

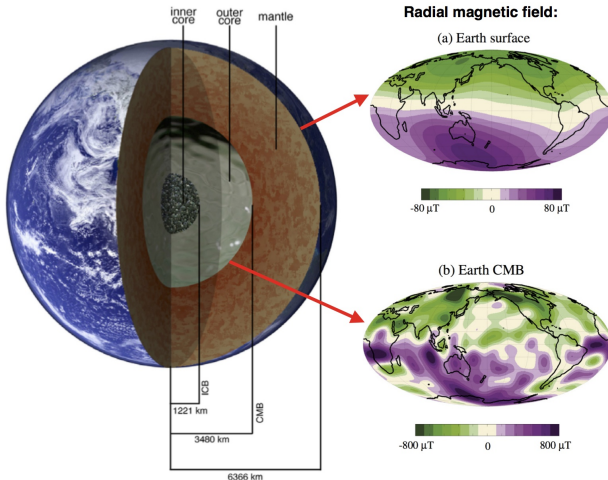
# Generation of magnetic fields by large-scale vortices in rotating convection

Céline Guervilly, David Hughes & Chris Jones

School of Mathematics, University of Leeds, UK



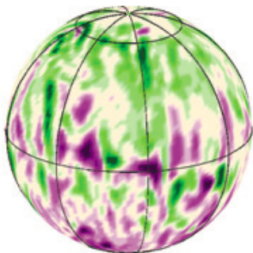
# Generation of the geomagnetic field in the Earth's outer core



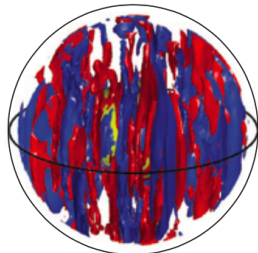
- Motions of liquid iron driven by convection.
- Dynamo: transforms kinetic energy into magnetic energy.
- Geomagnetic field: mainly dipolar, global polarity reversals

## Earth-like magnetic field in numerical models

- Global numerical models of the dynamics of the Earth's core:



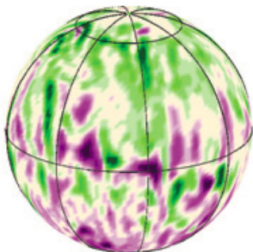
*radial magnetic field near the outer sphere*



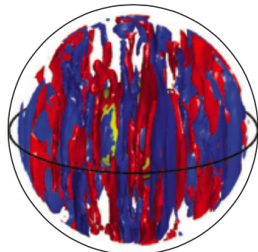
*iso-surfaces of the axial vorticity*  
(from Soderlund et al. (2012))

## Earth-like magnetic field in numerical models

- Global numerical models of the dynamics of the Earth's core:



*radial magnetic field near the outer sphere*



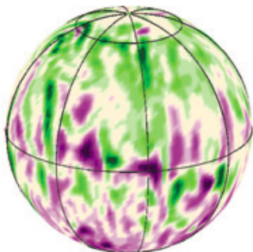
*iso-surfaces of the axial vorticity*

(from Soderlund et al. (2012))

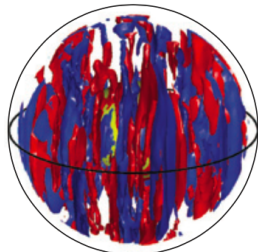
- Dipolar magnetic field are produced by columnar flows that are driven by convection.

## Earth-like magnetic field in numerical models

- Global numerical models of the dynamics of the Earth's core:



*radial magnetic field near the outer sphere*



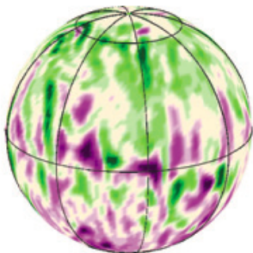
*iso-surfaces of the axial vorticity*

(from Soderlund et al. (2012))

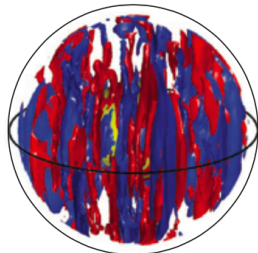
- Dipolar magnetic field are produced by columnar flows that are driven by convection.
- The size of the convective columns is set by viscosity ( $l_c \sim \mathcal{O}(Ek^{1/3})$ ).

## Earth-like magnetic field in numerical models

- Global numerical models of the dynamics of the Earth's core:



*radial magnetic field near the outer sphere*



*iso-surfaces of the axial vorticity*

(from Soderlund et al. (2012))

- Dipolar magnetic field are produced by columnar flows that are driven by convection.
- The size of the convective columns is set by viscosity ( $l_c \sim \mathcal{O}(Ek^{1/3})$ ).
- Viscosity is 10 orders of magnitude larger in the models than in the Earth's core → the convective columns appear on much larger scales in the models than in the Earth's core where  $l_c \approx 10\text{m}$ .

## Requirements for the geodynamo

- Rate of magnetic induction must be faster than the rate of magnetic dissipation:

$$Rm = \frac{UL}{\eta} > 1$$

## Requirements for the geodynamo

- Rate of magnetic induction must be faster than the rate of magnetic dissipation:

$$Rm = \frac{UL}{\eta} > 1$$

- In the Earth's core,  $U \sim 1\text{mm/s}$  and  $\eta = 1\text{m}^2/\text{s} \Rightarrow L > 1\text{km}$

## Requirements for the geodynamo

- Rate of magnetic induction must be faster than the rate of magnetic dissipation:

$$Rm = \frac{UL}{\eta} > 1$$

- In the Earth's core,  $U \sim 1\text{mm/s}$  and  $\eta = 1\text{m}^2/\text{s} \Rightarrow L > 1\text{km}$
- Flows at scale smaller than 1km cannot produce dynamo action in the Earth's core.

## Requirements for the geodynamo

- Rate of magnetic induction must be faster than the rate of magnetic dissipation:

$$Rm = \frac{UL}{\eta} > 1$$

- In the Earth's core,  $U \sim 1\text{mm/s}$  and  $\eta = 1\text{m}^2/\text{s} \Rightarrow L > 1\text{km}$
  - Flows at scale smaller than 1km cannot produce dynamo action in the Earth's core.
- $\Rightarrow$  An inviscid mechanism must be present to transfer energy from the small convective scales to larger scales where magnetic field can be generated:

## Requirements for the geodynamo

- Rate of magnetic induction must be faster than the rate of magnetic dissipation:

$$Rm = \frac{UL}{\eta} > 1$$

- In the Earth's core,  $U \sim 1\text{mm/s}$  and  $\eta = 1\text{m}^2/\text{s} \Rightarrow L > 1\text{km}$
  - Flows at scale smaller than 1km cannot produce dynamo action in the Earth's core.
- $\Rightarrow$  An inviscid mechanism must be present to transfer energy from the small convective scales to larger scales where magnetic field can be generated:
- Strong magnetic feedback forces acting on the flow (magnetoconvection:  $l_c \sim \mathcal{O}(1)$ ).

