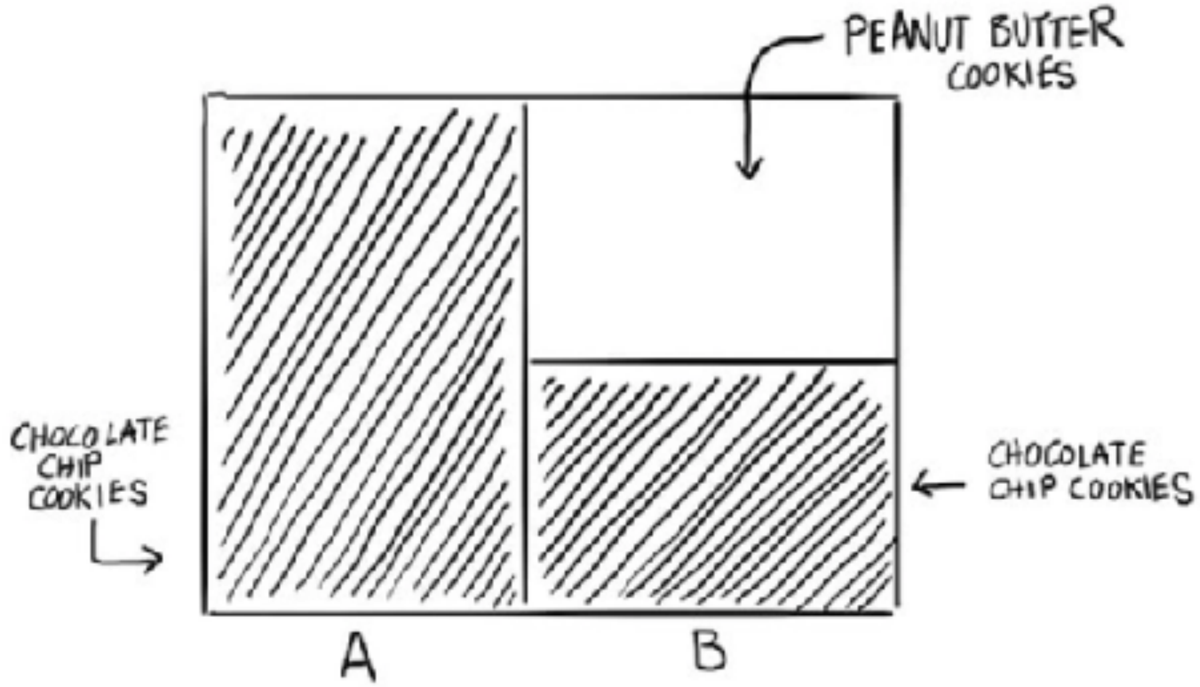
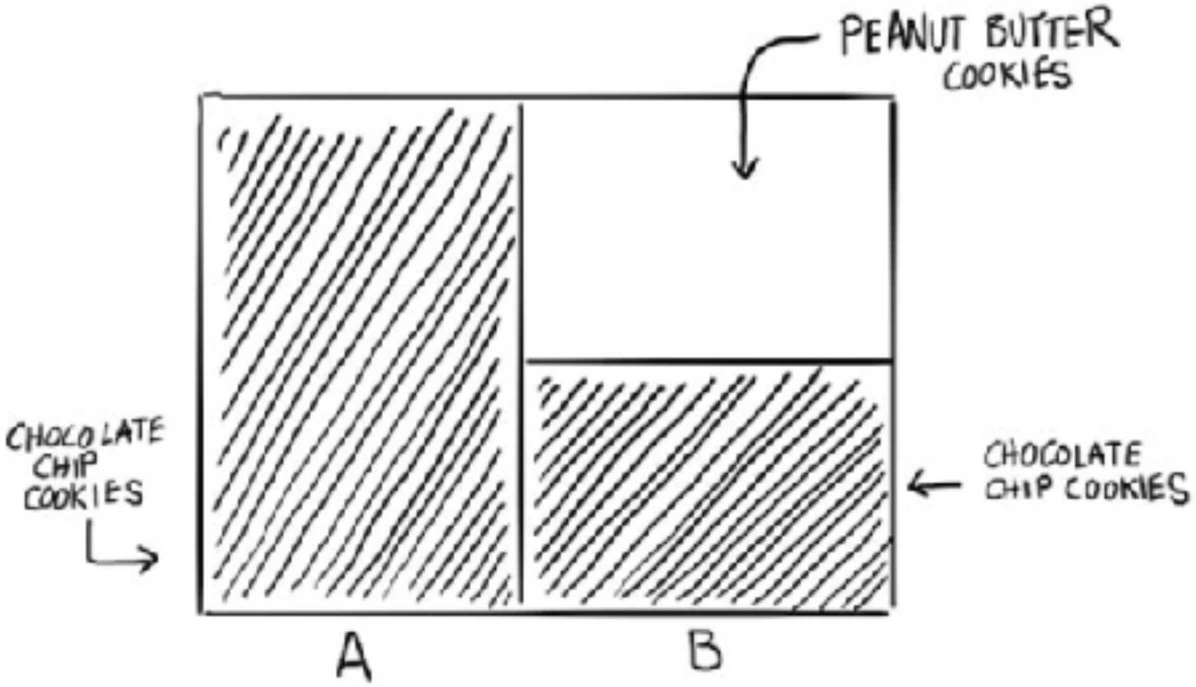
HOW IS PE COMPUTED?



$$P(\text{BOX A} \mid \text{CC COOKIE}) = \frac{\overset{1}{P(\text{CC COOKIE} \mid \text{BOX A})}\;\overset{1/2}{P(\text{BOX A})}}{\underset{3/4}{P(\text{CC COOKIE})}}$$

Es. from Bayes' Theorem:
a visual introduction for beginners

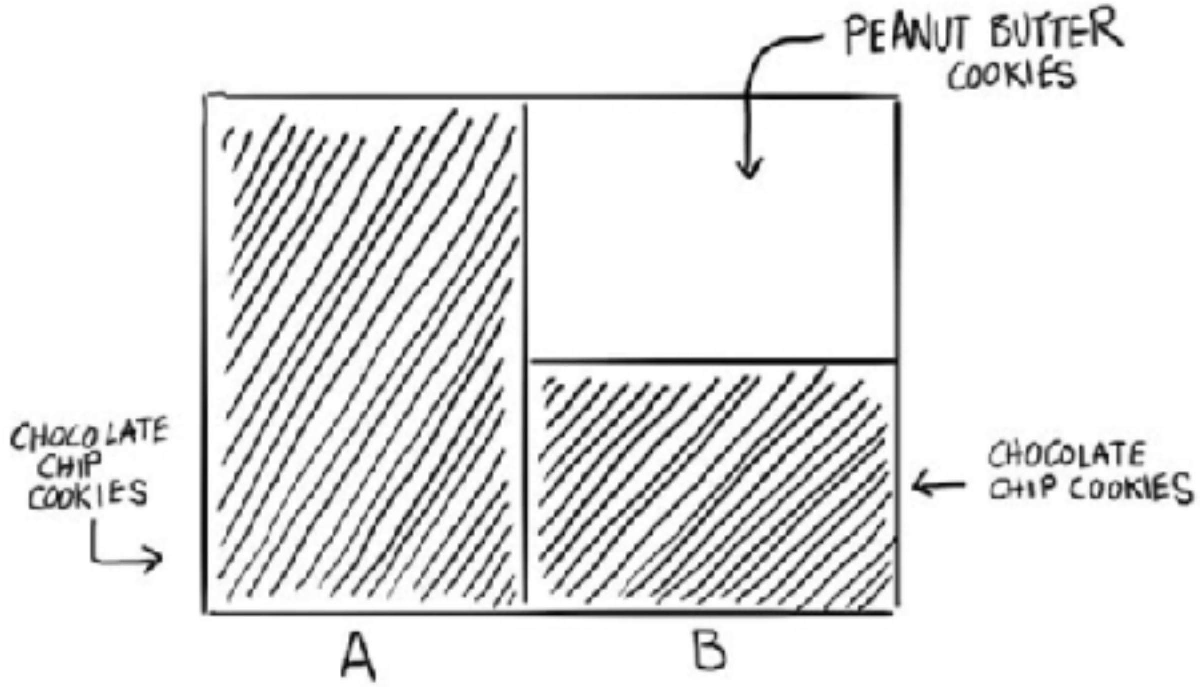
HOW IS PE COMPUTED?



$$\overset{2/3}{P(\text{BOX A} \mid \text{CC COOKIE})} = \frac{\overset{1}{P(\text{CC COOKIE} \mid \text{BOX A})}\ \overset{1/2}{P(\text{BOX A})}}{\underset{3/4}{P(\text{CC COOKIE})}}$$

Es. from Bayes' Theorem:
a visual introduction for beginners

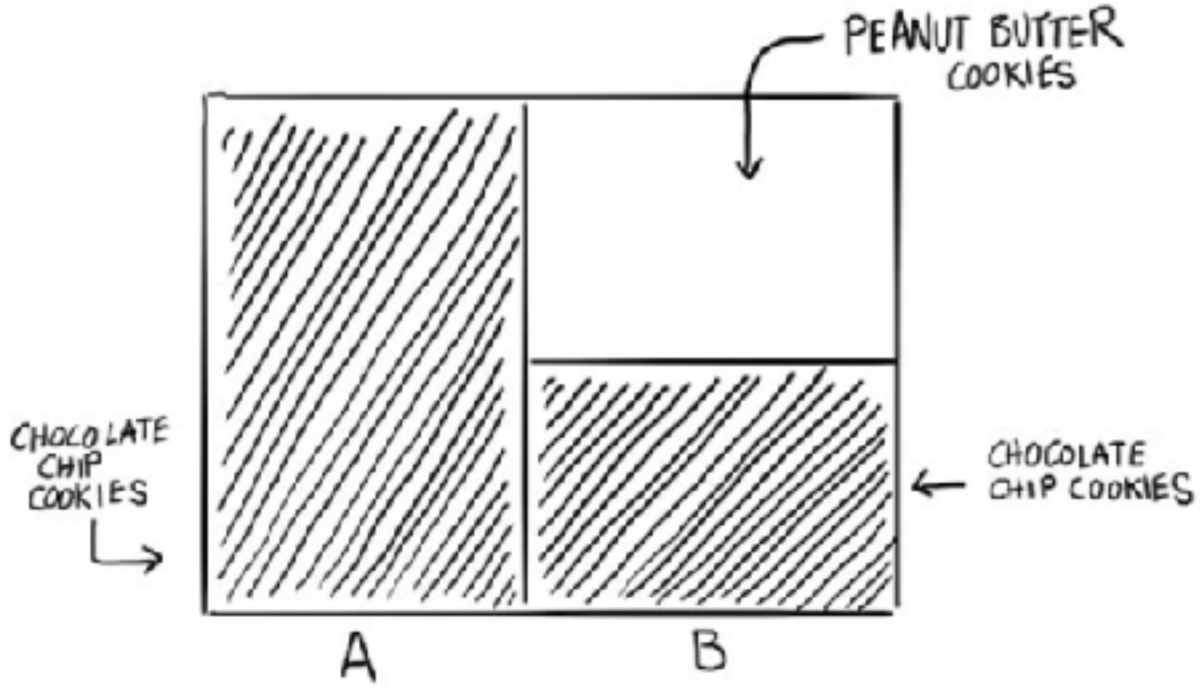
HOW IS PE COMPUTED?



$$\underbrace{P(\text{BOX A} \mid \text{CC COOKIE})}_{\text{Posterior}} = \frac{\overbrace{P(\text{CC COOKIE} \mid \text{BOX A})}^{\text{Likelihood}}\ \overbrace{P(\text{BOX A})}^{\text{Prior}}}{\underbrace{P(\text{CC COOKIE})}_{\text{Evidence}}}$$

Es. from Bayes' Theorem:
a visual introduction for beginners

HOW IS PE COMPUTED?



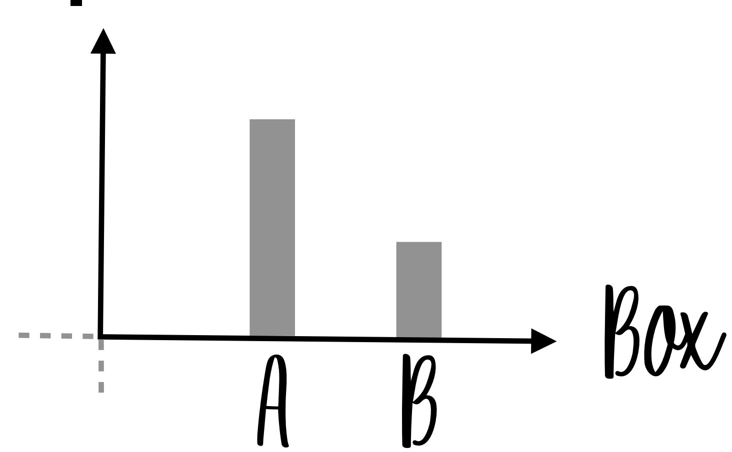
Likelihood Prior

Posterior

$$P(\text{BOX A} \mid \text{CC COOKIE}) = \frac{P(\text{CC COOKIE} \mid \text{BOX A})\, P(\text{BOX A})}{P(\text{CC COOKIE})}$$

Evidence

$P(\text{BOX} \mid \text{cc cookie})$



Es. from Bayes' Theorem:
a visual introduction for beginners

HOW IS PE COMPUTED?

BAYES' THEOREM

Likelihood Prior

Posterior

$$p(\bar{\theta}\,|\,\bar{d},H) = \frac{p(\bar{d}\,|\,\bar{\theta},H)\,p(\bar{\theta}\,|\,H)}{p(\bar{d}\,|\,H)}$$

Evidence

$\bar{d}$: data
$\bar{\theta}$: parameters
H : model

HOW IS PE COMPUTED?

BAYES' THEOREM

Posterior

Likelihood

Prior

$$p(\bar{\theta}\,|\,\bar{d},H) = \frac{p(\bar{d}\,|\,\bar{\theta},H)\,p(\bar{\theta}\,|\,H)}{p(\bar{d}\,|\,H)}$$

Evidence

$\bar{d}$: data
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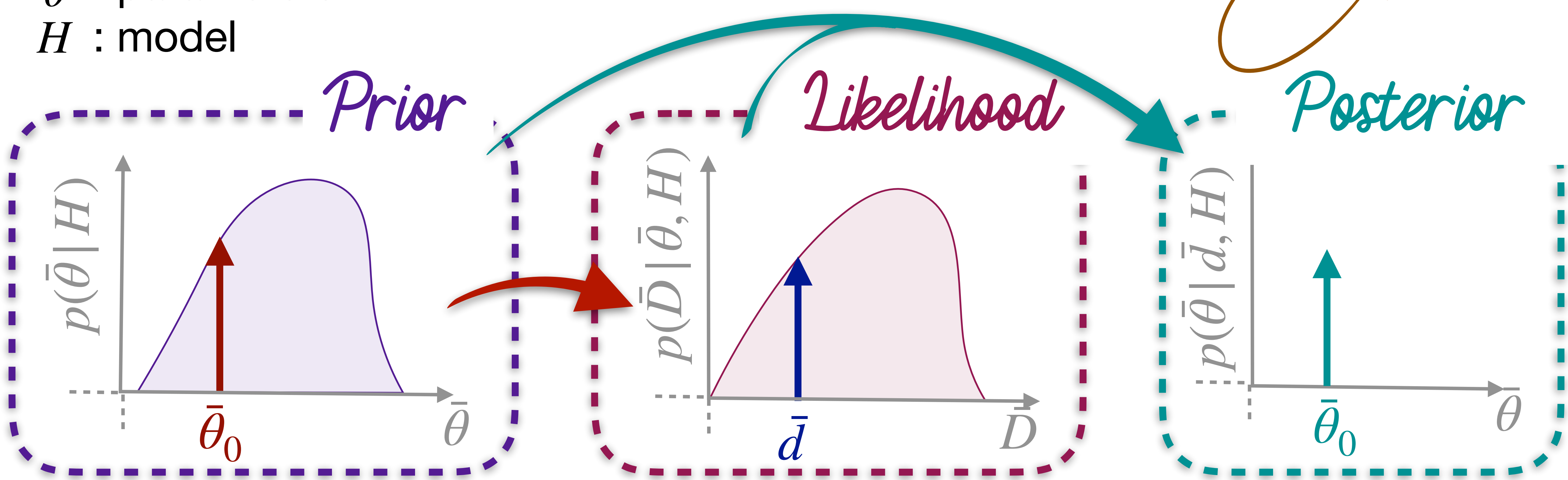

HOW IS PE COMPUTED?

BAYES' THEOREM

Likelihood

Prior

Posterior

$$p(\bar{\theta}\,|\,\bar{d},H)=\frac{p(\bar{d}\,|\,\bar{\theta},H)\,p(\bar{\theta}\,|\,H)}{p(\bar{d}\,|\,H)}$$

Evidence

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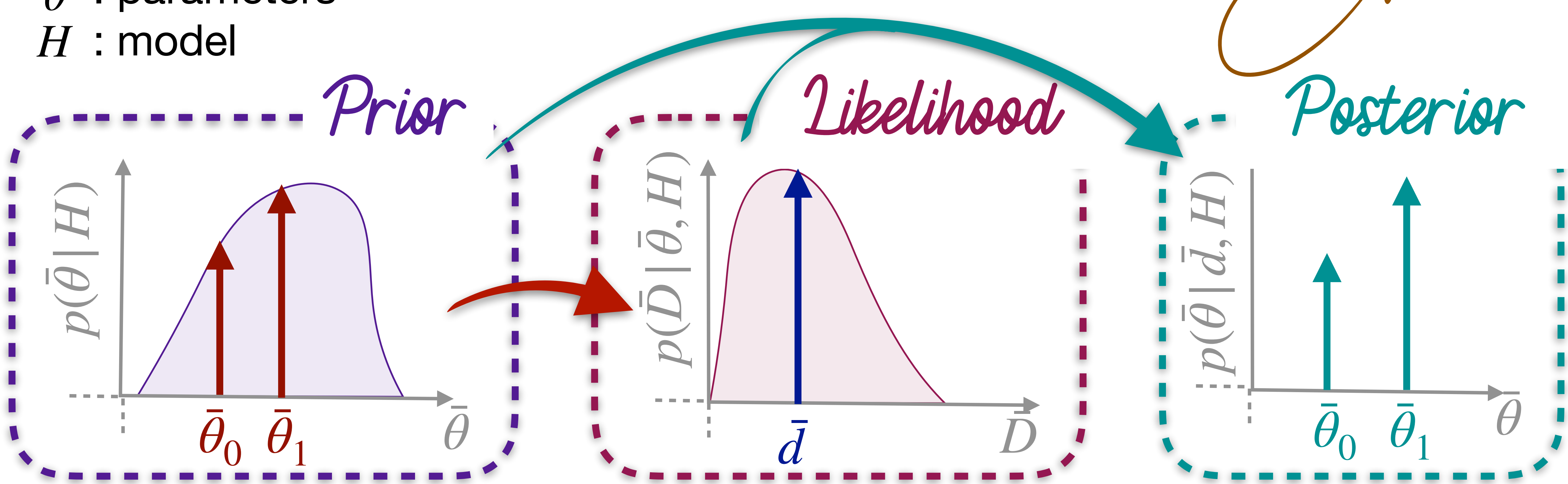
$\bar{\theta}$: parameters
H : model



COMPUTATIONAL CHALLENGE

Assuming GAUSSIAN NOISE:

0 mean; known variance

$$p(\bar{d}\,|\,\bar{\theta}, H) \propto \exp\left[-\delta f \sum_{i=0}^{N} \frac{2\,|\tilde{d}(f_i) - \tilde{h}(\bar{\theta}, f_i)\,|^2}{S_n(f_i)}\right]$$

