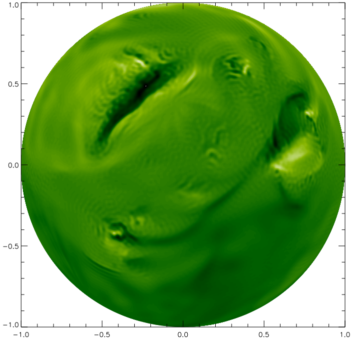
Electron-MHD Turbulence in Neutron Stars

Toby Wood
Newcastle University
(previously University of Leeds)



Wood, Hollerbach & Lyutikov 2014,
Physics of Plasmas 21, 052110

Science & Technology Facilities Council

Outline

- What is a neutron star?
- What is Hall / Electron MHD?
- What is the preferred field geometry in a neutron star?
- How does the field affect the star's evolution?

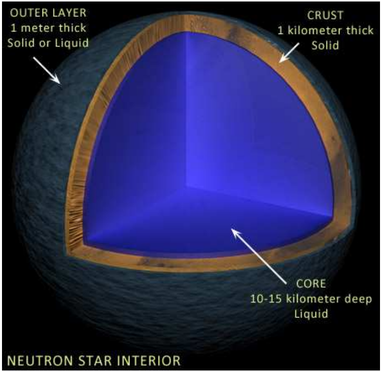
What is a neutron star?

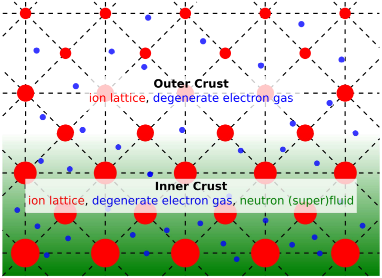
Types of dead star:

- $M \lesssim M_\odot$ white dwarf, supported by electron degeneracy pressure
- $M_\odot \lesssim M \lesssim 3M_\odot$ neutron star, supported by neutron degeneracy pressure
- $3M_\odot \lesssim M$ black hole

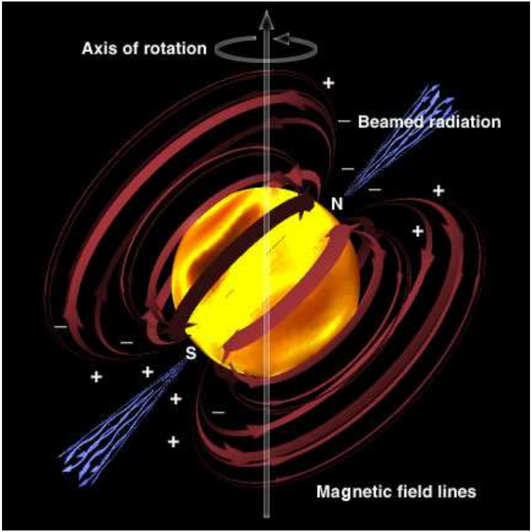
Neutron star structure

- $M \sim 1.5\,M_\odot$
- $R \sim 10\,\mathrm{km}$





Typical parameters



- $|\mathbf{B}| \lesssim 10^{15}$ G
- $|\boldsymbol{\Omega}| \lesssim 10^{3}$ s^{-1}
- $T \lesssim 10^{8}$ K

Do all neutron stars look like this?

Neutron star evolution

	Age	Rotation Period	Magnetic Field
Magnetars	$\sim 10^3$ years	2 – 10 s	10^{14-15} G
Classical Pulsars	10^{3-7} years	5 ms – 5 s	10^{11-13} G
Millisecond Pulsars	10^{8-10} years	1 – 10 ms	10^{8-9} G

Need to explain:

- magnetic field decay
- spindown?
- cooldown
- glitches
- magnetic spots?

Electron MHD / Hall MHD

- In a neutron star crust, ions are held fixed within the lattice, but electrons can still flow (EMHD).
- In Gaussian cgs units,

$$\frac{\partial \mathbf{B}}{\partial t} = -c\boldsymbol{\nabla} \times \mathbf{E}$$

$$\mathbf{J} = \frac{c}{4\pi}\boldsymbol{\nabla} \times \mathbf{B} = -\mathrm{e}n\mathbf{v}$$

$$m\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\mathbf{v}\right) = -\frac{1}{n}\boldsymbol{\nabla}P - \mathrm{e}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)$$

- Electromagnetism negligible on scales $\ll d$

$$d = \left(\frac{mc^2}{4\pi n\mathrm{e}^2}\right)^{1/2} \sim 10^{-11}\,\mathrm{cm}.$$

