IPAM Workshop “Mathematics of Turbulence”
Workshop IV: Turbulence in Engineering Applications
November 17-21, 2014

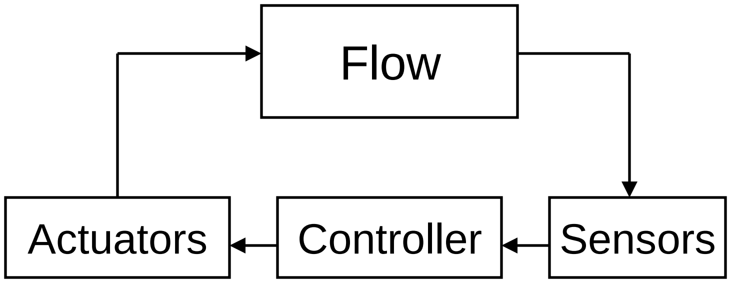
Some integral relationships and their applications to flow control

Koji Fukagata
Department of Mechanical Engineering
Keio University, Japan

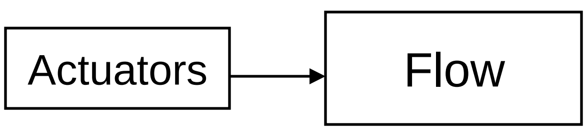
1. Introduction

Two major approaches for active control of turbulence

- **Feedback control**
  - Sensors, actuators, controller
  - Potentially effective
  - Big hurdle for hardware development (especially if the QSVs are targeted at)

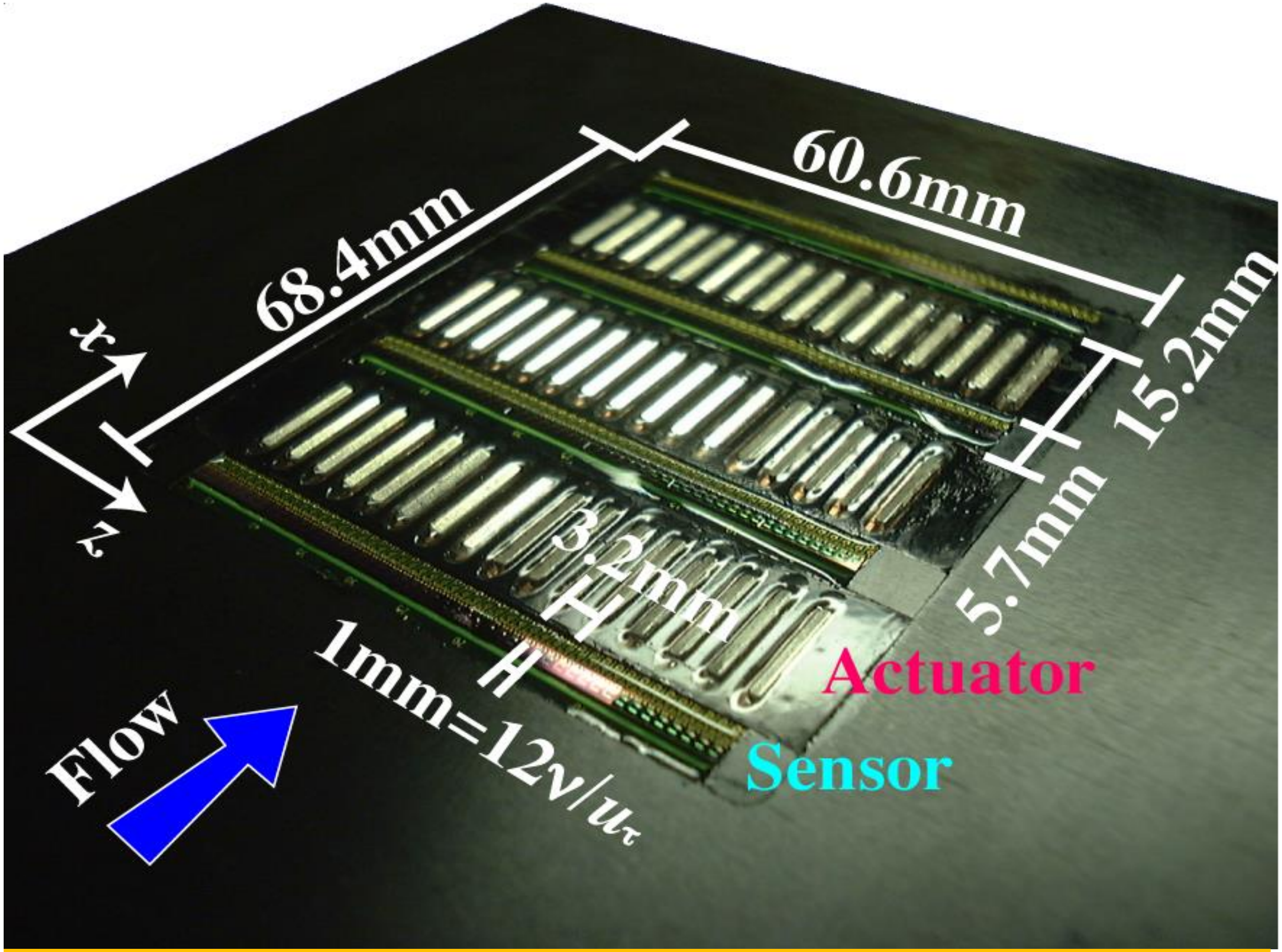


- **Predetermined control without sensors**
  - Less difficult to make hardware?
  - Suitable input is less clear (e.g. for friction drag reduction)



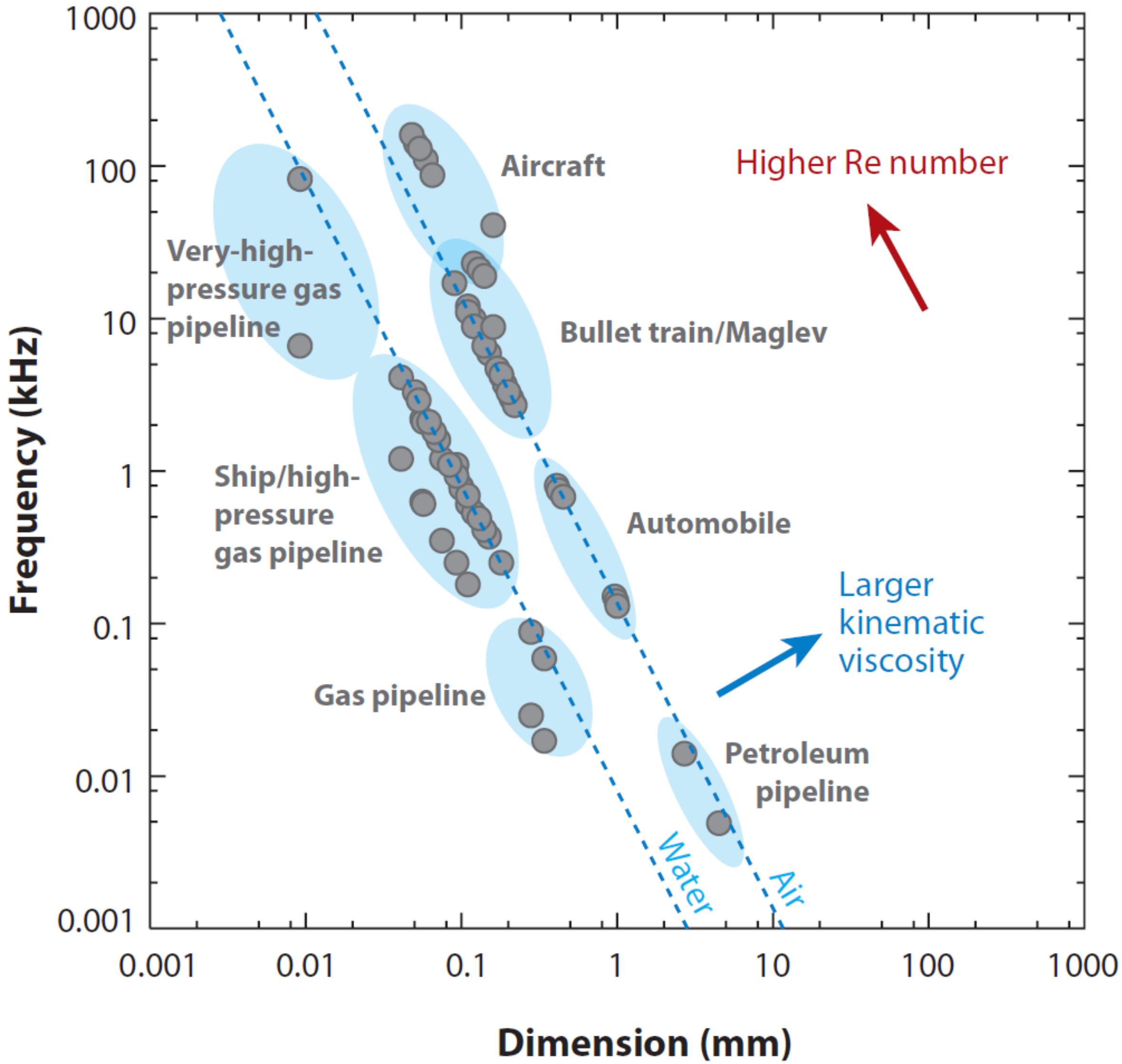
Feedback control (wind tunnel experiment)

(Yoshino et al., *J. Fluid Sci. Technol.*, 2009;
also introduced in Kasagi et al., *Annu. Rev. Fluid Mech.*, 2009)



Feedback system for turbulence control ･･･ 6% drag reduction

Big issue toward practical application: Physical length- and time-scales of QSV



(Kasagi, Suzuki, and Fukagata, *Annu. Rev. Fluid Mech.*, 2009)

2. Integral relationship between the skin friction drag and the turbulent statistics

Fundamental question in fluids engineering

- **Question: Take a straight pipe, for instance…**
  - Pressure gradient (or friction drag) given → Flow rate?
  - Flow rate given → Pressure gradient (or friction drag) ?

- **Answer:**
  - Laminar flow: Analytical solution (Hagen-Poiseuille)

$$C_f = \frac{16}{\mathrm{Re}_b} \qquad \text{where } C_f = \frac{\tau_w}{(1/2)\rho U_b^2}, \ \mathrm{Re}_b = \frac{U_b(2R)}{\nu}$$

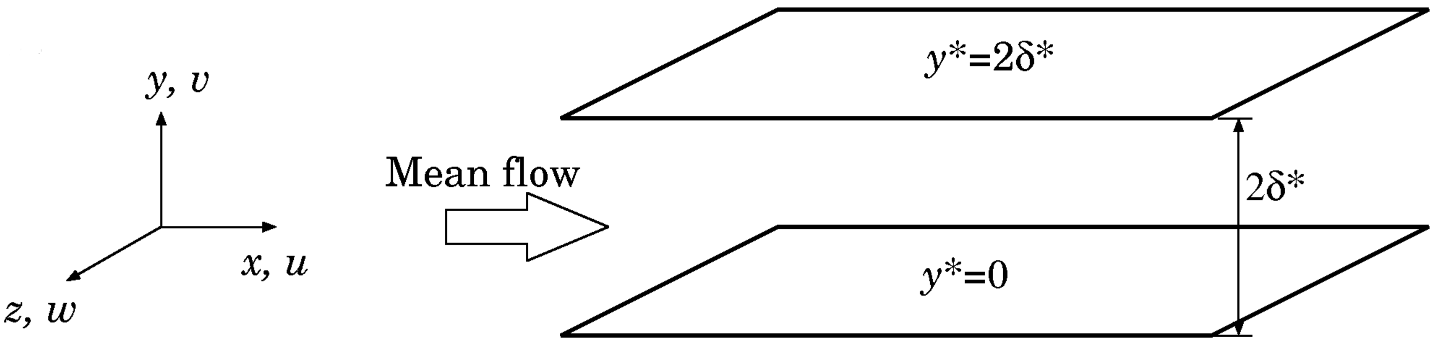
  - Turbulent flow
    - Rely on experiments, simulations, and (semi-)empirical formula
    - What is the relationship between turbulent statistics and drag?
    - How does it become when a control is applied?

***The same arguments hold for heat transfer problems**

Integral relationship between turbulent statistics and drag (1)

(Fukagata, Iwamoto, and Kasagi, *Phys. Fluids*, 2002)

- **Fully-developed channel flow (the simplest case)**



– Starting point: Reynolds-Averaged Navier-Stokes eq.

$$0 = -\frac{d\overline{p}}{dx} + \frac{d}{dy}\left[\frac{1}{\mathrm{Re}_b}\frac{d\overline{u}}{dy} + (-\overline{u'v'})\right]$$

$f = \overline{f} + f'$

$\overline{f}$: Mean

f' : Fluctuation

※Nondimensionalized by twice bulk-mean velocity $2U_b^*$ and channel half-width δ^*

Asterisk: dimensional quantity

$$\mathrm{Re}_b = \frac{2U_b^*\delta^*}{\nu^*}$$

Integral relationship between turbulent statistics and drag (2)

(Fukagata, Iwamoto, and Kasagi, *Phys. Fluids*, 2002)

- **Triple integration of Reynolds-Averaged N-S eq.**
  - 1st integration → Stress balance

$$\frac{1}{\mathrm{Re}_b}\frac{d\bar{u}}{dy} + (-\overline{u'v'}) = \frac{C_f}{8}(1-y)$$

Relationships below are used at each step

$$C_f = \frac{\tau_w^*}{\rho^* U_b^{*2}/2} = -8\frac{d\bar{p}}{dx} = \frac{8}{\mathrm{Re}_b}\left.\frac{d\bar{u}}{dy}\right|_{y=0}$$

  - 2nd integration → Mean velocity profile

$$\bar{u}(y) = \mathrm{Re}_b\left[\frac{C_f}{8}\left(y - \frac{y^2}{2}\right) - \int_0^y (-\overline{u'v'})dY\right]$$

$$u|_{y=0} = 0$$

$$(dY = dy)$$

  - 3rd integration → Bulk-mean velocity

$$\frac{1}{2} = \mathrm{Re}_b\left[\frac{C_f}{24} - \int_0^1\left\{\int_0^y (-\overline{u'v'})dY\right\}dy\right]$$

$$U_b = \frac{1}{2}$$

(since the velocity is nondimensionalized by $2U_b^*$)

Integral relationship between turbulent statistics and drag (3)

(Fukagata, Iwamoto, and Kasagi, *Phys. Fluids*, 2002)

