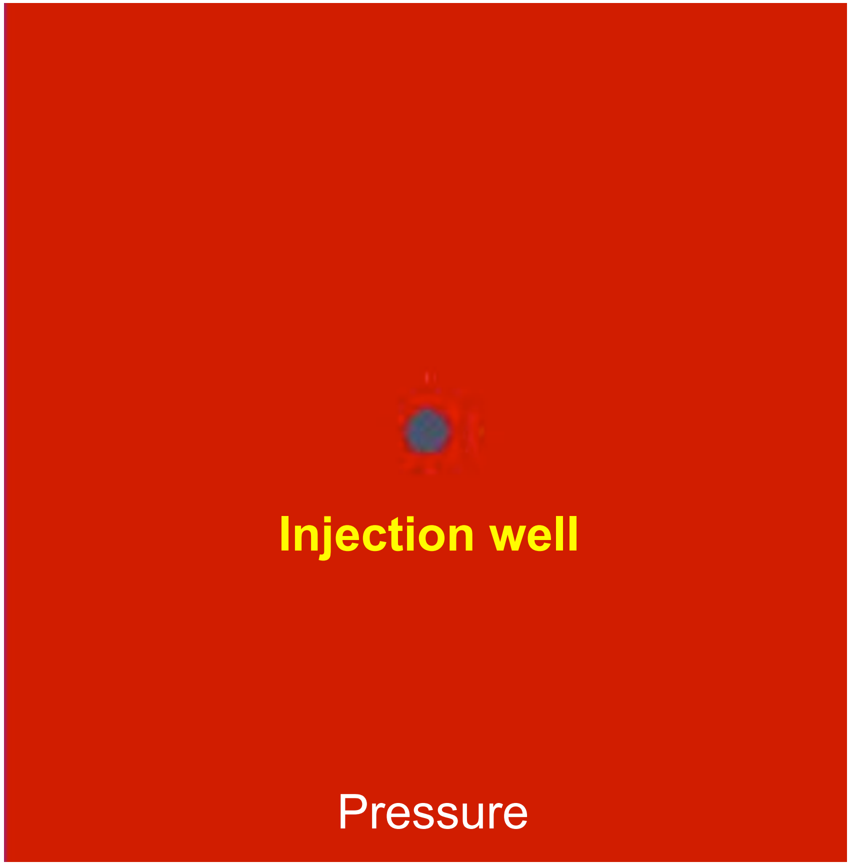
1. Crack propagation and the path

Multiple fluid filled fracture propagation **[L.-Wheeler-Wick 2016]**

- Fracture propagation



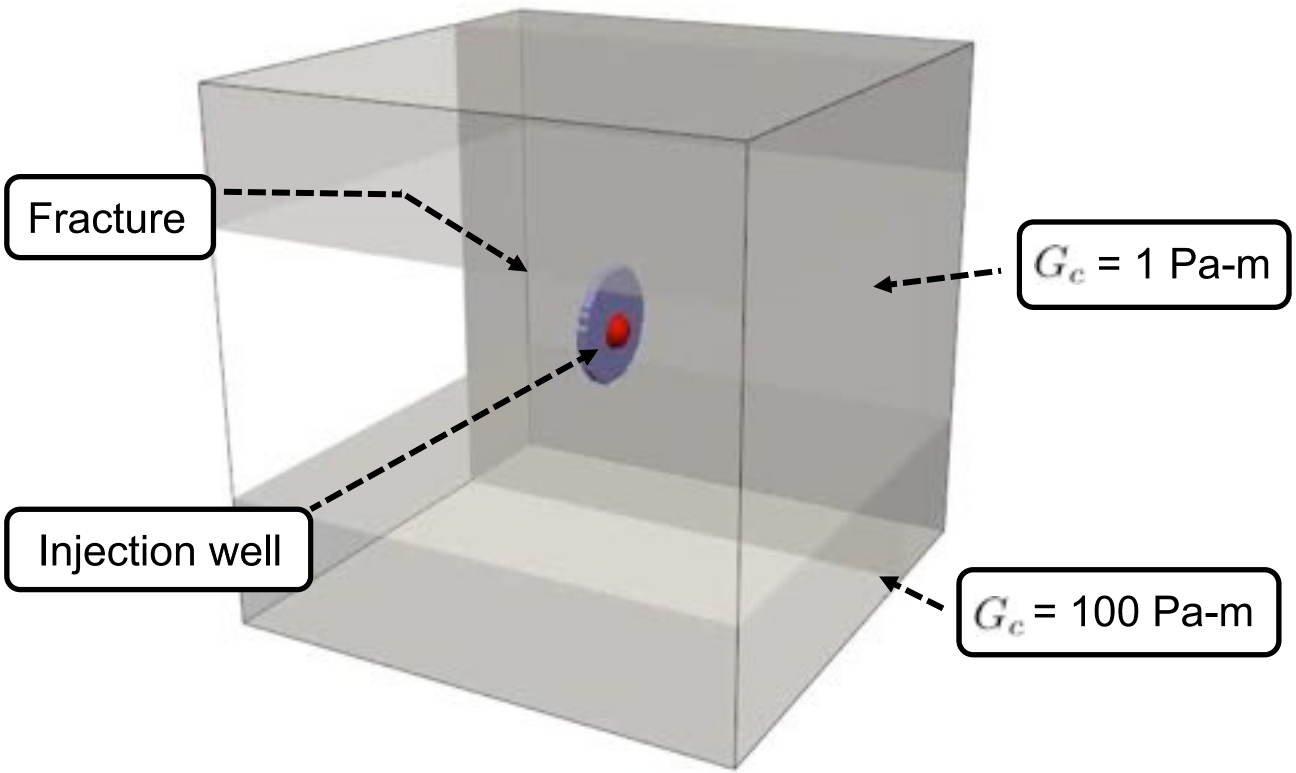
- Pressure distribution



Advantages of Phase Field Approach

1. Crack propagation and the path

Fluid Filled Penny Shape Fracture in a 3D Layered Media [L.-Wheeler-Wick 2016]



Advantages of Phase Field Approach

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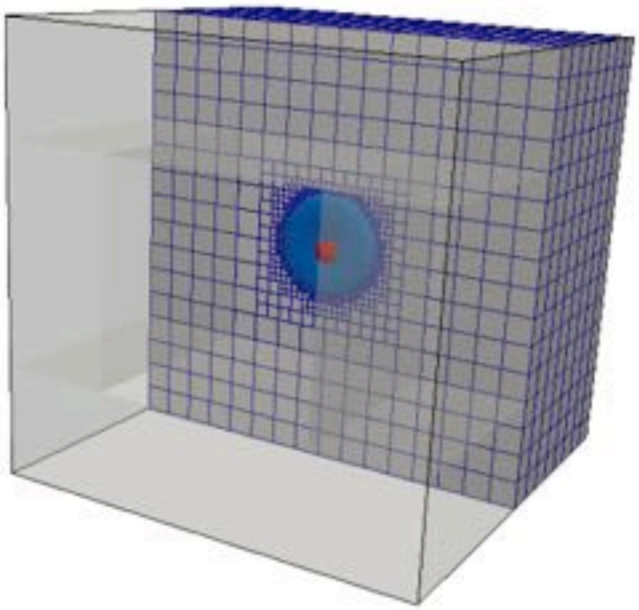
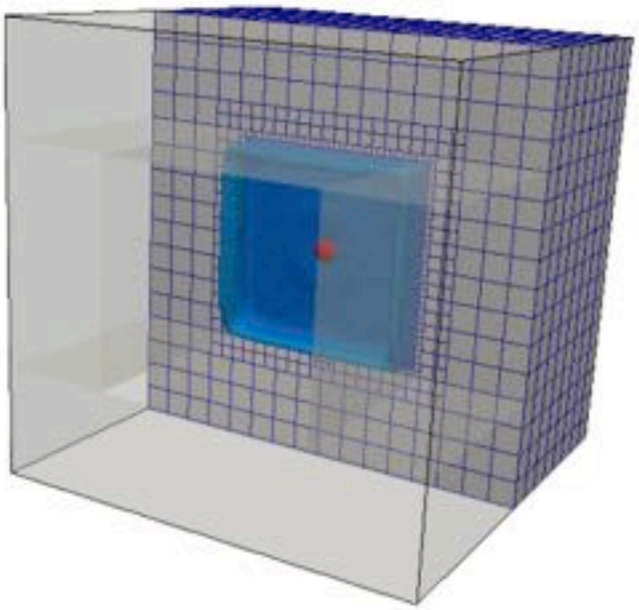


(a) k=1



(f) k=500

- Predictor-corrector dynamic mesh adaptivity [Heister-Wheeler-Wick 2015] [L.-Wheeler-Wick 2015]

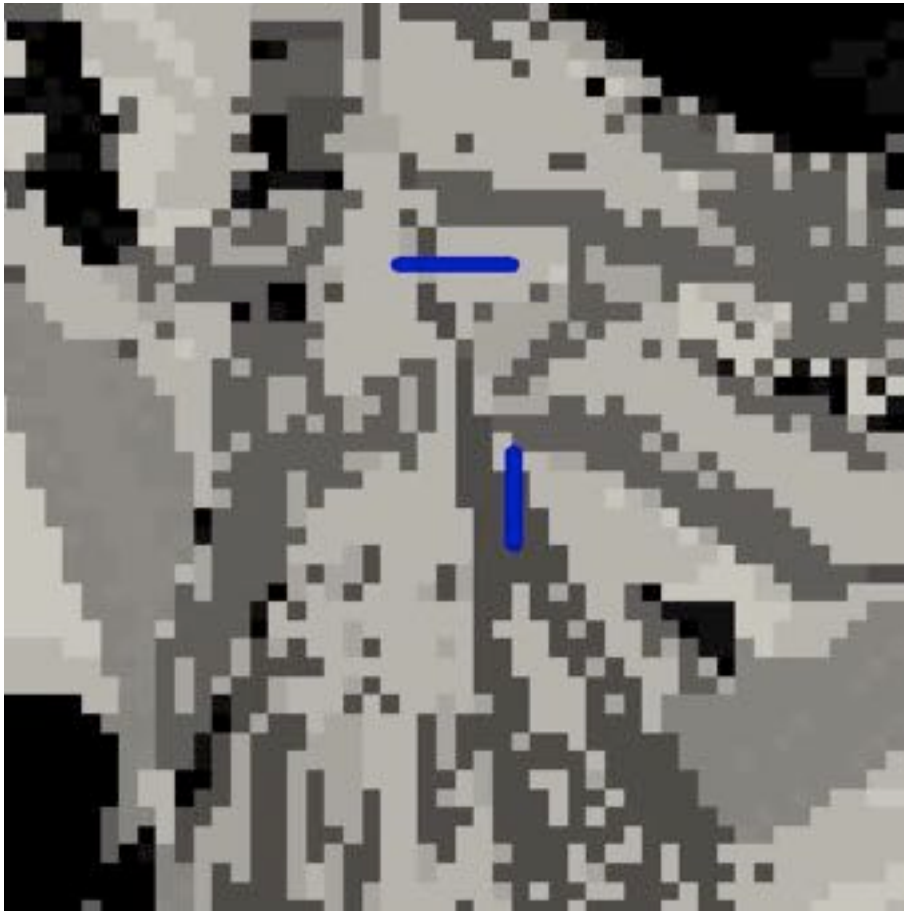
Advantages of Phase Field Approach

- ❑ 1. Crack propagation paths are automatically determined by energy minimization.
- ❑ **2. Joining and branching of non-planar cracks are simple to handle for two/three dimension.**
- ❑ 3. Computationally robust: verifications and validations are available.
- ❑ 4. Multiscale and multi-physical extensions are possible.

