Suppressing Plasma Instability Through Constrained Optimization

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Learning Models from Data for Multi-Fidelity Fusion Plasma Physics

IPAM, UCLA

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Joint with: Lukas Einkemmer (Innsbruck) Qin Li (UW-Madison) Yunan Yang (Cornell) Jingcheng Lu (Minnesota) Jeff Calder (Minnesota)

Advertisement

Learning-enhanced: light training, un-supervised

Structure-preserving: conservation of mass/momentum/energy; dissipation of entropy

Deterministic particle method: scalability in high dimensions

Collisional Plamsa models

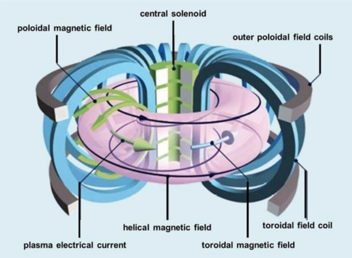
Check out **Yan Huang** 's poster

Outline

1. Problem set up
2. Open loop control
3. Dynamic feedback control
4. Conclusion & Discussion

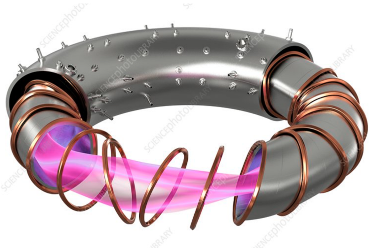
Outline

Nuclear Fusion



Basic tokamak components include the toroidal field coils (in blue), the central solenoid (in green), and poloidal field coils (in grey). The total magnetic field (in black) around the torus confines the path of travel of the charged plasma particles.

Image courtesy of EUROfusion



Challenges

- complexity of disruption causes
- high dimensional, high frequency control
- a wide range of plasma configurations

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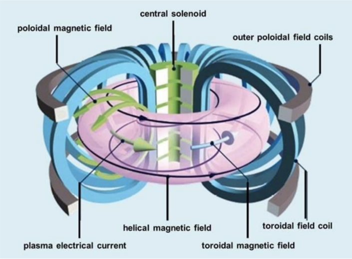

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Control

- Data driven chaotic control

LETTER

https://doi.org/10.1038/s41586-019-1116-4

Predicting disruptive instabilities in controlled fusion plasmas through deep learning

Julian Kates-Harbeck[1,2,3]*, Alexey Svyatkovskiy[4,5] & William Tang[3,4]

- "Active control"

Article

Magnetic control of tokamak plasmas through deep reinforcement learning

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Q: How to actively control the fusion process?

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- Data driven chaotic control

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Q: How to actively control the fusion process?

Our leverages

- Kinetic model & solver

	Magnetohydrodynamics	Two Fluids	Gyrokinetics	Kinetics	Everything
Description	The plasma is one continuous fluid - ions have all the mass, but electron carry all the current.	Break the ions & electrons into two continuous, mingling fluids.	Only track superparticles' straight motion - and ignore the corkscrewing.	Assign particles a speed and location based on a distribution. Track super particles through space.	Track every particle, at all times.
Strengthens	Easily solved.	Simple bulk effects like drift waves & reconnection can be understood.	Captures most of kinetic model, but much easier to solve - can model an entire Tokamak.	Many things captured, can get powerful results like the linear velocity-space instabilities.	Most accurate model possible.
Weakness	Most things not captured: most plasma waves, leakage, kinetic instabilities, structures etc.	Many things not captured: plasma instabilities, large effects & non-equilibrium effects. Assumes bell curves.	Non-physical behavior over long times: resonances & adiabatic invariants can be lost.	Tough to solve: hard to apply to full-size reactors. Loses some effects: like plasma microdensity and collective thomson scattering.	Typically impossible to solve.
Mathematics	Navier-stokes, Lorentz force, Maxwells' equations.	Navier-stokes, Lorentz force, Maxwells' equations.	Vlasov-Maxwell Expansion Equation	Vlasov-Maxwell Equation	Klimontovich Model

Plasma as a fluid (Chalkboard) — Plasma as a gas (Computer Required)

S i m p l i c i t y — D e t a i l

- PDE-constrained optimization
- Deep learning ?

