Modeling Compact Mergers in the Era of Regular Gravitational-Wave Observations

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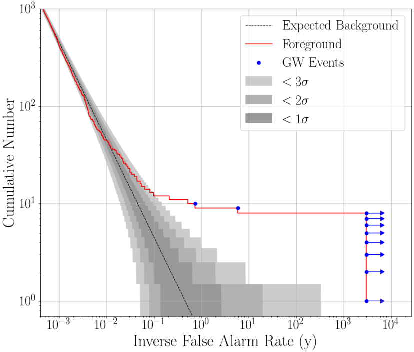
Computational Challenges in Gravitational Wave Astronomy

January 28, 2019

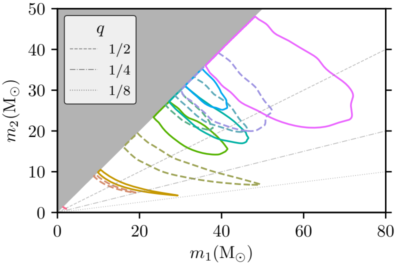
LIGO-G1900140-v2

Binary Merger Observations & Numerical Relativity

A Catalog of Compact Binary Mergers



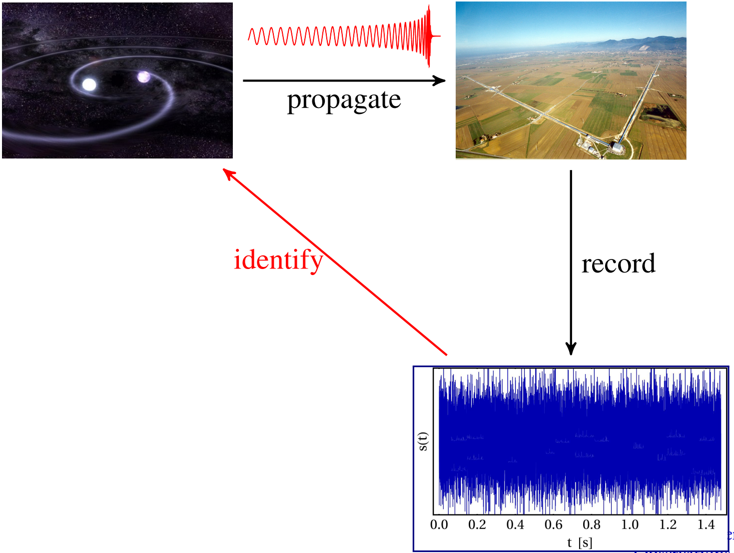
[LIGO+Virgo 1811.12907]