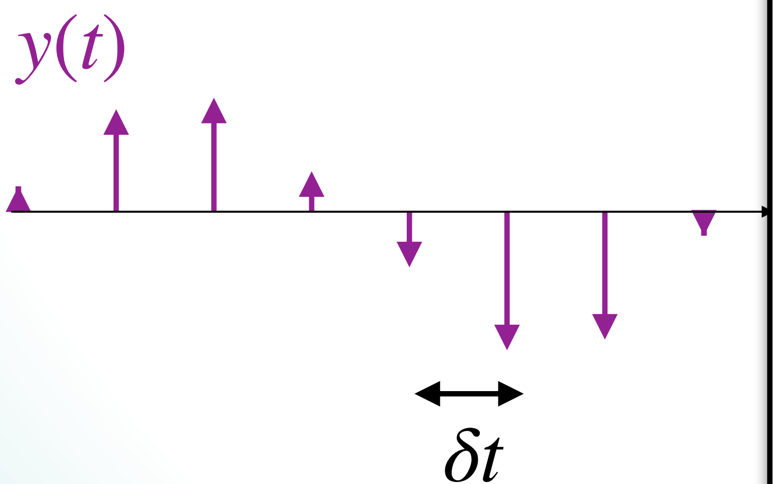
Evaluated $\sim 10^7$ times

$\bar{d}$: data
$\bar{\theta}$: parameters
H_s : model assuming signal
h : GW waveform
S_n : Power spectral density
f : frequency
~ : in f-domain

$$N = \frac{f_{max} - f_{min}}{\delta f}$$

NYQUIST THEOREM

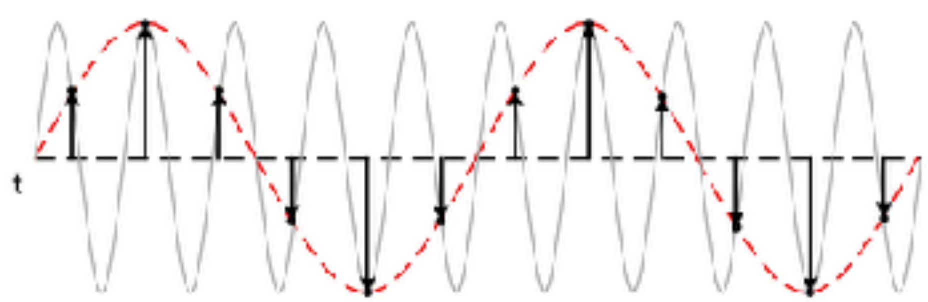
time domain

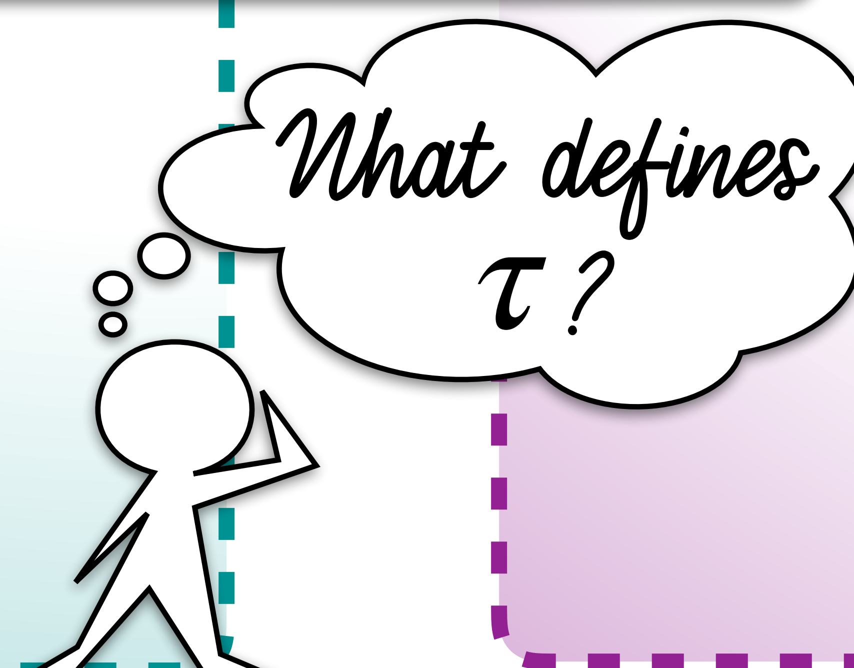


~ 2 samples per cycle

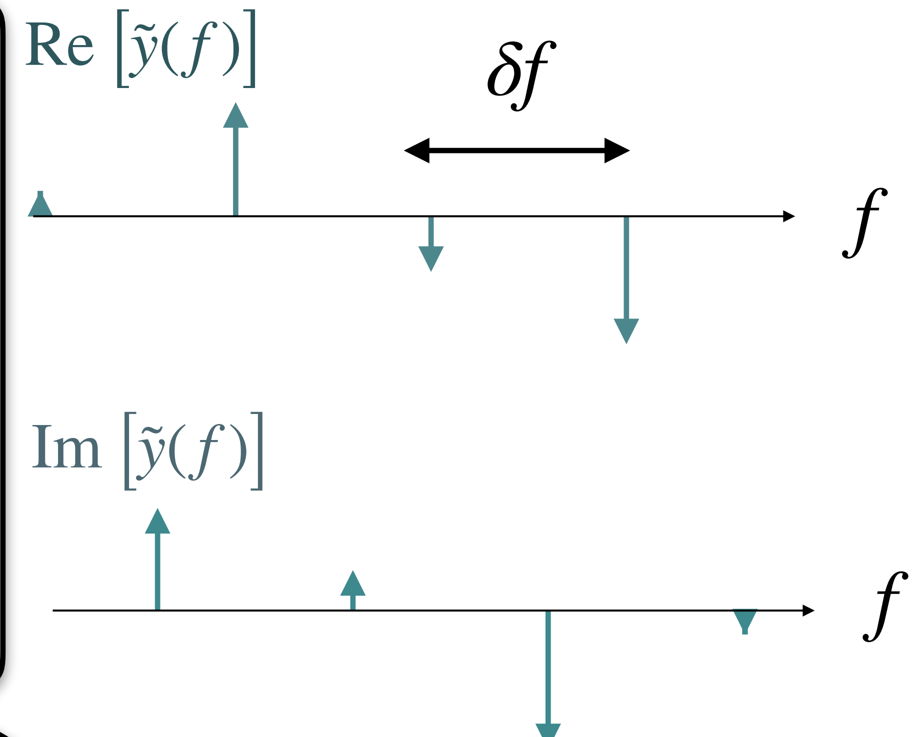
$$\delta t \leq (2f_{max})^{-1}$$

To avoid aliasing





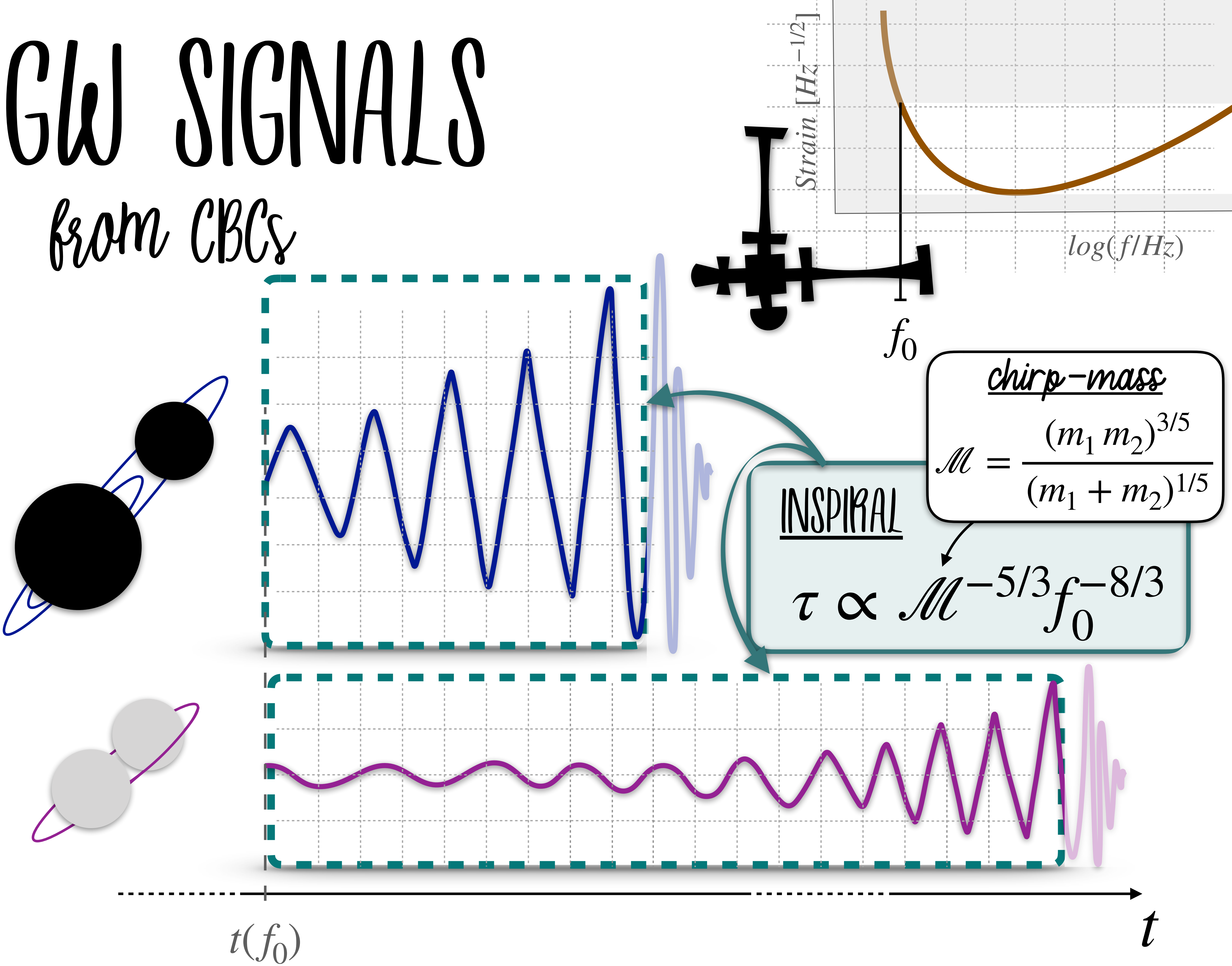
frequency domain

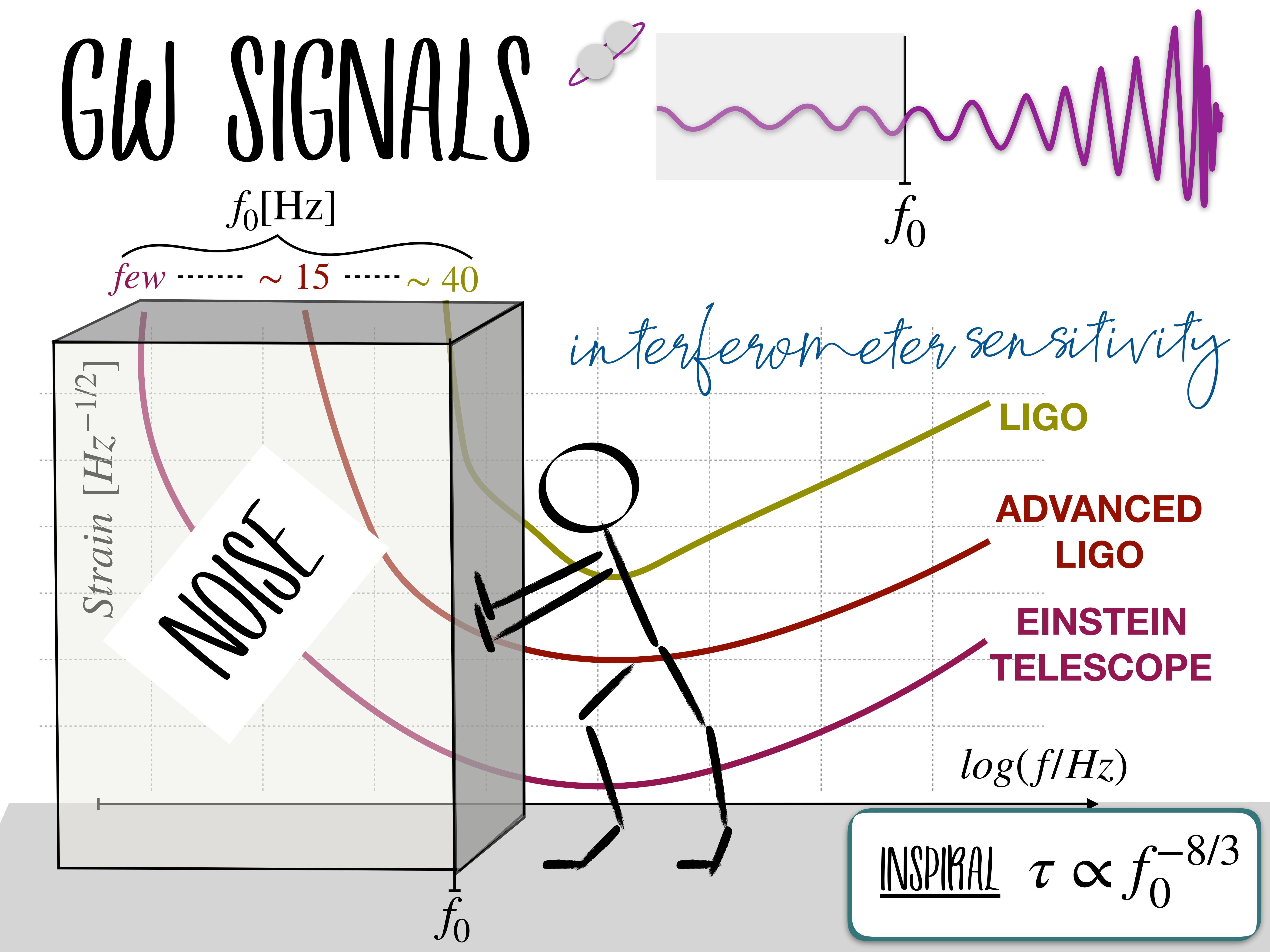


~ 1 sample per cycle

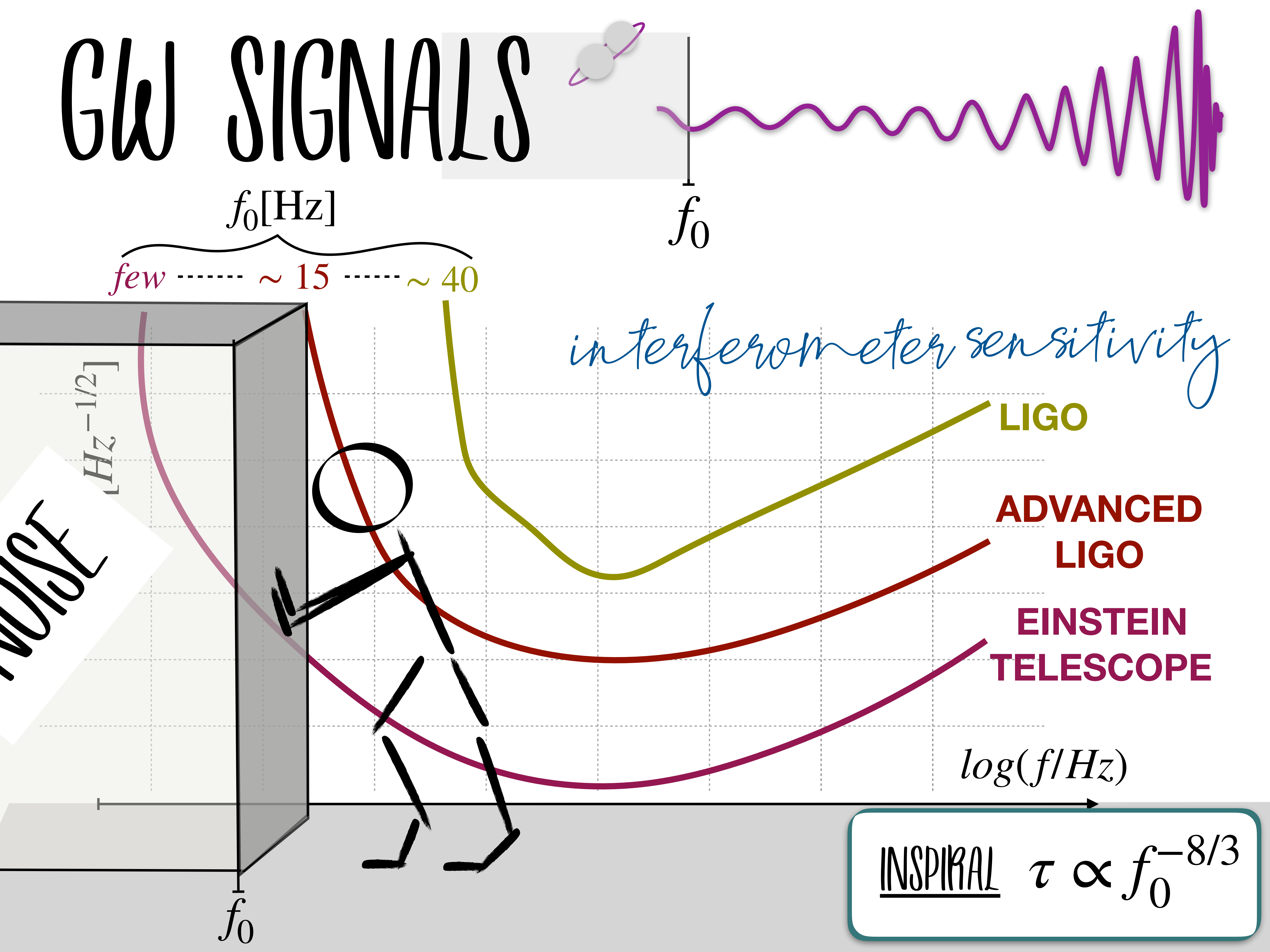
$$\delta f \leq \tau^{-1}$$

GW SIGNALS
from CBCs





GW SIGNALS
f_0
f_0[Hz]
few ------ ~ 15 ------ ~ 40
interferometer sensitivity
LIGO
ADVANCED LIGO
EINSTEIN TELESCOPE
Strain [$Hz^{-1/2}$]
NOISE
log(f/Hz)
f_0
INSPIRAL $\tau \propto f_0^{-8/3}$


GW SIGNALS
f_0
$f_0[\mathrm{Hz}]$
few ------- ~ 15 ------- ~ 40
interferometer sensitivity
LIGO
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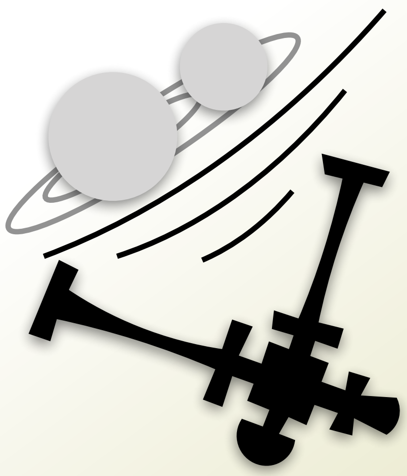
$[Hz^{-1/2}]$
NOISE
f_0
$log(f/Hz)$
INSPIRAL $\tau \propto f_0^{-8/3}$

HOW CAN WE OVERCOME THIS PROBLEM?

CONTEXT

MULTI-BANDING

DETECTION PIPELINE



gstlal

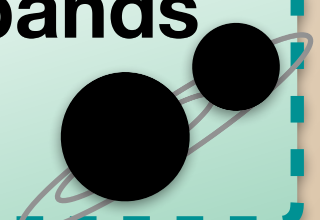
K. Cannon et al. 2012

MBTA

T. Adams et al. 2016

PE for LISA with 2 bands

E. K. Porter et al. 2014



MAIN PE ACCELERATION TECHNIQUE

REDUCED ORDER QUADRATURE

ROQ

$$L \propto \exp\left\{ 4\delta f \sum_{i=0}^{M} c_i(\bar{\theta})\bar{w}_i \right\}$$

$$\bar{w}_i = 4\delta f \sum_{k=0}^{M} \frac{\tilde{d}(f_k)B_i^*(f_k)}{S_n(f_k)}$$

Very large speed up factors

(~300 from f_0 ~20 Hz)

e.g.:

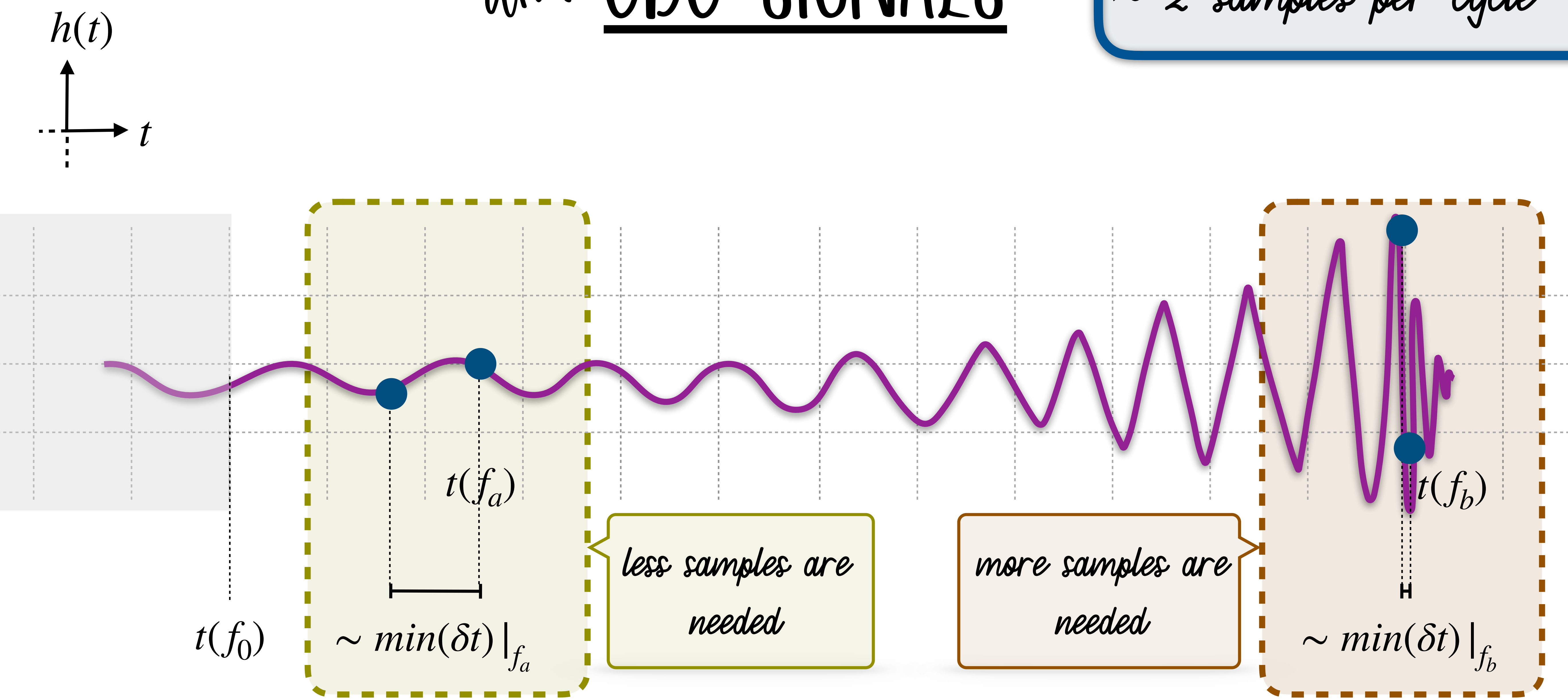
Canizares et al. 2013;
Canizares et al. 2015;
Smith et al. 2016