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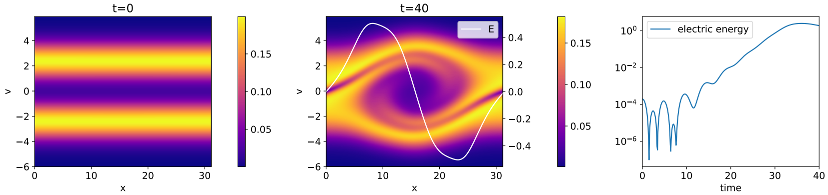
Plamsa instabilities

Vlasov-Poisson system

$$\begin{cases} \partial_t f + v \cdot \nabla_x f - \nabla_x V_f \cdot \nabla_v f = 0 \qquad (x,v) \in \mathbb{R}^{2d} \\ \Delta V_f = 1 - \rho_f(t,x) = 1 - \int f \mathrm{d}v \\ f(0,x,v) = f_0(x,v) = f^{\mathrm{eq}}(v) + \tilde{f}(x,v) \end{cases}$$



Plamsa instabilities

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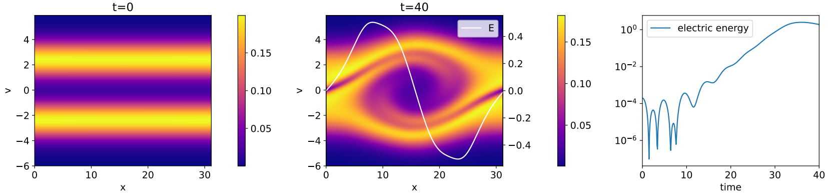

Control

$$\min_H J(f[H]) := \frac{1}{2}\|f[H](T) - f^{\mathrm{eq}}\|^2_{L^2(x,v)}$$

s.t. Vlasov-Poisson

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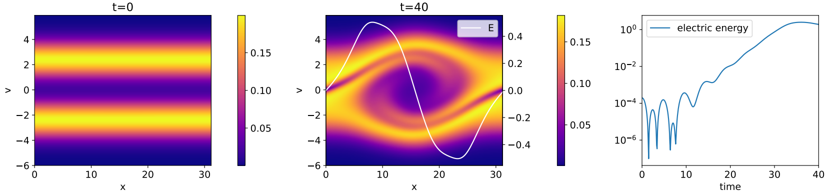

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Adjoint state method

$$\begin{cases} \min_H J(f) \\ \text{s.t.} \quad L(f,H) = 0 \end{cases} \quad \Longleftrightarrow \quad \min_H J(f[H])$$

Gradient descent

$$H^{k+1} = H^k - \nabla_H J(f[H^k])$$

$$\nabla_H J(f[H^k]) = \frac{\partial J}{\partial f}\frac{\partial f}{\partial H} \qquad ??$$

$$\nabla_H J(f[H^k])^\top \xi = \lim_{\epsilon \to 0} \frac{1}{\epsilon}\left(J(f[H^k + \epsilon\xi])) - J(f[H^k]))\right)$$ **too expensive!**

Adjoint state method

$$\tilde{J}(f,g,H) = J(f) + gL(f,H)$$

$$\nabla_H \tilde{J} = \frac{\partial \tilde{J}}{\partial f}\frac{\partial f}{\partial H} + \frac{\partial \tilde{J}}{\partial g}\frac{\partial g}{\partial H} + \frac{\partial \tilde{J}}{\partial H}$$

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Compute the gradient: continuous version

$$\min_H \quad J(f[H])$$

$$\text{s.t.} \quad \begin{cases} \partial_t f + v\partial_x f - (H + E[f])\partial_v f = 0 \\ E[f] = \partial_x G * (1 - \rho_f) \\ f(t=0,x,v) = f_0 = f^{\mathrm{eq}} + \tilde{f}, \end{cases}$$

- Lagrangian

$$L(f,H,g,\eta) = J(f) - \langle \partial_t f + v\partial_x f - (H + \partial_x G * (1-\rho_f))\partial_v f\,, g\rangle_{x,v,t} - \langle f(t=0) - f_0, \eta\rangle_{x,v}$$

