PDE-constrained optimization

In plasma simulation

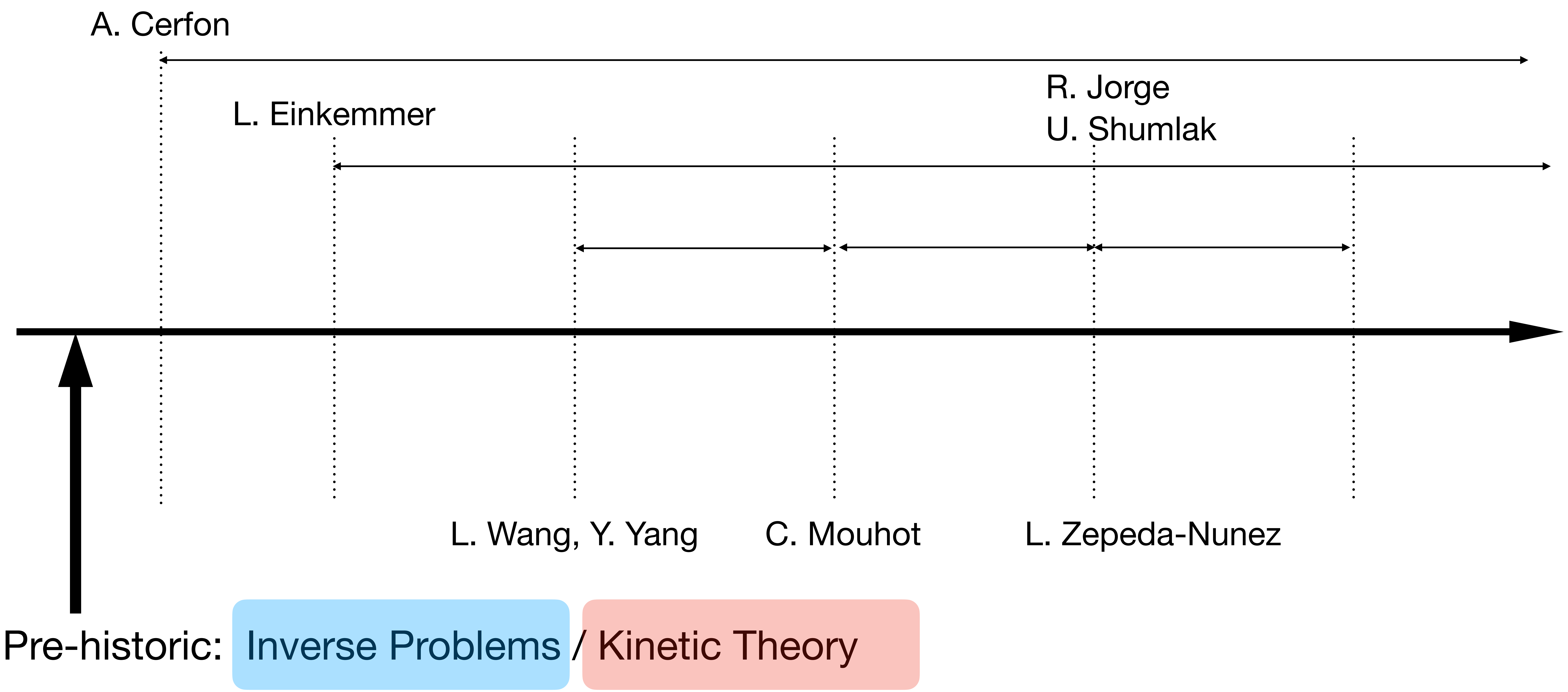
Qin Li, UW-Madison

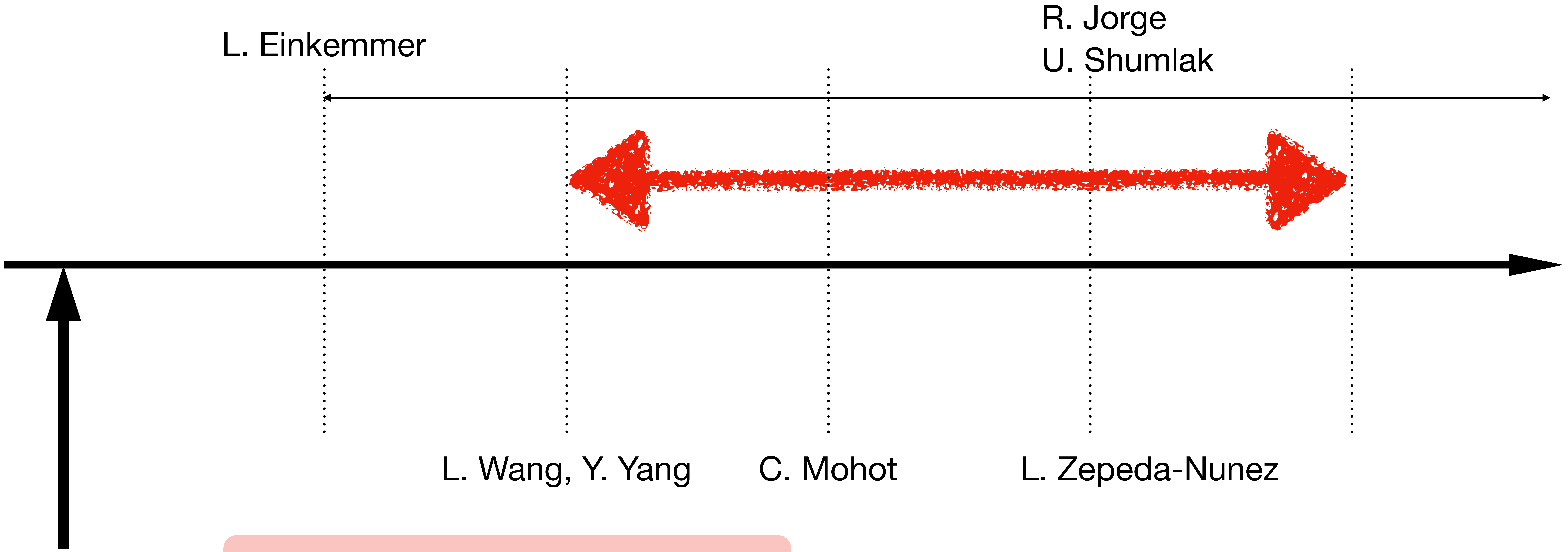
PDE-constrained optimization

In plasma simulation

10:15 - 11:00	Li Wang (University of Minnesota, Twin Cities) Suppressing Plasma Instability Through Constrained Optimization PDF

11:30 - 12:15	Qin Li (University of Wisconsin-Madison) Stabilizing plasma instabilities cast as PDE-constrained optimization


A. Cerfon
L. Einkemmer
R. Jorge
U. Shumlak
L. Wang, Y. Yang
C. Mouhot
L. Zepeda-Nunez
Pre-historic:
Inverse Problems / Kinetic Theory



Pre-historic: PDE-constrained optimization / Kinetic Theory

PDE-constrained optimization

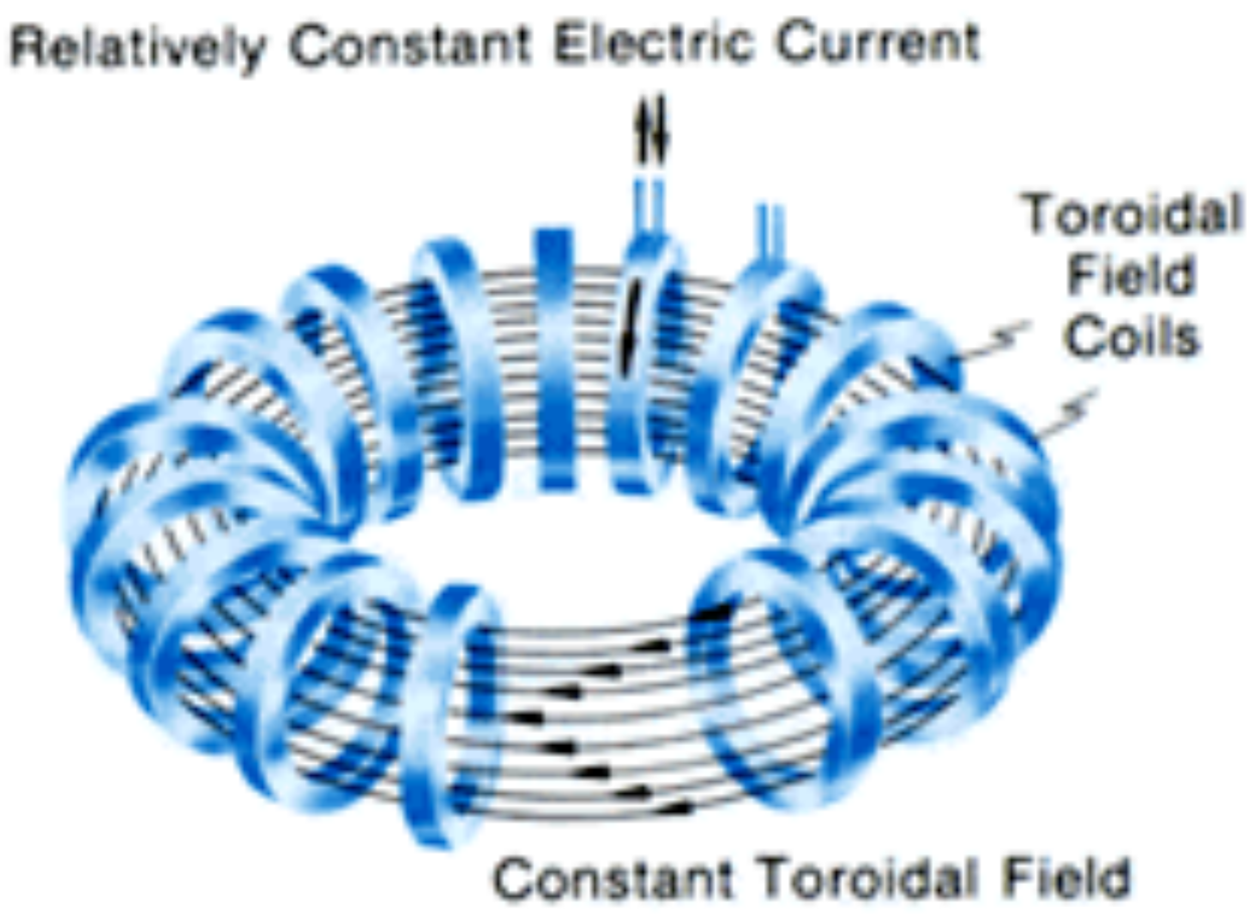
$$\min_E \mathcal{J}[f] \qquad \text{s.t.} \qquad \mathcal{N}[f; E] = S$$

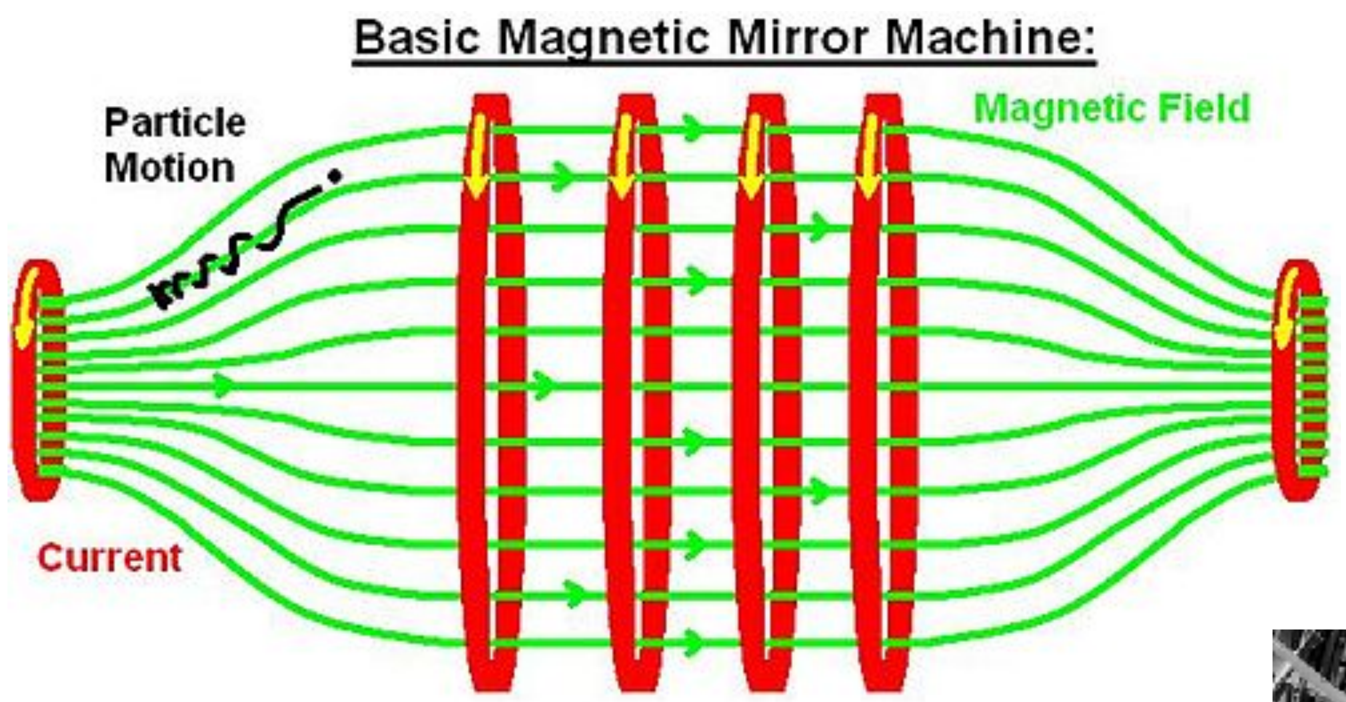
- Inverse problems
- Control problems

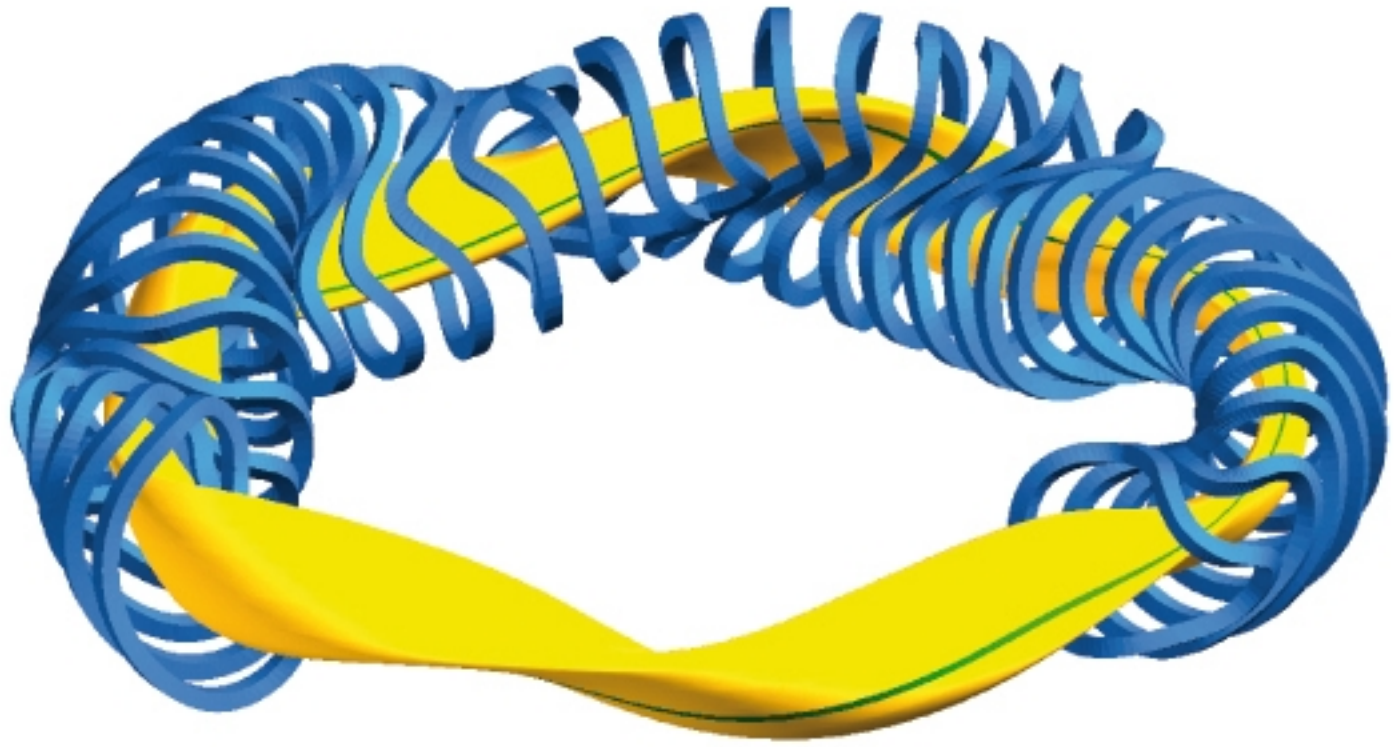
Inverse problems → Find parameter that minimizes mismatch

Control problems → Find parameter that achieves desired property

PDE-constrained optimization — control problems







$$\min_{E,Q} \mathcal{J}[f]$$

s.t. $\partial_t f + v \cdot \nabla_x f + E \cdot \nabla_v f = Q[f]$

PDE-constrained optimization

$\min_E \mathcal{J}[f]$ s.t. $\mathcal{N}[f; E] = S$

$$\mathcal{L}[f, E, g] = \mathcal{J}[f] - \langle \mathcal{N}[f; E] - S \, , g \rangle$$

$$\min \mathcal{L}[f, E, g]$$

$$\partial_{f,E,g} \mathcal{L} = 0$$

PDE-constrained optimization

$\min_E \mathcal{J}[f]$ s.t. $\mathcal{N}[f;E] = S$

$$\mathcal{L}[f,E,g] = \mathcal{J}[f] - \langle \mathcal{N}[f;E] - S\,, g \rangle$$

