Topology, Material, and Mechanisms Optimization: Level Set Methods

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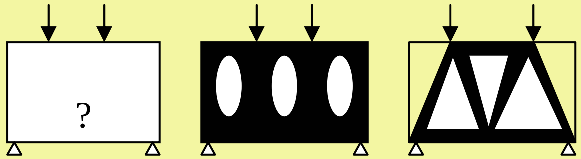
Outline

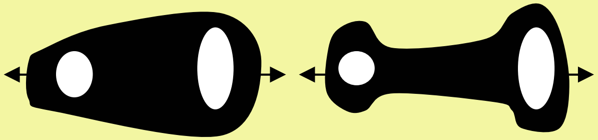
1. Topology Optimization Problems & Methods
2. Level-Set Concept
3. Shape Sensitivity and Velocity Function
4. Multi-Phase Level-Set Model
5. Optimization Algorithms
6. Examples:
    1. Topology Optimization with Multi-Materials
    2. Material Optimization
    3. Compliant Mechanism Design
7. Conclusions

Solid Structural Optimization

- Motivation
  - Traditional Design

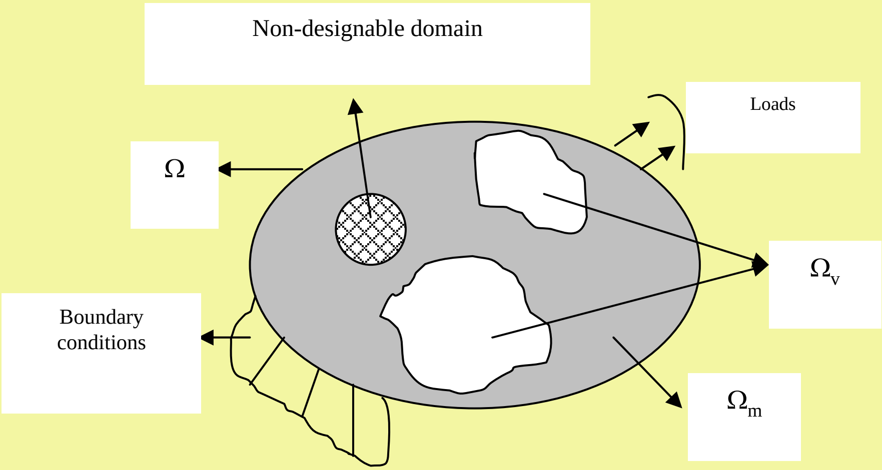


  - Shape Optimization



    - Homogenous Materials
    - Fixed Geometric Form
    - Limited Performance

- Topology Optimization

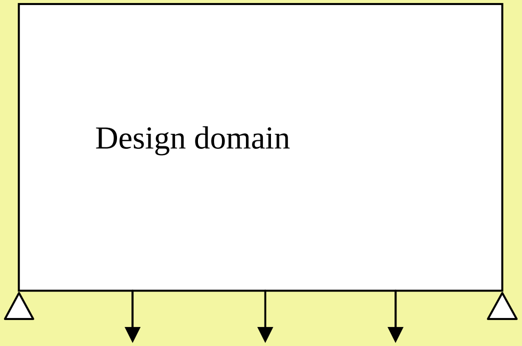


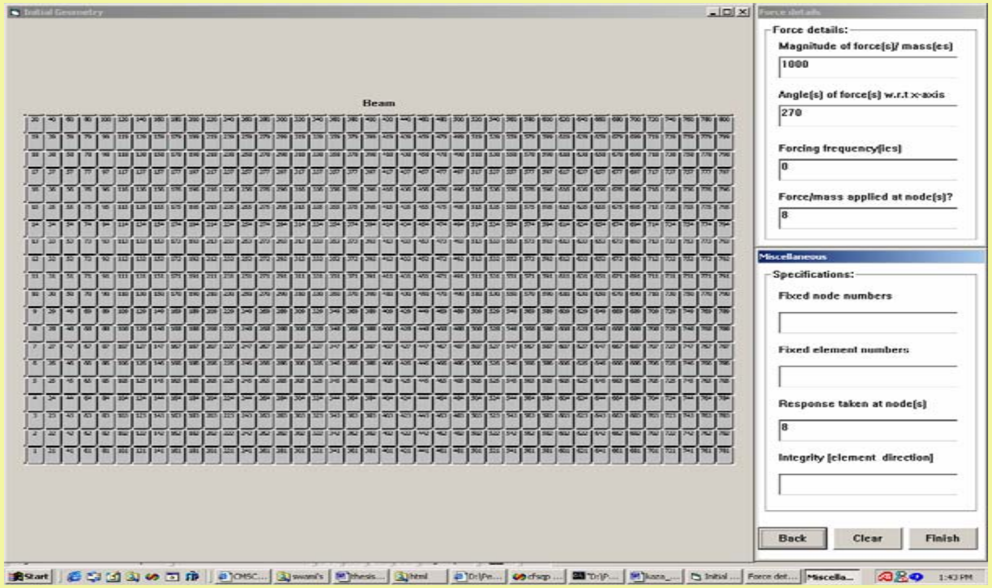
    - Shape & Topology
    - Heterogeneous Materials
    - Multi-Physics Domain
    - High Performance

Background

- Structural optimization – Michell structures (1904)
- FEM & shape optimization – Shape sensitivity and variations (Haug, Choi, Sokolowski, 70's and 80's)
  - Boundary Variation
  - Costly re-meshing
- Topology optimization – Trusses (Prager '80)
- Homogenization-based methods (Bendsoe '88)
- Simple Isotropic Material Penalty (SIMP) approach (Sigmund '90)
- Various evolutionary approach: GA, EA (Xie '95)

Homogenization Based Optimization





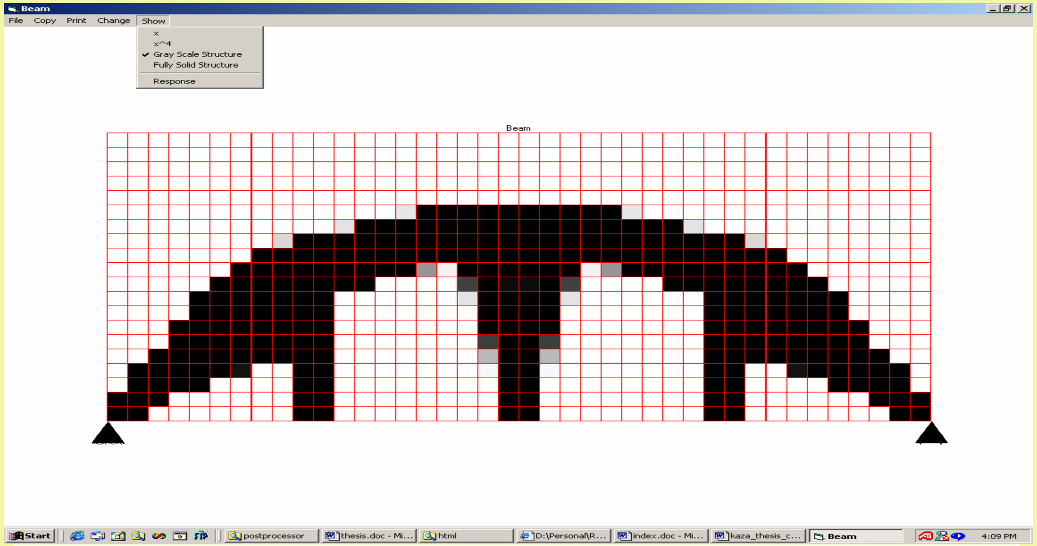


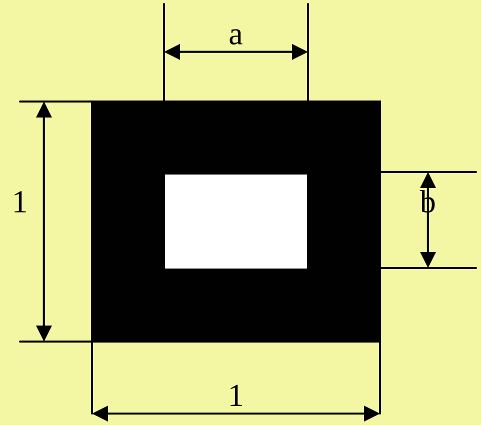
Figure 4.5: The Post-processor form with menu controls and results display

Topology Optimization

$\text{Min}_\rho \quad f(\rho)$ of a 'ground' structure

s. t $\quad \sum_{i=1}^{N} \rho_i v_i \le V^*$

$0 < \rho_i <= 1$

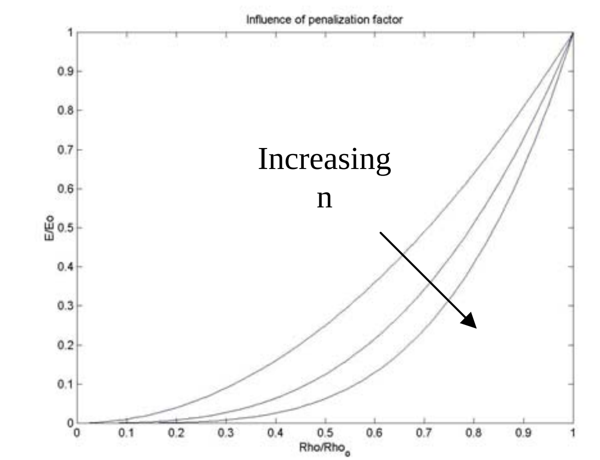


Microscopic Hole

Problem Relaxation:

- Homogenization-based method (Bendsoe 1988)
- Simple Isotropic Material with Penalty (SIMP)

$E = E_o * (\rho_i)^n \quad (n = 2, 3, 4)$



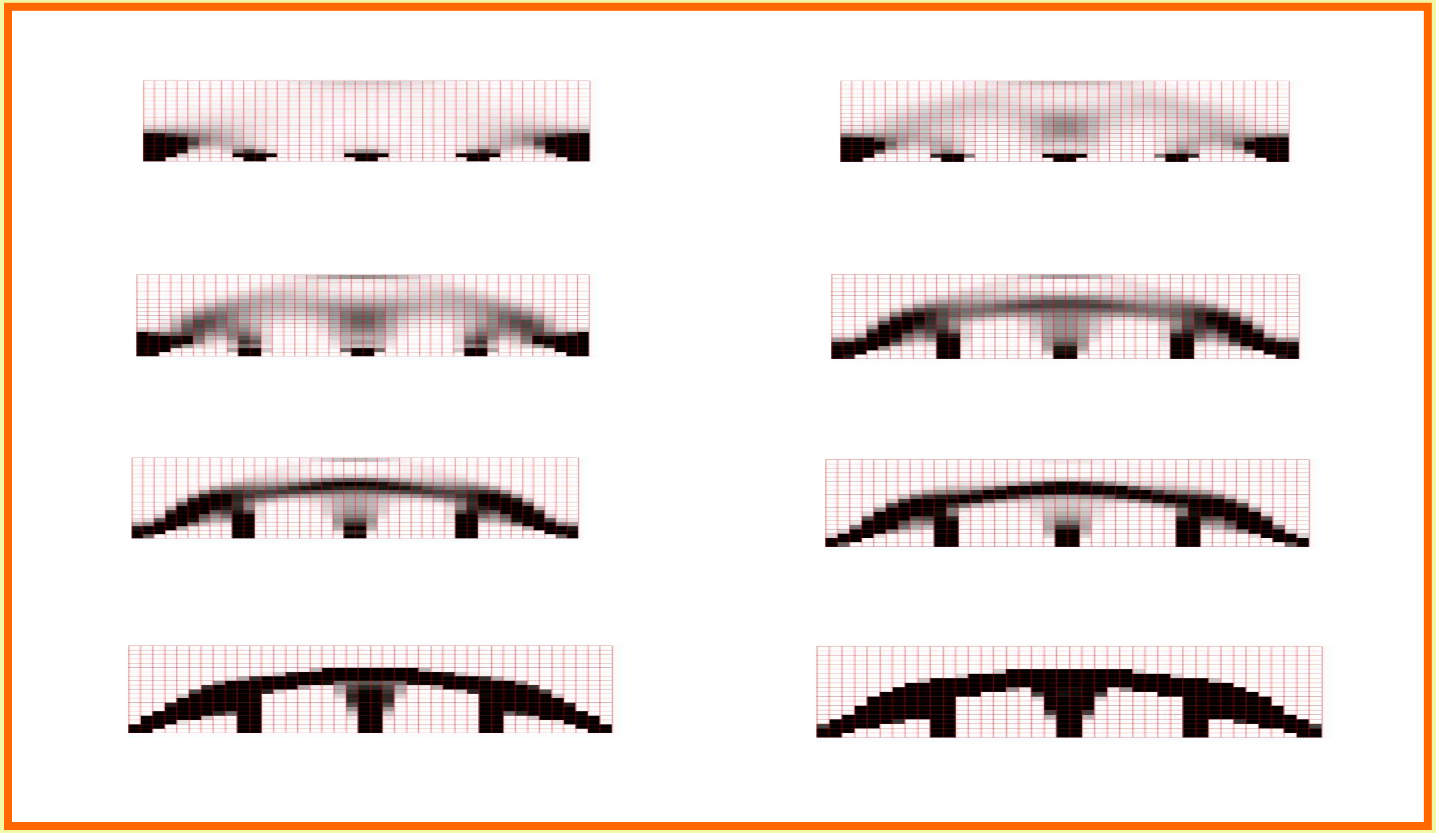
Influence of the penalization factor

Difficulties:

- Very large number of design variables N
- Various numerical instability & inaccuracy
- Lack of concise geometric boundary description

The Process

- Material starts accumulating first at the supports and loading points.
- Then it gradually spreads to other parts.



SIMP Optimization

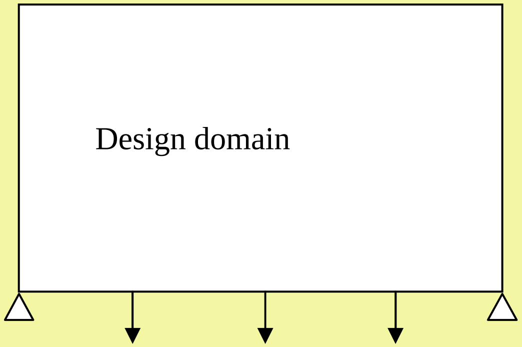


Figure 5.3a: Mean compliance case 1

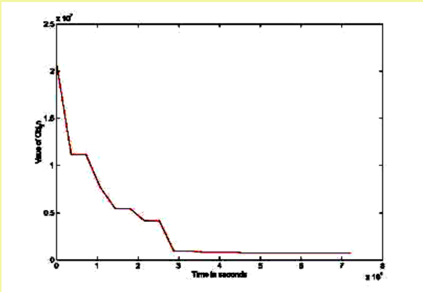


Figure 5.3b: Objective function Versus Time

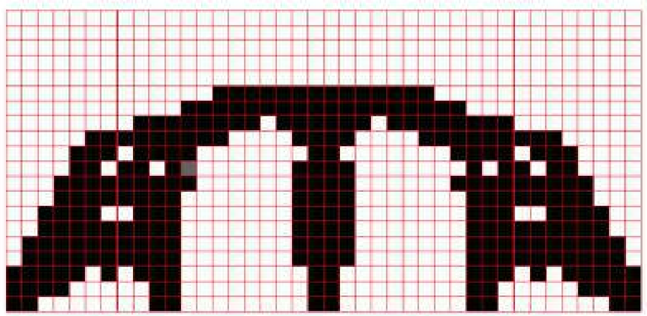


Figure 5.3c: Optimum structure

Checker-Board Problem

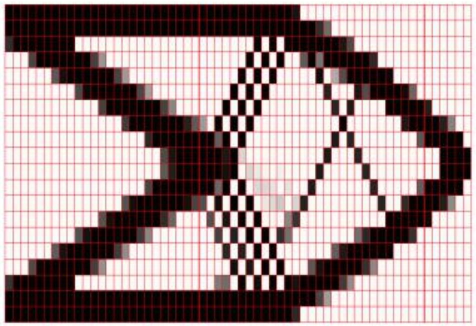


Figure 6.5a: Checker board

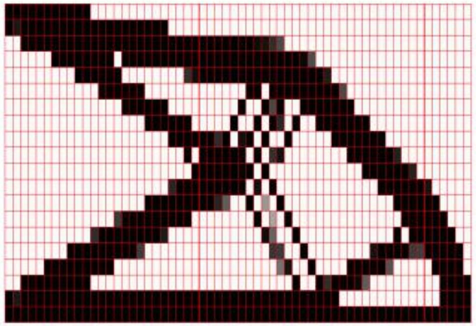


Figure 6.5b: Checker board

- Checker board is attributed to inaccuracy in FE modeling and homogenization relaxation.
- Checker board gives artificially high stiffness.