Advantages of Phase Field Approach

2. Joining and branching of non-planar fractures

Pressurized Multiple Fractures

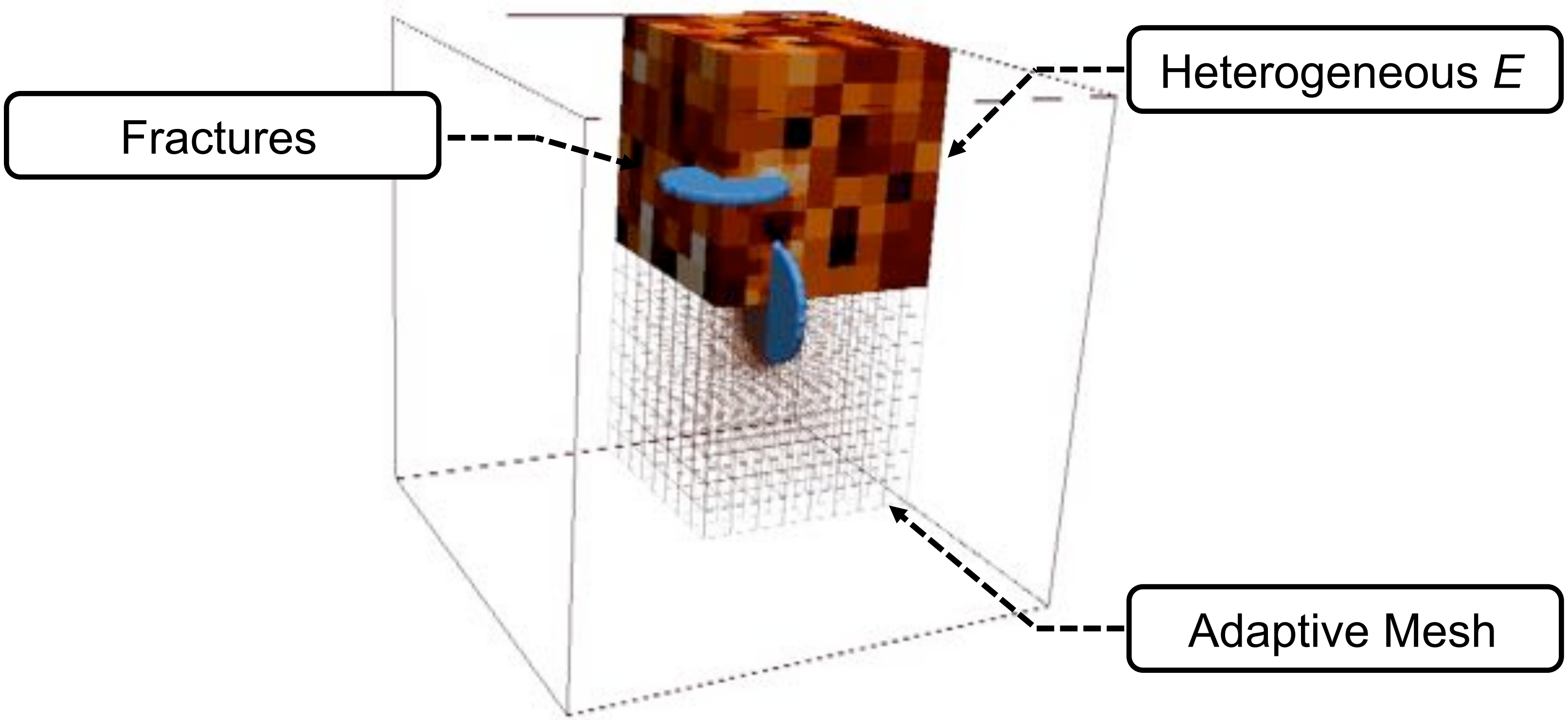


- Heterogeneous Young's modulus
- Data provided by Statoil
- Calculating stress intensity factors and remeshing along the crack path are embedded in the model. [Karma et al. 2001].

Advantages of Phase Field Approach

2. Joining and branching of non-planar fractures

Pressurized Multiple Penny Shape Fractures in a 3D Heterogeneous Media



Advantages of Phase Field Approach

2. Joining and branching of non-planar fractures

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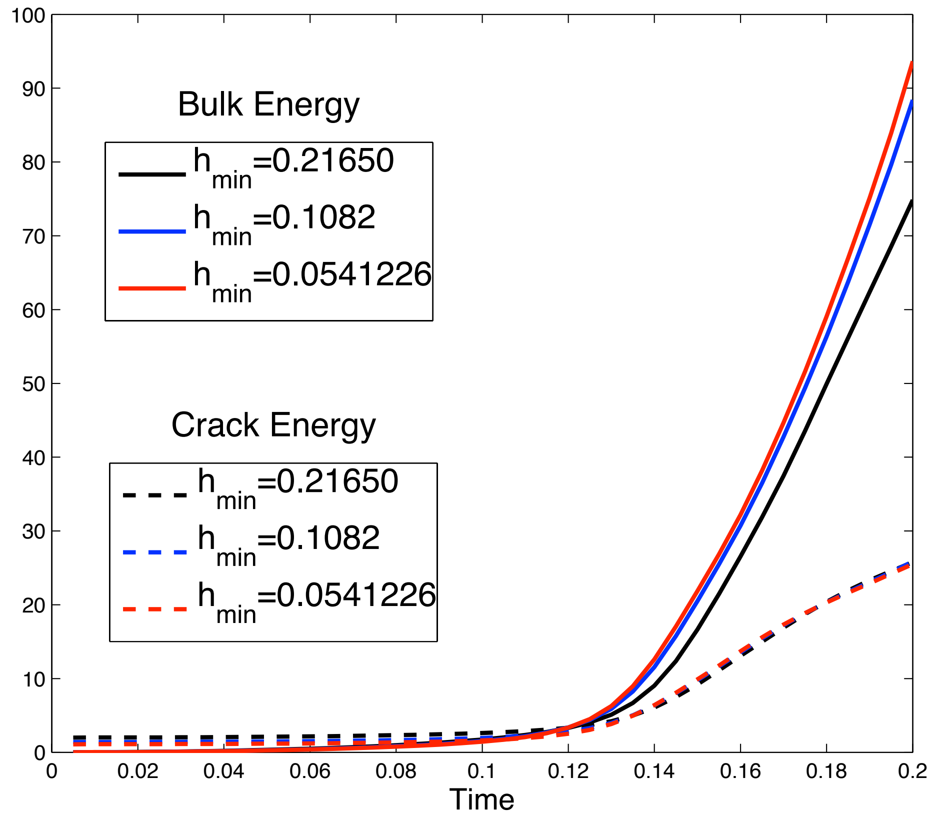
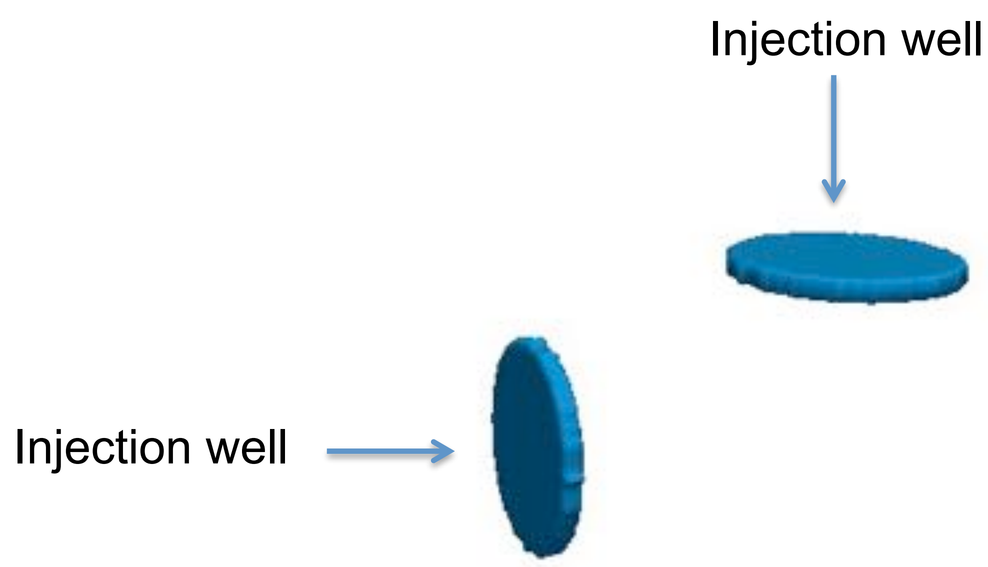


Advantages of Phase Field Approach

2. Joining and branching of non-planar fractures

Multiple Fluid Filled Penny Shape Fractures in a 3D Homogeneous Media



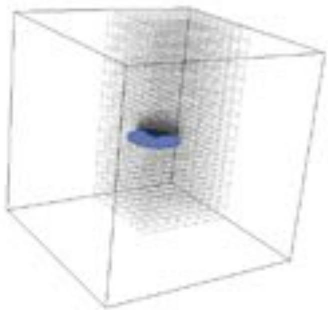
Advantages of Phase Field Approach

- ❑ 1. Crack propagation paths are automatically determined by energy minimization.
- ❑ 2. Joining and branching of non-planar cracks are simple to handle for two/three dimension.
- ❑ **3. Computationally robust: verifications and validations are available.**
- ❑ 4. Multiscale and multiphysical extensions are possible

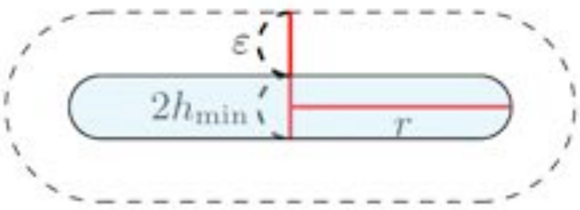
Advantages of Phase Field Approach

3. Validations – Pressurized Fractures

Sneddon's Test



(a) $\Lambda = (0, 10)^3$



(b) Center $(5, 5, 5)$, $\varepsilon = h_{\min}$

Crack Opening Displacement (COD) [Sneddon-Lowengrub 1969]

$$\mathbf{u}_y = \frac{4(1-\nu^2)pr}{\pi E} \approx \frac{1}{2}[\mathbf{u}\cdot\mathbf{n}^+].$$

$$V = \sqrt{\frac{64 G_c \pi r^5}{9E'}} \approx \frac{4}{3}\pi r^2 \mathbf{u}_y.$$

Here, $E = 1$, $\nu = 0.3$, $G_c = 1$.