$\partial_g \mathcal{L} = 0 \quad \Rightarrow \quad$ eqn

$\partial_f \mathcal{L} = 0 \quad \Rightarrow \quad$ adjoint

$\partial_E \mathcal{L} = 0 \quad \Rightarrow \quad$ KKT

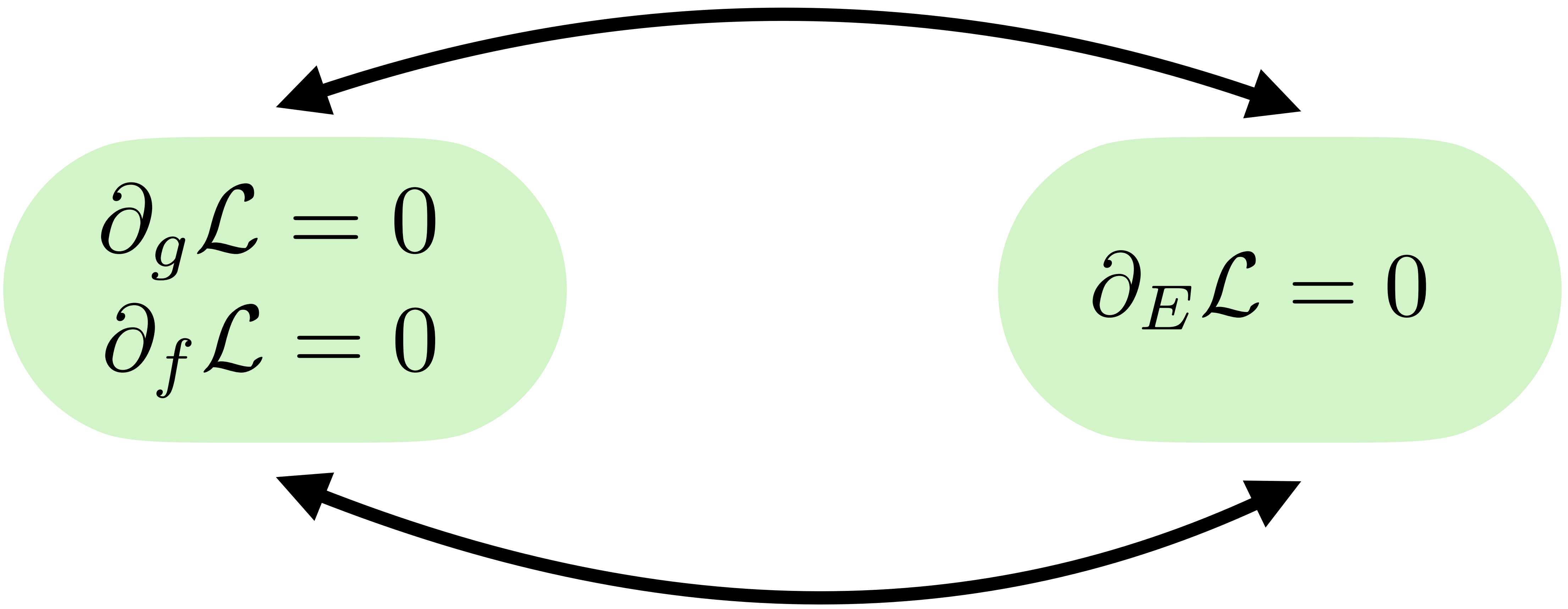
$$\partial_{f,E,g} \mathcal{L} = 0$$

PDE-constrained optimization

$$\min_E \mathcal{J}[f] \qquad \text{s.t.} \qquad \mathcal{N}[f;E] = S$$

$$\mathcal{L}[f,E,g] = \mathcal{J}[f] - \langle \mathcal{N}[f;E] - S\,, g \rangle$$

$\partial_g \mathcal{L} = 0$	$\Rightarrow$	eqn
$\partial_f \mathcal{L} = 0$	$\Rightarrow$	adjoint
$\partial_E \mathcal{L} = 0$	$\Rightarrow$	KKT



PDE-constrained optimization

$$\min_E \mathcal{J}[f]$$

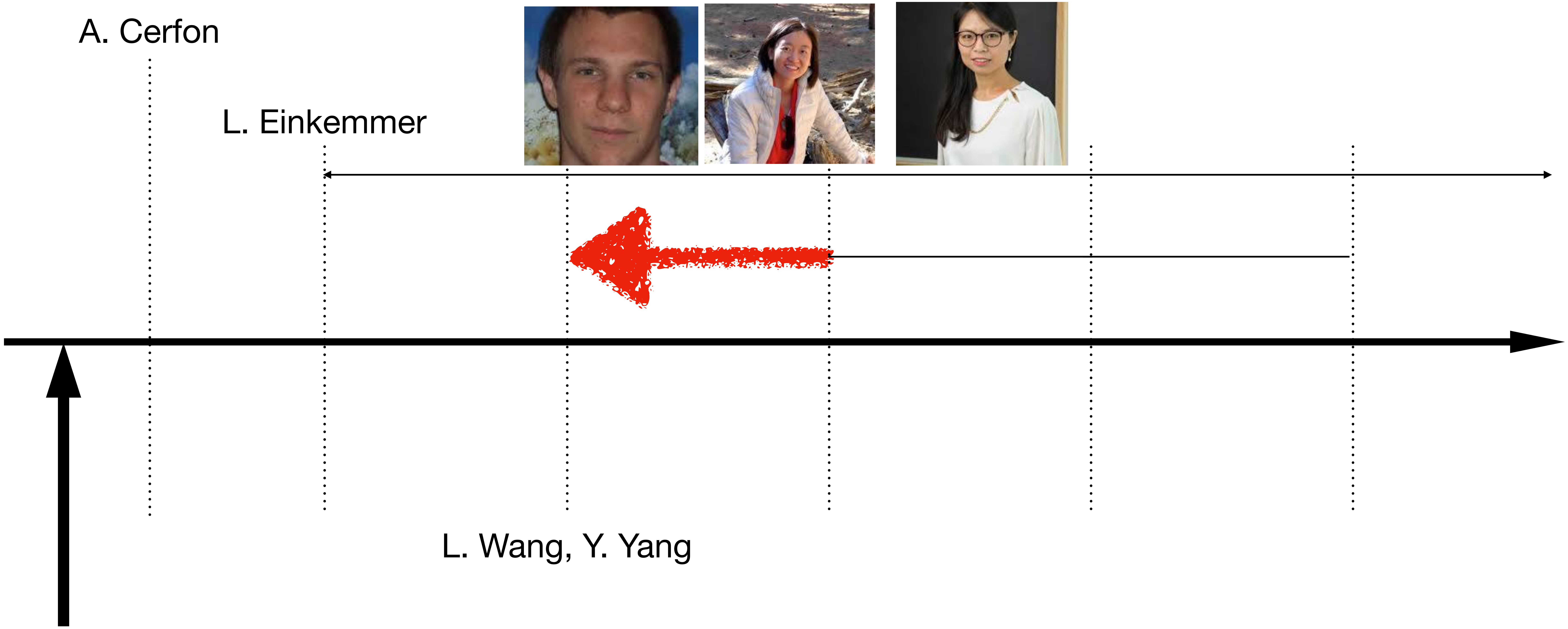
$$E^{n+1} = E^n - \eta \left.\frac{\delta \mathcal{J}}{\delta E}\right|_{E^n}$$

Forward solver + Adjoint (backward) solver

$$\mathcal{N}[f; E] = S$$

$$\mathcal{N}^*[g] = 0$$

PDE-constrained optimization in plasma



Pre-historic: PDE-constrained optimization / Kinetic Theory

- L. Einkemmer, Q. Li, L. Wang and Y. Yang. Suppressing instability in a Vlasov–Poisson system by an external electric field through constrained optimization, JCP, 498

PDE-constrained optimization — Vlasov stablization

$$\begin{cases} \partial_t f & +v\nabla_x f - (E+H)\nabla_v f = 0 \\ E & = \nabla_x V \\ \Delta V & = 1 - \int f dv \end{cases}$$

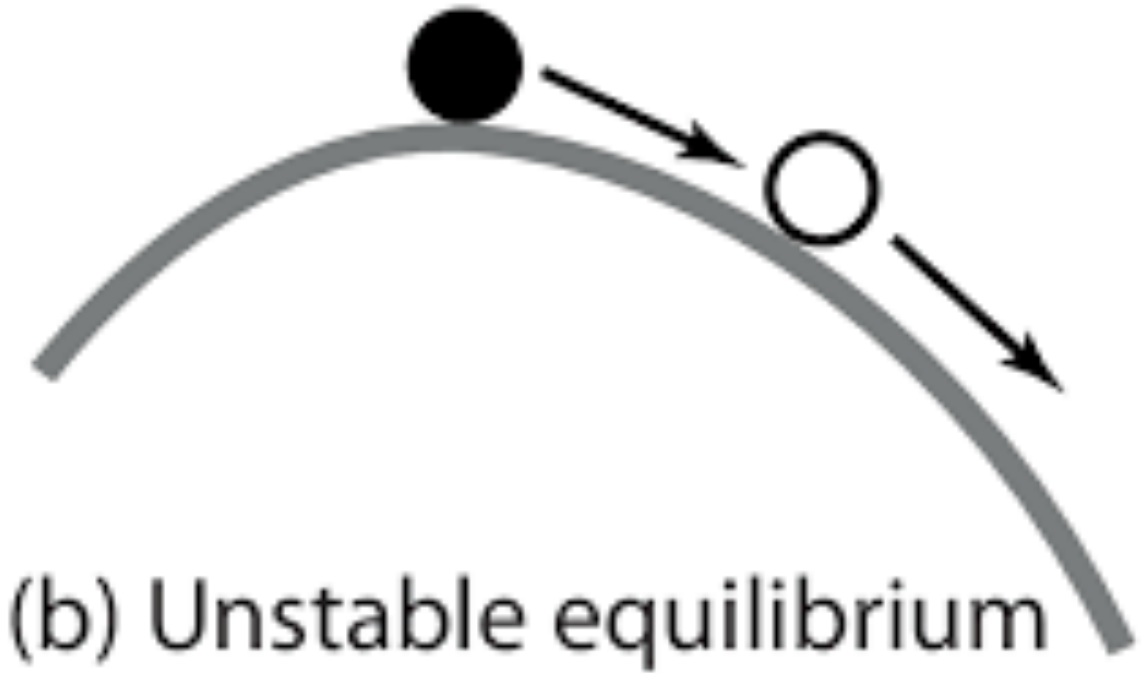
$$\partial_t f + \underbrace{v \cdot \nabla_x f}_{\dot{x}=v} - \underbrace{(\nabla_x G(x;y) * (1-\rho) + H) \cdot \nabla_v f}_{\dot{v}=-E-H} = 0$$

PDE-constrained optimization — Vlasov stablization

$$\partial_t f + \underbrace{v \cdot \nabla_x f}_{\dot{x}=v} - \underbrace{(\nabla_x G(x;y) * (1-\rho) + H) \cdot \nabla_v f}_{\dot{v}=-E-H} = 0$$

$$\text{if} \quad H = 0\,, \qquad f_{\text{eq}} = h(v) \; + \epsilon$$

(a) stable equilibrium



(b) Unstable equilibrium

PDE-constrained optimization — Vlasov stablization

$$\partial_t f + \underbrace{v \cdot \nabla_x f}_{\dot{x}=v} - \underbrace{(\nabla_x G(x;y) * (1-\rho) + H) \cdot \nabla_v f}_{\dot{v}=-E-H} = 0$$

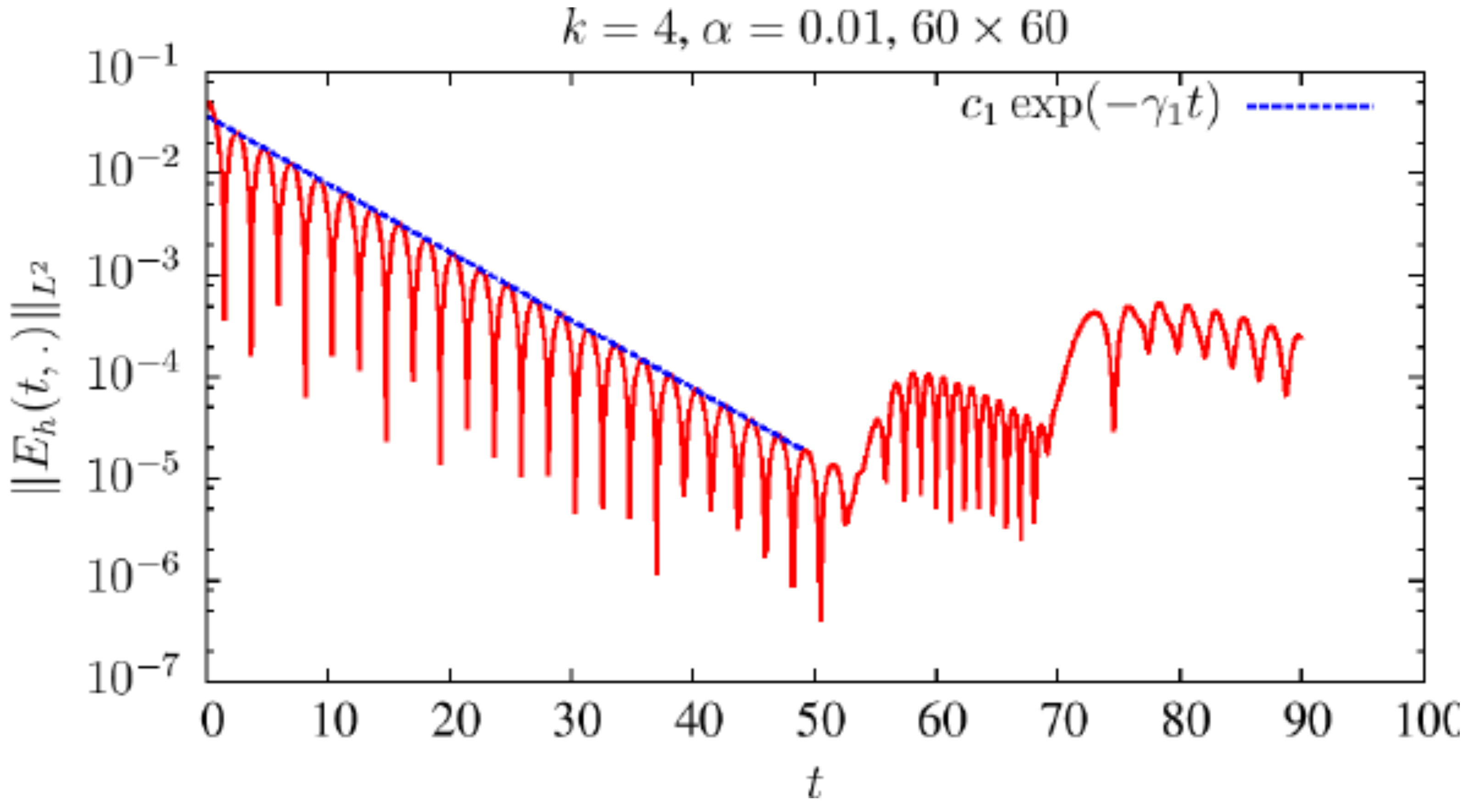
$$\text{if} \quad H = 0\,, \qquad f_{\mathrm{eq}} = h(v) \; {\color{red}+\epsilon}$$

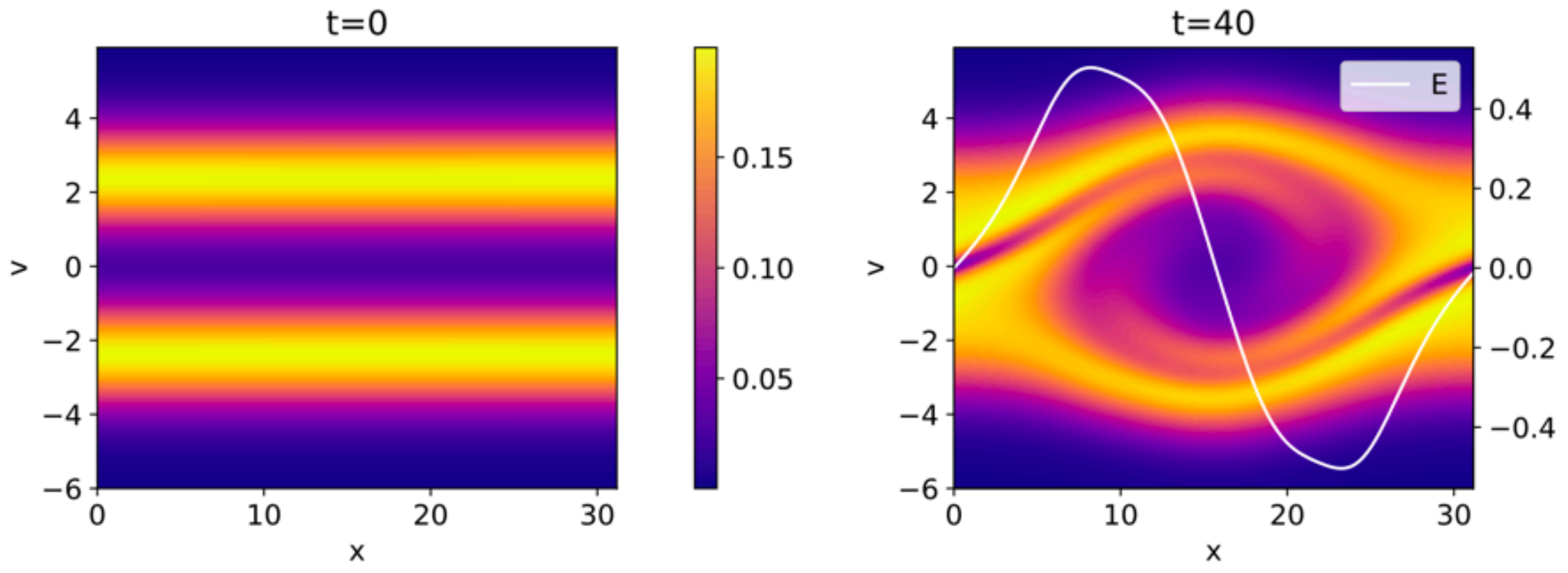
Stable

(Landau damping)

unstable

(two-stream instability)