FE Mesh Dependency

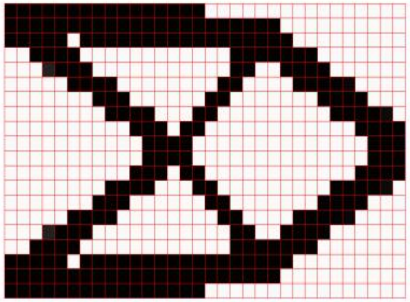


Figure 6.6a: 640 elements

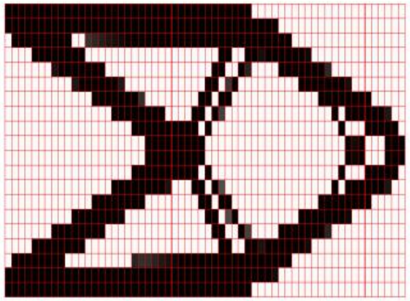


Figure 6.6b: 1200 elements

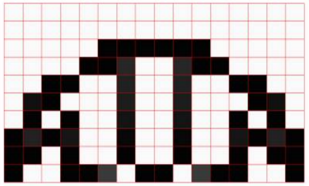


Figure 6.7a: 160 elements

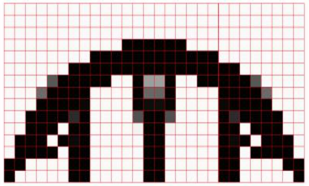


Figure 6.7b: 420 elements

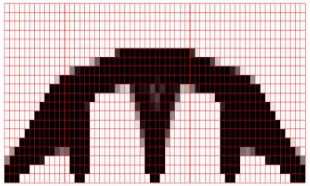


Figure 6.7c: 1200 elements

- Mesh changes the connectivity of the elements.
- Finer meshes tend to create more holes or internal boundaries.

"Gray-Scale" Structure Problem

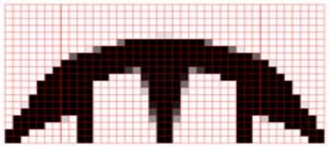


Figure 6.8a: eps = 0.01 optimized structure

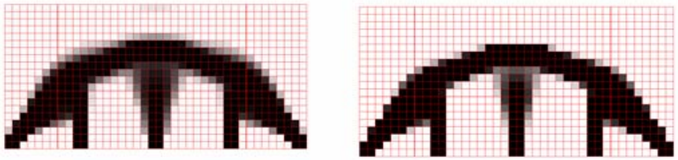


Figure 6.8b: eps = 0.07; optimized and gray scale structures

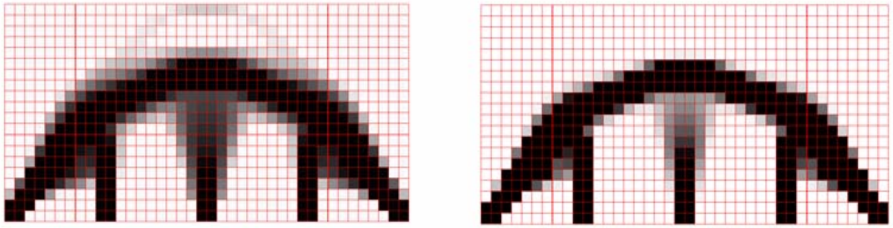


Figure 6.8c: eps = 0.1; optimized and gray scale structures

Class of Problems

Based on (Haber and Bendsoe 1998) & (Bendsoe 1999)

Class 1: Basic Problems	Class 2: Relaxed Problems	Class 3: Restricted Problems	Class 4: Evolutionary Problems
Discrete $\Omega \to \{0,1\}$ Ill-posed	Continuous $\Omega \to [0,1]$ "Regularized" Perforated Designs	Continuous $\Omega \to [0,1]$ Constraints on Admissible Space	Discrete $\Omega \to \{0,1\}$ Evolutionary Processes
	Homogenization SIMP	Perimeter, Slope, Filtering	"Greedy" method: Hard-Kill, Bi-directional

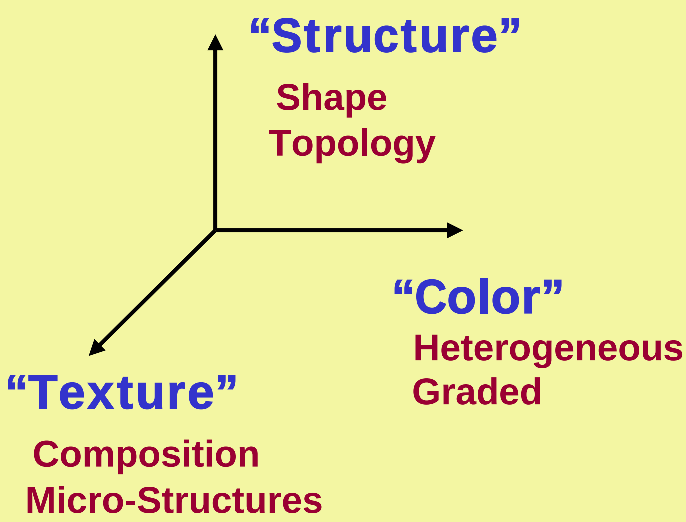
Geometric Models

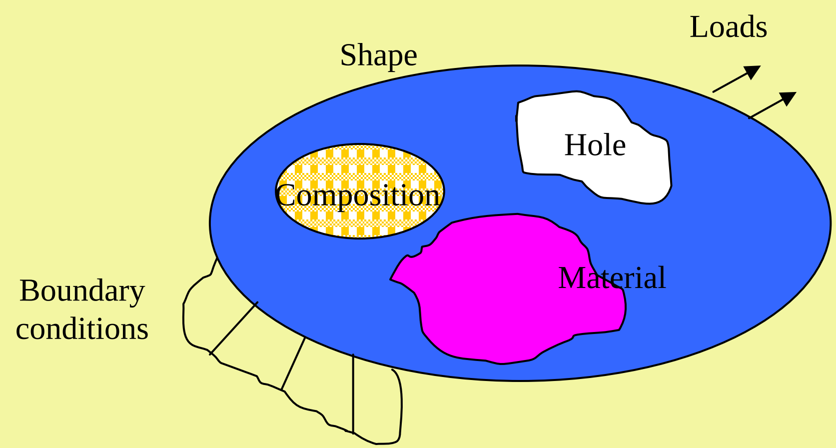
Ideal Characteristics	Homogenization Based	Evolution Based	Level-Set Based
Topological Flexibility and Robustness: to represent complex topologies and to evolve gracefully	✓	✓	✓
True Geometric Representation: boundary representation, geometric attributes (e.g., curvature, derivatives), few design DOF	×	×	✓
Continuous Parameterization: to avoid integer programming	✓	×	✓
Efficient Computation: with high accuracy and efficiency	×	×	✓
Separation from Analysis Representation: independent of FE mesh or coordinate system, independent accuracy control	×	×	✓
Adaptability: automatic refinements, adaptive algorithms	×	×	✓
Shape and Design Sensitivity: to link design derivative with shape derivatives	×	×	✓

"Color" & "Texture" in Structures

- Key Characteristics:
  - Function and Form Integration
    - e.g., micro-hinges
  - Multi-, Graded-, Textured-Materials & Composition
  - Multi-Physics Domains

- Key Attributes:
  - "Structure"
    – Material at right place
  - "Color"
    – Right material at right place
  - "Texture"
    – Right property for the right material at the right place





Major Challenges

1. Computer Representation:
   - Heterogeneous Materials in CAD (HKU, UM)
   - “Color” and “Rainbow” CAD
2. Design Methods:
   - Homogenization Methods in Mechanics
   - Optimization Methods for Design
3. Fabrication Technology:
   - Color SLA (HKU)
   - Shape Deposition Manufacturing (Stanford)
   - Multi-material Diffusion (SMU)

New Models

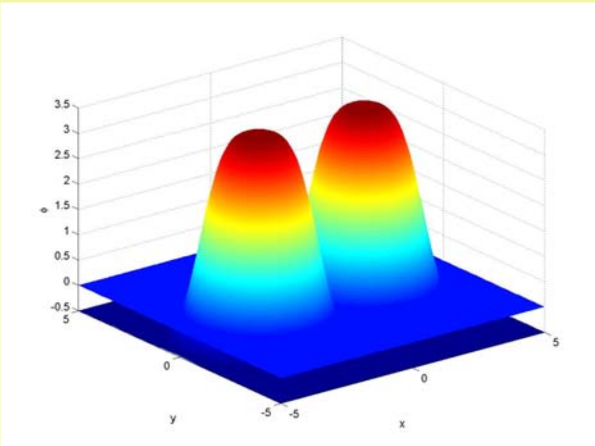
- Combining the Best of the Current Approaches:
  - Fixed mesh of homogenizastion
    - No re-meshing
    - Use adaptive meshing
  - General method of shape derivative
    - Any type of problem

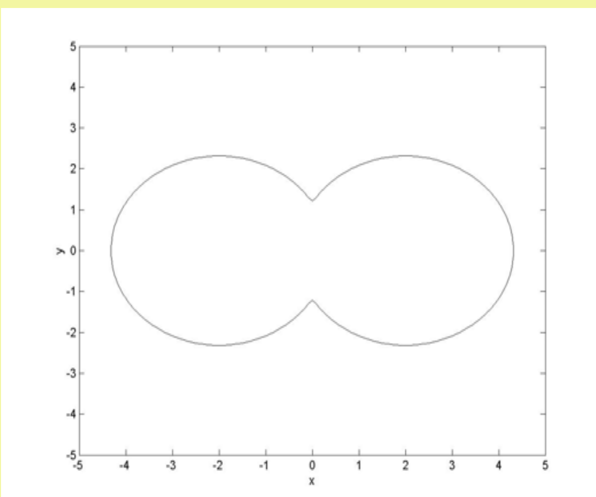
- Level Set Models
  - Kumar 1996
  - Sethian & Wiegmann 2000
  - Osher & Santosa 2001
  - Santosa 2000
  - Allaire et al. 2003
  - Wang et al. 2003

- Phase-Field Models
  - Bourdin & Chambolle 2003
  - Wang & Zhou 2003

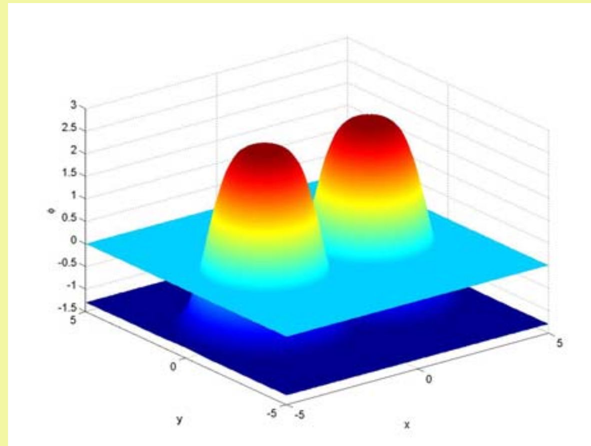
- Distance Fields
  - Shapiro (2002, 2003)
  - CAD community

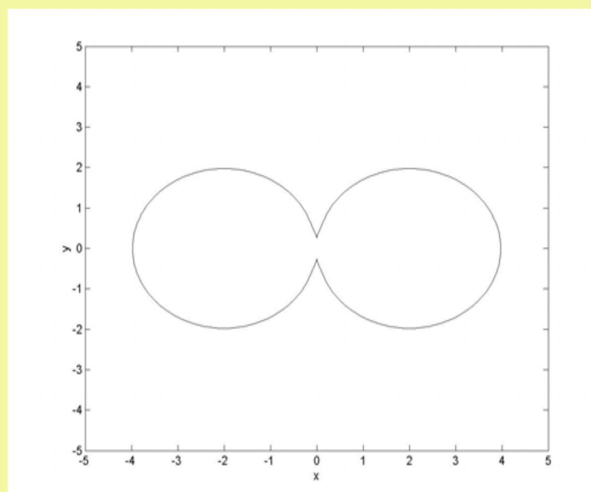
Concept of Level Sets



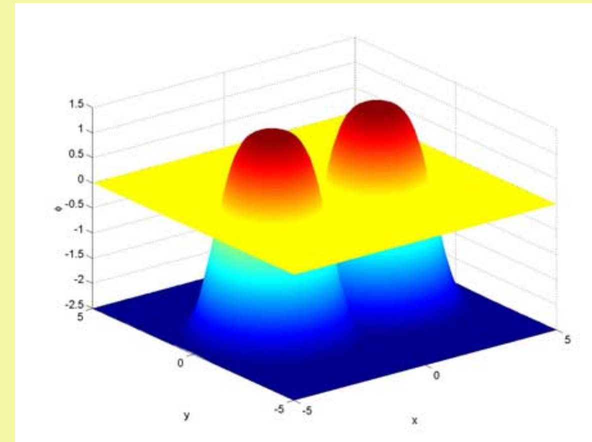


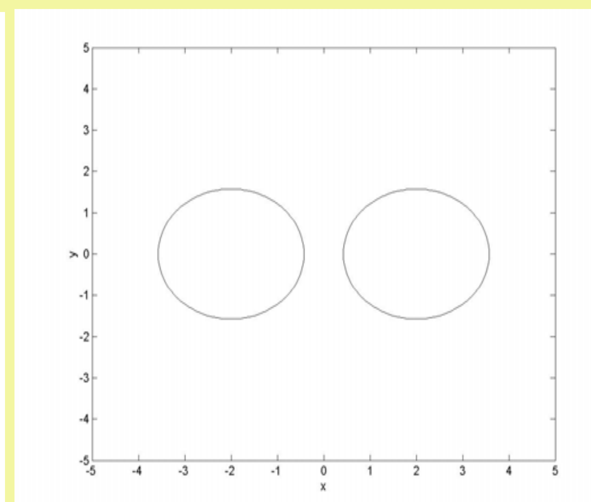
$Z = \phi(x, y, t_0)$





$Z = \phi(x, y, t_1)$





$Z = \phi(x, y, t_2)$

Level Set Model

- Embedding x into a Scalar Function ϕ of a Higher Dimension
- Boundary Γ is an "Iso-Surface" (a Level-Set)

$$\Gamma = \{x : \Phi(x) = 0\}$$

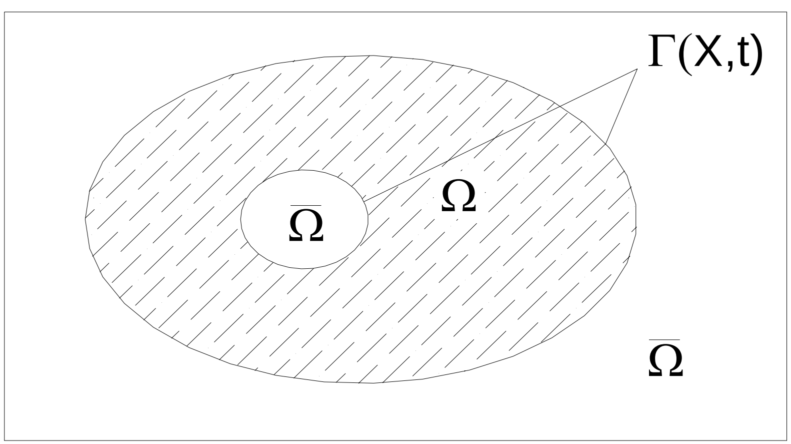
- Φ has Fixed Topology
- Γ has Variable Topology

