

Rotating dynamo turbulence: theoretical and numerical insights

Nathanaël Schaeffer¹, H.-C. Nataf¹, A. Fournier², J. Aubert², D. Jault¹, F. Plunian¹, P. Zitzke¹

1: ISTERre / CNRS / Université Joseph Fourier, Grenoble
2: IPGP, Paris

IPAM, 31 October 2014



Introduction

- Project *AVSGeomag*: *Assimilation of geomagnetic observations in dynamical models of the Earth's core.*
- *Need for a direct model for data assimilation of geomagnetic data.*
- *Current geodynamo models do not correctly reproduce rapid dynamics.*

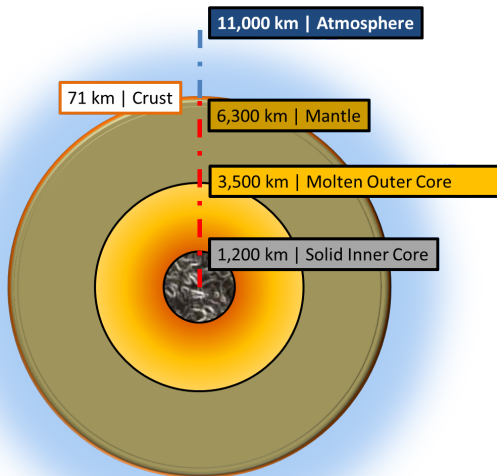
Goal

Can we obtain rapid variation of the geomagnetic field and the corresponding core dynamics from a full 3D numerical model ?

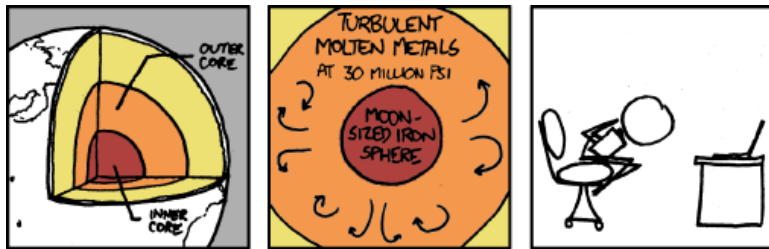
<http://avsgeomag.ipgp.fr/>

- 1 Turbulence in the Earth's core
 - The Earth's core
 - Inferring the turbulent state
- 2 Numerical Simulations
 - Overview
 - Jump 1: torsional oscillations
 - Jump 2: focus on turbulence
 - Force balance
 - Cartesian boxes
- 3 Summary

Structure of the Earth



Structure of the Earth



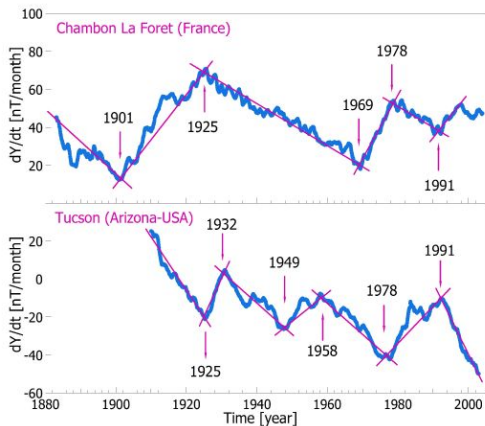
I FREAK OUT ABOUT FIFTEEN MINUTES INTO
READING ANYTHING ABOUT THE EARTH'S CORE
WHEN I SUDDENLY REALIZE IT'S *RIGHT UNDER ME.*

<http://www.xkcd.com/913/>

Cool facts about the Earth's core

- large scale motions at the top of the core has speeds around 10 km/year (0.3 mm/sec, turnover time is about 200 years)
- Magnetic field is dominated by a tilted dipole.
- Magnetic energy dominates kinetic energy by a factor 10^4 (4 mT estimated in the core, 0.5 mT or 5 gauss at the surface).
- Very low Ekman number $E \sim 10^{-15}$
- Very high Rayleigh number $Ra \gg 10^{20}$, probably many times critical.
- Very high Reynolds number $Re \gtrsim 10^8$.
- Magnetic Reynolds number $Rm \gtrsim 1000$ ($Pm \sim 10^5$)
- Very low Rossby number $Ro \sim 3 \times 10^{-6}$.
- Very low Lehnert number (Rossby based on Alfvén speed) $Le \sim 10^{-4}$.
- Heat flux extracted by the mantle is imposed (~ 10 TW, < 100 mW/m²).

Secular variation



Time-derivative of a magnetic field component recorded at a fixed observatory.

What turbulence in the Earth's core ?

- At what length- and time-scales is the turbulence regime dominated by rotation, by the magnetic field?
- Can we infer the evolution of flow velocity and small-scale magnetic field in these different regimes?
- How much energy is dissipated?
- What is the balance between viscous and ohmic dissipation?
- Which waves can propagate?
- Is it like MHD turbulence with imposed magnetic field ?
- How do the small scales produce or influence the production of magnetic field ?