Advantages of Phase Field Approach

3. Validations – Pressurized Fractures

Sneddon's Test

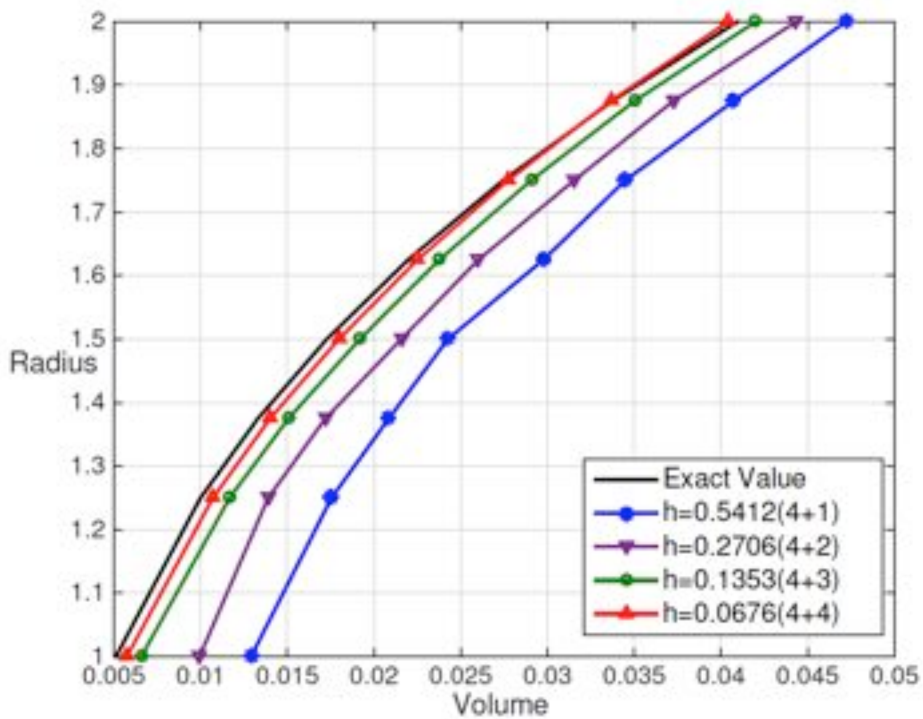


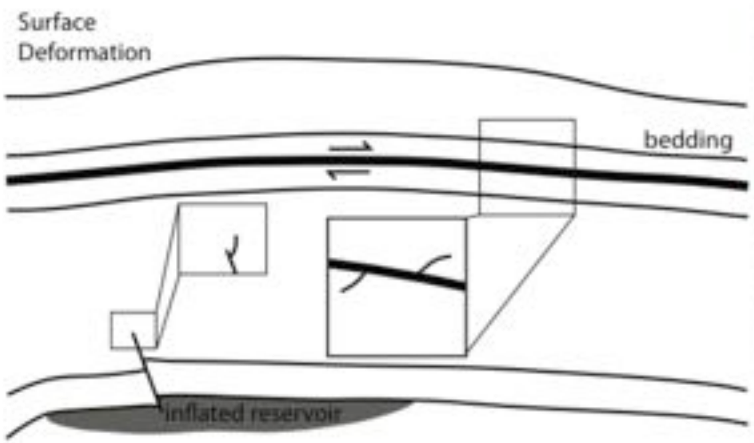
Figure: We observe the convergence of the value V under the spatial mesh refinement.

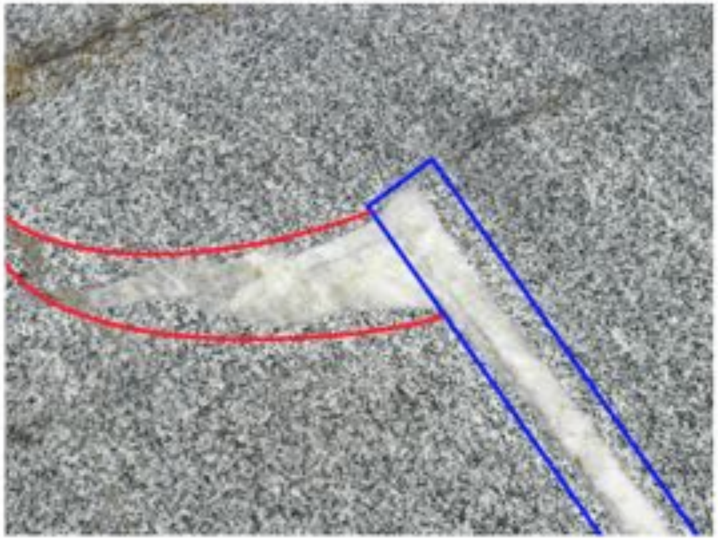
[Miehe et al 2012] [Gupta-Duarte 2014]

Advantages of Phase Field Approach

3. Validations − Elasticity

Wing crack formation compared with Gelatin experiments [L.-Reber-Hayman-Wheeler 2016 GRL]



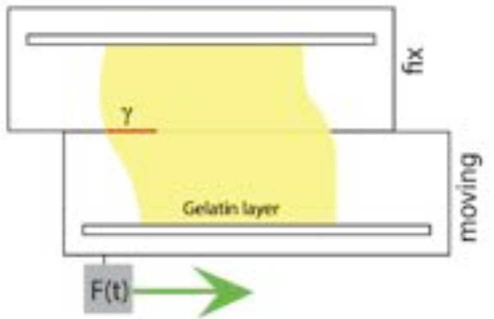


- Reservoir with leakage enhanced by wing crack formation around faults and flexed bedding planes [Verdon et al. 2013]
- Outcrop example of a wing crack, Lago Neves, Italy (courtesy Luc Lavier)

Advantages of Phase Field Approach

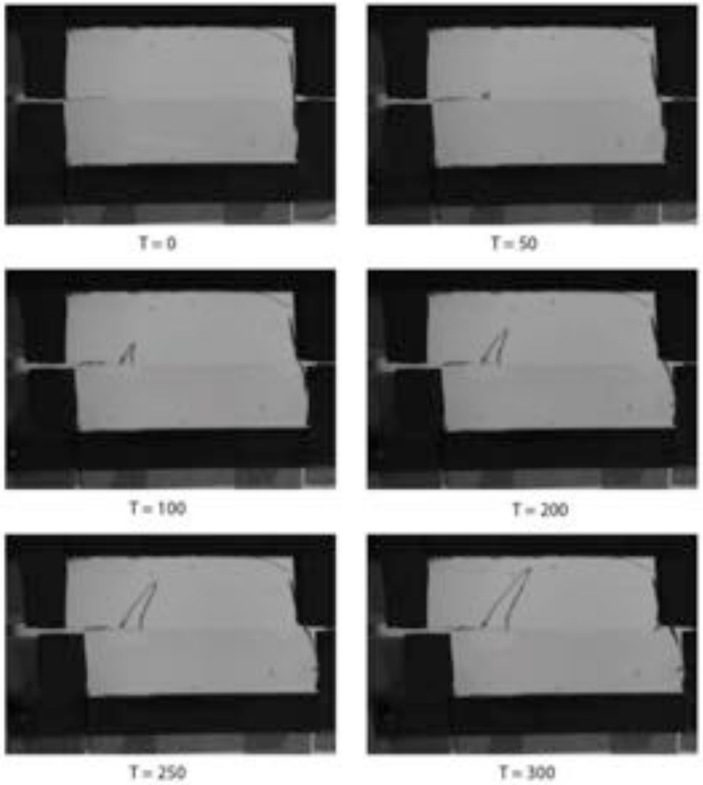
3. Validations – Elasticity

Wing crack formation compared with Gelatin experiments [L.-Reber-Hayman-Wheeler 2016 GRL]



- Great Lakes Gelatin, USA
- Mixed at 3 wt% with DI H20
- $E = 10^5$, $\mu = 0.48$
- $G_c = 1.96$, $p \equiv 0.$
- $\gamma = 30, 60, 90$mm

(a) Setup



(b) Experimental results $\gamma = 60$mm

Advantages of Phase Field Approach

3. Validations – Elasticity

Wing crack formation compared with Gelatin experiments [L.-Reber-Hayman-Wheeler 2016 GRL]

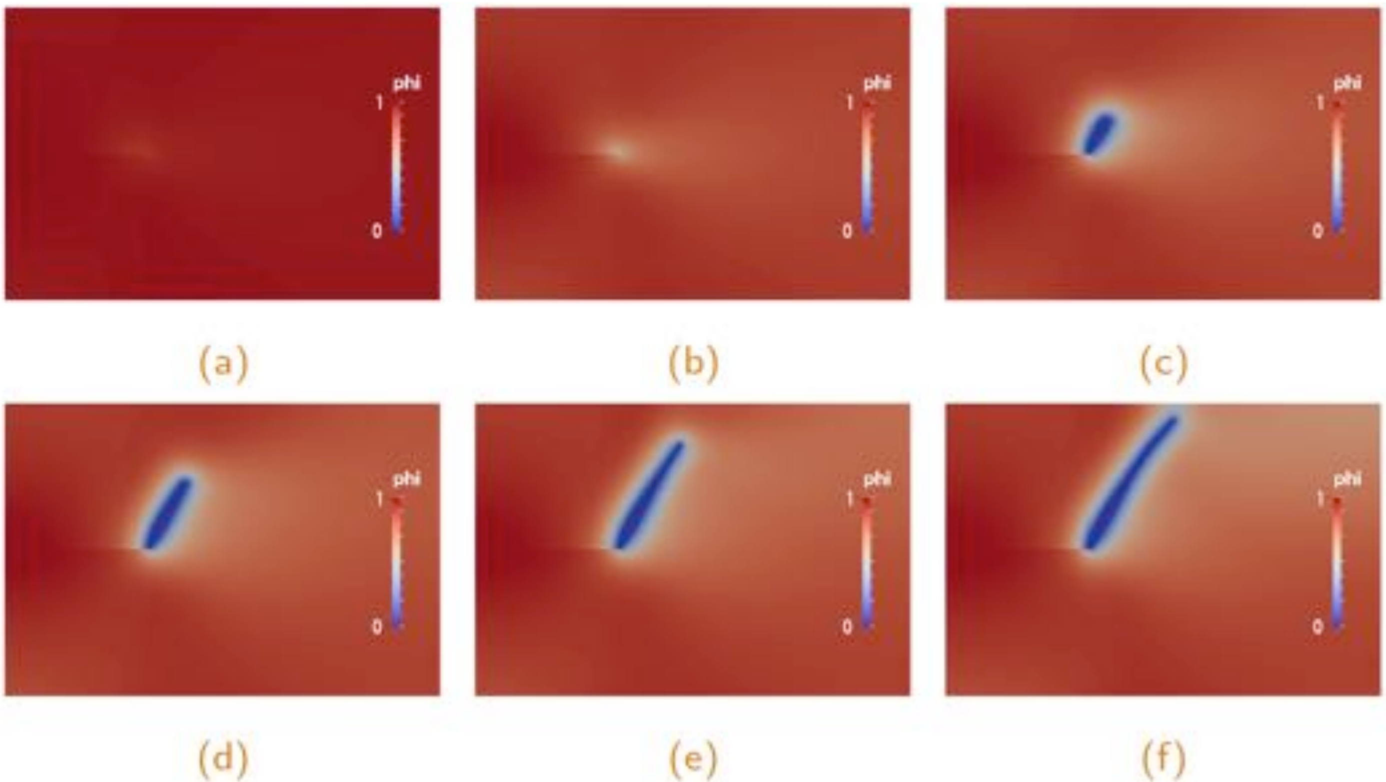


Figure: (From left to right top to bottom) red indicates the unbroken zone and the blue indicates the broken zone, fracture. $\gamma = 60$mm

Advantages of Phase Field Approach

3. Validations – Elasticity

Wing crack formation compared with Gelatin experiments [L.-Reber-Hayman-Wheeler 2016 GRL]