- **Convert double integral to single integral**

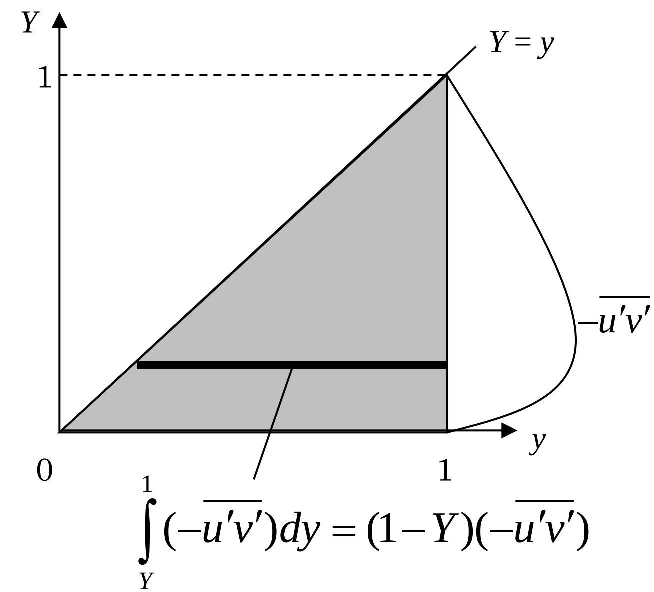
By integration by parts

$$\int_0^1 \left\{ \int_0^y (-\overline{u'v'})dY \right\} dy = \int_0^1 \left\{ 1 \int_0^y (-\overline{u'v'})dY \right\} dy$$

$$= \left[y \int_0^y (-\overline{u'v'})dY \right]_0^1 - \int_0^1 y(-\overline{u'v'})dy$$

$$= \int_0^1 (1-y)(-\overline{u'v'})dy$$

Or, by iterated integral (Yoshizawa, priv. commun., 2008)



- **Relationship for a fully-developed channel flow**

$$C_f = \frac{12}{Re_b} + 12\int_0^1 2(1-y)(-\overline{u'v'})dy$$

*Essentially the same relationship has been derived also by Bewley and Aamo (*JFM* 2004)

Contributions of different effects

- **More general form (for channel flow)**

$$C_f = \frac{12}{\mathrm{Re}_b} + 12\int_0^2 (1-y)(-\overline{u'v'})dy + (\mathrm{III}) + (\mathrm{IV}) + (\mathrm{V})$$

I II

I. **Laminar drag**

II. **Turbulent contribution**
= Weighted integral of the Reynolds shear stress

III. Contribution of spatio-temproral development
IV. Contribution of body force, additional stress (e.g., polymer)
V. Contribution from wall boundary (e.g., uniform blowing/suction)

Relationships in other geometries

(Fukagata, Iwamoto, and Kasagi, *Phys. Fluids*, 2002)

- **Pipe flow**

$$C_f = \frac{16}{\mathrm{Re}_b} + 16\int_0^1 2r\,\overline{u_r' u_z'}\,r\,dr$$

Nondimensionalized by $2U_b^*$ and pipe radius R^*

$$\mathrm{Re}_b = \frac{2U_b^* R^*}{\nu^*}$$

- **ZPG boundary layer**

$$C_f = \frac{4(1-\delta_d)}{\mathrm{Re}_\delta} + 4\int_0^1 (1-y)(-\overline{u'v'})dy - 2\int_0^1 (1-y)^2 \left(\frac{\partial \overline{u}\overline{u}}{\partial x} + \frac{\partial \overline{u}\overline{v}}{\partial y} \right) dy$$

Contribution of spatial development

Contribution of mean wall-normal flux

Nondimensionalized by free-stream velocity U_∞^* and 99% boundary layer thickness $\delta_{99\%}^*$

$$\mathrm{Re}_\delta = \frac{2U_\infty^* \delta_{99\%}^*}{\nu^*}$$

δ_d : Nondimensionalized displacement thickness

Example 1: Opposition-controlled pipe flow

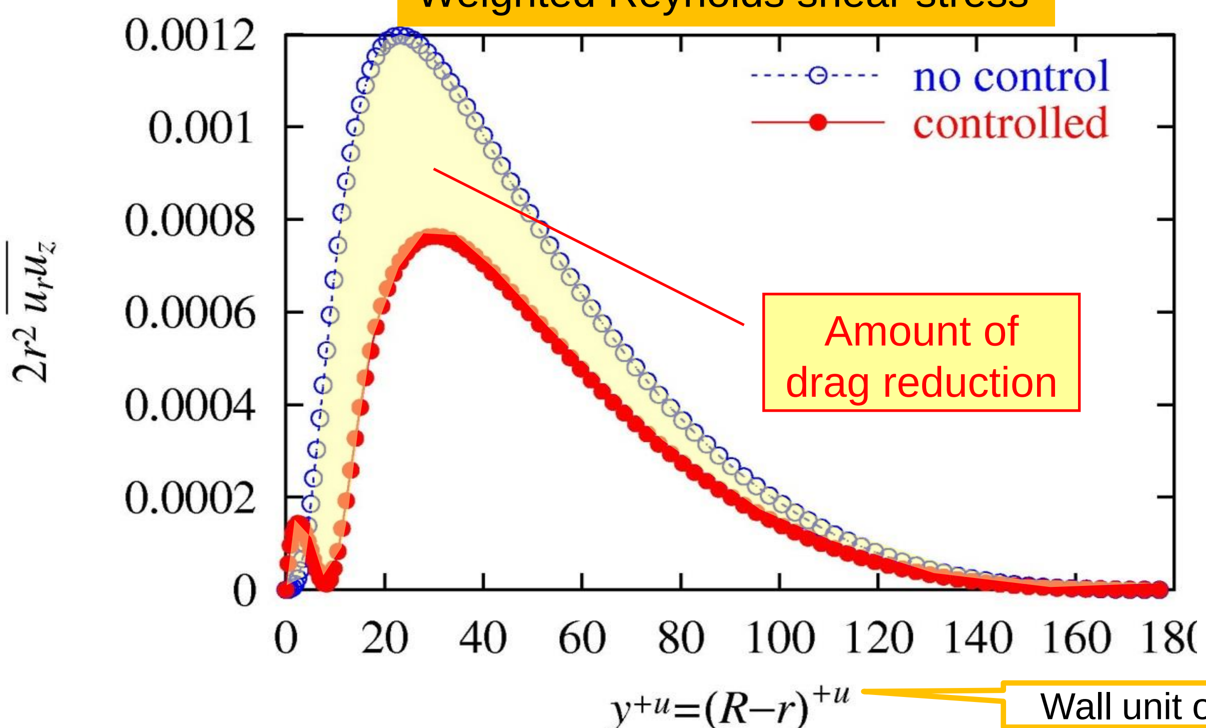
(Fukagata, Iwamoto, and Kasagi, *Phys. Fluids*, 2002)

- **Decomposition of contributions**

	I (Laminar drag)	**II** (Turb. Contrib.)	Sum (C_f)
No control	3.0×10^{-3}	6.3×10^{-3}	9.3×10^{-3}
Controlled	3.0×10^{-3}	4.0×10^{-3}	7.0×10^{-3}

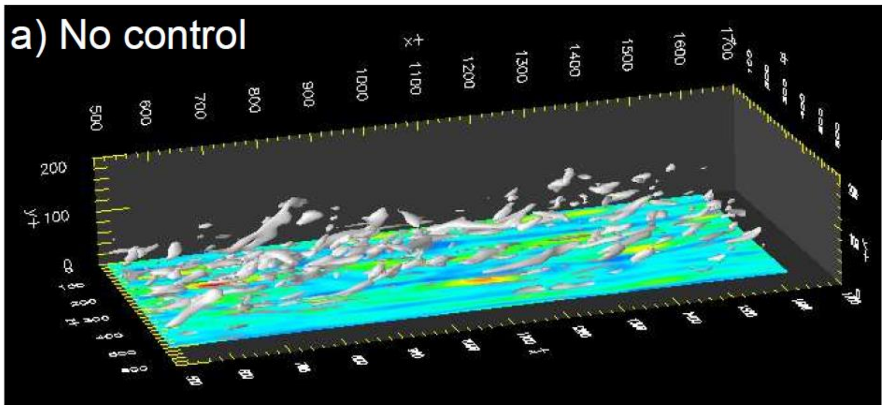
$$C_f = \underbrace{\frac{16}{Re_b}}_{\text{I}} + \underbrace{16\int_0^1 2r\,\overline{u_r' u_z'}\,r\,dr}_{\text{II}}$$

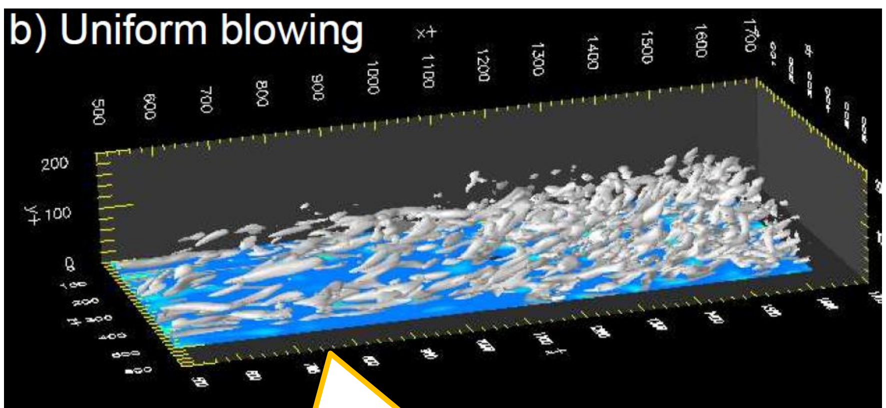
Weighted Reynolds shear stress



Example 2: Spatially-developing ZPG turbulent boundary layer with uniform blowing/suction

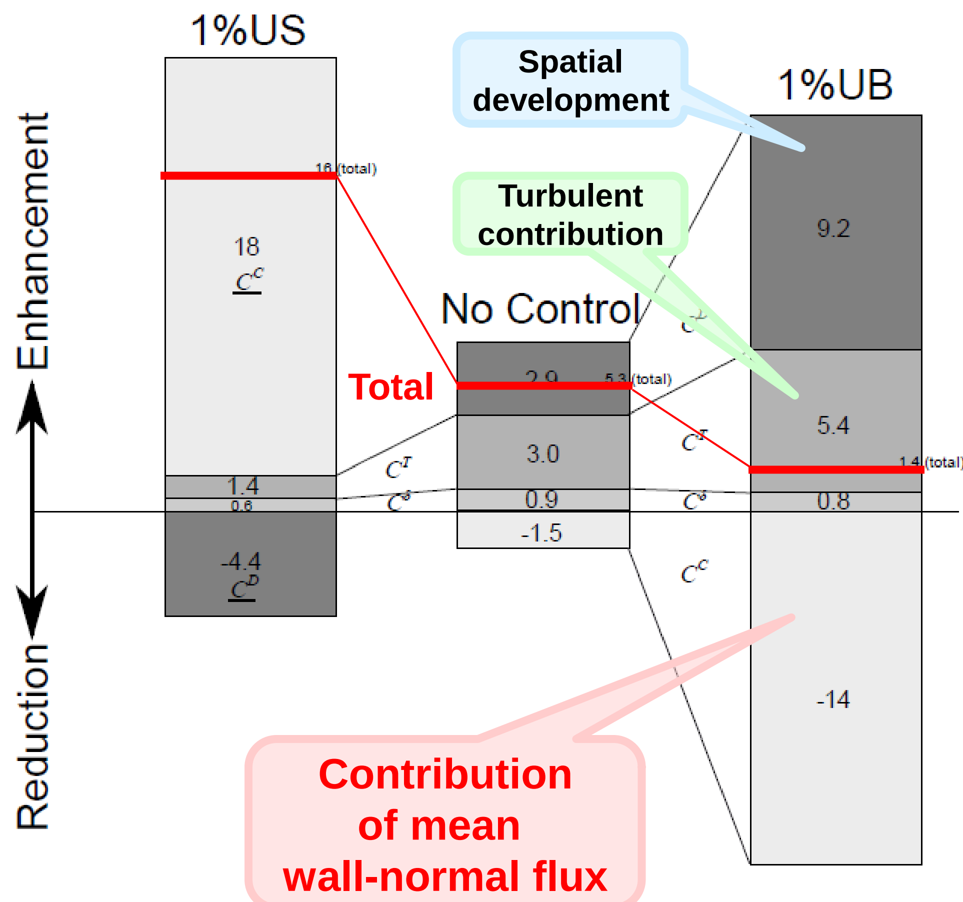
(Kametani & Fukagata, *J. Fluid Mech.*, 2011)





White: Vortex cores
Color: Wall shear stress

Decomposition of contributions



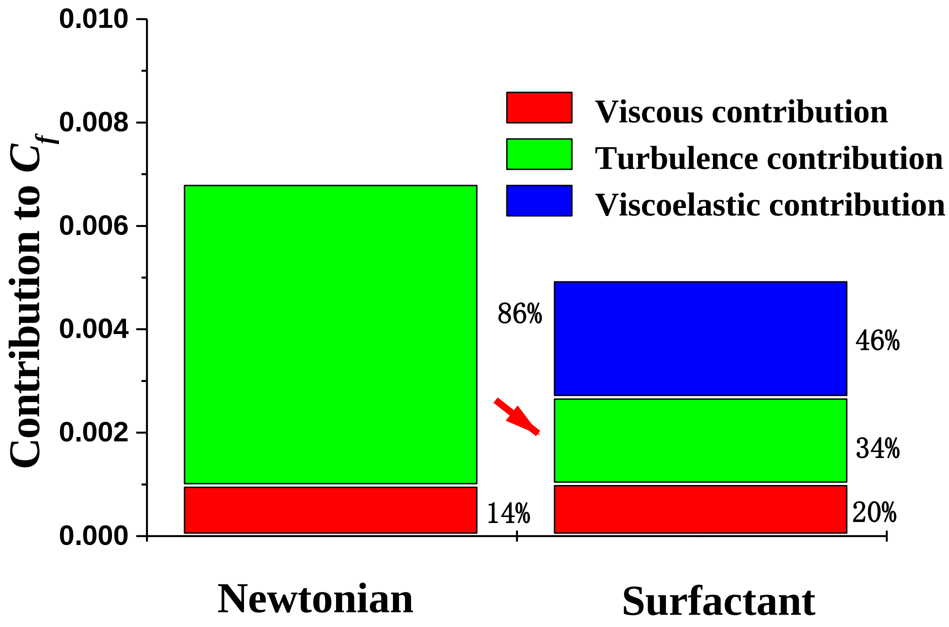
- With uniform blowing: Turbulence is enhanced, but drag is reduced!

