Qubits and quantum gates in optical lattices

- Qubits and quantum gates in optical lattices
- A path towards quantum logic in the lab
  - *Raman sideband cooling*
  - *one- and two-qubit control*
  - *ensemble measurement*
- Quantum state reconstruction
  - *measuring ρ in the ground hyperfine manifold*

The University of Arizona

OPTICAL SCIENCES CENTER

Experiment

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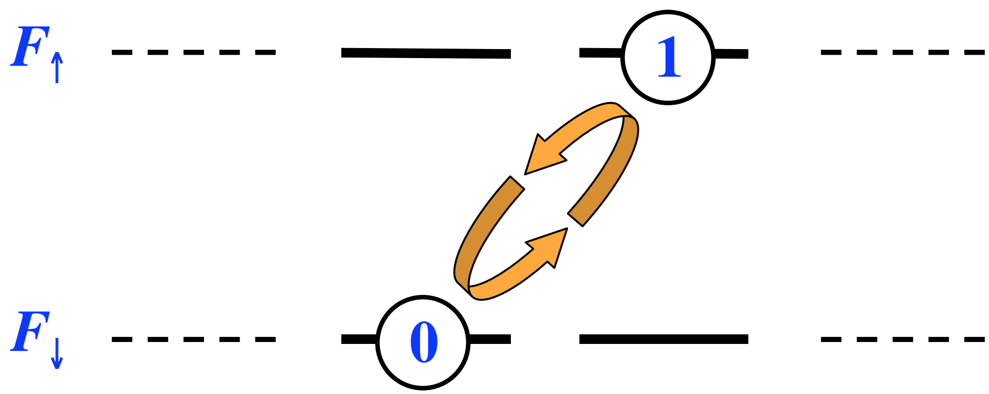
Quantum information and neutral atoms

Encoding qubits: *hyperfine ground states of alkali atoms*

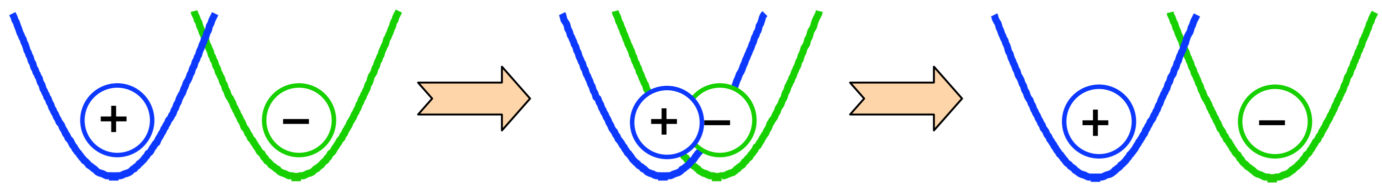


Advantages:
- *laser cooling and trapping*
- *weak coupling to environment*
- *long coherence times*

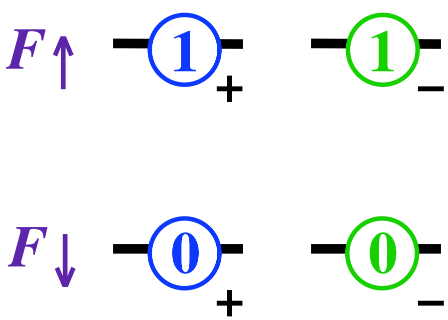
Disadvantages:
- *weak coupling between atoms*