EFFECTIVE SAMPLING

NYQUIST THEOREM and CBC SIGNALS

$$\delta t \leq (2f_{max})^{-1}$$

$\sim$ 2 samples per cycle

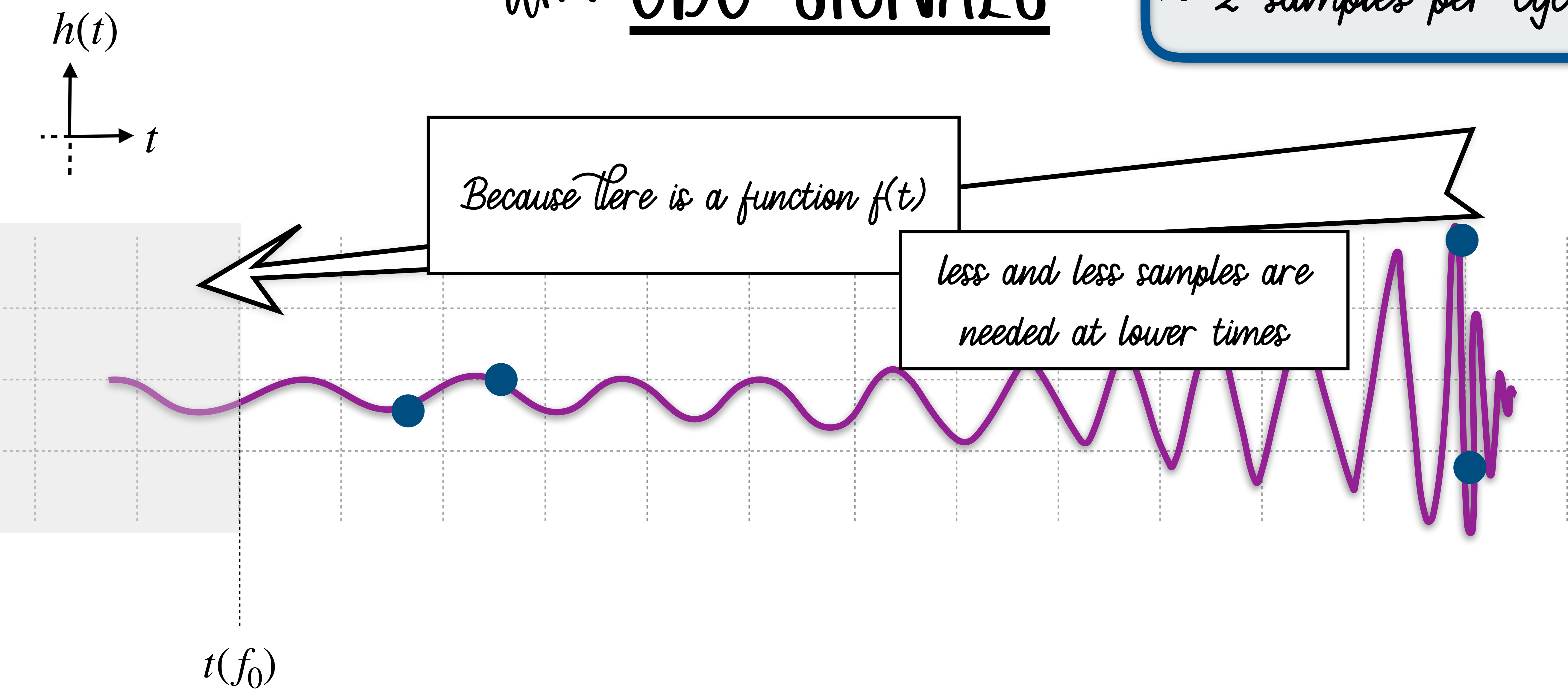


EFFECTIVE SAMPLING

NYQUIST THEOREM and CBC SIGNALS

$$\delta t \leq (2f_{max})^{-1}$$

$\sim$ 2 samples per cycle

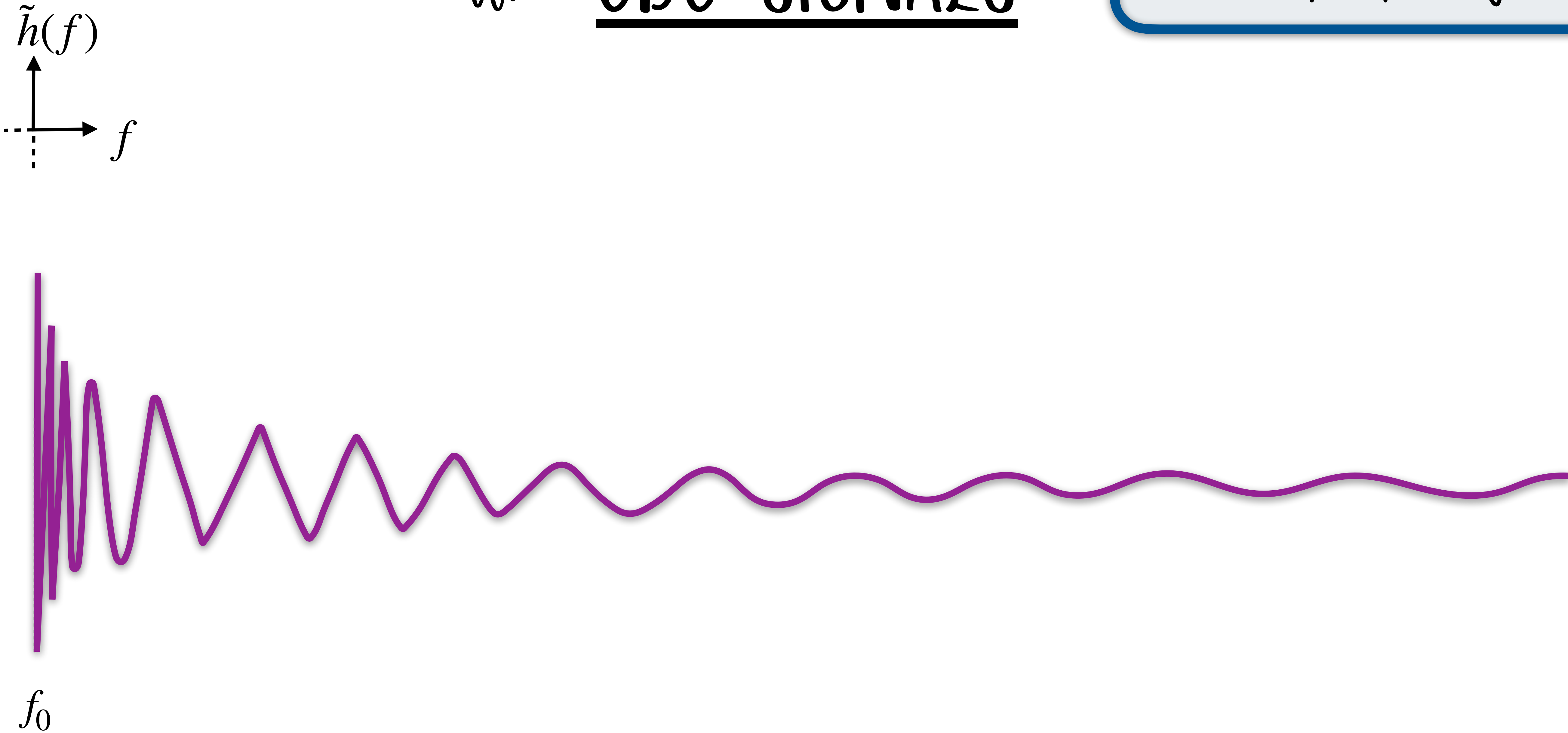


EFFECTIVE SAMPLING

NYQUIST THEOREM and CBC SIGNALS

$$\delta f \leq \tau^{-1}$$

$\sim$ 1 sample per cycle

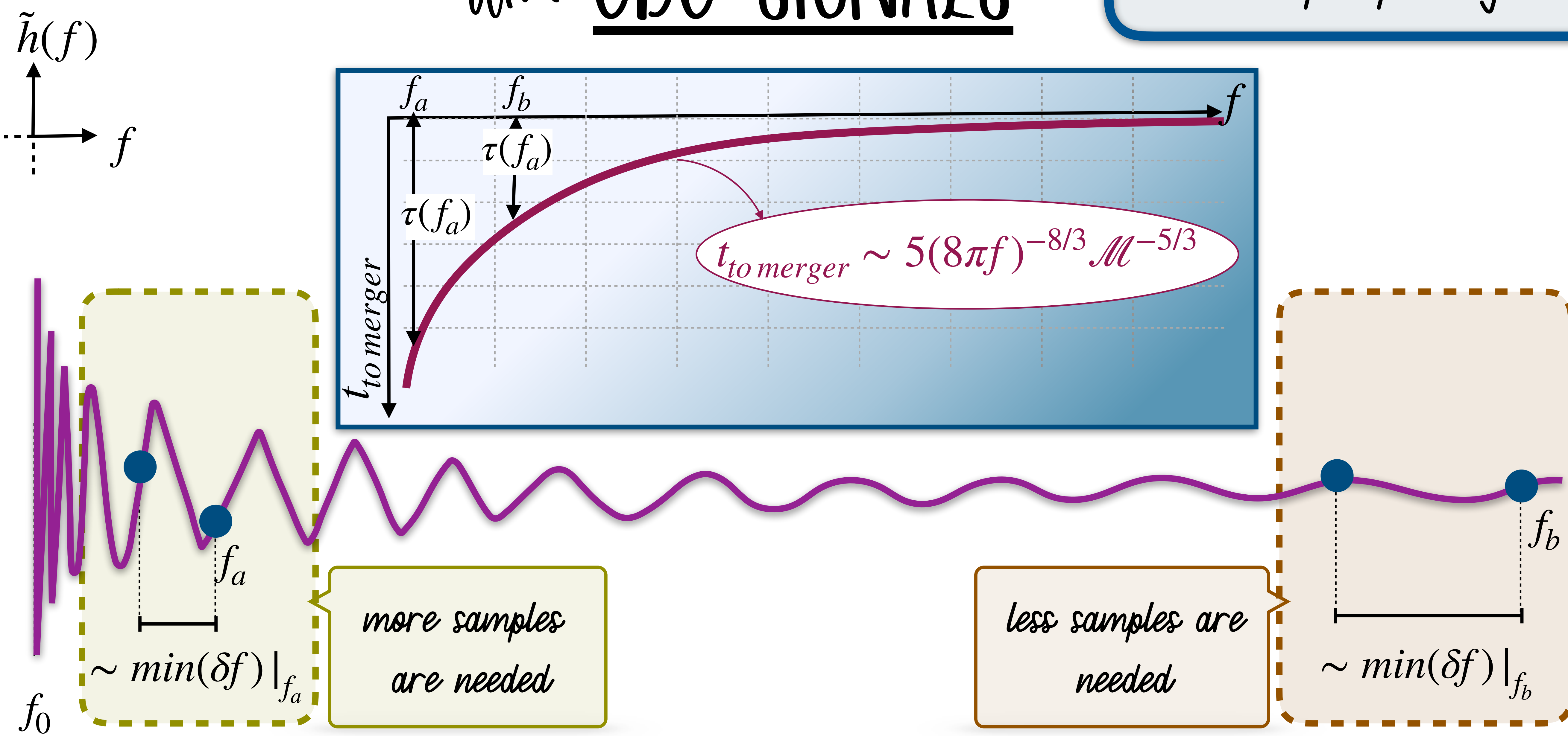


EFFECTIVE SAMPLING

NYQUIST THEOREM and CBC SIGNALS

$$\delta f \leq \tau^{-1}$$

$\sim$ *1 sample per cycle*

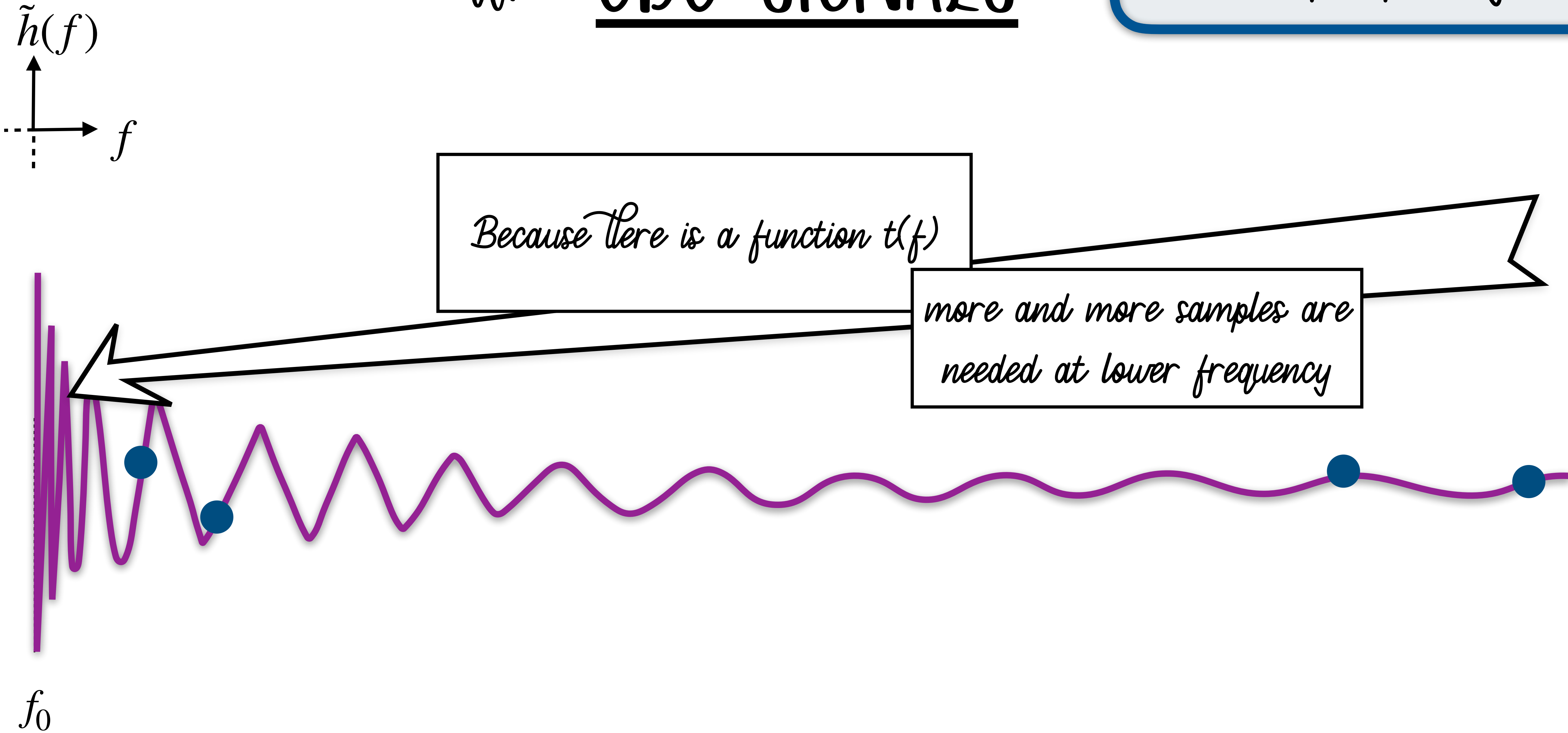


EFFECTIVE SAMPLING

NYQUIST THEOREM and CBC SIGNALS

$$\delta f \leq \tau^{-1}$$

$\sim$ 1 sample per cycle



POSSIBLE IMPROVEMENTS

$$N_{fix} = \int_{f_0}^{f_{max}} \delta f^{-1} \, df$$

$$N_{fix} \propto (f_{max} - f_0)\mathscr{M}^{-5/3} f_0^{-8/3}$$

$$\tau \sim t_{to\,merger}(f_0)$$

$$\sim 5\left(8\pi f\right)^{-8/3} \mathscr{M}^{-5/3}$$

$$\frac{N_{fix}}{N_{min}} \sim \frac{5}{3}\frac{f_{max}}{f_0}$$

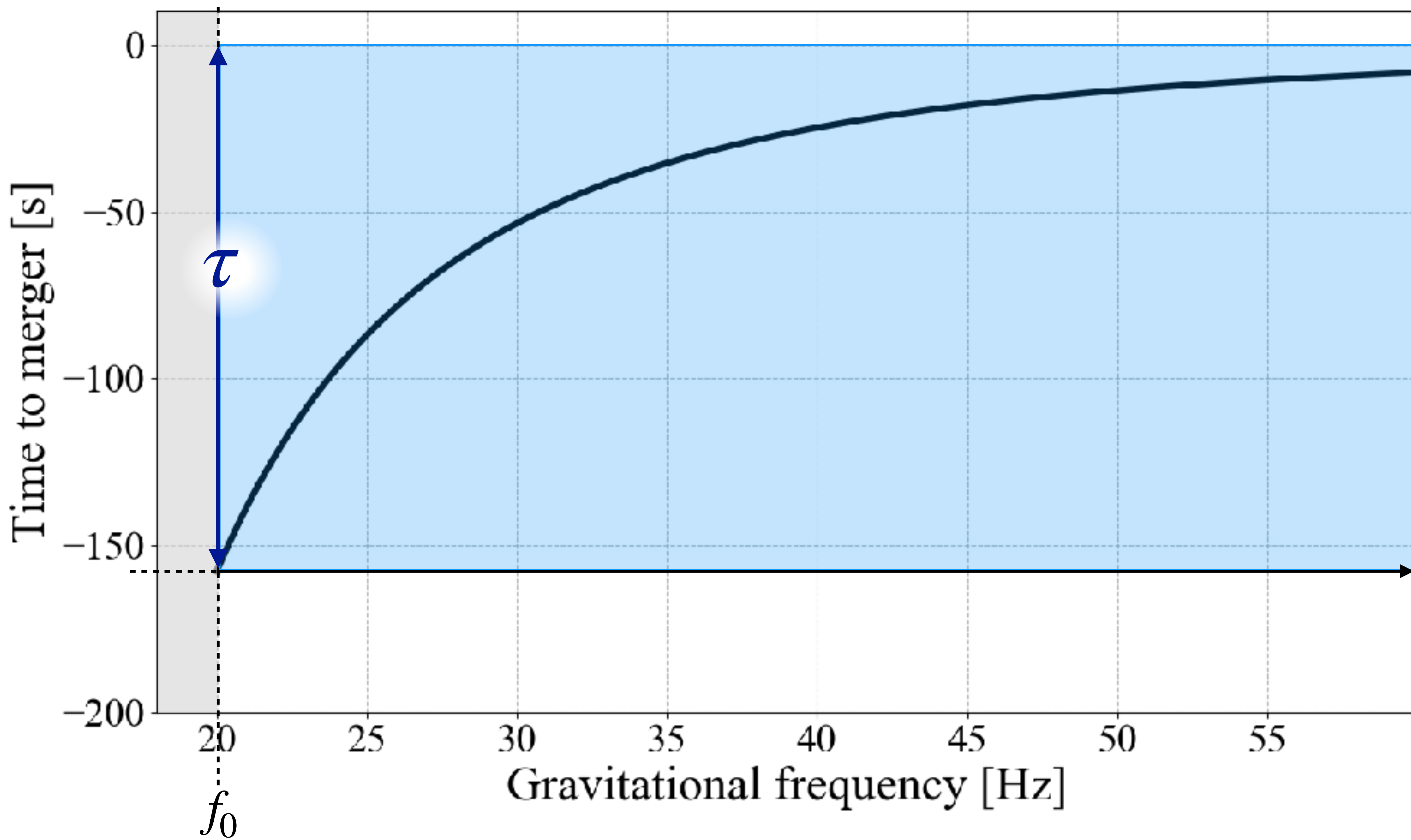
$$N_{min} = \int_{f_0}^{f_{max}} t_{to\,merger}(f) \, df$$