Example 3: Drag reduction by surfactant addition

(Yu, Li, and Kawaguchi, *Int. J. Heat Fluid Flow*, 2004)

- **Integral relationship for Giesekus fluid (channel flow)**

$$C_f = \frac{12}{\mathrm{Re}_b} + 24\left[\int_0^1 \left(-\overline{u'v'}\right)(1-y)dy + \int_0^1 \beta \frac{C_{xy}}{\mathrm{We}}(1-y)dy\right]$$



– A similar analysis can be made also for polymer addition

(White & Mungal, *Annu. Rev. Fluid Mech.*, 2008)

Some other extensions

- **Arbitrary-shaped straight duct (const. pres. grad.)**
  (Subragaglia & Sugiyama, *Physcia D*, 2007)

$$\langle u \rangle - \langle \tilde{u} \rangle = \langle (\boldsymbol{u}\boldsymbol{u}) : (\nabla \tilde{\boldsymbol{u}}) \rangle$$

Volume average — Stokes flow — Corresponding to $(1-y)$ weight

- **Compressible flow** (Gomez, Flutet, and Sagaut, *Phys. Rev. E*, 2009)

$$C_f = \underbrace{\frac{6}{\mathrm{Re}}}_{C_L} + \underbrace{6\int_{-1}^{0} z\langle\rho\rangle\{u''w''\}dz}_{C_T} + \underbrace{\frac{6}{\mathrm{Re}}\int_{-1}^{0} -z\langle\tilde{\mu}\rangle\frac{\partial\langle u\rangle}{\partial z}dz}_{C_C}$$

$$+ \underbrace{\frac{6}{\mathrm{Re}}\int_{-1}^{0} -z\left\langle \mu'\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right)\right\rangle dz}_{C_{CT}}.$$

Contribution from variable viscosity

Contribution from viscosity fluctuations (small)

Some other extensions (cont'd)

- **Relationship between wall heat flux and turbulent statistics** (Kasagi, Hasegawa, Fukagata, and Iwamoto, *J. Heat Transfer*, 2012)
  - Constant temperature difference condition

$$\mathrm{St} = \frac{2}{\mathrm{Pr}\,\mathrm{Re}_b} + \int_0^1 \left(-\overline{v'\theta'}\right) dy$$

Stanton number; Prandtl number; Dimensionless temperature

  - Constant heat flux condition

$$\frac{1}{\mathrm{St}} = \mathrm{Pr}\,\mathrm{Re}_b \left[\frac{17}{70} - \frac{1}{2}\int_0^1 (1-\phi)(-\overline{v'\theta'})\,dy - \frac{1}{2}\int_0^1 \{(y^3 - 3y^2 + 1)\phi_d - \phi_d^2\}dy\right] + \int_0^1 \overline{u'\theta'}\,dy$$

Partial flow rate; Partial flow rate deviation from laminar flow

  - Uniform heat generation condition (omitted here)

3. Application to drag reduction control

To start with predetermined control…

- **For a fully-developed incompressible channel flow (review)** (Fukagata et al., *Phys. Fluids*, 2002)

$$C_f = \underbrace{\frac{12}{\mathrm{Re}_b}}_{\text{laminar drag}} + \underbrace{24\int_0^1 (1-y)(-\overline{u'v'})dy}_{\text{turbulent contribution (=weighted integral of RSS)}}$$

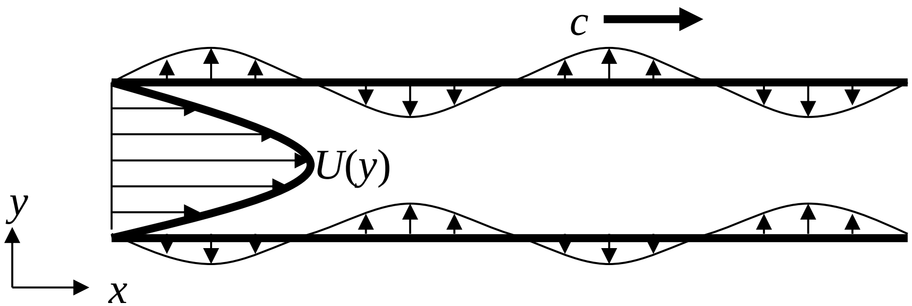
- **Even if we do not know anything about vortices, if we can reduce the RSS, then we can reduce drag!**
  - Feedback body force (Fukagata et al., Proc. SMART-6, 2005)
  - Upstream traveling wave-like blowing/suction (Min et al., *J. Fluid Mech.*, 2006)

→ Drag lower than laminar flow (sub-laminar drag*)

* Although re-laminarization is the best in terms of energy saving (Bewley, *J. Fluid Mech.*, 2009; Fukagata et al., *Physica D*, 2009)

Traveling wave-like blowing/suction

(Min, Kang, Speyer, and Kim, *J. Fluid Mech.*, 2006)



Drag normalized in a different way

$$D = \underbrace{2}_{\text{Laminar drag}} + \underbrace{\frac{3}{2}\mathrm{Re}_c \int_{-1}^{1} (-y)(-\overline{u'v'})dy}_{\text{"Turbulent" contribution, } \Delta D}$$

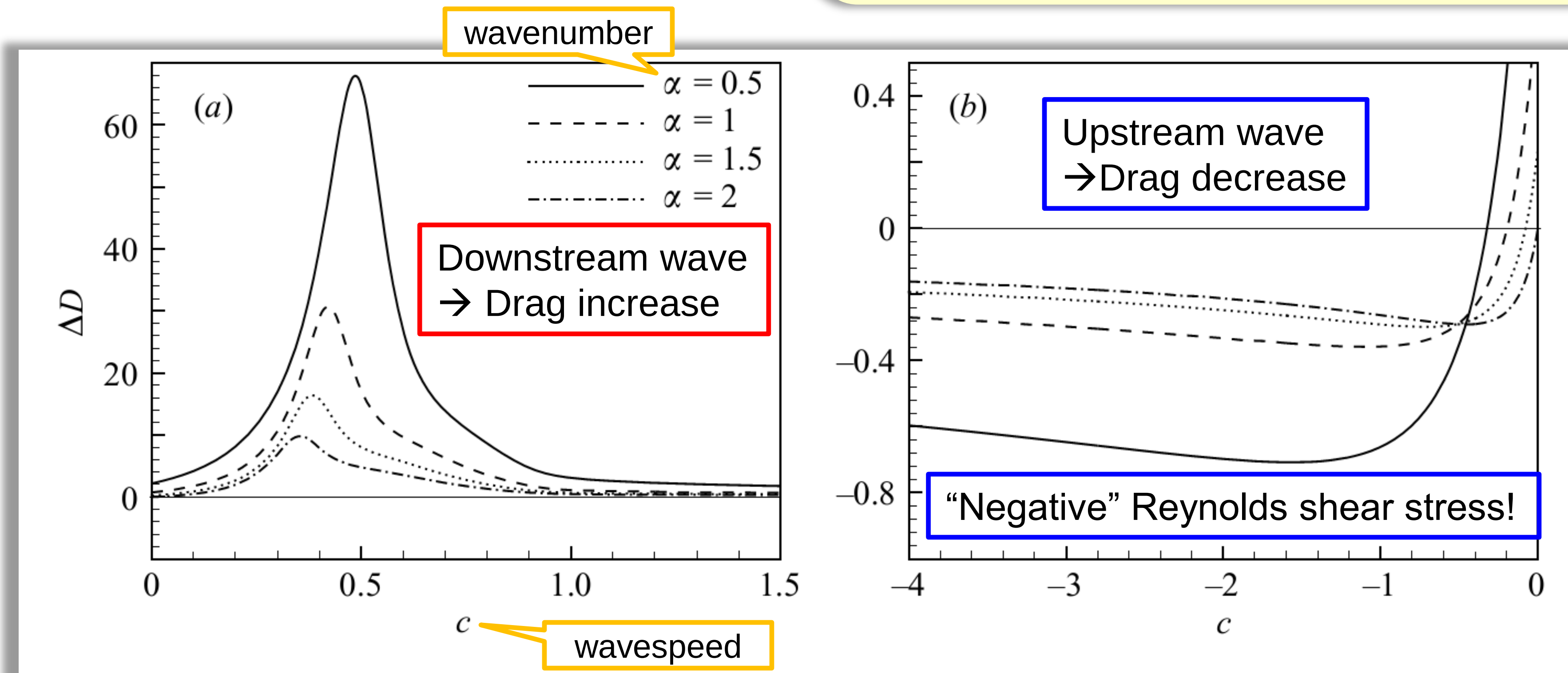
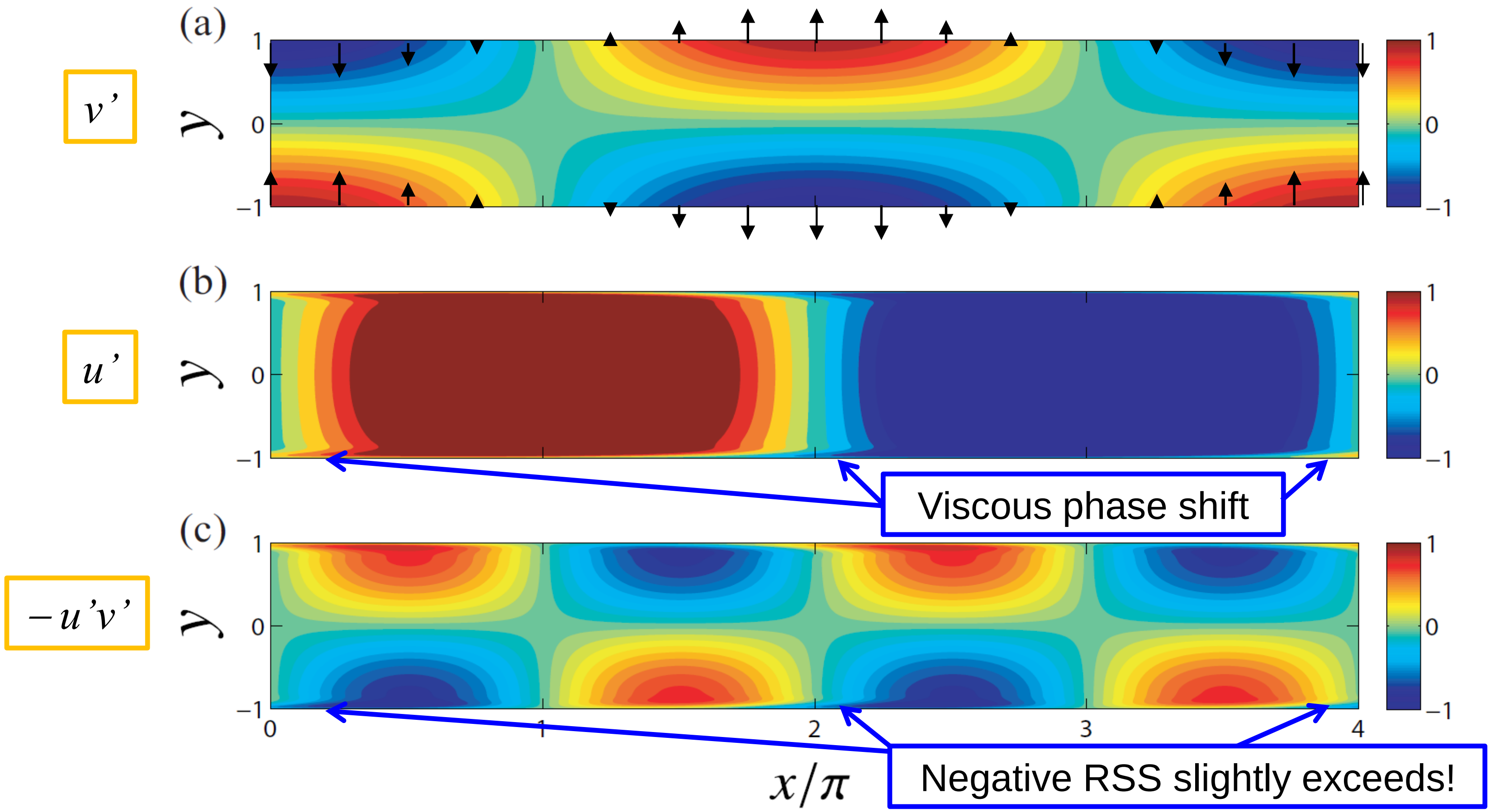


FIGURE 1. ΔD in steady state at $Re = 2000$ and $a = 0.1$: (*a*) downstream travelling waves ($c > 0$); (*b*) upstream travelling waves ($c < 0$).

“Negative” Reynolds shear stress (RSS) --- linear analysis

- **Away from the wall: *u*’ and *v*’ are orthogonal (same as inviscid)**
- **Near the wall: phase shift in *u*’ due to viscosity**

(Min et al., *J. Fluid Mech.*, 2006; Mamori et al., *Phys. Rev. E*, 2010)



(Mamori, Fukagata, and Hœpffner, *Phys. Rev. E*, 2010)

Primary drag reduction mechanism by traveling wave-like blowing/suction

- Wave of blowing/suction traveling in the <u>upstream</u> direction

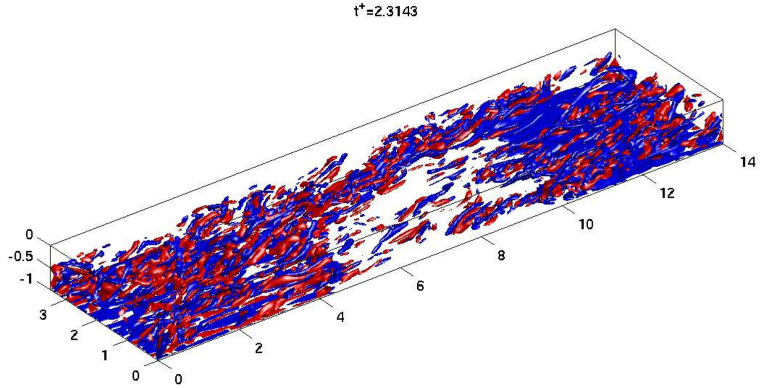
  → "Negative" Reynolds shear stress

  → Net flow in the <u>downstream</u> direction

  = "Pumping effect" (in the direction opposite to the wave)

External pressure gradient required to keep the flow rate (constant) is reduced

= "Drag reduction"



(+ Turbulence modification)