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Introducing $\tau - \ell$ regime diagrams.

H-C. Nataf and N. Schaeffer, Turbulence in the core, in Treatise on Geophysics, 2nd edition, to appear in 2015.

Assumptions

- Turbulence involves a wide range of scales
- Time-scales t and length-scales l are related by various physical processes
- Regime changes occur when $t(l)$ lines intersect
- Dimensionless numbers can be written as time-scale ratios
- The scale l at which a dimensionless number is 1 is more important than the value of that number at the integral scale
- Turbulent dynamics is controlled by the shortest time-scale process

Idea

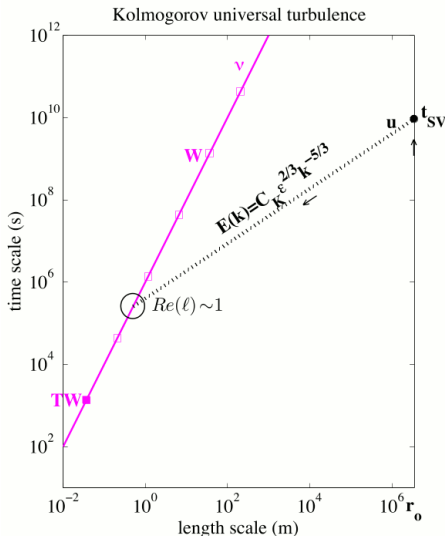
Translate the information contained in the power spectrum to more useful physical units

Example: K41 Turbulence (NOT the core)

- $t_\nu(\ell) = \ell^2/\nu$
- $u_\ell^2(\ell) = k E(k)$
- $u_\ell = \ell/\tau_u(\ell)$
- $t_u(\ell) = \epsilon^{-1/3}\ell^{2/3}$
- $Re(\ell) = u_\ell\ell/\nu$
- $P = MOC\epsilon = MOC\nu/t_\nu^2$

with:

- $\nu = 10^{-6}\text{m}^2/\text{s}$
- $t_{SV} = 300\text{years}$
- $r_o = 3480\text{km}$
- $MOC = 1.835\text{kg}$

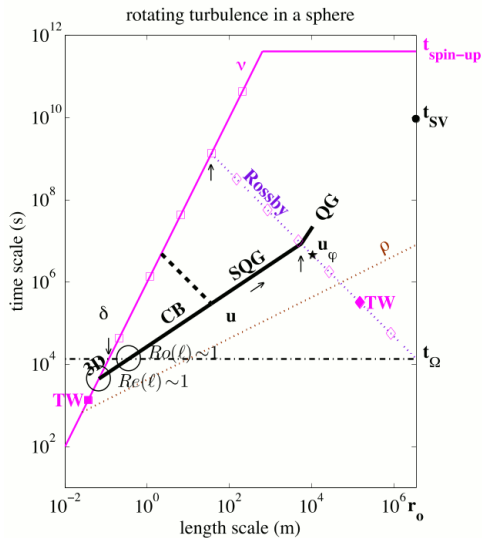


Example: rotating Turbulence (non-stratified)

- $t_\nu(\ell) = \ell^2/\nu$
- $\delta = \sqrt{\nu\Omega}$
- $t_{Rossby} = r_o/\Omega\ell$
- $t_{spin-up} = r_o/\sqrt{\nu\Omega}$
- $Ro(\ell) = u_\ell/\Omega\ell$

with:

- $\Omega = 2\pi/(1 \text{ day})$

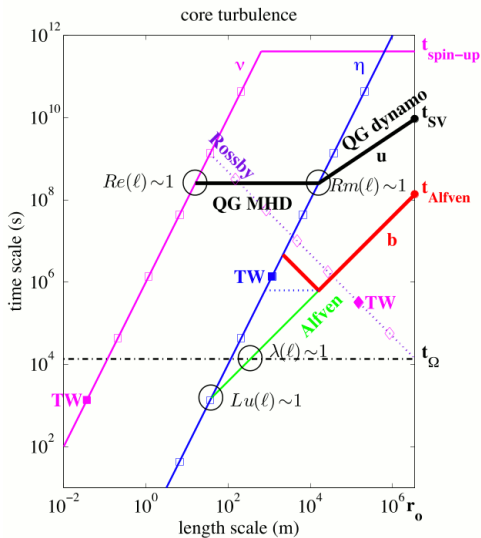


Turbulence in the Earth's core

- $t_{Alfven} = \ell / V_A$
- $Rm(\ell) = u_\ell(\ell) / \eta$
- $Lu(\ell) = t_\eta(\ell) / t_{Alfven}(\ell)$
- $\lambda(\ell) = t_\Omega / t_{Alfven}$

with:

- $\Omega = 2\pi / (1 \text{ day})$
- $\eta = 1 \text{ m}^2 / \text{s}$
- $t_{Alfven} = 4 \text{ to } 6 \text{ years}$



