Optimal design of microstructured optical fibers

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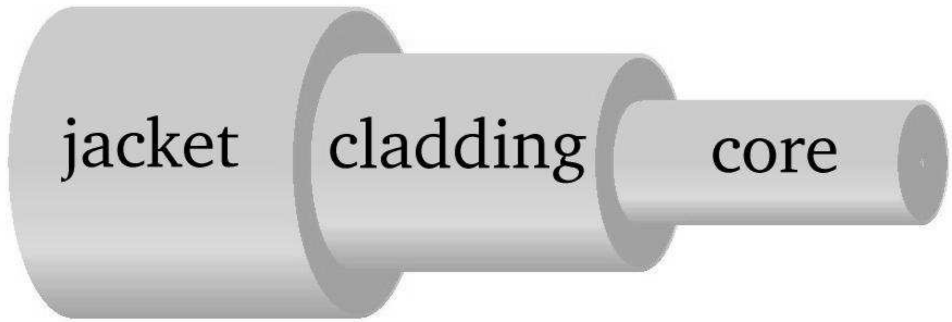
An optical fiber



Assumptions:

- The index of refraction: $n(x, y, z) = n(x, y)$.
- The cladding is infinite (no jacket).
- The core does not need to be circular.
- For some $R > 0$ and $x^2 + y^2 > R^2$, $n(x, y) = n_{clad}$ – the core is bounded.

The model

The Helmholtz equation

$$\Delta E + k^2 n^2 E = 0 \text{ in } \mathbb{R}^3.$$

Look for solutions propagating in the $z-$direction of the form

$$E(x, y, z) = e^{i\beta k z} u(x, y).$$

Obtain

$$-\Delta u + qu = \lambda u \text{ in } \mathbb{R}^2,$$

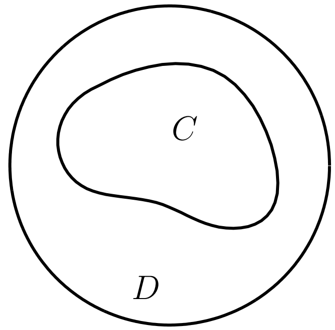
where

$$q(x, y) = k^2[n_{max}^2 - n(x, y)^2], \;\; \lambda = k^2(n_{max}^2 - \beta^2).$$

This as an *eigenvalue problem* in λ.

A *guided mode*: a solution $u(x, y)$ exponentially decaying as $x^2 + y^2 \to \infty$.

Solving for guided modes [Bonnet-Bendhia and Gmati]



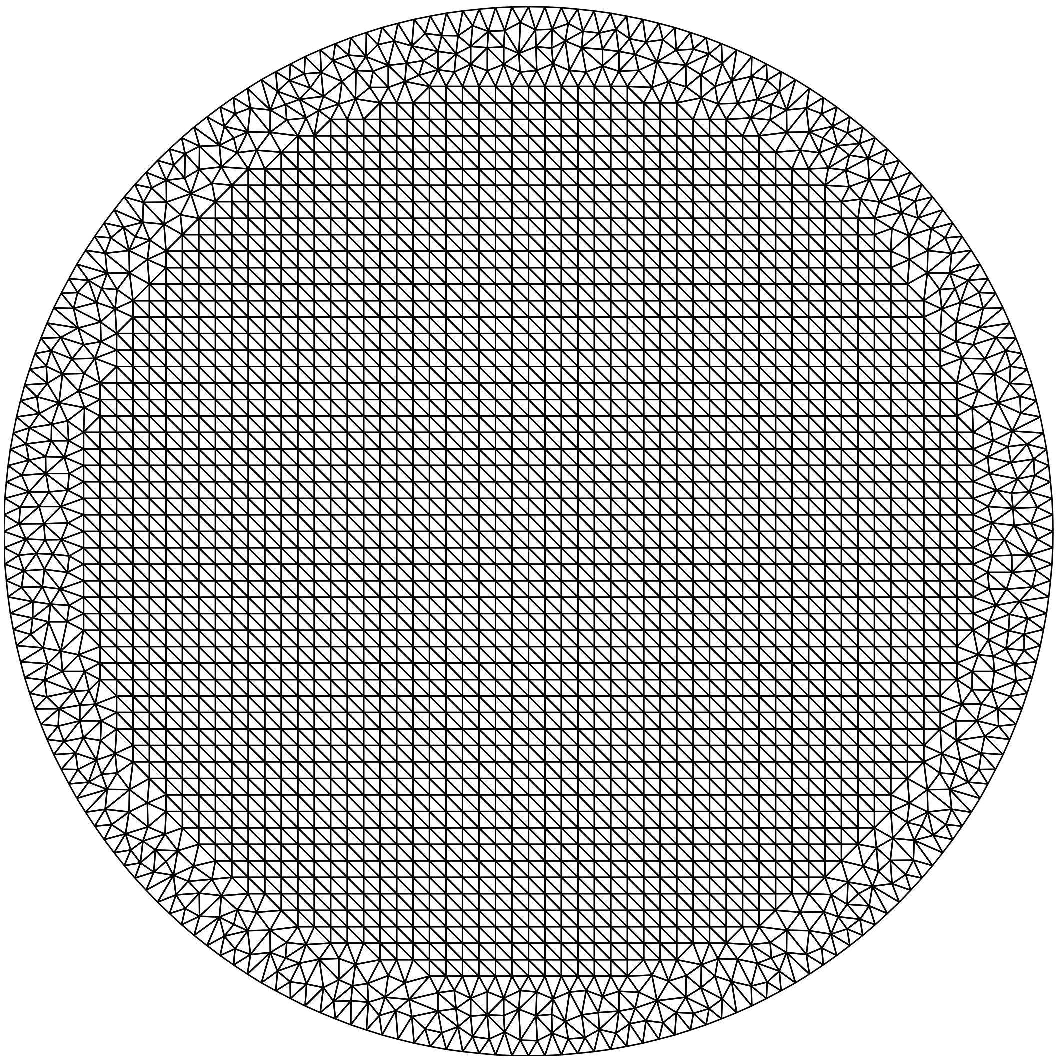
C – the fiber core, D – the domain of computation.