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$$\begin{cases} \partial_t g + v\partial_x g - H\partial_v g + [G' * (\rho_f - 1)]\partial_v g + G' * \langle f, \partial_v g\rangle_v = 0 & \Longleftarrow \quad \frac{\partial L}{\partial f} = 0 \\ g(T,x,v) = f(T) - f^{\mathrm{eq}} & \Longleftarrow \quad \frac{\partial L}{\partial f(T)} = 0 \end{cases}$$

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Discretizations

- DTO vs OTD [1]
  - Optimize-then-Discretize

$$\nabla_H J = \langle \partial_v f \, , g \rangle_{v,t}$$

  - Discretize-then-Optimize

$$H = (H(x_1), \cdots, H(x_m))$$
$$\nabla_H J = (\partial_{H_1} J, \cdots \partial_{H_m} J) \qquad \partial_{H_i} J = \langle \partial_v f \, , g \rangle_{v,t}(x_i)$$

- Semi-Lagrangian
- Strang splitting

[1] Hinze, Pinnau, Ulbrich, Optimization with PDE Constrains, 2018

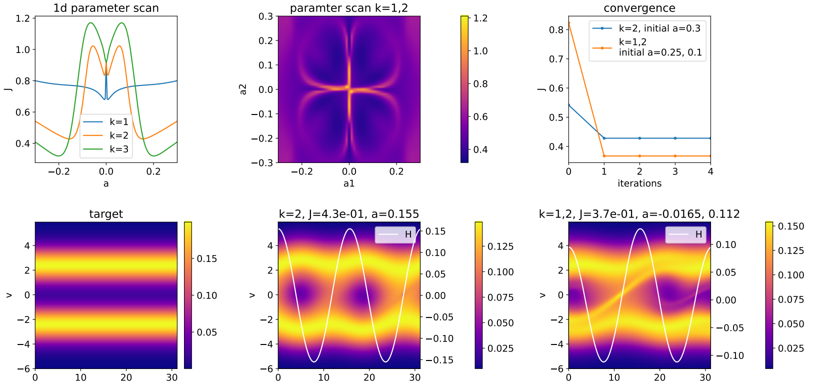
Suppress two-steam instability

Set up

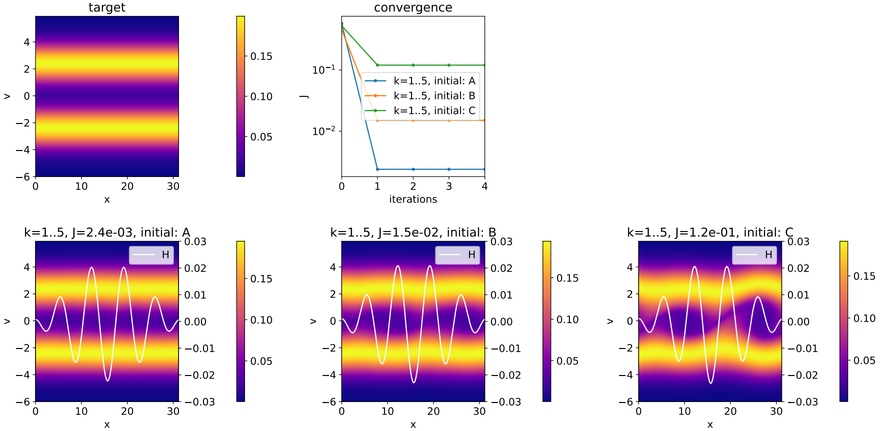
$$f(0,x,v) = (1+\alpha\cos(\beta x))f^{\mathrm{eq}}(v)\,, \quad (x,v)\in[0,2\pi/\beta]\times[-6,6]$$

$$f^{\mathrm{eq}}(v) = \frac{1}{2\sqrt{2\pi}}\left(\exp\left(-\frac{(v-\overline{v})^2}{2}\right)+\exp\left(-\frac{(v+\overline{v})^2}{2}\right)\right).$$

Opt landscape: $\min_H J(f) := \frac{1}{2}\|f(40,\cdot_x,\cdot_v) - f^{\mathrm{eq}}(\cdot_v)\|_2^2$, $H(x) = \sum a_k \cos(0.2kx)$