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MHD forced by compositional Boussinesq convection

$$\partial_t \mathbf{u} + \left(\frac{2}{E} \mathbf{e}_z + \nabla \times \mathbf{u} \right) \times \mathbf{u} = -\nabla p + \Delta \mathbf{u} + (\nabla \times \mathbf{b}) \times \mathbf{b} - Ra T \vec{r}$$

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \frac{1}{Pm} \Delta \mathbf{b}$$

$$\partial_t T + \mathbf{u} \cdot \nabla (T + T_0) = \Delta T$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\nabla \cdot \mathbf{b} = 0$$

with:

- 1 Ekman number $E = \nu / D^2 \Omega$
- 2 Rayleigh number $Ra = \Delta T \alpha g D^3 / \kappa \nu$
- 3 Magnetic prandtl number $Pm = \nu \mu_0 \sigma$
- 4 (Thermal) Prandtl number $Pr = 1$.
- 5 $T_0(r)$ is the conductive profile.

The code: XSHELLS

- **Spherical** shell geometry
- Fields expanded using $\mathbf{u} = \nabla \times (T\mathbf{r}) + \nabla \times \nabla \times (P\mathbf{r})$
- Spherical Harmonic expansion, with on-the-fly transform¹
- Finite differences in radius
- Semi-implicit scheme: diffusive terms are time-stepped using Cranck-Nicholson while other terms are treated explicitly with Adams-Bashforth scheme
- hybrid MPI/OpenMP
- max resolution so far: 2688 × 1344 × 1024 @ 512 cores (11 secs/step)

¹thanks to the SHTns library, <https://bitbucket.org/nschaeff/shtns>

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For reference:

1995: Glatzmaier and Roberts, 64 × 32 × 49 (the pioneers, with hyperviscosity)

2008: Kageyama et. al., 2048 × 1024 × 511 (Yin-Yang grid, E=1e-6, Re=700, Pm=1)

2009: Sakuraba and Roberts, 768 × 384 × 160 (Chebychev E=2e-6, Re=650, Pm=0.2)

2014: Hotta et. al., 4096 × 2048 × 512 (solar dynamo, Yin-Yang grid)

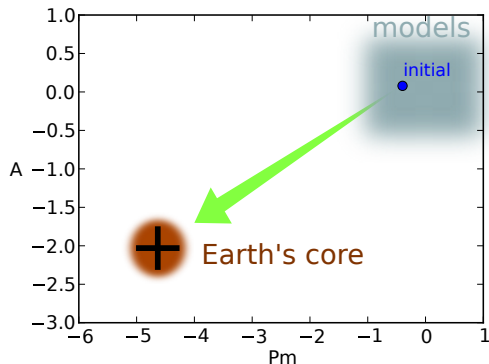
¹thanks to the SHTns library, <https://bitbucket.org/nschaeff/shtns>

- thermochemical convection (codensity, 75% chemical driving, Aubert *et al* 2009);
- including some secular cooling effect;
- no-slip, and fixed flux homogeneous boundary conditions
- **high rotation rate, low viscosity**
- **strong forcing** (more than 4000 times critical)

The simulations

The idea

- Keep super-criticality and Rm fixed.
- go to more Earth-like A and Pm .

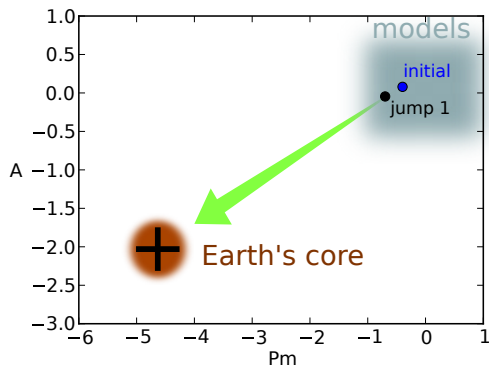


- **initial:** $E = 10^{-5}$,
 $Pm = 0.4$, $Ra = 6 \cdot 10^{10}$
 $\Rightarrow A = 1.5$ $F_\nu = 47\%$

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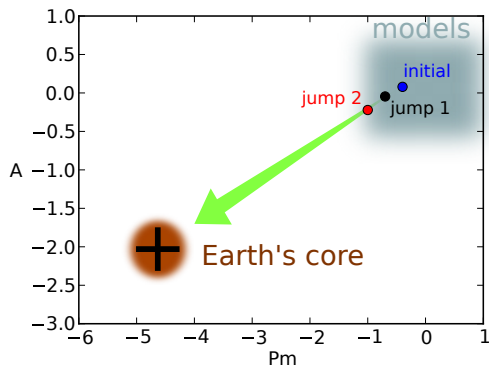