## Requirements for the geodynamo

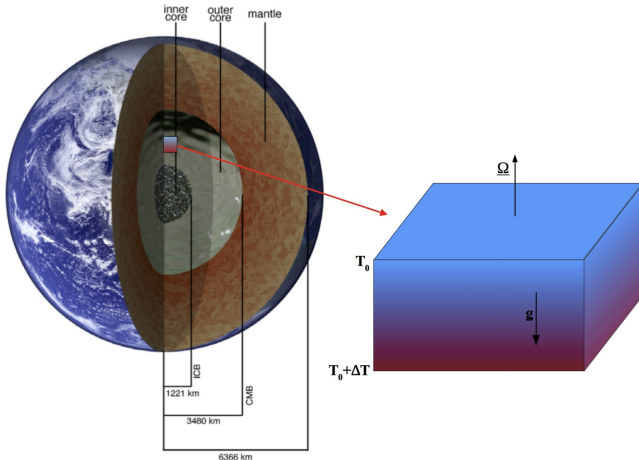
- Rate of magnetic induction must be faster than the rate of magnetic dissipation:

$$Rm = \frac{UL}{\eta} > 1$$

- In the Earth's core,  $U \sim 1\text{mm/s}$  and  $\eta = 1\text{m}^2/\text{s} \Rightarrow L > 1\text{km}$
  - Flows at scale smaller than 1km cannot produce dynamo action in the Earth's core.
- $\Rightarrow$  An inviscid mechanism must be present to transfer energy from the small convective scales to larger scales where magnetic field can be generated:
- Strong magnetic feedback forces acting on the flow (magnetoconvection:  $l_c \sim \mathcal{O}(1)$ ).
  - Nonlinear energy transfer in rapidly-rotating (non-magnetic) convection.

## Study of rotating turbulent convection

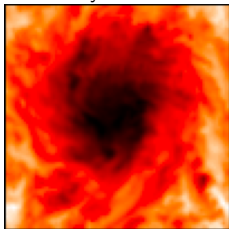
- Local Cartesian box in the outer core: faster computations allow to study more turbulent flows, less viscous fluid.
- No geometrical effects such as the curvature of the boundaries.



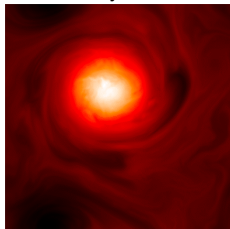
## Large-scale vortices in non-magnetic rotating convection

- First observed with **compressible** fluid (Chan 2007, Kapyla et al. 2011)

cyclone:



anticyclone:

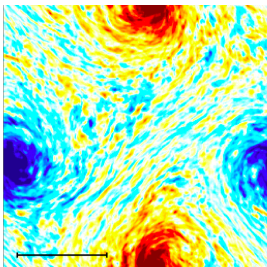


- long-lived, box-size vortices
- either cyclonic or anticyclonic
- large rotation rate (Rossby number  $< 1$ )

In the Earth's core, density scale height  $>$  core radius, so compressibility not particularly relevant.

## Large-scale vortices in non-magnetic rotating convection

- First observed with **compressible** fluid (Chan 2007, Kapyla et al. 2011)
- Also observed in a **reduced model of Boussinesq convection** valid in the limit of small Rossby number (Julien et al. 2012)



- cyclone and anticyclone pair
- depth-invariant

## Large-scale vortices in non-magnetic rotating convection

- First observed with **compressible** fluid (Chan 2007, Kapyla et al. 2011)
- Also observed in a **reduced model of Boussinesq convection** valid in the limit of small Rossby number (Julien et al. 2012)

Can they form in a simple model of **Boussinesq convection**: thermal convection between 2 parallel planes rotating about the vertical axis?

Favier, Silvers & Proctor (2014)

Guervilly, Hughes & Jones (2014)

# Outline

## In absence of magnetic field:

1. Structure of the large-scale vortices
2. Domain of existence in parameter space
3. Asymmetry cyclone/anticyclone
4. Energy transfer to large scales

*Guervilly, Hughes & Jones, 2014, JFM*

# Outline

## In absence of magnetic field:

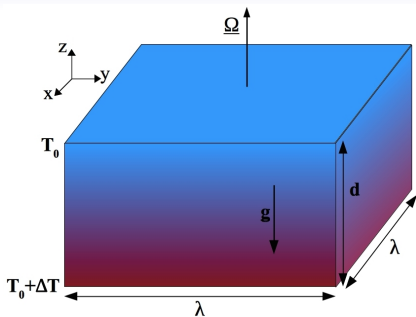
1. Structure of the large-scale vortices
2. Domain of existence in parameter space
3. Asymmetry cyclone/anticyclone
4. Energy transfer to large scales

*Guervilly, Hughes & Jones, 2014, JFM*

## After adding a seed magnetic field:

5. Magnetic field generated at large or small scales?
6. Feedback of magnetic field

## Rotating Rayleigh-Bénard convection



- 3D Cartesian layer of Boussinesq fluid
- periodic in the horizontal directions
- rotating about the vertical axis ( $z$ )
- vertical temperature difference:  $\Delta T$
- top and bottom boundary conditions:
  - stress-free and impenetrable
  - fixed temperature
  - perfect electrical conductor:
 
$$B_z = \partial_z B_x = \partial_z B_y = 0$$
- aspect ratio between horizontal/vertical box sizes:  $\lambda$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\Omega \mathbf{e}_z \times \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \alpha g \theta \mathbf{e}_z + \frac{1}{\rho \mu_0} \nabla \times \mathbf{B} \times \mathbf{B},$$