Neutral atom quantum logic

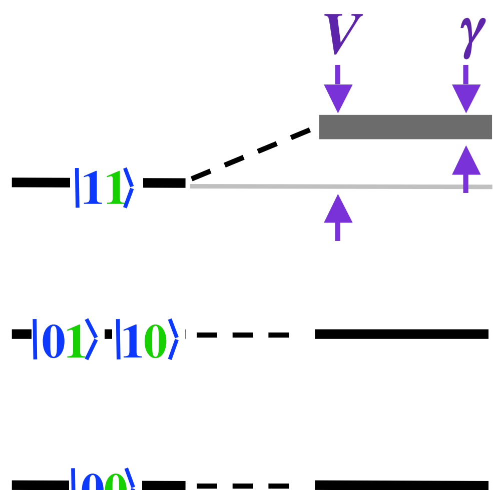
1st part: move/collide atomic qubits



2nd part: interaction



logical states



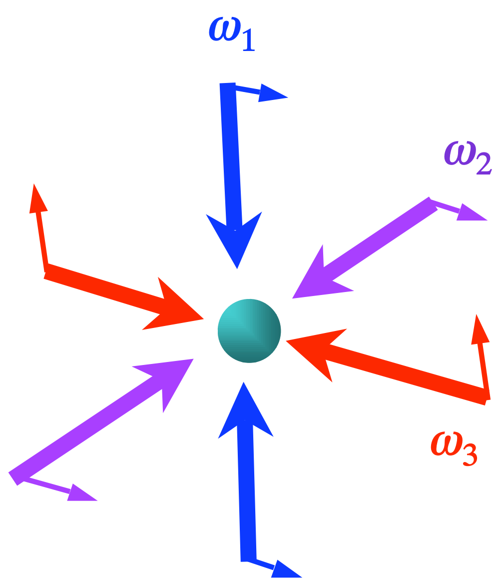
energy shifts

Apply V to accumulate π phase

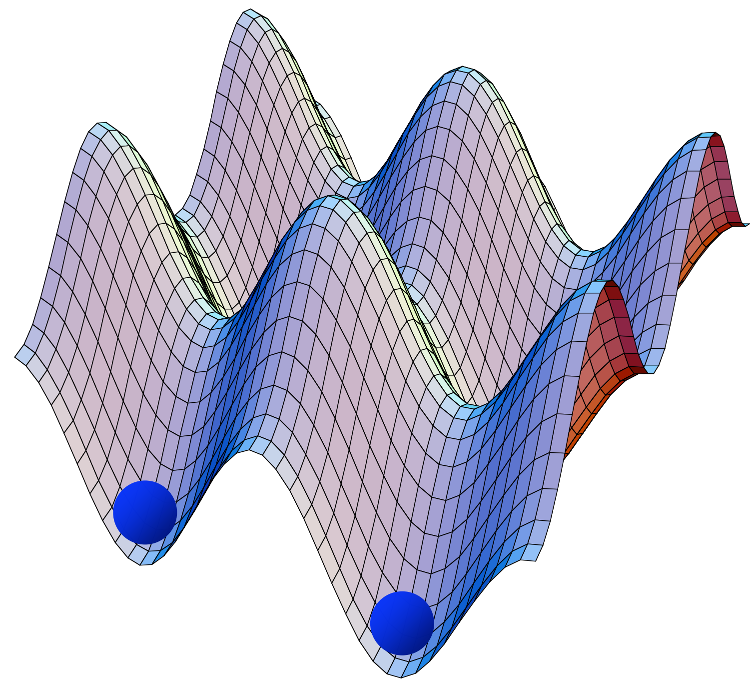
CPhase

1st part: 3D Optical Lattice

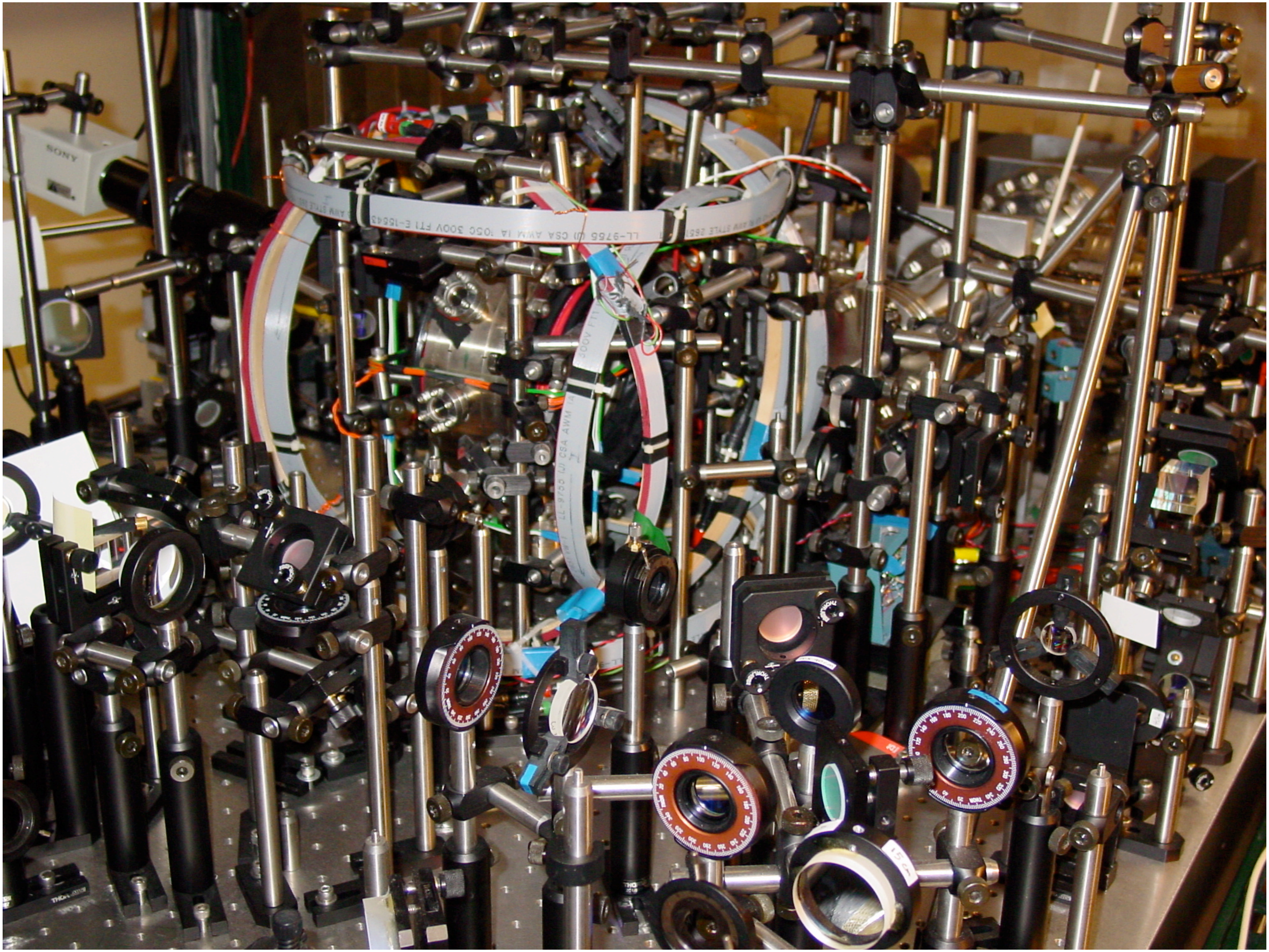
neutral atom +
laser standing wave



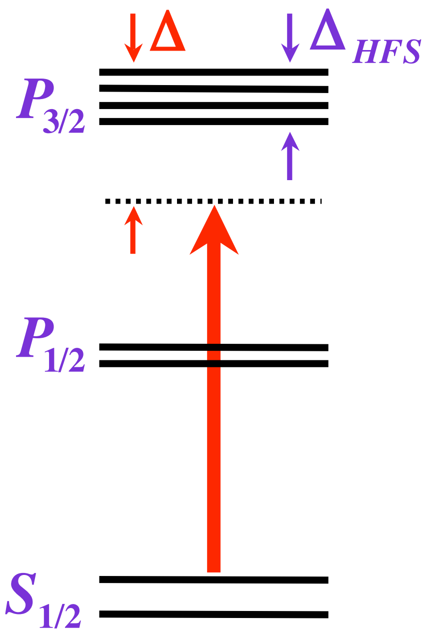
light shift potential



- *Non-dissipative*
- *Tight binding - like ion trap*
- *100% occupation difficult - but possible*
- *λ-scale separation makes addressing difficult*


SONY

Optical potentials for alkali atoms



Interaction $-\mathbf{d}\cdot\mathbf{E}(\mathbf{r})$

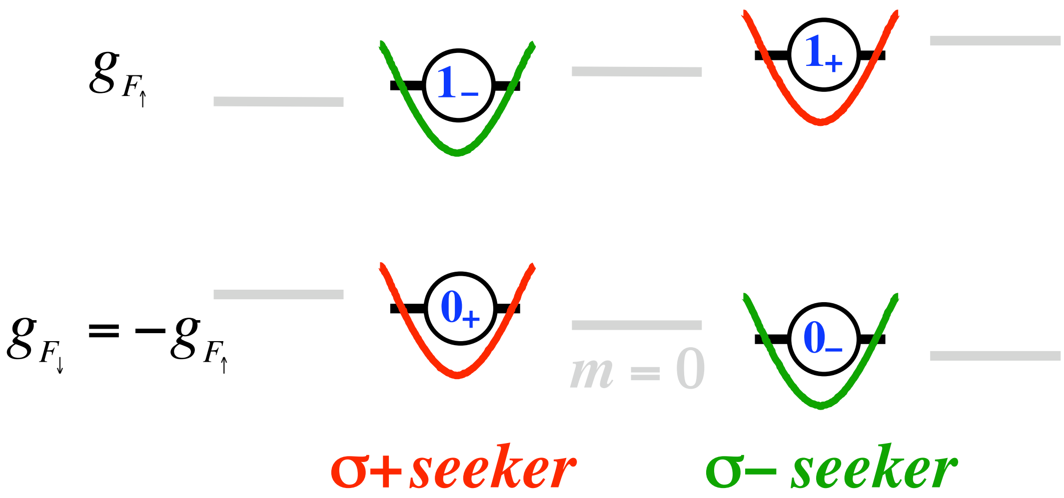
$$\hat{U}(\mathbf{x}) = U_J(\mathbf{x})\,\hat{I} + \mathbf{B}_{eff}(\mathbf{x})\cdot\frac{\hat{\mathbf{F}}}{F}$$

$$U_J(\mathbf{x}) \propto \left|\mathbf{E}_L(\mathbf{x})\right|^2$$

$$\mathbf{B}_{eff}(\mathbf{x}) \propto \mathbf{E}_L^*(\mathbf{x})\times\mathbf{E}_L(\mathbf{x})$$

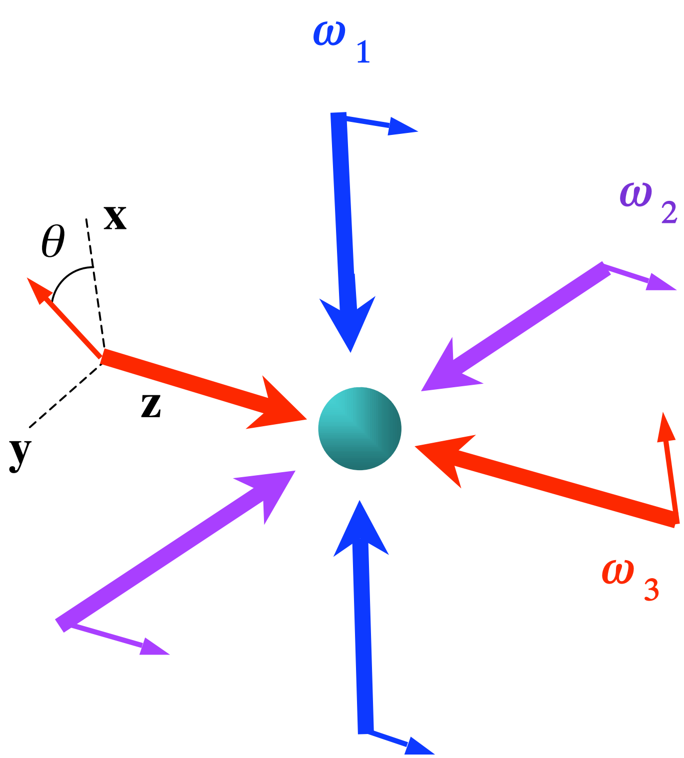
- scalar potential + effective magnetic field

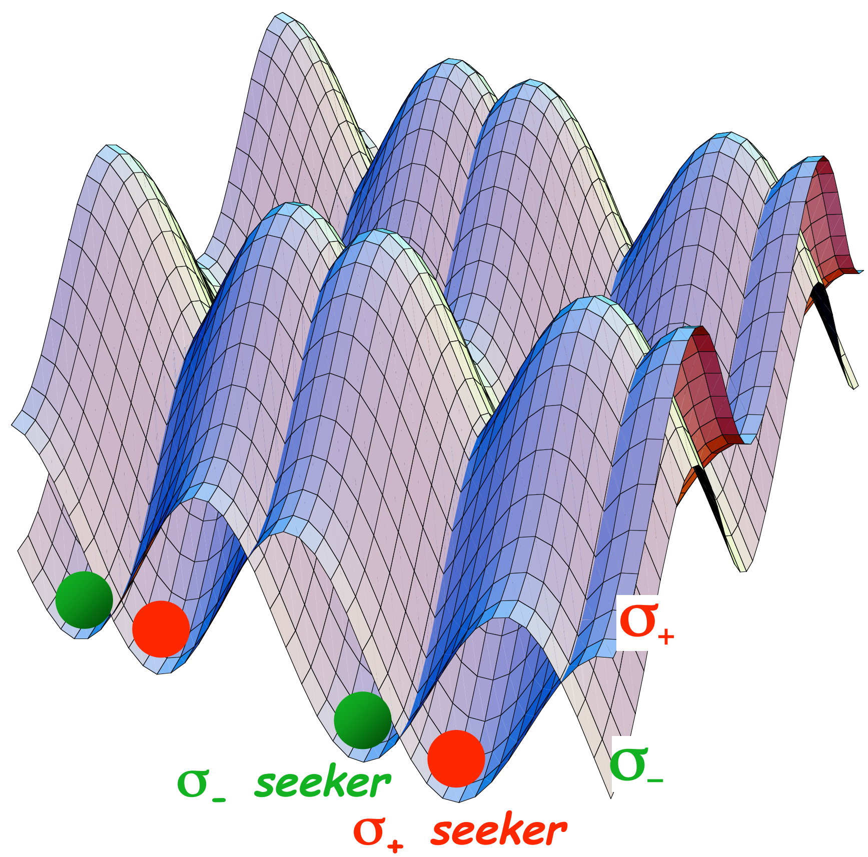
Two-species (DFS) encoding



- *insensitive to magnetic fields, laser fluctuations*
- *somewhat difficult to do single qubit rotations*
- *complex collision physics*