$$P : \begin{cases} -\Delta u + qu = \lambda u \text{ on } D, \\ \dfrac{\partial u}{\partial r} = A_\lambda(u|_{\partial D}) \text{ on } \partial D, \end{cases}$$

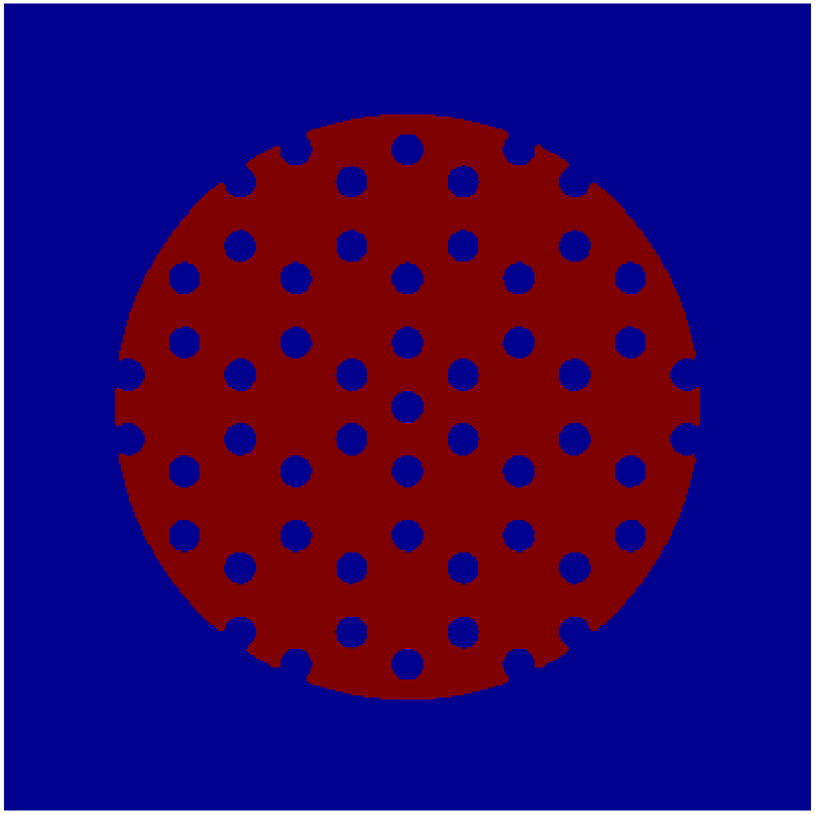
where for $\alpha \in (0, q_{max})$, $A_\alpha : H^{1/2}(\partial D) \to H^{-1/2}(\partial D)$,

$$v = \sum_{m\in\mathbb{Z}} v_m e^{im\theta} \to A_\alpha v = \sqrt{d^2 - \alpha} \sum_{m\in\mathbb{Z}} \frac{K_m'\left(R\sqrt{d^2-\alpha}\right)}{K_m\left(R\sqrt{d^2-\alpha}\right)} v_m e^{im\theta}.$$

The mesh – generated with *triangle* [J.R. Shewchuk]



The cross-section of a micro-structured optical fiber



The photonic bandgap effect

- There exist guided modes localized around the defect and decaying exponentially away from it.