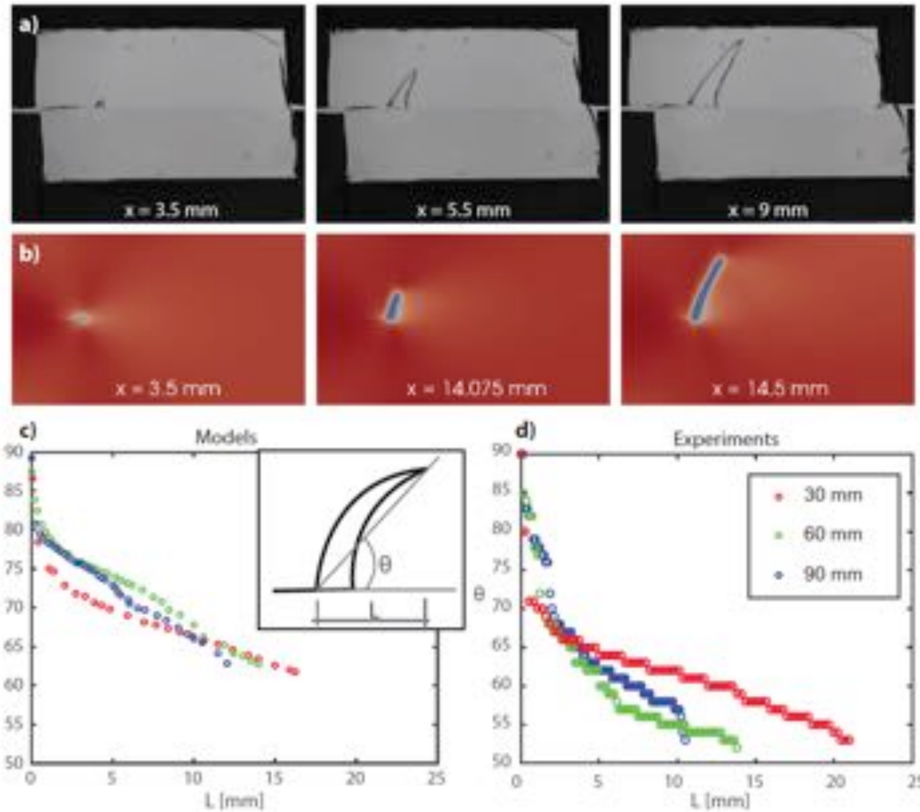
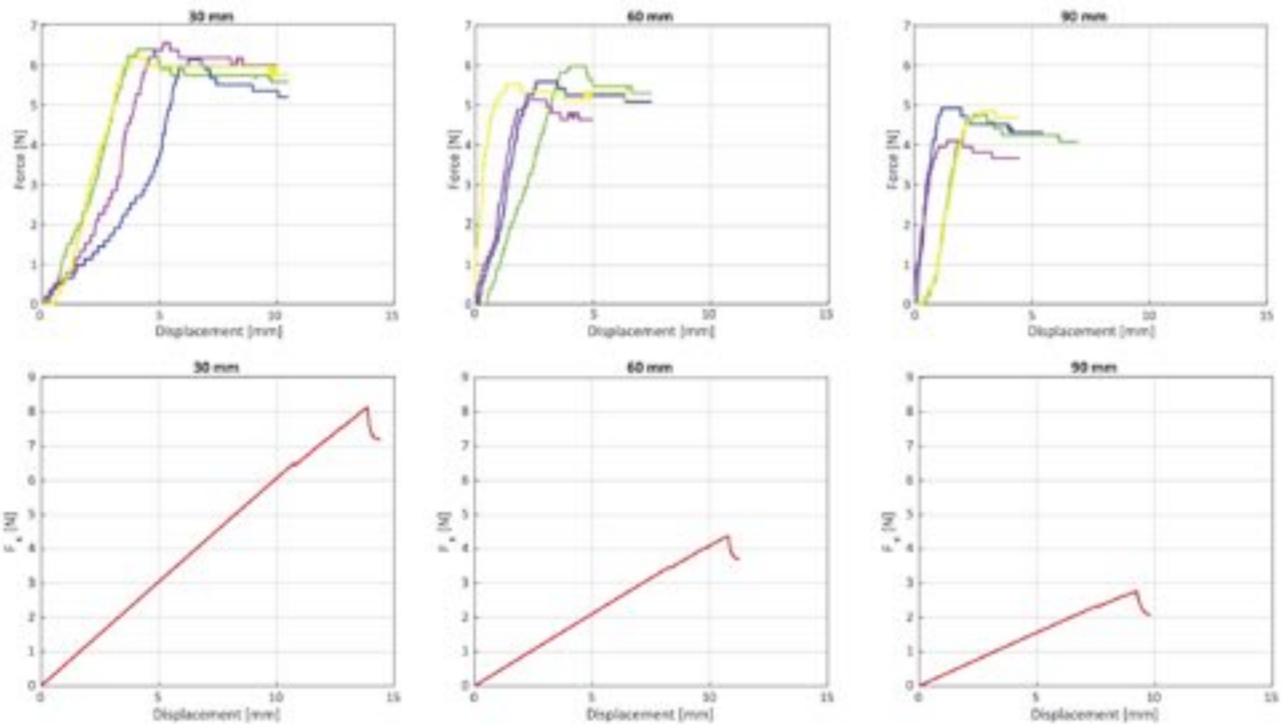


Figure: Comparison between the gel experiment and numerical simulation

Advantages of Phase Field Approach

3. Validations – Elasticity

Wing crack formation compared with Gelatin experiments [L.-Reber-Hayman-Wheeler 2016 GRL]



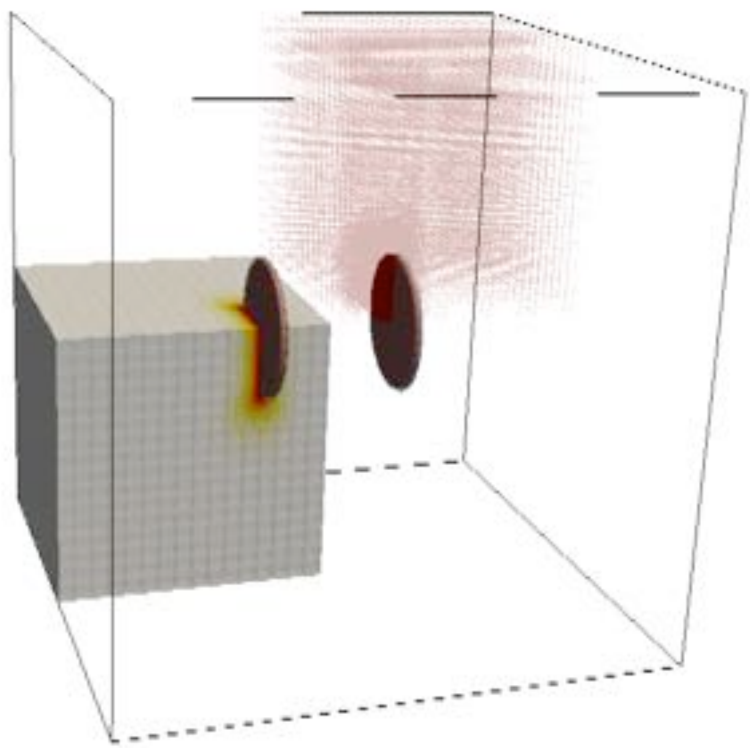
$$\gamma = 30, 60, 90\text{mm}$$

Figure: Comparison between the gel experiment and numerical simulation

Advantages of Phase Field Approach

3. Validations – stress shadowing effects

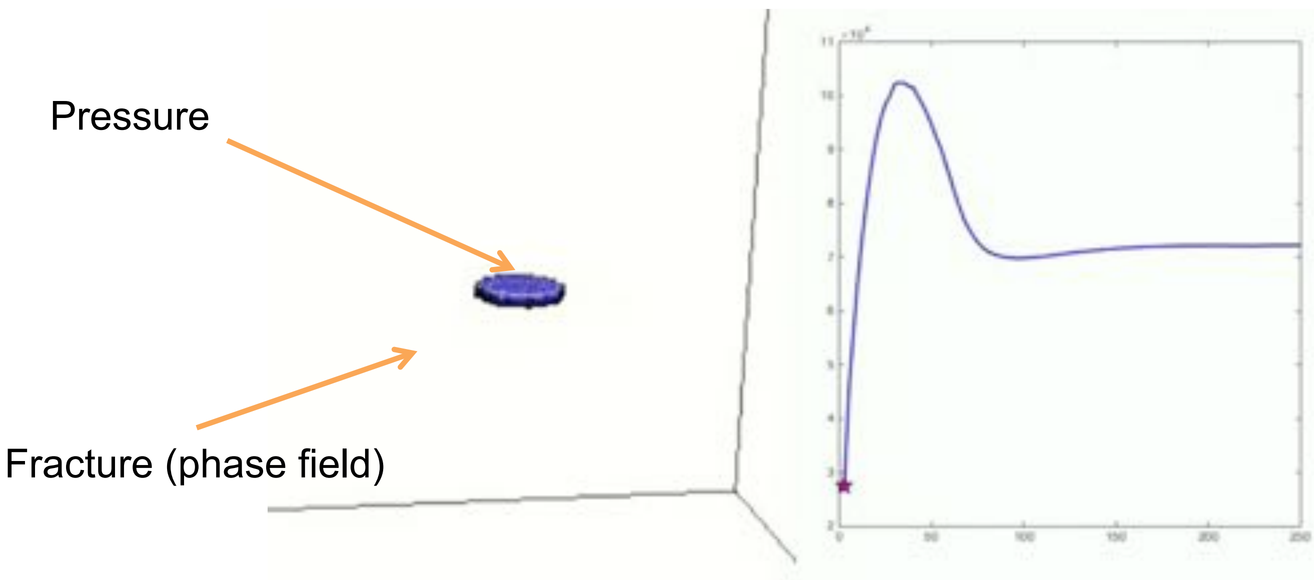
2 and 3 Parallel Fractures in 3D Domain [L.-Wheeler-Wick 2016]



Advantages of Phase Field Approach

3. Validations – pressure drops

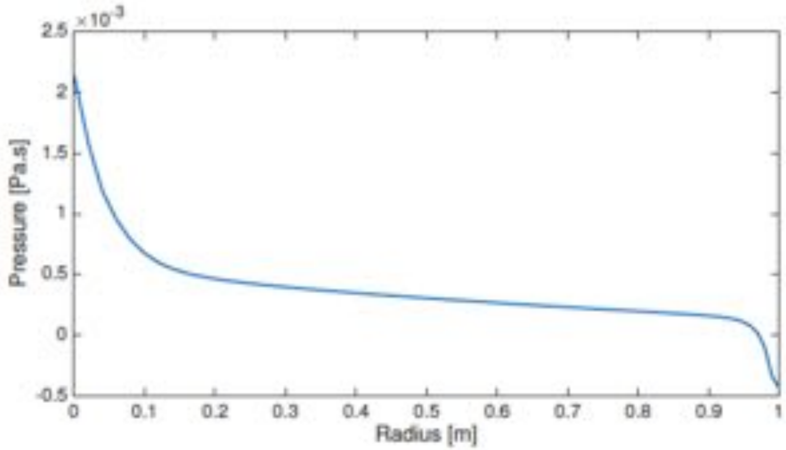
Fluid Filled Penny Shape Fracture [L.-Wheeler-Wick 2016]

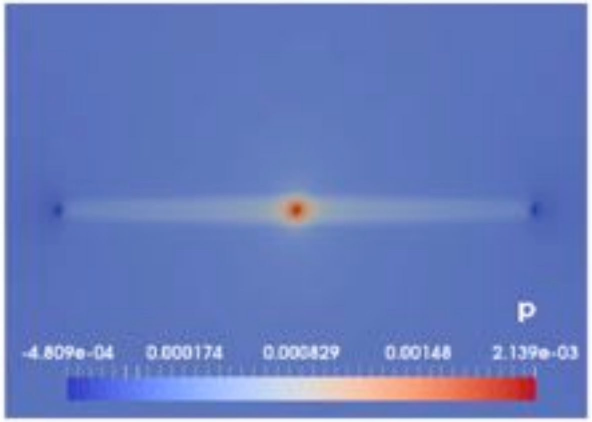


Advantages of Phase Field Approach

3. Validations

Fluid Lag : Negative pressure effect at the crack tip. **[L.-Wheeler-Wick 2016 JCAM]**





- [Savitski-Detournay 2002] [Lecampion et al 2013] [Lecampion et al 2017]
- We observe predicted negative pressure values at the beginning of an injection when the fracture is not yet completely filled with the fluid.