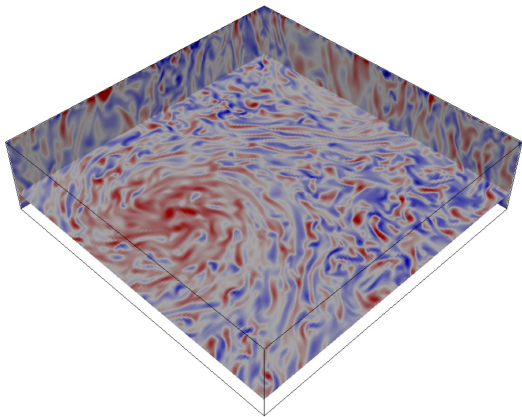
$$\nabla \cdot \mathbf{u} = 0,$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \frac{\Delta T}{d} u_z + \kappa \nabla^2 \theta,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0.$$

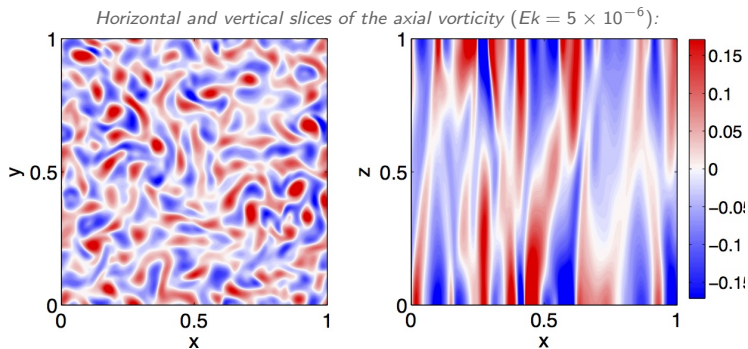
$Pr = \nu/\kappa = 1$ ,  $Pm = \nu/\eta$ ,  $10^{-4} \geq Ek \geq 10^{-6}$ ,  $Ra \leq 25Ra_c$  ( $Re \leq 3000$ )  
 Typical resolutions: (256, 256, 256)

## Large-scale vortices in non-magnetic convection



## Near the onset of convection

For  $Ra \approx Ra_c$ :



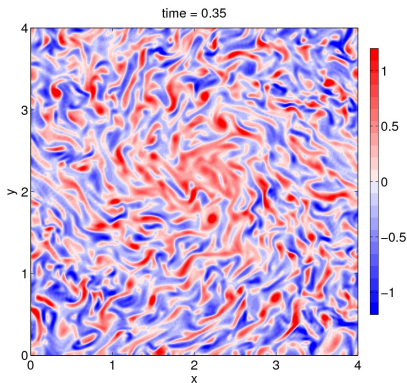
Convective structures near the onset:

- multitude of small vortices of either sign, driven directly by buoyancy
- elongated structures with horizontal size decreasing with Ekman number
- mid-plane antisymmetry of the axial vorticity

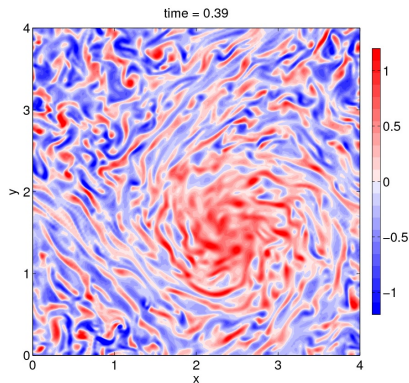
## Formation of large-scale vortices (LSV)

For  $Ra \approx 3Ra_c$ :

*During the growing phase:*

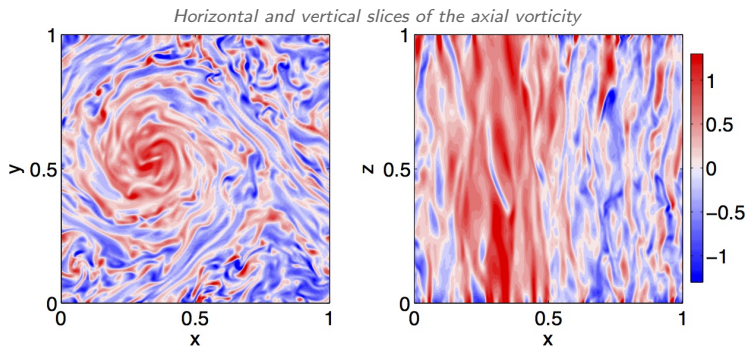


*During the saturated phase:*



- Clustering of small cyclonic vortices into a fast cyclonic circulation.
- Convective vortices advected by a slower anticyclonic circulation.
- Regions of intense shear: horizontal stretching of the convective vortices.

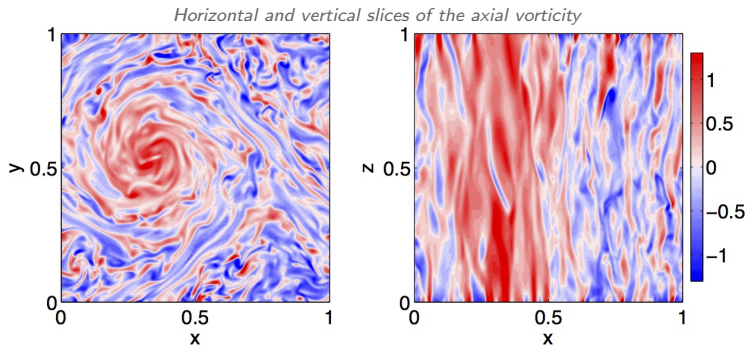
## Structure of the large-scale vortices (LSV)



Flow dominated by a cyclone:

- always grows to the box size
- mostly  $z$ -invariant
- essentially consists of horizontal motions (not a convective structure)

## Structure of the large-scale vortices (LSV)



Flow dominated by a cyclone:

- always grows to the box size
- mostly  $z$ -invariant
- essentially consists of horizontal motions (not a convective structure)

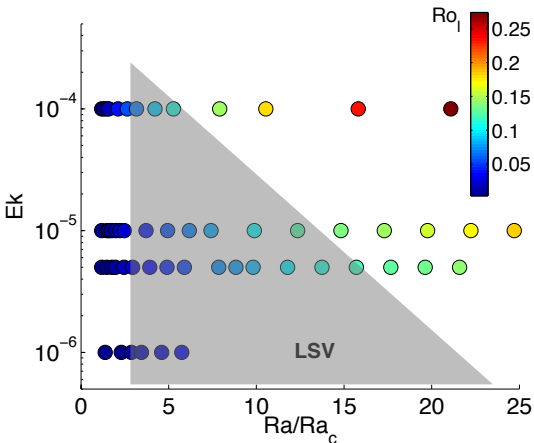
**Cyclone/anticyclone asymmetry** is common in turbulent 3D rotating systems:

- Anticyclones are unstable when locally  $\omega_z \sim -2\Omega$ .