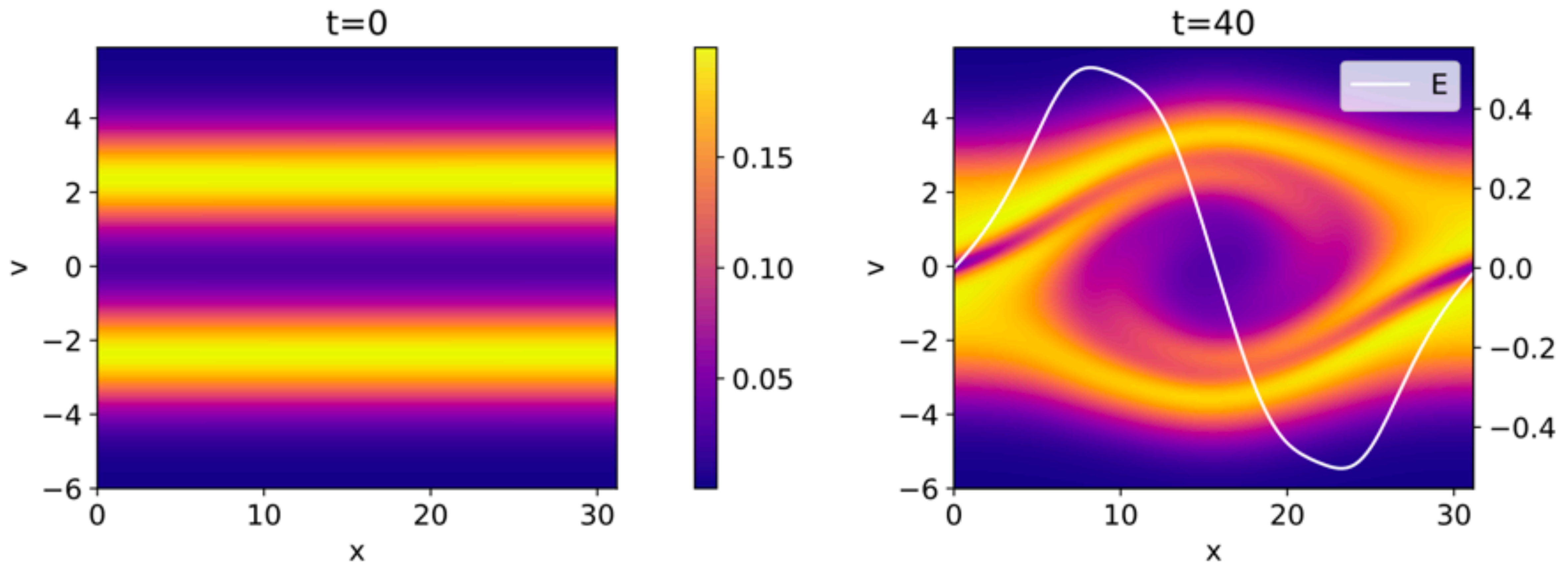
PDE-constrained optimization — Vlasov stablization

$$\partial_t f + \underbrace{v \cdot \nabla_x f}_{\dot{x}=v} - \underbrace{(\nabla_x G(x;y) * (1-\rho) + H) \cdot \nabla_v f}_{\dot{v}=-E-H} = 0$$

$$\text{if} \quad H = 0\,, \qquad f_{\text{eq}} = h(v) \; {\color{red}+\epsilon}$$

unstable
(two-stream instability)

Design H so to suppress instability



PDE-constrained optimization — Vlasov stablization

$$\min_E \mathcal{J}[f] \qquad \text{s.t.} \qquad \begin{cases} \partial_t f & +v\nabla_x f - (E+H)\nabla_v f = 0 \\ E & = \nabla_x V \\ \Delta V & = 1 - \int f dv \end{cases}$$

$$\mathcal{L} = \mathcal{J}[f] - \langle \text{eqn}\,, g\rangle - \langle \text{initial}\,, \eta\rangle$$

$$\partial_g \mathcal{L} = 0 \quad \Rightarrow \quad \text{eqn}$$

$$\partial_t g + v\partial_x g - H\partial_v g + [G' * (\rho_f - 1)]\partial_v g + G' * \langle f\partial_v g\rangle = -\frac{\partial J}{\partial f}(t,x,v)$$

$$\partial_E \mathcal{L} = 0 \quad \Rightarrow \quad \text{KKT}$$

$$\partial_{f,E,g}\mathcal{L} = 0$$

PDE-constrained optimization — Vlasov stablization

$$\min_H \mathcal{J}[f]$$

$$H^{n+1} = H^n - h \left. \frac{\delta \mathcal{J}}{\delta H} \right|_{H^n}$$

↑

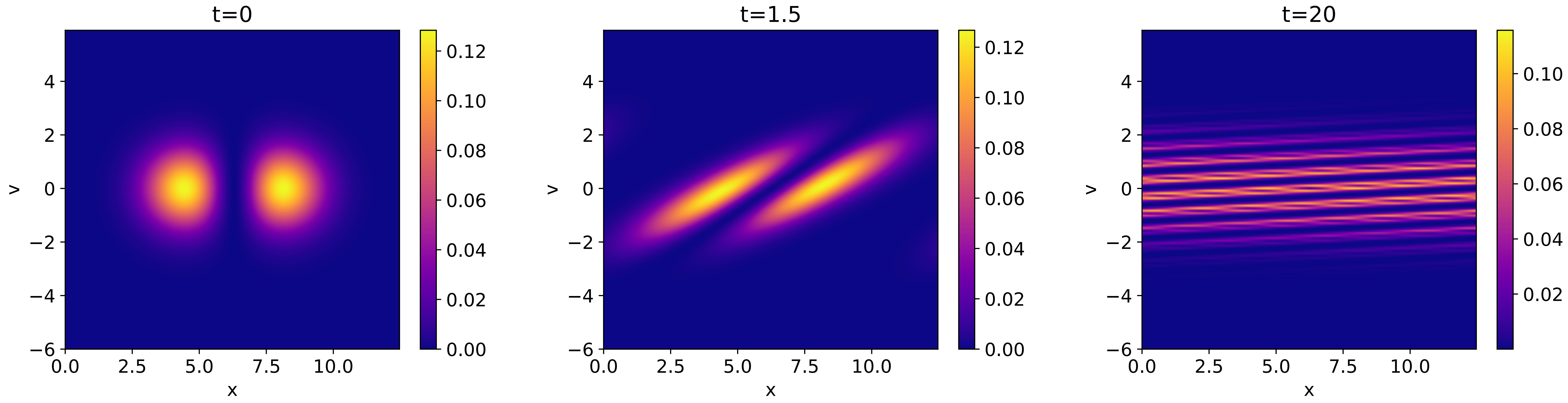
Forward solver + Adjoint (backward) solver

$$\partial_t f + v\partial_x g - H\partial_v f + [G' * (\rho_f - 1)]\partial_v f = 0$$

$$\partial_t g + v\partial_x g - H\partial_v g + [G' * (\rho_f - 1)]\partial_v g + G' * \langle f\partial_v g\rangle = -\frac{\partial J}{\partial f}(t,x,v)$$

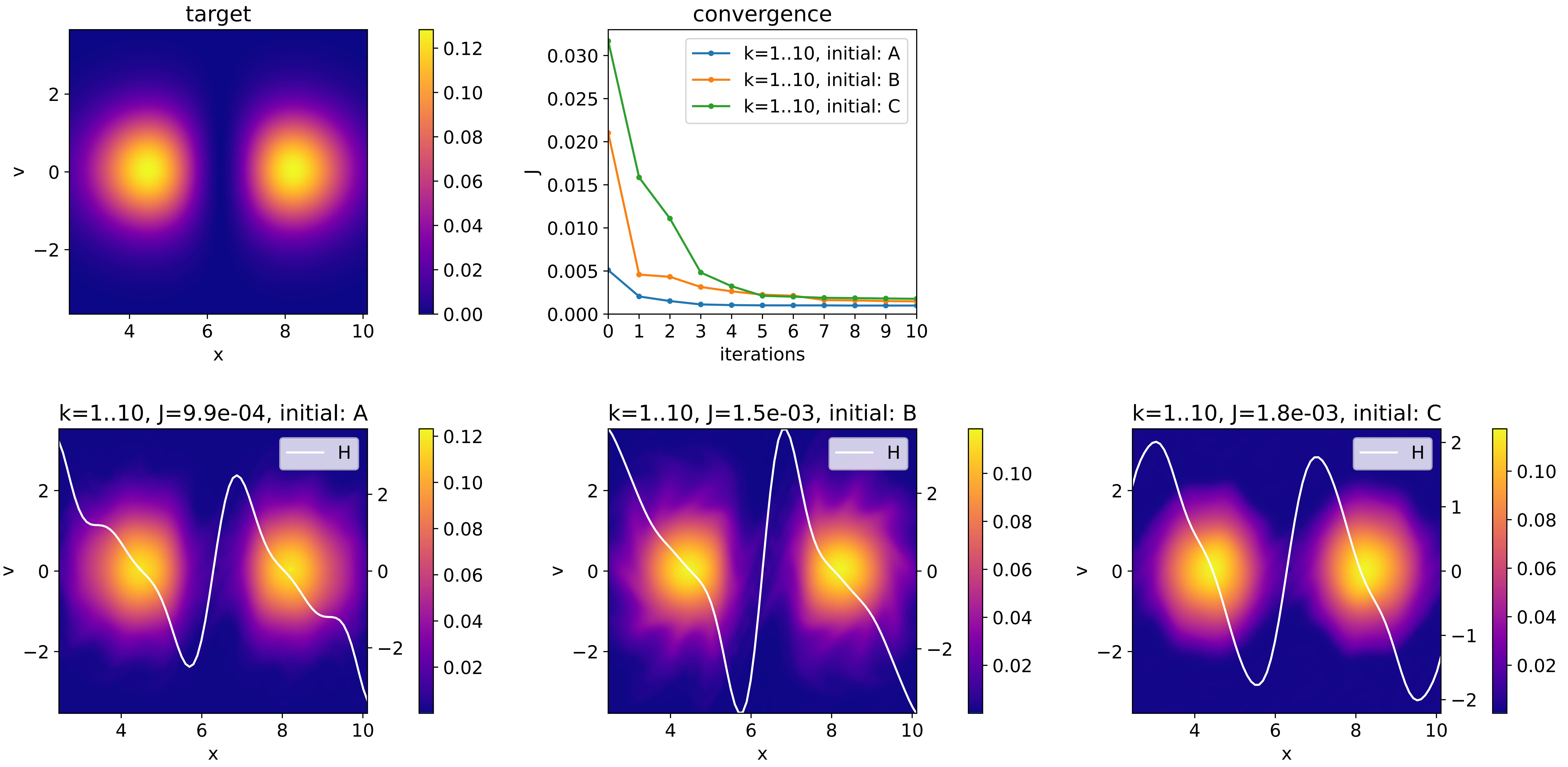
PDE-constrained optimization — Vlasov stablization

Focused beam



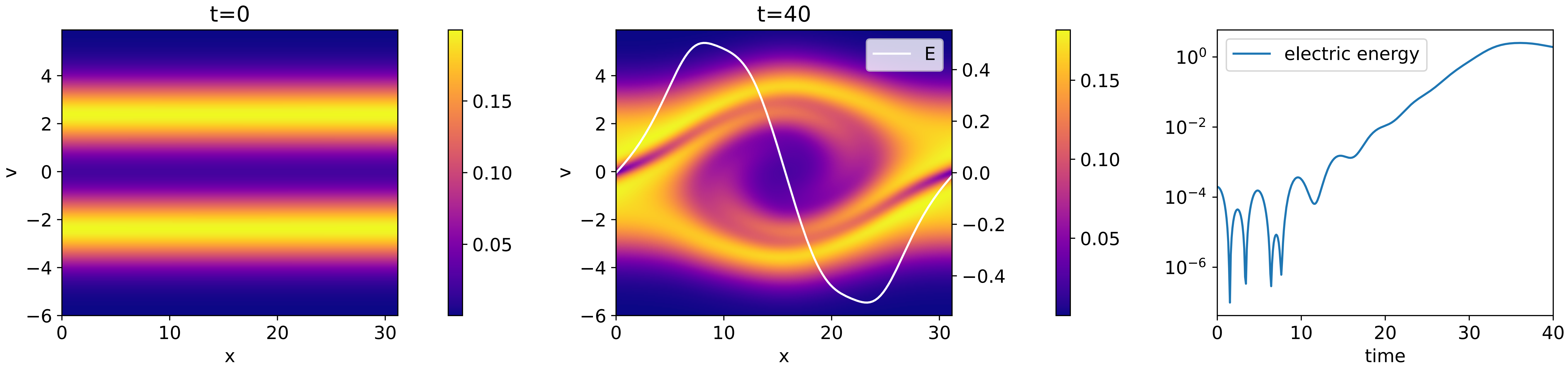
PDE-constrained optimization — Vlasov stablization

Focused beam



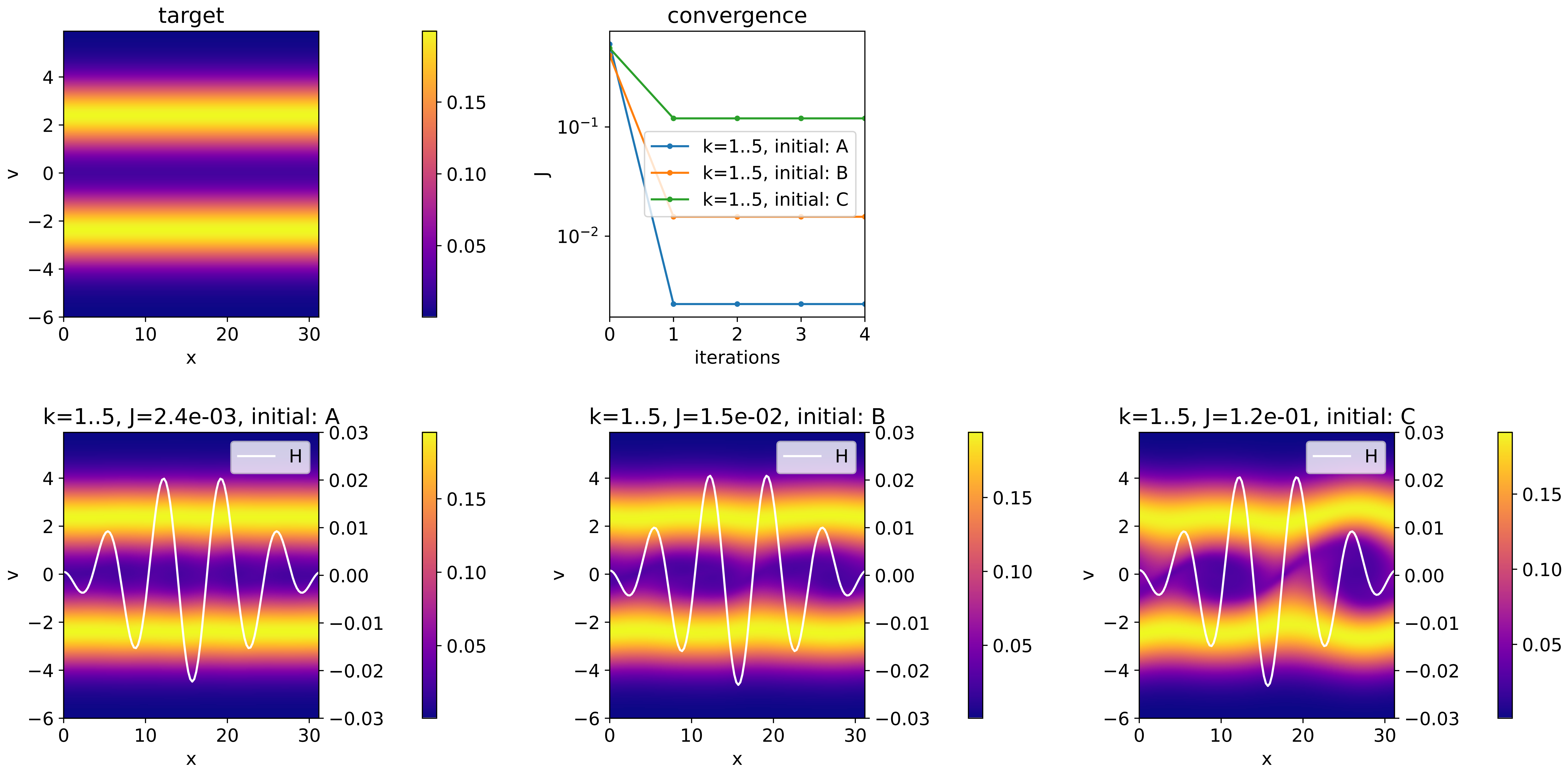
PDE-constrained optimization — Vlasov stablization