Moving qubits in 3D lattices





2nd part: coherent atom/atom coupling

Dipole-dipole interaction

G. K. Brennen et al., PRL 82, 1060 (1999)

level shift

$$V_{dd} \approx \frac{\langle \mathbf{d}_1 \rangle \cdot \langle \mathbf{d}_2 \rangle}{r_{12}^3}$$

superradiance

$$\Gamma_{tot} = \gamma_s + \Gamma_{dd} \leq 2\gamma_s$$

figure of merit for coherence

$$\kappa = \frac{V_{dd}}{\hbar \Gamma_{tot}} \approx \frac{1}{(k r_{12})^3}$$

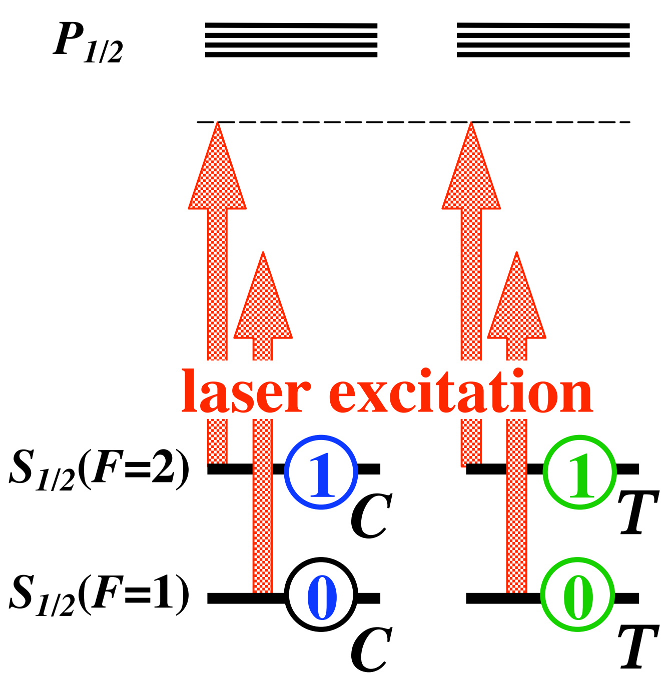
- Key parameter: atomic localization -

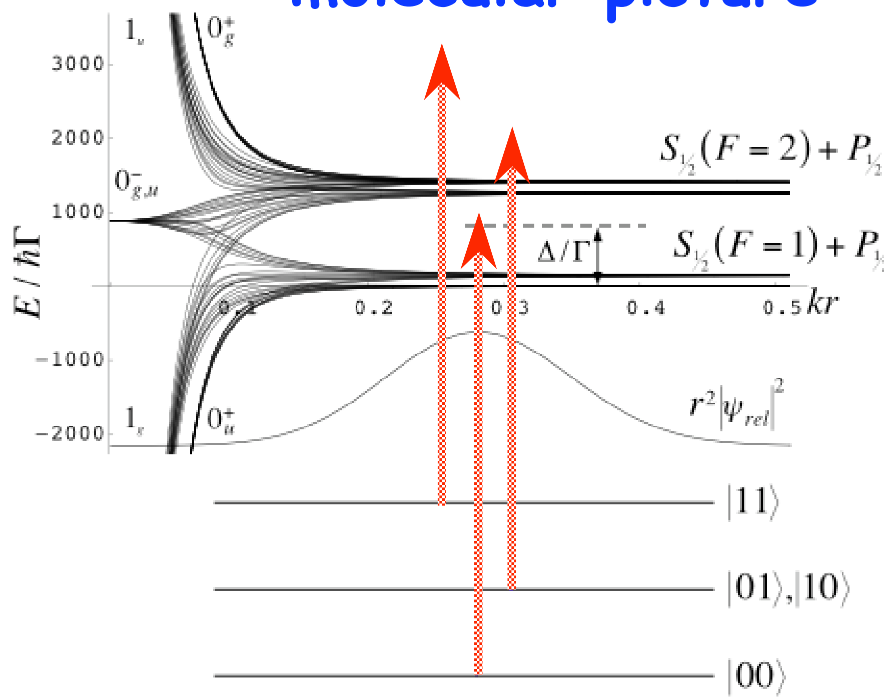
Many other ideas:

- cold collisions,
- Rydberg atoms,
- cavity coupling

Conditional dipole-dipole interaction

(Brennen et al, 1999, Brennen et al. 2002)





- off-resonance AC Stark shift ➡ differential phase shifts of logic states ➡ CPhase gate

Fidelity in 1st generation experiments?

- preliminary estimate $\mathcal{F} \sim 0.9\text{-}0.95$ -

Errors due to photon scattering

from coupling laser	$\mathcal{P}_C$
from lattice	$\mathcal{P}_L \propto \gamma_L \tau$

combined error probability

$$\mathcal{P}_{error} \sim \mathcal{P}_C + \mathcal{P}_L$$

Other errors

- hyperfine changing interactions (leakage)

- fidelity increases with tighter confinement ! -

Can fidelity be improved?

Error reduction in deeper lattices?

coupling	*lattice*	*total*
$\mathcal{P}_C \sim \frac{\pi}{\lvert\kappa\rvert} \propto \frac{\Delta_L^{3/4}}{I_L^{3/4}}$	$\mathcal{P}_L \propto \gamma_L \times \frac{1}{\omega_{osc}} \propto \frac{I_L^{1/2}}{\Delta_L^{3/2}}$	$\mathcal{P}_{error} \sim \mathcal{P}_C + \mathcal{P}_L$

- *choose optimum* Δ_L ➡ $\mathcal{P}_{error} \propto I_L^{-1/3}$

- *other sources of error may dominate, no simple dependence on localization*
- *we have not determined optimal working point*

Molecular control techniques?

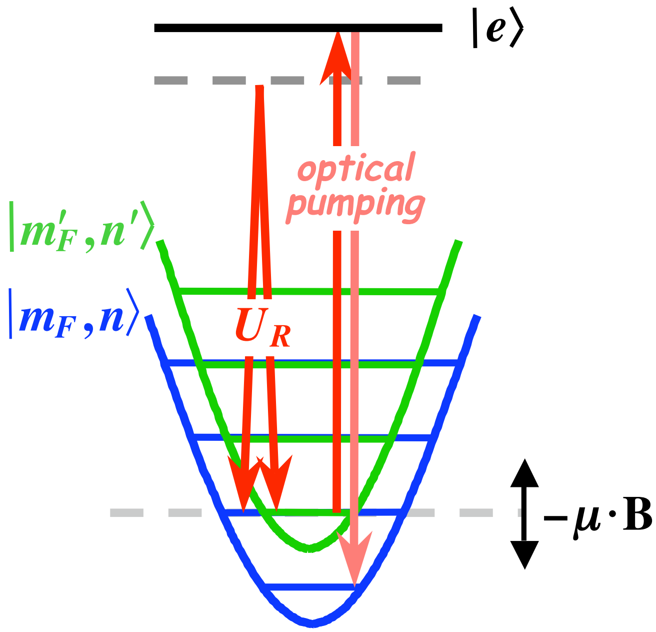
- *ask our friends in Albuquerque*

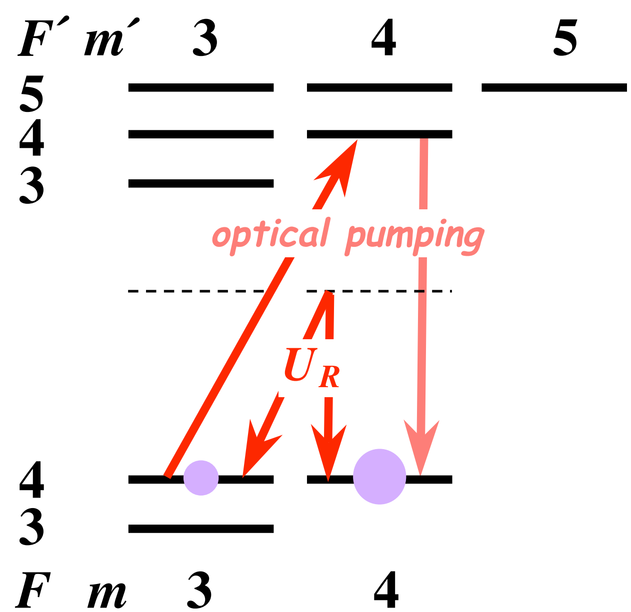
A path towards quantum logic in the laboratory

- Qubit initialization (Raman sideband cooling)
- Single-qubit control & readout
- 2-qubit gate demonstration & optimization
  - *evaluation by ensemble measurements*
  - *debug using quantum state and process tomography*

