Some Aspects of First Principle Models for Production Systems and Supply Chains

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Introduction

Topic: Overview of conservation law (traffic - like) models for large supply chains.

Joint work with....

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- D. Armbruster (ASU), P. Degond (Toulouse), M. Herty (Aachen)
- K. Kempf (INTEL Corp.)

Definition of a supply chain

One supplier takes an item, processes it, and hands it over to the next supplier.

Suppliers (Items):

- Machines on a factory floor (product item),
- Agent (client),
- Factory, many items,
- Processors in a computing network (information),

Example: Protocol for a Wafer in a Semiconductor Fab

	diffusion 1	diffusion 2	litho 1	etch clean	etch 1	ion impl	metal dep	litho 2	etch 2	. .	
step	a	b	c	d	e	f	g	h	i		
1				0.25							clean wafer
2	8.00										grow a layer
3			1.00								pattern it
4					1.00						etch away some
5		6.00									grow a layer
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OUTLINE$_{03}$

- Traffic flow like models
- Clearing functions: Quasi - steady state models - queueing theory. Similarities and differences to traffic flow models.
- First principle models for non - equilibrium regimes (kinetic equations and conservation laws.)
  - Stochasticity (transport in random medium).
  - Non Markov dynamics and fluctuations.
  - Hyperbolicity vs. diffusion. (???)
  - Policies and networks (traffic rules).

Traffic flow - like models

- Introduce the stage of the whole process as an artificial 'spatial' variable. Items enter as raw product at $x = 0$ and leave as finished product at $x = X$.
- Define microscopic rules for the evolution of each item.
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  - many body theory, large time averages
  - fluid dynamic models (conservation laws).
- Analogous to traffic flow models (items $\leftrightarrow$ vehicles).

Quasi - Steady State Models and Clearing functions $_{07}$

- A clearing function relates the expectation of the throughput time in steady state of each item for a given supplier to the expectation of the load, the 'Work in Progress'.
- Derived from steady state queuing theory.
- Yields a formula for the velocity of an item through the stages (Graves '96) and a conservation law of the form

$$\boxed{\partial_t \rho + \partial_x [v(x,\rho)\rho] = 0}$$

ρ: item density per stage,

$x \in [0, X]$: stage of the process.

Example: $M/M/1$ queues and simple traffic flow models

Arrivals and processing times governed by Markov processes:

$$v(x,\rho) = \frac{c(x)}{1+\rho}, \quad c(x) = \frac{1}{\langle \text{processing times} \rangle}$$

$c(x)$: service rate or capacity of the processor at stage x.

Simplest traffic flow model (Lighthill - Whitham - Richards)

$$v(x,\rho) = v_0(x)(1 - \frac{\rho}{\rho_{jam}})$$

- In supply chain models the density ρ can become arbitrarily large, whereas in traffic the density is limited by the space on the road ρ_{jam}.

phase velocity: $v_{phase} = \frac{\partial}{\partial \rho}[\rho v(x, \rho)]$

$$\boxed{v_{phase} = \frac{c(x)}{(1+\rho^2)} > 0, \quad v_{phase-traffic} = v_0(x)[1 - \frac{2\rho}{\rho_{max}}]}$$

- In supply chain models the propagation of information (shock speeds) is strictly forward $v_{phase} > 0$, whereas in traffic flow models shock speeds can have both signs.
- Problem: Queuing theory models are based on quasi - steady state regime. Modern production systems are almost never in steady state. (short product cycles, just in time production).
- Goal: Derive non - equilibrium models from first principles (first for automata) and then including stochastic effects.

First principle models for automata$_{12}$

- Assume processors work deterministically like automata. A processor located in the infinitesimal stage interval of length Δx needs a time $\tau(x) = \frac{\Delta x}{v_0(x)}$ to process an item.
- It cannot accept more than $c(x)\Delta t$ items per infinitesimal time interval Δt.

Theorem (Armbruster, CR SIAM.J.MMS'03): In the limit $\Delta x \to 0,\ \frac{\Delta t}{T} \to 0$. this yields a conservation law for the density ρ of items per stage of the form

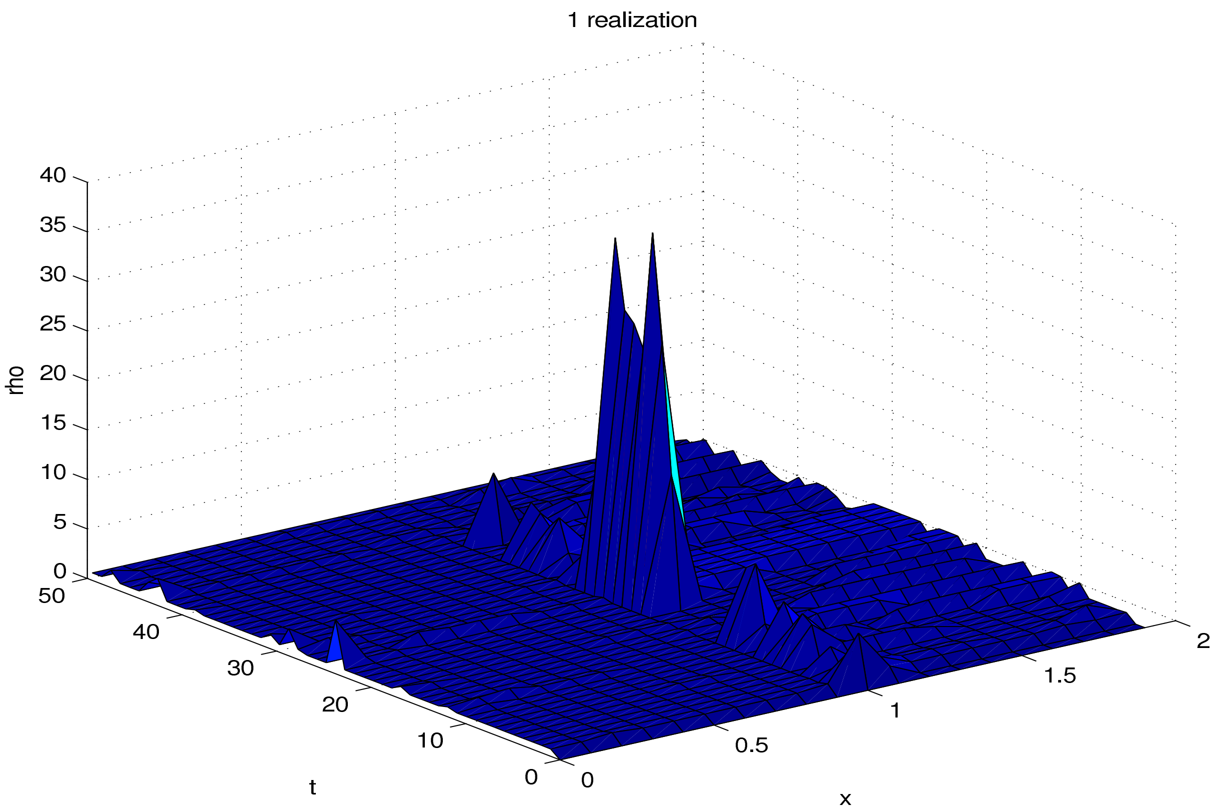
$$\boxed{\partial_t \rho + \partial_x F(x,\rho) = 0, \quad F(x,\rho) = \min\{c(x), v_0(x)\rho\}}$$