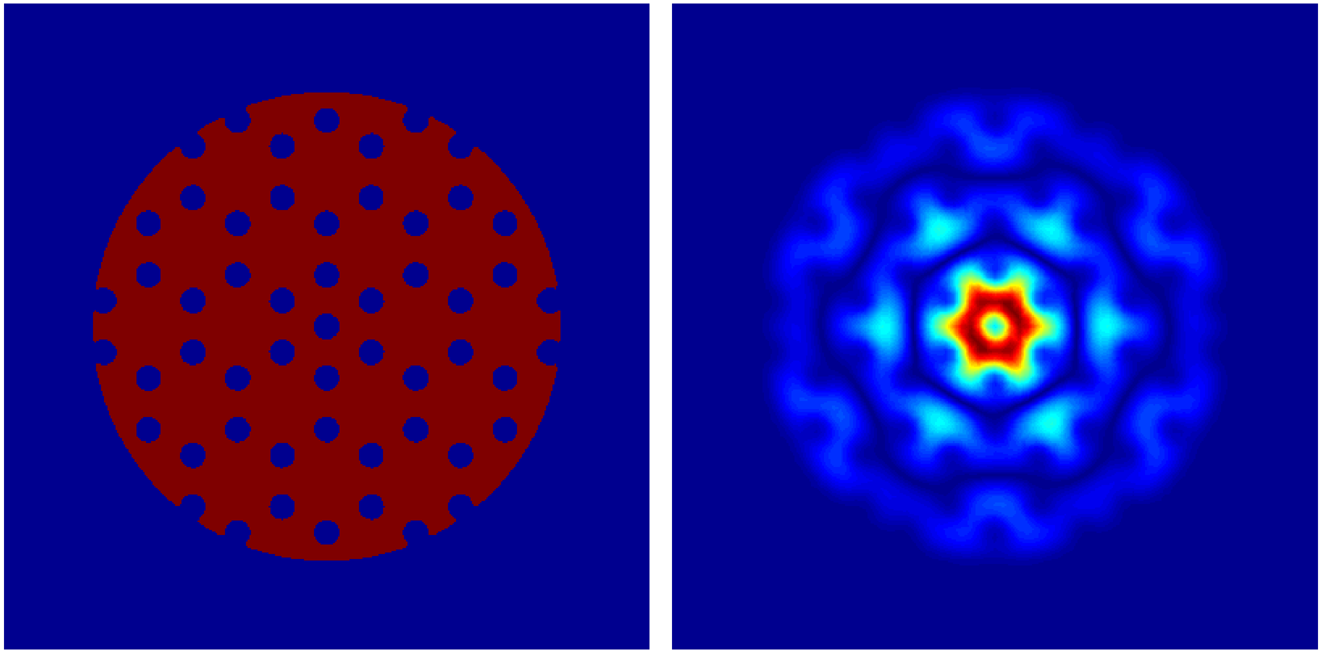


Figure 1: The cross-section of a micro-structured optical fiber and the corresponding field distribution.

The Photonic Bandgap Effect (continued)

- For those modes localized around the defect, it is not the difference in the indexes of refraction of the core and the cladding which is responsible for their exponential decay. These modes exist even if the core of the fiber is infinite, as long as the periodic structure is preserved.
- The defect is a perturbation in the periodic structure. The perturbation adds points of discrete spectrum to the otherwise continuous spectrum.
- The phenomenon of existence of such modes is called the *the photonic bandgap effect*.

The optimization problem

Let $k \in \mathbb{N}$ be such that the k-th mode of the optical fiber is localized around the defect. Denote that mode by u. Let $D \subset \mathbb{R}^2$ be the fiber cross-section. Try to make u be as localized as possible in the center of the fiber. Minimize

$$F(D) := \frac{\int\limits_{\mathbb{R}^2} (x^2 + y^2)\left[n(x,y)^2 - n_{clad}^2\right] u(x,y)^2 \, dx \, dy}{\int\limits_{\mathbb{R}^2} u(x,y)^2 \, dx \, dy},$$