5d parameter: hybrid optimizer



Improvements

- Better parameterization
- Improve optimization
- Reduce computational cost

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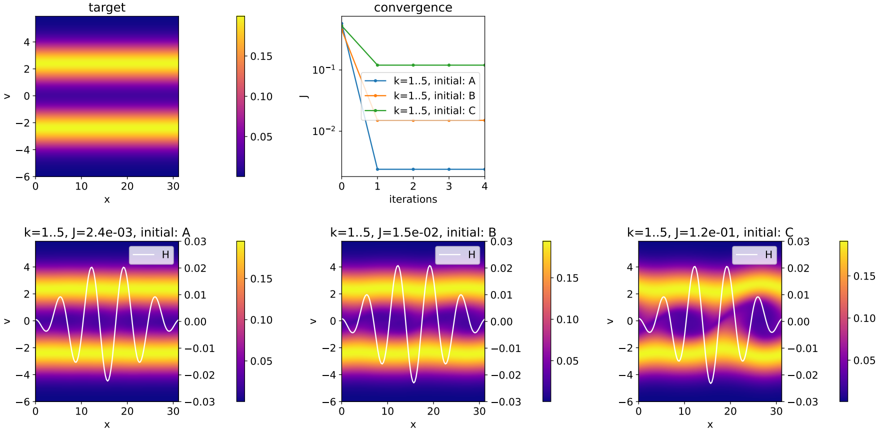

Improvements

- Better parameterization
- Improve optimization
- Reduce computational cost

Moment system

$$\begin{cases} \partial_t f + v\partial_x f + (E+H)\partial_v f = 0\,, \\ E = -\partial_x\phi, \quad \partial_x E = -\partial_x^2\phi = \rho - \rho_{ion}\,, \quad \rho = \int f \mathrm{d}v\,. \end{cases}$$

$$f(x,v,t) = \sum_{n=0}^{\infty} m_n(x,t)\widetilde{\mathrm{He}}_n(v)e^{-\frac{v^2}{2}}, \quad \widetilde{\mathrm{He}}_n(v) = \sqrt{\frac{1}{\sqrt{2\pi}n!}}(-1)^n e^{\frac{v^2}{2}}\frac{\mathrm{d}^n}{\mathrm{d}v^n}e^{-\frac{v^2}{2}}$$

- Let $m_n(x,t) = \int_{\mathbb{R}} f(x,v,t)\widetilde{\mathrm{He}}_n(v)\mathrm{d}v$, then

$$\begin{aligned} &\partial_t m_0 + \partial_x m_1 = 0, \\ &\partial_t m_1 + \partial_x m_0 + \sqrt{2}\partial_x m_2 = (E+H)m_0, \\ &\partial_t m_2 + \sqrt{2}\partial_x m_1 + \sqrt{3}\partial_x m_3 = \sqrt{2}(E+H)m_1, \\ &\vdots \\ &\partial_t m_N + \sqrt{N}\partial_x m_{N-1} + \sqrt{N+1}\partial_x m_{N+1} = \sqrt{N}(E+H)m_{N-1}\,. \end{aligned}$$

- $\int\int |f(x,v,t) - f^{\mathrm{eq}}(x,v)|^2 \mathrm{d}v\mathrm{d}x \le \sum_{n=0}^{\infty} ||m_n(\cdot,t) - \overline{m}_n(\cdot)||_2^2$ where $f^{\mathrm{eq}}(x,v) = \sum_{n=0}^{\infty}\overline{m}_n(x)\widetilde{\mathrm{He}}_n(v)e^{-\frac{v^2}{2}}$

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Quantifying errors

$$H(x;\alpha) = \sum_k \alpha_k \psi_k(x)$$

$$\min_\alpha \ \ \tfrac{1}{2}\sum_{n=0}^{N} ||m_n^N(T;\alpha) - \overline{m}_n||_2^2$$

$$\begin{cases} \partial_t \mathbf{m}_N^N + A_N \partial_x \mathbf{m}_N^N + \sqrt{N+1}\partial_x \overline{m}_{N+1}\mathbf{e}_{N+1} \\ \qquad\qquad = (E_N + H) D_N \mathbf{m}_N^N, \\ E_N = -\partial_x \phi_N, \quad -\partial_x^2 \phi_N = \rho_N - \rho_{ion} \\ \rho_N(x,t) = (2\pi)^{\frac{1}{4}} m_0^N(x,t), \end{cases}$$