$$N_{min} \propto \mathscr{M}^{-5/3}(f_{max}^{5/3} - f_0^{-5/3})$$

Multi-banding

NYQUIST THEOREM

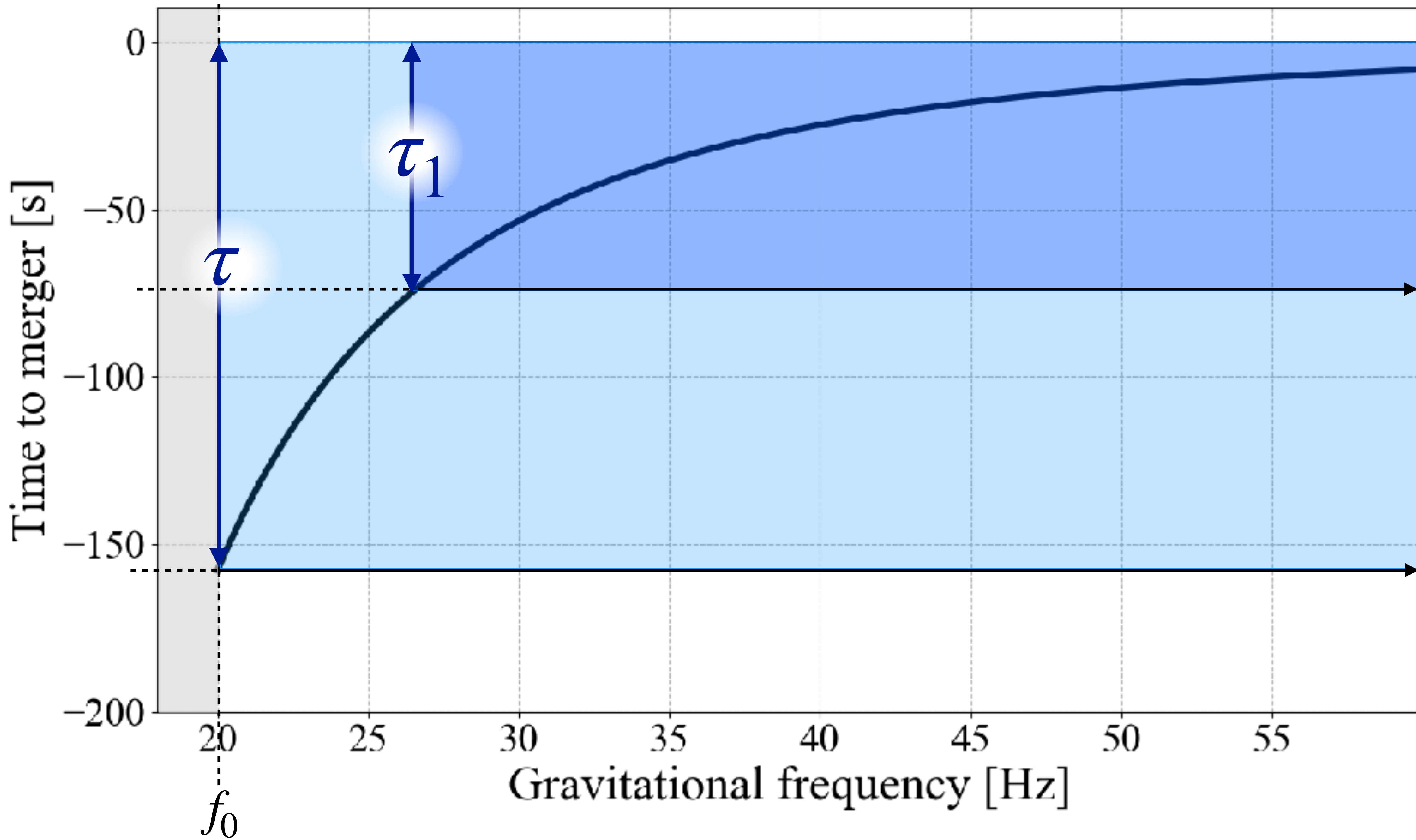
$$\delta f \leq \tau^{-1}$$



Multi-banding

NYQUIST THEOREM

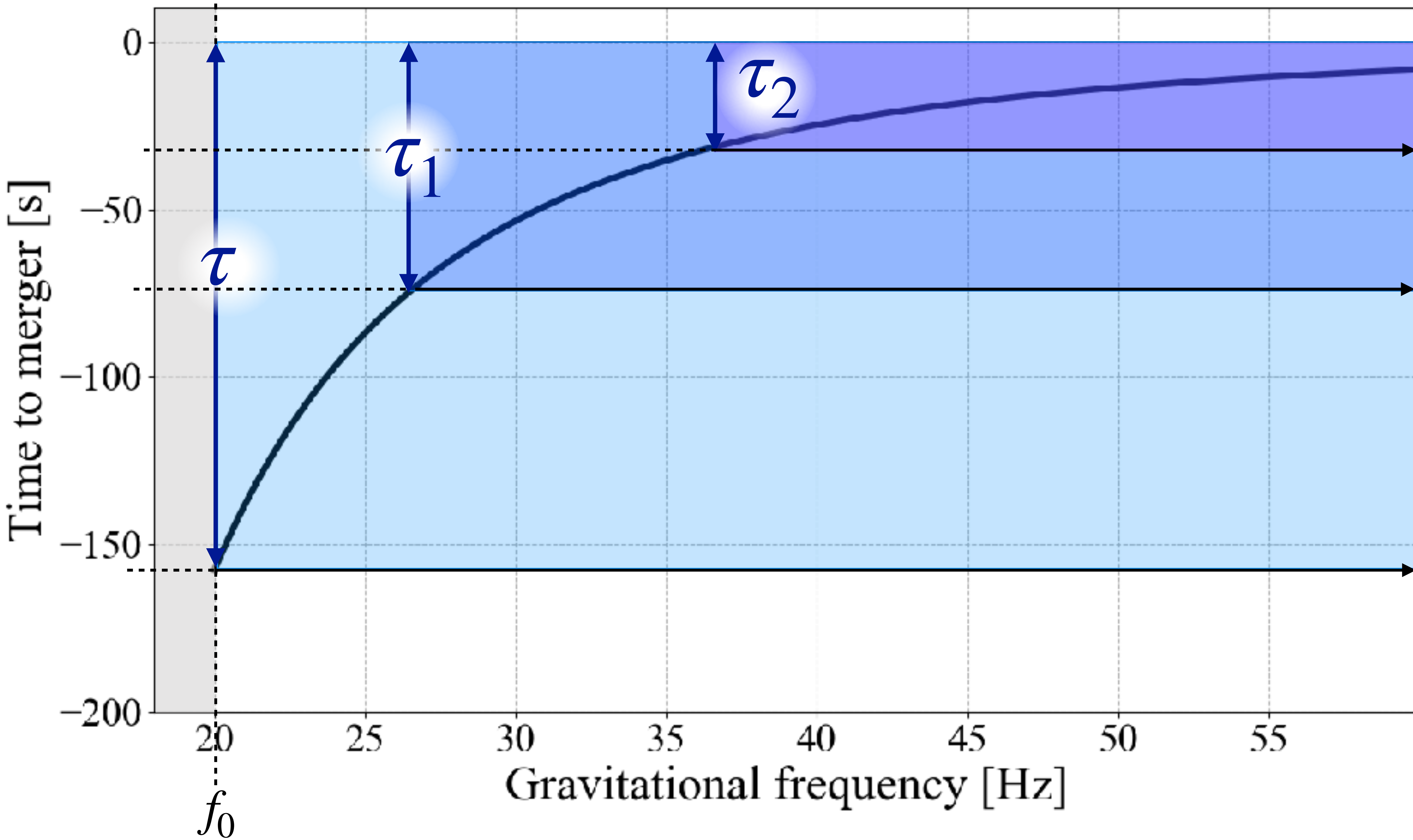
$$\delta f \le \tau^{-1}$$



Multi-banding

NYQUIST THEOREM

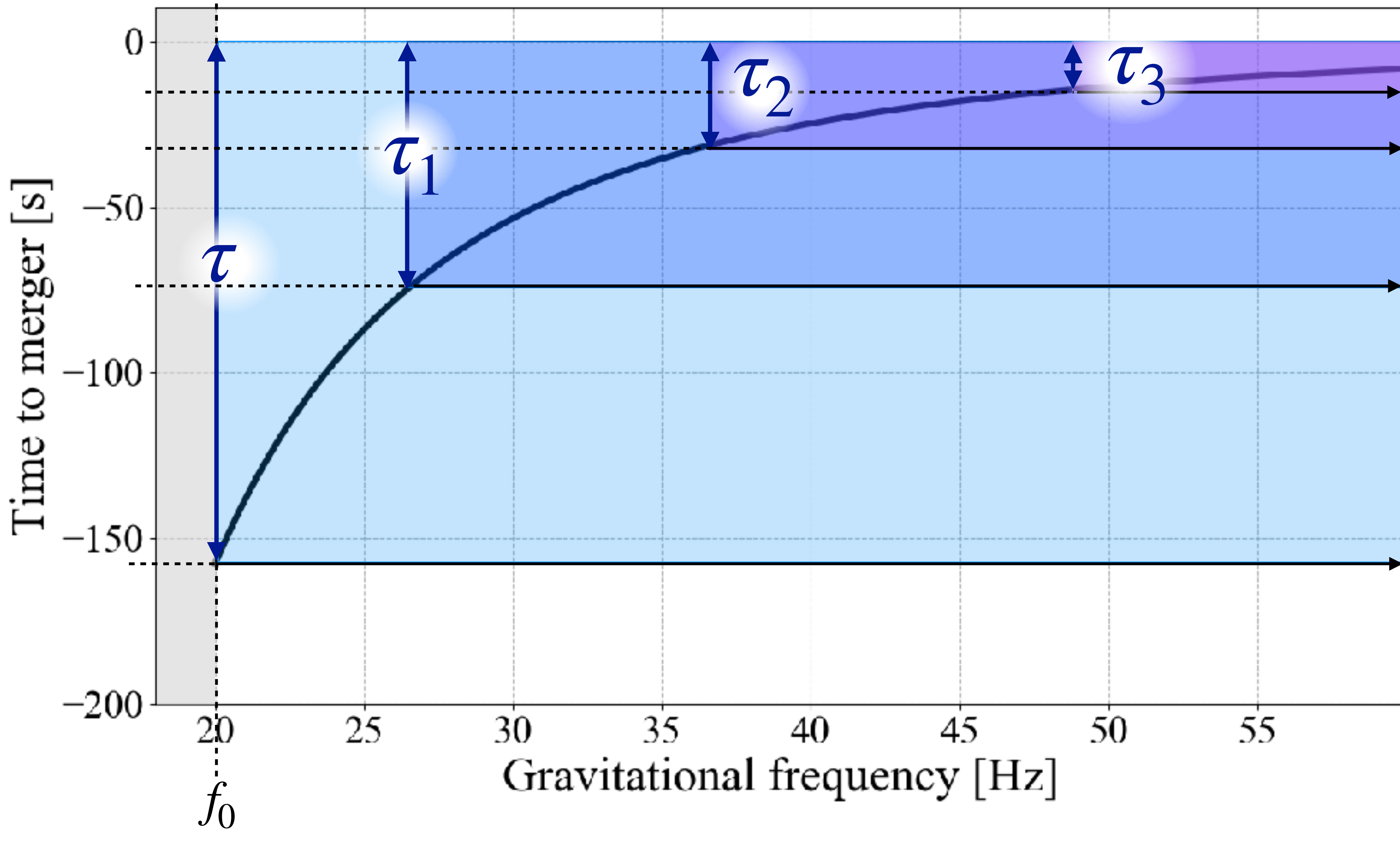
$$\delta f \leq \tau^{-1}$$



Multi-banding

NYQUIST THEOREM

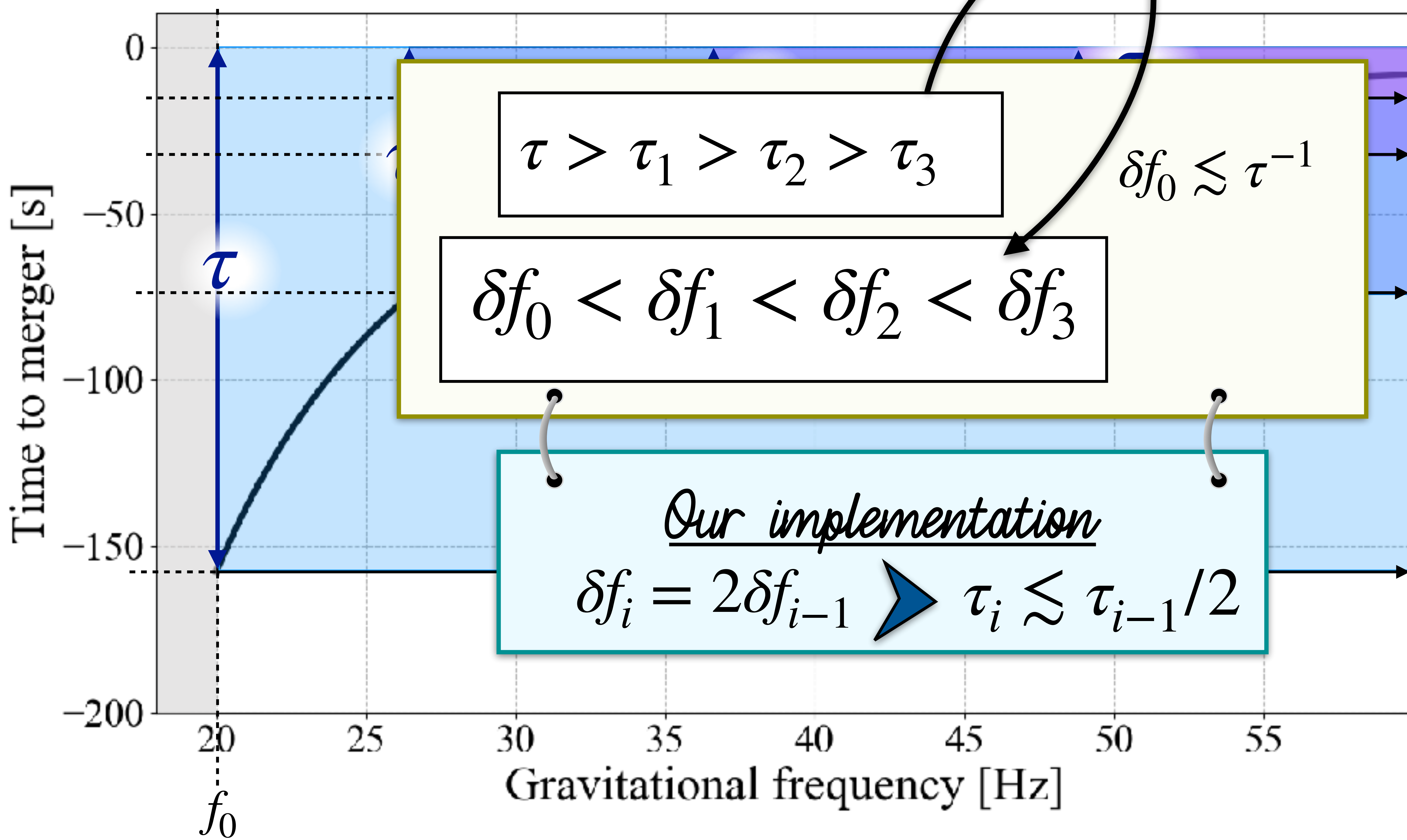
$$\delta f \leq \tau^{-1}$$



Multi-banding

NYQUIST THEOREM

$\delta f \leq \tau^{-1}$



$\tau > \tau_1 > \tau_2 > \tau_3$

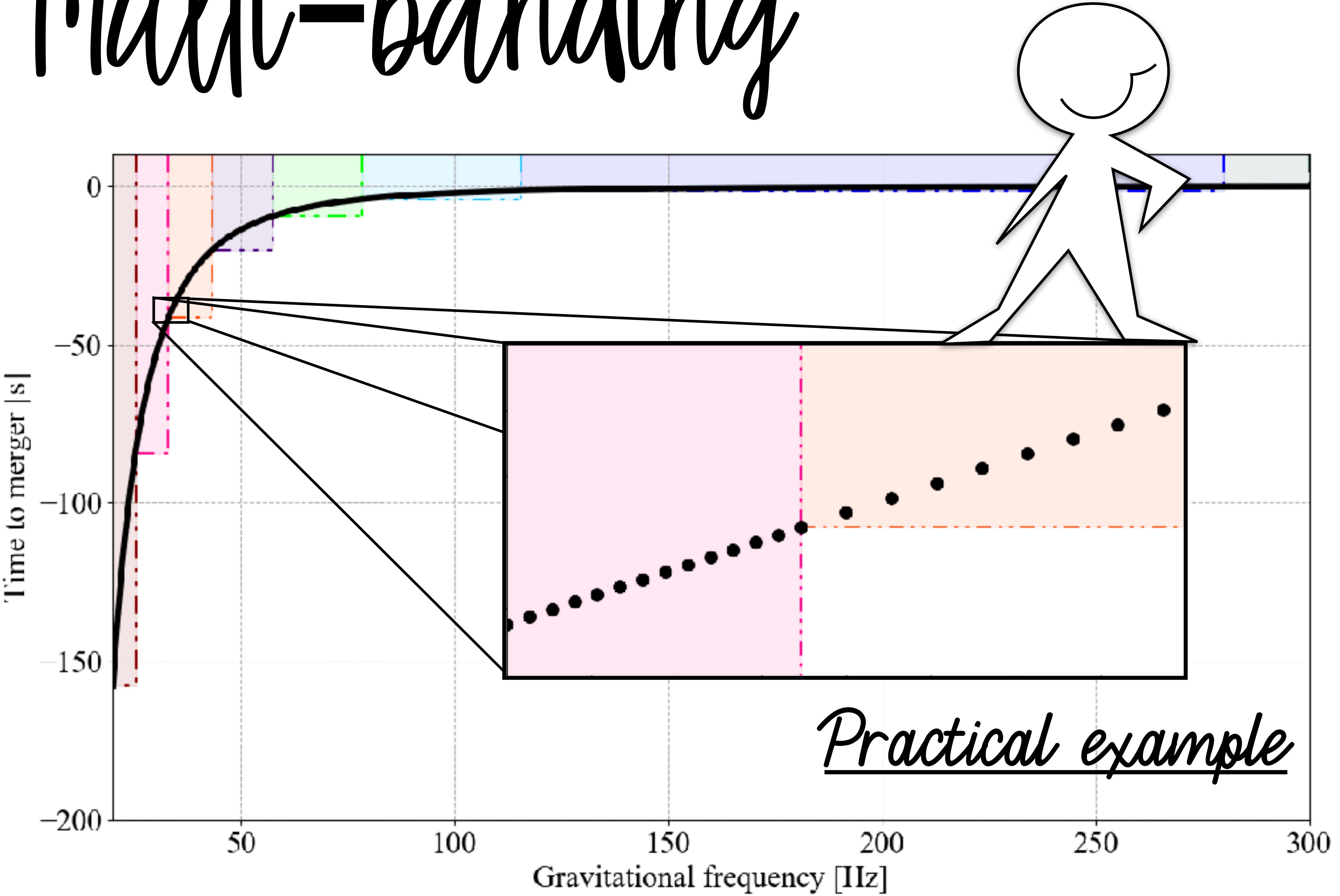
$\delta f_0 < \delta f_1 < \delta f_2 < \delta f_3$

