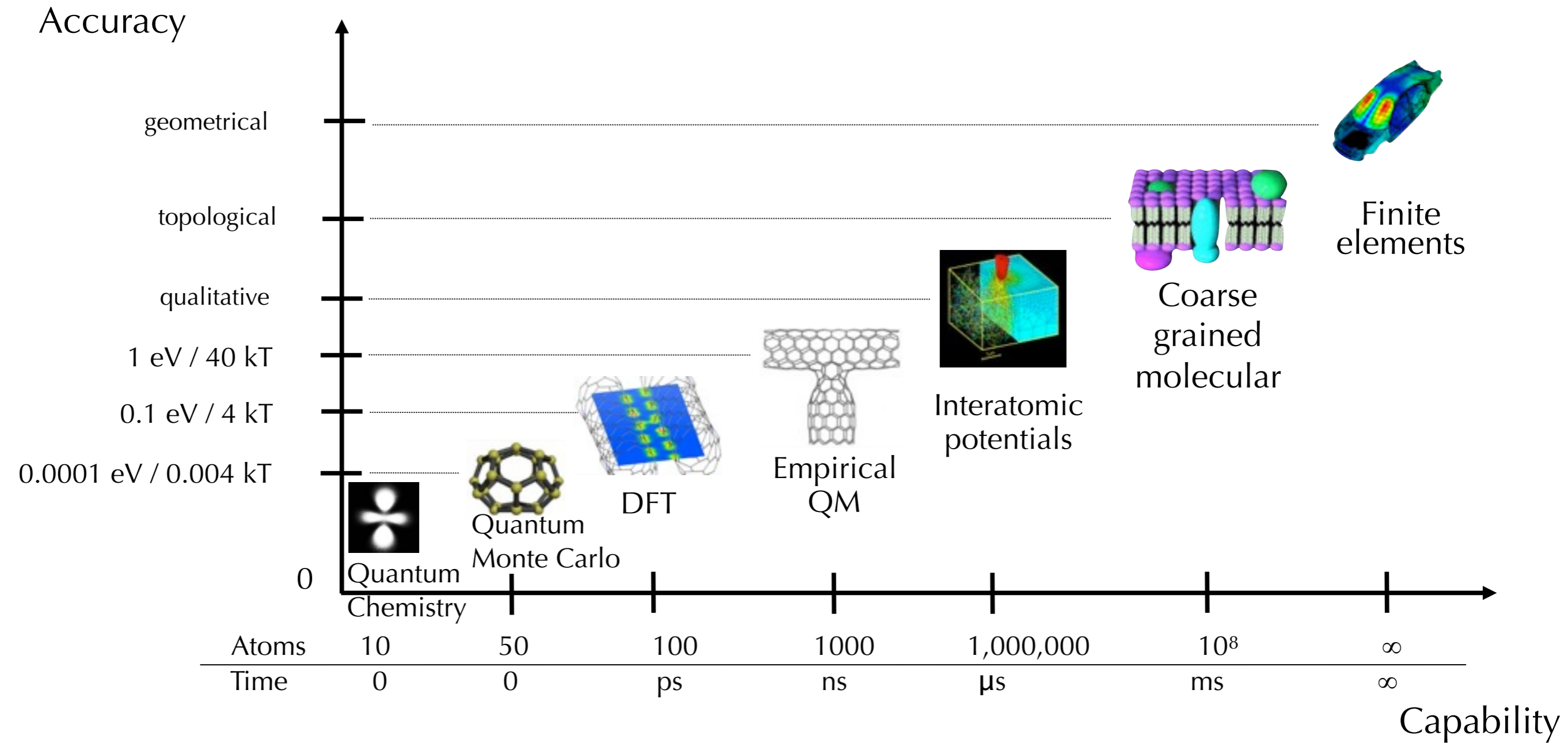


Representation and regression problems in molecular structure and dynamics

Gábor Csányi

Engineering Laboratory
 UNIVERSITY OF
CAMBRIDGE

Multiple scales of materials modelling



Multiple scales of materials modelling

Accuracy

geometrical

Quantum Mechanics

$$\hat{\mathcal{H}}\Psi = E\Psi$$

$\hat{\mathcal{H}}$

Nuclear positions, interaction of electrons and nuclei

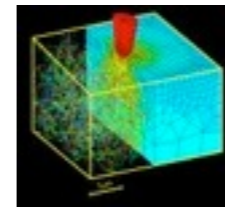
$\Psi(r_1 \dots r_N)$

Wavefunction of electrons

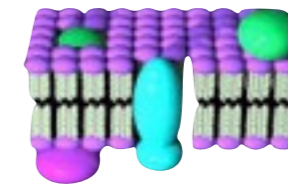
Unknown $3N$ dimensional function, unique for each nuclear configuration



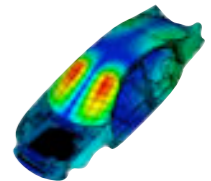
Empirical QM



Interatomic potentials



Coarse grained molecular



Finite elements

1000
ns

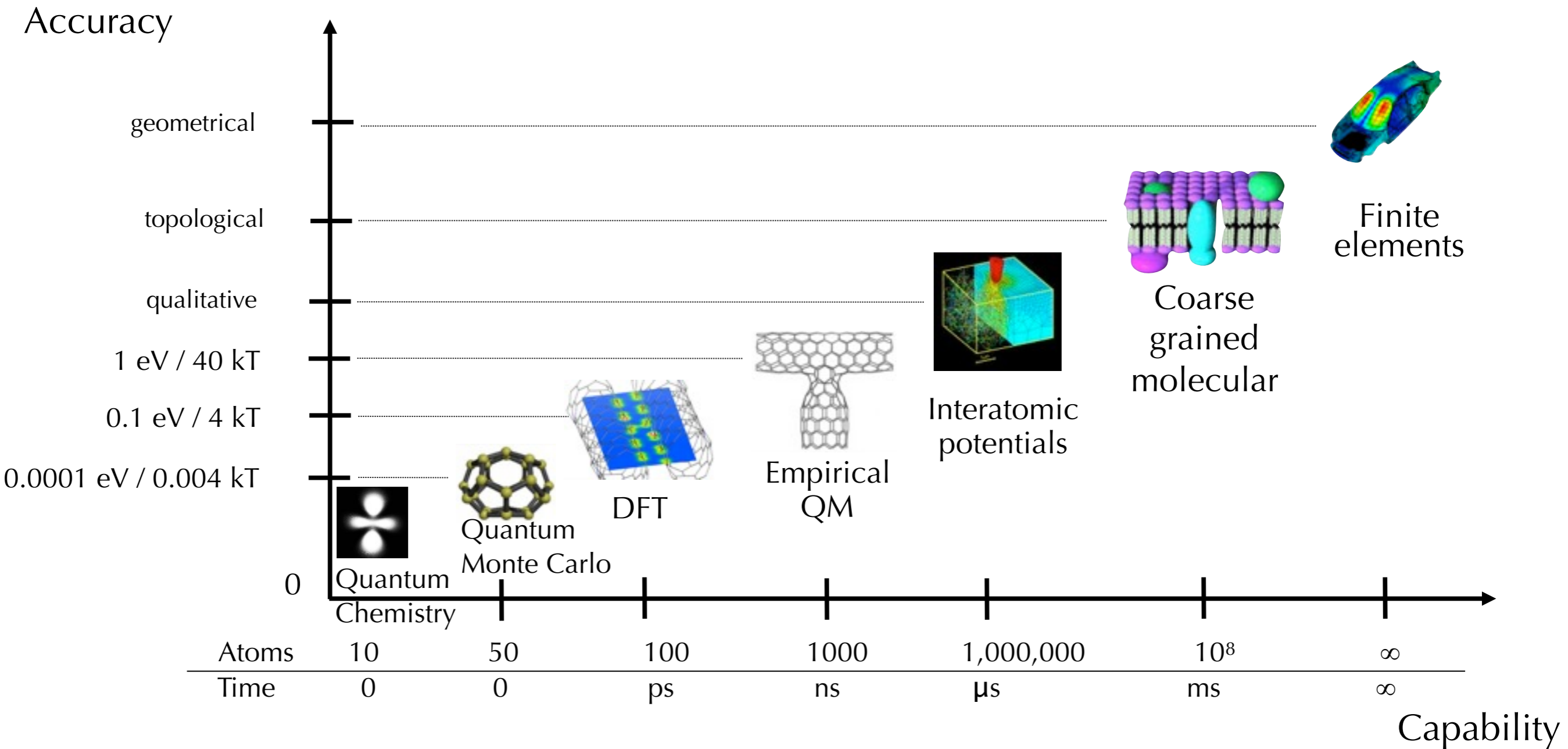
1,000,000
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10^8
ms

∞
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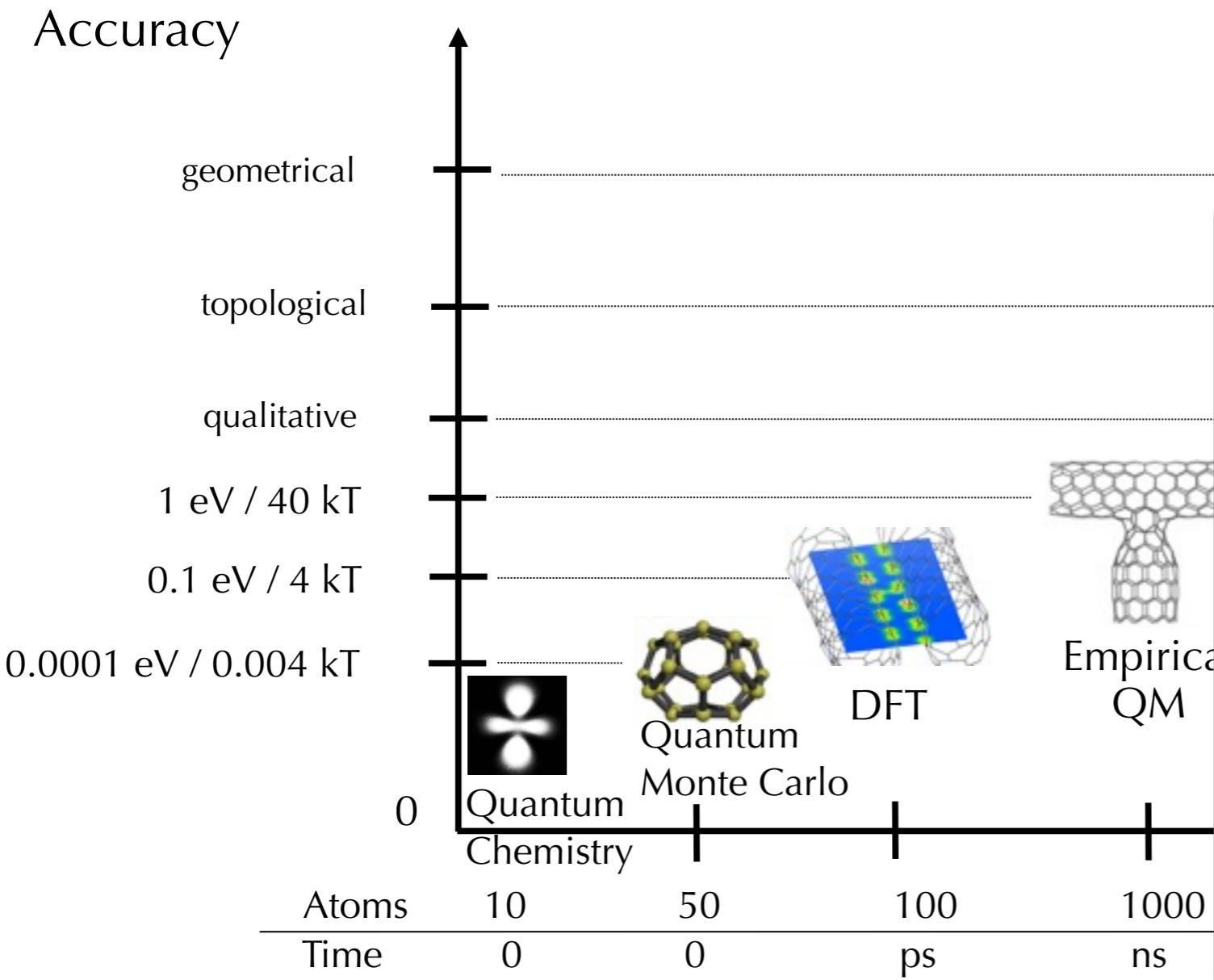
Capability

Multiple scales of materials modelling



Quantum Mechanics
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 $\hat{\mathcal{H}}$ Nuclear positions, interaction of electrons and nuclei
 $\Psi(r_1 \dots r_N)$ Wavefunction of electrons
 Unknown 3N dimensional function, unique for each nuclear configuration

Multiple scales of materials modelling



Empirical modelling

Minimal model to describe proposed mechanism

Understand experimentally observed phenomena

Capability

Quantum Mechanics

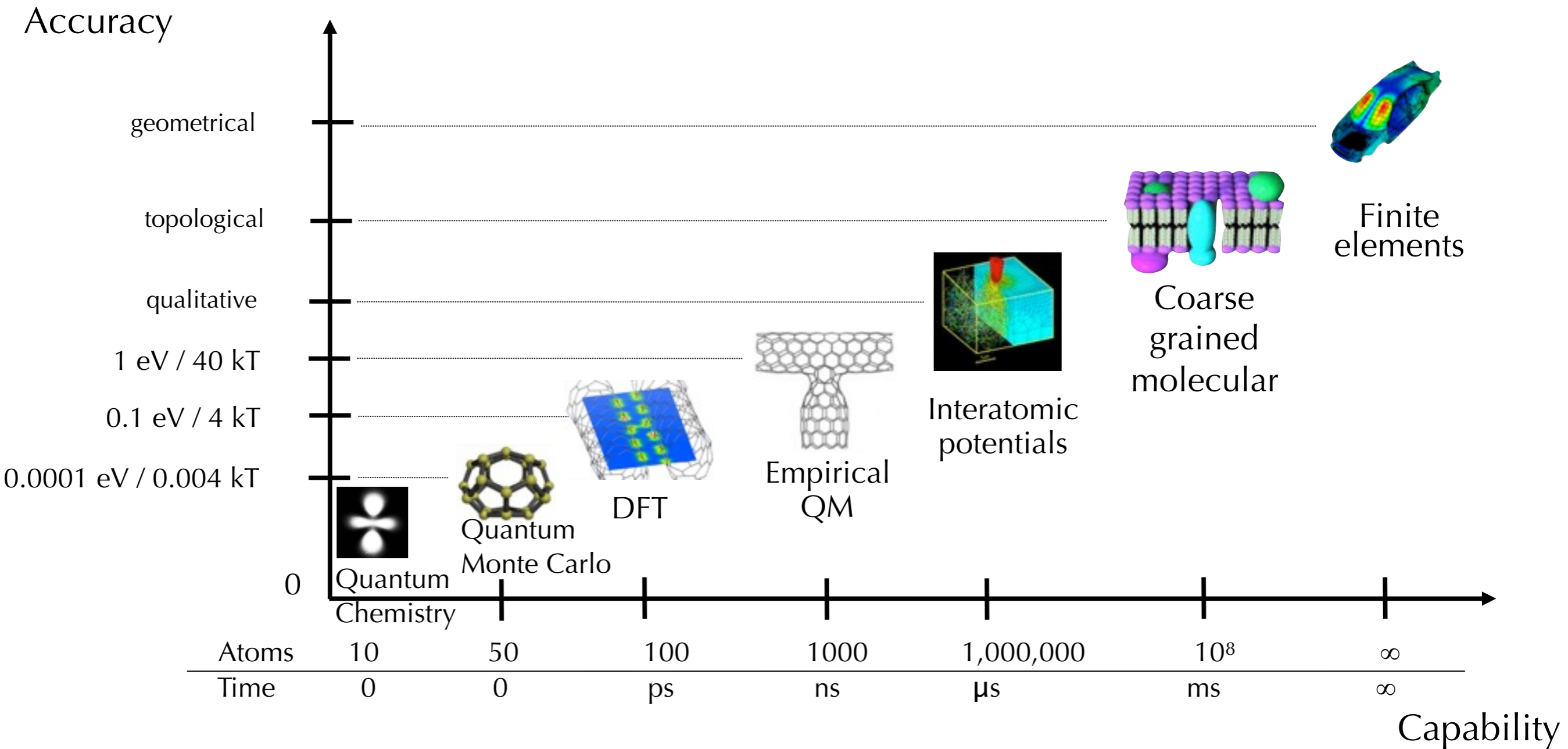
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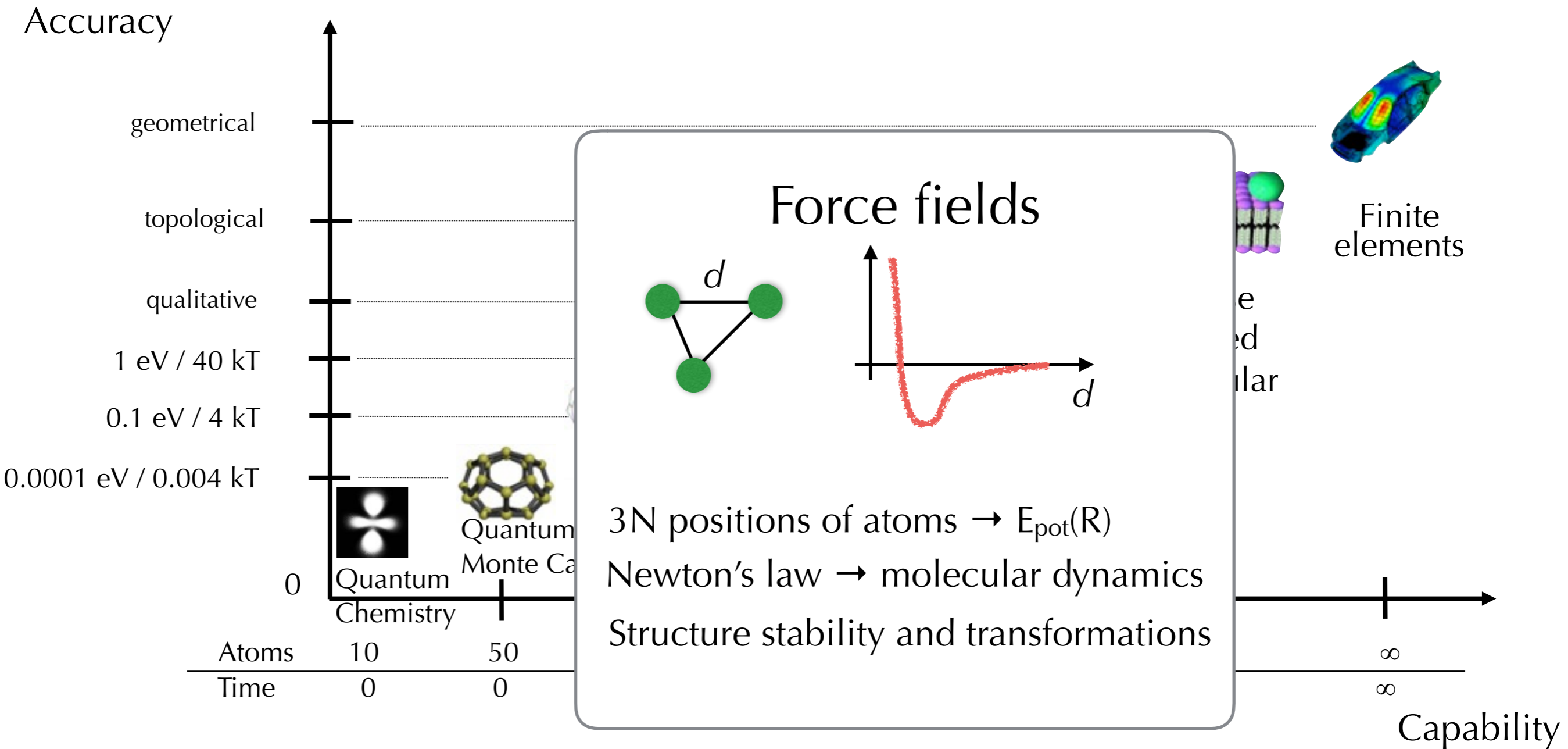
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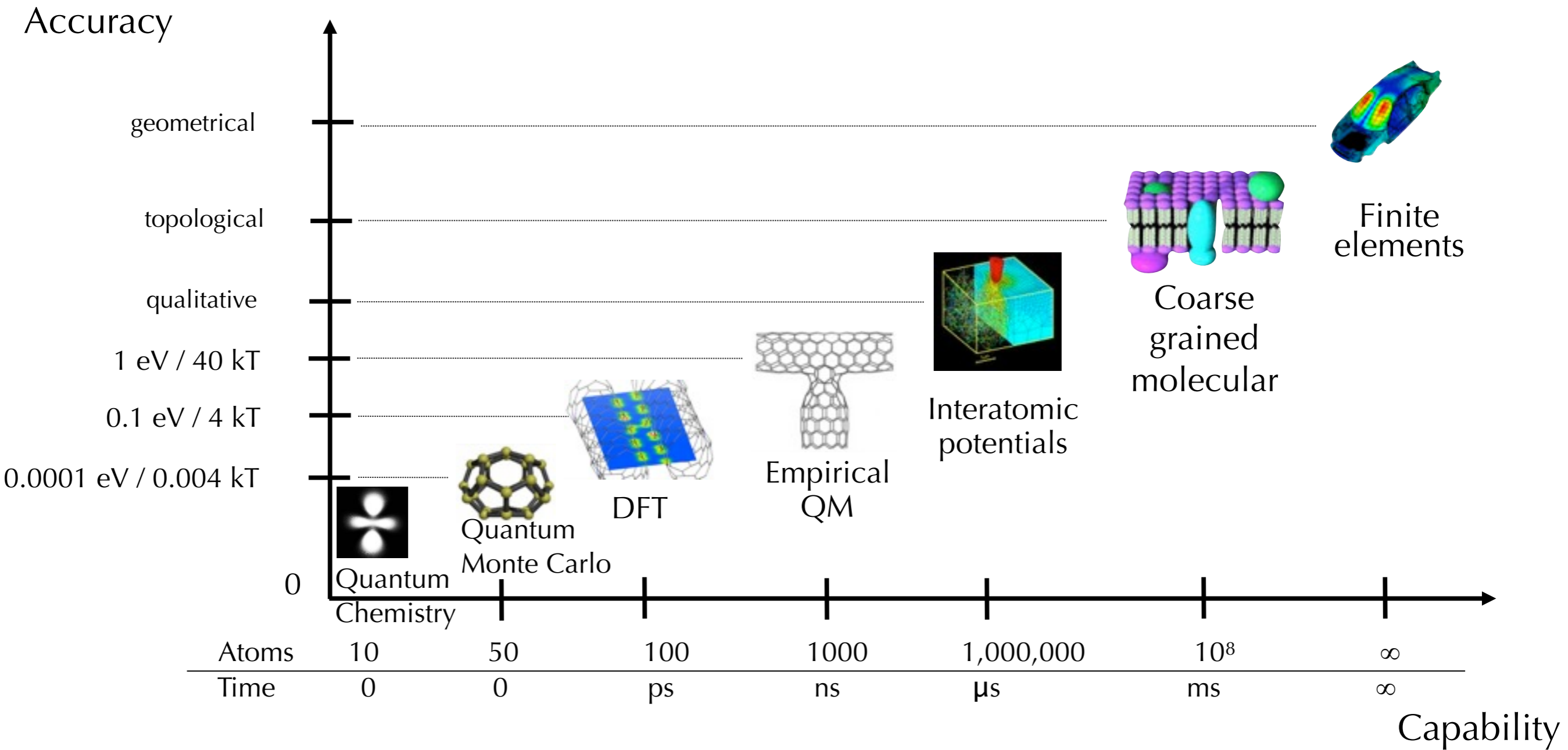
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Force fields

3N positions of atoms $\rightarrow E_{\text{pot}}(R)$

Newton's law \rightarrow molecular dynamics

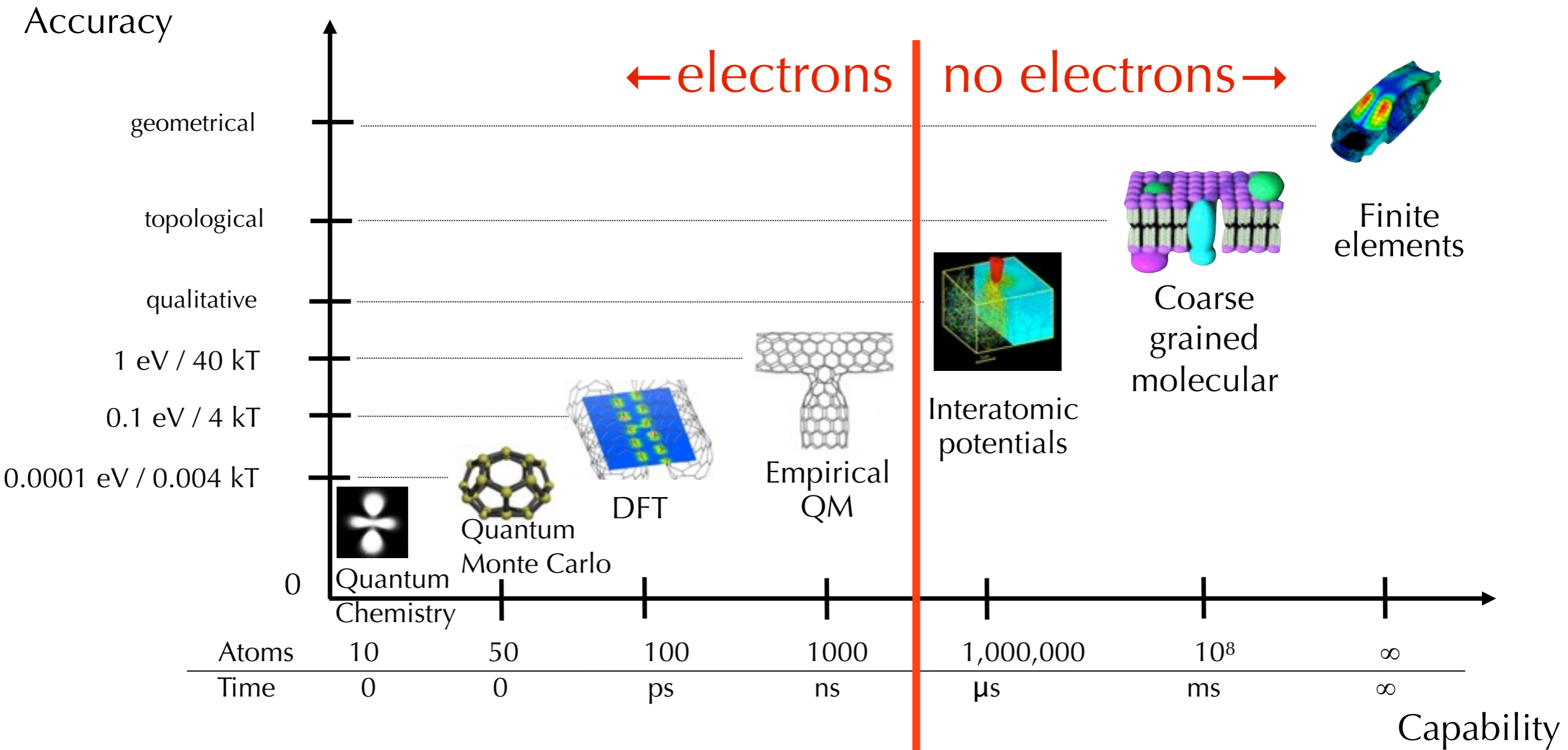
Structure stability and transformations

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$$-d - \uparrow$$

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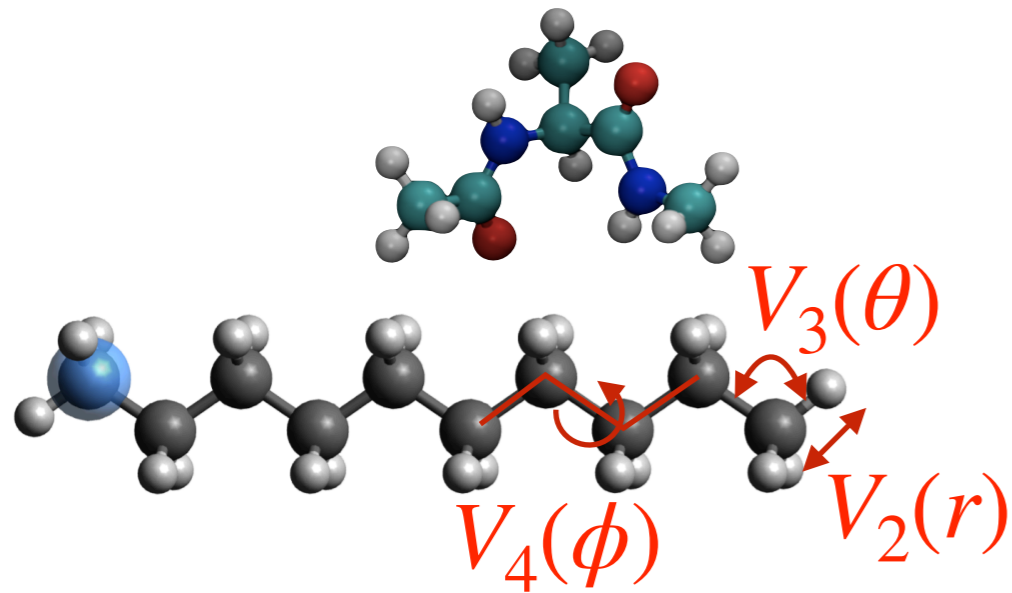
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GAP in accuracy and speed

Interatomic potentials (force fields)

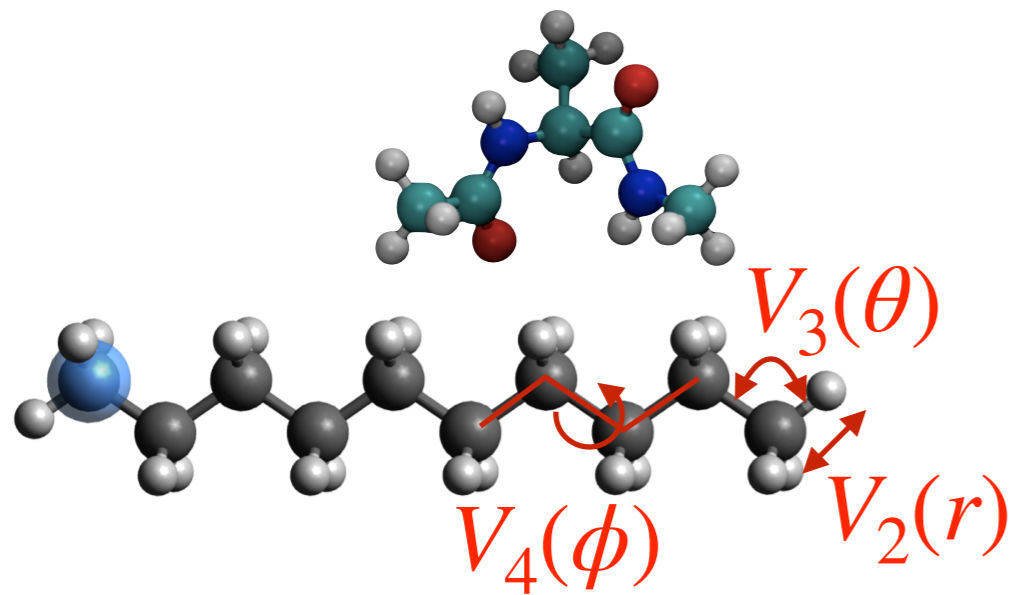


Transferability organic force fields

- Accurate for small displacements
- Typically no reactions (some exceptions)
- Ultimate fit is to macroscopic experimental properties

AMBER
CHARMM
COMPASS
OPLS
...