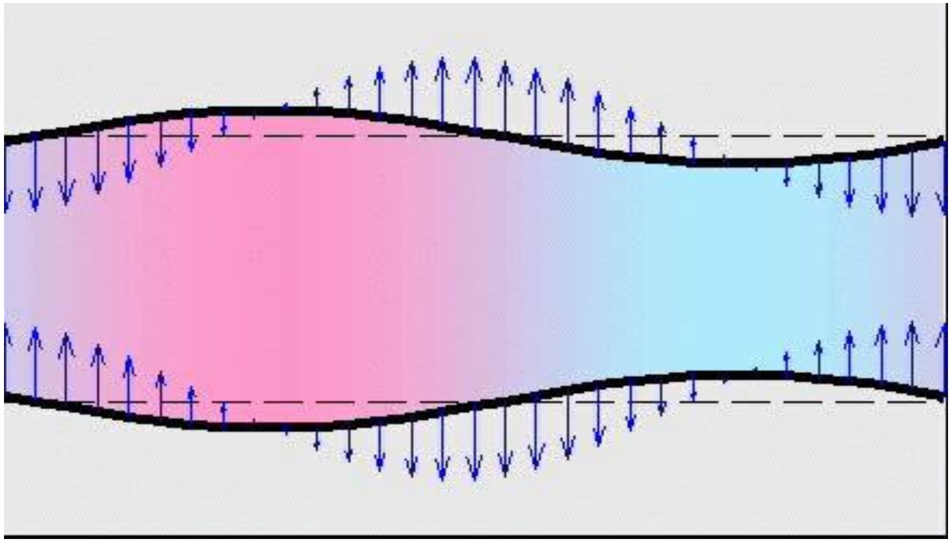
(Animation: Mamori, MEng Thesis, 2008)

But, traveling wave-like blowing/suction device is difficult to make in practice

- **Last sentence in Min et al (2006)'s paper**

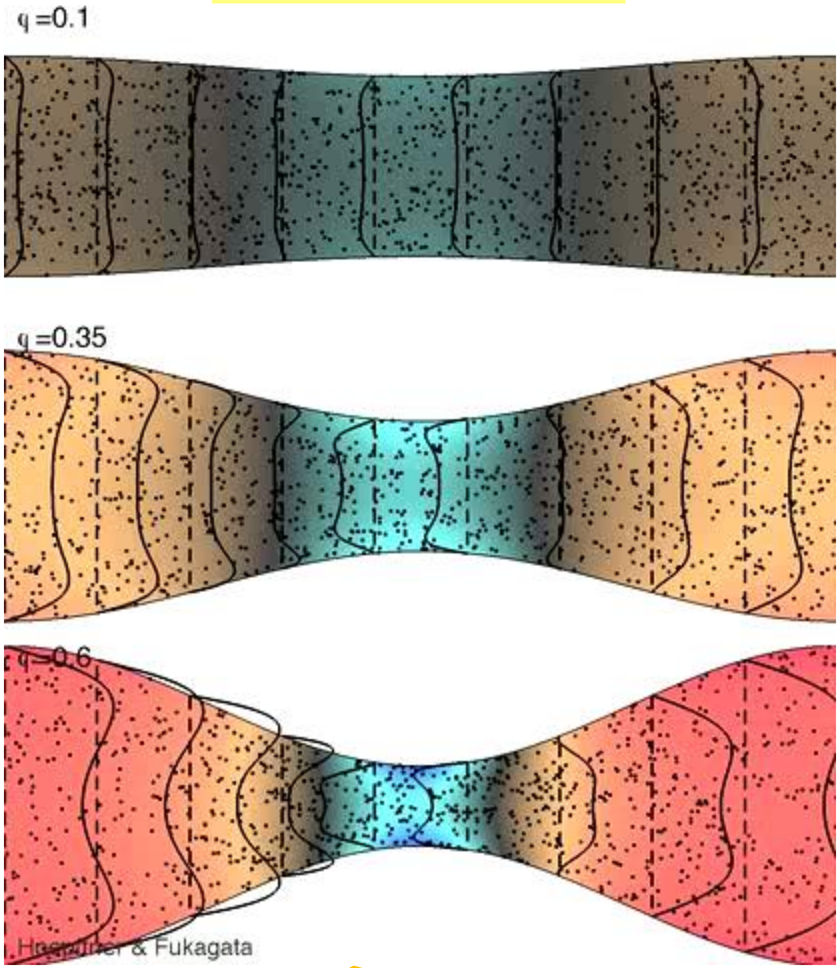
  However, a moving surface with wavy motion would produce a similar effect, since wavy walls with small amplitudes can be approximated by surface blowing and suction.

- **Question: Can it simply be substituted by wall deformation?**

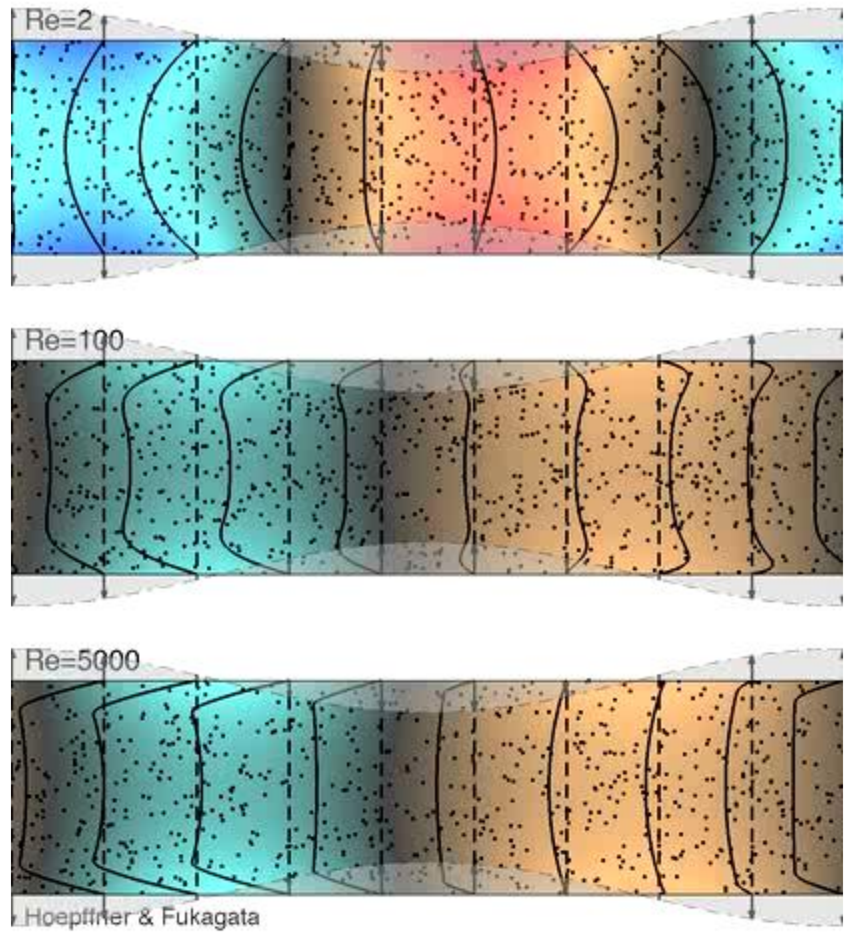


Blowing/suction vs wall deformation
(without external pressure gradient)

Wall deformation



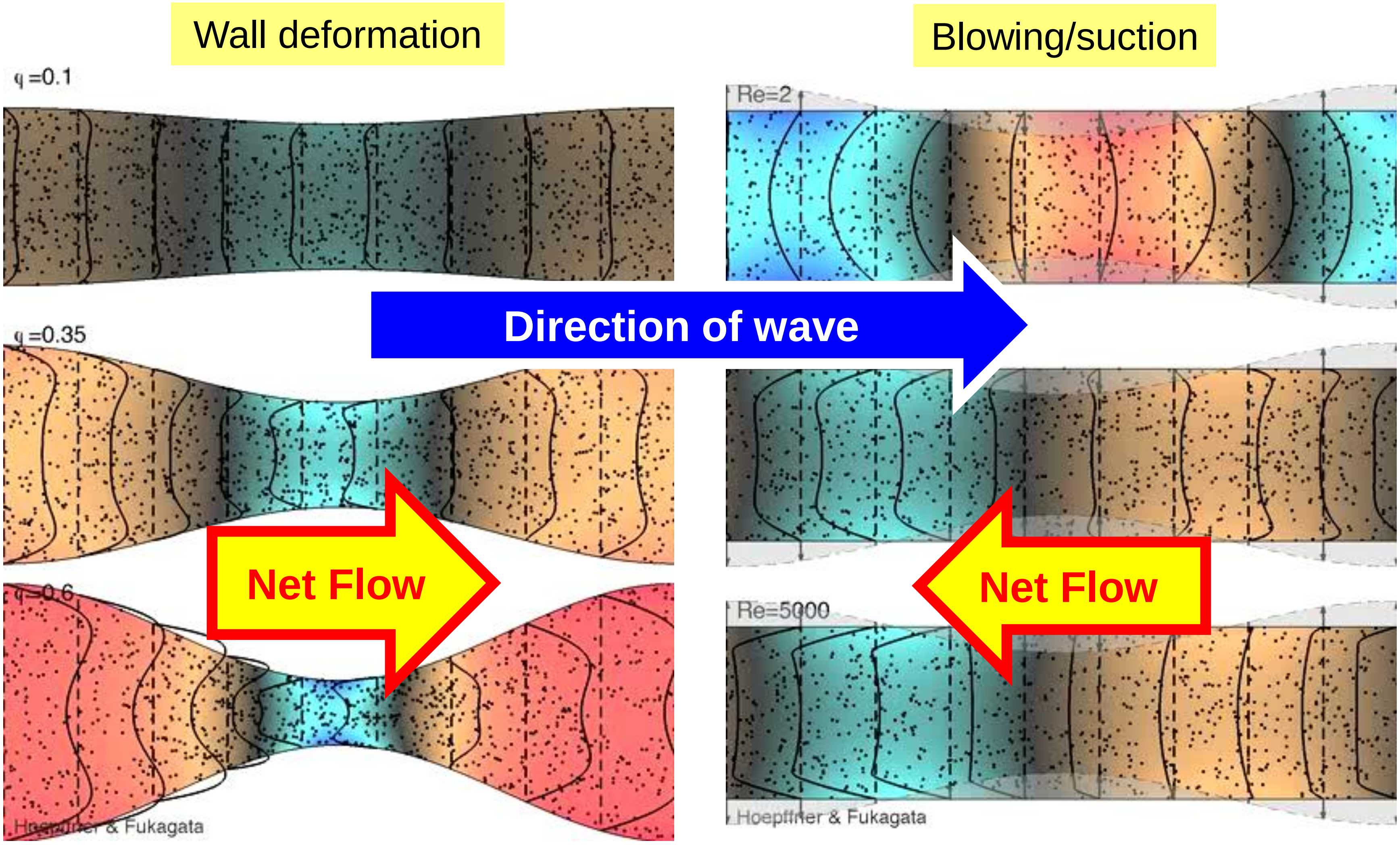
Blowing/suction



Black point: Fluid particle (marker)
Color: Pressure

(Hœpffner & Fukagata, *J. Fluid Mech.*, 2009)

Blowing/suction vs wall deformation
(without external pressure gradient)



(Hœpffner & Fukagata, *J. Fluid Mech.*, 2009)

Summary of existing knowledge and prediction

	Blowing/suction	Wall deformation
Pumping effect (Hœpffner & Fukagata, JFM 2009)	Direction opposite to the wave	Direction same as the wave
Stability (Lee et al., PoF 2008; Moarref & Jovanovic, JFM 2010)	Flow is stabilized by downstream wave of wall-normal velocity on the wall	
Drag reduction	1. Upstream wave (Min et al., JFM 2006) --- unstable? 2. Downstream wave (Mamori et al., PoF 2014) --- stable	**Downstream wave? --- stable?**

- **Question: Is the drag really reduced by a downstream traveling wave-like wall deformation?**

DNS with traveling wave-like wall deformation

(Nakanishi, Mamori, and Fukagata, *Int. J. Heat Fluid Flow*, 2012)

- **Boundary conditions**

$$u = w = 0, \qquad v = \frac{\partial \eta_d}{\partial t} = a\cos(k(\xi_1 - ct))$$

Deformation velocity

η_u , η_d: Displacements of walls (varicose mode)

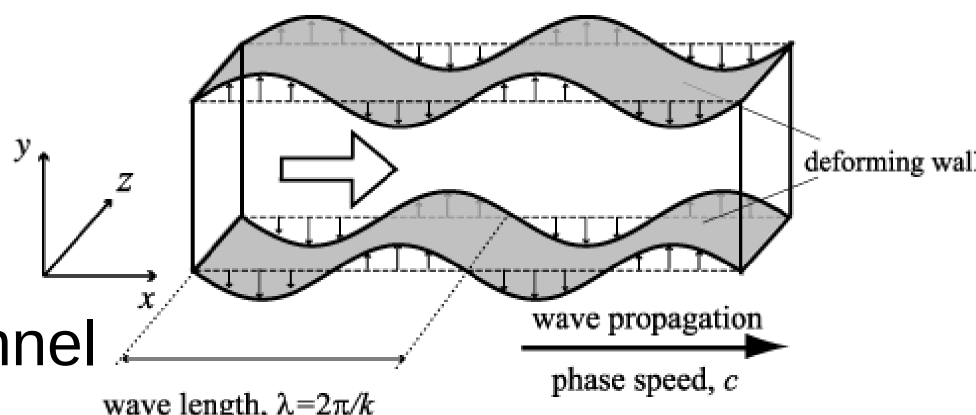
a : Velocity amplitude $(0.05 \ldots 0.3)$

c : Phase speed $(-3 \ldots 3)$

k : Wavenumber $(1 \ldots 4)$

All nondimensionalized by
Twice bulk-mean velocity $2U_b^*$
and channel half width δ^*

- Constant flow rate at
  $Re_b = 2U_b h/\nu = 5600$
- Initial field: Fully developed turbulent flow in plane channel



Keio University

DNS in a bit detail

(Nakanishi et al., *Int. J. Heat Fluid Flow*, 2012)

Coordinate transform (Kang & Choi, *Phys. Fluids*, 2000)

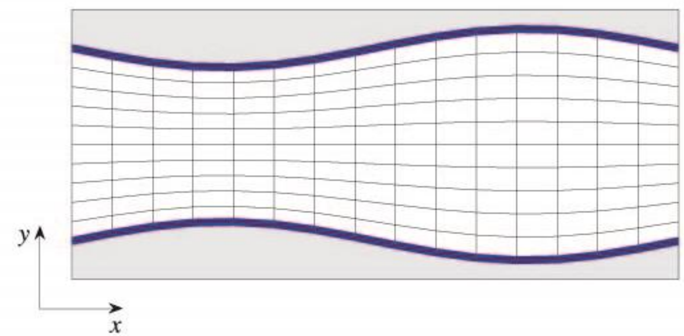
$$x_1 = \xi_1, \qquad x_2 = \xi_2(1+\eta_u) + \eta_d, \qquad x_3 = \xi_3$$

(streamwise) (wall-normal) (spanwise)

(η_u , η_d: displacements of upper and lower walls)