## Domain of existence of the LSV

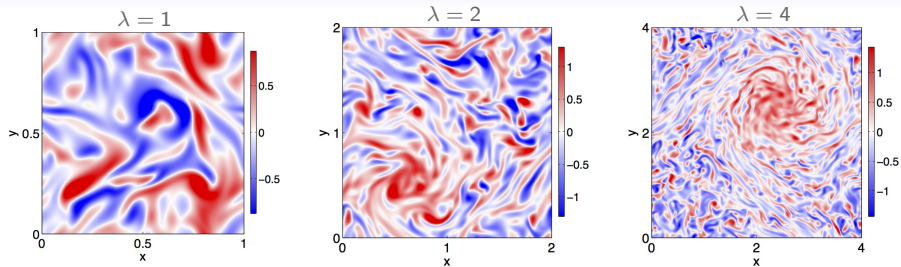
Large-scale vortices form in anisotropic convection:

- **sufficient level of convection-driven turbulence:** Reynolds number  $\geq 100$  ( $Ra/Ra_c \geq 3$ )
- **strongly dominated by the rotation:** local Rossby number  $< 0.1$

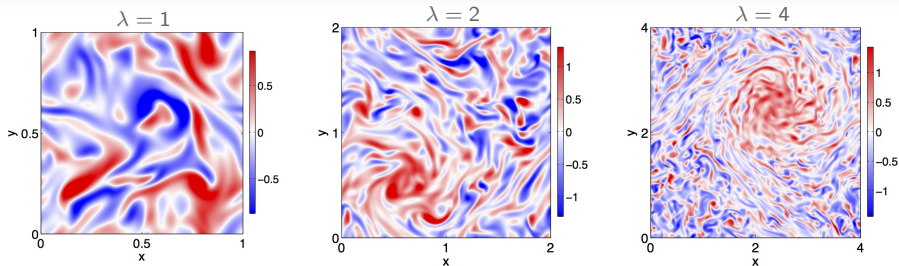


In the Earth's core:  $Ek \approx 10^{-15}$  and  $Ra \gg Ra_c$   
 $\Rightarrow$  conditions relevant for the Earth's core

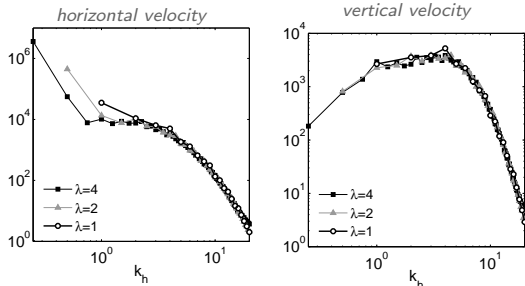
## Effect of the box aspect ratio on the amplitude of the LSV



## Effect of the box aspect ratio on the amplitude of the LSV



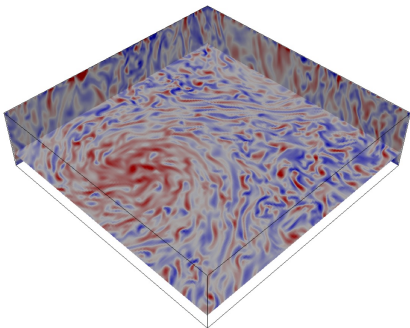
Horizontal power spectra  
of the kinetic energy:



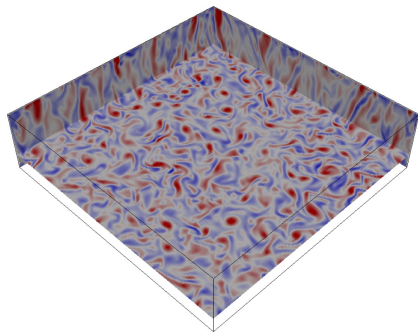
- Parameter window where LSV occurs depends on the box aspect ratio.

## Effect of the mechanical boundary conditions

Stress-free



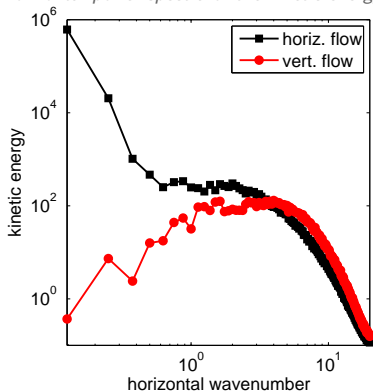
No-slip



- $Ek = 10^{-4}$ : viscous boundary friction damps the large-scale flows.

## Upscale energy transfer

Horizontal power spectra of the kinetic energy

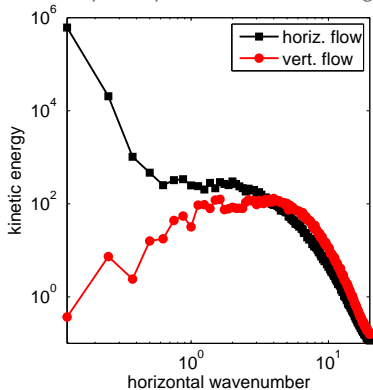


**Non-local energy transfer** from small convective scales to the large scale:

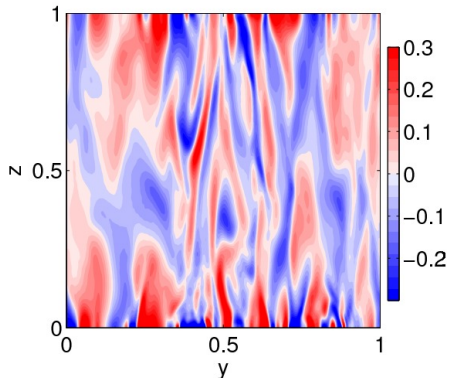
- Favier, Silvers & Proctor (2014): transfer functions of the kinetic energy
- Guervilly, Hughes & Jones (2014): filtration of spectral modes
- In agreement with small  $Ro$  Boussinesq model (Rubio et al. 2014)