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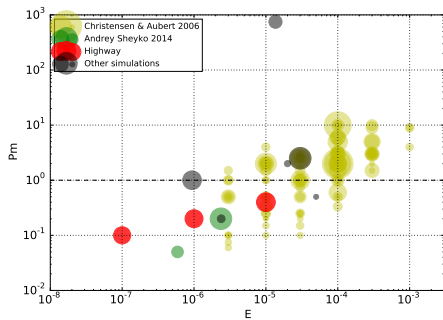
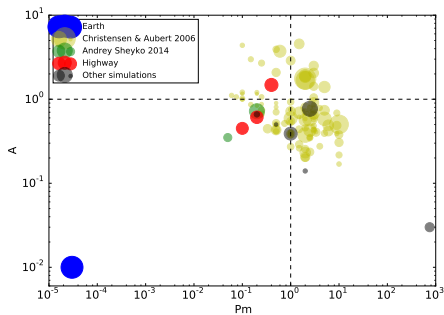
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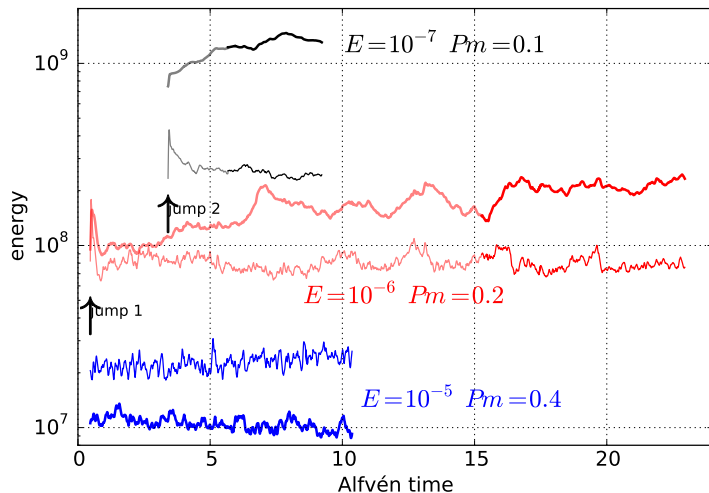
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- **jump 2:** $E = 10^{-7}$,
 $Pm = 0.1$, $Ra = 2.4 \cdot 10^{13}$
 $\Rightarrow A = 0.45$ $F_\nu = 17\%$

A selection of geodynamo simulations

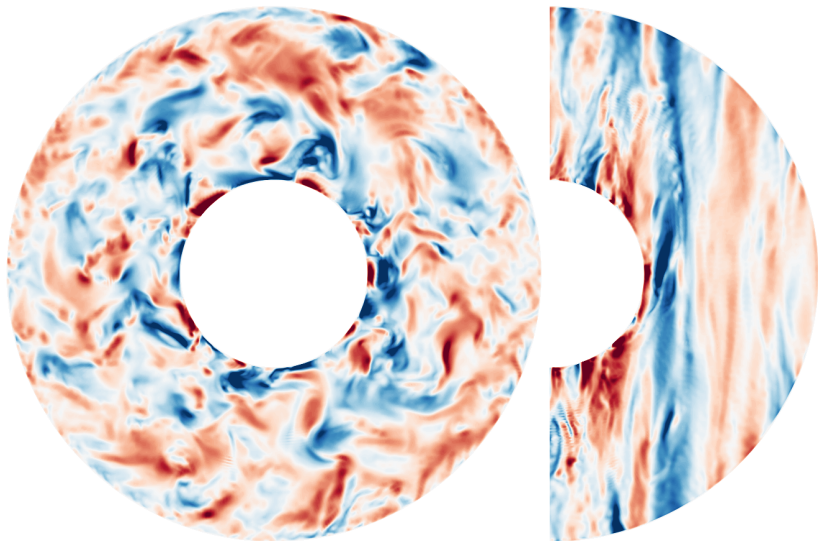


Surface of the discs is proportional to the magnetic Reynolds number (i.e. how strong the magnetic field generation is)

Energy vs Time

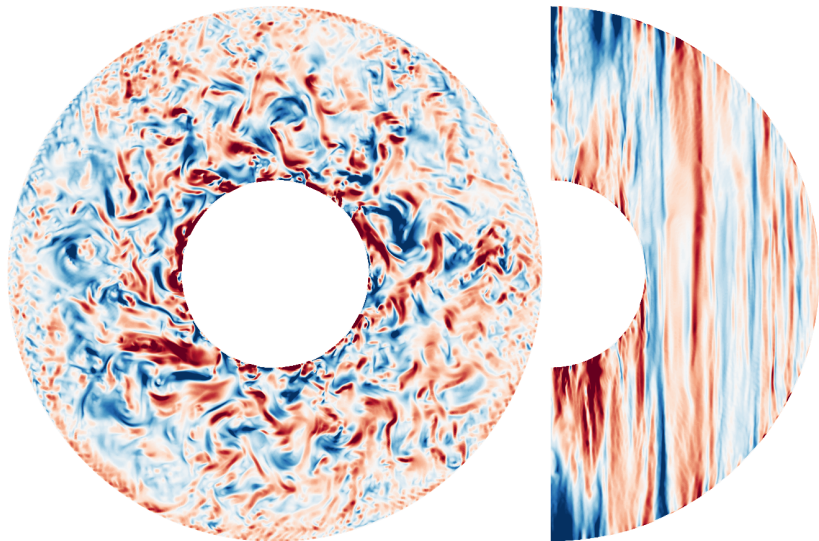


Snapshot: initial U_ϕ ($E = 10^{-5}$, $Pm = 0.4$, $A = 1.5$)