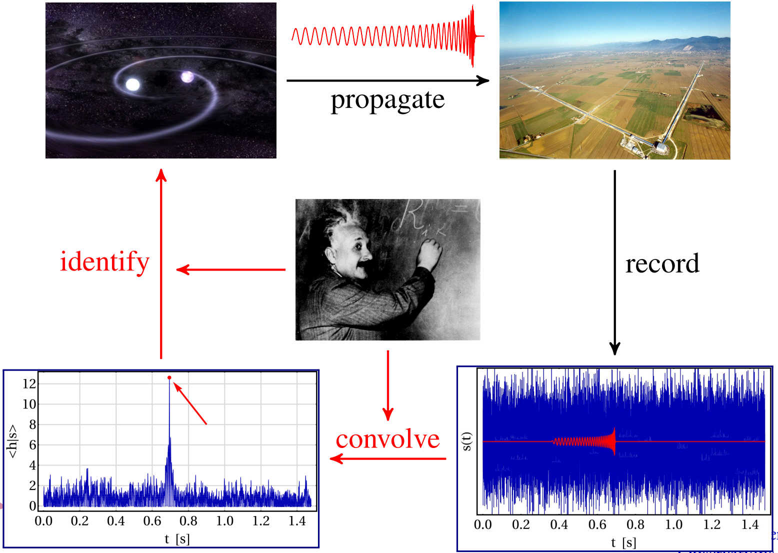


Data confronted with $> 10^7$ theoretically modeled signals

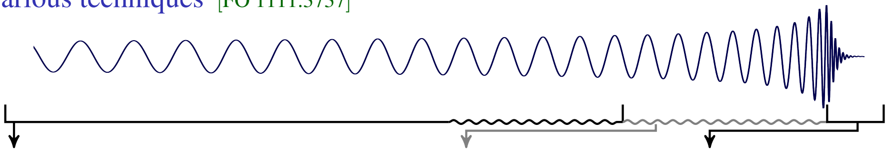
Binary Merger Observations & Numerical Relativity

The Analysis Challenge




propagate
record
identify
convolve
<h|s>
t [s]
s(t)
t [s]

Various techniques [FO 1111.3737]



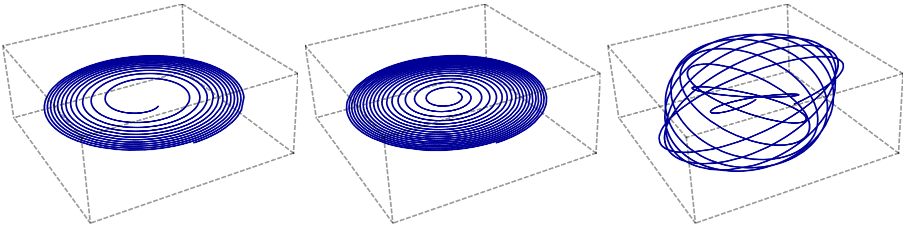
Inspiral
post-Newtonian theory
Effective One-Body (EOB)

Merger
no analyt. model

Ringdown
perturbation theory

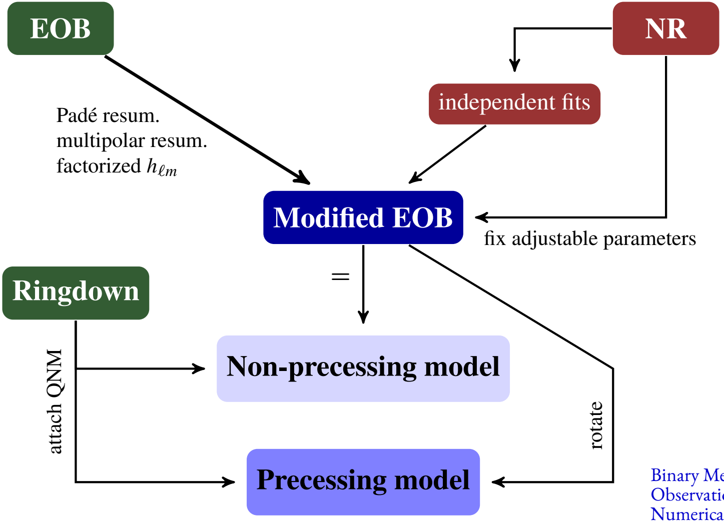
Numerical Relativity

Various effects



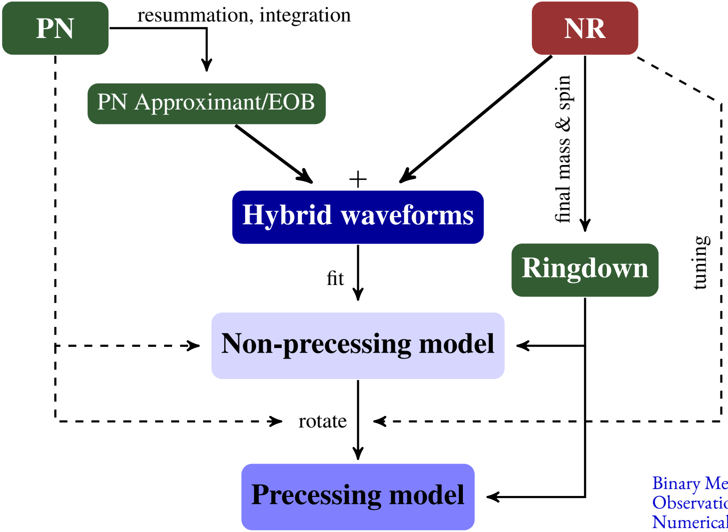
non-spinning   non-precessing   precessing