Bottlenecks

$$\boxed{\partial_t \rho + \partial_x F(x,\rho) = 0, \quad F(x,\rho) = \min\{c(x), v_0(x)\rho\}}$$

- No maximum principle (similar to pedestrian traffic with obstacles).
- The capacity $c(x)$ is discontinuous if nodes in the chain form a bottleneck.
- Flux F discontinuous $\Rightarrow$ density ρ distributional. (alternative model by Klar, Herty '04).
- Random server shutdowns $\Rightarrow$ bottlenecks shift stochastically.

A bottleneck in a continuous supply chain



Temporary overload of the bottleneck located at $x = 1$.

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Stochasticity: Random breakdowns and random media 15

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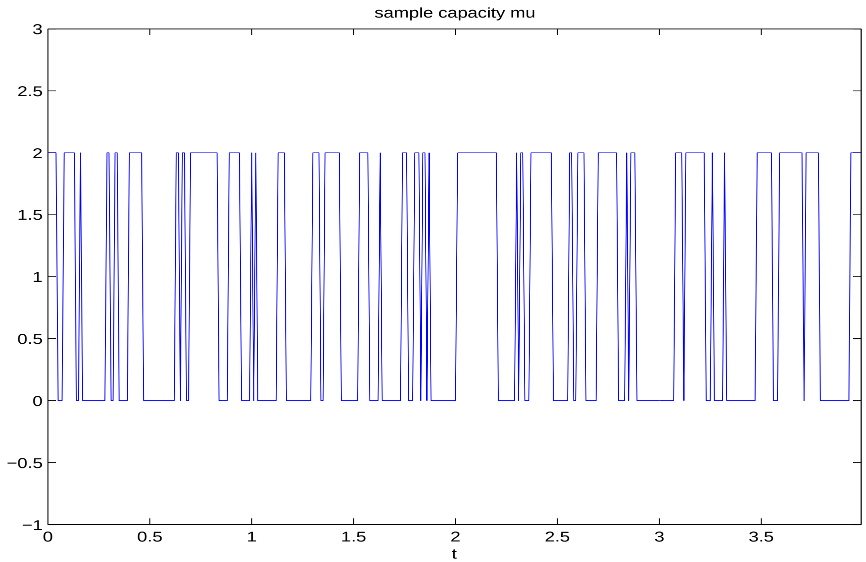
Availability

diffusion 1	diffusion 2	litho 1	etch clean	etch 1	ion impl	metal dep	litho 2	etch 2		
0.00	0.00	0.00	0.00	0.00	0.00	4.50	5.00	8.50		total hours required per lot
0.00	0.00	0.00	0.00	0.00	0.00	900.00	1000.00	1700.00		total hours needed per week
0.80	0.75	0.90	0.70	0.75	0.85	0.85	0.90	0.65		(average availability)
134.40	126.00	151.20	117.60	126.00	142.80	142.80	151.20	109.20		total hours available per machine per week
0.00	0.00	0.00	0.00	0.00	0.00	6.30	6.61	15.57		tools needed as time req / time avail
1.25	1.25	1.00	2.00	1.50	1.25	1.25	1.10	1.50		degree of constrainedness desired
0.00	0.00	0.00	0.00	0.00	0.00	7.88	7.28	23.35		number of tools needed
0	**0**	**0**	**0**	**0**	**0**	**8**	**8**	**24**		number of tools installed

Random capacities

Random breakdowns modeled by a Markov process setting the capacity to zero in random intervals.

$$\boxed{\partial_t \rho + \partial_x [\min\{c(x,t), v_0 \rho\}] = 0}$$



One realization of the capacity $c(x,t)$ for one processor x.