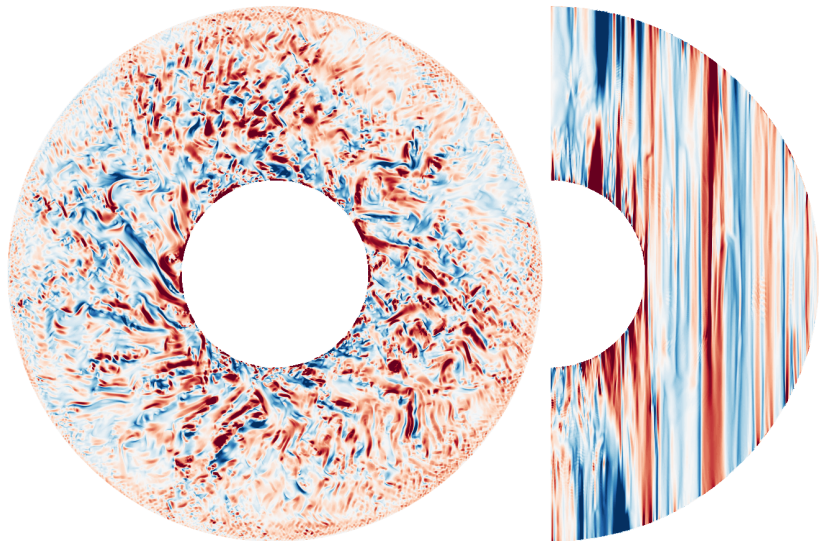
$NR = 224$, $L_{max} = 191$

Snapshot: jump 1 U_ϕ ($E = 10^{-6}$, $Pm = 0.2$, $A = 0.6$)



$NR = 512$, $L_{max} = 479$

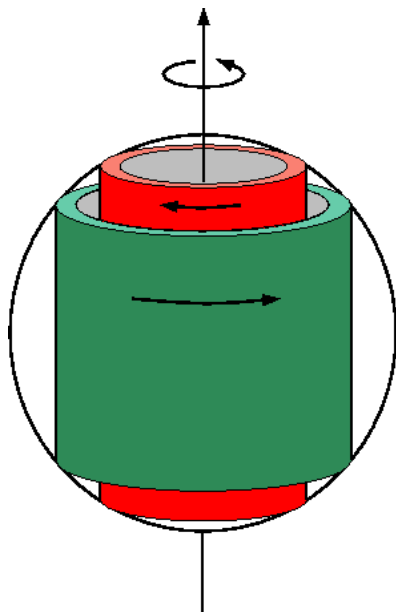
Snapshot: jump 2 U_ϕ ($E = 10^{-7}$, $Pm = 0.1$, $A = 0.45$)



$NR = 1024$, $L_{max} = 893$

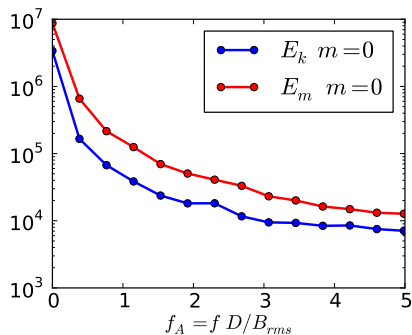
Torsional Alfvén Waves in the core

- Alfvén waves constrained by rotation can only propagate as geostrophic cylinders.
- Their speed is related to the integral over z and ϕ of B_s^2 .
- Measuring their speed gives information about the magnetic field inside the core.



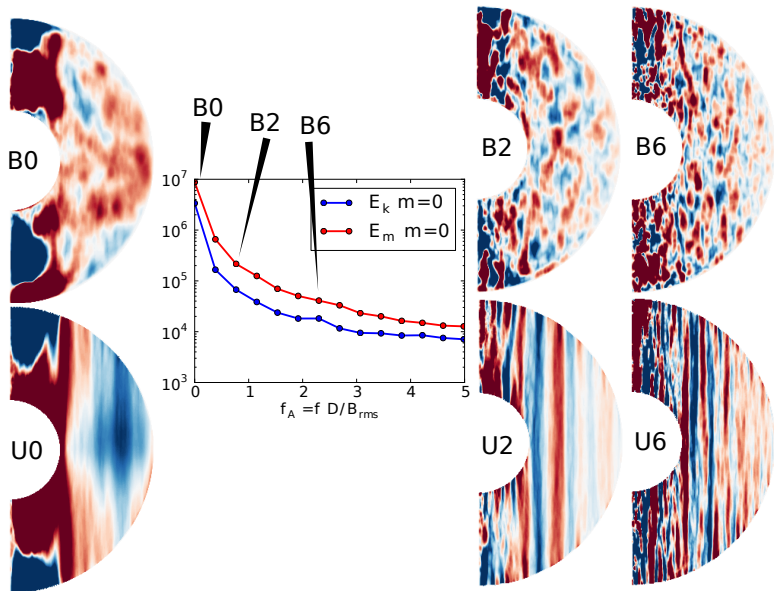
Jump 1: Fourier Analysis of axisymmetric fields