- **Continuity eq.**

$$\frac{\partial u_i}{\partial \xi_i} = \boxed{-S}$$



- **Navier-Stokes eq.**

$$\frac{\partial u_i}{\partial t} = -\frac{\partial (u_i u_j)}{\partial \xi_j} - \frac{\partial p}{\partial \xi_i} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u_i}{\partial \xi_j \partial \xi_j} - \frac{\partial P}{\partial \xi_i}\delta_{i1} \boxed{+ S_i}$$

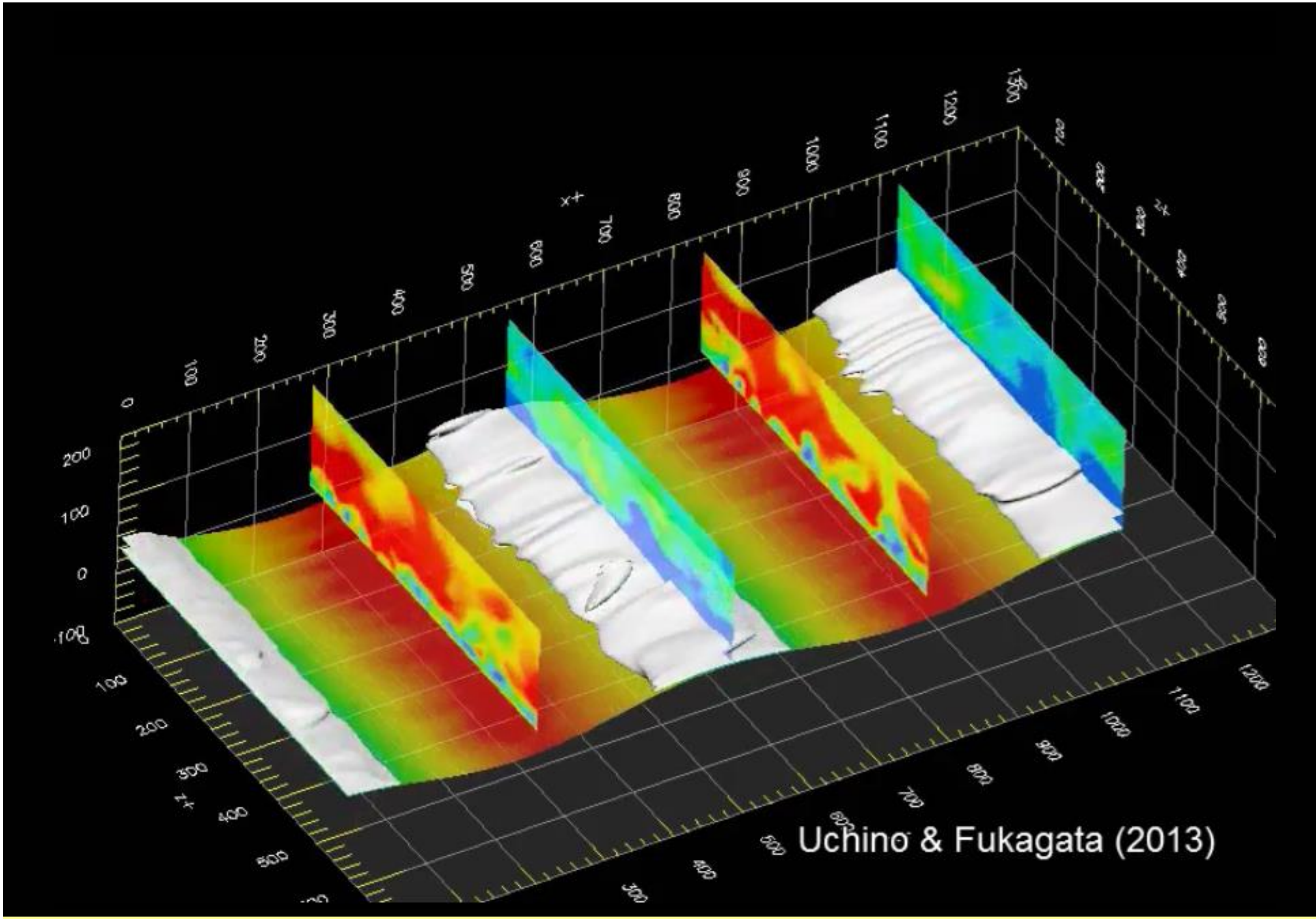
- **DNS Code: Based on the FDM code for a plane channel flow**
(Fukagata et al., *Phys. Fluids*, 2006)

Keio University

DNS with traveling wave-like wall deformation

(Nakanishi et al., *Int. J. Heat Fluid Flow*, 2012)

Animation: Uchino & Fukagata, CFD Symp, Japan, 2013

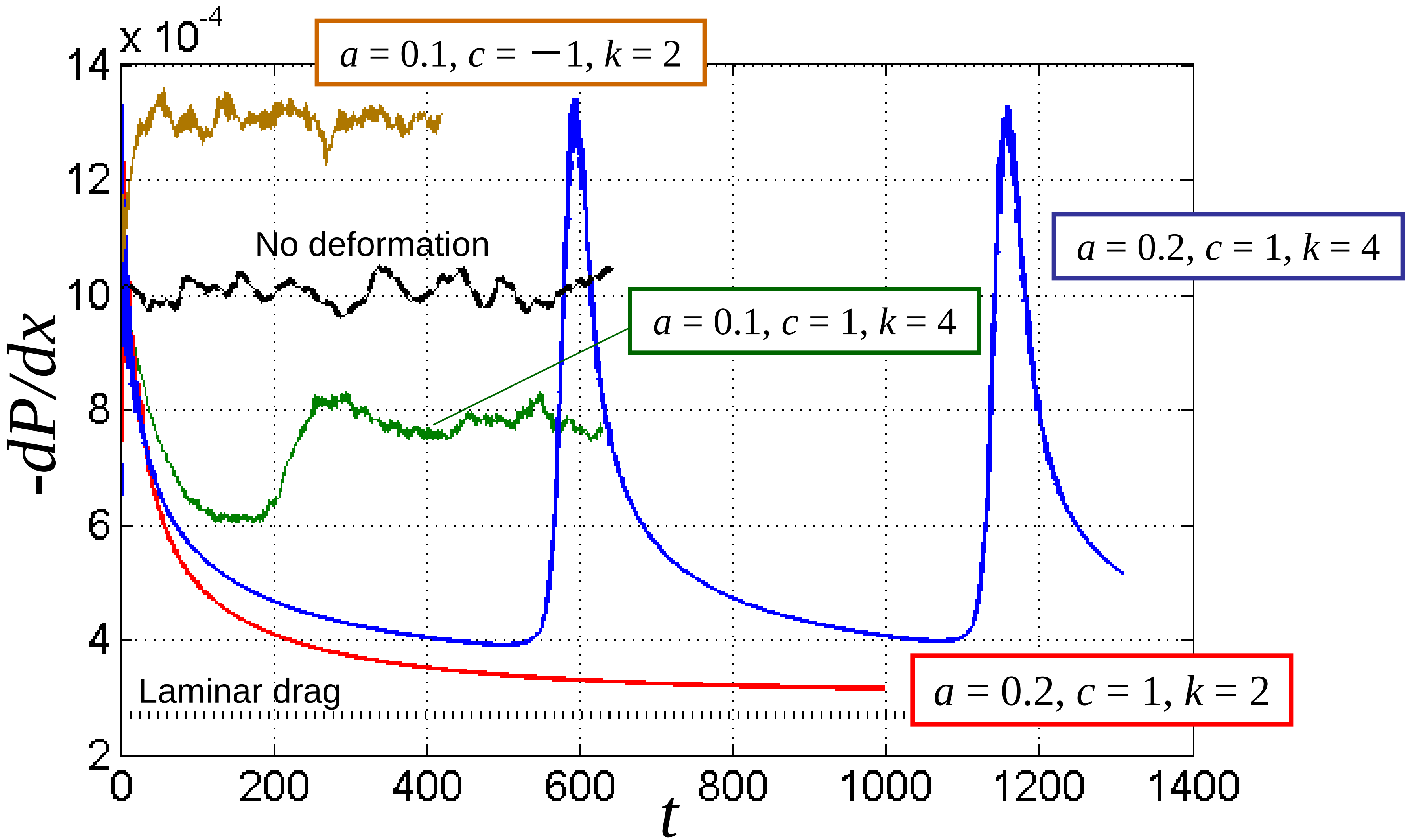


Relaminarization → About 70% drag reduction, 65% reduction in net power

Time trace of mean pressure gradient (=drag)

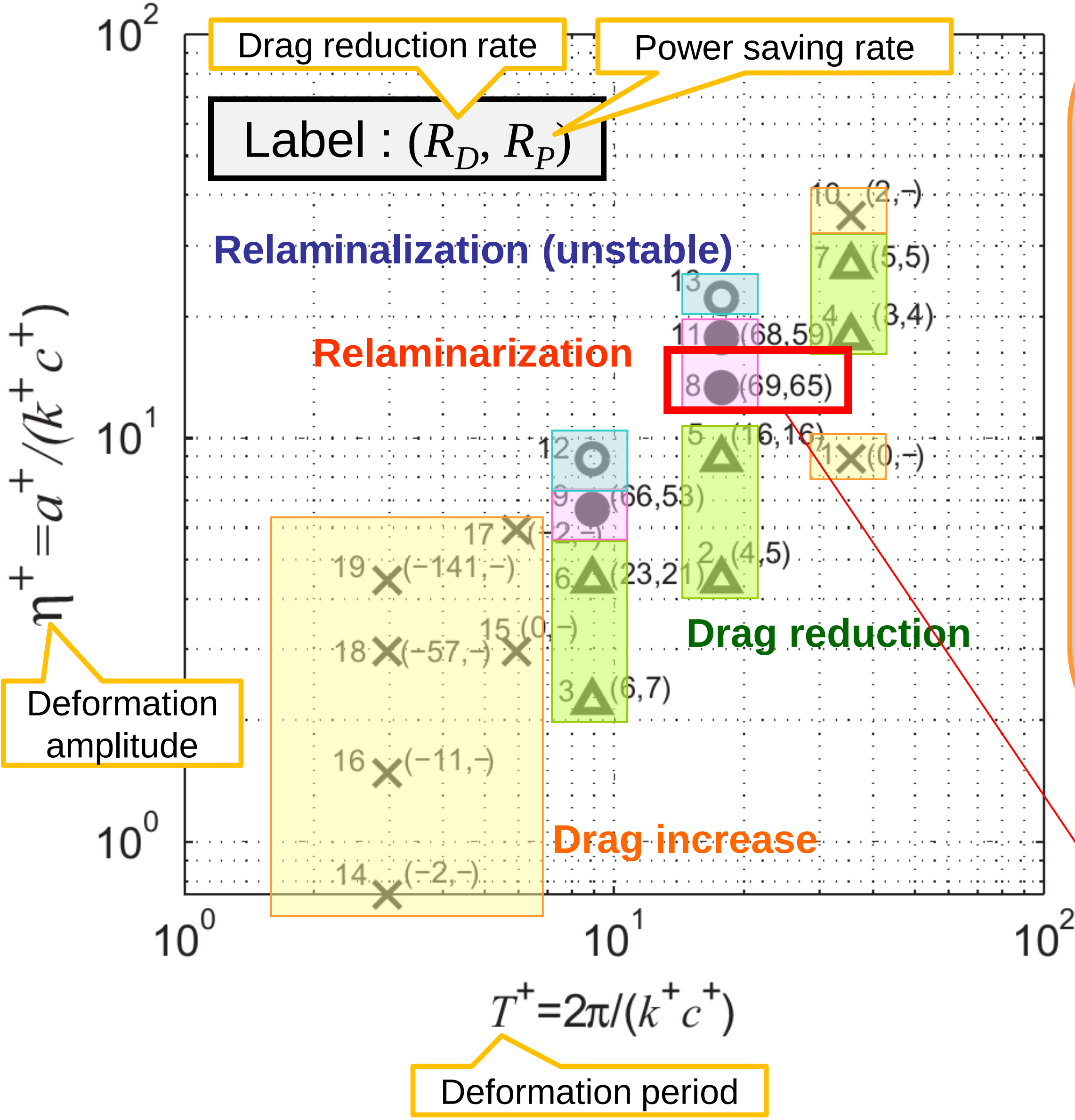
(Nakanishi et al., *Int. J. Heat Fluid Flow*, 2012)

- Upstream wave ($c < 0$): drag increase, or no change
- Downstream wave ($c > 0$): drag reduction



Drag reduction effects under different sets of parameters

(Nakanishi et al., *Int. J. Heat Fluid Flow*, 2012)



$$R_D = \frac{\left(-\left.\frac{dP}{dx}\right|_{\text{no control}}\right) - \left(-\frac{dP}{dx}\right)}{-\left.\frac{dP}{dx}\right|_{\text{no control}}} \times 100\%$$