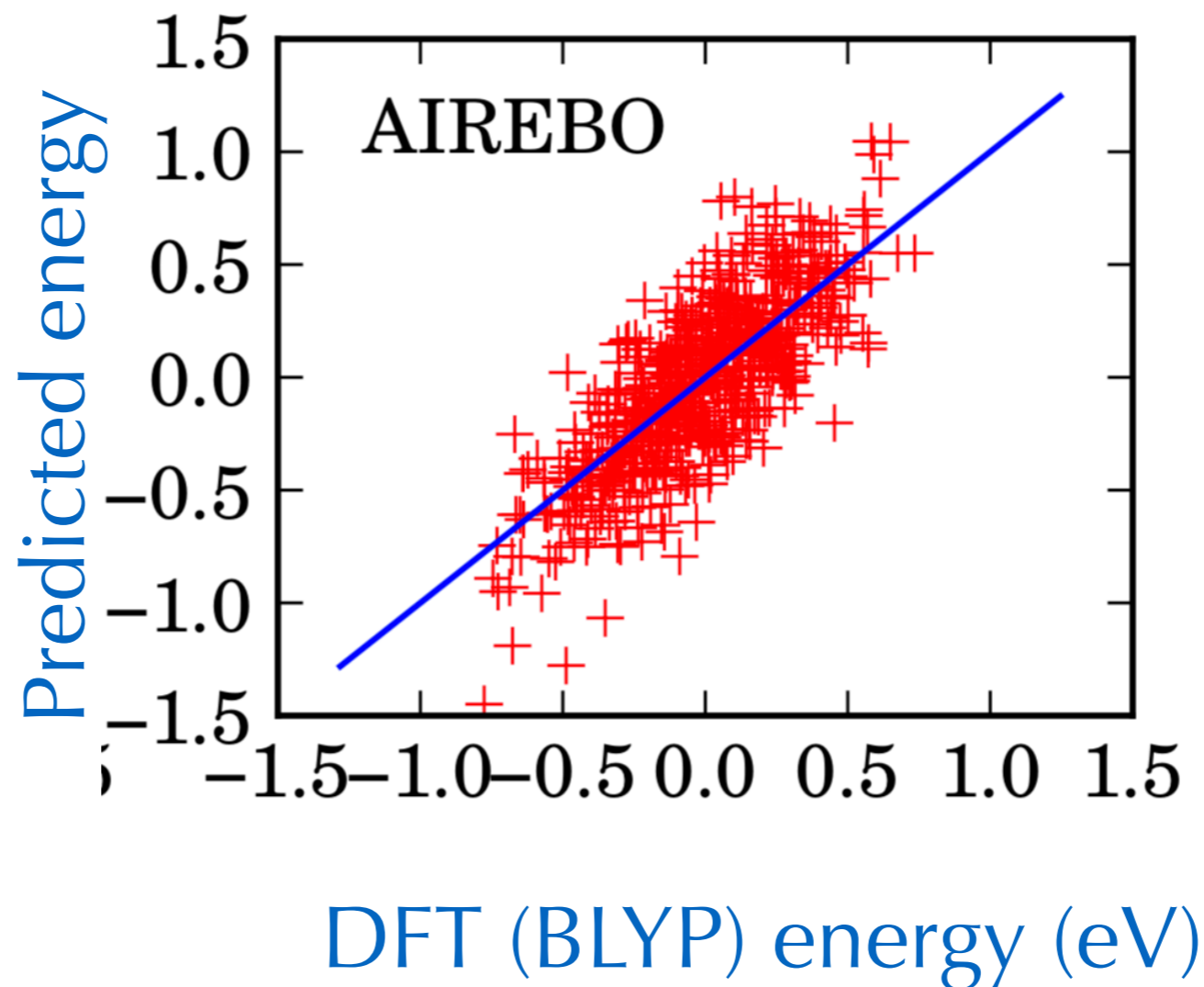
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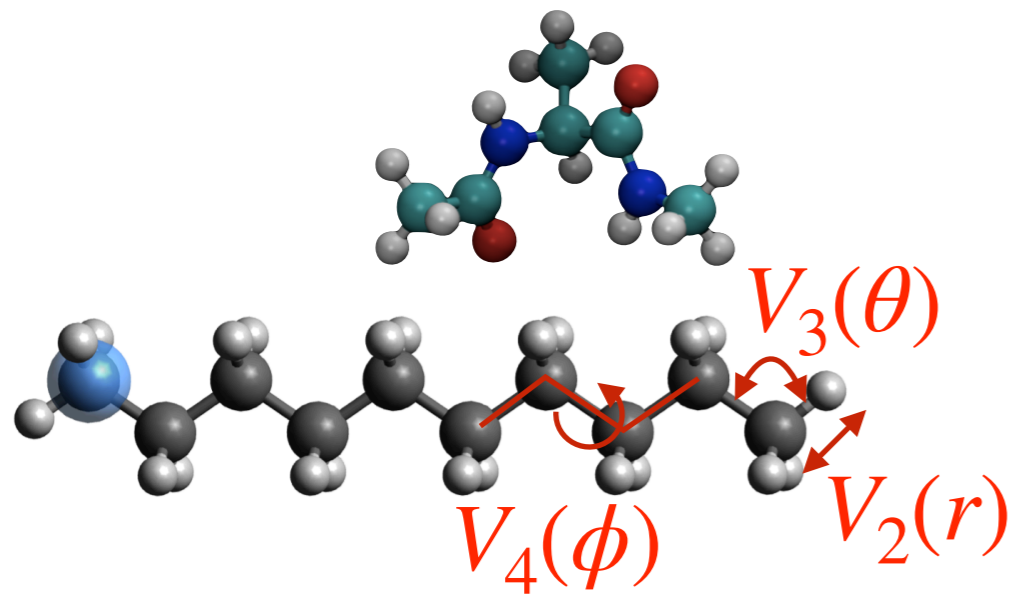
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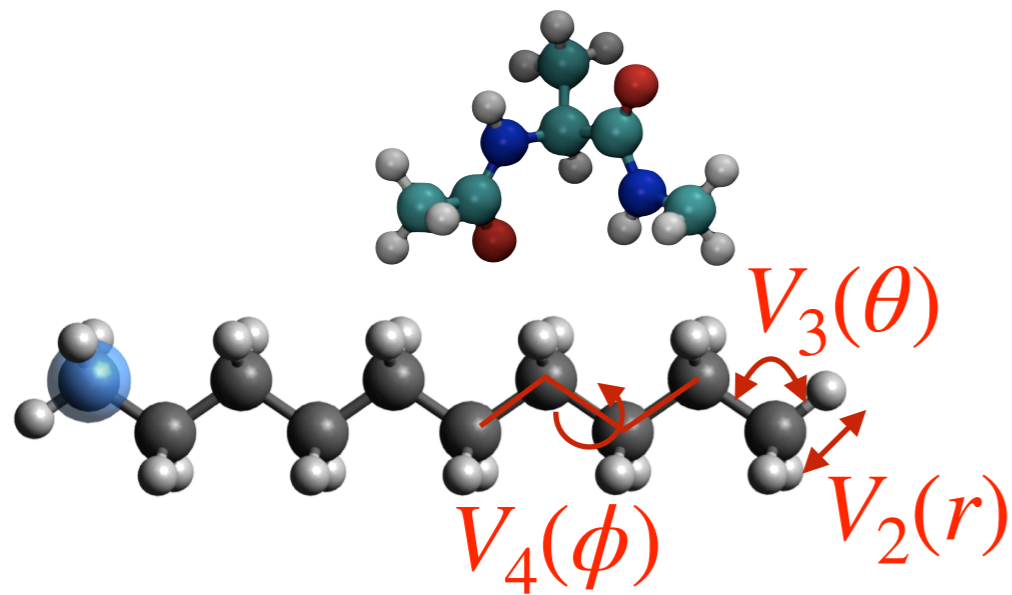


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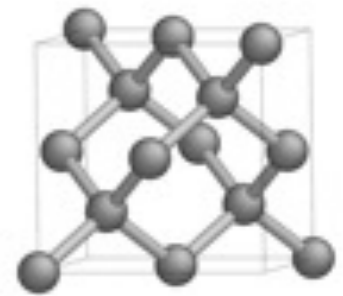
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$$V(\sum \rho(r))$$



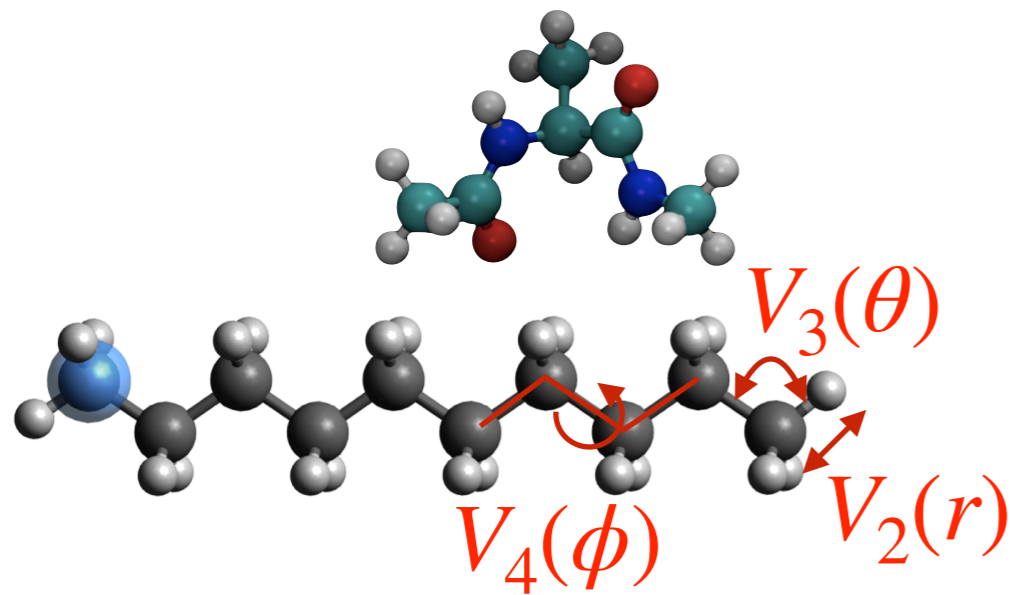
Reactive

solid state materials

- Physics-based functional forms
- Not systematic
- Simplest model to capture phenomena
- Mixture of fitting to experiment and ab initio data

Tersoff
EAM
BOP
...

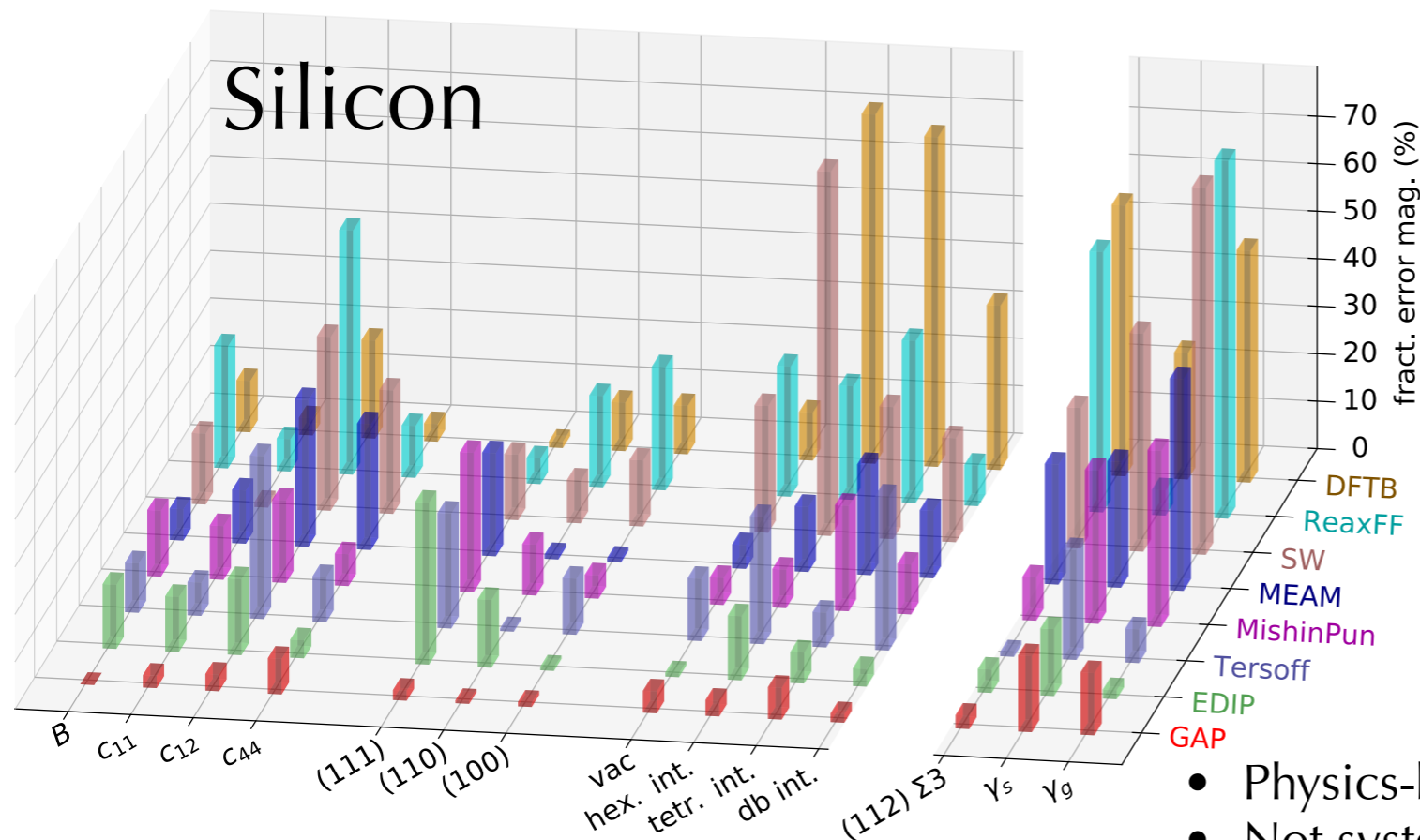
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Error %

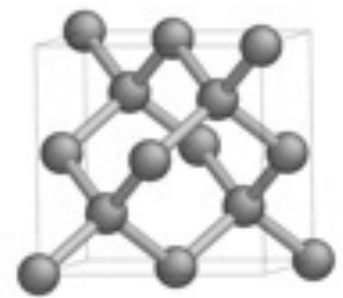
Properties

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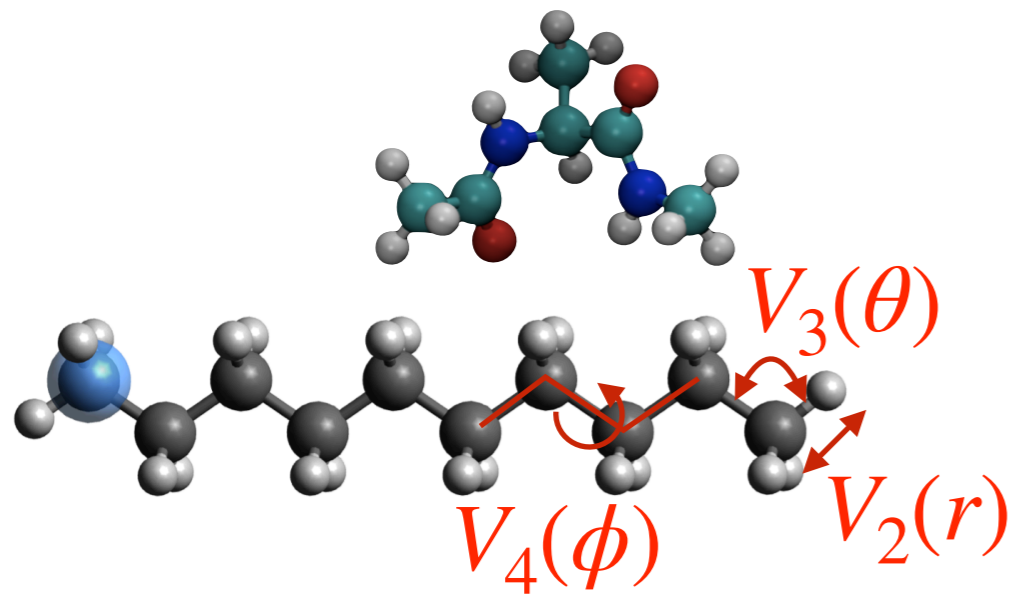
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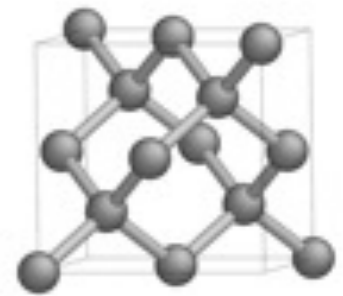
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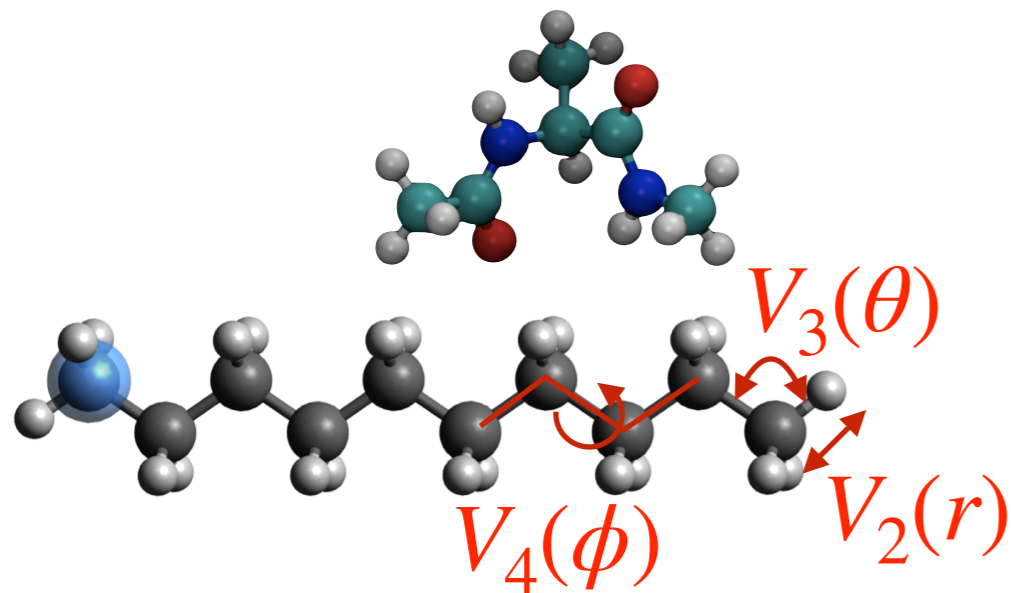
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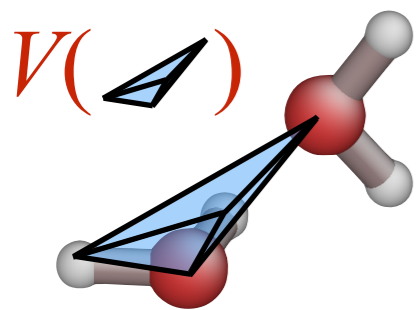


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gas phase molecules

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- Fit to ab initio data
- Exponential scaling in molecule size

Braams & Bowman

Szalewicz

Paesani

Thiel

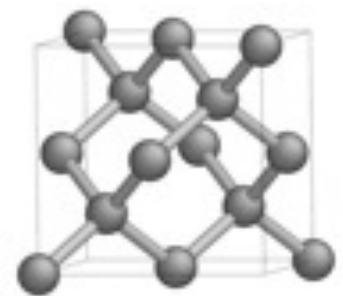
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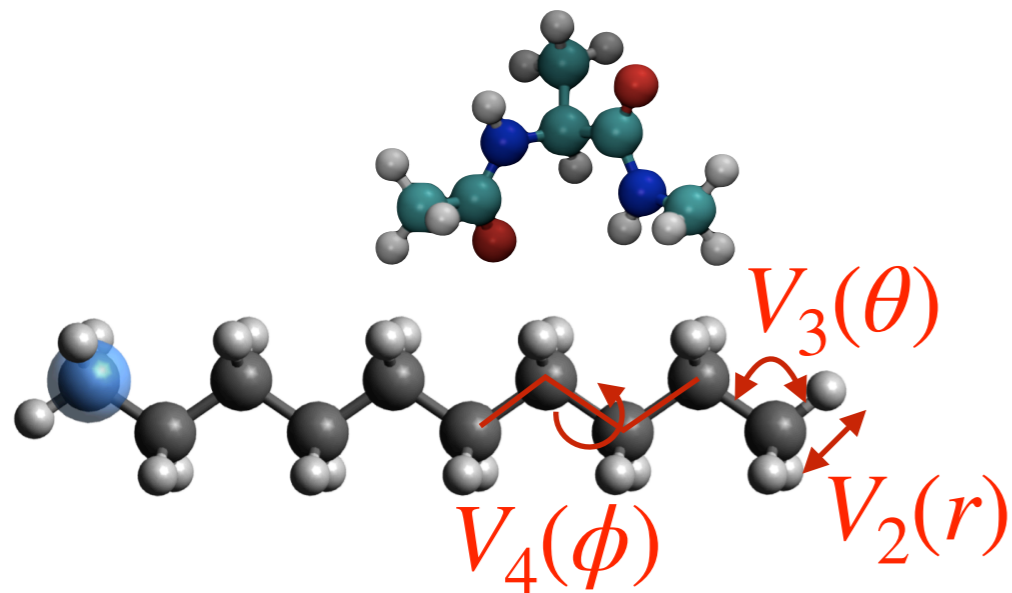
Tersoff

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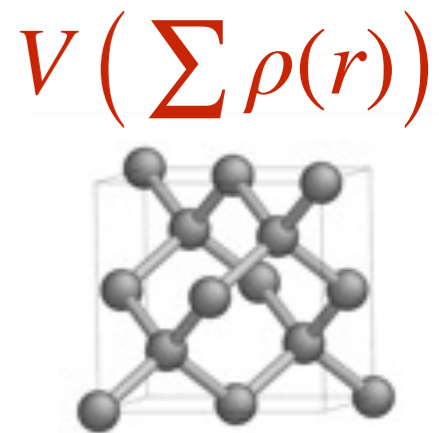
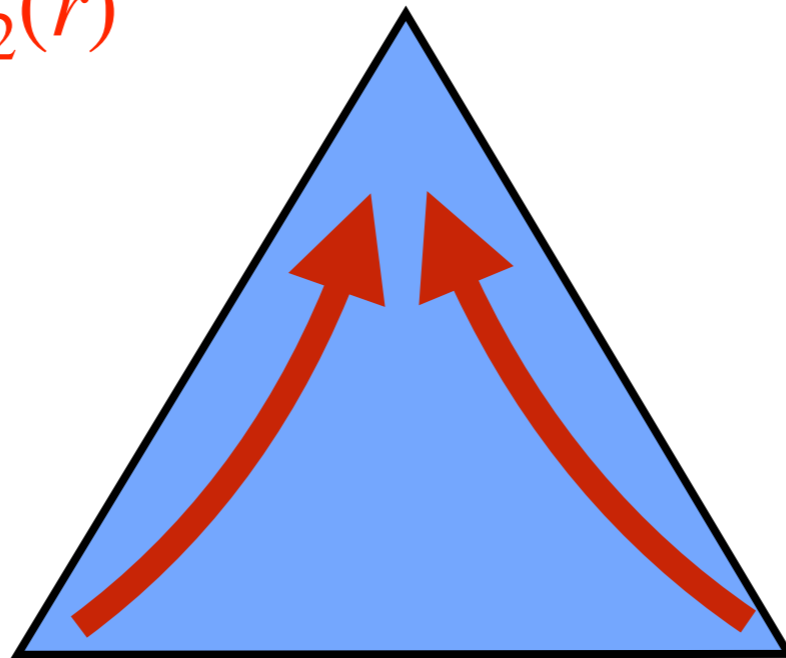
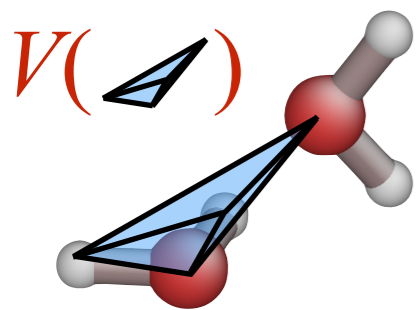


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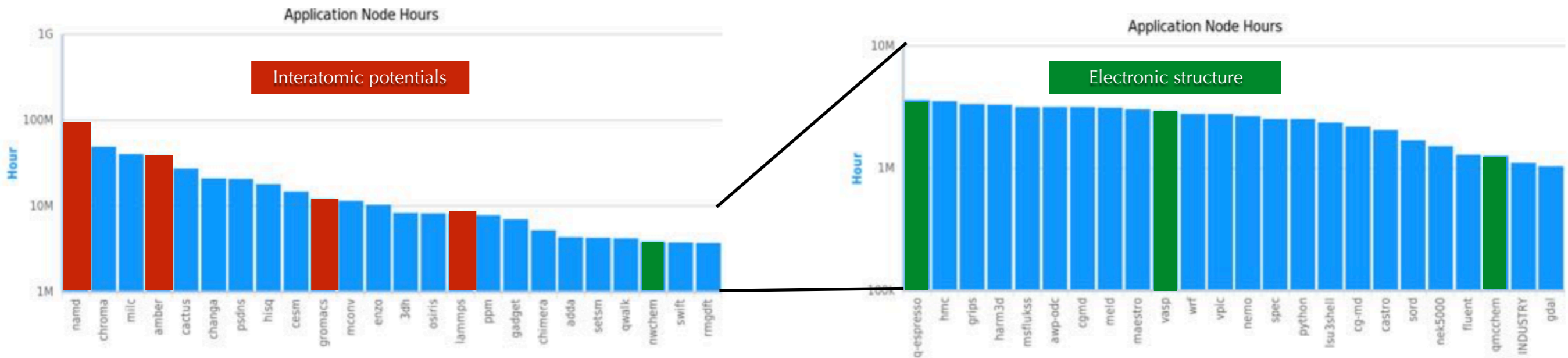
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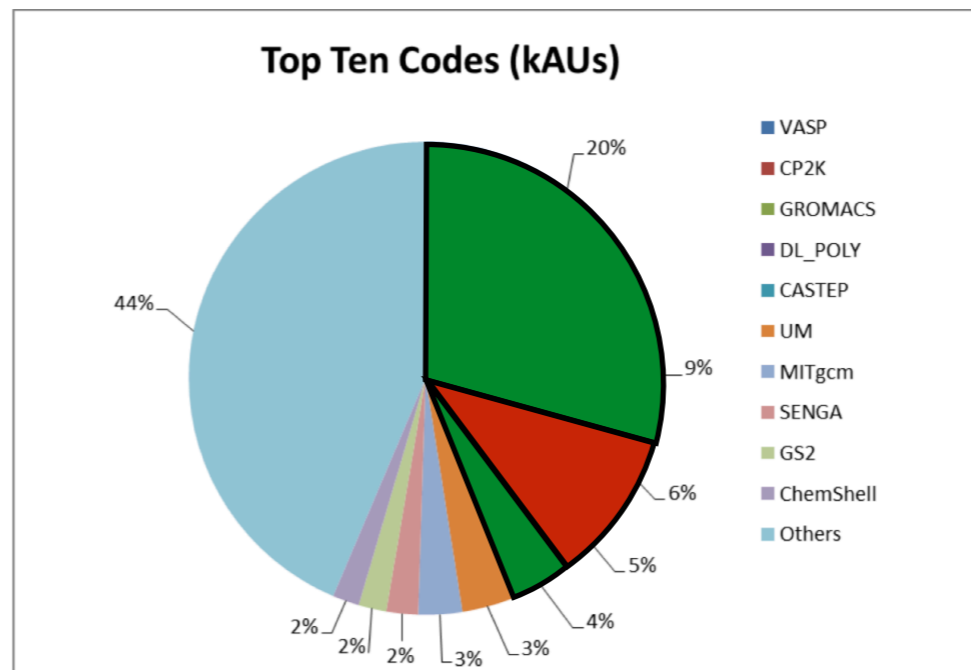
...

Significant HPC time

Blue Waters (USA) application workload analysis (2017 report)



UK national HPC
(Hector) 2012



Is it *possible* to create an accurate, reactive interatomic potential for covalently bound materials?

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- Separation of **short** and **long** range interactions

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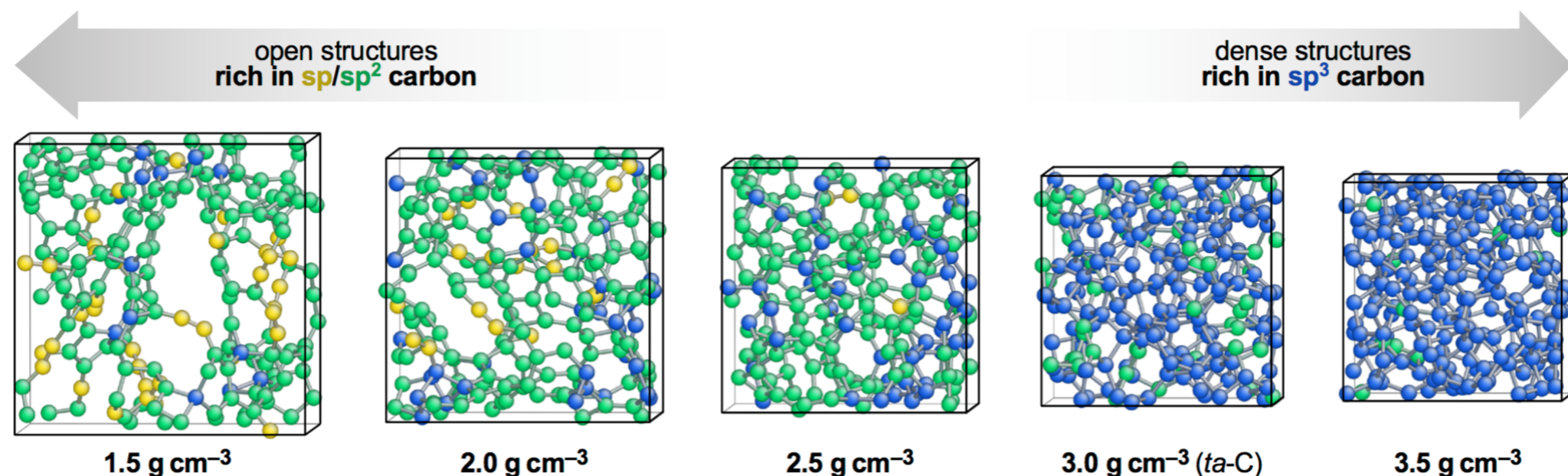
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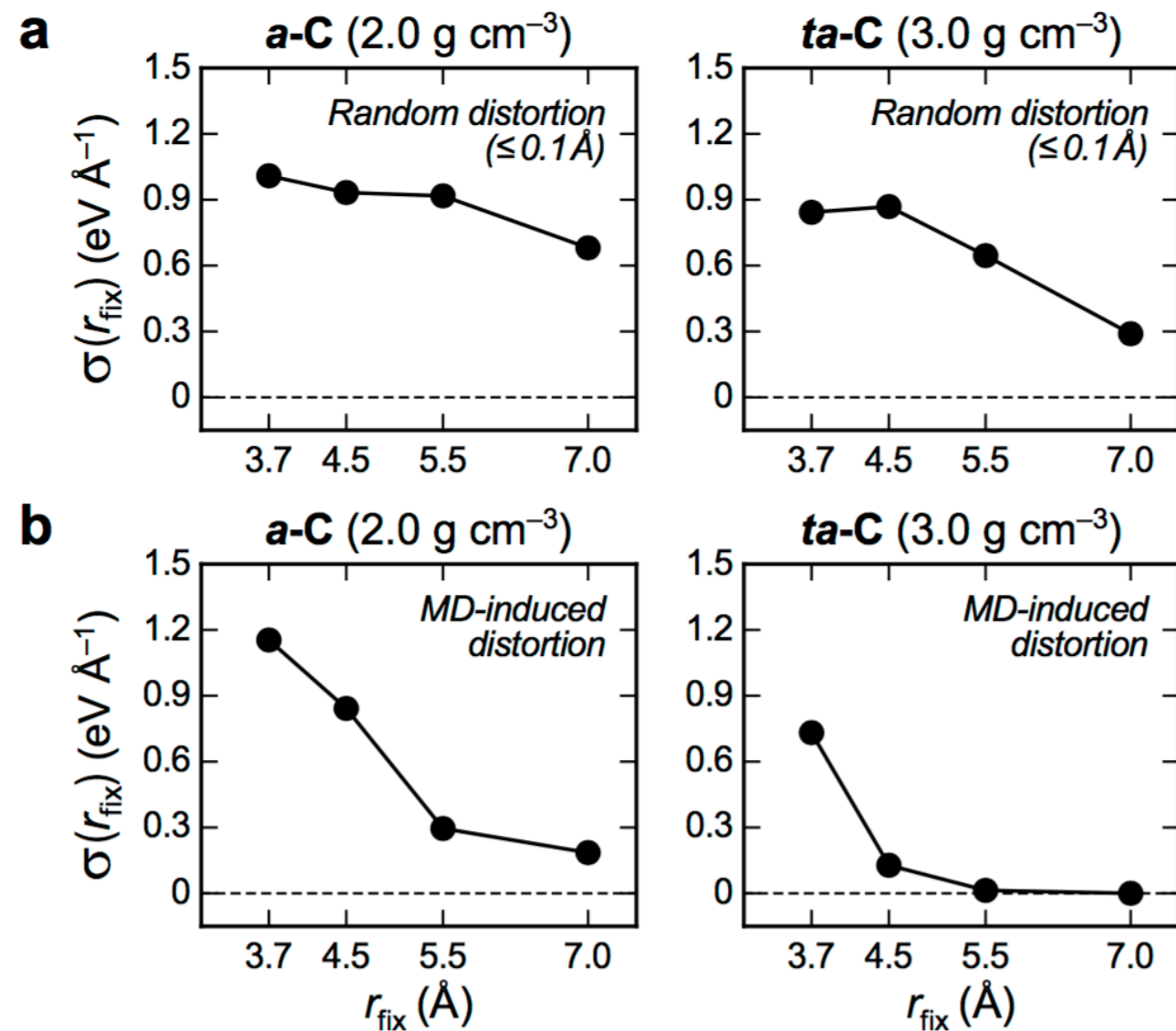
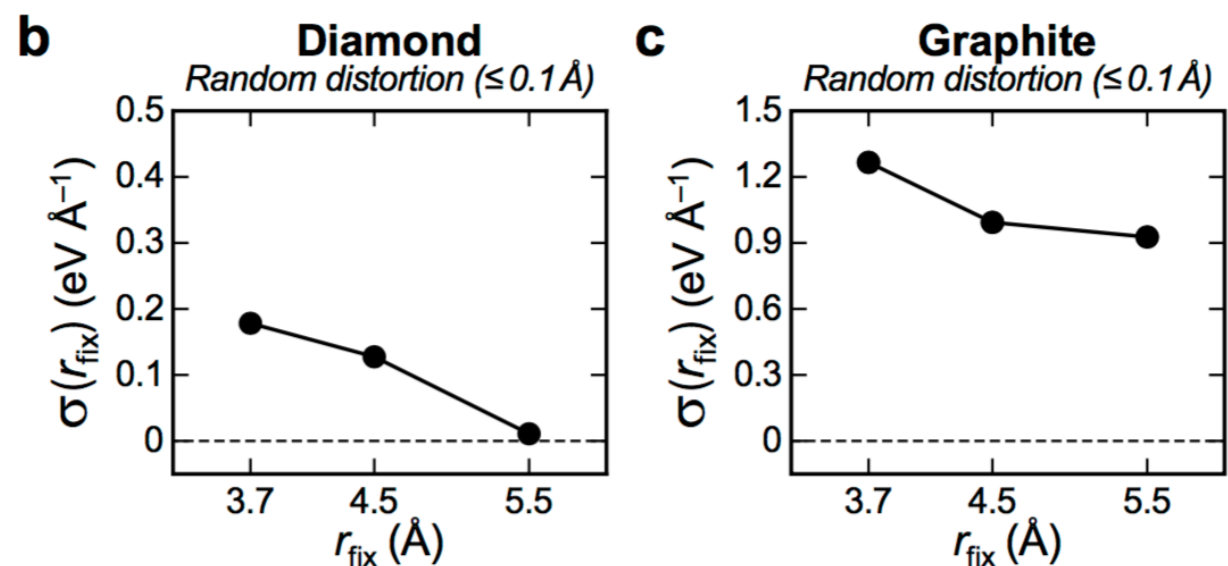
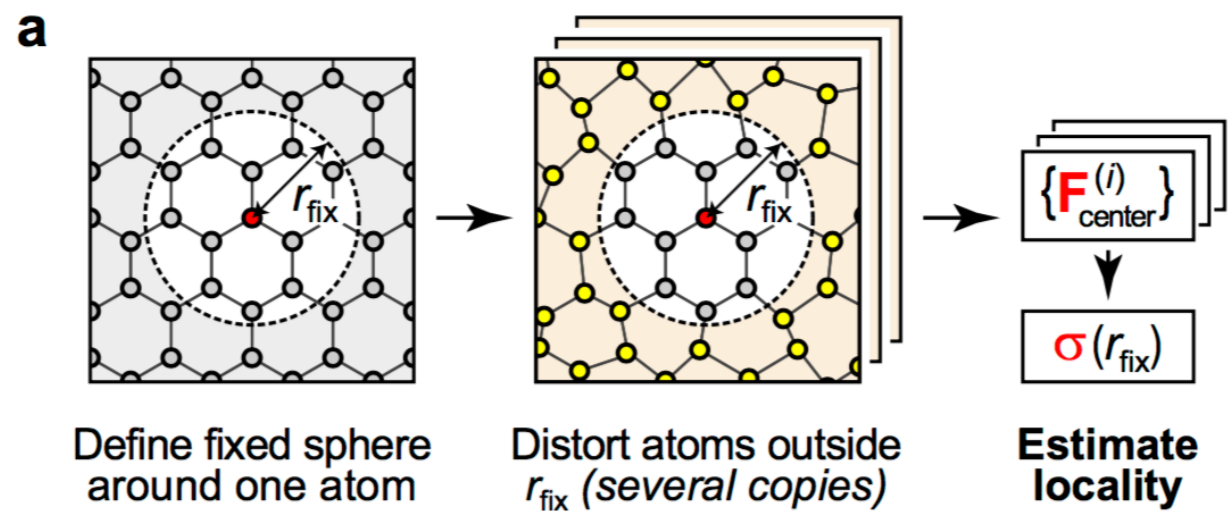
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- Three possibilities to deal with long range interactions
 - They are small enough to be ignored (beyond a cutoff of e.g. 5-6-8 Å)
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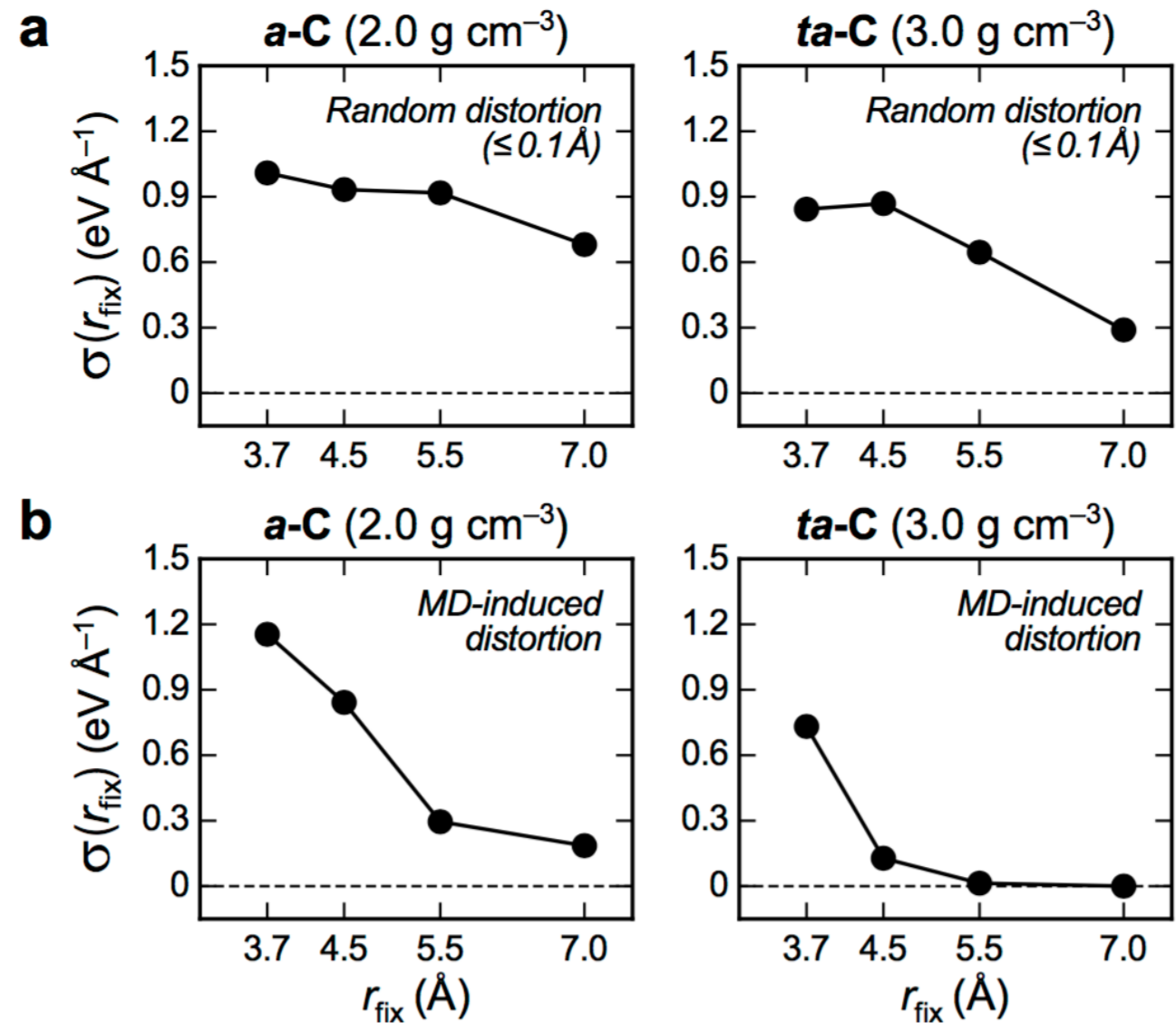
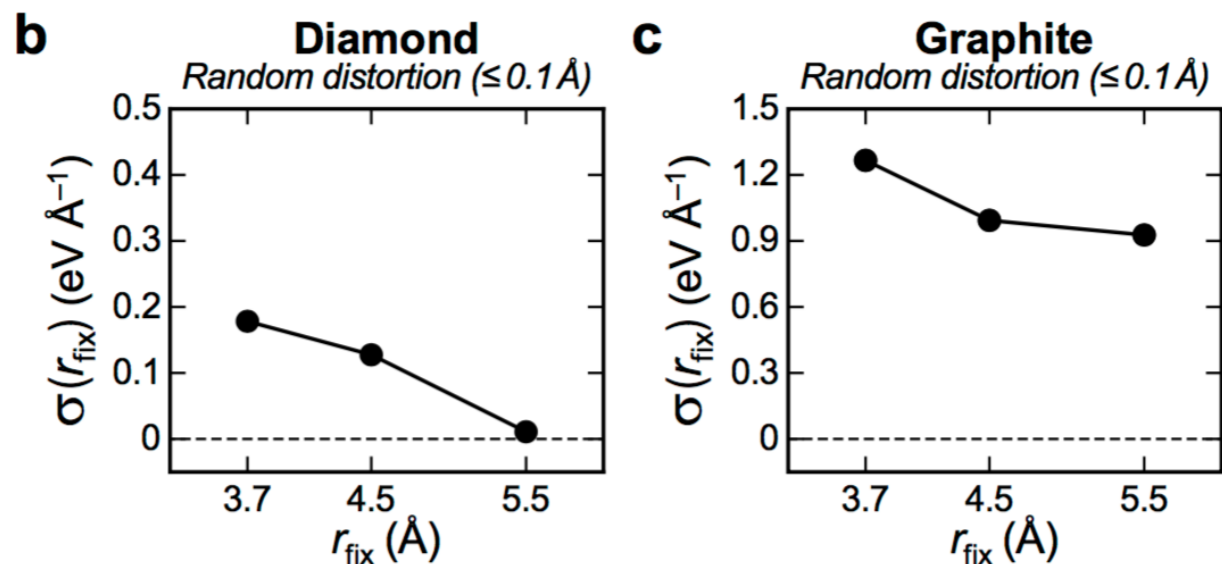
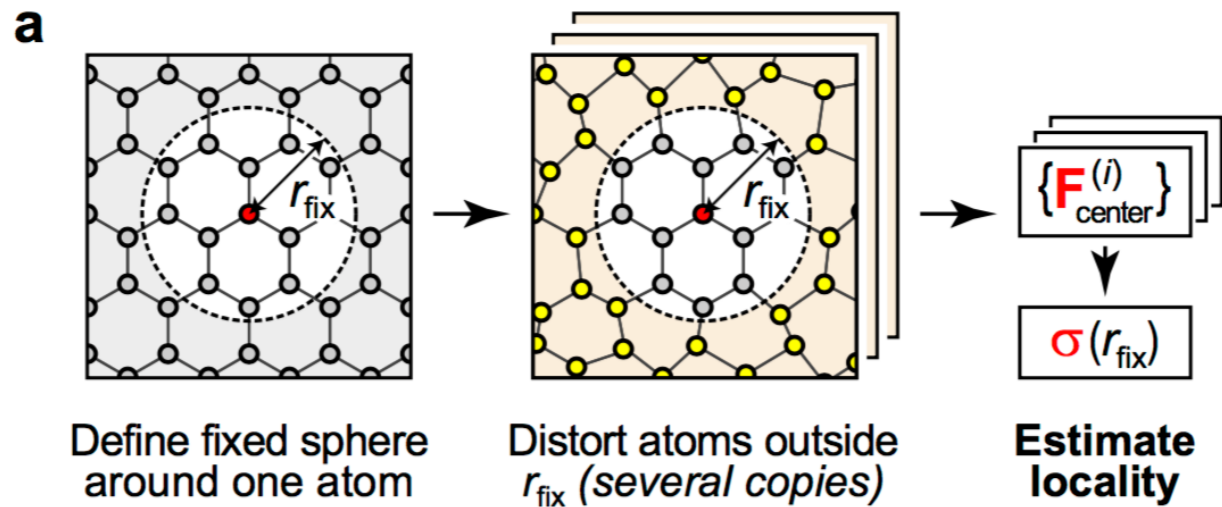
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- How local is the remainder?