2-stream



PDE-constrained optimization — Vlasov stablization

2-stream



Failed attempts

- External field needs to be parameterized

$$H(x) \quad \text{vs.} \quad H(x) = \sum_k a_k \sin(kx)$$

L. Einkemmer, Q. Li, L. Wang and Y. Yang. Suppressing instability in a Vlasov–Poisson system by an external electric field through constrained optimization, JCP, 498

PDE-constrained optimization — Vlasov stablization

Failed attempts

- External field needs to be parameterized
- Over-parameterization may help

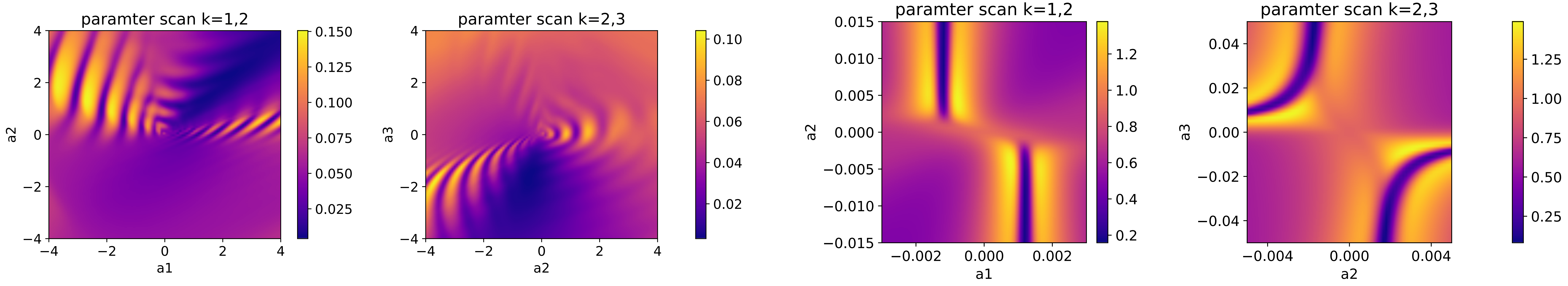
$$H(x) = \sum_{k=1}^{3} a_k \sin(kx) \qquad \text{vs.} \qquad H(x) = \sum_{k=1}^{10} a_k \sin(kx)$$

L. Einkemmer, Q. Li, L. Wang and Y. Yang. Suppressing instability in a Vlasov–Poisson system by an external electric field through constrained optimization, JCP, 498

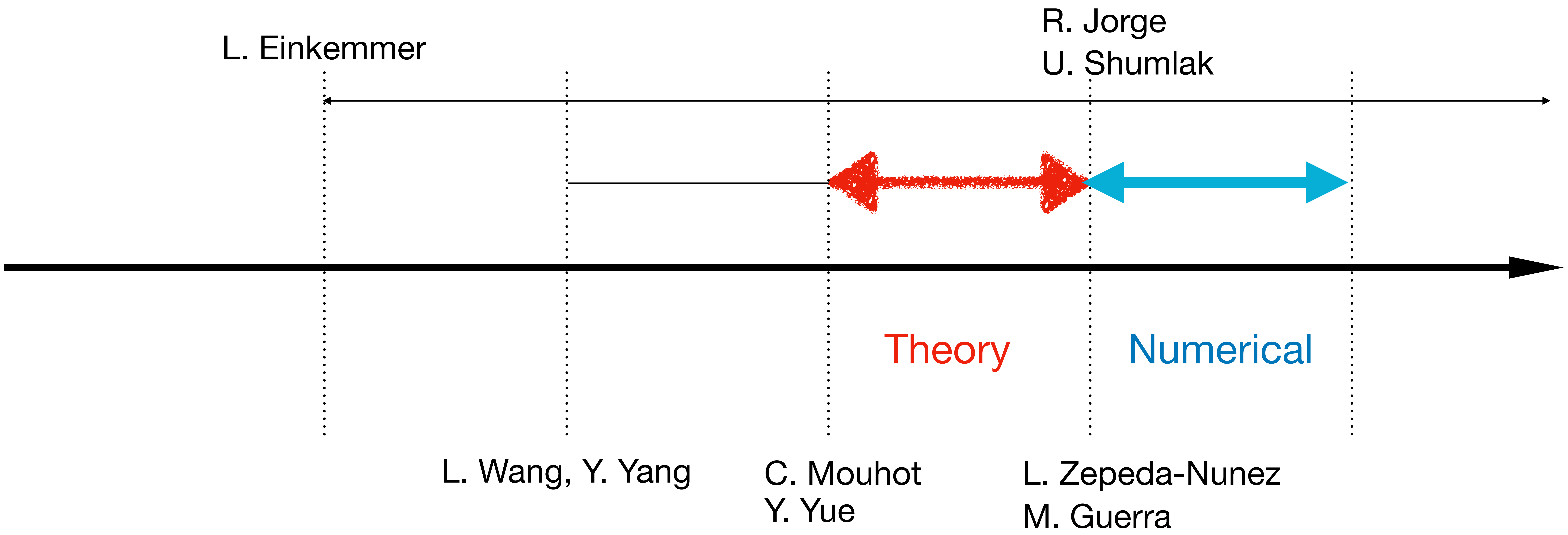
PDE-constrained optimization — Vlasov stablization

Failed attempts

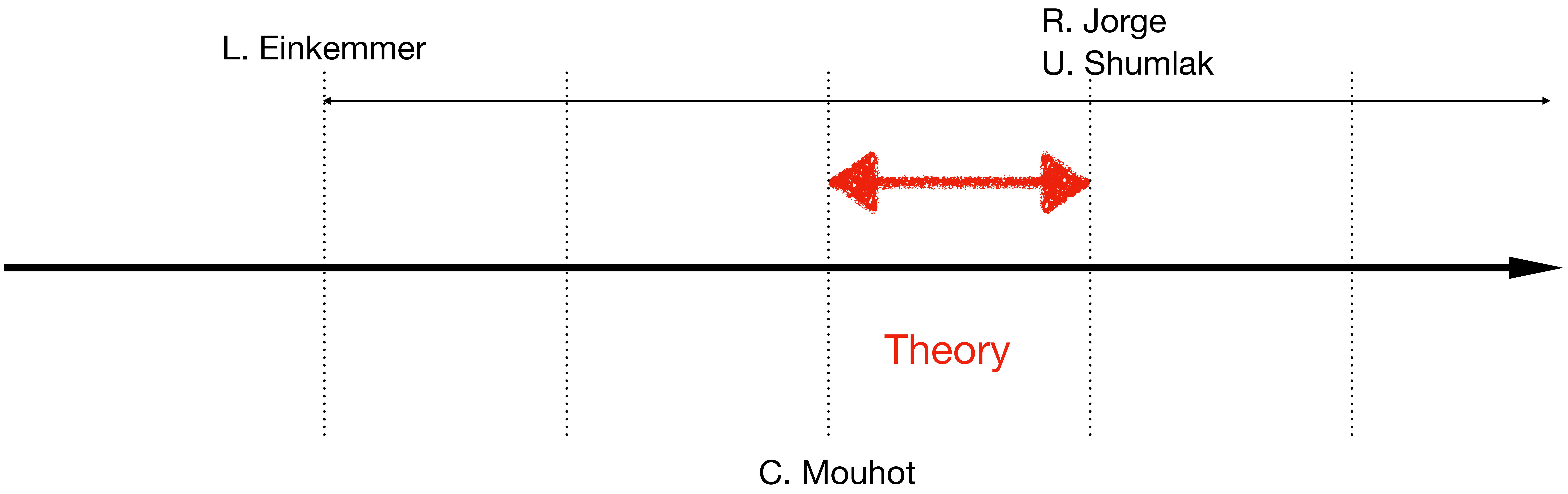
- External field needs to be parameterized
- Over-parameterization may help
- Objective function highly oscillatory, luck dependent



PDE-constrained optimization in plasma



PDE-constrained optimization in plasma



How did Landau Damping damp?

PDE-constrained optimization — Vlasov stablization, theory

$$\partial_t f + v\partial_x f - E[f]\partial_v f = 0$$

$$f = \mu + \delta f$$

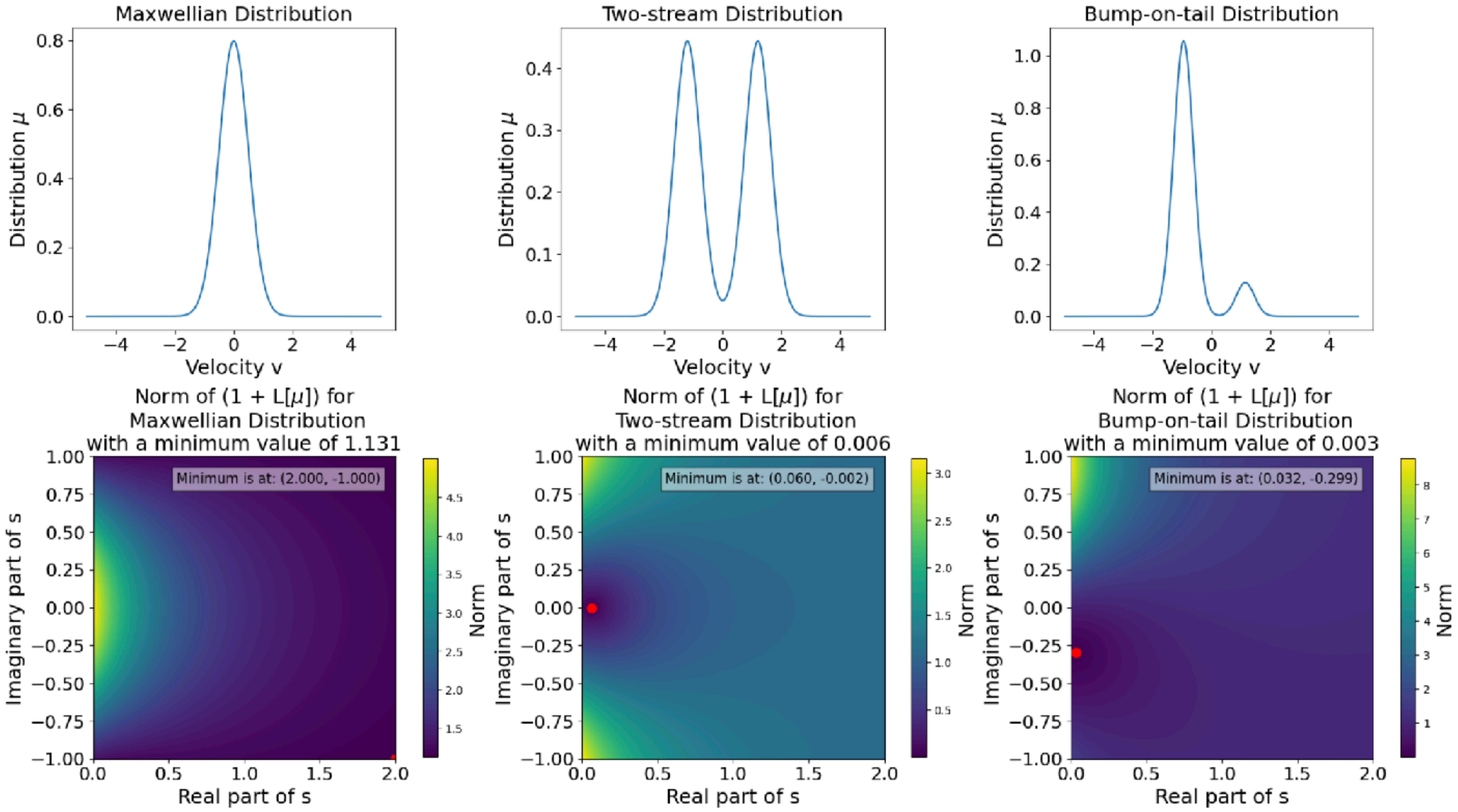
$$\partial_t f + v\partial_x f - E[f]\partial_v \mu = 0$$

$$\partial_t f = \mathcal{L}[f]$$

PDE-constrained optimization — Vlasov stablization, theory

$$\partial_t f + v\partial_x f - E[f]\partial_v \mu = 0$$

$$\partial_t f = \mathcal{L}[f]$$



PDE-constrained optimization — Vlasov stablization, theory

$$\partial_t f + v\partial_x f - E[f]\partial_v f = 0$$

$$\downarrow \quad f = \mu + \delta f$$

$$\partial_t f + v\partial_x f - E[f]\partial_v \mu = 0$$

$$\partial_t f + v\partial_x f - (E[f] + H)\partial_v f = 0$$

$$\downarrow \quad f = \mu + \delta f$$

$$\partial_t f + v\partial_x f - (E[f] + H)\partial_v \mu = 0$$

PDE-constrained optimization — Vlasov stablization, theory

$$\partial_t f + v\partial_x f - E[f]\partial_v \mu = 0$$

$$\partial_t f + v\partial_x f - (E[f] + H)\partial_v \mu = 0$$

$$\partial_t f = \mathcal{L}[f]$$

$$\partial_t f = \mathcal{L}_H[f]$$

Question: design H to remove positive roots

PDE-constrained optimization — Vlasov stablization, theory

$$\partial_t f + v\partial_x f - E[f]\partial_v \mu = 0$$

$$\partial_t f + v\partial_x f - (E[f] + H)\partial_v \mu = 0$$

$$L[\hat{\rho}(\cdot,k)](s)\left(1 + L[\hat{U}(\cdot,k)](s)\right) = L[\hat{S}(\cdot,k)](s)$$

Laplace Fourier Equilibrium Free-streaming

cannot have roots on the positive side

Penrose condition

PDE-constrained optimization — Vlasov stablization, theory

$$\partial_t f + v\partial_x f - E[f]\partial_v \mu = 0$$

$$\partial_t f + v\partial_x f - (E[f] + H)\partial_v \mu = 0$$

$$L[\hat{\rho}(\cdot,k)](s)\left(1 + L[\hat{U}(\cdot,k)](s)\right) = L[\hat{S}(\cdot,k)](s)$$

Laplace Fourier Equilibrium Free-streaming

$$L[\hat{\rho}(\cdot,k)](s)\left(1 + L[\hat{U}(\cdot,k)](s)\right) = L[\hat{S}(\cdot,k)](s) + ikL[\hat{U}(\cdot,k)](s)L[\hat{H}(\cdot,k)](s)$$

source canceling

PDE-constrained optimization — Vlasov stablization, theory

$$L[\hat{\rho}(\cdot,k)](s)\left(1+L[\hat{U}(\cdot,k)](s)\right)=L[\hat{S}(\cdot,k)](s)+ikL[\hat{U}(\cdot,k)](s)L[\hat{H}(\cdot,k)](s)$$

$$\left(L[\hat{\rho}]-L[\hat{S}]\right)\left(1+L[\hat{U}]\right)=L[\hat{U}]\left(-L[\hat{S}]+ikL[\hat{H}]\right)$$