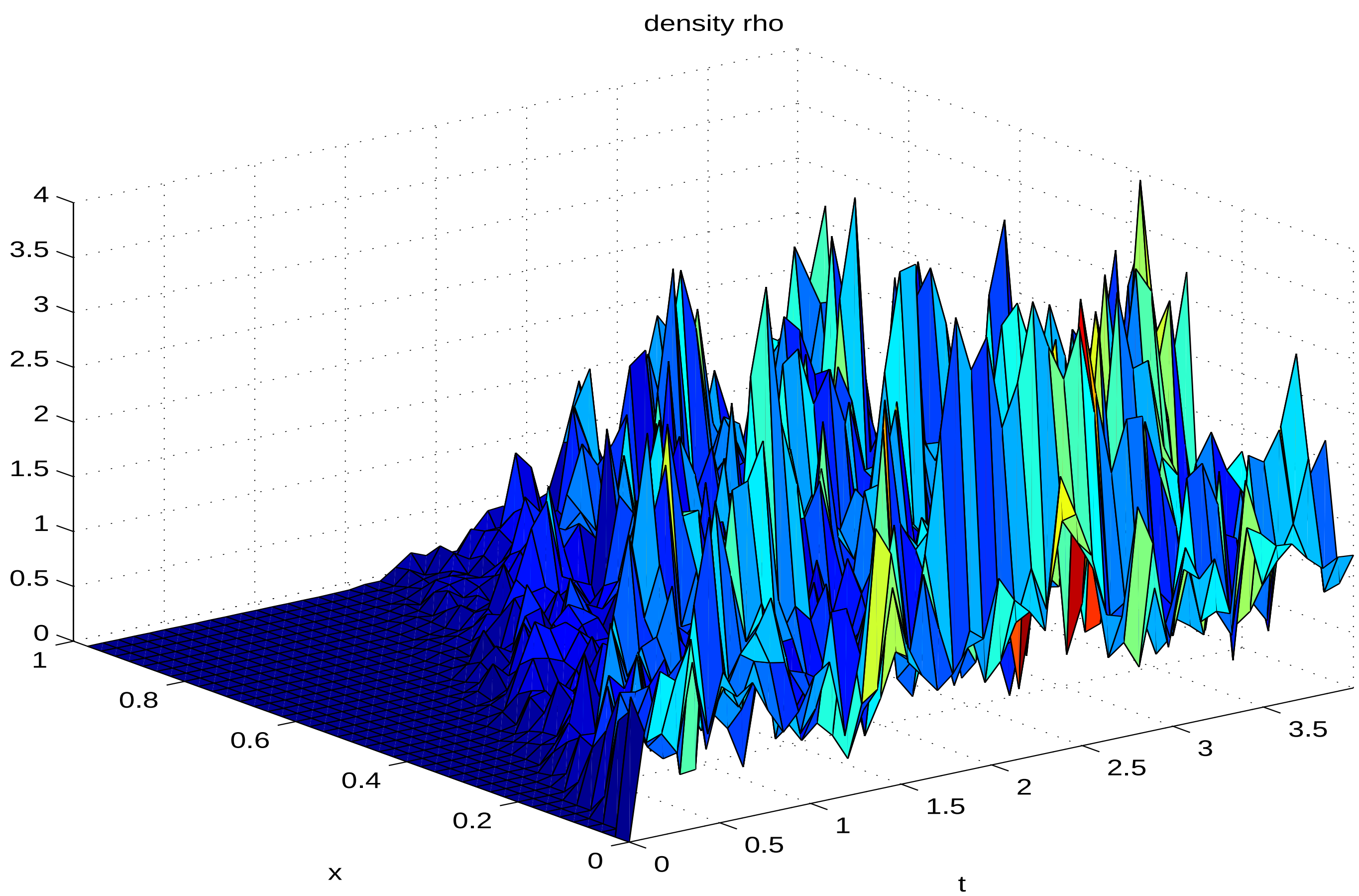
The Markov process: $_{17}$

$c(x,t)$ switches randomly between $c = c^{up}$ and $c = 0$

$$\boxed{c(x, t+\Delta t) = \begin{pmatrix} c(x,t) & prob = 1 - \Delta t \omega(x,c) \\ c^{up}(x) - c(x,t) & prob = \Delta t \omega(x,c) \end{pmatrix}}$$

- Frequency $\omega(x,c)$ given by mean up and down times of the processors.
- Particle moves in random medium given by the capacities.

One realization with flux $F = \min\{c, v\rho\}$ using a stochastic c



Goal: Derive equation for the evolution of the expectation $\langle\rho\rangle$ and the variance.

The many body problem $_{19}$

- Formulate deterministic model in Lagrangian coordinates. $\xi_n(t)$: position of part n at time t.
- A 'follow the leader' model:

$$\boxed{\frac{d}{dt}\xi_n = \min\{c(\xi_n, t)[\xi_n - \xi_{n-1}],\ v_0(\xi_n)\}}$$

- Particles move in a random medium, given by stochastic capacities $c(\xi_n, t)$

Kinetic equation for the many body probability density 20

$F(t, x_1, .., x_N, y_1, ., y_K)$: probability that

$\xi_1(t) = x_1, .., \xi_N(t) = x_N$ and $c_1(t) = y_1, .., c_K(t) = y_K$.

Satisfies a Boltzmann equation in high dimensional space.

$$\partial_t F(t, X, Y) + \nabla_X \cdot [V(X, Y)F] = Q[F]$$

$$Q[F] = \int K(X, Y, Y')F(t, X, Y')\Omega(X, Y')\, dY' - \Omega(X, Y)F$$

$X = (x_1, .., x_N)$: positions

$Y = (y_1, .., y_K),\ Y \in \{0, c^{up}\}^K$: kinetic variable (discrete velocity model).

$Q[F]$: interaction with random background.

- Discrete event simulation corresponds to solving the kinetic many body equation by Monte Carlo.
- Use methodology for many particle systems. Mean field theory, long time averages, Chapman - Enskog.

Mean field theory $_{23}$

- Assume identical particles and statistical independence (molecular chaos).
- Molecular chaos assumption for the conditional probability.

$$\boxed{F(t,X,Y) = F(t,X \mid Y)G(t,Y)}$$

$$G(t,Y) = \frac{d\mathcal{P}}{dY}[c_1 = y_1, .., c_K = y_K]$$

G: probability density of the state of the machines (independent of the ensemble).

$F(t,X \mid Y)$: conditional probability, given one realization of the machine background.

$$F(t, X \mid Y) = \frac{d\mathcal{P}}{dX}[\xi_1 = x_1, ..\xi_N = x_N \mid c_1 = y_1, .., c_K = y_k]$$

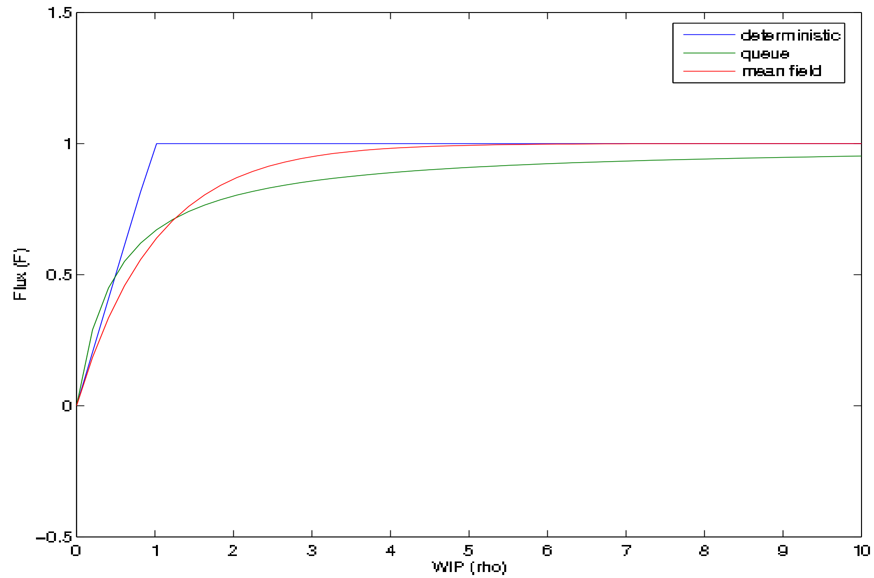
Molecular chaos ansatz for $F(t, X \mid Y)$:

$$\boxed{F(t, X \mid Y) = \prod_{n=1}^{N} f(t, x_n, Y)}$$

Theorem (Degond, CR, SIAP'06):$_{25}$

For a large ensemble ($N \to \infty$) the conditional density $f(t,x,Y)$ satisfies

$$\boxed{\partial_t f + \partial_x \Phi = Q[f], \quad \Phi(t,x,Y) = c(t,x,Y)[1 - \exp(-\frac{v_0 f}{c})]}$$