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- Target error: $\quad Ag := \partial_v g + vg$

$$\sum_{n=N+1}^{\infty} \|m_n - \overline{m}_n\|_{L_x^2}^2 \lesssim N^{-k}\|\mathrm{A}^k(f-\mu)\|^2_{w_v^{-1}L_x^2}$$

- Optimizer error [2]:

$$|\mathcal{L}[\widehat{H}_{mom}] - \mathcal{L}[\widehat{H}_{VP}]| \lesssim \frac{\sqrt{N+1}}{N^{k/2}}\|\mathrm{A}^k f_N\|_{w_v^{-1}L_x^2}$$

[2] Einkemmer, Li, Mouhot, Yue, 2025

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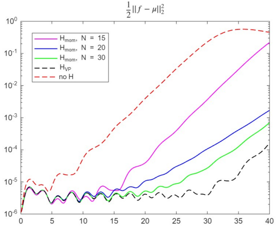
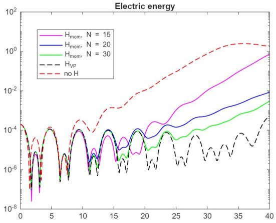
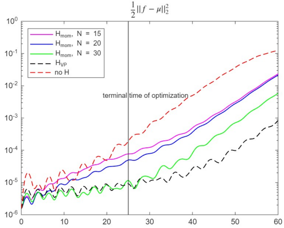
Results

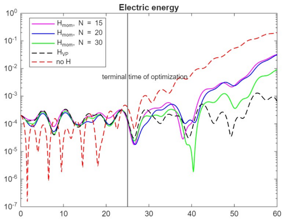
Two-stream instability





Bump-on-tail instability





Outline

Feedback control

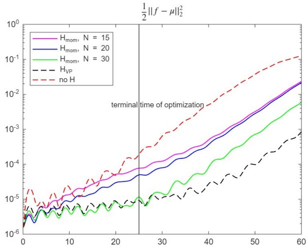
Main drawback

- Lose effect beyond T:

$$\min_H \int_0^T \cdots$$

- How to include time dependence?

$$H(x) = \sum_k \alpha_k \psi_k(x)$$



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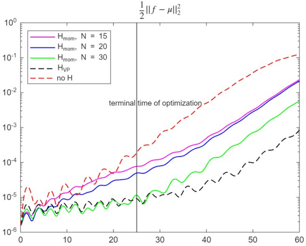

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Neural operators [3]

- DeepONet

$$\mathcal{P}_{NN}(u)(\boldsymbol{y};\theta,w,\xi) = \sum_{k=1}^{p} \underbrace{b_k^{NN}(u(\boldsymbol{x}_1),\cdots,u(\boldsymbol{x}_N);\theta,w)}_{\text{branch net}} \underbrace{t_k^{NN}(\boldsymbol{y};\xi)}_{\text{trunck net}}$$

- Fourier Neural Operators

$$g(\boldsymbol{x}) = \mathcal{F}^{-1}(R_\phi \cdot \mathcal{F}(u))(\boldsymbol{x}), \quad R_\phi(k,(\mathcal{F}u)(\boldsymbol{k})) := \phi_\theta(\boldsymbol{k},(\mathcal{F}u)(\boldsymbol{k}))$$

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$$Q = XW^Q, K = XW^K, V = XW^V, \quad \mathrm{Attn}(Q,K,V) = \mathrm{softmax}\left(\frac{QK^\top}{\sqrt{h}}\right)V$$

Q: Is there any structure we can leverage from the problem itself?