Advantages of Phase Field Approach

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- 3. Computationally robust: verifications and validations are available.
- **4. Multiscale and multiphysical extensions are possible.**

4. Initializing natural fractures with a probability map

Coupling with a probability map [L.-Wheeler-Wick-Srinivasan 2016]

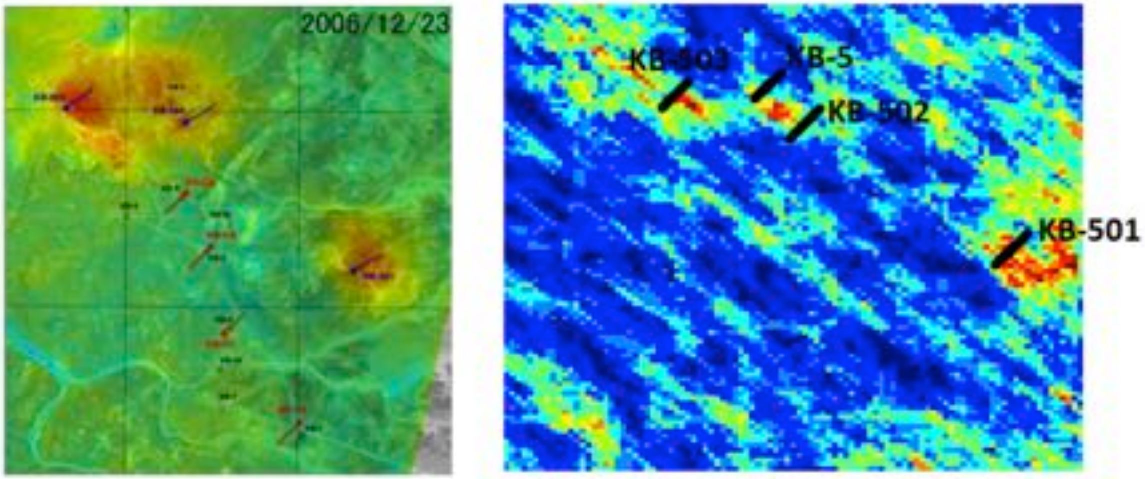


Figure: [Left] Surface deformation map (InSAR) at the In Salah. [Right] Generated posterior probability map of fractures based on the ensemble mean of the models in the selected cluster. [Nwachukwu 2015]

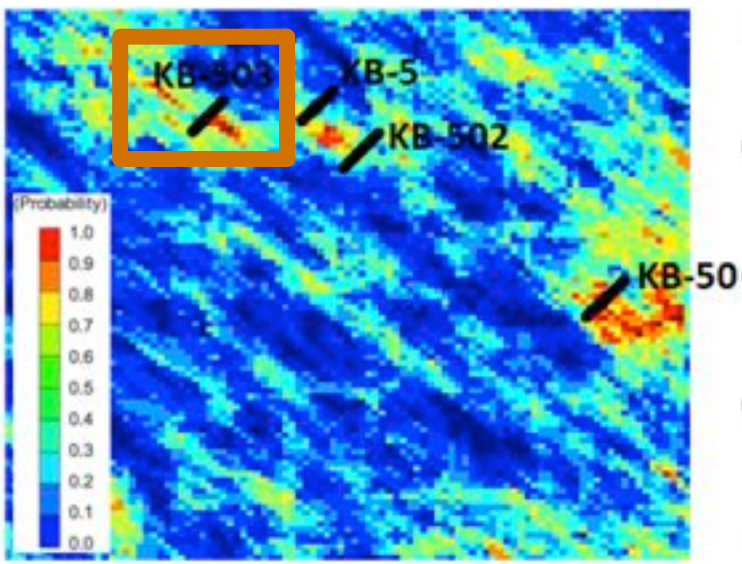
Basic Idea

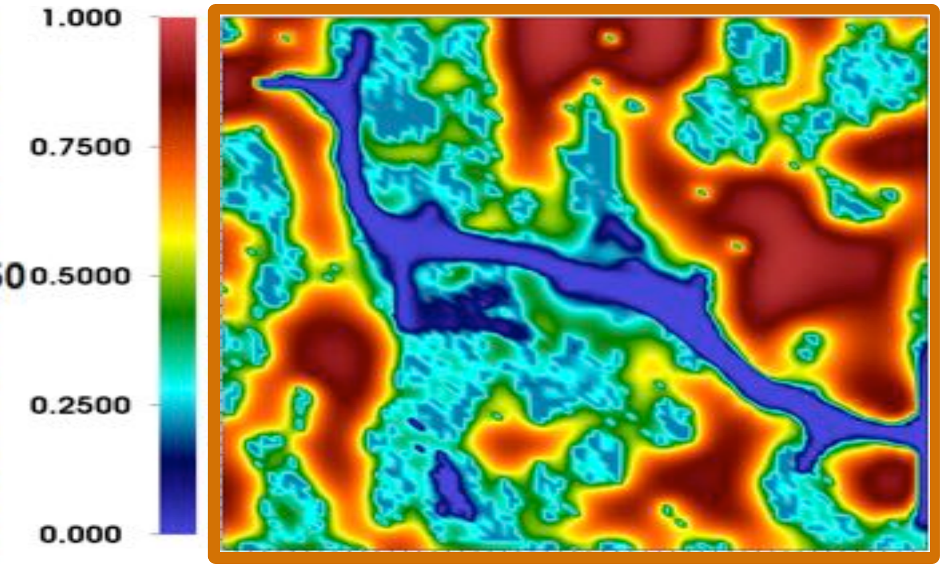
$$P_{\text{map}} = \begin{cases} 0, & 0\% \text{ of existence of fracture} \\ 1, & 100\% \text{ of existence of fracture} \end{cases} \longleftrightarrow \varphi = \begin{cases} 0, & \Omega_F \\ 1, & \Omega_R \end{cases}$$

Advantages of Phase Field Approach

4. Initializing natural fractures with a probability map

Coupling with a probability map [L.-Wheeler-Wick-Srinivasan 2016]





Probability map
InSAR (Surface Deformation Map)

Initialize hydraulic fractures.
Interactions with natural fractures.

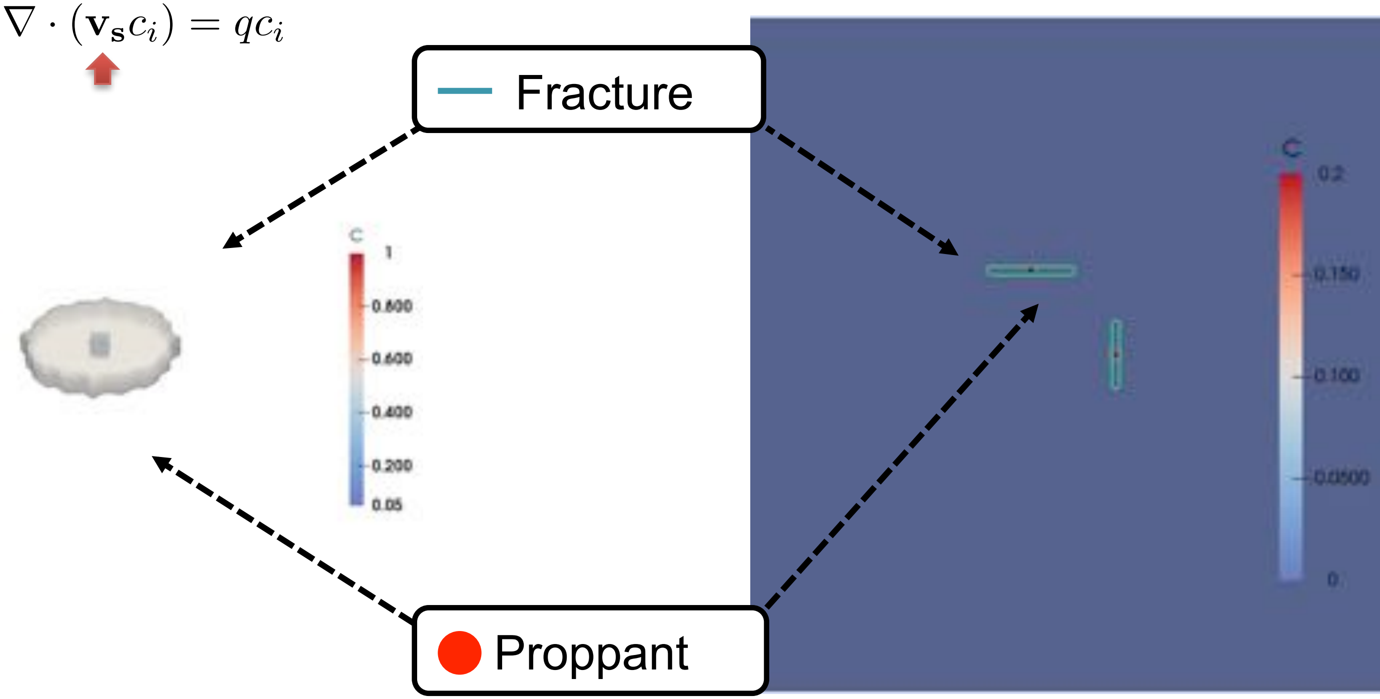
Advantages of Phase Field Approach

4. Coupling with transport systems : Ongoing Work

Proppant-filled fractures in a poroelastic medium [L.-Mikelic-Wheeler-Wick 2016]

- Enriched Galerkin FEM [L.-Lee-Wheeler 2016]

