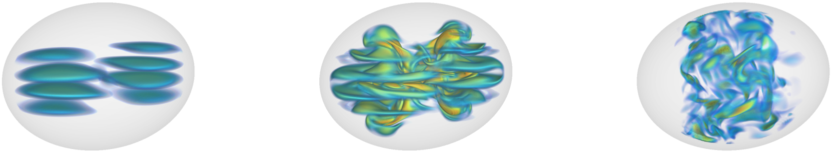
Turbulent flows in libration-driven ellipsoids

B. Favier

Collaborators: A.J. Barker, M. Le Bars, A. Grannan, J. Aurnou

October 27–31 2014
Mathematics of Turbulence: Geophysical and Astrophysical Turbulence
Institute for Pure and Applied Mathematics



Outline

Libration as a source of motion in planetary interiors

- Due to gravitational interactions, celestial bodies are affected by various mechanical forcings:
  - Precession
  - Tides
  - Libration

- These forcings can extract a fraction of the huge rotational energy and generate intense motions in the fluid layers
  - Waves
  - Zonal flows
  - Turbulence

Libration as a source of motion in planetary interiors

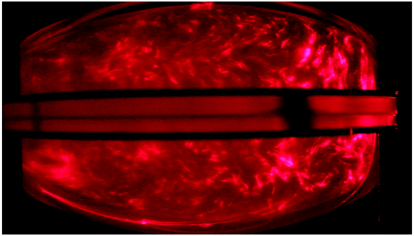
- Due to gravitational interactions, celestial bodies are affected by various mechanical forcings:
  - Precession
  - Tides
  - Libration
- These forcings can extract a fraction of the huge rotational energy and generate intense motions in the fluid layers
  - Waves
  - Zonal flows
  - Turbulence

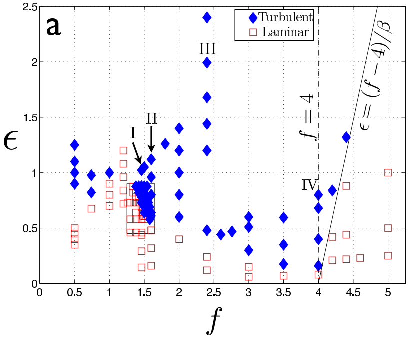
Previous studies about libration

- Zonal flows: nonlinear interactions in Ekman layers lead to zonal flows both in axisymmetric (Busse 2010, Calkins et al. 2010) and non-axisymmetric containers (Zhang et al. 2011, Grannan et al. 2014).

- Libration-driven elliptical instability: triadic resonance of two inertial modes with the base flow (Kerswell&Malkus 1998) shown both experimentally (Noir et al. 2012, Grannan et al. 2014) and numerically (Cébron et al. 2012, Wu& Roberts 2013, Zhang et al. 2013) and generalized to multi-polar deformations (Cébron et al. 2014)

Experiment by Grannan *et al.*



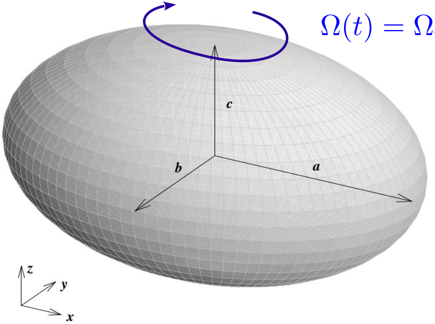


Grannan, Le Bars, Cébron & Aurnou, *Global-scale turbulent flows in libration-driven ellipsoids*, submitted to PoF.