The "locality test"



The "locality test"



Limit on force field accuracy: for a given cutoff, what force accuracy is achievable in principle?

A decade old promise. Where are we?

PRL **98**, 146401 (2007)

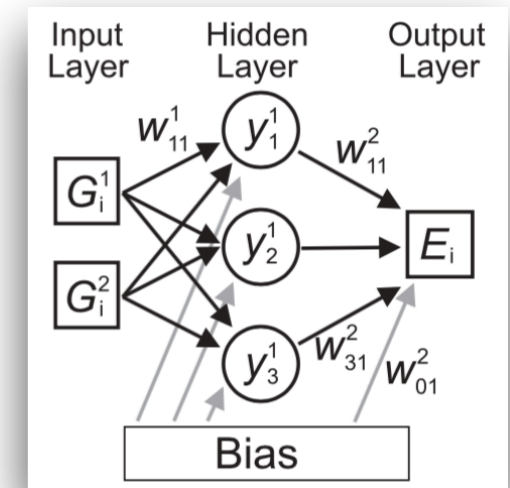
PHYSICAL REVIEW LETTERS

week ending
6 APRIL 2007

Generalized Neural-Network Representation of High-Dimensional Potential-Energy Surfaces

Jörg Behler and Michele Parrinello

Department of Chemistry and Applied Biosciences, ETH Zurich, USI-Campus, Via Giuseppe Buffi 13, CH-6900 Lugano, Switzerland
(Received 27 September 2006; published 2 April 2007)



Claim: short range QM regression problem is solved.

PRL **104**, 136403 (2010)

PHYSICAL REVIEW LETTERS

week ending
2 APRIL 2010

Gaussian Approximation Potentials: The Accuracy of Quantum Mechanics, without the Electrons

Albert P. Bartók and Mike C. Payne

Cavendish Laboratory, University of Cambridge, J J Thomson Avenue, Cambridge, CB3 0HE, United Kingdom

Risi Kondor

Center for the Mathematics of Information, California Institute of Technology, MC 305-16, Pasadena, California 91125, USA

Gábor Csányi

Engineering Laboratory, University of Cambridge, Trumpington Street, Cambridge, CB2 1PZ, United Kingdom

(Received 1 October 2009; published 1 April 2010)

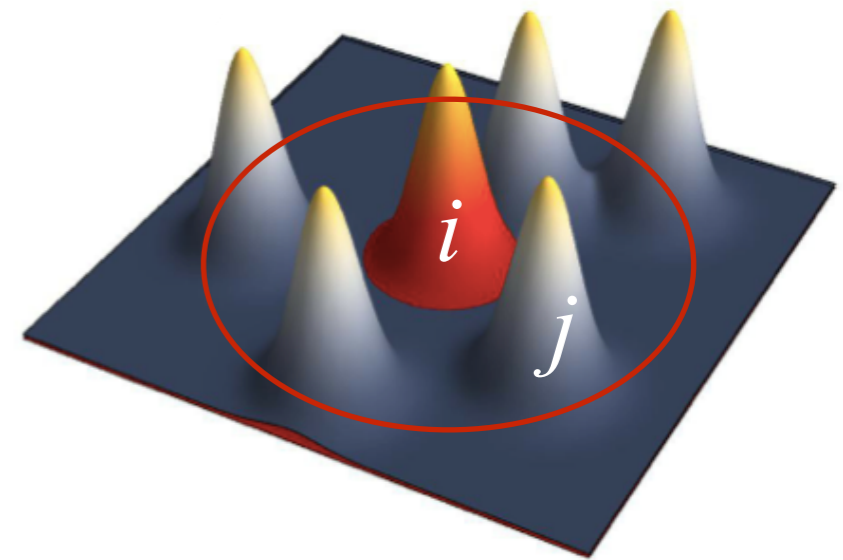
$$\begin{aligned}\mathbf{Q}_M &= \mathbf{C}_M + \mathbf{C}_{MN}(\mathbf{\Lambda} + \sigma^2\mathbf{I})^{-1}\mathbf{C}_{NM}, \\ \varepsilon_* &= \mathbf{k}_*^T \mathbf{Q}_M^{-1} \mathbf{C}_{MN}(\mathbf{\Lambda} + \sigma^2\mathbf{I})^{-1} \mathbf{y},\end{aligned}$$

Representation

Representation: atomic neighbour density

$$\rho^{(i)}(\mathbf{r}) = \sum_j G(\mathbf{r} - \mathbf{r}_{ij}) f_{\text{cut}}(|\mathbf{r}_{ij}|)$$

Convolution kernel $G(\mathbf{r}) = \begin{cases} \delta(\mathbf{r}) \\ e^{-|\mathbf{r}|^2/2\sigma^2} \end{cases}$



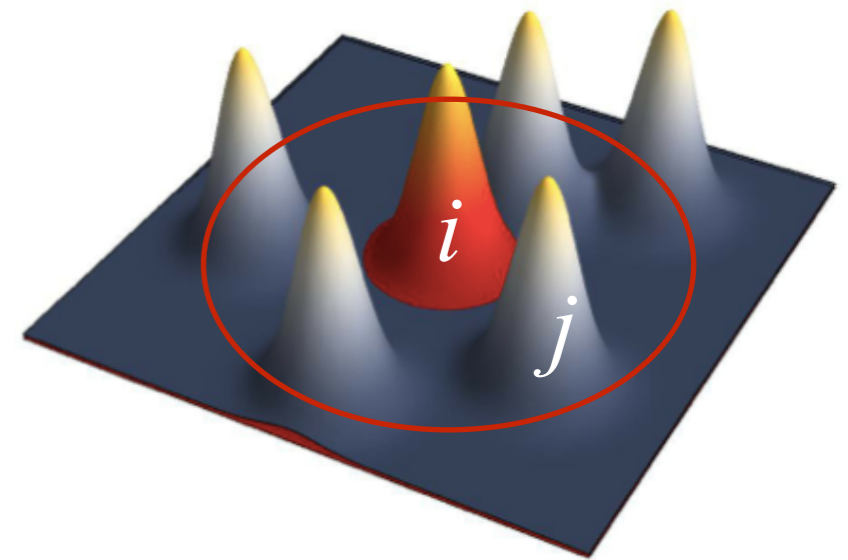
cutoff: compact support

ρ is permutational invariant, smooth and continuous

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Rotational invariance:

$$\rho(\mathbf{r}) = \sum_{nlm} c_{nlm} g_n(r) Y_{lm}(\hat{\mathbf{r}})$$

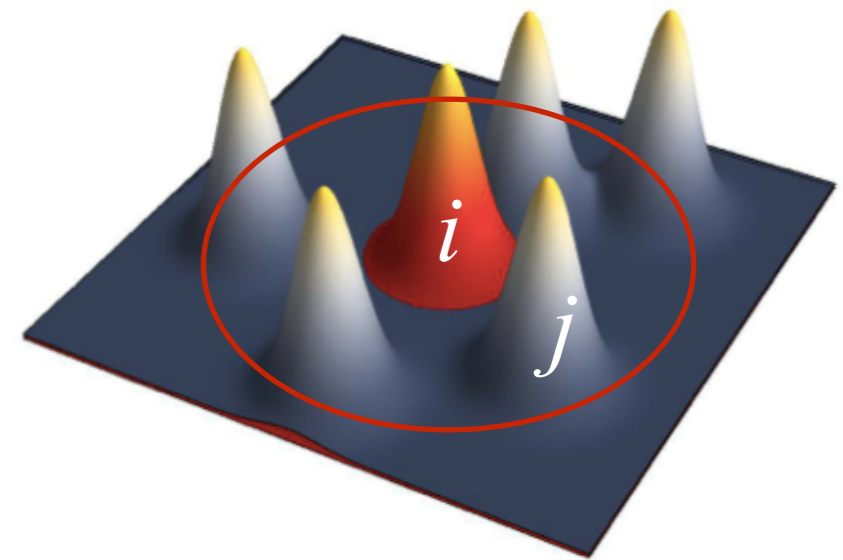
$$P_{nn'l} = \sum_m c_{nlm}^\dagger c_{n'lm}$$

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Smooth overlap of atomic positions (SOAP)

$$\mathbf{p} \cdot \mathbf{p}' = \int_{\hat{R} \in SO_3} d\hat{R} \left| \int d\mathbf{r} \rho(\mathbf{r}) \rho'(\hat{R}\mathbf{r}) \right|^2$$

SOAP kernel: $K(R, R') \equiv K(\rho, \rho') = |\mathbf{p} \cdot \mathbf{p}'|^\xi$

is the power spectrum, rotationally invariant

Density projections

We can think of the soap vector \mathbf{p} as the rotational invariant combination of projections of $\rho(\mathbf{r})$ onto basis functions:

$$b_{\alpha}(\mathbf{r}) = g_n(r)Y_{lm}(\hat{\mathbf{r}}) \quad \alpha = [nlm]$$

(Ceriotti bra-ket notation: $\langle nlm | \chi^{(2)} \rangle$)

Many other representations: ACSF of Behler, MBTR of Rupp, von Lilienfeld's histograms, Mallat's scattering transforms, Bispectrum in SNAP (Thompson) are **all density projections** with some choice of basis and convolution kernel.

Which is the best basis to project onto?

Other nice properties of the Y_{lm} basis

Other nice properties of the Y_{lm} basis

- Natural extension to basis functions to expand non-scalar properties, vectors and tensors (Grisafi, Wilkins, GC, Ceriotti PRL 2018)

$$k^\lambda(\rho, \rho') = \int d\hat{R} \mathbf{D}^\lambda(\hat{R}) \left| \int d\mathbf{r} \rho(r) \rho'(\hat{R}\mathbf{r}) \right|^2 \quad \mathbf{D} : \text{Wigner matrix}$$

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- Further invariants (Ralf Drautz, 2019)

$$\begin{aligned} & \sum \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} c_{n_1 l_1 m_1} c_{n_2 l_2 m_2} c_{n_3 l_3 m_3} \\ & \sum \begin{pmatrix} l_1 & l_2 & l_3 & l_4 \\ m_1 & m_2 & m_3 & m_4 \end{pmatrix} c_{n_1 l_1 m_1} c_{n_2 l_2 m_2} c_{n_3 l_3 m_3} c_{n_4 l_4 m_4} \\ & \vdots \end{aligned}$$

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- Variant to describe entire structures, rather than atomic neighbourhoods:

$$\bar{c}_{nlm} = \sum_i c_{nlm}^{(i)} \quad \text{average over all atoms, no cutoff}$$

$$\bar{P}_{nn'l} = \sum_m \bar{c}_{nlm}^\dagger \bar{c}_{nlm} \quad \text{or cutoff} > \text{periodic unit cell}$$

(Other ways to construct kernels for entire structures: De, Bartók, GC, Ceriotti PCCP 2016)

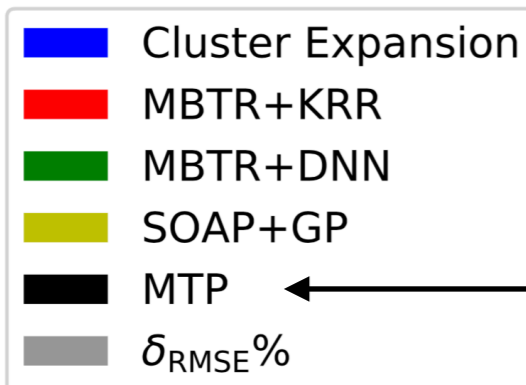
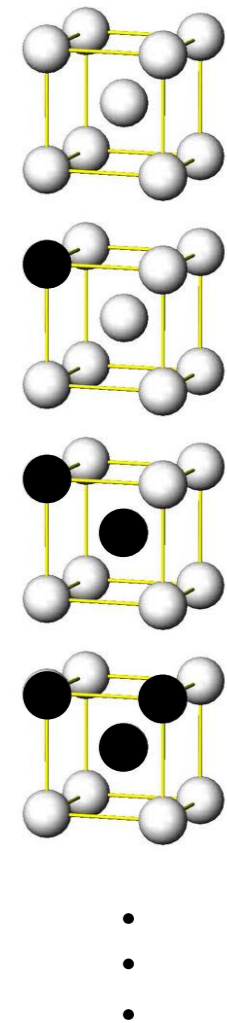
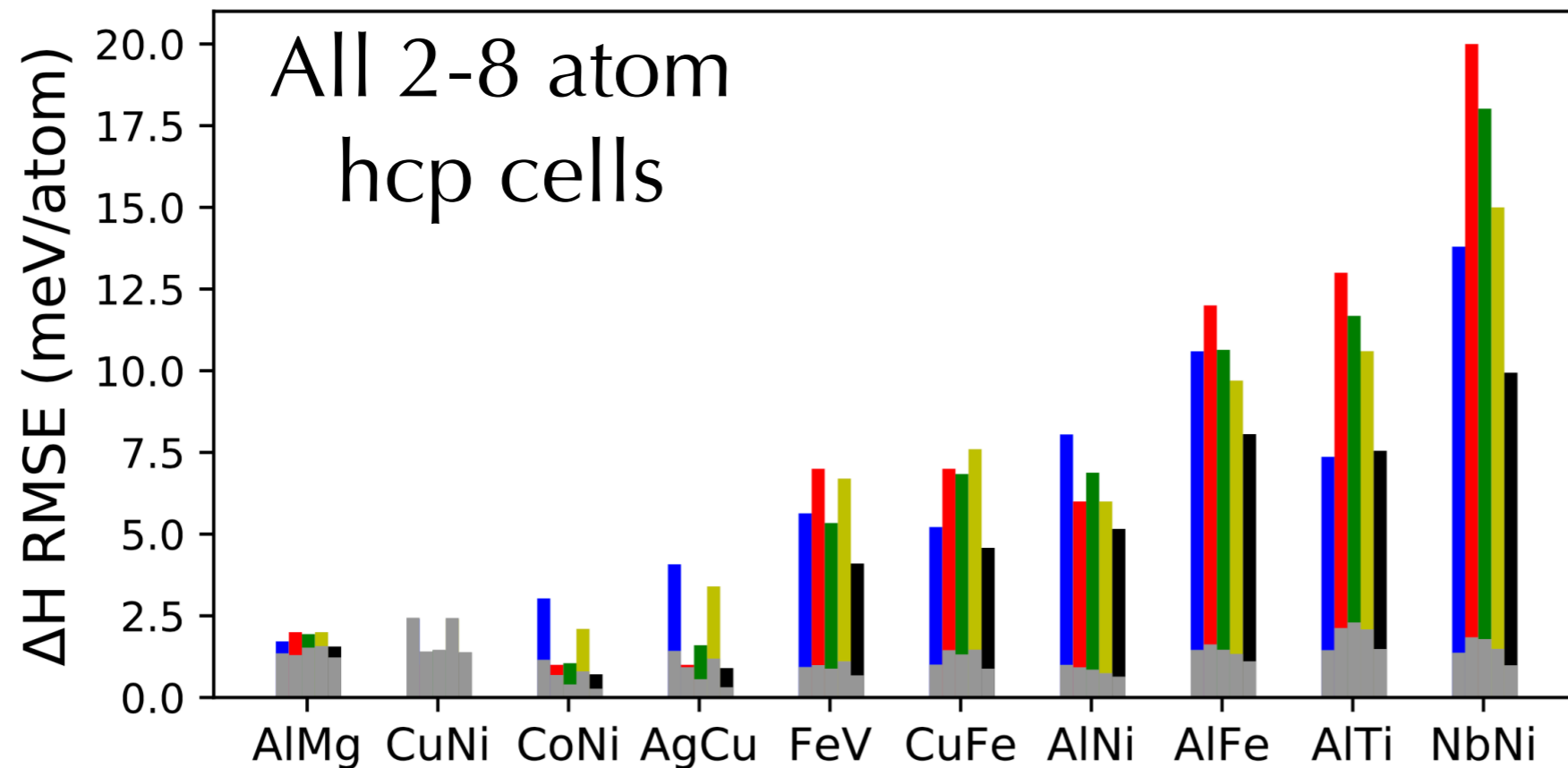
Method matrix

	SOAP/GAP	SNAP	BPNN	MTP	Drautz
Density	Smoothed	$\delta(\mathbf{r})$	$\delta(\mathbf{r})$	$\delta(\mathbf{r})$	$\delta(\mathbf{r})$
Basis set	Complete	Complete	Manual	Manual	Complete
Dimension	High	Medium	Medium	Low	Low
Regularised	Yes	No	Yes	No	Yes
Regression	Kernel	Linear	NN	Linear	Linear

Benchmarking

Metal alloys (with Gus Hart's group)

- For configurational prediction, Cluster Expansion is the successful method, meV accuracy
- Off-lattice is a problem, many components are difficult



Alex Shapeev's Moment Tensor Potentials
(body order expansion+polynomial fit)

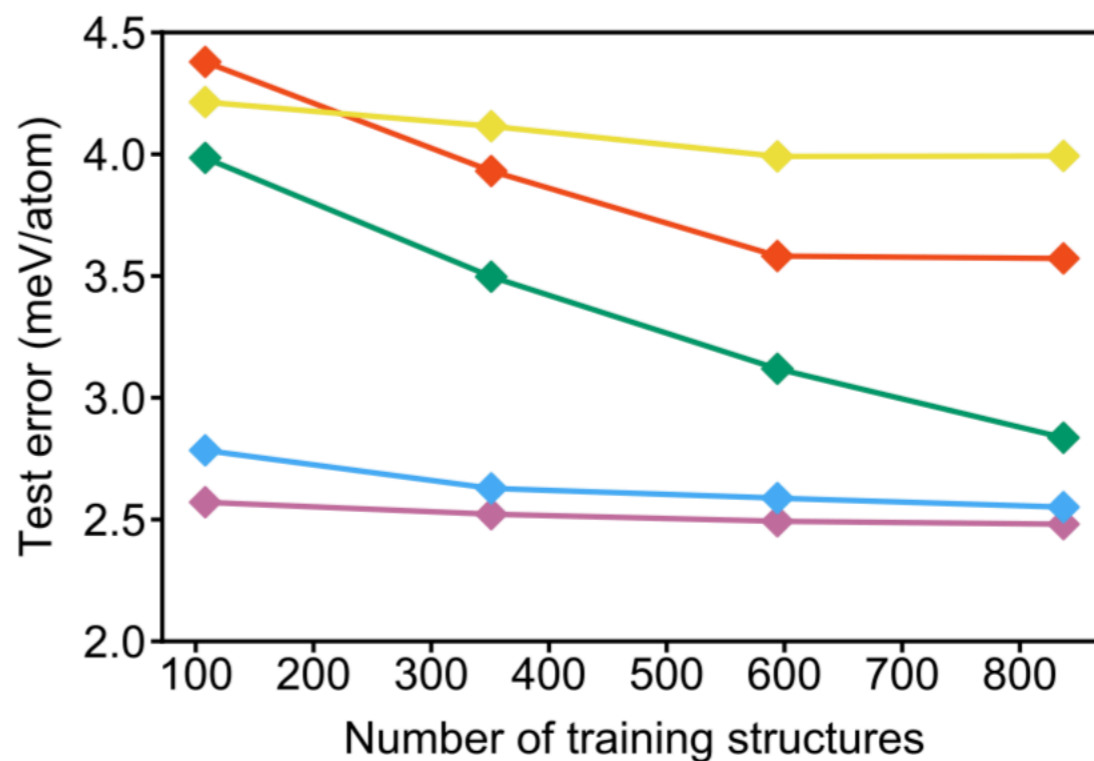
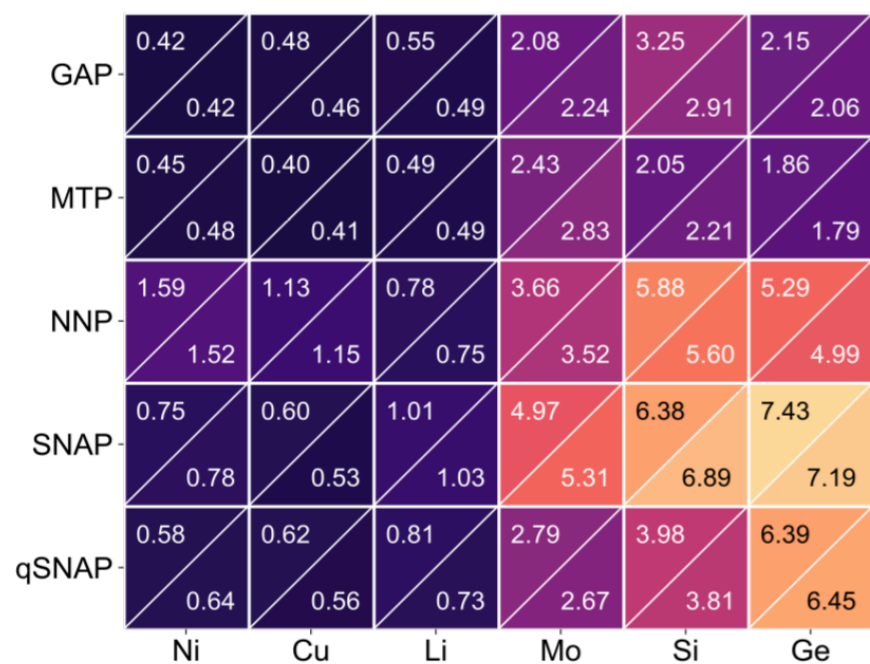
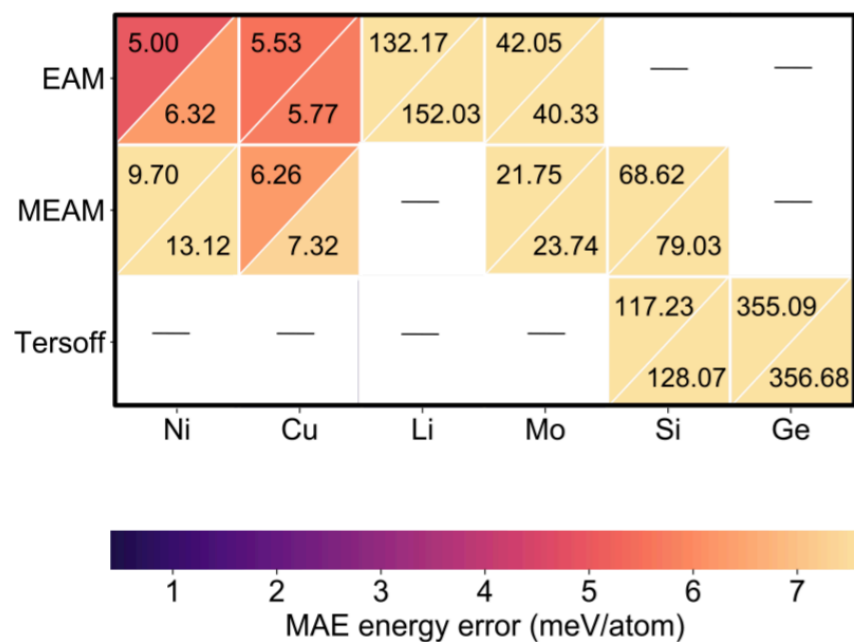


Figure 4: MAE in predicted (a) energies (



(a) Mean absolute errors in predicted energies

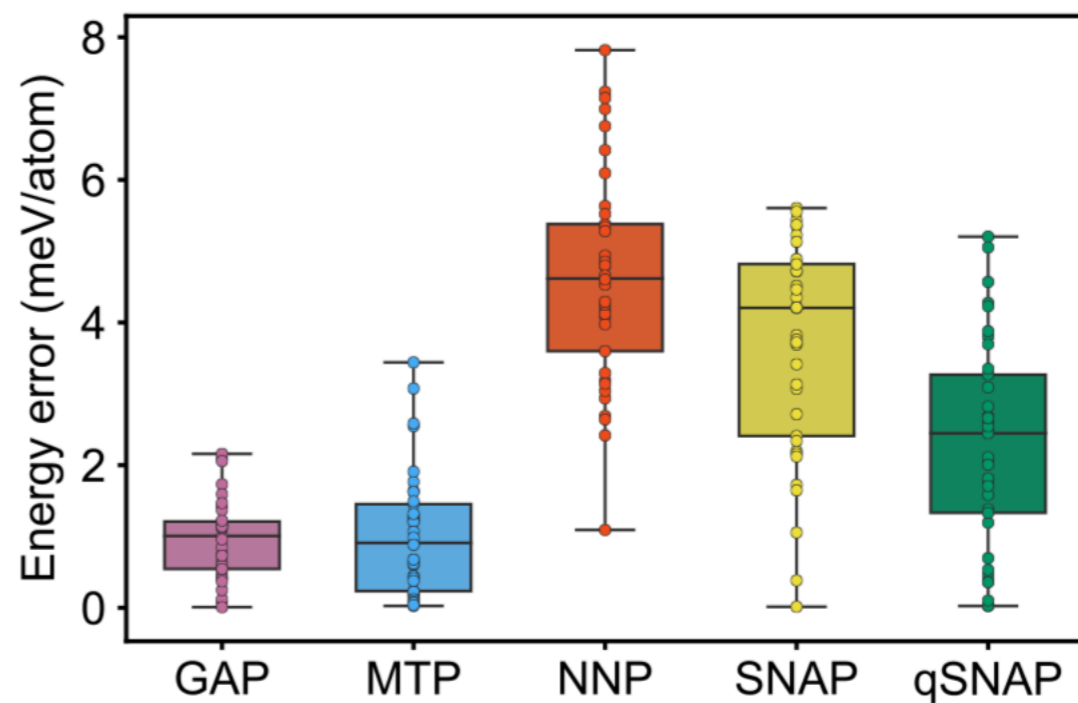
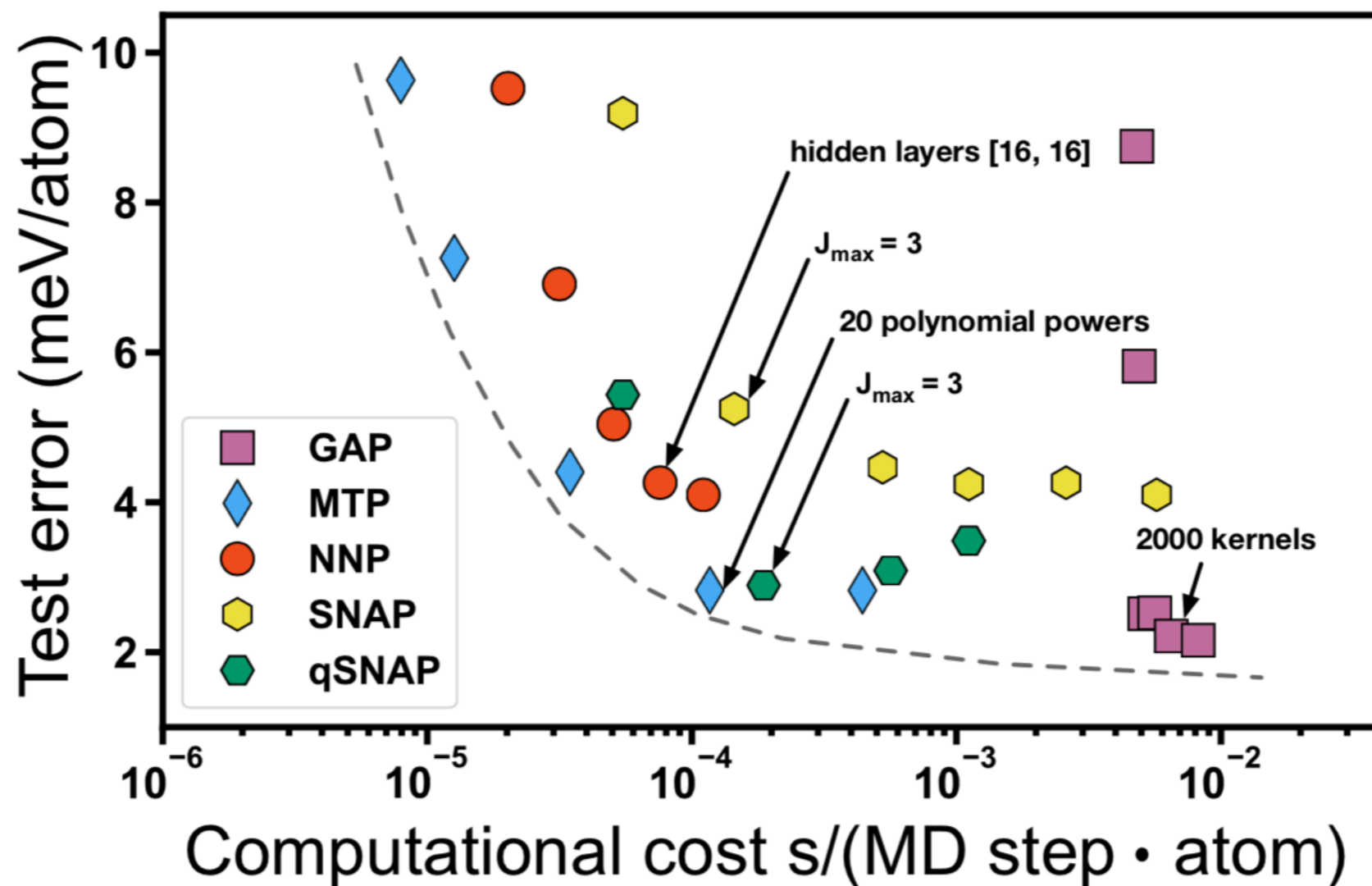