[3] Lu et al 2019
Li et al 2021
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Hint from the model

- Linearization: $f = f^{\mathrm{eq}} + \delta f$, $\|\delta f\| \ll \|f^{\mathrm{eq}}\|$

$$\mathcal{J}(\delta f, H) = \int_0^\infty \frac{1}{2}||\delta f(\cdot,\cdot,t)||^2_{x,v} + \frac{\gamma}{2}||H(\cdot,t)||^2_x \mathrm{d}t,$$

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- optimal control H^*

$$\begin{cases} \partial_t \lambda^* + v\partial_x \lambda^* - \langle \partial_{x_1} G(\cdot,x), \int \lambda^*(\cdot,v',t)\partial_v f^{\mathrm{eq}}(v')\mathrm{d}v' \rangle = -\delta f\,, \\ \gamma H^* - \int \lambda^* \partial_v f^{\mathrm{eq}} dv = 0\,. \end{cases}$$

$\Longrightarrow$ a single-layer low-rank neural operator

$$H[\delta f(t)](x;\theta) = \sum_{k=1}^{r} \phi_k(x;\theta_\phi) \iint \psi_k(y,v;\theta_\psi)\delta f(y,v,t)\mathrm{d}y\mathrm{d}v, \quad \theta = \{\theta_\phi, \theta_\psi\}.$$

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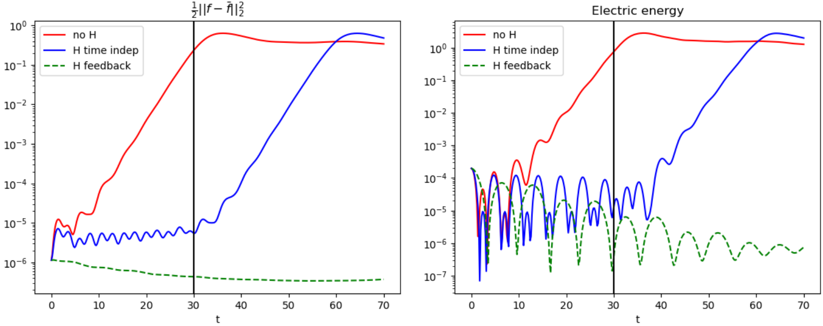
Two-stream instabilities

$$H_{indep}(x) = \sum_{k=1}^{15} \theta_k \sin(\tfrac{k}{5}x) + \sum_{k=0}^{15} \theta_{k+16} \cos(\tfrac{k}{5}x).$$



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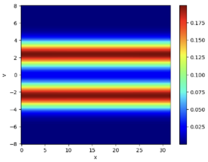
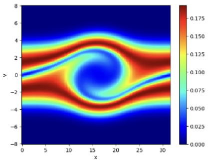
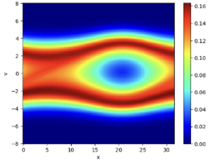


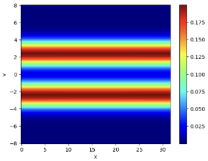
(a) no H, $t = 0$



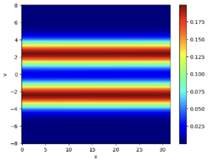
(b) no H, $t = 35$



(c) no H, $t = 70$



(d) H time-independent, $t = 0$



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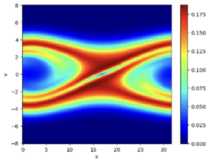
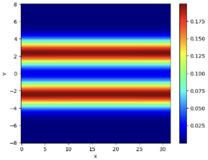
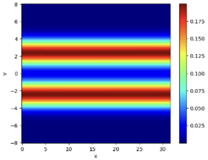


(f) H time-independent, $t = 70$



(g) H feedback, $t = 0$



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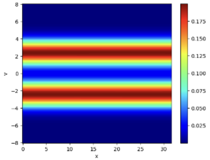
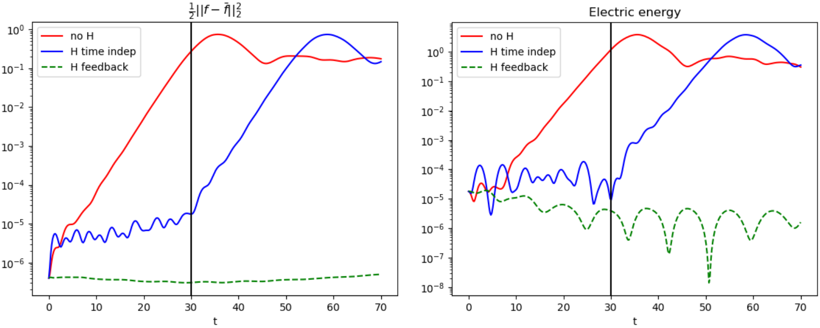