A non-linear induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\frac{c}{4\pi e n} \underbrace{\mathbf{B} \times (\nabla \times \mathbf{B})}_{\text{Hall term}} - \underbrace{\eta \nabla \times \mathbf{B}}_{\text{diffusion}} + \cancel{\frac{c}{en} \nabla P} \right]$$

Important parameter is $\quad \mathrm{R}_B = \dfrac{c|\mathbf{B}|}{4\pi e n \eta} \sim 10^2.$

Hall term has 2 derivatives, but is **non-dissipative**:

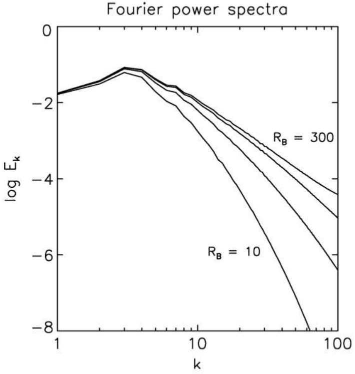
$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V \tfrac{1}{2} |\mathbf{B}|^2 = -\int_V \eta |\nabla \times \mathbf{B}|^2.$$

Could the Hall term enhance the magnetic diffusion?
(Goldreich & Reisenegger 1992)

EMHD turbulence

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times \left[\mathbf{B} \times (\boldsymbol{\nabla} \times \mathbf{B}) \; - \; \frac{1}{\mathrm{R}_B} \boldsymbol{\nabla} \times \mathbf{B} \right]$$

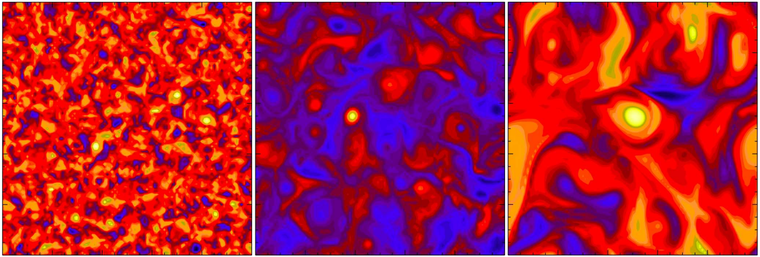
- Well-defined energy spectrum and forward cascade



- But no dissipative cut-off
- ... and interactions are non-local in spectral space

EMHD turbulence

- AND temporal coherence in real space



Wareing & Hollerbach 2009, 2010

- Is this really turbulence at all?
- Is the system even really unstable??

EMHD stability

Are there instabilities in EMHD?

- Yes, for nonuniform density (Gordeev & Rudakov 1968)
- Yes, for finite resistivity (Gordeev 1970)
- Yes, for finite inertia (Bulanov et al. 1992)

What about for uniform density, zero resistivity, and zero inertia?

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times [\mathbf{B} \times (\boldsymbol{\nabla} \times \mathbf{B})]$$

- Yes. Drake et al. (1994)
- Yes. Rheinhardt & Geppert (2002)
- No. Lyutikov (2013)

EMHD stability

Regular MHD:

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times [\mathbf{u} \times \mathbf{B}]$$
$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = (\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} - \boldsymbol{\nabla} P.$$

For a mode with growthrate λ,
$$\lambda^2 \int \mathrm{d}V \, |\boldsymbol{\xi}|^2 = \int \mathrm{d}V \, \boldsymbol{\xi}^\star \cdot \mathcal{F}(\boldsymbol{\xi}).$$

Stable iff $\mathcal{F}$ is negative definite (Lundquist 1951).

Electron MHD:

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times [\mathbf{B} \times (\boldsymbol{\nabla} \times \mathbf{B})]$$

For a mode with growthrate λ,
$$(\lambda + \lambda^\star) \int \mathrm{d}V \, \boldsymbol{\xi}^\star \cdot \mathcal{F}(\boldsymbol{\xi}) = 0.$$

Stable if $\mathcal{F}$ is negative definite.

So electron MHD is "at least as stable" as regular MHD (Wood, Hollerbach & Lyutikov 2014).