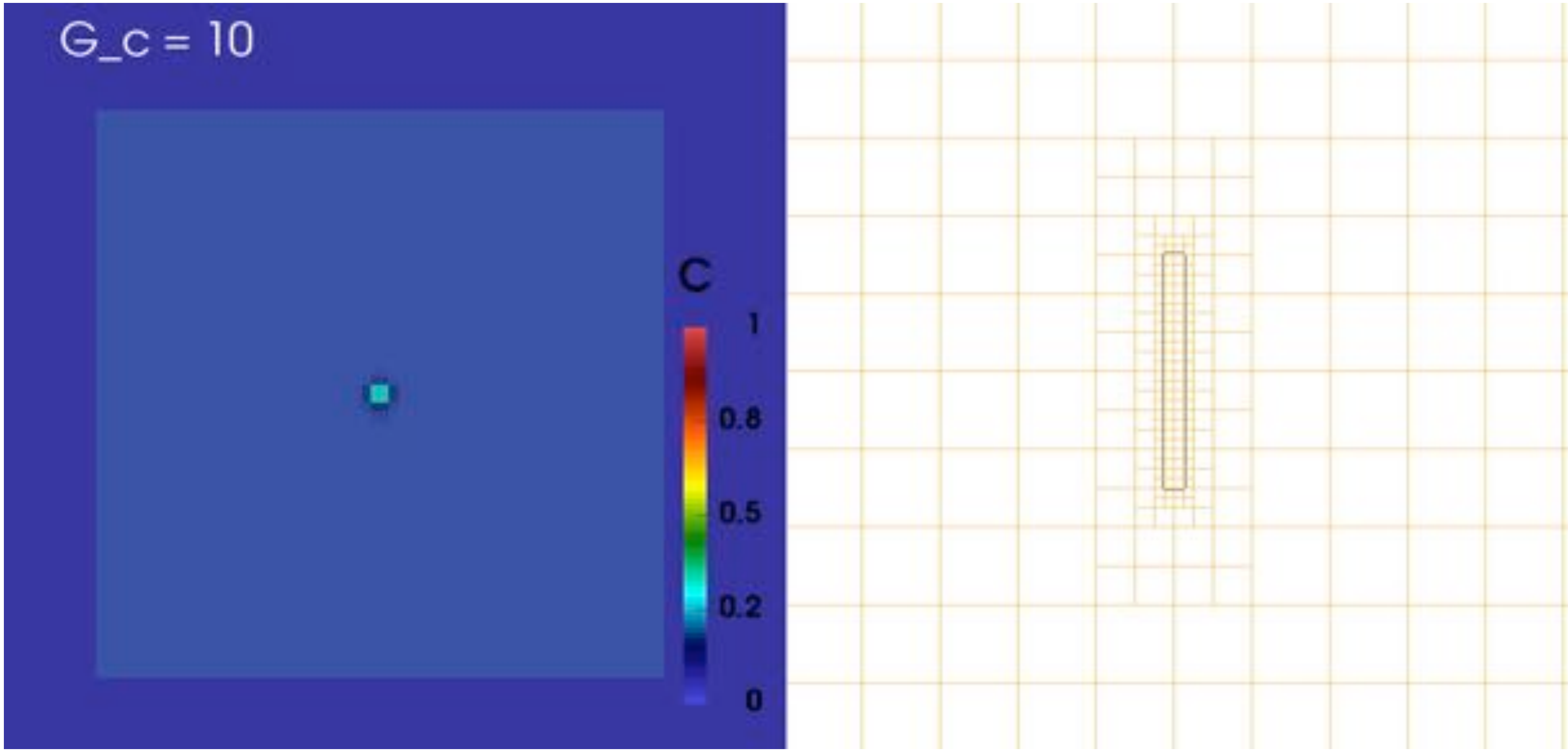
$$\frac{\partial}{\partial t}(\phi^* \rho c_i) + \nabla \cdot (\mathbf{v_s} c_i) = q c_i$$



4. Coupling with transport systems : Ongoing Work

Proppant-filled fractures in a layered medium with a gravitational acceleration. [L.-Shiozawa-Wheeler]

- Enriched Galerkin FEM [L.-Lee-Wheeler 2016]



Advantages of Phase Field Approach

4. Coupling with transport systems : Ongoing Work

Enriched Galerkin FEM [Sun-Liu2009] [L.-Lee-Wheeler SIAM SISC 2016] [L.-Wheeler JCP 2017]

Advantages for EG and Entropy Viscosity

- Takes the advantages of the interior penalty discontinuous Galerkin methods.
  - Flexibility of mesh and rough coefficients.
  - Locally conservative.
- Higher order finite elements with stability and convergence analysis for coupled flow and transport.
  - Identical bilinear(linear) formulations with interior penalty Galerkin methods.

(a) CG_1 (b) DG_1 (c) EG_1

- Less degrees of freedom : half and quarter of that for DG in 2D and 3D, respectively (linear case).
- Multigrid solver with jump coefficients. [L.-Lee-Wheeler 2016]
- Dynamic mesh adaptivity. [L.-Lee-Wheeler 2016] [L.-Wheeler 2016]
- Entropy residual stabilization (entropy viscosity) for higher order advection transport equation. [Guermond-Pasquetti 2008]

Advantages of Phase Field Approach

4. Coupling with transport systems : Ongoing Work

Enriched Galerkin FEM [Sun-Liu2009] [L.-Lee-Wheeler SIAM SISC 2016] [L.-Wheeler JCP 2017]

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- Takes the advantages of the interior penalty discontinuous Galerkin methods.
  - Flexibility of mesh and rough coefficients.
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- Higher order finite elements with stability and convergence analysis for coupled flow and transport.
  - Theorem (Error Convergence [L.-Lee-Wheeler SIAM SISC 2016])

    Let P^n and $p(\cdot, t^n)$ be the discrete and analytic solutions at the time level t^n, respectively. Assume that $p(t^0) \in H^s(\Omega)$ and $p_t \in L^1([0,\mathrm{T}], H^s(\Omega))$ with $s \geq 2$, and $p_{tt} \in L^1([0,\mathrm{T}], H^1(\Omega))$. We then obtain the following error estimate:

    $$\|p(t^n) - P^n\|_{EG} \lesssim h^{\min\{k_p+1,s\}-1}\left(\|p(t^0)\|_{s,\Omega} + \int_0^{t_n} \|p_t\|_{s,\Omega} d\gamma\right)$$

- Le… ively (linear case).

Theorem (Error Convergence [L.-Lee-Wheeler])

Let c be the solution and let C be the solution to the discrete problem. The following estimate holds:

$$|||c - C||| \lesssim h^{k_c}\left(\|\mathbf{D}\|_{\infty,\Omega}^{1/2} + \|\mathbf{u}\|_{0,\infty,\Omega}^{1/2} h^{1/2} + \|\varrho\|_{0,\infty,\Omega}^{1/2} h\right),$$

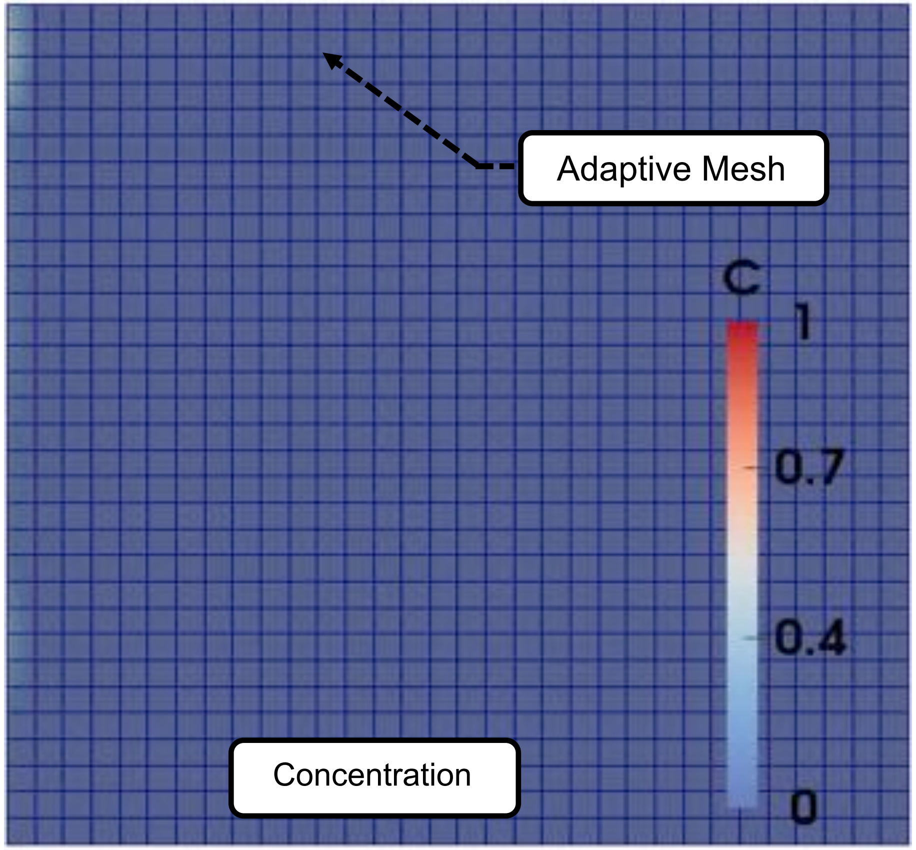
- Mu
- Dy
- En… tion.

  [Gu

Advantages of Phase Field Approach

4. Coupling with transport systems : Ongoing Work

Enriched Galerkin FEM [L.-Lee-Wheeler SIAM SISC 2016] [L.-Wheeler JCP 2017]



- **Miscible displacement** in a random heterogeneous media.
- **EG-Q1 FEM** for **flow and convection-diffusion-dispersion transport.**
- Entropy residual stabilization
- Dynamics mesh adaptivity
- Efficient solver

Advantages of Phase Field Approach

4. Coupling with transport systems : Ongoing Work

Enriched Galerkin FEM **[L.-Lee-Wheeler SIAM SISC 2016] [L.-Wheeler JCP 2017]**

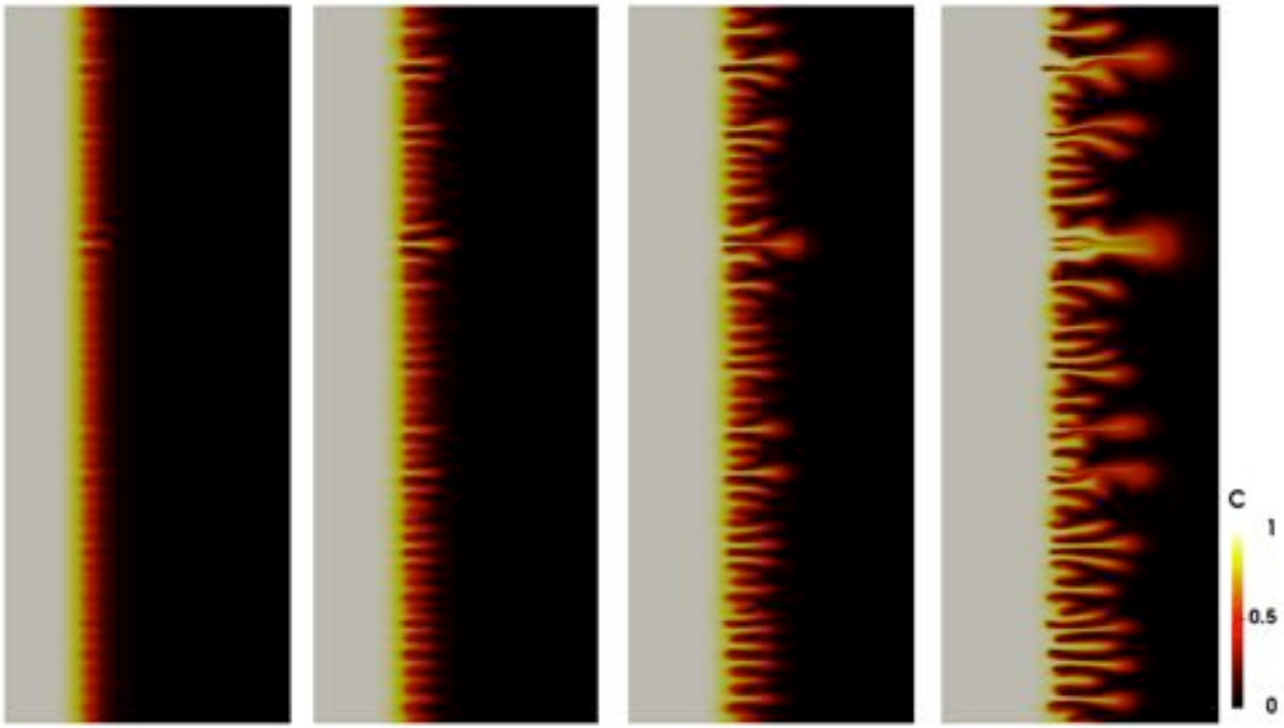
Viscous fingering in a Hele-Shaw cell with viscosity ratio 150.