(i) H feedback, $t = 70$

Bump-on-tail instabilities

$$f^{\mathrm{eq}}(v) = \frac{0.9}{\sqrt{2\pi}} \exp\left(-\frac{(v+2)^2}{2}\right) + \frac{0.1}{\sqrt{0.5\pi}} \exp\left(-\frac{(v-3.5)^2}{0.5}\right)$$



Robust control under noisy feedback

$$\delta f^{\sigma}(x,v,t) = \delta f(x,v,t) + \sigma \mathcal{N}_{x,v,t} \,.$$

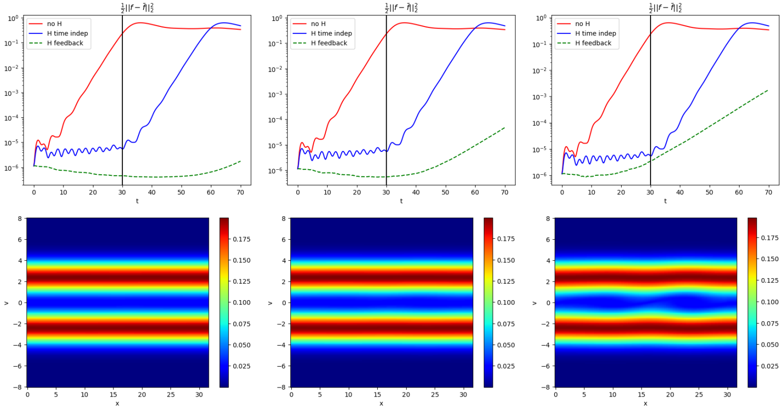


Figure: Left $\sigma = 2\times 10^{-5}$ $(0.1 \cdot ||\delta f_0||_\infty)$, middle: $\sigma = 5\times 10^{-5}$ $(0.25 \cdot ||\delta f_0||_\infty)$, right: $\sigma = 1\times 10^{-4}$ $(0.5 \cdot ||\delta f_0||_\infty)$.

A quasi-optimal universal control

- Perturbation $\delta f := f - f^{\mathrm{eq}}$ satisfies

$$\partial_t \delta f + v\partial_x \delta f + \overline{E}\partial_v \delta f + (\delta E + H)\partial_v f = 0$$

$$\delta E[\delta f(t)](x) = -\partial_x(-\partial_x^2)^{-1}\delta\rho(x,t), \quad \delta\rho(x,t) = \int \delta f(x,v,t)\mathrm{d}v.$$

- Design the electric field

$$H[\delta f(t)](x) = \underbrace{-\delta E[\delta f(t)](x)}_{\text{cancel self-generated } \delta E} + \underbrace{\delta H[\delta f(t)](x)}_{\text{additional dissipation}}$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}||\delta f(t)||_2^2 = -\int \delta H(x,t)\Big[\int \delta f(x,v,t)\partial_v f^{\mathrm{eq}}(x,v)\mathrm{d}v\Big]\mathrm{d}x.$$

$$\Longrightarrow \quad \delta H[\delta f(t)](x) = \gamma \int \delta f(x,v,t)\partial_v f^{\mathrm{eq}}(x,v)\mathrm{d}v, \quad \gamma > 0$$

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$$\delta E[\delta f(t)](x) = -\partial_x(-\partial_x^2)^{-1}\delta\rho(x,t), \quad \delta\rho(x,t) = \int \delta f(x,v,t)\mathrm{d}v.$$

- Design the electric field

$$H[\delta f(t)](x) = \underbrace{-\delta E[\delta f(t)](x)}_{\text{cancel self-generated } \delta E} + \underbrace{\delta H[\delta f(t)](x)}_{\text{additional dissipation}}$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}||\delta f(t)||_2^2 = -\int \delta H(x,t)\Big[\int \delta f(x,v,t)\partial_v f^{\mathrm{eq}}(x,v)\mathrm{d}v\Big]\mathrm{d}x.$$

$$\Longrightarrow \quad \delta H[\delta f(t)](x) = \gamma \int \delta f(x,v,t)\partial_v f^{\mathrm{eq}}(x,v)\mathrm{d}v, \quad \gamma > 0$$