subject to

$$G(D) := \text{area}(D) - K = 0,$$

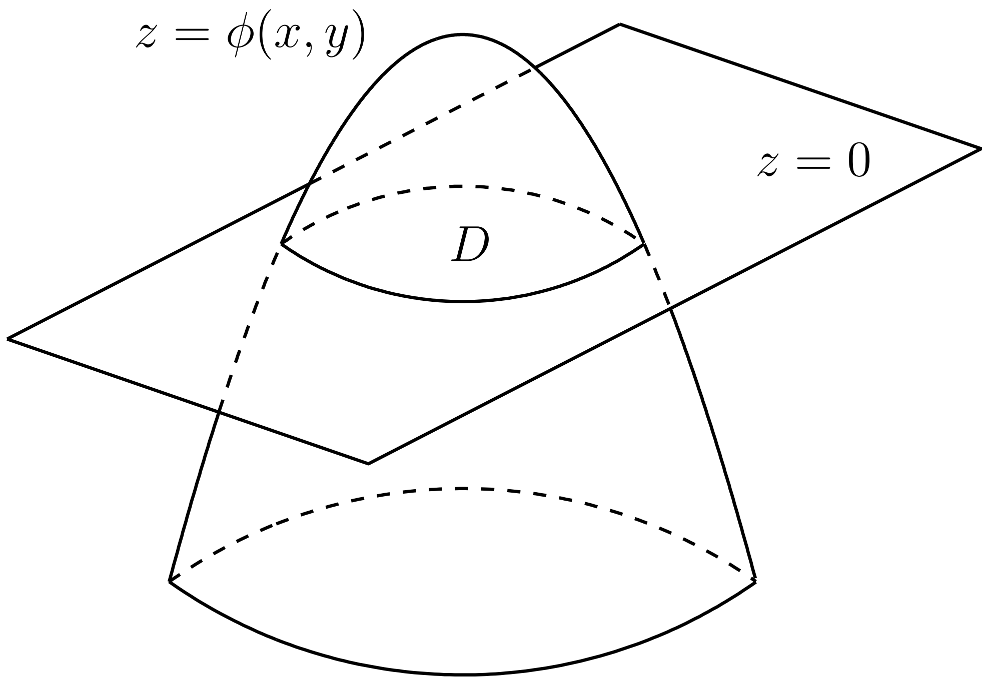
for some $K > 0$.

The level set method

Represent the *shape* D as the level set of a function ϕ :

$$D = \{(x, y) \in \mathbb{R}^2 : \phi(x, y) > 0\}.$$

Work with ϕ instead of D.



The steepest descent method [Osher and Santosa]

The optimization problem becomes

$$\min F(\phi) \text{ subject to } G(\phi) = 0.$$

To solve it, start with the initial guess ϕ_0, and at every iteration take a step in the direction v (that is, $\phi_{n+1} = \phi_n + \alpha\, v$, with $\alpha \in \mathbb{R}$) in which

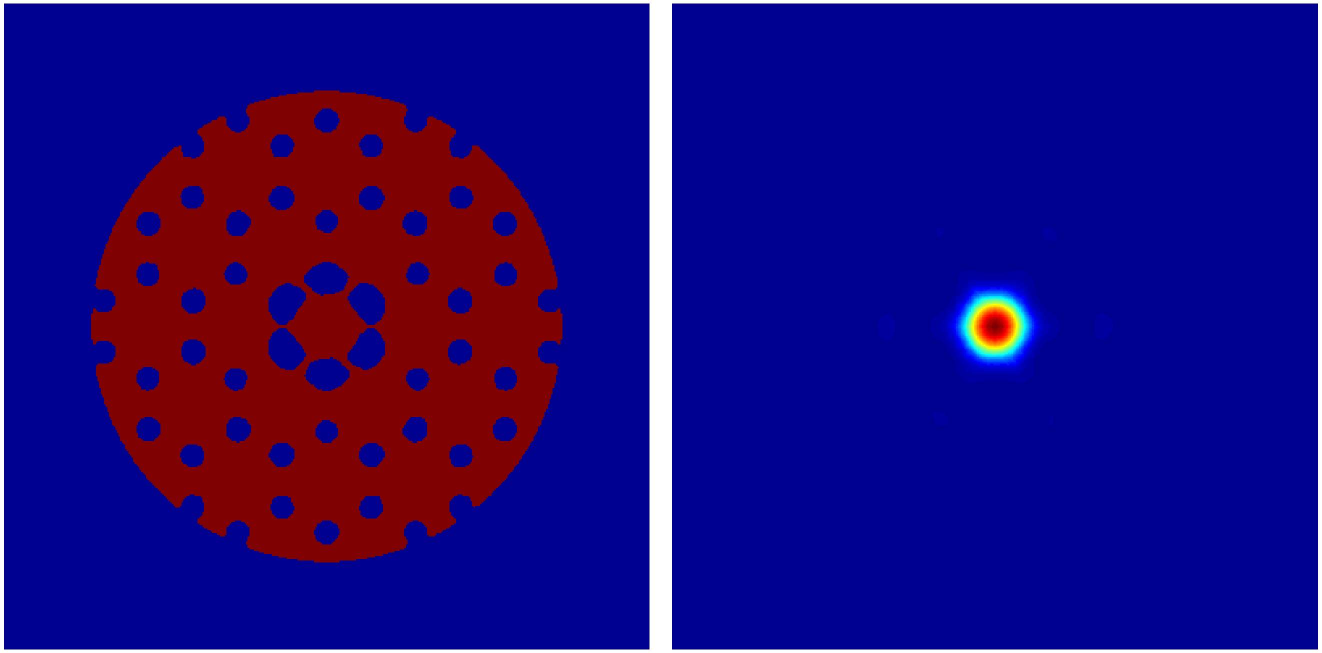
$$L(\phi, \nu) = F(\phi) + \nu G(\phi)$$