Outline

Librating ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



$$\Omega(t) = \Omega_0 + \Delta\phi\omega_l \sin(\omega_l t)$$

- No-slip boundary conditions
- Incompressible fluid with constant ν

Governing equations and parameters

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \underbrace{2\left[1 + \epsilon \sin(ft)\right] \hat{\boldsymbol{z}} \times \boldsymbol{u}}_{\text{Coriolis}} = -\nabla p + E \nabla^2 \boldsymbol{u} - \underbrace{\epsilon f \cos(ft)\ \hat{\boldsymbol{z}} \times \boldsymbol{r}}_{\text{Poincaré}}$$

$$\nabla \cdot \boldsymbol{u} = 0$$

- Eccentricity: $\beta = \dfrac{a^2 - b^2}{a^2 + b^2} = 0.34$
- Aspect ratio: $\dfrac{c}{b} = 1$
- Librating frequency: $f = \omega_l / \Omega_0 = 4$
- Libration amplitude: $\epsilon = \Delta\phi\ f = 0.8$
- Ekman number: $E = \dfrac{\nu}{\Omega_0 a} \approx 10^{-3} - 10^{-5}$

Governing equations and parameters

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \underbrace{2\left[1+\epsilon\sin(ft)\right]\hat{\boldsymbol{z}}\times\boldsymbol{u}}_{\text{Coriolis}} = -\nabla p + E\nabla^2\boldsymbol{u} - \underbrace{\epsilon f\cos(ft)\;\hat{\boldsymbol{z}}\times\boldsymbol{r}}_{\text{Poincaré}}$$

$$\nabla\cdot\boldsymbol{u} = 0$$

- Eccentricity: $\beta = \frac{a^2-b^2}{a^2+b^2} = 0.34$
- Aspect ratio: $\frac{c}{b} = 1$
- Librating frequency: $f = \omega_l/\Omega_0 = 4$
- Libration amplitude: $\epsilon = \Delta\phi\; f = 0.8$
- Ekman number: $E = \frac{\nu}{\Omega_0 a} \approx 10^{-3} - 10^{-5}$

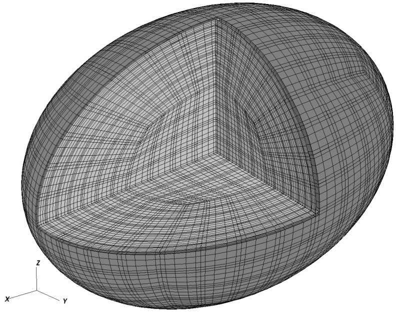
Growth rate of the elliptical instability:

$\approx \epsilon\beta - K\sqrt{E}$

Numerical approach

Spectral element code Nek5000

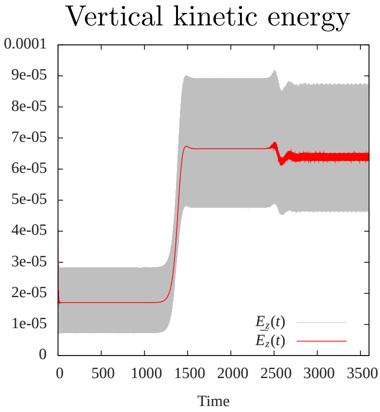
`http://nek5000.mcs.anl.gov`



- E isoparametric hexahedral elements
- N^3 tensor-product Gauss-Lobatto Legendre collocation points
- Algebraic convergence with E
- Exponential convergence with N
- 3^{rd} order explicit Adams-Bashforth scheme for convective terms
- 3^{rd} order implicit Backward Differentiation scheme for diffusive and pressure terms