Figure 6: Error distributions in (a) predicted dynamics

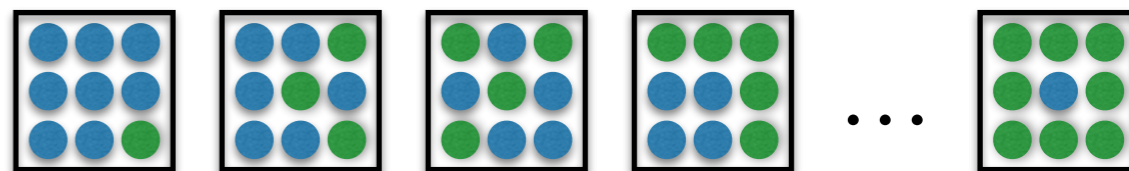
Performance

Several are in LAMMPS now :

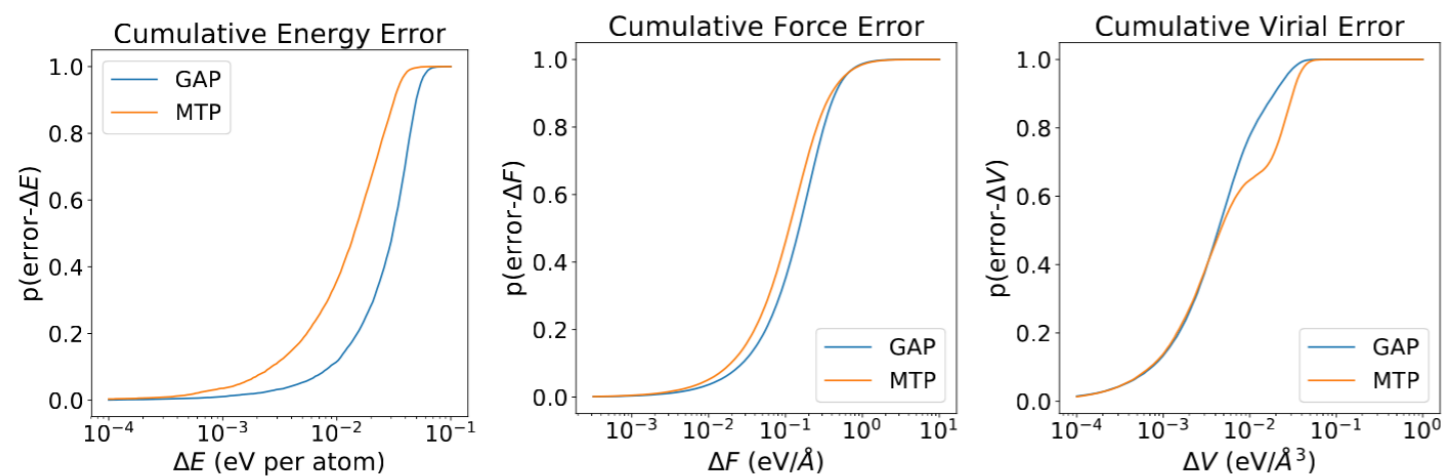


Extrapolation: SOAP/GAP vs polynomials (MTP)

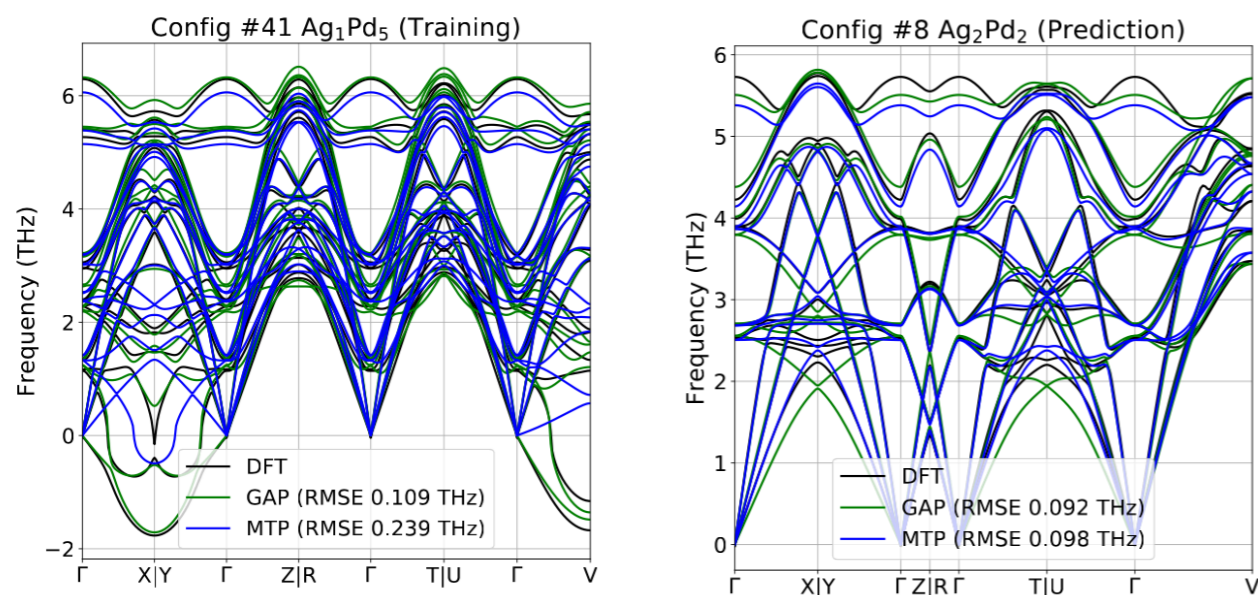
AgPd bcc crystal dataset (actively learned)



Liquid test

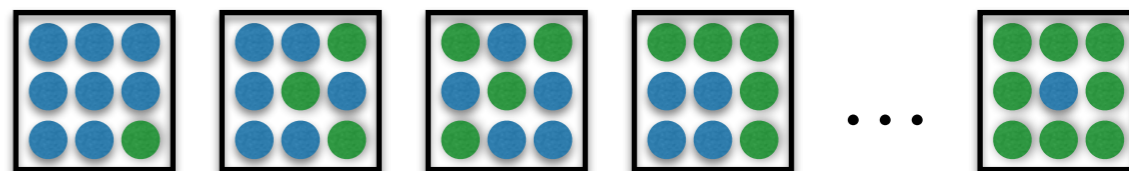


Phonon spectra

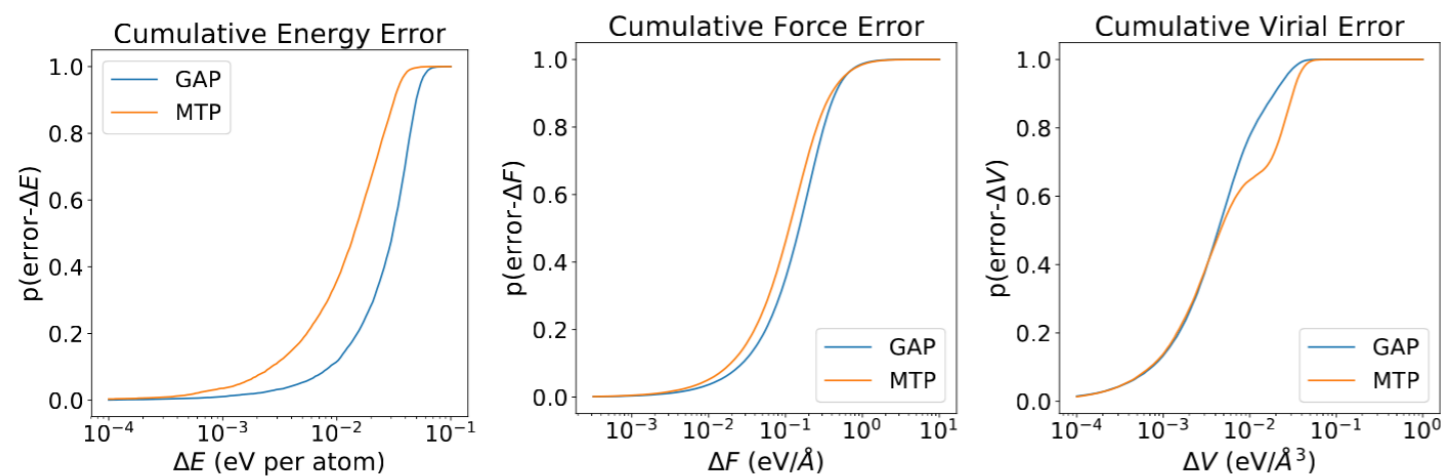


Extrapolation: SOAP/GAP vs polynomials (MTP)

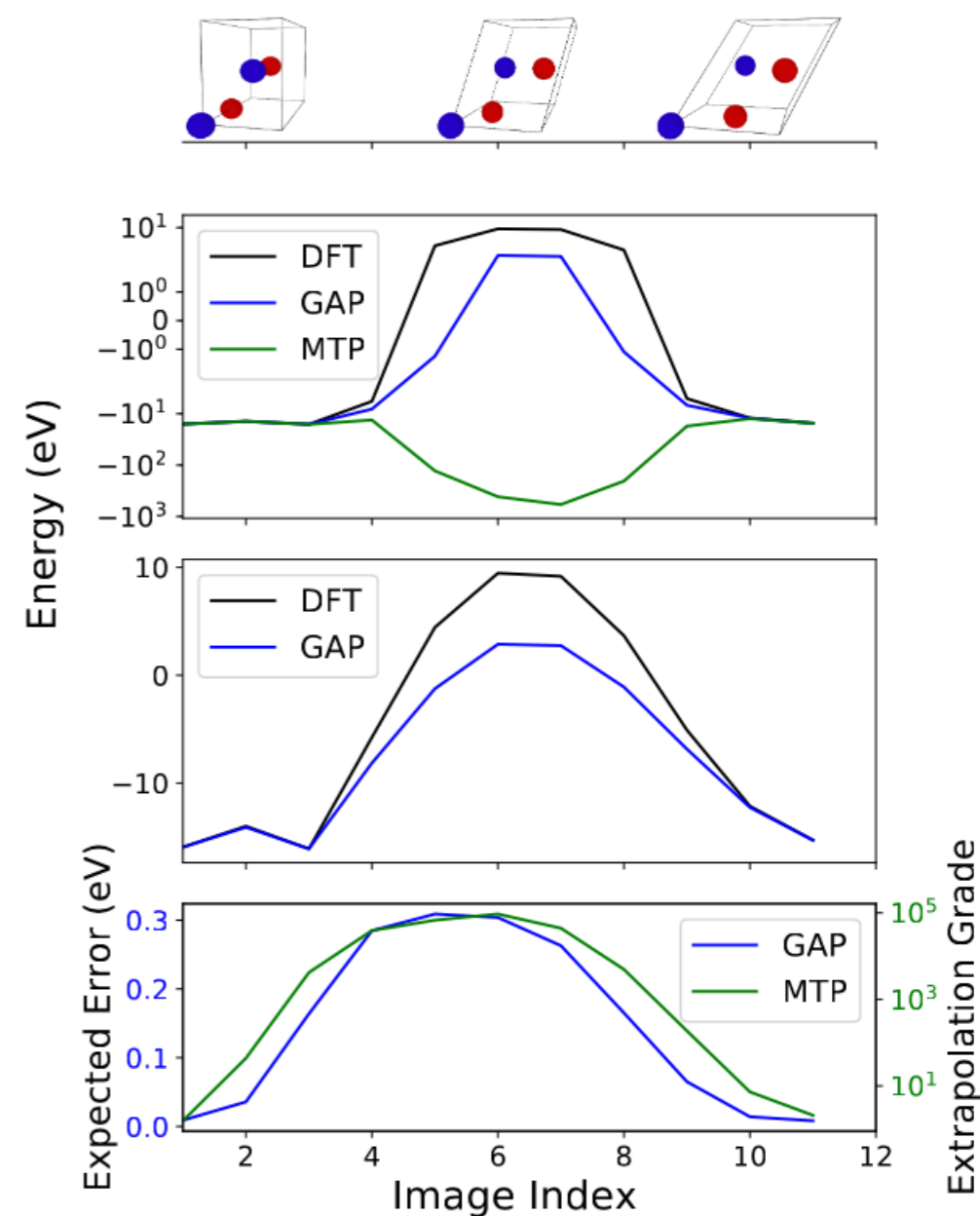
AgPd bcc crystal dataset (actively learned)



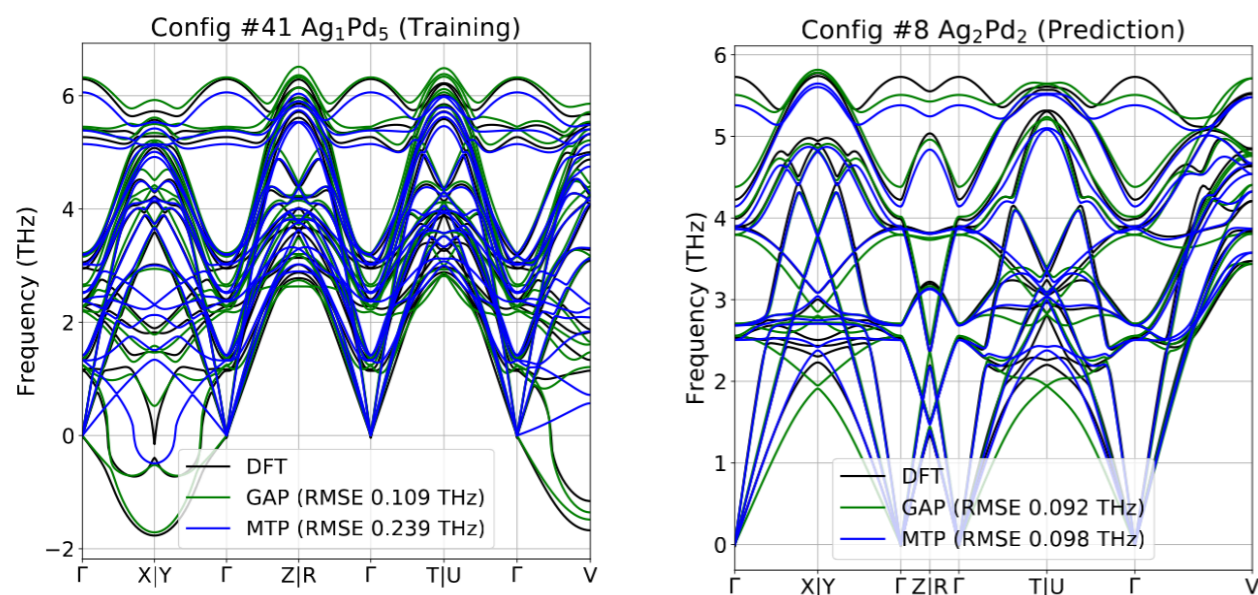
Liquid test



Transition Pathway

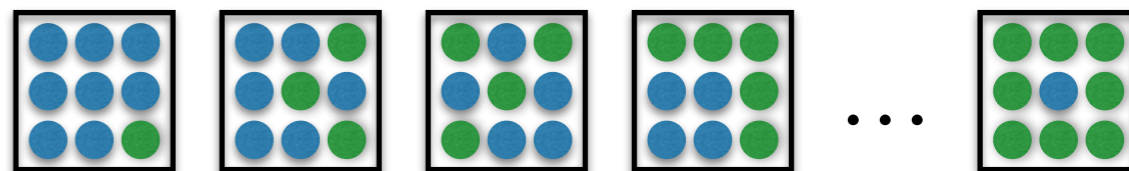


Phonon spectra

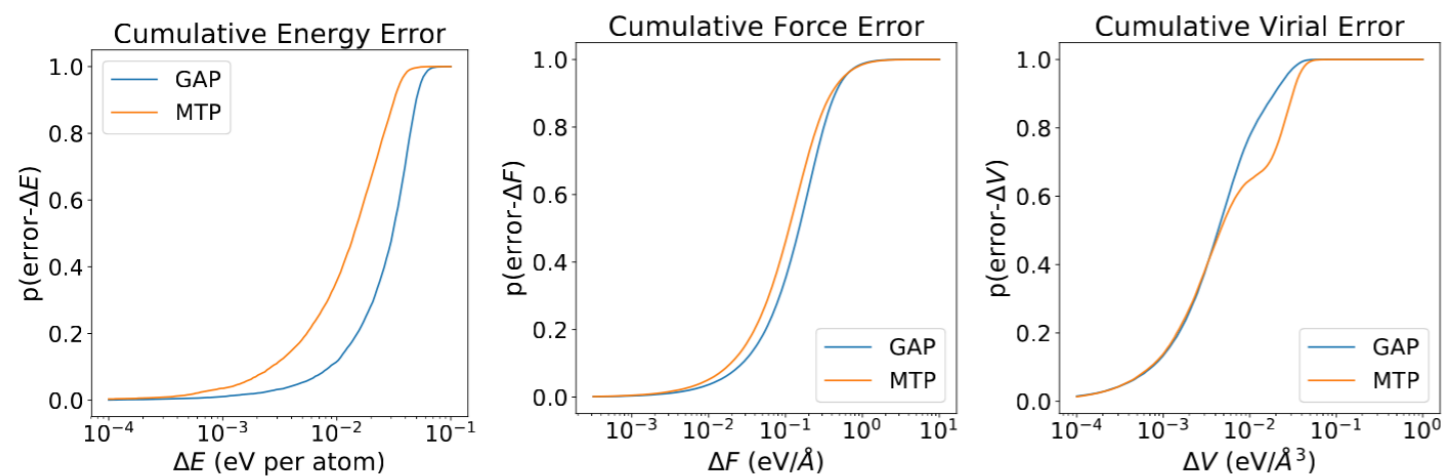


Extrapolation: SOAP/GAP vs polynomials (MTP)

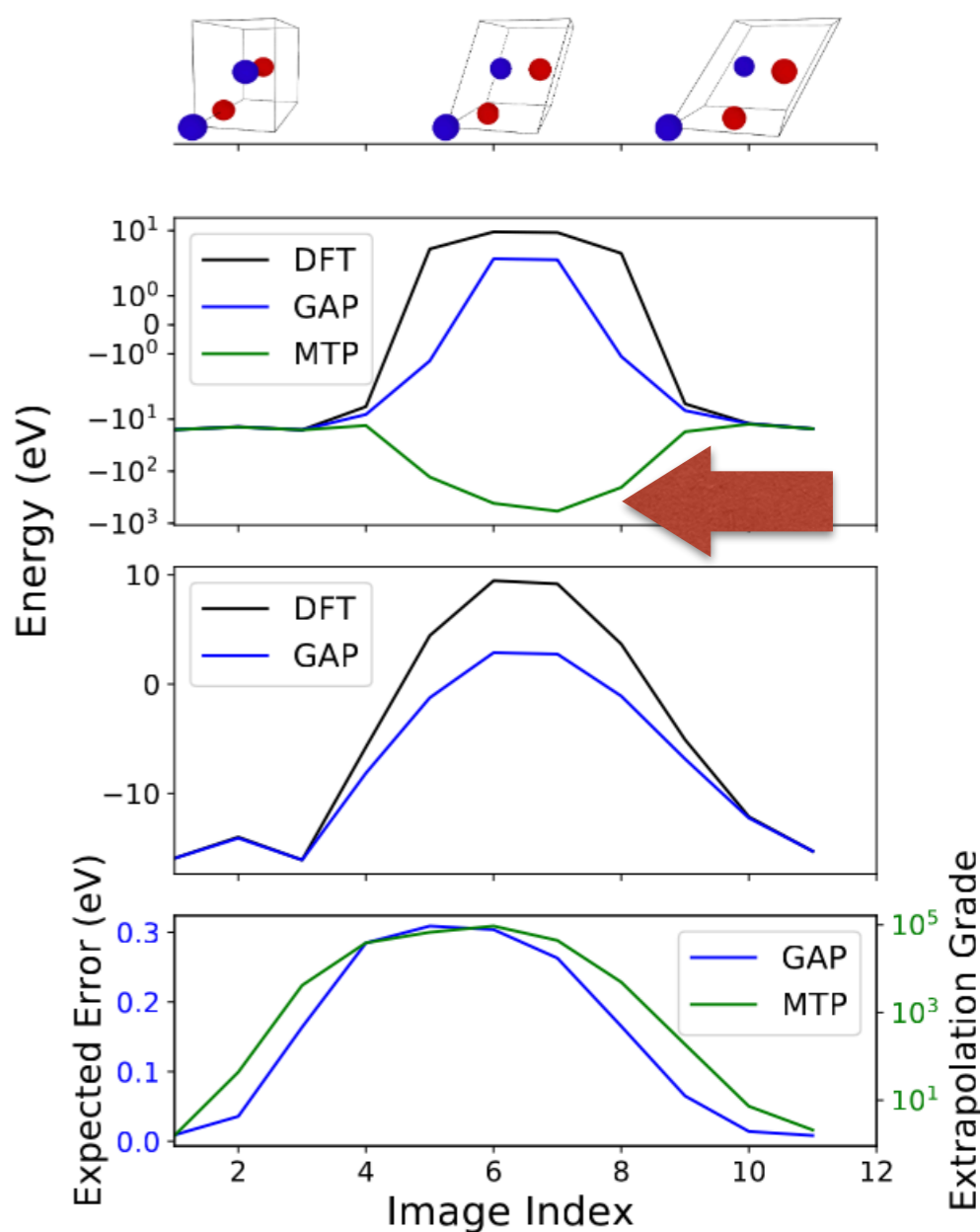
AgPd bcc crystal dataset (actively learned)



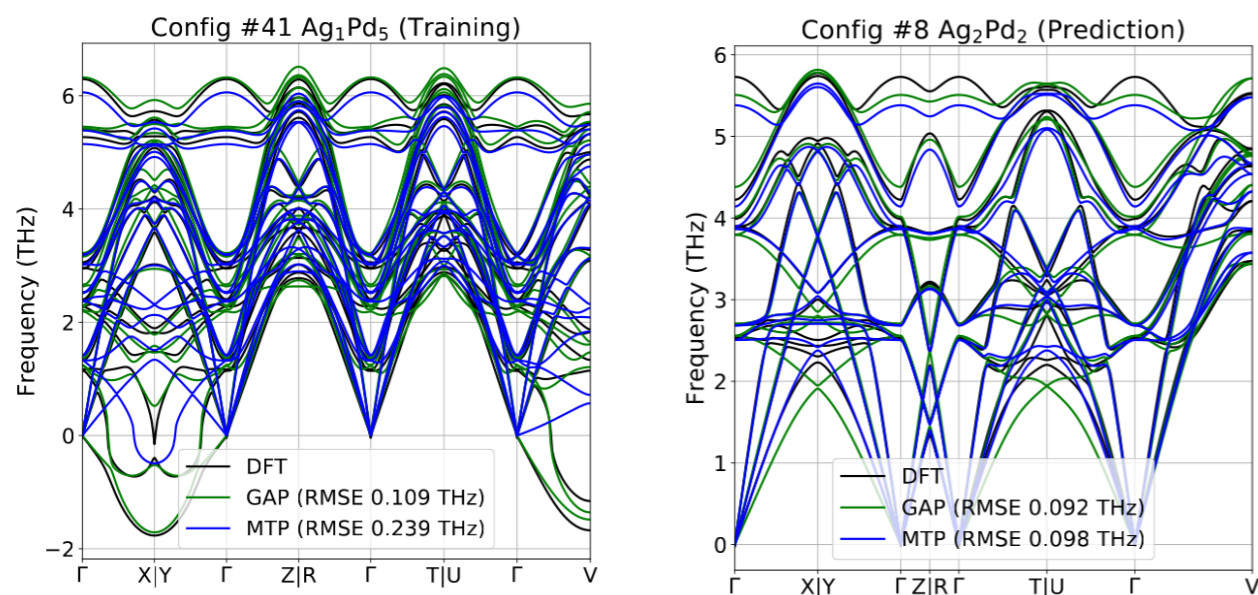
Liquid test



Transition Pathway



Phonon spectra



Applications

Successful *applications* to hard materials - 2018

With Volker Deringer
(now at Oxford)



Successful *applications* to hard materials - 2018

With Volker Deringer
(now at Oxford)



C



Structure and growth
mechanism of
amorphous carbon
films

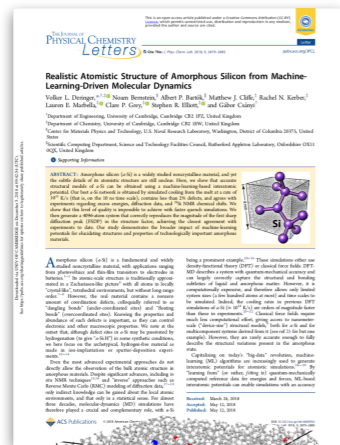
Successful *applications* to hard materials - 2018

With Volker Deringer
(now at Oxford)



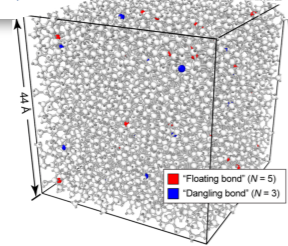
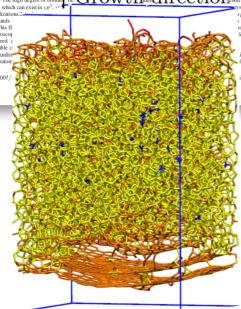
C

Si



Structure and growth mechanism of amorphous carbon films

Realistic structure of slow-quenched amorphous silicon



Successful applications to hard materials - 2018

With Volker Deringer
(now at Oxford)



C

Si

B

Growth Mechanism and Origin of High β Content in Tetrahedral Amorphous Carbon
Miguel A. Caro,^{1,2} Volker Deringer,^{1,2} Tom Lazar,^{1,2} and Gilbert Chabot¹
¹Department of Applied Physics and Materials, York University, Toronto, Ontario M3J 1P3, Canada
²Department of Chemistry, University of Cambridge, Lensfield Road, Cambridge CB2 3RQ, United Kingdom
³Department of Chemistry and Materials Science, York University, Toronto, Ontario M3J 1P3, Canada

Abstract
We study the kinetics of tetrahedral amorphous carbon (ta-C) film growth from molecular dynamics simulations based on a machine-learning potential based on density functional theory data. We find that the rate of growth is highly dependent on the structure of the ta-C network, which is in turn dependent on the growth mechanism. High β content is observed in the ta-C network, which is a consequence of the growth mechanism. We show that the growth mechanism is highly dependent on the growth rate, and that the growth mechanism is highly dependent on the growth rate. We show that the growth mechanism is highly dependent on the growth rate, and that the growth mechanism is highly dependent on the growth rate.

Realistic Atomic Structure of Amorphous Silicon from Machine-Learning-Driven Molecular Dynamics
Volker Deringer,^{1,2} Tom Lazar,^{1,2} Miguel A. Caro,^{1,2} and Gilbert Chabot¹
¹Department of Applied Physics and Materials, York University, Toronto, Ontario M3J 1P3, Canada
²Department of Chemistry, University of Cambridge, Lensfield Road, Cambridge CB2 3RQ, United Kingdom
³Department of Chemistry and Materials Science, York University, Toronto, Ontario M3J 1P3, Canada

Abstract
Amorphous silicon (a-Si) is a widely used semiconductor material, and its atomic structure is of great interest for both fundamental and applied research. In this work, we present a realistic atomic structure of a-Si obtained from machine-learning-driven molecular dynamics simulations. The structure is compared to experimental data and to previous theoretical models. We show that the machine-learning-driven structure is in excellent agreement with experimental data and provides a more realistic picture of the atomic structure of a-Si than previous theoretical models.

Data-Driven Learning of Total and Local Energies in Elemental Boron
Volker Deringer,^{1,2} Chao J. Pickard,^{1,2} and Gilbert Chabot¹
¹Department of Applied Physics and Materials, York University, Toronto, Ontario M3J 1P3, Canada
²Department of Chemistry, University of Cambridge, Lensfield Road, Cambridge CB2 3RQ, United Kingdom
³Department of Chemistry and Materials Science, York University, Toronto, Ontario M3J 1P3, Canada

Abstract
Elemental boron presents a number of complex crystal structures, and its atomic structure is of great interest for both fundamental and applied research. In this work, we present a data-driven learning of the total and local energies in elemental boron. The structure is compared to experimental data and to previous theoretical models. We show that the machine-learning-driven structure is in excellent agreement with experimental data and provides a more realistic picture of the atomic structure of boron than previous theoretical models.