, $f << 1 \Rightarrow \Phi \approx v_0 f, \;\; f >> 1 \Rightarrow \Phi \approx c$

Large time asymptotics $_{27}$

$$c(x,t+\Delta t) = \begin{pmatrix} c(x,t) & prob = 1 - \Delta t\omega(x,c) \\ c^{up}(x) - c(x,t) & prob = \Delta t\omega(x,c) \end{pmatrix}$$

Consider time scales $>> \frac{1}{\omega}$

$$\partial_t f(t,x,Y) + \partial_x \Phi = \frac{1}{\varepsilon} Q[f],$$

Chapman - Enskog expansion

$$f(t,x,Y) = \psi(\rho(t,x),Y,\varepsilon), \quad \rho = \int \psi(\rho,Y,\varepsilon)\, dY \ \forall \rho$$

ψ: shape function, parameterized by its mean in Y. expand

$$\psi(\rho,Y,\varepsilon) = \psi_0(\rho,y) + \varepsilon\psi_1(\rho,Y) + \ldots$$

Equation for mean and variance

$$\boxed{\partial_t \rho(t,x) + \partial_x \{a c_{up}(x)[1 - \exp(-\frac{v_0 \rho}{c_{up}})] - a\varepsilon D(\rho)\mathcal{V}^2 \partial_x \rho\} = 0}$$

$a(x)$: availability $= \frac{\langle T_{up}\rangle}{\langle T_{up}\rangle + \langle T_{down}\rangle}$, $(\langle T\rangle = \frac{1}{\omega})$

$\mathcal{V} = \frac{\sigma}{\langle T\rangle}$: variation coefficient ($= 1$ for a Markov process).

compare to

$$\boxed{\partial_t f + \partial_x \Phi = \frac{1}{\varepsilon} Q[f], \quad \Phi(t,x,Y) = c(t,x,Y)[1 - \exp(-\frac{v_0 f}{c}))]}$$

- c replaced by c_{up}
- Flux multiplied by the average availability a.
- Fluctuations enter as a (higher order) diffusive term.

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Non - Markov processes $_{30}$

Problem:

$$c(x,t+\Delta t) = \begin{pmatrix} c(x,t) & prob = 1 - \Delta t \omega(x,c) \\ c^{up}(x) - c(x,t) & prob = \Delta t \omega(x,c) \end{pmatrix}$$

- No memory of how long the processor has been running or down.
- T_{up}, T_{down} distributed according to exponential distributions $\Rightarrow \mathcal{V} = 1$.
- Analogy to 'intelligent particles' with memory (drivers).
- Model an arbitrary stochastic process, by enlarging the space.

A different game

τ: time elapsed since the last change of state of the processor.

$$c(x, t + \Delta t) = \begin{pmatrix} c(x,t) & prob = 1 - \Delta t \omega(x, c, \tau) \\ c^{up}(x) - c(x,t) & prob = \Delta t \omega(x, c, \tau) \end{pmatrix}$$

$$\tau(x, t + \Delta t) = \begin{pmatrix} \tau(x,t) + \Delta t & prob = 1 - \Delta t \omega(x, c, \tau) \\ 0 & prob = \Delta t \omega(x, c, \tau) \end{pmatrix}$$

- Larsen ('07) (Radiative transfer models in clouds)

Lemma $_{32}$

$u(x,c,s)\,ds$: distribution of the time between scattering events.

$$\boxed{u(x,c,s)\,ds = d\mathcal{P}[\tau = s] = \omega(x,c,s)\exp[-\textstyle\int_0^s \omega(x,c,q)\,dq]\,ds}$$

$\Rightarrow$ given any kind of probability distribution $u(x,c,\tau)\,d\tau$ of up / down times, we define a (τ dependent) frequency ω

$$\boxed{\omega(x,c,\tau) = \frac{u(x,c,\tau)}{1-\int_0^\tau u(x,c,s)\,ds}}$$

giving

$$\omega(x,c,\tau)\exp[-\int_0^\tau \omega(x,c,s)\,ds] = u(x,c,\tau)$$

This is a generalization of the Markov process, since, for $\omega(x,c,\tau) = \omega(x,c)$ we have

$$\omega(x,c)\exp[-\tau\omega(x,c)] = u(x,c,\tau)$$

Using this trick, everything stays the same, except for the fact that the conditional probability (in the mean field assumption) satisfies

$$\boxed{\partial_t f(t,x,Y,\overrightarrow{\tau}) + \partial_x \Phi = \frac{1}{\varepsilon} Q[f],}$$

$$\boxed{Q[f] =}$$

$$\boxed{-\nabla_{\overrightarrow{\tau}} f + \sum_j \delta(\tau_j) \int K(x,Y,Y')\Omega f(t,x,Y',\tau')\, dY'\tau' - \Omega(x,Y,\tau)}$$

CE for the modified process

Large time asymptotics (Chapman - Enskog) gives the same result

$$\boxed{\partial_t \rho(t,x) + \partial_x \{ a c_{up}(x)[1 - \exp(-\frac{v_0 \rho}{c_{up}})] - a\varepsilon D(\rho) \mathcal{V}^2 \partial_x \rho \}}$$

- $\mathcal{V}$: Variation coefficient for the up and down times of the general underlying switching process.

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Diffusion vs. Hyperbolicity $_{35}$

The basic problem:

- A diffusion equation (as a result of Chapman - Enskog) propagates information in both directions.
- Parts (or drivers in traffic flow) do not react to what is happening behind them.
- This is an artifact of the Chapman - Enskog procedure which transforms diffusion (arising from the random fluctuations in the flow) in velocity into spatial diffusion in a macroscopic limit.
- General problem for directional flows and fluctuations.

- In practice, the equation

$$\boxed{\partial_t \rho(t,x) + \partial_x \{a c_{up}(x)[1 - \exp(-\frac{v_0 \rho}{c_{up}})] - a\varepsilon D(\rho)\mathcal{V}^2 \partial_x \rho\}}$$

needs a boundary condition at $x = 1$ and there is no 'physics' to determine this condition.