Qubit initialization = Raman sideband cooling

Basic idea





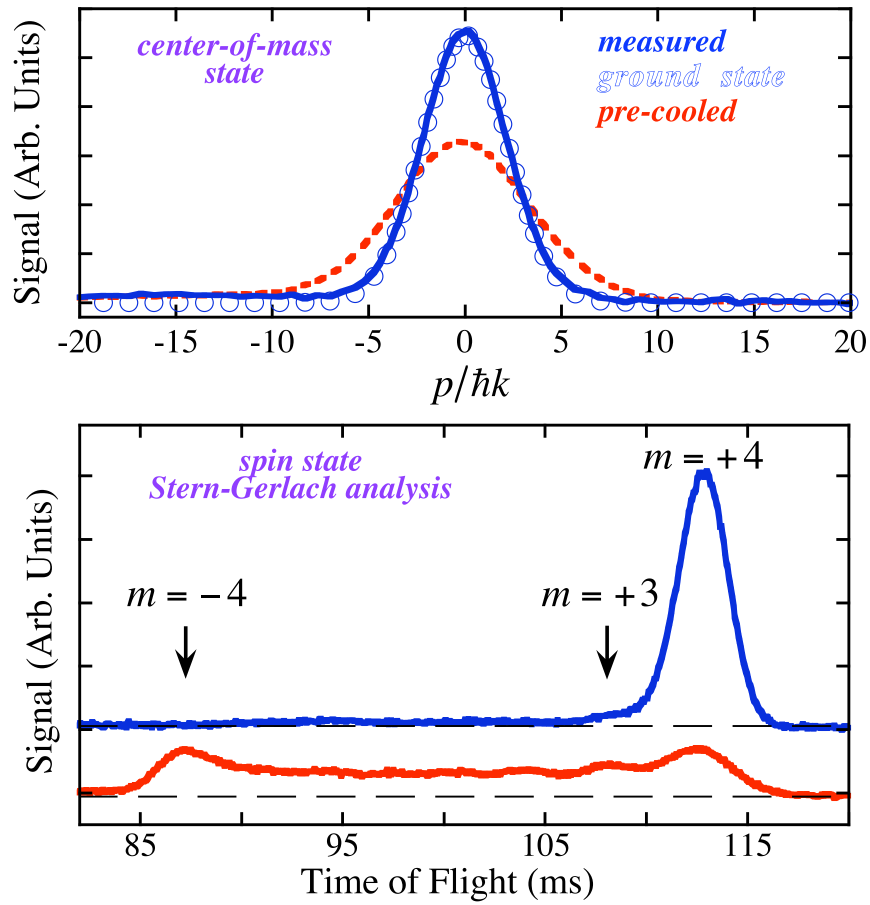
Design Raman coupling into optical lattice

$$\hat{U}(\mathbf{x}) = U_J(\mathbf{x})\hat{I} + \mathbf{B}_{eff}(\mathbf{x})\cdot\frac{\hat{\mathbf{F}}}{F}$$

$$U_R = \langle m_F', n' | \hat{U}(\mathbf{x}) | m_F, n \rangle$$

1998: Cesium in 2D lattice

S. E. Hamann et al., PRL 80, 4149 (1998)

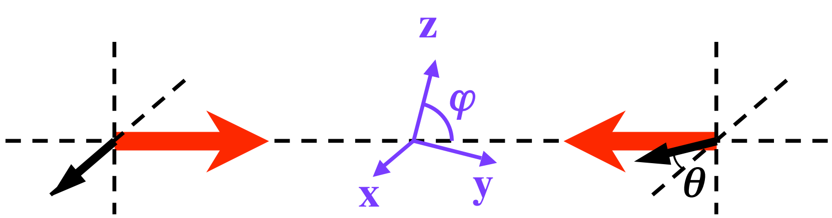


ground state population

$\pi_0 = 0.984(31)$

A flexible approach to sideband cooling

Cooling in 1D lin-θ-lin

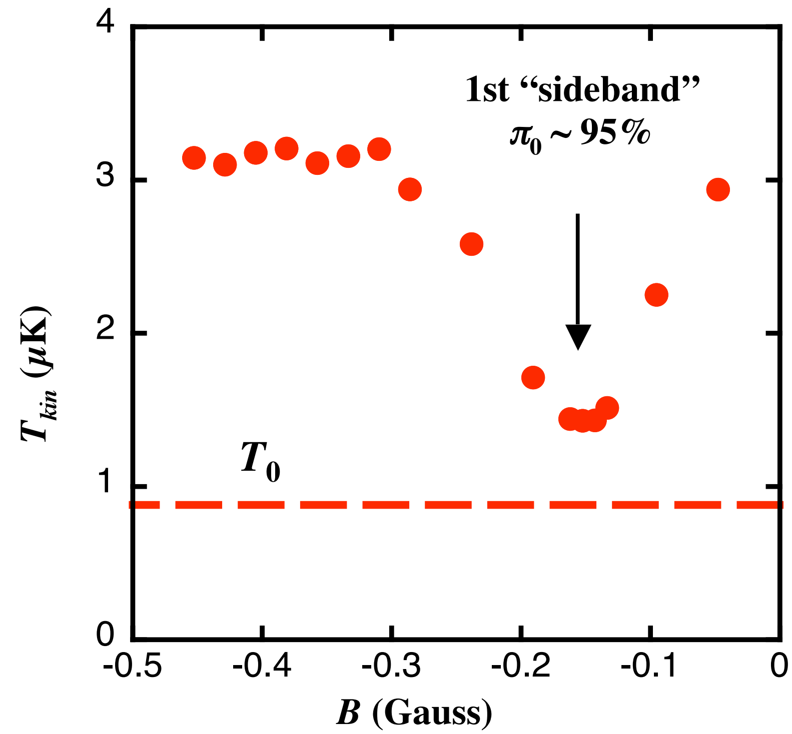


- Raman coupling can be controlled by adjusting
  - *polarization*
  - *quantization axis*

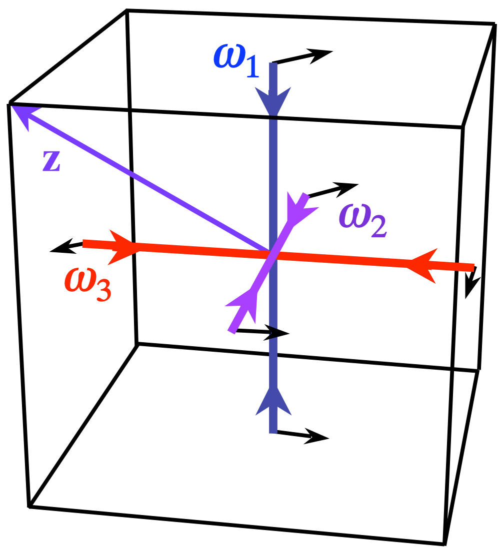
$$U_R \propto \frac{\sin(\theta)\sin(\varphi)}{\sqrt{4+\cos^2(\theta)\tan^2(\varphi)}}$$

Note: U_R =0 for φ =0 or θ =0

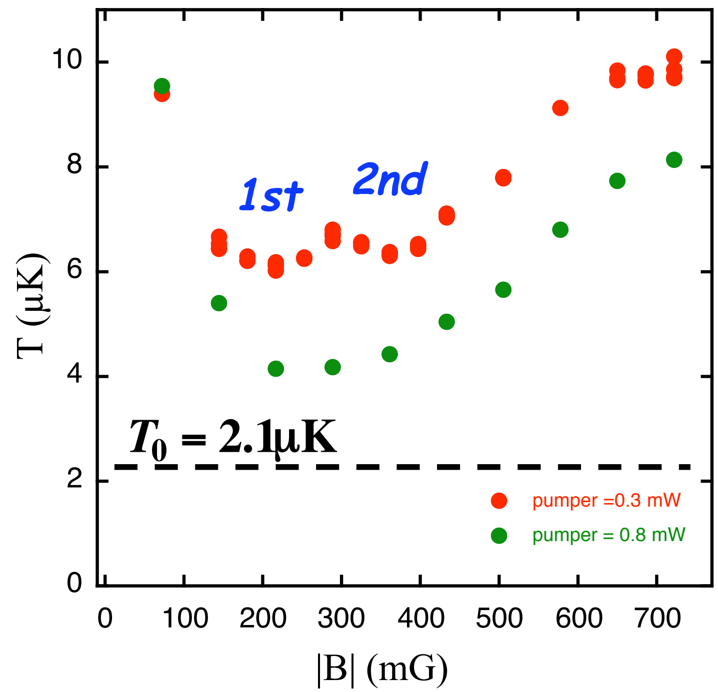
Exploratory 1D lattice experiment



3D extension



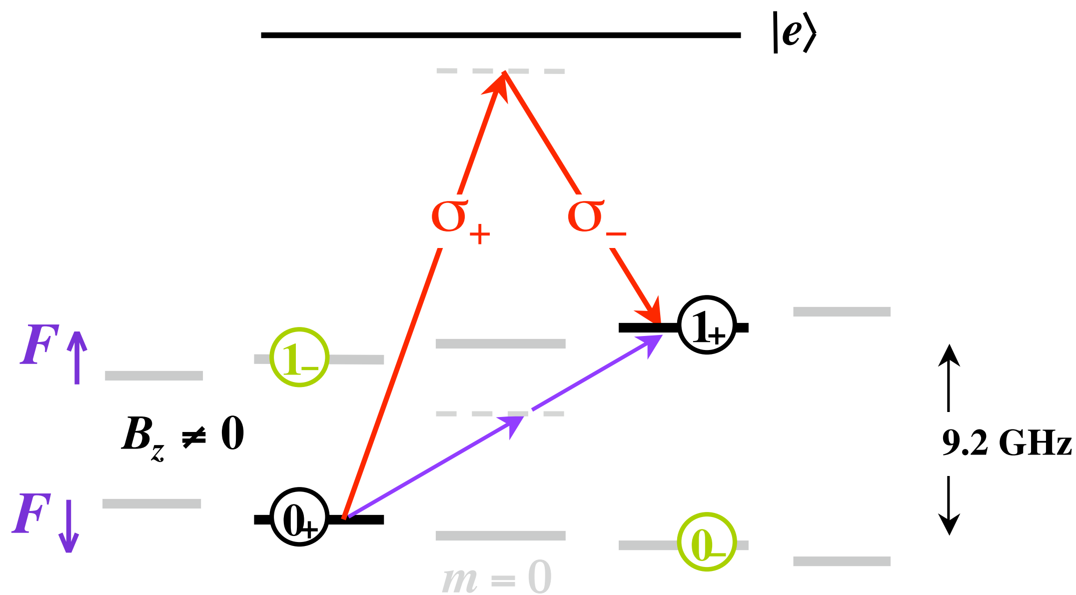
First results



- Preliminary result: π_0=45%
- Random filling of sites at ~1% occupation
- SB cooling at high density or load from BEC
  Weiss, Chu
  I. Bloch et all.