Structure and growth mechanism of amorphous carbon films

First potential for boron - the search for the elusive β phase

Realistic structure of slow-quenched amorphous silicon

Successful applications to hard materials - 2018

With Volker Deringer
(now at Oxford)



C

Si

B

P

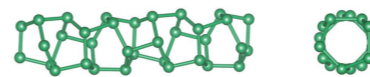


Structure and growth mechanism of amorphous carbon films

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Crystal structure search, fibrous phosphorous



Successful applications to hard materials - 2018

With Volker Deringer
(now at Oxford)



C

Si

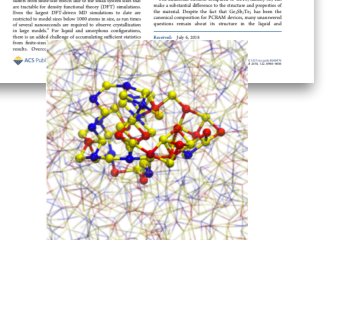
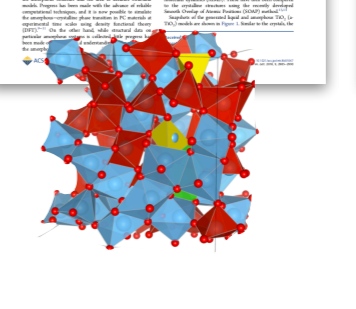
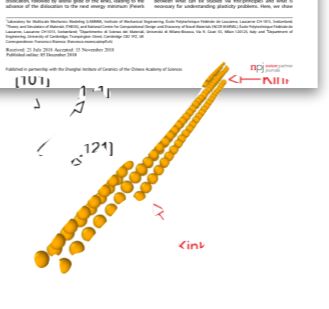
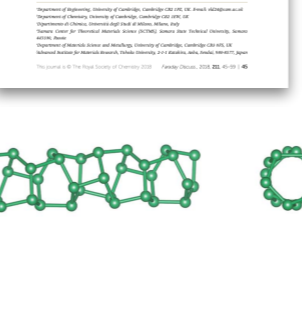
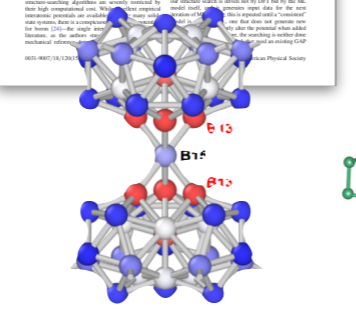
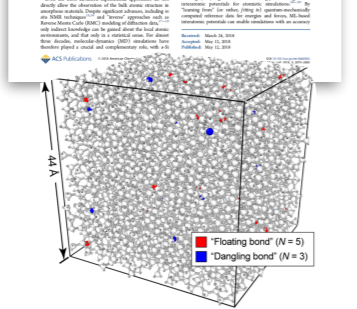
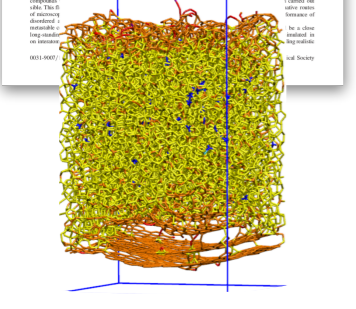
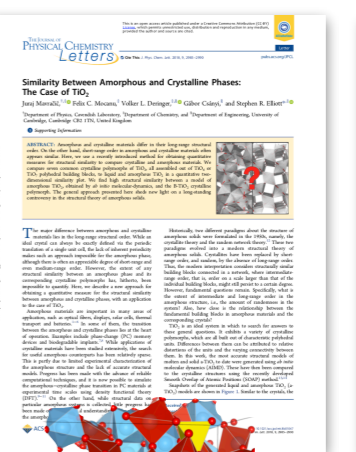
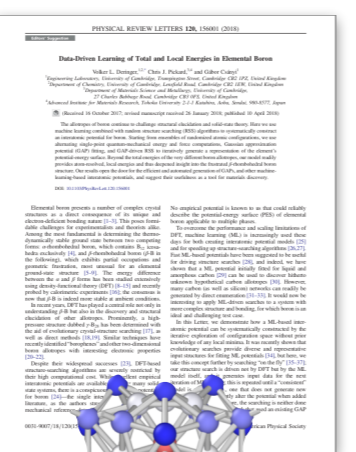
B

P

Fe

TiO₂

GeSbTe



Structure and growth mechanism of amorphous carbon films

First potential for boron - the search for the elusive β phase

Screw dislocation glide in bcc iron

Medium-range order in the phase change material GST

Realistic structure of slow-quenched amorphous silicon

Crystal structure search, fibrous phosphorous

Crystalline and amorphous phases of titanium oxide

The Silicon Challenge

PHYSICAL REVIEW X

Highlights Recent Subjects Accepted Collections Authors Referees Search

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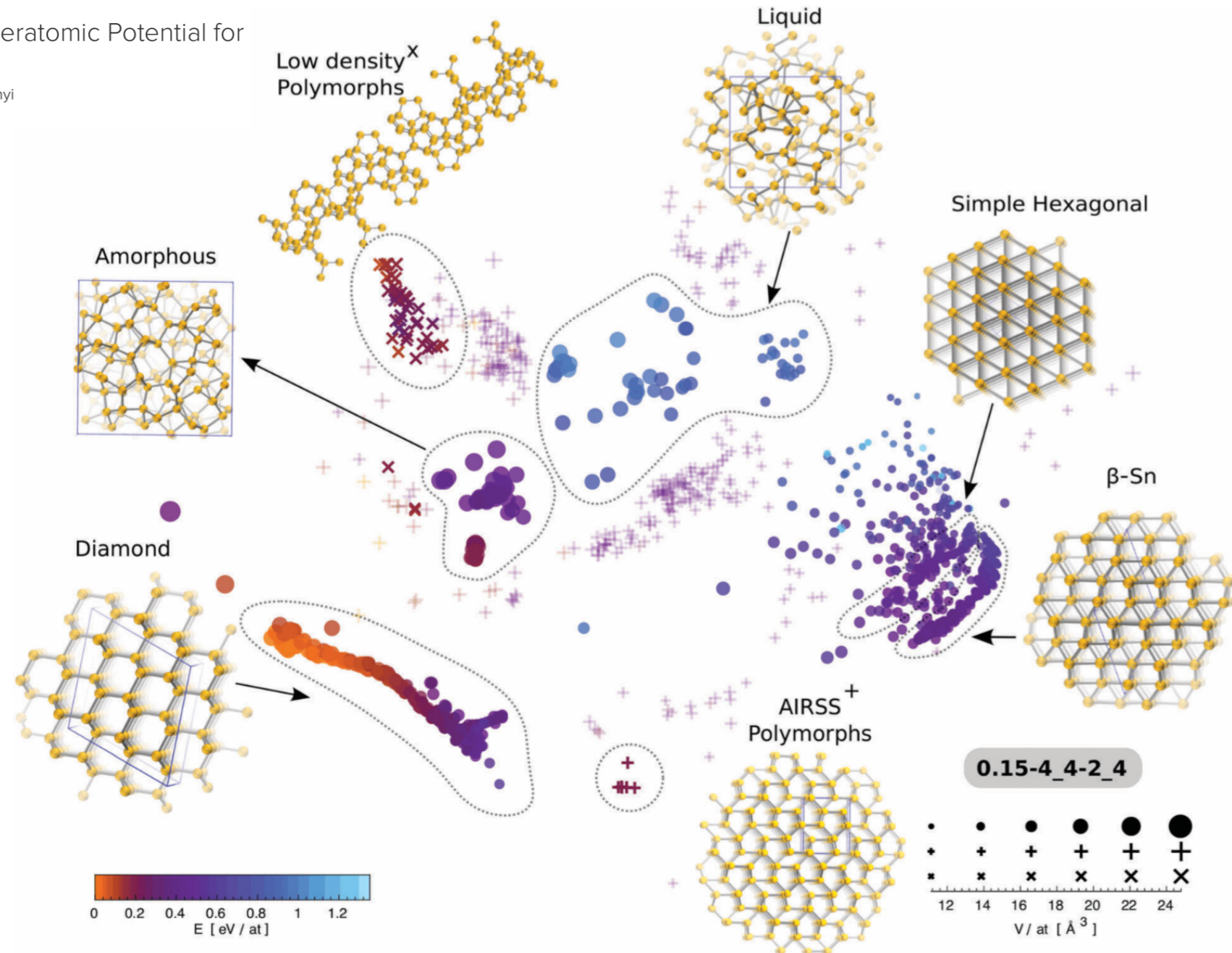
Cite this: *Phys. Chem. Chem. Phys.*, 2016, 18, 13754

Comparing molecules and solids across structural and alchemical space†

Sandip De,^{ab} Albert P. Bartók,^c Gábor Csányi^c and Michele Ceriotti^{*ab}

Machine Learning a General-Purpose Interatomic Potential for Silicon

Albert P. Bartók, James Kermode, Noam Bernstein, and Gábor Csányi
Phys. Rev. X 8, 041048 – Published 14 December 2018

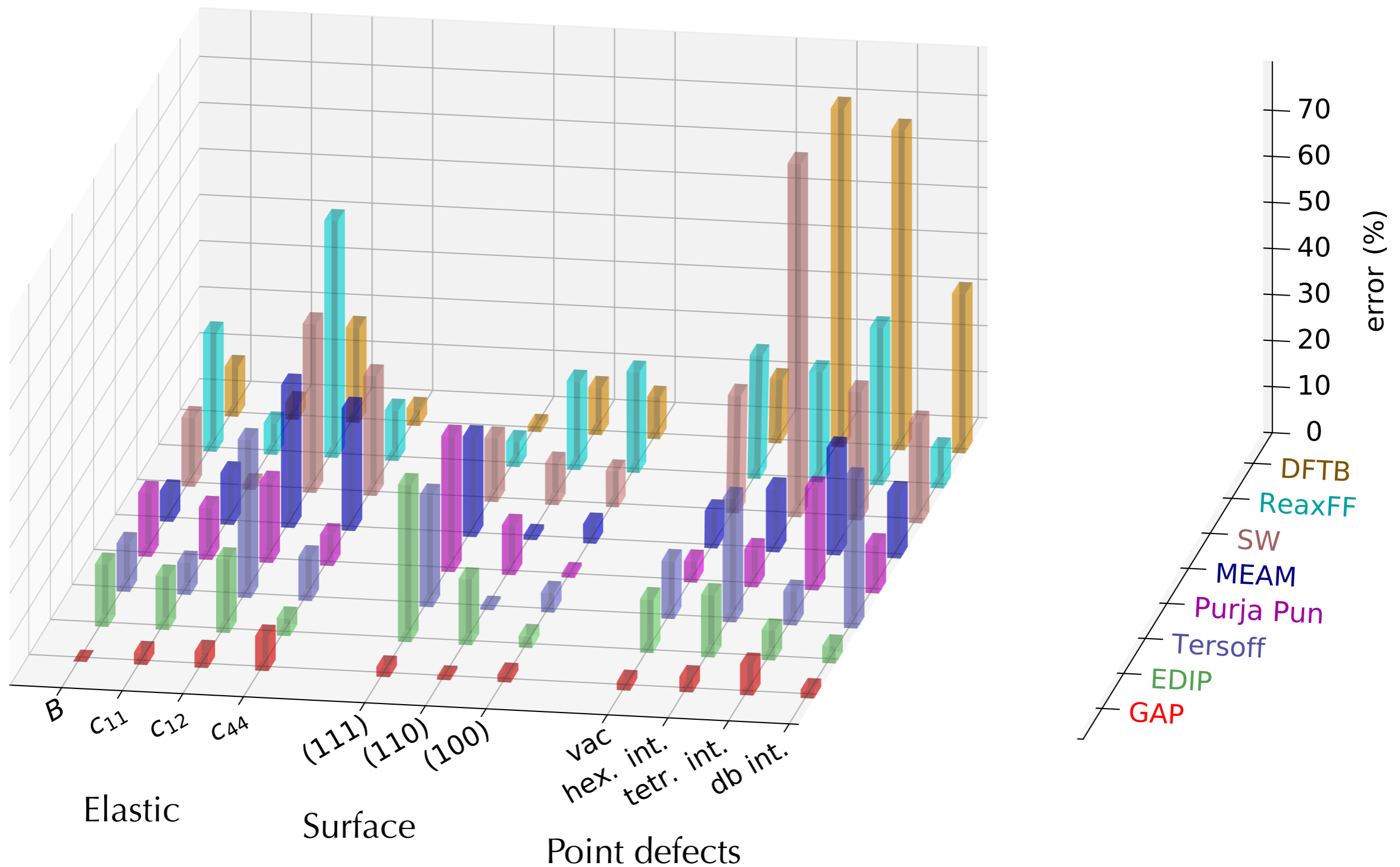


Can we cover **all** relevant configurations?

The monster database

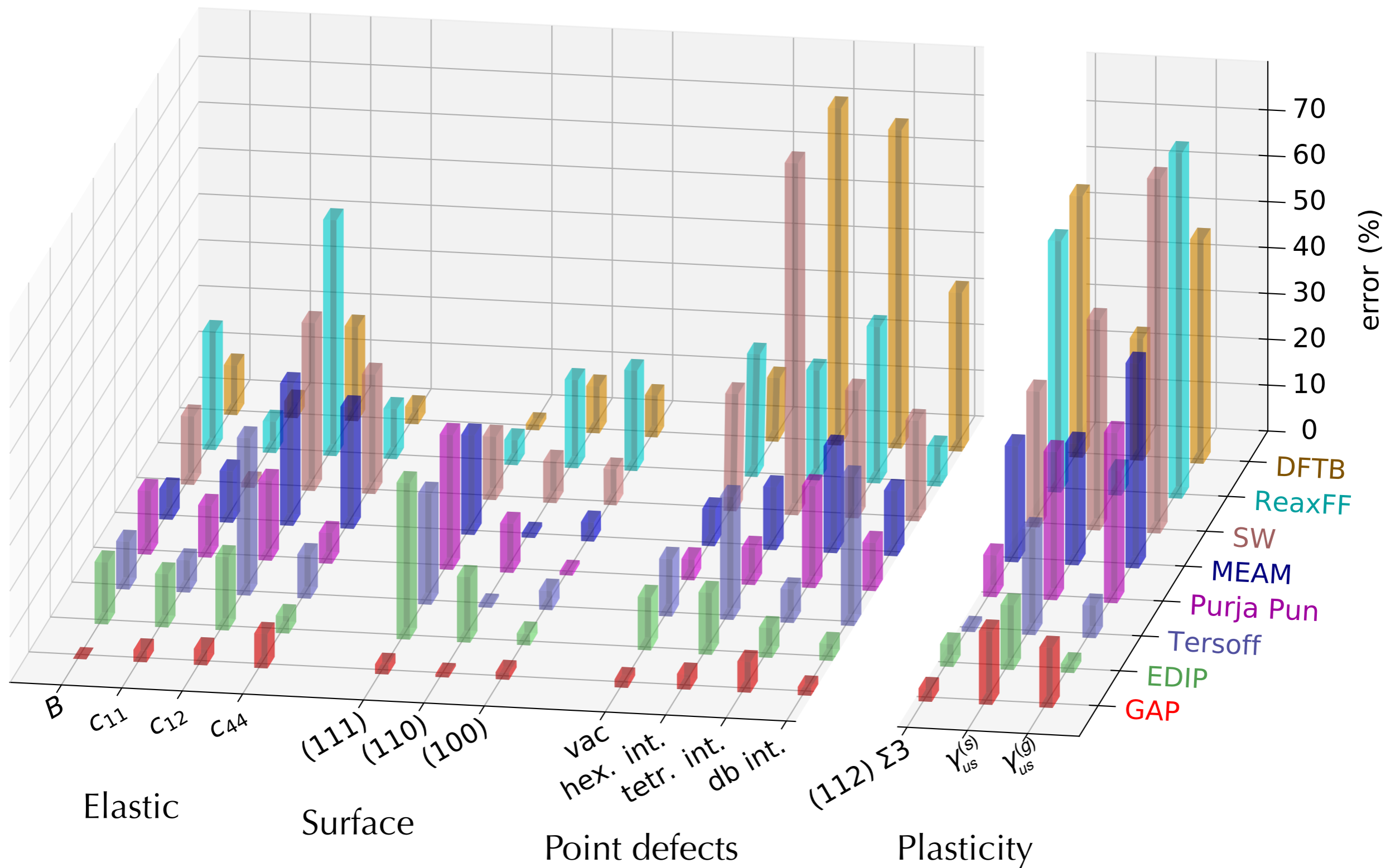
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isolated atom	1	1	1	1			
diamond	2	104	208	6			
	16	220	3520	53			
	54	110	5940	58			
	128	55	7040	92			
β -Sn	2	60	120	32			
	16	220	3520	51			
	54	110	5940	66			
	128	55	7040	157			
simple hexagonal	1	110	110	13			
	8	30	240	15			
	27	30	810	42			
	64	53	3392	89			
bcc	2	49	392	40			
bc8	8	49	88	66			
fcc	4	49	196	46			
hcp	2	49	88	28			
st12	12	49	588	94			
liquid	64	69	4416	1114	0.003	0.15	0.2
	128	7	896	323			
amorphous	64	31	1984	231	0.01	0.2	0.4
	216	128	27648	1719			
diamond surface (001) decohesion	144	29	4176	514			
	32	11	352	28			
diamond surface (110) decohesion	108	26	2808	338			
	16	11	176	8			
diamond surface (111) decohesion	24	11	264	10			
unreconstructed	96	47	4512	573			
adatom	146	11	1606	62			
Pandey reconstruction	96	50	4800	632			
DAS 3x3 unrelaxed	52	1	52	6			
diamond vacancy	63	100	6300	168			
	215	111	23865	405			
diamond divacancy	214	78	16692	416			
diamond interstitial	217	115	24955	605			
small (110) crack tip	200	7	1400	130			
small (111) crack tip	192	10	1920	185			
screw dislocation core	144	19	2736	124			
sp ² bonded	8	51	408	61			
sp bonded	4	100	400	392	0.01	0.2	0.4
Total							

Summary of material properties



Properties **targeted** by the training database

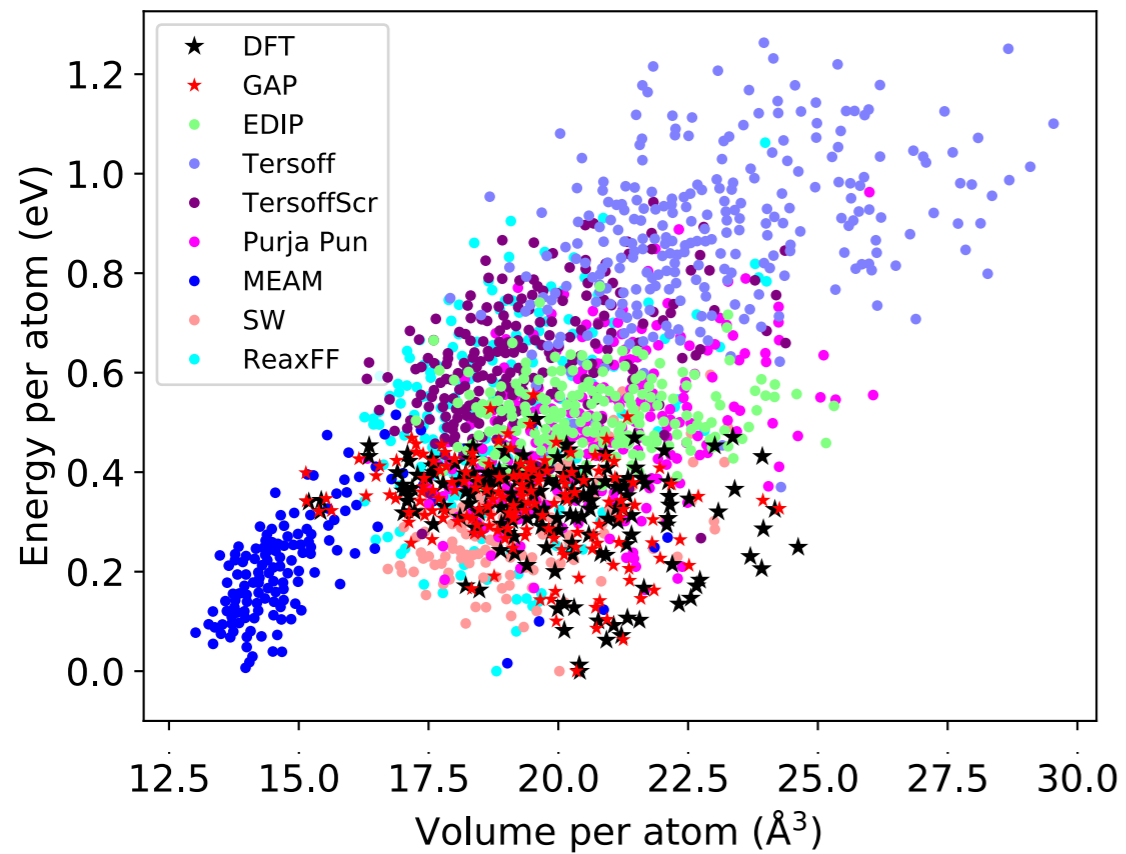
Summary of material properties



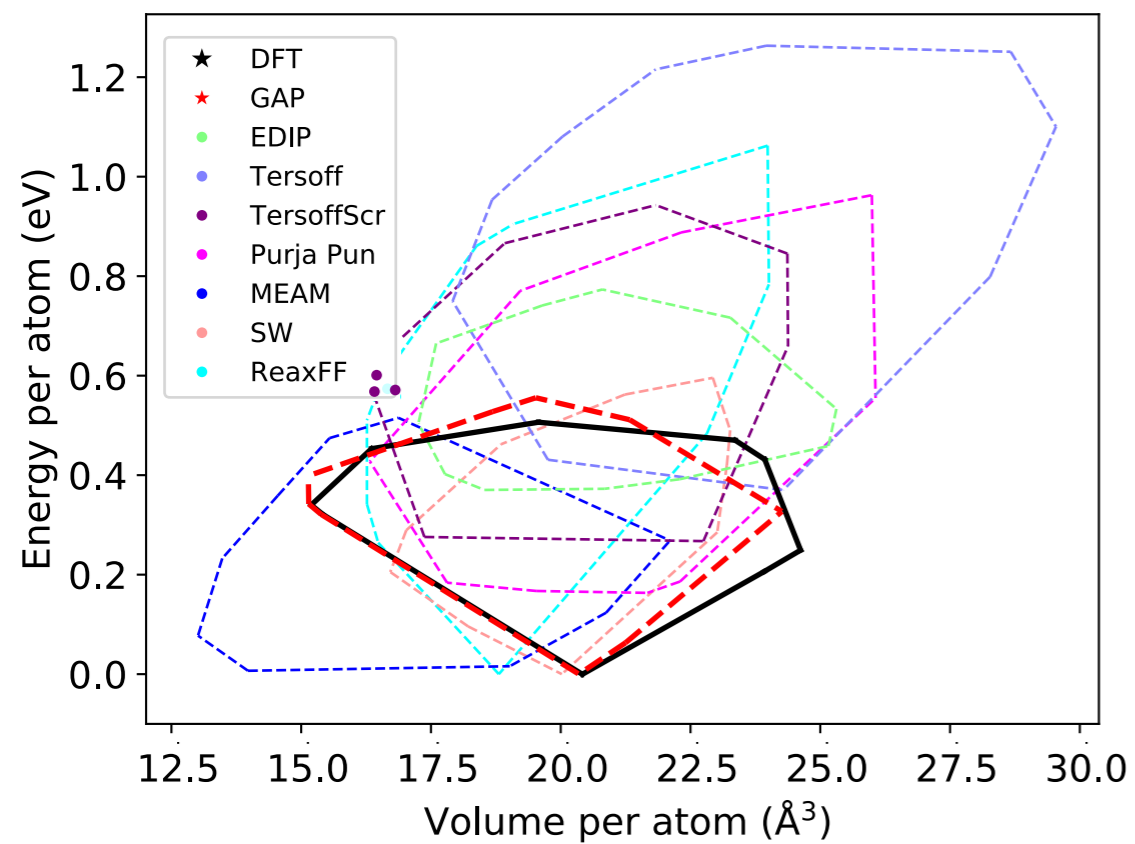
Properties **targeted** by the training database

Untargeted properties

Stringent tests: crystal structure search and phases

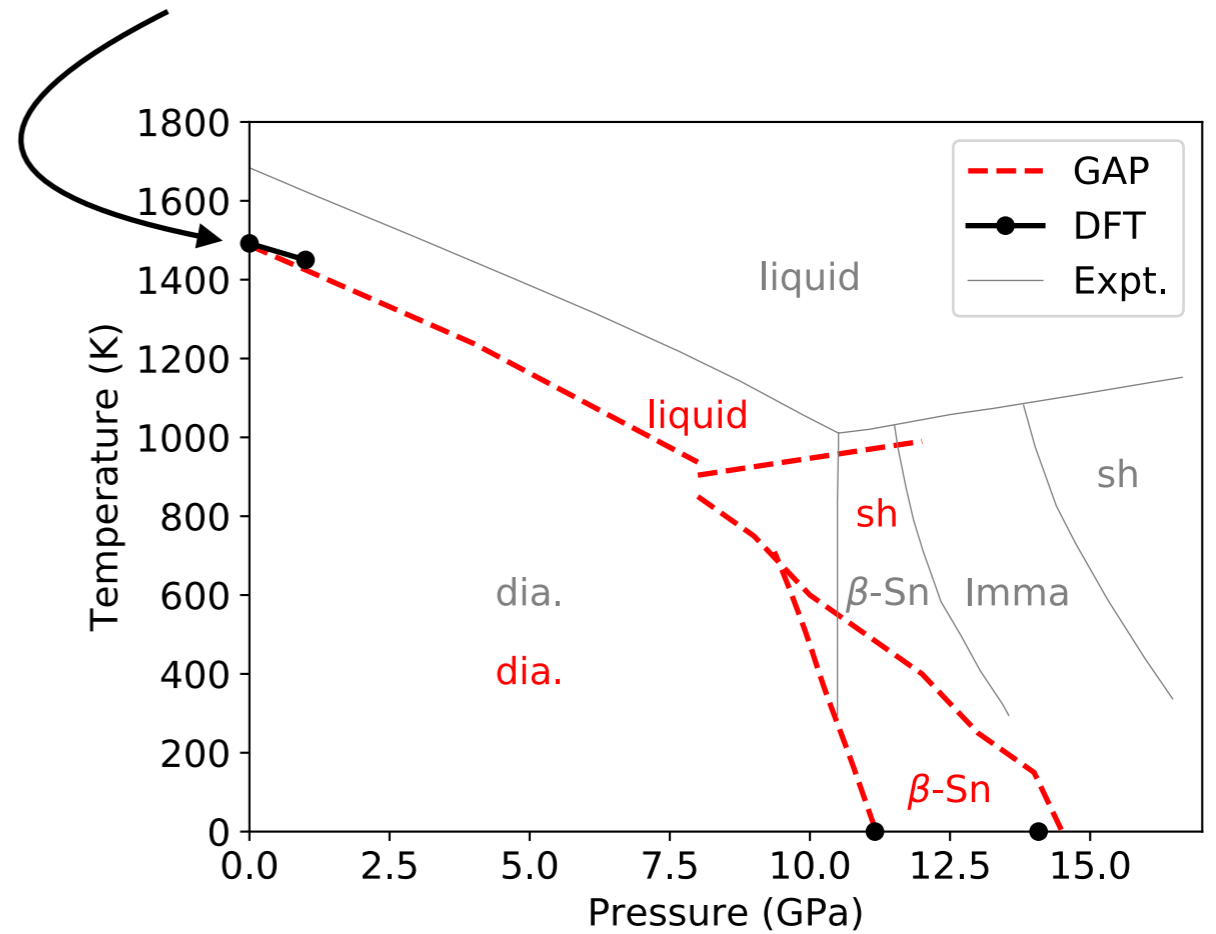
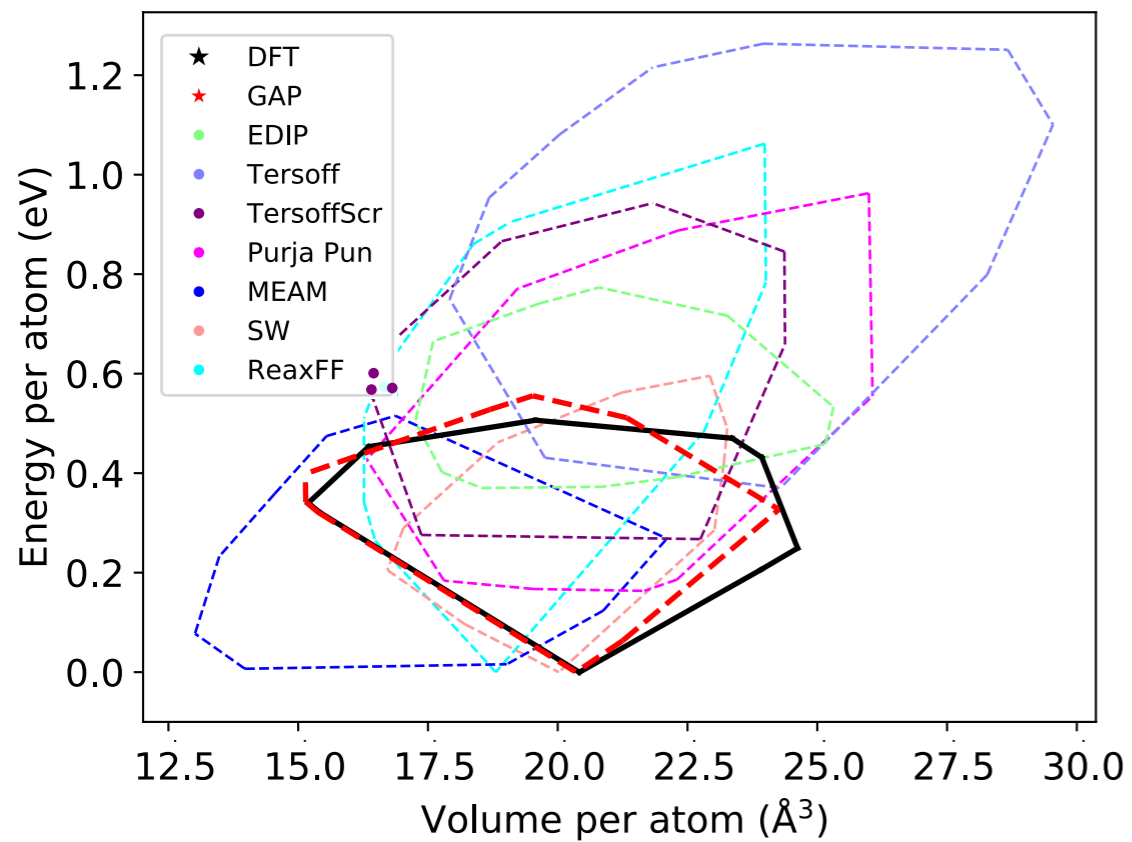


Stringent tests: crystal structure search and phases



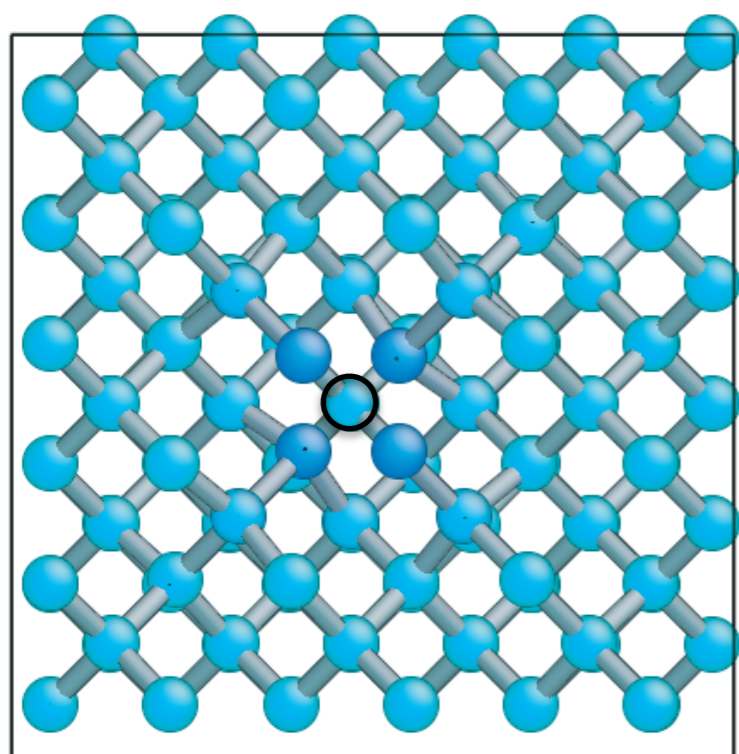
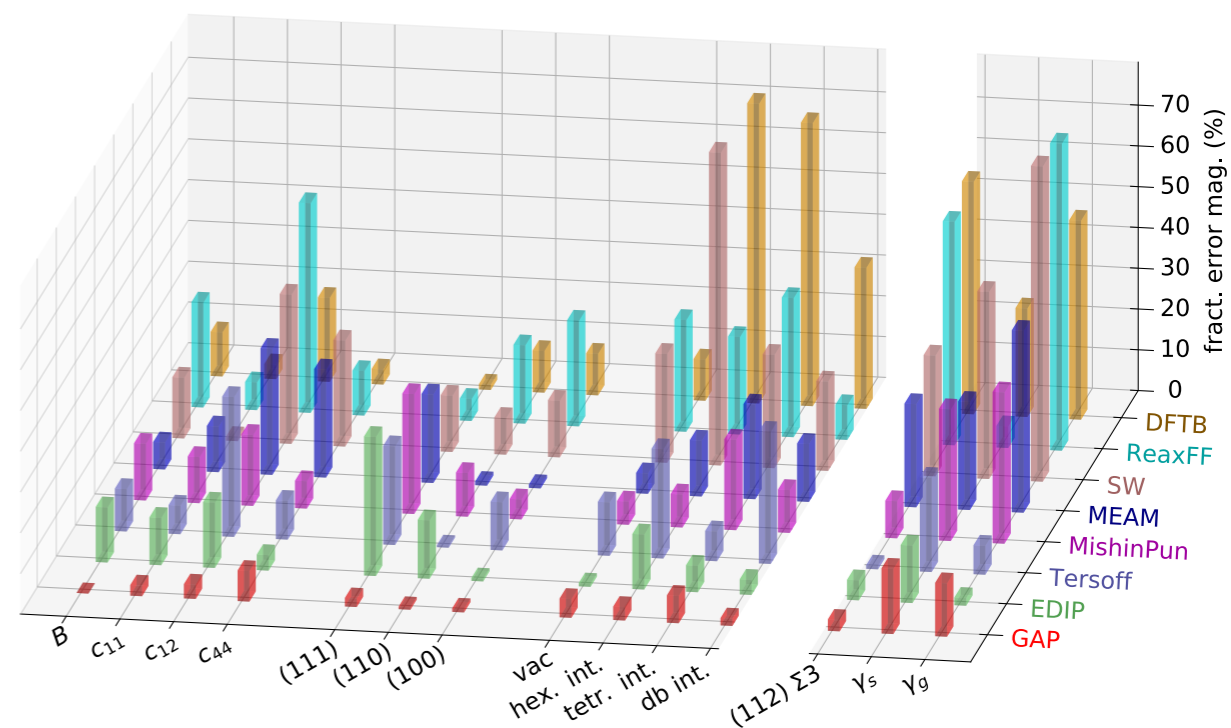
Stringent tests: crystal structure search and phases

DFT : Many months on a supercomputer



Predicted errors from equivalent Gaussian Process

Predicted error² \sim Variance of GP



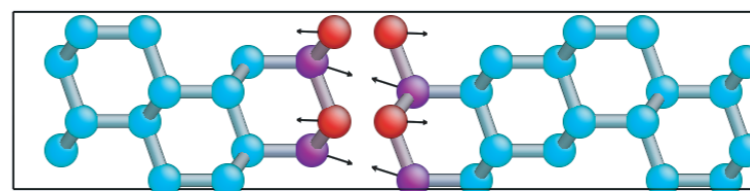
Vacancy

> 0.005 eV

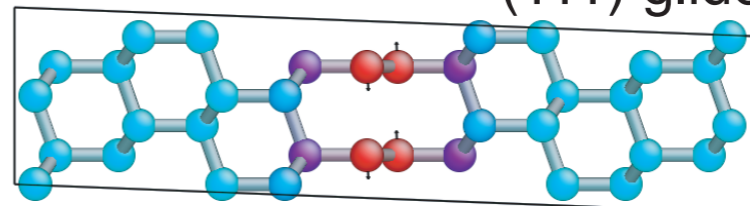


< 0.001 eV

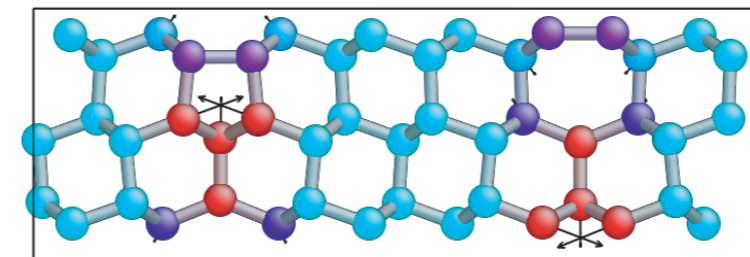
(111) shuffle



(111) glide



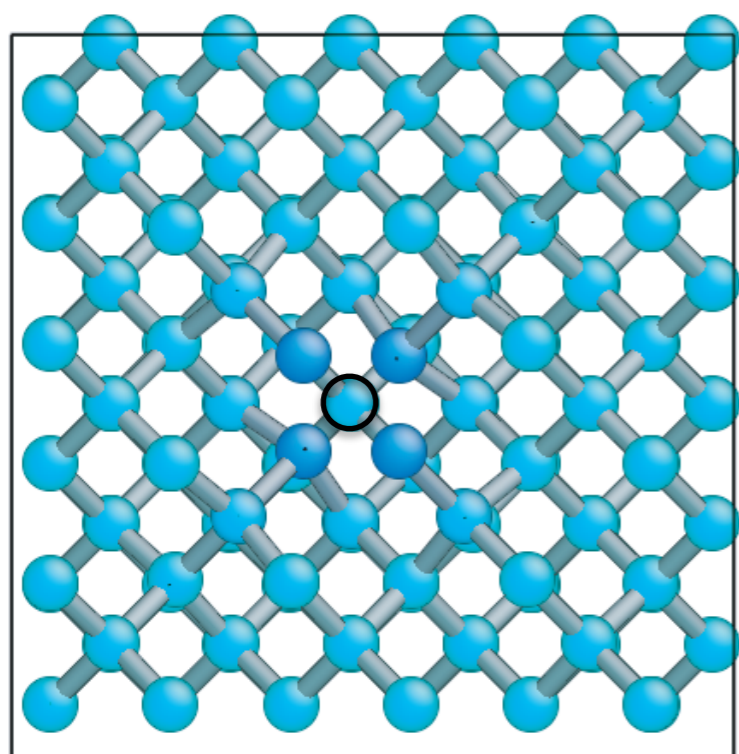
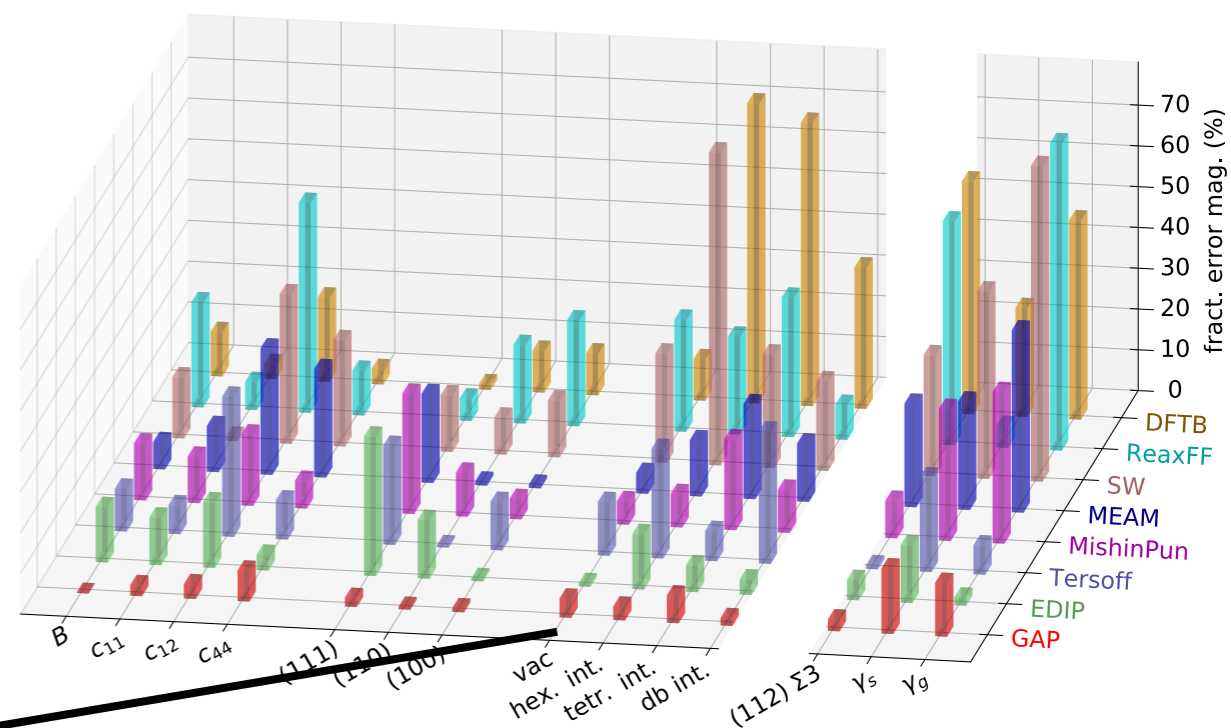
(112) Σ3