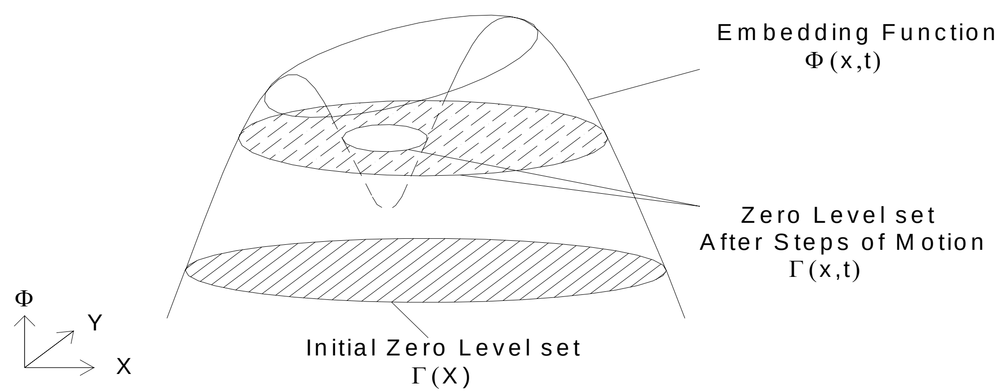
- Regional Representation (R-rep):
  - Semi-Explicit
  - Global
  - Inside & Outside Regions:

$$\Phi(x) > 0 \quad \forall x \in \Omega \setminus \partial\Omega$$

$$\Phi(x) = 0 \quad \forall x \in \partial\Omega$$

$$\Phi(x) < 0 \quad \forall x \in \overline{\Omega} \setminus \Omega$$





Level-Set Propagation

- "Iso-Surface":

$$\Gamma(t) = \{x(t) : \Phi(x(t), t) = 0\}$$

- Level-Set Equation:

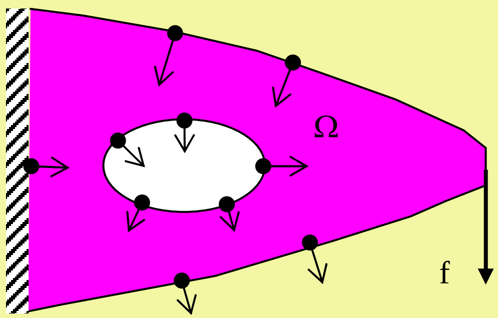
$$\frac{\partial \Phi(x)}{\partial t} = -\nabla\Phi(x)\frac{dx}{dt} \equiv -\nabla\Phi(x)\cdot V(x)$$

- Hamilton-Jacobi Eq. for Front Propagation:

$$\frac{\partial \Phi}{\partial t} = V_n|\nabla\Phi|$$

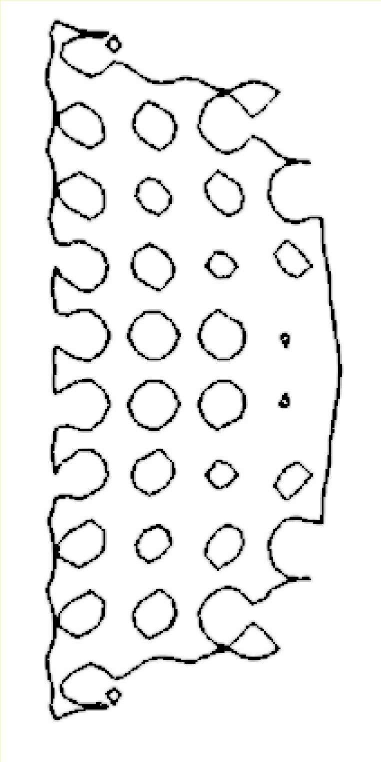
- "Normal Velocity" :
  - Effective for changes in normal direction only
  - Tangential changes are for re-parameterization only

$$V_n(x)$$



Geometric Evolution

- Boundary Capturing vs. Boundary Tracking:
  - *Implicit Method (Euler Method) vs.*
  - *Explicit Method (Lagrange Method)*
- Topologically Flexible:
  - Variable Topology in Γ
  - Fixed Topology in Φ
- Geometrically Concise:
  - True Geometric Model
  - Normal, Tangent, Curvatures
- Evolution Process for Optimization:
  - $V_n(x)$ is the link



Optimization of Elastic Structures

Optimization:

$$\underset{\Phi}{Minimize} \quad J(u,\Phi) = \int_{\bar{\Omega}} F(u) H(\Phi) d\Omega$$

$$subject\ to:$$

$$a(u,v,\Phi) = L(v,\Phi) \ \text{ for all } \ v \in U$$

$$u = u_0 \ \text{ on } \ \Gamma_{\mathrm{u}}$$

$$\mathrm{V}(\Phi) = \int_{\bar{\Omega}} H(\Phi) d\Omega \le \mathrm{V}_{\max}$$

Lagrange Equation:

$$\underset{\Phi}{Minimize}\ \bar{J}(u,\Phi) = J(u,\Phi) + \lambda_{+} \cdot \left(\mathrm{V}(\Phi) - \mathrm{V}_{\max}\right)$$

$$subject\ to:$$

$$a(u,v,\Phi) = L(v,\Phi)\ ,\ u\big|_{\partial D_u} = u_0 \ \text{ for all } \ v \in U$$

$$\lambda_{+} \cdot \left(\mathrm{V}(\Phi) - \mathrm{V}_{\max}\right) = 0$$

$$\lambda_{+} \ge 0$$

$$a(u,v,\Phi) = \int_{\bar{\Omega}} E_{ijkl} \varepsilon_{ij}(u) \varepsilon_{kl}(v) H(\Phi) d\Omega$$

$$L(v,\Phi) = \int_{\bar{\Omega}} p v H(\Phi) d\Omega + \int_{\bar{\Omega}} \tau v \delta(\Phi) \left|\nabla \Phi\right| d\Omega$$

$$H(\Phi) = \begin{cases} 1 & \text{if } \ \Phi \ge 0 \\ 0 & \text{if } \ \Phi < 0 \end{cases}$$

$$\delta(\Phi) = \frac{dH}{d\Phi}$$

Problem Formulation

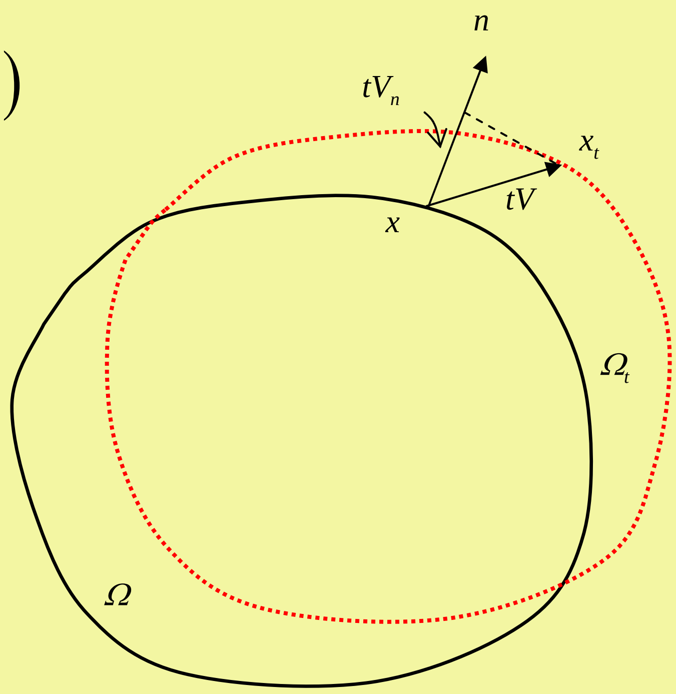
- Existence Theory
  1. Generally, the minimization problem has no solution.
  2. Solution exists with additional conditions:
     - Perimeter constraint
     - Topology constraint
- Two Main Elements:
  1. Euler derivative of the objective functional with respect to shape changes – Shape Derivatives
  2. Transport equations (PDE) for shape evolution and minimization

Shape Sensitivity

- Shape Transformation – Perturbation in Diffeomorphism: $\Omega_t = (I+\psi)\Omega$

$$T : x \to x_t(x),\ x \in \Omega, \qquad x_t = T(x,t) \qquad \Omega_t = T(\Omega,t)$$

- Transformation Velocity: $$\frac{dx_t}{dt} = V(x_t, t)$$



- Transformation Identity:

$$x_t = T(x,t) = x + tV(x)$$

$$V(x) = V(x,0)$$

- Well-Established Methods:
  - Murat & Simon 70's, Haug & Choi 80's, Sokolowski & Zolesio 90's

Frechet Derivatives

- Material Derivative
  - For a given velocity vector in the shape transformation, the *material derivative* is defined by

$$\dot{u}(x;V) = \lim_{t\to 0}\frac{1}{t}\left[u(x+tV)-u(x)\right]$$

- Shape derivative is the Frechet derivative on the boundary $\partial\Omega$, depending only on V_n

$$J'(\Omega)(V\cdot n)$$

- Lemmas (Haug 1986):

Lemma 1: For a regular function

$$\psi_1 = \int_\Omega f(x)d\Omega$$

the material derivative is given by

$$\psi_1' = \int_\Omega f'(x)d\Omega + \int_\Gamma f(x)(V\cdot n)d\Gamma$$

Lemma 2: For

$$\psi_2 = \int_\Gamma g(x)d\Gamma$$

the material derivative is given by

$$\psi_2' = \int_\Gamma g'(x)d\Gamma + \int_\Gamma (\nabla g\cdot n + \kappa g(x))(V\cdot n)d\Gamma$$

Lemma 3: For

$$\psi_3 = \int_\Gamma g(x)\cdot n d\Gamma$$

the material derivative is given by

$$\psi_3' = \int_\Gamma \left(g'(x)\cdot n + \mathrm{div}\, g(x)(V\cdot n)\right)d\Gamma$$

Shape Derivative

- Shape Derivative in V_n

$$\frac{d\bar{J}(u,\Phi)}{d\Phi} = \int_{\bar{\Omega}} \delta(\Phi)\left(\beta(u,w,\Phi) + \lambda_+\right) V_n \, d\Omega$$

$$\beta(u,w,\Phi) = F(u) + pw - \tau w \kappa - E_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(w)$$

- The Kuhn-Tucker Condition of Optimal Solution:

$$\beta(u,w,\Phi) + \lambda_+ \big|_{\partial\Omega} = 0$$

$$\lambda_+ \cdot \left(\mathrm{V}(\Phi) - \mathrm{V}_{\max}\right) = 0$$

$$\lambda_+ \geq 0$$

Gradient Descent/Projected Gradient Method

- Construction of the Velocity Field:

$$V_n(x) = -[\beta(u, w, \Phi) + \lambda_+]$$

$$\lambda_+ = -\int_{\partial\bar{\Omega}} \beta(u, w, \Phi) d\Gamma \Big/ \int_{\partial\bar{\Omega}} d\Gamma$$

- Gradient-Descent Optimization Process:

$$\frac{d\bar{J}(u, \Phi)}{d\Phi} = -\int_{\bar{\Omega}} \delta(\Phi) V_n^2 d\Omega \leq 0$$

- Level-Set Evolution:

$$\frac{\partial\Phi}{\partial t} = V_n |\nabla\Phi| \quad \text{and} \quad \left.\frac{\partial\Phi}{\partial N}\right|_{\partial\bar{\Omega}} = 0$$

MY Wang, et. al. “A level method for structural topology optimization,” Computer Methods in Applied Mechanics & Engineering, 192, 227-246, January 2003.

Perimeter Regularization

- Perimeter measure: $E(\Omega) \equiv |\partial\Omega| = \int_\Gamma d\Gamma$
- Euler derivative:

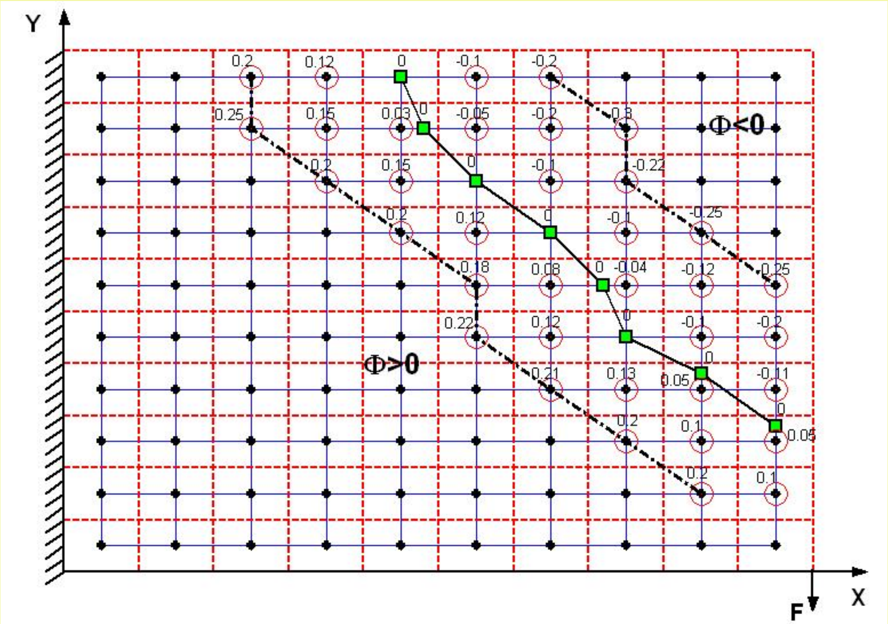
$$E'(\Omega) \equiv dE(\Omega)/dt = \int_\Gamma \kappa (V \cdot n) d\Gamma = \int_\Omega \delta(\Phi)\kappa(\Phi)V_n |\nabla\Phi| d\Omega$$

- Curvature flow: $V_n = V \cdot n = -\kappa$
- Geometric heat equation: $E'(\Omega) = -\int_\Omega \delta(\Phi)\kappa^2 |\nabla\Phi| d\Omega \le 0$
- Weighted Total-Variation Scheme: $E_{TV}(\Phi) = \int_{\overline{D}} I(x)|\nabla\Phi| d\Omega$

$$I(x) = \frac{c_1}{1 + c_2 {V_N}^2(x)}$$

Computations

- PDEs on Rectilinear Grid:
  - Finite Difference Methods
  - Interface Embedded
- Finite Element Method for Mechanics:
  - Independent FD Grid and FE Mesh
- Boundary Recovery:
  - Marching-Cube Methods in Computer Graphics