Single-qubit control

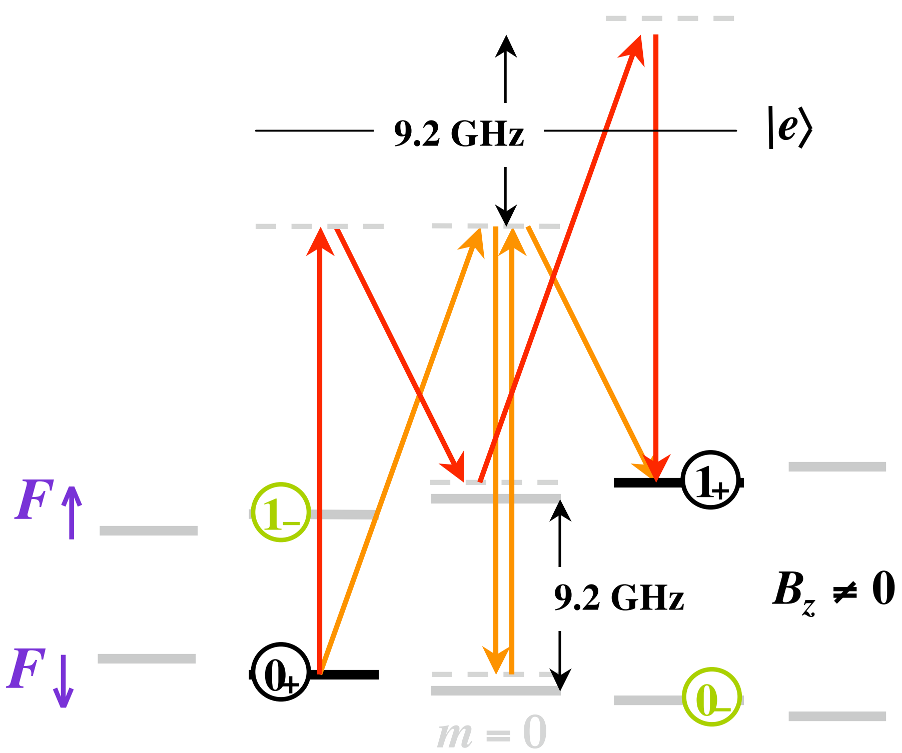
Goal: arbitrary rotation on Bloch sphere
independently for each qubit species



Problem: *2 photon Raman coupling = 0*
simple RF pulse not species selective
individual qubit addressing very hard!

4-photon Raman transitions

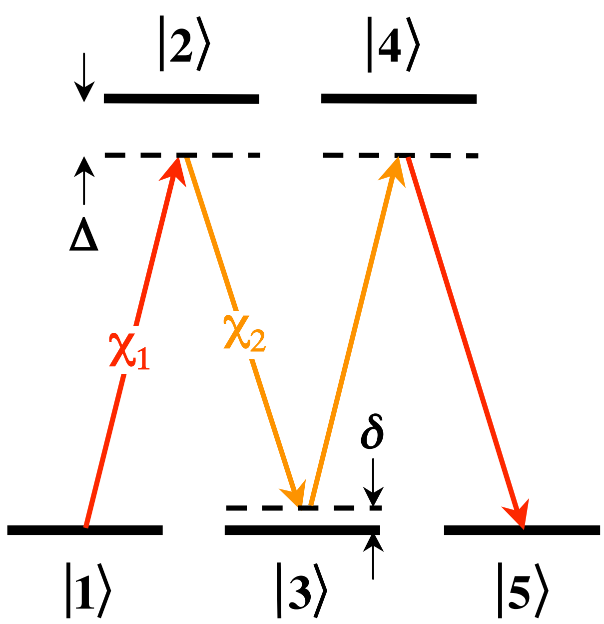
- *4-photon coupling ≠ 0*
- *distinguish species by polarization*



- *Not as straightforward as we would like!*
  Better way? Perhaps we can work with qudits?

Coherence of 4-photon transitions

Simple model



$$H = \begin{pmatrix} 0 & \chi_1 & 0 & 0 & 0 \\ \chi_1 & -i\Gamma/2+\omega_0 & \chi_2 & 0 & 0 \\ 0 & \chi_2 & 0 & \chi_2 & 0 \\ 0 & 0 & \chi_2 & -i\Gamma/2+\omega_0 & \chi_1 \\ 0 & 0 & 0 & \chi_1 & 0 \end{pmatrix}$$

Solution

- *adiabatically eliminate states 2,3,4*

  requires $\Delta >> \chi_1, \chi_2 \quad \delta >> \frac{\chi_1 \chi_2}{\Delta}$

- *solve remaining 2-level equations*

$$\rho_{11/55} = \tfrac{1}{2} e^{-\gamma t} [1 \pm \cos(\Omega t)]$$

$$\rho_{15} = \tfrac{i}{2} e^{-\gamma t} \sin(\Omega t)$$

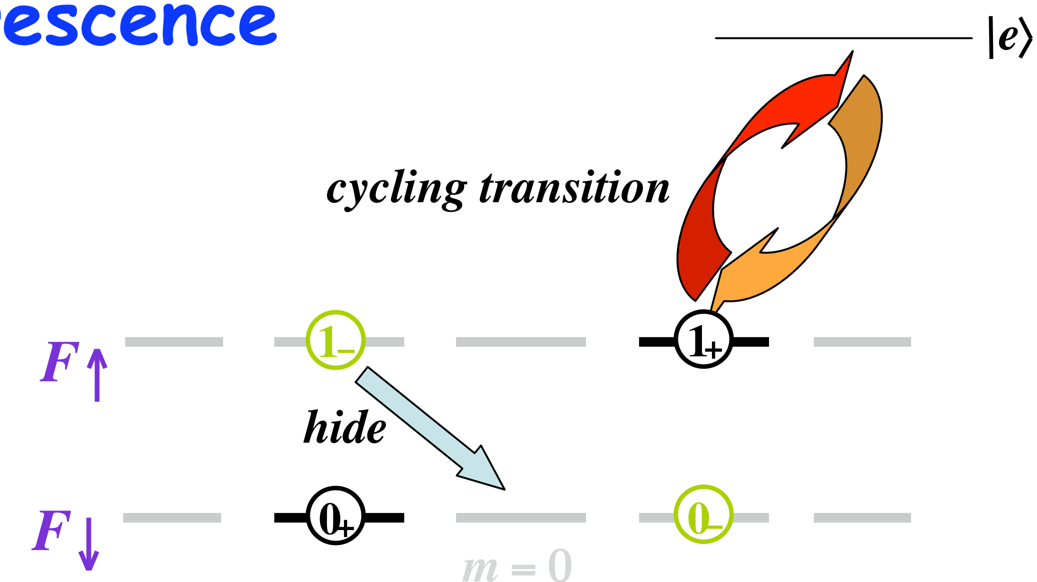
where $\Omega = \frac{\chi_1^2 \chi_2^2}{8\delta\Delta^2} \quad \gamma = \frac{\chi_1^2 \Gamma}{8\Delta^2}$

- *coherent if* $\Omega >> \gamma$

  requires $\Delta >> \frac{\chi_2^2}{\delta} >> \Gamma$

Qubit readout

Fluorescence

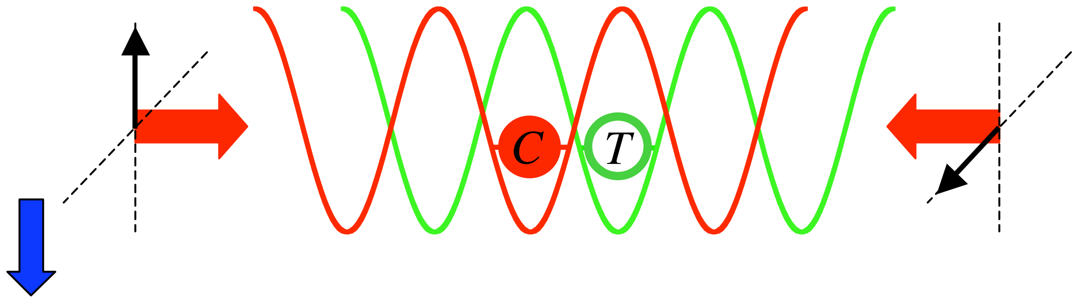


- *easy to get enough sensitivity for ensemble measurements*
- *very hard to spatially resolve individual atoms and do projective measurements*