A (somewhat simplified) sketch of EOBNR



Binary Merger Observations & Numerical Relativity

A (somewhat simplified) sketch of Phenomenological models



Binary Merger Observations & Numerical Relativity

Complexity, Efficiency

Numerical Relativity

- Einstein's Equation
- Coupled PDEs
- Time integration

EOBNR

- Hamiltonian eq.
- ODEs
- Time integration

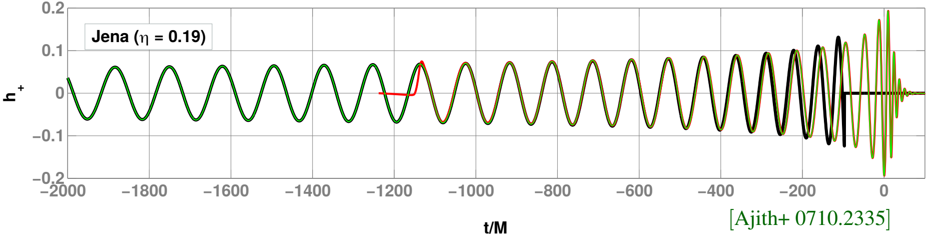
Phenomenological

- Fitting formulae
- Explicit closed form
- Frequency domain

complexity ←

efficiency →

The beginning: non-spinning signals



[Ajith+ 0710.2335]

Analytical form

$$h(f) = \sum_{\ell,m} {}^{-2}Y_{\ell m}\, h_{\ell m} \approx {}^{-2}Y_{22}\, h_{22} = A(f)\, e^{i\Psi(f)}$$

$$\Psi(f) = 2\pi f t_0 + \varphi_0 + \sum_{k=0}^{7} \psi_k f^{(k-5)/3}$$

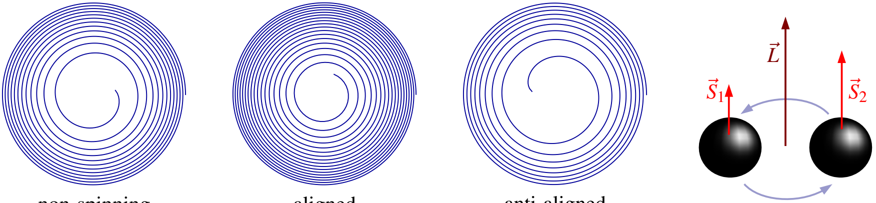
$$A(f) = \mathcal{C} \begin{cases} (f/f_m)^{-7/6} & \text{if } f < f_m \\ (f/f_m)^{-2/3} & \text{if } f_m \le f \le f_{\mathrm{RD}} \\ \omega \mathcal{L}(f) & \text{if } f_{\mathrm{RD}} \le f < f_{\mathrm{cut}} \end{cases}$$

Restrictions

- no spins
- dominant harmonic
- no eccentricity
- 7 parameter space points (mass ratio ≤ 4)

Numerical Relativity

Spinning, non-precessing signals



non-spinning | aligned | anti-aligned

Additions

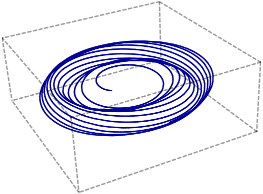
- Dominant spin effect: $\chi_{\text{eff}} = \frac{m_1\chi_1 + m_2\chi_2}{m_1+m_2}$
- Extreme mass-ratio limit (PhenB)
  [Ajith..FO+ 0909.2867]
- Fourier-domain hybridization (PhenC)
  [Santamaría, FO+ 1005.3306]
- Smooth (tanh) transitions