$\delta f_0 \lesssim \tau^{-1}$

Our implementation

$\delta f_i = 2\delta f_{i-1}$ ➤ $\tau_i \lesssim \tau_{i-1}/2$

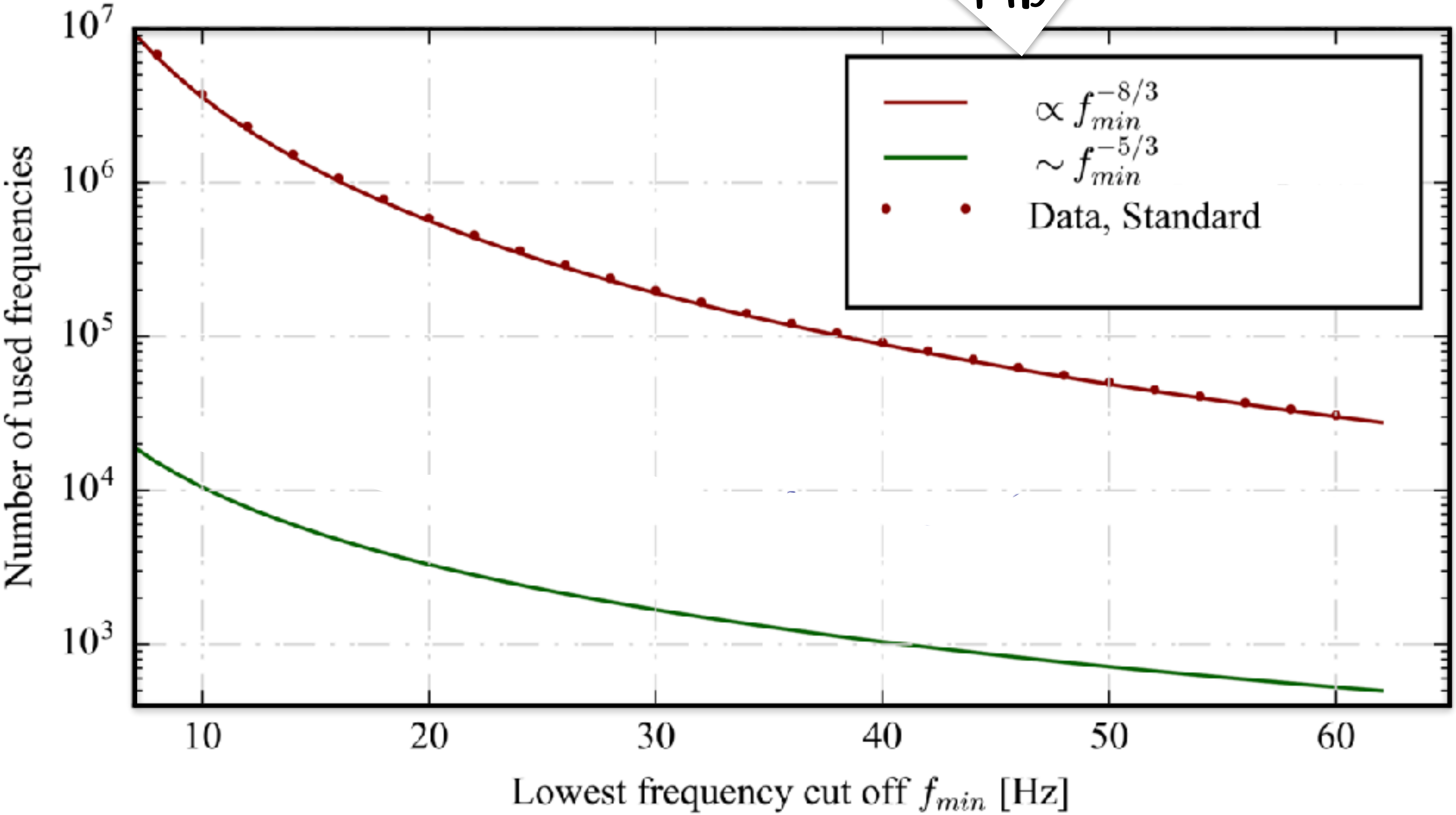
Multi-banding



Practical example

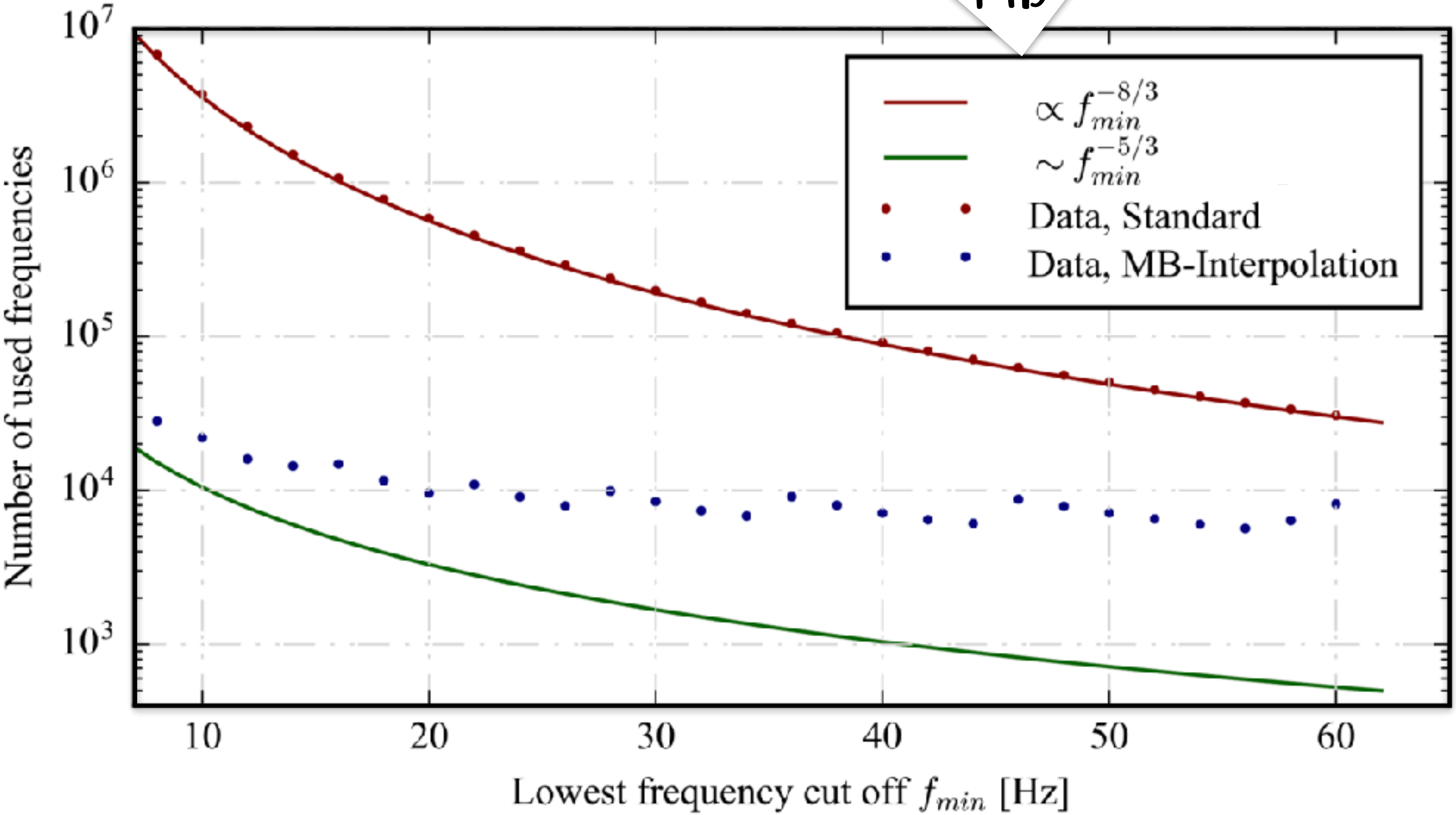
Multi-banding

MB



Multi-banding

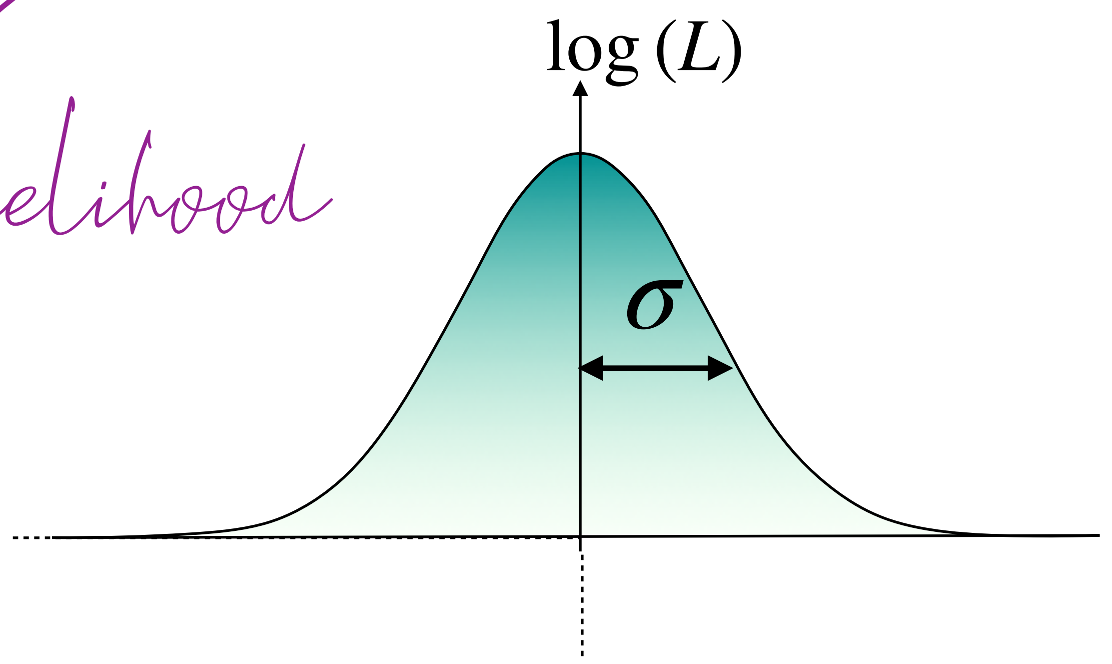
MB



ACCURACY REQUIRED FOR PE

$$\delta \log\left(L_{MB}\right) \ll \sigma_{\log\ L}$$

Likelihood



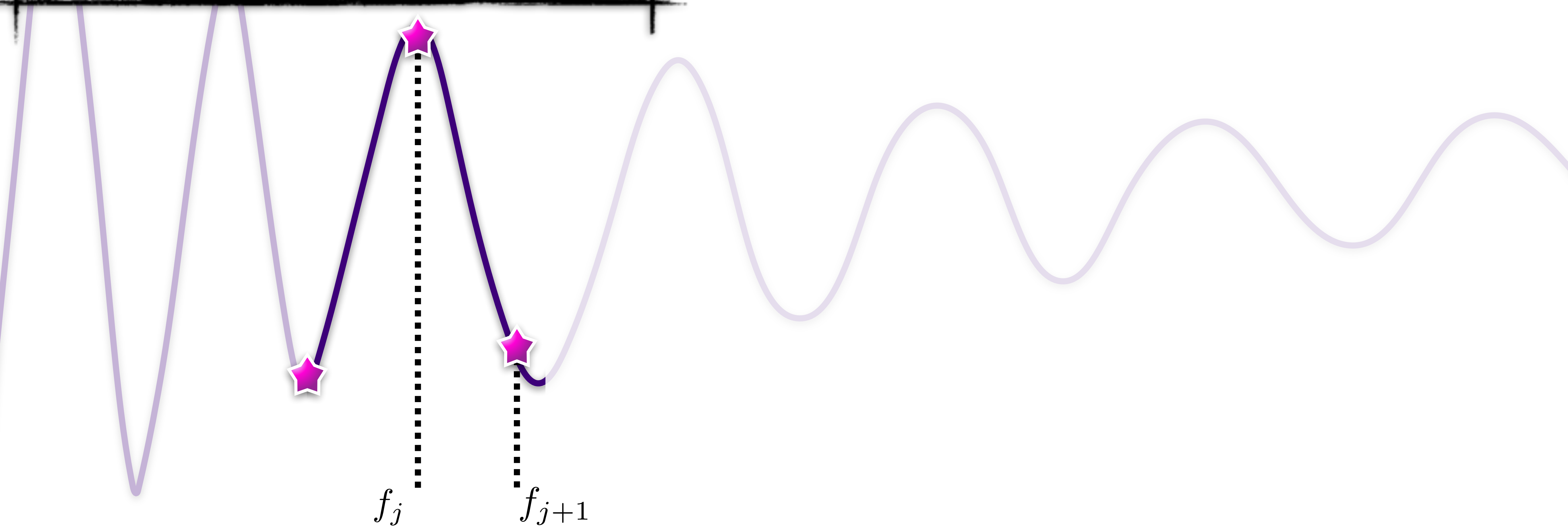
WHAT TO DO TO REACH THE REQUIRED ACCURACY?

WHAT TO DO TO REACH THE REQUIRED ACCURACY?

INTERPOLATION

INT

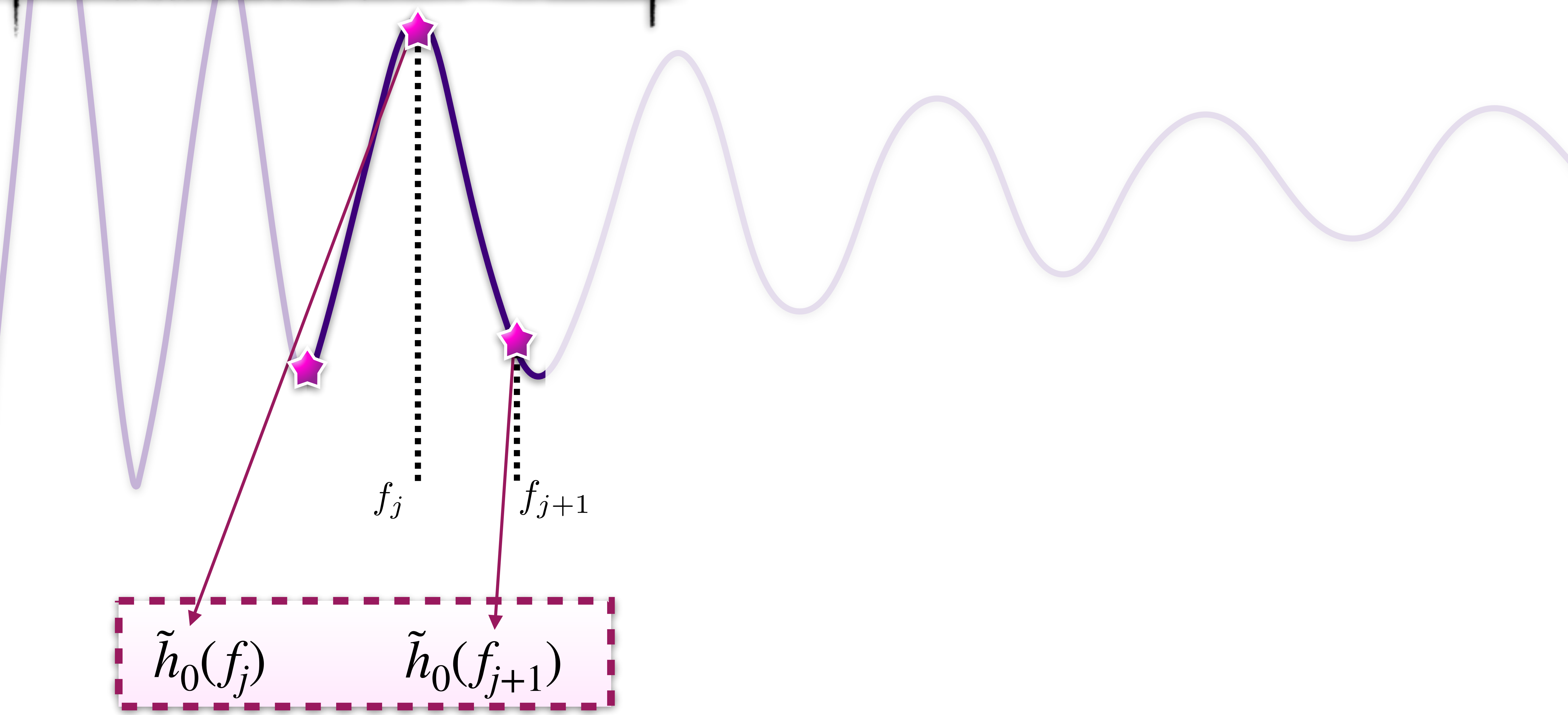
$$\tilde{h}(f) = A(\theta, f)e^{-i\psi(\theta, f)}$$



INTERPOLATION

INT

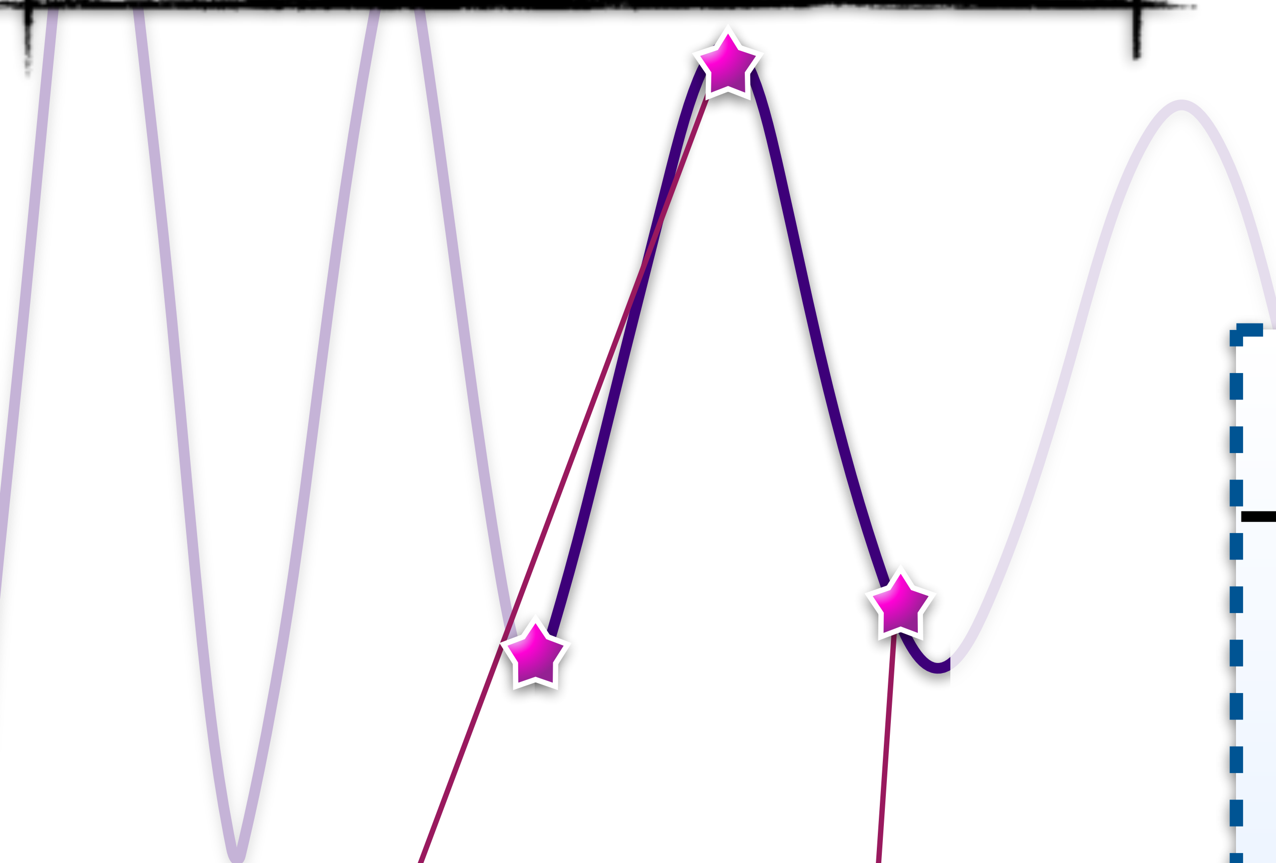
$$\tilde{h}(f) = A(\theta, f)e^{-i\psi(\theta, f)}$$



INTERPOLATION

INT

$$\tilde{h}(f) = A(\theta, f)e^{-i\psi(\theta, f)}$$



$\tilde{h}_0(f_j)$ $\quad$ $\tilde{h}_0(f_{j+1})$

IN

$$g(f) = \psi(f), A(f)$$

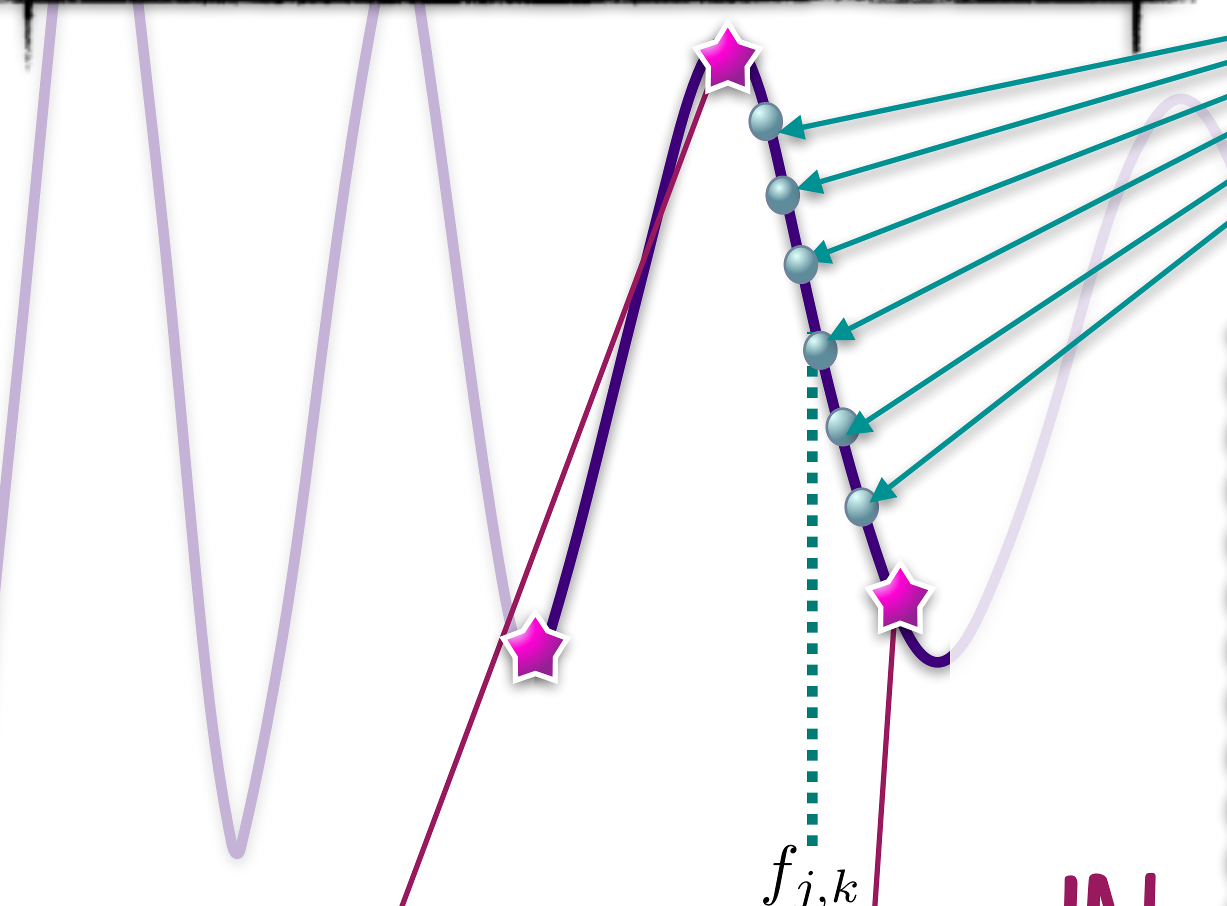
Linear interpolation $\Delta g(f) = \int_{f_j}^{f} \frac{dg}{d\hat{f}} d\hat{f}$

$$\overbrace{g(f_j + k\delta f_0)}^{g_{j,k}} \simeq g(f_j) + \overbrace{\frac{g(f_{j+1}) - g(f_j)}{f_{j+1} - f_j} k\delta f_0}^{\Delta g_{j,k}}$$

INTERPOLATION

INT

$$\tilde{h}(f) = A(\theta, f)e^{-i\psi(\theta, f)}$$



IN

$\tilde{h}_0(f_j)$ $\tilde{h}_0(f_{j+1})$

$$g(f) = \psi(f), A(f)$$

<u>Linear interpolation</u> $\Delta g(f) = \int_{f_j}^{f} \frac{dg}{d\hat{f}} d\hat{f}$

$$\overbrace{g(f_j + k\delta f_0)}^{g_{j,k}} \simeq g(f_j) + \overbrace{\frac{g(f_{j+1}) - g(f_j)}{f_{j+1} - f_j} k\delta f_0}^{\Delta g_{j,k}}$$

OUT

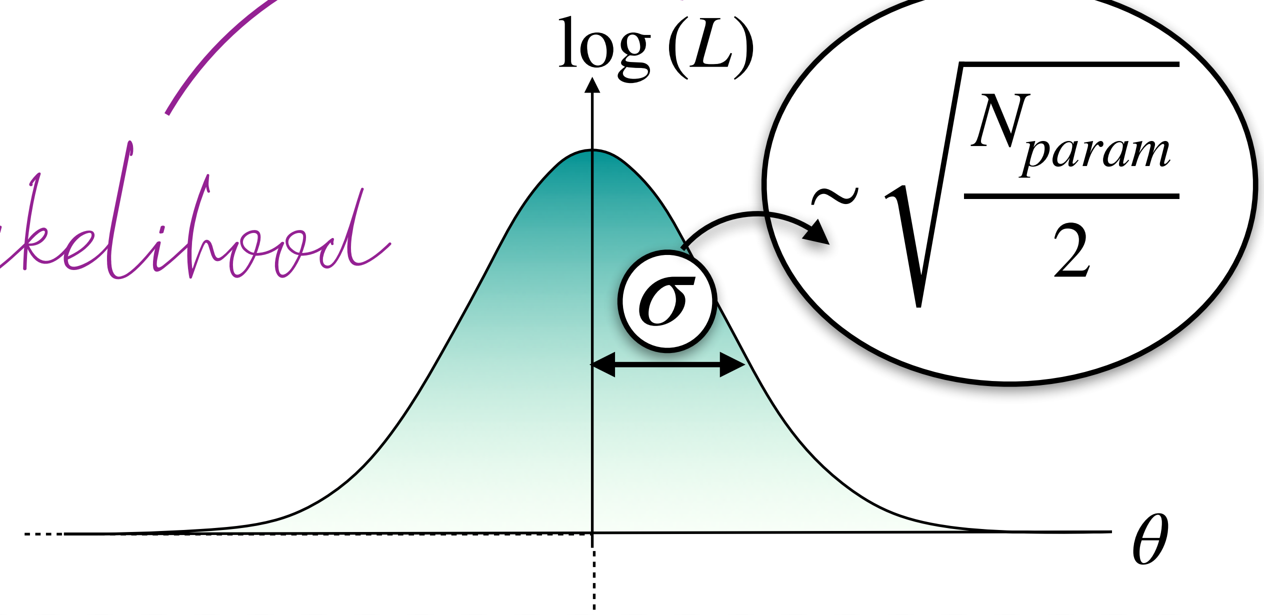
$$\tilde{h}_{j,k} \sim \tilde{h}_0(f_k)$$

$$\tilde{h}_{j,k} = (A_j + \Delta A_{j,k})e^{-i(\psi_j + \Delta\psi_{j,k})}$$

ACCURACY REQUIRED FOR PE

$$\delta \log\left(L_{MB}\right) \ll \sigma$$

Likelihood



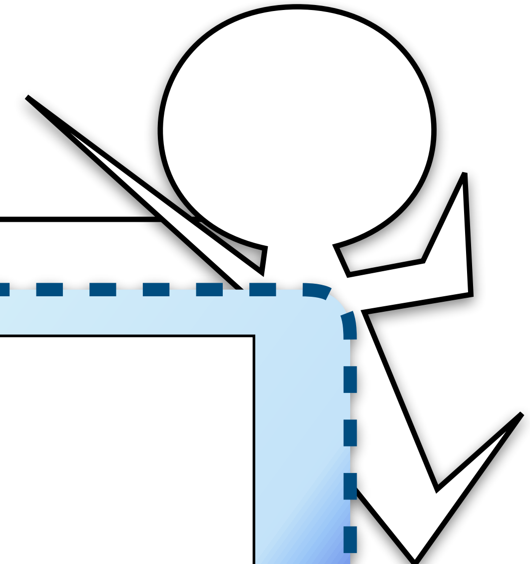
$$\frac{\langle h-h_0 \,|\, h-h_0\rangle}{\langle h_0 \,|\, h_0\rangle} \ll \frac{\sqrt{2N_{param}}}{SNR^2} \sim 10^{-3}$$