## Upscale energy transfer

Horizontal power spectra of the kinetic energy



Axial vorticity in a  $yz$ -plane next to the LSV

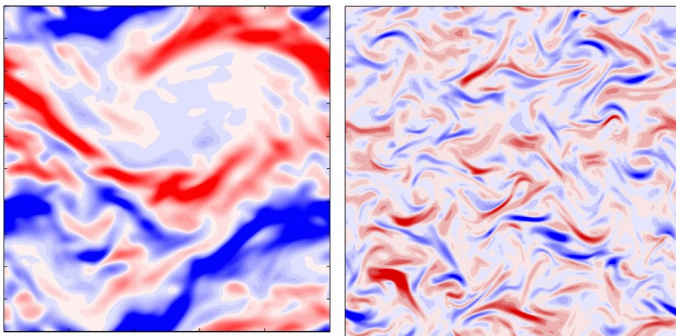


**Non-local energy transfer** from small convective scales to the large scale:

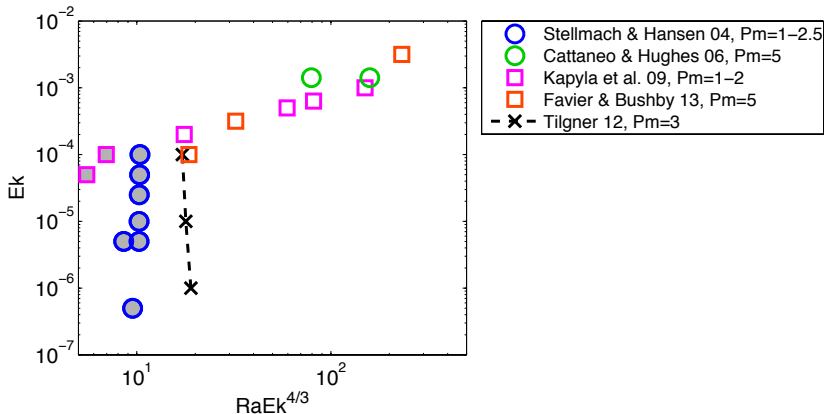
- Favier, Silvers & Proctor (2014): transfer functions of the kinetic energy
- Guervilly, Hughes & Jones (2014): filtration of spectral modes
- In agreement with small  $Ro$  Boussinesq model (Rubio et al. 2014)

Thermal plumes ejected from the top and bottom thermal boundary layers are mostly cyclonic by vortex stretching: increased likelihood for LS cyclone

## Generation of magnetic fields by the LSV

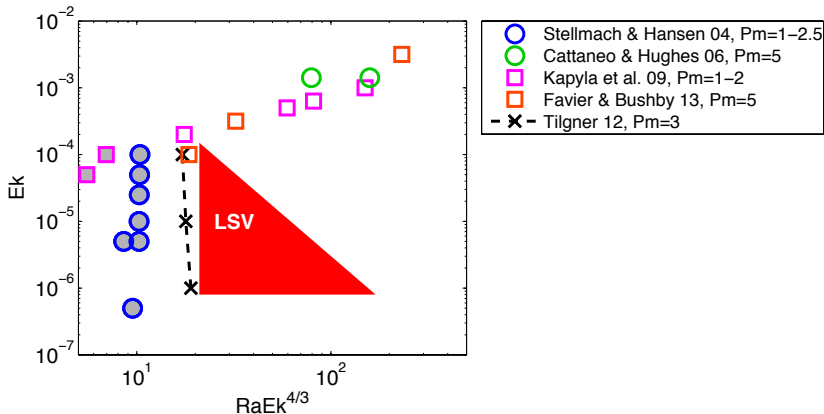


## Convective dynamos in planar geometry



- circles = Boussinesq; squares = compressible
- gray face = generation of a significant horizontal field ( $k_x, k_y$ ) = (0, 0)
- $RaEk^{4/3}$  = Rayleigh number rescaled by its  $Ek$ -dependence at the convection onset

## Convective dynamos in planar geometry



- circles = Boussinesq; squares = compressible
- gray face = generation of a significant horizontal field  $(k_x, k_y) = (0, 0)$
- $RaEk^{4/3}$  = Rayleigh number rescaled by its  $Ek$ -dependence at the convection onset

Can the LSV offer an alternative route to the generation of large-scale magnetic fields?

## Magnetic Prandtl number

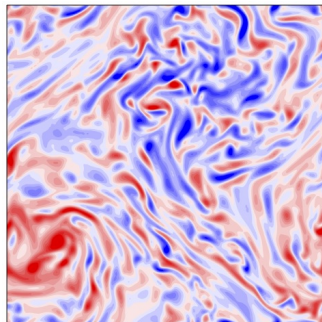
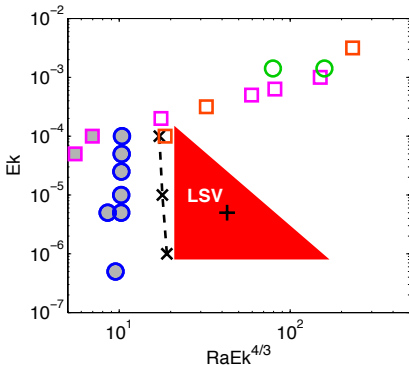
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Pm} \nabla^2 \mathbf{B}$$

- $Pm = \nu/\eta$  controls the dynamo threshold for fixed  $(Ra, Ek, Pr, \lambda)$ .
- $Pm \approx 10^{-6}$  in the Earth's core, but  $Pm \approx \mathcal{O}(1)$  in numerical models.

## Magnetic Prandtl number

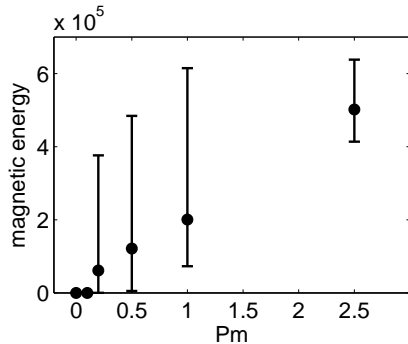
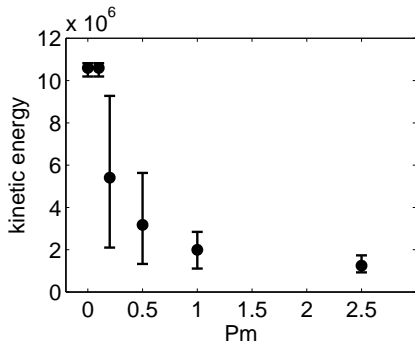
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Pm} \nabla^2 \mathbf{B}$$

- $Pm = \nu/\eta$  controls the dynamo threshold for fixed  $(Ra, Ek, Pr, \lambda)$ .
- $Pm \approx 10^{-6}$  in the Earth's core, but  $Pm \approx \mathcal{O}(1)$  in numerical models.