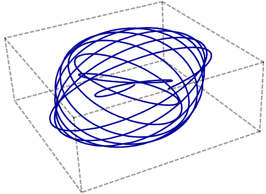
Restrictions

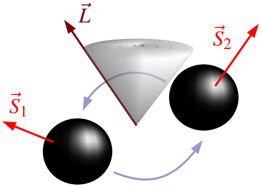
- no precession
- dominant harmonic
- no eccentricity
- 24 NR signals (mass ratio ≤ 4, $|\chi_{\text{eff}}| \leq 0.85$)

Precessing signals

[Hannam..FO+ 1308.3271]







Separation of complex dynamics

- Precession dominated by χ_p (larger in-plane spin component)

  [Schmidt, FO, Hannam 1408.1810]

- Full signal = non-precessing $\times$ rotation

$$h^{\mathrm{prec}}_{\ell m} = e^{-im\alpha} \sum_{|m'|\leq \ell} e^{im'\epsilon} d^{2}_{m'm}(-\iota)\, h^{\mathrm{np}}_{\ell m'}$$

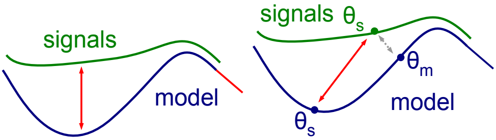
[Schmidt, Hannam, Husa 1207.3088]

Restrictions

- dominant spin effects
- dominant harmonic
- no eccentricity
- single-spin precession, not NR-tuned

Observations & Numerical Relativity

Signal geometry



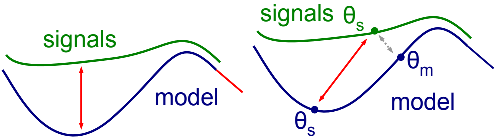
$$\langle h_1 \,|\, h_2 \rangle = 4\,\mathrm{Re} \int \frac{h_1(f)\, h_2^*(f)}{S_n(f)}\, df$$

$$O = \frac{\langle h_1 \,|\, h_2 \rangle}{\sqrt{\langle h_1 \,|\, h_1 \rangle \langle h_2 \,|\, h_2 \rangle}}$$

- signals missed: $(\max O)^3$
- biased characterization at SNR 10: $1 - \mathcal{O} > 0.5\%$

Binary Merger Observations & Numerical Relativity

Signal geometry



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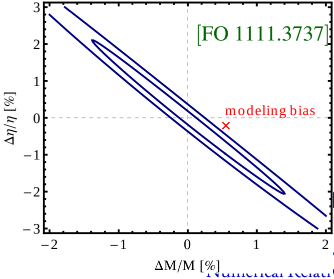
- signals missed: $(\max O)^3$
- biased characterization at SNR 10: $1 - \mathcal{O} > 0.5\%$

Results

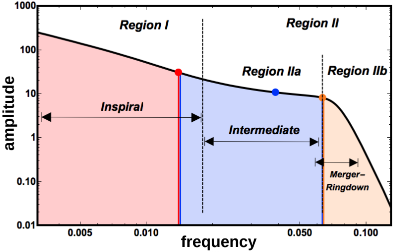
[FO+ 1107.0996]

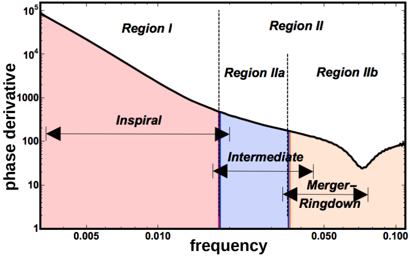
origin	$1 - O$
Hybridization	$< 0.02\%$
NR	$< 0.1\%$
Interpolation	$\lesssim 3\%$
PN inspiral	$\sim \mathcal{O}(10\%)$



Overhaul of the fit

[Husa..FO+ 1508.07250, Khan..FO+ 1508.07253]





