Encoded Universality – adapting quantum processing to physical interactions

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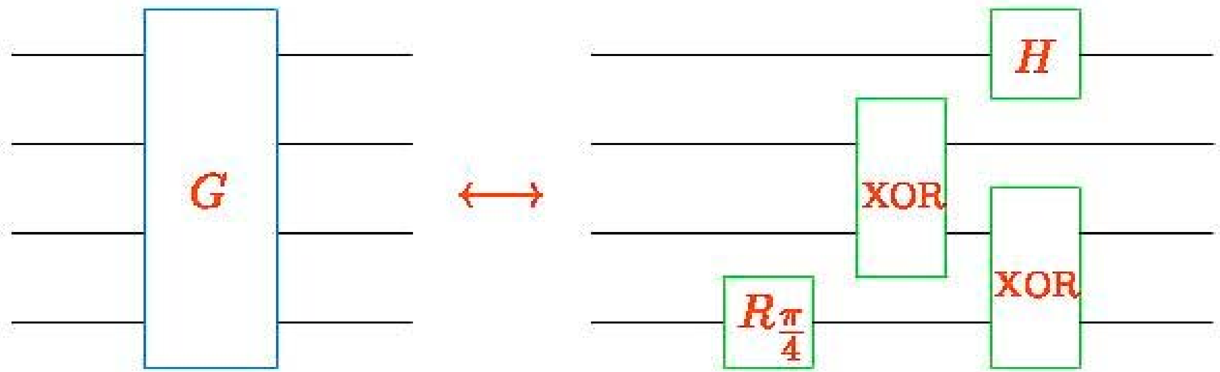
Overview

- Universal quantum computation - some history
- Change of paradigm
- Example : exchange-only qc
- General: Lie algebra formalism for encoded universal computation
- solid state: exchange-based qc

  - Heisenberg, symmetric "XY", asymmetric "XY", with crossterms, ...
- gas phase: scalable ion trap qc

Quantum circuits

Quantum circuit: $(G \in U(16))$

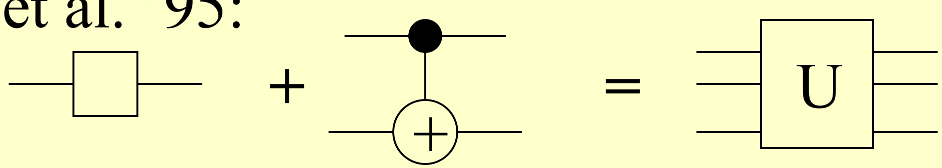


Theorem [DiV95,BMPRV99]:

Every transformation on n-qubit decomposes into transformations on 1-qubit and 2-qubit.

$\Longrightarrow$ Universal family.

Barenco et al. ’95:



MANTRA

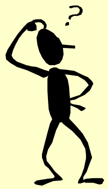
Single-qubit gates and CNOT $\rightarrow$ every unitary transformation

The problem

"Easy" and "hard" interactions (system-dependent)

"Easy": intrinsic interactions "natural" to the system, easy to tune, rapid
"Hard": slower, require higher device complexity, high decoherence

System	"Easy"	"Hard"
photon-qubits	Single qubit operations (linear optics)	Two qubit operations (non-linearity, non-deterministic qc)
spins in solid state	Two qubit operations (electrically tuned J)	Single qubit operations (local focused B-fields)
atomic states	Single qubit operations (atomic transition)	Two qubit operations

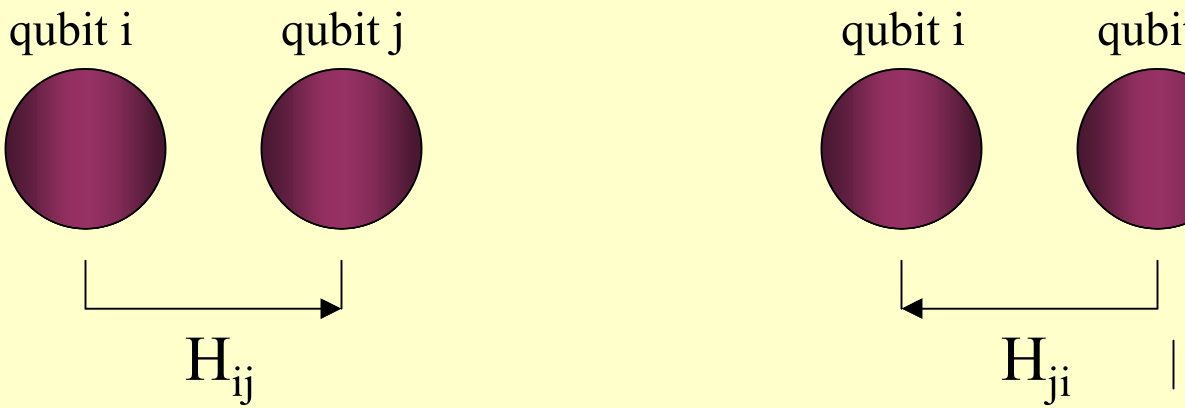


Can we avoid "hard" interactions?

"Almost" every interaction is universal

Deutsch et al.('95), Lloyd ('95) :

Almost any interaction on two qubits is universal



In the generic sense

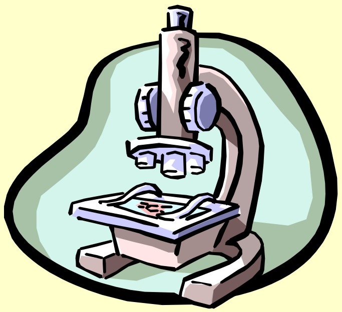
but ...

Nature is not generic!

Encoded Universality

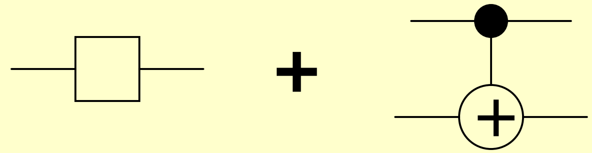
Traditionally:

manipulate the physical system



$H_1, H_2, \ldots$

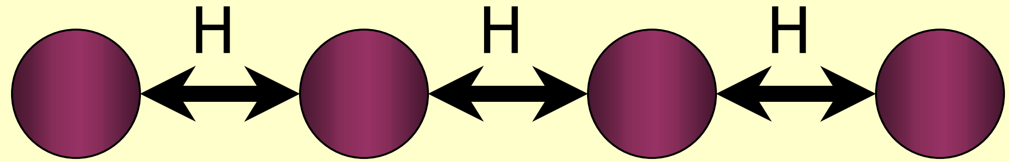
to produce



Universal encoded computation:

interactions *given* by the physical system

Encoding?



find a way to **make these universal**

language of Hamiltonians

U(t) = exp(iHt) which *interactions* are universal?

given $\mathcal{H}=\{H_1, H_2, \ldots, H_n\}$ can one generate any unitary transformation? (exact or approximate)

$$e^{it_1 H_1} \cdot e^{it_2 H_2} \cdot e^{it_3 H_3} \cdot \ldots \approx U$$

$\mathcal{H}$ has to generate the Lie algebra *su(N)* of the unitary group *SU(N)*

1) $$U(a) = e^{iaH}$$ *scalar multiple*

2) $$e^{i(aH_1 + bH_2)} = \lim_{p \to \infty} \{e^{iaH_1/p} e^{ibH_2/p}\}^p$$ *linear combination*

3) $$e^{i[H_1, H_2]} = \lim_{p \to \infty} \{e^{-iH_1/\sqrt{p}} e^{iH_2/\sqrt{p}} e^{iH_1/\sqrt{p}} e^{-iH_2/\sqrt{p}}\}^p$$ *Lie bracket*

e.g. Heisenberg exchange, E

$$E_{ij} = \sum_{\alpha=x,y,z} \sigma_\alpha^i \cdot \sigma_\alpha^j$$

(Pauli matrices)

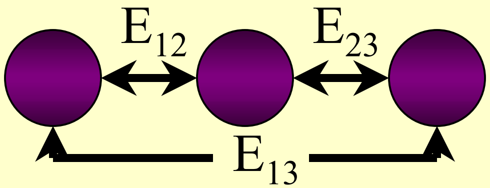
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|0\rangle|1\rangle \xleftrightarrow{E} |1\rangle|0\rangle$$

- **omnipresent in solid state physics (« easy »)**
- **is not universal: preserves the total spin of the qubits**

Lie algebra of E :

On three qubits:



$$[H_0, H_i] = 0$$

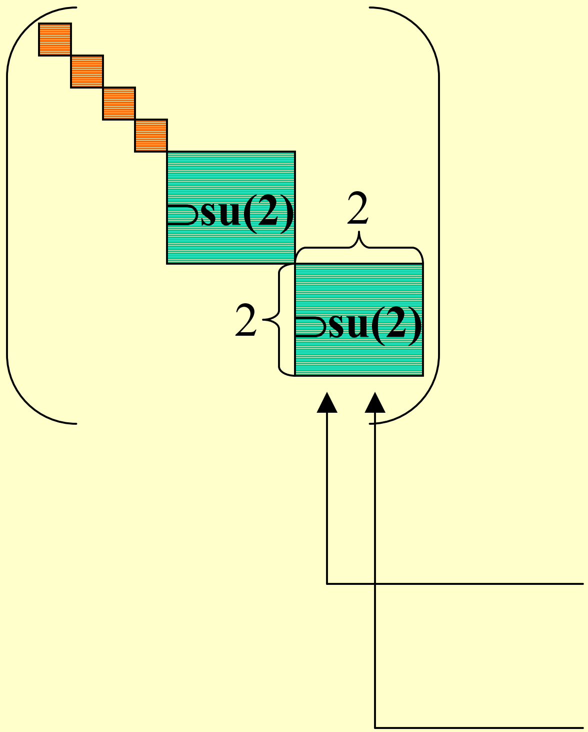
$$[H_i, H_j] = i\varepsilon_{ijk} H_k$$

$$\left.\begin{array}{l} H_0 := E_{12} + E_{13} + E_{23}; \ H_1 := \frac{1}{4\sqrt{3}}(E_{13} - E_{23}) \\ H_3 := \frac{1}{12}(-2E_{12} + E_{23} + E_{13}); \ H_2 := i[H_3, H_1] \end{array}\right\} \quad \{H_1, H_2, H_3\} \Longrightarrow \text{su}(2)$$

the algebra *L(E)* of E (on 3 qubits)

the algebra $L^3(E)$ splits as:

