Locally Indistinguishable Ground States
Monday, August 30, 2021 9:01 AM

Part I. Implication from QEC.

Q) "TQO" in 1D ?

$$H = -\sum h_j , \quad h_j^2 = h_j = h_j^\dagger$$
$$[h_j, h_{j'}] = 0$$

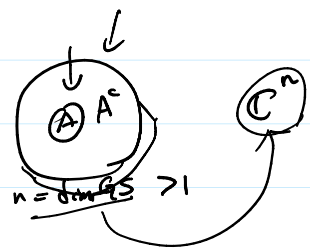
$$\Pi_{GS} = \prod_j h_j$$

Tool: Q. Info Π_{GS} TFAE, Given $A \subseteq \Lambda$

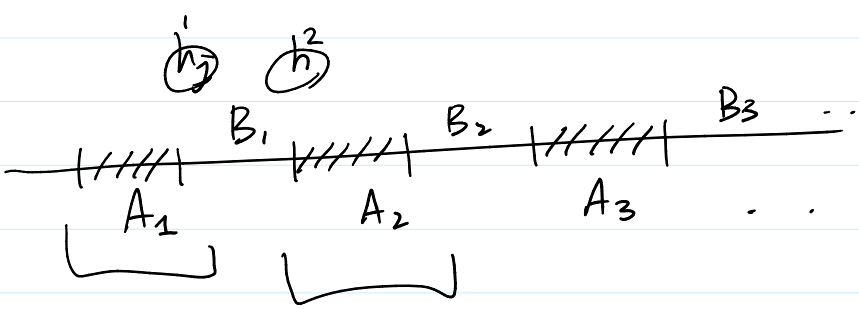
1. (TQO) $\forall O_A \quad \Pi_{GS} O_A \Pi_{GS} = c(O_A)\Pi_{GS}$, $c(O_A) \in \mathbb{C}$

   $\langle \varphi | O_A | \psi \rangle = 0$
2. (Decoupling) $I_\rho(A : R) = 0$
3. (Cleaning) $\forall$ logical U (i.e. $[U, \Pi_{GS}] = 0$)

   $\exists V^{A^c}$ s.t. $(V^{A^c} - U)\Pi_{GS} = 0$



arXiv: [1610.06169] Limits on the storage of quantum information in a volume of space (arxiv.org)



$$\Pi_{GS} \, O_{A_1} \, \Pi_{GS} \, O_{A_2} \, \Pi_{GS} = c(O_{A_1})\, c(O_{A_2}) \, \Pi_{GS}$$

$\Rightarrow$ If A_1, A_2 are "correctable", so is $A_1 \cup A_2$.

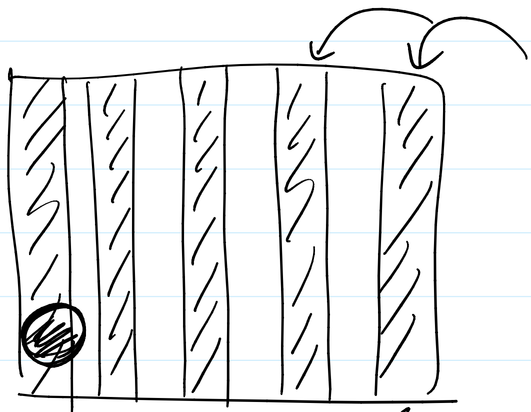
$\bigcup_i A_i$ is corr.

$\bigcup_i B_i$ is corr.

Thm in 1D $\nexists$ TQO.