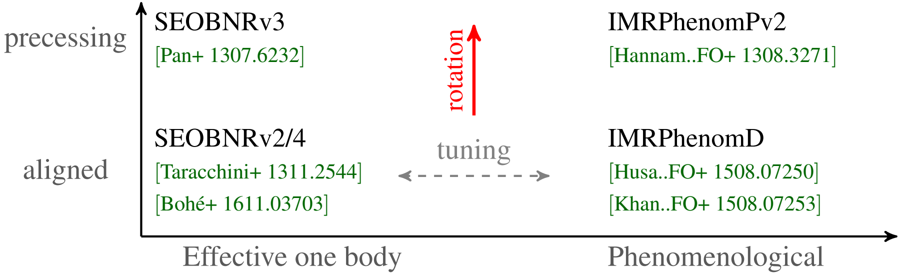
Novelties

- EOB hybrids, PN-inspired fit
- two spins in inspiral and ringdown
- C^1 continuous, step transitions
- vastly improved fit with 17 independent phenomenological parameters

Restrictions

- precession as before
- dominant harmonic
- no eccentricity
- 19 NR signals (mass ratio ≤ 18, $|\chi_{\rm eff}| \leq 0.85/0.98$)

Main models used in most recent GW observations



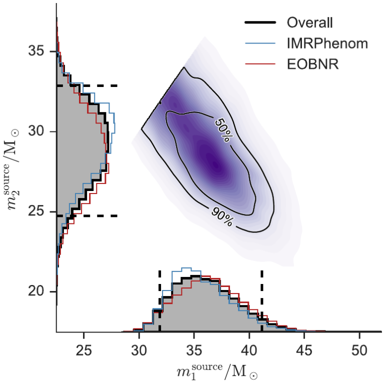
Notes (some details later in this talk)

- More flavors, including optimized versions, of those models are in use
- Alternatives exist that avoid construction of analytical model altogether

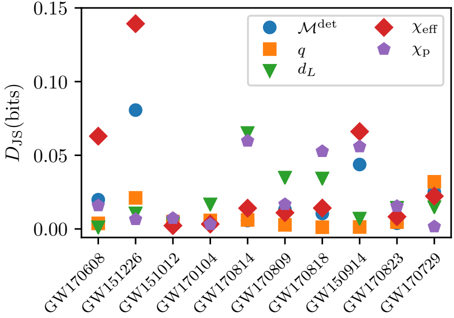
Numerical Relativity

Observation + model → posterior distribution

GW150914 [LIGO+Virgo 1602.03840]



GWTC-1 [LIGO+Virgo 1811.12907]

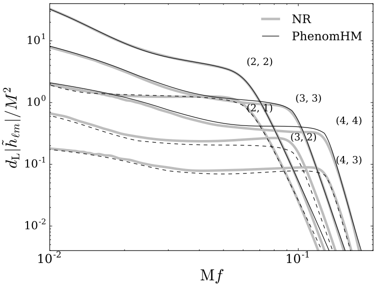


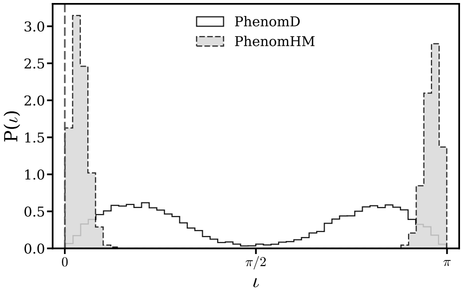
(Jensen-Shannon divergence)

Note: Sources have been in the easiest-to-model part of the parameter space.

Adding higher multipoles

[London..FO+ 1708.00404]





A simple mapping

$$h(f) = \sum_{\ell m} {}^{-2}Y_{\ell m}\, h_{\ell m} = \sum_{\ell m} {}^{-2}Y_{\ell m}\, A_{\ell m} e^{i\Psi_{\ell m}}$$

$$A_{\ell m}(f) = \beta_{\ell m}(f)\, A_{22}(f_{22})$$

$$\Psi_{\ell m}(f) = \Psi_{22}(f_{22}) + \Delta_{\ell m}$$

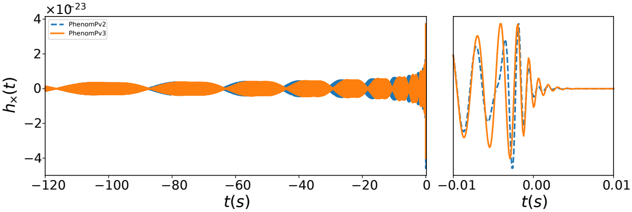
f_{22}: linear transition from $(2f/m)$ to $f^{\mathrm{RD}}_{\ell m}$