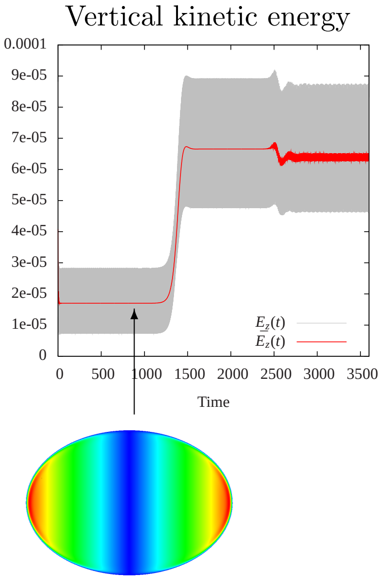
Outline

Example close to threshold: $E = 5 \times 10^{-4}$

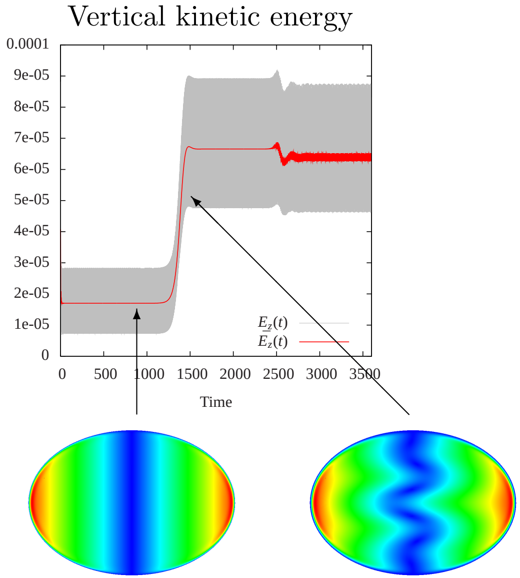


$$\bar{A}(t) = \frac{1}{T}\int_t^{t+T} A(\tau)\mathrm{d}\tau$$

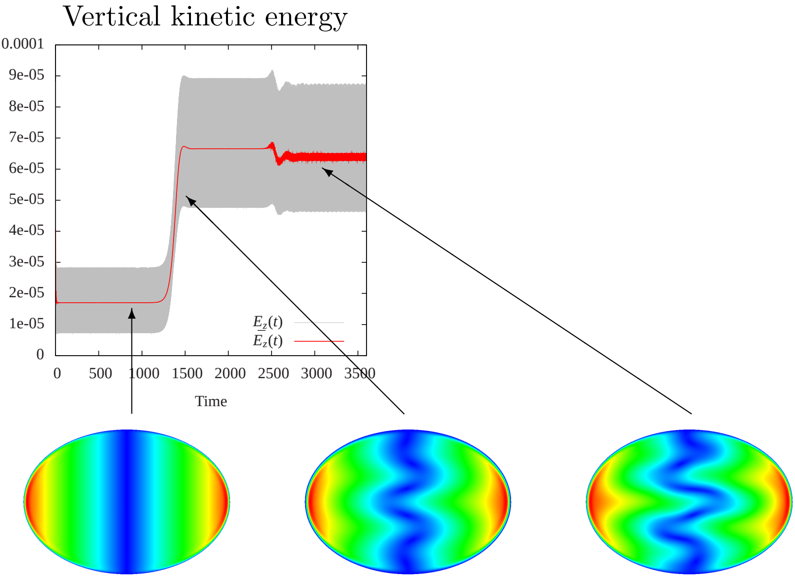
Example close to threshold: $E = 5 \times 10^{-4}$



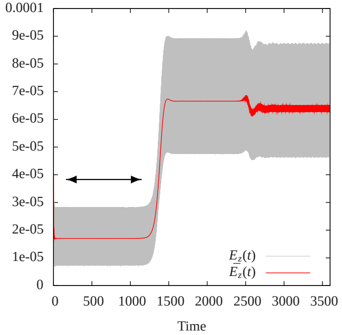
Example close to threshold: $E = 5 \times 10^{-4}$