A quasi-optimal universal control

- Perturbation $\delta f := f - f^{\mathrm{eq}}$ satisfies

$$\partial_t \delta f + v \partial_x \delta f + \overline{E} \partial_v \delta f + (\delta E + H) \partial_v f = 0$$

$$\delta E[\delta f(t)](x) = -\partial_x (-\partial_x^2)^{-1} \delta\rho(x,t), \quad \delta\rho(x,t) = \int \delta f(x,v,t) \mathrm{d}v.$$

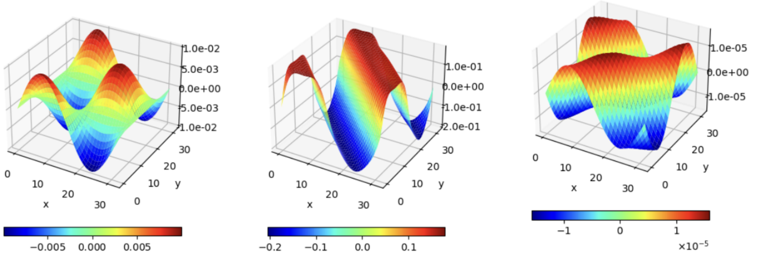
- Design the electric field

$$H[\delta f(t)](x) = \underbrace{-\delta E[\delta f(t)](x)}_{\text{cancel self-generated } \delta E} + \underbrace{\delta H[\delta f(t)](x)}_{\text{additional dissipation}}$$

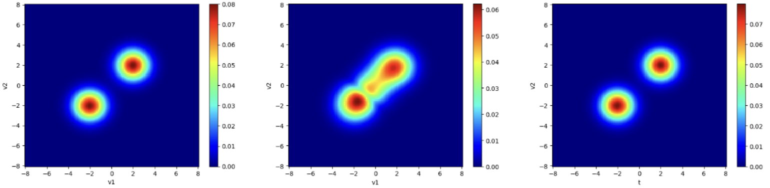
$$\frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} ||\delta f(t)||_2^2 = -\int \delta H(x,t) \Big[\int \delta f(x,v,t) \partial_v f^{\mathrm{eq}}(x,v) \mathrm{d}v \Big] \mathrm{d}x.$$

$$\Longrightarrow \delta H[\delta f(t)](x) = \gamma \int \delta f(x,v,t) \partial_v f^{\mathrm{eq}}(x,v) \mathrm{d}v, \quad \gamma > 0$$

A 2D example



(a) $t = 0$ (b) $t = 30$, no H (c) $t = 30$, H cancellation-based



(d) $t = 0$ (e) $t = 30$, no H (f) $t = 30$, H cancellation-based

Outline

Conclusion & Discussion

Conclusion

- kinetic equation constrained optimization
- moment control
- feedback control

Limitations & Future work

- Model
  - Magnetic effects
  - Unknown boundary conditions, actuator delays, turbulence
- Control:
  - δf is impossible to measure
  - time delay, better generalization
- Optimization
  - parameterization, objective function, optimizer
  - analyze the optimization landscape $\Longrightarrow$ better parameterization

Reference

- L. Einkemmer, Q. Li, LW and Y. Yang, Suppressing instability in a Vlasov-Poisson system by an external electric field through constrained optimization, J. Comput. Phys. 2024.
- J. Lu, LW, J. Calder, Controlling instability in the Vlasov-Poisson system through moment-based optimization, arXiv:2508.18412
- J. Lu, LW, J. Calder, Dynamical feedback control with operator learning for the Vlasov-Poisson system, arXiv:2509.23063.

Thank You

Reference

- L. Einkemmer, Q. Li, LW and Y. Yang, Suppressing instability in a Vlasov-Poisson system by an external electric field through constrained optimization, J. Comput. Phys. 2024.
- J. Lu, LW, J. Calder, Controlling instability in the Vlasov-Poisson system through moment-based optimization, arXiv:2508.18412
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Thank You