Hyperbolic Relaxation Models - Basic Idea[38]

- Solve the kinetic equation by a moment closure, taking additional (not conserved) moments.
- $\Rightarrow$ a hyperbolic system, still containing ε.
- Close the moment hierarchy by an ansatz, such that $\varepsilon \longrightarrow 0$ asymptotics on the macroscopic level would reproduce the diffusion picture.
- Don't do it. Use the hyperbolic model instead.
- Natalini, Jin, Slemrod ('95): Regularization of the Burnett and super - Burnett equations.

One more moment

$$\boxed{\partial_t \rho + \partial_x(u\rho) = 0, \quad \partial_t(u\rho) + \partial_x[u^2\rho + P\rho] = \frac{\rho}{\varepsilon}(u_0 - u)}$$

P: pressure (from the closure).

Asymptotics on the hyperbolic level:

$$u(x,t) = u_0 - \frac{\varepsilon}{\rho}\partial_x[P\rho]$$

- Close with the asymptotic form given by Chapman - Enskog:

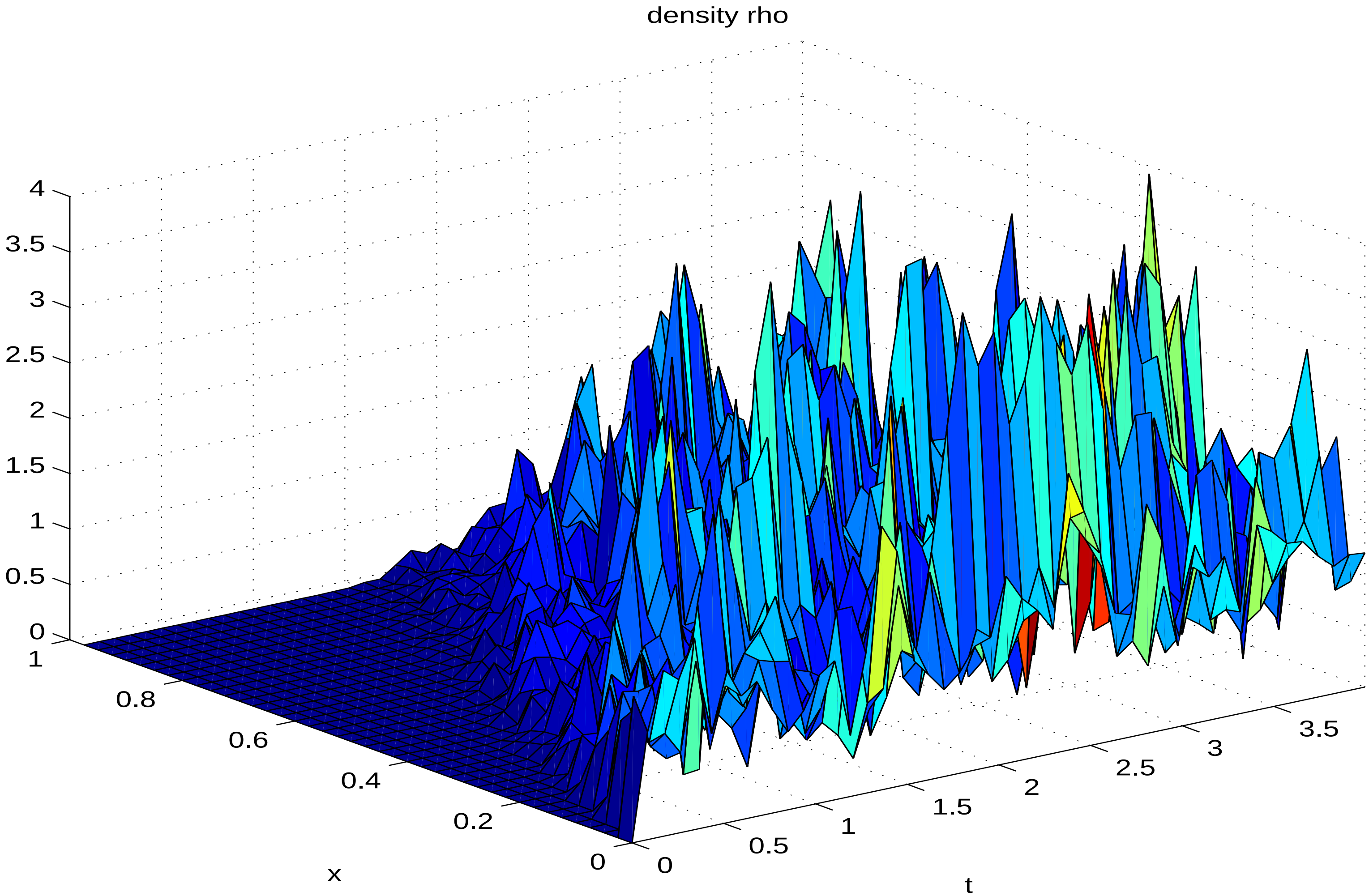
$$\rho u^2 + \rho P = \int V^2 \psi_\varepsilon(\rho, Y, t)\, dY$$

- Show that characteristic speeds ≥ 0 (at least for $\varepsilon << 1$).