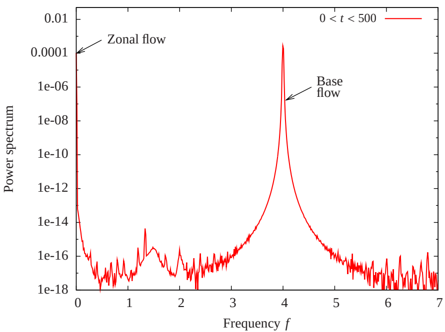


Example close to threshold: $E = 5 \times 10^{-4}$

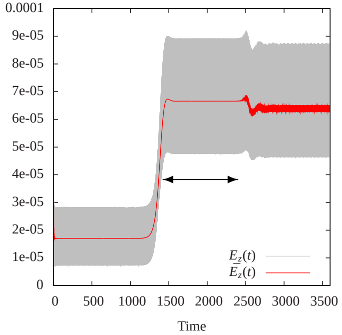


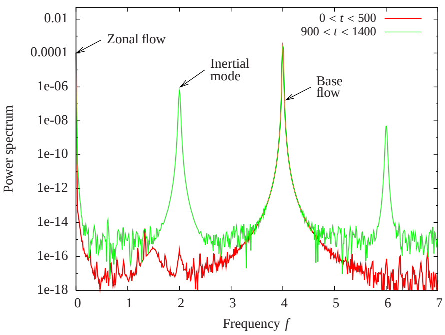
Spectral analysis



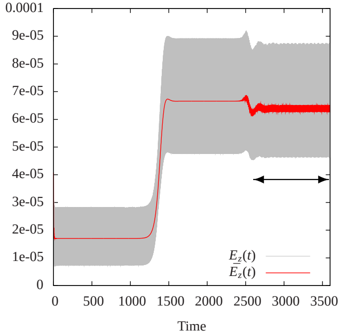


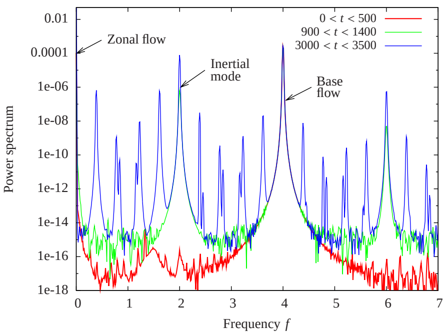
Spectral analysis





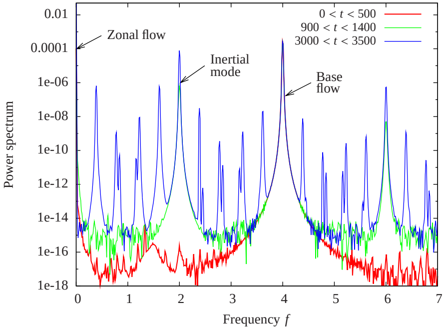
Spectral analysis

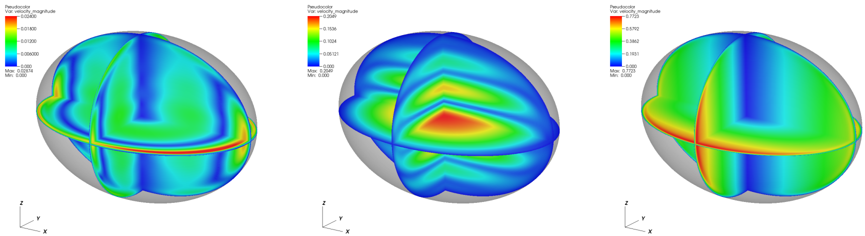




Spectral analysis

$$\hat{\boldsymbol{u}}(\omega_f, \boldsymbol{x}) \approx \int_{t_i}^{t_f} \boldsymbol{u}(\boldsymbol{x}) e^{i\omega_f (t-t_i)} \mathrm{d}t$$

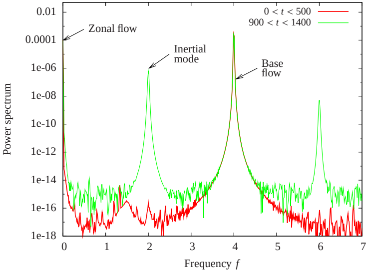


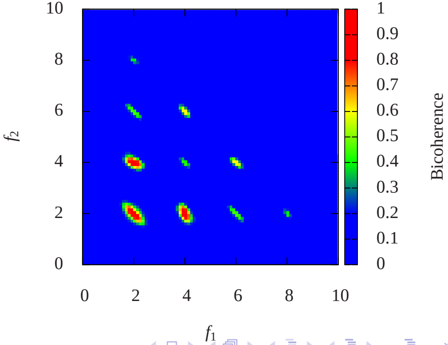


High-order spectral analysis

In order to detect quadratic nonlinearities, we compute the *bicoherence* of the time signals:

$$b^2(k,l) = \frac{\left| \sum_{i=1}^{N} u_i(k) u_i(l) u_i^*(k+l) \right|^2}{\sum_{i=1}^{N} |u_i(k)u_i(l)|^2 \sum_{i=1}^{N} |u_i(k+l)|^2}$$