decreases fastest. To keep the constraint $G(\phi) = 0$ pick ν such that

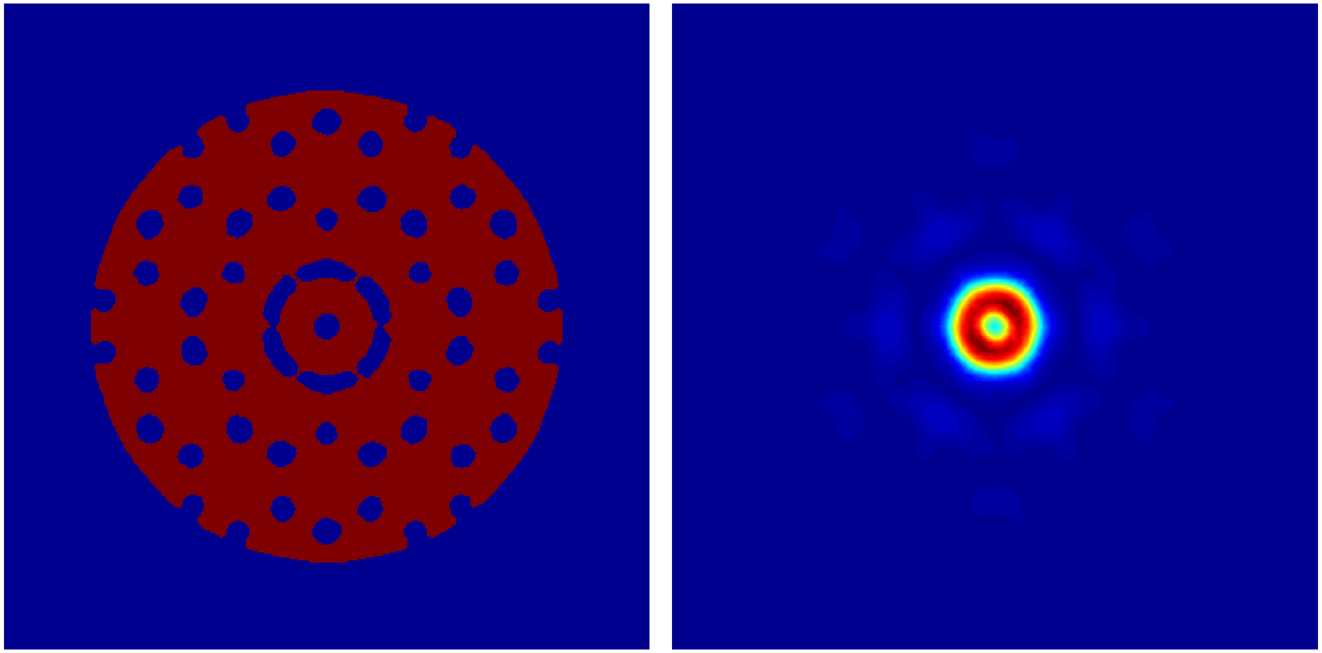
$$D_\phi G(\phi) \cdot v = 0.$$

Some corrections to ν might be necessary now and then; obtain the corrections by using Newton's method.

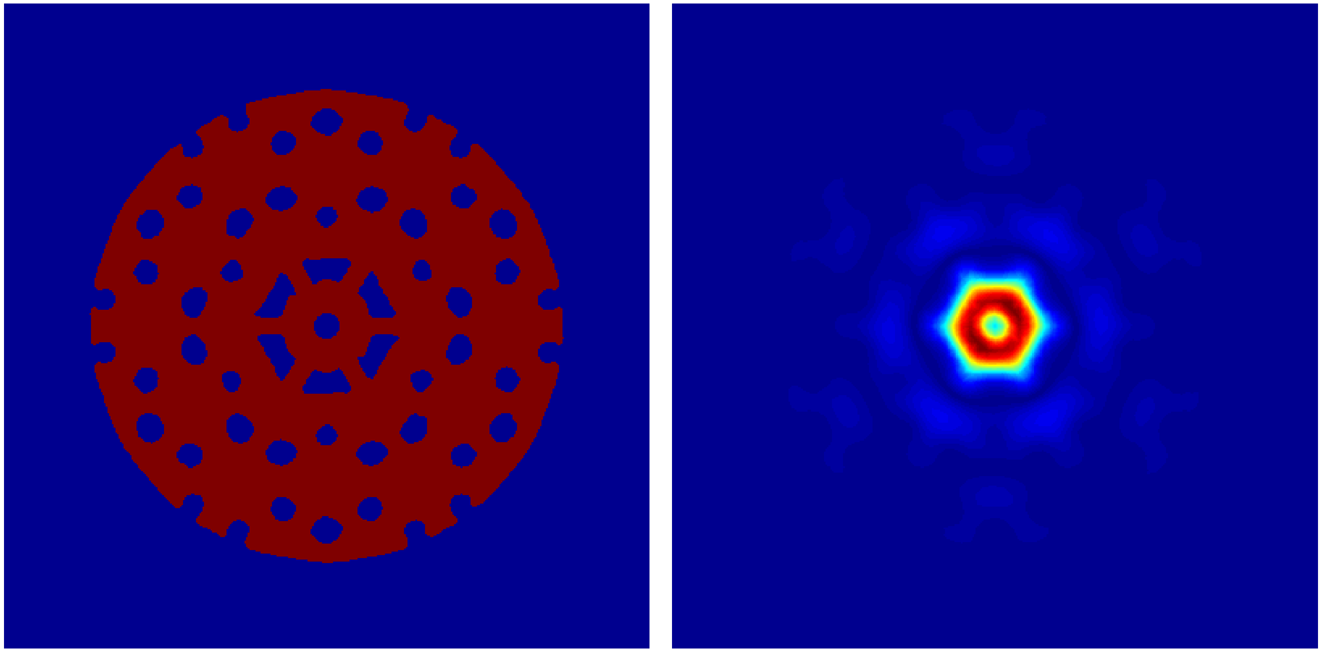
The result of optimization – no constraints