difference against free-streaming

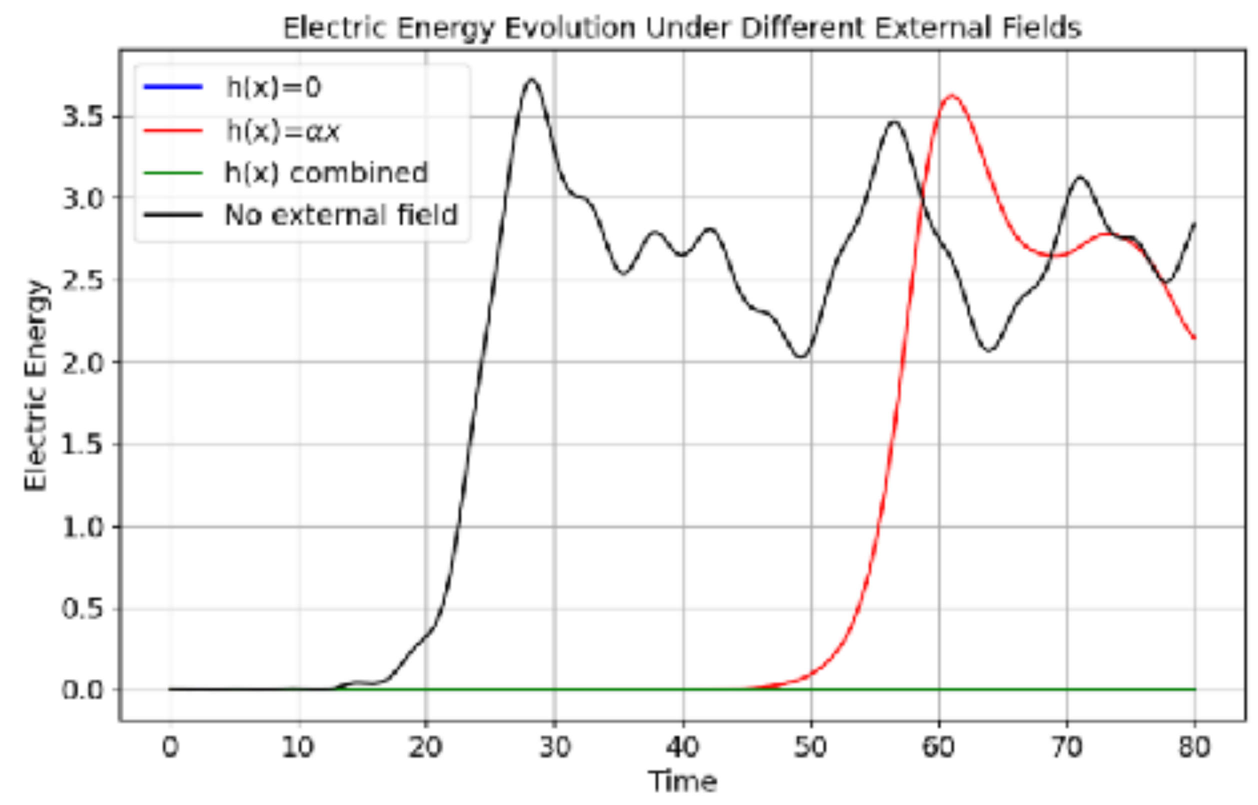
control

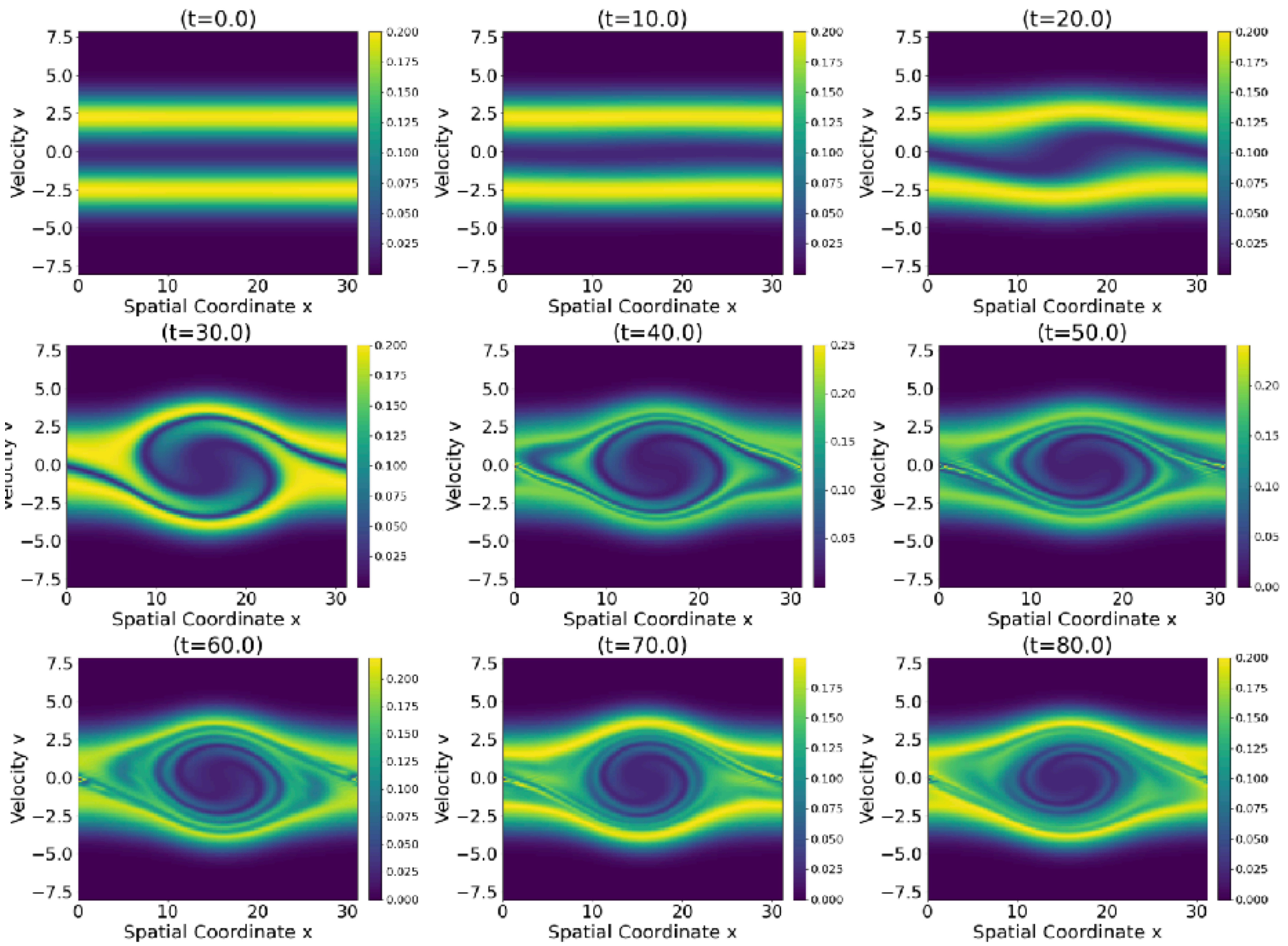
Special case: control = 0

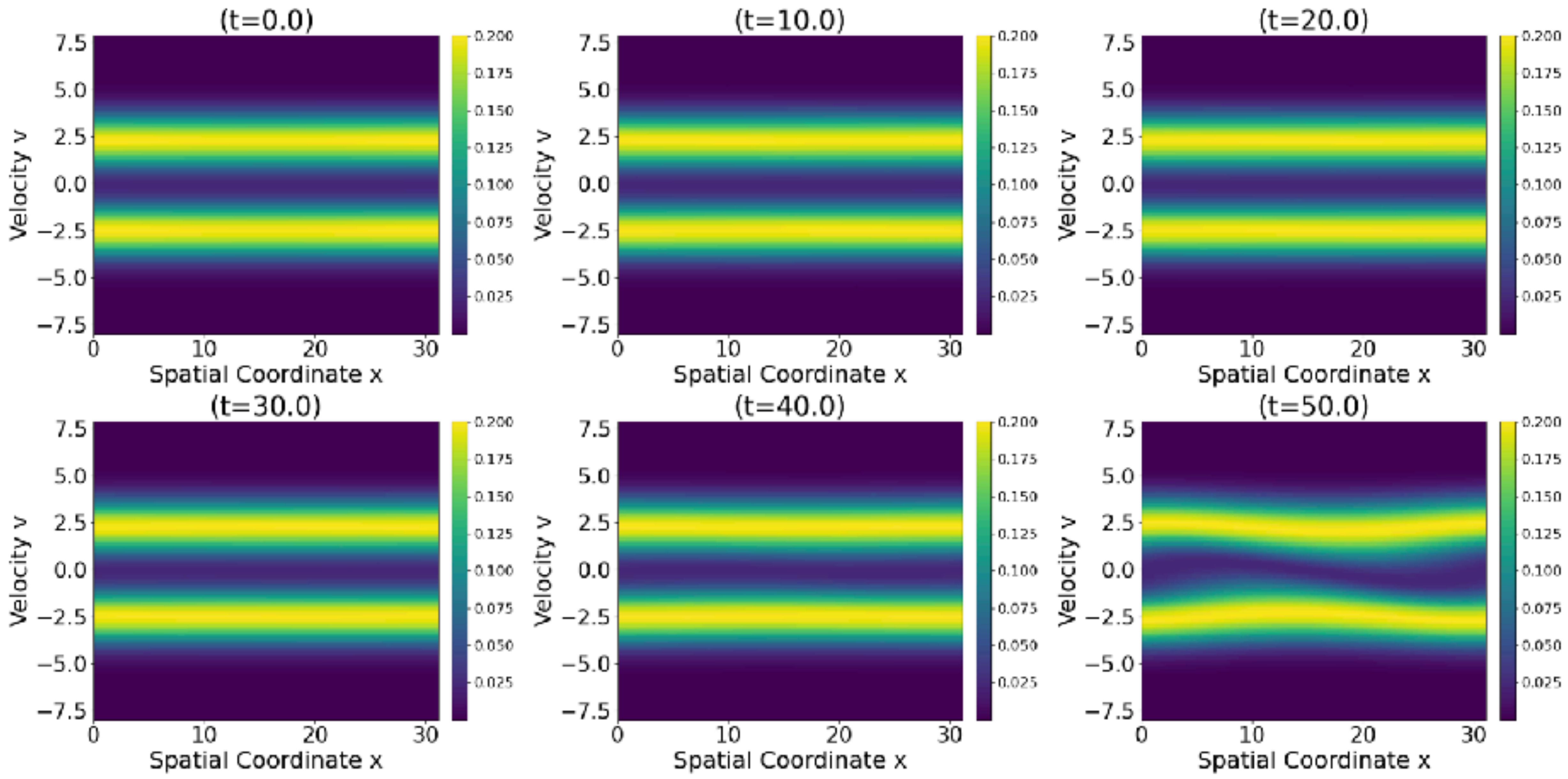
$$\partial_t f+v\partial_x f-\cancel{(E[f]+H)\partial_v f}=0$$

Theorem: free-streaming solution is stable.

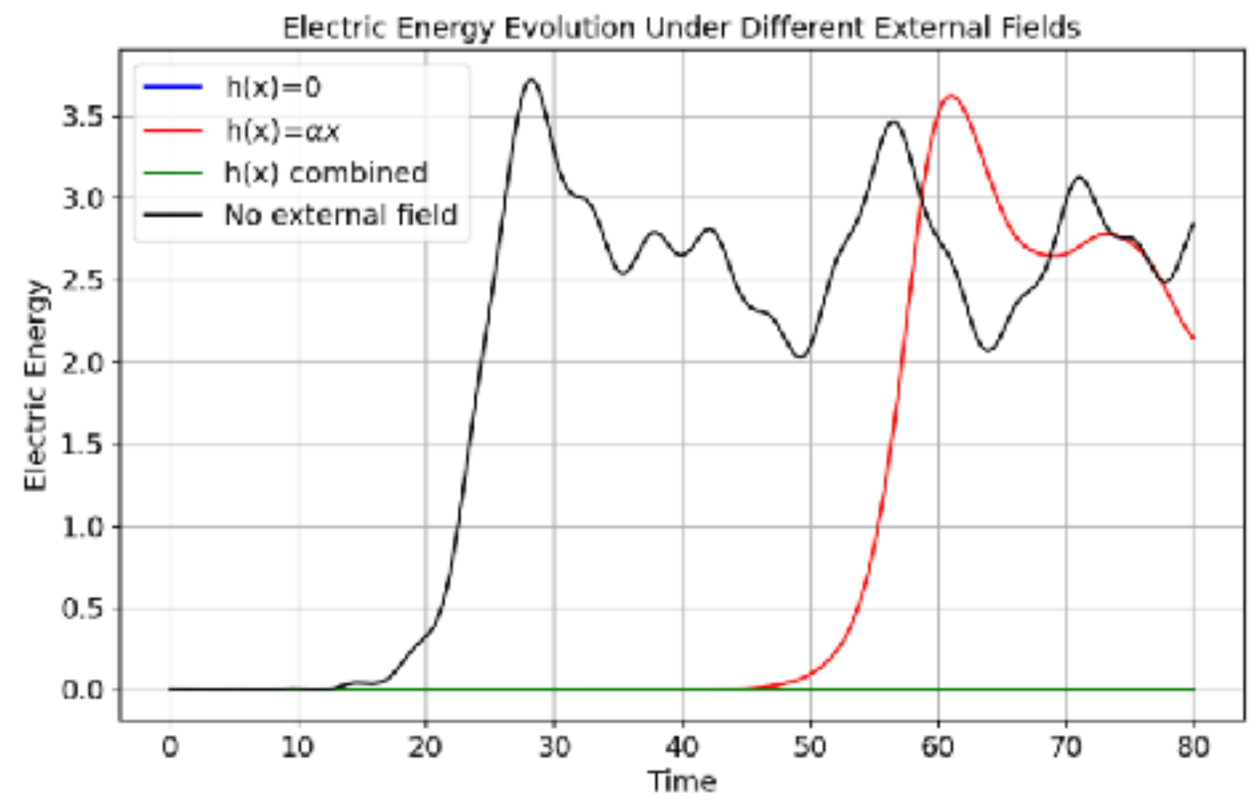
PDE-constrained optimization — Vlasov stablization, theory