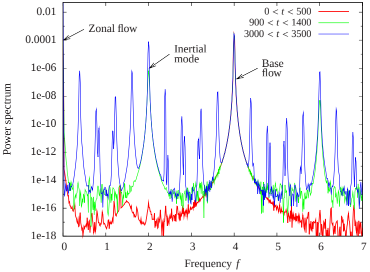


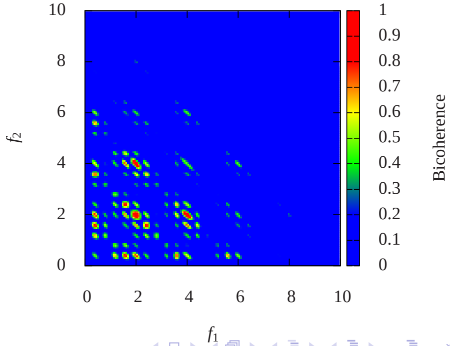


High-order spectral analysis

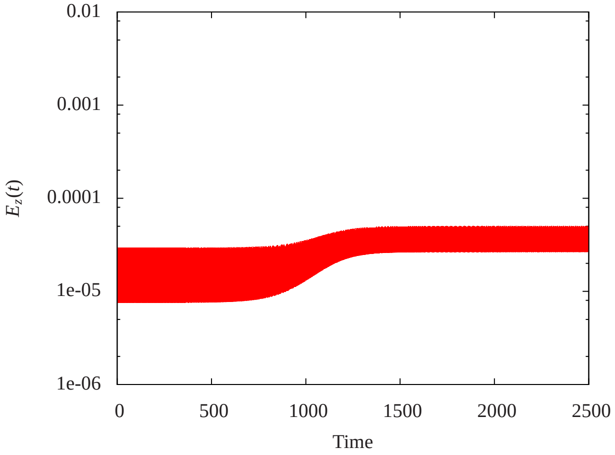
In order to detect quadratic nonlinearities, we compute the *bicoherence* of the time signals:

$$b^2(k,l) = \frac{\left|\sum_{i=1}^{N} u_i(k)u_i(l)u_i^*(k+l)\right|^2}{\sum_{i=1}^{N}\left|u_i(k)u_i(l)\right|^2 \sum_{i=1}^{N}\left|u_i(k+l)\right|^2}$$