[Scovazzi-Wheeler-Mikelic-L. 2017 JCP]

Advantages of Phase Field Approach

4. Coupling with transport systems : Ongoing Work

Enriched Galerkin FEM [L.-Lee-Wheeler SIAM SISC 2016] [L.-Wheeler JCP 2017]



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Advantages of Phase Field Approach

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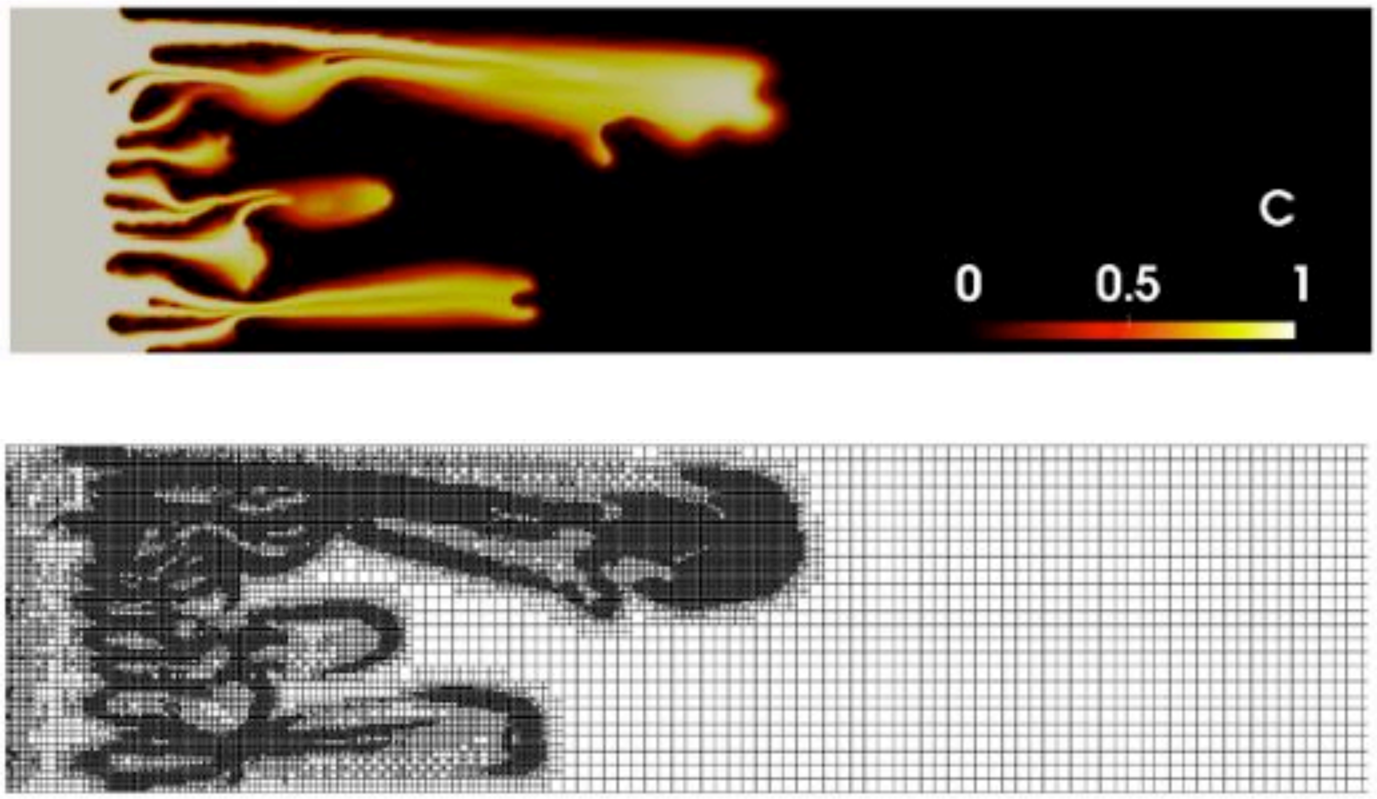

Advantages of Phase Field Approach

4. Coupling with transport systems : Ongoing Work

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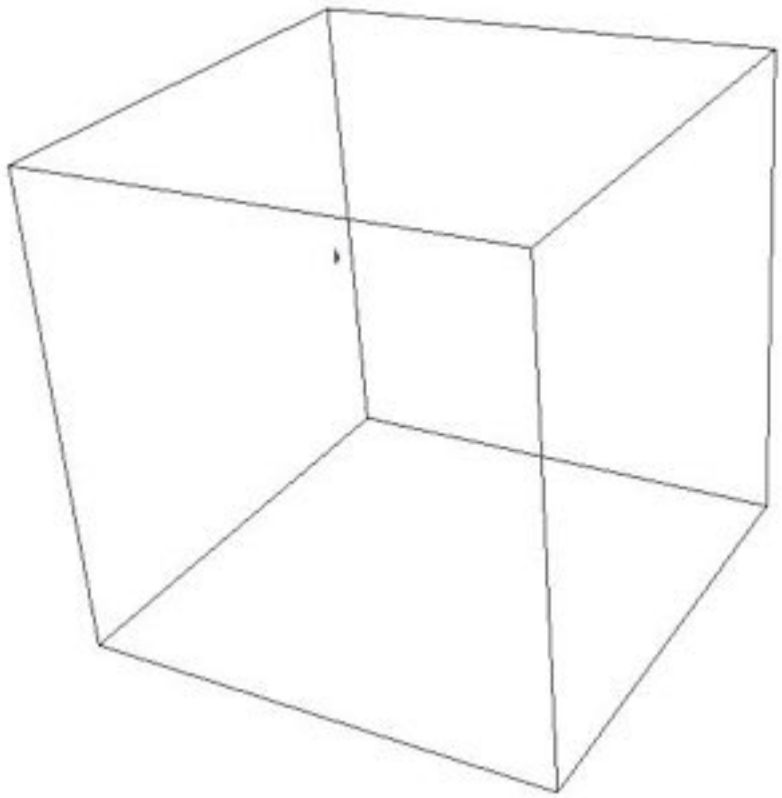
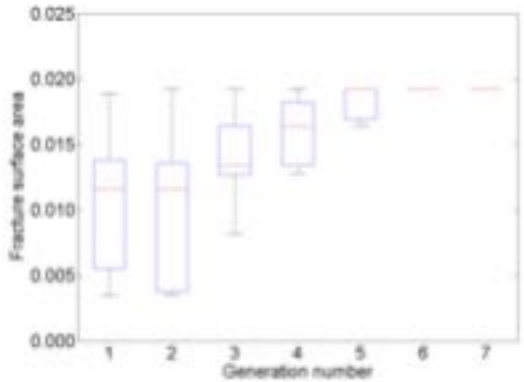


Optimization

Optimization

Coupling Phase Field



Optimization

THE WALL STREET JOURNAL.

U.S. Edition ▾ | March 31, 2017 | Today's Paper

LOW OIL PRICE USHERS IN SHALE 2.0

Texas fracker EOG pioneers way to turn profit by extracting crude faster and at less cost

By Erin Ailworth

MIDLAND, Texas—Using a proprietary app called iSteer, Brian Tapp, a geologist for EOG Resources Inc., dashed off instructions to a drilling rig 100 miles away. This tool is among the reasons the little-known Texas company says it pumps more oil from the continental U.S. than Exxon Mobil Corp.—or any other producer.

A rig worker received Mr. Tapp's iPhone alert and tweaked the trajectory of a drill bit thousands of feet underground to land more squarely in a sweet spot of rock filled with West Texas crude.

U.S. shale drillers transformed the energy industry over the past decade with hydraulic fracturing and horizontal drilling, in the early days using brute force to unleash a torrent of oil and gas that altered the balance of power among oil-producing nations and triggered a global glut.

Now, with oil currently trading near $50 a barrel, these producers are trying to unleash fracking 2.0, the next step in the technological transformation of the sector that is aimed at extracting oil even faster and less expensively to eke out profits at that level.

The promise of this new phase is po-

Please see SHALE page A11

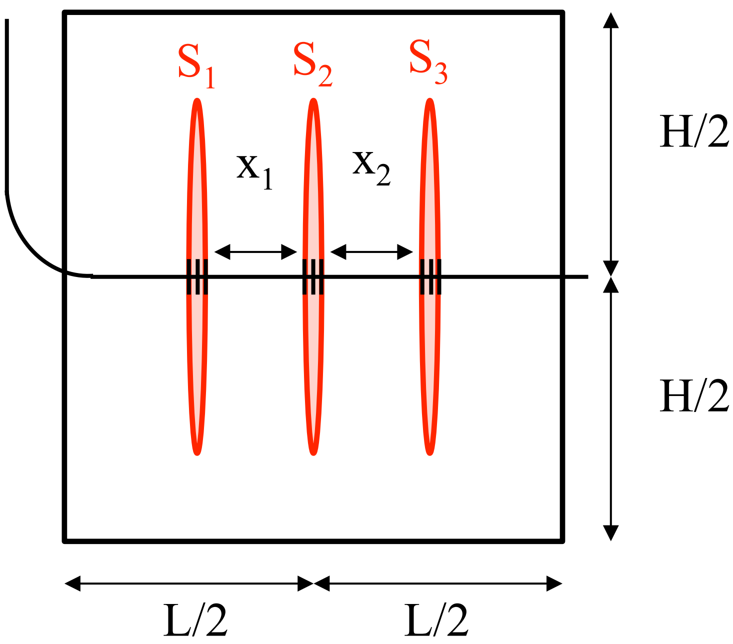
Next step in the technological transformation of the sector that is aimed at extracting oil even faster and less expensively..

The shale sector must shift toward finding cost-effective ways to get to oil and gas

Optimization: Coupling with Phase Field

Example 1. A single well: maximize the fracture volume in a homogeneous domain

3 Parallel Fluid Filled Fractures in a Homogeneous Domain [L.-Min-Wheeler]



C2Frac model [Cheng-Bunger-Peirce 2016]

Goal : maximize

the fracture volume (≈ production).

i.e. argmax $f(\mathbf{x})$ = argmax $f(x_1, x_2)$

= argmax $(S = S_1 + S_2 + S_3)$,

where x: spacing, S: fractured surface area.