Moderately loud signals
$SNR \sim \text{few} \times 10$

inner product

$$\langle a \,|\, b\rangle \sim 2\delta f \sum_{k>0} \frac{\tilde{a}^*(f_k)\tilde{b}(f_k) + \tilde{b}^*(f_k)\tilde{a}(f_k)}{S_n(f_k)}$$

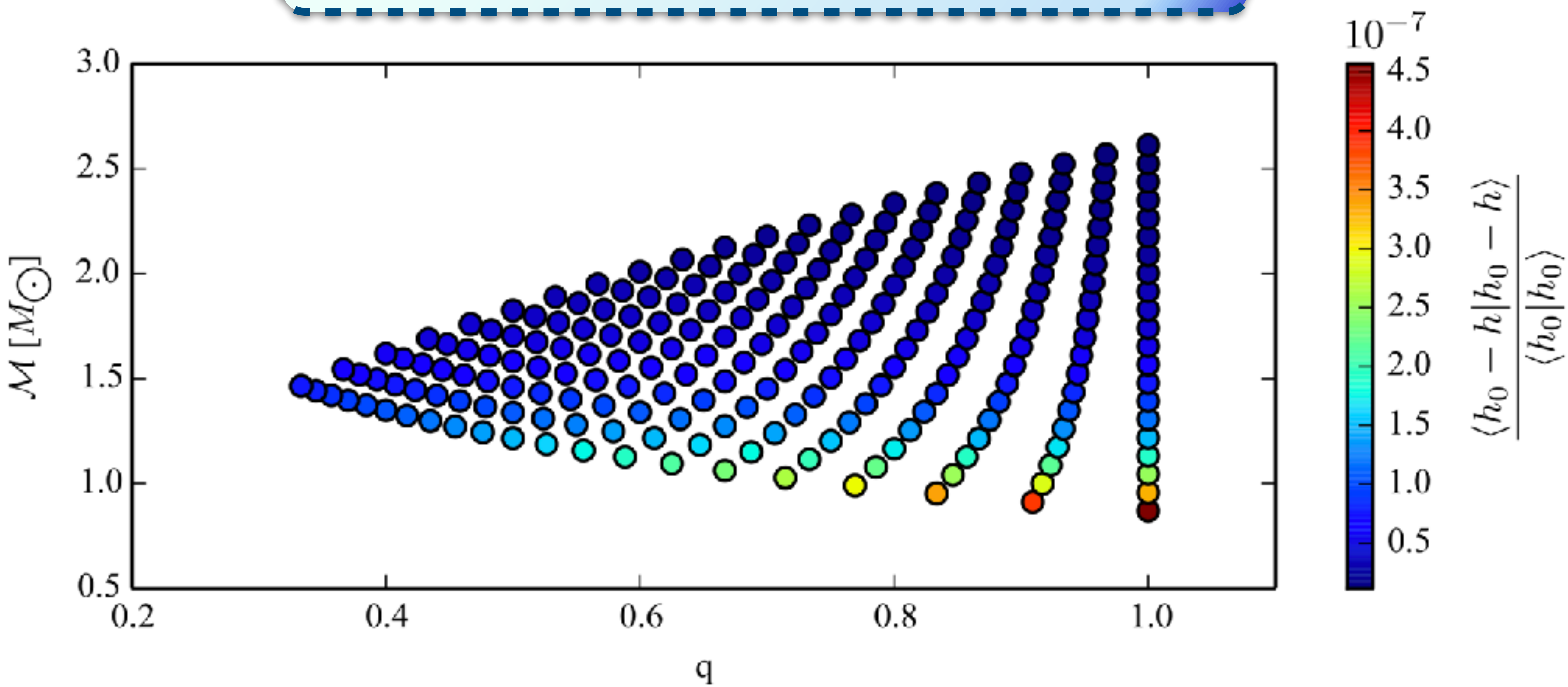
MB-INT ACCURACY

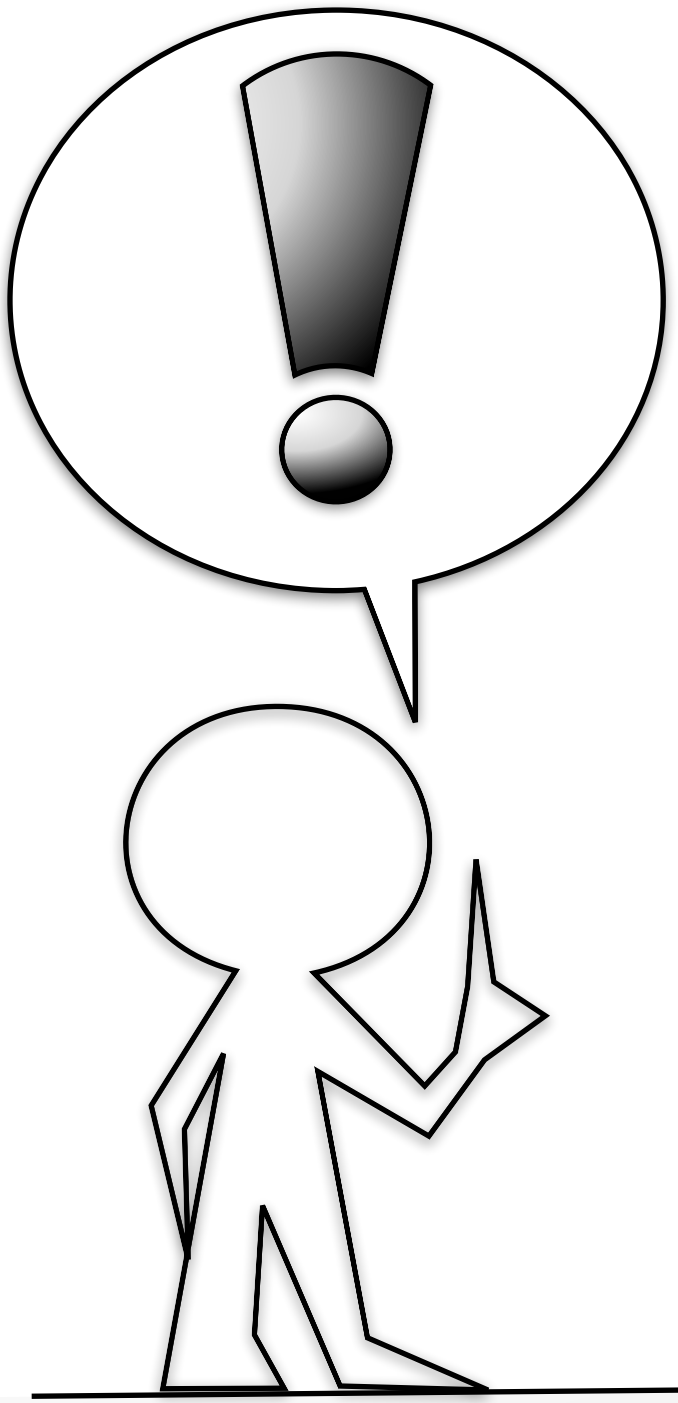


$$\frac{\langle h - h_0 | h - h_0 \rangle}{\langle h_0 | h_0 \rangle} \ll 10^{-3}$$

MB-INT ACCURACY

$$\frac{\langle h - h_0 | h - h_0 \rangle}{\langle h_0 | h_0 \rangle} \ll 10^{-3}$$





... CAVEAT

waveform generation times:

Time for interpolation

Time for calculate $\frac{\Delta g}{\Delta f}$

$$T_{std} \sim N_{fix} \cdot t^{w}$$

$$T_{MB-INT} \sim N_{fix} \cdot t^{i} + N_{MB} \cdot (t^{w} + \delta t^{i})$$

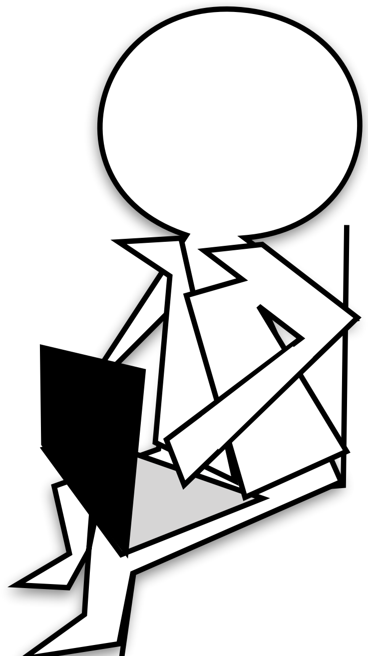
$$N_{fix} \propto f_0^{-8/3}$$

Time required to generate waveform at a single Fourier component

for $f_0 \to \varepsilon$:

$$N_{fix} \cdot t^{i} \gg N_{MB} \cdot (t^{w} + \delta t^{i}) \quad \Rightarrow \quad G_{template} = \frac{T_{std}}{T_{MB-INT}} \to \frac{t^{w}}{t^{i}}$$

RESULTS



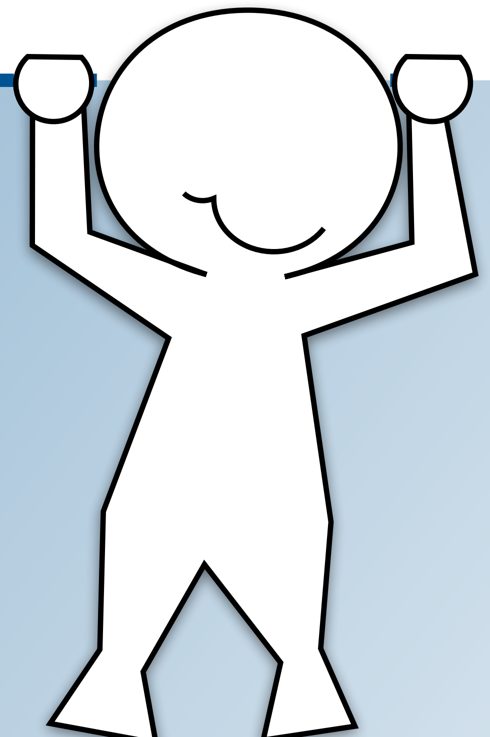
SPEED UP WAVEFORM GENERATION

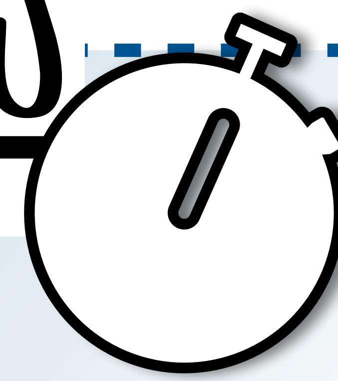
PE

CONSISTENCY TEST

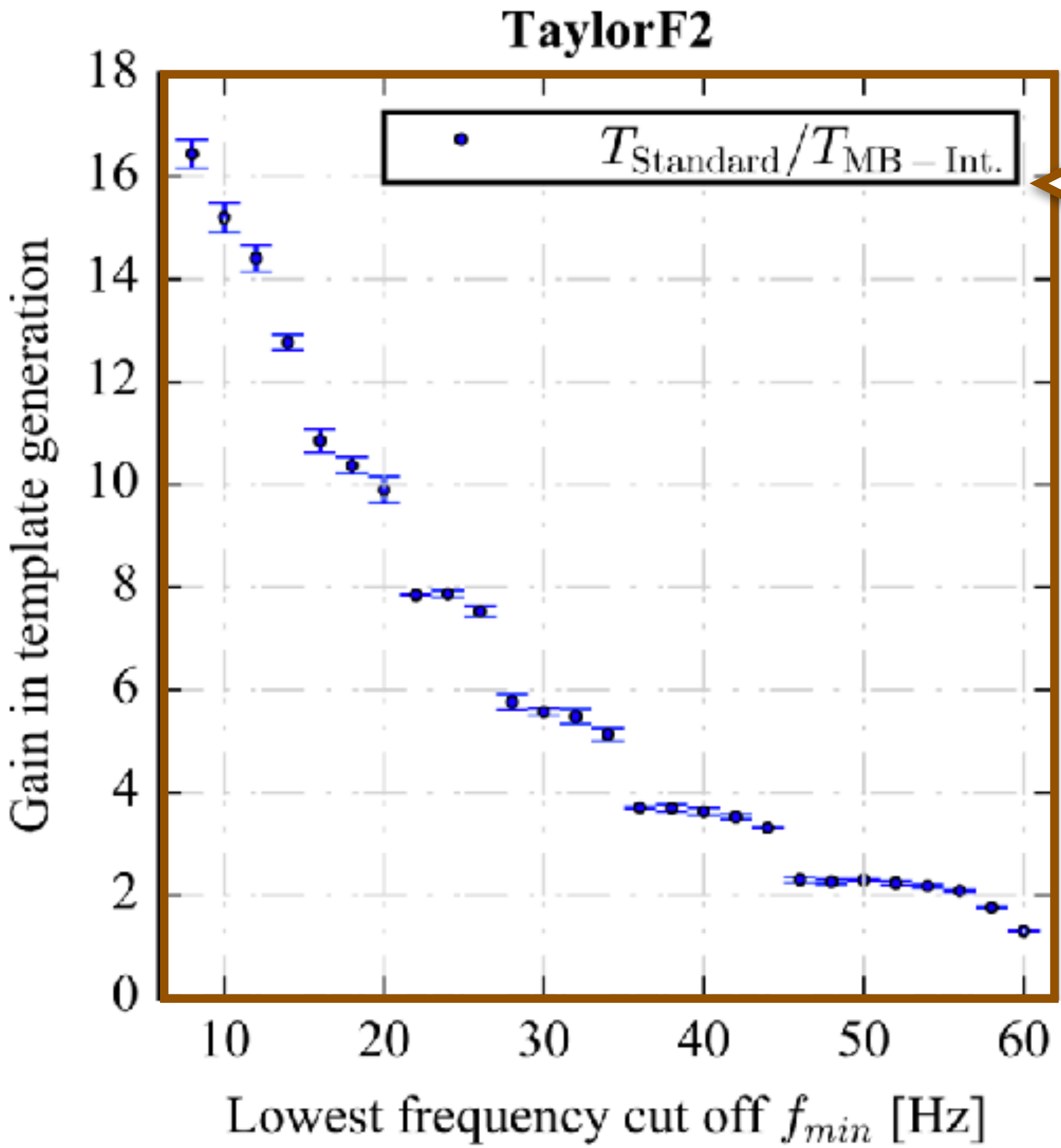
SPEED UP PE

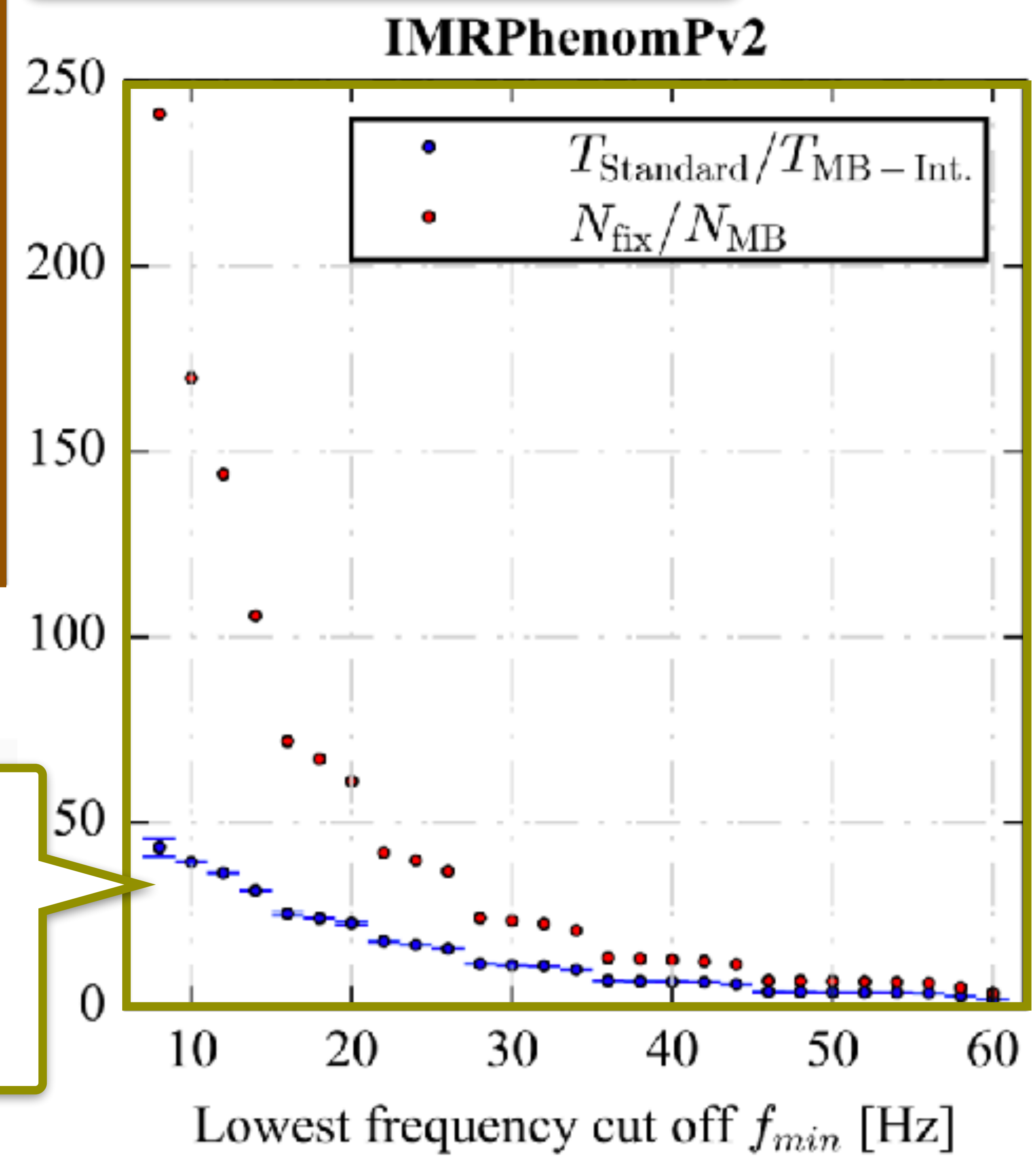
RESULTS





SPEED UP WAVEFORM GENERATION

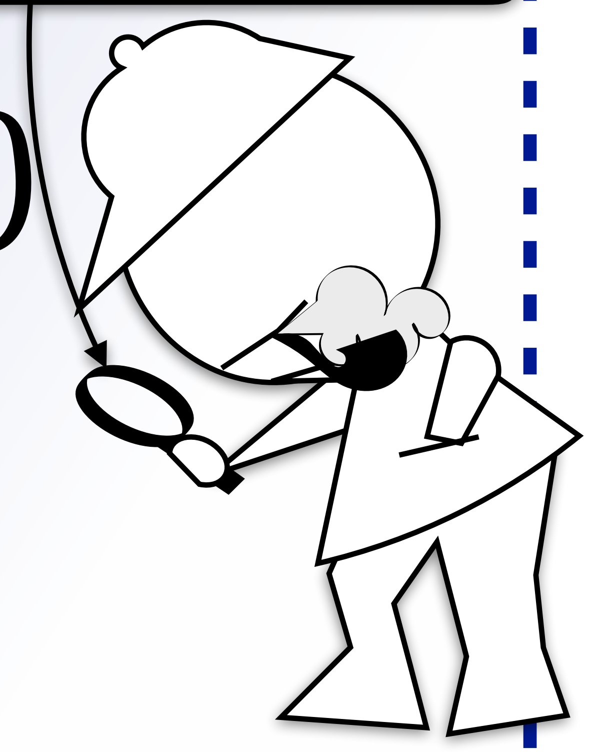

TaylorF2
Gain in template generation
$T_{\text{Standard}}/T_{\text{MB - Int.}}$
Lowest frequency cut off f_{min} [Hz]
ONE OF THE SIMPLEST WAVEFORM MODELS


IMRPhenomPv2
$T_{\text{Standard}}/T_{\text{MB - Int.}}$
$N_{\text{fix}}/N_{\text{MB}}$
Lowest frequency cut off f_{min} [Hz]
MORE SOPHISTICATED WAVEFORM MODEL