Numerical Schemes

- Robust Schemes:
  - Up-wind Scheme of Entropy Solution (Osher & Sethian '88)

$$\phi_{ijk}^{n+1} = \phi_{ijk}^{n} - \Delta t[\max(V_{N_{ij}},0)\nabla^{+} + \min(V_{N_{ij}},0)\nabla^{-}]$$

$$\begin{aligned}\nabla^{+} = [\; & \max(D_{ijk}^{-x},0)^2 + \min(D_{ijk}^{+x},0)^2 \\ & + \max(D_{ijk}^{-y},0)^2 + \min(D_{ijk}^{+y},0)^2 \\ & + \max(D_{ijk}^{-z},0)^2 + \min(D_{ijk}^{+z},0)^2 \;]^{1/2}, \\ \nabla^{-} = [\; & \max(D_{ijk}^{+x},0)^2 + \min(D_{ijk}^{-x},0)^2 \\ & + \max(D_{ijk}^{+y},0)^2 + \min(D_{ijk}^{-y},0)^2 \\ & + \max(D_{ijk}^{+z},0)^2 + \min(D_{ijk}^{-z},0)^2 \;]^{1/2}\end{aligned}$$

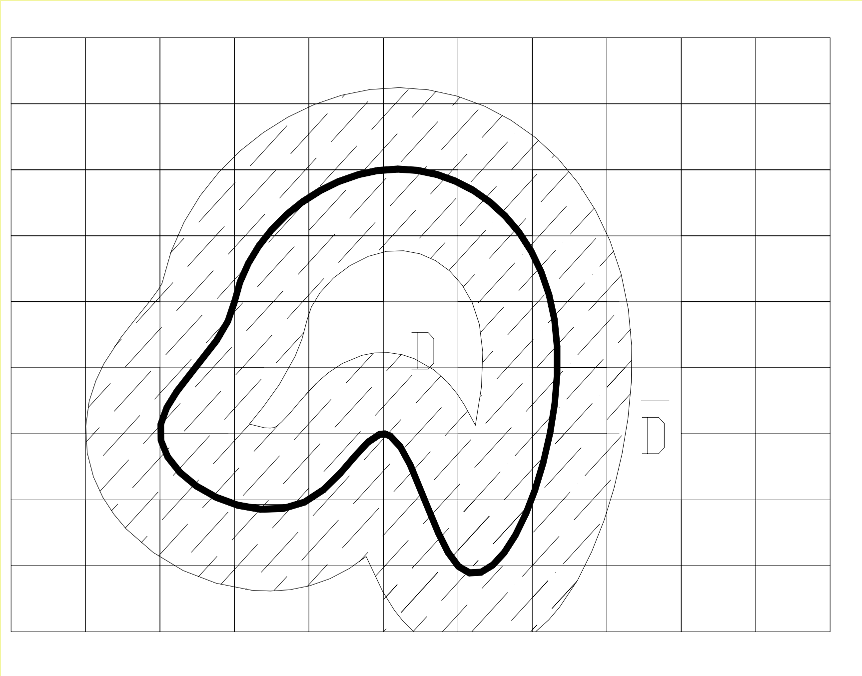
  - ENO & TVD-RK High Order Schemes (Shu & Osher '88)

Level-Set Numerics

- Narrow-Band Schemes (Sethian '99)
- Velocity Extensions
- Re-Initialization
  - Signed Distance Function Schemes (Peng '99)
- Linear Complexity of Boundary Only:

$$O(N(\Gamma))$$

- Well-Documented:
  - Osher & Fedkiw 2003



Enhancements

- Conjugate Velocity Mapping:
  - Wang *et al.* '03

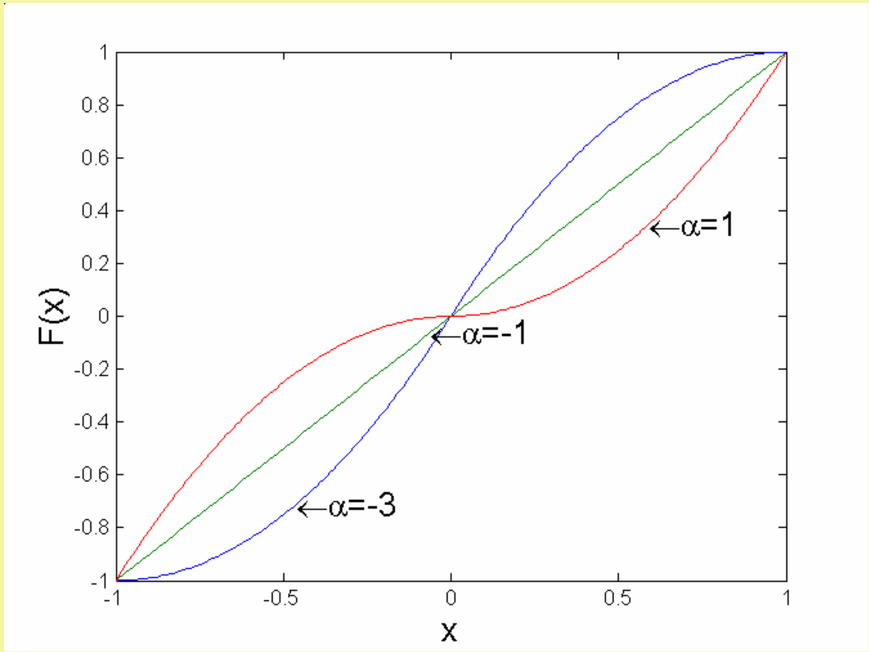
$$\frac{\partial \Phi}{\partial t} = f(V_n)|\nabla \Phi|$$

$$\left.\frac{\partial \Phi}{\partial N}\right|_{\Gamma} = 0$$

$$f(V_n) = F(V_n) - \mu \quad \text{for} \quad V_n \in T$$

$$F(r) = r\left(\frac{1-\alpha}{2} + \frac{1+\alpha}{2}|r|\right)$$

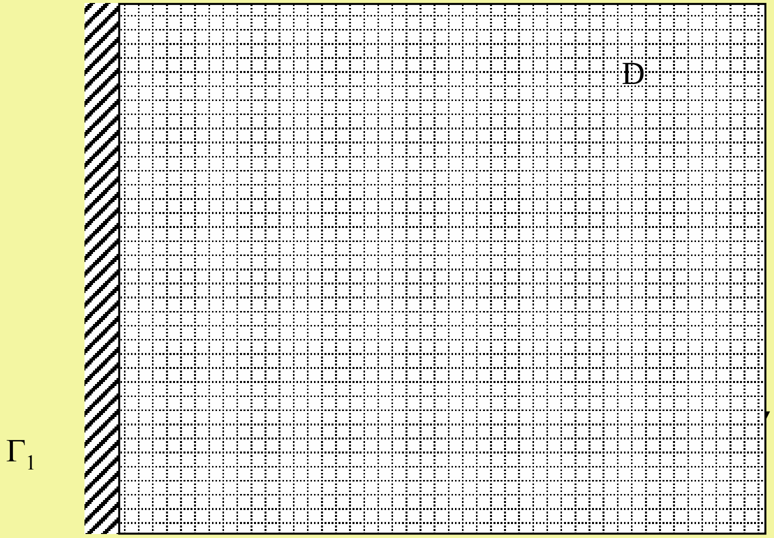
$$\mu = \int_{\partial\bar{\Omega}} F(V_n)\,d\Gamma \Big/ \int_{\partial\bar{\Omega}} d\Gamma$$



- Adaptive Oct-tree Schemes with Semi-Lagrange Method:
  - Strain '01

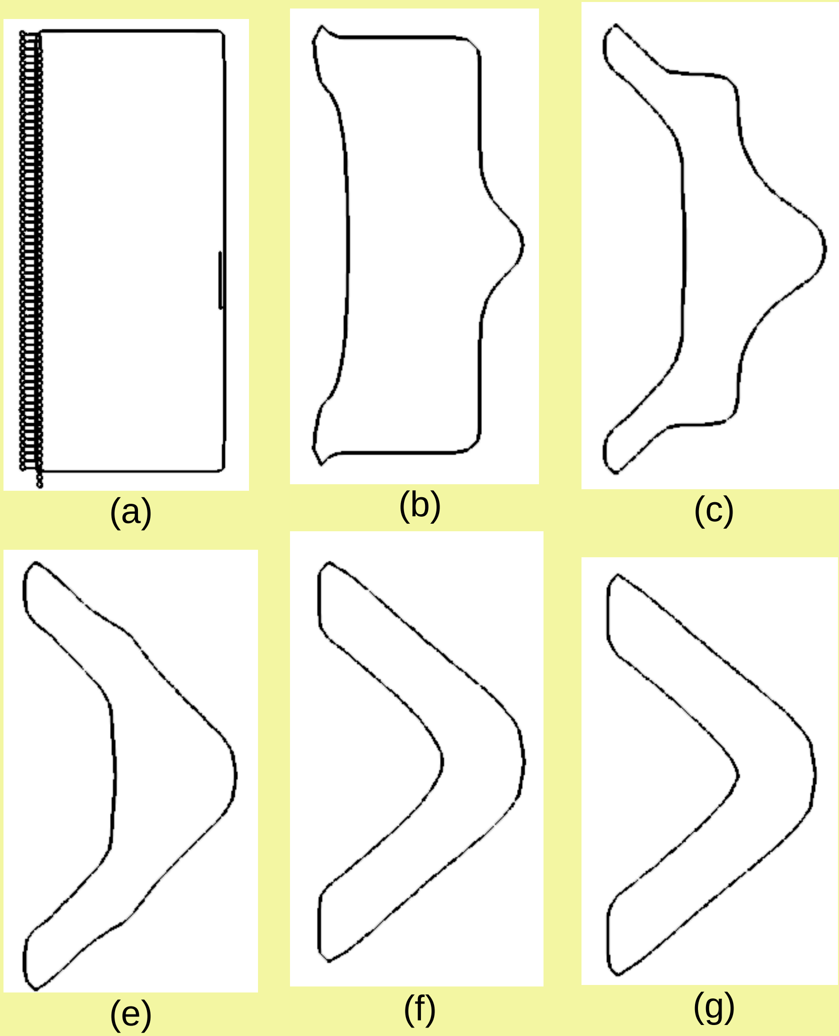
Algorithm

- *Step 1:* Iinitialize the level set function for an initial design in terms of its boundary.
- *Step 2:* Compute the displacement field and the adjoint displacement field through the linear elastic system.
- *Step 3:* Calculate the “speed function” on the surface along the normal direction.
- *Step 4:* Solve the level set equation to update the embedding function.
- *Step 5:* Check if a termination condition is satisfied. Repeat Steps 2 through 5 until convergence.

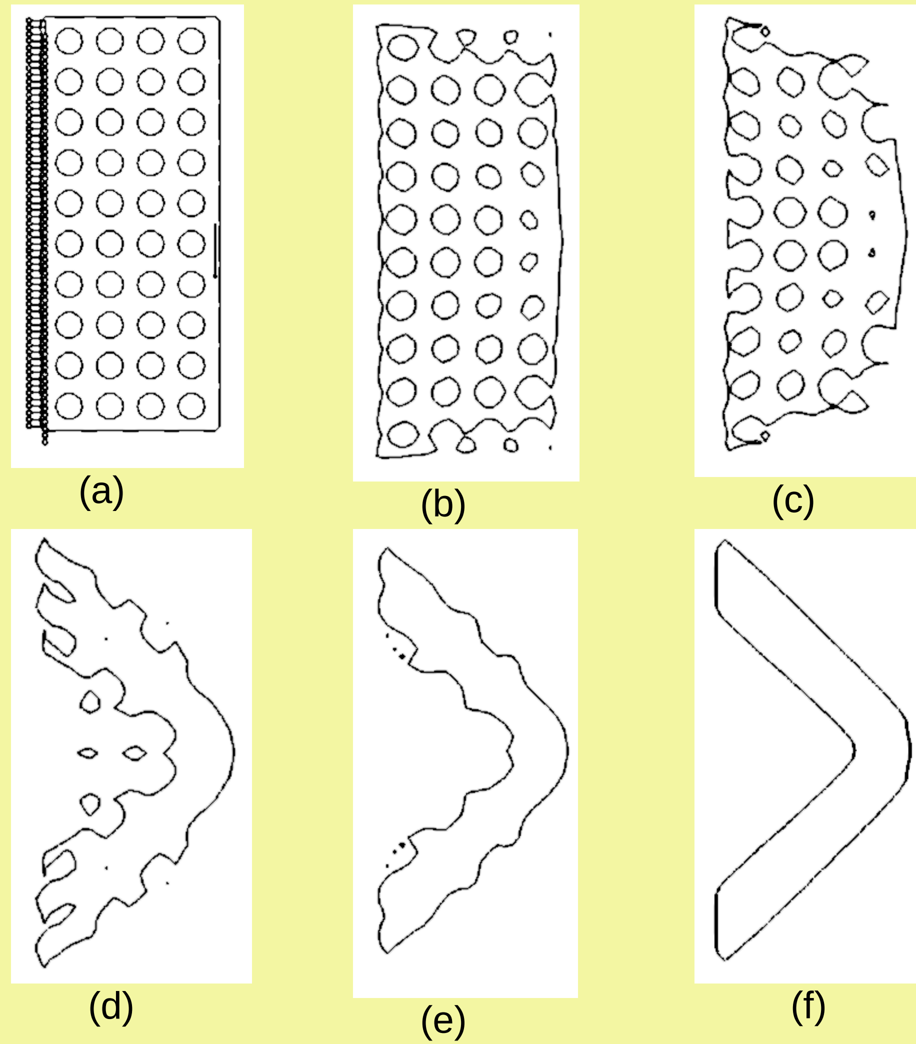


$$J(u) = \int_D E_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(u)d\Omega$$

Two-Bar Example

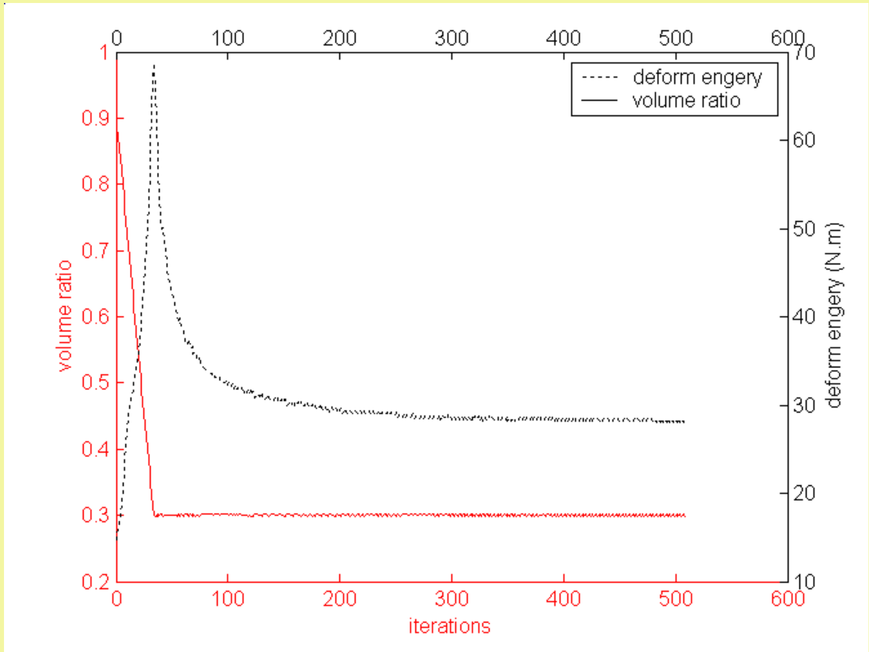


(1) Initially 90%

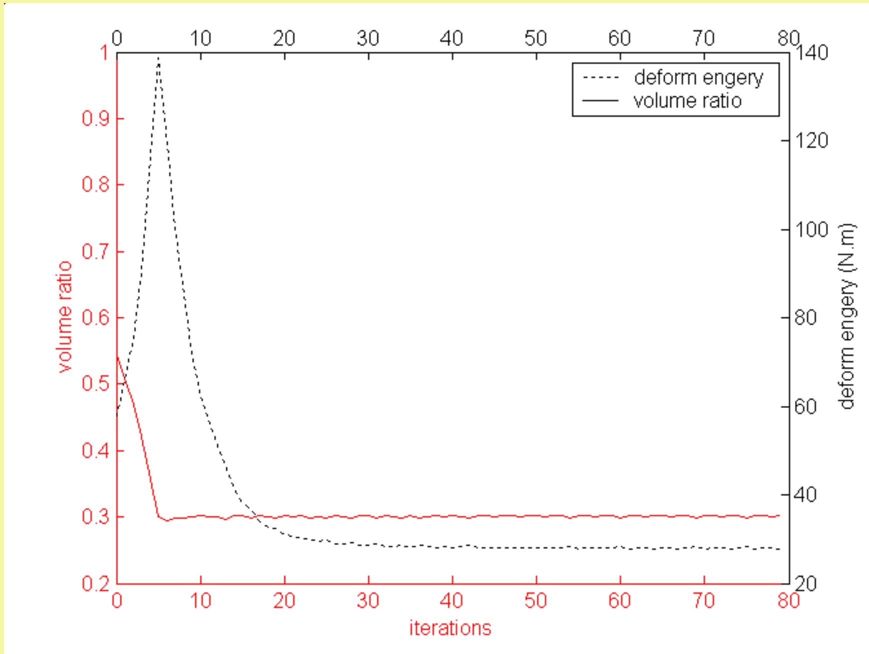


(2) Initially 55%

Two-Bar (cont.)