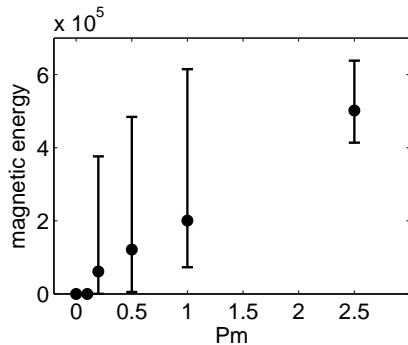
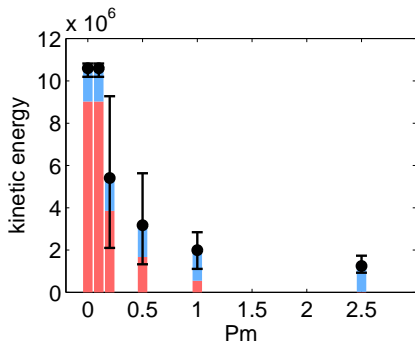
$Pm = 0$  (non-magnetic),  $Re = 1345$

## Dynamo threshold



- dynamo threshold  $Pm \geq 0.2$ .

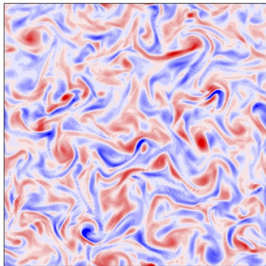
## Dynamo threshold



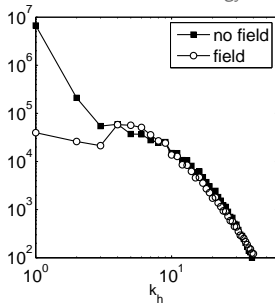
- dynamo threshold  $Pm \geq 0.2$ .
- decrease of the kinetic energy corresponds to the suppression of the LSV.
- amplitude of the convective flows does not change.

## Dynamo at $Pm = 2.5$

*axial vorticity  
(horizontal slice)*

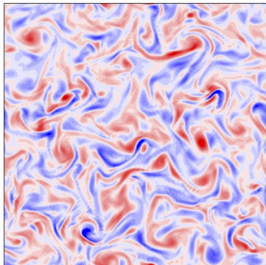


*Horizontal power spectra of the  
horizontal kinetic energy*

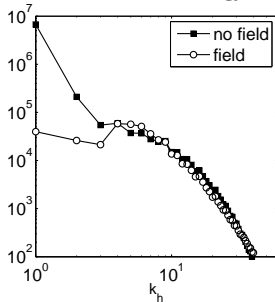


## Dynamo at $Pm = 2.5$

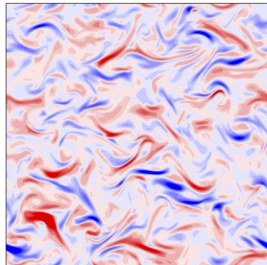
axial vorticity  
(horizontal slice)



Horizontal power spectra of the  
horizontal kinetic energy



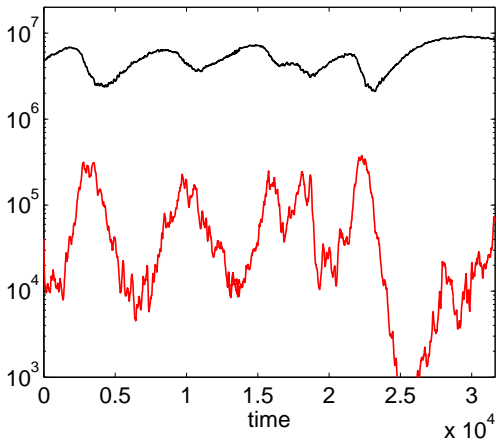
$B_x$   
(horizontal slice)



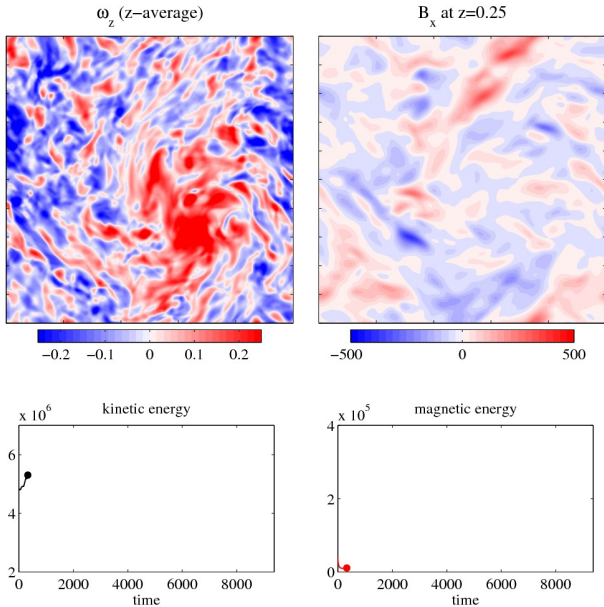
- Small-scale magnetic field: dominated by scales close to the convective scale.
- Small-scale field impedes the transport properties of the flow for large enough  $Rm$  ( $= RePm$ ) (Cattaneo & Vainshtein 1991, Tobias et al. 2007).
- Magnetic energy  $<$  kinetic energy: magnetic field does not need to be strong to suppress the large-scale cyclone.