$L^3(E) \cong$



$$L^3(E) \cong \boldsymbol{S}_1 \otimes \mathrm{I}_4 \oplus \boldsymbol{S}_2 \otimes \mathrm{I}_2$$

$$\mathrm{su}(2) \subset \boldsymbol{S}_2$$

Encoded qubit ?

$$|0_L\rangle = \tfrac{1}{\sqrt{2}}\left(|010\rangle - |100\rangle\right)$$

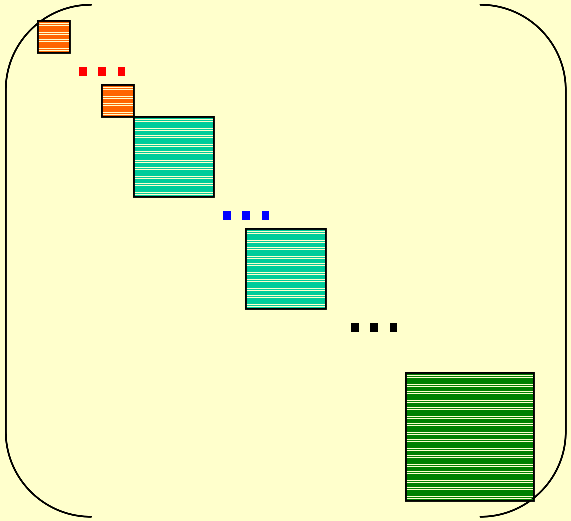
$$|1_L\rangle = \tfrac{1}{\sqrt{6}}\left(2|001\rangle - |010\rangle - |100\rangle\right)$$

simulation of all operations of one qubit (su(2)) with $L^3(E)$ on the encoded qubit !

the algebra $L^n(E)$ of E (on n qubits)

the algebra $L^n(E)$ splits as:

$L^n(E) \cong$



$$L^n(E) \cong \bigoplus_{j=0\,(1/2)}^{n/2} \boldsymbol{S}_{n_j} \otimes I_{2j+1}$$

commutant L' of $L^n(E)$: $\quad L' = \{A : [A, L] = 0 \quad \forall L \in L^n(E)\}$

L' is generated by $S_\alpha = \sum_{i=1}^{n} \sigma_\alpha^n$ (« spin » algebra su(2))

$$L' \cong \bigoplus_{j=0\,(1/2)}^{n/2} I_{n_j} \otimes S'_{2j+1}$$

as a Lie algebra, L' splits into irreducible representations of su(2)

useful theorem...

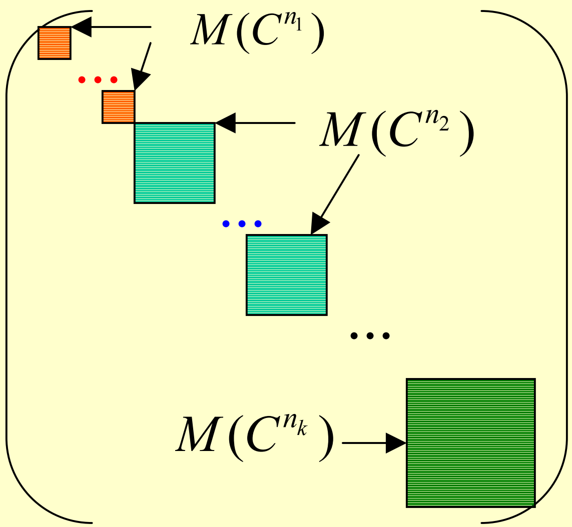
Let S be a †-closed algebra closed under ***multiplication*** *and* ***linear combination****. Then the underlying space H is isomorphic to*

$$\mathrm{H} \cong \bigoplus_{j \in \mathcal{J}} C^{n_j} \otimes C^{d_j}$$

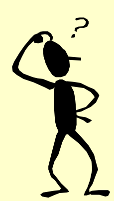
such that S and its commutant S' split as:

$$\mathrm{S} \cong \bigoplus_{j \in \mathcal{J}} M(C^{n_j}) \otimes I_{d_j} \qquad \mathrm{S}' \cong \bigoplus_{j \in \mathcal{J}} I_{n_j} \otimes M(C^{d_j})$$

where $M(C^d)$ $(M(C^n))$ is the algebra of all matrices on C^n)



$$su(n_j) \subset M(C^{n_j})$$



universal computation "for free"?

BUT...

For S the interaction algebra, e.g.,

$$H = \sum_{\alpha} S_{\alpha} \otimes B_{\alpha}, \ \ \{S_{\alpha}\} \to S$$

the full multiplicative algebra *S* is not at our disposition, only the Lie algebra *L*

however the Lie algebra splits into irreducible components in the same basis:

$$\mathcal{L} \simeq \bigoplus_{j \in \mathcal{J}} \boldsymbol{S}_{n_j} \otimes I_{d_j}$$

central problem of "Encoded Universality"

Given an ensemble of generators **H** *with Lie algebra L(***H***) which splits as*

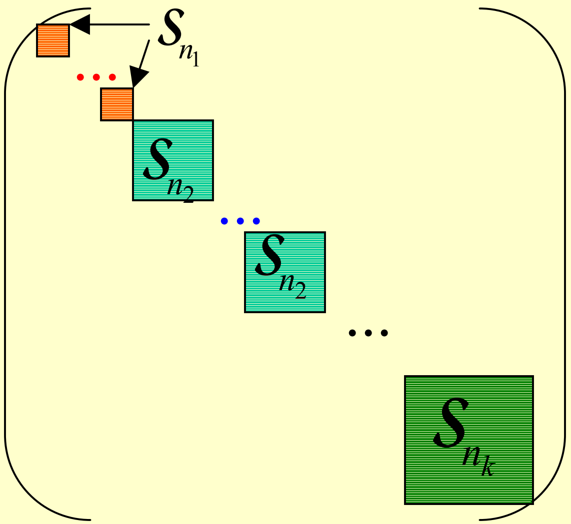
$$\mathcal{L} \cong \bigoplus_{j \in \mathcal{J}} \mathcal{S}_{n_j} \otimes I_{d_j}$$

can one find a component $\mathcal{S}_{n_j} \otimes I_{d_j}$ *s.t.* $\mathcal{S}_{n_j}$ *contains* $su(n_j)$*?*

$su(n_j) \in \mathcal{S}_{n_j}$?

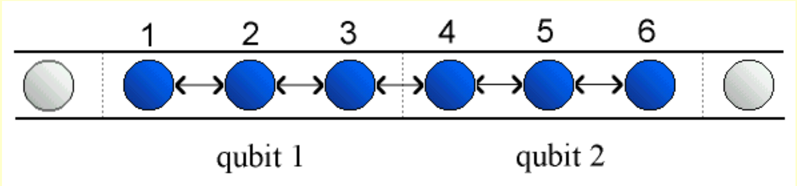
↓ Yes

encode the quantum information into the corresponding sub-space of dimension: n_j



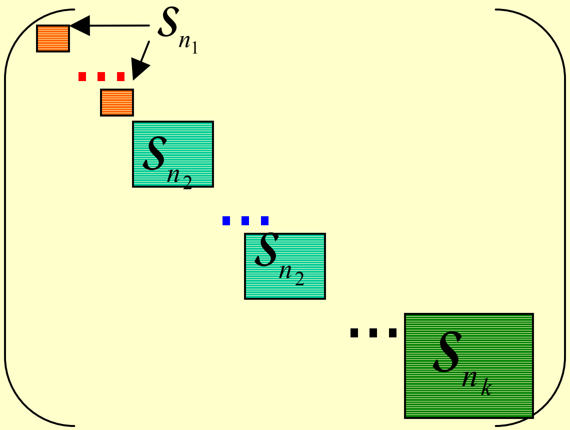
D. Bacon, J. Kempe, D.P. DiVincenzo, D.A. Lidar, K.B. Whaley, *" Encoded Universality in Physical Implementations of a Quantum Computer"* **Proceedings of IQC '01**, Rinton Press, Australia, (2001); also quant-ph/010240

Conjoining – a useful tensor structure

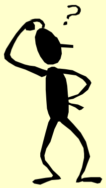


introduce a cutoff that defines a single "qudit".

In principle:



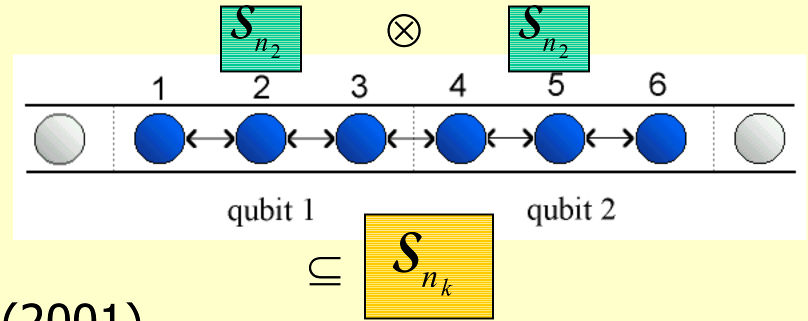
for larger n, find larger component S_{n_k} with better encoding ratio r?



$$r = \log_2(\dim S_{n_k})/n$$

need to guarantee uniformity of quantum circuits!
("form" of circuit should not depend on size of problem)
introduce cutoff -> tensor product structure

conjoining subsystems:

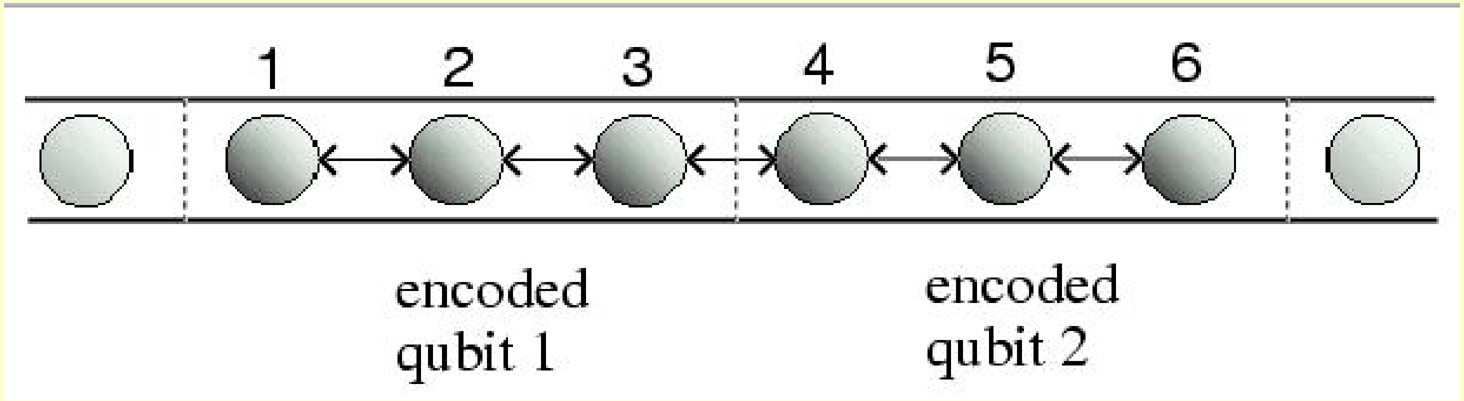


Kempe et al., Qu. Inf. & Comp. 1, 33 (2001)

Heisenberg interaction (isotropic exchange)

$$\mathbf{E}_{i,j} = \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- E is universal with encoding*
- introduce tensor structure, eg. blocks with 3 qubits**

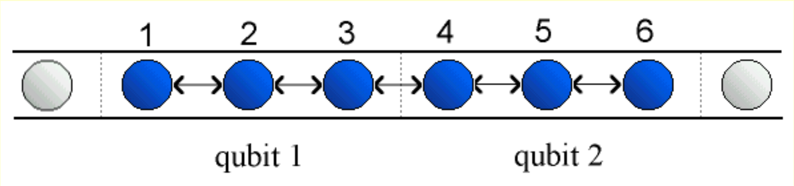


efficient implementation of encoded gates found by numerical search**

serial - 19 operations for CNOT, 4 operations for 1-qubit
parallel - 7 operations for CNOT, 3 operations for 1-qubit

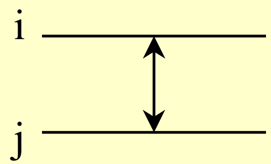
*Kempe, Bacon, Lidar, Whaley, Phys. Rev. A 63:042307 (2001)
DiVincenzo, Bacon, Kempe,Whaley, NATURE **408, 339 (2000)

exchange-only CNOT

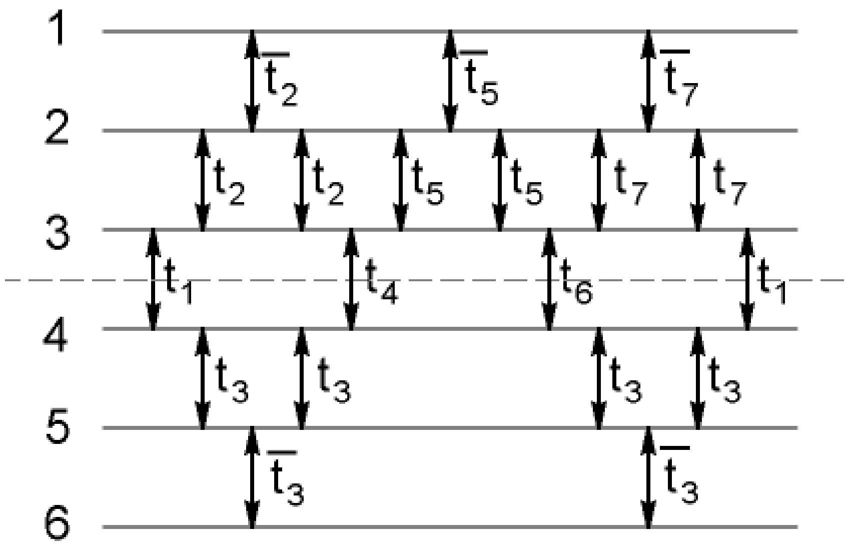


nearest neighbor exchange coupling

exchange gate



$$= \exp\left(iJE_{i,j}t\right)$$



t_1=0.410899(2) t_5=0.414720(10)

t_2=0.207110(20) t_6=0.147654(12)

t_3=0.2775258(12) t_7=0.813126(12)

t_4=0.640505(8) $\tan(\pi t)\tan(\pi\bar{t}) = -2$

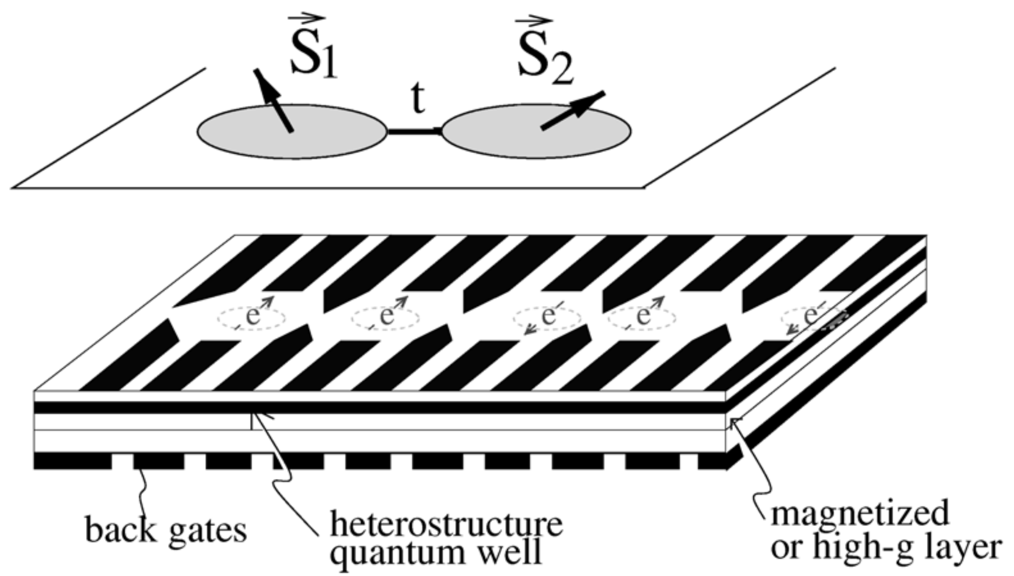
DiVincenzo, Bacon, Kempe, Burkard, Whaley, *Nature* **408**, 339 (2000)

Tradeoffs

factor of 3 in space (encoding)

factor of ~ 10 in time

Spintronic Quantum Computer



D.P. DiVincenzo, G. Burkard, D. Loss, E.V. Sukhorukov, in *Quantum Mesoscopic Phenomena And Mesoscopic Devices In Microelectronics,* eds. I.O. Kulik and R. Elliatioglu (NATO Advanced Study Institute, Turkey) 1999

qubit: spin of single electron

use **exchange interaction** between qubits for two-qubit gates
tune by raising/lowering barrier between dots to control overlap
coupling strength: on $\approx$0.1 meV, off $\approx$0

single qubit gates needed to supplement exchange $\Rightarrow$ high demands on g-factor engineering, strong inhomogeneous magnetic fields, slow microwave manipulations, ...

Physical Systems

Solid state systems with spin-spin interaction

isotropic exchange

$H_I(t) = J(t)\, \mathbf{S_1} \cdot \mathbf{S_2}$

Loss, DiVincenzo,
PRA **57**,120 (1998)

spin-orbit coupling anisotropy

$H_{SO}(t) = \mathbf{b}(t)\, (\mathbf{S_1} \times \mathbf{S_2}) + \mathbf{S_1} . B(t) . \mathbf{S_2}$

Moriya, Phys.Rev. **120**, 91 (1960)
Bonesteel et al., PRL **87**, 207901 (2001)

$$H(t) = H_I(t) + H_{SO}(t) + H_D(t)$$

cross terms
$\sigma_{1x}\sigma_{2y}$, etc.

$J(t)(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y} + \sigma_{1z}\sigma_{2z})$

dipole-dipole interaction

$H_D(t,R) = d(t, R)\, (\mathbf{S_1} . \mathbf{S_2} - 3\, \mathbf{S_1} . \Gamma(t) . \mathbf{S_2})$

Burkard, Loss, PRL **88**, 047903

Qubits coupled via cavity modes

$$H(t) = J(t)\, (\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y})$$

Imamoglu et al., PRL **83**, 4204 (1999)

Anisotropic Exchange (XY)

$$\left(H_{XY}\right)_{ij}=\frac{J_{ij}}{2}\left(\sigma_x^i\sigma_x^j+\sigma_y^i\sigma_y^j\right)\equiv J_{ij}A_{ij}$$ "XY"-interaction

encode into qutrit: $|0_L\rangle=|100\rangle$ $\rightarrow$ operators $A_{12}^L, A_{23}^L, A_{31}^L$

$|1_L\rangle=|010\rangle$

$|2_L\rangle=|001\rangle$

$$e.g.,\quad A_{12}^L=\begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix}$$

construct $\left[A_{\alpha\beta}^L, A_{\gamma\delta}^L\right]\rightarrow su(3)$ 1-qutrit operations

- conjoin qutrits: $|0_L\rangle\otimes|1_L\rangle\rightarrow$ 9D subspace
- H_{XY} generates su(9) on this subspace

via commutant of H_{XY}

"Truncated qubit": use $|0_L\rangle=|100\rangle$ and $|1_L\rangle=|010\rangle$ only