$$R_P = \frac{W_0 - (W_p + W_a)}{W_0} \times 100\%$$

W_0 : Pumping power for plane channel flow

W_p : Pumping power

W_a : Actuation power

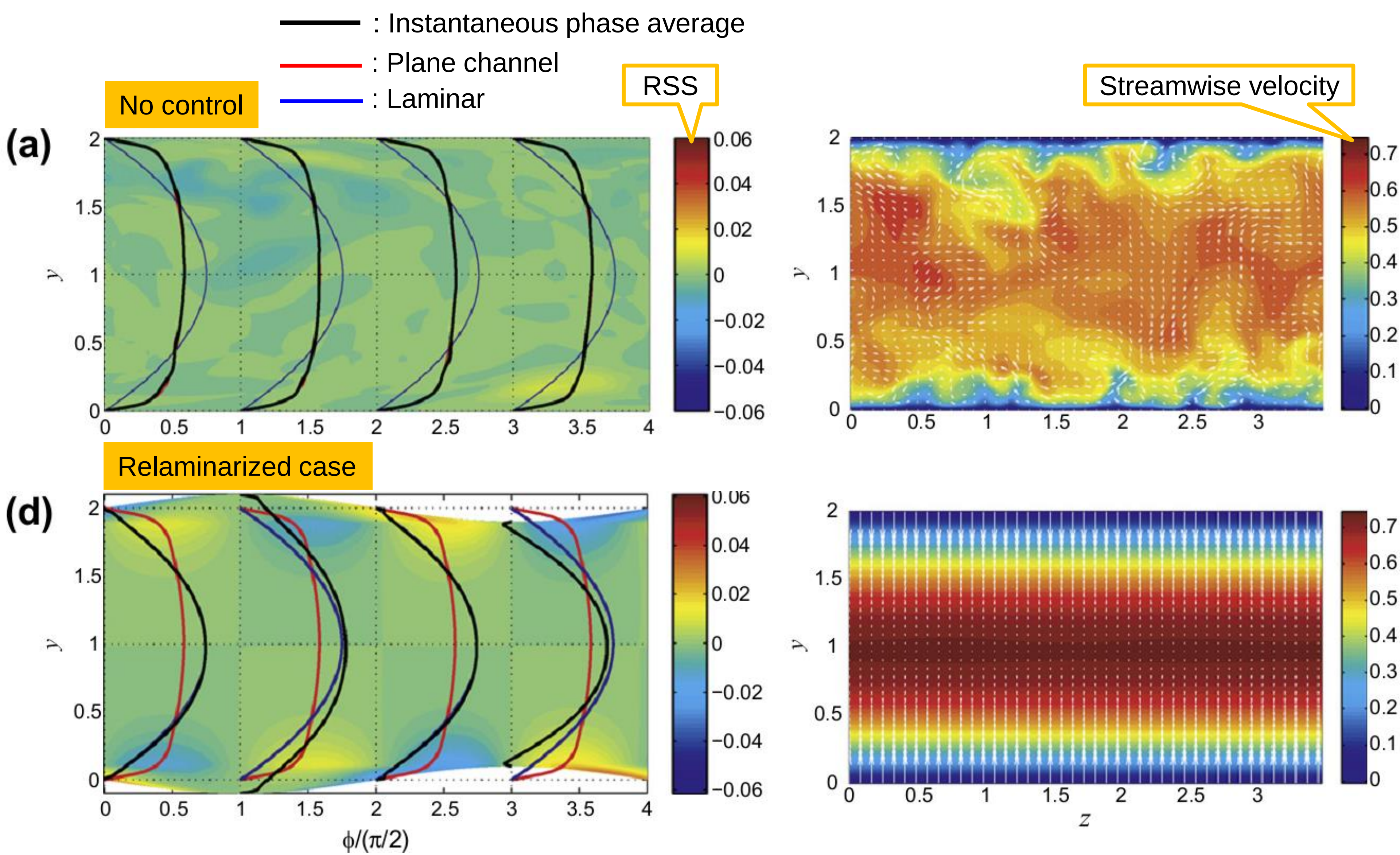
Drag reduction: 69%

Power saving: 65%

Flow field $(a = 0.2,\ c = 1,\ k = 2)$

(Nakanishi et al., *Int. J. Heat Fluid Flow*, 2012)

$a^+ \simeq 6,\ c^+ \simeq 30,\ \lambda^+ \simeq 560$

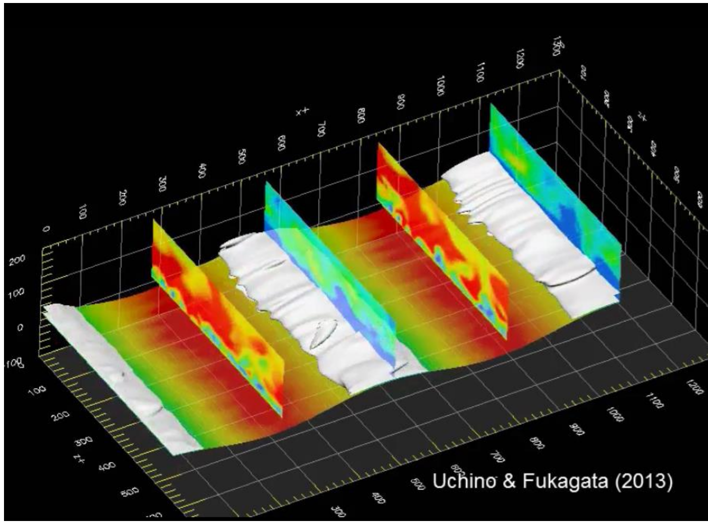


Drag reduction mechanism by downstream traveling wave-like wall-deformation

(Nakanishi et al., *Int. J. Heat Fluid Flow*, 2012)

- **Negative RSS**

  → "Pumping" in the same direction as the wave



External pressure gradient required to keep the flow rate (constant) is reduced

= "Drag reduction"

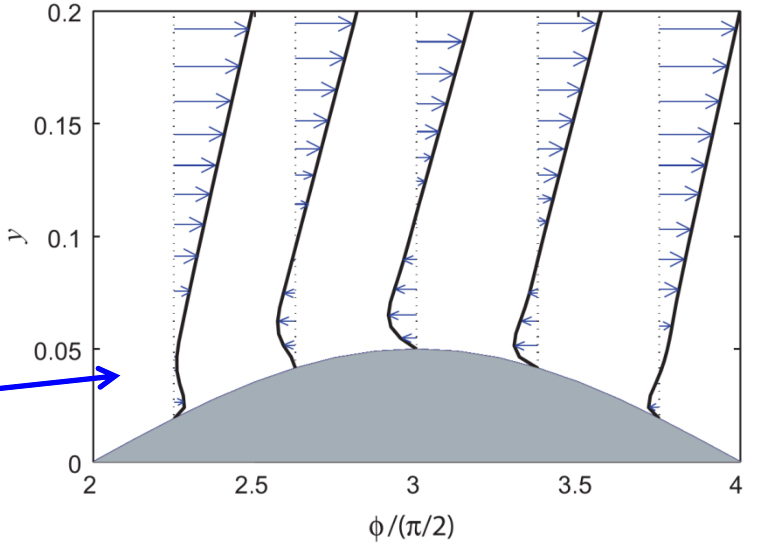
+

- **Stabilization → Relaminraization**

  or

- **Destabilization (inflectional instability) at larger deformation amplitude**

  → Periodic oscillation

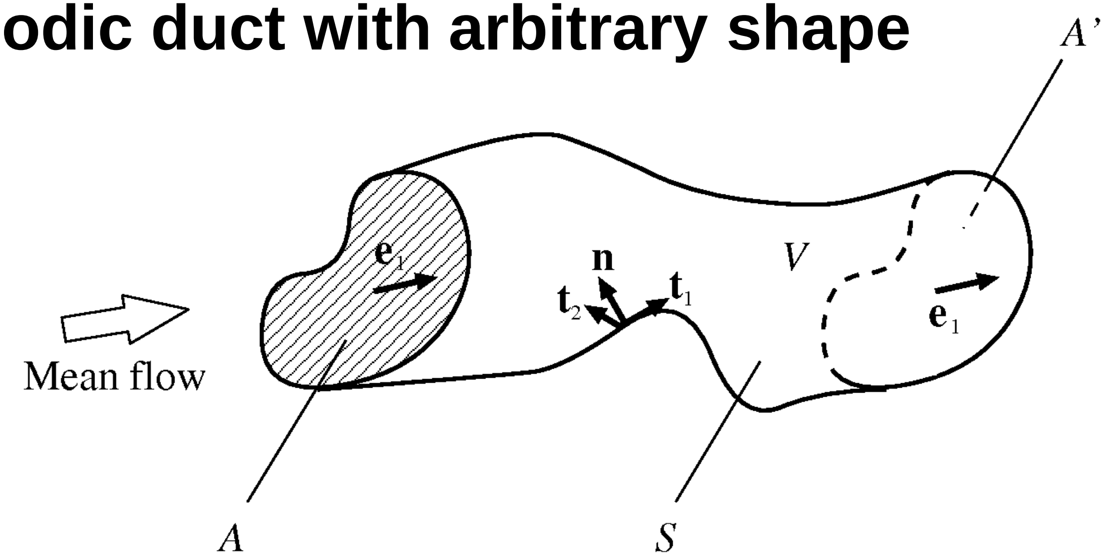


4. Integral relationships on dissipation (or power balance)

Power balance in internal flows

(Fukagata, Sugiyama, and Kasagi, *Physica D*, 2009)

- **Periodic duct with arbitrary shape**



- **Derivation of power balance**
  - Starting point: streamwise momentum equation
    - Zero-net-flux blowing/suction
    - Zero-net body force
  - Multiplication by streamwise velocity
  - Integration in volume + vector/tensor operations

Integral relationship on dissipation

(Fukagata, Sugiyama, and Kasagi, *Physica D*, 2009)

$$W_p + W_a = \underbrace{\frac{2}{Re_b}\int_V \bar{\mathbf{s}}^S : \bar{\mathbf{s}}^S \, \mathrm{d}V}_{\text{(I)}} + \underbrace{\frac{2}{Re_b}\int_V \bar{\mathbf{s}}^D : \bar{\mathbf{s}}^D \, \mathrm{d}V}_{\text{(II)}} + \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}' : \mathbf{s}'} \, \mathrm{d}V}_{\text{(III)}} + \underbrace{2\int_S \bar{p}^S \, \bar{\phi} \, \mathrm{d}S}_{\text{(IV)}} .$$

- W_p: Pumping power $W_p = \left(\int_A \overline{p\mathbf{u}} \cdot \mathbf{e}_1 \, \mathrm{d}A - \int_{A'} \overline{p\mathbf{u}} \cdot \mathbf{e}_1 \, \mathrm{d}A'\right) = AU_b \Delta P$
- W_a: Actuation power $W_a = \int_S \left[\frac{1}{2}\overline{\phi^3} + \overline{p'\phi} + \frac{2}{Re_b}(\nabla \cdot \mathbf{n})\,\overline{\phi^2}\right] \mathrm{d}S + \int_V \overline{\mathbf{u} \cdot \mathbf{b}} \, \mathrm{d}V.$
- (I): Dissipation from the velocity profile of the Stokes flow at the same flow rate
- (II): Dissipation due to the mean deviation from the Stokes profile (non-negative)
- (III): Dissipation due to the fluctuating velocities (non-negative)
- (IV): Additional term for variable-curvature ducts (zero for constant-curvature ducts)

Lower bound of net power

(Fukagata, Sugiyama, and Kasagi, *Physica D*, 2009)

- **For straight channel, straight pipe, channel/pipe with constant curvature**

$$W_p + W_a = \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}}^S : \overline{\mathbf{s}}^S \,\mathrm{d}V}_{\text{(I)}} + \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}}^D : \overline{\mathbf{s}}^D \,\mathrm{d}V}_{\text{(II)}} + \underbrace{\frac{2}{Re_b}\int_V \overline{\mathbf{s}' : \mathbf{s}'} \,\mathrm{d}V}_{\text{(III)}} .$$

Pumping power (W_p)

Actuation power (W_a)

(I) Stokes dissipation

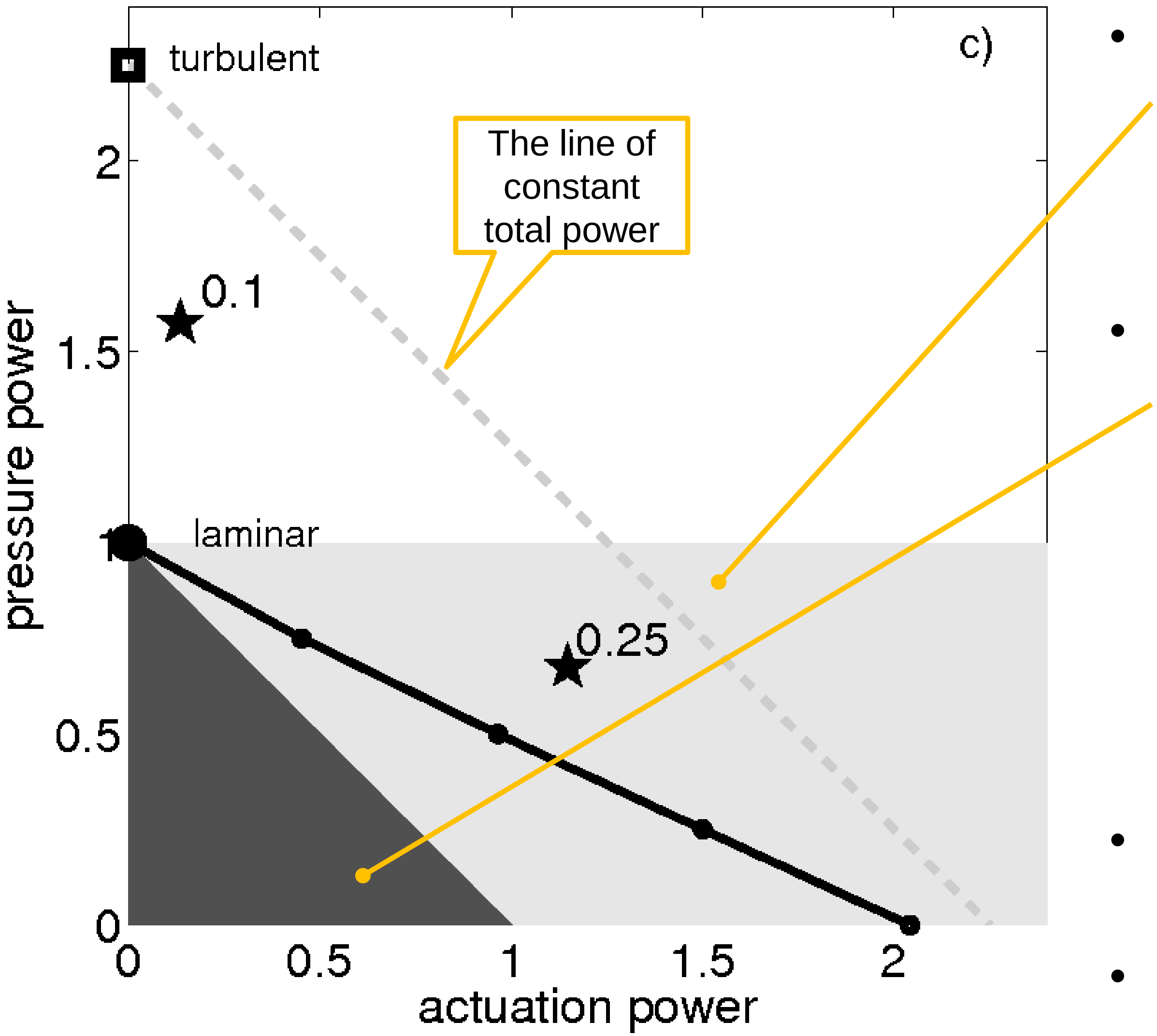
(II) Deviation from Stokes

(III) Fluctuations

"Lowest net power achievable in a controlled duct flow is the pumping power of the Stokes flow at the same flow rate" (not always the same as "laminar flow" !)