The result of optimization – the central hole constraint



The result of optimization – a rough constraint preventing the holes from merging



The problem

Find a way to do shape optimization while keeping the shape topology unchanged. That is, do not allow the holes in the shape to merge, split, or disappear.

The solution

Introduce a *penalty* functional. Most of the time its value will be negligible, but it will grow extremely large when the shape to be optimized is close to violating the constraints.

The neighborhood of the boundary

Let $d > 0$ and $l > 0$. Define

$$I_d = \{x + d\nabla\phi(x) : x \in \partial D\},$$

$$E_l = \{x - l\nabla\phi(x) : x \in \partial D\}.$$

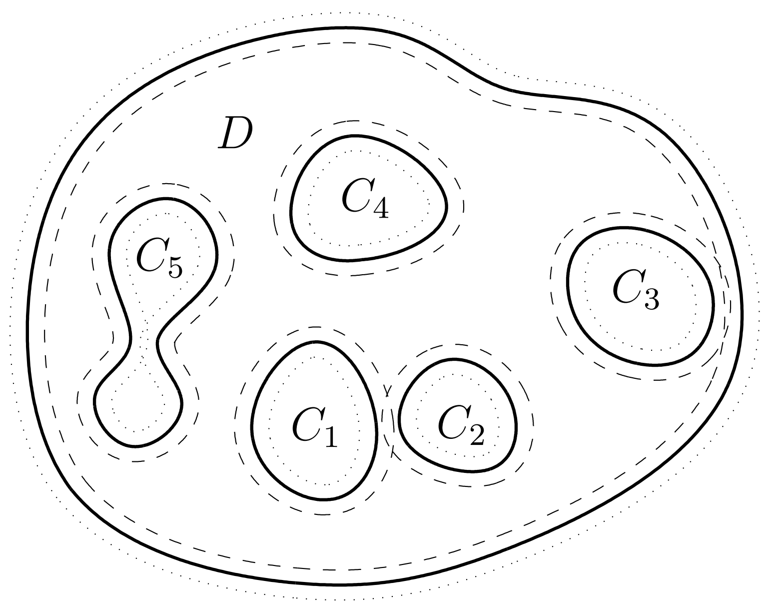


Figure 2: I_d: the dashed curves; E_l : the dotted curves.

The idea

Keep I_d *inside* of D, while E_l *outside* of D. Equivalent to requiring

$$\phi > 0 \text{ on } I_d,$$

and

$$\phi < 0 \text{ on } E_l.$$

The penalty functional

$$H(\phi) = -\int_{\partial D} \log\left[\phi\big(x + d\nabla\phi(x)\big)\right] ds - \int_{\partial D} \log\left[-\phi\big(x - l\nabla\phi(x)\big)\right] ds.$$

Let $0 < \varepsilon \ll 1$. Minimize

$$F(\phi) + \varepsilon H(\phi), \text{ subject to } G(\phi) = 0.$$

A simple example

Find the shape with minimal perimeter while keeping its area fixed.