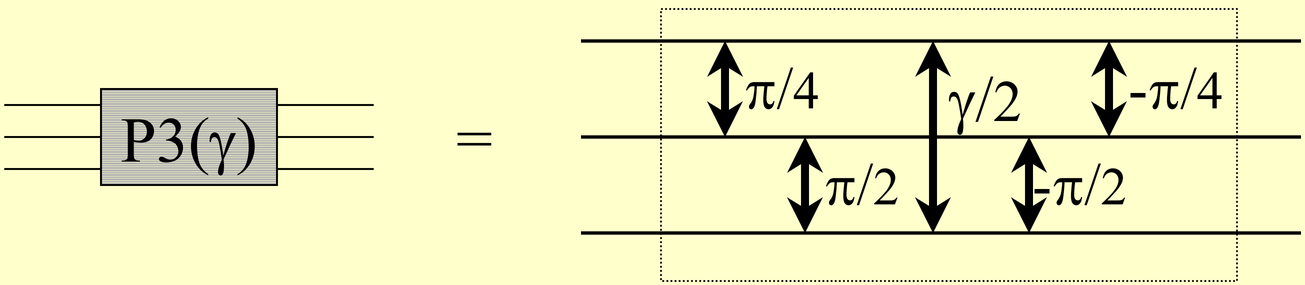
effectively $\begin{array}{l}|0_L\rangle=|10\rangle\\|1_L\rangle=|01\rangle\end{array}$ with an ancillary qubit $|0\rangle$ for 1-qubit gates

J. Kempe, D. Bacon, D.P. DiVincenzo, K.B. Whaley, "*Encoded universality from a single physical interaction*", in **«Quantum Information and Computation»;** Special Issue, Vol. 1, 33 (2001)

Exact Gates for XY

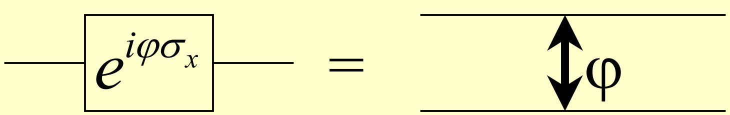
Gate sequences: 7 operations for single qubit operations (serial)
5 operations for Sqrt (-ZZ) (≡ controlled phase)

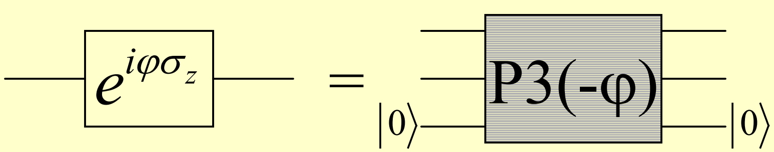
"P3"-gate:



Truncated qubit: $|0_L\rangle = |10\rangle \quad |1_L\rangle = |01\rangle$

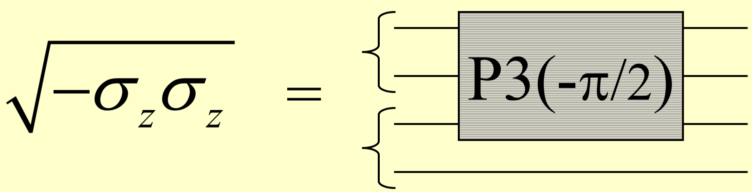
Single qubit operations:





$$U = e^{i\varphi_1\sigma_x} \circ e^{i\varphi_2\sigma_z} \circ e^{i\varphi_3\sigma_x} \quad \text{(Euler angles)}$$

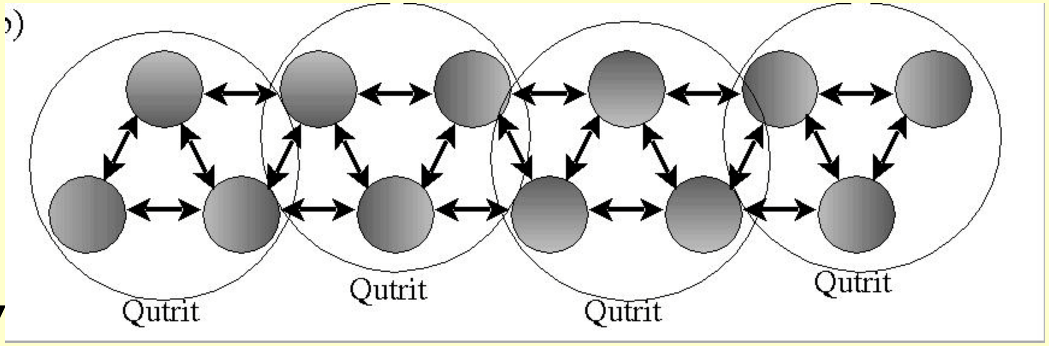
Two-qubit operation:



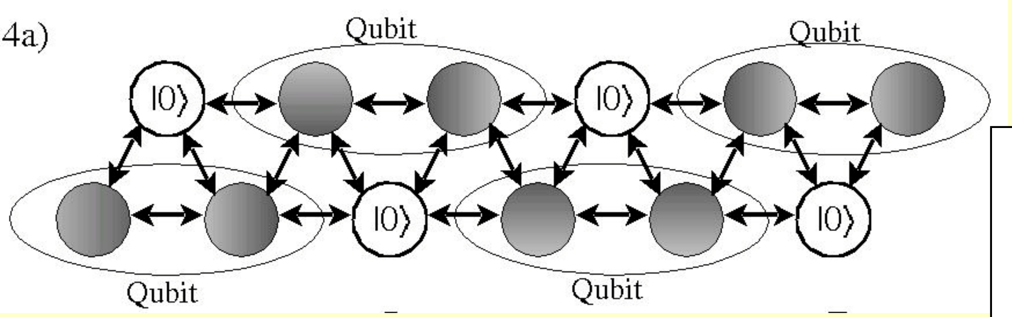
J. Kempe and K.B.Whaley,
Phys. Rev. A 65, 052330 (2002)

Layout – Anisotropic Exchange

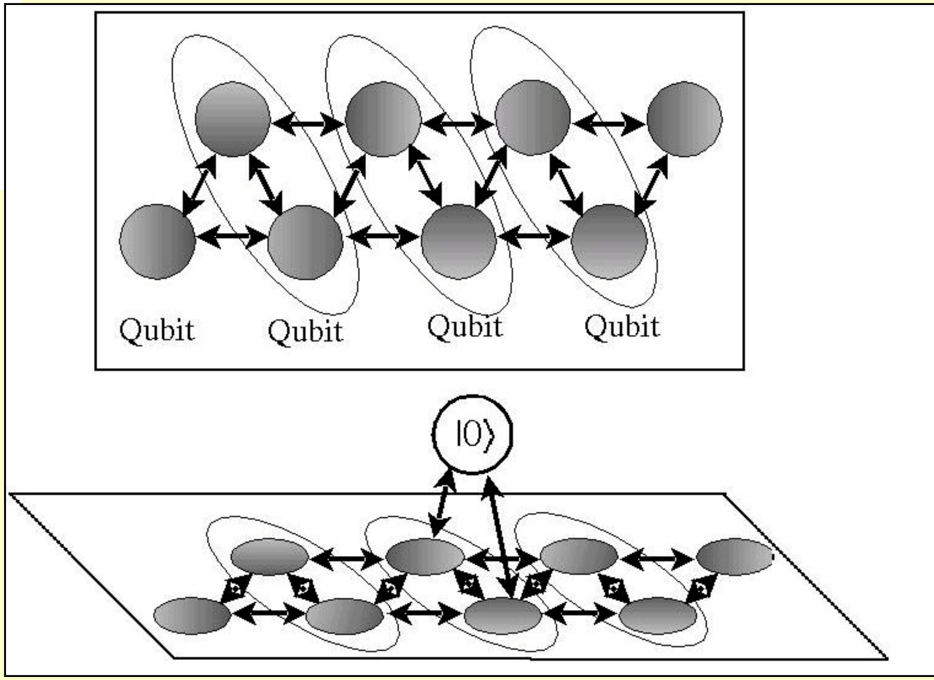
a) triangular array (qutrit)



b) “truncated qubit”



or



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Moriya, Phys.Rev. **120**, 91 (1960)
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Burkard, Loss, PRL **88**, 047903

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Generalized Anisotropic Exchange

$$2J \equiv (J_{ij}^{x} + J_{ij}^{y})$$ symmetric

$$2j \equiv (J_{ij}^{x} - J_{ij}^{y})$$ antisymm

Two contributions to H (in 2D):

1) symmetric term, which couples the physical qubit states |01> and |10>

$$\mathbf{H_{ij} = J\,(\sigma_{x,i}\,\sigma_{x,j} + \sigma_{y,i}\,\sigma_{y,j}) + K\,(\sigma_{x,i}\,\sigma_{y,j} - \sigma_{y,i}\,\sigma_{x,j})}$$

2) antisymmetric term, coupling physical qubit states |00> and |11>

$$\mathbf{h_{ij} = j\,(\sigma_{x,i}\,\sigma_{x,j} - \sigma_{y,i}\,\sigma_{y,j}) + k\,(\sigma_{x,i}\,\sigma_{y,j} + \sigma_{y,i}\,\sigma_{x,j})}$$

Matrix representation

in the basis $\mathcal{B} = \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$$\begin{pmatrix} 0 & 0 & 0 & j+ik \\ 0 & 0 & J+iK & 0 \\ 0 & J-iK & 0 & 0 \\ j-ik & 0 & 0 & 0 \end{pmatrix}$$

Vala & Whaley, PRA **66**, 022304 (2002)

3-qubit encoding

Algebraic approach: commutant of H is spanned by

$\mathbf{Z} = \otimes_k \mathbf{s}_{z,k}$ and $\mathbf{X} = \otimes_k \mathbf{s}_{x,k}$, for $k = 1, 2, 3$