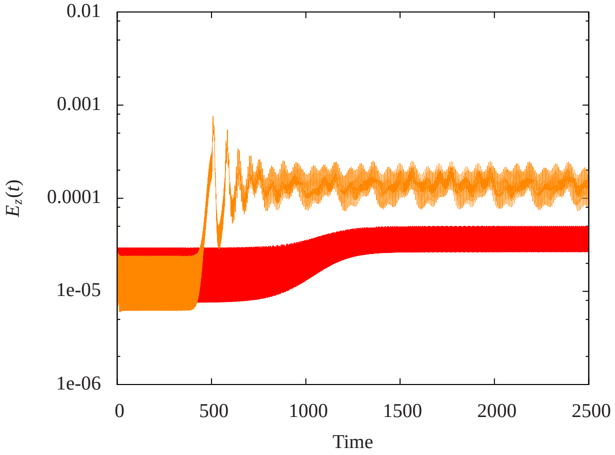


Decreasing the Ekman number



$E = 5 \times 10^{-4}$

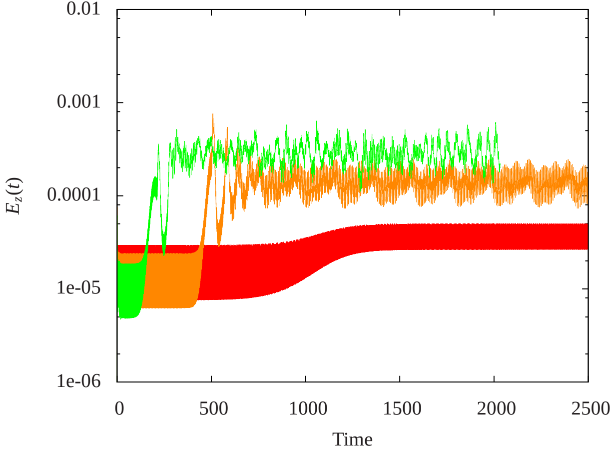
Decreasing the Ekman number



$E = 5 \times 10^{-4}$

$E = 3.5 \times 10^{-4}$

Decreasing the Ekman number

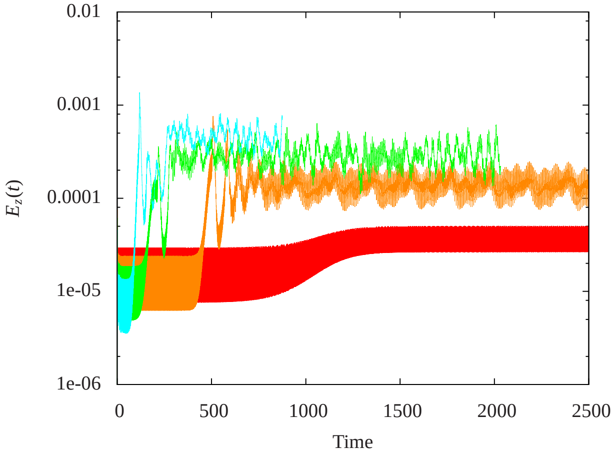


$E = 5 \times 10^{-4}$

$E = 3.5 \times 10^{-4}$

$E = 2 \times 10^{-4}$

Decreasing the Ekman number



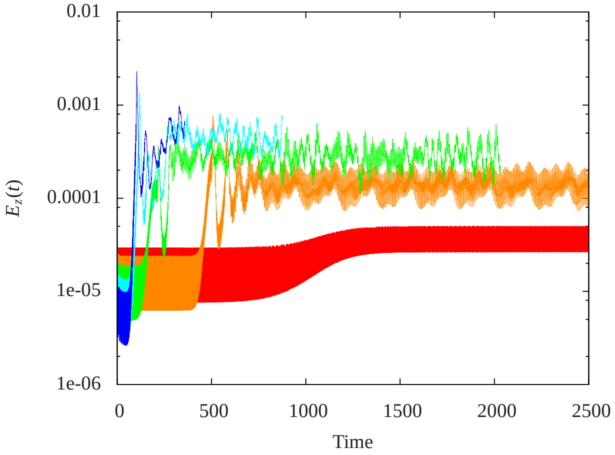
$E = 5 \times 10^{-4}$

$E = 3.5 \times 10^{-4}$

$E = 2 \times 10^{-4}$

$E = 10^{-4}$

Decreasing the Ekman number



$E = 5 \times 10^{-4}$

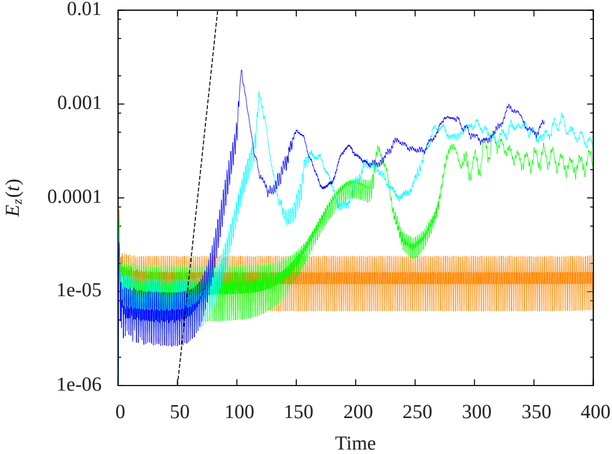
$E = 3.5 \times 10^{-4}$

$E = 2 \times 10^{-4}$

$E = 10^{-4}$

$E = 5 \times 10^{-5}$

