# Kinetic Theory for Hamilton-Jacobi PDEs


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# Outline

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# Motivation

## (Stochastic) Growth Models

In many models of interest we encounter an interface that separates different phases and is evolving with time. The interface at a location $x$ and time $t$ changes with a rate that depends on $(x, t)$, and the inclination of the interface at that location. If the interface is represented by a graph of a function $u : \mathbb{R}^d \times [0, \infty) \to \mathbb{R}$, then a natural model for its evolution is a Hamilton-Jacobi PDE:

$$u_t + H(x, t, u_x) = 0, \qquad u(x, 0) = g(x).$$

(In discrete setting some of the variables $x, t$ or $u$ are discrete; examples SEP, HAD, etc.) $H$ is often random (hence $u$ is random), and we are interested in various scaling limits of solutions.

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## A Natural Question/Strategy

Select $g$ according to a (reasonable) probability measure $\mu^0$.

Let us write $\mu^t$ for the law of $u(\cdot, t)$ at time $t$. Note: If $\Phi_t$ is the flow (in other words $u(\cdot, t) = (\Phi_t g)(\cdot)$), then $\mu^t = \Phi_t^* \mu^0$.

Question: Can we find a nice/tractable/explicit evolution equation for $\mu^t$?

We may also keep track of $\rho = u_x$ (more natural). The law of $\rho(\cdot, t)$ is denoted by $\nu^t$. Equilibrium Measure: $\nu^t = \nu^0$.

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# Some Examples

- Some exactly solvable discrete models are determinantal: The finite dimensional marginals of $\nu^t$ can be expressed as a determinant of an explicit matrix. Example: TASEP A. Borodin, P. L. Ferrari, M. Prähofer, T. Sasamoto (2007), G. M. Schütz (1997)
- $d = 1$, $H(x, t, p) = p^2/2$, $\rho(\cdot, 0)$ is a Lévy process. Then $\rho(\cdot, t)$ is also a Lévy process (Bertoin 1998). Associated Lévy measures solve a kinetic-type equation (Smoluchowsky Equation with additive kernel).
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- $b(\cdot, t)$ solves $b_t(\rho, t) = -H''(\rho) b(\rho, t)^2$. Trivially solved. Note that if $b^0 \geq 0$, then no blow up.
- $f(\rho_-, \rho_+, t)$ solves a kinetic equation (resembles Smoluchowski but far more complicated) of the form

$$f_t + C(f) = Q(f, f) = Q^+(f, f) - Q^-(f, f),$$

$C(f)$ a first order differential operator (transport type).

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# Main Result I

## Setting

- Assume $d = 1$, $H(x,t,p)$ is convex in $p$. The function $\rho(x,t)$ solves

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(Or $\rho = u_x$, and $u$ solves $u_t + H(x,t,u_x) = 0$.)

- Assume that $\rho^0$ is a Markov process with a drift $b^0(\rho, x)$ and a jump rate $f^0(\rho_-, \rho_+; x)\, d\rho_+$. This means $x \mapsto \rho^0(x)$ solves and ODE $\dot{\rho}^0(x) = b^0(\rho^0(x), x)$, except at stochastic jump locations. When a jump occurs at $a$, it changes from $\rho_-$ to $\rho_+ \in (-\infty, \rho_-)$, with a rate $f^0(\rho_-, \rho_+; x, t)\, d\rho_+$.

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- $b$ satisfies the linear PDE:

$$b_t + \{H, b\} + H_{\rho\rho} b^2 + 2H_{\rho x} b + H_{xx} = 0,$$

where $\{H, b\} = H_\rho b_x - H_x b_\rho$. The solution $b$ may blow up in finite time. We will discuss an important class of examples with no blowup.

- The function $f(\rho_-, \rho_+; x, t)$ satisfies a kinetic (integro-)PDE

$$f_t + (vf)_x + C(f) = Q(f, f),$$

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# Main Result II

## A scenario with no blowup

We now describe an important class of examples for which $b$ is already determined and there is never a blowup. In this case, even the kinetic equation for $f$ simplifies! Recall that the job of $b(\rho; x, t)$ was to produce a classical solution in between jump discontinuities. A natural candidate for a classical solution is the fundamental solution:

Given a pair $(y, g)$, define a fundamental solution associated with $(y, g)$ by

$$w(x, t; y, g) = w(x, t) = g + \inf \left\{ \int_0^t L(\dot{z}(s), z(s), s)\, ds : \; z(t) = x \right\}$$

where $v \mapsto L(v, x, t)$ is the Legendre conjugate of $p \mapsto H(p, x, t)$.