2-by-2 block diagonal structure in the basis set

B = {|000>, |111>,|110>, |001>,|101>, |010>,|011>,|100>}

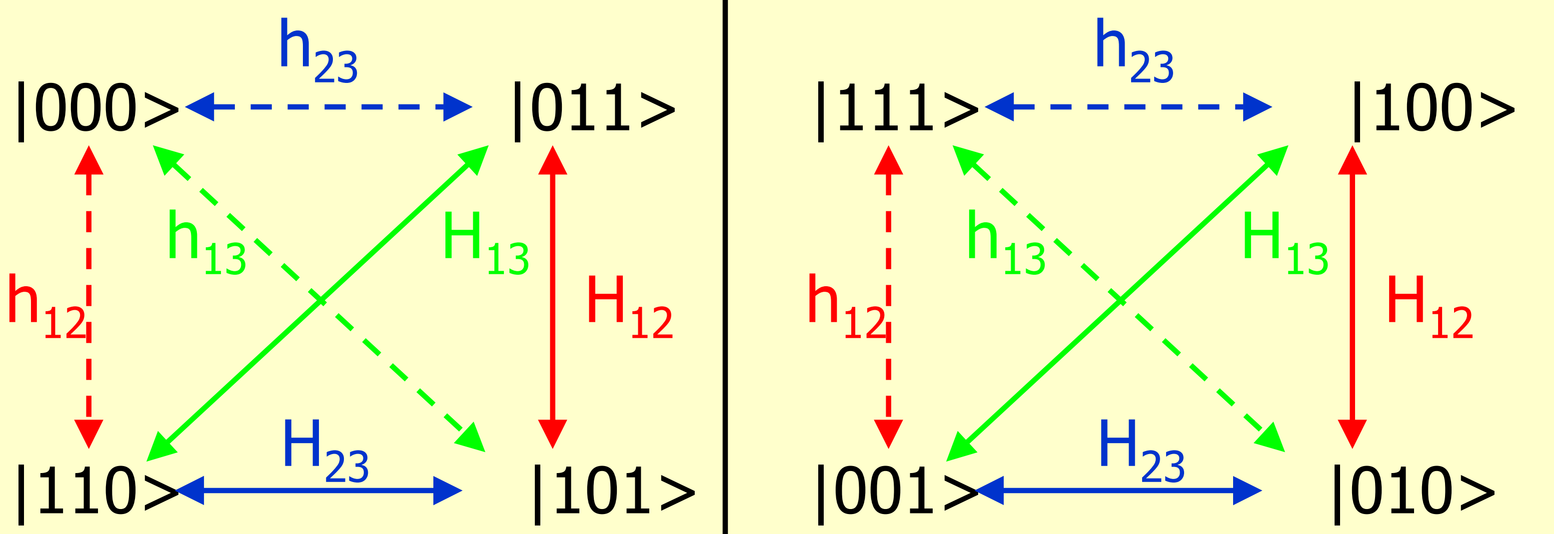
the Lie algebra of H and Hilbert space split as

$\mathbf{L} = \mathbf{L}(4) \otimes I_2$ and $\mathbf{H} = C^4 \otimes C^4$

and code spaces are defined as

(I) {|000>,|110>,|101>,|011>}
(II) {|111>,|001>,|010>,|100>}

Action of the Hamiltonian



Code I　　**Code II**

Hamiltonian can be expressed in either basis

{|000>,|110>,|101>,|011>} or {|111>,|001>,|010>,|100>}:

$$\begin{pmatrix} 0 & h & 0 & 0 \\ h & 0 & 0 & 0 \\ 0 & 0 & 0 & H \\ 0 & 0 & H & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & h & 0 \\ 0 & 0 & 0 & H \\ h & 0 & 0 & 0 \\ 0 & H & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & h \\ 0 & 0 & H & 0 \\ 0 & H & 0 & 0 \\ h & 0 & 0 & 0 \end{pmatrix}$$

encoded logical qubit

the complete su(2) can be generated over subsets

{|110>,|101>,|011>} and {|001>,|010>,|100>}

of code spaces

{|000>,|110>,|101>,|011>} and {|111>,|001>,|010>,|100>}

→ the qubit can be represented by any pair of states from one subset, e.g.,

$|0_L\rangle = |110\rangle \quad |1_L\rangle = |011\rangle$ ...

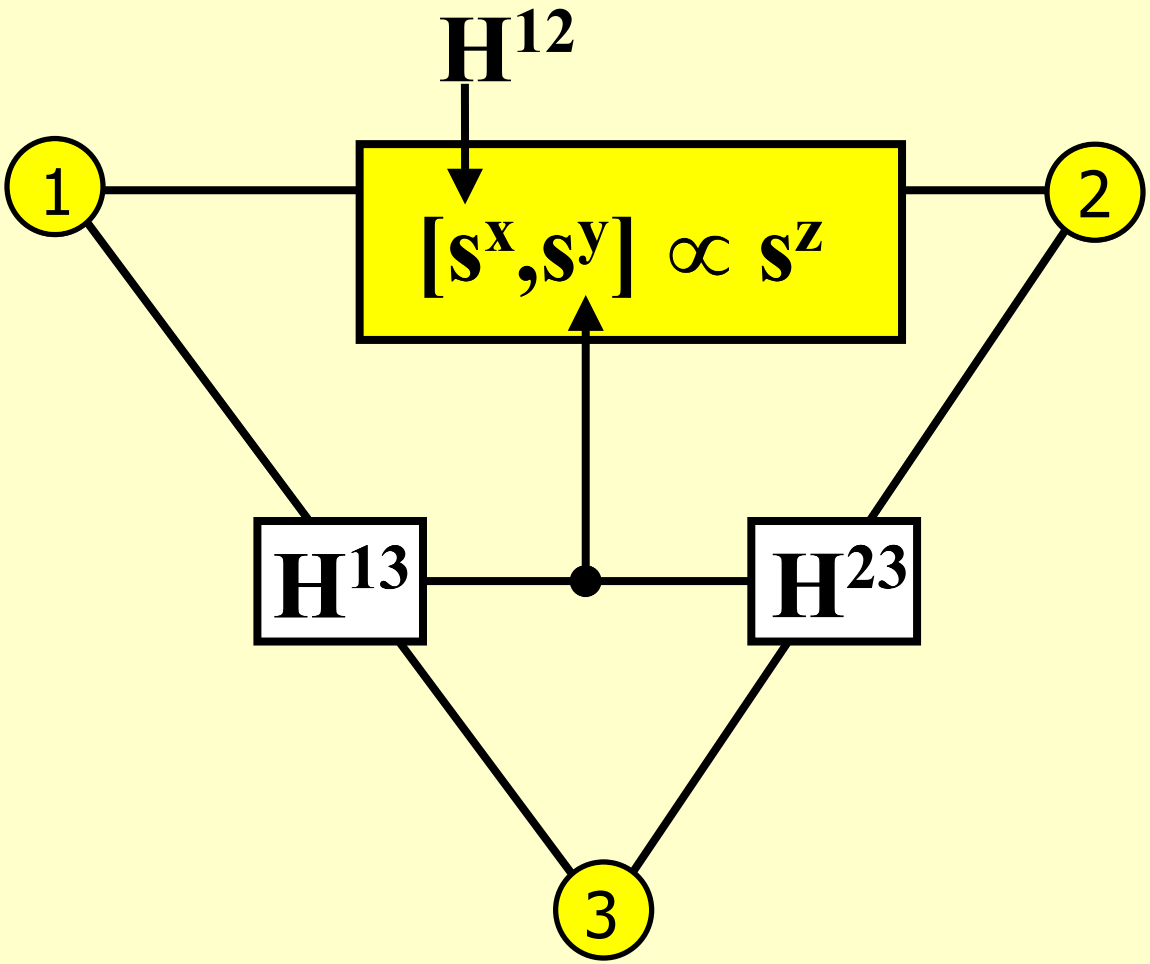
$|0_L\rangle = |001\rangle \quad |1_L\rangle = |100\rangle$ ...

TRIANGULAR LAYOUT ...

Single-Qubit Gates

the full su(2) algebra over a single logical qubit is generated via the commutation relations between exchange interactions over physical qubits:

e.g.



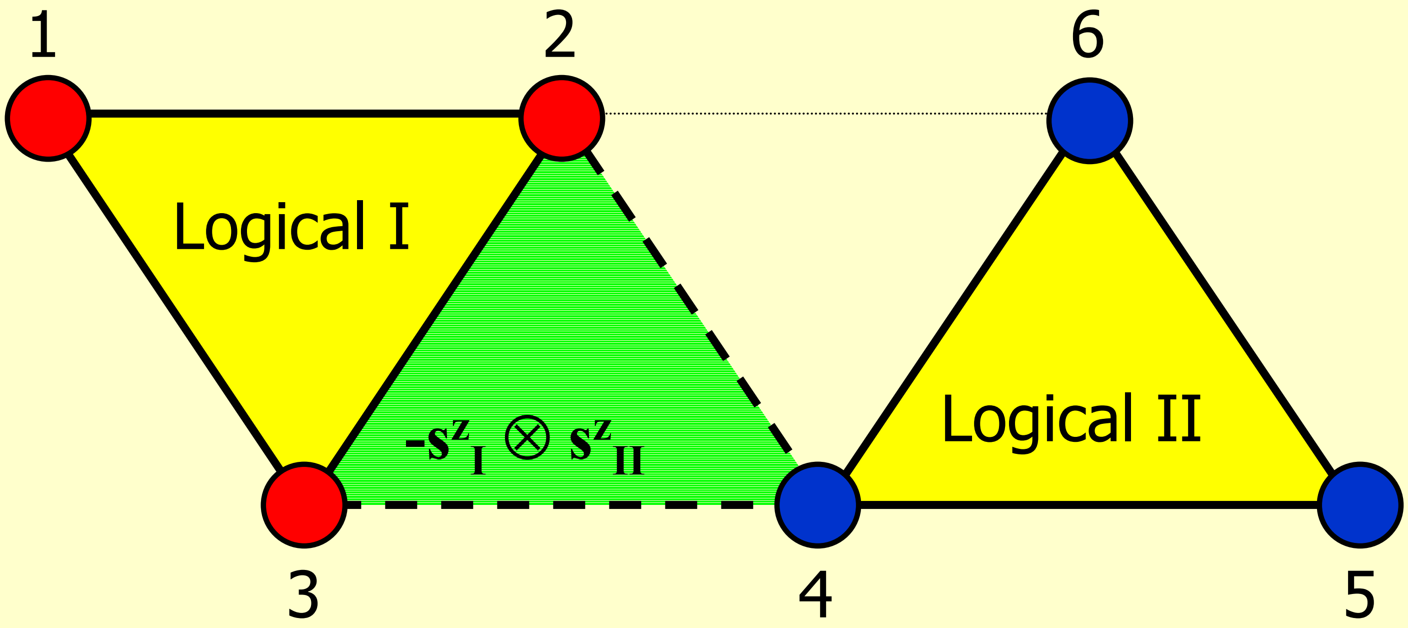
$[H^{13},H^{23}] = i\,(J^2 - j^2)\, s^{y,12}$

and

$[H^{12}, s^{y,12}] = i\, 2\, J\, s^{z,12}$

Two-Qubit Gates

Entangling two-qubit operation C(Z) results from application of the encoded s_z operation onto the physical qubits 2-3-4 in the triangular architecture, plus single-qubit operations