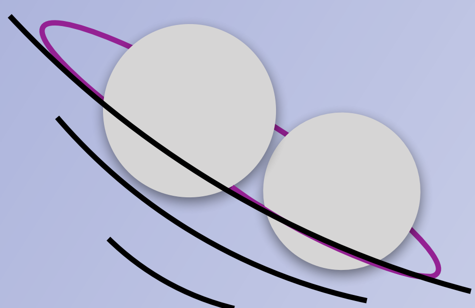
RESULTS

LALInference

J. Veitch Phys. Rev. D, 91:042003, 2015.

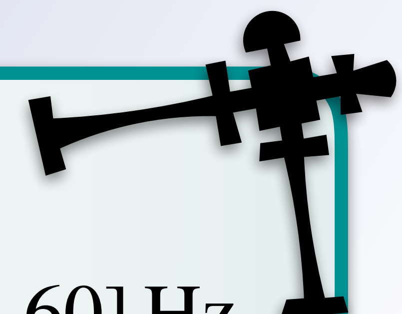


PE TESTS – SET UP



- $f_{max} = 2048\,\mathrm{Hz}$
- $f_0 = [20, 30, 40, 60]\,\mathrm{Hz}$
- *sensitivity*

  Advanced LIGO & Advanced Virgo



INJECTION

- *Stationary gaussian noise*
- $D_L \sim 200\,\mathrm{Mpc}$

  so that: $\mathrm{SNR}_{f_0=40Hz} = 15$
- $m_1 = m_2 = 1.4\,\mathrm{M}_\odot$

PE RUNS

Only TaylorF2

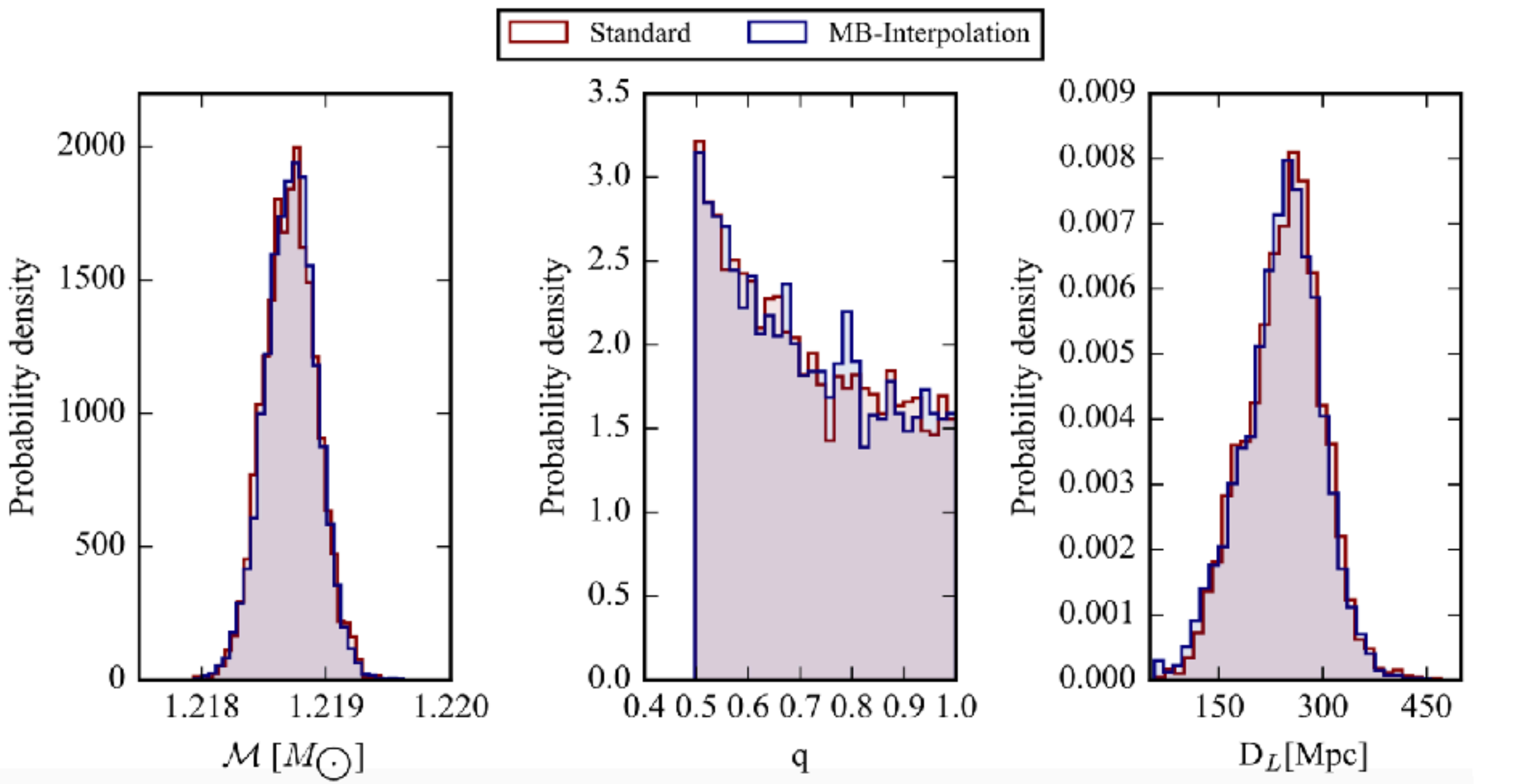
- *Companion masses prior*:

  uniform in range $1 - 3\,\mathrm{M}_\odot$
- *Distance prior*:

  uniform in volume up to $500\,\mathrm{Mpc}$

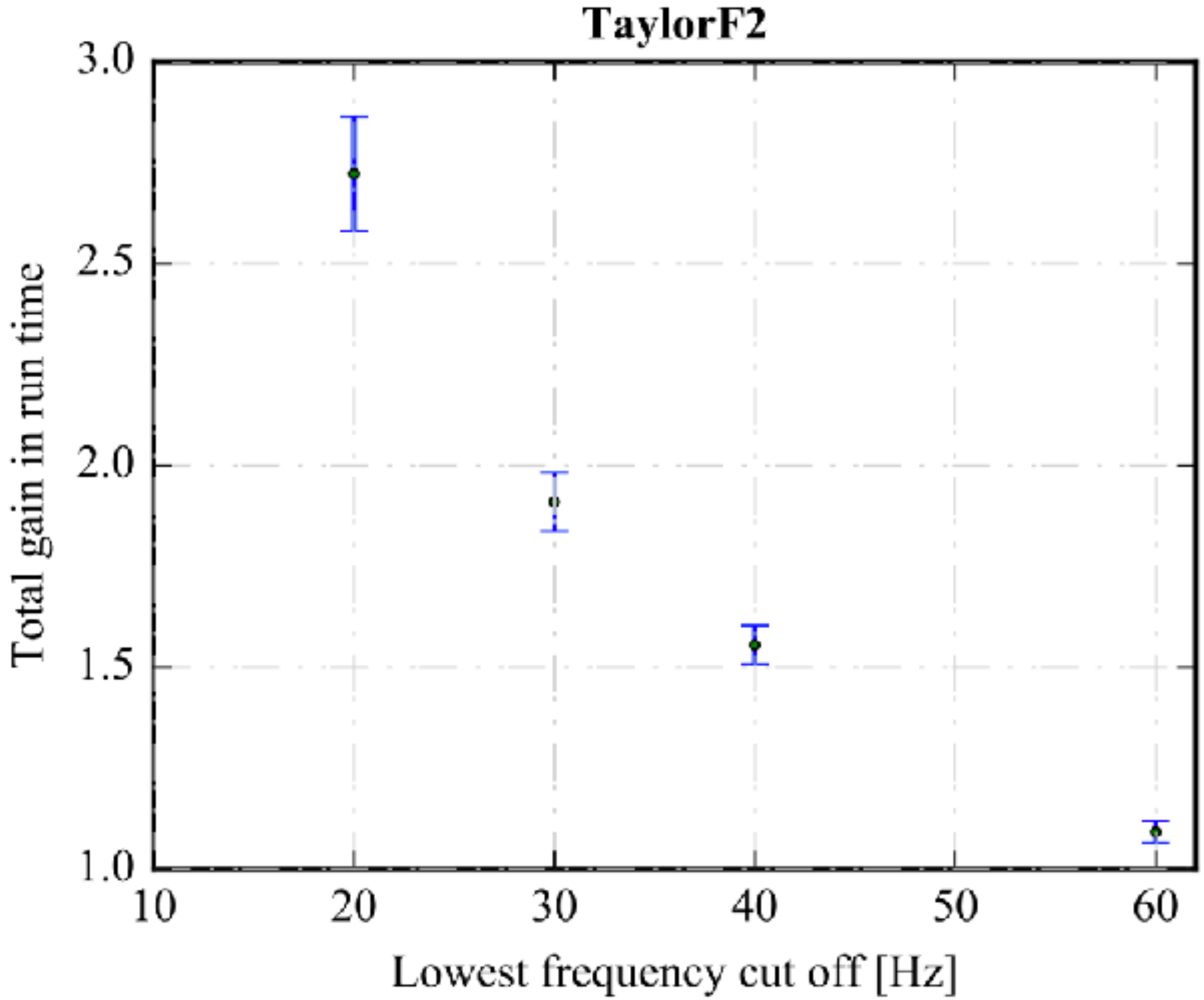
RESULTS

PE CONSISTENCY TEST



RESULTS

SPEED UP PE



$\mathbf{f}_{\rm min}[Hz]$	$\delta\mathbf{f}_0[Hz]$	$\mathbf{G}_{\rm PE}^{\rm TF2}$	$\mathbf{G}_{\rm template}^{\rm TF2}$	$\mathbf{N}_{\rm fix}/\mathbf{N}_{\rm MB}$	$\mathbf{N}_{\rm fix}/\mathbf{N}_{\rm min}$
60	1/16	1.09 ± 0.03	1.31 ± 0.01	3.76	55.4
40	1/64	1.56 ± 0.05	3.8 ± 0.1	12.82	83.8
30	1/128	1.91 ± 0.07	5.5 ± 0.1	23.40	112.2
20	1/300	2.72 ± 0.14	8.8 ± 0.2	61.01	169.1

CONCLUSION AND REMARKS

Current and future instruments

Physics of low mass systems mergers

Important to speed up the parameter estimation

One of the main bottlenecks of the analysis:

Waveform computation for sophisticated models

Possible solution inspired by search methods: **MULTI-BANDING**

(+ **INTERPOLATION** for accuracy)

Already good results with speed-up up to ~ **50** for tested sophisticated model at lowest tested f_0

CONCLUSIONS AND REMARKS

MB-INT

No set up costs

Flexible: can be easily applied to waveforms with higher dimensionality parameter space

Compared to ROQ:

- Higher speed up factors
- Requires high computational and memory set up costs
- **ROQ + MB-INT** could accelerate and save memory for initial set up

General validity of the method (also outside the GW field)

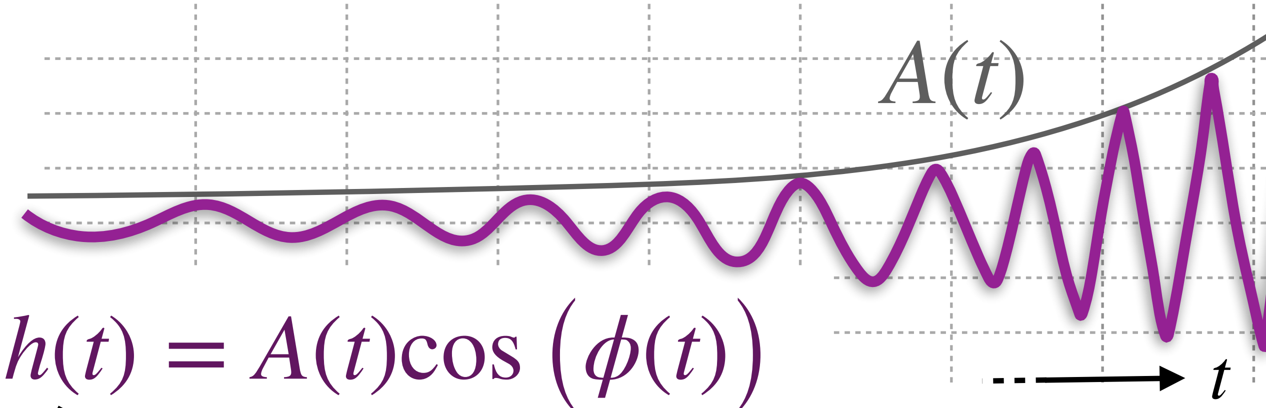
Thank you

GraWIToN
GW Initial Training Network
MARIE CURIE ACTIONS

EXTRA SLIDES

GW SIGNALS: from CBCs

NEWTONIAN ORDER



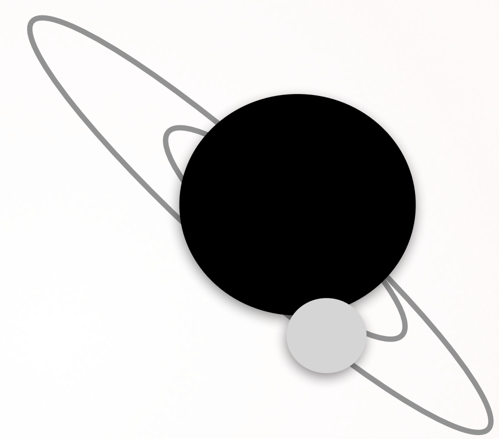
$h(t) = A(t)\cos\left(\phi(t)\right)$

f domain

$t \propto f^{-8/3} \mathscr{M}^{-5/3}$

$A_t(t) \propto f^{2/3} \mathscr{M}^{5/3}$

STATIONARY PHASE APPROXIMATION



for $h(t) = A(t)\cos\left(\phi(t)\right)$

if $\phi''(t) \ll \phi'(t)$

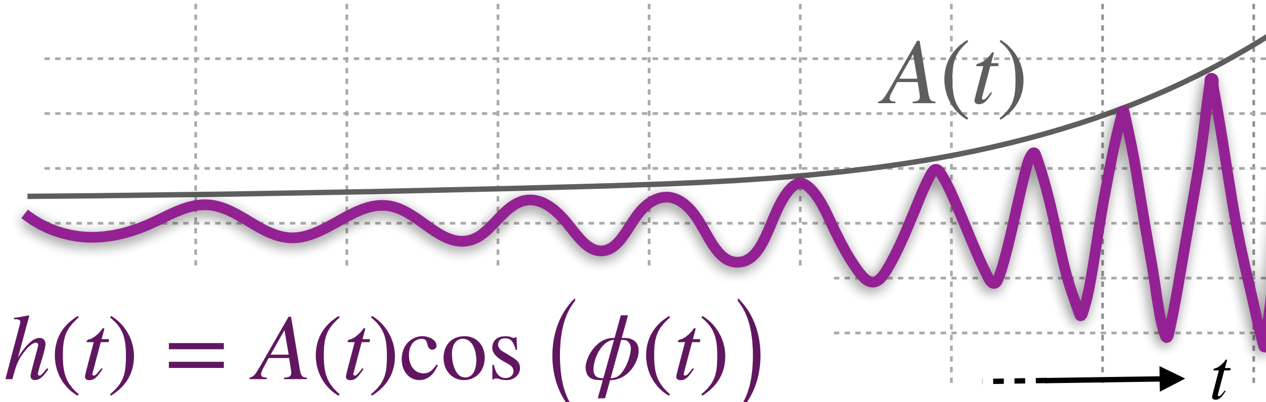
$\log A(t)' \ll \phi'(t) \sim f(t)$

then $\tilde{h}(f) = \frac{1}{2} A_t(t) \sqrt{\frac{dt}{df}} \exp\left(i\psi(f)\right)$

with $\psi(f) = 2\pi f t_c - \phi(t) - \pi/4$

GW SIGNALS: from CBCs

NEWTONIAN ORDER



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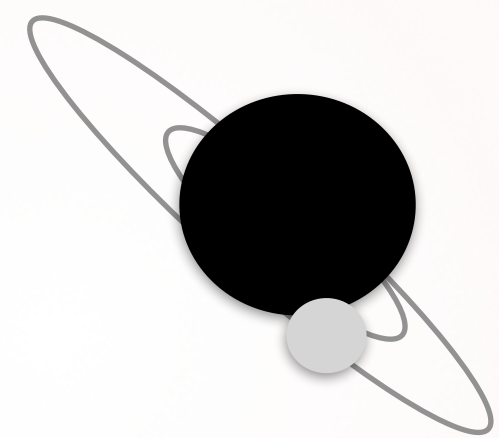

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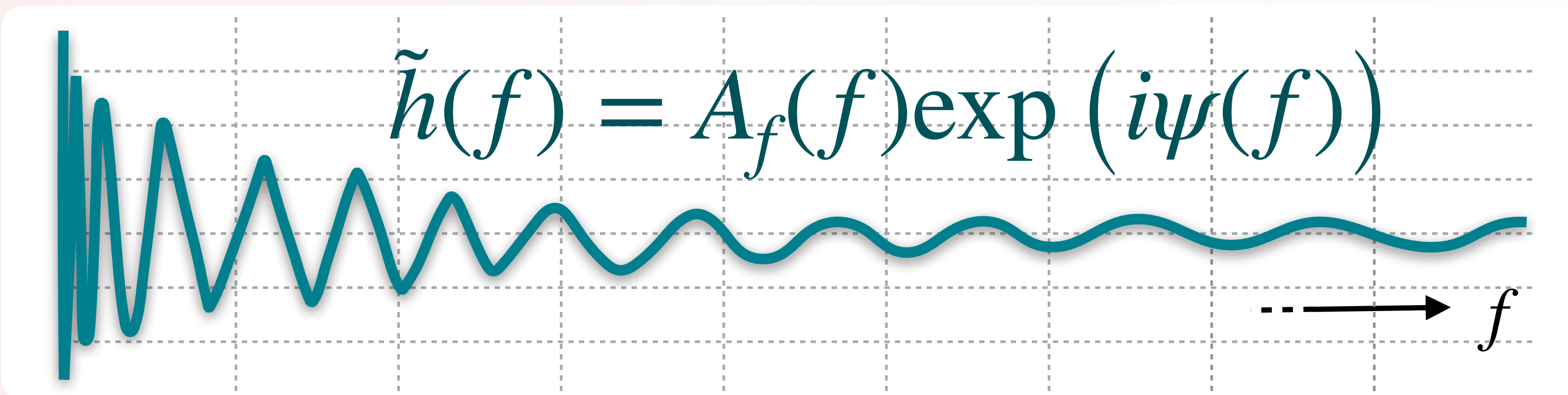


then $$\tilde{h}(f) = \underbrace{\frac{1}{2}A_t(t)\sqrt{\frac{dt}{df}}}_{A_f(f) \propto \mathscr{M}^{5/6} f^{-7/6}}\exp\big(i\psi(f)\big)$$

$\phi(t_c) - \phi(t) \propto \mathscr{M}^{-5/3} f^{-5/3}$

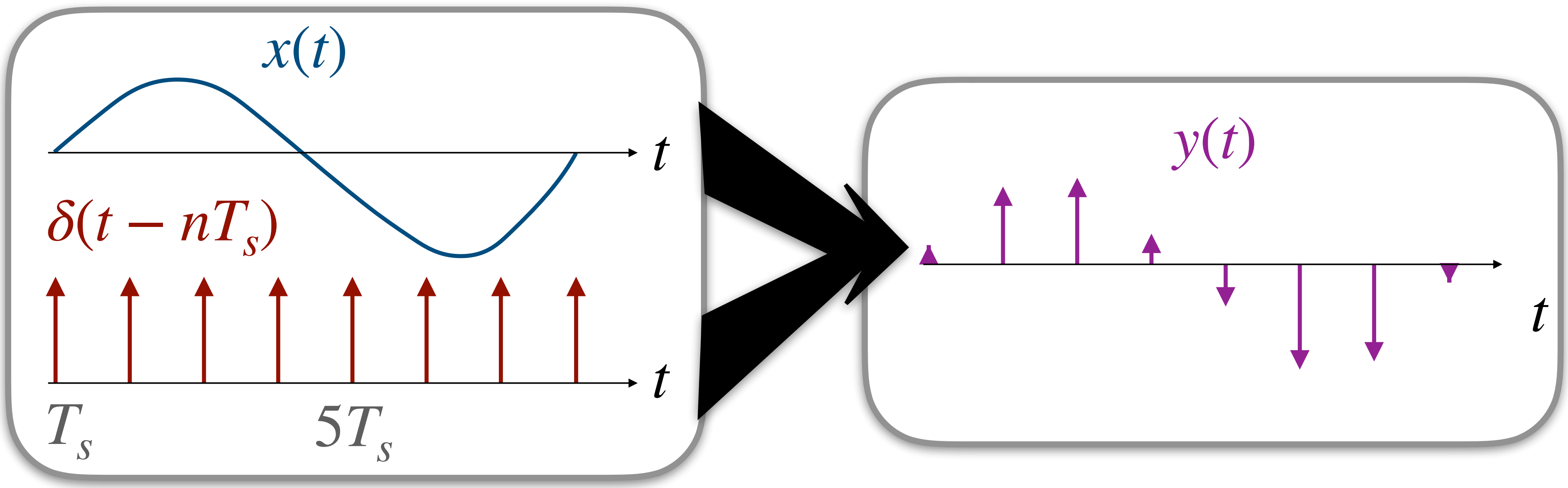
with $\psi(f) = 2\pi f t_c - \phi(t) - \pi/4$