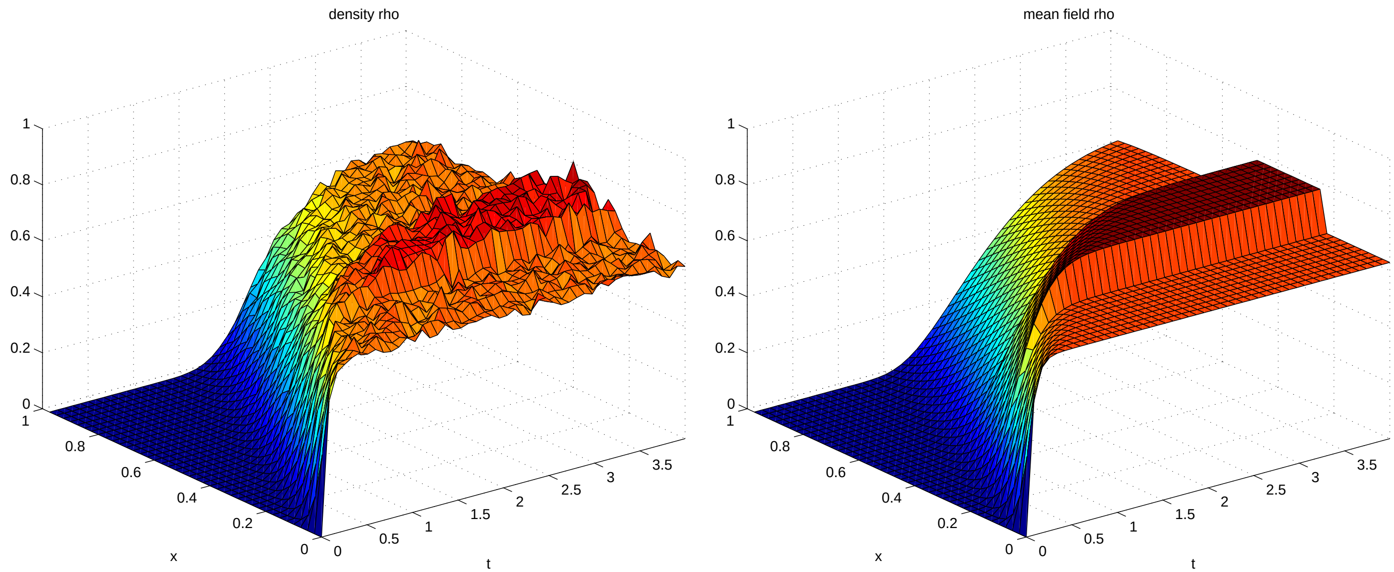
Open Problem

- Doable for linear problems $V(\rho, Y) = V(y)$. (Armbruster, Unver, CR, Proc. AMS '08)
- Only for small densities ρ in the nonlinear case.
- Derive a closure for which the phase velocities have the right sign (> 0) in the nonlinear setting.
- Need some form of unidirectional diffusion model.
- Same problem in traffic flow if random fluctuations in driver behavior are considered.

One realization



The steady state case



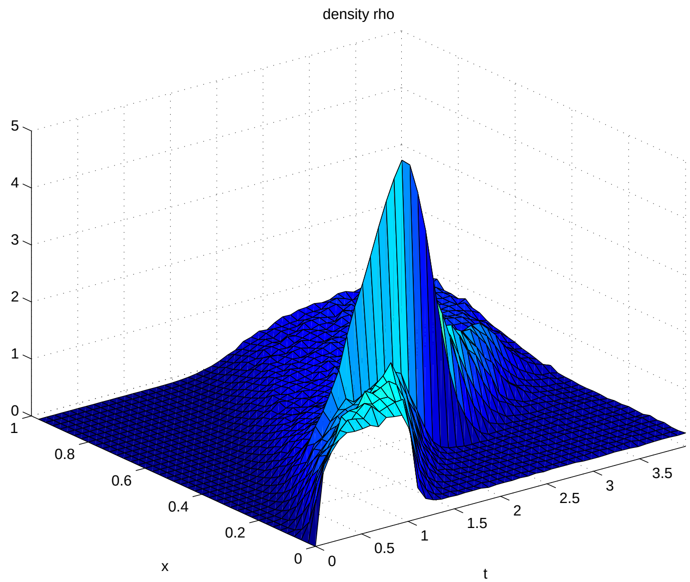
60 stages, bottleneck processors for $0.4 < x < 0.6$

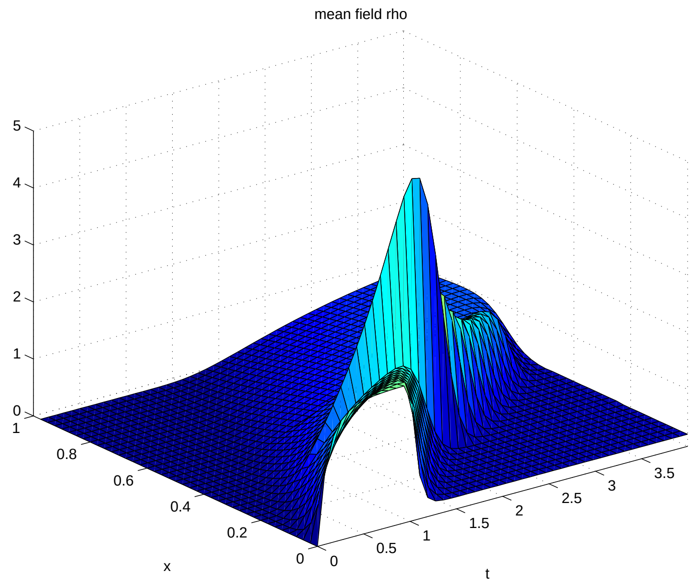
Constant influx;

$F(x = 0) = 0.5\times$ bottleneck capacity

Left: DES (100 realizations), Right: mean field equations

Verification of the transient case





Influx $F(x = 0)$ temporarily at $2.0\times$ the bottleneck capacity

Left: 500 realizations, Right: mean field equations

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Re - entrant Networks and Scheduling Policies 40

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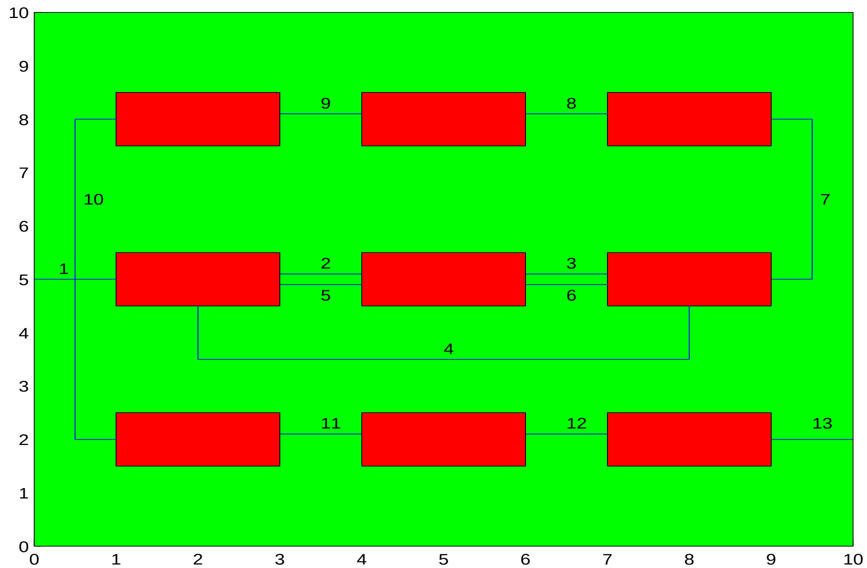
Re - entrant manufacturing lines: One and the same tool is used at different stages of the manufacturing process.