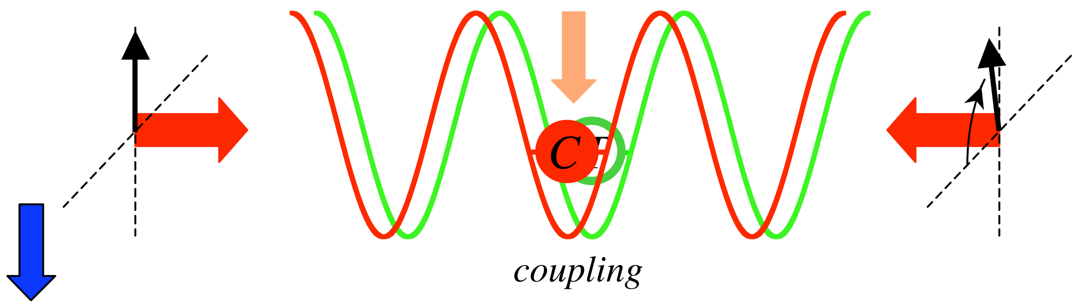
● ***We can demonstrate & optimize logic protocols using ensemble measurements only!***

Exploring two-qubit logic gates

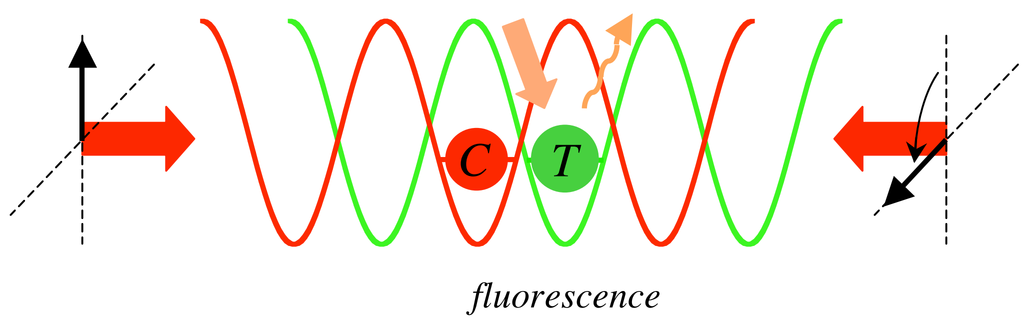
Prepare lattice with +/- species in pairs



Bring together, **CNOT = π/2-CPhase-π/2**



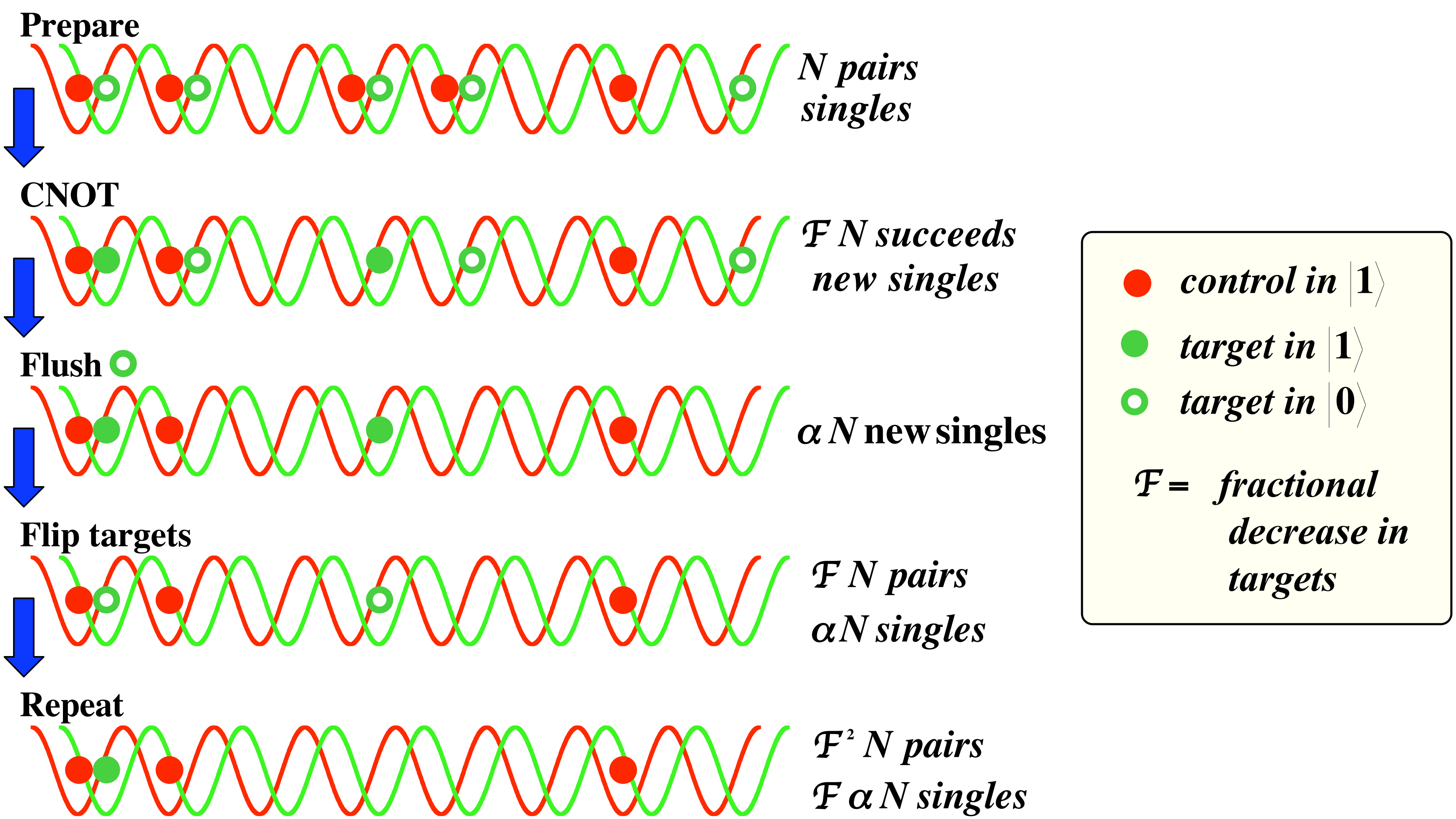
Read out state of target



Experimental strategy

- *work with ensembles of qubit pairs*
- *use quantum gate to identify & isolate pairs*
- *test and optimize logic protocol*

Ensemble measurement of fidelity

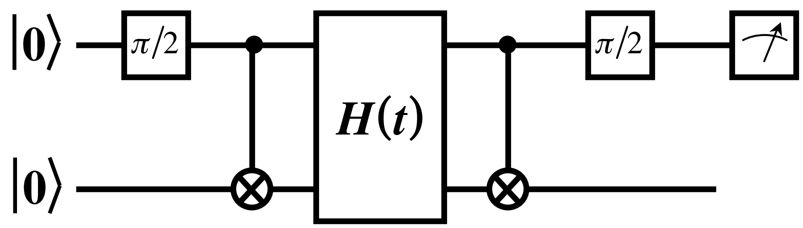


Evaluating *quantum* logic

Problem: *ensemble measurements cannot observe correlations*

- instead we access coherences between 2-qubit states

quantum circuit



probes evolution of state

$$\frac{1}{\sqrt{2}}\left(|00\rangle + e^{-i2\omega t}|11\rangle\right)$$

Wineland, Nature 404 (2000)

Similar approach

classical CNOT
single-qubit rotations
ensemble measurements

➡ *reconstruction of ρ in* $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$ *basis*

various NMR experiments

Quantum state reconstruction for the complete ground hyperfine manifold

Original motivation:

Better tools to observe spinor wavepackets

New application:

Evaluation of error modes in quantum logic

- In analogy to

light fields

motion of trapped ions

molecular vibrations

free atomic waves

coupled spin-1/2 (NMR)

Spin-1/2:

density matrix

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

3 indep. real numbers

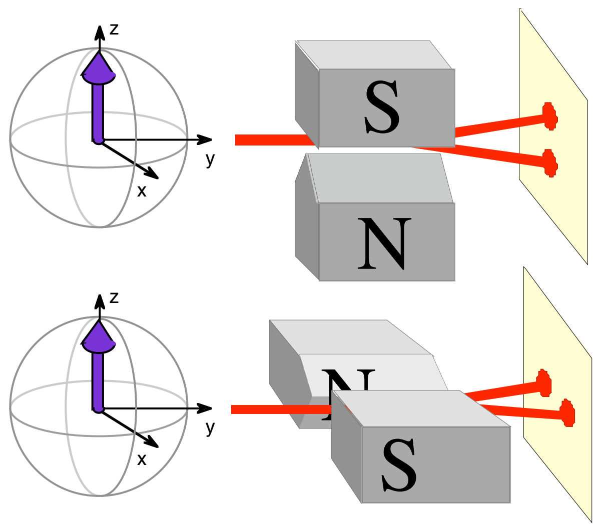
$Tr[\rho] = 1$

$\rho = \rho^{+}$

ρ_{11} | $\mathrm{Re}[\rho_{12}]$ | $\mathrm{Im}[\rho_{12}]$

Stern-Gerlach measurement:

Q-Axis | *Apparatus* | *Observable* | *Information*



$\hat{S}_z \rightarrow \rho_{11}, \rho_{22}$

$\hat{S}_x \rightarrow \mathrm{Re}[\rho_{12}]$

similarly

$\hat{S}_y \rightarrow \mathrm{Im}[\rho_{12}]$

Note: *can equally well rotate system 3 ways & measure* $\hat{S}_z$

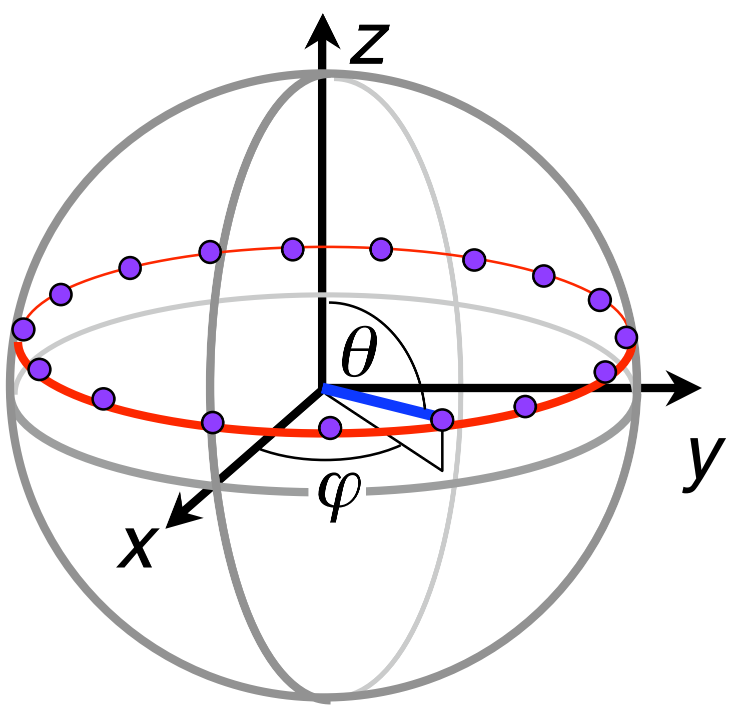
Large Angular Momenta

(Cesium: *ground* (F = 4) *hyperfine manifold*)

Measure: $\hat{F}_z$ $(2F+1=9$ *populations*)

use $4F+1=17$ *geometrical rotations*

Newton & Young 1968



Each rotation:

$$\hat{\rho}'(\theta,\varphi)=\hat{R}\hat{\rho}\hat{R}^{\dagger} \qquad \hat{R}=\hat{R}(\theta,\varphi)$$

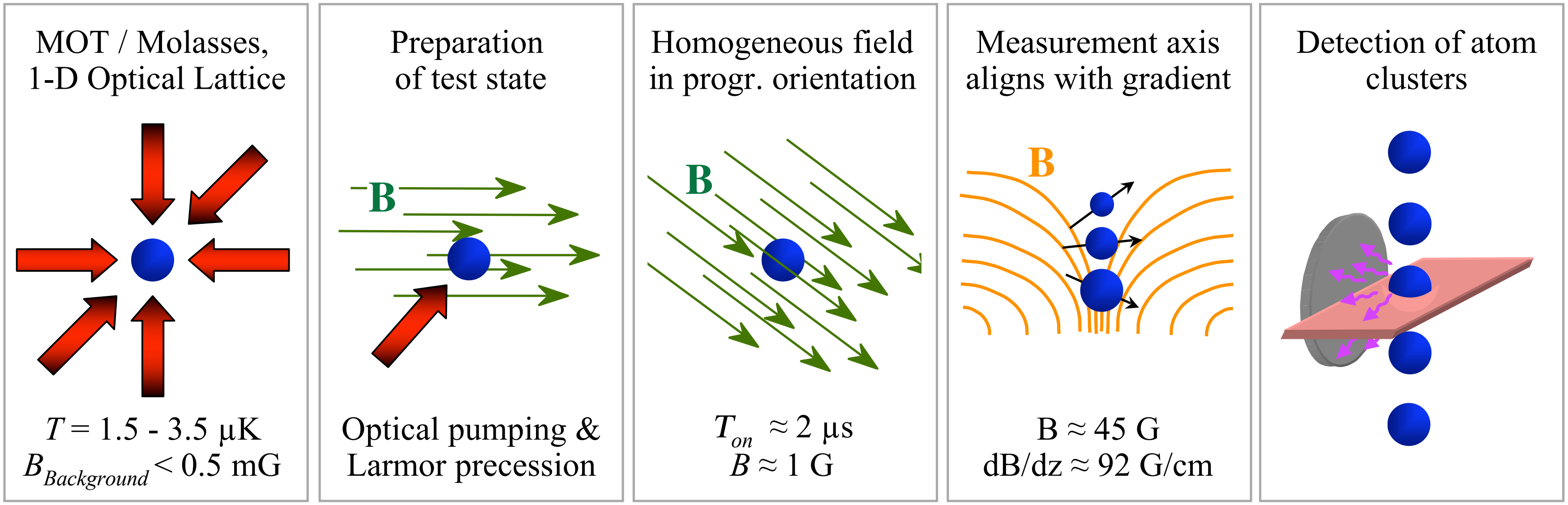
$$\rho'_{ii}=\sum_{j,k}R_{ij}R^{*}_{ik}\rho_{jk}$$

Pseudo-inverse solution

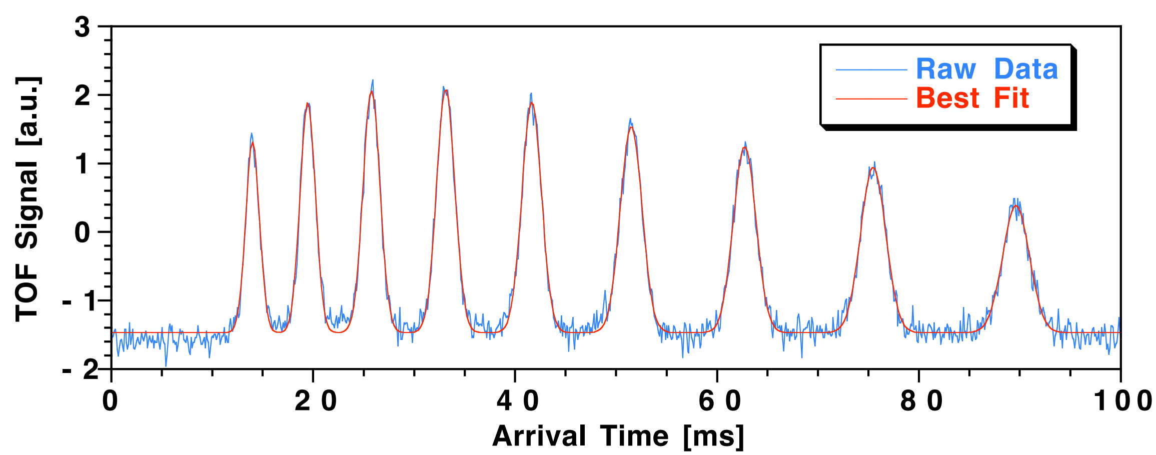
$$\vec{\pi}=\mathbf{M}\cdot\vec{\rho} \;\Rightarrow\; \vec{\rho}=\mathbf{M}^{+}\cdot\vec{\pi}$$