0.5 eV/Å \rightarrow

Predicted errors from equivalent Gaussian Process

Predicted error² \sim Variance of GP



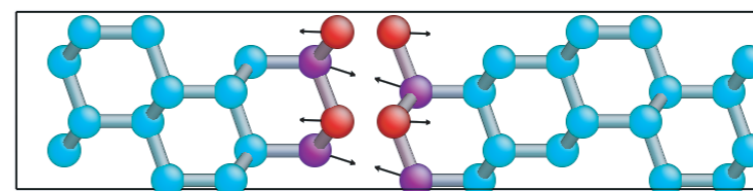
Vacancy

> 0.005 eV

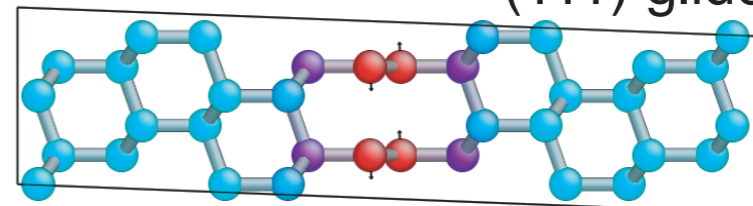
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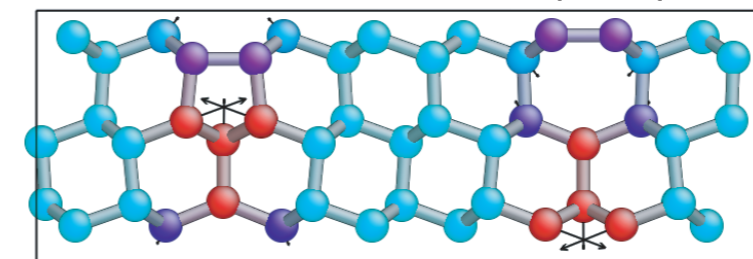
(111) shuffle



(111) glide



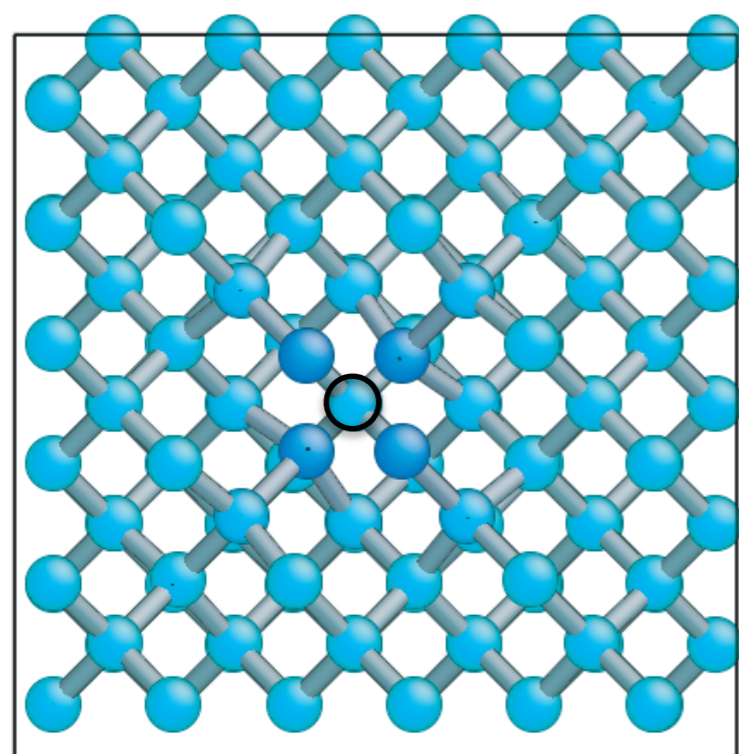
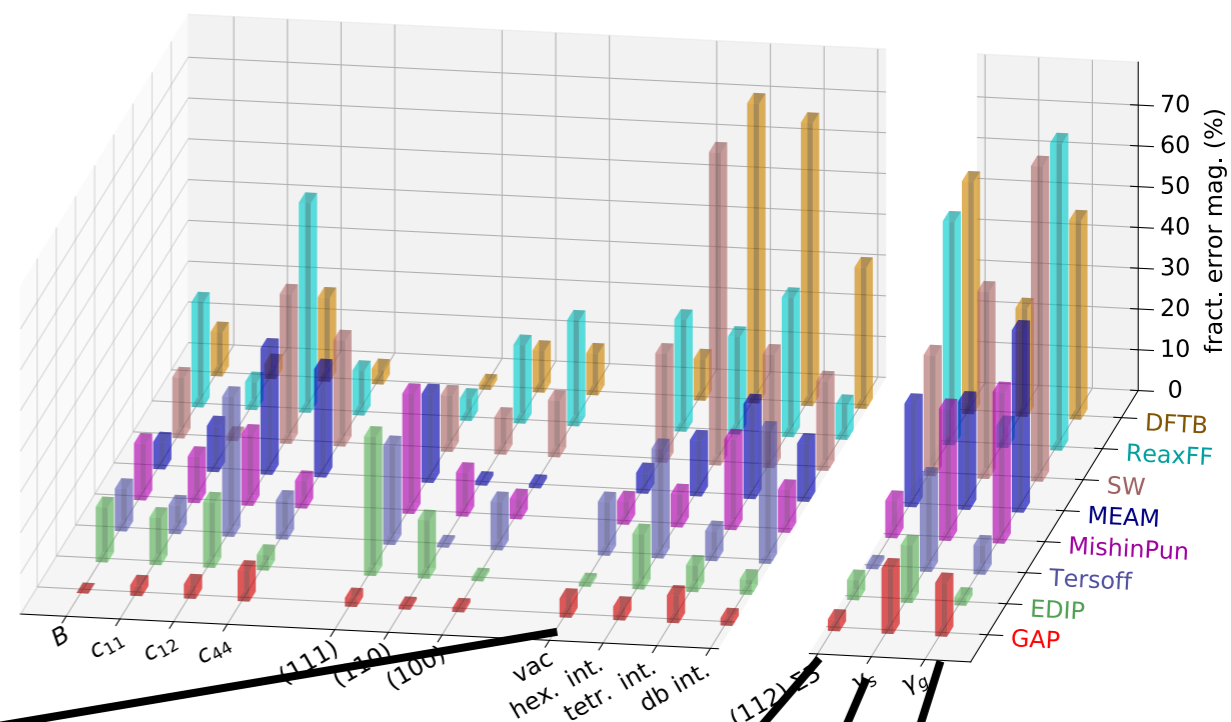
(112) Σ3



0.5 eV/Å \rightarrow

Predicted errors from equivalent Gaussian Process

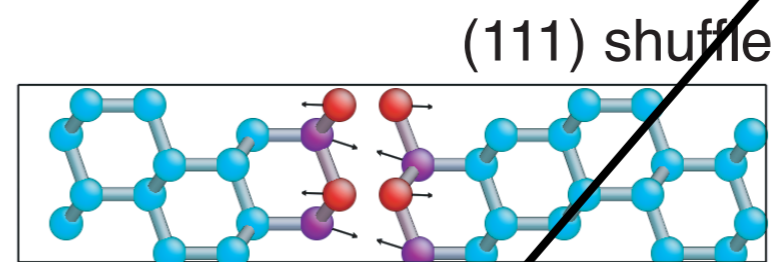
Predicted error² \sim Variance of GP



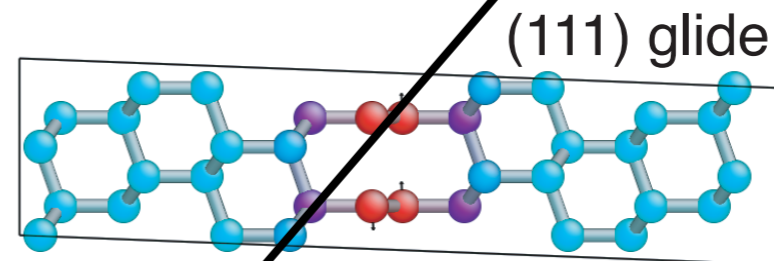
Vacancy

> 0.005 eV

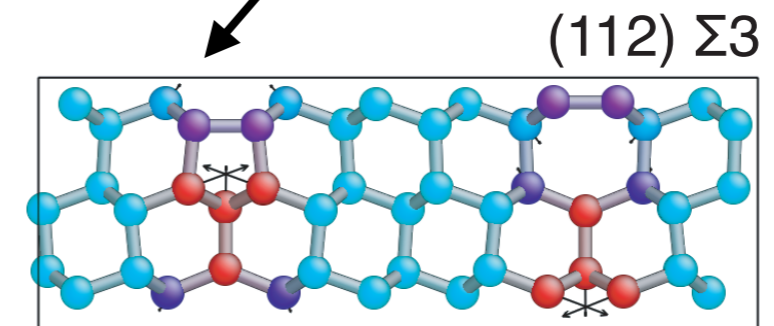
< 0.001 eV



(111) shuffle



(111) glide



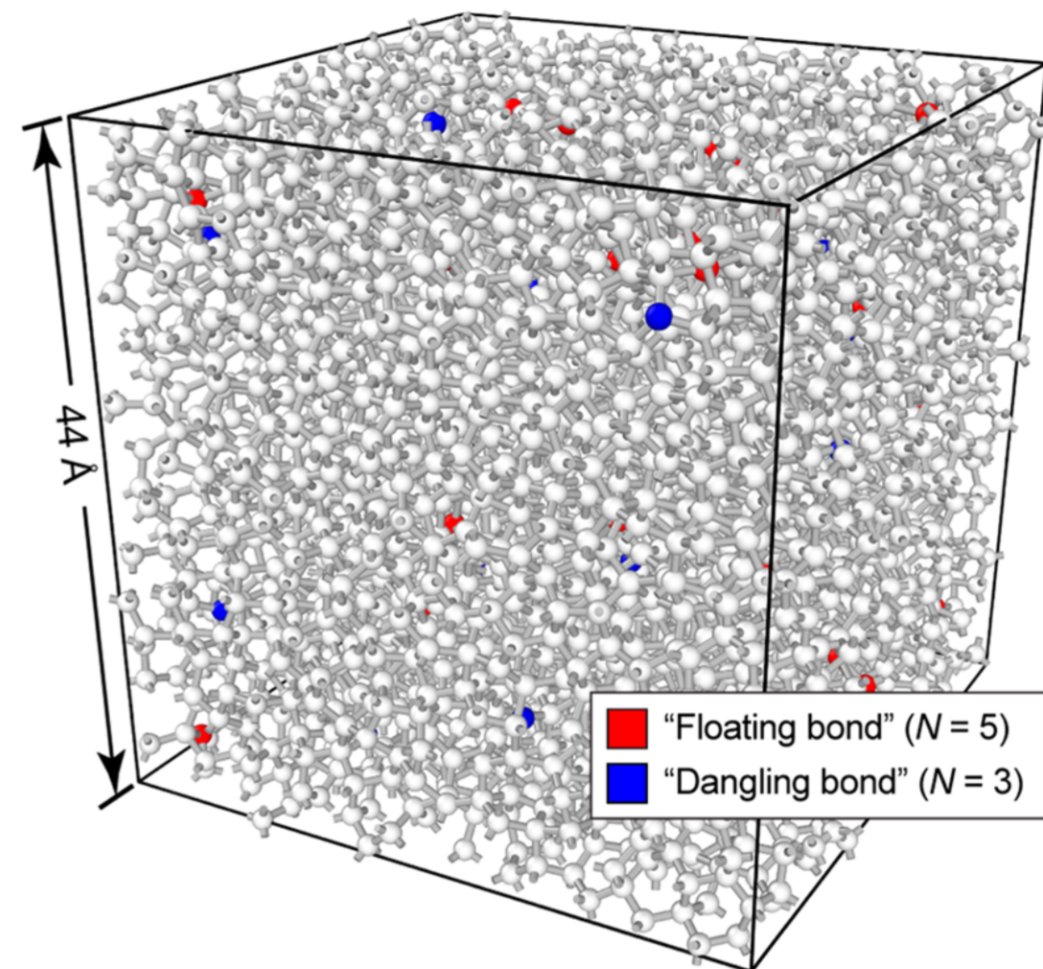
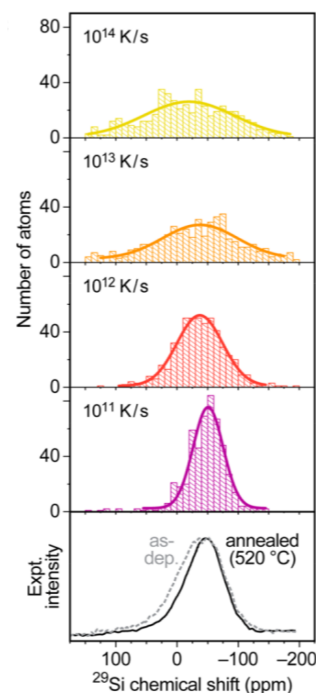
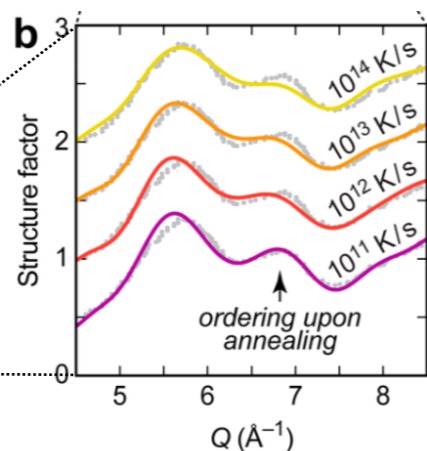
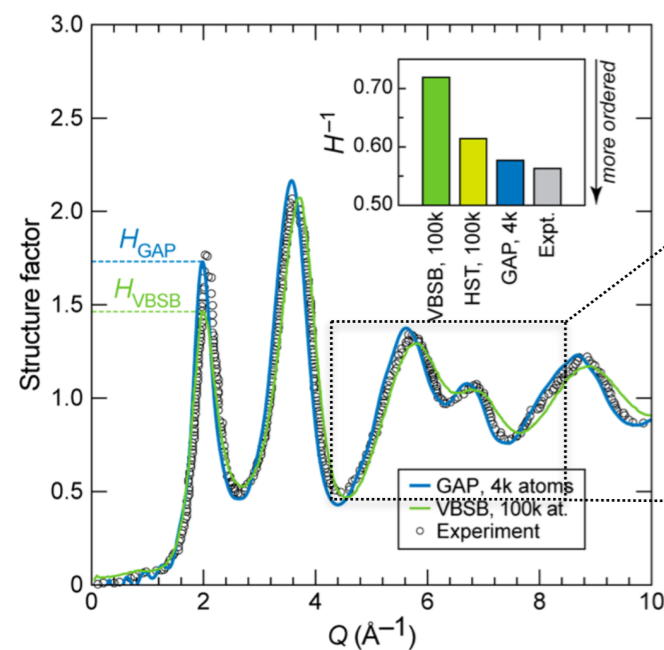
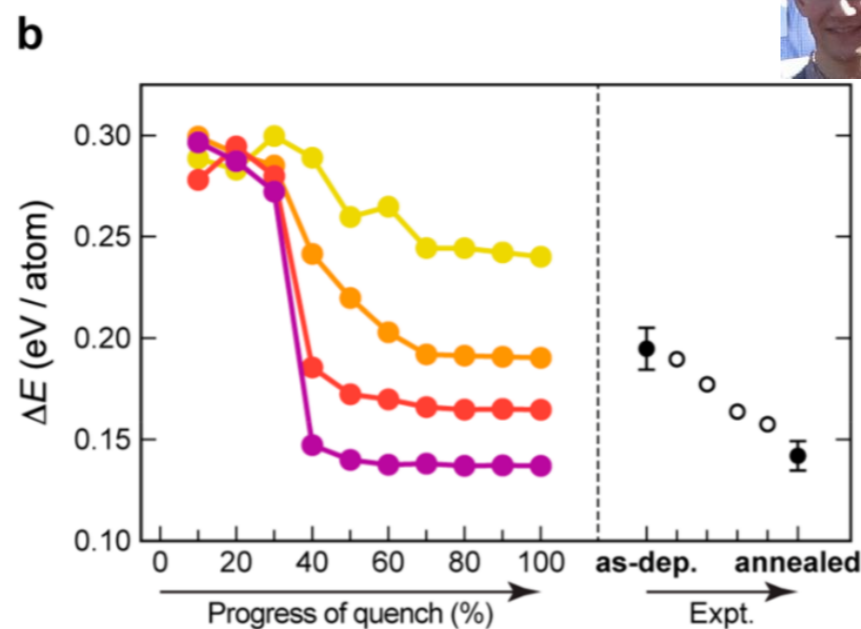
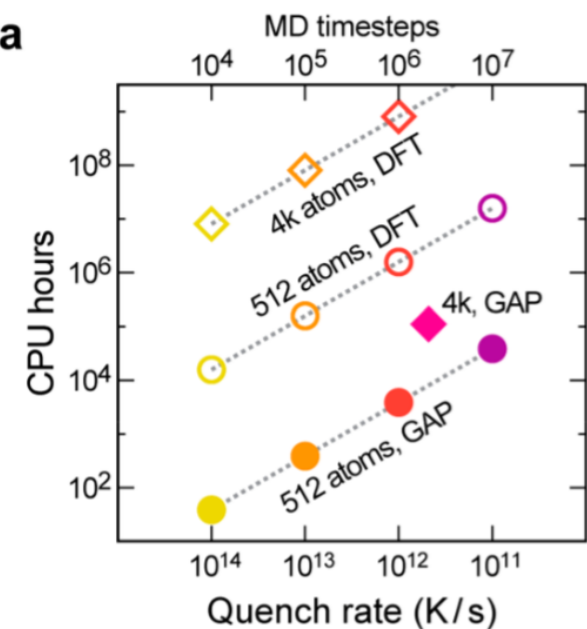
(112) Σ3

0.5 eV/Å \rightarrow

Realistic Atomistic Structure of Amorphous Silicon from Machine-Learning-Driven Molecular Dynamics



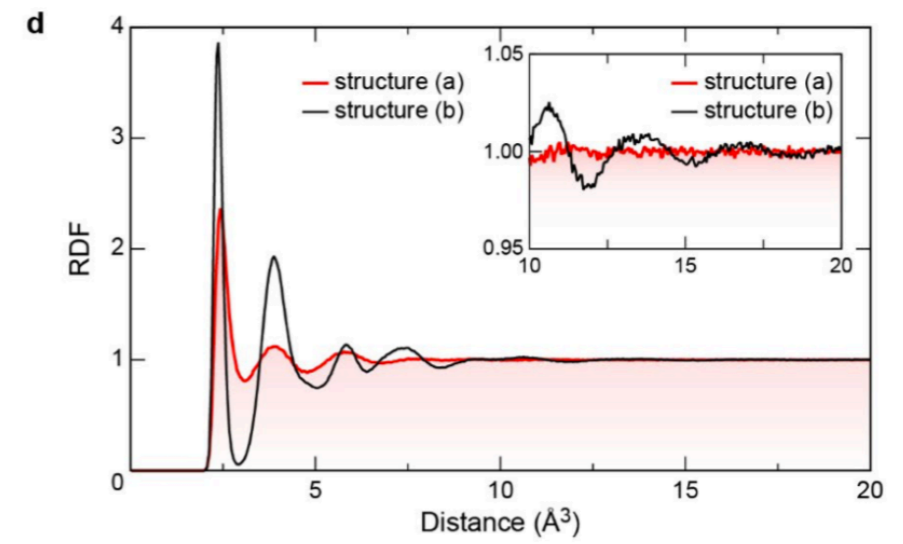
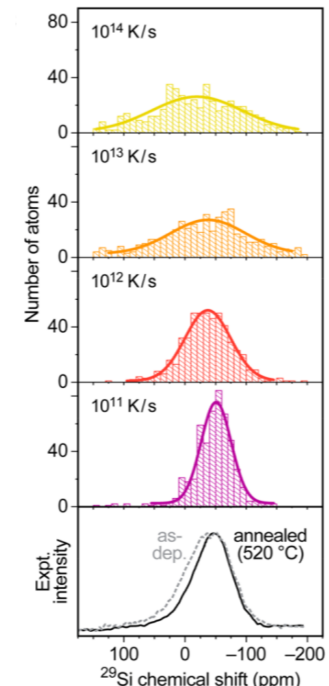
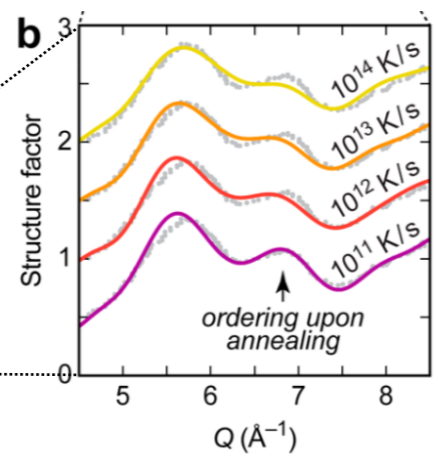
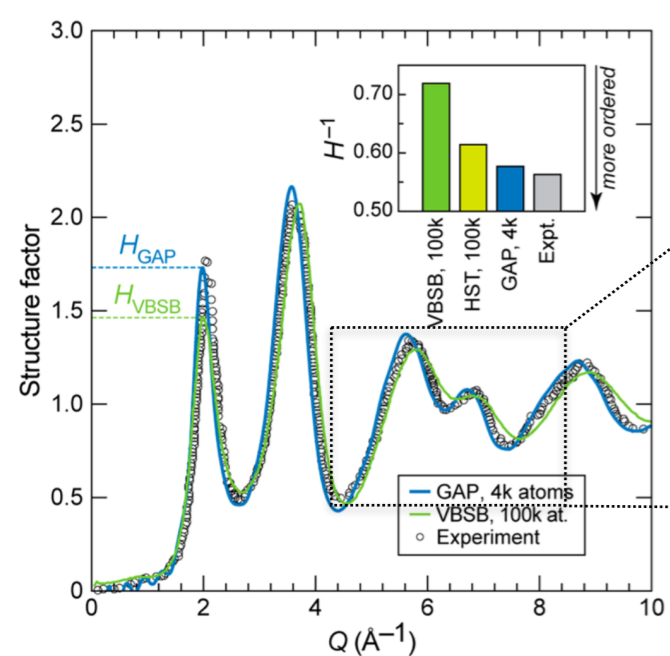
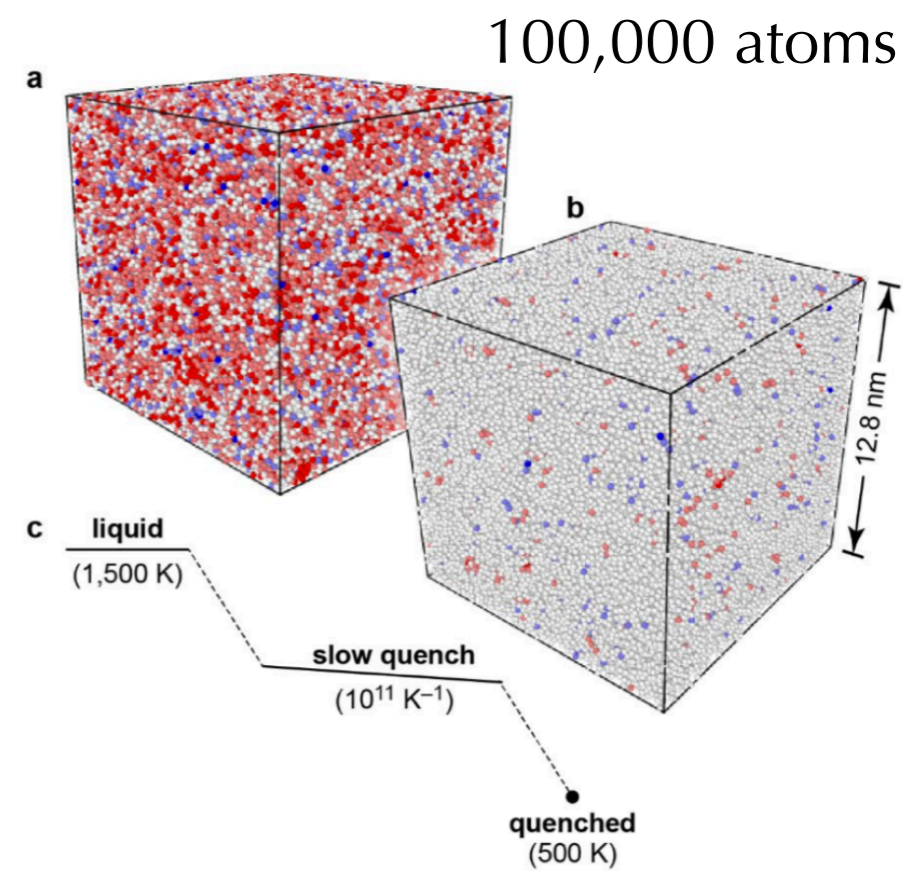
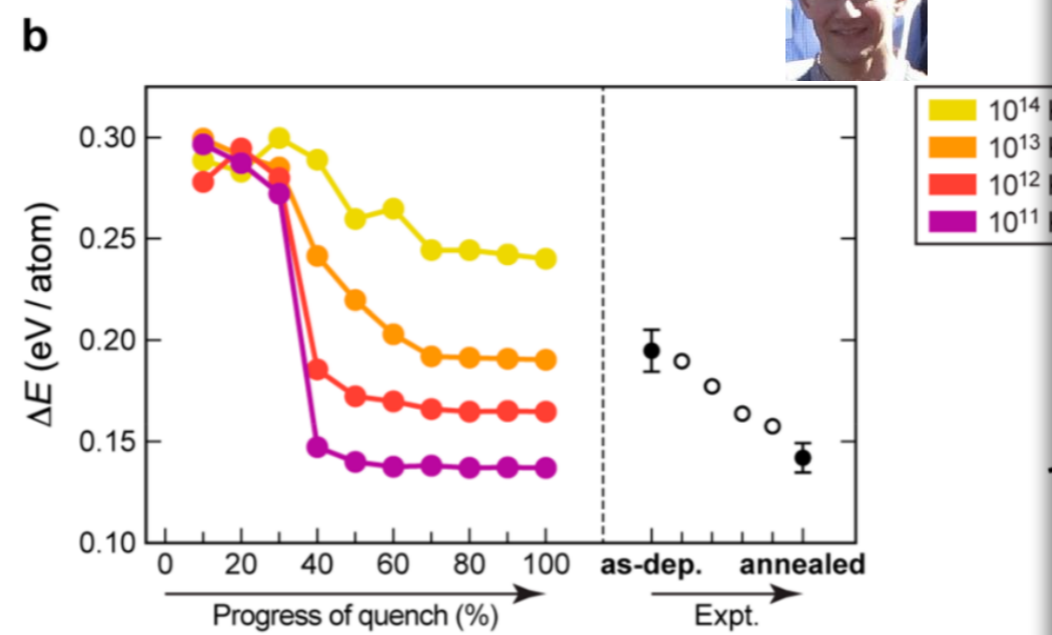
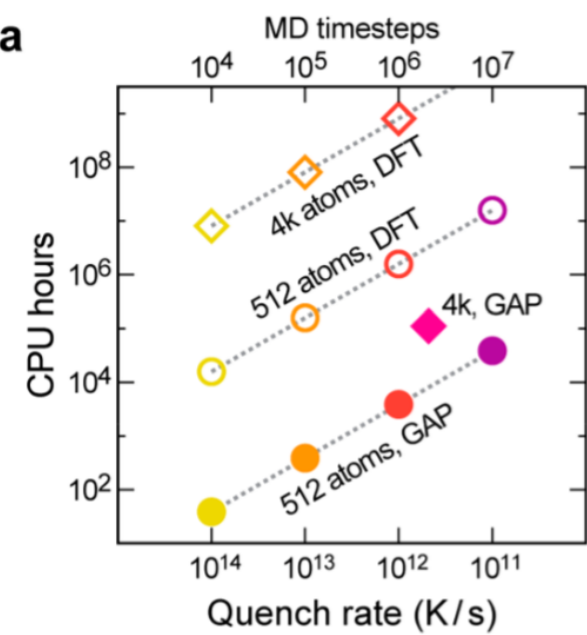
Volker L. Deringer,^{*,†,‡,§} Noam Bernstein,[§] Albert P. Bartók,^{||} Matthew J. Cliffe,[‡] Rachel N. Kerber,[‡] Lauren E. Marbella,^{‡,§} Clare P. Grey,^{‡,§} Stephen R. Elliott,^{‡,§} and Gábor Csányi[†]



Realistic Atomistic Structure of Amorphous Silicon from Machine-Learning-Driven Molecular Dynamics



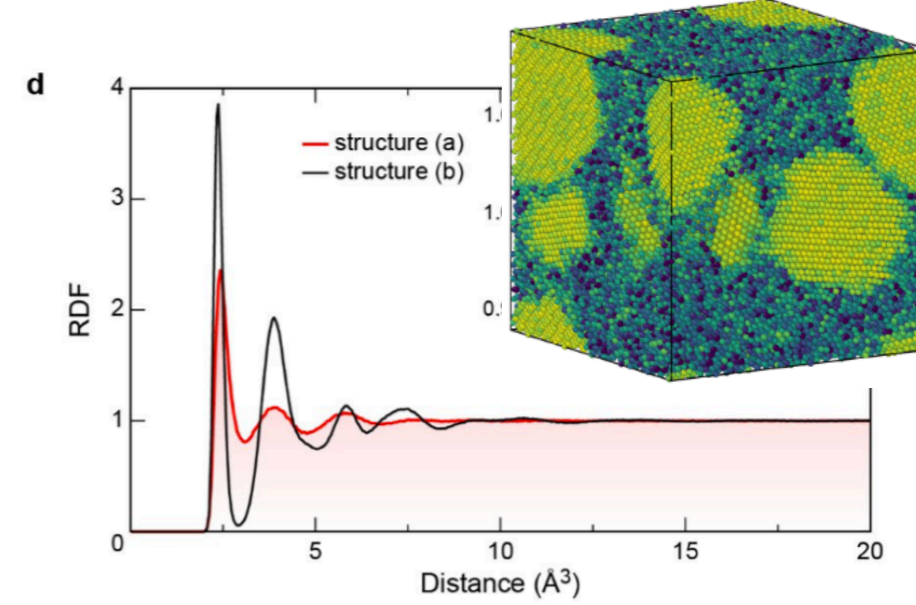
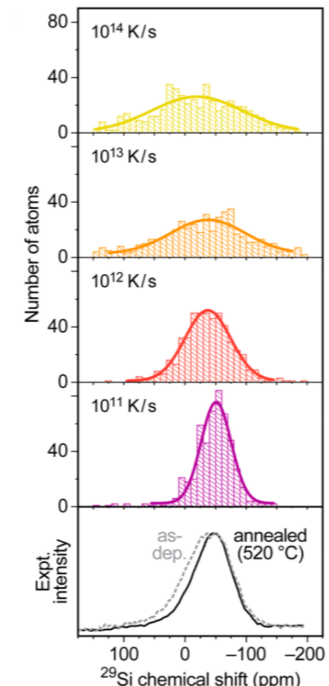
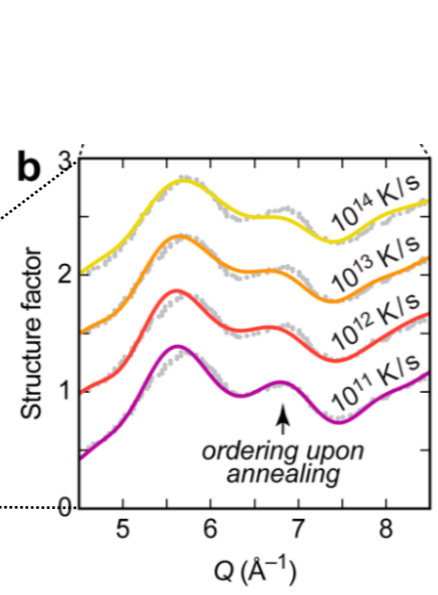
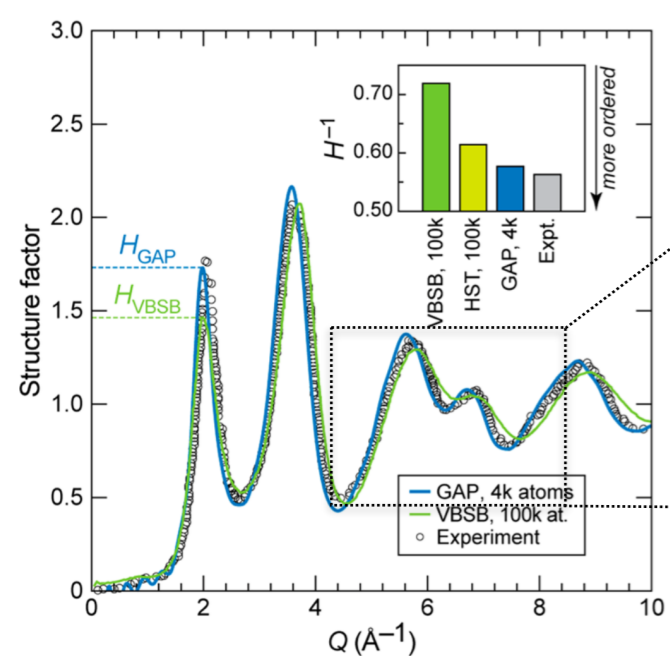
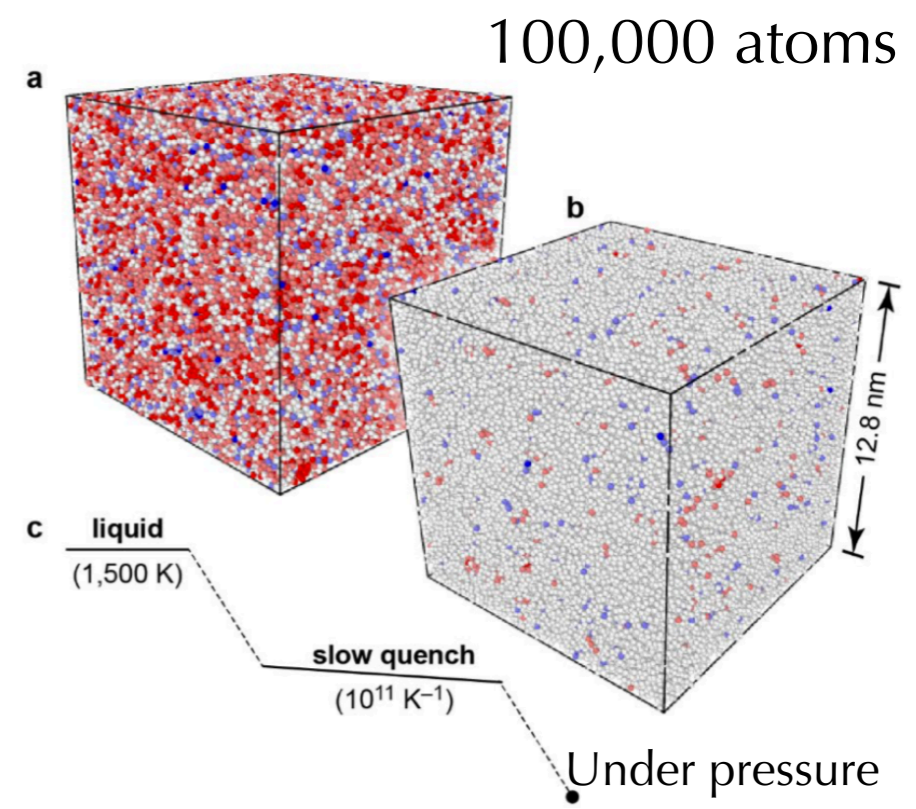
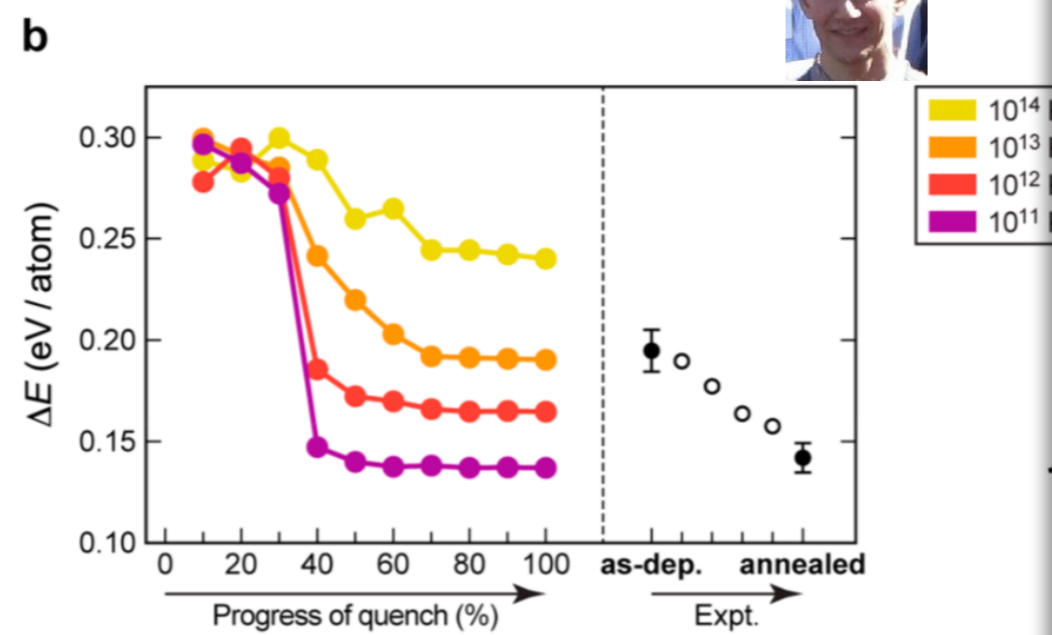
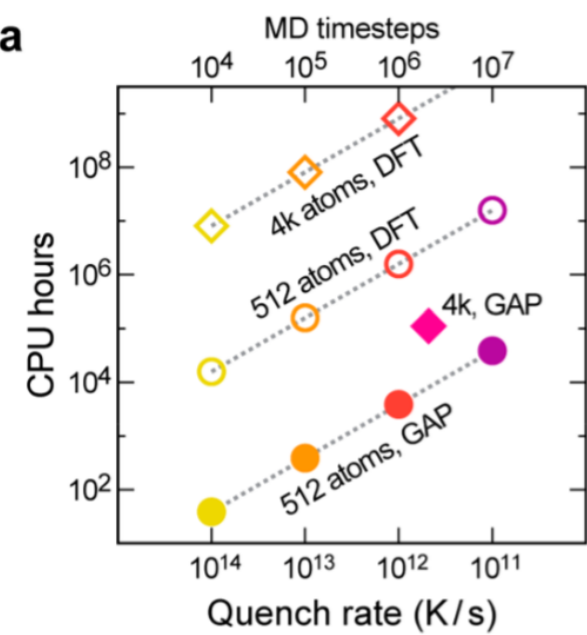
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Fluid methane

- Dominated by weak dispersion interactions
- 2-body enough or need many-body effects?
- What level of quantum mechanics is required?