- We perform an FFT of the axisymmetric component over about 3 Alfvén times (defined with B_{rms}).
- Modal analysis similar to Figueroa *et al* (2013)



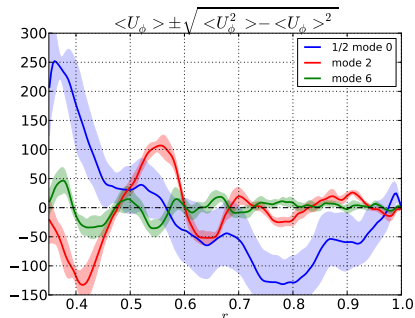
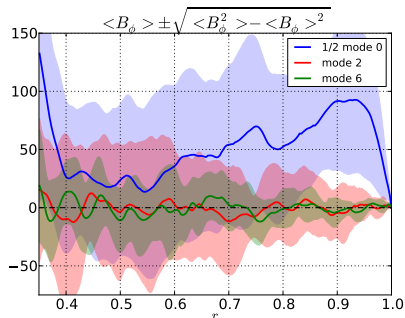
A. Figueroa *et al*, *Modes and instabilities in magnetized spherical Couette flow*, *JFM*, 2013.

Jump 1: Fourier modes of axisymmetric U_ϕ and B_ϕ



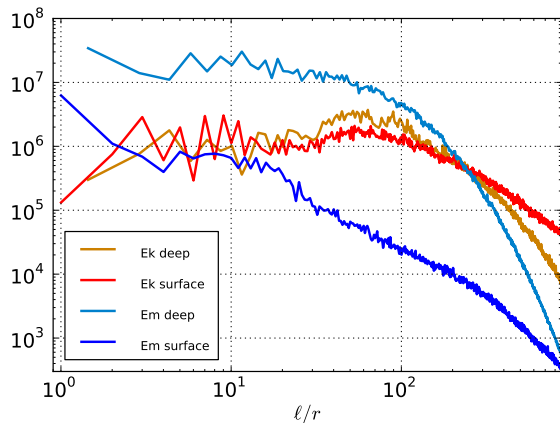
Jump 1: Fourier modes: z-averages and standard deviation

We quantify the z-invariance of the magnetic and velocity fields for mode 0 ($f_A = 0$), mode 2 ($f_A = 0.77$), and mode 6 ($f_A = 2.3$).



Evidence for **geostrophy of intermediate frequency modes**
($0.5 < T < 10$ years, axisymmetric)

Jump 2: spectra

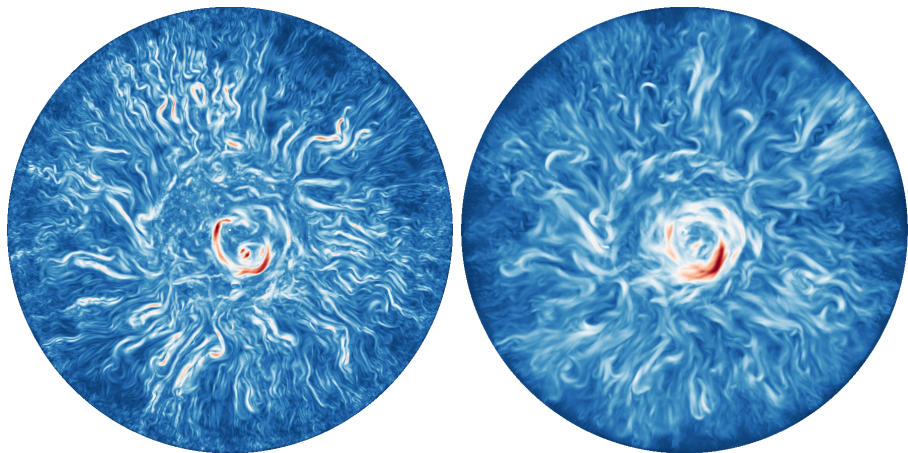


$$E = 10^{-7}$$
$$Pm = 0.1$$
$$Ra = 2.4 \cdot 10^{13}$$
$$Rm = 600$$
$$A = 0.45$$
$$\Lambda = 1.2$$
$$F_\nu = 17\%$$