– For a fully-developed plane channel flow, this is exactly the same as Bewley (JFM 2009)'s argument because "laminar" = "Stokes" in that case

Example 1: Flow in a straight channel with Min et al. (2006)'s traveling-wave-like blowing/suction

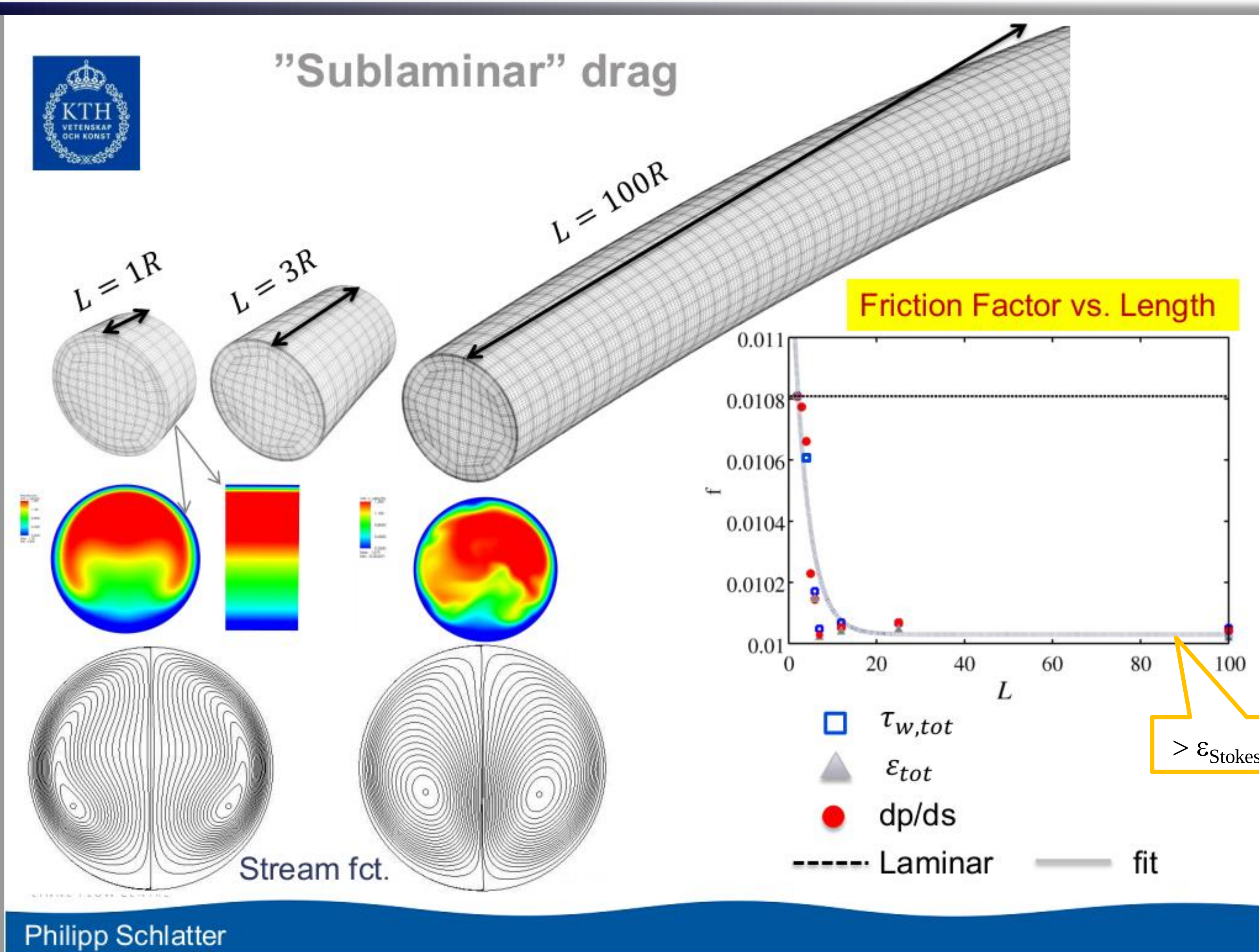


(Hœpffner & Fukagata, *J. Fluid Mech.*, 2009)

- Light gray: Region not allowed by "Bewley's conjecture"
  (Bewley, *Prog. Aerosp. Sci.*, 2001)
- Dark gray: Region not allowed by the correct theorem
  (Fukagata et al., *Physica D*, 2009; Bewley, *J. Fluid Mech.*, 2009)
- Star: Turbulent (DNS, Min et al. (2006)'s condition)
- Circle: Laminar (2D-DNS)

Example 2: Sublaminar drag in uncontrolled flow in a pipe with a constant curvature

(Noorani & Schlatter, submitted for publication)

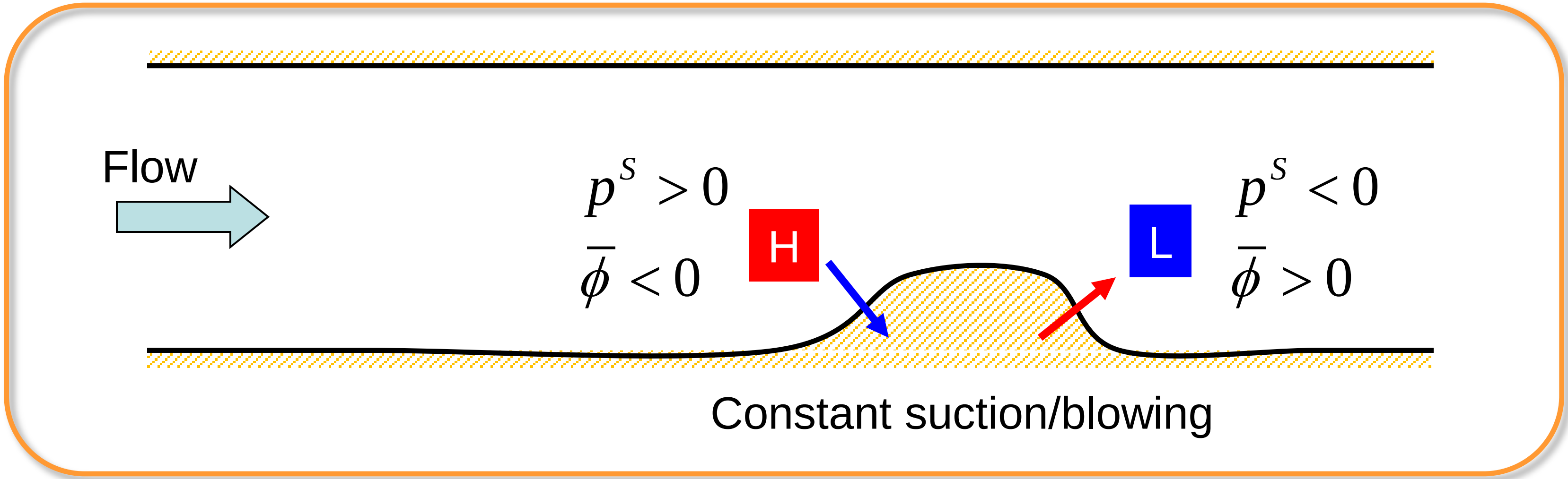


$> \varepsilon_{Stokes} \approx 0.005$

For variable curvature ducts: Possibility of "Lower-than-Stokes" net power?

- **Possible situation of**

$$(\text{IV}) = 2\int_S p^S \overline{\phi}\, dS < 0$$



- **Gives a hint to clarify the theoretical limitation of bluff body control, which seems also unclear** (Choi et al., *Annu. Rev. Fluid Mech.*, 2008) **?**

Integral relationship on dissipation in an external flow

(Naito & Fukagata, *Phys. Rev. E*, 2014)

- **For a flow around a circular cylinder**

$$\varepsilon = -\int_V \frac{\partial\left(\frac{1}{2}\mathbf{u}' \cdot \mathbf{u}'\right)}{\partial t}\, dv + U_\infty (F_{DP} + F_{DF} + F_{D\phi}) + W_{id},$$

Deviation from freestream velocity

where

$$\varepsilon = \frac{2}{\mathrm{Re}} \int_V \mathbf{s} : \mathbf{s}\, dv,$$

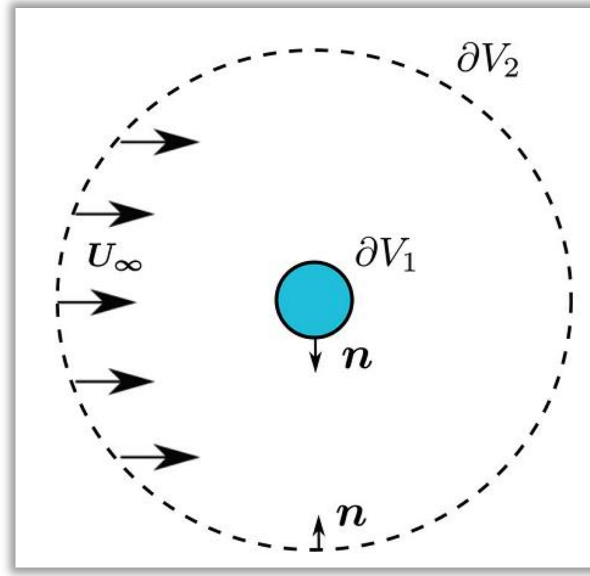
Dissipation

$$F_{DP} = \int_{\partial V_1} (-p \cos\theta)\, ds,$$

Pressure drag

$$F_{DF} = \frac{2}{\mathrm{Re}} \int_{\partial V_1} \mathbf{n} \cdot \mathbf{s} \cdot \mathbf{e}_x\, ds,$$

Friction drag



Additional drag

$$F_{D\phi} = \int_{\partial V_1} (-\phi^2 \cos\theta)\, ds,$$

Ideal actuation power

$$W_{id} = \int_{\partial V_1} \left[\left(p + \frac{1}{2}\phi^2\right)\phi + \frac{2}{\mathrm{Re}}\frac{1}{R}\phi^2\right] ds.$$

Suboptimal control minimizing dissipation

(Naito & Fukagata, *Phys. Rev. E*, 2014)

- **Cost function $\mathcal{J}$: Dissipation expressed by the surface quantities only (neglecting the time derivative)**

neglect

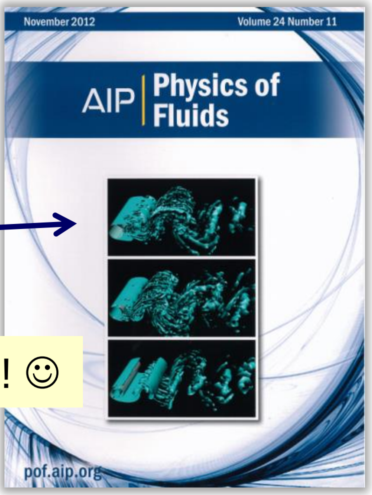
$$\mathcal{J} = -\int_V \frac{\partial\left(\frac{1}{2}\mathbf{u}' \cdot \mathbf{u}'\right)}{\partial t} dv + U_\infty(F_{DP} + F_{DF} + F_{D\phi}) + W_{id},$$

- **Control input (blowing/suction) to minimize the cost function $\mathcal{J}$: Obtained using the suboptimal procedure**

(Lee et al., *J. Fluid Mech.*, 1998; Min & Choi, *J. Fluid Mech.*, 1999)

- **DNS of a flow around a cylinder: Validated in our previous work**

(Naito & Fukagata, *Phys. Fluids*, 2012)

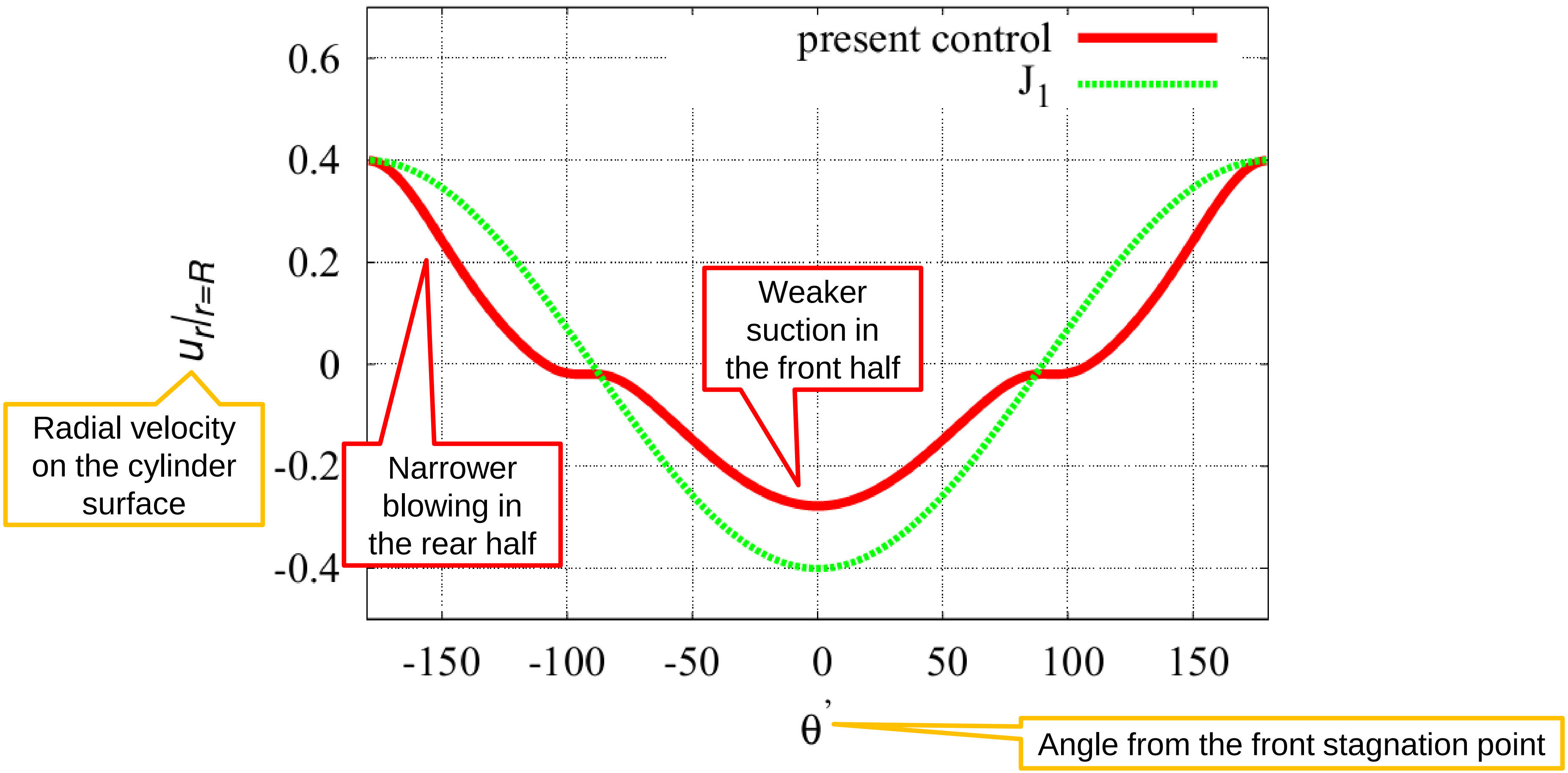


Thanks, John! ☺

Suboptimal control minimizing dissipation (cont'd)