This argument adopts an idea from [0810.1983] A no-go theorem for a two-dimensional self-correcting quantum memory based on stabilizer codes (arxiv.org)

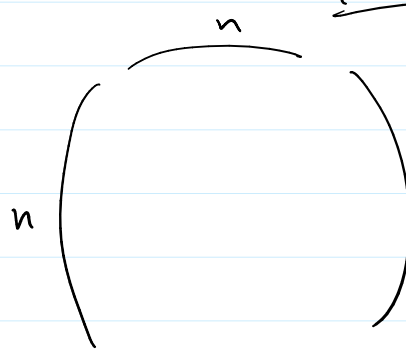
2D



$\bigcup A_i$

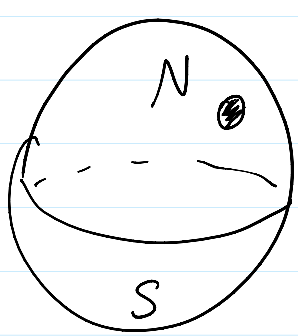
$\bigcup B_i$

$\Pi_{GS} \overset{\text{linear}}{\simeq} \underline{\sum_{i=1}^{n} |i\rangle\langle i|}$



φ must support some "logical" op.

How large can Π_{GS} be?



dim $\Pi_{GS} = n$

$\overset{\circlearrowright}{LR}\,\mathbb{C}^n$

$$-\underbrace{S_{NR}}_{= S_S} + S_N + S_R = 0$$

$$-\underbrace{S_{SR}}_{= S_N} + S_S + S_R = 0$$

$$+)\quad \overline{\qquad\qquad\qquad\qquad}$$

$2S_R = 0$

$\dim \mathcal{H}_{GS} = n$

pure. $|\psi\rangle = \sum_i |\psi_i\rangle |i\rangle / \sqrt{n}$

$\Downarrow$

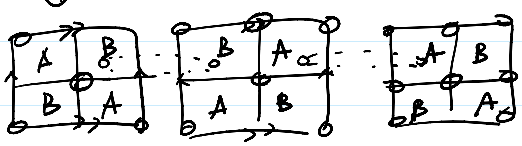
$S_R = \log n$.

$+)$

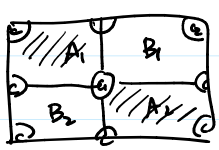
$2 S_R = 0$

$\Rightarrow n = 1$

$\Downarrow$



$\log n \leq g \cdot (\#1)$



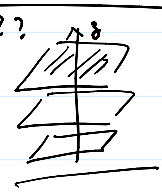
$$S_A + S_R = S_{AR} = S_{BC} \leq S_B + S_C$$

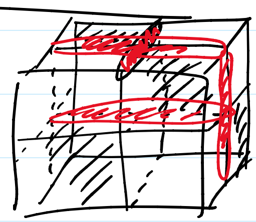
$$+)\quad S_B + S_R = S_{BR} = S_{AC} \leq S_A + S_C$$

$$2 S_R \leq 2 S_C$$

$$\log n \leq \frac{\mathrm{vol}(C)}{O(1)}$$

3D ??





//// : A

○ : B

$$\log n \leq \mathrm{vol}(C) \sim L$$

[0909.5200] Tradeoffs for reliable quantum information storage in 2D systems (arxiv.org)
[2009.13551] A degeneracy bound for homogeneous topological order (arxiv.org)

Part 2 : Laurent polys for Pauli Ham.

$$\Lambda = \mathbb{F}_2[\mathbb{Z}^2] \ni f = \text{subset in } \Lambda$$

$$\mathbb{F}_2[x^{\pm}, y^{\pm}]$$

$\mathbb{F}_2[x^{\pm}, y^{\pm}]$

$$\begin{array}{c} ① \to \\ x^a y^b \\ ② \to \end{array} \begin{pmatrix} 1 + x^{-1} \\ \\ 1 + y^{-1} \end{pmatrix} = \sigma$$

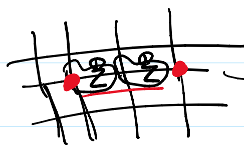
$$\begin{array}{c} ① \to \\ \\ ② \to \end{array} \begin{pmatrix} 1 + y^{+1} \\ \\ 1 + x^{+1} \end{pmatrix}$$