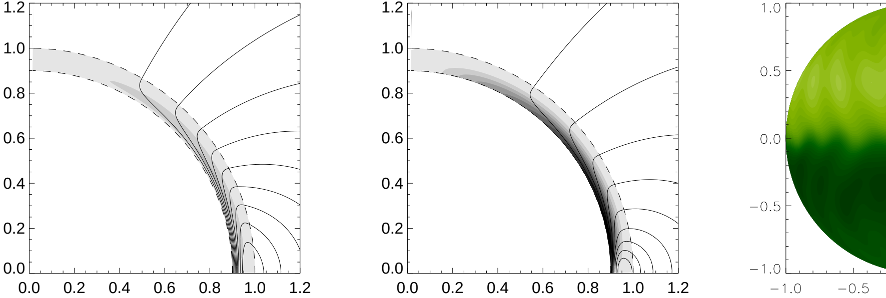
EMHD stability

- For nonuniform density, EMHD is "at least as stable" as anelastic MHD.
- So the density-gradient instability in EMHD must have an anelastic analog.
- In fact, this is the anelastic "zero-gravity magneto-buoyancy instability"!

More realistic numerical modeling

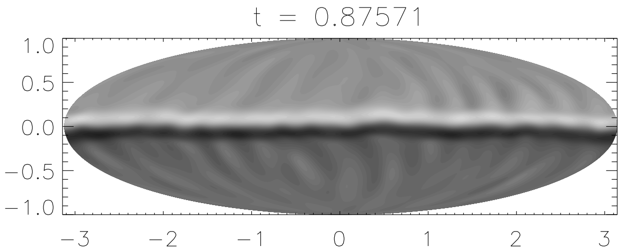
$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times \left[\frac{c}{4\pi e n(r)} \mathbf{B} \times (\boldsymbol{\nabla} \times \mathbf{B}) - \eta(r) \boldsymbol{\nabla} \times \mathbf{B} \right]$$

A “Hall attractor” in 2D.

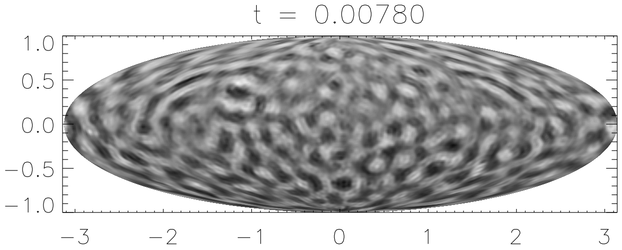


Is this stable to 3D perturbations? Wood & Hollerbach (in prep.)

More realistic numerical modeling

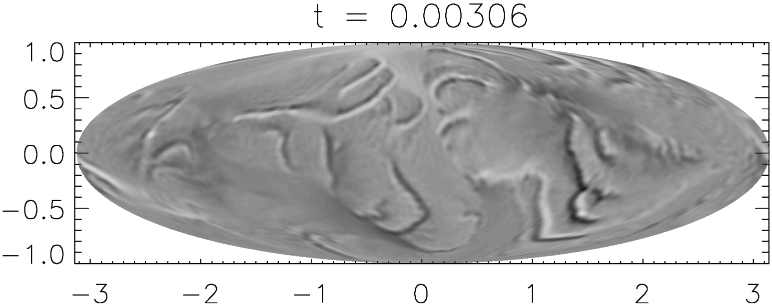


$\mathrm{R}_B = 50$



$\mathrm{R}_B = 200$

More realistic numerical modeling

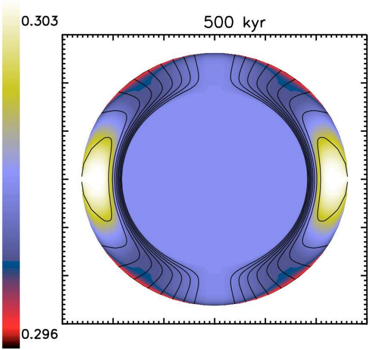


Coupled magneto-thermal evolution

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\frac{1}{n} \mathbf{B} \times (\nabla \times \mathbf{B}) \; - \; \eta \nabla \times \mathbf{B} \; + \; S \nabla T \right]$$

$$0 = \nabla \cdot \left(k \frac{\nabla T + (1/n\eta)^2 (\mathbf{B} \cdot \nabla T) \mathbf{B} + (1/n\eta) \mathbf{B} \times \nabla T}{1 + (1/n\eta)^2 |\mathbf{B}|^2} \right)$$

- Induction equation has a source term from electron baroclinicity.
- Temperature diffusion is inhibited across $\mathbf{B}$ field lines.
- Can lead to electron convection.



Pons et al. (2009)

Implications and future work

- Non-uniform density can drive EMHD instability in neutron stars.
- But realistic 3D simulations generally converge to a large-scale equilibrium.
- Strong jets of warm electrons near the surface.
- Possibility of electron convection — automatic dynamo!