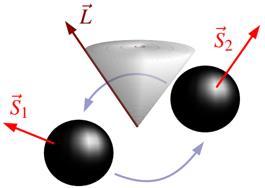
Restrictions

- no eccentricity
- no additional NR tuning
- no mode mixing in ringdown

Improved precession

[Khan..FO+ 1809.10113]





Analytical solution for precession

- Analytical solution of orbit-averaged spin dynamics [Kesden+ 1411.0674]
- Multiple scale analysis using $T_{\rm prec} \ll T_{rr}$ $\to$ closed-form evolution of precession angles [Chatziioannou+ 1606.03117, 1703.03967]

Restrictions

- no eccentricity
- inspiral precession, no NR tuning

Binary Merger Observations & Numerical Relativity

Ongoing developments

Precessing higher modes

- Combine analytical precession with higher multipole mapping

[Khan+]

- Tune precession angles to NR

[Hamilton+]

Eccentric model

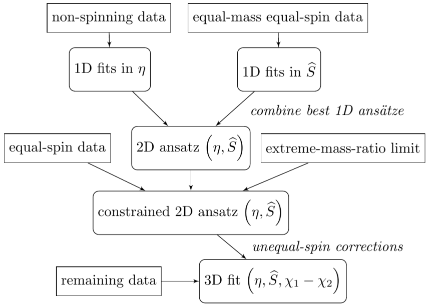
- 70 eccentric NR waveforms eccentricities ≤ 0.5, mass ratio ≤ 4, non-precessing
- PN+NR hybrids

[Ramos Buades, Husa, Haney]]

PhenomX

[Pratten, Husa+]

- Automated NR processing
- Test particle Kerr dynamics
- Higher-multipole hybridization and tuning



[Jiménez-Forteza+, 1611.00332]

Numerical Relativity

Effective-One-Body

Phenomenological

complexity

Binary Merger
Observations &
Numerical Relativity

Complexity and Efficiency again

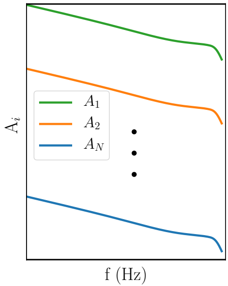
Numerical Relativity

Effective-One-Body

Phenomenological

Binary Merger Observations & Numerical Relativity

Surrogate models



Basis representation

$$A_i \approx \sum_{k=1}^{n} c_{ik}\, \hat{A}_k \qquad (n < N)$$

Interpolation

$$A(\boldsymbol{\theta}) \approx \sum_{k=1}^{n} \tilde{c}_k(\boldsymbol{\theta})\, \hat{A}_k$$

Binary Merger
Observations &
Numerical Relativity

Example: Singular Value Decomposition

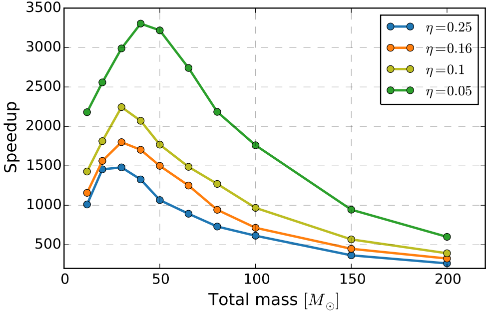
$$\boxed{A = U\Sigma V^T}$$

- A : gravitational-wave amplitudes or phases
- V : singular (basis) vectors
- Σ : diagonal matrix of singular values
- $U\Sigma$: projection coefficients