► Conservation laws on graphs. Implies that the velocity is computed non - locally. (Different stages of the process correspond to the same physical node.)

► Requires the use of a policy governing in what sequence to serve different lines ('the right of way': FIFO, FISFO, PULL, PUSH).

Priority scheduling $_{42}$

- Equip each part with an attribute vector $y \in \mathbb{R}^K$.
- Define the priority of the part by $p(y) : \mathbb{R}^K \longrightarrow \mathbb{R}$
- The velocity of the part is determined by all the parts using the same tool at the same time with a higher priority.
- Leads to a (nonlocal) kinetic model for stages and attributes (high dimensional).
- Recover systems of conservation laws by using multi - phase approximations for level sets in attribute space.

Kinetic model (Degond, Herty, CR '07)

(Vlasov - type)

$$\boxed{\partial_t f(x,y,t) + \partial_x[v(\phi(x,p(y)))f] + \nabla_y[Ef] = 0}$$

f: kinetic density of parts at stage x with attribute y.

$p(y)$: priority of parts with attribute y.

$\phi(x,q)$: cumulative density of parts with priority higher than q.

$$\boxed{\phi(x,q) = \int H(p(y)-q) f(x,y,t)\, dy}$$

or (for re-entrant systems):

$$\boxed{\phi(x,q) = \int H(p(y)-q) K(x,x') f(x',y,t)\, dx' dy}$$

Choices:

- Attributes y;
- $p(y)$ determines the policy;
- the velocity $v(\phi)$ (the flux model).

Example: $y \in \mathbb{R}^3$

y_1: cycle time (time the part has spent in the system).

y_2: time to due date.

y_3: type of the part (integer valued).

$$\partial_t f + \partial_x [vf] + \nabla_y \cdot [Ef] = 0 \Rightarrow E = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Policies:

- FISFO: $p(y) = y_1$
- Due date scheduling: $p(y) = -y_2$.
- Combined policy (c.f. for perishable goods)

$$\boxed{p(y) = y_1 H(y_1 - d(y_3)) - y_2 H(d(y_3) - y_1)}$$

The phase velocity:

The microscopic velocity $v(\phi)$ has to be chosen as the phase velocity $\frac{\partial F(\phi)}{\partial \phi}$ of a macroscopic conservation law.

Theorem (CR, CMMS '09)$_{29}$: The total density of parts (with all attributes) $\rho(x,t) = \int f(x,y,t)\, dy$ satisfies the conservation law

$$\boxed{\partial_t \rho + \partial_x F(\rho) = 0, \quad F(\rho) = \int_{-\infty}^{\rho} v(\phi)\, d\phi}$$

Decide on an over all flux model $F(\rho)$. Set $v(\phi) = \partial_\phi F(\phi)$.

Example: Deterministic flux model.

$$\boxed{F(\rho) = \min\{c, v_0 \rho\} \ \Rightarrow v(\phi) = v_0 H(c - v_0 \phi)}$$

Multi - phase approximations

- Leads to high dimensional kinetic equation. Reduce to conservation laws via a multi - phase ansatz.
- Approximate $f(x, y, t)$ by a combination of $\delta-$ measures in y.

$$\boxed{f(x, y, t) = \sum_n \rho_n(x, t)\delta(y - Y_n(x, t))}$$

- Derive conservation laws for the number densities $\rho_n(x, t)$ with attributes $y = Y_n(x, t)$.

Standard approach (Jin, Li 03): Moment closures

Level Sets $_{48}$

Almost all information about the microscopic transport picture is contained in the evolution of the level sets of parts with equal priority

$$\Lambda(x,q,t) = \int \delta(p(y)-q) f(x,y,t)\, dy = -\partial_q \phi(x,q,t)$$

Level set equation:

$$\boxed{\partial_t \Lambda(x,q,t) + \partial_x[v(\phi(x,q))\Lambda] + \partial_q A[f] = 0, \quad \Lambda = -\partial_q \phi}$$

$$A[f](x,q,t) = \int \delta(q-p) f[\partial_t p + v(\phi(x,p))\partial_x p + E\nabla_y p] dy$$

The Riemann Problem

The multi - phase approximation implies for the level sets $\Lambda(x, q, t)$ and the cumulative densities $\phi(x, q, t)$

$$\boxed{\Lambda(x, q, t) = \sum_n \rho_n(x, t)\delta(P_n - q)}$$

$$\boxed{\phi(x, q, t) = \sum_n \rho_n(x, t)H(P_n - q),}$$

with $P_n(x, t) = p(Y_n(x, t)) \in \mathbb{R}^1$.

The cumulative density $\phi(x, q, t)$ is piecewise constant in q $\Rightarrow$ solve a Riemann problem for ϕ and compute the motion of $p(Y_n)$ from the Rankine - Hugoniot condition for the shock speeds.

$$\boxed{\frac{d}{dt}P_n + v_n \partial_x P_n = A_n(Y), \quad v_n = \lim_{\varepsilon \to 0} \frac{F(\phi(P_n+\varepsilon)) - F(\phi(P_n-\varepsilon))}{\phi(P_n+\varepsilon) - \phi(P_n-\varepsilon)}}$$