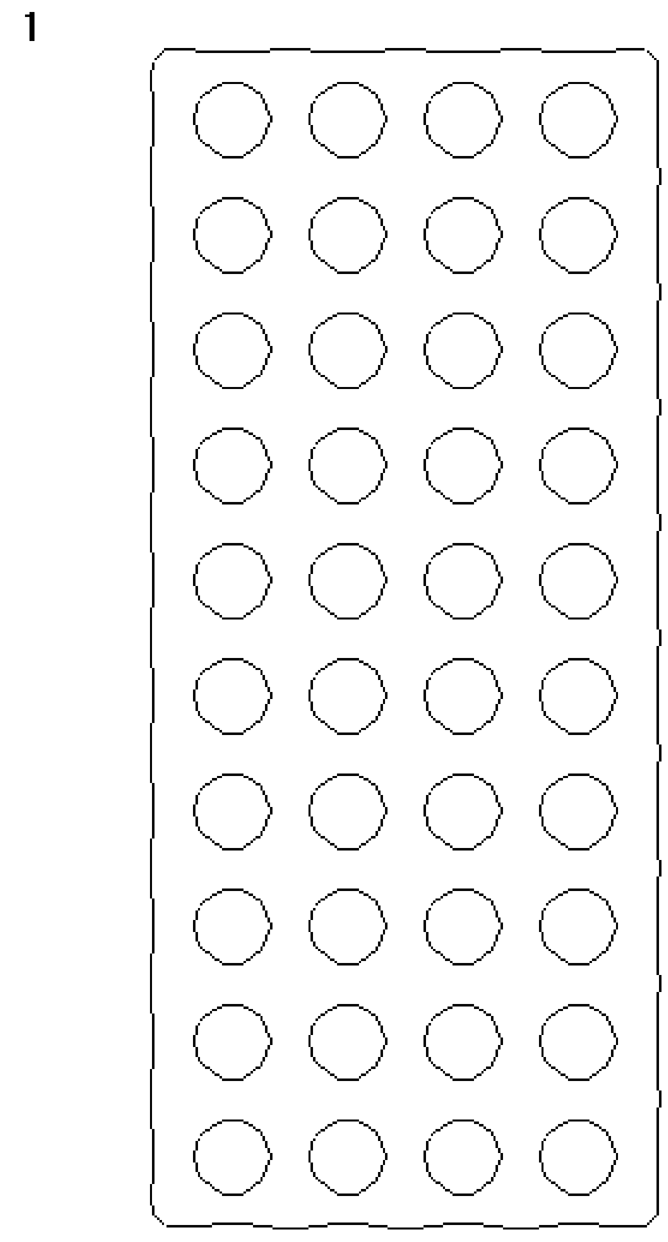


deform engery
volume ratio
volume ratio
deform engery (N.m)
iterations
0
100
200
300
400
500
600
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
70
60
50
40
30
20
10

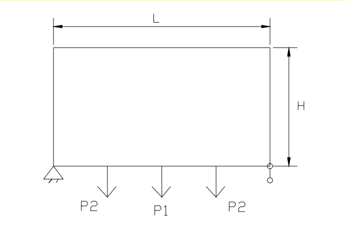


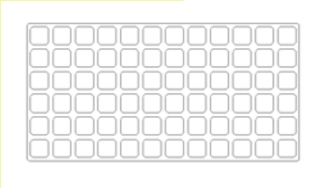
deform engery
volume ratio
volume ratio
deform engery (N.m)
iterations
0
10
20
30
40
50
60
70
80
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
140
120
100
80
60
40
20

Two-Bar (Video)

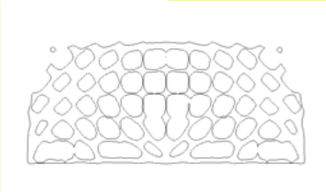


Multiple Loading

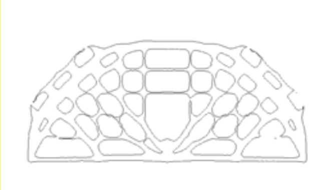




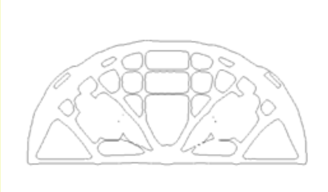
(a)



(b)

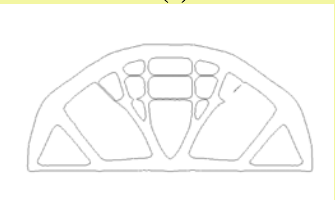


(c)

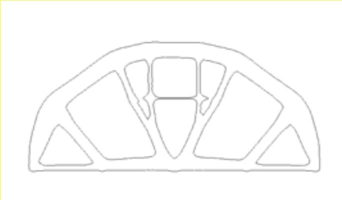


(d)

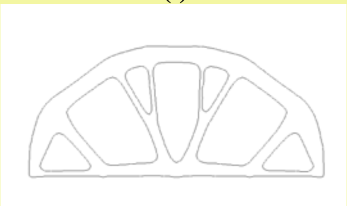
(e)



(f)



(g)



(h)

Fig.17.3 (a) The Third Michell Type Structure

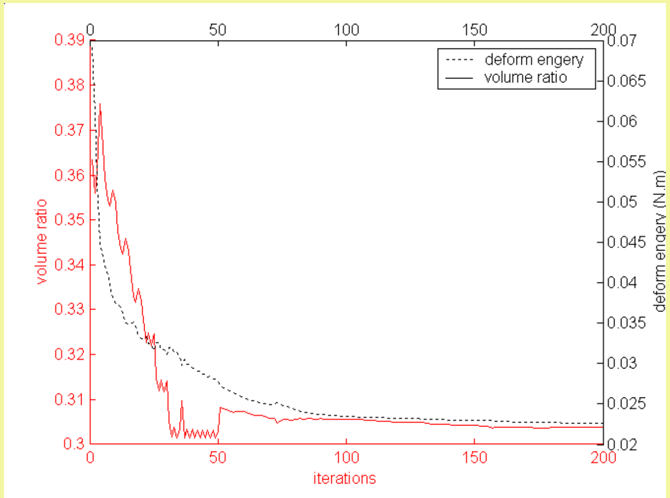
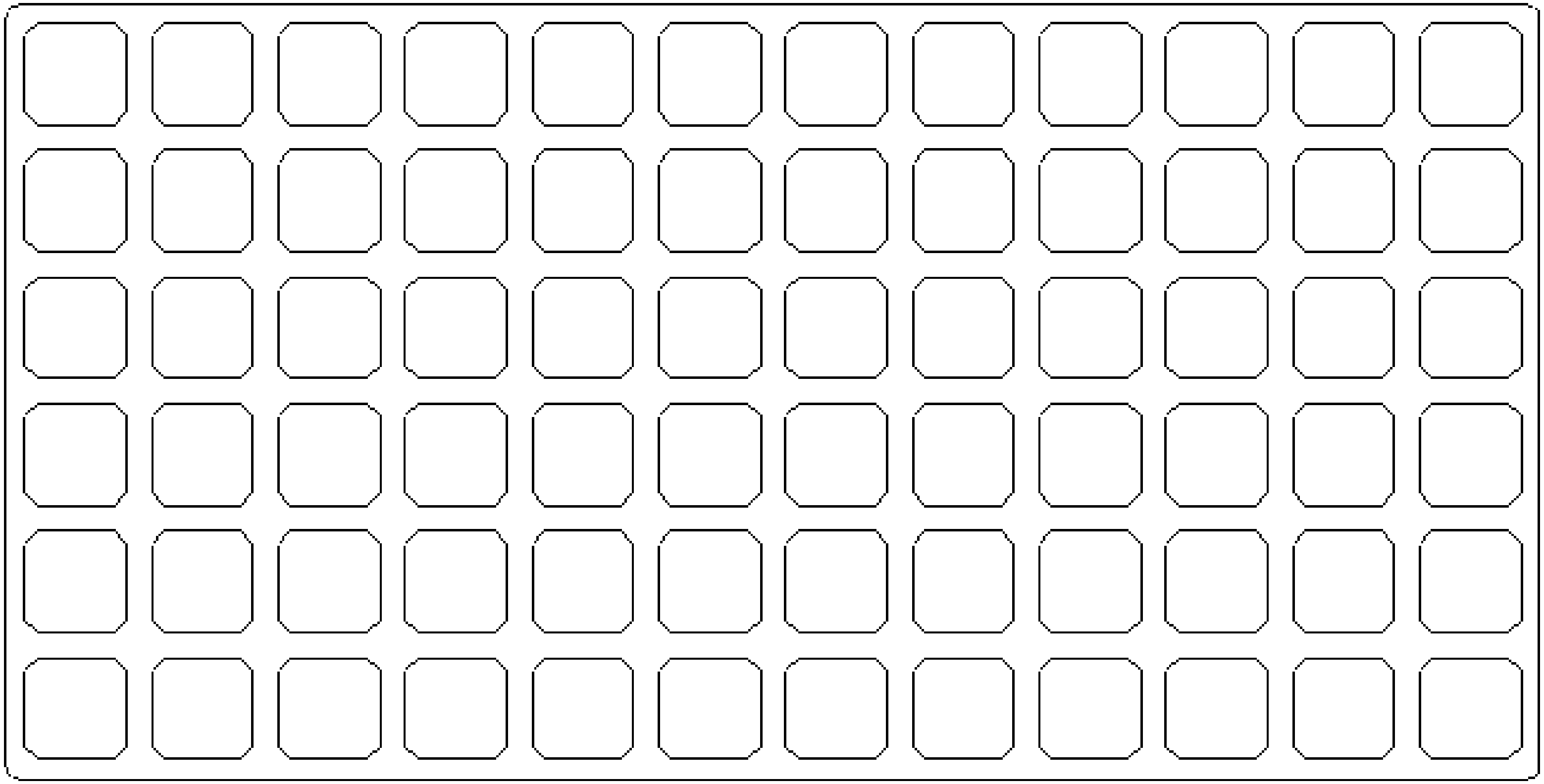
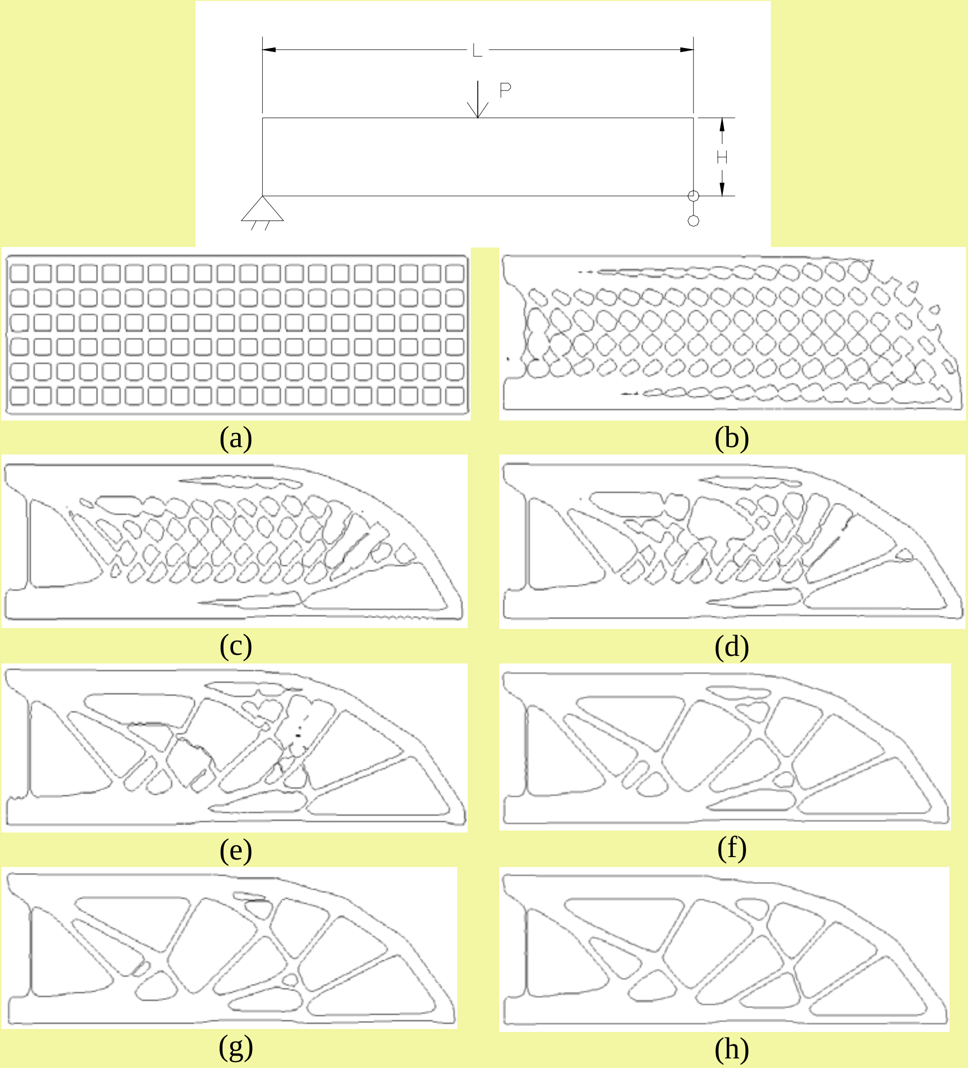