Decreasing the Ekman number



$E = 5 \times 10^{-4}$

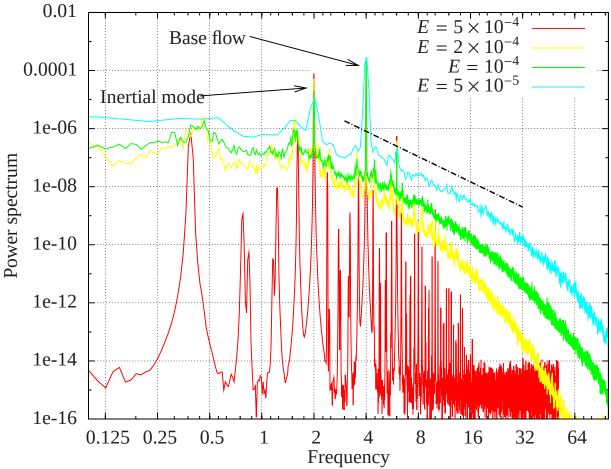
$E = 3.5 \times 10^{-4}$

$E = 2 \times 10^{-4}$

$E = 10^{-4}$

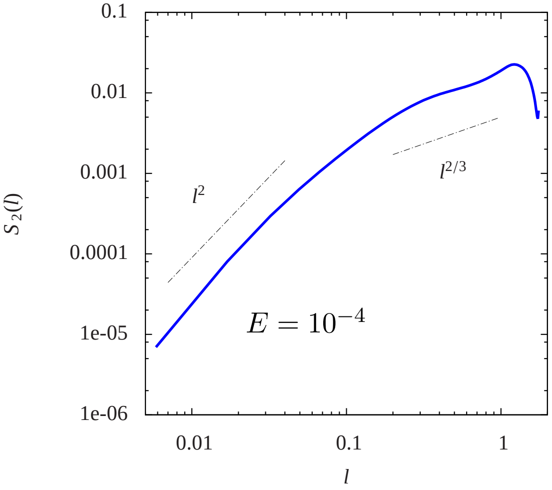
$E = 5 \times 10^{-5}$

Power spectra



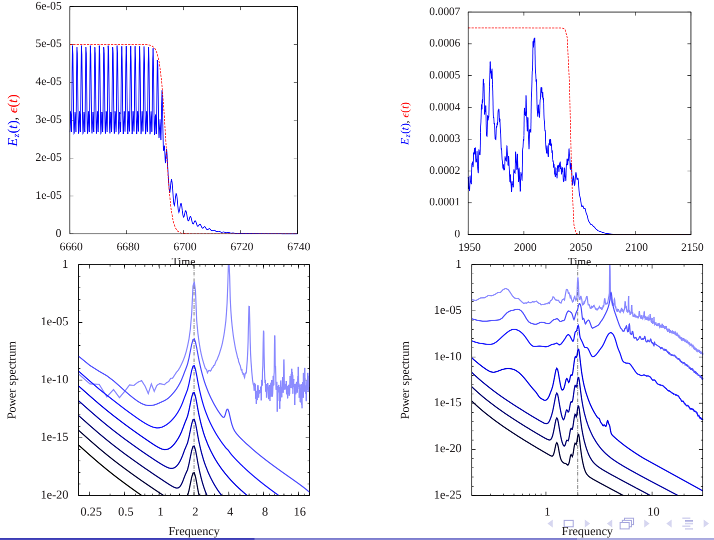
Structure functions

2^{nd} order structure function: $S_2(l) = \left\langle \left[\hat{\boldsymbol{r}} \cdot (\boldsymbol{u}(\boldsymbol{x}) - \boldsymbol{u}(\boldsymbol{x} + l\boldsymbol{r})) \right]^2 \right\rangle$

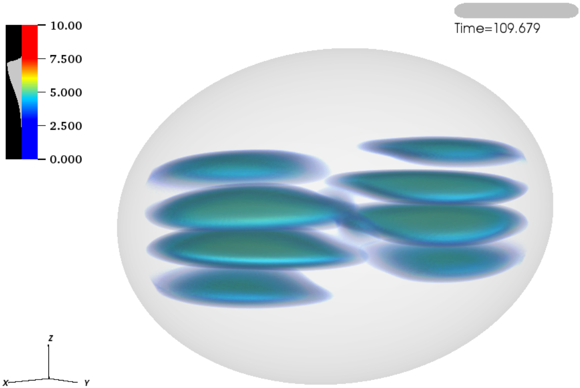


Decaying simulations

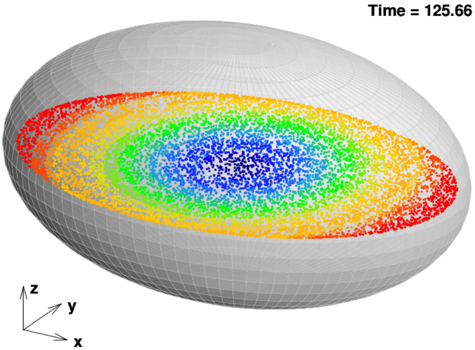
After reaching a quasi-steady state, the libration amplitude is gradually reduced.