Gate Sequences

Vala & Whaley, PRA **66**, 022304 (2002)

For the Hamiltonian

$$\mathbf{H_{ij} = J\,(\,\sigma_{x,i}\,\sigma_{x,j} + \sigma_{y,i}\,\sigma_{y,j}\,) + j\,(\,\sigma_{x,i}\,\sigma_{x,j} - \sigma_{y,i}\,\sigma_{y,j}\,)}$$

commutation relations are applied through conjugation

$$U(\sigma^{y,12},\phi) = \exp(-i\,\sigma^{y,12}\,\phi) = \exp(iH^{13}\Theta/2)\,\exp(iH^{23}\phi/2J)\,\exp(-iH^{13}\Theta/2)$$

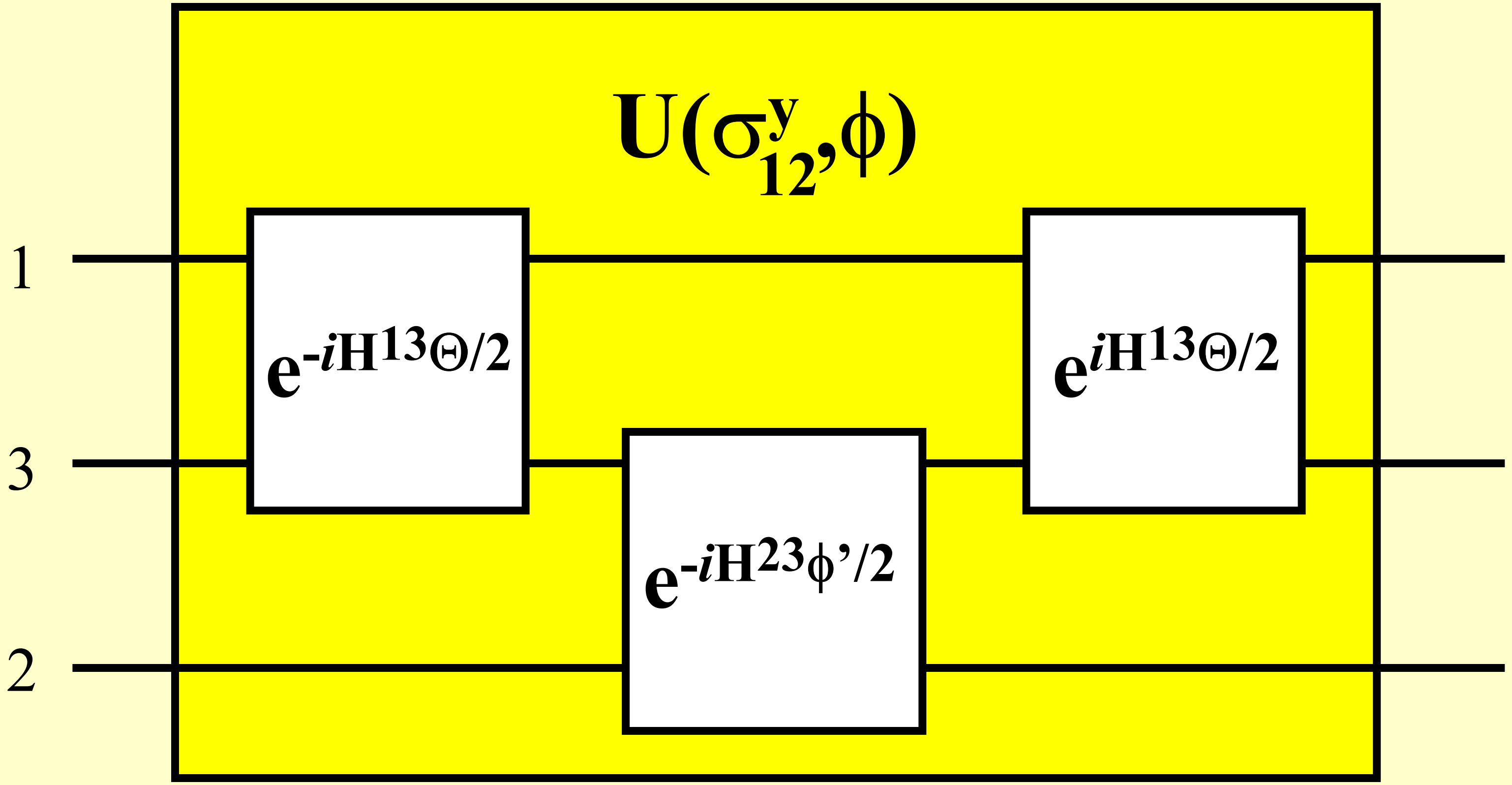
with condition on the duration of conjugation:

$$\Theta = 0\ (\mathrm{mod}\ \pi)/j = (\pi/2)\ (\mathrm{mod}\ \pi)\ /\ J$$

conjugation turns off the antisymmetric terms (~j)

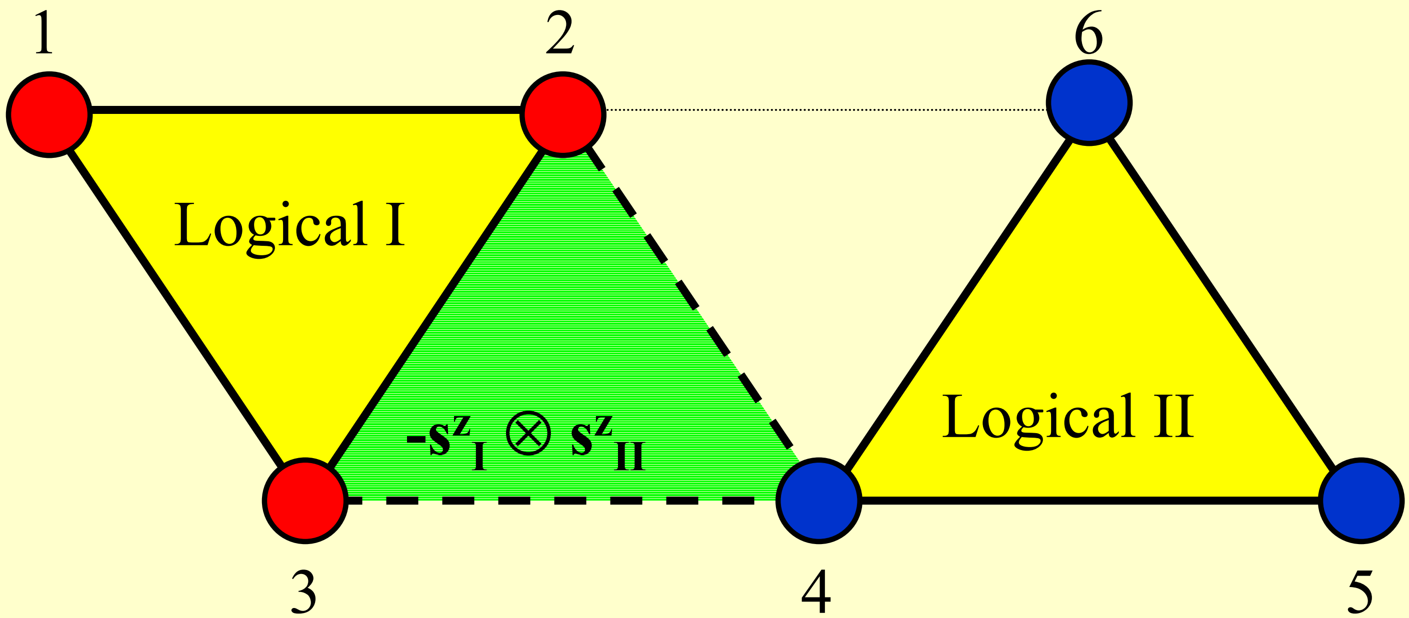
a similar construction is valid for the general anisotropic interaction with cross-product terms

quantum circuit for a commutator



Implementing generalized anisotropic exchange:

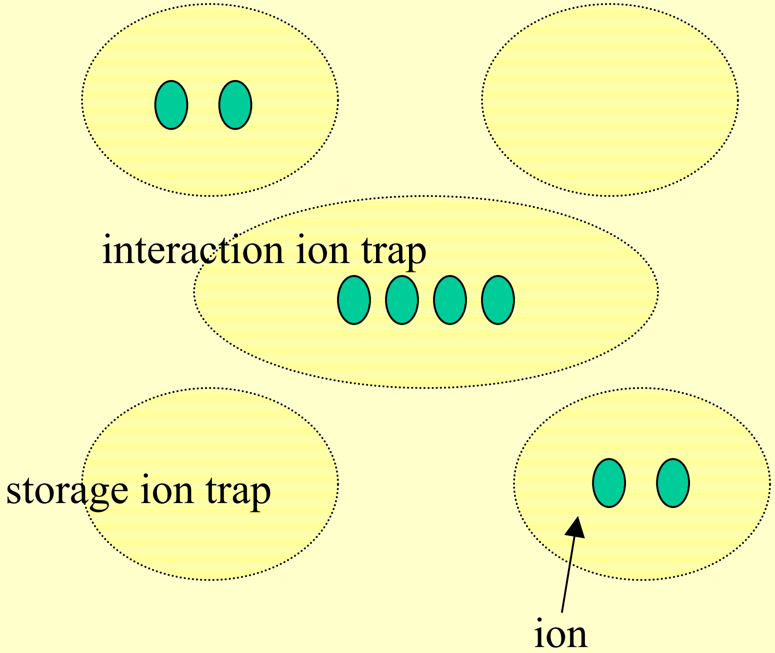
encode logical qubit into 3 physical qubits, e.g. $|0_L> = |110>$ $|1_L> = |011>$