Fig.17.3 (a) The Third Michell Type Structure

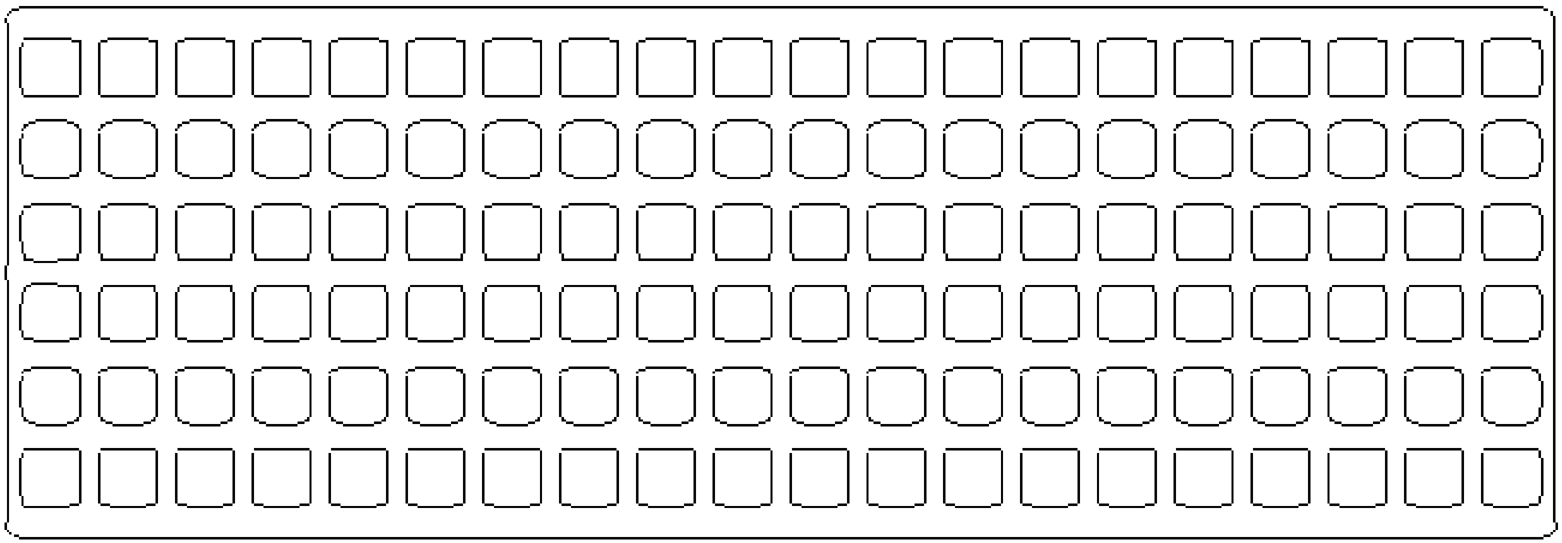
Multi-Load (Video)



MBB Beam

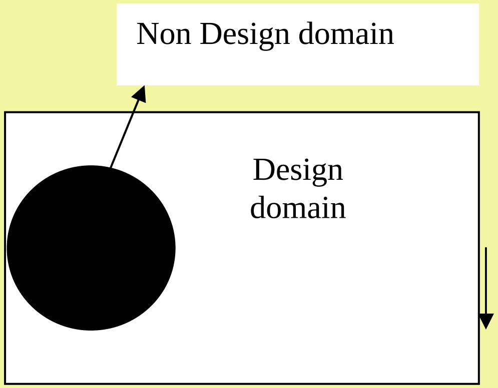


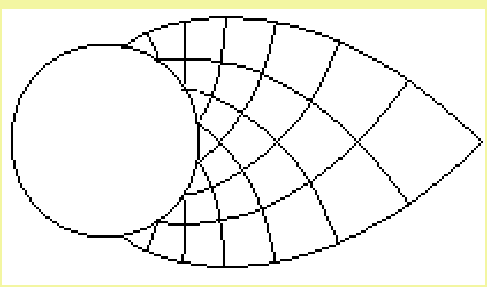
MBB Beam (Video)

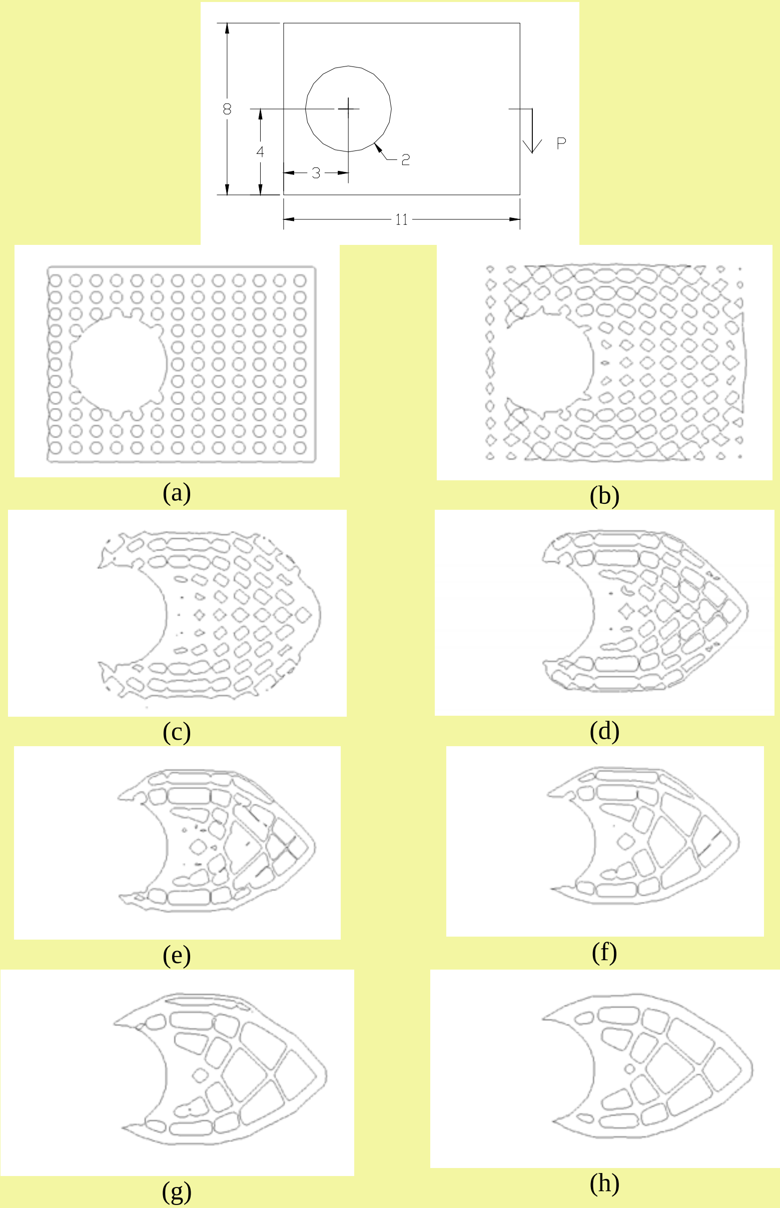


Michell Truss

- Michell Solution (1904)

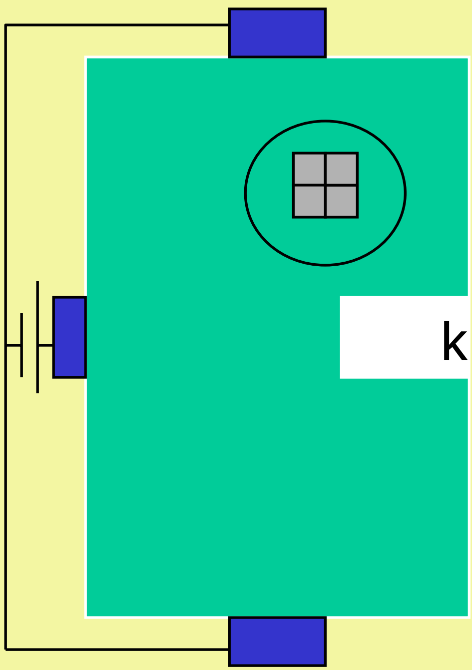






Micro-Gripper

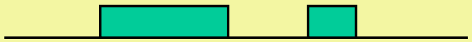
Design Domain



Micro-Models

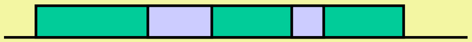
1. Single Material:

$$E = xE_1$$



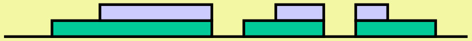
2. Two Material Mixture:

$$E = x_1(E_1 + x_2(E_2 - E_1))$$



3. Material Reinforcement:

$$E = x_1(E_1 + x_2E_2)$$



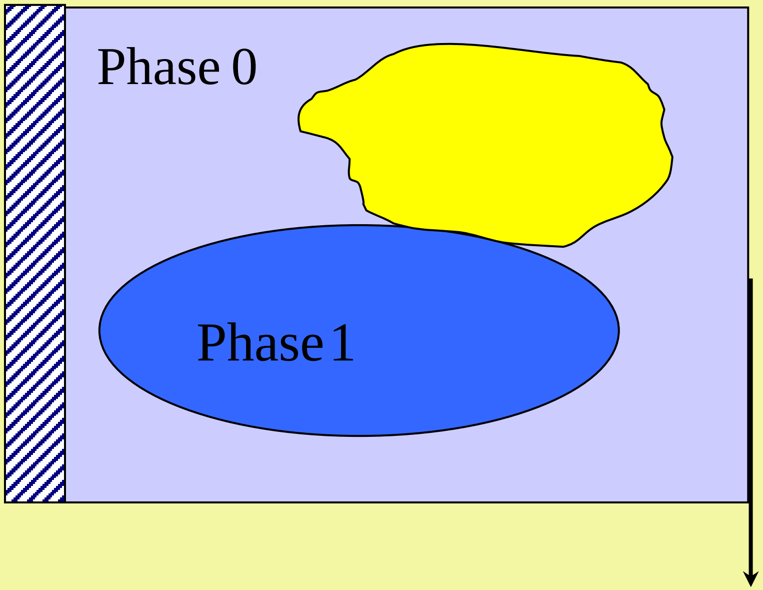
Multiphase Materials

- Partitioning level-set model:
  - Phase 1 by Level-set 1
  - Phase 2 by Level-set 2
- Domain partition by m Level-sets for m phases

$$\phi_i \quad (i = 1,\ldots,m)$$

- Overlap problem:
  - T-Junction
- Enforcing constraint:

$$\sum_{i=1}^{n} H\left(\phi_i(x)\right) = 1$$



$$\Omega = \bigcup_{i=1}^{n} \Omega_i$$

$$\Omega_i \cap \Omega_j = \varnothing$$

$$i \neq j$$

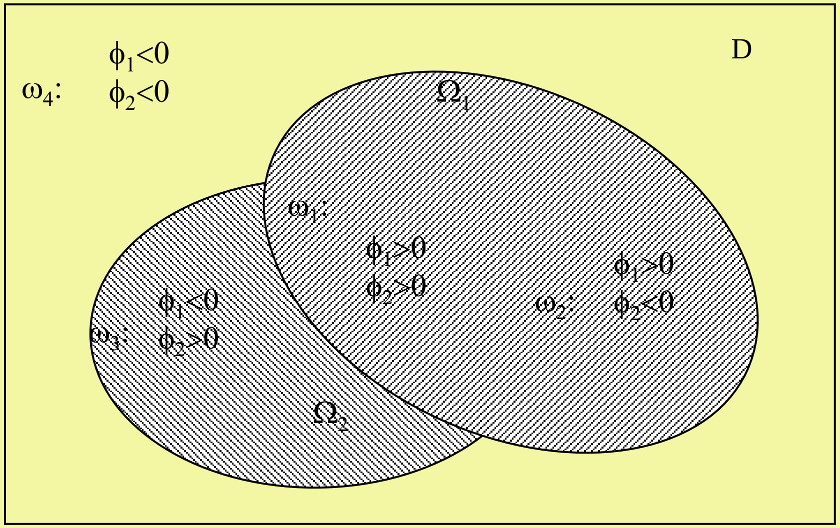
"Color" Level-sets Model

- m level-set functions

  (Chan & Vase '02)

$$\Phi = [\phi_1, \phi_2, \cdots, \phi_m]$$

$$\mathrm{H}(\Phi) = [H(\phi_1), H(\phi_2), \cdots, H(\phi_m)]$$



$$(m = 2, n = 4)$$

- n material Phases:

$$\omega_k = \{x : H(\Phi(x)) = \text{constant vector}, x \in D\}$$

$$n = 2^m$$

$$D = \bigcup_{k=1}^{n} \omega_k \qquad \omega_k \cap \omega_l = \varnothing$$

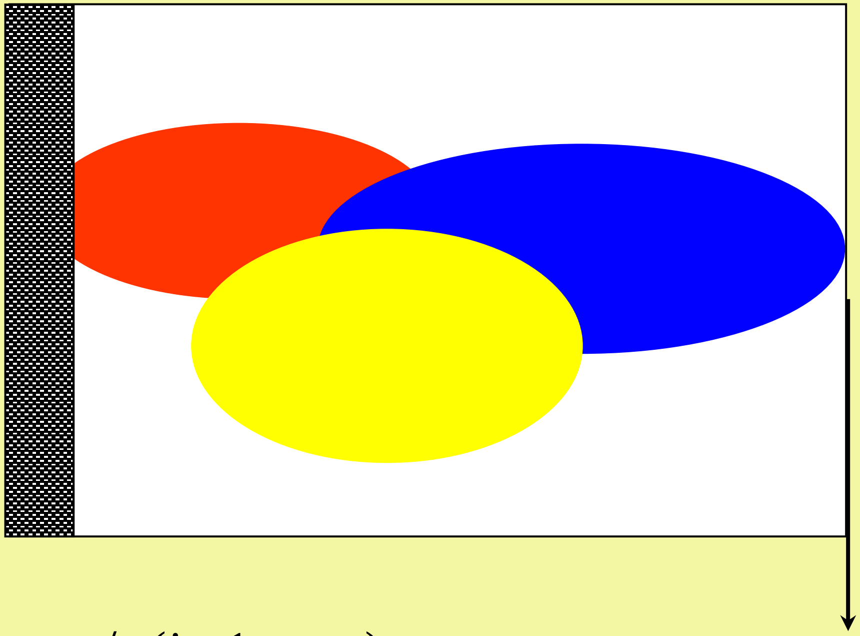
$$k \neq l$$

- (Chan & Vase '02) for Image Segmentation

Optimization with "Color" Level-Set Model

- Phase Characteristic Function:

$$\chi_k(x)=\begin{cases}1 & \text{if } \ x\in\omega_k\\ 0 & \text{otherwise}\end{cases} \qquad \chi_k(\Phi)=\prod_{i=1}^{m} H_i^{I_i^k}$$



- Optimization with Color Level-sets:

$$\underset{\Phi}{Minimize} \quad J(u,\Phi)=\sum_{k=1}^{n}\int_D F^k(u)\chi_k(\Phi)d\Omega \qquad \phi_i \ (i=1,\cdots,m)$$

$$subject\ to: \quad G_j(u,\Phi)=\sum_{k=1}^{n}\int_D g_j^k(u)\chi_k(\Phi)d\Omega\le 0 \quad (j=1,\cdots,r)$$

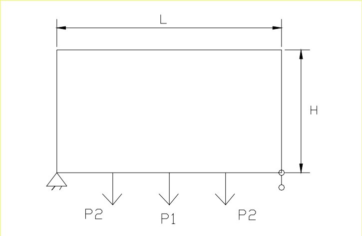
- Level-set PDEs:

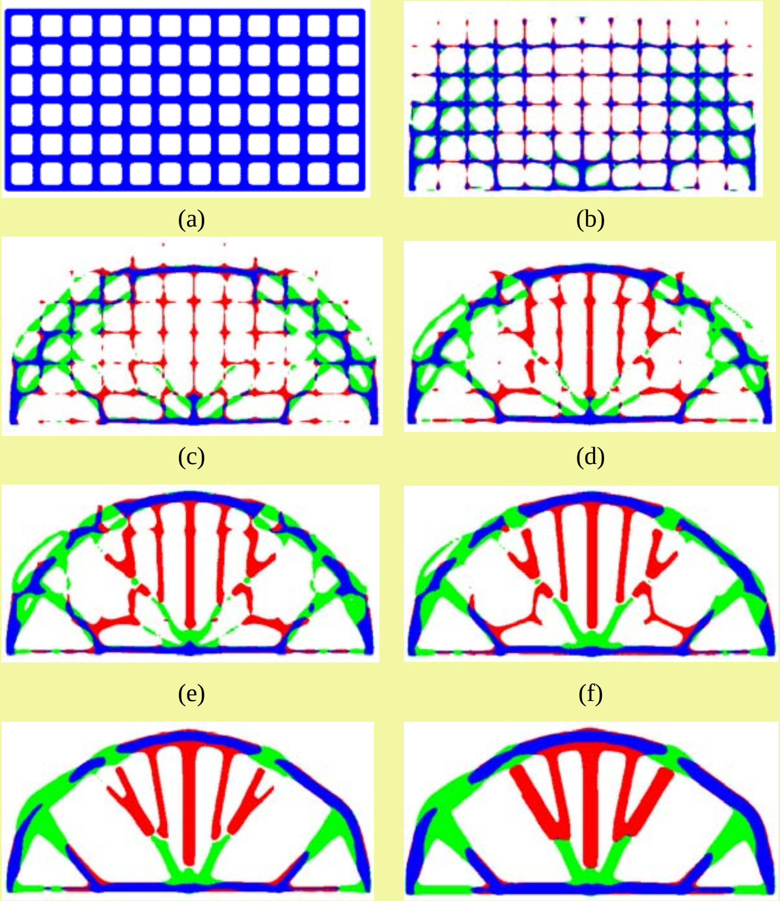
$$\frac{\partial\phi_i}{\partial t}=-P_i(\Phi)\left|\nabla\phi_i\right|, \quad x\in D\setminus\partial D$$

$$\frac{\partial\phi_i}{\partial n}=0 \quad \text{on } \ \partial D$$

Three-Materials

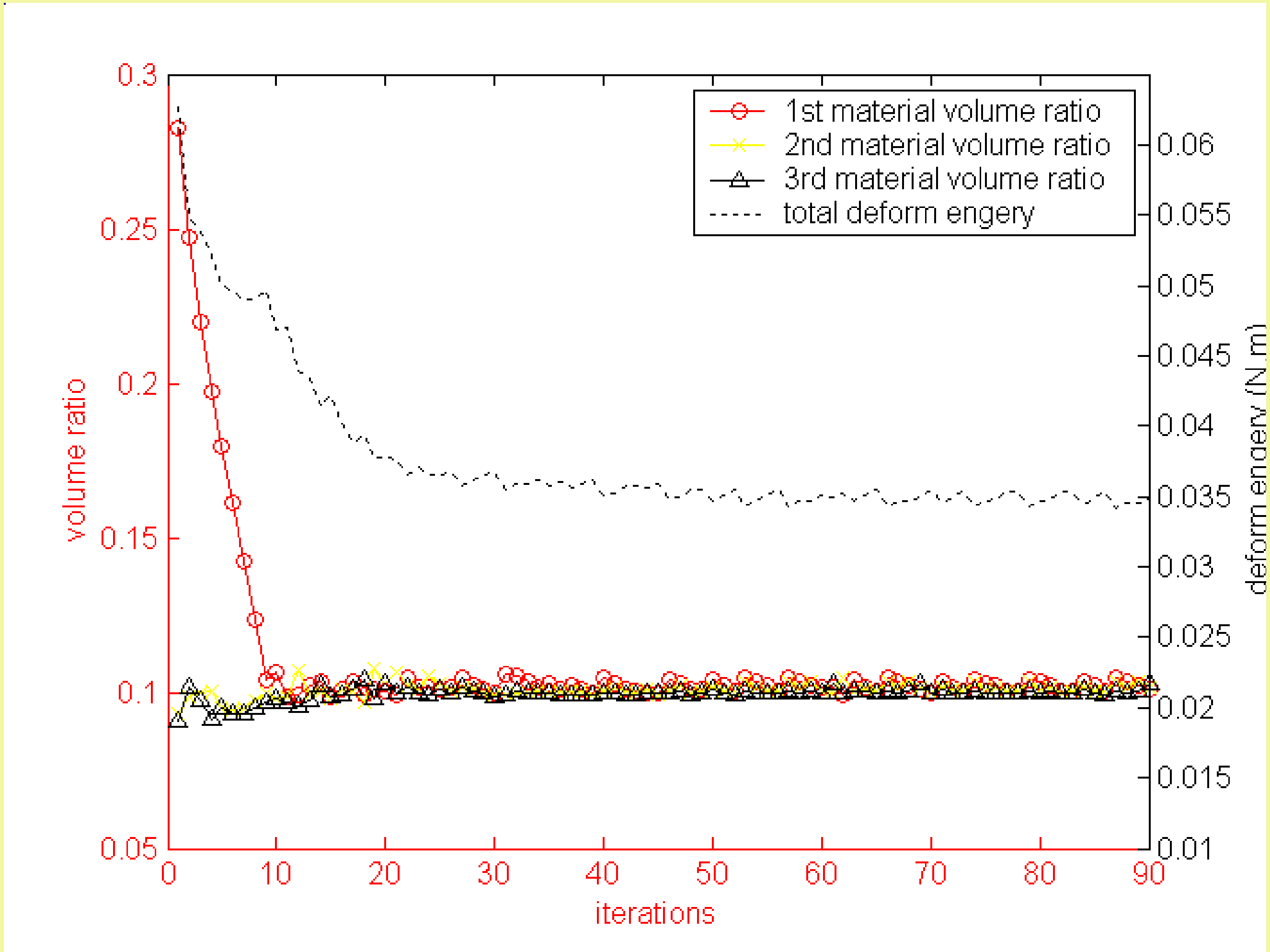
- Three Phases Plus Void
- P = 30, 15
- Each phase of 10%
- E = 200, 100, 50





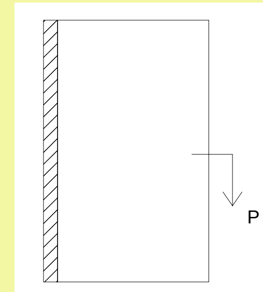
(a) (b) (c) (d) (e) (f) (g) (h)

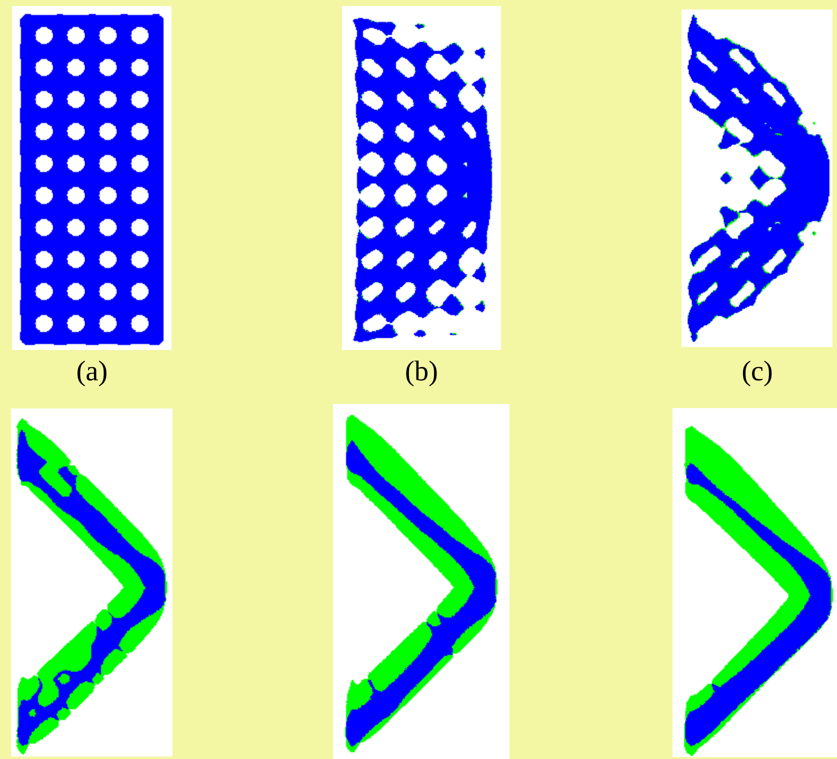
Convergence



Two Materials

- Two Phases Plus Void
- P = 80
- Volume = 10%, 20%
- E = 200, 100

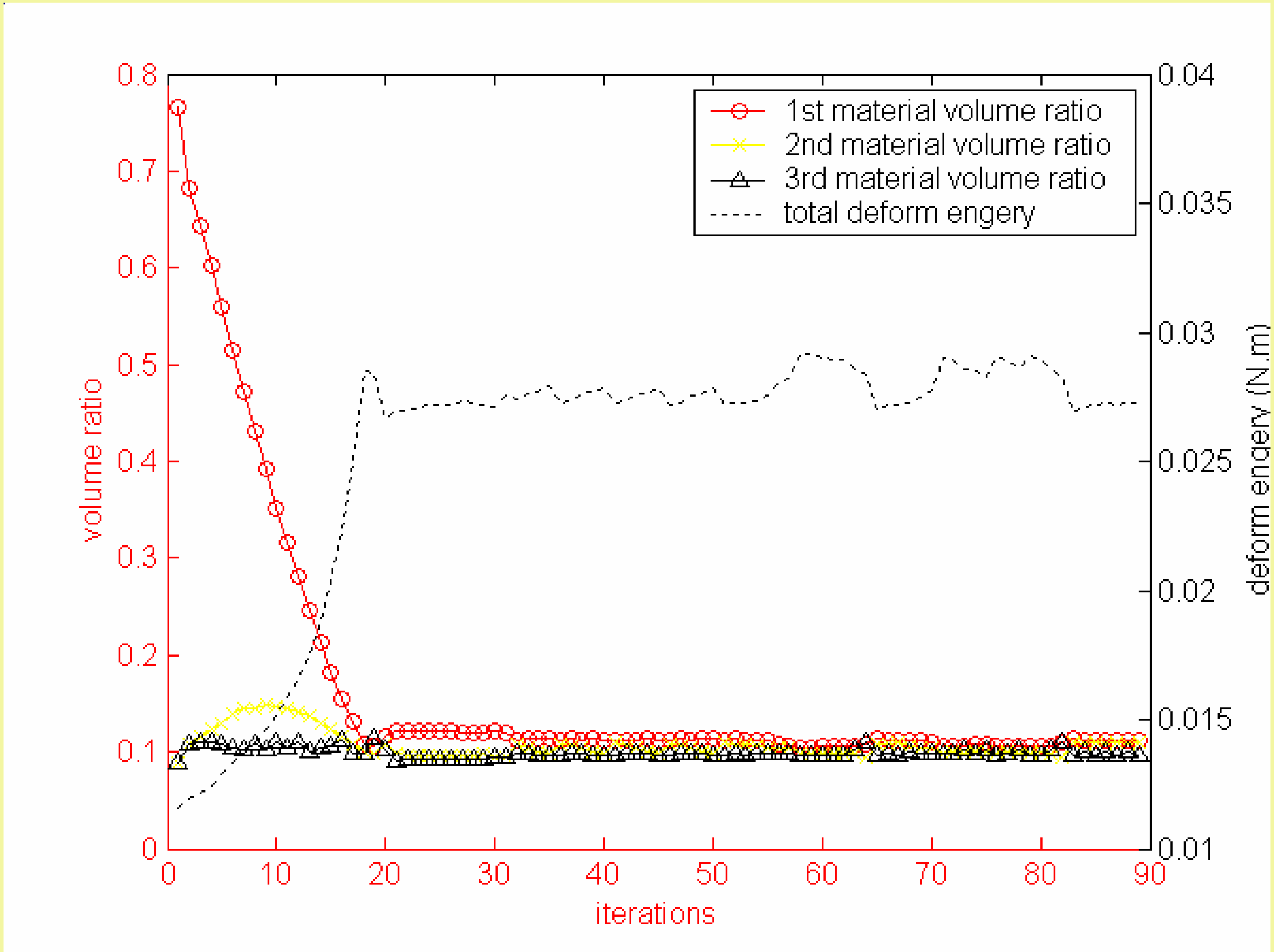




(a) (b) (c)

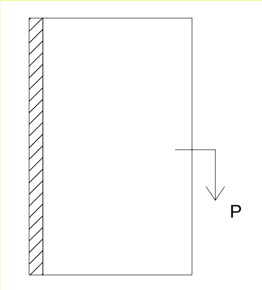
(d) (e) (f)