Fracture volume (area) :

$$S = \int_{\Omega_F} 1 \, dx$$

$$\Omega_F := \{x \mid \varphi(x) \leq c_v\}, \;\; c_v = 0.1$$

Variables : x_1 **and** x_2

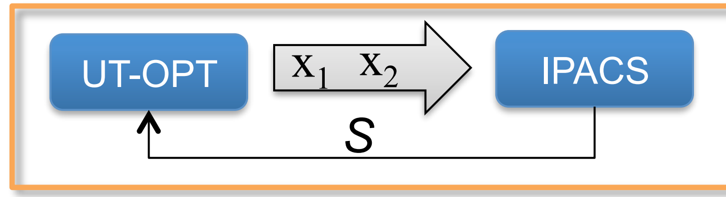
Optimization: Coupling with Phase Field

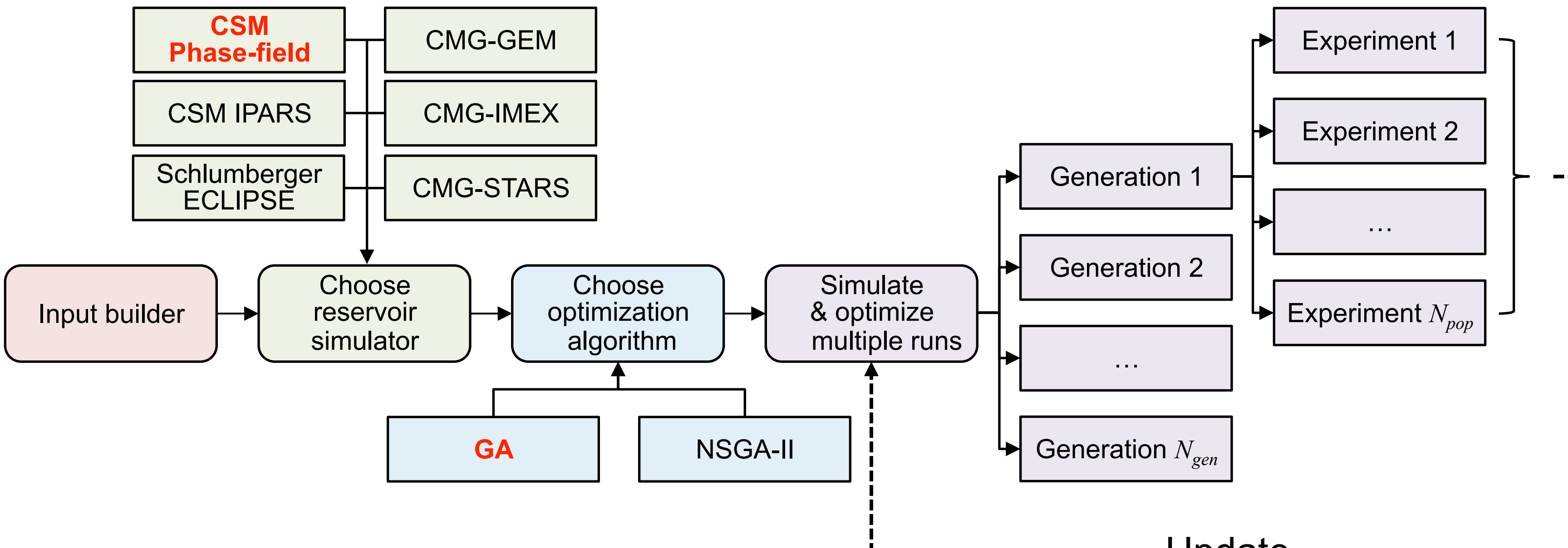
- **Coupling UT-OPT** [Min-Wheeler-Sun 17] **with the phase field approach** [L.-Min-Wheeler]

Inverse model: Genetic Algorithm [Goldberg, 1989]

Goal: I) $\operatorname{argmax} f(\mathbf{x})$: Example 1, 2, and 4

II) $\operatorname{argmin} f(\mathbf{x})$: Example 3





Optimization: Coupling with Phase Field

Example 1. A single well: maximize the fracture volume in a homogeneous domain

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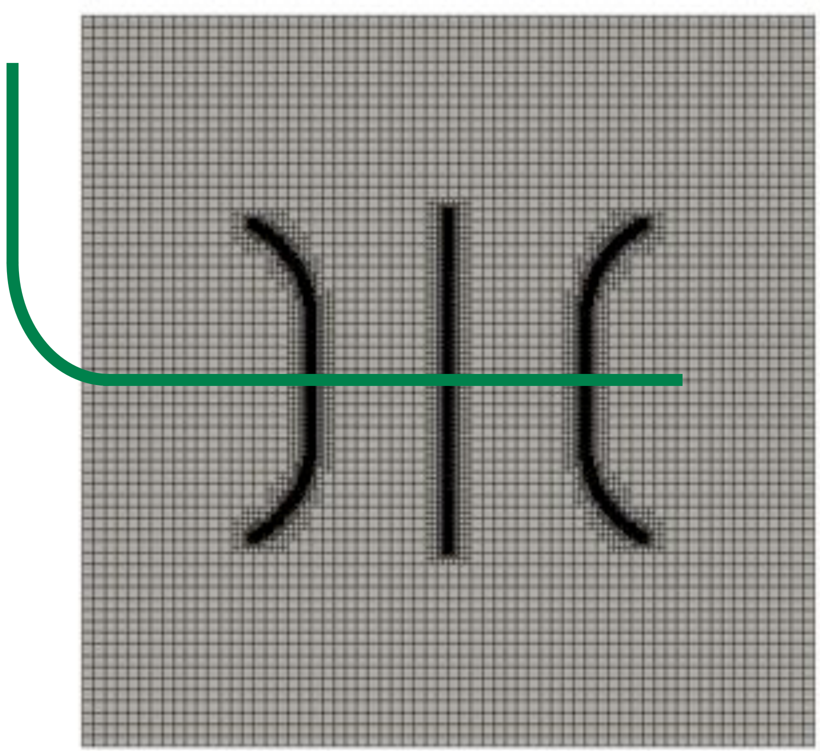
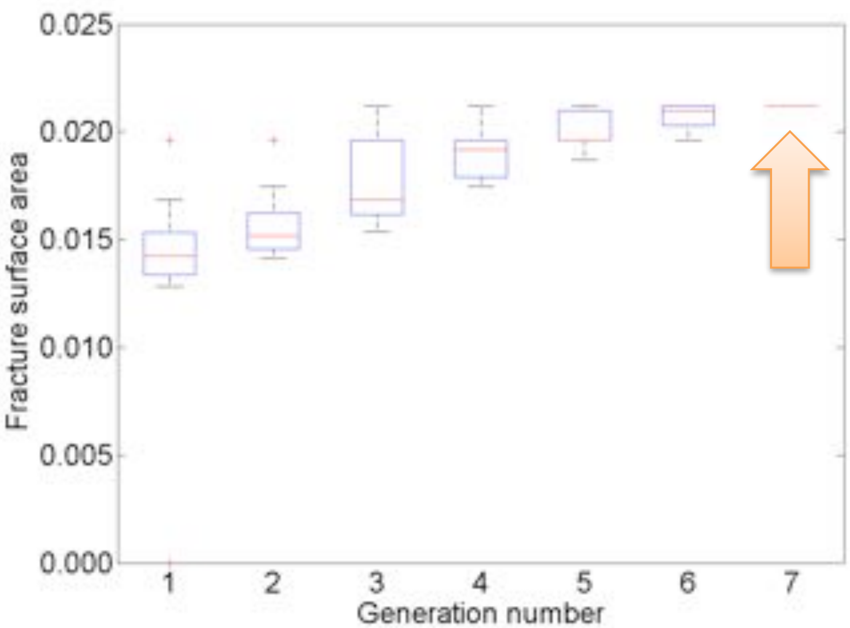




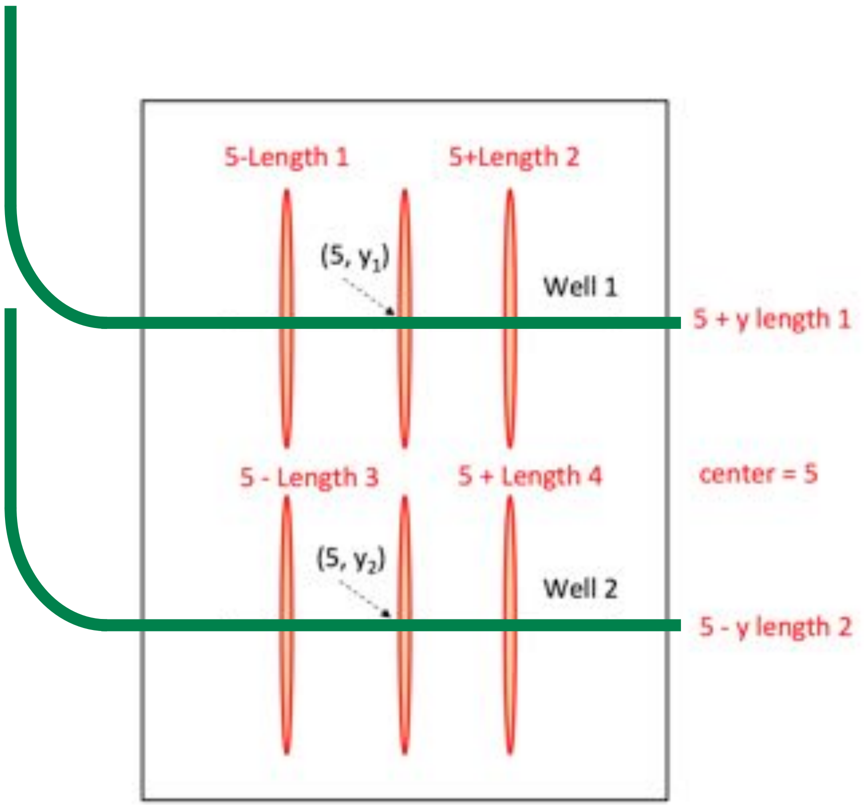
$$(x_1, x_2) = (0.75\text{m}, 0.75\text{m})$$

$$S = 0.0212261\text{m}^2$$

Optimization: Coupling with Phase Field

Example 4. Two wells: maximize the fracture volume (on-going work)

Parallel Fluid Filled Fractures in a Homogeneous Domain



Goal : maximize

the production ($\approx$ fracture volume).

i.e. argmax $f(\mathbf{x})$

$= \text{argmax } f(x_1, x_2, x_3, x_4, y_1, y_2)$

$= \text{argmax } (S_1 + S_2 + S_3 + S_4 + S_5 + S_6)$,

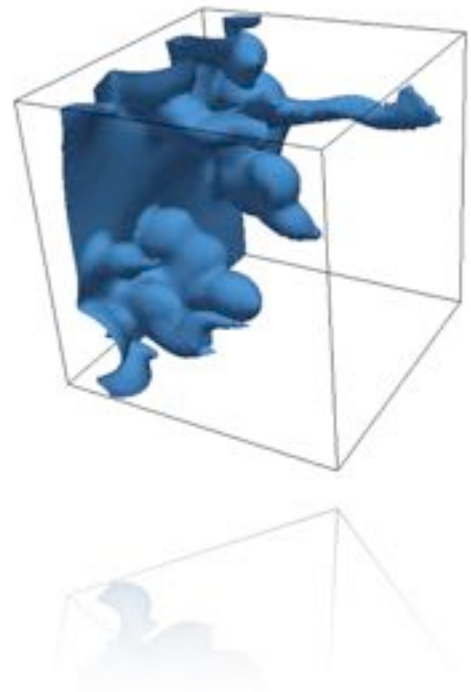
where x,y: spacing, S: fractured surface area.

More variables : more x and additional y

Conclusion

Conclusion

future works



Conclusion

Objectives	Develop a multi physics and fluid flow driven **fracture propagation** method employing the phase field approach in a **poroelastic medium**.

- Phase field fracture propagation simulator.
  (IPACS: Integrated Phase field Advanced Crack Simulator)
- Biot model with dynamic mesh adaptivity on a parallel framework.
- Fixed stress split iterative coupling scheme / Fully coupled scheme.
- High order finite element methods (Enriched Galerkin)

	Demonstrate the potential of the phase field for treating practical reservoir engineering applications by providing numerical examples.