• one-qubit gates generated from application of exchange interactions and their commutators to physical qubits within a single logical qubit

• entangling two-qubit operation C(Z) from application of encoded s_z operation onto the physical qubits 2-3-4 in the triangular architecture, plus one-qubit operations

Scalable Array-Based Ion Trap Quantum Computation - with protection



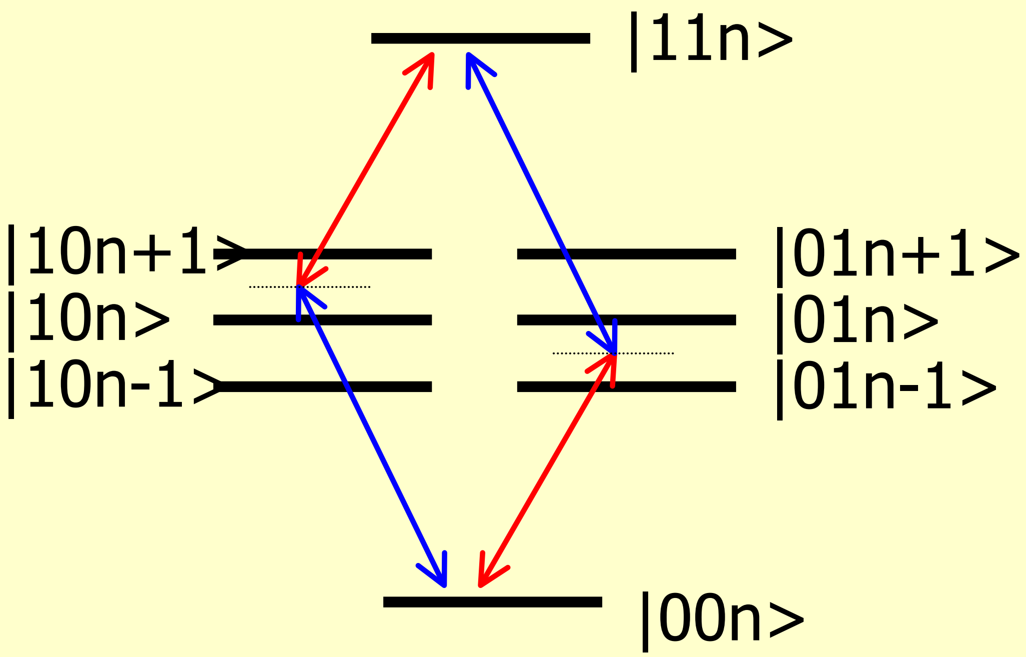
- ions stored in pairs (the encoded qubit)
- Sorensen-Molmer gates provide all single qubit rotations
- pairs of encoded qubits are moved into an interaction ion trap in order to perform two qubit operations
- the encoded states are also a DFS that protects the qubits from collective dephasing
- collective dephasing errors result from ambient magnetic field fluctuations and moving ions

Brown, Vala & Whaley

quant-ph/0207155

Sorensen-Molmer Entangling Operation

Sorenson & Molmer, PRL **82**, 1971 (1999)



$\omega = \eta^2 \Omega^2 / \Delta$

$\eta =$ Lamb Dicke parameter

$\Omega =$ single ion Rabi frequency

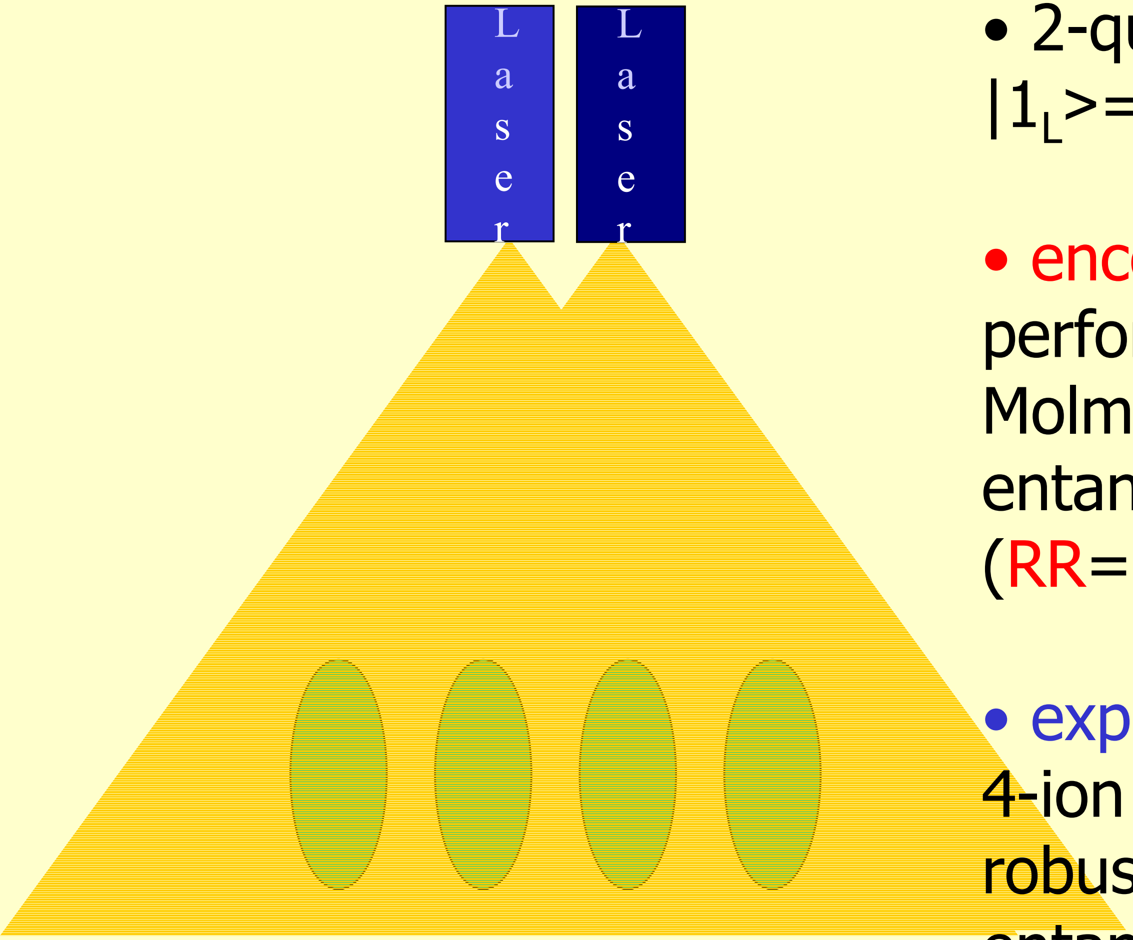
- 2 lasers are applied to 2 ions and the ions entangled by a virtual interaction with a common motional mode
- 1 laser blue detuned, 1 laser red detuned, by common frequency, Δ
- changing the phase of each laser $\rightarrow$ perform two body Hamiltonian:

$H = \omega\, R(\theta) R(\phi)$

$R(\theta) = \cos(\theta) X + i \sin(\theta) Y$

encoded 2-qubit operations: I

Kielpinski, Monroe, Wineland
Nature **417** 709 (2002)

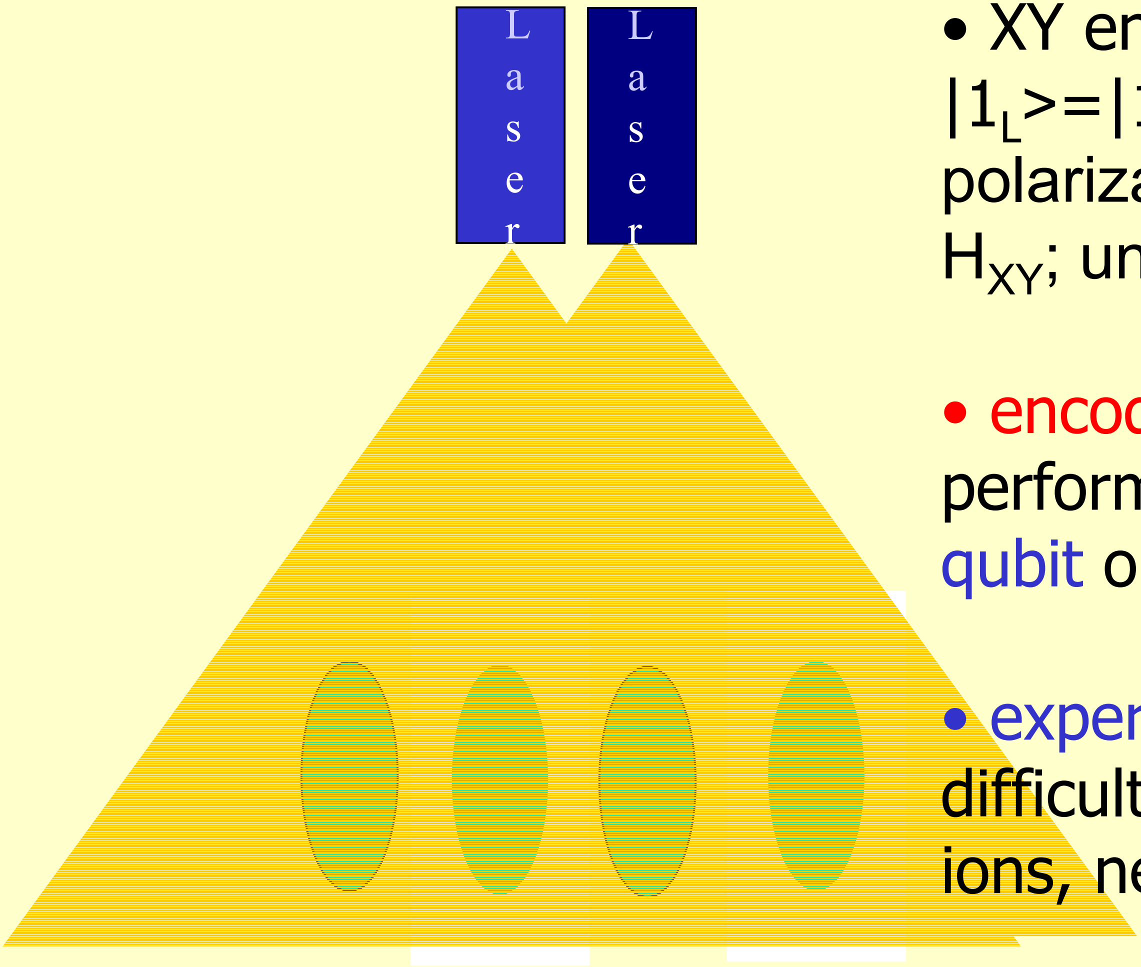


• 2-qubit encoding $|0_L\rangle = |01\rangle$, $|1_L\rangle = |10\rangle$

• encoded 2-qubit operations performed using Sorensen-Molmer single physical 4-qubit entangling operation (RR=RRRR)