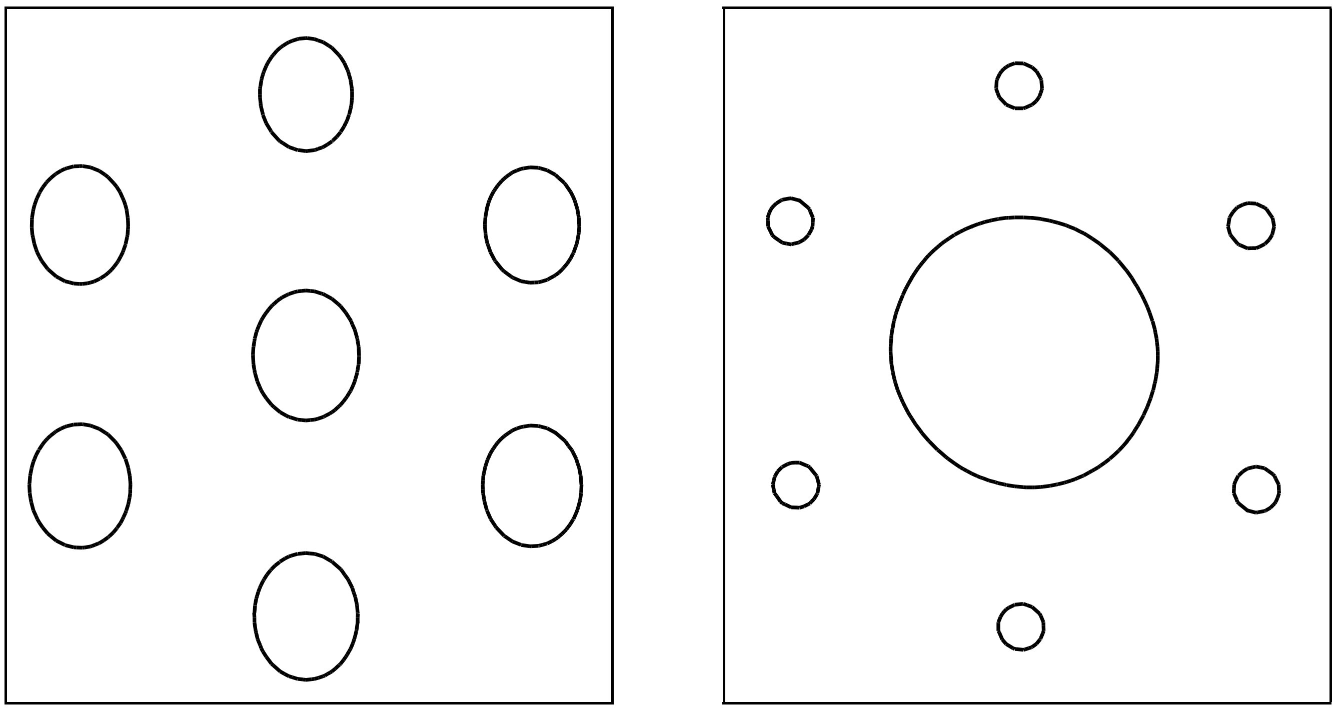
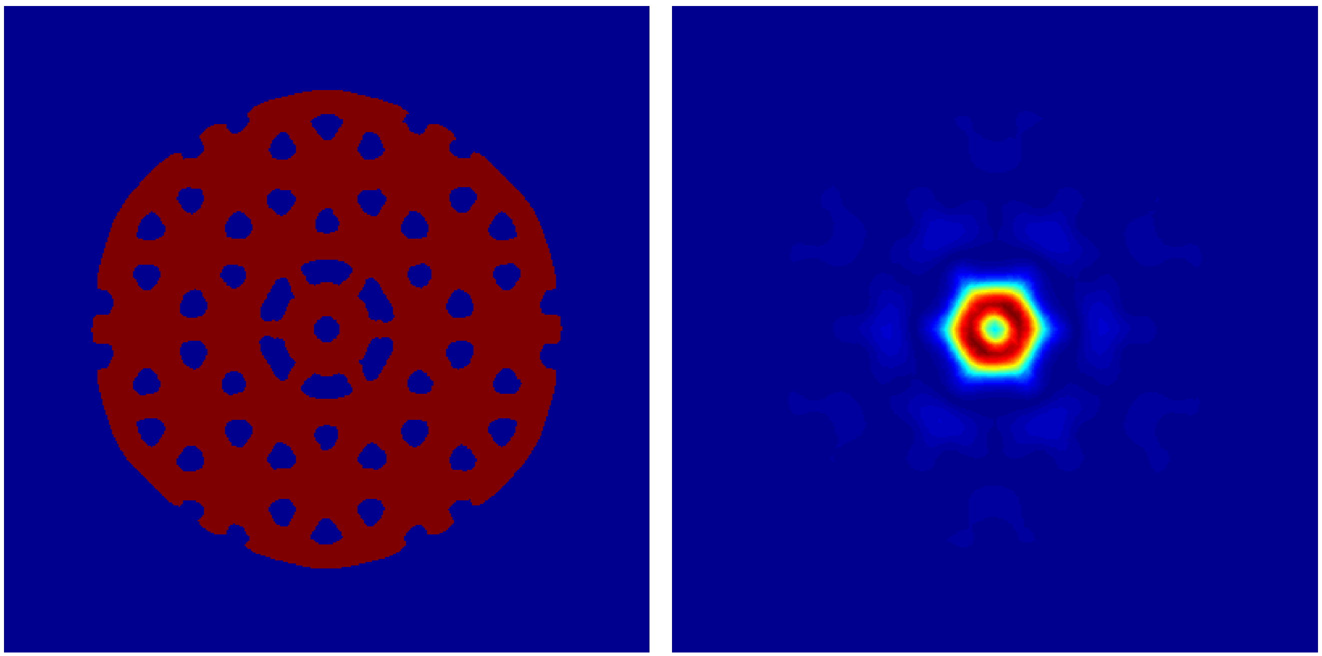
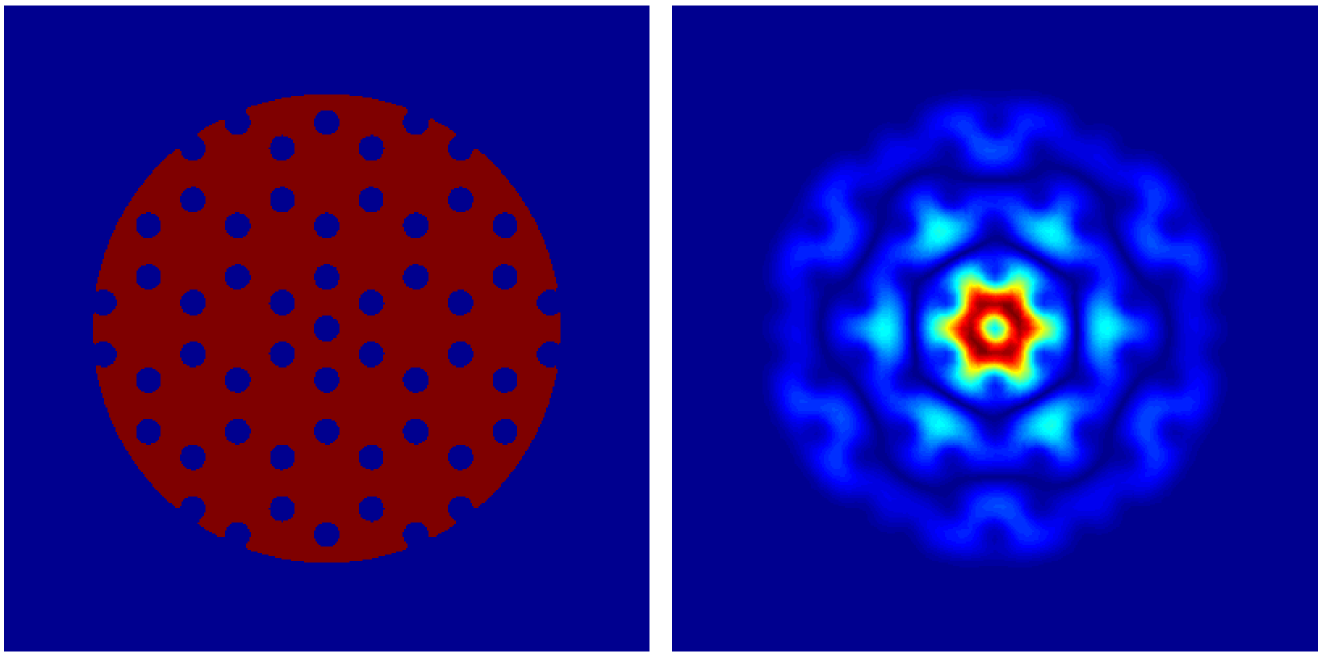