Volume rendering of the enstrophy for $E = 10^{-4}$

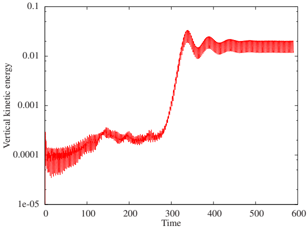


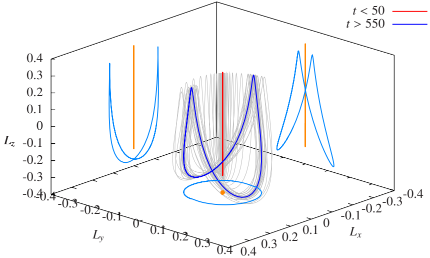
Lagrangian particles for $E = 10^{-4}$

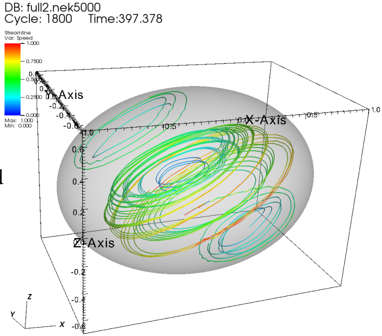


Outline

Varying the librating frequency: $f = 2.4$

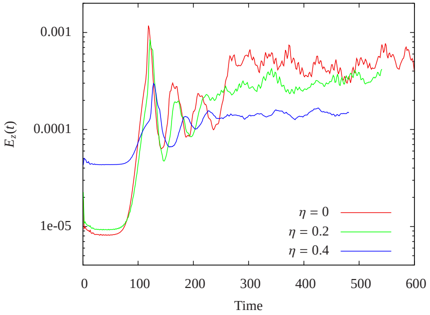


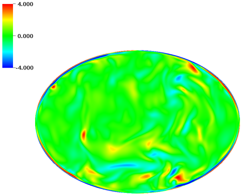


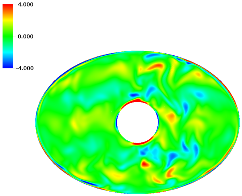


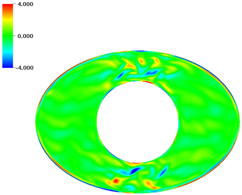
Similar behaviours observed by Grannan et al. 2014

Spherical inner core

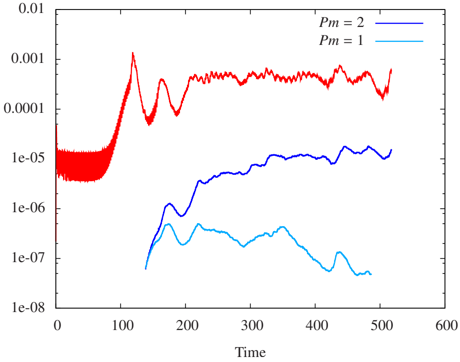


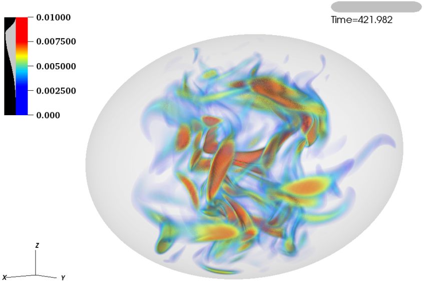






Dynamo action?





Conclusions

- At low Ekman numbers, the saturation of the instability leads to turbulence. What is the difference between the sustained and intermittent regime? (similarity with shearing-box simulations? see Barker&Lithwick 2013).
- Kinematic dynamo action is very likely (see Wu&Roberts 2013) although this remains to be properly checked in our case! Saturation of such a libration-driven dynamo?
- Stress-free simulations, extension to other mechanical forcings (in particular tides) and of course comparisons with the experiment.

Thank you for your attention!