NYQUIST THEOREM

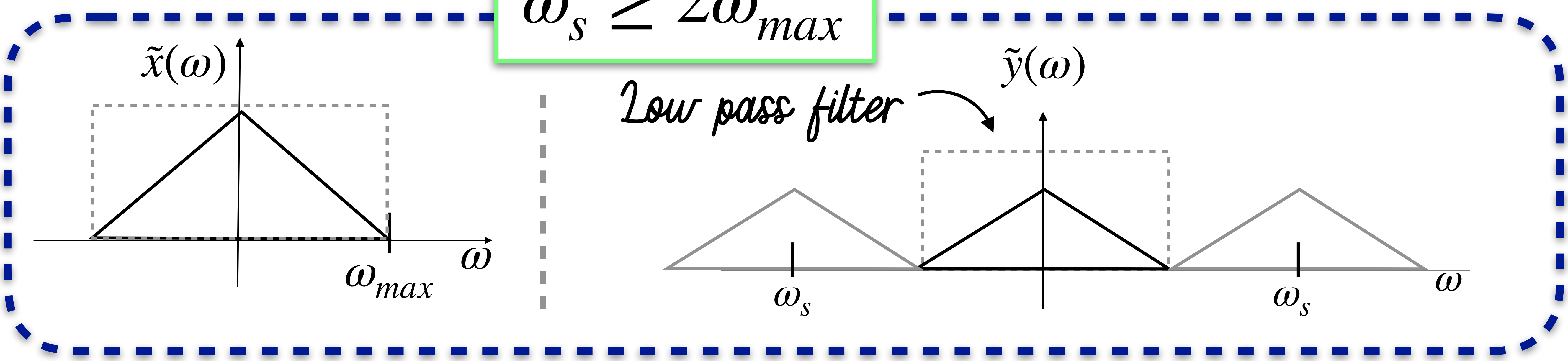
$$\omega_s = \frac{2\pi}{T_s}$$

~ : Fourier Tranform (FT)

time domain



$$\boxed{\omega_s \geq 2\omega_{max}}$$



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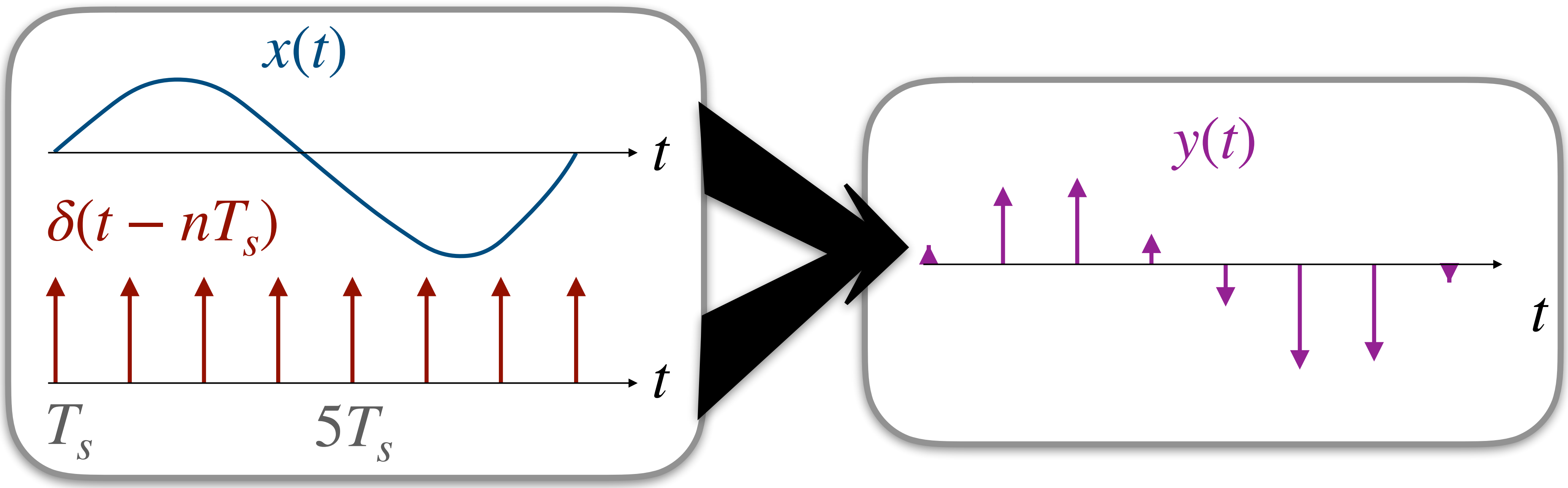
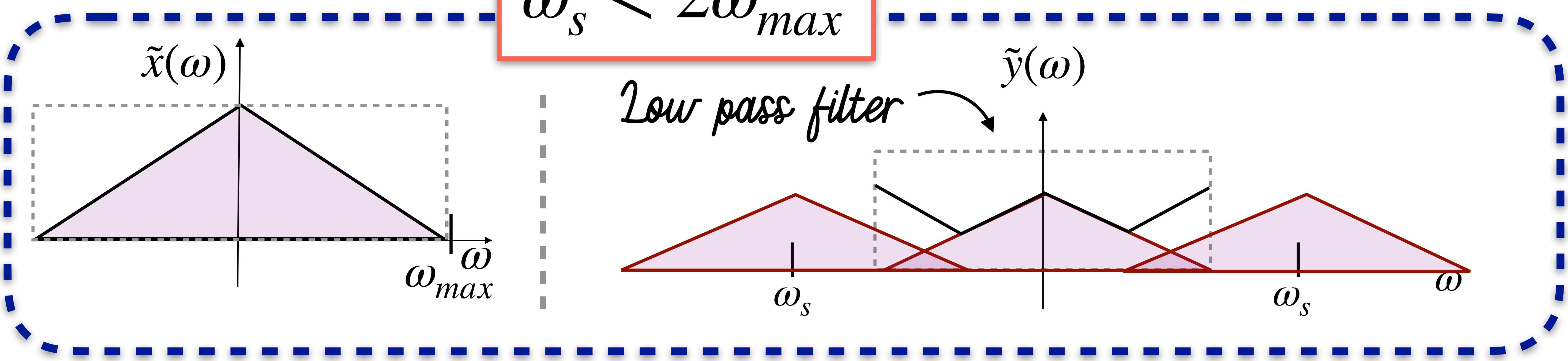


$$\boxed{\omega_s < 2\omega_{max}}$$



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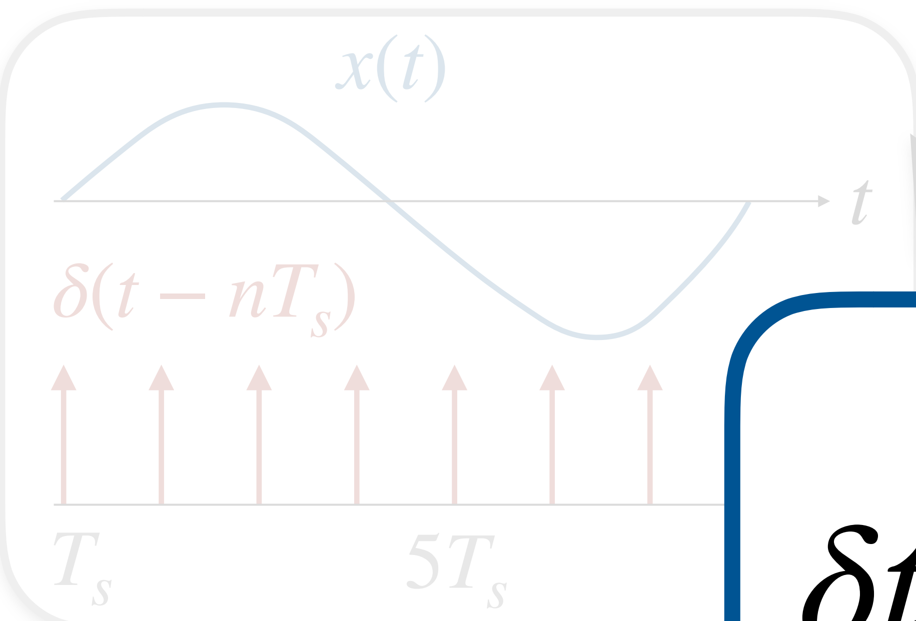
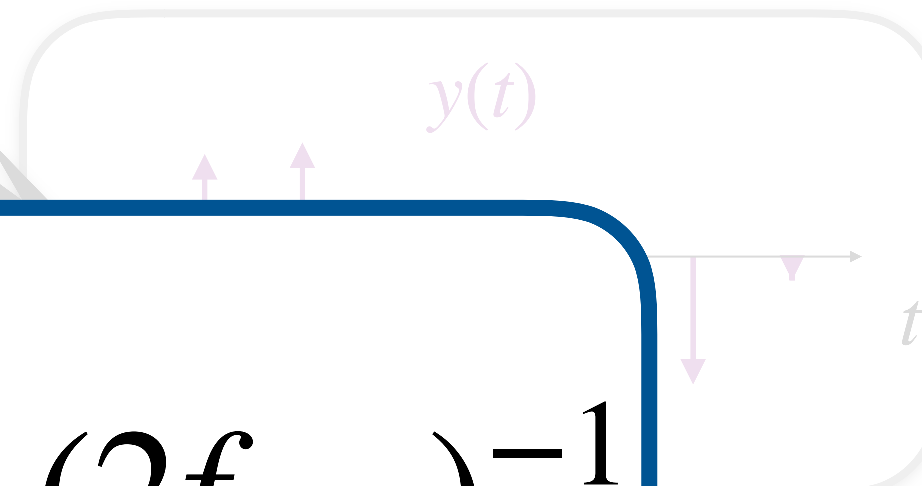
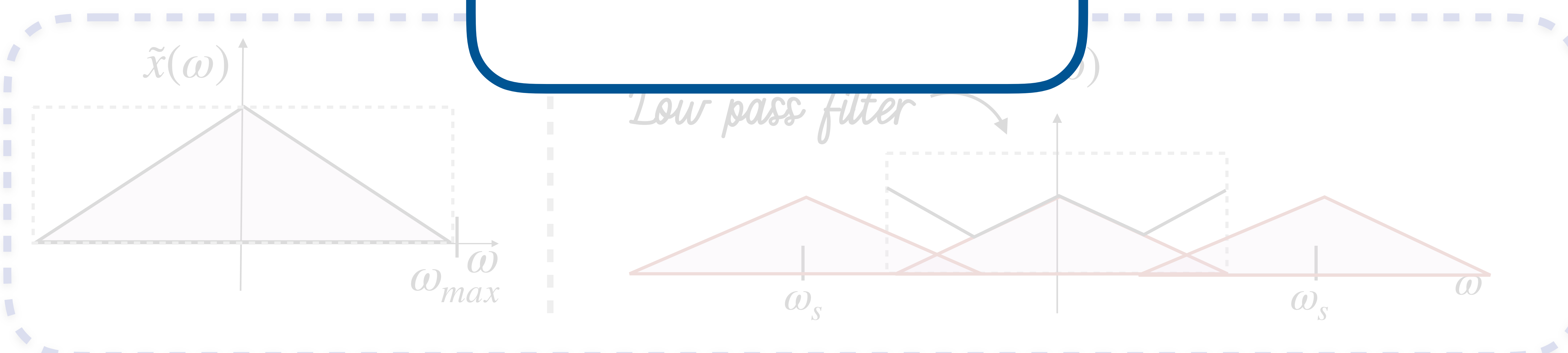


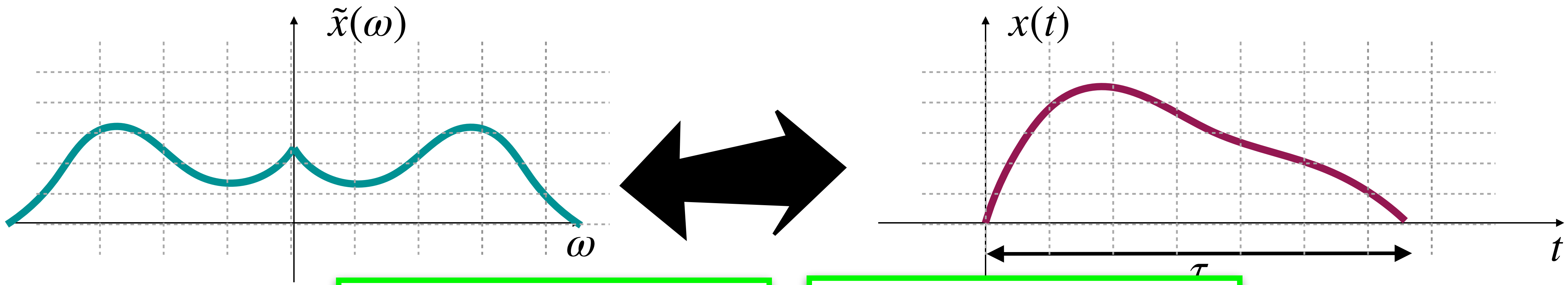


$$\boxed{\delta t \leq (2f_{max})^{-1}}$$



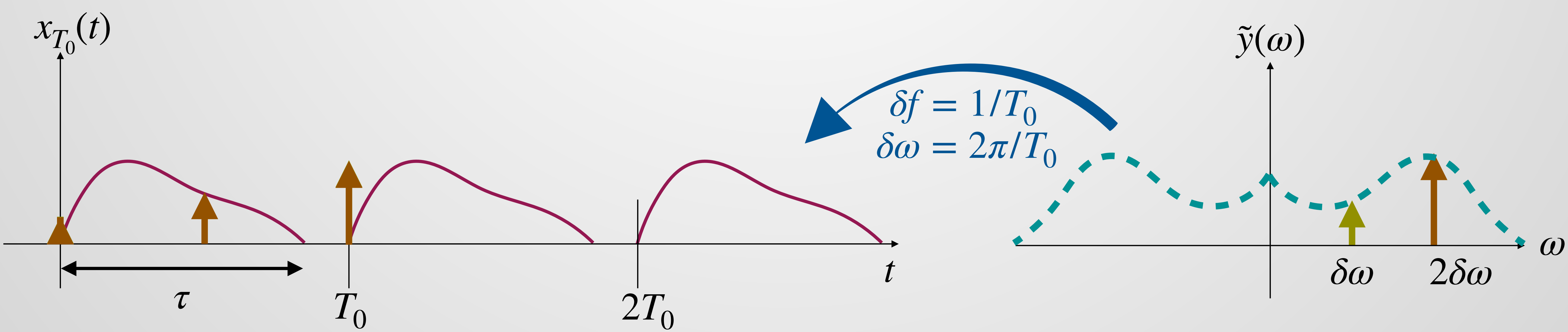
NYQUIST THEOREM

frequency domain



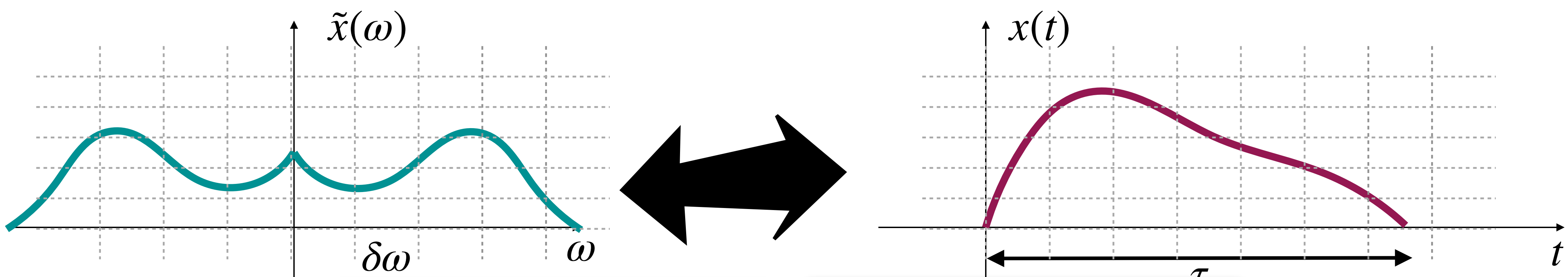
$$T_0 < \tau$$

$$\delta f > \tau^{-1}$$



NYQUIST THEOREM

frequency domain



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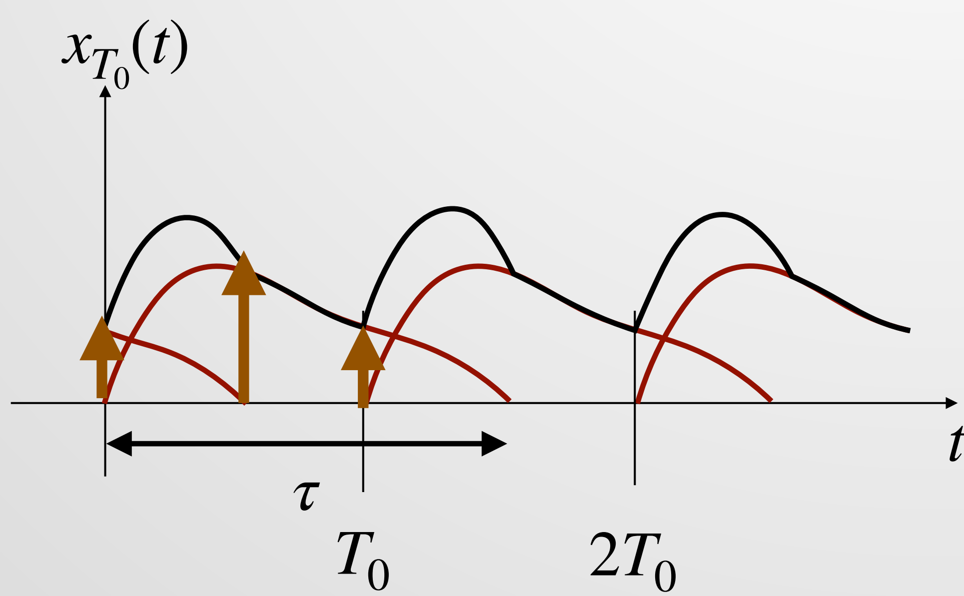

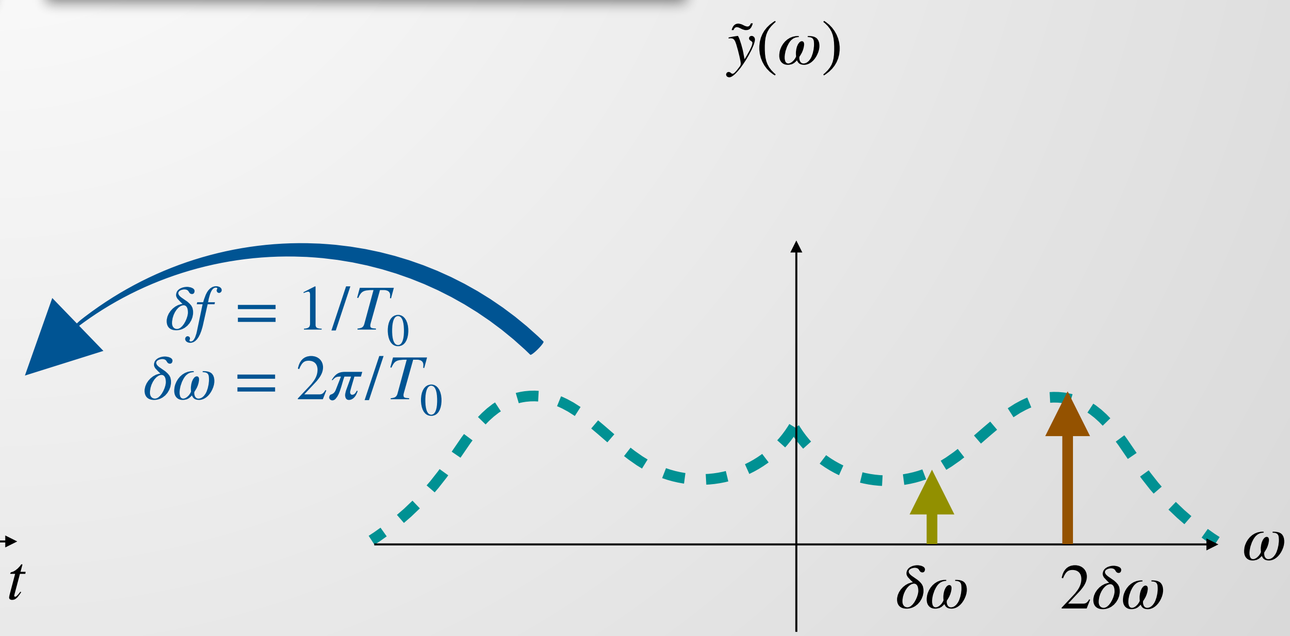

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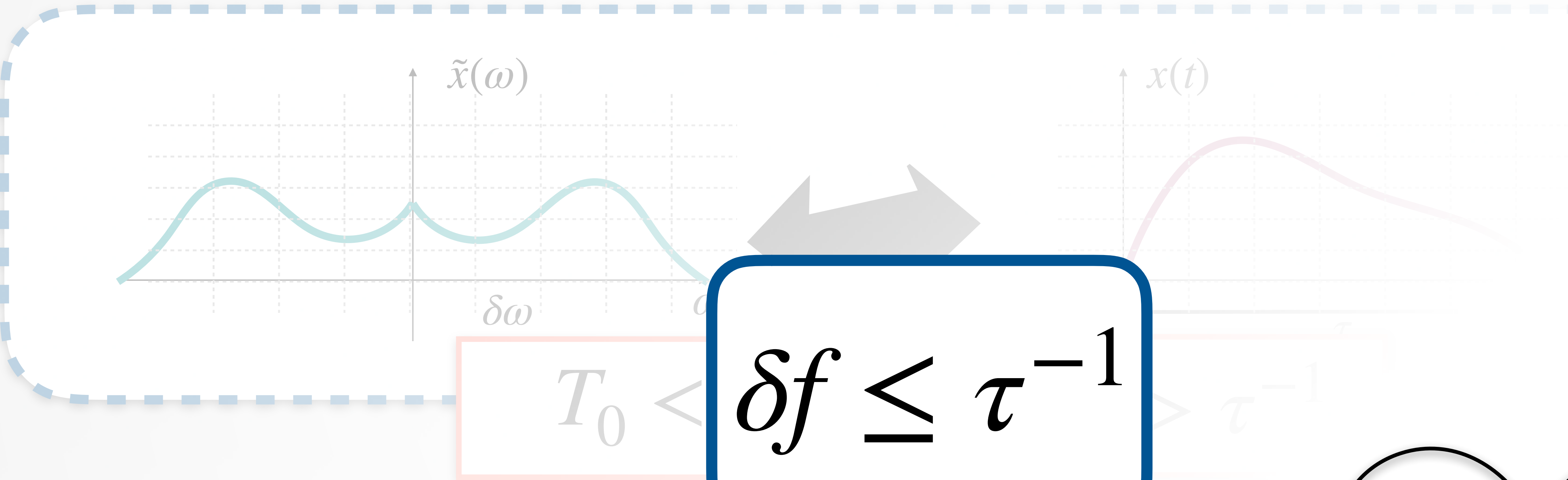

$$\boxed{\delta f \leq \tau^{-1}}$$