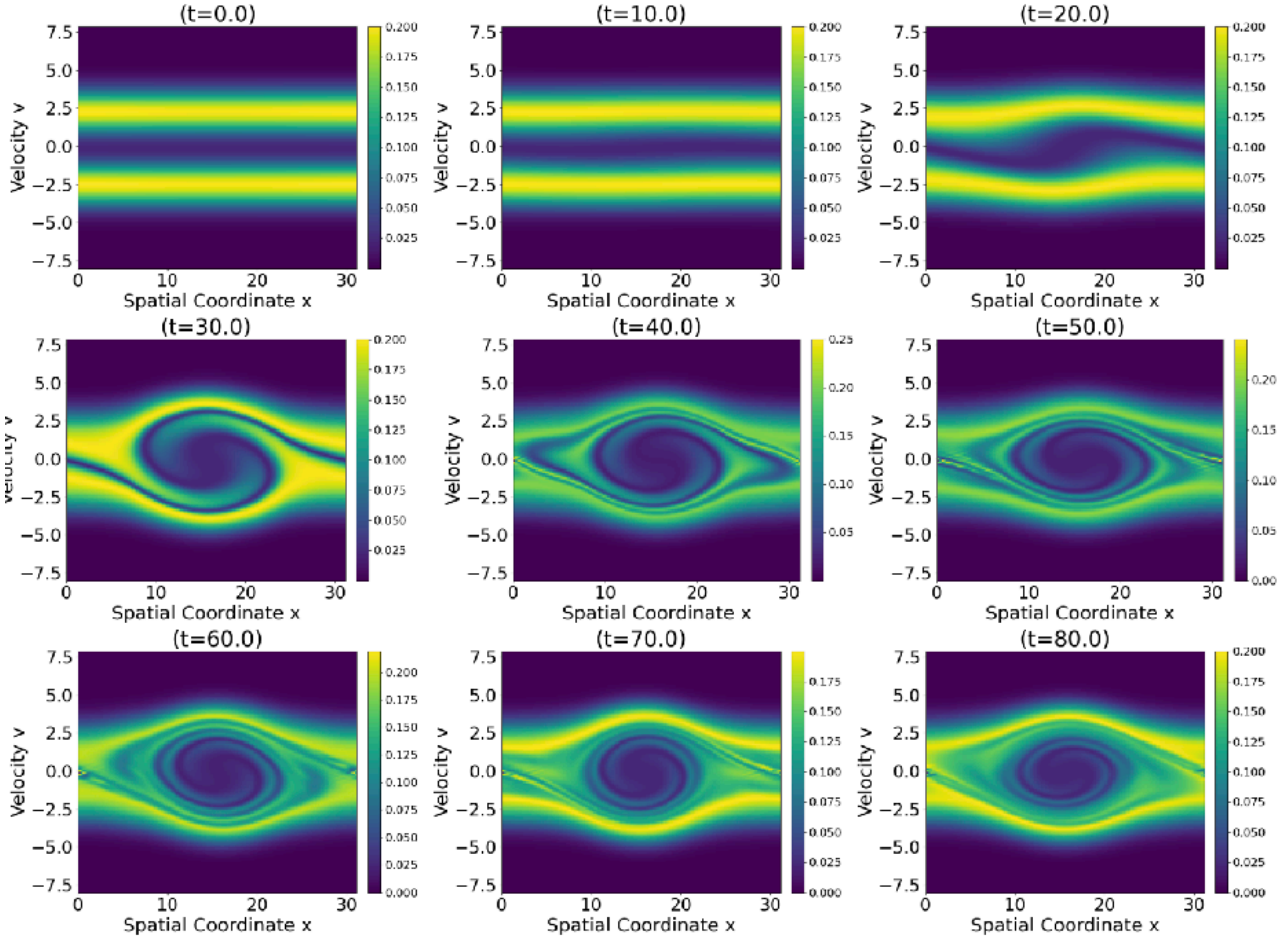


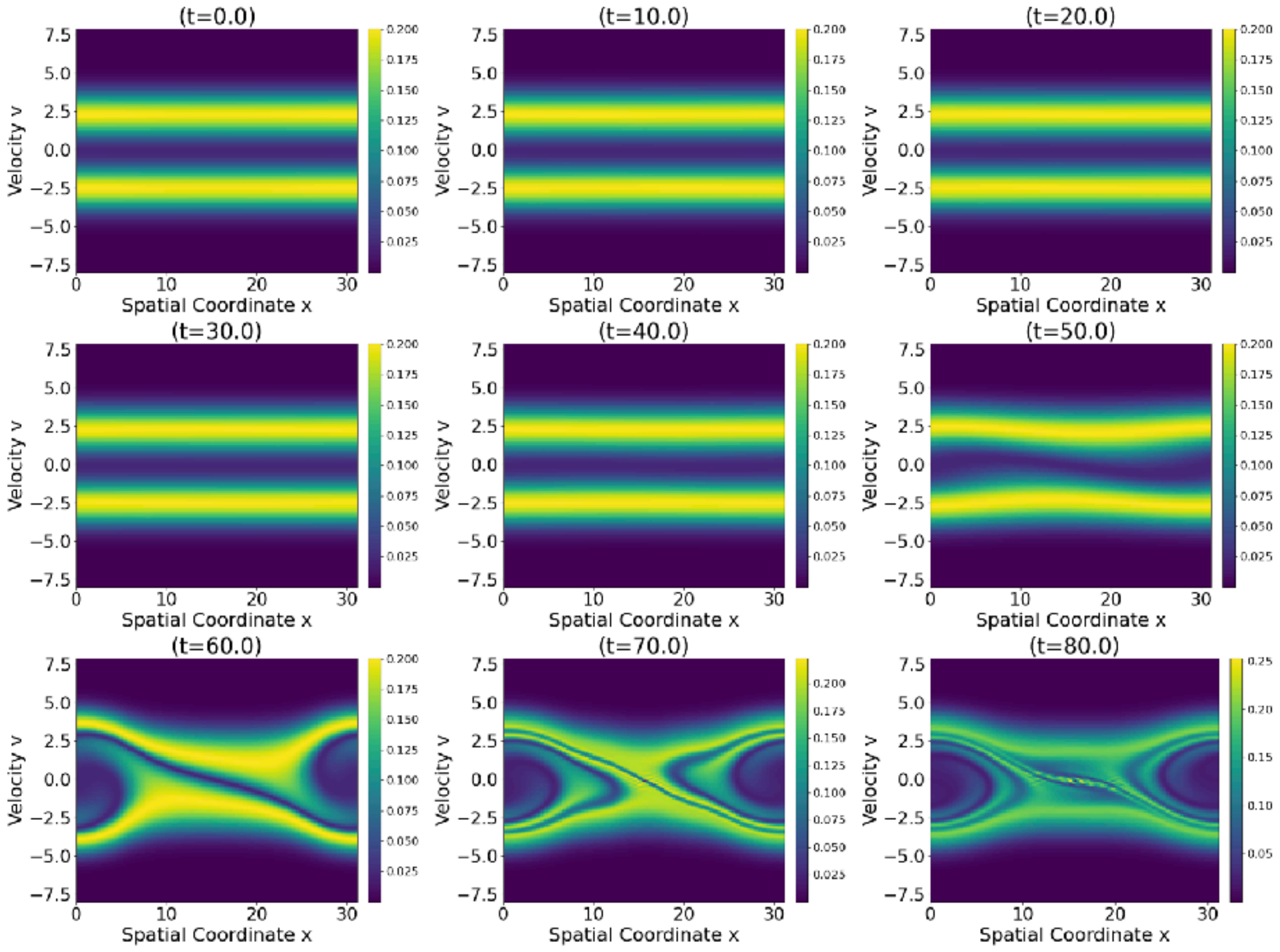




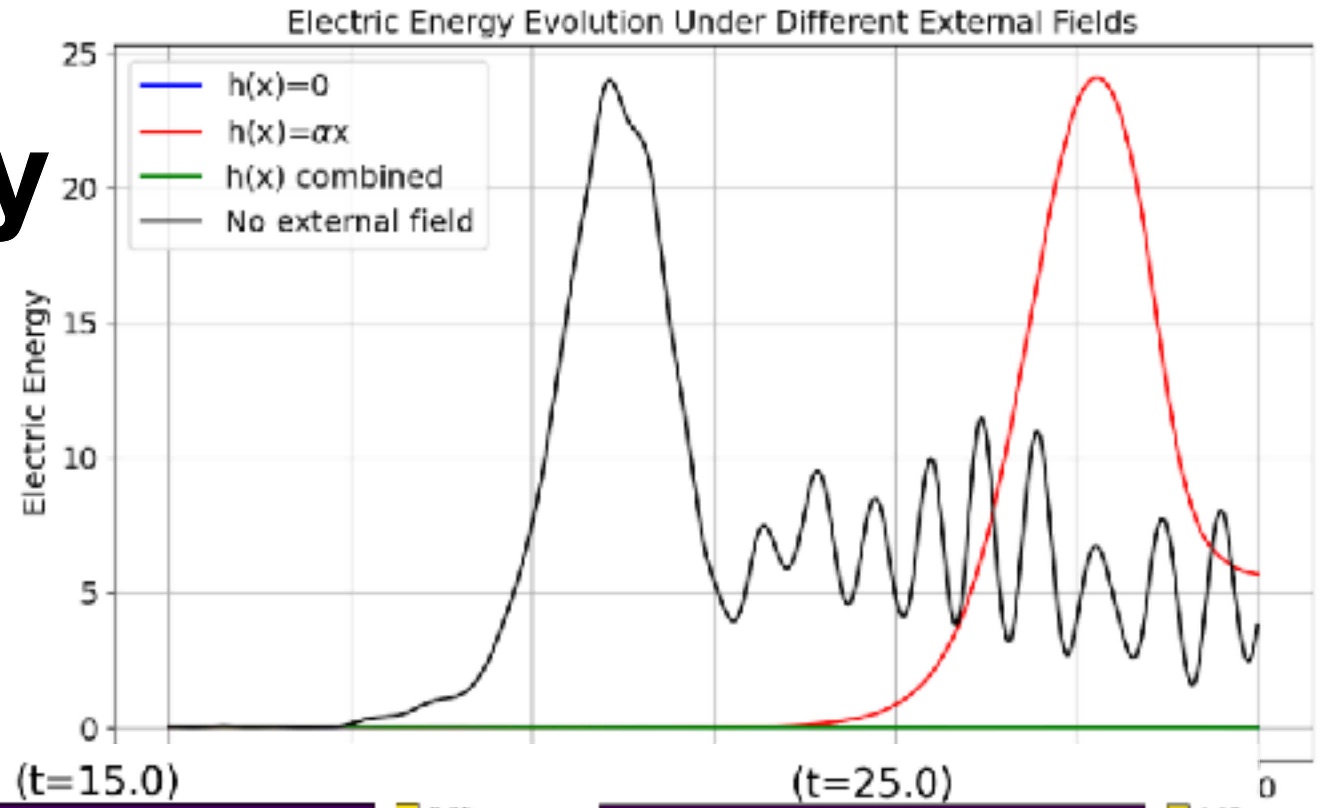
PDE-constrained optimization — Vlasov stablization, theory

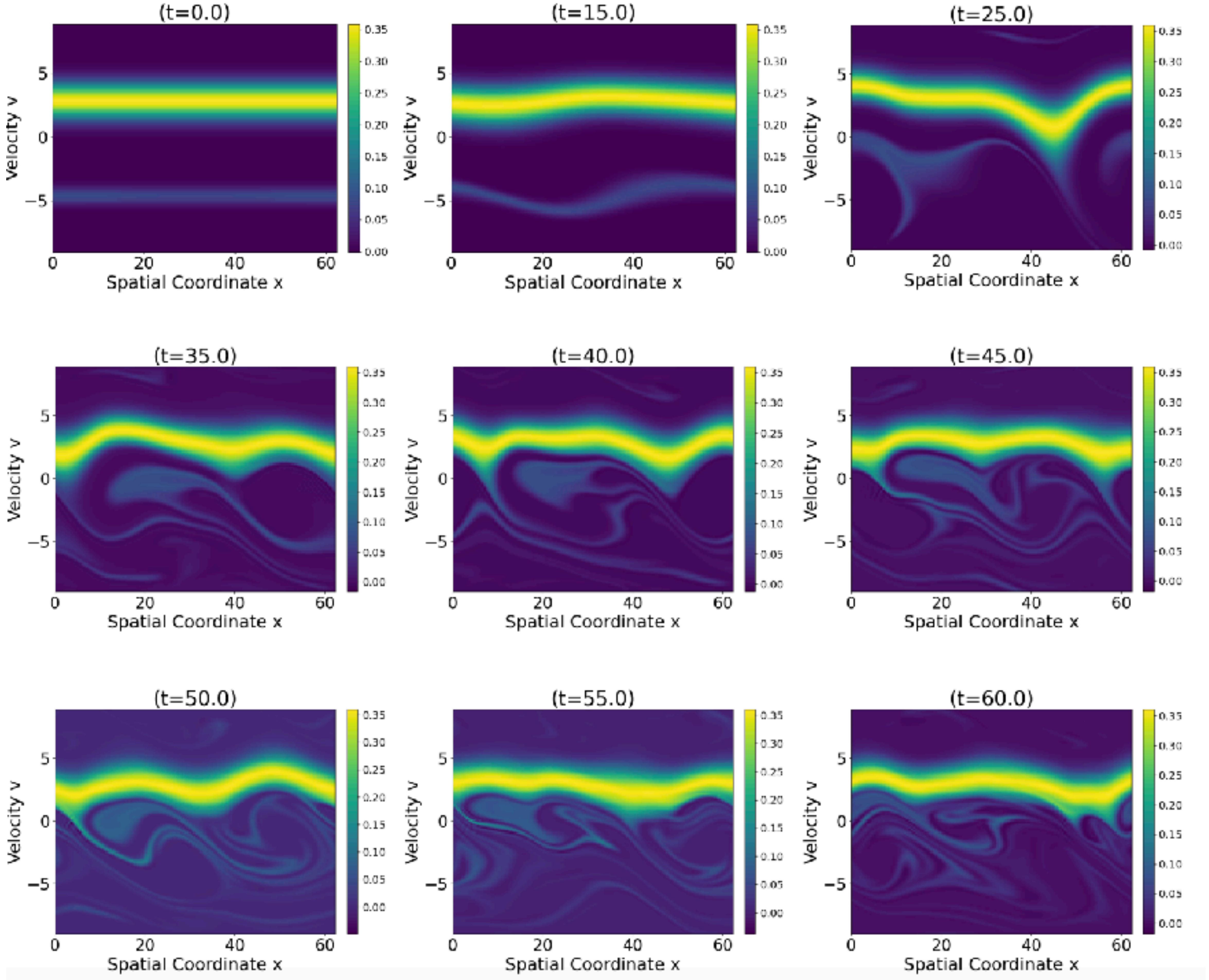


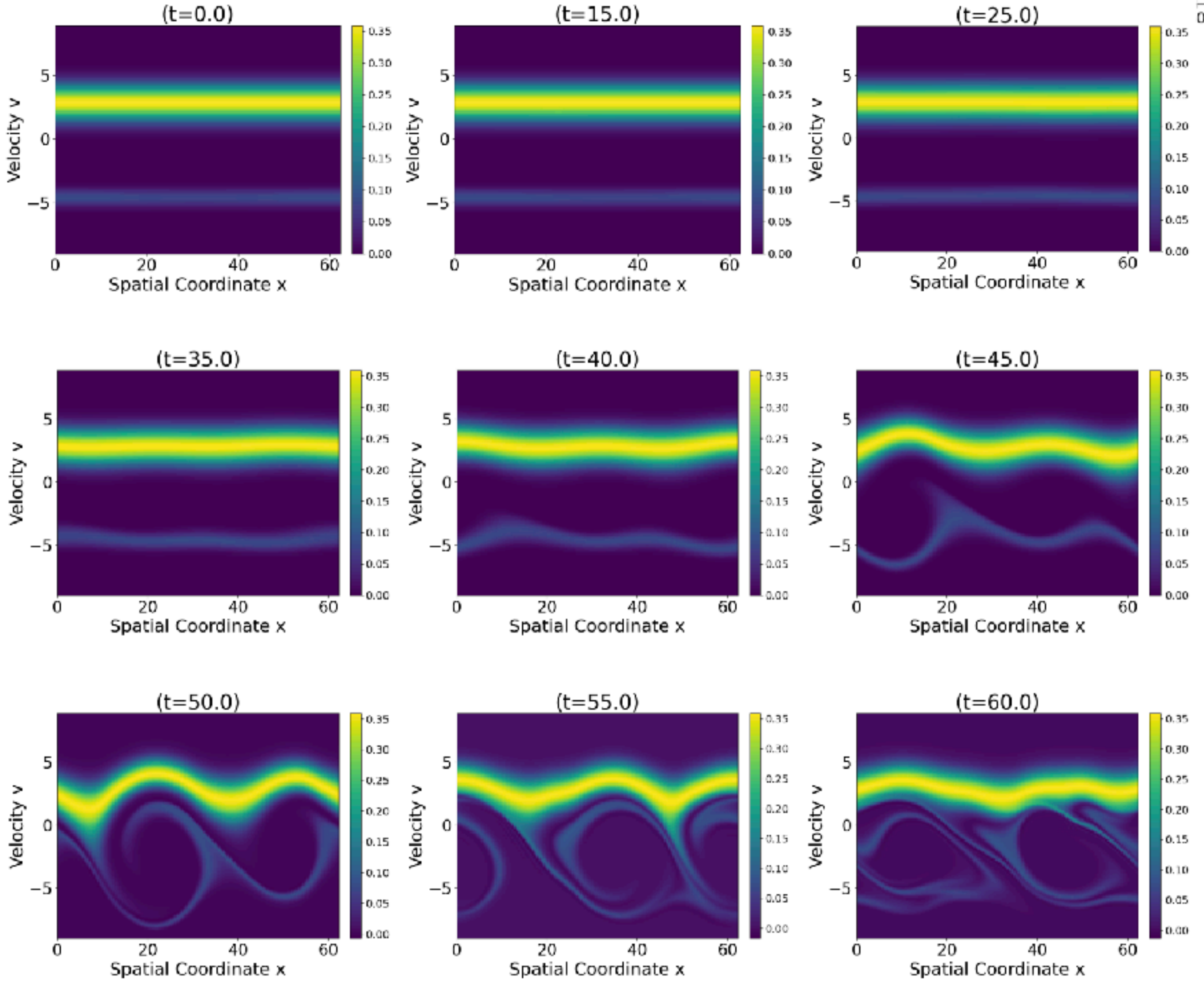




PDE-constrained optimization — Vlasov stablization, theory







L. Einkemmer, Q. Li, C. Mouhot and Y. Yue, Control of instability in a Vlasov-Poisson system through an external electric field, JCP, 530

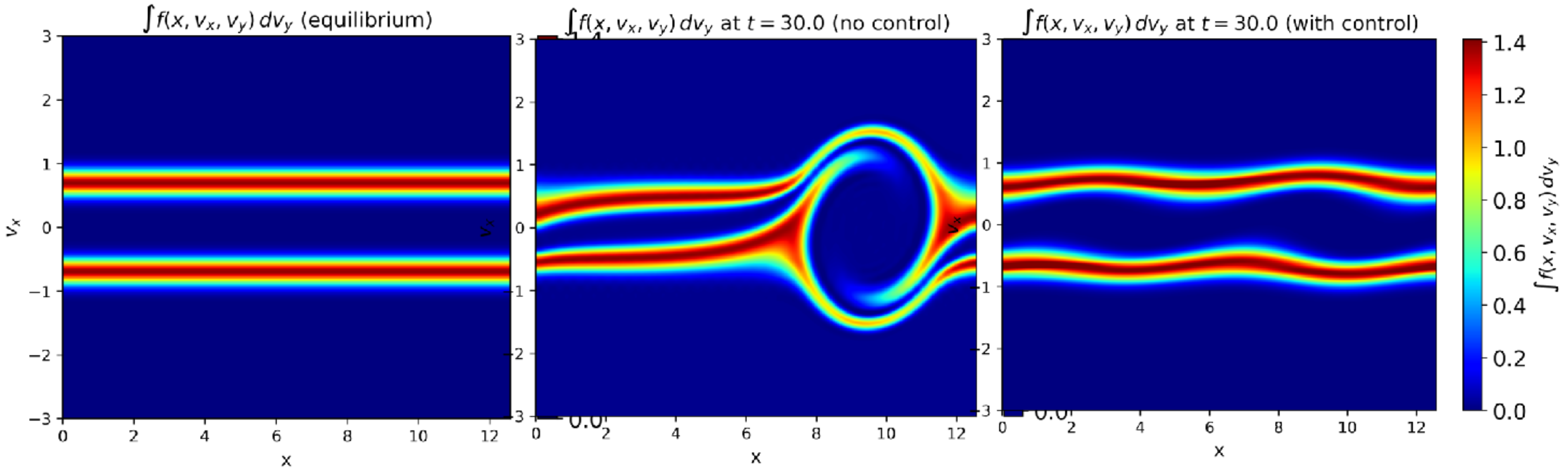
PDE-constrained optimization — Vlasov stablization, theory

Control of kinetic plasma instabilities by laser fields, 1D2V Vlasov–Maxwell

N. Crouseilles, L. Einkemmer, Q. Li, U. Shumlak, Y. Yue

$$
\begin{cases}
\partial_t f + v_x \partial_x f + (E_x + v_y B_z)\, \partial_{v_x} f + (E_y - v_x B_z)\, \partial_{v_y} f = 0\,, \\
\partial_x E_x = \rho - 1\,, \\
\partial_t E_y = -\partial_x B_z - (j_y - \langle j_y \rangle), \\
\partial_t B_z = -\partial_x E_y,
\end{cases}
$$