$$NR = 1024$$
$$L_{max} = 893$$

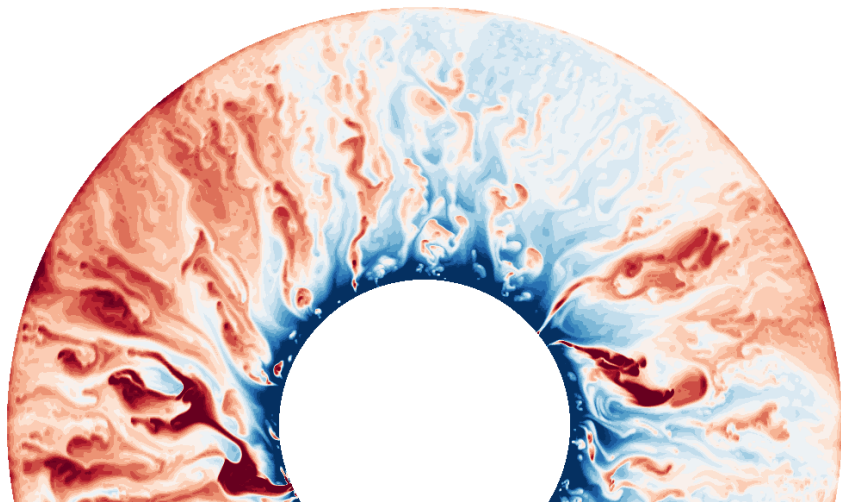
- Magnetic field dominates deep in the core but not near the surface.
- Velocity spectrum nearly flat at the surface but increasing deep down.

Jump 2: z-averaged energy densities



z-averaged equatorial energy densities, left: $\langle U_{eq}^2 \rangle$, right: $\langle B_{eq}^2 \rangle$.
 $E = 10^{-7}$, $Pm = 0.1$, $Rm = 600$, $A \sim 0.45$.

Jump 2: Temperature field



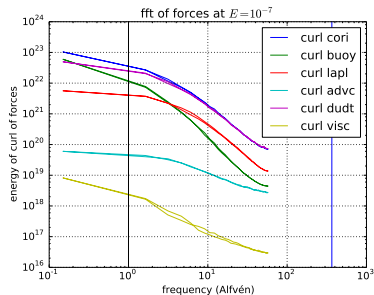
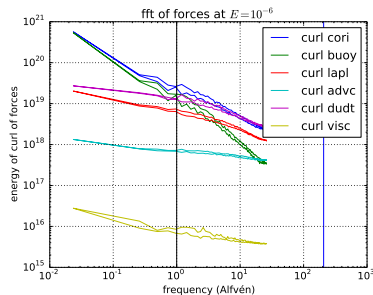
Mean temperature of each shell has been removed.

Method

- Fields up to $l_{\max}=30$ stored periodically for significant time-spans.
- Post-processing of these provide force balances at different time-scales.
- Boundary layers removed.

Problem: large storage requirement for full fields (only up to $l_{\max}=30$ here)

Force balance vs frequency (large scales only)

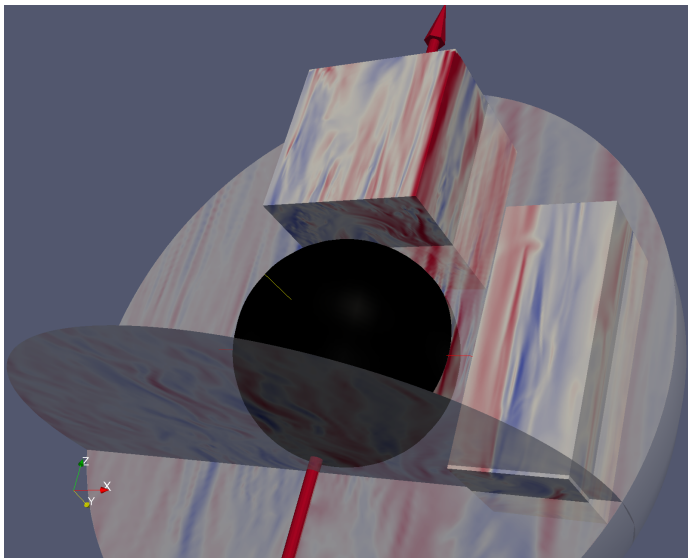


- Geostrophic balance removed by curl.
- For low frequency: Coriolis-Buoyancy balance.
- For high frequency: Coriolis-inertia balance (inertial waves).
- Laplace force overtakes Buoyancy at high frequency.
- contribution of small scales ignored.

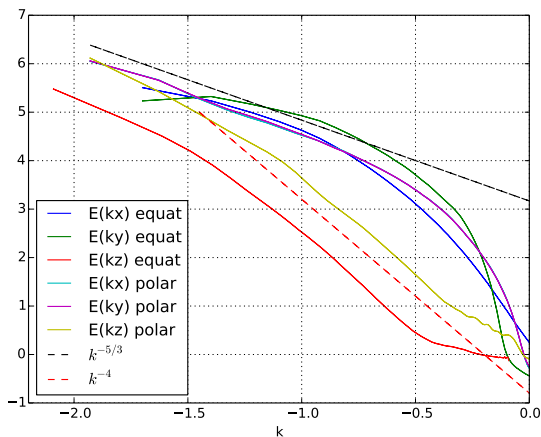
Can we relate to the cartesian boxes ?