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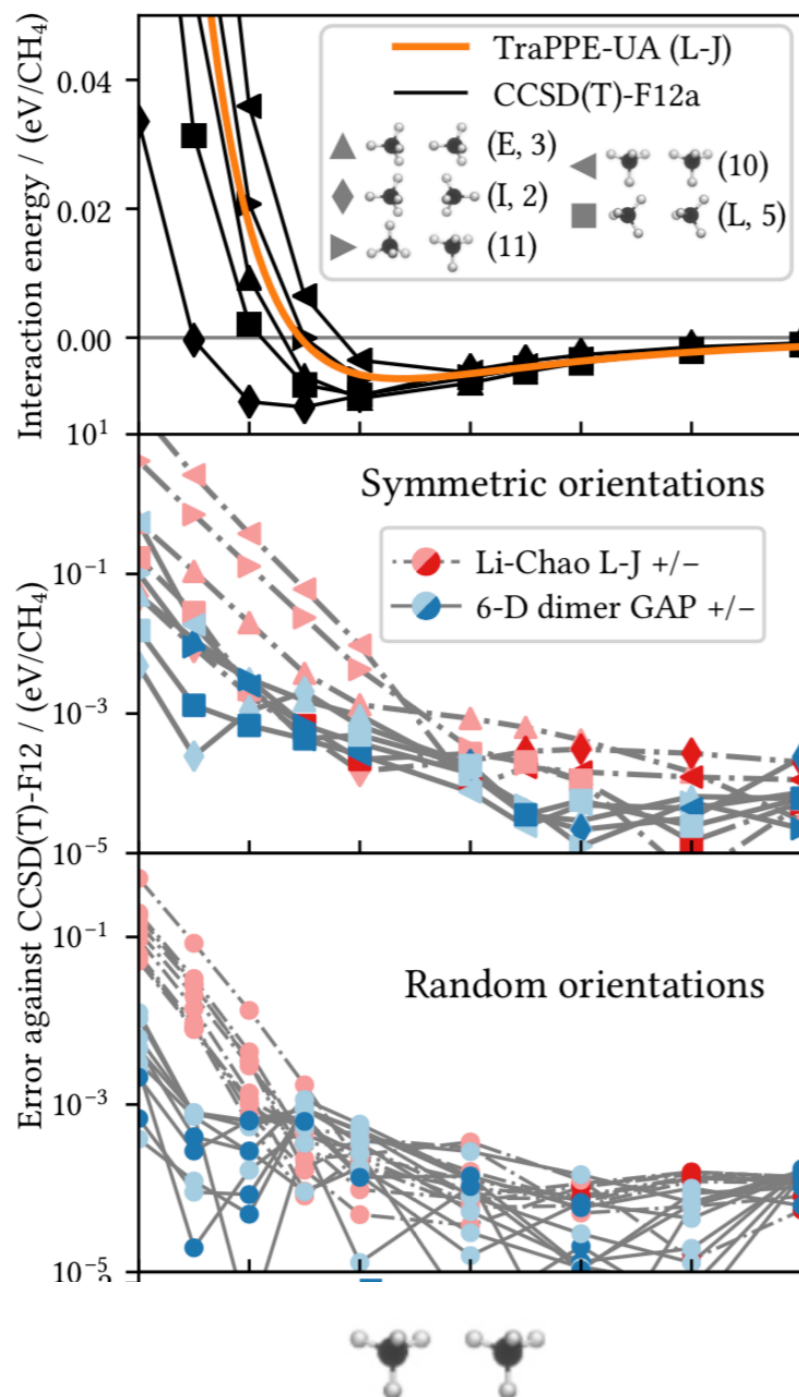


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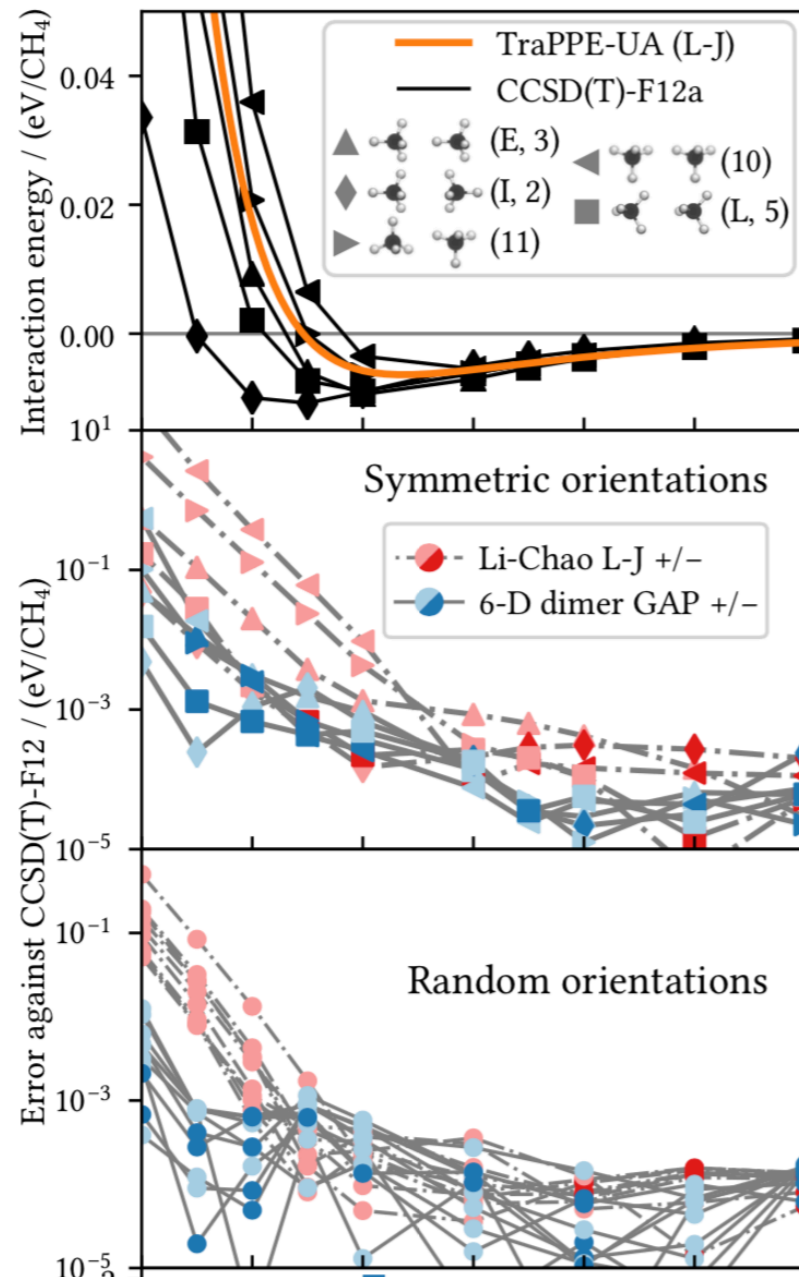
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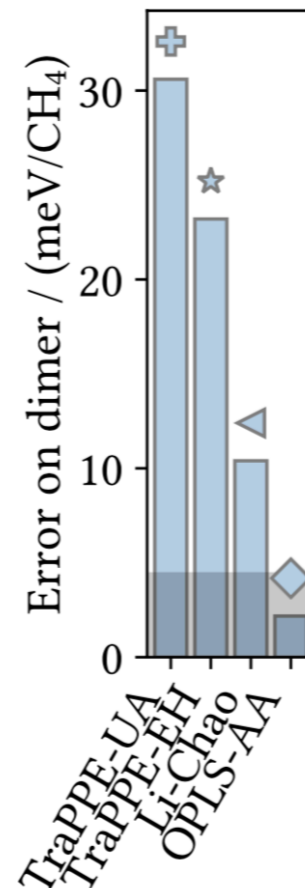
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Empirical models



Fluid methane

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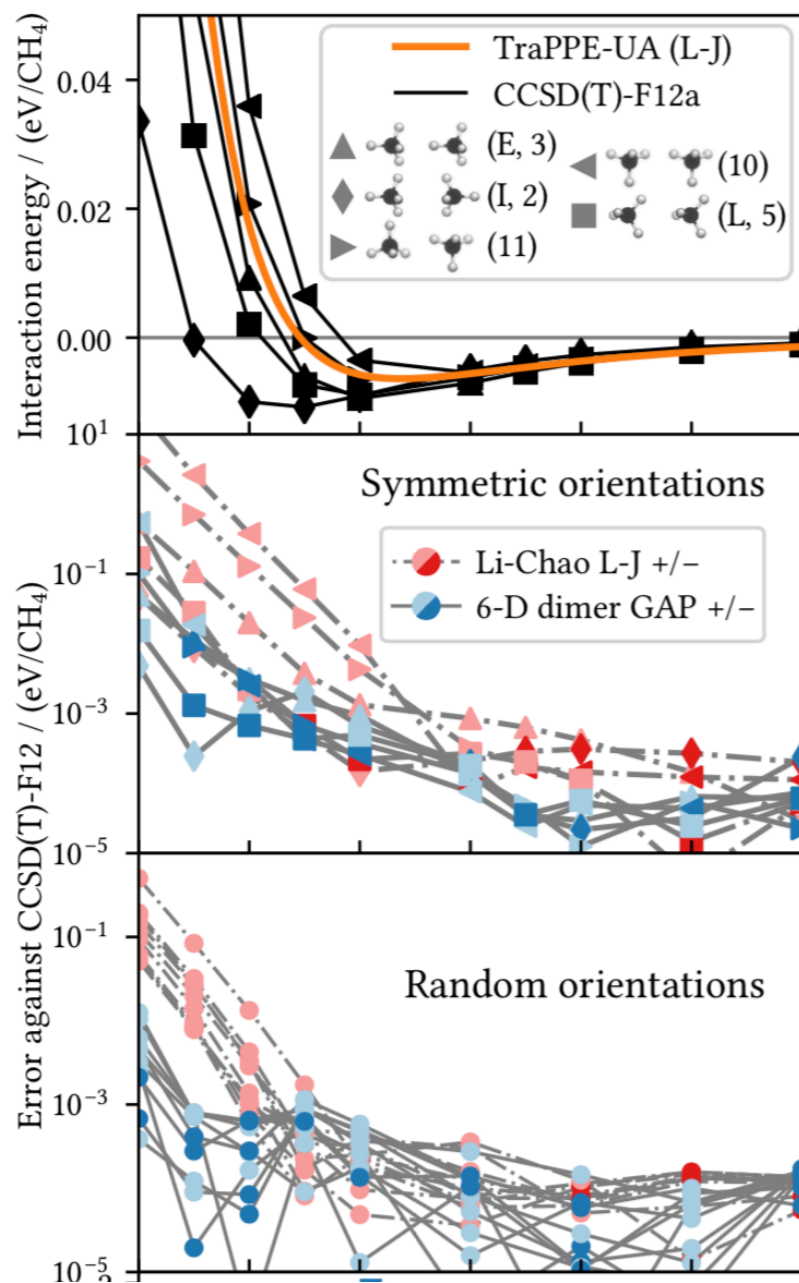
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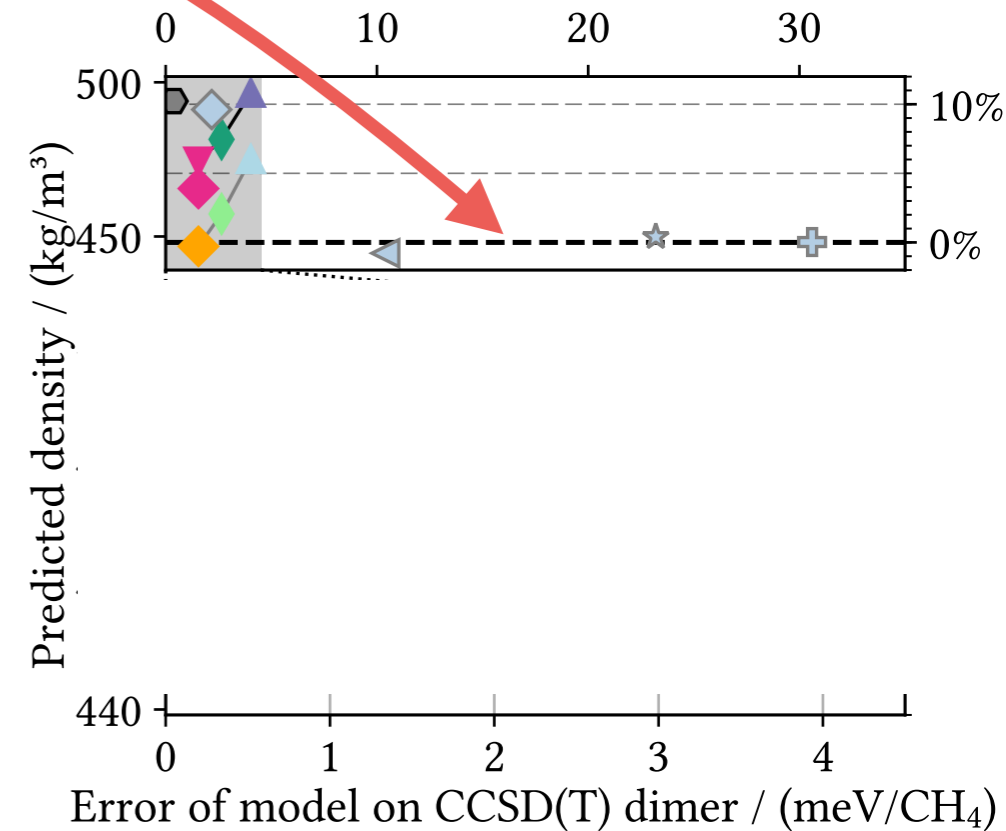
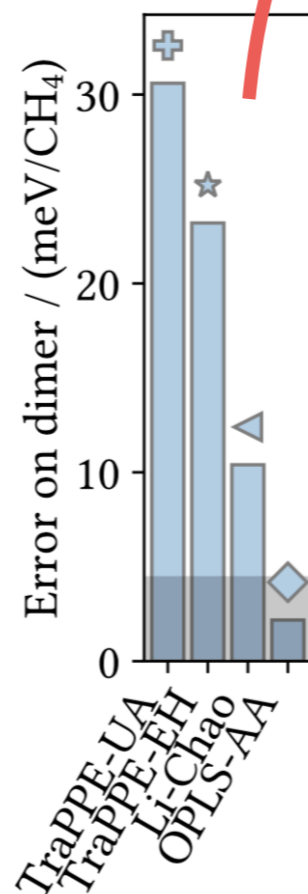
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Empirical models



Density error

Fluid methane

- Dominated by weak dispersion interactions
- 2-body enough or need many-body effects?
- What level of quantum mechanics is required?



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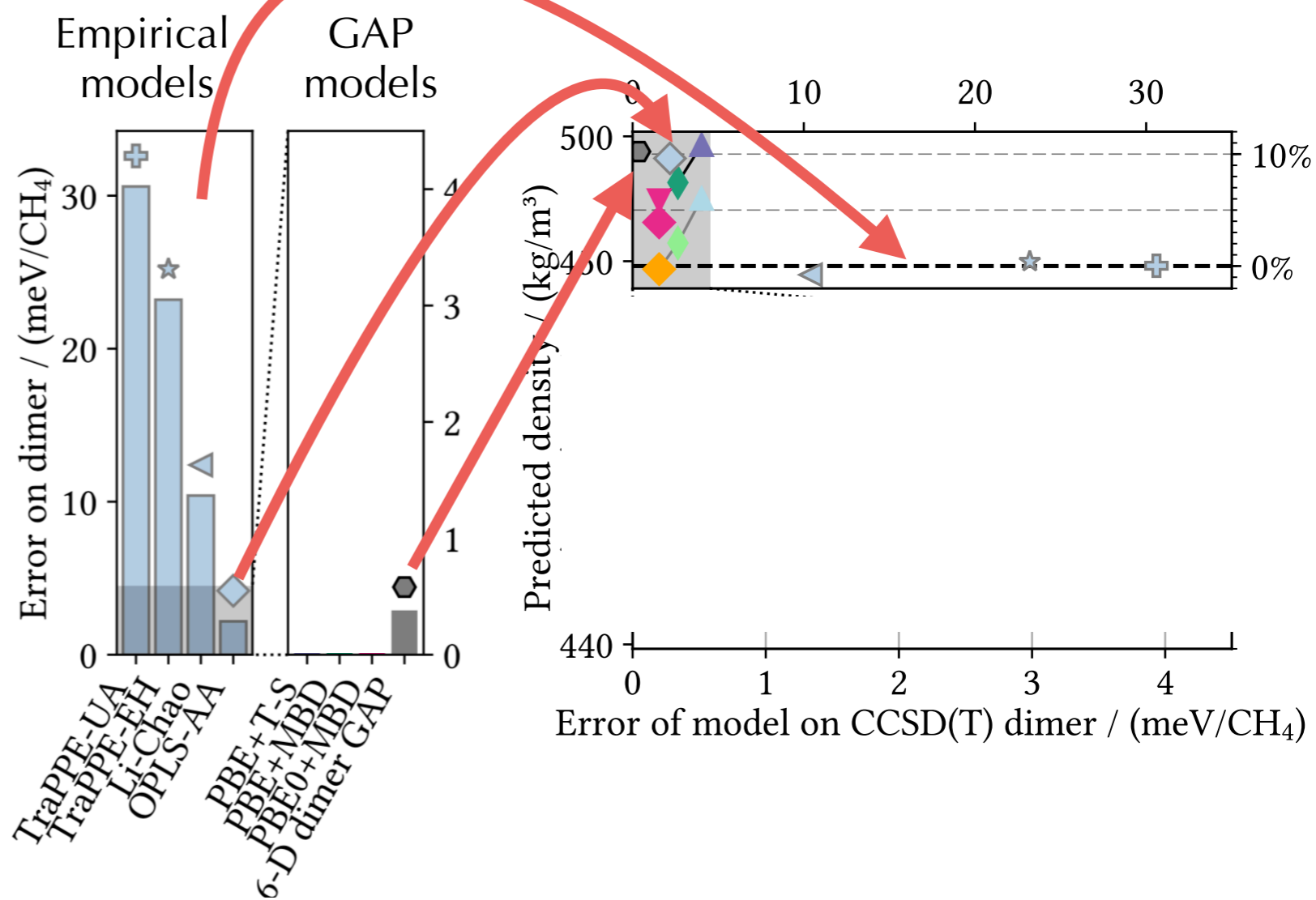
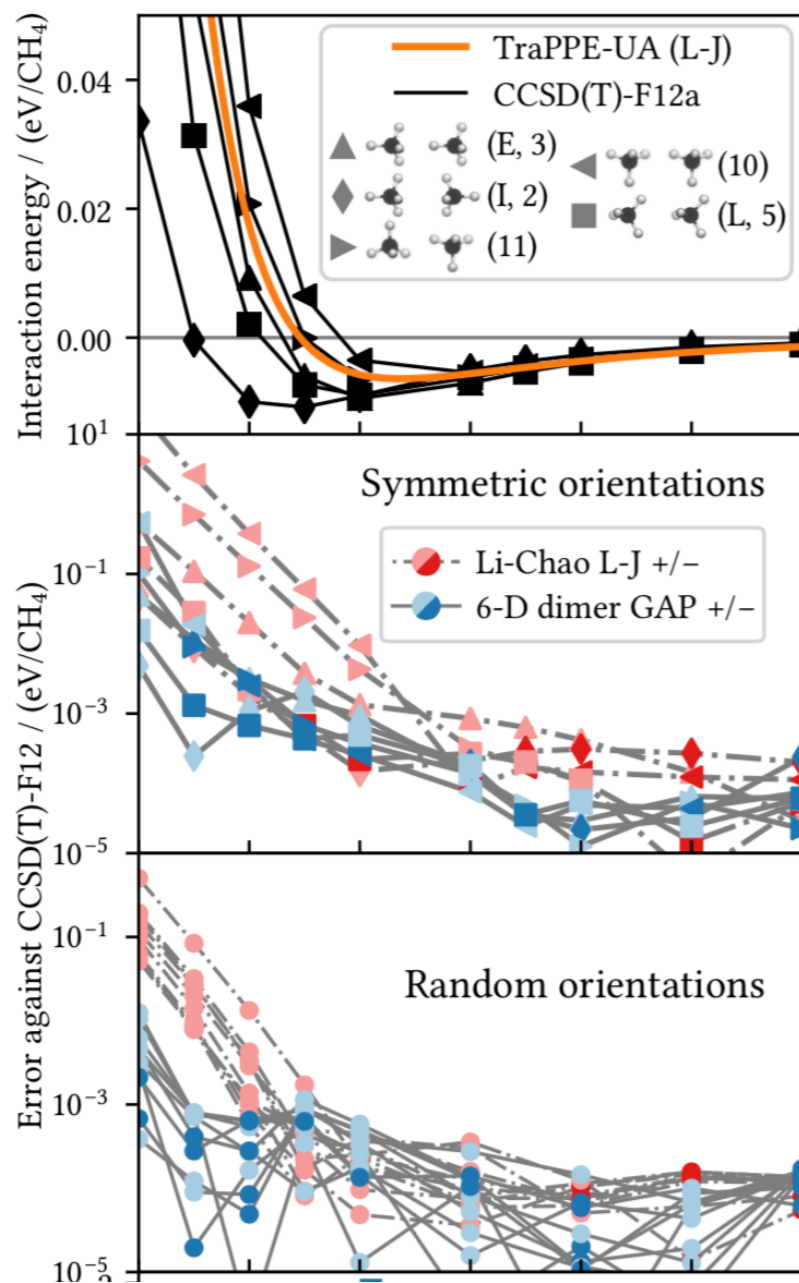
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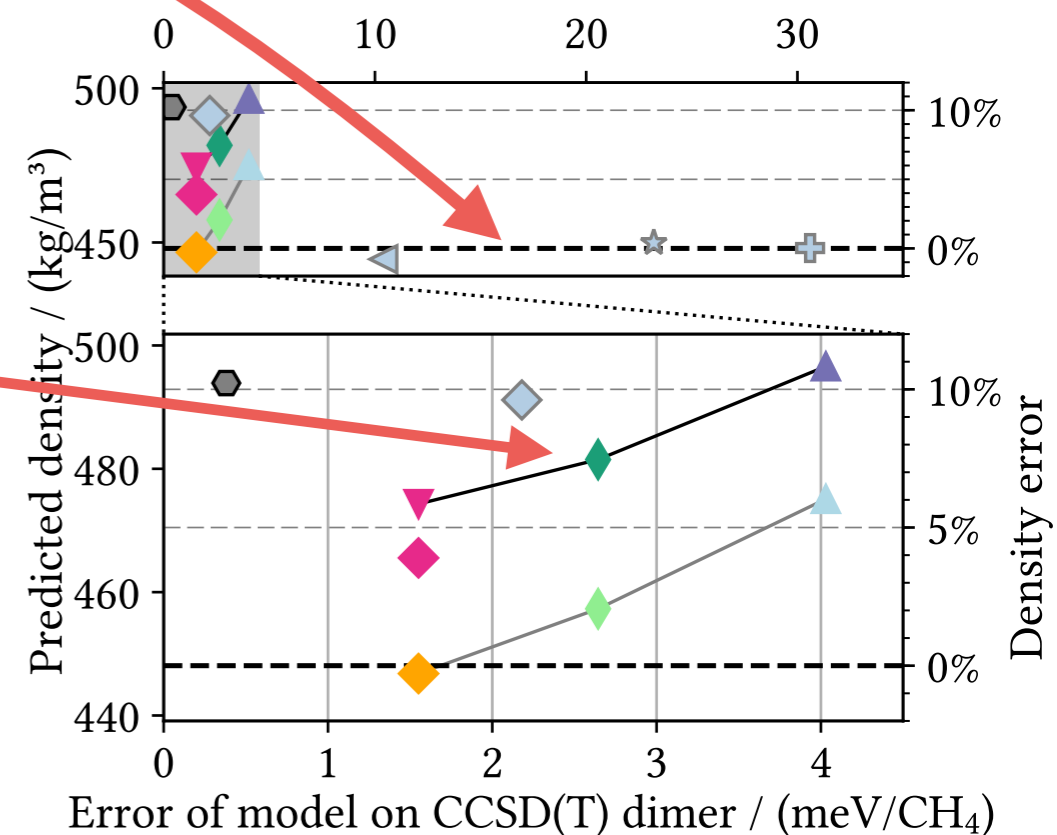
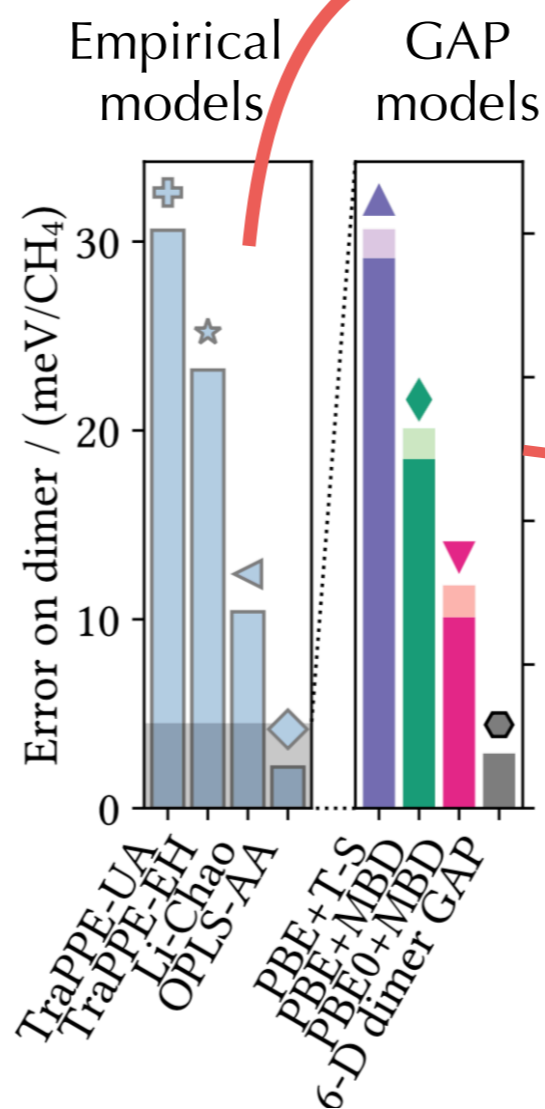
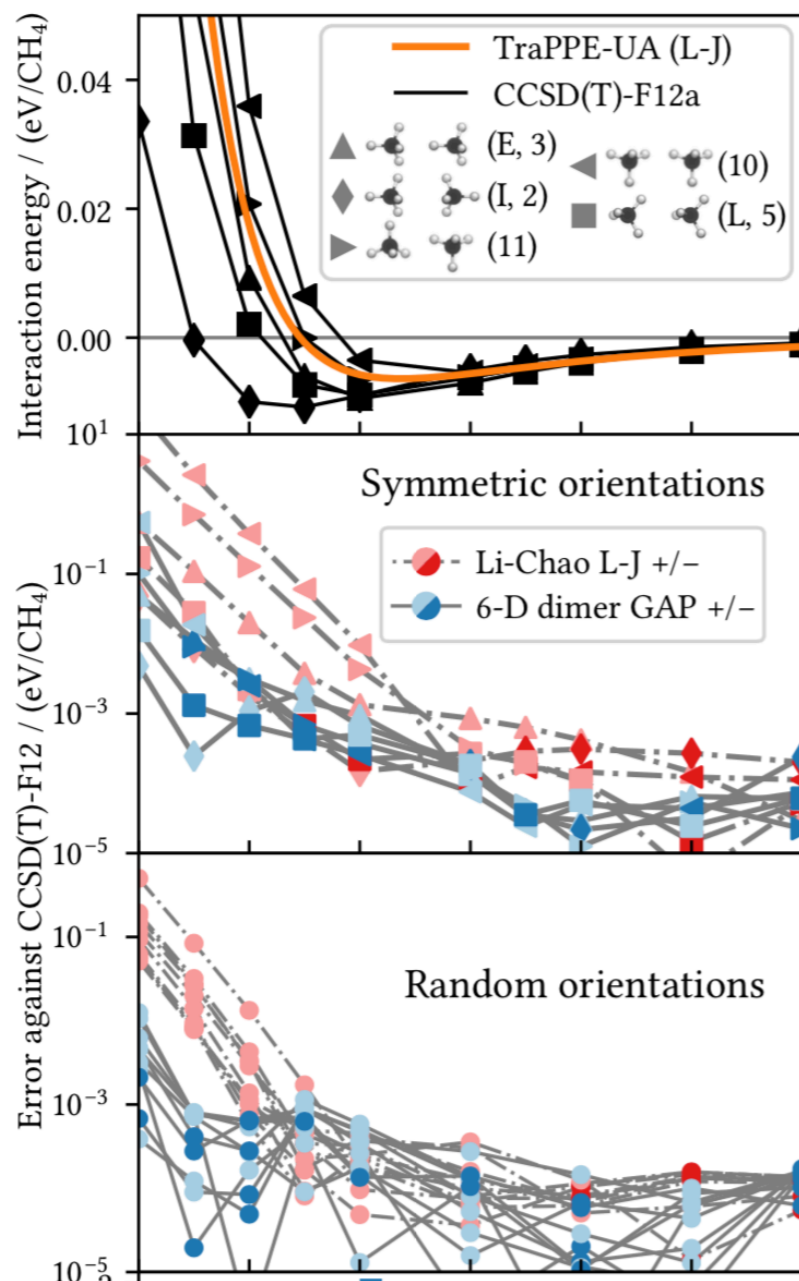
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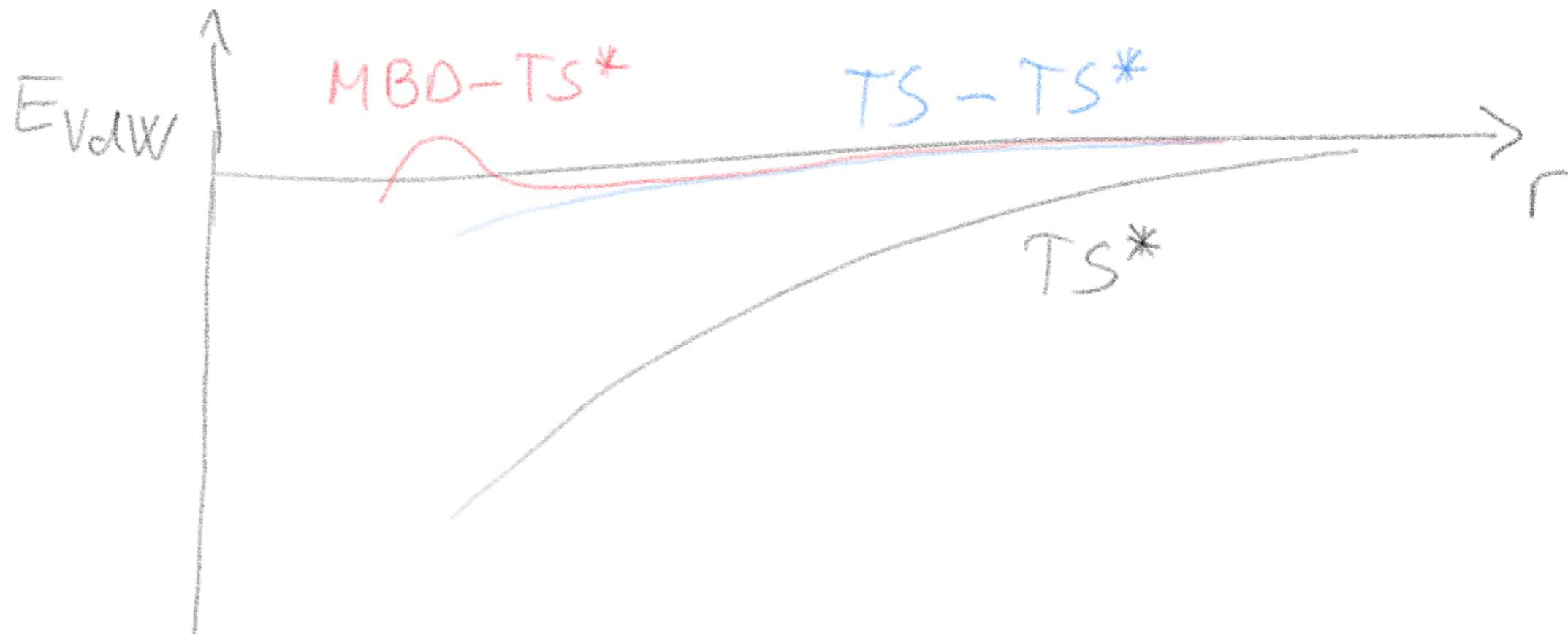