$+\mathbf{B}, +\mathbf{E}_{\text{laser}}$



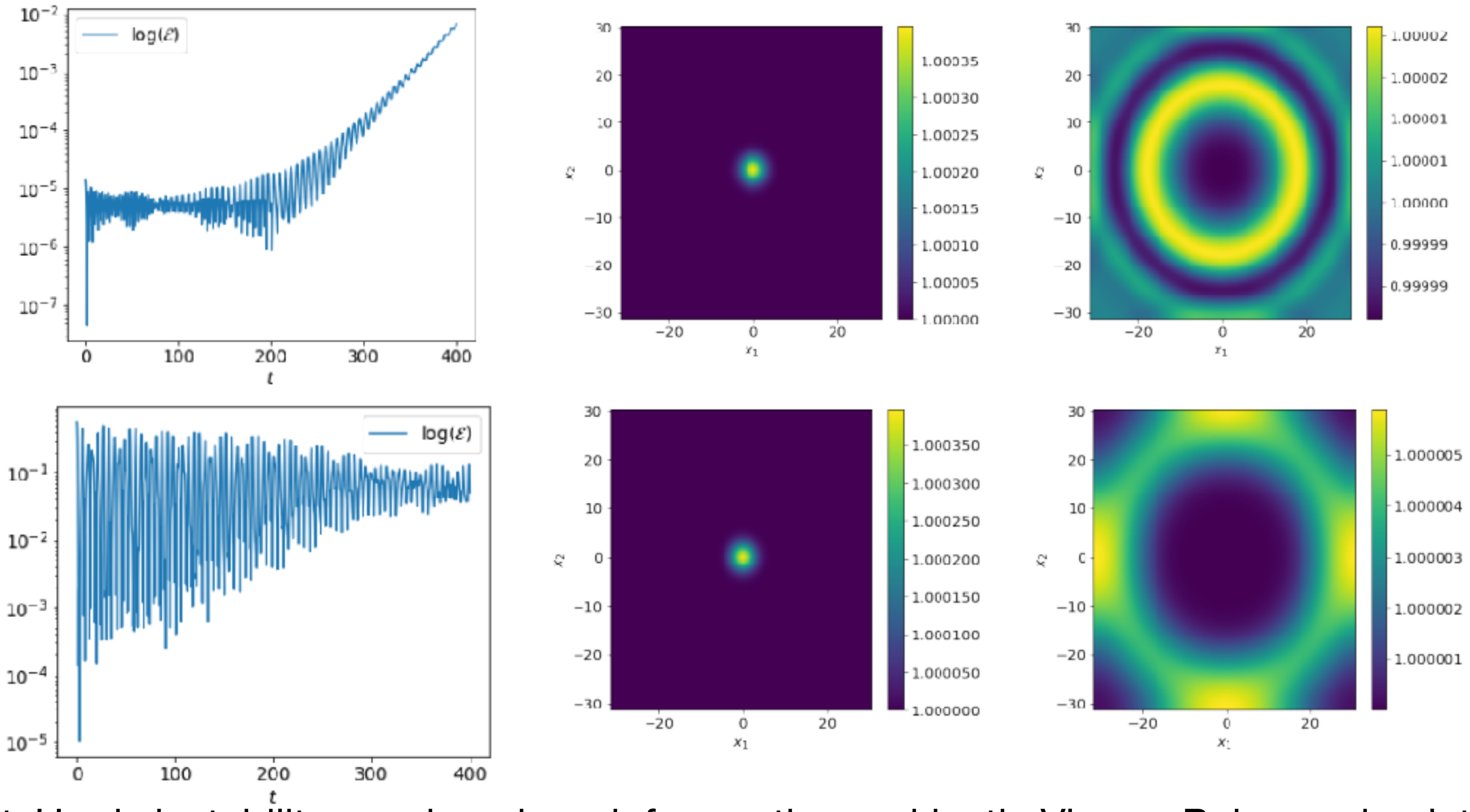
PDE-constrained optimization — Vlasov stablization, theory

Control of a Uniformly Magnetized Plasma with External Electric Fields, 2D2V Vlasov–Poisson

P. Chen, R. Jorge, Q. Li, Y. Yue

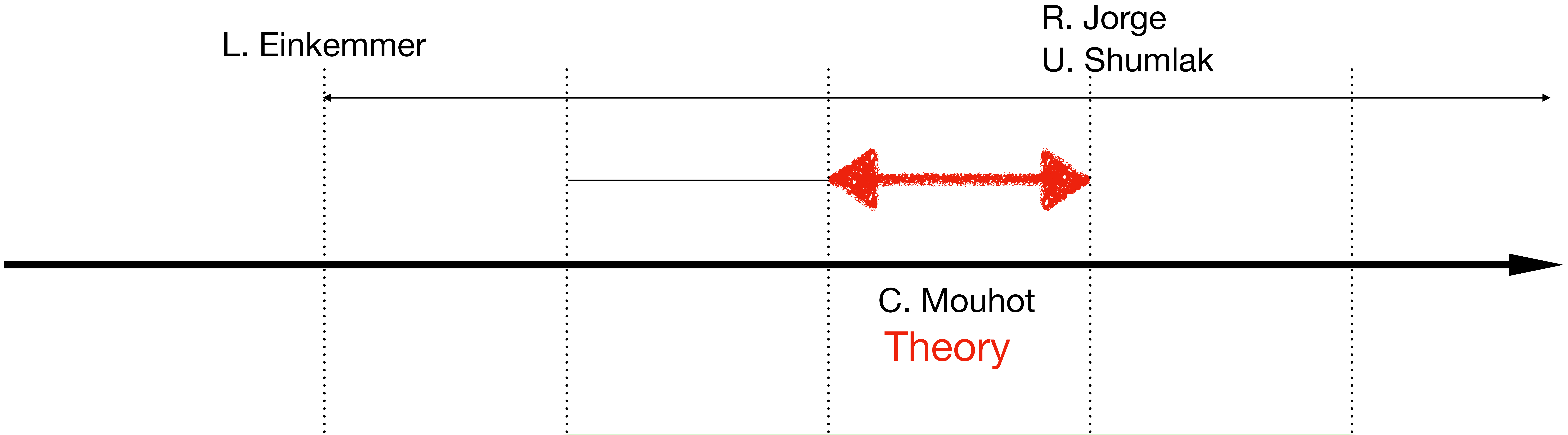
$$\begin{cases} \partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = 0\,, \\ \nabla^2_{\mathbf{x}} V = 1 - \int_{\mathbb{R}^2} f \, d\mathbf{v}\,, \quad \mathbf{E} = -\nabla_{\mathbf{x}} V\,. \end{cases}$$

$$+\mathbf{E}_{\text{ext}}$$



Vogman, G., Colella, P., Shumlak, U.: Dory–Guest–Harris instability as a benchmark for continuum kinetic Vlasov–Poisson simulations of magnetized plasmas. J. Comput. Phys. 277,

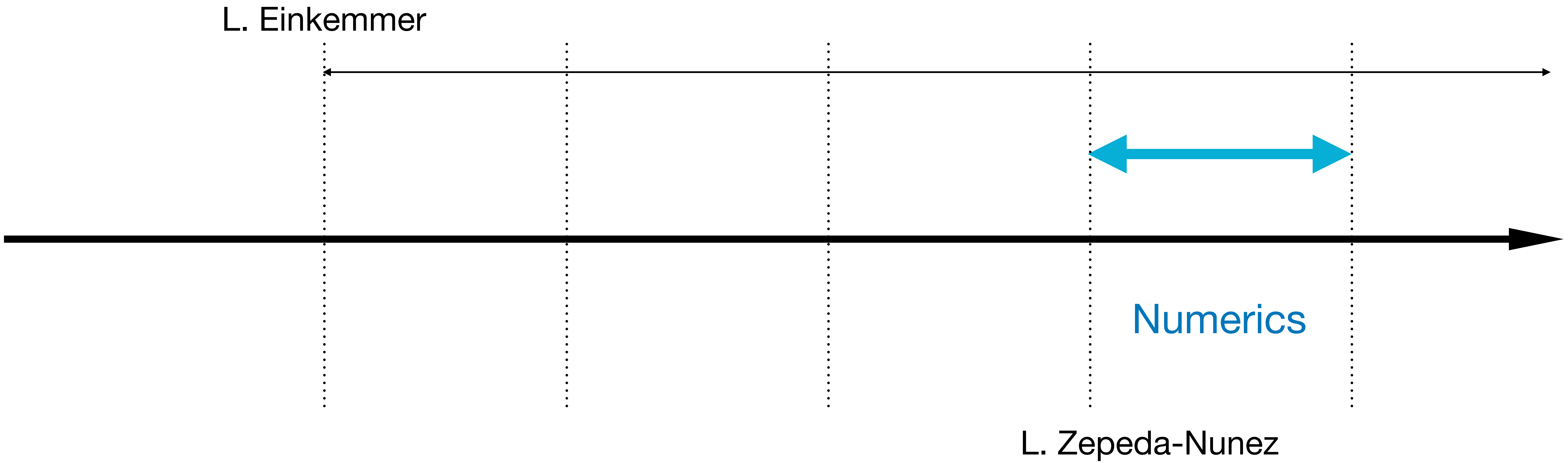
PDE-constrained optimization in plasma



How did Landau Damping damp?

- Mathematical machinery very systematic
- Linear solver for a warm start —> PDE-constrained opt

PDE-constrained optimization in plasma



Any way to smooth objective landscape?

PDE-constrained optimization in plasma

WORKSHOP

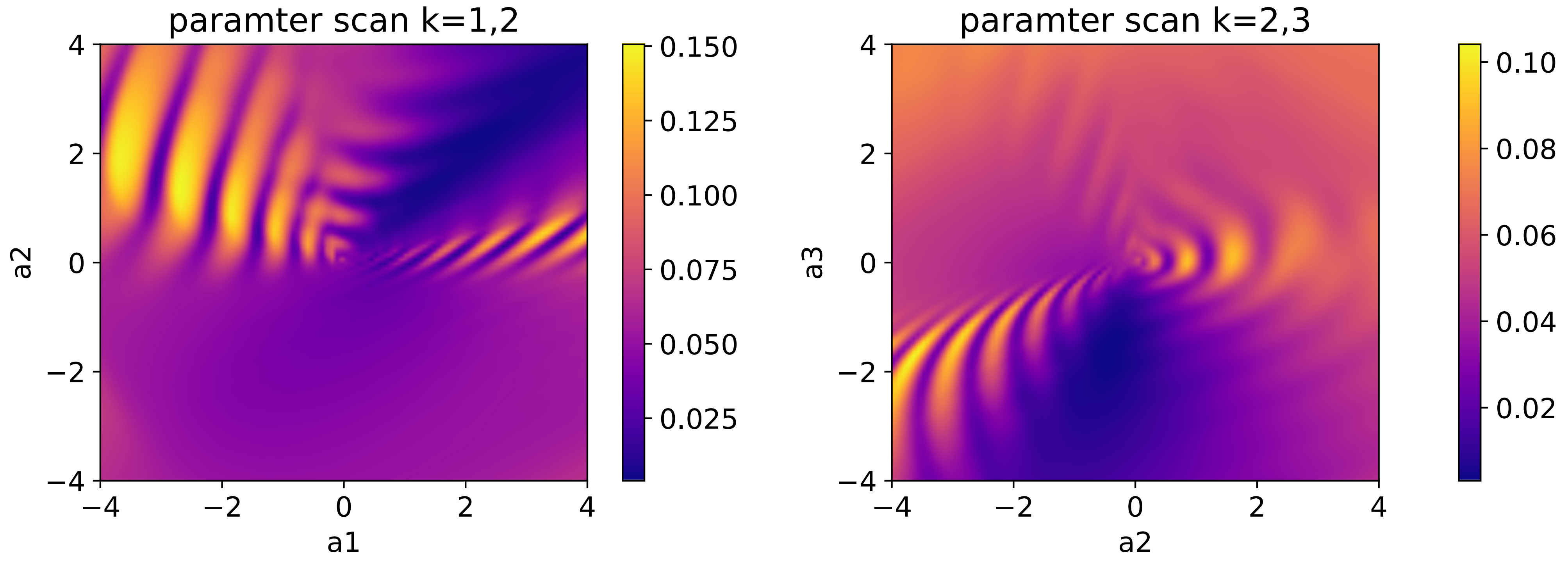
Climate & Sustainability (Archived)

Computational Challenges and Optimization in Kinetic Plasma Physics

February 19 — 22, 2024

Numerics

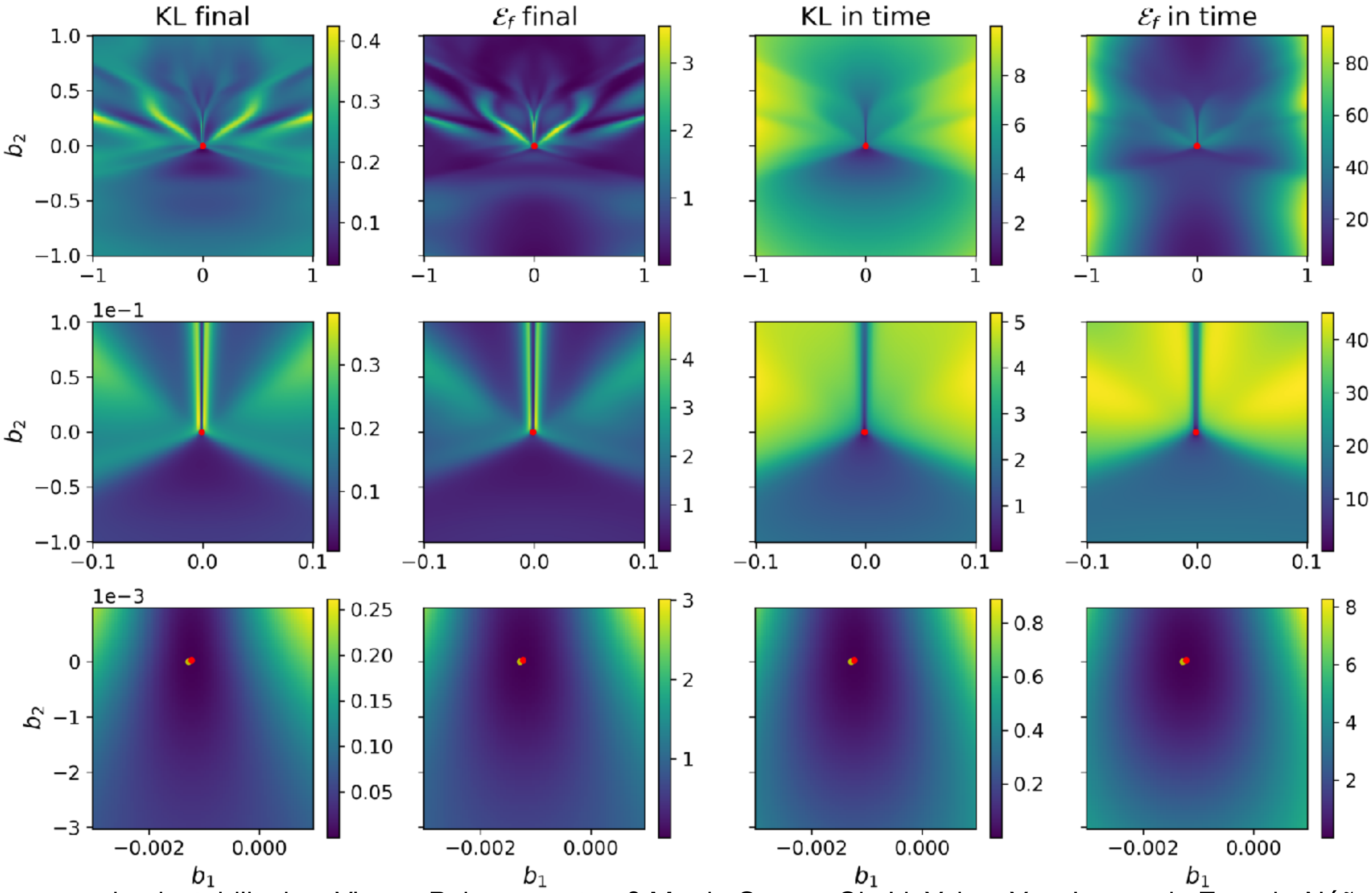
L. Zepeda-Nunez



$$J[f] = ||f - \mu||_2$$

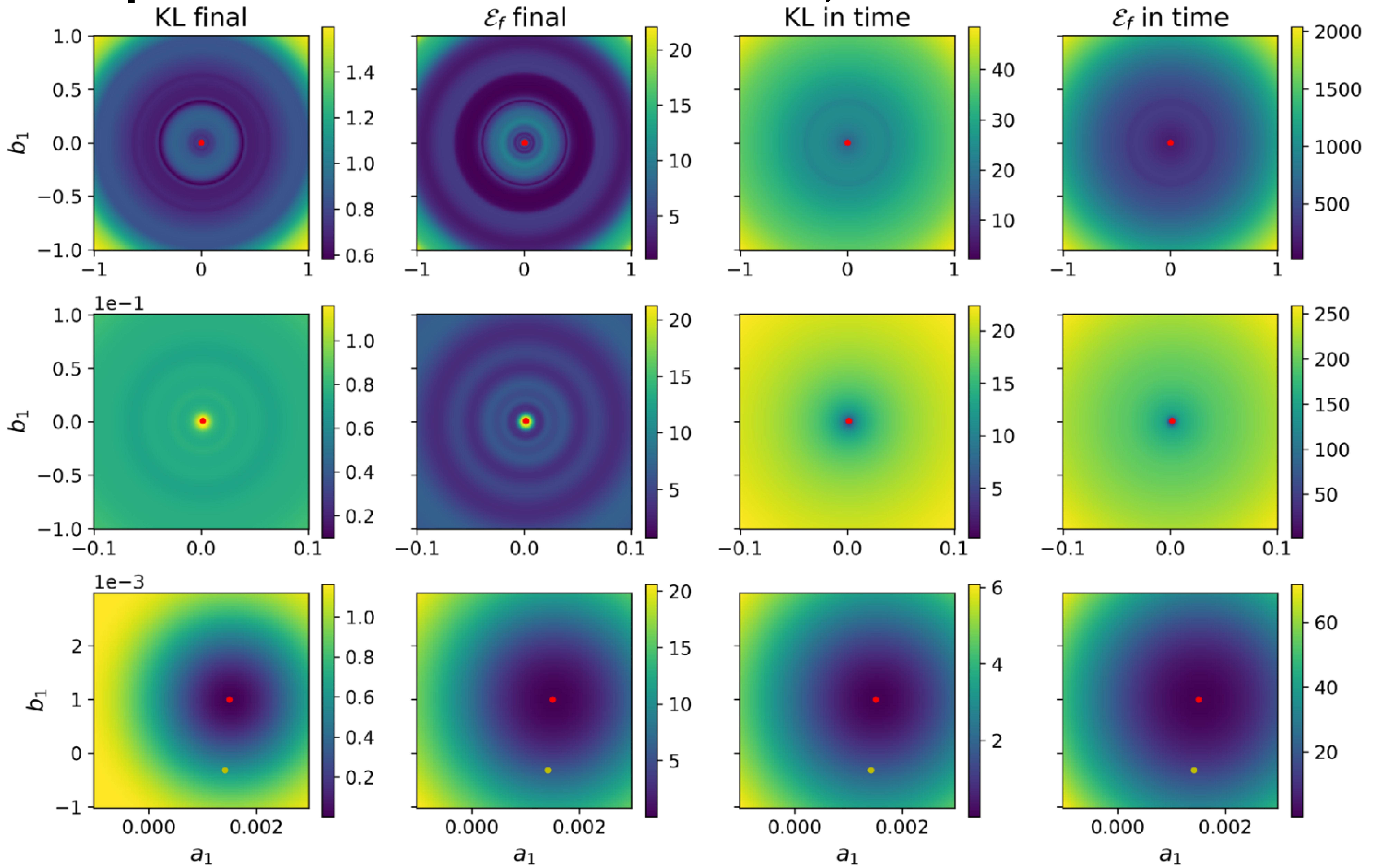
Any way to smooth objective landscape?