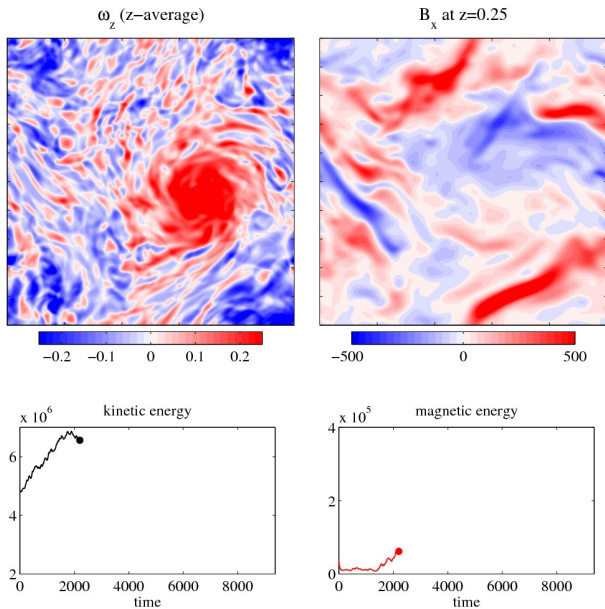
## Dynamo at $Pm = 0.2$

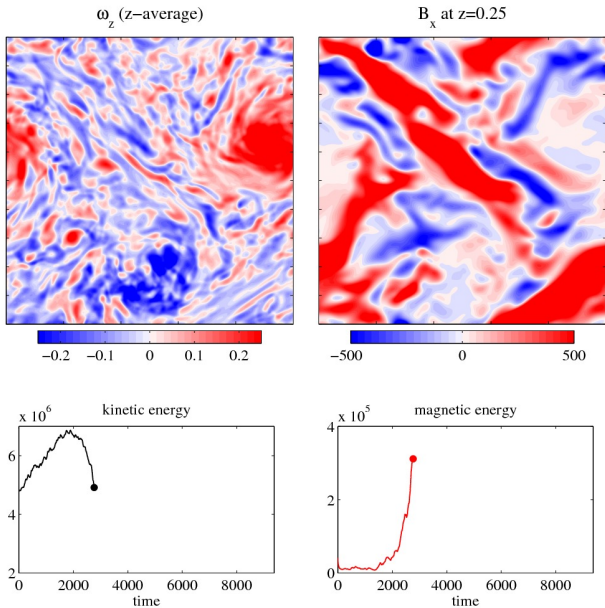
Evolution of the kinetic and magnetic energies

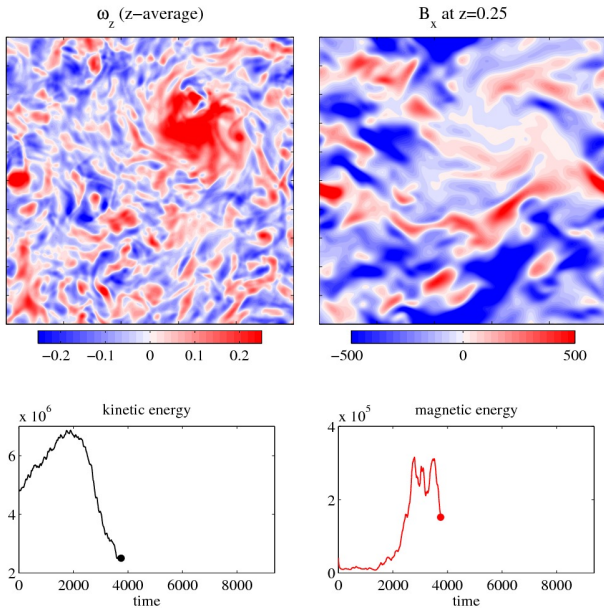


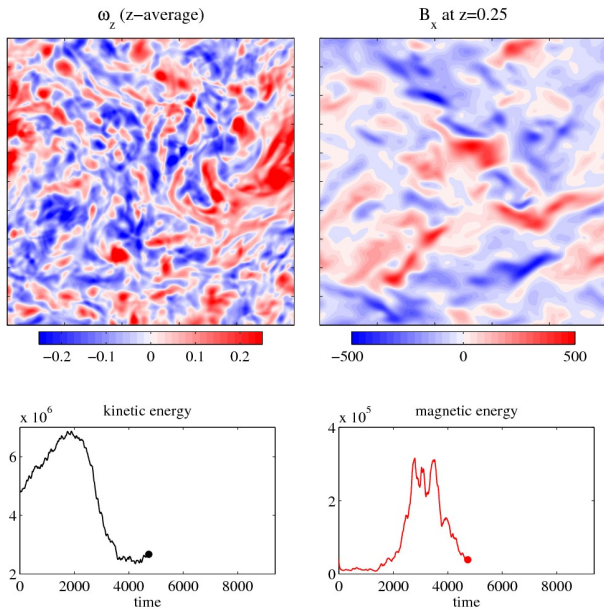
- Large anti-correlated fluctuations of the kinetic and magnetic energies.

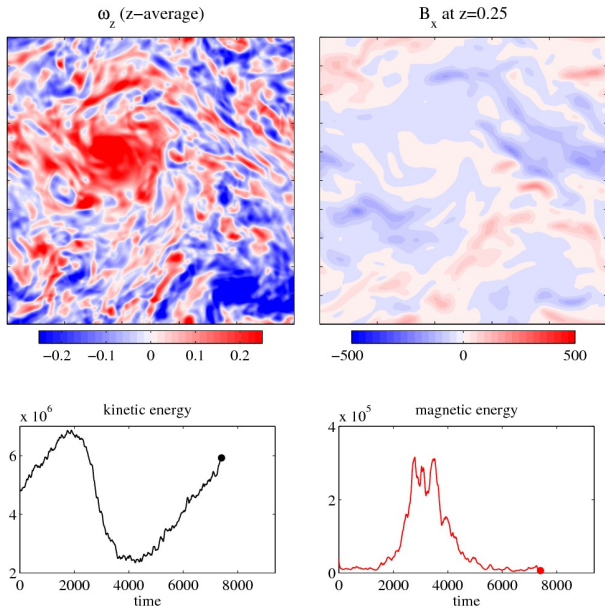
Fluctuations of the LSV and magnetic field for  $Pm = 0.2$ 

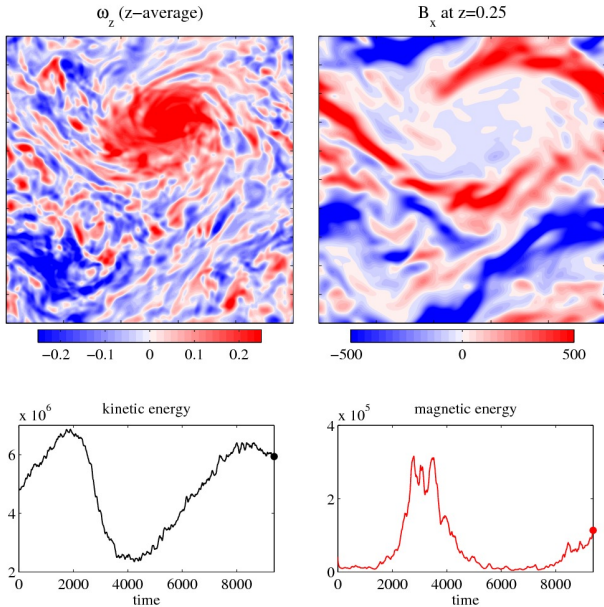
Fluctuations of the LSV and magnetic field for  $Pm = 0.2$ 