• experimental challenges: 4-ion entangling gates are less robust and slower than 2-ion entangling gates.

encoded 2-qubit operations: II



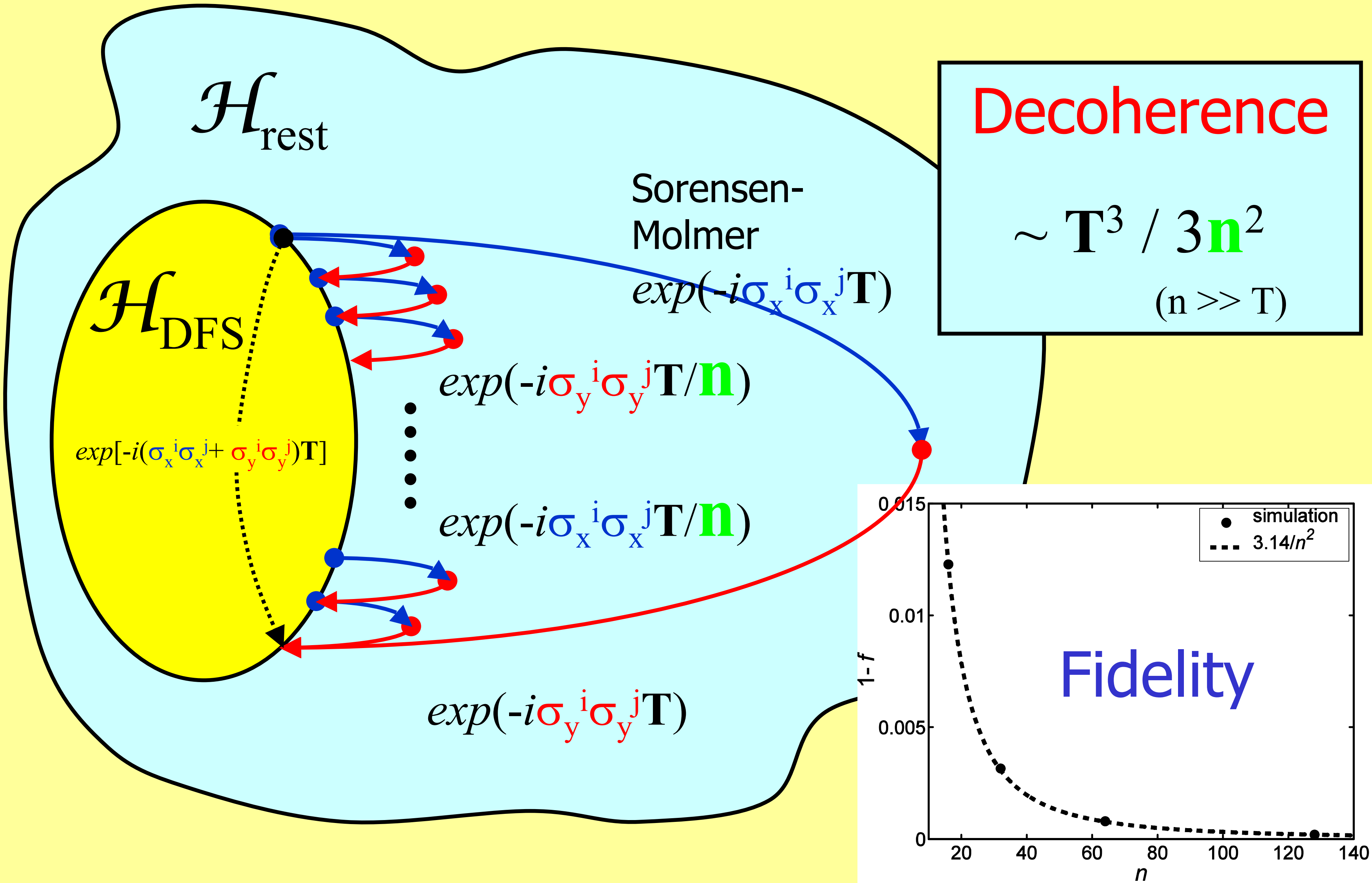
Brown, Vala, Whaley quant-ph/0270115

- XY encoding $|0_L\rangle = |010\rangle$, $|1_L\rangle = |100\rangle$; rapid alternation of polarization creates effective H_{XY}; universal on this encoding

- encoded 2-qubit operations performed using 5 physical 2-qubit operations (conjugation)

- experimental challenges: difficult to address individual ions, need to alternate pulses

Decoherence Minimization



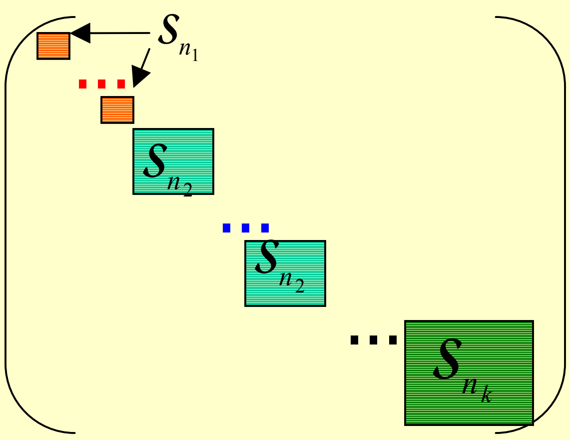
Is encoded universality always possible?

NO!

- non-interacting fermions (Valiant, Terhal&DiVincenzo, Knill ’01)
- nearest-neighbor XY-interaction
- linear optics quantum computation

Criterion: Kempe, Bacon et al. Qu. Comp. & Inf. 1, 33 (2001)

If a set of Hamiltonians (over n qubits) allows for (encoded) universal computation then the Lie algebra L(H) contains exponentially many linearly independent elements



some component S_{n_k} has to contain $su(n_k)$ where $\log_2(n_k)$ is a polynomial function of n

e.g., $H = \{\sigma_z^i, \sigma_x^i \sigma_x^{i+1}\}$ is not universal with any encoding

Summary of results:

I. we can do quantum computing
(all required logic operations, i.e. universal QC)
with only the exchange operation,
via encoded universality

- general algebraic framework (`00, `01)
- universality of various exchange interactions proven ('00-`02)
- encodings found (`00-`02)
- gate-sequences for encoded gates found (`00,`02)
- robustness and fidelity of these gate-sequences studied (`02)

II. we can map exchange-based encoded universality to other physical systems

Conclusions

- Encoding into sub-spaces allows to make certain interactions universal
- Representation theory of Lie groups - powerful tool
- E and XY (symm, asymm, generalized) *alone* are universal - important simplification of physical implementations

References

J. Kempe and K.B.Whaley, *"Exact gate-sequences for universal quantum computation using the XY-interaction alone ",* **Phys. Rev. A 65,** 052330 (2002)

J. Kempe, D. Bacon, D.P. DiVincenzo, K.B. Whaley, *" Encoded universality from a single physical interaction"*, in **«Quantum Information and Computation»;** Special Issue, Vol. 1, 33, (2001); quant-ph/0112013

D. Bacon, J. Kempe, D.P. DiVincenzo, D.A. Lidar, K.B. Whaley, *" Encoded Universality in Physical Implementations of a Quantum Computer"*, **Proceedings of IQC '01**, Rinton Press, Australia (2001); quant-ph/0102140

D.P. DiVincenzo, D. Bacon, J. Kempe, K.B. Whaley, *" Universal Quantum Computation with the Exchange Interaction"*, **NATURE 408**, 339 (2000), quant-ph/0005116

J. Vala and K.B. Whaley*, "Encoded Universality with Generalized Anisotropic Exchange Interactions", **Phys. Rev. A 66**, 022304 (2002)*

Earlier related work on universal QC on DFS:

J. Kempe, D. Bacon, D. Lidar,K.B. Whaley, **Phys. Rev. A 63**, 042307 (2001)

D. Bacon, J. Kempe, D. Lidar, K.B. Whaley, **Phys. Rev. Lett. 85**, 1758 (2000)

- tensor product of encoded qubits

  conjoined codes   Bacon et al., *PRL* **85**, 1758 (2000)

  Bacon et al., quant-ph/0102140

- find entangling operations

  Lie algebraic analysis   Kempe et al., *PRA* **63**, 042307 (2001)

  Kempe et al., JQIC (2001)

- efficient implementation

  numerical search   e.g. Heisenberg exchange

serial coupling - 19 operations for CNOT, 4 operations for 1-qubit
parallel coupling - 7 operations for CNOT, 3 operations for 1-qubit

DiVincenzo, Bacon, Kempe, Burkard, Whaley, *Nature* **408**, 339 (2000)