PDE-constrained optimization — Vlasov stablization, numerical



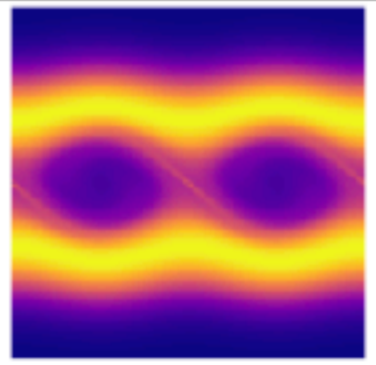
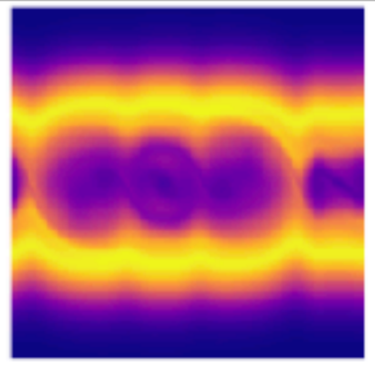
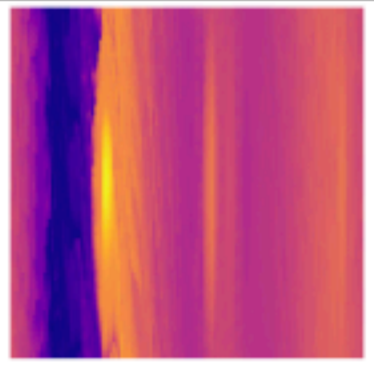
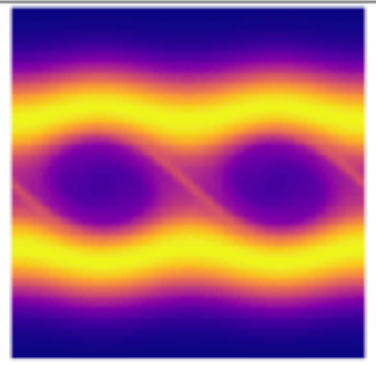
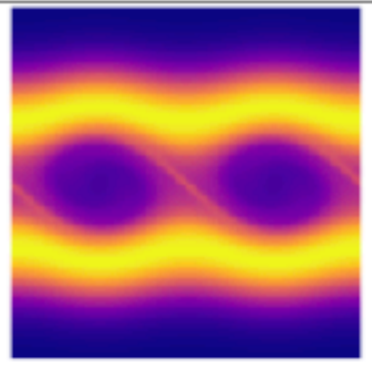
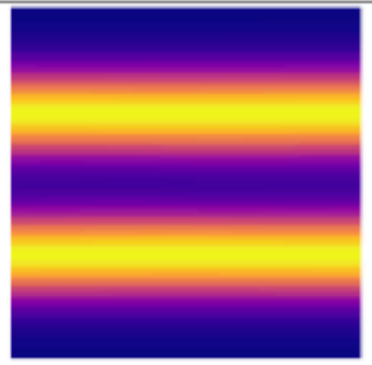
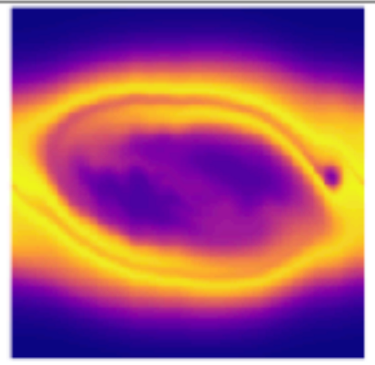
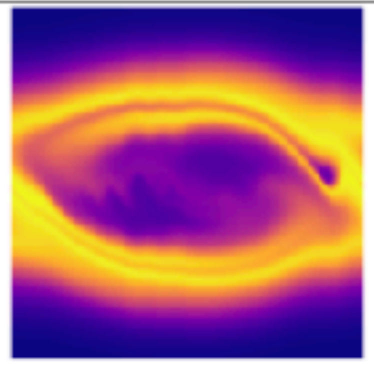
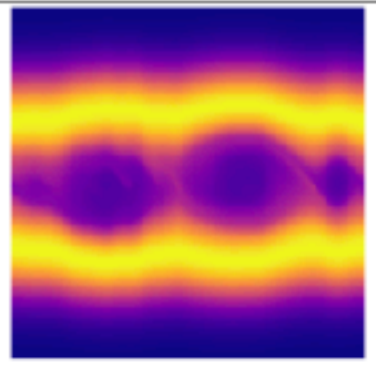
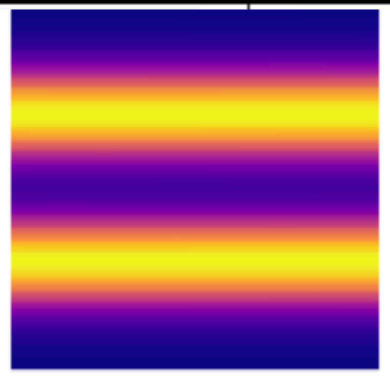
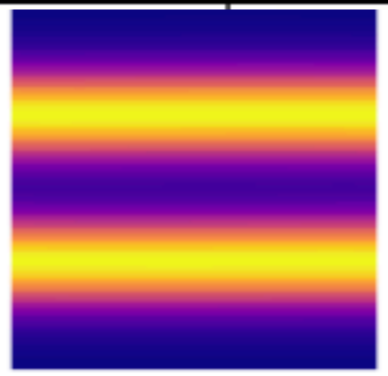
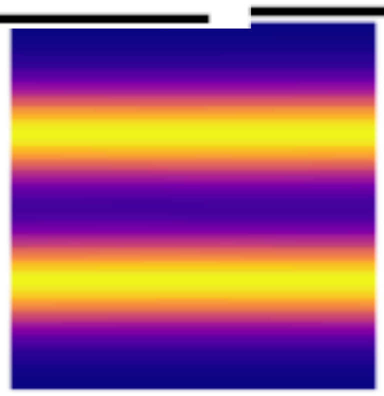
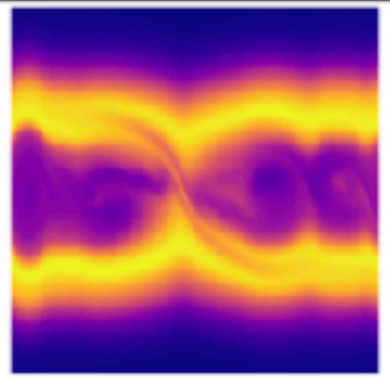
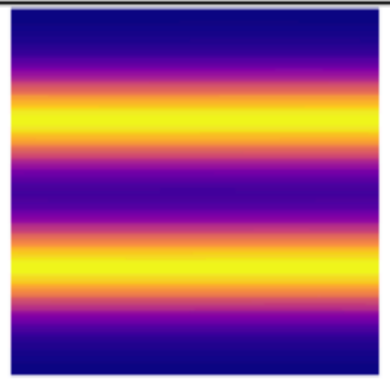
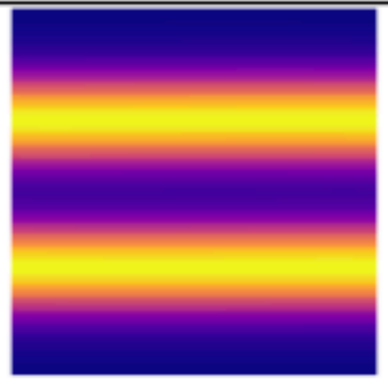
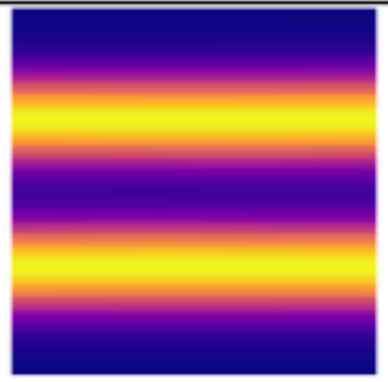
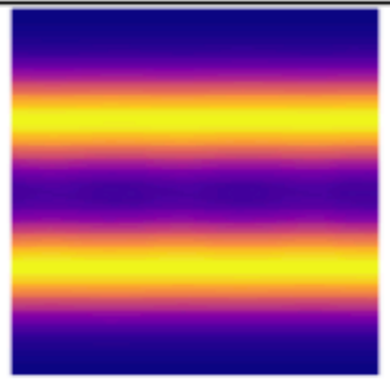
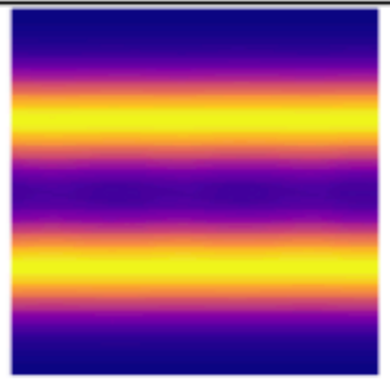
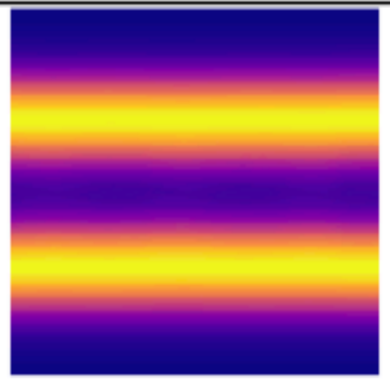
What metric to optimize for suppressing instability in a Vlasov-Poisson system? Martin Guerra, Qin Li, Yukun Yue, Leonardo Zepeda-Núñez

PDE-constrained optimization — Vlasov stablization, numerical



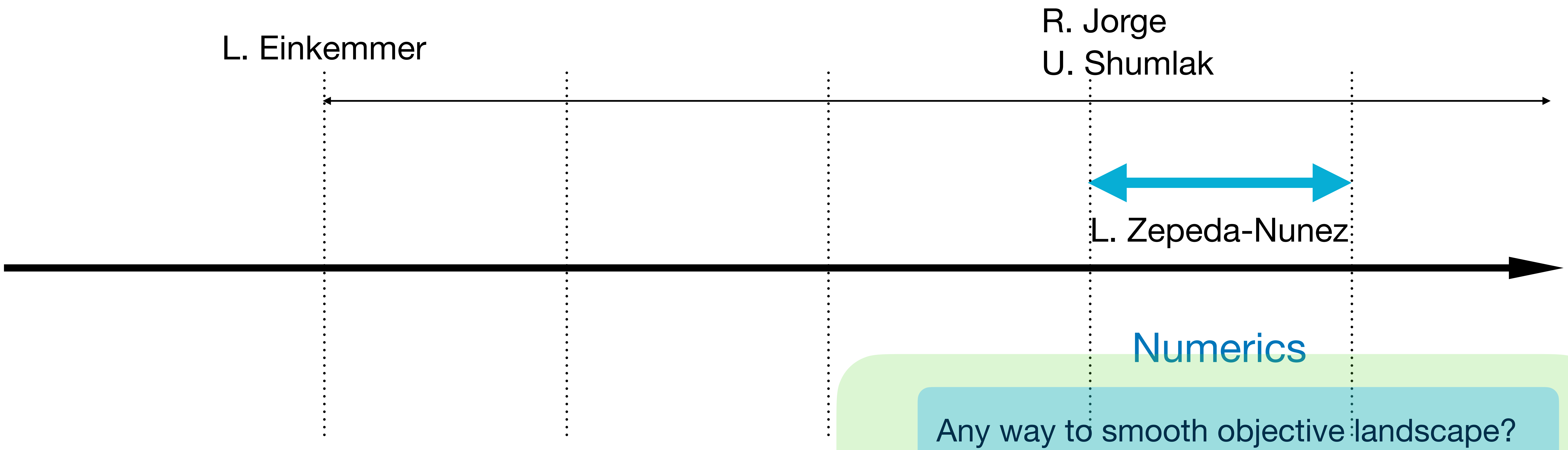
What metric to optimize for suppressing instability in a Vlasov-Poisson system? Martin Guerra, Qin Li, Yukun Yue, Leonardo Zepeda-Núñez

PDE-constrained optimization — Vlasov stablization, numerical

		Under-parametrized (30)			Over-parametrized (32)		
Init.	Step	(KL)	(EE)	(EET)	(KL)	(EE)	(EET)
Mid	Adaptive	 (Fig. C.14)	(Fig. C.20)	(Fig. C.26)	 (Fig. C.32)	 (Fig. C.38)	 (Fig. C.44)
	Local	 (Fig. C.15)	 (Fig. C.21)	 (Fig. C.27)	 (Fig. C.33)	 (Fig. C.39)	 (Fig. C.45)
Near	Adaptive	 (Fig. C.16)	 (Fig. C.22)	 (Fig. C.28)	 (Fig. C.34)	(Fig. C.40)	(Fig. C.46)
	Local	 (Fig. C.17)	 (Fig. C.23)	 (Fig. C.29)	 (Fig. C.35)	 (Fig. C.41)	 (Fig. C.47)

What metric to optimize for suppressing instability in a Vlasov-Poisson system? Martin Guerra, Qin Li, Yukun Yue, Leonardo Zepeda-Núñez

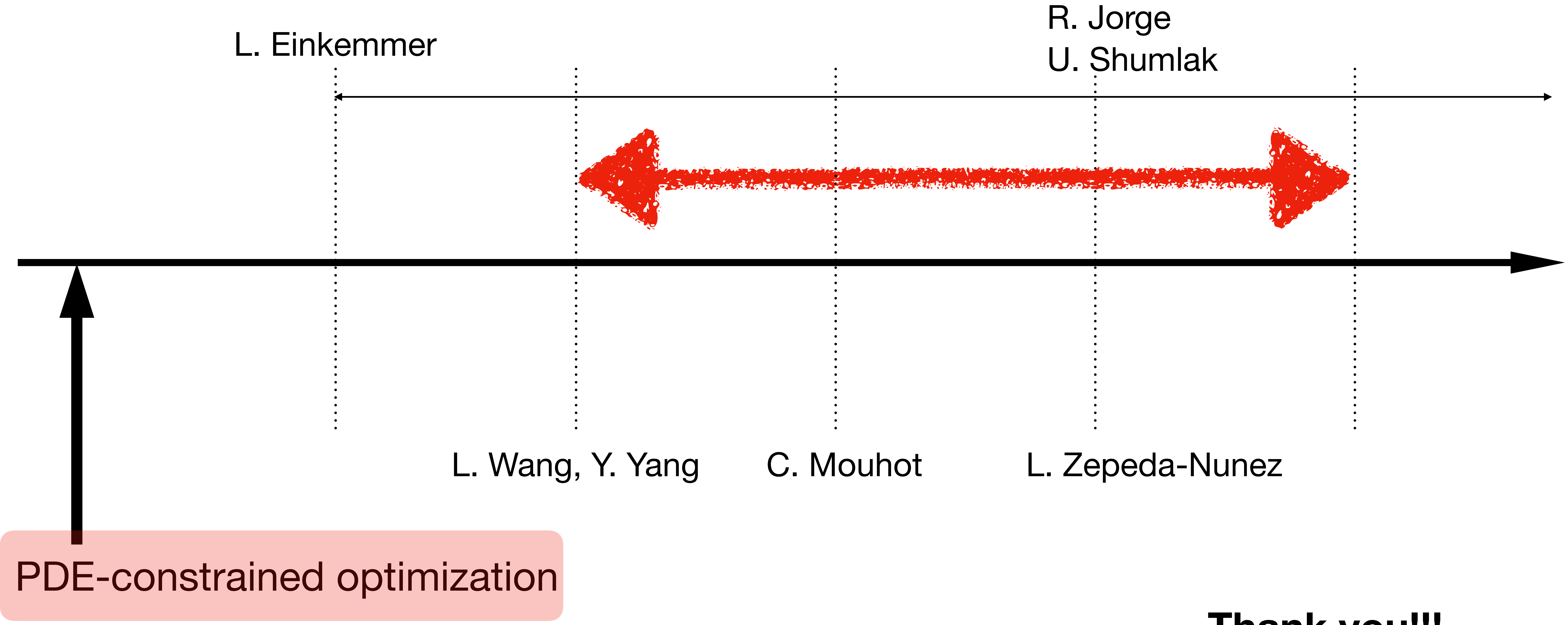
PDE-constrained optimization in plasma



Numerics

Any way to smooth objective landscape?

- In-time measurement better
- Warm start important — local convex


R. Jorge
U. Shumlak
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L. Wang, Y. Yang
C. Mouhot
L. Zepeda-Nunez
PDE-constrained optimization

Thank you!!!