The densities ρ_n are evolved according to

$$\boxed{\partial_t \rho_n + \partial_x (v_n \rho_n) = 0}$$

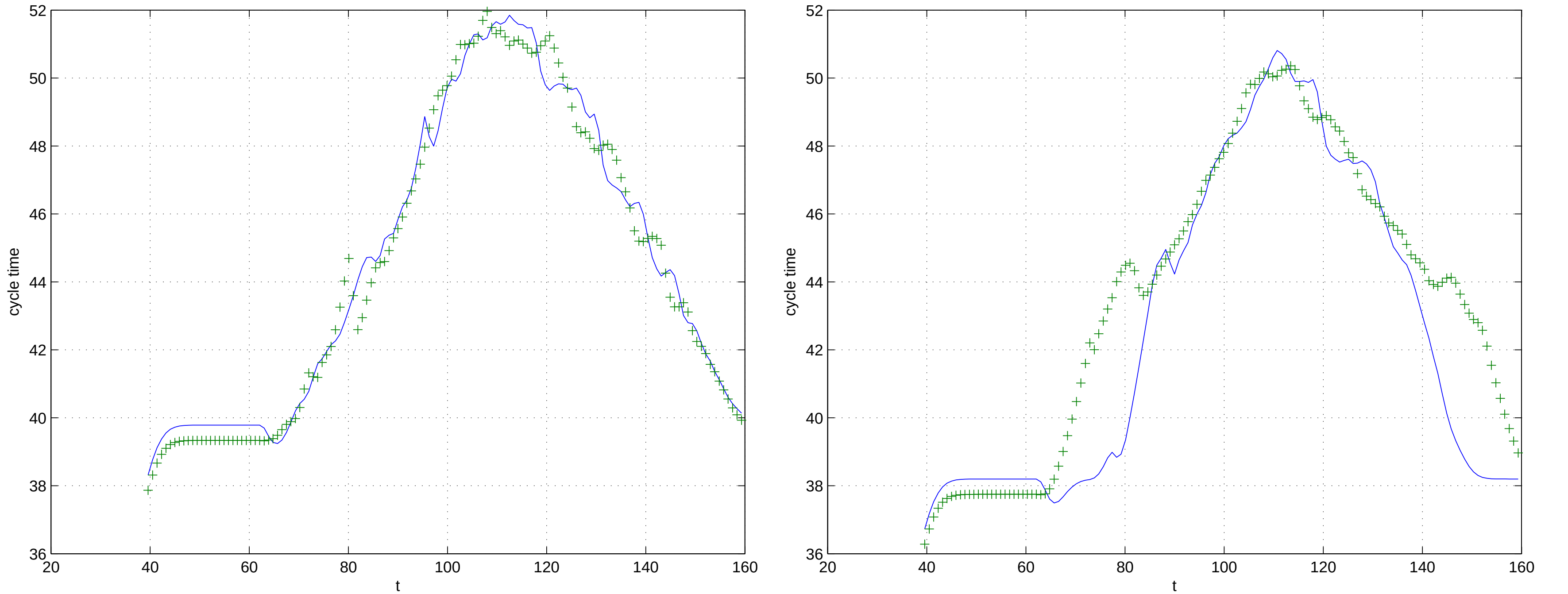
For $y \in \mathbb{R}^1$, $Y_n = P_n$ this is an exact (weak) solution of the kinetic transport equation.

For more than one dimensional attributes the actual attributes Y_n are evolved according to

$$\boxed{\partial_t Y_n + v_n \partial_x Y_n - E = 0, \quad v_n = \lim_{\varepsilon \to 0} \frac{F(\phi(P_n+\varepsilon)) - F(\phi(P_n-\varepsilon))}{\phi(P_n+\varepsilon) - \phi(P_n-\varepsilon)}}$$

within the level set - subject to the constraints $p(Y_n) = P_n$ (enforced by a projection method).

Policy effect on cycle time

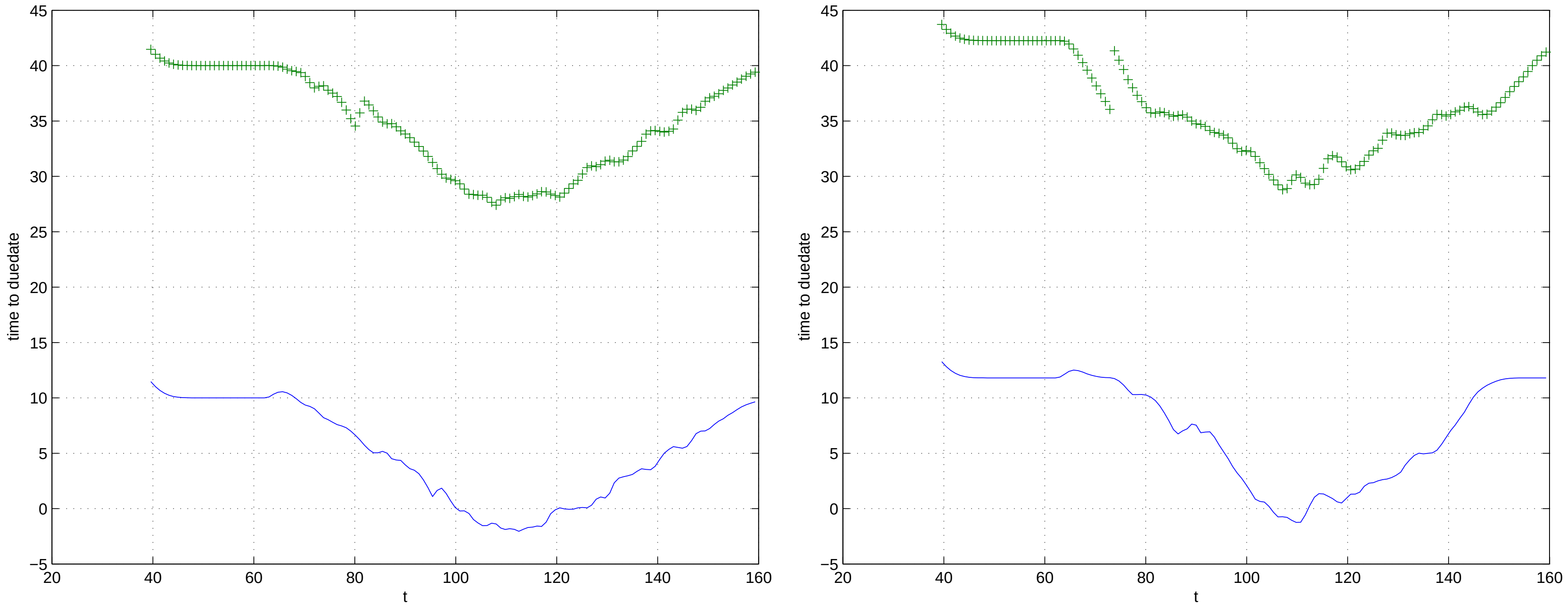


2 products with 2 different delivery due dates.

$+$:slow lots, $-$ hot lots.

Left: FISFO, Right: PERISH

Time to due date at exit



2 products with 2 different delivery due dates.

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Conclusions $_{50}$

- Value of PDE models: Intermediate tool between heuristic, fluid and DES - models. (Includes more detail, but still a conservation law.)
- Less versatile than DES, but amenable to optimization.
- Conservation laws on graphs (nonlocal constitutive relations).
- Issues:
  - Stochastic fluctuations on a macro level $\Rightarrow$ Non - Markovian behavior.
  - Directional flows and 'infinite' propagation speeds.
  - More complicated graphs (branching and downbinning).

Center for Scientific Computation And Mathematical Modeling

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an NSF Focus Research Group

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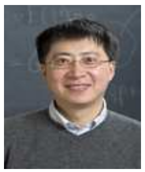

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