Sometimes yields unphysical $\rho \Rightarrow$
Fit closest physical ρ *to data*

Stern-Gerlach analysis of laser-cooled atoms



well separated arrival times

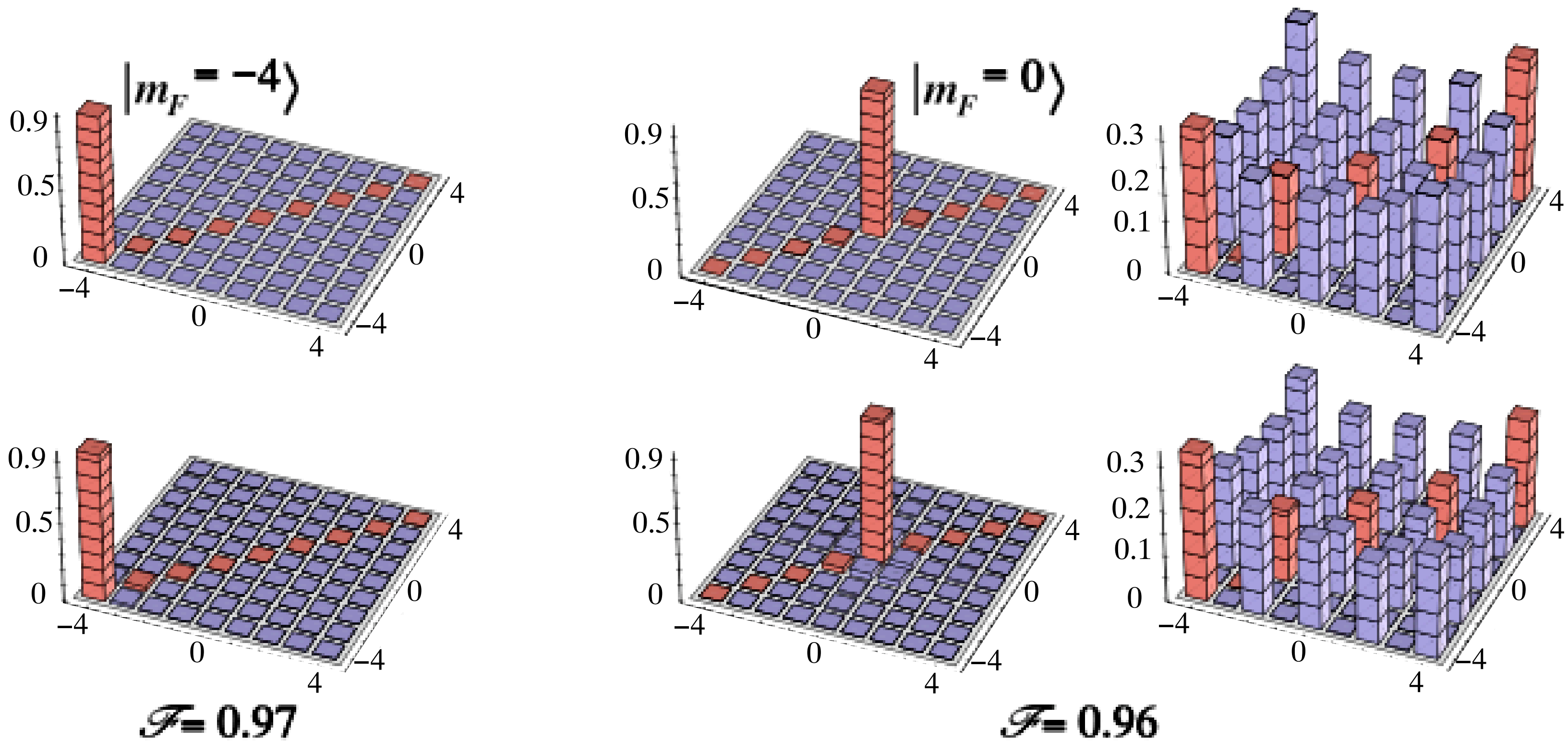


Quantum State Reconstruction

Test with known input states

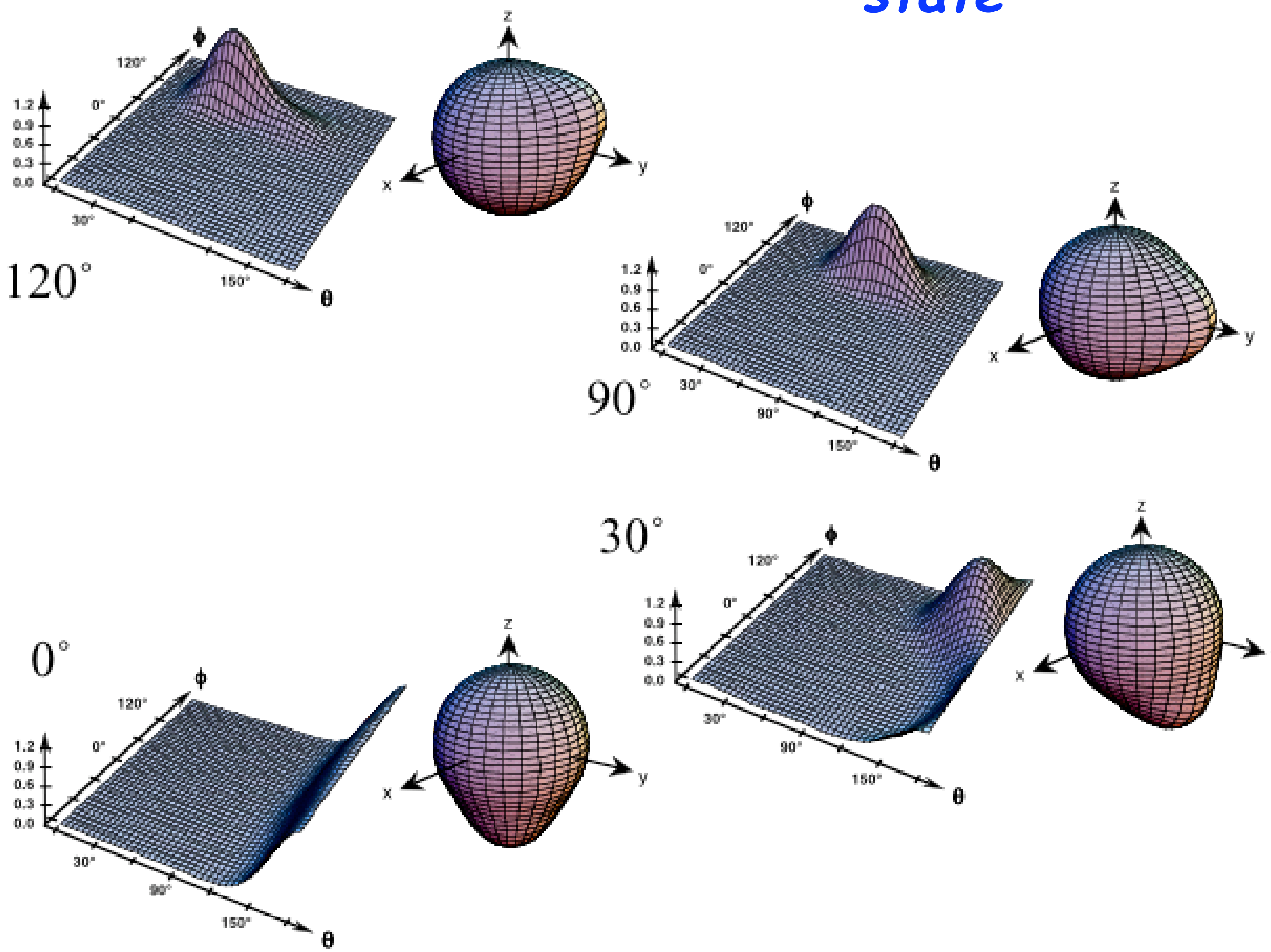
(Cs 6S1/2(F=4) ground state)

Klose, Smith & Jessen
Phys. Rev. Lett. 86, 4721 (2001)



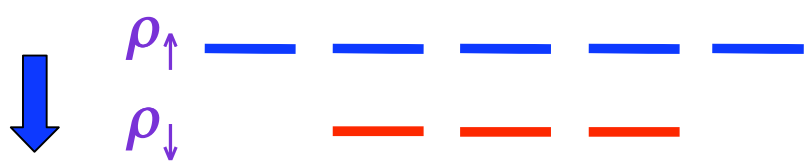
Angular Momentum Wigner Functions

precession of a spin-coherent state

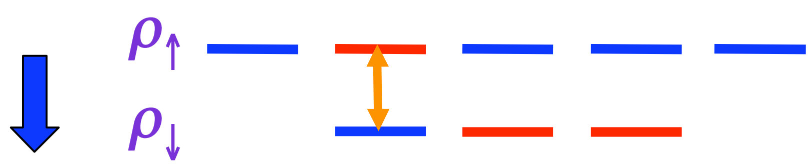


Extension to complete ground manifold

- *measure $\rho_\uparrow$ and $\rho_\downarrow$*



- *swap pair of states*



- *measure $\rho_\uparrow$ and $\rho_\downarrow$*

- *repeat...*

5 swaps required in this example

ρ_{11}	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{16}	ρ_{17}	ρ_{18}
ρ_{21}	ρ_{22}	ρ_{23}	ρ_{24}	ρ_{25}	ρ_{26}	ρ_{27}	ρ_{28}
ρ_{31}	ρ_{32}	ρ_{33}	ρ_{34}	ρ_{35}	ρ_{36}	ρ_{37}	ρ_{38}
ρ_{41}	ρ_{42}	ρ_{43}	ρ_{44}	ρ_{45}	ρ_{46}	ρ_{47}	ρ_{48}
ρ_{51}	ρ_{52}	ρ_{53}	ρ_{54}	ρ_{55}	ρ_{56}	ρ_{57}	ρ_{58}
ρ_{61}	ρ_{62}	ρ_{63}	ρ_{64}	ρ_{65}	ρ_{66}	ρ_{67}	ρ_{68}
ρ_{71}	ρ_{72}	ρ_{73}	ρ_{74}	ρ_{75}	ρ_{76}	ρ_{77}	ρ_{78}
ρ_{81}	ρ_{82}	ρ_{83}	ρ_{84}	ρ_{85}	ρ_{86}	ρ_{87}	ρ_{88}

ρ_{55}	ρ_{52}	ρ_{53}	ρ_{54}	ρ_{51}	ρ_{56}	ρ_{57}	ρ_{58}
ρ_{25}	ρ_{22}	ρ_{23}	ρ_{24}	ρ_{21}	ρ_{26}	ρ_{27}	ρ_{28}
ρ_{35}	ρ_{32}	ρ_{33}	ρ_{34}	ρ_{31}	ρ_{36}	ρ_{37}	ρ_{38}
ρ_{45}	ρ_{42}	ρ_{43}	ρ_{44}	ρ_{41}	ρ_{46}	ρ_{47}	ρ_{48}
ρ_{15}	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{11}	ρ_{16}	ρ_{17}	ρ_{18}
ρ_{65}	ρ_{62}	ρ_{63}	ρ_{64}	ρ_{61}	ρ_{66}	ρ_{67}	ρ_{68}
ρ_{75}	ρ_{72}	ρ_{73}	ρ_{74}	ρ_{71}	ρ_{76}	ρ_{77}	ρ_{78}
ρ_{85}	ρ_{82}	ρ_{83}	ρ_{84}	ρ_{81}	ρ_{86}	ρ_{87}	ρ_{88}

- potentially powerful tool to debug -

error modes of quantum gates

Summary

- *Qubits can be encoded in atoms trapped in optical lattices*
- *One- and two-qubit gates can be implemented via Raman transitions and dipole-dipole interactions*
- *Logic protocols can be evaluated and optimized in ensemble experiments*
- *Good progress towards 3D Raman sideband cooling*
- *Powerful control & measurement tools available, in tradition of quantum optics and laser spectroscopy*