- Would allow to compute *real* spectra.
- Can be easily related to many results in periodic boxes.
- Requires some special treatment (apodization) to allow fft.
- Some kind of interpolation is required.

Can we relate to the cartesian boxes ?

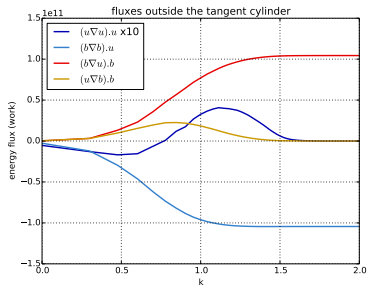
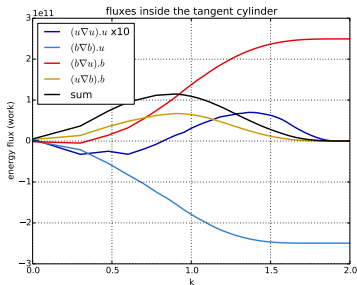


1D spectra of kinetic energy



Strong anisotropy due to rotation (along z -axis), very steep spectra in k_z .

Fluxes and transfers



- Reynolds: inverse cascade and forward cascade, but very small !
- Largest production of B happens between forward and inverse cascades.
- Transfer of energy between scales happens mostly through the Magnetic field.

Summary

Turbulence in the Earth's core (dynamo magnetic field + strong rotation) is not well understood.

As we go toward more turbulent simulations

- Velocity field peaks at smaller and smaller scales !
- Torsional waves are excited.
- Improved z-invariance.

Open questions:

- Can we really forget about $u \nabla u$?
- How does the magnetic field affect the flow ?
- Ongoing work on extracting cartesian boxes (fluxes, ...).
- The data is available if someone wants to look.

N. Schaeffer, Efficient Spherical Harmonic Transforms aimed at pseudo-spectral numerical simulations, Gcubed, 2013.

Thank you !

Breaking news

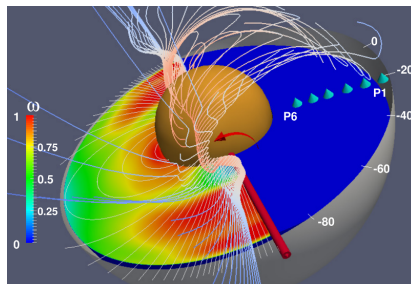
PRL 113, 184501 (2014)

PHYSICAL REVIEW LETTERS

week ending
31 OCTOBER 2014

Turbulence Reduces Magnetic Diffusivity in a Liquid Sodium Experiment

Simon Cabanes, Nathanaël Schaeffer, and Henri-Claude Nataf^{*}



Some numbers

| | definition | initial | jump 1 | jump 2 | Earth's core |
|-----------|--------------------------------------|-------------------|---------------------|---------------------|--------------------|
| N_r | | 224 | 512 | 1024 | |
| L_{max} | | 191 | 479 | 893 | |
| Ek | $\nu/D^2\Omega$ | 10^{-5} | 10^{-6} | 10^{-7} | $3 \cdot 10^{-15}$ |
| Ra | $\Delta T \alpha g D^3 / \kappa \nu$ | $6 \cdot 10^{10}$ | $1.2 \cdot 10^{12}$ | $2.4 \cdot 10^{13}$ | $10^{30} ?$ |
| Pm | ν/η | 0.4 | 0.2 | 0.1 | $3 \cdot 10^{-5}$ |
| Pr | ν/κ | 1 | 1 | 1 | 0.1 - 10 |
| Rm | UD/η | 700 | 650 | 600 | 2000 |
| A | $\sqrt{\mu\rho}U/B$ | 1.5 | 0.6 | 0.45 | 0.01 |
| Re | UD/ν | 1770 | 3240 | 5960 | $2 \cdot 10^8$ |
| Ro | $U/D\Omega$ | 0.018 | $3.2 \cdot 10^{-3}$ | $6 \cdot 10^{-4}$ | $3 \cdot 10^{-6}$ |
| Le | $B/\sqrt{\mu\rho}D\Omega$ | 0.012 | $5 \cdot 10^{-3}$ | $1.3 \cdot 10^{-3}$ | 10^{-4} |
| Λ | $B^2/\eta\Omega$ | 5.8 | 5.7 | 1.7 | 1 - 10 |
| F_ν | $D_\nu/(D_\eta + D_\nu)$ | 47% | 24% | 17% | ? |
| F_η | $D_\eta/(D_\eta + D_\nu)$ | 53% | 76% | 83% | ? |

Table 1: Various input and output parameters of our simulations, where D is the shell thickness, U the rms velocity and B the rms magnetic field.