Figure 3: The shape before and after optimization. Without the logarithmic barrier constraint the optimal shape would have been a circle.

The result of optimization –require that the central hole be preserved

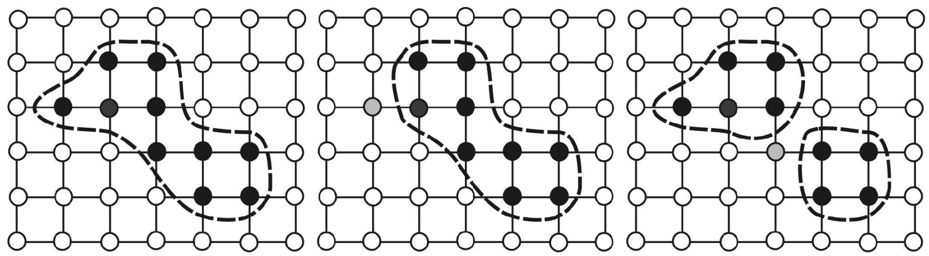


The unoptimized profile



Differences with another approach constraining the level set method to preserve the topology

[Han, Xu, and Prince, June 2003]



The authors of this paper are able to detect that a shape is about to change topology, only when certain dimensions of the shape are of size comparable to the grid size. For example, in this picture the “neck” of the shape is smaller than $\sqrt{2}$ times the mesh size. Our method allows for topological constraints independent of the grid size.

Conclusion

We have constructed a version of the level set method which preserves the topology of the shape to be optimized. We achieve that by using a penalty functional. The method we suggest is applicable to a wide range of optimization problems.