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Medium-range physics

- Best many-body QM we can do: Hybrid DFT + VdW
- VdW is long range : $\sim 1/r^6$
- Hierarchy of models:
 - Tkatchenko-Scheffler pair potential with fixed coefficient: TS*
 - TS correction with on-the-fly-dft-computed coefficients
 - MBD (many-body dispersion also by Tkatchenko et al), also needs DFT-computed coefficients

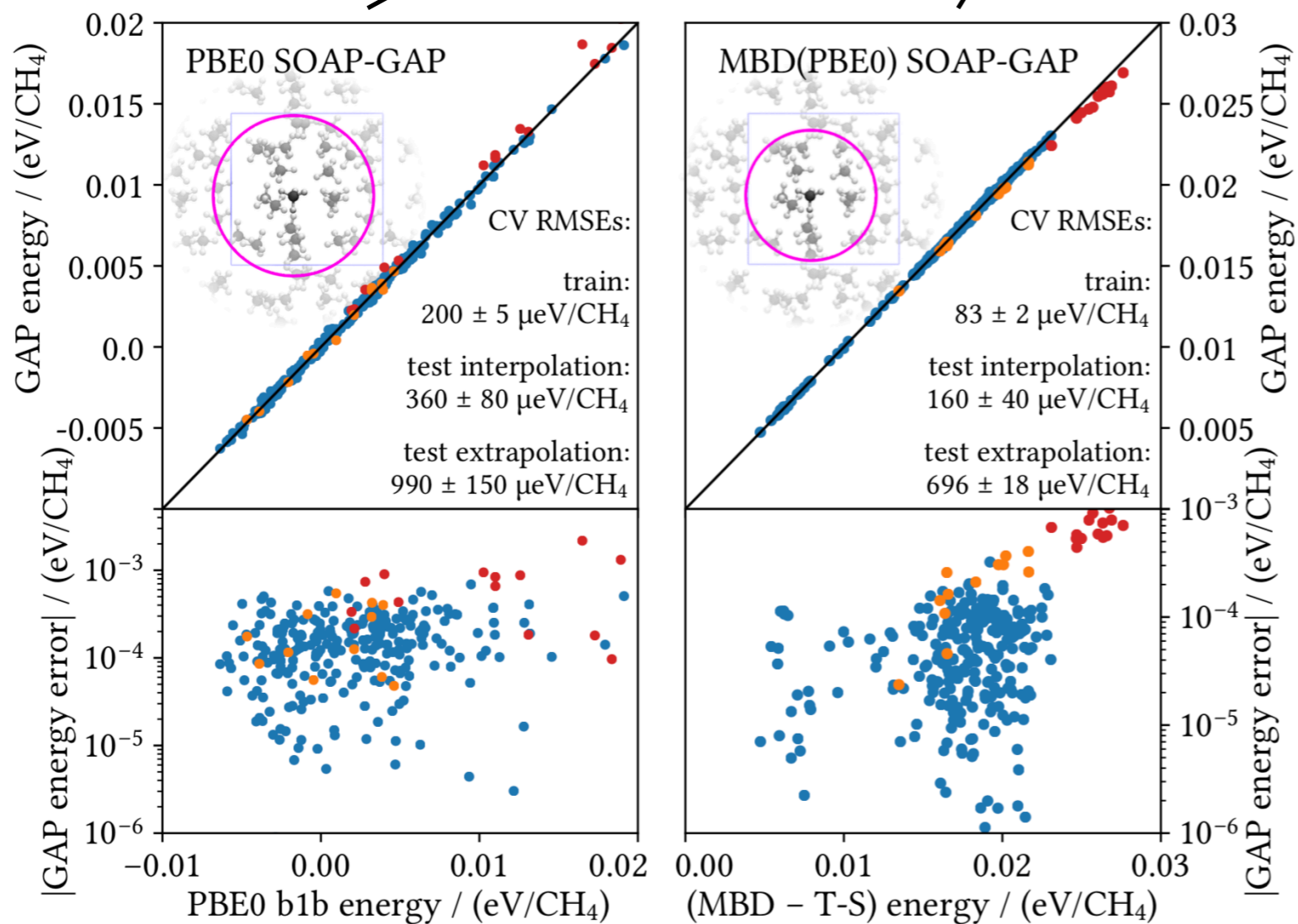


Many-body SOAP-GAP model

$$E_{\text{potential}} = N E_{\text{CH}_4} + E_{\text{short-range}} + E_{\text{vdW}}(\text{MBD-TS}^*) + E_{\text{vdW}}(\text{TS}^*)$$

Many-body SOAP-GAP model

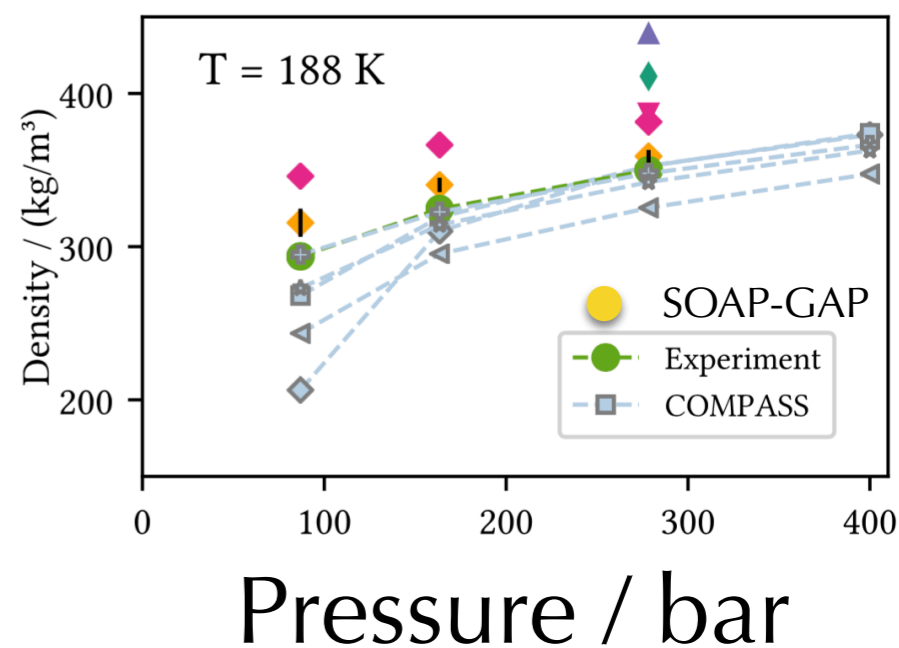
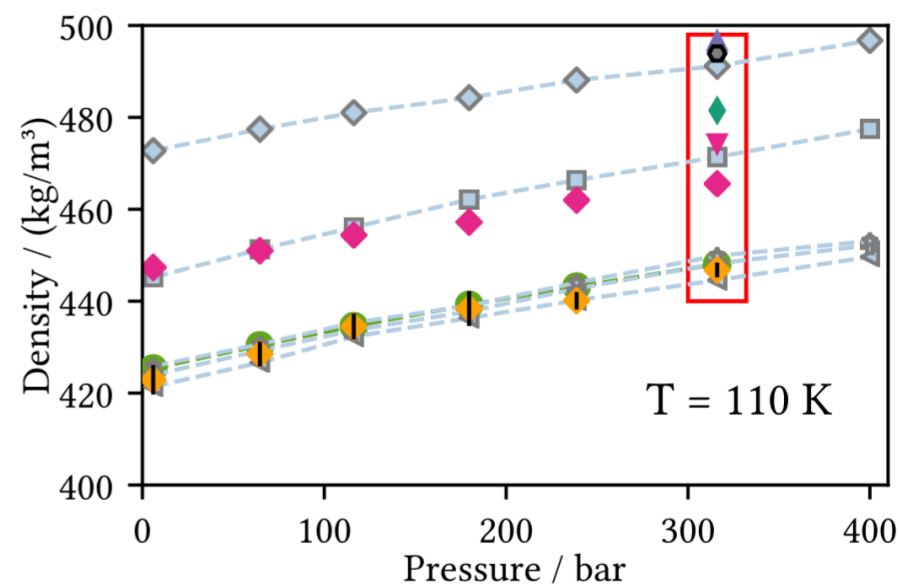
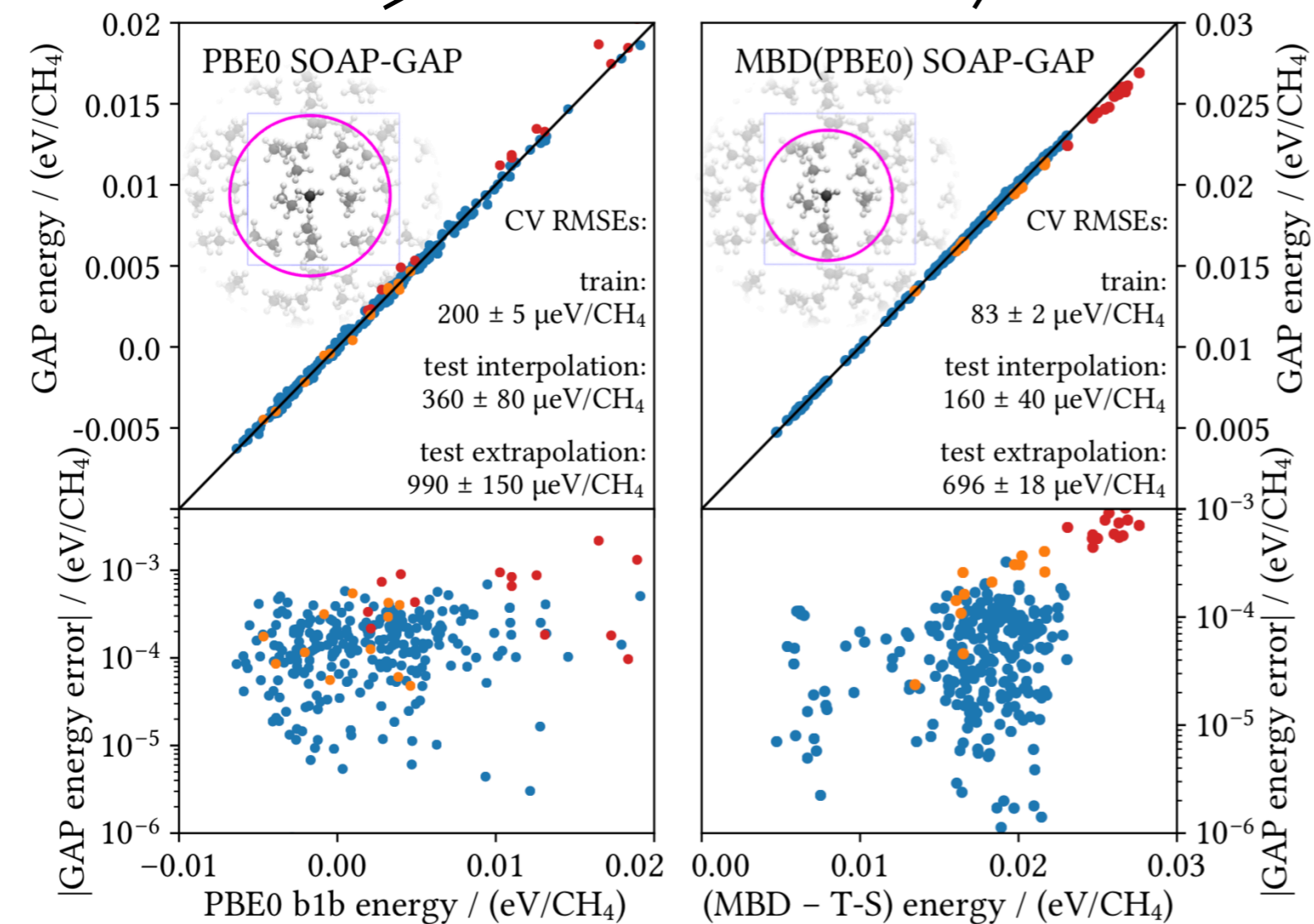
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Many-body SOAP-GAP model

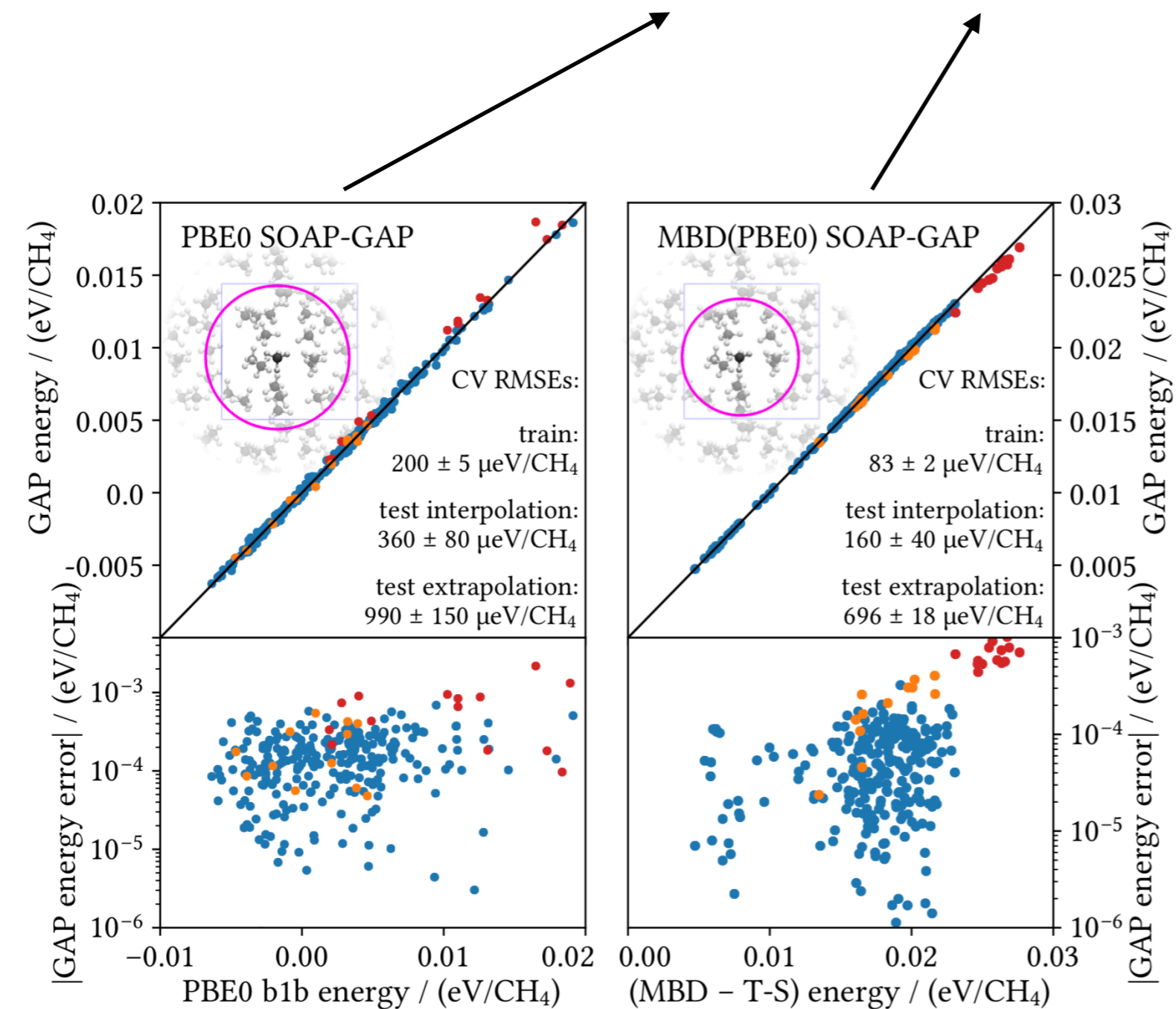
$$E_{\text{potential}} = N E_{\text{CH}_4} + E_{\text{short-range}} + E_{\text{VdW}}(\text{MBD-TS}^*) + E_{\text{VdW}}(\text{TS}^*)$$

Density prediction

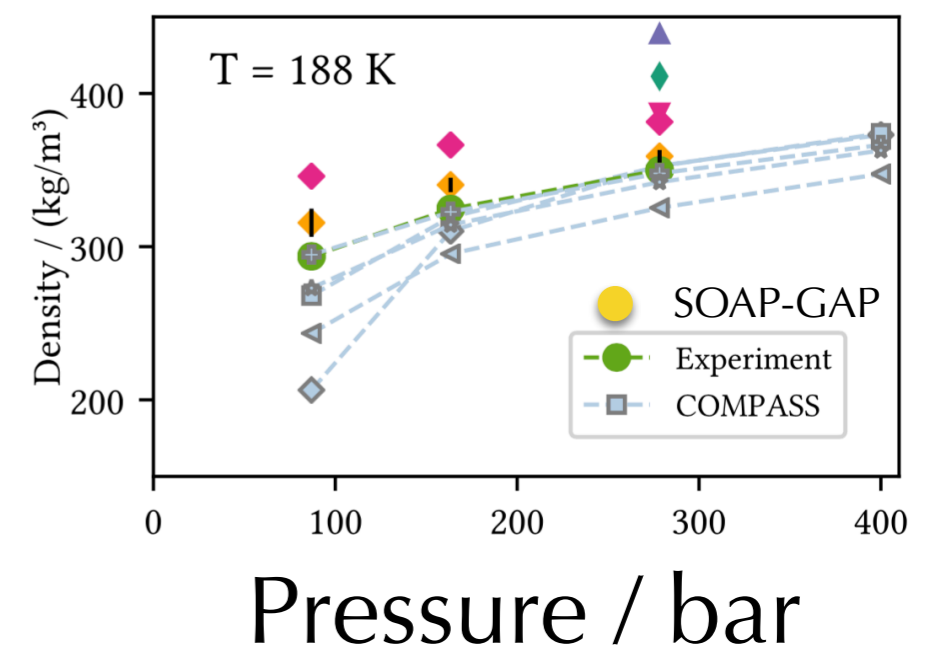
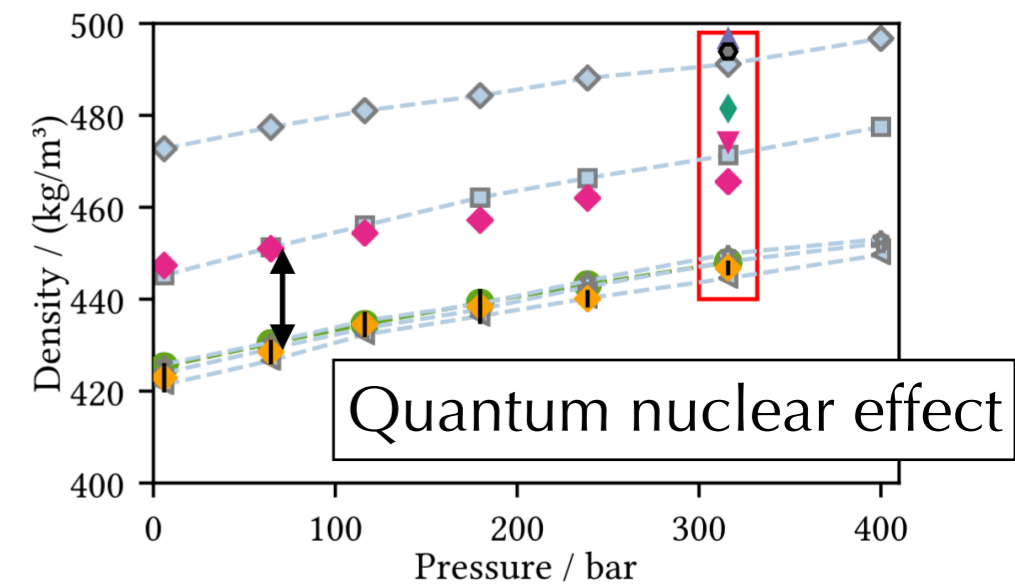


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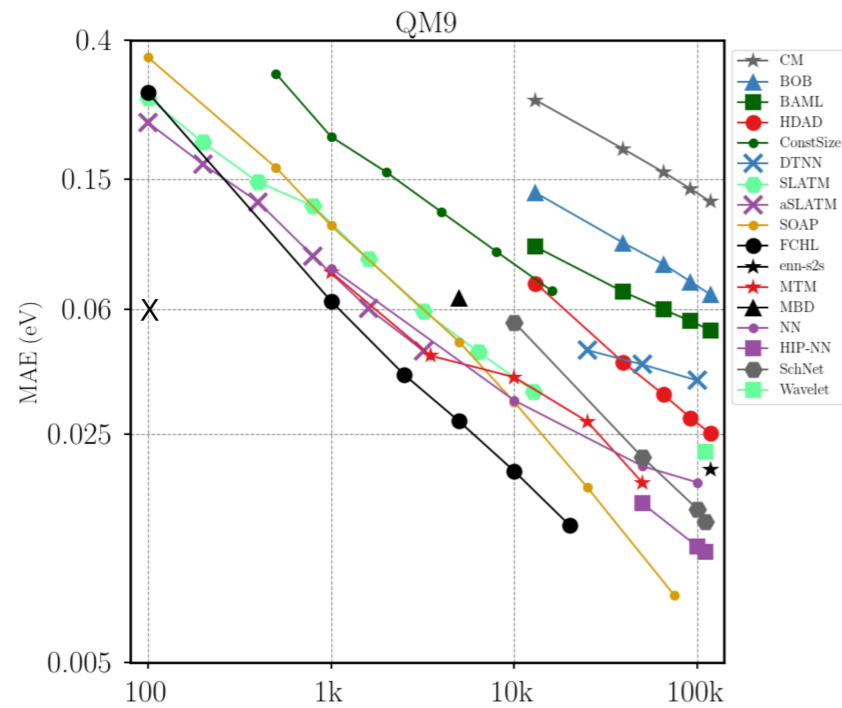


Density prediction



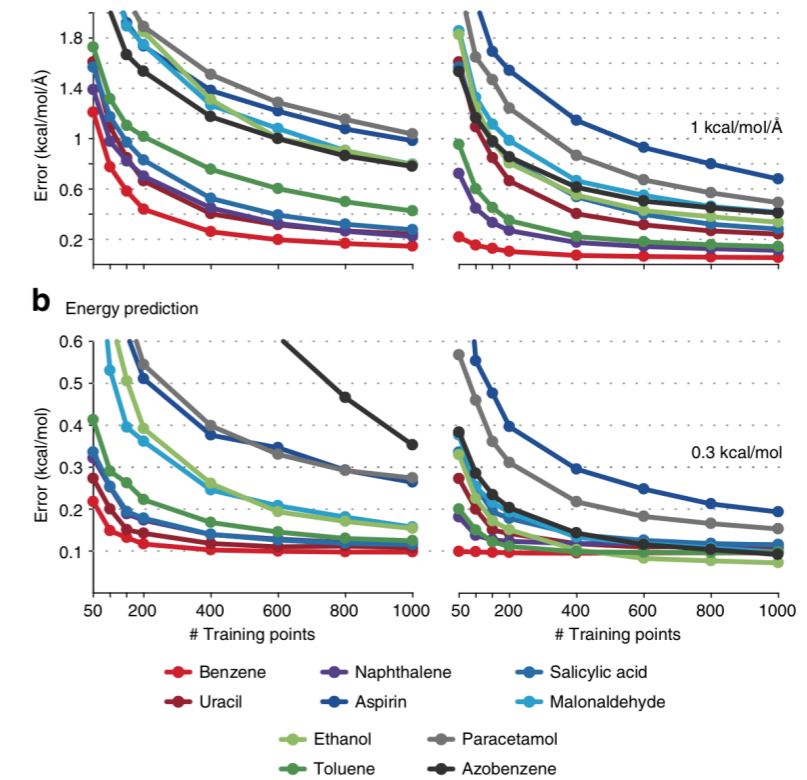
Molecular successes (of others, mostly)

QM9 (local minima)

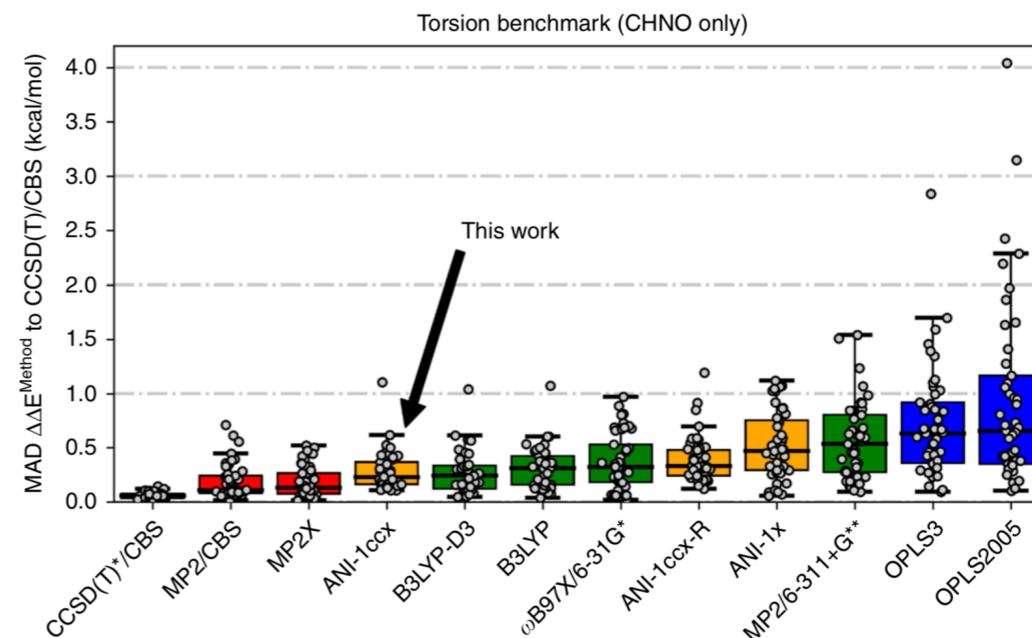


(Courtesy of O. A von Lilienfeld)

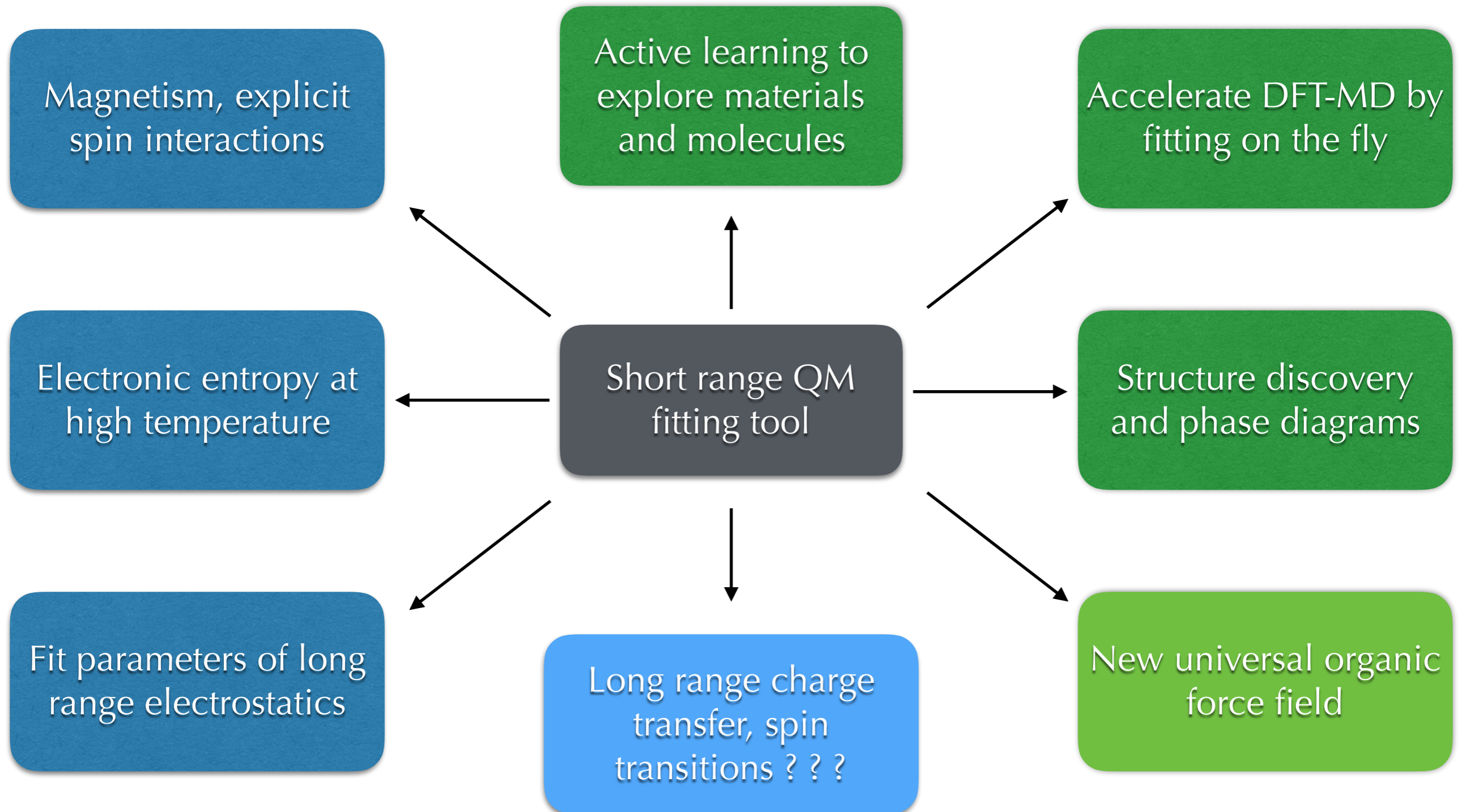
(s)GDML custom force fields of Tkatchenko et al.
(KRR model on interatomic distances)



ANI-CC model
of Isayev, Roitberg and
coworkers for arbitrary molecules
(2-3b descriptors, 3-layer ANN)



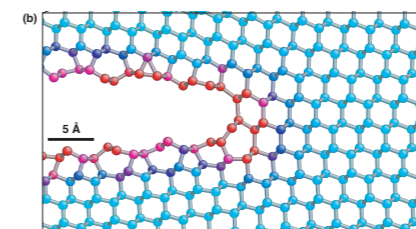
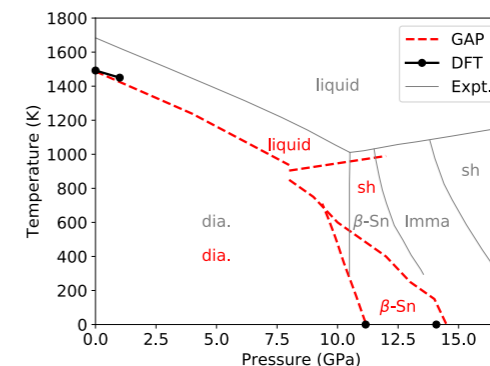
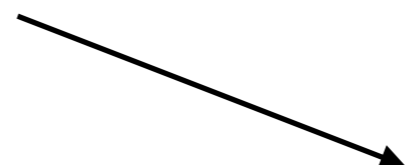
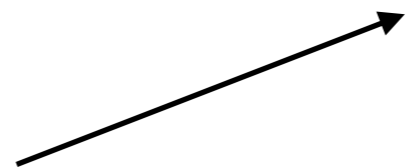
Go back to doing physics and materials modelling



Challenge: rigorous approximation

- How do we characterise relationship between fitting database and accuracy of predicted results?
- How do we give **global** accuracy guarantees?
- Inverse problem: what fitting database do we need to predict a given property ?

Structure type	# atoms	# structures	# environments	# representative pts	σ_{energy} σ_{force} σ_{virial}		
					default values: 0.001 0.1 0.05		
isolated atom	1	1	1	1			
diamond	2	104	208	6			
	16	220	3520	53			
	54	110	5940	58			
β -Sn	128	55	7040	92			
	2	60	120	32			
	16	220	3520	51			
simple hexagonal	54	110	5940	66			
	128	55	7040	157			
	1	110	110	13			
bcc	8	30	240	15			
	27	30	810	42			
	64	53	3392	89			
bc8	2	49	392	40			
	8	49	88	66			
	4	49	196	46			
fcc	2	49	88	28			
	12	49	588	94			
	64	69	4416	1114	0.003	0.15	0.2
liquid	128	7	896	323			
	64	31	1984	231	0.01	0.2	0.4
	216	128	27648	1719			
amorphous	144	29	4176	514			
	32	11	352	28			
	108	26	2808	338			
diamond surface (110) decohesion	16	11	176	8			
	24	11	264	10			
	96	47	4512	573			
diamond surface (111) unreconstructed	146	11	1606	62			
	96	50	4800	632			
	52	1	52	6			
diamond surface (111) Pandey reconstruction	63	100	6300	168			
	215	111	23865	405			
	214	78	16692	416			
diamond surface (111) DAS 3x3 unrelaxed	217	115	24955	605			
	200	7	1400	130			
	192	10	1920	185			
small (110) crack tip	144	19	2736	124			
	8	51	408	61			
	4	100	400	392	0.01	0.2	0.4
sp ² bonded							
sp bonded							
Total							



???