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## A scenario with no blowup

Given a discrete set $\{(y_i, g_i) : \ i \in I\}$, consider a solution of the form

$$u(x,t) = \inf_{i \in I} \ w(x,t;y_i,g_i).$$

Example: If $H(x,t,p) = H(p)$, we simply have

$$w(x,t;y,g) = g + tL\left(\frac{x-y}{t}\right).$$

Important Remark: For each $t$, there exists $I(t) \subseteq I$ such that

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We can show that for each $t$, there are

$$\cdots < x_i(t) < x_{i+1}(t) < \ldots, \qquad \cdots < y_i(t) < y_{i+1}(t) < \ldots,$$

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## Main Result

**Theorem**

*The process $x \mapsto \rho(x,t)$ is Markov if this is the case initially. At a discontinuity point $x_i(t)$, the position $y_i$ jumps to $y_{i+1} \in (y_i, \infty)$ stochastically with rate $\hat{f}(y_i, y_{i+1}; x_i, t)\, dy_{i+1}$.*

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- The function $\hat{f}(y_-, y_+; x, t)$ satisfies a kinetic PDE

$$\hat{f}_t + (\hat{v}\hat{f})_x = \hat{Q}(\hat{f}, \hat{f}),$$

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## Main Result

- The function $\hat{f}(y_-, y_+; x, t)$ satisfies a kinetic PDE

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# Kinetic Description in Dimension One

Since $H$ is convex in momentum variable, one may use variational techniques to study the solutions. However for our results, we use a different approach.

Suppose $\rho$ is a classical solution and solves an ODE associated with $b$. The compatibility of the two equations

$$\rho_t = -H(\rho, x, t)_x, \qquad \rho_x = b(\rho, x, t),$$

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CLAIM: The picture we have initially persists at later times. The PDE reduces to an interacting particle system!

## Particles Configuration

There are particles $\mathbf{q}(t) = \{(x_i(t), \rho_i(t)) : \ i \in \mathbb{Z}\}$ with $x_i(t) < x_{i+1}(t)$ (we may replace $\mathbb{Z}$ with a finite set).
$x_i(t)$ represents the location of the $i$-th particle.
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## Dynamics

- **q motion** We can set up a collection of ODEs for the evolution of $\mathbf{q}(t)$. For example

$$\frac{dx_i}{dt} = v(\rho_i^-(t), \rho_i(t), x_i(t), t),$$

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- Coagulation/Loss of Particle When two particles meet i.e. $x_i(t) = x_{i+1}(t)$, kill the $i$-particle, and relabel particles to its right.
- The Birth of a Particle At each blowup of $b$, a particle is created. How? Details! Can be worked out in some cases.

## Our Results

- Our two results avoid particle births.
- If there is creation of particles (blowup of $b$), the kinetic equation for $f$ must be modified. When $H$ is also random, we need to add a term representing the creation.
- For a variant of our model, when a particle is created, it fragments into two particles.

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- If there is creation of particles (blowup of $b$), the kinetic equation for $f$ must be modified. When $H$ is also random, we need to add a term representing the creation.
- For a variant of our model, when a particle is created, it fragments into two particles.

## Dynamics

- Coagulation/Loss of Particle When two particles meet i.e. $x_i(t) = x_{i+1}(t)$, kill the $i$-particle, and relabel particles to its right.
- The Birth of a Particle At each blowup of $b$, a particle is created. How? Details! Can be worked out in some cases.

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# Kinetic Description in Higher Dimension

## In Progress, Joint work with Mehdi Ouaki

### Moral

Assume $d = 1$. For a solution of the form $u(x,t) = \inf_{i \in I(t)} w(x,t;y_i,g_i)$, there are two ways to examine it:

(1) Examine the set $\{(y_i,g_i) : i \in I(t)\} \in \mathcal{I}(t)$. As $t$ increases, the state space $\mathcal{I}(t)$ is changing with time. All particles $(y_i,g_i)$ stay put. Occasionally a particle dies, because it becomes redundant. Or put it differently, because the set of allowed particles change with time. This point of view is not mathematically tractable.

(2) Instead, we may switch to $\{(y_i,x_i) : i \in I(t)\} \in \mathcal{J}$ with $x_i$'s representing the locations of discontinuities. $y_i$ stays put but $x_i$ changes with time. Though the state space no longer changes with time ($x_i$'s and $y_i$'s are ordered). This is the point of view that we have successfully adopted in dimension one. We now have a billiard! Disappearance of a particle means that state has reached the boundary to jump to another component of state space.

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# Kinetic Description in Higher Dimensions

### Higher Dimensions

What are the analogs of $x_i$'s in higher dimensions?

### Answer:

There is a Voronoi type tessellation initially that evolves to a Laguerre type tessellation at a later time.

The vertices of this tessellation play the role of $x_i$'s. Each particle has a velocity. When two particles collide, two things can happen (different from what we had in the case of $d = 1$):

- They gain new velocities.
- They kind of coagulate! (For example, when $d = 2$, a triangular face collapses to a vertex; a particle dies.)

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