Fluctuations of the LSV and magnetic field for  $Pm = 0.2$ 

Fluctuations of the LSV and magnetic field for  $Pm = 0.2$ 

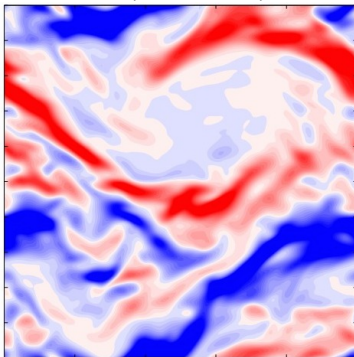
Fluctuations of the LSV and magnetic field for  $Pm = 0.2$ 

Fluctuations of the LSV and magnetic field for  $Pm = 0.2$ 

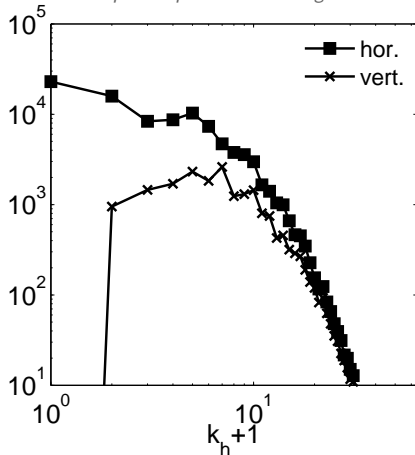
Fluctuations of the LSV and magnetic field for  $Pm = 0.2$ 

## Generation of large-scale magnetic field for $Pm = 0.2$

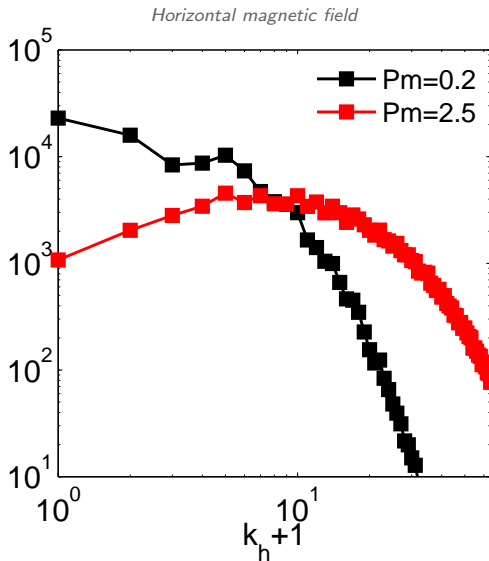
$B_x$  (horizontal slice)



Horizontal power spectra of the magnetic field

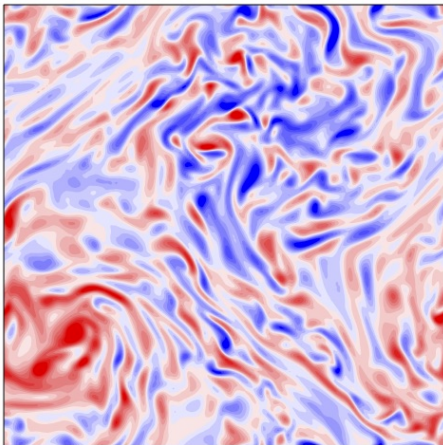


- Formation of bands of strong magnetic field in the shear layers.
- Horizontal magnetic field dominated by largest horizontal wavenumbers  $k_h \leq 1$ .

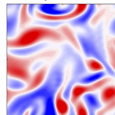
Comparison of the dynamos at  $Pm = 0.2$  and  $Pm = 2.5$ 

## Small-scale dynamo threshold

$\lambda = 1$  (*non-magnetic*)

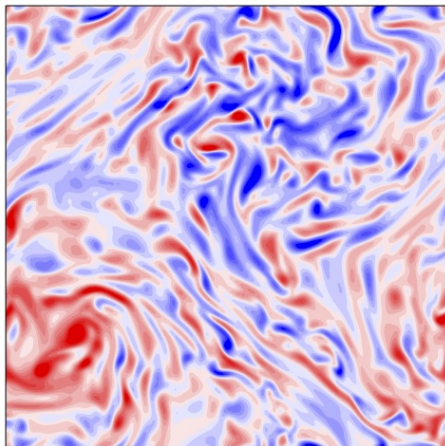


$\lambda = 0.25$  (*non-magnetic*)

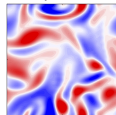


## Small-scale dynamo threshold

$\lambda = 1$  (non-magnetic)



$\lambda = 0.25$  (non-magnetic)



- Small aspect ratio  $\lambda = 0.25$ : dynamo threshold  $Pm \geq 1$ .
- No small-scale dynamo driven by the convective flows for  $Pm < 1$ .
- Dynamo for  $0.2 \leq Pm < 1$  when  $\lambda = 1$  entirely relies on the presence of the LSV.

## Summary

System-size magnetic fields produced in rotating convection by a two-step dynamo mechanism:

## Summary

System-size magnetic fields produced in rotating convection by a two-step dynamo mechanism:

- **1 - Formation of large-scale vortices from turbulent small-scale flows:**
  - Convective flows need to be anisotropic (dominated by rotation) and have sufficient velocity to merge (Reynolds numbers  $> 100$ ).
  - Wide enough computational domain.
  - Stress-free top and bottom boundary conditions ( $10^{-4} \geq Ek \geq 10^{-6}$ ).

## Summary

System-size magnetic fields produced in rotating convection by a two-step dynamo mechanism:

- **1 - Formation of large-scale vortices from turbulent small-scale flows:**
  - Convective flows need to be anisotropic (dominated by rotation) and have sufficient velocity to merge (Reynolds numbers  $> 100$ ).
  - Wide enough computational domain.
  - Stress-free top and bottom boundary conditions ( $10^{-4} \geq Ek \geq 10^{-6}$ ).
- **2 - Large-scale vortices produce magnetic fields of similar size:**
  - For small  $Pm$ , below the threshold for small-scale dynamo.
  - Horizontal magnetic field concentrated in the shear layers.
  - Fluctuations correspond to cycles of re-generation/suppression of the LSV.

If  $Pm >$  threshold for small-scale dynamo action, the LSV is destroyed and no large-scale field is produced.