(Naito & Fukagata, *Phys. Rev. E*, 2014)

- **Resultant mean blowing/suction profile, as compared to J_1 control** (= minimizing the pressure drag: Min & Choi, *JFM* 1999)

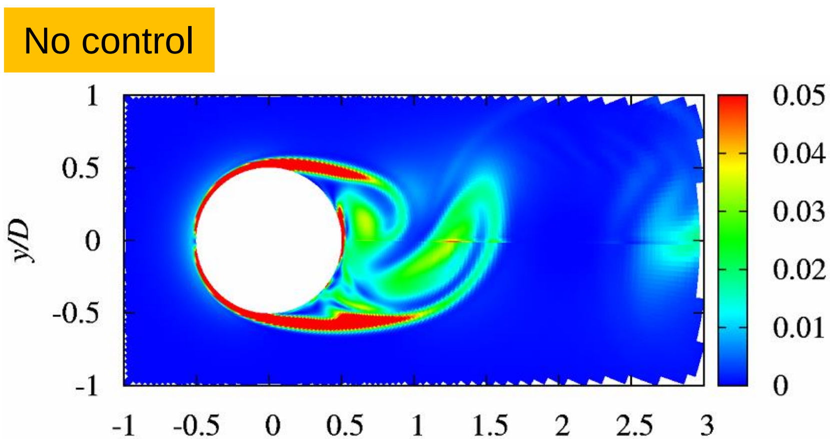


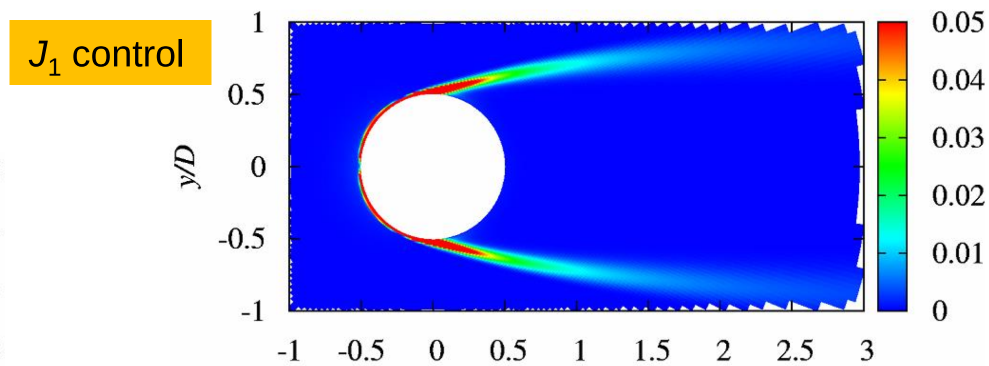
Suboptimal control minimizing dissipation (cont'd)

(Naito & Fukagata, *Phys. Rev. E*, 2014)

TABLE I. Drag, dissipation, input power, and energy efficiencies (Re = 1000, two-dimensional flow).

	$\overline{C_D}$	$\overline{F_D}$	$\overline{\varepsilon}$	$\overline{W_{id}}$	$\overline{W_a}$	$\overline{\eta_{id}}$	$\overline{\eta_a}$
No control	1.545	0.773	0.773				
J_1 control	1.037	0.518	0.378	−0.140	0.324	−1.807	0.784
Present suboptimal control ($\Delta t_c/T = 0.75$)	0.864	0.432	0.328	−0.104	0.202	−3.326	1.680
Present predetermined control	0.864	0.432	0.328	−0.105	0.203	−3.249	1.676
Localized predetermined control (Sec. VE)	0.940	0.470	0.367	−0.103	0.122	−2.945	2.479





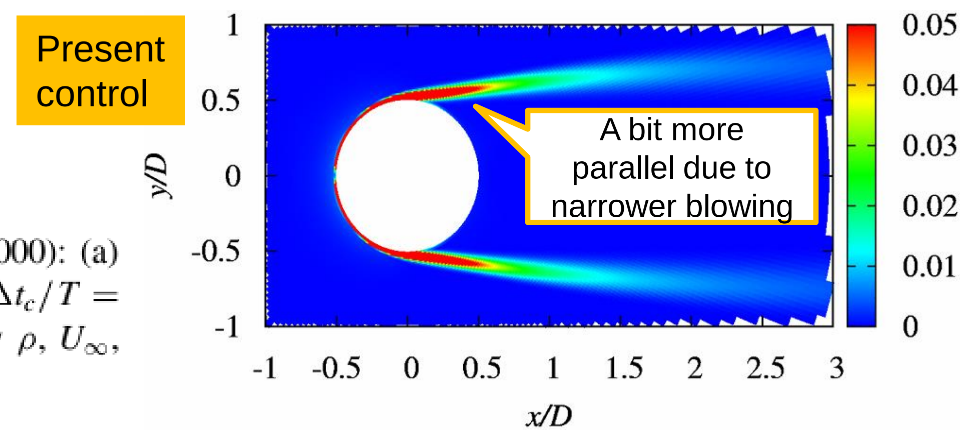


FIG. 7. Instantaneous energy dissipation field (Re = 1000): (a) no control; (b) J_1 control; (c) present suboptimal control ($\Delta t_c/T = 0.75$). The local dissipation rate is made dimensionless by ρ, U_∞, and D.

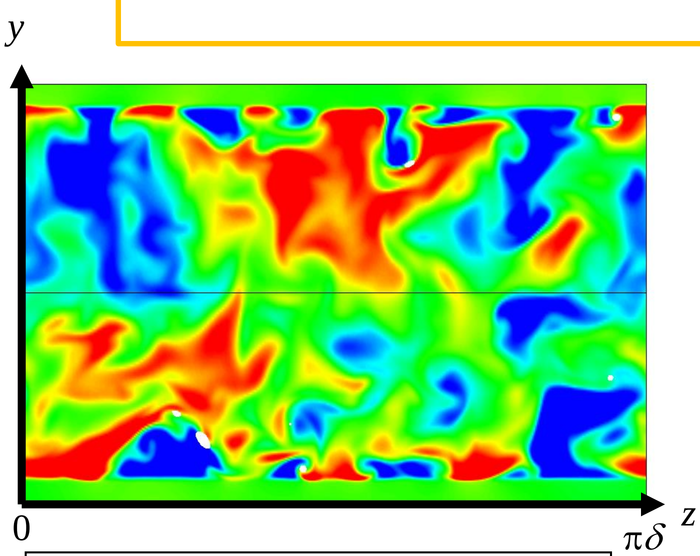
5. Some other useful relationships for turbulence control

Prediction of drag reduction rate in channel flow by ideal damping of near-wall fluctuations

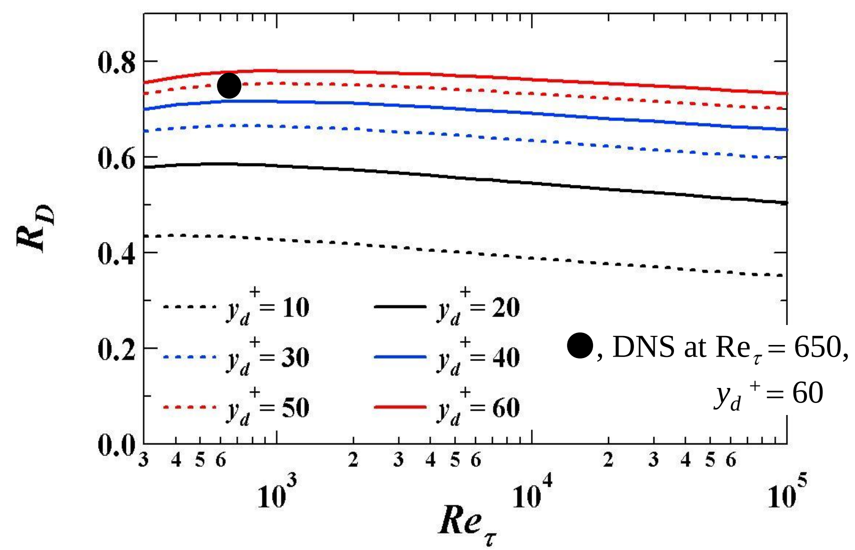
(Iwamoto, Fukagata, Kasagi, and Suzuki, *Phys. Fluids*, 2005)

- Constant flow rate; ideal damping in $0 < y < y_d$
  → Implicit formula among $\mathrm{Re}_{\tau 0}$, R_D, and y_d/δ

$$\frac{1}{\kappa}\ln \mathrm{Re}_{\tau 0} + F = \frac{1}{2}(y_d/\delta)^2(1-y_d/3\delta)(1-R_D)\mathrm{Re}_{\tau 0} + (y_d/\delta)(1-y_d/2\delta)(1-y_d/\delta)(1-R_D)\mathrm{Re}_{\tau 0} + (1-y_d/\delta)^{3/2}(1-R_D)^{1/2}\left[\frac{1}{\kappa}\ln\left\{(1-y_d/\delta)^{3/2}(1-R_D)^{1/2}\mathrm{Re}_{\tau 0}\right\}+F\right]$$



Damping near-wall region ($y^+ < 60$)



Prediction of drag reduction rate of flow in a channel with superhydrophobic surfaces

(Fukagata, Kasagi, and Koumouotsakos, *Phys. Fluids*, 2006)

- Implicit formula among l_x, l_z, $\mathrm{Re}_{\tau 0}$, and R_D

$$\frac{1}{\kappa}\ln \mathrm{Re}_{\tau 0} + F_0 = (1-R_D)l_x^{+0} + \frac{\sqrt{1-R_D}}{\kappa}\ln\left(\sqrt{1-R_D}\,\mathrm{Re}_{\tau 0}\right) + \sqrt{1-R_D}\,F\left(\sqrt{1-R_D}\,l_z^{+0}\right)$$

where

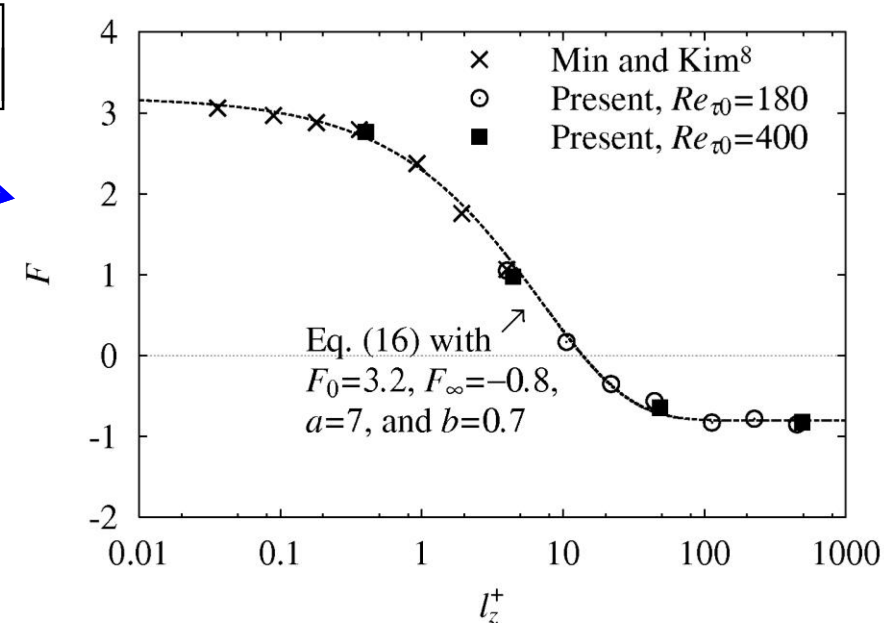
$$F(l_z^+) = F_\infty + \left(F_0 - F_\infty\right)\exp\left[\left(-l_z^+/a\right)^b\right]$$

(Original)

or an improved model

$$F(l_z^+) = F_\infty + \frac{(F_0 - F_\infty)^2}{(F_0 - F_\infty) + l_z^+}$$

(Busse & Sandham, *Phys. Fluids*, 2012)



6. Summary

Summary

- **Integral relationships**
  - "Formal" solutions to the fundamental questions in fluids engineering
    - Connection between drag and turbulent statistics
    - Connection among pumping power, actuation power, and dissipation
  - Convenient tools for turbulence control studies
    - Analysis of control effect
      - Quantification of major contributor(s)
    - Proposal of new control methods
      - Method to reduce RSS, such as traveling waves
      - Good affinity with (sub-)optimal control theory
- **Future directions**
  - Connection to "dynamics"

Acknowledgments

- **Financial support**
  - **Institute for Pure and Applied Mathematics (IPAM)**, for invitation to this workshop
  - Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan Society for the Promotion of Science (JSPS), Japan Aerospace Exploration Agency (JAXA), Railway Technical Research Institute (RTRI) of Japan
- **Co-workers (current affiliation)**
  - Univ. Tokyo: Drs. Nobuhide Kasagi (JST), Yuji Suzuki, Kaoru Iwamoto (TUAT), Yosuke Hasegawa, Kazuyasu Sugiyama (Osaka Univ.)
  - ETH: Drs. Petros Koumoutsakos, Stefan Kern, Philippe Chatelain (Louvain Catholique Univ.)
  - KTH: Drs. Henrik Alfredsson, Antonio Segalini, Philipp Schlatter, Ramis Örlü
  - Keio Univ.: Drs. Shinnosuke Obi, Jerôme Hœpffner (UPMC), Yoshitsugu Naka (Tokyo Tech), Hiroya Mamori (TUAT), Simon J. Illingworth (Univ. Melbourne), Hiroshi Naito (JAXA), Yukinori Kametani (KTH), Ms. Rio Nakanishi (Chubu Electric Power), Messrs. Kosuke Higashi (Kobelco), Nobuhito Tomiyama (Daikin), Sho Watanabe (Komatsu), Taichi Igarashi (Kobelco), Keisuke Uchino, Ryosuke Kidogawa, and all the former and current students
- **Research community**
  - Drs. John Kim (UCLA), Tom Bewley (UCSD), Haecheon Choi (Seoul National Univ.), Godfrey Mungal (Stanford/SCU), Pierre Sagaut (UPMC), Yasuo Kawaguchi (Tokyo Univ. Sci.), Bo Yu (China Univ. Petroleum), Shu Takagi (Univ. Tokyo), Yuichi Murai (Hokkaido Univ.), Michael Leschziner (Imperial College), Bettina Frohnapfel (KIT), Maurizio Quadrio (Politec Milano), Mihailo Jovanovic (Minnesota Univ.), Ivan Marusic (Univ. Melbourne), Pierre Ricco (Univ. Sheffield), Azad Noorani (KTH), and actually many other people!
- **And, of course, the workshop organizers and all the participants today --- Thank you very much!**