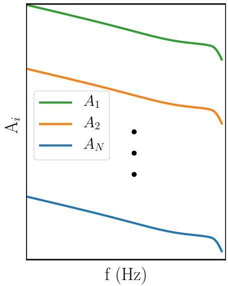
Application

Successfully implemented in combination with tensor spline interpolation in `SEOBNR_ROMv2/4`

[Pürrer 1512.02248]



Enhancing models on the way



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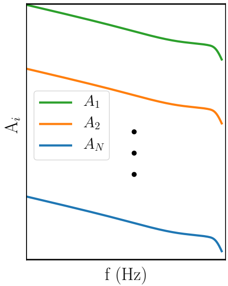
Binary Merger Observations & Numerical Relativity

Enhancing models on the way



Basis representation

$$A_i \approx \sum_{k=1}^{n} c_{ik}\,\hat{A}_k \qquad (n < N)$$

New information

Interpolation

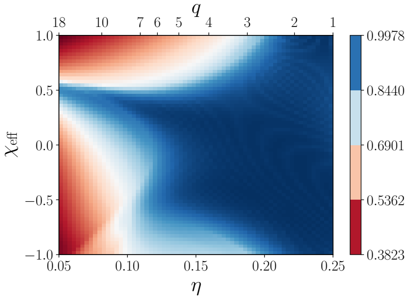
$$A(\boldsymbol{\theta}) \approx \sum_{k=1}^{n} \tilde{c}_k(\boldsymbol{\theta})\,\hat{A}_k$$

Binary Merger
Observations &
Numerical Relativity

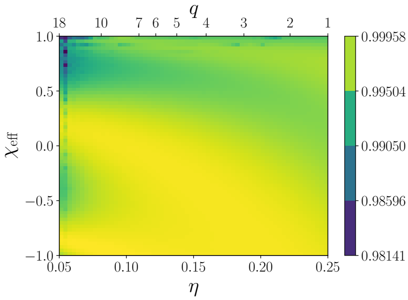
Proof of principle: PhenomB → PhenomD

[Setyawati, FO, Khan, 1810.07060]

Approximate basis



Enriched model

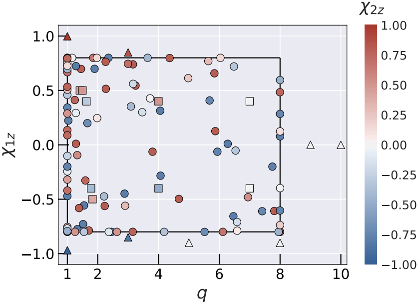


- Enriching of approximate model through few(er) accurate models
- No manual re-tuning
- Idea first sketched by [Cannon+ 1211.7095]

Optimal placement: greedy basis

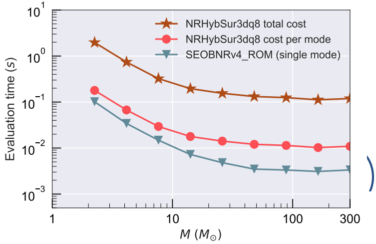
Greedy strategy

1. Project test signals onto basis
2. Add signal with highest deviation from its basis projection to the basis
3. Repeat until projection error sufficient [Field+ 1101.3765]



Variants for NR

- Use PN as proxy to find greedy points in parameter space [Blackman+ 1701.00550, Varma+ 1812.07865]
- Use Gaussian Process Regression to estimate error [Doctor+ 1706.05408]



Conclusion

- So far, waveform models for compact binaries lived up to the challenge of gravitational-wave astronomy thanks to a combinations of many techniques (NR, analytical information, reduced-order interpolation)
- More frequent observations $\rightarrow$ efficient models (simplified, hierarchical, optimized)
- Observations with higher signal-to-noise $\rightarrow$ accurate, complex models
- Need ways to efficiently incorporate new NR data
- How can we gain confidence in a surprising measurement?
  (Keep independent, alternative modeling approaches.)
- Promising early developments regarding tidal effects, eccentricity, alternative theories