Pauli Z

$$\downarrow$$

$$\varepsilon = \sigma^{\dagger} = \begin{pmatrix} 1+x & 1+y \end{pmatrix} \begin{pmatrix} 1+x \\ 0 \end{pmatrix}$$

$$= 1+2x+x^2 = 1+x^2$$



[1204.1063] Commuting Pauli Hamiltonians as maps between free modules (arxiv.org)

position of plaquettes

$$R \xleftarrow{\varepsilon} R^2 \xleftarrow{\sigma} R \to 0$$

position of $+$

$R = \mathbb{F}_2[x^{\pm}, y^{\pm}]$

$$\text{coker } \varepsilon = R / \text{im } \varepsilon = \mathbb{F}_2[x^{\pm}, y^{\pm}] / (1+x,\ 1+y)$$

$$\cong \mathbb{F}_2$$

zero divisor

$$\text{torsion } T(M) = \{ \tau \in M/\varphi \mid \exists\, r \in R \setminus \{0\} \ \ r\tau = 0 \}$$

$$(1+x)\, e^{\in \text{coker } \varepsilon} = 0$$

$$e + xe$$

T coker ε

$\Downarrow$

Topological charge

If $\exists$ binomial zero-divisor,

$x^a + y^b$

$$(\underbrace{x^a + y^3}_{r})\, e = \mathcal{E}(p)$$

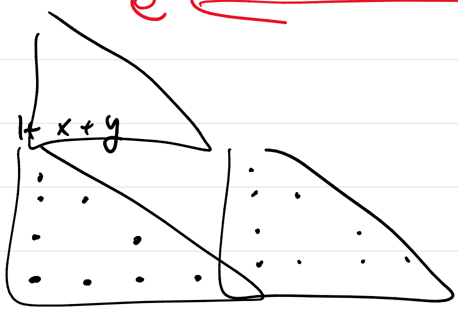
$(a+b)^{2^m}$

$= a^{2^m} + b^{2^m}$

$$\mathcal{E}\left(\underline{r^{2^m-1}} p\right) = \underline{r^{2^1} e} \quad r^{2^{m-1}} \quad r^{2^{m-2}} \quad r^4 r^2 \underline{r} \; \underline{p}$$

$e \qquad\qquad e$

$r = 1 + x + y$



Part 3 : Fractions

$$\mathcal{E} = \textcircled{R}\left(\underline{1 + x + y + z}\,,\ \underline{1 + xy + yz + zx}\right)$$

Claim: $\nexists$ binomial zero-divisor on coker $\mathcal{E}$

Pf)

$$\begin{cases} 1 + x + y + z = 0 \\ 1 + \ldots = 0 \end{cases}$$

F_4

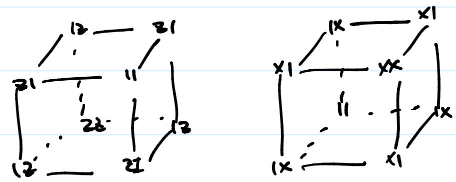
$$\begin{cases} x = 1 + t \\ y = 1 + \omega t \\ z = 1 + \omega^2 t \end{cases} \quad (\omega^2 + \omega + 1 = 0) \quad (\omega \leftrightarrow \omega^2)$$

$(1 + x^a y^b z^c) = u(1 + x + y + z) + v(1 + xy + yz + zx)$

$$(1+x^a y^b z^c) = u\,\underbrace{(1+x+y+z)}_{=0} + v\,\underbrace{(1+xy+yz+zx)}_{=0}$$

$$(1+t)^a(1+\omega t)^b(1+\omega^2 t)^c = 1$$

[1305.6973] Lattice quantum codes and exotic topological phases of matter (arxiv.org) contains this proof of no mobile excitation.
[1101.1962] Local stabilizer codes in three dimensions without string logical operators (arxiv.org) where this model is written first has more elementary argument.

$\Rightarrow$ 

$$R^1 \oplus R^3$$

$$\mathcal{E} = \begin{pmatrix} 1+x & 1+y & 1+z \\ 0 & 1+\bar z & 1+\bar y \\ 1+\bar z & 0 & 1+\bar x \\ 1+\bar y & 1+\bar x & 0 \end{pmatrix}$$