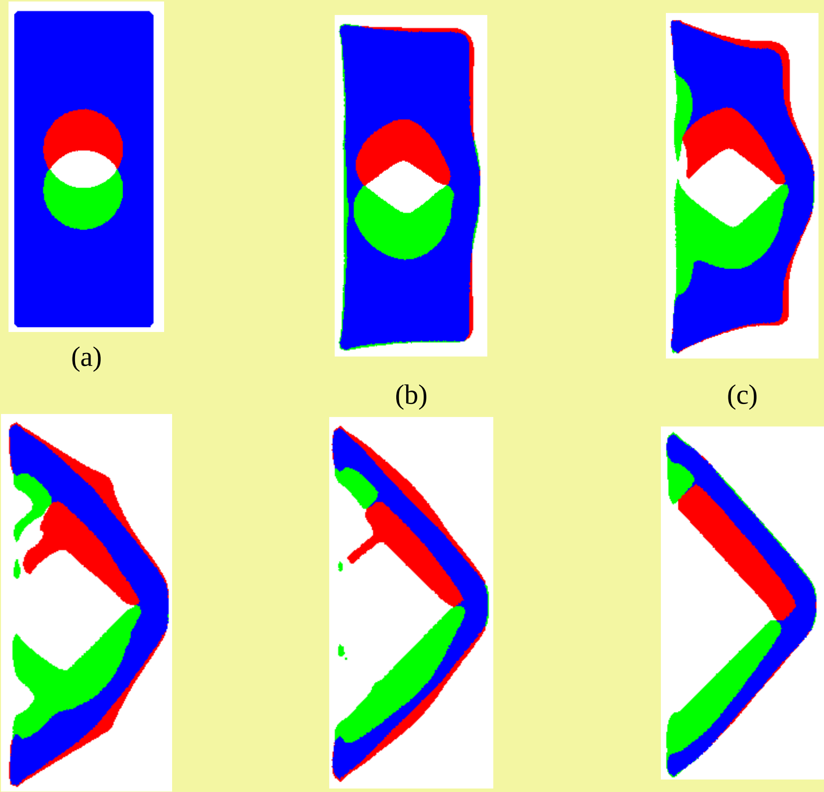
Convergence



Three Materials

- Three Phases Plus Void
- P = 80
- Each phase of 10%
- E = 200, 100, 50

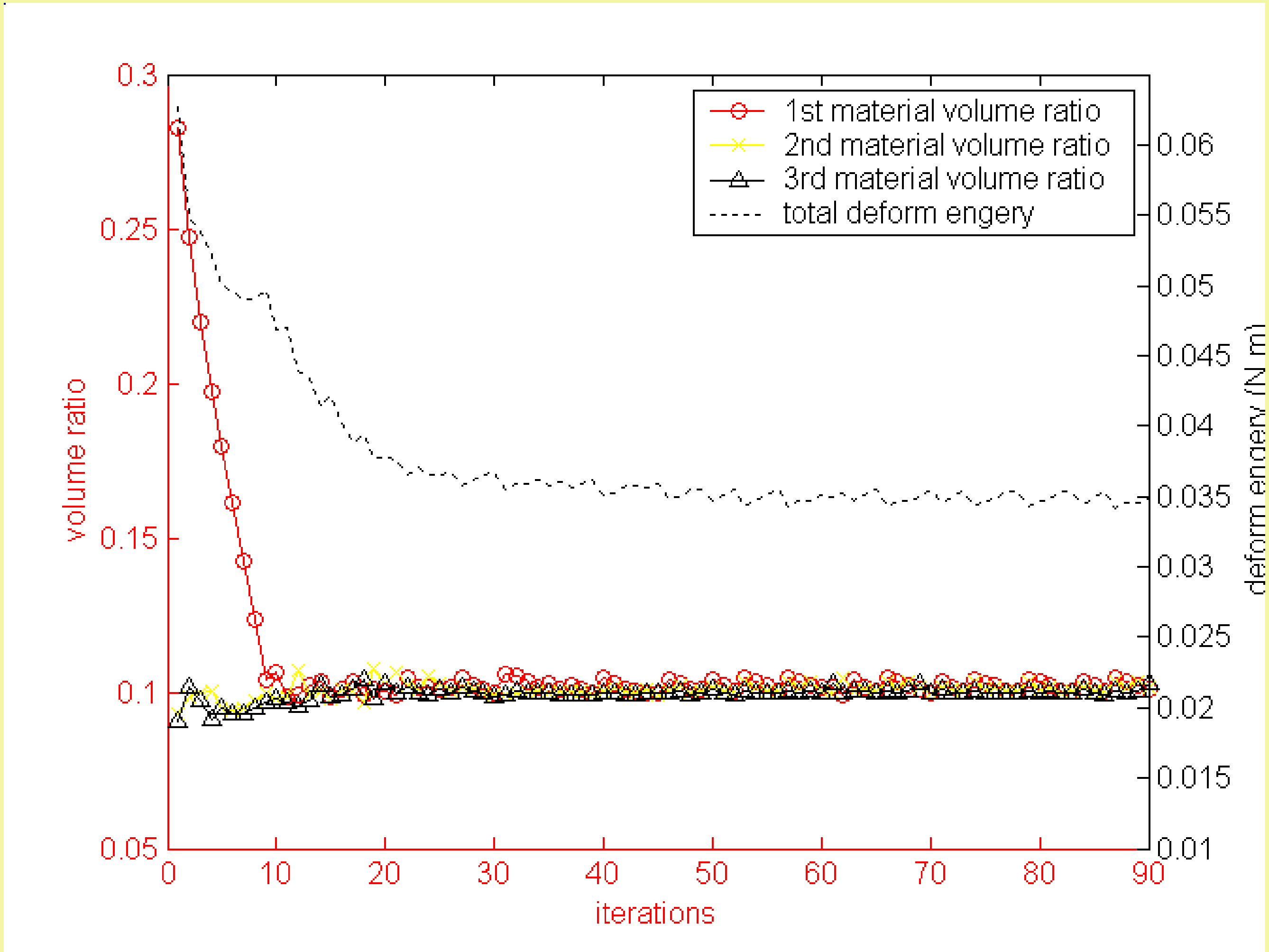




(a) (b) (c)

(d) (e) (f)

Convergence



Material Design

- Base Material: $E^{(1)}_{1111} = E^{(1)}_{2222} = 1.0 \quad E^{(1)}_{1122} = 0.3$
  - Poisson's ratio = 0.3
- Designed Material: $E^{*}_{1111} = E^{*}_{2222} = 0.2(GPa) \quad E^{*}_{1122} = -0.1$
  - Poisson's ratio = –0.5
- Optimization: $min\,imize\ \left(E^{H}_{1111} - E^{*}_{1111}\right)^2 - \left(E^{H}_{2222} - E^{*}_{2222}\right)^2 + \left(E^{H}_{1122} - E^{*}_{1122}\right)^2$
- Y-periodic Cellular Material:

$$E^{H}_{ijkl} = \frac{1}{|Y|}\int_Y \left(E_{ijkl} - E_{ijpq}\frac{\partial \kappa_p^{kl}}{\partial y_q}\right) dY$$

$$\int_Y E_{ijpq}\frac{\partial \kappa_p^{kl}}{\partial y_q}\frac{\partial v_i}{\partial y_j}\, dY = \int_Y E_{ijkl}\frac{\partial v_i}{\partial y_j}\, dY$$

Material Cell

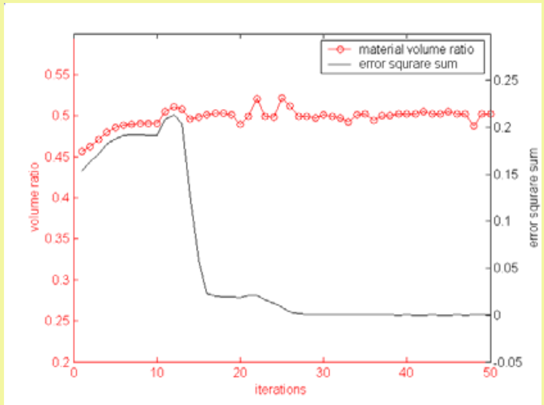
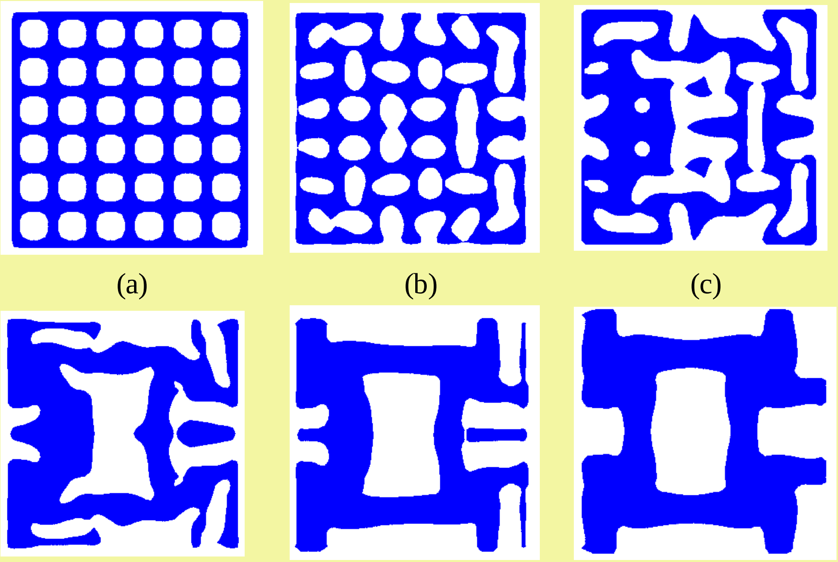


Fig.2 Isotropic Microstructure with Poisson's Ratio Equal to –0.5



(a) (b) (c)

(d) (e) (f)

Fig.3 Isotropic Microstructure with Poisson's Ratio Equal to –0.5

Compliant Mechanisms

- Micro-Griper/Clamp
- MEMS Device
- Flexure Hinges

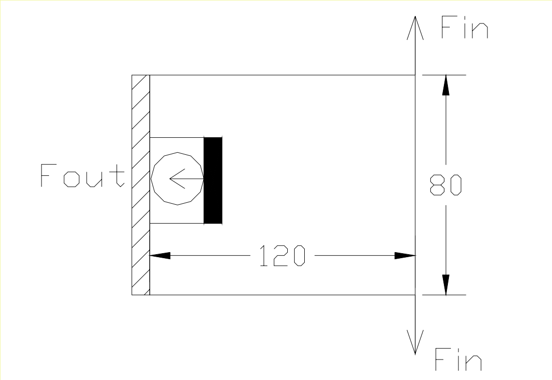


Fig. (4) Design domain for flexible mechanism

Micro-Gripper

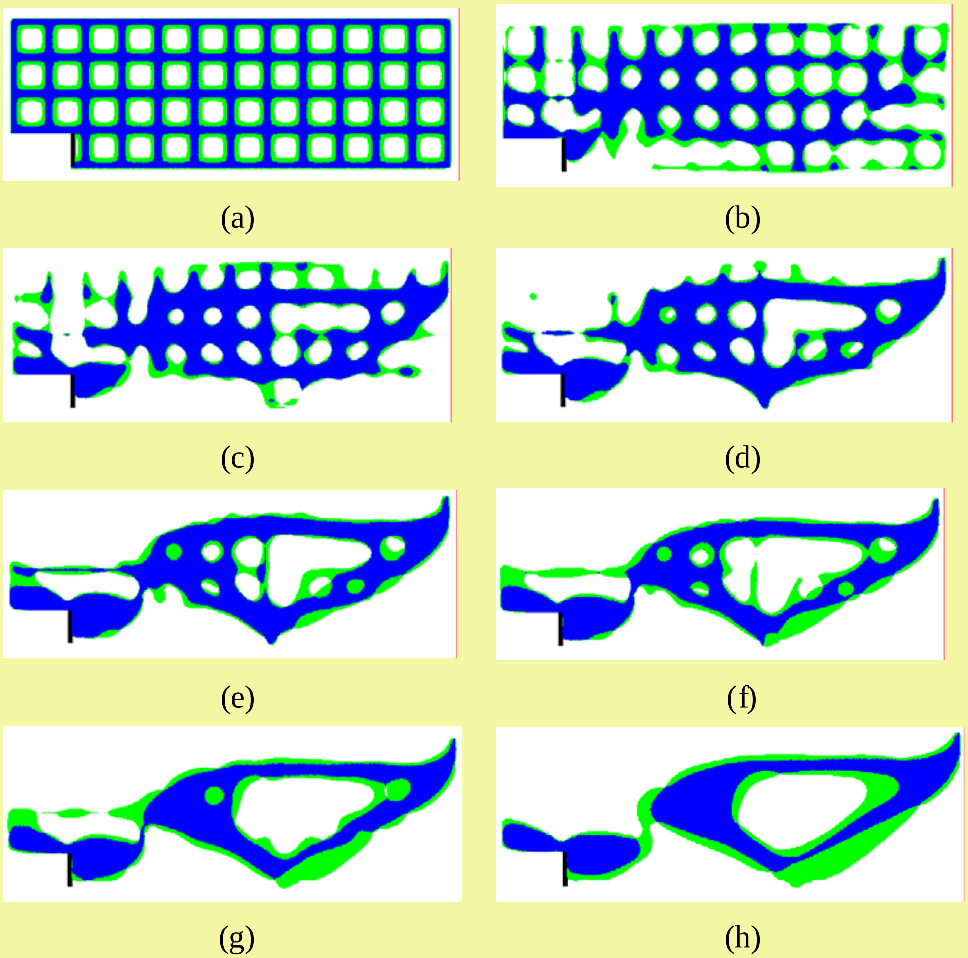
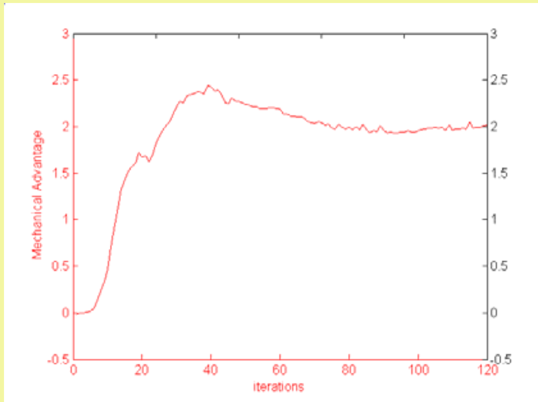
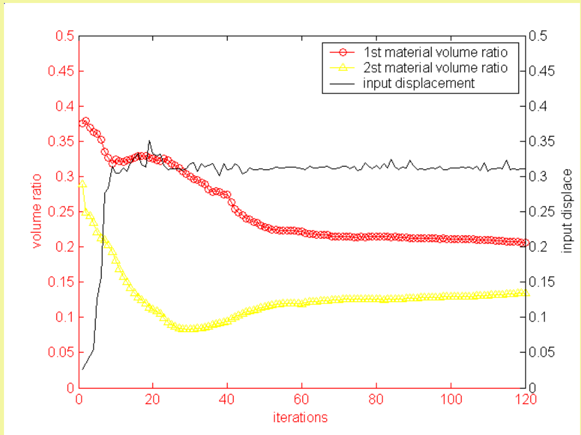


Fig. 6 Evolution Procedure of the Pulled Crunching Mechanism

Convergence





(a) The Mechanical Advantage (b) The material and displacement constraints

Fig.5 The pulled Crunching Mechanism

- Variational Problems of Free-Discontinuities:
  - Mumford-Saha Model (1989):

$$J_{MS}(f,\Gamma)=\int_{\Omega}F(f)dx+\alpha\int_{\Omega\backslash\Gamma}\varphi(|\nabla f(x)|)dx+\beta\int_{\Gamma}dS$$

    - Multiple, distinct regions, each with a continuous (or constant) variable (Material regions)
    - Separated by interfaces (Boundaries)

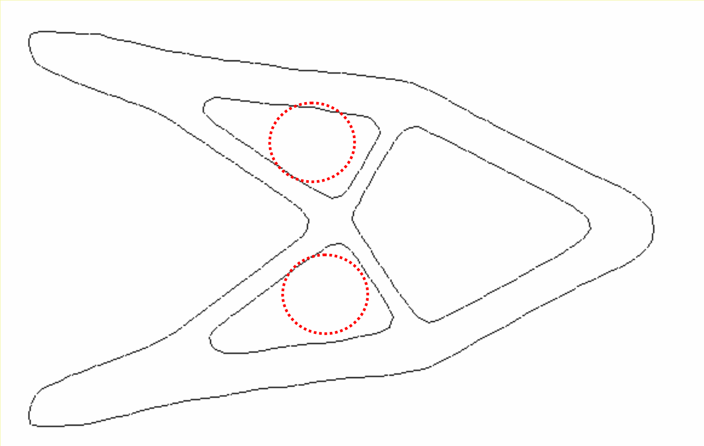
- PDE-driven Geometro-Physical Evolution
  - Level-Set PDE Models:

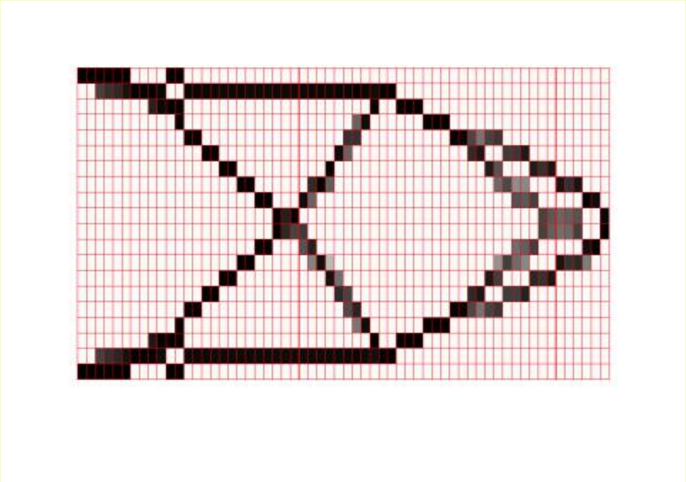
$$\frac{\partial\Phi(x,t)}{\partial t}+\nabla\Phi(x,t)\cdot V(x)=0$$

    - Physical optimization
    - Regularization on material domain
    - Regularization on geometric domain Γ $\qquad V_n=V\cdot n=-\kappa$
    - Topology and geometry control

Constraints

- Geometric Constraints:
  - Curvature Constraint
    - Circular Holes
  - Gradient Constraints
- Topology Constraints:
  - Topology-Preserving
- Manufacturing Constraints:
  - Regional
- Homogenization-Based Methods:
  - Image-Based Post-Processing
  - Difficult to Constrain "Pixels"





Conclusions

1. Level-Set Models and Computation:
   - Concise Boundary & Interior Representation
   - Topologically Flexible & Geometrically Accurate
   - Powerful PDE & Variational Numerical Methods
2. Multi-Phase Level-Sets for Optimization:
   - Heterogeneous phases
   - Efficient and Concise Representation
   - Structure, Material and Mechanism Design
3. On-going Research:
   - Geometry-dependent Loading (e.g., Pressure loading)
   - Variational Methods for Topology Optimization
   - Solid Free-Form Design as a Geometro-Physical Evolution Process for Heterogeneous Systems

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- Contributions
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  - Prof. DM Guo
    (Dalian Univ. of Tech.)
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